TRANSACTIONS

OF THE

American Institute of Electrical Engineers

Vol. 51 MARCH, 1932 No. 1

TABLE OF CONTENTS

Electrical Equipment for Oil Field Operations, by H. C. Hill and J. B. SeLegue 1
Discussion 10
Calorimeter Measurement of Stray-Load Losses of a 55,000-Kva. Generator, by G. D. Floyd and J. R. Dunbar 12
Discussion 18
Power Transmission and Distribution from the Mokelumne River Development of the Pacific Gas and Electric Company, by E. M. Wright and B. D. Dexter 20
The Mokelumne River Development of the Pacific Gas and Electric Company, by E. A. Crellin 28
Tie-Line Control of Interconnected Networks, by T. E. Purcell and C. A. Powel 40
Discussion 47
Tuned Power Lines, by H. H. Skilling 51
Discussion 54
Intercontinental Radiotelephone Service from the United States, by J. J. Pilliod 56
Discussion 61
San Francisco-Los Angeles Section of the Pacific Coast Telephone Cable Network, by E. M. Calderwood and D. F. Smith 63
Discussion 73
The Kindling of Electric Sparkover Based on Lichtenberg Figures, by C. Edward Magnusson 74
Discussion 79
Radio Coordination, by C. C. Campbell and H. N. Kalb 81
Discussion 89
Backfires in Mercury Arc Rectifiers, by J. Slepian and L. R. Ludwig 92
Overvoltage on Transmission Systems Caused by Dropping Load, by E. J. Burnham 105
Discussion 112
Interconnection of the 25- and 60-Cycle System of the Union Electric Light and Power Company, by L. V. Nelson and E. L. Hough 113
Discussion 119
Automatic Control for Variable-Ratio Frequency Converters, by E. K. Shively and G. S. Whitlow 121
Discussion 127
Forecasting Population for Engineering Purposes, by J. N. Holsen 128
Discussion 133
Tape-Armored Telephone Toll Cable, by C. W. Nystrom 135
Discussion 139
The Time Factor in Telephone Transmission, by O. B. Blackwell 141
Discussion 147
Impulse Voltage Tests on a 4,800-Volt Distribution Substation, by H. W. Collins, E. E. Piepho, and J. J. Torok 148
Discussion 153
Corona Energy Loss, by W. D. Weidlein 154
The Expulsion Fuse, by J. Slepian and C. L. Denault 157
Discussion 165
The Theory of Oil-Blast Circuit Breakers, by D. C. Prince 166
Discussion 191
Practical Application of the Oil-Blast Principle of Circuit Interruption, by R. M. Spurck 171
Discussion 191
Recent Developments in Arc Rupturing Devices, by R. C. Van Sickle and W. M. Leeds 177
Discussion 191
Extinction of A-C. Arcs in Turbulent Gases, by T. E. Browne, Jr 185
Discussion 191
High-Voltage Bridge for Measurements of Cables with Grounded Sheaths, by C. L. Dawes and A. F. Daniel 198
A High Sensitivity Power Factor Bridge, by W. B. Kouwenhoven and A. Baños, Jr 202
Discussion 210
Power Factor Measurement by the Capacitance Bridge, by R. P. Siskind 214
An Automatic Oscillograph, by C. M. Hathaway and R. C. Buell 222
Discussion 225
The Photoelectric Recorder, by C. W. LaPierre 226
Discussion 232
Interconnection of Primary Lightning Arrester Ground and the Grounded Neutral of the Secondary Main, by C. F. Harding and C. S. Sprague 234
Discussion 271
Lightning Protection for Distribution Transformers, by K. B. McEachron and L. Saxon 239
Discussion 271
Lightning Protection for Distribution Transformers, by A. M. Opsahl, A. S. Brookes and R. N. Southgate 245
Discussion 271
Studies in Lightning Protection on 4,000-Volt Circuits—III, by D. W. Roper 252
Discussion 271
Lightning Protection for Distribution Transformers, by T. H. Haines and C. A. Corney 259
Discussion 271
Distribution System Lightning Studies by Philadelphia Electric Company, by H. A. Dambly, H. N. Ekvall, and H. S. Phelps 265
Discussion 271

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PREFACE

This issue, Number 1 of Volume 51, contains the papers presented at the Pacific Coast Convention held at Lake Tahoe, California, August 25-28, 1931, and at the South West District Meeting, Kansas City, Missouri, October 22-24, 1931.

In addition to the above papers those for the first three sessions of the Winter Convention, held in New York, N. Y., January 25-29, 1932, are included in this issue. The remainder of the Winter Convention papers will be included in the next issue, Number 2, of the TRANSACTIONS.

The complete Table of Contents is published on the first page and the usual Index of Authors will be found at the end of the book.

TRANSACTIONS
OF THE
American Institute of Electrical Engineers

Vol. 51 JUNE, 1932 No. 2

TABLE OF CONTENTS

Equivalent Circuits—I, by Frank M. Starr...... 287
Discussion.... 321
Transient Oscillations of Mutually Coupled Windings, by L. V. Bewley.... 299
Discussion.... 321
Geometrical Circuits of Electrical Networks, by Ronald M. Foster.... 309
Discussion.... 321
Vector Theory of Circuits Involving Synchronous Machines, by I. H. Summers.... 318
Discussion.... 321
Auxiliary Drive for Steam Power Stations, by F. H. Hollister.... 329
Discussion.... 341
Steam-Driven Auxiliaries for Power Plants, by W. Poole Dryer.... 331
Discussion.... 341
Electrically Driven Auxiliaries for Steam Power Stations, by Lyle W. Smith.... 337
Discussion.... 341
Standard Decrement Curves, by W. C. Hahn and C. F. Wagner.... 353
Discussion.... 361
Generalized Stability Solution for Metropolitan Type Systems, by S. B. Griscom, W. A. Lewis, and W. R. Ellis.... 363
Discussion.... 373
Stability of Conowingo Hydroelectric Station, by R. A. Hentz and J. W. Jones.... 375
Discussion.... 384
Radio Interference from Insulator Corona, by F. O. McMillan.... 385
Predetermination of the A-C. Characteristics of Dielectrics, by J. B. Whitehead and Alfredo Baños, Jr.... 392
Discussion.... 402
Breakdown of Glass with Alternating Potentials, by N. D. Kenney, A. M. Luery, and J. D. Moriaty.... 404
Discussion.... 408
Magnet Steels and Permanent Magnets, by K. L. Scott.... 410
Discussion.... 417
An Induction Motor with Paralleled Rotor and Stator, by A. G. Conrad and R. G. Warner... 418
Discussion.... 422
Some Considerations in the Design of Damper Windings for Synchronous Motors, by C. C. Shutt.... 424
Discussion.... 430
Film Characteristics of Electrolytic Condensers, by F. W. Godsey, Jr.... 432
Discussion.... 449
Power Losses in Electrolytic Condensers, by F. W. Godsey, Jr.... 439
Discussion.... 449
Electrical Precipitation, by A. W. Simon and L. C. Kron.... 451
Discussion.... 462
Commutation Considered as a Switching Phenomenon, by R. E. Hellmund and L. R. Ludwig.. 465
Discussion.... 468
Equalizing Currents in the Armature of a D-C. Machine, by Robert M. Baker.... 473
Discussion.... 476
Calculation of No-Load Damper Winding Loss in Synchronous Machines, by E. I. Pollard.... 477
Discussion.... 481
Sine-Wave Generators, by Lloyd P. Shildneck... 484
Discussion.... 488
Engineering Features of Three-Power Locomotives, by F. H. Brehob and F. H. Craton..... 489
Discussion.... 495
Motors for 3,000-Volt D-C. Multiple Unit Cars, by John C. Aydelott.... 498
Temperature Rise of Ventilated Railway Motor Armatures, by D. A. Lightband.... 502
Discussion.... 509
High Capacity Rectifier Efficiency Improves by Sectionalizing, by A. L. Atherton.... 511
Discussion.... 515
Preparing Substations for Electric Operation, by L. B. Curtis.... 518
Time and Its Measurements, by Ernest W. Brown 523
Discussion.... 550
Modern Developments in Precision Clocks, by Alfred L. Loomis and W. A. Marrison..... 527
Discussion.... 550
Time Service of the U. S. Naval Observatory, by J. Frederick Hellweg.... 538
Discussion.... 550
Time Services of the Telegraph Companies, by G. W. Janson.... 541
Discussion.... 550
Synchronous Electric Time Service, by H. E. Warren.... 546
Discussion.... 550
Forces of Electric Origin in the Iron Arc, by F. Creedy, R. O. Lerch, P. W. Seal, and E. P. Sordon.... 556
Discussion.... 563
An Improved A-C. Arc Welder, by A. M. Candy.. 567
Discussion.... 569
Recent Developments in Design of Arc Welding Generators, by K. L. Hansen.... 570
Discussion.... 575

PUBLISHED QUARTERLY BY THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
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PREFACE

This issue of the QUARTERLY TRANSACTIONS, Number 2 of Volume 51, contains the balance of the 1932 Winter Convention papers, the first of which were published in Number 1 of this volume. This completes the publication of the Winter Convention program with the exception of two papers, *viz.*: *Surge Proof Transformers* and *Effect of Transient Voltages on Power Transformer Design—IV*. The discussions on these two papers were received too late to permit their inclusion in this volume. They will appear in the Number 3 QUARTERLY TRANSACTIONS in September, 1932.

TRANSACTIONS

OF THE

American Institute of Electrical Engineers

Vol. 51 SEPTEMBER, 1932 No. 3

TABLE OF CONTENTS

Surge-Proof Transformers, by H. V. Putman 579
- Discussion 584

Effect of Transient Voltages on Power Transformer Design—IV, by K. K. Palueff and J. H. Hagenguth 601
- Discussion 616

The Proximity Effect, by Edward Bennett 621

Toll Switching Plan for Wisconsin, by W. C. Lallier 628
- Discussion 633

Weather Resistant Covering for Line Wires, by C. F. Harding, L. L. Carter, and J. W. Olson 635
- Discussion 639

Electrical Design Features of Waukegan Station, by E. C. Williams 644
- Discussion 651

115,000-Kw. Turbo-Alternator, by R. B. Williamson 652
- Discussion 657

The Mercury Arc Rectifier Applied to A-C. Railway Electrification, by Othmar K. Marti 659
- Discussion 664

Insulator Sparkover, by W. L. Lloyd, Jr 669
- Discussion 697

An Improved Type of Limiting Gap, by A. O. Austin 676
- Discussion 697

Suspension Insulator Assemblies, by J. J. Torok and C. G. Archibald 682
- Discussion 697

Normal Frequency Arcover Values of Insulators as Affected by Size and Humidity, by H. A. Frey and K. A. Hawley 690
- Discussion 697

The Parallel Type Inverter, by Frederick N. Tompkins 707
- Discussion 714

A General Theory of Systems of Electric and Magnetic Units, by Vladimir Karapetoff 715
- Discussion 727

A Proposal to Abolish the Absolute Electrical Unit Systems, by Ernst Weber 728

Performance Calculations on Induction Motors, by C. G. Veinott 743
- Discussion 754

Measurement of Stray Load Loss in Polyphase Induction Motors, by C. J. Koch 756
- Discussion 761

Torque-Angle Characteristics of Synchronous Machines Following System Disturbances, by S. B. Crary and M. L. Waring 764
- Discussion 773

Field Tests to Determine the Damping Characteristics of Synchronous Generators, by F. A. Hamilton, Jr 775

Design of Capacitor Motors for Balanced Operation, by P. H. Trickey 780
- Discussion 784

The Flexible Progressive Traffic Signal System, by H. I. Turner 786

The Electro-Matic Traffic Dispatching System, by H. A. Haugh, Jr 790

A Recent Development in Traffic Control, by H. W. Vickery and V. W. Leonard 796

The Traffic Flow Regulator, by C. H. Bissell and J. G. Hummel 801

Double Conductors for Transmission Lines, by H. B. Dwight and E. B. Farmer 803
- Discussion 821

Three-Phase Multiple-Conductor Circuits, by Edith Clarke 809
- Discussion 821

Solution of Circuits Subjected to Traveling Waves, by Harold L. Rorden 824

PUBLISHED QUARTERLY BY THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
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PREFACE

This issue of the QUARTERLY TRANSACTIONS, Number 3, Volume 51, contains the last two papers from the Winter Convention; also those papers presented at the Great Lakes District Meeting, Milwaukee, Wisconsin, and the Northeastern District Meeting, Providence, Rhode Island, which were approved for publication. The papers are, as usual, arranged chronologically. This is the only publication of the Providence Meeting papers in full, as no separate copies have been printed.

TRANSACTIONS
OF THE
American Institute of Electrical Engineers

Vol. 51 DECEMBER, 1932 No. 4

TABLE OF CONTENTS

Engineering Subjects, Electrical and Cognate, in the Four-Year College Program of Electrical Engineering, by Alfred H. Lovell ... 839
Discussion ... 842
Educational Aspects of Engineering and Management, by R. E. Doherty ... 848
Discussion ... 857
Combined Reliability and Economy in Operation of Large Electric Systems
Part I—The Detroit Edison Company, by A. P. Fugill ... 859
Part II—The Edison Electric Illuminating Company of Boston, by R. E. Dillon ... 866
Part III—Philadelphia Electric Company System, by J. W. Anderson and Herbert Estrada ... 872
Part IV—Chicago District, by Leslie L. Perry and Frank V. Smith ... 879
Discussion ... 888
Electrical Equipment for Precipitation Service, by H. Speight and V. G. Rydberg ... 892
Discussion ... 898
Oxide Coatings on Aluminum, by J. D. Edwards, Martin Tosterud, and H. K. Work ... 900
Calculation of Inductance and Current Distribution in Low-Voltage Connections to Electric Furnaces, by C. C. Levy ... 903
New Application of Non-Linear Circuits to Relay and Control Problems, by C. G. Suits ... 914
Discussion ... 921
The Coordination of Transformer Insulation with Line Insulation, by V. M. Montsinger and W. M. Dann ... 923
Discussion ... 929
Insulation Coordination of Distribution Transformers, by E. D. Treanor and W. H. Cooney ... 925
Discussion ... 929
Characteristics of Surge Generators for Transformer Testing, by P. L. Bellaschi ... 936
Discussion ... 945
Characteristics of Load Ratio Control Circuits for Changing Transformer Ratio Under Load, by L. F. Blume ... 952
Discussion ... 956
Influence on Commutation of Brush Contact Drop, by L. R. Ludwig and R. M. Baker ... 959
Discussion ... 963
Single-Phase Short-Circuit Torque of a Synchronous Machine, by C. A. Nickle, C. A. Pierce, and M. L. Henderson ... 966
Discussion ... 971
Wire Communication Aids to Air Transportation, by H. H. Nance ... 974
Discussion ... 980
Characteristics of Electromagnetic Radiation from Aircraft in Flight, by Tom C. Rives and James C. Coe ... 981
Discussion ... 986
Vertically Cut Sound Records, by H. A. Frederick and H. C. Harrison ... 987
Discussion ... 992
Adequate Wiring of Buildings, An Essential for Good Illumination, by G. H. Stickney and Walter Sturrock ... 993
Discussion ... 997
Induction Motor Versatility, by E. W. Henderson ... 1001
Discussion ... 1006
Dynamic Braking of Synchronous Machines, by C. E. Kilbourne and I. [illegible] Terry ... 1007
Discussion ... 1010
Current Propagation in Electric Railway Propulsion Systems, by John Riordan ... 1011
Discussion ... 1019
The Boric Acid Fuse, by A. P. Strom and H. L. Rawlins ... 1020
Discussion ... 1025
The Torque Balance Telemeter, by A. J. Johnston ... 1027
Application and Performance of Automatic Equipment at Stations and Substations on the American Gas and Electric Company System, by Philip Sporn, Basil Lanphier, and Harold E. Turner ... 1031
Discussion ... 1045
Vibration and Fatigue in Electrical Conductors, by A. E. Davison, J. A. Ingles, and V. M. Martinoff ... 1047
Discussion ... 1076
Stress-Strain Studies of Transmission Line Conductors, by G. W. Stickley ... 1052
Discussion ... 1076
Vibration of Overhead Transmission Lines, by R. A. Monroe and R. L. Templin ... 1059
Discussion ... 1076
Transmission Line Vibration Due to Sleet, by J. P. Den Hartog ... 1074
Discussion ... 1076
Technical Committee Reports
Research, L. W. Chubb, Chairman ... 1087
Electrical Machinery, P. L. Alger, Chairman ... 1090
Protective Devices, R. Bailey, Chairman ... 1093
Instruments and Measurements, E. J. Rutan, Chairman ... 1095
Transportation, E. L. Moreland, Chairman ... 1097
Applications to Marine Work, R. A. Beekman, Chairman ... 1099
Production and Application of Light, W. T. Blackwell, Chairman ... 1100
Applications to Mining Work, D. E. Renshaw, Chairman ... 1106
Transmission and Distribution, P. H. Chase, Chairman ... 1107
Power Generation, J. R. Baker, Chairman ... 1108
General Power Applications, C. W. Drake, Chairman ... 1109
Iron and Steel Production, A. C. Cummins, Chairman ... 1109
Communication, H. S. Osborne, Chairman ... 1112

PUBLISHED QUARTERLY BY THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
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PREFACE

This is the final number of Vol. 51 of the QUARTERLY TRANSACTIONS of the American Institute of Electrical Engineers and completes the Institute's published technical record for the year 1932. It embraces the Institute's 48th annual Summer Convention held at Cleveland, Ohio, June 20-24, 1932, and includes all technical committee reports, technical papers, and associated discussions formally presented at that convention.

In keeping with custom, a subject index and an author index covering all four quarterly issues for 1932 are published in this number; this year in an improved and extended form through which subjects may be located with greater facility.

As an explanation of inevitable inconsistencies that may be noted in this and the next subsequent numbers, it is significant to note that a change has been made in the basic reference dictionary by which the editorial staff is guided in the preparation of copy for publication. Beginning January 1, 1933, "Webster's International" dictionary (G. & C. Merriam) will be the editors' guide and arbiter for all published matter. Material composed for publication after January 1, will follow the new guide, but for economical reasons standing material will not be changed. Thus the two styles may appear side by side until all standing material has been published.

to each other and the carrier that harmonic distortion as well as crosstalk between channels is eliminated.

A successful demonstration of two-way radiotelephony on an 18-cm. wavelength was given across the English Channel by the International Telephone and Telegraph Laboratories. This system, called "micro-ray," employs an antenna only 2 cm. in length. The greater directivity of these short waves makes it possible to employ the same wavelength for a number of separate channels closely spaced as to geographical separation, without mutual interference. Because of this, and the wide frequency band thus opened up, the commercial application of the "micro-ray" would make available nearly a quarter of a million additional radio channels.

Improvements have been made during the year in controllers for pre-timed traffic signals. One type is arranged so that signals may be automatically operated on different time cycles at different periods of the day or week. Another type is primarily intended to operate in connection with a flexible progressive system, but may be also used to meet the special requirements of detached street intersections. This provides for several functions in addition to the usual automatic stop-and-go control: manual control, flashing amber on one street and flashing red on the cross streets, lengthened green on the main street, pedestrian period, fire lane at one or more of the intersections and varying go periods at different intersections, while preserving the total cycle for progression.

1932 Index—A.I.E.E. TRANSACTIONS

Papers and reports contained in the 4 quarterly issues of the 1932 Transactions are covered in this annual reference index which is published herewith in an enlarged and improved form. This index embraces all material presented at the Pacific Coast (Lake Tahoe) convention, August 25–28, 1931; the South West District (Kansas City) meeting, October 22–24, 1931; the winter convention, New York, January 25–29, 1932; the Great Lakes District (Milwaukee) meeting, March 14–16, 1932; the North Eastern District (Providence) meeting, May 4–7, 1932; and the summer convention, Cleveland, June 20–24, 1932. Papers informally presented at these meetings (and not published in the Transactions) are included, with references to issues of Electrical Engineering containing these papers in whole or part.

Page Allocation by Issues

March	1–286
June	287–578
September	579–838
December	839–1124

Subject Index

A

A-C Networks in Portland, Oregon. Clark. Electrical Engineering, August 1931, 644
Adequate Wiring of Buildings, an Essential for Good Illumination. Stickney, Sturrock. (Dec.)............ 993
Advance Planning of Long Distance Telephone Facilities. (Abs.) McFarland. Electrical Engineering, November 1931, 902
Air Transportation, Wire Communication Aids to. Nance. (Dec.)...... 974
Aircraft in Flight, Characteristics of Electromagnetic Radiation From. Rives, Coe. (Dec.)............... 981
Alternator, 115,000-Kw Turbo. Williamson. (Sept.)...................... 652
Aluminum, Oxide Coatings on. Edwards, Tosterud, Work. (Dec.).... 900
(American Gas and Elec. Co.) The Application and Performance of Automatic Equipment. Sporn, Lanphier, Turner. (Dec.).......... 1031
Application and Performance of Automatic Equipment, The. Sporn, Lanphier, Turner. (Dec.)......... 1031
Application of High Speed Relays. (Abs.) Gerell. Electrical Engineering, June 1932, 411
Application of Primary Distribution Fuses. (Abs.) Sanford. Electrical Engineering, November 1931, 903
Arc, Forces of Electric Origin in the Iron. Creedy, Lerch, Seal, Sordon. (June)............................ 556
Arc Rupturing Devices, Recent Developments in. Van Sickle, Leeds. (March)........................... 177
Arc Welder, An Improved A-C. Candy. (June)............................ 567
Arc Welding Generators, Recent Developments in Design of. Hansen. (June)............................ 570
Arc Welding in Building Construction, Electric. Vinther. Electrical Engineering, October 1931, 785
Arcover Values of Insulators as Affected by Size and Humidity, Normal Frequency. Frey, Hawley. (Sept.). 690
Arcs and Glow Discharges, Cathode Drop in. (Abs.) Mackeown. Electrical Engineering, September 1931, 755
Arcs in Turbulent Gases, Extinction of A-C. Browne. (March).......... 185
Armatures, Temperature Rise of Ventilated Railway Motor. Lightband. (June)............................ 502
Automatic Control for Variable-Ratio Frequency Converters. Shively, Whitlow. (March)............... 121
(Automatic) Developments in Two-Wire Supervisory Systems. (Abs.) Oliver. Electrical Engineering, June 1931, 412
Automatic Equipment, The Application and Performance of. Sporn, Lanphier, Turner. (Dec.).......... 1031
Automatic Oscillograph, An. Hathaway, Buell. (March).................. 222
Automatic Railway Substations, Mercury Arc Rectifier Versus Rotary Converter. (Abs.) Ward. Electrical Engineering, March 1932, 191
Auxiliaries for Power Plants, Steam-Driven. Dryer. (June).......... 331
Auxiliaries for Steam Power Stations, Electrically Driven. Smith. (June)............................ 337
Auxiliary Drive for Steam Power Stations. Hollister. (June).......... 329
(Aviation) Electrical Solutions of Problems of Regular Scheduled Flight. Green. Electrical Engineering, August 1931, 654

B

Backfires in Mercury Arc Rectifiers. Slepian, Ludwig. (March)........ 92
Boric Acid Fuse, The. Strom, Rawlins. (Dec.).......................... 1020
Braking of Synchronous Machines, Dynamic. Kilbourne, Terry. (Dec.).......................... 1007
Breakdown of Glass With Alternating Potentials. Kenney, Luery, Moriaty. (June)................ 404
Bridge, A High Sensitivity Power Factor. Kouwenhoven, Banos. (March)........................ 202
Bridge for Measurements of Cables With Grounded Sheaths, High-Voltage. Dawes, Daniel. (March)......... 198

Bridge, Power Factor Measurement by the Capacitance. Siskind. (March) 214
Brush Contact Drop, Influence on Commutation of. Ludwig, Baker. (Dec.) 959
Building Construction, Electric Arc Welding in. Vinther. Electrical Engineering, October 1931, 785
Buildings, an Essential for Good Illumination, Adequate Wiring of. Stickney, Sturrock. (Dec.) 993

C

Cable Network, The San Francisco-Los Angeles Section of the Pacific Coast Telephone. Calderwood, Smith. (March) 63
Cable, Tape-Armored Telephone Toll. Nystrom. (March) 135
Cables With Grounded Sheaths, High Voltage Bridge for Measurements of. Dawes, Daniel. (March) 198
Calculating Load Division in Distribution Systems. (Abs.) Almquist. Electrical Engineering, November 1931, 900
Calculation of Inductance and Current Distribution in Low-Voltage Connections to Electric Furnaces. Levy. (Dec.) 903
Calculation of No-Load Damper Winding Loss in Synchronous Machines. Pollard. (June) 477
Calculation of Short-Circuits on Power Systems. (Abs.) Wagner, Wright. Electrical Engineering, February 1932, 131
Calculations on Induction Motors, Performance. Veinott. (Sept.) 743
Calorimeter Measurement of Stray-Load Losses of a 55,000-Kva Generator. Floyd, Dunbar. (March) 12
Capacitance Bridge, Power Factor Measurement by the. Siskind. (March) 214
Capacitor Motors for Balanced Operation, Design of. Trickey. (Sept.) 780
Cathode Drop in Arcs and Glow Discharges. (Abs.) Mackeown. Electrical Engineering, September 1931, 755
Characteristics of Electromagnetic Radiation From Aircraft in Flight. Rives, Coe. (Dec.) 981
Characteristics of Load Ratio Control Circuits for Changing Transformer Ratio Under Load. Blume. (Dec.) 952
Characteristics of Surge Generators for Transformer Testing. Bellaschi. (Dec.) 936
Chicago District. Perry, Smith. (Dec.) (See Reliability) 879
(Chronometers) Modern Developments in Precision Clocks. Loomis, Marrison. (June) 527
(Chronometers) Synchronous Electric Time Service. Janson. (June) 546
(Chronometers) Time and Its Measurement. Brown. (June) 523
(Chronometers) Time Service of the U. S. Naval Observatory. Hellweg. (June) 538
(Chronometers) Time Services of the Telegraph Companies. Janson. (June) 541
(Circuit Breakers) Recent Developments in Arc Rupturing Devices. Van Sickle, Leeds. (March) 177
Circuit Breakers, The Theory of Oil-Blast. Prince. (March) 166
Circuit Interruption, Practical Application of the Oil-Blast Principle of. Spurck (March) 171
(Circuits) Calculation of Inductance and Current Distribution in Low-Voltage Connections to Electric Furnaces. Levy. (Dec.) 903
Circuits, Equivalent. Starr. (June) 287
Circuits for Changing Transformer Ratio Under Load, Characteristics of Load Ratio Control. Blume. (Dec.) 952
Circuits Involving Synchronous Machines, Vector Theory of. Summers. (June) 318
Circuits of Electrical Networks, Geometrical. Foster. (June) 309
Circuits Subjected to Traveling Waves, Solution of. Rorden. (Sept.) 824
Circuits, Three-Phase Multiple-Conductor. Clarke. (Sept.) 809
Circuits to Relay and Control Problems, New Applications of Non-Linear. Suits. (Dec.) 914
Clocks, Modern Developments in Precision. Loomis, Marrison. (June) 527
(Clocks) Synchronous Electric Time Service. Warren. (June) 546
(Clocks) Time and Its Measurement. Brown. (June) 523
(Clocks) Time Service of the U. S. Naval Observatory. Hellweg. (June) 538
(Clocks) Time Services of the Telegraph Companies. Janson. (March) 541
College Program of Electrical Engineering, Engineering Subjects, Electrical and Cognate in the Four-Year. Lovell. (Dec.) 839
Combined Reliability and Economy in Operation of Large Electric Systems. (Dec.) (See Reliability)
(Communication) Advance Planning of Long Distance Telephone Facilities. (Abs.) McFarland. Electrical Engineering, November 1931, 902
Communication Aids to Air Transportation, Wire. Nance. (Dec.) 974
(Communication) Characteristics of Electromagnetic Radiation From Aircraft in Flight. Rives, Coe. (Dec.) 981
(Communication) Current Propagation in Electric Railway Propulsion Systems. Riordan. (Dec.) 1011
(Communication) Geometrical Circuits of Electrical Networks. Foster. (June) 309
(Communication) Intercontinental Radio-Telephone Service From the United States. Pilliod. (March) 56
Communication, Police Teletypewriter. Pierce. Electrical Engineering, August 1932, 550
(Communication) Radio Coordination. Campbell, Kalb. (March) 81
(Communication) Tape-Armored Telephone Toll Cable. Nystrom. (March) 135
(Communication) The San Francisco-Los Angeles Section of the Pacific Coast Telephone Cable Network. Calderwood, Smith. (March) 63
(Communication) The Time Factor in Telephone Transmission. Blackwell. (March) 141
(Communication) Toll Switching Plan for Wisconsin. Lallier. (Sept.) 628
(Communication) Vertically Cut Sound Records. Frederick, Harrison. (Dec.) 987
Commutation Considered as a Switching Phenomenon. Hellmund, Ludwig. (June) 465
Commutation of Brush Contact Drop, Influence on. Ludwig, Baker. (Dec.) 959
Condensers, Film Characteristics of Electrolytic. Godsey. (June) 432
Condensers, Power Losses in Electrolytic. Godsey. (June) 439
Conductors for Transmission Lines, Double. Dwight, Farmer. (Sept.) 803
Conductors, Stress-Strain Studies of Transmission Line. Stickley. (Dec.) 1052
Conductors, Vibration and Fatigue in Electrical. Davison, Ingles, Martinoff. (Dec.) 1047
Conowingo Hydroelectric Station, Philadelphia Electric Company System, Stability of. Hentz, Jones. (June) 375
Control, A General Method of Gaseous Tube. (Abs.) Stansbury. Electrical Engineering, March 1932, 188
(Control) Developments in Two-Wire Supervisory Systems. (Abs.) Oliver. Electrical Engineering, June 1932, 412
(Control) Electric Instruments in the Gas Industry. (Abs.) Schmidt. Electrical Engineering, March 1932, 189
Control for Variable-Ratio Frequency Converters, Automatic. Shively, Whitlow. (March) 121
Control Problems, New Applications of Non-Linear Circuits to Relay and. Suits. (Dec.) 914
(Control) The Application and Performance of Automatic Equipment. Sporn, Lanphier, Turner. (Dec.) 1031
Converter Automatic Railway Substations, Mercury Arc Rectifier versus Rotary. (Abs.) Ward. Electrical Engineering, March 1932, 191
Converters, Automatic Control for Variable-Ratio Frequency. Shively, Whitlow. (March) 121
(Converters) Interconnection of the 25- and 60-Cycle System of the Union Electric Light and Power Company. Nelson, Hough. (March) 113
Coordination of Transformer Insulation With Line Insulation. Montsinger, Dann. (Dec.) 923
Coordination, Radio. Campbell, Kalb. (March) 81
Corona Energy Loss. Weidlein. (March) 154
Corona, Radio Interference From Insulator. McMillan. (June) 385
Correlation of Induction Motor Design Factors. (Abs.) Hoover. Electrical Engineering, September 1931, 754
(Cottrell) Electrical Equipment for Precipitation Service. Speight, Rydberg. (Dec.) 892
(Cottrell Smoke Precipitators) Electrical Precipitation. Simon, Kron. (June) 451
Current Propagation in Electric Railway Propulsion Systems. Riordan. (Dec.) 1011
Cut-Outs—Their Design and Application for A-C Distribution Circuits, Fuse. (Abs.) Newton. Electrical Engineering, November 1931, 903

D

Damper Windings for Synchronous Motors, Some Considerations in the Design of. Shutt. (June) 424
Damping Characteristics of Synchronous Generators, Field Tests to Determine the. Hamilton. (Sept.) 775
Decrement Curves for Specific Systems. (Abs.) Hahn. Electrical Engineering, February 1932, 131
Decrement Curves, Standard. Hahn, Wagner. (June) 353
Definitions of Terms Used in Power System Terms, Proposed. (Com. rept.) (Sels) Electrical Engineering, February 1932, 106
Design of Capacitor Motors for Balanced Operation. Trickey. (Sept.) 780
Detroit Edison Co. Fugill. (Dec.) (See Reliability) 859
Development of the Waukegan Station of the Pub. Serv. Co. of No. Ill. Hecht. Electrical Engineering, June 1932, 391

Developments in Two-Wire Supervisory Systems. (Abs.) Oliver. Electrical Engineering, June 1932, 412
(Dielectrics) A High Sensitivity Power Factor Bridge. Kouwenhoven, Banos. (March) 202
(Dielectrics) Breakdown of Glass With Alternating Potentials. Kenney, Luery, Moriaty. (June) 404
(Dielectrics) High-Voltage Bridge for Measurements of Cables With Grounded Sheaths. Daws, Daniel. (March) 198
(Dielectrics) Power Factor Measurement by the Capacitance Bridge. Siskind. (March) 214
Dielectrics, Predetermination of the A-C Characteristics of. Whitehead, Banos. (June) 392
Discharges, Cathode Drop in Arcs and Glow. (Abs.) Mackeown. Electrical Engineering, September 1931, 755
(Distribution) A-C Networks in Portland, Oregon. Electrical Engineering, August 1931, 644
(Distribution) Decrement Curves for Specific Systems. (Abs.) Hahn. Electrical Engineering, February 1932, 131
(Distribution) Equivalent Circuits. Starr. (June) 287
(Distribution) Fuse Cut-Outs—Their Design and Application for A-C Distribution Circuits. (Abs.) Newton. Electrical Engineering, November 1931, 903
Distribution Fuses, Application of Primary. (Abs.) Sanford. Electrical Engineering, November 1931, 903
(Distribution) Generalized Stability Solution for Metropolitan Type Systems. Griscom, Lewis, Ellis. (June) 363
(Distribution) Geometrical Circuits of Electrical Networks. Foster. (June) 309
(Distribution) Interconnection of Primary Lightning Arrester Ground and the Grounded Neutral of the Secondary Main. Harding, Sprague. (March) 234
(Distribution) Studies in Lightning Protection on 4,000-Volt Circuits. Roper. (March) 252
Distribution Substation, Impulse Voltage Tests on a 4,800-Volt. Collins, Piepho, Torok. (March) 148
Distribution System Lightning Studies by Philadelphia Electric Company. Dambly, Ekvall, Phelps. (March) 265
Distribution Systems Calculating Load Division in. (Abs.) Almquist. Electrical Engineering, November 1931, 900
(Distribution) The Coordination of Transformer Insulation With Line Insulation. Montsinger, Dann. (Dec.) 923
Distribution Transformers, Insulation Coordination of. Treanor, Cooney. (Dec.) 925
Distribution Transformers, Lightning Protection for.
  McEachron, Saxon. (March) 239
  Opsahl, Brookes, Southgate. (March) 245
  Haines, Corney. (March) 259
(Distribution) Weather Resistant Covering for Line Wires. Harding, Carter, Olson. (Sept.) 635
Double Conductors for Transmission Lines. Dwight, Farmer. (Sept.) 803
Dynamic Braking of Synchronous Machines. Kilbourne, Terry. (Dec.) 1007

E

Economy in Operation of Large Electric Systems, Combined Reliability and. (Dec.) (See Reliability)
Edison Elec. Illum. Co. of Boston. Dillon. (Dec.) (See Reliability) 866
(Education) Engineering Subjects, Electrical and Cognate, in the Four-Year College Program of Electrical Engineering. Lovell. (Dec.) 839
(Education) The Engineer in a Changing Society. Wickenden. Electrical Engineering, July 1932, 465
Educational Aspects of Engineering and Management. Doherty. (Dec.) 848
Effect of Transient Voltages on Power Transformer Design. Palueff, Hagenguth. (Sept.) 601
Effect of Wave Form on Operation of Induction Type Protective Relays. (Abs.) Robinson. Electrical Engineering, November 1931, 900
(Efficiency) Calculation of No-Load Damper Winding Loss. Pollard. (June) 477
(Efficiency) Calorimeter Measurement of Stray-Load Losses of a 55,000 Kva Generator. Floyd, Dunbar. (March) 12
(Efficiency) Corona Energy Loss. Weidlein. (March) 154
(Efficiency) Double Conductors for Transmission Lines. Dwight, Farmer. (Sept.) 803
(Efficiency) Power Losses in Electrolytic Condensers. Godsey. (June) 439
(Efficiency) Three-Phase Multiple-Conductor Circuits. Clarke. (Sept.) 809
Electric and Magnetic Units, A General Theory of Systems of. Karapetoff. (Sept.) 715
Electric Arc Welding in Building Construction. Vinther. Electrical Engineering, October 1931, 785
Electric Furnaces, Calculation of Inductance and Current Distribution in Low-Voltage Connections to. Levy. (Dec.) 903
Electric Instruments in the Gas Industry. (Abs.) Schmidt. Electrical Engineering, March 1932, 189
Electric Power in the Wood-Products Industry. (Abs.) Carey, Howe. Electrical Engineering, September 1931, 753
Electrical Design Features of Waukegan Station. Williams. (Sept.) 644
Electrical Engineering, Engineering Subjects, Electrical and Cognate, in the Four-Year College Program of. Lovell. (Dec.) 839
Electrical Equipment for Oil Field Operations. Hill, SeLegue. (March) 1
Electrical Equipment for Precipitation Service. Speight, Rydberg. (Dec.) 892
Electrical Measurements of Sound-Absorption. (Abs.) Albert, Bullis. Electrical Engineering, September 1931, 755
Electrical Precipitation. Simon, Kron. (June) 451
Electrical Solutions of Problems of Regular Scheduled Flight. Green. Electrical Engineering, 1931, 654
Electrical Unit Systems, A Proposal to Abolish the Absolute. Weber. (Sept.) 728
Electrically Driven Auxiliaries for Steam Power Stations. Smith. (June) 337
Electrification, The Mercury Arc Rectifier Applied to A-C Railway. Marti. (Sept.) 659
Electrolytic Condensers, Film Characteristics of. Godsey. (June) 432
Electrolytic Condensers, Power Losses in. Godsey. (June) 439
Electro-Matic Traffic Dispatching System, The. Haugh. (Sept.) 790
(Electronics) A General Method of Gaseous Tube Control. (Abs.) Stansbury. Electrical Engineering, March 1932, 188
(Electronics) The Parallel Type Inverter. Tompkins. (Sept.) 707
(Electronics) The Torque Balance Telemeter. Johnston. (Dec.) 1027
Engineer in a Changing Society, The. Wickenden. Electrical Engineering, July 1932, 465
Engineering and Management, Educational Aspects of. Doherty. (Dec.) 848
Engineering Features of Three-Power Locomotives. Brehob, Craton. (June) 489
Engineering Subjects, Electrical and Cognate, in the Four-Year College Program of Electrical Engineering. Lovell. (Dec.) 839
Equalizing Currents in the Armature of a D-C Machine. Baker. (June) 473
Equivalent Circuits. Starr. (June) 287
Expulsion Fuse, The. Slepian, Denault. (March) 157
Extinction of A-C Arcs in Turbulent Gases. Browne. (March) 185

F

Fatigue in Electrical Conductors, Vibration and. Davison, Ingles, Martinoff. (Dec.) 1047
(Fatigue) Stress-Strain Studies of Transmission Line Conductors. Stickley. (Dec.) 1052
(Fatigue) Transmission Line Vibration Due to Sleet. DenHartog. (Dec.) 1074
(Fatigue) Vibration of Overhead Transmission Lines. Monroe, Templin. (Dec.) 1059
Field Tests to Determine the Damping Characteristics of Synchronous Generators. Hamilton. (Sept.) 775
Film Characteristics of Electrolytic Condensers. Godsey. (June) 432
Flexible Progressive Traffic Signal System, The. Turner. (Sept.) 786
Forces of Electric Origin in the Iron Arc. Creedy, Lerch, Seal, Sordon. (June) 556
Forecasting Population for Engineering Purposes. Holsen. (March) 128
Forecasting, Snow Surveys and Hydro-. (Abs.) Boardman. Electrical Engineering, September 1931, 756
Frequency Converters, Automatic Control for Variable-Ratio. Shively, Whitlow. (March) 121
(Frequency Converters) Interconnection of the 25- and 60-Cycle System of the Union Electric Light and Power Company. Nelson, Hough. (March) 113
Fuse Cut-Outs—Their Design and Application for A-C Distribution Circuits. (Abs.) Newton. Electrical Engineering, November 1931, 903
Fuse, The Boric Acid. Strom, Rawlins. (Dec.) 1020
Fuse, The Expulsion. Slepian, Denault. (March) 157
Fuses, Application of Primary Distribution. (Abs.) Sanford. Electrical Engineering, November 1931, 903

G

Gas Industry, Electric Instruments in the. (Abs.) Schmidt. Electrical Engineering, March 1932, 189
General Method of Gaseous Tube Control, A. (Abs.) Stansbury. Electrical Engineering, February 1932, 188
General Theory of Systems of Electric

and Magnetic Units, A. Karapetoff. (Sept.) 715
Generalised Stability Solution for Metropolitan Type Systems. Griscom, Lewis, Ellis. (June) 363
(Generation) Electrical Design Features of Waukegan Station. Williams. (Sept.) 644
Generator, Calorimeter Measurement of Stray-Load Losses of a 55,000-Kva. Floyd, Dunbar. (March) 12
(Generators) Calculation of No-Load Damper Winding Loss in Synchronous Machines. Pollard. (June) 477
(Generators) Commutation Considered as a Switching Phenomenon. Hellmund, Ludwig. (June) 465
(Generators) Equalizing Currents in the Armature of a D-C Machine. Baker. (June) 473
Generators, Field Tests to Determine the Damping Characteristics of Synchronous. Hamilton. (Sept.) 775
Generators for Transformer Testing, Characteristics of Surge. Bellaschi. (Dec.) 936
(Generators) Influence on Commutation of Brush Contact Drop. Ludwig, Baker. (Dec.) 959
(Generators) Overvoltage on Transmission Systems Caused by Dropping Load. Burnham. (March) 105
Generators, Recent Developments in Design of Arc Welding. Hansen. (June) 570
Generators, Sine-Wave. Shildneck. (June) 484
(Generators) Single-Phase Short-Circuit Torque of a Synchronous Machine. Nickle, Pierce, Henderson. (Dec.) 966
(Generators) Torque-Angle Characteristics of Synchronous Machines Following System Disturbances. Crary, Waring. (Sept.) 764
(Generators) Turbo-Alternator, 115,000. Williamson. (Sept.) 652
(Generators) Vector Theory of Circuits Involving Synchronous Machines. Summers. (June) 318
Geometrical Circuits of Electrical Networks. Foster. (June) 309
Grounded Neutral of the Secondary Main, Interconnection of Primary Lightning Arrester Main. Harding, Sprague. (March) 234

H

(Heat) Electric Instruments in the Gas Industry. (Abs.) Schmidt. Electrical Engineering, March 1932, 189
(Heating) The Proximity Effect. Bennett. (Sept.) 621
High Capacity Rectifier Efficiency Improves by Sectionalizing. Atherton. (June) 511
High Sensitivity Power Factor Bridge, A. Kouwenhoven, Banos. (March) 202
High Voltage Bridge for Measurements of Cables With Grounded Sheaths. Dawes, Daniel. (March) 198
(Human Relations) Educational Aspects of Engineering and Management. Doherty. (Dec.) 848
(Human Relations) The Engineer in a Changing Society. Wickenden. Electrical Engineering, July 1932, 465
Hydro-Forecasting, Snow Surveys and. (Abs.) Boardman. Electrical Engineering, September 1931, 756
Hydroelectric Station, Phila. Elec. Co. System, Stability of Conowingo. Hentz, Jones. (June) 375
(Hydroelectric) The Mokelumne River Development of the Pacific Gas and Elec. Co. Crellin. (March) 28

I

Illumination, Adequate Wiring of Buildings, an Essential for Good. Stickney, Sturrock. (Dec.) 993
Improved A-C Arc Welder, An. Candy. (June) 567
Improved Type of Limiting Gap for Protecting Station Apparatus, An. Austin. (Sept.) 676
Impulse Voltage Tests on a 4,800-Volt Distribution Substation. Collins, Piepho, Torok. (March) 148
Induction Motor Versatility. Henderson. (Dec.) 1001
Induction Motor With Paralleled Rotor and Stator, An. Conrad, Warner. (June) 418
Induction Motors, Measurement of Stray Load Loss in Polyphase. Koch. (Sept.) 756
Induction Motors, Performance Calculations. Veinott. (Sept.) 743
(Industrial) Dynamic Braking of Synchronous Machines. Kilbourne, Terry. (Dec.) 1007
(Industrial) Electrical Equipment for Oil Field Operations. Hill, SeLegue. (March) 1
Industry, Electric Power in the Wood-Products. (Abs.) Carey, Howe. Electrical Engineering, September 1931, 753
Influence on Commutation of Brush Contact Drop. Ludwig, Baker. (Dec.) 959
(Instruments) A High Sensitivity Power Factor Bridge. Kouwenhoven, Banos. (March) 202
(Instruments) An Automatic Oscillograph. Hathaway, Buell. (March) 222
(Instruments) Electrical Solutions of Problems of Regular Scheduled Flight. Green. Electrical Engineering, August 1931, 654
(Instruments) High Voltage Bridge for Measurements of Cables With Grounded Sheaths. Dawes, Daniel. (March) 198
Instruments in the Gas Industry, Electric. (Abs.) Schmidt. Electrical Engineering March 1932, 189
(Instruments) Power Factor Measurement by the Capacitance Bridge. Siskind. (March) 214
(Instruments) The Photoelectric Recorder. LaPierre. (March) 226
(Instruments) The Torque Balance Telemeter. Johnston. (Dec.) 1027
(Insulation) A High Sensitivity Power Factor Bridge. Kouwenhoven, Banos. (March) 202
(Insulation) An Improved Type of Limiting Gap for Protecting Station Apparatus. Austin. (Sept.) 676
(Insulation) Breakdown of Glass With Alternating Potentials. Kenney, Luery, Moriaty. (June) 404
Insulation Coordination of Distribution Transformers. Treanor, Cooney. (Dec.) 925
(Insulation) High-Voltage Bridge for Measurements of Cables With Grounded Sheaths. Dawes, Daniel. (March) 198
(Insulation) Power Factor Measurement by the Capacitance Bridge. Siskind. (March) 214
(Insulation) Predetermination of the A-C Characteristics of Dielectrics. Whitehead, Banos. (June) 392
(Insulation) Weather Resistant Covering for Line Wires. Harding, Carter, Olson. (Sept.) 635
Insulation With Line Insulation, The Coordination of Transformer. Montsinger, Dann. (Dec.) 923
Insulator Assemblies, Their Design and Economic Selection, Suspension. Torok, Archibald. (Sept.) 682
Insulator Corona, Radio Interference From. McMillan. (June) 385
Insulator Sparkover. Lloyd. (Sept.) 669
Insulators as Affected by Size and Humidity, Normal Frequency Arcover Values of. Frey, Hawley. (Sept.) 690
Interconnected Networks, Tie-Line Control of. Purcell, Powel. (March) 40
(Interconnection) Automatic Control for Variable-Ratio Converters. Shively, Whitlow. (March) 121
Interconnection of Primary Lightning Arrester Ground and the Grounded Neutral of the Secondary Main. Harding, Sprague. (March) 234
Interconnection of the 25- and 60-Cycle System of the Union Elec. Lt. and Pwr. Co. Nelson, Hough. (March) 113
Intercontinental Radiotelephone Service From the United States. Pilliod. (March) 56
(Interference) Current Propagation in Electric Railway Propulsion Systems. Riordan. (Dec.) 1011
Interference From Insulator Corona, Radio. McMillan. (June) 385
(Interference) Radio Coordination. Campbell, Kalb. (March) 81
Inverter, The Parallel Type. Tompkins. (Sept.) 707

K

Kindling of Electric Sparkover Based on Lichtenberg Figures, The. Magnusson. (March) 74
(Kirchoff's Laws) Calculating Load Division in Distribution Systems. (Abs.) Almquist. Electrical Engineering, November 1931, 900

L

Lichtenberg Figures, The Kindling of Electric Sparkover Based on. Magnusson. (March) 74
Lightning Arrester Ground and the Grounded Neutral of the Secondary Main, Interconnection of Primary. Harding, Sprague. (March) 234
(Lightning) Characteristics of Surge Generators for Transformer Testing. Bellaschi. (Dec.) 936
Lightning Disturbances on Transmission Lines. (Abs.) Jackson, Warner. Electrical Engineering, November 1931, 903
(Lightning) Effect of Transient Voltages on Power Transformer Design. Palueff, Hagenguth. (Sept.) 601
(Lightning) Impulse Voltage Tests on a 4,800-Volt Distribution Substation. Collins, Piepho, Torok. (March). 148
(Lightning) Insulation Coordination of Distribution Transformers. Treanor, Cooney. (Dec.) 925
(Lightning) Insulator Sparkover. Lloyd. (Sept.) 669
(Lightning) Normal Frequency Arcover Values of Insulators as Affected by Size and Humidity. Frey, Hawley. (Sept.) 690
(Lightning) (Protection) An Improved Type of Limiting Gap for Protecting Station Apparatus. Austin. (Sept.) 676
Lightning Protection for Distribution Transformers.
  McEachron, Saxon. (March) 239
  Opsahl, Brookes, Southgate. (March) 245
  Haines, Corney. (March) 259
Lightning Protection on 4,000-Volt Circuits, Studies in. Roper. (March) 252
(Lightning) Solution of Circuits Sub-

jected to Traveling Waves. Rorden. (Sept.) 824
Lightning Studies by Philadelphia Electric Company. Dambly, Ekvall, Phelps. (March) 265
(Lightning) Surge-Proof Transformers. Putman. (Sept.) 579
(Lightning) Suspension Insulator Assemblies. Torok, Archibald. (Sept.) 682
(Lightning) The Coordination of Transformer Insulation With Line Insulation. Montsinger, Dann. (Dec.) 923
Locomotives, Engineering Features of Three-Power. Brehob, Craton. (June) 489
Loss, Corona Energy. Weidlein. (March) 154
Loss in Polyphase Induction Motors, Measurement of Stray Load. Koch. (Sept.) 756
Loss in Synchronous Machines, Calculation of No-Load Damper Winding. Pollard. (June) 477
Losses in Electrolytic Condensers, Power. Godsey. (June) 439
Losses of a 55,000-Kva Generator, Calorimeter Measurement of Stray-Load. Floyd, Dunbar. (March) 12

M

Magnet Steels and Permanent Magnets. Scott. (June) 410
(Magnetic) A Proposal to Abolish the Absolute Electrical Unit Systems. Weber 728
Magnetic Units, A General Theory of Systems of Electric and. Karapetoff. (Sept.) 715
Management, Educational Aspects of Engineering and. Doherty. (Dec.) 848
Measurement by the Capacitance Bridge, Power Factor. Siskind. (March) 214
Measurement of Stray Load Loss in Polyphase Induction Motors. Koch. (Sept.) 756
Measurement of Stray Load Losses of a 55,000-Kva Generator, Calorimeter. Floyd, Dunbar. (March) 12
Measurement, Time and Its. Brown. (June) 523
(Measurements) A High Sensitivity Power Factor Bridge. Kouwenhoven, Banos. (March) 202
(Measurements) An Automatic Oscillograph. Hathaway, Buell. (March) 222
Measurements of Cables With Grounded Sheaths, High Voltage Bridge for. Dawes, Daniel. (March) 198
Measurements of Sound-Absorption, Electrical. (Abs.) Albert, Bullis. Electrical Engineering, September 1931, 755
(Measurements) The Photoelectric Recorder. LaPierre. (March) 226
(Measurements) The Torque Balance Telemeter. Johnston. (Dec.) 1027
Mercury Arc Rectifier Applied to A-C Railway Electrification, The. Marti. (Sept.) 659
(Mercury Arc) Rectifier Efficiency Improved by Sectionalizing, High Capacity. Atherton. (June) 511
Mercury Arc Rectifier versus Rotary Converter Automatic Railway Substations. (Abs.) Ward. Electrical Engineering, March 1932, 191
Mercury Arc Rectifiers, Backfires in. Slepian, Ludwig. (March) 92
Milwaukee Elec. Ry. and Lt. Co., The 60-Cycle Primary Transmission System of the. Brown, Hatz. Electrical Engineering, October 1932, 708
Modern Developments in Precision Clocks. Loomis, Marrison. (June) 527
Mokelumne River Development of the Pacific Gas and Elec. Co., The. Crellin. (March) 28
Mokelumne River Development of the Pacific Gas and Elec. Co., Power Transmission and Distribution From the. Wright, Dexter. (March) 20
Motor Armatures, Temperature Rise of Ventilated Railway. Lightband. (June) 502
Motor Design Factors, Correlation of Induction. (Abs.) Hoover. Electrical Engineering, September 1931, 754
Motor Versatility, Induction. Henderson. (Dec.) 1001
Motor With Paralleled Rotor and Stator, An Induction. Conrad, Warner. (June) 418
(Motors) Calculation of No-Load Damper Winding Loss in Synchronous Machines. Pollard. (June). 477
(Motors) Commutation Considered as a Switching Phenomenon. Hellmund, Ludwig. (June) 465
(Motors) Dynamic Braking of Synchronous Machines. Kilbourne, Terry. (Dec.) 1007
(Motors) Equalizing Currents in the Armature of a D-C Machine. Baker. (June) 473
Motors for Balanced Operation, Design of Capacitor. Trickey. (Sept.) 780
Motors for 3,000-Volt D-C Multiple Unit Cars. Aydelott. (June) 498
(Motors) Influence on Commutation of Brush Contact Drop. Ludwig, Baker. (Dec.) 959
Motors, Measurement of Stray Load Loss in Polyphase Induction. Koch. (Sept.) 756
Motors, Performance Calculations on Induction. Veinott. (Sept.) 743
(Motors) Single-Phase Short-Circuit Torque of a Synchronous Machine. Nickle, Pierce, Henderson. (Dec.) 966
Motors, Some Considerations in the Design of Damper Windings for Synchronous. Shutt. (June) 424
(Motors) Torque-Angle Characteristics of Synchronous Machines Following System Disturbances. Crary, Waring. (Sept.) 764
(Motors) Vector Theory of Circuits Involving Synchronous Machines. Summers. (June) 318
Multiple Unit Cars, Motors for 3,000-Volt D-C. Aydelott. (June) 498

N

Naval Observatory, Time Service of the U. S. Hellweg. (June) 538
(Networks) Equivalent Circuits. Starr. (June) 287
Networks, Geometrical Circuits of Electrical. Foster. (June) 309
Networks in Portland, Oregon, A-C. Clark. Electrical Engineering, August 1931, 644
Networks, Tie-Line Control of Interconnected. Purcell, Powel. (March) 40
New Applications of Non-Linear Circuits to Relay and Control Problems. Suits. (Dec.) 914
New High Speed Distance Relay, A. Goldsborough, Lewis. Electrical Engineering, March 1932, 157
New High-Speed Reactance Relay, A. (Abs.) Warrington. Electrical Engineering, June 1932, 410
(Noise) Electrical Measurements of Sound-Absorption. (Abs.) Albert, Bullis. Electrical Engineering, September 1931, 755
(Nomenclature) Proposed Definitions of Terms Used in Power System Studies. (Com. rept.) (Sels) Electrical Engineering, February 1932, 106
Normal Frequency Arcover Values of Insulators as Affected by Size and Humidity. Frey, Hawley. (Sept.) 690

O

Oil-Blast Circuit Breakers, The Theory of. Prince. (March) 166
Oil-Blast Principle of Circuit Interruption, Practical Application of the. Spurck. (March) 171
Oil Field Operations, Electrical Equipment for. Hill, SeLegue. (March) 1
Operating Experience With Supervisory Control. Pastoret. Electrical Engineering, November 1932, 776
Operation of Large Electric Systems, Combined Reliability and Economy in. (Dec.) (See Reliability)
Oscillations of Mutually Coupled Windings, Transient. Bewley. (June) 299
Oscillograph, An Automatic. Hathaway, Buell. (March) 222
Overvoltage on Transmission Systems Caused by Dropping Load. Burnham. (March) 105
Oxide Coatings on Aluminum. Edwards, Tosterud, Work. (Dec.) 900

P

Pacific Gas and Elec. Co., Power Transmission and Distribution From the Mokelumne River Development of the. Wright, Dexter. (March) 20
Pacific Gas and Elec. Co., The Mokelumne River Development. Crellin. (March) 28
Parallel Type Inverter, The. Tompkins. (Sept.) 707
(Pennsylvania RR.) Preparing Substations for Electric Operation. Curtis. (June) 518
Performance Calculations on Induction Motors. Veinott. (Sept.) 743
Performance of Automatic Equipment, The Application and. Sporn, Lanphier, Turner. (Dec.) 1031
Permanent Magnets, Magnet Steels and. Scott. (June) 410
Philadelphia Elec. Co., Distribution System Lightning Studies by. Dambly, Ekvall, Phelps. (March) 265
(Philadelphia Elec. Co.) Stability of Conowingo Hydroelectric Station. Hentz, Jones. (June) 375
Philadelphia Elec. Co. System. Anderson, Estrada. (Dec.) 872
Photoelectric Recorder, The. LaPierre. (March) 226
Pilot-Wire Relay System, A Telegraphic. Frier. Electrical Engineering, October 1931, 824
Police Teletypewriter Communication. Pierce. Electrical Engineering, August 1932, 550
Polyphase Induction Motors, Measurement of Stray Load Loss. Koch. (Sept.) 756
Population for Engineering Purposes, Forecasting. Holsen. (March) 128
Power Factor Bridge, A High Sensitivity. Kouwenhoven, Banos. (March) 202
Power Factor Measurement by the Capacitance Bridge. Siskind. (March) 214
Power Losses in Electrolytic Condensers. Godsey. (June) 439
Power Systems, Calculation of Short-Circuits on. (Abs.) Wagner, Wright. Electrical Engineering, February 1932, 131
Power Transmission and Distribution From the Mokelumne River Development of the Pacific Gas and Elec. Co. Wright, Dexter. (March) 20
Practical Application of the Oil-Blast Principle of Circuit Interruption. Spurck. (March) 171

Precipitation, Electrical. Simon, Kron. (June) 451
Precipitation Service, Electrical Equipment for. Speight, Rydberg. (Dec.) 892
Predetermination of the A-C Characteristics of Dielectrics. Whitehead, Banos. (June) 392
Preparing Substations for Electric Operation. Curtis. (June) 518
Proposal to Abolish the Absolute Electrical Unit Systems, A. Weber. (Sept.) 728
Proposed Definitions of Terms Used in Power System Studies. (Com. rept.) (Sels) Electrical Engineering, February 1932, 106
(Protection) A New High Speed Distance Relay. Goldsborough, Lewis. Electrical Engineering, March 1932, 157
(Protection) A New High-Speed Reactance Relay. (Abs.) Electrical Engineering, June 1932, 410
(Protection) A Telegraphic Pilot-Wire Relay System. Frier. Electrical Engineering, October 1931, 824
(Protection) Application of High Speed Relays. (Abs.) Gerell. Electrical Engineering, June 1932, 411
(Protection) Application of Primary Distribution Fuses. (Abs.) Sanford. Electrical Engineering, November 1931, 903
(Protection) Effect of Wave Form on Operation of Induction Type Protective Relays. (Abs.) Robinson. Electrical Engineering, November 1931, 900
Protection for Distribution Transformers, Lightning.
  McEachron, Saxon. (March) 239
  Opsahl, Brookes, Southgate. (March) 245
  Haines, Corney. (March) 259
(Protection) Fuse Cut-Outs—Their Design and Application for A-C Distribution Circuits. (Abs.) Newton. Electrical Engineering, November 1931, 903
(Protection) Impulse Voltage Tests on 4,800-Volt Distribution Substation. Collins, Piepho, Torok. (March) 148
(Protection) Interconnection of Primary Lightning Arrester Ground and the Grounded Neutral of the Secondary Main. Harding, Sprague. (March) 234
(Protection) New Applications of Non-Linear Circuits to Relay and Control Problems. Suits. (Dec.) 914
Protection on 4,000-Volt Circuits, Studies in Lightning. Roper. (March) 252
(Protection) Practical Application of the Oil-Blast Principle of Circuit Interruption. Spurck. (March) 171
(Protection) Recent Developments in Arc Rupturing Devices. Van Sickle, Leeds. (March) 177
(Protection) Standard Decrement Curves. Hahn, Wagner. (June) 353
(Protection) The Boric Acid Fuse. Strom, Rawlins. (Dec.) 1020
(Protection) The Expulsion Fuse. Slepian, Denault. (March) 157
(Protection) The Theory of Oil-Blast Circuit Breakers. Prince. (March) 166
Proximity Effect, The. Bennett. (Sept.) 621
Pub. Serv. Co. of No. Ill., Development of the Waukegan Station of the. Hecht. Electrical Engineering, June 1932, 391
(Pub. Serv. Co. of No. Ill.) Electrical Design Features of Waukegan Station. Williams. (Sept.) 644
(Pub. Serv. Co. No. Ill.) Turbo-Alternator, 115,000-Kw. Williamson. (Sept.) 652

R

(Radio) Characteristics of Electromagnetic Radiation From Aircraft in Flight. Rives, Coe. (Dec.) 981
Radio Coordination. Campbell, Kalb. (March) 81
Radio Interference From Insulator Corona. McMillan. (June) 385
Radiotelephone Service From the United States, Intercontinental. Pilliod. (March) 56
(Radio) Vertically Cut Sound Records. Frederick, Harrison. (Dec.) 987
(Railroads) Engineering Features of Three-Power Locomotives. Brehob, Craton. (June) 489
(Railroads) Motors for 3,000-Volt D-C Multiple Unit Cars. Aydelott. (June) 498
(Railroads) Operating Experience With Supervisory Control. Pastoret. Electrical Engineering, November 1932, 776
(Railroads) Preparing Substations for Electric Operation. Curtis. (June) 518
Railway Electrification, The Mercury Arc Rectifier Applied to A-C. Marti. (Sept.) 659
Railway Motor Armatures, Temperature Rise of. Lightband. (June) 502
Railway Propulsion Systems, Current Propagation in Electric. Riordan. (Dec.) 1011
Railway Substations, Mercury Arc Rectifier versus Rotary Converter Automatic. (Abs.) Ward. Electrical Engineering, March 1932, 191
(Reading-Philadelphia Electrification) Operating Experience With Supervisory Control. Pastoret. Electrical Engineering, November 1932, 776
Recent Development in Traffic Control, A. Vickery, Leonard. (Sept.) 796
Recent Developments in Arc Rupturing Devices. Van Sickle, Leeds (March) 177
Recent Developments in Design of Arc Welding Generators. Hansen. (June) 570
Recorder, The Photoelectric. LaPierre. (March) 226
(Recording) Vertically Cut Sound Records. Frederick, Harrison. (Dec.) 987
Rectifier Applied to A-C Railway Electrification, The Mercury Arc. Marti. (Sept.) 659
Rectifier Efficiency Improves by Sectionalizing, High Capacity. Atherton. (June) 511
Rectifier versus Rotary Converter Automatic Railway Substations, Mercury Arc. (Abs.) Ward. Electrical Engineering, March 1932, 191
Rectifiers, Backfires in Mercury Arc. Slepian, Ludwig. (March) 92
(Regulation) Automatic Control for Variable-Ratio Frequency Converters. Shively, Whitlow. (March) 121
(Regulation) Overvoltage on Transmission Systems Caused by Dropping Load. Burnham. (March) 105
(Regulation) Tie-Line Control of Interconnected Networks. Purcell, Powel. (March) 40
Regulator, The Traffic Flow. Bissell, Hummel. (Sept.) 801
Relay, A New High Speed Distance. Goldsborough, Lewis. Electrical Engineering, March 1932, 157
Relay, A New High-Speed Reactance. (Abs.) Warrington. Electrical Engineering, June 1932, 410
Relay and Control Problems, New Applications of Non-Linear Circuits to. Suits. (Dec.) 914
Relay System, A Telegraphic Pilot-Wire. Frier. Electrical Engineering, October 1931, 824
Relays, Application of High Speed. (Abs.) Gerell. Electrical Engineering, June 1932, 411
Relays, Effect of Wave Form on Operation of Induction Type Protective. (Abs.) Robinson. Electrical Engineering, November 1931, 900
Reliability and Economy in Operation of Large Electric Systems, Combined. (Dec.)
  The Detroit Edison Co. Fugill 859
  The Edison Elec. Illum. Co. of Boston. Dillon 866
  Philadelphia Elec. Co. System. Anderson, Estrada 872
  Chicago District. Perry, Smith 879
(Reproducing) Vertically Cut Sound Records. Frederick, Harrison. (Dec.) 987
(Research) Breakdown of Glass With Alternating Potentials. Kenney, Luery, Moriaty. (June) 404
(Research) Calculation of Inductance and Current Distribution in Low-Voltage Connections to Electric Furnaces. Levy. (Dec.) 903
(Research) Cathode Drop in Arcs and Glow Discharges. (Abs.) Mackeown. Electrical Engineering, September 1931, 755
(Research) Electrical Equipment for Precipitation Service. Speight, Rydberg. (Dec.) 892
(Research) Electrical Precipitation. Simon, Kron. (June) 451
(Research) Extinction of A-C Arcs in Turbulent Gases. Browne. (March) 185
(Research) Forces of Electric Origin in the Iron Arc. Creedy, Lerch, Seal, Sordon. (June) 556
(Research) Magnet Steels and Permanent Magnets. Scott. (June) 410
(Research) New Applications of Non-Linear Circuits to Relay and Control Problems. Suits. (Dec.) 914
(Research) Oxide Coatings on Aluminum. Edwards, Tosterud, Work. (Dec.) 900
(Research) Predetermination of the A-C Characteristics of Dielectrics. Whitehead, Banos. (June) 392
(Research) Radio Interference From Insulator Corona. McMillan. (June) 385
(Research) The Boric Acid Fuse. Strom, Rawlins. (Dec.) 1020
(Research) The Kindling of Electric Sparkover Based on Lichtenberg Figures. Magnusson. (March) 74
Rotor and Stator, An Induction Motor With Paralleled. Conrad, Warner. (June) 418

S

Safe Harbor-Westport 230-Kv Transmission Line. Hansson. Electrical Engineering, December 1932, 834
San Francisco-Los Angeles Section of the Pacific Coast Telephone Cable Network, The. Calderwood, Smith. (March) 63
Short-Circuit Torque of a Synchronous Machine Single-Phase. Nickle, Pierce, Henderson. (Dec.) 966
Signal System, The Flexible Progressive Traffic. Turner. (Sept.) 786
Sine-Wave Generators. Shildneck. (June) 484
Single-Phase Short-Circuit Torque of a Synchronous Machine. Nickle, Pierce, Henderson. (Dec.) 966

Snow Surveys and Hydro-Forecasting. (Abs.) Boardman. Electrical Engineering, September 1931, 756
Solution of Circuits Subjected to Traveling Waves. Rorden. (Sept.) 824
Some Considerations in the Design of Damper Windings for Synchronous Motors. Shutt. (June) 424
Sound-Absorption, Electrical Measurements of. (Abs.) Albert, Bullis. Electrical Engineering, September 1931, 755
Sound Records, Vertically Cut. Frederick, Harrison. (Dec.) 987
Sparkover Based on Lichtenberg Figures, The Kindling of Electric. Magnusson. (March) 74
Sparkover, Insulator. Lloyd. (Sept.) 669
(Stability) Calculation of Short-Circuits on Power Systems. (Abs.) Wagner, Wright. Electrical Engineering, February 1932, 131
(Stability) Decrement Curves for Specific Systems. (Abs.) Hahn. Electrical Engineering, February 1932, 131
(Stability) Field Tests to Determine the Damping Characteristics of Synchronous Generators. Hamilton. (Sept.) 775
Stability of Conowingo Hydroelectric Station, Philadelphia Electric Company System. Hentz, Jones. (June) 375
(Stability) Overvoltage on Transmission Systems Caused by Dropping Load. Burnham. (March) 105
(Stability) Single-Phase Short-Circuit Torque of a Synchronous Machine. Nickel, Pierce, Henderson. (Dec.) 966
Stability Solution for Metropolitan Type Systems, Generalized. Griscom, Lewis, Ellis. (June) 363
(Stability) Standard Decrement Curves. Hahn, Wagner. (June) 353
(Stability) Torque-Angle Characteristics of Synchronous Machines Following System Disturbances. Crary, Waring. (Sept.) 764
Standard Decrement Curves. Hahn, Wagner. (June) 353
Stator, An Induction Motor With Paralleled Rotor and. Conrad, Warner. (June) 418
(Steam Plants) Development of the Waukegan Station of the Pub. Serv. Co. of No. Ill. Hecht. Electrical Engineering, June 1932, 391
(Steam Plants) Electrical Design Features of Waukegan Station. Williams. (Sept.) 644
(Steam Plants) 115,000-Kw Turbo-Alternator. Williamson. (Sept.) 652
(Steam Plants) Steam-Driven Auxiliaries for Power Plants. Dryer. (June) 331
(Steam) Power Plants, Steam-Driven Auxiliaries. Dryer. (June) 331
Steam Power Stations, Auxiliary Drive for. Hollister. (June) 329
Steam Power Stations, Electrically Driven Auxiliaries for. Smith. (June) 337
Steels and Permanent Magnets, Magnet. Scott. (June) 410
Stress-Strain Studies of Transmission Line Conductors. Stickley. (Dec.) 1052
Studies in Lightning Protection on 4,000-Volt Circuits. Roper. (March) 252
Substation, Impulse Voltage Tests on a 4,800-Volt Distribution. Collins, Piepho, Torok. (March) 148
Substations for Electric Operation, Preparing. Curtis. (June) 518
(Substations) The Application and Performance of Automatic Equipment. Sporn, Lanphier, Turner. (Dec.) 1031
Supervisory Control, Operating Experience With. Pastoret. Electrical Engineering, November 1932, 776
Supervisory Systems, Developments in Two-Wire. (Abs.) Oliver. Electrical Engineering, June 1932, 412
Surge Generators for Transformer Testing, Characteristics of. Bellaschi. (Dec.) 936
Surge Proof Transformers. Putman. (Sept.) 579
Suspension Insulator Assemblies, Their Design and Economic Selection. Torok and Archibald. (Sept.) 682
Switching Phenomenon, Commutation Considered as a. Hellmund, Ludwig. (June) 465
Synchronous Electric Time Service. Warren. (June) 546
Synchronous Generators, Field Tests to Determine the Damping Characteristics of. Hamilton. (Sept.) 775
Synchronous Machine, Single-Phase Short-Circuit Torque of a. Nickle, Pierce, Henderson. (Dec.) 966
Synchronous Machines, Calculation of No-Load Damper Winding Loss in. Pollard. (June) 477
Synchronous Machines, Dynamic Braking of. Kilbourne, Terry. (Dec.) 1007
Synchronous Machines Following System Disturbances, Torque-Angle Characteristics of. Crary, Waring. (Sept.) 764
Synchronous Machines, Vector Theory of Circuits Involving. Summers. (June) 318
Synchronous Motors, Some Considerations in the Design of Damper Windings for. Shutt. (June) 424

T

Tape-Armored Telephone Toll Cable. Nystrom. (March) 135
Telegraph Companies, Time Services of the. Janson. (June) 541
Telegraphic Pilot-Wire Relay System, A. Frier. Electrical Engineering, October 1931, 824
Telemeter, The Torque Balance. Johnston. (Dec.) 1027
Telephone Facilities, Advance Planning for Long Distance. (Abs.) McFarland. Electrical Engineering, November 1931, 902
Telephone Toll Cable, Tape-Armored. Nystrom. (March) 135
Telephone Transmission, The Time Factor in. Blackwell. (March) 141
Teletypewriter Communication, Police. Pierce. Electrical Engineering, August 1932, 550
(Teletypewriter) Wire Communication Aids to Air Transportation. Nance. (Dec.) 974
Temperature Rise of Ventilated Railway Motor Armatures. Lightband. (June) 502
Tests on a 4,800-Volt Distribution Substation, Impulse Voltage. Collins, Piepho, Torok. (March) 148
Theory of Oil-Blast Circuit Breakers, The. Prince. (March) 166
Three-Phase Multiple Conductor Circuits. Clarke. (Sept.) 809
Tie-Line Control of Interconnected Networks. Purcell, Powel. (March) 40
Time and Its Measurement. Brown. (June) 523
Time Factor in Telephone Transmission, The. Blackwell. (March) 141
(Timekeepers) Modern Developments in Precision Clocks. Loomis, Marrison. (June) 527
Time Service of the U. S. Naval Observatory. Hellweg. (June) 538
Time Service, Synchronous Electric. Warren. (June) 546
Time Services of the Telegraph Companies. Janson. (June) 541
Toll Switching Plan for Wisconsin. Lallier. (Sept.) 628
Torque-Angle Characteristics of Synchronous Machines Following System Disturbances. Crary, Waring. (Sept.) 764
Torque Balance Telemeter, The. Johnston. (Dec.) 1027
Torque of a Synchronous Machine, Single-Phase Short-Circuit. Nickle, Pierce, Henderson. (Dec.) 966
Traffic Control, A Recent Development in. Vickery, Leonard. (Sept.) 796
Traffic Dispatching System, The Electro-Matic. Haugh. (Sept.) 790
Traffic Flow Regulator, The. Bissell, Hummel. (Sept.) 801
Traffic Signal System, The Flexible Progressive. Turner. (Sept.) 786
Transformer Design, Effect of Transient Voltages on Power. Palueff, Hagenguth. (Sept.) 601
Transformer Insulation With Line Insulation, The Coordination of. Montsinger, Dann. (Dec.) 923
Transformer Ratio Under Load, Characteristics of Load Ratio Control Circuits for Changing. Blume. (Dec.) 952
Transformer Testing, Characteristics of Surge Generators for. Bellaschi. (Dec.) 936
(Transformers) Equivalent Circuits. Starr. (June) 287
Transformers, Insulation Coordination of Distribution. Treanor, Cooney. (Dec.) 925
Transformers, Lightning Protection for Distribution.
  McEachron, Saxon. (March) 239
  Opsahl, Brookes, Southgate. (March) 245
  Haines, Corney. (March) 259
Transformers, Surge-Proof. Putman. (Sept.) 579
(Transformers) Transient Oscillations of Mutually Coupled Windings. Bewley. (June) 299
Transient Oscillations of Mutually Coupled Windings. Bewley. (June) 299
(Transmission) An Improved Type of Limiting Gap for Protecting Station Apparatus. Austin. (Sept.) 676
Transmission and Distribution From the Mokelumne River Development of the Pacific Gas and Electric Company. Wright, Dexter. (March) 20
(Transmission) Corona Energy Loss. Weidlein. (March) 154
(Transmission) Decrement Curves for Specific System. (Abs.) Hahn. Electrical Engineering, February 1932, 131
(Transmission) Equivalent Circuits. Starr. (June) 287
(Transmission) Generalized Stability Solution for Metropolitan Type Systems. Griscom, Lewis, Ellis. (June) 363
(Transmission) Geometrical Circuits of Electrical Networks. Foster. (June) 309
(Transmission) Insulator Sparkover. Lloyd. (Sept.) 669
(Transmission) Interconnection of the 25- and 60-Cycle System of the Union Electric Light and Power Company. Nelson, Hough. (March) 113
Transmission Line Conductors, Stress-Strain Studies of. Stickley. (Dec.) 1052
Transmission Line, Safe Harbor-Westport 230-Kv. Hansson. Electrical Engineering, December 1932, 834
Transmission Line Vibration Due to Sleet. DenHartog. (Dec.) 1074
Transmission Lines, Double Conductors for. Dwight, Farmer. (Sept.) 803
Transmission Lines, Lightning Disturbances on. (Abs.) Jackson,

Warner. Electrical Engineering, November 1931, 903

Transmission Lines, Vibration of Overhead. Monroe, Templin. (Dec.).. 1059

(Transmission) Normal Frequency Arc-over Values of Insulators as Affected by Size and Humidity. Frey, Hawley. (Sept.)........ 690

(Transmission) Radio Interference From Insulator Corona. McMillan. (June) 385

(Transmission) Solution of Circuits Subjected to Traveling Waves. Rorden. (Sept.)........ 824

(Transmission) Standard Decrement Curves. Hahn, Wagner. (June).. 353

(Transmission) Suspension Insulator Assemblies, Their Design and Economic Selection. Torok, Archibald. (Sept.)........ 682

Transmission System of the Milwaukee Elec. Ry. and Lt. Co., The 60-Cycle Primary. Brown, Hats. Electrical Engineering, October 1932, 708

Transmission Systems Caused by Dropping Load, Overvoltage on. Burnham. (March)........ 105

(Transmission) The Coordination of Transformer Insulation With Line Insulation. Montsinger, Dann. (Dec.)........ 923

(Transmission) Three-Phase Multiple-Conductor Circuits. Clarke. (Sept.) 809

(Transmission) Tie-Line Control of Interconnected Networks. Purcell, Powel. (March)........ 40

(Transmission) Tuned Power Lines. Skilling (March)........ 51

(Transmission) Vibration and Fatigue in Electrical Conductors. Davison, Ingles, Martinoff. (Dec.)........ 1047

(Transportation) Engineering Features of Three-Power Locomotives. Brehob, Craton. (June)........ 489

(Transportation) Motors for 3,000-Volt D-C Multiple Unit Cars. Aydelott. (June)........ 498

(Transportation) Preparing Substations for Electric Operation. Curtis. (June)........ 518

(Transportation) The Mercury Arc Rectifier Applied to A-C Railway Electrification. Marti. (Sept.)... 659

Transportation, Wire Communication Aids to Air. Nance. (Dec.)...... 974

Tube Control, A General Method of Gaseous. (Abs.) Stansbury. Electrical Engineering, March 1932, 188

(Tubes) The Parallel Type Inverter. Tompkins. (Sept.)........ 707

Tuned Power Lines. Skilling. (March) 51

U

(Union Elec. Lt. and Pwr. Co.) Application of High Speed Relays. (Abs.) Gerell. Electrical Engineering, June 1932, 411

Unit Systems, A Proposal to Abolish the Absolute Electrical. Weber. (Sept.)........ 728

Units, A General Theory of Systems of Electric and Magnetic Units. Karapetoff. (Sept.)........ 715

V

Vector Theory of Circuits Involving Synchronous Machines. Summers. (June)........ 318

Vertically Cut Sound Records. Frederick, Harrison. (Dec.)........ 987

Vibration and Fatigue in Electrical Conductors. Davison, Ingles, Martinoff. (Dec.)........ 1047

Vibration Due to Sleet, Transmission Line. DenHartog. (Dec.)........ 1074

Vibration of Overhead Transmission Lines. Monroe, Templin. (Dec.) 1059

(Vibration) Stress-Strain Studies of Transmission Line Conductors. Stickley. (Dec.)........ 1052

W

Waukegan Station, Electrical Design Features of. Williams. (Sept.)... 644

Waukegan Station of the Pub. Serv. Co. of No. Ill., Development of the. Hecht. Electrical Engineering, June 1932, 391

(Waukegan) 115,000-Kw Turbo-Alternator. Williamson. (Sept.)....... 652

Wave Form on Operation of Induction Type Protective Relays, Effect of. (Abs.) Robinson. Electrical Engineering, November 1931, 900

Waves, Solution of Circuits Subjected to Traveling Waves. Rorden. (Sept.)........ 824

Weather Resistant Covering for Line Wires. Harding, Carter, Olson. (Sept.)........ 635

Welder, An Improved A-C Arc. Candy. (June)........ 567

(Welding) Forces of Electric Origin in the Iron Arc. Creedy, Lerch, Seal, Sordon. (June)........ 556

Welding in Building Construction, Electric Arc. Vinther. Electrical Engineering, October 1931, 785

Welding, Recent Developments in Design of Arc. Hansen. (June)........ 570

(Welding) The Proximity Effect. Bennett. (Sept.)........ 621

Westport 230-Kv Transmission Line, Safe Harbor. Hansson. Electrical Engineering, December 1932, 834

Winding Loss in Synchronous Machines, Calculation of No-Load Damper. Pollard. (June)........ 477

Windings for Synchronous Motors, Some Considerations in the Design of Damper. Shutt. (June)........ 424

Windings, Transient Oscillations of Mutually Coupled. Bewley. (June) 299

Wire Communication Aids to Air Transportation. Nance. (Dec.)........ 974

Wires, Weather Resistant Covering for Line. Harding, Carter, Olson. (Sept.)........ 635

Wiring of Buildings, An Essential for Good Illumination. Stickney, Sturrock. (Dec.)........ 993

Wisconsin, Toll Switching Plan for. Lallier. (Sept.)........ 628

Wood-Products Industry, Electric Power in the. (Abs.) Carey, Howe. Electrical Engineering, September 1931, 753

Author Index

A

Albert, A. L.; Bullis. Electrical Measurements of Sound-Absorption. (Abs.) Electrical Engineering, September 1931, 755

Almquist, Carl T. Calculating Load Division in Distribution Systems. (Abs.) Electrical Engineering, November 1931, 900

Anderson, J. W.; Estrada. Combined Reliability and Economy in Operation of Large Electric Systems. (Phila. Elec. Co. System) (Dec.) 872

Archibald, C. G.; Torok. Suspension Insulator Assemblies, Their Design and Economic Selection. (Sept.).. 682

Atherton, A. L. High Capacity Rectifier Efficiency Improves by Sectionalizing. (June)........ 511

Austin, A. O. An Improved Type of Limiting Gap for Protecting Station Apparatus. (Sept.)........ 676

Aydelott, John C. Motors for 3,000-Volt D-C Multiple Unit Cars. (June)........ 498

B

Baker, R. M. Equalizing Currents in the Armature of a D-C Machine. (June)........ 473

Baker, R. M.; Ludwig. Influence on Commutation of Brush Contact Drop. (Dec.)........ 959

Banos, Alfredo, Jr.; Kouwenhoven. A High Sensitivity Power Factor Bridge. (March)........ 202

Banos, Alfredo, Jr.; Whitehead. Predetermination of the A-C Characteristics of Dielectrics. (June).... 392

Bellaschi, P. L. Characteristics of Surge Generators for Transformer Testing. (Dec.)........ 936

Bennett, Edward. The Proximity Effect. (Sept.)........ 621

Bewley, L. V. Transient Oscillations of Mutually Coupled Windings. (June) 299

Bissell, C. H.; Hummel. The Traffic Flow Regulator. (Sept.)........ 801

Blackwell, O. B. The Time Factor in Telephone Transmission. (March) 141

Blume, L. F. Characteristics of Load Ratio Control Circuits for Changing Transformer Ratio Under Load. (Dec.)........ 952

Boardman, Horace P. Snow Surveys and Hydro-Forecasting. (Abs.) Electrical Engineering, September 1931, 756

Brehob, F. H.; Craton. Engineering Features of Three-Power Locomotives. (June)........ 489

Brookes, A. S.; Southgate, Opsahl. Lightning Protection for Distribution Transformers. (March)...... 245

Brown, C. D.; Hats. The 60-Cycle Primary Transmission System of the Milwaukee Elec. Ry. and Lt. Co. Electrical Engineering, October 1932, 708

Brown, Ernest W. Time and Its Measurement. (June)........ 523

Browne, T. E., Jr. Extinction of A-C Arcs in Turbulent Gases. (March) 185

Buell, R. C.; Hathaway. An Automatic Oscillograph. (March)........ 222

Bullis, W. R.; Albert. Electrical Measurements of Sound-Absorption. (Abs.) Electrical Engineering, September 1931, 755

Burnham, E. J. Overvoltage on Transmission Systems Caused by Dropping Load. (March)........ 105

C

Calderwood, E. M.; Smith. The San Francisco-Los Angeles Section of th Pacific Coast Telephone Cable Network. (March)........ 63

Campbell, C. C.; Kalb. Radio Coordination. (March)........ 81

Candy, A. M. An Improved A-C Arc Welder. (June)........ 567

Carey, C. E.; Howe. Electric Power in the Wood-Products Industry. (Abs.) Electrical Engineering, September 1931, 753

Carter, L. L.; Olson, Harding. Weather Resistant Covering for Line Wires. (Sept.)........ 635

Clark, S. B. A-C Networks in Portland, Oregon. Electrical Engineering, August 1931, 644

Clarke, Edith. Three-Phase Multiple-Conductor Circuits. (Sept.)....... 809

Coe, James C.; Rives. Characteristics of Electromagnetic Radiation From Aircraft in Flight. (Dec.).......... 981

Collins, H. W.; Piepho, Torok. Impulse Voltage Tests on a 4,800-Volt Distribution Substation. (March) 148

Conrad, A. G.; Warner. An Induction Motor With Paralleled Rotor and Stator. (June)................... 418

Cooney, W. H.; Treanor. Insulation Coordination of Distribution Transformers. (Dec.).................. 925

Corney, C. A.; Haines. Lightning Protection for Distribution Transformers. (March)................ 259

Crary, S. B.; Waring. Torque-Angle Characteristics of Synchronous Machines Following System Disturbances. (Sept.)............... 764

Craton, F. H.; Brehob. Engineering Features of Three-Power Locomotives. (June).................... 480

Creedy, F.; Lerch, Seal, Sordon. Forces of Electric Origin in the Iron Arc. (June)......................... 556

Crellin, E. A. The Mokelumne River Development of the Pacific Gas and Elec. Co. (March)............... 28

Curtis, L. B. Preparing Substations for Electric Operation. (June)....... 518

D

Dambly, H. A.; Ekvall, Phelps. Distribution System Lightning Studies by Phila. Elec. Co. (March)....... 265

Daniel, A. F.; Dawes. High Voltage Bridge for Measurements of Cables With Grounded Sheaths. (March) 198

Dann, W. M.; Montsinger. The Coordination of Transformer Insulation With Line Insulation. (Dec.) 923

Davison, A. E.; Ingles, Martinoff. Vibration and Fatigue in Electrical Conductors. (Dec.)............. 1047

Dawes, C. L.; Daniel. High Voltage Bridge for Measurements of Cables With Grounded Sheaths. (March) 198

Denault, C. L.; Slepian. The Expulsion Fuse. (March)................. 157

DenHartog, J. P. Transmission Line Vibration Due to Sleet. (Dec.).... 1074

Dexter, B. D.; Wright. Power Transmission and Distribution From the Mokelumne River Development of the Pacific Gas and Elec. Co. (March)......................... 20

Dillon, R. E. Combined Reliability and Economy in Operation of Large Electric Systems. (The Edison Elec. Illum. Co. of Boston.) (Dec.).............................. 866

Doherty, R. E. Educational Aspects of Engineering and Management. (Dec.).............................. 848

Dryer, W. Poole. Steam-Driven Auxiliaries for Power Plants. (June).... 331

Dunbar, J. R.; Floyd. Calorimeter Measurement of Stray-Load Losses of a 55,000-Kva Generator Having Enclosed System of Ventilation. (March)......................... 12

Dwight, H. B.; Farmer. Double Conductors for Transmission Lines. (Sept.)............................ 803

E

Edwards, J. E.; Tosterud, Work. Oxide Coatings on Aluminum. (Dec.).............................. 900

Ekvall, H. N.; Phelps, Dambly. Distribution System Lightning Studies by Phila. Elec. Co. (March)...... 265

Ellis, W. R.; Griscom, Lewis. Generalized Stability Solution for Metropolitan Type Systems. (June) 363

Estrada, Herbert; Anderson. Combined Reliability and Economy in Operation of Large Electric Systems. (Phila. Elec. Co. System.) (Dec.) 872

F

Farmer, E. B.; Dwight. Double Conductors for Transmission Lines. (Sept.)........................... 803

Floyd, G. D.; Dunbar. Calorimeter Measurement of Stray-Load Losses of a 55,000-Kva Generator Having Enclosed System of Ventilation. (March)........................... 12

Foster, Ronald M. Geometrical Circuits of Electrical Networks. (June)........................... 309

Frederick, H. A.; Harrison. Vertically Cut Sound Records. (Dec.)....... 987

Frey, H. A.; Hawley. Normal Frequency Arcover Values of Insulators as Affected by Size and Humidity. (Sept.)............................ 690

Frier, C. H. A Telegraphic Pilot-Wire Relay System. Electrical Engineering, October 1931, 824

Fugill, A. P. Combined Reliability and Economy in Operation of Large Electric Systems. (The Detroit Edison Co.) (Dec.).............. 859

G

Gerell, G. W. Application of High Speed Relays. (Abs.) Electrical Engineering, June 1932, 411

Godsey, F. W.
  Film Characteristics of Electrolytic Condensers. (June).............. 432
  Power Losses in Electrolytic Condensers. (June)..................... 439

Goldsborough, S. L.; Lewis. A New High Speed Distance Relay. Electrical Engineering, March 1932, 157

Green, Charles F. Electrical Solutions of Problems of Regular Scheduled Flight. Electrical Engineering, August 1931, 654

Griscom, S. B.; Lewis, Ellis. Generalized Stability Solution for Metropolitan Type Systems. (June).... 363

H

Hagenguth, J. H.; Palueff. Effect of Transient Voltages on Power Transformer Design. (Sept.)........... 601

Hahn, W. C. Decrement Curves for Specific Systems. (Abs.) Electrical Engineering, February 1932, 131

Hahn, W. C.; Wagner. Standard Decrement Curves. (June)........... 353

Haines, T. H.; Corney. Lightning Protection for Distribution Transformers. (March)............... 259

Hamilton, F. A., Jr. Field Tests to Determine the Damping Characteristics of Synchronous Generators. (Sept.).................... 775

Hansen, K. L. Recent Developments in Design of Arc Welding Generators. (June)..................... 570

Hansson, Edwin. Safe Harbor-Westport 230-Kv Transmission Line. Electrical Engineering, December 1932, 834

Harding, C. F.; Carter, Olson. Weather Resistant Covering for Line Wires. (Sept.)........................... 635

Harding, C. F.; Sprague. Interconnection of Primary Lightning Arrester Ground and the Grounded Neutral of the Secondary Mains. (March)......................... 234

Harrison, H. C.; Frederick. Vertically Cut Sound Records. (Dec.)....... 987

Hathaway, C. M.; Buell. An Automatic Oscillograph. (March)..... 222

Hatz, E. W.; Brown. The 60-Cycle Primary Transmission System of the Milwaukee Electric Railway and Light Company. Electrical Engineering, October 1932, 708

Haugh, H. A., Jr. The Electro-Matic Traffic Dispatching System. (Sept.) 790

Hawley, K. A.; Frey. Normal Frequency Arcover Values of Insulators as Affected by Size and Humidity. (Sept.)........................... 690

Hecht, J. L. Development of the Waukegan Station of the Pub. Serv. Co. of No. Ill. Electrical Engineering, June 1932, 391

Hellmund, R. E.; Ludwig. Commutation Considered as a Switching Phenomenon. (June)................ 465

Hellweg, J. Frederick. Time Service of the U.S. Naval Observatory. (June)............................ 538

Henderson, E. W. Induction Motor Versatility. (Dec.)............. 1001

Henderson, M. L.; Nickle, Pierce. Single-Phase Short-Circuit Torque of a Synchronous Machine. (Dec.) 966

Hentz, R. A.; Jones. Stability of Conowingo Hydroelectric Station, Phila. Elec. Co. System. (June)......... 375

Hill, Harold C.; SeLegue. Electrical Equipment for Oil Field Operations. (March)......................... 1

Hollister, F. H. Auxiliary Drive for Steam Power Stations. (June).... 329

Holsen, James N. Forecasting Population for Engineering Purposes. (March).......................... 128

Hoover, Vaino. Correlation of Induction Motor Design Factors. (Abs.) Electrical Engineering, September 1931, 754

Hough, E. L.; Nelson. Interconnection of the 25- and 60-Cycle System of the Union Elec. Lt. and Pwr. Co. (March)........................... 113

Howe, K. L.; Carey. Electric Power in the Wood-Products Industry. (Abs.) Electrical Engineering, September 1931, 753

Hummel, J. G.; Bissell. The Traffic Flow Regulator. (Sept.).......... 801

I

Ingles, J. A.; Martinoff, Davison. Vibration and Fatigue in Electrical Conductors. (Dec.).............. 1047

J

Jackson, D. C., Jr.; Warner. Lightning Disturbances on Transmission Lines. (Abs.) Electrical Engineering, November 1931, 903

Janson, G. W. Time Services of the Telegraph Companies. (June).... 541

Johnston, A. J. The Torque Balance Telemeter. (Dec.).............. 1027

Jones, J. W.; Hentz. Stability of Conowingo Hydroelectric Station, Phila. Elec. Co. System. (June)......... 375

K

Kalb, H. N.; Campbell. Radio Coordination. (March).............. 81

Karapetoff, Vladimir. A General Theory of Systems of Electric and Magnetic Units. (Sept.)................. 715

Kenney, N. D.; Luery, Moriaty. Breakdown of Glass With Alternating Potentials. (June)........... 404

Kilbourne, C. E.; Terry. Dynamic Braking of Synchronous Machines. (Dec.)... 1007
Koch, C. J. Measurement of Stray Load Loss in Polyphase Induction Motors. (Sept.)... 756
Kouwenhoven, W. B.; Banos. A High Sensitivity Power Factor Bridge. (March)... 202
Kron, L. C.; Simon. Electrical Precipitation. (June)... 451

L

Lallier, W. C. Toll Switching Plan for Wisconsin. (Sept.)... 628
Lanphier, Basil; Turner, Sporn. The Application and Performance of Automatic Equipment. (Dec.)... 1031
LaPierre, C. W. The Photoelectric Recorder. (March)... 222
Leeds, W. M.; Van Sickle. Recent Developments in Arc Rupturing Devices. (March)... 177
Leonard, V. W.; Vickery. A Recent Development in Traffic Control. (Sept.)... 796
Lerch, R. O.; Seal, Sordon, Creedy. Forces of Electric Origin in the Iron Arc. (June)... 556
Levy, C. C. Calculation of Inductance and Current Distribution in Low-Voltage Connections to Electric Furnaces. (Dec.)... 903
Lewis, W. A.; Ellis, Griscom. Generalized Stability Solution for Metropolitan Type Systems. (June)... 363
Lewis, W. A.; Goldsborough. A New High Speed Distrance Relay. Electrical Engineering, March 1932, 157
Lightband, D. A. Temperature Rise of Ventilated Railway Motor Armatures. (June)... 502
Lloyd, W. L., Jr. Insulation Sparkover. (Sept.)... 669
Loomis, Alfred L.; Marrison. Modern Developments in Precision Clocks. (June)... 527
Lovell, Alfred H. Engineering Subjects, Electrical and Cognate, in the Four-Year College Program of Electrical Engineering. (Dec.)... 839
Ludwig, L. R.; Baker. Influence on Commutation of Brush Contact Drop. (Dec.)... 959
Ludwig, L. R.; Hellmund. Commutation Considered as a Switching Phenomenon. (June)... 465
Ludwig, L. R.; Slepian. Backfires in Mercury Arc Rectifiers. (March). 92
Luery, A. M.; Moriaty, Kenney. Breakdown of Glass With Alternating Potentials. (June)... 404

M

Mackeown, S. S. Cathode Drop in Arcs and Glow Discharges. (Abs.) Electrical Engineering, September 1931, 755
Magnusson, C. Edward. The Kindling of Electric Sparkover Based on Lichtenberg Figures. (March)... 74
Marrison, W. A.; Loomis. Modern Developments in Precision Clocks. (June)... 527
Marti, Othmar K. The Mercury Arc Rectifier Applied to A-C Railway Electrification. (Sept.)... 659
Martinoff, V. M.; Davison, Ingles. Vibration and Fatigue in Electrical Conductors. (Dec.)... 1047
McEachron, K. B.; Saxon. Lightning Protection for Distribution Transformers. (March)... 239
McFarland, C.C. Advance Planning of Long Distance Telephone Facilities. (Abs.) Electrical Engineering, November 1931, 902
McMillan, F. O. Radio Interference From Insulator Corona. (June)... 385
Monroe, R. A.; Templin. Vibration of Overhead Transmission Lines. (Dec.)... 1059
Montsinger, V. M.; Dann. The Coordination of Transformer Insulation With Line Insulation. (Dec.) 923
Moriaty, J. D.; Kenney, Luery. Breakdown of Glass With Alternating Potentials. (June)... 404

N

Nance, H. H. Wire Communication Aids to Air Transportation. (Dec.) 974
Nelson, L. V.; Hough. Interconnection of the 25- and 60-Cycle System of the Union Elec. Lt. and Pwr. Co. (March)... 113
Newton, E. G. Fuse Cut-Outs—Their Design and Application for A-C Distribution Circuits. (Abs.) Electrical Engineering, November 1931, 903
Nickle, C. A.; Pierce, Henderson. Single-Phase Short-Circuit Torque of a Synchronous Machine. (Dec.) 966
Nystrom, C. W. Tape-Armored Telephone Toll Cable. (March)... 135

O

Oliver, J. H. Developments in Two-Wire Supervisory Systems. (Abs.) Electrical Engineering, June 1932, 412
Olson, J. W.; Harding, Carter. Weather Resistant Covering for Line Wires. (Sept.)... 635
Opsahl, A. M.; Brookes, Southgate. Lightning Protection for Distribution Transformers. (March)... 245

P

Palueff, K. K.; Hagenguth. Effect of Transient Voltages on Power Transformer Design. (Sept.)... 601
Pastoret, J. E. Operating Experience With Supervisory Control. Electrical Engineering, November 1932, 776
Perry, Leslie, L.; Smith. Combined Reliability and Economy in Operation of Large Electric Systems. (Chicago District.) (Dec.)... 879
Phelps, Howard S.; Dambly, Ekvall. Distribution System Lightning Studies by Phila. Elec. Co. (March) 265
Piepho, E. E.; Torok, Collins. Impulse Voltage Tests on a 4,800-Volt Distribution Substation. (March)... 148
Pierce, C. A.; Henderson, Nickle. Single-Phase Short-Circuit Torque of a Synchronous Machine. (Dec.) 966
Pierce, R. E. Police Teletypewriter Communication. Electrical Engineering, August 1932, 550
Pilliod, J. J. Intercontinental Radiotelephone Service From the United States. (March)... 56
Pollard, E. I. Calculation of No-Load Damper Winding Loss in Synchronous Machines. (June)... 477
Powel, C. A.; Purcell. Tie-Line Control of Interconnected Networks. (March)... 40
Prince, D. C. The Theory of Oil-Blast Circuit Breakers. (March)... 166
Purcell, T. E.; Powel. Tie-Line Control of Interconnected Networks. (March)... 40
Putman, H. V. Surge-Proof Transformers. (Sept.)... 579

R

Rawlins, H. L.; Strom. The Boric Acid Fuse. (Dec.)... 1020
Riordan, John. Current Propagation in Electric Railway Propulsion System. (Dec.)... 1011
Rives, Tom C.; Coe. Characteristics of Electromagnetic Radiation From Aircraft in Flight. (Dec.)... 981
Robinson, P. H. Effect of Wave Form on Operation of Induction Type Protective Relays. (Abs.) Electrical Engineering, November 1931, 900
Roper, D. W. Studies in Lightning Protection on 4,000-Volt Circuits. (March)... 252
Rorden, Harold L. Solution of Circuits Subjected to Traveling Waves. (Sept.)... 824
Rydberg, V. G.; Speight. Electrical Equipment for Precipitation Service. (Dec.)... 892

S

Sanford, Frank E. Application of Primary Distribution Fuses. (Abs.) Electrical Engineering, November 1931, 903
Saxon, L.; McEachron. Lightning Protection for Distribution Transformers. (March)... 239
Schmidt, Edwin X. Electric Instruments in the Gas Industry. (Abs.) Electrical Engineering, March 1932, 189
Scott, K. L. Magnet Steels and Permanent Magnets. (June)... 410
Seal, P. W.; Sordon, Creedy, Lerch. Forces of Electric Origin in the Iron Arc. (June)... 556
SeLegue, J. B.; Hill. Electrical Equipment for Oil Field Operations. (March)... 1
Sels, H. K. Proposed Definitions of Terms Used in Power System Studies. Electrical Engineering, February 1932, 106
Shildneck, Lloyd P. Sine-Wave Generators. (June)... 484
Shively, E. K.; Whitlow. Automatic Control for Variable Ratio Frequency Converters. (March)... 121
Shutt, C. C. Some Considerations in the Design of Damper Windings for Synchronous Motors. (June)... 424
Simon, A. W.; Kron. Electrical Precipitation. (June)... 451
Siskind, Robert P. Power Factor Measurement by the Capacitance Bridge. (March)... 214
Skilling, H. H. Tuned Power-Lines. (March)... 51
Slepian, J.; Denault. The Expulsion Fuse. (March)... 157
Slepian, J.; Ludwig. Backfires in Mercury Arc Rectifiers. (March).. 92
Smith, D. F.; Calderwood. The San Francisco-Los Angeles Section of the Pacific Coast Telephone Cable Network. (March)... 63
Smith, Frank V.; Perry. Combined Reliability and Economy in Operation of Large Electric Systems. (Chicago District.) (Dec.)... 879
Smith, Lyle W. Electrically Driven Auxiliaries for Steam Power Stations. (June)... 337
Sordon, E. P.; Creedy, Lerch, Seal. Forces of Electric Origin in the Iron Arc. (June)... 556
Southgate, R. N.; Opsahl, Brookes. Lightning Protection for Distribution Transformers. (March)... 245

Speight, H.; Rydberg. Electrical Equipment for Precipitation Service. (Dec.)............................ 892

Sporn, Philip; Lanphier, Turner. The Application and Performance of Automatic Equipment. (Dec.).... 1031

Sprague, C. S.; Harding. Interconnection of Primary Lightning Arrester Ground and the Grounded Neutral of the Secondary Main. (March).. 234

Spurck, R. M. Practical Application of the Oil-Blast Principle of Circuit Interruption. (March)............ 171

Stansbury, Carroll. A General Method of Gaseous Tube Control. (Abs.) Electrical Engineering, March 1932, 188

Starr, Frank M. Equivalent Circuits. (June)............................ 287

Stickley, G. W. Stress-Strain Studies of Transmission Line Conductors. (Dec.)............................ 1052

Stickney, G. H.; Sturrock. Adequate Wiring of Buildings, an Essential for Good Illumination. (Dec.).... 993

Strom, A. P.; Rawlins. The Boric Acid Fuse. (Dec.)............... 1020

Sturrock, Walter; Stickney. Adequate Wiring of Buildings, an Essential for Good Illumination. (Dec.)....... 993

Suits, C. G. New Applications of Non-Linear Circuits to Relay and Control Problems. (Dec.)........... 914

Summers, I. H. Vector Theory of Circuits Involving Synchronous Machines. (June)................... 318

T

Templin, R. L.; Monroe. Vibration of Overhead Transmission Lines. (Dec.)............................ 1050

Terry, I. A.; Kilbourne. Dynamic Braking of Synchronous Machines. (Dec.)............................ 1007

Tompkins, Frederick N. The Parallel Type Inverter. (Sept.).......... 707

Torok, J. J.; Archibald. Suspension Insulator Assemblies, Their Design and Economic Selection. (Sept.).. 682

Torok, J. J.; Collins, Piepho. Impulse Voltage Tests on a 4,800-Volt. (March)............................ 148

Tosterud, Martin; Work, Edwards. Oxide Coatings on Aluminum. (Dec.)............................ 900

Treanor, E. D.; Cooney. Insulation Coordination of Distribution Transformers. (Dec.)................. 925

Trickey, P. H. Design of Capacitor Motors for Balanced Operation (Sept.)............................ 7

Turner, Harold E.; Sporn, Lanphier. The Application and Performance of Automatic Equipment. (Dec.) 1031

Turner, H. I. The Flexible Progressive Traffic Signal System. (Sept.).... 786

V

Van Sickle, R. C.; Leeds. Recent Developments in Arc Rupturing Devices. (March)................... 177

Veinott, C. G. Performance Calculations on Induction Motors. (Sept.) 743

Vickery, H. W.; Leonard. A Recent Development in Traffic Control. (Sept.)............................ 796

Vinther, P. N. Electric Arc Welding in Building Construction. Electrical Engineering, October 1931, 785

W

Wagner, C. F.; Hahn. Standard Decrement Curves. (June)............. 353

Wagner, C. F.; Wright. Calculation of Short-Circuits on Power Systems. (Abs.) Electrical Engineering, February 1932, 131

Ward, O. M. Mercury Arc Rectifier versus Rotary Converter Automatic Railway Substations. (Abs.) Electrical Engineering, March 1932, 191

Waring, M. L.; Crary. Torque-Angle Characteristics of Synchronous Machines Following System Disturbances. (Sept.)................... 764

Warner, R. G.; Conrad. An Induction Motor With Paralleled Rotor and Stator. (June)................... 418

Warner, R. W.; Jackson. Lightning Disturbances on Transmission Lines. (Abs.) Electrical Engineering, November 1931, 903

Warren, H. E. Synchronous Electric Time Service. (June)............ 546

Warrington, A. R. van C. A New High-Speed Reactance Relay. (Abs.) Electrical Engineering, June 1932, 410

Weber, Ernst. A Proposal to Abolish the Absolute Electrical Unit Systems. (Sept.)................... 728

Weidlein, Wm. D. Corona Energy Loss. (March)................... 154

Whitehead, J. B.; Banos. Predetermination of the A-C Characteristics of Dielectrics. (June)............ 392

Whitlow, Geo. S.; Shively. Automatic Control for Variable-Ratio Frequency Converters. (March)....... 121

Wickenden, William E. The Engineer in a Changing Society. Electrical Engineering, July 1932, 465

Williams, E. C. Electrical Design Features of Waukegan Station. (Sept.) 644

Williamson, R. B. 115,000-Kw Turbo-Alternator. (Sept.)............ 652

Work, H. K.; Edwards, Tosterud. Oxide Coatings on Aluminum. (Dec.)............................ 900

Wright, E. M.; Dexter. Power Transmission and Distribution From the Mokelumne River Development of the Pacific Gas and Electric Company. (March)................... 20

Wright, S. H.; Wagner. Calculation of Short-Circuits on Power Systems. (Abs.) Electrical Engineering, February 1932, 131

Electrical Equipment for Oil Field Operations

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INTRODUCTION

In the application of electricity to the work of drilling for oil and pumping the wells, many problems peculiar to this industry have been encountered. This paper presents some of the more interesting features of these applications and is confined to a discussion of the duty involved and of the design of the apparatus for the purpose.

Electricity for oil field operations was first used in the Eastern fields over 30 years ago, but the extensive use of this form of power did not begin until 10 or 12 years later when it was introduced in the California fields. Most of the new developments which have since been made in the design and use of electrical apparatus for oil well drilling and pumping have first been tried and perfected in the California fields before being adopted elsewhere. This has been due not only to the fact that field conditions encountered there have resulted in many advances in the art, but also to the availability of dependable central station power service in all the fields.

GROWTH OF THE INDUSTRY

Since the first well was completed near Titusville, Pa. in August 1859 the oil industry has had a tremendous growth, oil now being produced in nineteen states from 330,000 wells.

In 1906 there were less than 800 million dollars invested in the oil industry. Today the investment exceeds twelve billions of dollars.

The records of the petroleum industry during the last fifteen or twenty years indicate that it is probably the most highly competitive of American industries, and it is therefore natural that the oil operators are interested not only in costs, but also in the development of the most efficient methods of drilling and producing.

Of the tremendous connected horsepower in the industry in this country, it is understood that only 7 per cent is electrical. In California alone there is one and one-half million horsepower installed, of which only 15 per cent is electrical, which indicates the potential possibilities for electrification.

CABLE TOOL DRILLING

For the past fifteen years there has been an average of 24,000 oil wells drilled per year in the United States by the cable tool and rotary methods, and although the latter is now the predominating one, the cable tool method was the original one employed and is still extensively used. Drilling by cable tools is accomplished by percussion, the bit being suspended from a walking-beam by a manila or steel cable and cutting the formation by the vertical reciprocating motion imparted to it by the rocking motion of the beam. The latter is actuated by a crank and the crank shaft is belt-, gear- or chain-driven. All work of handling the drilling tools, bailing and swabbing the hole and inserting casing is handled by a hoisting mechanism which is a part of the drilling rig.

Nearly all cable tool drillers claim that, as the depth of the well changes, a very close adjustment of the number of strokes of the bit per minute must be made to accord with the natural period of the drilling line and thus obtain the most effective results, and it has therefore been necessary to provide such control in the electrical equipment. This proved to be the chief feature of operation which could not be readily accomplished with strictly standard motor equipment.

Two different methods have been devised to meet this requirement. One of these employs the standard wound-rotor induction motor with two controllers and resistors, the main controller being of standard design for coarse speed adjustment and the auxiliary controller being specially designed with the three secondary phases separate, so that it could be cut in between any two points of the main controller and thus act as a vernier adjustment of speed. The total number of control points thus made available is equal to the product of the number of points in each controller, and is usually from 135 to 160.

The other method makes use of two oil well pumping motors of the two-speed wound-rotor induction type, mechanically installed to operate in multiple to handle the drilling operations, and so arranged that one motor and control may be left permanently at the well to pump it after it is "brought in" as a producer. Each of these motors has its individual control equipment, consisting essentially of a controller, a resistor and a pole-changing switch. The controllers are both operated by ropewheels from the driller's position at the derrick, and these wheels are arranged for separate operation to give individual speed control of each motor, or may be locked together to give simultaneous control of both. When a small change in speed of the twin-motor drive is needed during drilling, one controller is moved one or more points beyond the position of the other, thus shifting part of the load from one motor to the other. This changes the resultant slip of the combination and gives the change in speed desired. On hoisting duty, for which the power demand is much higher, the ropewheels are locked together and

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the motors are controlled as a single unit, dividing the load evenly.

Some oil companies have succeeded in drilling with cable tools by using only a single two-speed motor equipment with less than ten points of control available, and they have claimed the operation was satisfactory, but so few control points would not be acceptable to most drillers and so an auxiliary control unit must be added. Such an equipment may be left at the rig permanently for the purpose of pumping the well after it is drilled provided that the size of motor required for pumping is somewhere near that necessary for drilling. The auxiliary control unit is moved to the next new location.

There is one other speed characteristic which cable tools require to drill most effectively, that being an almost free drop of the tools on the down stroke, followed by a slower pick-up. This is accomplished without special design of apparatus, by operating the motor with secondary resistance in circuit when drilling, thus giving a wide speed regulation with change in load. The best motion is obtained if the motor is fairly heavily loaded, but the drilling load is moderate in comparison with the hoisting load and does not necessarily increase with the depth. Therefore, when the hoisting duty is heavy because of the high speeds at which it is desired to handle heavy loads, and so calls for a motor of large capacity, it has been proposed to use a two-speed motor with ample capacity for drilling on the low-speed connection with secondary resistance in circuit. The full capacity of the high-speed connection would then be available for the heavy work.

A brief analysis of the motor characteristics suitable for cable tool drilling begins with the fact that it is first necessary, as already indicated, to adjust the secondary resistance of the motor or motors to get an average speed in strokes per minute to accord with the natural period of the drilling line. The time for one complete stroke then becomes temporarily fixed, and the best motion of the bit is obtained when the motor responds quickest to the changes in load during that stroke. On the down stroke, the rate of increase in speed is retarded by rotor inertia and limited by the available torque to accelerate. On the up stroke, the rate of decrease in speed is retarded by the rotor inertia and is also affected by the amount of torque available in excess of that required to lift the total weight of drilling tools and suspended line. Thus, in either case, low armature inertia is essential for best results. On the down stroke, high torque is desirable, and on the up stroke, low torque over and above that required to lift the load.

These requirements are well met with a motor having fairly high torque and low $W R^2$, and so geared or belted to the load as to be run on a reduced speed point of the controller when drilling, and under those conditions loaded up as much as possible consistent with its thermal capacity. With a motor assumed to be thermally large enough for the service, a measure of its effectiveness on actual drilling is the maximum torque in lb-ft., divided by ($W R^2 \times$ drilling r. p. m. of motor).

A capacity of 50 hp. is usually sufficient for drilling to a depth of 1,500 ft. and, under some conditions, to 2,000 ft. With a 75-hp. motor, wells have been successfully drilled to as great a depth as 7,300 ft. but more capacity is desirable for deep drilling to enable tools and bailer to be removed from the hole at high speeds. For this reason a rating of 125/45 hp. has been under consideration for the proposed two-speed motor already mentioned.

ROTARY DRILLING

Rotary drilling is accomplished, as its name implies, by a rotary motion of the drilling bit. The bit is of course designed for boring, and is supported at the end of a column of screwed-joint drill pipe which is suspended by blocks and tackle from the top of the derrick and raised and lowered by a hoist mechanism located at the edge of the derrick floor and known as the draw works. The drill pipe is turned by a rotary table through an opening in which the pipe is suspended. The table is mounted on the derrick floor and is driven from the draw works.

The rotary drilling process requires the circulation of a heavy mixture of mud and water down through the drill pipe, out through holes in the bit and back to the surface, carrying the cuttings with it and "mudding off" the walls of the hole, preventing cave-ins and holding back high pressure gas. Drilling is thus a continuous process except when it is necessary to replace worn bits or recover pipe which has twisted off. The pipe is then hoisted out in lengths which can be stacked in the derrick and is afterward replaced in the hole for the resumption of drilling.

Steam or internal combustion engines or electric motors are used for motive power, the draw works and each of the two mud pumps being driven by separate power units.

Draw Works Drive. A good idea of the range of conditions to be met by the power equipment may be gained from the following approximate figures, which are mentioned as a matter of interest, but a complete discussion of which is prevented by lack of space.

Ordinary drilling loads are from 30 to 100 hp. at speeds of 50 to 120 r. p. m. of the table. The operations of coring and reaming require low speeds, from 15 to 40 r. p. m. and the horsepower required is from 10 to 25 hp.

Under hoisting duty, the same engine or motor must deliver from 100 to 250 hp., with peaks in extreme cases up to about 600 hp., over a range of hook speeds from 90 to 180 ft. per min. The load of hoisting the empty traveling block varies from about 10 to 165 hp. with speeds up to 350 ft. per min.

The steam engine has been popular for many years because its speed and torque characteristics make it admirably fitted to perform all the necessary operations in rotary drilling. Particular attention has therefore been given toward duplicating or improving upon steam engine characteristics in devising electrical equipment to do the work. This has been done with motors and control departing very little from standard electrical design, but a few interesting features are worthy of mention.

Standard wound-rotor induction motors up to 200 hp. have been used for moderate and even deep drilling, the speed and torque characteristics meeting the requirements quite well when the motor is provided with suitable control equipment. Deep drilling requires relatively large horsepower capacity for the heavy hoisting duty, but the rotary table loads do not increase proportionately with depth. As satisfactory control of an induction motor is difficult to obtain under light loads and slow speeds, special means have been devised to get good results. In some cases, additional resistance has been inserted in the secondary circuit of the motor by means of an auxiliary controller, but two more efficient and generally better methods are now being used.

One method is to use two motors in multiple, driving the draw works through a special twin-drive gear reduction unit. One motor is used for drilling, coring, reaming and other relatively light duty, and the two motors together are used for hoisting. Magnetic control equipment for each motor, suitably interconnected and interlocked, makes the handling of the equipment very simple for the driller. Not only is the speed control at light loads thus materially improved, but the twin motors have the advantage of low total armature inertia, which permits rapid acceleration and reversals. Installations of two 125-hp. and also of two 150-hp. motors have been made.

The other method is the use of a Y-delta wound-rotor motor, six stator leads being brought out to enable the motor to be run on the Y-connection for light work and on the delta connection for heavy work. Motors of this type developed for this service are rated 250/85 hp. and 250/125 hp., and are designed for low armature inertia.

Both of these methods have worked out very well in rotary drilling service and have overcome the difficulty of satisfactorily controlling the standard wound-rotor motor equipment in the large sizes.

Electrically, the control equipment for draw works motors follows the standard practise in industrial motor applications except that provision must be made in the design of the secondary resistor for running the motor at very low speeds under some light load conditions, such as rotating the drill pipe at low speed with the bit raised off bottom, which is a practise generally followed when changing tours, and, in some fields, when the crew is eating lunch. The smaller sizes of motors are governed by manually-operated controllers, and the larger sizes by full magnetic control with a master switch. The controller or master switch is operated by a ropewheel or handwheel located on the draw works post at the driller's position, the main control equipment being set back behind the draw works or even at some distance from the well if gas conditions make it advisable.

Acceleration of the motor by magnetic control is automatically governed by either time-limit or current-limit relays, each having its advantages and the preference of some operators. Both prevent the operator from cutting out the secondary resistance too rapidly for the best rate of acceleration, and they reduce the current peaks and the possibility of damage to the mechanical equipment. The definite time relays are generally set for a total time delay of about 8 seconds, with 2½ seconds allowed at the point giving maximum motor torque, so as to insure that this point is not passed until the load is partially accelerated. When current-limit relays are used, they are adjusted to suit the load conditions encountered and keep the current peaks from exceeding reasonable limits. In general, the use of accelerating relays in conjunction with magnetic control has been one of the chief factors in making a pronounced success of electric drive for deep rotary drilling.

Mud Pumps. Fully as important as any other part of the rotary drilling rig are the mud pumps, and they are subjected to some of the greatest abuses. These pumps are of the positive displacement piston type, and for electric drive include a reduction gear which is usually V-belt driven from the motor. The speed of the crank shaft varies from 20 to 50 r. p. m. and is desired to be reduced occasionally to the point of stalling. The fluid pressure for 7,000-ft. depths will run as high as 700 lb. per sq. in. and it is estimated that with increased depths, pressures in excess of 900 lb. will be required for normal circulation of the mud in the well. Higher pressures are occasionally required in emergencies in some localities, these being obtained by compounding the pumps, the discharge of one being piped to the intake of the other. Each rig is equipped with two or more pumps, all but one being stand-by units.

Only a few years ago the requirements averaged from 60 to 100 hp. per unit, but today the tendency is to use larger pumps requiring up to 250 hp.

The flow of mud is frequently retarded or stopped by plugging of the holes in the bit or restrictions elsewhere along the line of flow, and it is preferable, in order to avoid breakage from high pressure developed by the pump under such conditions, and also to warn the driller of the trouble developing, that the speed be reduced automatically, even to the stalling point, as the pressure increases. Steam pumps have this desirable speed-pressure characteristic, and electric drive may be controlled to do likewise. A standard type of wound-rotor induction motor is employed, but as normally the pump is run at full speed, the torque of the motor with short-circuited collector rings would, in case of clogging of the mud line, increase sometimes beyond the breaking strength of the pump without a sufficient reduction in speed to avoid such damage. Such a reduction may be made by secondary resistance, and recently one of the electrical manufacturers, in cooperation with a California oil company, made an installation in which a permanent block of resistance and reactance was put in the rotor circuit to cause the motor to drop in speed and stall at approximately 250 per cent of full-load torque. The results are reported to be very satisfactory from an operating standpoint. It is considered that the protection thus afforded to the pump and mud lines is well worth the additional electrical losses entailed, although oil companies to whom this scheme was suggested several years ago were not then ready to accept the idea, as they hoped that some means could be devised which would have a higher efficiency. Forced ventilation of the motor from a separate motor-driven blower has been resorted to under the circumstances, to prevent overheating during long periods of stalling which may occur.

With this arrangement it should now be possible to successfully compound two induction motor-driven pumps, which has heretofore been considered impracticable.

AUTOMATIC FEED

Electric drive has afforded the opportunity for development of means for automatically feeding the drilling bit, and several methods, differing considerably in principle, have been devised. In the differential electric drive, of which there are two types, the rate of feed is balanced against the torque required to turn the drill pipe, so that the feed is automatically regulated to maintain a constant torque. One design of this type of drive makes use of two wound-rotor induction motors which drive the draw works hoist drum and the rotary table through a differential gear unit. Each motor drives one main differential gear of the cluster, and one of the motors also drives the rotary table through suitable gearing. The ring gear of the differential cluster is connected to drive the hoist drum which feeds the bit. The two motors run in opposite directions, so that when they are at the same speed the ring gear and therefore the hoist drum do not turn. Normally the speed of the motor which drives the rotary table is set higher than that of the other motor, and this causes the ring gear and the hoist drum to turn in the direction which will pay out line and thus feed the bit. Any change in torque required to drive the drill pipe will affect the speed of the driving motor and thus increase or decrease the difference between the speeds of the two motors, which consequently changes the speed of the ring gear and the rate of feed correspondingly. Heavy torque will thus slow down the feed to standstill or even reverse it to retrieve the bit, and stalling of the drill pipe will result in instant retrieval until the stalled condition is relieved, after which feeding is automatically resumed.

The two motors are therefore both a power drive and a means of regulation of feed and torque. For hoisting work in removing the drill pipe and bit from the hole, both motors are run in the same direction, thus locking the differential gears, which then merely serve to transmit the power.

In another design of differential drive the power unit, which may be a motor or an engine, drives the ring gear of the differential cluster. One main differential gear drives the rotary table, and the other drives the draw works through a two-speed sprocket and chain transmission. The balance of torque against feed is thus entirely mechanical, and is adjusted by changing the transmission ratio to the hoist drum. With two transmission speeds and three draw works speeds, the total number of adjustments is six, and the speed of the entire drive is regulated by control of the motor or engine.

The action of these two differential drives in controlling the torque and feed is very similar, although the two-motor arrangement provides more flexibility of control. A large number of differential drives has been installed, particularly those of the two-motor type.

During the past few years, methods of obtaining straight holes have been uppermost in the minds of oil company engineers and officials, and thousands of dollars have been spent in endeavoring to determine the causes of crooked holes and methods of preventing them. This has led to the present quite wide acceptance of the theory that the maintenance of proper pressure of the bit on the bottom of the hole is one of the most certain ways of keeping the hole straight. As a result, two somewhat different electrical methods of automatic control of the weight on the bit have recently been developed and put into service.

One of these methods employs electricity only in an auxiliary capacity, but is nevertheless of interest although yet in a developmental stage. It acts on the principle of automatically releasing and setting the draw works brake to feed the bit, but does not provide for its retrieval. In one layout which has been used, which illustrates the principle of operation, the brake lever is attached to the piston rod of a double-acting steam cylinder, and steam is admitted to one side or the other of the piston by solenoid-operated valves, the solenoids being energized by adjustable electric switch contacts opened or closed by hydraulic pressure. The hydraulic pressure which is imparted to this switch is established by a diaphragm which is so attached to a bight in the anchored or "dead" end of the drilling line that the pressure of the diaphragm against the liquid behind it varies with the tension in the line, and hence inversely with the amount of weight resting on the drilling bit. An indicating gage measures the hydraulic pressure.

The position of the contacts in the pressure switch is readily adjusted by turning a button, and they may be placed in a neutral position for any desired pressure. In practise, they are set for as much lower pressure, below that representing the entire weight of the string of tools, as corresponds to the amount of weight desired to be carried on the bit. This closes the contact which causes release of the draw works brake, and the decrease in hydraulic pressure, then resulting from setting part of the weight on bottom, causes the pressure switch to close the other contact and set the brake. This automatic feeding action occurs as frequently as necessary to maintain the weight on the bottom at or above that permitted by the setting of the switch.

The other method is accomplished by equipment entirely independent of the draw works and its drive, and it provides both automatic feed and retrieval. Its main element is an auxiliary induction motor-driven hoist unit, which is installed on the opposite side of the derrick floor from the draw works, or at the side where the calf-wheel is usually placed. The "dead" end of the drilling line is spooled on the hoist drum of this unit. The driving motor is a two-speed wound-rotor induction motor as designed for oil well pumping. It is equipped with a solenoid brake and drives the hoist drum through a transmission gear and a worm-gear reduction unit. The thrust of the worm shaft produced by the tension of the drilling line is carried on a diaphragm which imparts a corresponding pressure to liquid behind it, this being piped to an automatic pressure switch actuated by another diaphragm and functioning to open and close electrical contacts to start, stop and reverse the motor. The hydraulic pressure on this diaphragm in the pressure switch is balanced by air pressure on the other side, established by a small hand pump and regulated by a hand-valve.

As the thrust of the worm shaft is proportional to the weight suspended in the derrick and varies inversely with the amount of this weight set on bottom, the variation in this weight produces a corresponding change in the hydraulic pressure on one side of the switch diaphragm and this results in movement of the diaphragm which actuates the switch. Increase in pressure will cause the switch to start the motor in the direction to feed the bit, while a decrease will result in starting the motor in the reverse direction and thus retrieve the bit. The action of feeding or retrieving reestablishes the required weight on the bit and thus brings the hydraulic pressure back to its original value, when the switch stops the motor and the solenoid brake prevents its coasting.

The amount of weight to be automatically carried on the bit is adjusted by the driller by reducing the air pressure in the switch the necessary amount below that corresponding to the entire weight of the string of tools. The switch functions to set weight on bottom until the hydraulic and air pressures are equalized.

The electrical circuits are so arranged that the two-speed motor runs on its low speed connection for feeding the bit and on its high speed connection for retrieving it.

In field service this equipment functions to maintain a tension in the drilling line varying only about 10 lb. at the drum. The corresponding variation of weight on the bit is determined by the number of lines in the tackle, which is usually from 4 to 10. The actual weight variation is therefore the product of this number by the 10-lb. variation in the tension of the fast line at the drum.

To make this weight control equipment of general utility, electrical control is incorporated to permit the driller to pick the bit up off bottom at any time, or to both feed and retrieve entirely by hand control. By means of a transfer switch, all automatic feed features may be disconnected and the hoist operated from a manual controller for use in rigging-up or for emergency removal of the drill pipe from the hole in case of failure of any part of the draw works or its drive. In this service the motor operates on its high-speed connection. The automatic control feature may also be utilized to maintain and not exceed a predetermined pull on a string of tools or casing which may have become stuck in the hole and perhaps requires all the pull the derrick can withstand to dislodge it.

So far this type of automatic weight control equipment has been used to drill over 16,600 ft. of hole in California oil fields, and very successful results are being obtained.

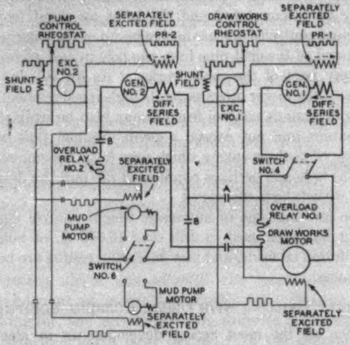
Rotary Drilling with Direct-Current Equipment

One of the most recent developments in electric drive for rotary drilling and mud pump operation is the application of d-c. motors operated by generator voltage control from engine-driven generators or motor-generator sets. In working out a suitable system, of which two are in successful use, the designing engineers have had recourse to the years of experience gained in applying similar equipment to elevators, electric shovels and steel mill drives, and have been able to make an interesting adaptation to fulfill the unusual requirements of rotary drilling service.

A d-c. rotary drilling equipment usually consists of two or three duplicate generators with direct-connected exciters, driven by duplicate internal combustion engines or by an a-c. motor, one or two motors for driving the draw works, two motors for the mud pumps and suitable control and protective equipment. One generator supplies power to the draw works motor for rotating the drill pipe during drilling, and the other at the same time supplies power to one of the mud pump motors. When it is necessary to withdraw the drill pipe and bit from the hole, the mud pump is shut down and the power from the two generators is combined to supply the heavy demand from the draw works motor.

One of the systems mentioned makes use of differentially compound wound generators, each provided with a separately excited shunt-wound field opposed by a series field, resulting in a drooping characteristic of the generator voltage with increase in load. One object accomplished by this characteristic is to limit the generator output to the maximum which the engine can deliver, and thus automatically prevent stalling the engine. This also prevents the current exceeding the maximum safe value for the electrical equipment, but permits high torque to be developed by the draw works motor at low speed, and high speed at light loads.

Another object which this generator characteristic accomplishes is to cause the mud pump motor to slow down with increase in load and stall at a maximum torque which does not exceed the strength of the pump. As has already been pointed out, this is the ideal way to operate the pump. The motor when stalled will not be damaged if it is separately ventilated by a blower, as is customary on the d-c. drilling equipments for the chief purpose of excluding inflammable gases.



	Closed	Open
Drilling	*B*, No. 4, No. 6	*A*
Normal hoisting	*A*, No. 4	*B*
Emergency hoisting with gen. No. 1	*B*, No. 4	*A*
Emergency hoisting with gen. No. 2	*A*	*B*, No. 4

Resistor $P\ R_1$—$P\ R_2$ are not used with general shunt fields in series

FIG. 1—ELEMENTARY DIAGRAM FOR D-C. VARIABLE VOLTAGE CONTROL FOR OIL WELL ROTARY DRILLING

Standard arrangement with one draw works motor

The scheme of connections is shown in Fig. 1 for a system with two generators, one draw works motor and two mud pump motors. The positions of the various switches for the several drilling operations are indicated in the legend. For drilling, generator No. 1 supplies power to the draw works motor and generator No. 2 to either of the mud pump motors. For normal hoisting, the generators are in parallel to supply power to the draw works motor, and the mud pump is shut down, while for emergency hoisting, either generator may be used. The arrows in the field circuits show the relative polarity of the separately excited and series fields of the generators.

For the sake of clearness the field transfer connections have been omitted from the diagram, but it should be noted that when the generators are in parallel, their separately excited fields are connected in series, insuring the same degree of excitation to both. When they operate separately, each generator has a block of permanent resistance in its separately excited field circuit equal to the resistance of the field.

It usually is a difficult matter to operate constant voltage d-c. generators in parallel when driven by internal combustion engines, but the differentially wound series fields and the series connection of the separately excited fields result in stable operation and in a division of load approximately in proportion to the generator speeds. Tests on two 75-kw. generators in service showed the respective loads to be 410 and 385 amperes with respective speeds of 1,050 and 975 r. p. m. In all tests where the speeds were the same, the load was evenly divided.

For successful drilling service it is desirable that the engine governors give a speed regulation of 8 per cent or better, and it is necessary that the engines be equipped with flywheels and that their governors have the same characteristics.

Of course this type of drive is not limited to the use of engines as primary power, but may be utilized to full advantage where central station power is available, by the use of synchronous or induction motor drive for the generators. Under these circumstances it would be possible to employ regenerative braking in conjunction with the mechanical brake for lowering pipe, thus reducing the wear on the mechanical brake. This type of electrical braking is impracticable with engine drive, as the only means of absorbing the pump-back power would be by permitting the engines to speed up, which is usually not allowable. However the counter-torque method has been used very successfully on a gas-engine-driven d-c. drilling equipment, by running the draw works motor on the first or second hoisting control point when lowering pipe into the hole. To stop the load, the generator voltage is increased. The mechanical brake is then set when the load comes to rest, and the motor is released.

The curves in Fig. 2 illustrate the volt-ampere characteristic of the differentially compound-wound generator and its relation to the corresponding curve representing the maximum output of the engine. The generator curve does not cross the engine curve; consequently the engine cannot be stalled by any possible load on the generator. This characteristic is fully obtained in actual drilling service.

In Fig. 3 are given the approximate characteristic curves of a 150-hp., 230-volt motor when used with a 175-kw., 1,000 r. p. m. differentially compound-wound generator. The shape of these curves is not only typical for this type of equipment, but is very desirable for rotary drilling work. It should be noted that the maximum horsepower output of the motor is obtained at the value of current at which the generator curve is tangent to the engine curve, and is therefore the point where the engine is required to develop maximum output. On either side of this point the engine load is lighter.

The curves in Figs. 2 and 3 are characteristic of the mud pump drive as well as the draw works drive. In comparison with the speed-ampere curve, which is of the most interest in considering mud pump operation, the volume-pressure curve (curve S) of a steam pump is shown. These curves touch at the point of maximum horsepower output of the pump. The steam pump curve is theoretically a straight line, but there is reason to believe that it actually curves due to throttling, and so very closely follows the curvature of the motor-driven pump characteristic, which of course is due to the effect of saturation in the generator fields. Thus it is evident that with the proper design of generator the d-c. motor-driven pump will equal the steam pump in performance.

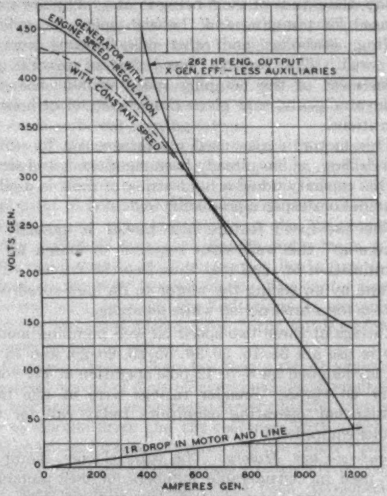


FIG. 2—ESTIMATED GENERATOR CHARACTERISTICS FOR DIFFERENTIALLY COMPOUND WOUND GENERATOR DRIVEN BY INTERNAL COMBUSTION ENGINE

In the other system of d-c. drive the practical operation is similar to the one described but the generators used have been shunt wound, separately excited, and the motors compound wound with separate excitation for the shunt winding. When the combined power of the generators is required for hoisting drill pipe, their armatures are connected in series and their separately excited fields in parallel, the two field controllers being mechanically arranged to operate together on corresponding points under this condition.

The series connection of the two generators doubles the line voltage, and hence the draw works motor is designed to operate on 250 volts when working from one generator and 500 volts when working from both. For hoisting conditions with power from both generators, approximately the same range of speed and torque is possible as with the first scheme described.

With the generators operating either singly or in series, increase of the torque load on the motor not only reduces its speed because of its compound winding, but also increases the torque load on the engine. With an engine of suitable size having a drooping speed characteristic at heavy loads, the resulting drop in voltage will reduce the motor speed to the stalling point without stalling the engine.

The economy and flexibility of d-c. drive with power from internal combustion engines will often justify the cost of such equipment, especially for wildcat drilling in remote locations. For instance, this has been found to be the case in the Mississippi delta region, where steam drilling is almost prohibitively expensive, largely due to the difficulty of providing fuel and water. There the saving effected by the use of a Diesel-electric rig over a steam rig was enough to more than pay for the power plant in the drilling of two wells.

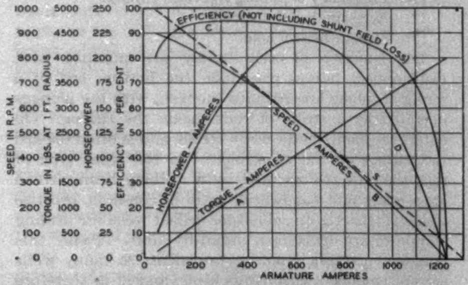


FIG. 3—ESTIMATED CHARACTERISTIC CURVES FOR 150-HP. 450-R. P. M. 230-VOLT SHUNT-WOUND MOTOR, OPERATING FROM GENERATOR WITH CURVE AS IN FIG. 2

The following is a summary of the advantages of d-c. drilling equipment from the standpoint of the operating engineer.

a. Its characteristics are so designed that it is impossible to impose loads on the engines greater than they can carry, thus definitely preventing their being stalled.

b. It combines the fineness and flexibility of control of d-c. motors with the economy of the internal combustion engine.

c. It has speed characteristics like the steam engine, thus giving a wide range in speed and torque for the draw works and a positive indication of any restriction or stoppage in the mud pump line.

d. It makes available the combined capacity of the mud pump and draw works power supply for heavy hoisting duty when "coming out of" and "going into" the hole.

e. It provides individual control of drilling and mud pump operations, centralizing this control at the driller's stand.

f. It readily provides low table speeds, quick reversals, fast or slow pick-up as required on heavy or light loads, and speeds of the empty hook as high as the crew can handle.

WELL PUMPING

A wider variety of types of motors is used for well pumping than for drilling, because of the numerous methods of pumping and types of pumping rigs employed, and also because of variable well conditions encountered. A glance at the following items will indicate the many things to be considered in the selection of equipment.

1. Maximum depth of well.
2. Daily production.
3. Gravity of oil.
4. Number of hours pumped.
5. Weight of rods and tubing.
6. Length of stroke.
7. Strokes per minute.
8. Depth of working barrel and fluid level.
9. Amount of sand and gas produced.
10. Percentage of water produced.
11. Frequency of pulling rods and tubing.
12. Accessibility of the well.
13. Straightness of hole.

The majority of wells today are pumped with displacement pumps placed on the bottom end of a string of tubing lowered into the well, with the plunger operated by a string of rods inside the tubing. Oil enters the pump on the down stroke and is lifted toward the surface on the up stroke. On the older type of rigs, the rods are operated by a walking beam driven by a crank and bandwheel, as it is common practise to install this rig on cable-tool drilled wells for drilling and on rotary drills for a stand-by unit. Today this pumping practise is not so prevalent, as many wells are not equipped with bandwheels, and it is therefore necessary to install some form of pumping unit and hoist.

Some wells do not require frequent hoisting of rods and tubing, or "pulling," as it is commonly termed, while others have to be pulled as often as every 10 days. Some operators prefer to have their equipment arranged for both pumping and pulling whereas others desire it only for pumping. The latter then must have a portable pulling outfit which is either gas or electric driven.

No two wells are exactly alike in operating conditions, and the latter may change from week to week or at longer intervals. The fluid level may go down, the gas decrease, water come in, or sometimes the wells sand up. To meet these conditions, it is often necessary to change the stroke, or the speed, or both.

Two-Speed Oil Well Pumping Motors. When motors were first applied to the standard walking-beam type of pumping rig, they replaced steam or gas engines and were required to perform all the operations at the well. The motor had to be fast enough to pull the well and slow enough to pump it, and this resulted in the development of the special double rated, two-speed wound-rotor induction motor which admirably fits these conditions and continues to be a popular drive today, thousands of them being in use. The low speed is used for pumping and the high speed for pulling, bailing, swabbing, and other miscellaneous work at the well. The variable speed feature permits the adjustment of the pumping speed without changing pulleys or gears, and gives proper control of hoisting operations.

This motor is also used advantageously for cable-tool drilling, as has already been mentioned, and serves in this capacity when a light string of tools is used to clean out or deepen a producing well.

The two-speed feature also makes it possible to "shake-up" the well when required to loosen up an accumulation of sand and thus free the valves. This is done by switching the motor to its high-speed connection for a brief period while pumping.

Ratings of these two-speed oil well pumping motors now in use are 35/15, 50/20, 55/25, 65/25, and 75/35 hp., all designed for 6- or 12-pole operation at 440 volts, 50 or 60 cycles. Smaller ratings were at one time popular but operating conditions today call for the heavier equipment.

Squirrel-Cage Motors. The squirrel-cage motor is now used in considerable numbers for well pumping, and is popular because of its low cost and high efficiency. The standard type, the low-starting-current type and the high-starting-torque type all find their field of usefulness for which they are best adapted. Some wells are hard to start and high starting torque is needed, and so the high torque motor is obviously the best. If however as a matter of expediency or availability a normal-torque motor is applied, the well may be started by "rocking" the walking beam if necessary, which requires a reversing switch for the motor. The ratings of motors used run from 7½ to 25 hp., 8 or 10 poles.

In many cases the squirrel-cage pumping motor is connected by chain or V-belt to the shaft of the engine installed at the well, the engine crank being disconnected. The engine is used for pulling the well, and for this purpose the crank is temporarily reconnected and the chain or belt removed. Such an installation requires probably the least investment of any to pump electrically, but a more efficient and compact drive is obtained by employing a small reduction gear unit instead of using the engine shaft, and either installing a motor large enough for pulling or using a portable hoist equipment for that work.

Some operators prefer not to have many different ratings of motors on each lease, owing to the complication of stock and maintenance problems. However, they appreciate the advantages of high efficiency and power factor, and so to meet their requirements, the Y-delta type of motor is furnished, either of the squirrel-cage or wound-rotor type, usually rated 40/15 or 50/20 hp., 6 or 8 poles. Either the Y or the delta connection is used as the load may require, the advantage of the Y connection being improved efficiency and power factor on light pumping loads. When the starting torque required is considerable, even though the well may be a light pumper, an advantage is gained in starting on the delta connection and running on the Y. This eliminates some of the bad effects of overmotoring in order to obtain sufficient starting torque.

The Y-delta squirrel-cage motor has also been applied for both pumping and pulling, requiring a primary resistor to give a cushioning effect on the first point of hoisting. The second point cuts out this resistance and runs the motor on the Y-connection, giving sufficient torque to handle light loads. The delta connection is used on heavier loads, and is obtained on the third control point. The motor used in this way of course has its limitations in comparison with the two-speed wound-rotor motor, but is somewhat cheaper and more efficient.

Pumping Units. There has been a steady advance in the development of more efficient mechanical equipment for well pumping, encouraged largely by the adaptability of electric drive and the readiness with which the efficiency can be determined by electric power measurements. Compact self-contained electric-driven pumping units of many designs have been developed for wells approximately 5,000 ft. or less in depth. For greater depths such units are less often used owing to their high cost and bulkiness. The horsepower requirements range from 2 to 25 and squirrel-cage motors for full voltage starting are generally used.

One particularly interesting unit of this character recently developed by one of the electrical manufacturers includes a triple reduction gear unit with a crank on the low-speed shaft for operating a walking-beam. The driving motor is a squirrel-cage machine and both the motor and control box are built into and form an integral part of the gear unit. This electrical equipment is of weatherproof construction, and so no housing is necessary. Provision is made for readily changing gears to change the pumping speed. Such an equipment represents the maximum in simplicity and the minimum in installation work.

Some pumping units have some flywheel effect or are equipped with small flywheels, and for these the high-resistance rotor type of squirrel-cage motor is theoretically best adapted, although there is a question whether under well pumping conditions there is any decrease in power consumption. The motor efficiency is of course lower than that of a normal design of squirrel-cage motor, but the current peaks on the pumping cycle are reduced, and the motor may thus run on a better point of the efficiency curve. Any gain which might thus be possible in reducing the size of motor required is usually opposed by the preference of most operators for a larger motor than initial pumping conditions require to provide for the fact that a year or two later the load is likely to increase as the level of fluid in the well subsides, the gas decreases, or water encroachment increases.

Reduction Gear Units. The old standard pumping rig with belted bandwheel to drive the crankshaft is inefficient at best and the bandwheel and belts are now quite often replaced by single- or double-reduction gear units with a crank on the low-speed shaft. This has not materially affected the application of electric drive but has been brought about by the desire and necessity for better equipment to meet modern conditions. Electric drive has made it possible to detect the source of losses in the old equipment and is thus in a way considerably responsible for these improvements. Provision is made on these gear units to drive a tubing hoist for pulling operations.

Such an arrangement is now frequently used to drive the calf-wheel for a stand-by hoist on a rotary drilling rig, and is usually equipped with a two-speed pumping motor. When the well is completed, this equipment is left installed at the same location for pumping the well, the tubing hoist and a walking beam being added, as well as a good mechanical counterbalance for the crank or beam.

Central Power Jacks. Provided the terrain permits, the pumping of small producers, usually shallow wells, is accomplished in groups from a central plant, called a "power," when it becomes uneconomical to pump them individually. The central power consists of one or more large eccentrics on a vertical shaft, the collar of the eccentric being attached to rods which extend to the wells and pump them by bell-cranks called pumping jacks. The central power shaft is engine- or motor-driven. As many as 20 or 25 wells are thus pumped from one plant. Hook-ups of 10 to 25 wells require approximately 35 to 40 hp., but as the conditions vary larger motors are often used, from 40 to 75 hp. in some cases. The starting torque is usually heavy and may be obtained with a high-torque squirrel-cage motor, but is sometimes done by overmotoring. Constant-speed wound-rotor motors are used if there is no friction clutch on the rig, as slow starting is usually a necessary practise to avoid rod breakage.

CONTROL UNITS

One of the rather unique developments in oil field electrification is the assembly of all control apparatus for a motor on welded structural steel or pipe framework at the factory, where it is completely wired before shipment. This practise has been applied to drilling equipments, which must of necessity be frequently moved from one location to another, and to the two-speed pumping motor equipments, which include several items of control apparatus. The resulting advantages to the user are obvious, and are particularly important in an industry involving the difficult conditions of operation as encountered in the production of oil. The frameworks for these units are sturdily constructed to withstand abusive handling, and mounted on skids to facilitate frequent moving.

PROTECTED EQUIPMENT

In many installations of pumping equipment, no house is provided. Open type motors are usually covered with a small sheet metal "dog house," but the totally enclosed fan-cooled type of motor is becoming more popular.

There is a demand for a reasonable amount of protection of the electrical equipment against hazardous gas conditions, particularly for drilling equipments where gas is more likely to be prevalent. Motors of the open type are usually acceptable to the oil companies but collector rings are specially enclosed by a suitable housing. Electrical contacts of the control apparatus are immersed in oil or are built in an enclosure specially designed for the purpose. In general, the design of such enclosures follows the recommendations of the U. S. Bureau of Mines for equipment permissible for use in gaseous mines, or of the Underwriters Laboratories for equipment to operate in Class 1, Group D hazardous conditions.

Where it is possible, as on drilling rigs, to set the control apparatus back at a safe distance from the well, this is sometimes done and standard equipment is then employed.

Another method of protection which has been applied to both motors and control is to enclose them suitably and ventilate the enclosures with fresh air from a separate blower, the air being piped from a sufficiently distant point to be free from hazardous quantities of gas. This method is a good one for magnetic control equipments, which, if oil immersed, are considerably more expensive and give less satisfactory service on heavy duty.

AIR-GAS LIFT

When wells stop natural flowing, many of them can be made to flow artificially by compressed air or air forced down between the casing and tubing. The compressors are usually driven by squirrel-cage motors, 75 to 200 hp., 8 poles, with close coupled belt drive. Hand starting compensators are generally used although, due to gaseous conditions, magnetic compensators placed in a separate building are often preferred. An oil immersed push button is located near the motor. Synchronous motors are being considered more and more for this service, due to penalty and bonus clauses in central station power rates. Intermittent blowing is quite popular in certain fields and this is easily accomplished by an electric timer and electric solenoid valves. Frequently the valve controlling the flow of gas freezes up with ice due to the expansion of the gas and electrically heated valves have solved the problem.

CONCLUSION

Though the petroleum industry still relies largely on engine power, great strides have been made during the past ten years in the development and application of electrical apparatus and in the extension of central station power in the fields, and oil companies are giving an increasing amount of attention to its use. There are many other uses for electricity in oil production, such as for gathering and pipe line pumps, welding, dehydration, illumination, heating and refining, but these are beyond the scope of this paper.

ACKNOWLEDGMENTS

The published works of others in the electrical and petroleum industries have been consulted for data and facts in the preparation of this paper, including papers or articles by Messrs. H. E. Dralle and L. J. Murphy of the Westinghouse Electric and Mfg. Company, Thomas Fleming, Jr., of the Oil Well Supply Company, B. L. Moore of the Humble Oil and Refining Company, and O. B. Goldman of The Drillometer Company. Unpublished papers also consulted include one on "The Practical Application of Electricity to Rotary Drilling," by Mr. W. H. Skivington, Electrical Engineer of the Doheny-Stone Drill Company, several by Messrs. W. G. Taylor and B. M. Mills of the Industrial Engineering Dept., General Electric Company, and one read before the Pacific Coast Electrical Bureau by Mr. J. H. Fenton of the Westinghouse Electric and Manufacturing Company. To all of these the authors of the present paper are indebted for information, as well as to the Westinghouse Electric and Mfg. Company for permission to include material concerning a recent development not previously announced.

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Discussion

David Hall: The paper calls attention to the fact that the oil industry is only 7 per cent electrified, which fact cannot be attributed in any measure to the inadequacy of electric power, but rather to the "inertia" of steam practise, the familiarity of operators with steam, and to the lower initial cost of steam equipment. Builders of steam machinery have jealously noted any introductions of electric power and have met electrical competition by larger engines and higher steam pressures and it is probable that the introduction of electricity in the oil fields has hastened the improvements in steam machinery.

This evolution is still progressing and the paper refers to recent accomplishments in mud pump drive, the results of which have not yet been published. This drive, which was introduced by the Westinghouse Electric & Manufacturing Co. and tried out through the cooperation of one of the large oil companies in Southern California, consists of wound rotor induction motors with permanent resistance and reactance in the rotor circuit. The real reason for this arrangement is to provide a prime mover which will slow down under increasing loads and actually stall before any injury is done to the pumps. This arrangement successfully operates pump, singly or in tandem, or in parallel and the entire equipment permits of stalling and the maintaining of the stalled torque for operations when low speed and high pressures are necessary.

The use of permanent resistance in the rotor with the necessary losses may provoke criticism if the equipment is narrowly viewed, but a broader vision of the over-all efficiency will clarify this question. For example, the normal cost of operating a full drilling outfit is roughly $600 for twenty-four hours. If electrically equipped the cost of power will not exceed $20 or $30 per day, of which $2 per day might be charged to the resistance, without which the entire operation would be less safe.

Rotary Drilling. The paper has given a concise résumé of rotary drilling, which is the universal method used in California. As an example of progress it might be mentioned that the deepest well in the world, drilled to a depth of 10,030 ft. in the Ventura district, uses a completely electrified rig of the Hild differential type, and two motors as described in the paper.

The drilling equipment was supplied with two 25/65-hp. motors having a total pull-out torque equivalent to approximately 600 hp. In this differential drilling arrangement, the full power of both motors is available for either drilling or hoisting.

As a further example of the increased demands for more power for drilling these deep holes and the desire for faster drilling, single-motor sizes have been increased from 50 hp., as of less than ten years ago, to the present *Y*-delta equipment referred to in the paper and it is well to note that this equipment has an even greater maximum torque capacity than the 600 hp. mentioned above.

Direct Current. Our reason for introducing direct current in the oil fields was to meet a demand for the use of electricity in the drilling of wildcat wells. Since the electric power must be generated at the well site, the simplest and most flexible equipment could be made with direct current.

The first equipment was set up in the Los Angeles district, particularly for close observation, and after successfully drilling three wells, it was moved to an isolated district in Colorado, where it is now in use. The success of this equipment has been followed by the sale of additional equipments and the problem of wildcat drilling has been demonstrated. The use of such equipment will probably be expanded as the economies of this method become more generally known.

The authors have not only pointed out the use of electricity in drilling but have called attention to the problem of pumping. This problem is demanding more attention probably than any other in the oil fields. Much total power is involved on account of the many wells, some 350,000 now active, and efficiency is a major concern. Numerous schemes have been devised and many are on trial; the final answer is not in sight and the great depths now encountered have increased the difficulties and interest in this subject.

F. E. Terman: I would inquire as to whether any serious efforts have been made to develop electrical pumps in which the motors can be dropped down into the casing. In the case of deep wells I understand that the gas pressure is relied upon to lift the oil part way to the surface, and that the pumping methods now employed will not satisfactorily pump the well if the gas pressure fails. This is because the stretch and inertia of the pumping rods are such that when the length is in the order of two miles impulses applied by a motor at the surface must be considered as traveling by wave propagation, and this introduces many complications. I have heard that several attempts have been made to develop some sort of a centrifugal pump and motor unit that could be dropped down the well casing, and would appreciate hearing a discussion of the merits and disadvantages of such pumps, and whether there is any serious effort now being made toward the development of such equipment.

H. C. Hill and J. B. SeLegue: As far as submersible motors are concerned, a number of attempts has been made to solve this problem in order to produce a more efficient method of pumping and one which could operate on crooked holes and at great depths. These experiments have been along the line of centrifugal pumps as well as reciprocating pumps but it is the opinion of the writers that although some success has been obtained in wells to a depth of 3,000 ft., it is not practical as yet to pump wells of 7,000 ft. or more by this method. In the first place, the smallest motor so far constructed has an outside diameter to fit in a 6⅝ in. casing and this of course would not fit in the bottom of some of the deeper holes. It is also very difficult to seal the motor so that it will not absorb moisture at the tremendous pressures which are encountered. Furthermore, the cable to conduct the power from the surface to the motor is very expensive, difficult to handle, and not of long life. There are very few of these submersible pumps in operation in the oil industry although several of them have been used in water well pumping of shallow depths. More experience has been had on these in the mid-continent field than in the California field but we understand that very few of them have been used over 2,000 ft. in depth.

More detailed description of the design and operation, as well as some of the results obtained is contained in an article by E. S. Haury, page 126, of the 1931 issue of the *Petroleum Engineer Handbook* published by the Petroleum World. It might be interesting at this point to add that as far as pumping with sucker rods at great depths is concerned, satisfactory results have been obtained in wells at Santa Fe Springs with the pump located 7,801 ft. from the surface. This matter is covered in an article by Mr. B. H. Robinson in the proceedings of the Twelfth Annual Meeting of the American Petroleum Institute, November, 1931

Calorimeter Measurement of Stray-Load Losses of a 55,000-Kva. Generator

Having Enclosed System of Ventilation

BY G. D. FLOYD[*]
Member, A. I. E. E.

and

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Member, A. I. E. E.

***Synopsis.**—The paper covers a number of thermal or calorimeter tests which were made on a 55,000-kva. generator at the Queenston generating station of the Hydro-Electric Power Commission of Ontario. The objects of the tests were to determine stray-load losses on short circuit and under rated load, and to obtain some information regarding the accuracy of this type of test. The paper includes descriptions of the methods followed and of the actual tests made. The details of the calculations are given in the appendix.*

The results of the tests show that the stray-load losses can be determined with fair accuracy by this method on large units when an enclosed system of ventilation is used. They also seem to show that the difference between the stray-load losses under load conditions and under short-circuit conditions is slight, and that a test under load conditions is justified only when a very accurate determination of the losses is required. For most purposes, a test under short-circuit conditions seems to be sufficient.

INTRODUCTION

THIS paper covers a number of thermal or calorimeter tests which were made on a large hydraulic turbine-driven electric generator at the Queenston generating station of the Hydro-Electric Power Commission of Ontario. The primary object of the tests was to determine stray-load losses on short circuit and under rated load, and information was also desired on the accuracy of this method of determining the stray-load losses.

DESCRIPTION OF UNIT

The generator tested is known as No. 10 unit. The nominal rating of this machine is 55,000 kva., 12,000 volts, 80 per cent power factor, three-phase, 25-cycle, 187½ r. p. m. It is practically a duplicate of the 45,000-kva. units that have been previously described.[1]

The tenth unit differs from the preceding ones in that it has a closed ventilating system. Fig. 1 shows, in diagrammatic form, the ventilating arrangement of the first machines installed. The air is drawn in through a duct, either from the outside or from the generator room. After passing through the windings and core it is discharged through the generator frame into an enclosure between two floors of the power house. It is then discharged to the roof of the power house through exhaust fans, or if it is desired, the air can be recirculated to heat the generator room. A diaphragm is located across the wheel pit just below the generator coupling for the purpose of keeping the ventilating air clear of the turbine floor.

Fig. 2 shows, diagrammatically, the ventilating arrangement used on No. 10 unit. It will be noticed that the ventilating system of the new unit is completely closed. The heated air discharged from the unit passes through two air coolers each having a water requirement of 250 gallons per min. with cooling water at 20 deg. cent. The two coolers will absorb 1,090 kw. from the air, with a rated air flow of 115,000 cu. ft. per min.

CALCULATED LOSS DATA

It is possible to calculate the losses of a synchronous machine with a very close degree of accuracy. The only losses about which there is any great uncertainty are those defined in the A. I. E. E. Standards as the stray-load losses.

Several writers[2,5,6,7,8,9] have attempted to develop methods for calculating these losses but, as far as the writers know, no complete determination has been published. Eddy current losses in the armature copper can be calculated with a fair amount of accuracy but the extra iron losses are practically impossible of calculation.

In Holmes'[4] paper it is shown that the armature eddy current loss was of the order of 50 per cent of the total stray-load losses in a certain machine. Since the loss that can be readily calculated is such a small percentage of the stray-load losses, the practical method of determining them before the machine is tested is to compare the machine with others of similar construction that have been tested, and to estimate the stray-load losses from the test data on the previous machines.

The calculated ratio of the a-c. resistance to the d-c. resistance, following Gilman's method,[8] was as follows:

Top coil side	1.16
Bottom coil side	1.08
Average for winding	1.12

A comparison with tests on other similar machines indicated that the total stray-load losses would be approximately five to six times the eddy current loss in the armature copper. Since the calculated value of armature $I^2 R$ loss at 75 deg. cent. was 205 kw. the

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1. For references see Bibliography.

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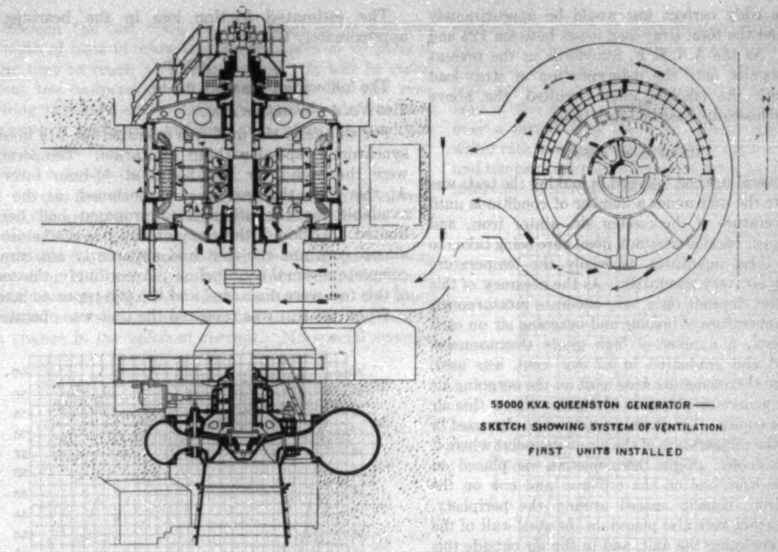
N
55000 KVA QUEENSTON GENERATOR —
SKETCH SHOWING SYSTEM OF VENTILATION
FIRST UNITS INSTALLED

FIG. 1—DIAGRAM TO ILLUSTRATE VENTILATING ARRANGEMENTS OF FIRST UNITS INSTALLED AT QUEENSTON

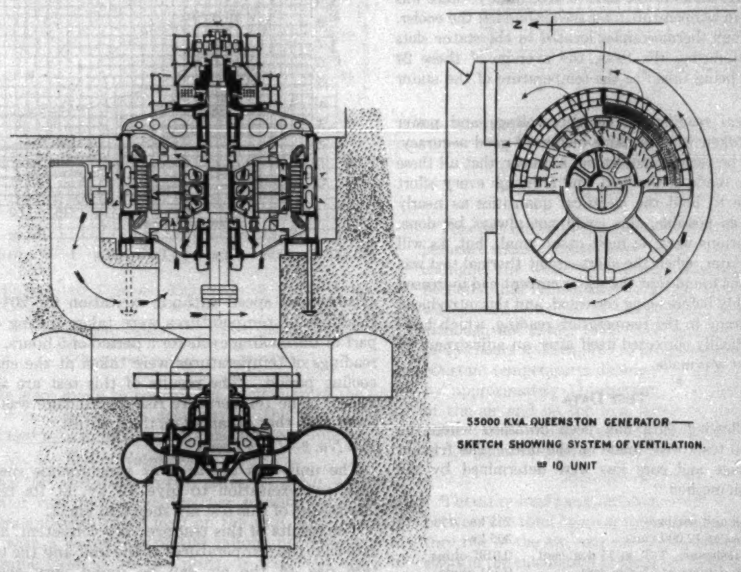
N
55000 KVA QUEENSTON GENERATOR —
SKETCH SHOWING SYSTEM OF VENTILATION.
№ 10 UNIT

FIG. 2—DIAGRAM TO ILLUSTRATE VENTILATING ARRANGEMENTS OF NO. 10 UNIT AT QUEENSTON

armature eddy current loss would be approximately 25 kw. and the total stray-load losses between 125 and 150 kw. As the A. I. E. E. Standards at the present time recognize only the determination of stray-load losses with the machine short-circuited, the above estimated losses are based on such tests.

METHOD OF TESTS

The general method followed in making the tests was to operate the unit under a number of conditions until the temperature of the cooling air, stator iron, and stator copper became constant, great care being taken to measure these quantities, especially the temperature of cooling air, very accurately. As the accuracy of this type of test depends on a very accurate measurement of the temperature of ingoing and outgoing air on each of the tests, a number of high grade thermometers calibrated and graduated to 0.2 deg. cent. was used. Ten of the thermometers were used on the outgoing air from the generator placed at the point where this air enters the cooler. Ten thermometers were also used to measure the temperature of the air at the point where it leaves the cooler. Eight thermometers was placed on the stator iron, one on the top iron and one on the bottom iron, equally spaced around the periphery. Thermometers were also placed on the steel wall of the chamber enclosing the unit, and in the air outside this chamber. A number of thermometers was placed in the inlet air ducts to the unit to determine if there was any drop in temperature after the air had left the cooler. Twenty-four thermocouples located in the stator slots were read during the tests, the average of these 24 detectors being taken as the temperature of the stator copper.

Electrical readings of current, voltage, and power were all taken with portable meters of good accuracy. It should be clearly understood, however, that all these tests were made in the field, and although every effort was made to hold the electrical quantities as nearly constant as possible, this could not always be done. The variations were, in most cases, small, but, as will be noted later, when the short-circuit thermal test was made it was found that the stator current had increased considerably before being corrected, and this introduced an inaccuracy in the temperature reading, which however practically corrected itself after an adjustment of the current was made.

TEST DATA

The following data had been obtained when the acceptance tests were made on the unit. The friction and windage and core loss were determined by the retardation method.[10]

Friction and windage	215 kw.
Core loss at 12,000 volts	355 kw.
Stator resistance, T-T, at 75 deg. cent	0.0197 ohms.
Rotor resistance at 75 deg. cent	0.341 ohms.

The estimated friction loss in the bearings was approximately 65 kw.

TESTS

The following tests were made:

Test No. 1—Windage Test.

For this test, the unit was operated for 9½ hours at synchronous speed without excitation. Temperatures were then read for 2½ hours at ½-hour intervals. At this time the test was discontinued, as the time available for the other tests proposed had become limited. Although the temperature rise of the air was almost constant the unit had apparently not come to complete thermal equilibrium. Accordingly, the results of this test were discarded and the test repeated later.

When the test was repeated the unit was operated at synchronous speed without excitation for 20½ hours. Readings of temperatures were taken during the first part of the cooling cycle for a period of 5 hours, and two readings of temperatures were taken at the end of the cooling period. The results of this test are shown in Fig. 3. The temperature rise on windage was slightly lower than that obtained on the first test.

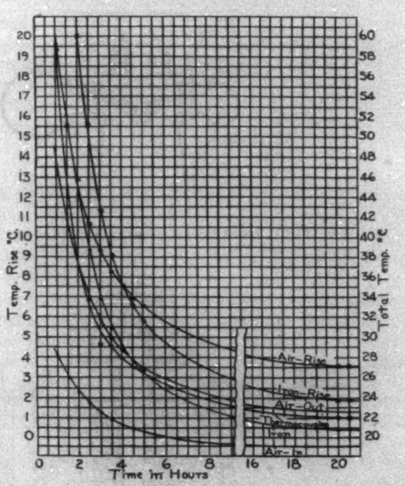

FIG. 3—COOLING CURVES—TEST NO. 1—WINDAGE ONLY

Test No. 2—Core Loss.

The unit was operated at synchronous speed with sufficient excitation to give 12 kv. at its terminals. The results of this test are shown in Fig. 4.

The results of this test are very consistent, and indicate that the temperatures of the unit and the temperature rise on the air can be found very accurately, although the test must be continued for a sufficient length of time to allow the temperatures of all parts of the unit to reach a constant value. It will be noted that the temperature of ingoing air to the unit rises along with temperature of the unit itself; air, iron, and copper temperature arriving at a constant condition at the same time.

Test No. 3—Short-Circuit Test.

On this test the unit was operated at its rated current of 2,650 amperes under three-phase short circuit. The results of this test are shown in Fig. 5.

It was found difficult to hold the stator current constant on the unit, as it has a direct-connected shunt wound exciter, so that small variations in the speed caused a corresponding variation in the excitation and a change in the value of current. No special arrangements were made to control the excitation of the unit, and it is believed the accuracy of the test could be increased by a small degree if this had been done.

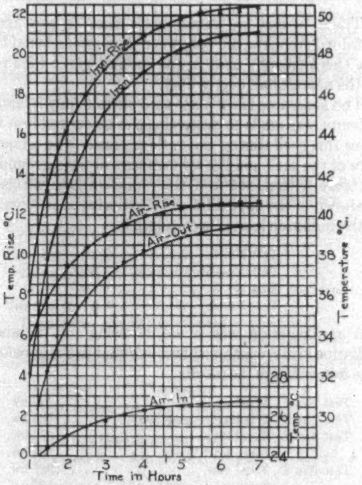

FIG. 4—HEATING CURVES—TEST NO. 2—CORE LOSS AND WINDAGE

Terminal voltage....... 12,000 volts
Field current........... 314 amperes
Collector ring voltage.... 98 volts

Test No. 4—Test at Rated Load.

This test was made in two parts, Test No. 4(a) with a water rate of 500 gallons per minute through the coolers and with rated load on the unit, and Test No. 4(b) with a water rate of 400 gallons per minute with the same load as on the previous test. In order to complete the test in a minimum time, the unit was operated for 8 hours at approximately rated load during which no temperature readings were taken. The load was then set accurately as close to rated conditions of load, power factor, and voltage as could be obtained and held at this value. Temperature readings were taken over a period of 3 hours at ½-hour intervals. The water rate was then adjusted to 400 gallons per minute, and temperature readings taken for 4½ hours at ½-hour intervals. The results of this test are shown in Fig. 6.

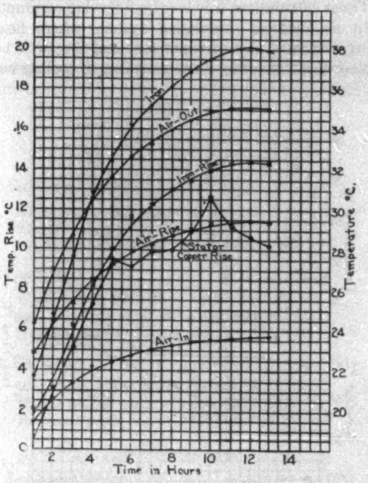

FIG. 5—HEATING CURVES—TEST NO. 3—SHORT-CIRCUIT LOSS AND WINDAGE

Armature current........ 2,650 amperes
Field current............ 320 amperes
Collector ring voltage.... 98 volts

This test indicates the accuracy with which the temperature measurements were made, and the effect of changing the water rate through the coolers. It will be noted that immediately following the adjustment of water rate, changes in temperature occurred which although not great, were quite definite, and a period of 4½ hours was required to bring the unit back to new constant temperatures differing from the previous ones by approximately 1½ degrees. The temperature rise of the air and on the iron, however, returned to the same values which they had had with a water rate of 500 gallons per min.

RESULTS OF TESTS

The stray-load losses are determined by the difference between the total losses as calculated from the temperature rise of the air, and the known losses. The actual steps in the calculation of the stray-load losses are given in detail in the appendix.

The following is the method used in working up the results:

The first test gives the temperature rise on windage only. The second test gives the rise due to windage and core loss. The difference between these two temperature rises gives the rise due to core loss only. Since the core loss is known from other tests, a constant *K* may be calculated giving the kilowatt loss per degree rise. The kilowatt loss may then be determined for any other operating condition, from the air rise.

These calculations involve the following assumption:

In an enclosed ventilating system, where heat is being generated at a constant rate and the air is being cooled in coolers using a constant flow of cooling water, the kilowatts converted into heat are proportional to the temperature rise of the air, after all conditions have become constant.

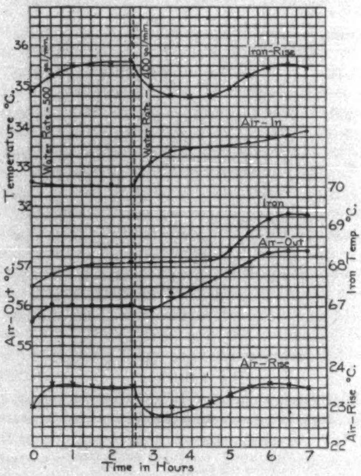

FIG. 6—HEATING CURVES—TEST NO. 4—RATED LOAD TESTS

	1st Period	2nd Period
Water rate	500 gal. per min.	400 gal. per min.
Armature current	2,580 amperes	2,610 amperes
Terminal voltage	12,070 volts	11,900 volts
Kw. load	43,400 kw.	43,200 kw.
Field current	608 amperes	612 amperes
Collector ring voltage	229 volts	234 volts

It is readily seen that the crux of the method is that all conditions must be constant. Among other things, this requires that the humidity and barometric pressure must not change during a series of tests. If the barometer were to drop appreciably during a series of tests, the constant *K* would be affected, and it would be necessary to repeat the "windage only" and the "windage and core loss" tests, in order to obtain reliable results. Also, a constant *K*, determined under certain atmospheric conditions, cannot be used with the results of tests under different atmospheric conditions.

The following are the losses, as computed from the tests.

Test No. 1	Windage	152 kw.
Test No. 2	Windage + core loss	538 kw.
Test No. 3	Total losses on short circuit	483 kw.
	Stray-load losses, by difference	108 kw.
Test No. 4	Total losses under load conditions	990 kw.
	Stray-load losses, by difference	138 kw.

A check on these losses was obtained from the heat absorbed by the cooling water.

The details of the calculations are shown in the appendix. The following are the results:

Test No. 2 Heat absorbed by cooling water = 475 kw.
Test No. 3 Heat absorbed by cooling water = 475 kw.
Test No. 4(a) Heat absorbed by cooling water = 1,030 kw.
Test No. 4(b) Heat absorbed by cooling water = 1,010 kw.

These figures are remarkably close to the figures obtained from the air rise, when it is remembered that the thermometers used in the cooling water were read to the nearest half degree only.

The generator has a plate shell surrounding it, and an attempt was made to determine the amount of heat lost from this shell during the various tests. The temperature of the shell and the temperature of the surrounding air were determined and the radiation loss calculated as shown in the appendix. The following are the values of the radiation losses, as calculated.

Test No. 1	None
Test No. 2	7 kw.
Test No. 3	4 kw.
Test No. 4	16 kw.

A recalculation was made, allowing for the above radiation losses, which is shown in detail in the appendix. The following are the results.

Test No. 1	Windage	149 kw.
Test No. 2	Windage + core loss	535 kw.
Test No. 3	Total loss	479 kw.
	Stray-load losses	104 kw.
Test No. 4	Total loss	987 kw.
	Stray-load losses	135 kw.

This calculation indicates that the radiation loss makes a very small difference in the stray-load loss measured by this test. It is probably less than the experimental error associated with the type of test.

Bearing friction gives a further remarkably close check, as follows:

Friction and windage loss, retardation test	215 kw.
Windage loss, calorimeter test	152 kw.
Friction loss, by difference	63 kw.
Estimated friction loss	65 kw.

CONCLUSION

The results of this test indicate that, for this particular type of unit, the stray-load losses under short circuit and at rated load are approximately equal.

The stray-load losses are about 50 per cent of the stator copper loss at rated load. The results also indicate that if sufficient care is taken in the measurements of temperatures, the stray-load losses can be determined with fair accuracy on large units when an enclosed system of ventilation is used. The great drawback appears to be the time necessary to make a complete series of tests, although it is possible that a compromise test over a short period of time may be made and the final temperature rise of the air determined by calculation.

The difference between the stray-load losses under load conditons and under short-circuit conditions is slight, but these tests seem to show that the losses under load conditions are a little higher than those determined under short-circuit conditions.

The results obtained by Laffoon and Calvert[11] by calorimeter tests on turbo-generators also show the stray-load losses under load conditions to be slightly higher. In other words, if an accurate determination of the losses is required, a test under load conditions is justified, but for most purposes, a test under short-circuit conditions seems to be sufficient.

Appendix

CALCULATIONS

For the benefit of those who wish to check the calculations in detail, the actual figures are given here.

1. To Determine Constant K

Test No. 1
Temperature rise due to windage
= 3.6 deg. cent.
Test No. 2
Temperature rise due to windage + core loss
= 12.75 deg. cent.
Temperature rise due to core loss only
= 9.15 deg. cent.

Core loss = 355 kw.
Field copper loss = 31 kw.

Total loss heating air 386 kw.
Constant K = kw. per deg. cent. rise = 42.19

2. To Determine Total Losses

Test No. 1 Temperature rise = 3.6 deg. cent.
Total loss = 152 kw.
Test No. 2 Temperature rise = 12.75 deg. cent.
Total loss = 538 kw.
Test No. 3 Temperature rise = 11.45 deg. cent.
Total loss = 483 kw.
Test No. 4 Temperature rise = 23.5 deg. cent.
Total loss = 990 kw.

3. To Determine Stray-Load Losses, by Difference

Test No. 3 Total loss = 483 kw.
Windage loss = 152 kw.
Field copper = 31 kw.
Stator copper = 192 kw.

Total, by summation = 375 kw.

Stray-load losses, by difference...108 kw.

Test No. 4 Total loss = 990 kw.
Core loss = 355 kw.
Windage = 152 kw.
Field copper = 145 kw.
Stator copper = 200 kw.

Total, by summation............852 kw.

Stray-load losses, by difference...138 kw.

COOLING WATER CHECK

500 Gallons of water per minute
= 5,000 lb. per minute
= 5,000 × 0.4536 = 2,268. Kg. per min.
1 kg. calories = 4.186 kw-sec.

Therefore 1 deg. cent. rise in 500 gals. of water per minute is caused by 2,268 kg. calories per minute, or by

$$\frac{2268}{60} \times 4.186 = 158.2 \text{ kw.}$$

Test No. 2 and Test No. 3

Water rate = 500 gallons per minute.
Rise of cooling water = 3 deg. cent.
Hence, heat abstracted = 3 × 158.2 = 475 kw.

Test No. 4 (a)

Water rate = 500 gallons per minute.
Rise of cooling water = 6.5 deg. cent.
Heat abstracted = 6.5 × 158.2 = 1030 kw.

Test No. 4 (b)

Water rate = 400 gallons per minute.
Rise of cooling water = 8 deg. cent.
Heat abstracted = 8 × 0.8 × 158.2 = 1010 kw.

RADIATION LOSS

Radiation constant = 1.77 B. t. u. per hr. per deg. fahr.
Radiating surface = 1,140 sq. ft.
Radiation from complete casing per deg. cent.

$$= \frac{1.77 \times 1140 \times 1054 \times 1.8}{3600 \times 1000} = 1.063 \text{ kw.}$$

Test No. 2

Temperature difference between shell and surrounding air = 6.5 deg. cent.
Radiation loss = 6.5 × 1.063 = 7 kw.

Test No. 3

Temperature difference = 3.8 deg. cent.
Radiation loss = 3.8 × 1.063 = 4 kw.

Test No. 4

Temperature difference = 14 deg. cent.
Radiation loss = 14 × 1.063 = 15 kw.

RECALCULATION, ALLOWING FOR RADIATION

1. To Determine Constant K

Temperature rise due to core loss only = 9.15 deg. cent.
Kw. core loss + field loss = 386 kw.
Radiation loss = 7 kw.
Loss heating air = 379 kw.
Constant K = 41.4

2. *To Determine Total Losses*

Test No.	Temp. rise	*K* × temp. rise	Radiation loss	Total loss
1	3.6° C	149	0	149
2	12.75° C	528	7	535
3	11.45° C	475	4	479
4	23.5° C	972	15	987

3. *To Determine Stray-Load Losses, by Difference*

Test No. 3	Total losses	479
	Losses, by summation	375
	Stray-load losses	104
Test No. 4	Total losses	987
	Losses, by summation	852
	Stray-load losses	135

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Discussion

E. A. Crellin and F. I. Lawson: During recent years more and more attention is being directed to the matter of waterwheel efficiencies. Since the only practical method of determining the efficiency of a modern prime mover is by calibrating the generator as a dynamometer and carefully measuring its output while it is assuming the power system load, hence the accuracy of hydro unit efficiency tests depends on the accuracy of the generator calibration.

Most generator losses can be determined with refinements well within the range of accuracy of the hydraulic input measurements. Stray load losses, however, fall into the classification of residual losses which cannot be readily isolated. Following the best information available the Institute has set up rules for the allocation of stray load loss based on measurements made under the very low magnetic density existing in the machine during a steady short-circuit state. This has often aroused the natural inquiry as to whether or not such a determination completely includes all the losses which may be present during load conditions involving much higher and differently distributed magnetic densities. The fact that the Institute rules concerning this point have been changed within comparatively recent years to recognize higher losses bears witness to the uncertainty regarding these stray load losses.

For these reasons the information presented by the authors, giving an independent determination of stray load losses, is a distinct contribution to the hydraulic as well as the electrical engineer.

Within the past eight or nine years efficiency tests have been made on quite a number of hydroelectric units on the Pacific Gas and Electric system. The following tabulation of generator losses obtained by the retardation method during some of these tests may be of interest.

The Stanislaus unit has been used during recent years as somewhat of a proving ground for waterwheel buckets. Tests on the unit when equipped with different types of buckets on the two overhung wheels could not be verified by tests using each wheel singly. The single wheel tests of course involve half load on the generator for full load on the wheel, whereas when using both wheels the loading becomes complete on the generator as well as the wheels. Since the generator is of a rather early design (1907) and was later rebuilt for a higher rating, its stray load losses under full load conditions come under suspicion as an uncertain factor.

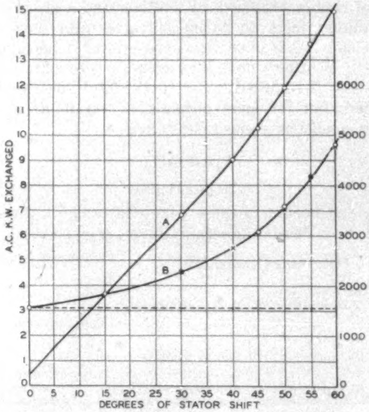

FIG. 1

Curve *A*—Kw. exchanged at 100 per cent power factor—200 volts
Curve *B*—D-c. armature watts (*o*) and sum of losses (*x*)

In an attempt to investigate the matter of these stray load losses advantage was taken of a small laboratory unit in a rather novel way. The unit used consists of a d-c. motor driving two identical 15-kw., three-phase, 110-220-volt generators mounted on the same shaft, one of which can have its stator shifted at will with respect to the other. By paralleling the two generators while driven by the motor and then shifting the stator of one of them, an electrical phase shift is introduced which forces the flow of power from one generator to the other. This power is returned to the first machine through the common shaft and hence the d-c. motor is only called upon to furnish the losses in the set.

Measurements of friction, windage, core loss, I^2R and stray load losses were made by d-c. input in the customary Institute manner. The a-c. machines were then loaded into each other and the total d-c. input measured for various amounts of interchanged load. Fig. 1 shows how very closely the measured losses under actual loading correspond with the sum of the losses determined in the manner prescribed by the Institute rules.

Tests of this kind on a small, rather specially designed machine may at first sight appear to be incomparable with results to be expected on large commercial units, and yet the subdivision of conductors introduced on large, modern machines should accomplish somewhere near the same degree of eddy current isolation as exists in the laboratory machine.

Name of plant	Kva. of generator	Date of mfr.	Voltage	Rated power factor	R.p.m.	Type of generator	Friction and windage	Core loss	Stray and I^2R	I^2R	Stray
Drum No. 3	15,000	1916	6,600	0.80	360	Horizontal shaft	202.0	130.9	160.5		
Drum No. 4	15,000	1928	6,600	0.80	257	" "	107.7	89.5	120.0		
Pit No. 3	27,000	1924	11,000	0.90	225	Vertical	304.2	187.4	163.7	99.7	64
Pit No. 1	35,000	1921	11,000		257	"	200	302	220	126	94
Melones	13,500	1927	6,600	0.90	277	"	60*	80	120.5	78.5*	42*
Stanislaus	8,500	1907	4,000	1.00	400	Horizontal	80	75.3	94.3		
Electra	4,000	1905	2,400	1.00	400	"	82	68	32	16.8	15.2

*Factory value

Power Transmission and Distribution From the Mokelumne River Development of the Pacific Gas and Electric Company

BY E. M. WRIGHT[1] and B. D. DEXTER[1]
Member, A. I. E. E. Member, A. I. E. E.

Synopsis.—*The additional power generated on the Mokelumne River has a peak capacity in excess of 150,000 kw. It is transmitted over a double-circuit 220-kv. transmission line approximately 109 miles to Newark substation where it is fed into the existing 110-kv. and 60-kv. networks of the Pacific Gas and Electric Company.*

Newark substation has rapidly grown into the most important substation on the system and numerous alterations and additions have been made from time to time to provide increased capacity. The addition of 150,000 kw. of peak capacity into this station has necessitated a major rebuilding and it has now become a twin station, divided into two major parts for the purpose of reducing oil circuit breaker interrupting duties and lessening the concentration of power on a single bus.

In addition to the two 220-kv. Mokelumne lines, there are sixteen 110-kv. and three 60-kv. transmission circuits radiating from Newark substation. With the final addition of the third transformer bank for incoming Mokelumne power there will be 249,000 kva. of transformer capacity and 75,000 kva. of synchronous condenser capacity connected to the bus.

The Mokelumne River Development consists of two major storage reservoirs and four power houses. From this development it is necessary to carry the power over 100 miles to a central distribution point. The existing lines from this territory consist of one 60-kv. wood pole line. It is, therefore, necessary to consider transmission lines and substation facilities which would take care of the entire development. The storage reservoirs and power houses are described in a companion paper, "Hydroelectric Development on the Mokelumne River," by E. A. Crellin, see page 28.

THE transmission line will extend in two circuits from Tiger Creek power house, the major plant, in a southeasterly direction for approximately 109 mi. (175 km.) to Newark, the position chosen for the substation, at the south end of San Francisco Bay.

En route, the line crosses the ultimate 220-kv. twin-circuit tower line from the Feather River hydroelectric plants, to the San Joaquin Light and Power Corporation's proposed steam plant near Fresno. Both of the Mokelumne River circuits are tapped through a switching station at this point of crossing, and feed south on one circuit on the No. 2 position of this tower line, about 100 mi. (161 km.) to the San Joaquin Light and Power Corporation's substation near Fresno. The Mokelumne lines also generally parallel for the last half of their lengths, two separate 110-kv. twin-circuit tower lines, one from the Spaulding-Drum hydroelectric development in the northwest and the other from the Stanislaus hydroelectric development in the eastern part of the company's territory.

An extended study, with due regard to the above conditions as well as for future ties to the company's existing system, showed that a voltage of 220 kv. would deliver the power with less cost per kilowatt-hour than would 110 kv. The receiving voltage at Newark was set at 200 kv. to permit future interconnection with Vaca-Dixon substation and the lines from the Pit River development. Sending voltage was set at 215 kv. so that charging current would be supplied from the receiving end and generators would operate at unity or lagging power factor and thereby require high excitation essential for good stability.

110-Kv. Line

In addition to the 220 kv., there is a single circuit line from Salt Springs to Tiger Creek power house. On account of the smaller amount of power available at the Salt Springs power house, 110 kv. was found to be best suited to this circuit.

The 110-kv. line is approximately 17 mi. (27 km.) long and generally parallels the Mokelumne River at an elevation of 3,800 ft. (1,158 m.) at Salt Springs, and 2,400 ft. (732 m.) at Tiger Creek, reaching 5,000 ft. (1,524 m.) about half-way between. Due to the snowfall in this territory, the circuit is in flat configuration, 14½ ft. (4.42 m.) between conductors, on snow-type towers with concrete foundations. (Fig. 2.) The conductor is 4/0, 7-strand medium hard-drawn bare copper except in some long spans where 4/0 "Hitenso" copper is used for the additional strength required. All splices are the drawn-joint type and were put on in the field with a portable draw-bench built for the purpose. Standard strength suspension type insulators in strings of eight 10-in. (25.4 cm.) units were used at all suspension points with nine units at most dead-ends. High strength insulators in strings of nine 11-in. (28 cm.) units were used at several dead-end points where the strain was excessive, such as at high points in the line. The normal span is 600 ft. (183 m.) but a number of spans are between 1,500 ft. (457 m.) and 2,100 ft. (640 m.). Transpositions are the rolling type without dead-end, and are completed in three normal spans by carrying one outside wire over the top of the other two on extensions attached to the top of two standard towers. A minimum angle is maintained at all insulator supports by offsetting the transposition towers. (Fig. 3.)

220-Kv. Towers

The 220-kv. transmission line starts from Tiger

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[1] Pacific Gas & Electric Co., San Francisco, Calif.

Presented at the Pacific Coast Convention of the A. I. E. E., Lake Tahoe, California, August 25-28, 1931.

Creek power house at an elevation of 2,400 ft. (731 m.) and ends at Newark at an elevation of 7 ft. (1.1 m.) above sea level. It consists of two general types of towers and two types of conductors, and is divided into three types of loading and also three zones of insulation. Neither the types of towers, conductors, loading or insulation sections coincide with each other exactly so that an explanation of each must be given separately.

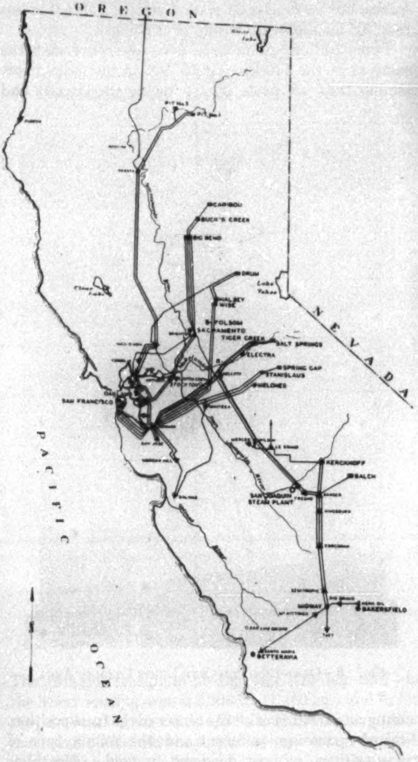
OREGON
NEVADA
PACIFIC
OCEAN
CARIBOU
BUCK'S CREEK
BIG BEND
DRUM
Lake Tahoe
HALSEY
WISE
FOLSOM
SACRAMENTO
TIGER CREEK
SALT SPRINGS
ELECTRA
SPRING GAP
STANISLAUS
MELONES
STOCKTON
SAN FRANCISCO
SAN JOSE
KERCKHOFF
BALCH
SAN JOAQUIN STEAM PLANT
MIDWAY
BAKERSFIELD
BETTERAVIA

FIG. 1—110, 165 AND 220-KV. TRANSMISSION SYSTEM

In the heavy loading area, which provides for ½-in. (12.7 mm.) ice, and 6 lb. per sq. ft. (29.3 kg. per sq. m.) wind at 0 deg. fahr. (– 17.8 deg. cent.) and covers 16 mi. (25.7 km.) of line, there are two lines of single-circuit snow-type towers with concrete foundations. The circuit is in flat configuration with a horizontal separation between adjacent conductors of 20 ft. (6.09 m.) (Fig. 4).

The medium loading area provides for ¼ in. (6.35 mm.) of ice and 6 lb. per sq. ft. (29.3 kg. per sq. m.) wind at 0 deg. fahr. (– 17.8 deg. cent.) and covers a line distance of 11 mi. (17.7 km.). In this section of line there are twin-circuit steel towers on concrete foundations. These towers permit a semi-vertical configuration for both circuits with the center conductor offset 5 ft. (1.52 m.) outward giving 15 ft. (4.57 m.) separation between the conductors vertically, 28 ft. (8.53 m.) horizontal separation between the top wires and bottom wires of the different circuits and 38 ft. (11.58 m.) horizontal separation between the middle wires of the different circuits. (Fig. 5.)

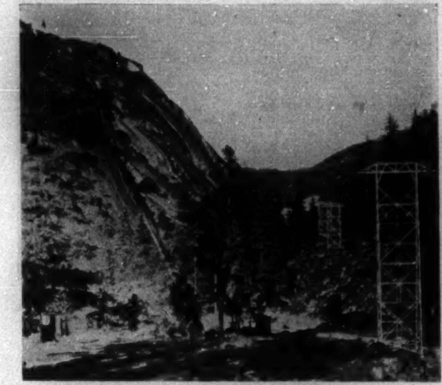

FIG. 2—110-KV. TOWER LINE NEAR SALT SPRINGS POWER HOUSE

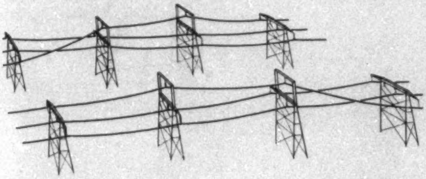

FIG. 3—TYPES OF TRANSPOSITION STRUCTURES AND METHOD OF TRANSPOSING WIRES IN SNOW TERRITORY ON 110-KV. AND 220-KV. LINES

The structures in the light loading area permit a loading of an 8 lb. per sq. ft. (39.1 kg. per sq. m.) wind at 25 deg. fahr. (– 3.9 deg. cent.). This area covers the remaining distance to Newark 82 mi. (132 km.) but is divided into two sections as far as towers are concerned. The first section, 53 mi. (85 km.) covers the entire valley from Valley Springs to Tracy and consists of towers the same as in the previous section except that the center conductors are not offset. (Fig. 6.) The remaining section 29 mi. (46.6 km.) is considered fog area, and contains structures having the same steel, below the crossarm, as the previous sections. The crossarms are longer and farther apart and provide a vertical separation between conductors of 18 ft. (5.48 m.) and a horizontal separation between circuits of 33 ft. (10.06 m.). These larger spacings are used to permit increased insulation. (Fig. 7.)

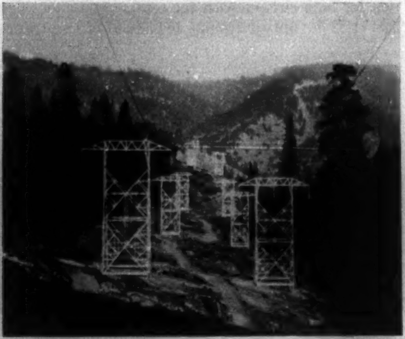

FIG. 4—220-KV. TOWER LINE NEAR TIGER CREEK POWER HOUSE

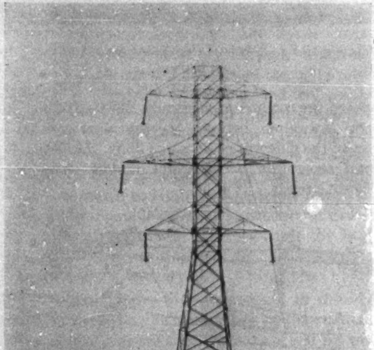

FIG. 5—220-KV. TOWER LINE IN MEDIUM LOADING AREA, SHOWING TRANSPOSITION SCHEME

The normal span on the snow-type towers is 600 ft. (183 m.) and on the twin-circuit towers 800 ft. (244 m.) but there are many spans from 1,000 ft. (305 m.) to 1,800 ft. (549 m.) in length. Towers of the same type but of increased strength are used where necessity requires, but no dead-ends or semi-strains are used unless required for angles, up-strains or crossings.

Transpositions on the snow-type towers are made the same as in the 110-kv. section but in the twin-circuit section they are made by separating the circuits and adding an extra tower at the center of the transposition. Normal spans are sufficient and the rolling type of transpositions, without dead-end, is used. (See Figs. 5 and 8.)

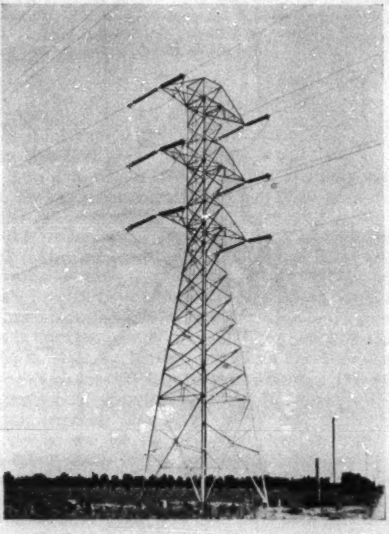

FIG. 6—DEAD-END TOWER IN LIGHT LOADING AREA

CONDUCTORS

Preceding the choice of conductors, extensive tests were made at Ryan Laboratory, Stanford University, on a three-phase, 220-kv. line under various spacings, positions and weather conditions, to determine corona losses for cables of various diameter. As a result of these tests two different types of cable were chosen. (TRANS. A. I. E. E., March 1931, Vol. 50, p. 36.)

Between Tiger Creek and Valley Springs, 27 mi.

(43.5 km.), the heavy and medium loading area, where higher strength was required due to snow and sleet, aluminum cable steel reinforced was chosen. This cable has 518,000 cir. mils (334.2 cir. mm.) of aluminum and an outside diameter of 1 in. (2.54 cm.). The stranded steel core is approximately ⅝ in. (15.9 mm.) in diameter. Standard, Aluminum Company of America, compression joints were pressed on in the field for all splices. The stringing tension at 60 deg. fahr. (15 deg. cent.) in the heavy loading area is 3,200 lb. (1,460 kg.) and in the medium loading area it is 4,200 lb. (1,920 kg.). Under maximum conditions the tension is approximately 7,000 lb. (3,200 kg.) in both cases.

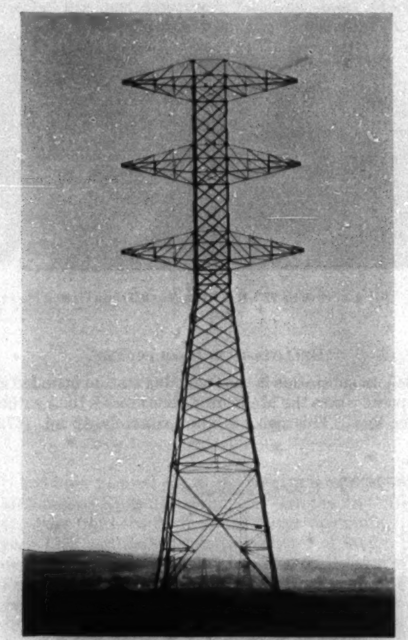

FIG. 7—220-KV. TOWER IN FOG SECTION

Between Valley Springs and Newark Substation, 82 mi. (132 km.) where conductivity as well as low corona loss was of primary importance, hollow-core copper conductor was chosen. This conductor is 500,000 cir. mils, 1 in. (2.54 cm.) outside diameter and has two layers of 0.097 in. (2.46 mm.) diameter wires laid over a twisted copper I beam. The splices are screw joints and consist of threaded lugs drawn on the ends of the conductor in the factory and connected in the field by means of a sleeve nut. The entire joint is similar to a pipe union and, on account of right and left hand threads, is completed in the field without the necessity of twisting the conductor on either side. Where necessary to make an emergency splice in the field, either during construction or in case of failure during operation, a single piece sleeve is provided, to be drawn on with a portable draw-bench kept on hand for that purpose (Fig. 9). This drawing on process is done in the field the same as in the factory and differs only in respect to the portability of the machine used. The stringing tension at 60 deg. fahr. (15 deg. cent.) is 6,100 lb. (2,800 kg.) and for maximum conditions is about 7,200 lb. (3,300 kg.)

HARDWARE

The suspension clamps and dead-end clamps were made up specially and correspond very closely to the standard *J* and *U* bolt clamps used on many of the transmission lines. Particular attention was paid to the fitting of the clamp around the conductor so that the conductor would be in contact for not less than 80 per cent of its circumference. The standard suspension clamps are of the J-bolt type and are used at all suspension points except special railroad and important crossings. At these points the same type of clamp is used with U-bolt so that in case of failure the wire will not slip. U-bolt clamps were used at all dead-end points, thus eliminating any cutting of the wire.

INSULATION

The insulation is divided into three sections: standard, light fog, and heavy fog. All insulators are ball socket type. In the first section from Tiger Creek to a point near Tracy, a distance of 79 mi. (127 km.) there are 13 standard strength 10-in. (25.4 cm.) suspension type insulator units with 5.5-in. (14 cm.) spacing in the suspension strings and 13 high-strength 11-in. (28 cm.) units with 7-in. (17.78 cm.) spacing in the dead-end strings, on both circuits.

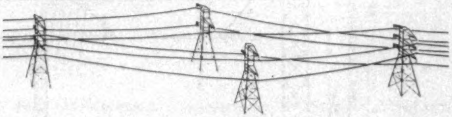

FIG. 8—TYPES OF TRANSPOSITION STRUCTURES AND METHOD OF TRANSPOSING WIRES ON 220-KV. TWIN CIRCUIT TOWERS

The second section, in the light fog area from Tracy to Livermore, 11 mi. (17.7 km.), has 17 standard units as above in the suspension strings on the No. 2 circuit and 17 high-strength units as above in all dead-end strings on both circuits. The No. 1 circuit for the first half of the distance has 12 units of fog type insulators 10 in. in diameter with 6¼-in. (15.87 cm.) spacing and having a 3¼-in. (8.25 cm.) shell droop. This insulator is smooth outside and has one petticoat on the inside. The last half of this light fog section has 11 units of fog type insulators 10 in. (25.4 cm.) in diameter and 7½-in. (19 cm.) spacing with three petticoats on the outside and smooth on the inside.

The heavy fog section from Livermore to Newark, a distance of 18 mi. (29 km.) has 20 standard units in suspension on the No. 2 circuit and 20 high-strength units in dead-end on both circuits. The No. 1 circuit for the first half of this distance has 13 of the 7½-in. (19 cm.) spaced fog units in suspension and for the last half of the distance has 14 of the 6¼-in. (15.87 cm.) spaced fog units in suspension.

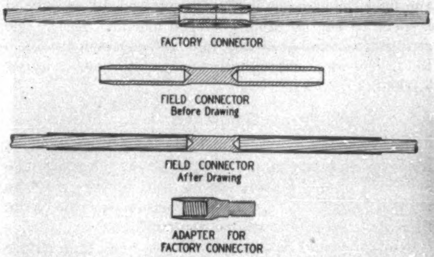

Fig. 9—Types of Connectors Used on Hollow-Core Conductor

In all of the fog area, standard insulator units are used on one circuit and fog type units of two different designs are used on the other circuit, the length of strings of each type being in proportion to the recommended insulating value. From an experimental standpoint this should give some very useful information as to the relative values of the standard and fog type units under operating conditions.

A corona shield is installed on the line end of all insulator strings on the 220 kv. This shield is a round sheet of pressed steel curved down at the edge to protect the line hardware and connections from excessive corona losses.

Fig. 10—Section of 220-Kv. Line Near Tiger Creek Power House

Bellota-San Joaquin Tie

Bellota substation is the switching station installed at the point where the Mokelumne line crosses the Feather River line. This point is approximately 42 mi. (67.5 km.) from Tiger Creek. From Bellota substation one circuit extends on the existing tower line to the San Joaquin Light and Power Corporation substation near Fresno, approximately 100 mi. (161 km.). The conductor is bellow-core copper the same as on the Mokelumne circuits. The insulation is 14 standard units in suspen-

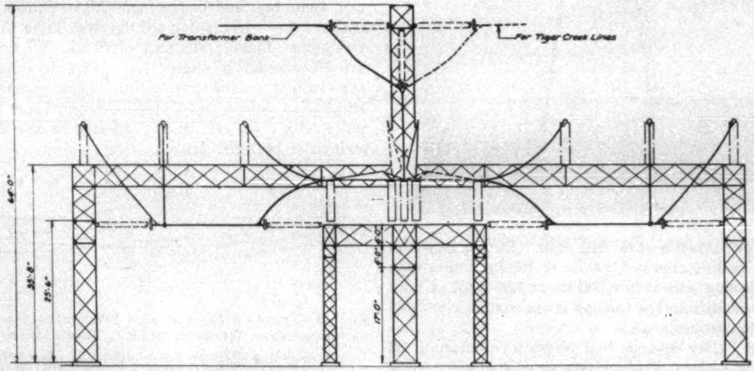

Fig. 11—Sections of 220-Kv. Bus—Newark Substation

sion and 14 high-strength units in dead-end of the same types and spacing as on the Mokelumne circuits. All of the switching of the Bellota substation will be handled by remote control from Stockton, a distance of about 20 mi. (32.2 km.).

NEWARK SUBSTATION

Newark, the terminal station of the 220-kv. transmission line just described, has long been the distributing point of power from the Mokelumne River.

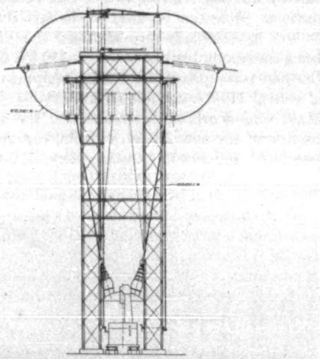

FIG. 12—220-KV. OIL CIRCUIT BREAKER AND DISCONNECTING SWITCH STRUCTURE—NEWARK SUBSTATION

It has been pointed out in the companion paper on the Mokelumne River Development* that the first delivery of power from the Mokelumne River was made in the year 1902 and that it was transmitted at 60 kv. The most distant delivery point was Martin Station in San Francisco, but among the more important intermediate switching stations was that located at Mission San Jose, some 15 mi. (24.14 km.) north of the City of San Jose. Some of the original equipment installed at this substation was in use up to 1919, when it was found that more modern facilities must be provided, and as space requirements could not be satisfied at this location it was decided to abandon Mission San Jose substation and build a new station about four miles further south and near the town of Newark.

The facilities at Newark substation in 1919 provided for the continued reception of Mokelumne River power at 60 kv. but as this source of supply was somewhat diminished due to the growth of load between Electra power house and the San Francisco Bay region it was necessary to deliver power to Newark substation at 100 kv. from the Spaulding-Drum hydroelectric development. In 1930 growth of load in the territory supplied from Newark substation required still larger amounts of power and delivery of same is now being made at 220 kv. from the Mokelumne River Development, thus marking the use of a still higher voltage and the handling of larger amounts of power at this important load center.

In order that an economical and yet flexible switching arrangement might be provided for handling the 220-kv. circuits it was decided to use a double bus with bus selector air switches and a single oil circuit breaker for each transmission line and transformer bank. With this arrangement a circuit can be connected to both buses or to either bus as switching requirements may dictate. This bus structure provides switching facilities for the operation of two transmission lines, three transformer banks, bus paralleling oil circuit breaker and carrier-current telephone coupling equipment.

As shown in Fig. 11, the buses and selector switches are placed well above ground and are supported on a galvanized steel structure. The general dimensions of the structure are: length, 354 ft. (107.9 m.), width, 116 ft. (35.35 m.), height of center dead-end frame, 64 ft. (19.5 m.). The bus conductor consists of 650,000-cir. mils hard-drawn bare copper cable, concentric lay with 61 strands. The six conductors comprising the double bus are in tension and dead-ended on each end of the bus structure and supported on the cross frames with pillar type insulators at each bay.

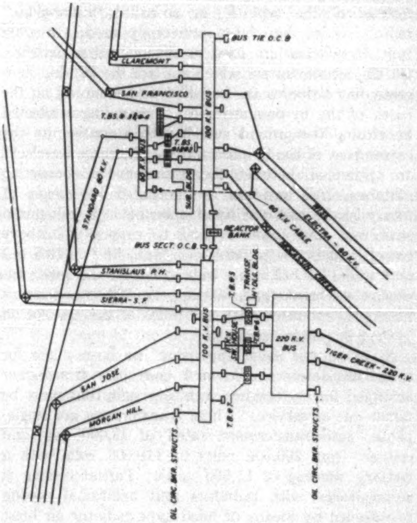

FIG. 13—GENERAL ARRANGEMENT OF NEWARK SUBSTATION

The selector switches are rated at 400 amperes and are of the vertical break type. Each single-pole assembly consists of two single-pole air switches mounted on a common structural steel base, five insulator posts per pole being considered as most suitable. The three poles of each selector switch are actuated by a single manually-operated control handle located at the bottom of the supporting column adjacent to the selector switch assembly.

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*Loc. cit.

Connections from the bus structure to the oil circuit breakers are made at an elevation of 64 ft. (19.5 m.) and the center lines of the oil circuit breakers are located 98 ft. (29.87 m.) from the center of the bus structure. The two incoming 220-kv. lines and the bus tie breaker are on one side of the bus and the three transformer banks are on the opposite side.

The arrangement of a 220-kv. oil circuit breaker complete with by-pass and disconnecting switches and supporting structures is shown in Fig. 12. The arrangement is similar to that employed at other 220-kv. stations of the Pacific Gas and Electric Company, with exception of the use of a vertical break by-pass switch instead of the type P, or so-called "roller-skate" switch, which has been previously used. Inverted type P switches are used as disconnecting switches. On the switch towers which control the 220-kv. lines grounding switches are installed and mounted on the bases of the by-pass switches. Grounding is effected by closing the ground switch into a contact on the center post of the by-pass switch. Suitable interlocks are provided between the by-pass and disconnecting switches. All switches are actuated by means of rotary shafts and are hand operated. A galvanized structural steel tower is used to support the above switch and is 60 ft. (18.3 m.) high, 64 ft. (19.5 m.) long and 17 ft. (5.5 m.) wide. Within the tower enclosure a 6-break type, 600-ampere, 220-kv. oil circuit breaker is mounted with the poles of the breaker on 14 ft., 3 in. (4.3 m.) centers.

As mentioned above, provision has been made for three transformer banks and one spare transformer arranged for exchanging with any unit that must be taken out of service. These transformers are single-phase, auto-transformers, rated at 18,000 kva. and convert from 200,000 volts to 110,000 volts with a tertiary winding of 11,500 volts. Partial cooling is accomplished with radiators and additional cooling is obtained by means of hood type radiator air blast equipment. The units weigh approximately 200,000 lb. (90,720 kg.) each. The transformer output at 110 kv. is delivered to the 110-kv. bus and use is made of the 11-kv. winding for the operation of synchronous condensers. To facilitate the handling of transformers both for the initial installation and any subsequent maintenance work a permanent handling house has been constructed. This building is equipped with a traveling crane and all necessary tackle for untanking.

The 110-kv. bus is of the double-bus, single-breaker type and the switching arrangement is the same as that used on the 220-kv. bus. The buses and selector switches are mounted on a rigid steel framework with all live parts well isolated by elevation. The bus conductors consist of 2 in. (5.08 cm.) outside diameter, 14 gage (2.108 mm.) copper tubing and the bus and selector insulators are pillars made of two 20-in. (50.8 cm.) high, 4-part units per post. This bus is divided into two sections and provision is made for controlling 12 circuits in each section.

Due to the large concentration of power at Newark, particularly with the advent of power from the Mokelumne project, it was considered advisable to install current limiting reactors in one bus between the bus sections. Attention is called to the fact that this substation is essentially two stations, the dividing line being the reactor bank between the 110-kv. bus sections. One part is comprised of the older equipment consisting of several 110-kv. circuits, 60-kv. circuits, transformer banks and synchronous condensers, the second part consists of the new 220-kv. circuits, together with the associated 220/110-kv. transformers, 110-kv. circuits and synchronous condensers. The three current-limiting reactors mentioned above are rated as follows: 5,400-kva., single-phase, 60 cycles, outdoor-type, 110,000 volts, reactance 60 ohms, 300 amperes continuous capacity with an overload capacity of 1,060 amperes for 3 sec. Externally operated tap changers are provided. These are the largest oil-filled reactors ever built, the approximate weight of each is about 85,000 lb. (38,500 kg.).

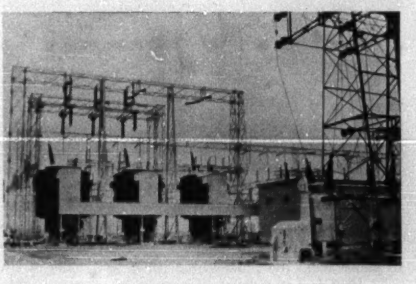

Fig. 14—Rear Elevation 220/110/11-Kv. Transformer Bank Newark Substation

Consistent with the dual station principle, it was necessary to make the switchboard in two sections, each section arranged to control its respective group of equipment. Due to the large number of circuits involved it was found necessary to use a benchboard and two vertical boards, one for the indicating instruments and the second for relays. In order to reduce the over-all length of the switchboard, two circuits per panel were found necessary, one circuit being above the other, this arrangement forcing the use of 100-in. (254 cm.) high panels in place of the conventional 90-in. (228.6 cm.) panels. The entire switchboard is constructed of ⅛-in. (3.17 mm.) stretcher-leveled steel.

The synchronous condenser equipment for the new station consists of two machines each rated as follows: 25,000 kva., 720 r. p. m., 11,500 volts, three-phase, 60-cycle. These machines are of the totally enclosed ventilated type equipped with surface air coolers. Pedestal bearings are provided with passage for cooling water in the bearing shells. Fire extinguisher manifolds are installed at the ends of the stator frame around the air-gap. Water at 25 lb. per sq. in. (1.76 kg/sq. cm.) will be used for fire fighting. Each condenser is provided with a main and sub-exciter and one machine is built with a 1,500-hp. synchronous driving motor. This latter device is required for line and relay testing. This feature is particularly applicable to this station where three main transformer banks are to be used, for with this grouping a bank can be released for build-up tests without great impairment of capacity.

The switching equipment used in connection with the operation of the condensers is of the metal enclosed type using truck type oil circuit breakers and motor-operated disconnecting switches. This equipment is housed in a small building adjacent to the 220-kv. transformers and underground lead-covered cable connections are made between the switching equipment and the condensers.

In unifying the existing and new control equipment in a new switchboard room considerable planning was necessary in order to arrange conduits in such a way that the transfer of control circuits could be made from the old station to the new and service maintained at all times. This was facilitated to a large extent by the use of a control circuit terminal room located directly beneath the switchboard room. All switchboard wires and control circuits are brought to terminal blocks and all cut-over work and testing procedure was readily executed.

Carrier current telephone equipment has been installed which will provide for communication over the 220-kv. lines to Tiger Creek power house and also by means of the 110-kv. circuits to Oakland, where a wire line extension is made to the Load Dispatcher's office. The coupling condensers on the 110-kv. and 220-kv. buses are so connected that the dispatcher may communicate directly with Tiger Creek power house without assistance from the Newark operator.

As pointed out above, Newark substation as it is now constituted is the result of many additions. Other stations previously chosen as transmission line centers and designed with the idea of future expansion have each in turn failed to develop their expected importance due to the shifting of load centers or to changes in transmission voltage. This station, however, not originally considered as likely to be greatly expanded, has grown step by step into the largest and most important transmission center on the system and it is worthy of comment that in spite of the many additions and the changing importance of its position, the flexibility of the unit structures used has allowed it to develop into a rational arrangement. Except for the old switchboard just replaced, every structure, whether installed in 1919 or in 1931 is functioning in its logical location as part of a modern substation.

In view of the fact that it is impossible to make accurate predictions as to the location of future load growth and the routing of transmission circuits to supply this load, it becomes necessary to design and construct major transmission substations in such a manner that they can be readily extended in any direction without the necessity of moving existing equipment or giving the appearance of a patched up job. Satisfactory operation requires a simple and easily followed arrangement of equipment. Proper thought given to the design of the original installation will permit the achievement of these aims even though the station grows far in excess of any preconceived ideas as to its probable future importance.

The Mokelumne River Development
of the Pacific Gas and Electric Company

BY E. A. CRELLIN*
Fellow, A. I. E. E.

***Synopsis.**—Studies of topography and run-off have made possible a comprehensive development of the power resources on the Mokelumne River in California. Originally developed in 1902 to yield 100 million kw-hr. annually, additional storage and increased generating facilities will produce a ten-fold increase in annual output. Two major storage reservoirs and four power houses, one of which replaces the original plant completed in 1902, are included in the program.*

The entire project is expected to be completed by the summer of 1935 at which time the total fall of more than 5,000 feet available in the Mokelumne River drainage area will have been put to the most economical usage.

EACH growing power system must from time to time add new generating capacity to care for increasing load demands. A new steam plant is added here, or a new hydro plant there, as will best meet the physical and economic requirements of the system. Usually these new plants feed into existing facilities which are reinforced to carry the additional burden. Not often does such an addition of generating equipment involve all of the features of a complete power system from the study of water storage and usage through generation and transmission to the step-down substation. It is this aspect of the Mokelumne River Development of the Pacific Gas and Electric Company which makes it of more than passing interest. This paper will treat of the hydroelectric features of the project while a companion paper will cover the transmission and substation design.

The Mokelumne River is one of the pioneer power streams of California. It has its source in Alpine County approximately twenty miles south of Lake Tahoe at an elevation of 9,700 ft. (2,950 m.) above sea level and flows in a general southwesterly direction for some 120 mi. (190 km.) where its waters join the San Joaquin River on their way to the Pacific Ocean through San Francisco Bay. The junction of the Mokelumne and San Joaquin Rivers is only a few feet above sea level and the lower 60 mi. (95 km.) of the river is quite flat lying at an elevation of less than 200 ft. (60 m.) above sea level.

The discovery of gold and its accompanying need for water for mining operations brought about the first developments on the Mokelumne River. In the fall of 1857 a company was formed which built canals and diversion works to deliver water to the miners. This company and its successors continued in the water business until the properties were acquired by the Standard Electric Company of California in the late nineties for the purpose of generating power for transmission to San Francisco, 110 miles (177 km.) away.

Electra power house and its associated 60-kv. transmission line were placed in service May 6th, 1902, one of the pioneer hydroelectric and high-voltage transmission projects of the world. The original installation of five 2,000-kw. units was increased by the addition of two 5,000-kw. generators in 1907 and the plant has been in continuous operation since it was first placed in service.

Water storage facilities on the upper reaches of the Mokelumne totaled approximately 25,000 acre-ft. (31,000,000 cu. m.) divided among several small reservoirs located at relatively high altitudes. This water, released as needed to augment the natural flow of the river, was diverted in the vicinity of the junction of Tiger Creek and the Mokelumne River into two canals each approximately 20 miles (32 km.) in length and with a combined capacity of some 200 cu. ft. per sec. (5.66 cu. m.) for delivery into the forebays above Electra power house. The annual output of power from the system was approximately 100 million kw-hr. The foregoing introductory remarks are intended to disclose the conditions existing at the time studies for the development of additional power on the Mokelumne River were begun.

That portion of the Mokelumne River which lies upstream from a point near its junction with Tiger Creek was relatively inaccessible and only occasionally visited by sportsmen who were obliged to carry their equipment on pack animals. As a result of this condition, actual water record data for the upper portion of the water shed of the river were not available. From 1906 to the present time, complete records of stream flow have been kept of readings taken at gaging stations located near the diversion into the conduit supplying water to Electra power house and beginning with the early spring of 1924 additional gaging stations were installed at strategic locations on the upper river and certain tributary streams.

The information obtained from these later stations was studied in connection with the long time record of the original stations and the run-off and storage possibilities analyzed for twenty-four consecutive seasons. Run-off characteristics for the upper and lower portions

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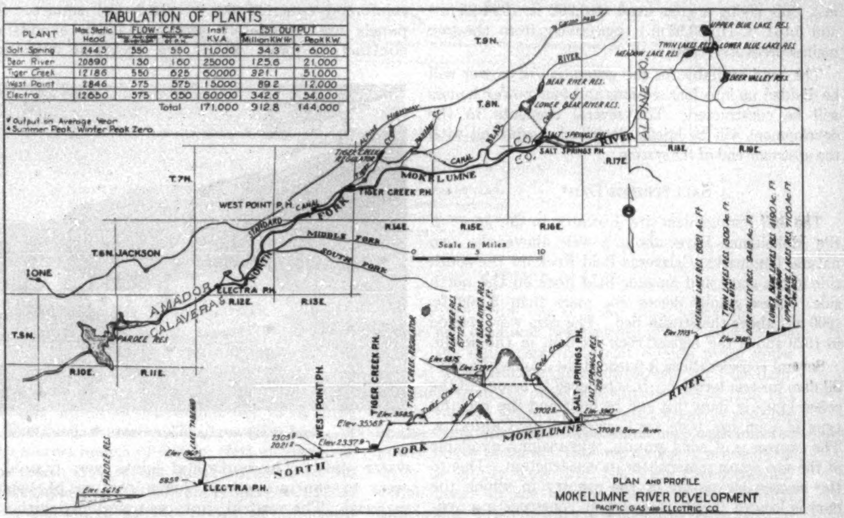
of the river vary during the different months of the year. During the winter and early spring the precipitation on most of the upper basin is in the form of snow, while that on the lower portion of the development is rain. This means that two groups of run-off curves must be studied, those developed from run-off due to melting snow in the late spring and early summer months, and those derived from the autumn and winter rains. This study of precipitation and run-off has involved a vast amount of detail work and the results derived have withstood the test of actual observations during the past two seasons when it was possible to compare the stream flow readings of the upper river and its tributaries as sent in by the gage readers in the field with the predicted results as obtained from the formulas derived from the studies of the lower river records. This check on calculated data has inspired considerable confidence in the accuracy of the assumptions made as to the run-off from the several component parts of the drainage area.

From the water supply studies and accurate topographic maps it was possible to determine the most advantageous storage sites and to plan a comprehensive development for the most economical utilization of the water. A tabulation was prepared to show the probable use of storage water in maintaining a more or less uniformly regulated flow at the forebays of the various plants. Certain assumptions were made regarding the method of operation based on a workable plan which would permit the use of storage facilities and physical features to good advantage in meeting the demand for energy. Calculations of energy output were then made covering the conditions of average rainfall and of the maximum and minimum twelve consecutive months of rainfall for the twenty-four seasons for which records were available. It was found that the average year would yield approximately 913 million kw-hr., with 1,038 million for a year of full supply and 489 million kw-hr. for the dry year period.

Consideration was given to the necessity of having peak output sufficient to meet system load conditions at all times, it being assumed that on dry years peaks would be met by hydro power with steam power carrying the base load. Economic studies showed that the Mokelumne River development required back up steam capacity to the order of about 35 per cent of the peak, the hydro capacity representing 65 per cent of the peak. Likewise consideration was given to having a maximum release of stored water during the summer months, in order to give the most benefit to irrigation interests on the lower reaches of the river, which was consistent with maintaining peak demand throughout the late fall and winter. When the project is entirely completed it will involve a total storage of some 200,000 acre-ft. (247,000,000 cu. m.) of water and the generation of a billion kw-hr. annually, a ten-fold increase over the original Electra development.



TABULATION OF PLANTS

PLANT	Max. Static Head	FLOW-C.F.S.		Inst. KVA	EST. OUTPUT	
		[illegible]	[illegible]		Million KW Hr	Peak KW
Salt Spring	2445	550	550	11,000	34.3	* 6000
Bear River	20890	130	160	25000	125.6	21,000
Tiger Creek	12186	550	625	60000	321.1	51,000
West Point	2846	575	575	15000	89.2	12,000
Electra	12650	575	650	60000	342.6	54,000
			Total	171,000	912.8	144,000

* Output on Average Year
* Summer Peak, Winter Peak Zero

UPPER BLUE LAKE RES.
TWIN LAKES RES.
LOWER BLUE LAKE RES.
MEADOW LAKE RES.
DEER VALLEY RES.
BEAR RIVER RES.
LOWER BEAR RIVER RES.
SALT SPRINGS RES.
SALT SPRINGS PH.
ALPINE CO.
RIVER
MOKELUMNE
TIGER CREEK P.H.
TIGER CREEK REGULATOR
WEST POINT P.H.
CANAL
STANDARD
NORTH
FORK
MIDDLE FORK
SOUTH FORK
ELECTRA PH.
AMADOR
CALAVERAS
PARDEE RES.
IONE
T.5N. JACKSON
T.5N.
T.6N.
T.7N.
T.8N.
T.9N.
R.10E.
R.11E.
R.12E.
R.13E.
R.14E.
R.15E.
R.16E.
R.17E.
R.18E.
R.19E.
Scale in Miles
0
5
10
Elev. 5675
Elev. 3585
Elev. 3556
Elev. 2337
Elev. 3942
Elev. 5875
Elev. 5797
3708 Bear River
3702
2305
2021
585
WEST POINT PH.
TIGER CREEK PH.
SALT SPRINGS PH.
ELECTRA P.H.
PLAN AND PROFILE
MOKELUMNE RIVER DEVELOPMENT
PACIFIC GAS AND ELECTRIC CO.

FIG. 1—PLAN AND PROFILE OF MOKELUMNE RIVER DEVELOPMENTS

Two major storage reservoirs, one known as Salt

Springs reservoir, and located on the Mokelumne River; the other known as Lower Bear River reservoir, and located on one of the principal tributaries of the upper Mokelumne are being created to impound the run-off waters from their respective drainage areas. The regulation of flow in the Mokelumne River made possible by these two storage reservoirs, together with the smaller reservoirs already in service is the principal factor contributing to the increased power output. The spillway elevation at Salt Springs dam is 3,947 ft. (1,203.04 m.) above sea level; that of the Bear River dam 5,816 ft. (1,772.72 m.). The center line elevation of the nozzles of the waterwheel units to be installed at Electra power house, which is the down stream end of the power development on the river, is 685 ft. (208.79 m.), thus giving a gross head of 3,262 ft. (994.25 m.) and 5,131 ft. (1,563.93 m.) respectively from the two main sources of storage.

The head available for the generation of power will be divided up into four sections and four power houses will be constructed. The several elements in the development will be briefly described, beginning with the upstream end of the system.

SALT SPRINGS DAM

The Salt Springs dam site is located in the gorge of the Mokelumne River about a mile above the two natural landmarks, Calaveras Bald Rock on the south side of the gorge and Amador Bald Rock on the north side. These granite domes rise more than 3,000 ft. (900 m.) above the stream bed. The dam was started in 1929 and is the highest rock fill dam in the world.

Several considerations influenced the choice of a rock fill dam for this location. It is believed to be one of the safest kinds of dam that can be built and the rock fill type of dam has been very successful in California. The canyon is of solid granite, thus making available at the site ample material for its construction. Due to the inaccessible nature of the country in which the dam is located it was necessary to construct approximately 50 miles of roadway suitable for heavy trucking and to haul all materials 50 mi. (80 km.) by motor transport. The high cost of concrete materials at such a site made the estimated cost of a rock fill dam approximately two million dollars less than for a suitable concrete dam. Modern and large scale construction equipment was used on the job with electrically-operated shovels and storage battery operated locomotives hauling rock cars.

Rock for the dam was obtained from quarries at different levels near the north abutment of the dam and from the spillway excavation on the south bank. Quarrying methods followed, in general, usual practise and featured the breaking down of large volumes of rock with large charges of dynamite. Several "big shots" were made each of which required in excess of 50 tons (45,000 kg.) of powder and brought down approximately 200,000 cu. yd. (150,000 cu. m.) of granite.

The dam rises 328 ft. (100 m.) in height above the stream bed with a crest length of 1,300 ft. (396 m.). It is 900 ft. (274 m.) through at the base and contains approximately 3,000,000 cu. yd. (2,300,000 cu. m.) of material, of which approximately 225,000 cu. yd. (172,000 cu. m.) are in the derrick placed rock section having a uniform thickness of 15 ft. (4.57 m.) over the entire area of the upstream face of the dam. This placed rock forms a foundation for the outer facing of reinforced concrete which covers the entire upstream slope, varying in thickness from 36 in. (91.44 cm.) at the bottom to 12 in. (30.48 cm.) at the top. Concrete was placed in continuously poured panels of heavily reinforced concrete 60 ft. (18.3 m.) square. These panels are completely separated with expansion or contraction joints which are provided with copper water seals. The horizontal joints were poured concrete to concrete except for a coating of asphaltic material. The vertical joints were provided with a 1-in. (2.54 cm.) spacing which was filled with asphaltic compounds to permit lateral movement.

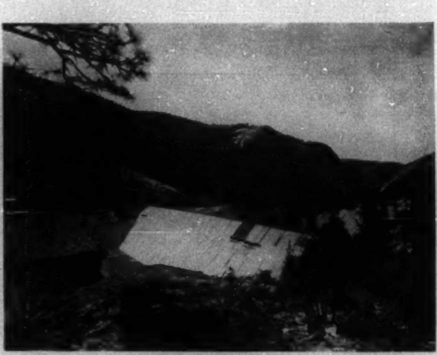
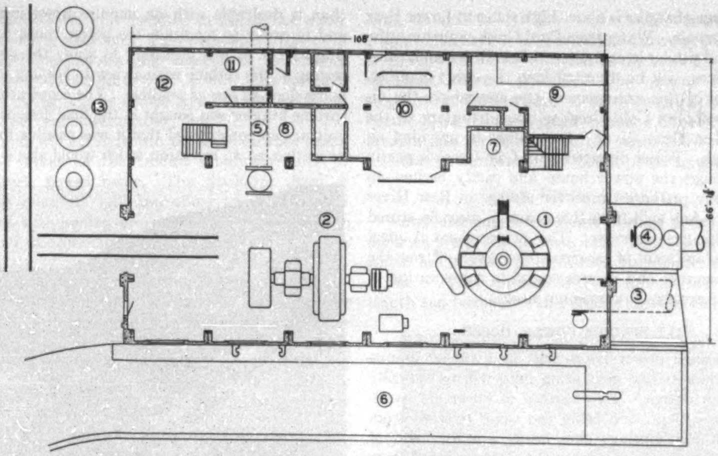


FIG. 2—UPSTREAM FACE OF SALT SPRINGS DAM

Due to the probable settlement of rock fill in a structure of this kind, the crest of the dam was crowned, the maximum crown being six to seven ft. (1.83 to 2.13 m.) above the theoretical crest line. In order to take care of the downstream settlement due to water pressure from the reservoir, the upstream face was designed and constructed with an upstream convexity in plan. For similar reasons and to prevent possible buckling of the concrete face, the face was concave to the water surface in vertical section. The reinforcing steel does not extend through the joints. These provisions were made in order to provide as flexible a facing as possible. The facing contains about 30,000 cu. yd. (23,000 cu. m.) of concrete and has an area of about 9.2 acres (37,200 sq. m.). It is anchored to con-



Main Floor Plan of Salt Springs Power House

1. Vertical shaft reaction-type generating unit
2. Horizontal shaft impulse-type generating unit
3. Penstock for No. 1
4. By-pass penstock
5. Penstock for No. 2
6. Tail-race conduit
7. Governor oil pump
8. Governor oil pump
9. 440-volt bus structure
10. Bench board
11. Air compressor
12. Machine shop
13. Single-phase transformers

crete cut-off walls extending vertically from 5 to 25 ft. (1.5 to 7.5 m.) into solid granite. Grout holes extend some 50 ft. (15 m.) below the bottom of the cut-off wall.

Salt Springs reservoir has a capacity of 130,000 acre-ft. (160,000,000 cu. m.) is 4 mi. (6.4 km.) long and has a maximum width of three-quarters of a mile (1.2 km.). The flooded area is 925 acres (375 ha.). A spillway with a crest length of 625 ft. (190 m.) is channeled in the south side of the gorge adjacent to the dam. It has a capacity of 48,000 cu. ft. per sec. (1,360 cu. m/sec.) or 300 cu. ft. per sec. per sq. mi. of drainage area (89 cu. m/sec. per sq. km.). The freeboard under this condition is 8 ft. (2.44 m.).

The outlet works are at the north end of the dam and consist of two 10-ft. (3.05 m.) diameter steel pipes inserted into the downstream end of a 19-ft. (5.79 m.) diameter concrete-lined tunnel built originally as a diversion tunnel to facilitate construction of the dam. Each pipe is provided with a 10-ft. 9-in. (3.28 m.) diameter hydraulic-operated butterfly valve and is then extended on into the river channel where it ends in a 78-in. (1.98 m.) diameter butterfly valve designed for free discharge. These valves are provided to permit unwatering the reservoir at the rate of approximately 10,000 acre-ft. (12,300,000 cu. m.) per day in case of emergency, and are designed for a maximum head of 275 ft. (83.8 m.) making them the highest head free discharge butterfly valves yet installed.

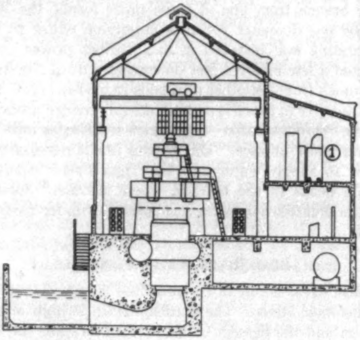
A branch from one of these pipes forms the 8-ft. (2.438 m.) diameter penstock supplying water to the generating unit installed in Salt Springs power house located a few hundred feet downstream from the dam. A branch from the other pipe ends in a 54-in. (1.37 m.) needle valve in which is incorporated an energy absorber of the "through baffle" type which discharges into the power house tailrace. This bypass line is necessary in order to supply water into the Tiger Creek conduit, which heads in the tailrace, when the Salt Springs unit is shut down or when it is operating under reduced head.

Bear River Water Supply

Bear River is one of the principal tributaries of the Mokelumne River. The drainage basin is high in elevation and the descent of the stream is rapid, the fall in the last 6 mi. (9.66 km.) above the junction with the Mokelumne being more than 2,500 ft. (762 m.). It is planned to construct a dam about 200 ft. (61 m.) in height in Bear River some 2.5 mi. (4 km.) below the existing dam which forms Upper Bear River reservoir. The new reservoir will have a capacity of 34,000 acre-ft. (42,000,000 cu. m.). A tunnel approximately 2.5 mi. (4 km.) in length will terminate in a penstock which leads to Salt Springs power house. The junction of the penstock and tunnel is formed in a surge chamber which will be excavated in the rock. The elevation of the top of this surge chamber is above high water in Lower Bear River reservoir. Water from Cold Creek, a stream adjacent to and similar in character to Bear River but lacking storage sites, will be diverted into this surge chamber. By means of this arrangement the area above the diversion point on Cold Creek is made tributary to the reservoir on Bear River. No storage is provided on Cold Creek. Water diverted from Cold Creek is partly used through the power house and partly backed up through the pressure tunnel for storage in Bear River reservoir. Any spill from these sources may be stored in Salt Springs reservoir. The arrangement is ideal from the standpoint of ease in using the run-off from the different sources and insures complete conservation of all water resources to maximum usage.

SALT SPRINGS POWER HOUSE

Salt Springs power house will be a rather unique plant in that its two generating units will be radically different in design. Two sources of water are available to this plant, one being the water released from storage in Salt Springs dam under a static head of 244.5 ft. (74.52 m.); the other the water released from storage in Lower Bear River reservoir, which will be under a static head of 2,089 ft. (636.73 m.). In the first case the head and water quantities are such as to require the installation of a Francis type turbine, while the latter can only be utilized through an impulse wheel. Many problems in plant design were occasioned by the different characteristics of these two prime movers. In keeping with the policy of the company, it was desired to secure a plant arrangement which would afford the greatest operating convenience in order to keep the personnel for plant operation at the minimum possible number of men. In general a vertical turbine with its draft tube setting requires a power house floor level considerably higher above tail water elevation than is desirable with an impulse wheel installation, and in order to maintain the power house floor on a single level, it was necessary to study the design and setting of the turbine with a view to keeping the power house floor as low as possible. The cooperation of the turbine builder was sought in the final designs and the unit was so constructed that it was possible to arrange its setting at an elevation which would also accommodate the impulse wheel. The high head unit will be installed at a future date following the completion of the dam and tunnel on Bear River but it was necessary to make considerable progress in the preliminary design of the impulse wheel unit in order to determine with some degree of accuracy what its probable dimensions would be. This also required cooperation with the waterwheel builders and preliminary designs were all based on the desire to secure a plant which would make for convenience in operation.



SECTION OF SALT SPRINGS POWER HOUSE

1. 11-kv. oil circuit breakers

FIG. 3—SALT SPRINGS POWER HOUSE AND DOWNSTREAM FACE OF SALT SPRINGS DAM

The Francis type turbine for utilizing the water released from Salt Springs dam is a vertical shaft unit designed to pass 550 cu. ft. per sec. (15.7 cu. m/s.) at heads as low as 175 ft. (53.34 m.) in order to accommodate the lowered level of water behind the dam as storage is withdrawn. In effect, this requires a turbine which must be operated at part gate under maximum head conditions with gate openings increasing as the head falls in order to maintain full load on the generator. At heads below 175 ft., it will be necessary to bypass water around the turbine into the tailrace in order to maintain the conduit flow at its full capacity. The turbine is rated at 13,500 hp., 300 r. p. m. It drives a vertical shaft waterwheel generator rated at 11,000 kva., 11,000 volts, 60 cycle, 85 per cent power factor. The generator is mounted on a concrete pedestal above the turbine and is totally enclosed with surface air coolers arranged at four points around the circumference of the generator. Contrasted to this will be the horizontal shaft unit installed to utilize the water from Bear River reservoir. This waterwheel will be a single overhung impulse unit capable of developing approximately 35,000 hp. at 300 r. p. m. The probable generator rating will be 30,000 kva., 11,000 volts, 60 cycles, 85 per cent power factor.

Both units will disharge into the same tailrace which forms the intake for the conduit supplying water to Tiger Creek power house. The discharge from a turbine is relatively still and offers no problems in connection with feeding into such a conduit. On the other hand, the discharge from an impulse wheel upon the rejection of load must be carefully handled if it is to be confined into the narrow limits of such a tailrace and designs have accordingly been laid out for the incorporation in the power house foundations of an energy absorbing device of the "through baffle" type. In this device the jet of water which has been deflected from the waterwheel runner by means of the governor actuated sleeve surrounding the nozzle tip impinges on a splitter which turns the water back on itself, thus destroying the energy of the jet and permitting it to flow outward freely into the tailrace. Such devices have been used in previous high-head installations on the Pacific Coast and have proven thoroughly reliable and successful.

The capacity of the Tiger Creek conduit is 550 cu. ft. per sec. (15.57 cu. m/s.), and it is desired to maintain this flow practically constant in order to supply water to Tiger Creek power house down stream. At full load the turbine unit will pass 550 cu. ft. per sec. and deliver power up to the 11,000-kva. rating of the generator. On the other hand, the impulse unit will pass approximately 180 cu. ft. per sec. (5.1 cu. m/s.) and deliver power up to the full rating of its generator which will be 30,000 kva. or slightly above. Thus it is seen that by properly adjusting the load between the two generating units in the plant it will be possible to maintain a constant flow in the conduit at plant outputs which will vary between the limits of 11,000 kva. or less, depending upon the height of water behind Salt Springs dam and 37,500 kva., as may best suit the load requirements of the system.

The low-tension switching arrangement in the plant is simple. Each generator is connected to a main bus through a single truck type oil circuit breaker. Two additional truck type breakers feed power from the main bus into the station power transformer bank. The two breakers supplying the station transformers are arranged so as to be interchangeable with the main generator breakers. While this required extra large breakers for the service demands the added cost was greatly offset by the simplicity of the arrangement and the possibility of taking any oil circuit breaker out of service for inspection or repairs without any loss in plant output.

From the main 11-kv. bus, leads are run direct to the step-up transformer bank consisting of three 12,000-kva., 11-kv. to 110-kv. water-cooled transformers. A spare transformer is provided and arranged to permit of its substitution for any unit in the bank by means of air-break selector switches. The output from the step-up transformer bank is fed through a 115-kv. oil circuit breaker into a single-circuit steel tower transmission line for delivery to the main 220-kv. step-up transformer banks at Tiger Creek power house.

TIGER CREEK CONDUIT

Water discharged from Salt Springs power house is diverted from the tailrace into Tiger Creek conduit which has a capacity of 550 cu. ft. per sec. (15.57 cu. m/sec.). This conduit is 17.3 mi. (27.84 km.) in length and terminates in a regulating reservoir on Tiger Creek. From the regulator a conduit of 625 cu. ft. per sec. (17.7 cu. m/sec.) capacity extends 2.6 mi. (4.18 km.) to a small forebay at the head of the penstock to Tiger Creek power house. The conduit is composed chiefly of reinforced concrete flume with a water section of 84 sq. ft. (7.8 sq. m.) and a water depth of 6 ft. (1.83 m.). The grade of the portion from the intake to the regulator is 0.8 ft. (24.38 cm.) per 1,000 ft. (304.8 m.) and the grade of the portion between the regulator and the forebay 1.0 ft. (30.48 cm.) per 1,000 ft. (304.8 m.). Part of the flume is elevated but in the main it is of the bench type. In addition to the flume there are two steel siphons, 93 in. (2.36 m.) in diameter totaling in length 0.23 mi. (370 m.), and 2.5 mi. (4 km.) of unlined tunnel with a section 10 ft.

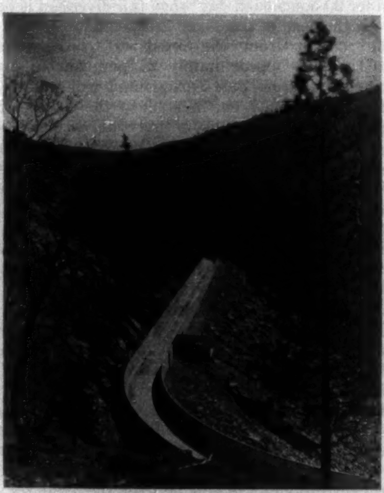


FIG. 4—A SECTION OF THE SALT SPRINGS-TIGER CREEK CONDUIT

(3.05 m.) wide by 11.3 ft. (3.44 m.) high on a grade of 3.5 ft. (1.07 m.) per 1,000 ft. (304.8 m.).

The topography of the country in the vicinity of the site chosen for the construction of Tiger Creek power house did not offer any suitable location for a forebay which would have sufficient capacity for the regulation of flow in the conduit and the carrying of short duration peaks and a compromise was made by the creation of a pond about 2.6 mi. (4.18 km.) up stream from the forebay site. This pond is known as the Tiger Creek regulator and is formed by a buttressed slab concrete dam approximately 100 ft. (30.48 m.) high on Tiger Creek. The upper 18 ft. (5.49 m.) of the reservoir thus formed provides about 175 acre-ft. (216,000 cu. m.) of capacity. This is sufficient to permit peaking the Tiger Creek plant for 20 hours each day when full diversion is being made at the head of the Tiger Creek conduit.

The conduit from the regulator to the forebay has a capacity approximately 25 per cent in excess of the conduit from Salt Springs and will deliver 625 cu. ft. per sec. (17.7 cu. m/s.) into the Tiger Creek forebay. This forebay is entirely an artificial pond and was created by excavating the top of a knoll on the ridge above Tiger Creek power house. About 200,000 cu. yd. (153,000 cu. m.) of material was moved to provide a forebay of 40 acre-ft. (49,500 cu. m.) capacity. About thirty minutes are required for water released at the regulator to reach the forebay and the available capacity in the forebay is sufficient to provide peaking during this interval.

Automatic gates are provided at the outlet of the regulator to maintain a constant flow in the conduit. These gates operate from a float control mechanism which will maintain a constant level of water in the conduit regardless of the height of water behind the gates discharging into the canal. The controls have been carried to the switchboard room in Tiger Creek power house and the operator may adjust the floats at the regulator to maintain any desired level of water and consequently any rate of flow between the regulator and forebay. The design of this automatic gate control equipment was perfected several years ago in connection with the control of water discharged from the afterbay at Pit River power house No. 3 where it was necessary to maintain the flow in Pit River below the power house at an amount equal to the natural flow in the river at that season of the year and at the same time permit peaking of the plant. No difficulty whatever has been experienced in the operation of this equipment and it has performed in every respect exactly as planned. Therefore, much confidence is placed in the ability of the operator at Tiger Creek power house to maintain complete and accurate control of the water being fed from the regulator into Tiger Creek forebay.

TIGER CREEK POWER HOUSE

From the forebay above Tiger Creek power house, water enters the penstock which consists of a single steel pipe 4,750 ft. (1,448 m.) in length varying in diameter from 102 in. (2.59 m.) at the top to 72 in. (1.83 m.) at the wye outside the power house. About 4,000 ft. (1,220 m.) of the upper portion of the pipe is of riveted steel with rivets countersunk on the inside of the pipe. The lower portion is of seamless pipe. At the wye the pipe branches into two 52-in. (132.08 cm.) diameter lines and again into four 36-in. (91.44 cm.) diameter lines, a separate line leading to each of the four impulse wheels of the two double overhung units in the power house.

Tiger Creek power house is the controlling station for the 220-kv. transmission lines constructed to receive the power generated by the waters of the Mokelumne River and transmit it to the load centers at Newark and Herndon. To charge the 109 mi. (175 km.) of 220-kv. line between Tiger Creek and Newark requires approximately 25,000 kva., which made it desirable that the minimum capacity for each generator be not less than this amount in order that a single unit could charge the transmission line. Experience has indicated that it is much better to be able to charge a transmission line with a single unit than to attempt this function with two units in parallel. The extremely low excitation of the generator at times of charging an unloaded transmission line renders the parallel operation of generators unstable and makes their control exceedingly difficult. For this reason plant operation is very greatly facilitated if a line which is taken out of service

FIG. 5—GENERAL VIEW OF TIGER CREEK POWER HOUSE AND MOKELUMNE RIVER

for any reason, can be charged from a single unit. Also it renders unnecessary the unloading of a second unit in order to make it available for line charging purposes. The static head and water quantities available at Tiger Creek indicated that this plant should have installed in it approximately 60,000 kva. in generator capacity and it was, therefore, at once evident that a two-unit layout was the proper one.

Each waterwheel unit was designed to deliver 36,000 hp. at 225 r. p. m. under a net effective head of 1,190 ft. (362.71 m.), each of the overhung impulse wheels being rated at 18,000 hp. The generators are rated 30,000 kva., 11,000 volts, 60 cycles, 85 per cent power factor. These waterwheel units are well up towards the maximum size practical for the head and water quantities available at Tiger Creek and in physical dimensions rank among the largest impulse wheels ever constructed.

Many refinements have been incorporated in the design of the wheels. A straight flow needle nozzle has been designed which introduces a minimum disturbance in the water flow, thus giving a uniform, compact jet delivering maximum power to the waterwheel buckets. The power needles are operated by the governor servo-motors and are arranged for slow closure upon load rejection as a water conservation measure in much the same manner that the relief valve closes on a large hydraulic turbine installation. Linked with the governor and needle operating mechanism is a jet deflector or stream bender in the form of a collar surrounding the jet which operates to remove water from the wheel instantly upon load rejection. The combination of jet deflector and slow closing needle is ideal for an impulse wheel installation in that it makes it possible to divert the water from the wheel quickly without disturbing the flow in the penstock with its attendant surge. This permits economies in the installation through reduced penstock cost and the omission of a surge chamber.

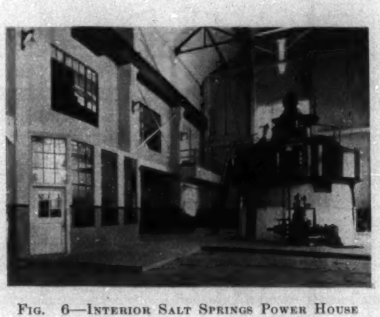


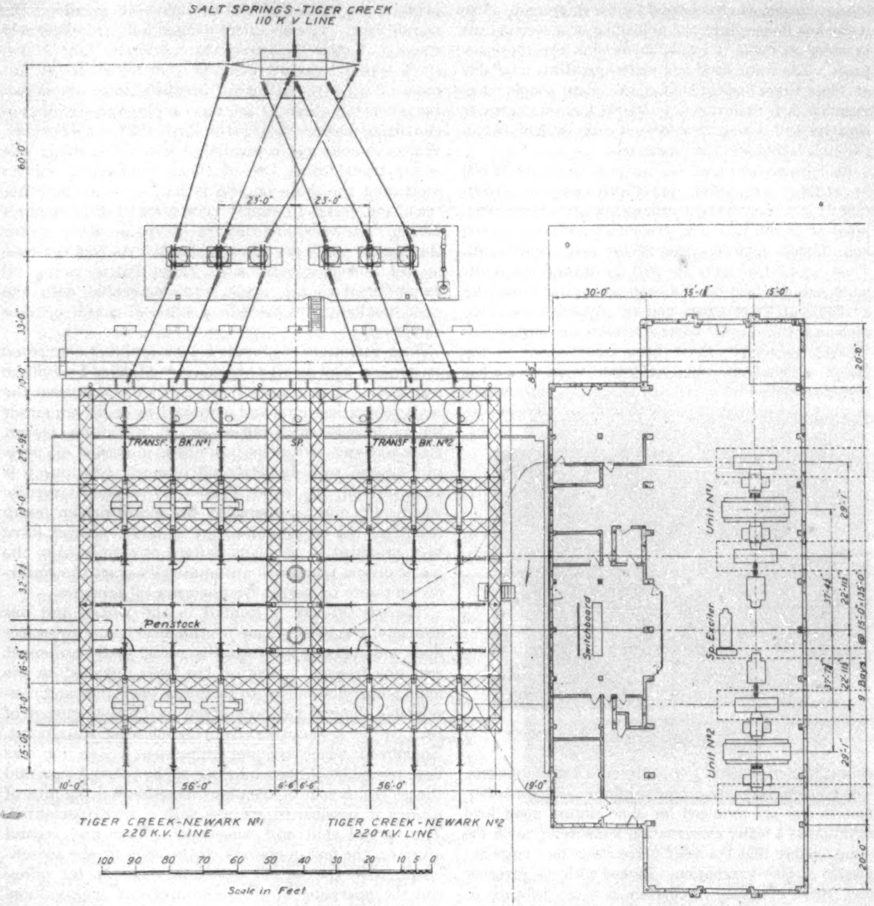
Fig. 6—Interior Salt Springs Power House

Simplicity of plant layout was the aim throughout the design of Tiger Creek power house. Each generator with its associated step-up transformer bank is considered as a unit and there is no oil circuit breaker between the generator and the low-voltage side of the transformer. The electrical connections are extremely simple. A steel bus structure containing four 11-kv. truck type oil circuit breakers is installed in the bus room. Two of the oil circuit breakers serve to connect the generators to an 11-kv. bus for the purpose of synchronizing before closing the main 220-kv. breakers. When the units are in parallel on the high-voltage side of the transformers one of these breakers is opened separating the units on the generator side, the other remaining closed to supply power for station requirements. The remaining two truck type breakers are for the station power transformer bank. As was the case at Salt Springs power house, these station power oil circuit breakers are made interchangeable with the main breakers, thus providing spare oil circuit breaker equipment.

Each generator is provided with a direct-connected exciter and pilot exciter which are controlled by voltage regulators. A spare exciter set is installed between the main generating units and arranged to be driven either by an impulse waterwheel or an induction motor. Each waterwheel has its individual governor oil pressure system with duplicate oil pumps. One pump is motor-driven, the other driven by a small impulse wheel. For normal operation the motor-driven pump maintains the oil pressure, but suitable controls have been provided so that any failure of supply from the motor-driven pump will automatically start the water-driven pump to reestablish governor oil pressure.

The switchboard is located in the switch and bus section of the power house on the level of the generator floor and midway between the units. All equipment requiring frequent attention has been installed on the main power house floor to facilitate operation and considerable thought has been given to the convenience of the location of equipment from an operating standpoint. Equipment which requires infrequent inspection has been placed on the second floor of the power house and the general layout of the plant has followed the idea of making it possible to operate with a small personnel. The normal shift will consist of a first and second operator, the first operator's duty being at the switchboard, with the second operator available for oiling and the operation of the hand-operated high-voltage disconnecting switches in the outdoor switch yard.

The step-up transformers consist of two main banks with one spare unit arranged for physical interchange in case of damage to an operating unit. These transformers are provided with combined two-winding and auto-transformer windings. The auto-transformer winding has a rating of 10,000 kva. per unit and will step the incoming power from Salt Springs power house from 107.5 to 215 kv. The tertiary winding has a rating of 10,000 kva. per unit, giving 30,000 kva. per bank, sufficient to care for one Tiger Creek generator. The transformer banks, therefore, will have a capacity of 60,000 kva. each, 30,000 kva. for one Tiger Creek


SALT SPRINGS-TIGER CREEK
110 K V LINE
60'-0"
23'-0"
23'-0"
33'-0"
10'-0"
27'-9½"
13'-0"
32'-7¼"
16'-5½"
13'-0"
15'-0½"
TRANSF. BK. Nº1
SP.
TRANSF. BK. Nº2
Penstock
10'-0"
56'-0"
6'-6"
6'-6"
56'-0"
19'-0"
8'-5"
TIGER CREEK-NEWARK Nº1
220 K.V. LINE
TIGER CREEK-NEWARK Nº2
220 K.V. LINE
100 90 80 70 60 50 40 30 20 10 5 0
Scale in Feet
30'-0"
35'-1⅛"
15'-0"
20'-0"
Unit Nº1
29'-1"
37'-4½"
22'-11⅛"
Switchboard
Sp. Exciter
9 Bays @ 15'-0"=135'-0"
37'-7½"
22'-11⅛"
Unit Nº2
29'-1"
20'-0"

Plan of Tiger Creek Power House and Bus Structure

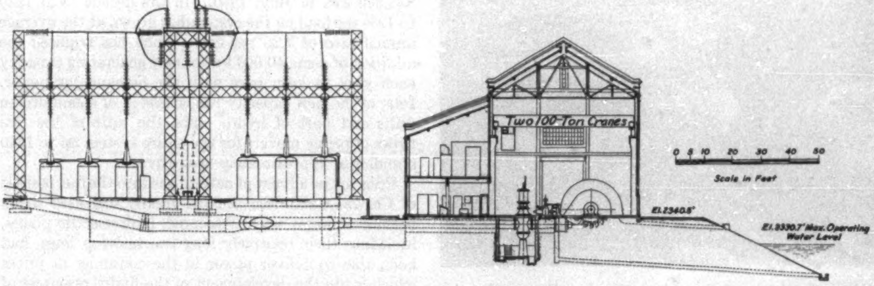
unit and 30,000 kva. from Salt Springs. Each bank is capable of carrying the normal Salt Springs output as well as the output from one Tiger Creek unit.

In order to re-regulate the water discharged from Tiger Creek plant for diversion to the next lower plant an afterbay has been provided. This was formed by the construction of a concrete arch dam about 100 ft. (30.48 m.) in height in the Mokelumne River, some 2 mi. (3.2 km.) below the plant. This dam creates a reservoir with a flooded area of about 100 acres (400,000 sq. m.). The top 3 ft. (91 cm.) of this pond, with a capacity of 300 acre-ft. (370,000 cu. m.) will be used to re-regulate the Tiger Creek peaks as well as some of the irregularities in the natural run-off tributary to the lower plant which are not controlled by the storage on the system.

West Point Power House

The last of the four power houses to be constructed on the Mokelumne River, but the one immediately downstream from Tiger Creek will be known as West Point power house. The existence of the present Electra power house has influenced the location and construction of this plant and it will be necessary to make brief mention of the factors involved.

At the present time Electra is supplied with water through two conduits known as the Upper Standard Canal and the Lower Standard Canal. The lower conduit was constructed during the early mining days and delivers water into Lake Tabeaud which forms the forebay for Electra power house. At a later date the upper conduit was constructed, some 200 ft. (61 m.) higher in elevation than the lower conduit and terminating in Petty reservoir, an enlarged section of the conduit forming a second forebay at Electra with 200 ft. (61 m.) extra head. The combined capacity of these conduits is approximately 210 cu. ft. per sec. (5.95 cu. m/sec.). Both have grades of 2.5 ft. (76.2 cm.) per 1,000 ft. (304.8 m.) for flumes and 1.5 ft. (45.7 cm.) per 1,000 ft. (304.8 m.) for ditch sections which are excessive for the enlarged capacity now contemplated. The construction of a new conduit above the existing conduits has several disadvantages, the most important of which are the lack of a forebay site at the Electra end and the impossibility of keeping the conduits free from debris while excavating a new location immediately above. The plan finally adopted for the development of this head was to build a new conduit below the two existing canals beginning at the high water level of Tabeaud reservoir and rising on a grade of 0.8 ft. (24.38 cm.) per 1,000 ft. (304.8 m.) until the grade line intersects the river gradient. This new conduit will be about 13.7 mi. (22 km.) long, intersecting the river in the vicinity of the West Point road crossing and about 4 mi. (6.4 km.) below Tiger Creek afterbay dam. It will have a capacity of 575 cu. ft. per sec. (16.28 cu. m/sec.). An advantage of this plan is that the existing canals and power house may be operated without interference during the period of construction of the conduit. At the time construction is started on West Point, the new Electra will have been completed thus diverting water from the river into the new conduit and freeing the Upper Standard Canal for construction work.



Section Through Tiger Creek Power House Parallel to Penstock

The diversion of water for West Point will be made from Tiger Creek afterbay about 3 ft. (91 cm.) from the top of the dam and will follow roughly the location of the present Upper Standard Canal for about 4 mi. (6.4 km.) where it will join the West Point penstock. No forebay facilities being available, the plant will be designed to operate at 100 per cent load factor. The static head will be about 285 ft. (86.87 m.) and the maximum flow 575 cu. ft. per sec. (16.28 cu. m/sec.). The installation will consist of a single vertical turbine driving a 15,000-kva., 11,000-volt generator and the plant will be laid out for full automatic operation with certain controls carried back to Tiger Creek power house. The generated power will be stepped up to 60 kv. to supply the demands now served by the present Electra plant. West Point power house will be the only plant on the Mokelumne System not regularly feeding into the 220-kv. transmission system. Provision will be made at Electra however to absorb this power into the 220-kv. system at such times as there is available sufficient energy at 60-kv. from another source to make the output from West Point in excess of requirements. A 60-kv. wood pole line will be constructed from West Point to Electra to tie in with the existing 60-kv. lines.

ELECTRA POWER HOUSE

Construction of the new Electra power house and conduit will follow immediately upon the completion of the Salt Springs-Tiger Creek development, probably in the fall of 1931. The static head to be developed at Electra is 1,265 ft. (385.57 m.) and the water flow 575 cu. ft. per sec. (16.28 cu. m/sec.). The head and water quantity are so nearly the same as at Tiger Creek that the plant will be almost an exact duplicate. The two waterwheels will have a slightly greater horsepower rating being designed for 37,500 hp. each at 225 r. p. m. but will have almost identical physical dimensions which will permit the construction of a duplicate power house. One factor is present in the Electra designs which was absent at Tiger Creek and that is the presence of an excellent forebay in Lake Tabeaud with a capacity of 1,158 acre-ft. (1,428,000 cu. m.). This will permit of considerable peaking at Electra and all designs are being made so that a third unit for peaking purposes may be added at a later date if economies and system load requirements point to the desirability of its installation.

Power generated at Electra will be stepped up through two banks of transformers to 220-kv. for connection to the Tiger Creek-Newark transmission lines. It will also be necessary to provide for supplying power to the 60-kv. and 17-kv. systems now taking the output from the present plant. As pointed out above, this demand will normally be met by West Point power house.

FIG. 7—GENERAL VIEW SALT SPRINGS POWER HOUSE

Showing penstock (one for turbine and one for by-pass) tail canal and Tiger Creek conduit main quarry (upper right) for dam material and gorge of the Mokelumne River

The old Electra power house will be continued in operation during the construction of the new power house and upon completion of the new plant will be completely demolished. There is a remarkable contrast between the thirty-year old pioneer plant and its modern successor and brief mention of the old plant will be of interest.

The power house is built of corrugated iron on a steel frame and houses seven generating units, five 2,000-kw. Stanley inductor type generators and two 5,000-kw. Stanley revolving field generators. Excitation is at 70 volts and the generator voltage is 2,300, three-phase, 60 cycles. The five 2,000-kw. units are set quite close together with their shafts at an angle of 45 deg. with the main axis of the generator room. The 5,000-kw. units are installed with their shafts parallel to the long axis of the room. Power is stepped up through six main banks of Stanley single-phase transformers, five banks feeding into the 60-kv. transmission system and the sixth bank furnishing 17-kv. power to the gold mines along the Mother Lode.

The 20,000-kva. installation, which at one time was one of the important hydroelectric plants of the country, is now completely obsolete and will be scrapped in its entirety. In its place will stand a modern steel frame, reinforced concrete building housing two generating units each of which is capable of delivering one and one-half times as much power as the combined output of the seven replaced units. The transmission voltage has risen from the 60-kv. record of 1902 to the 220-kv. commonplace of thirty years later giving marked evidence of the rapid changes in the art.

GENERAL

The peak load on the system of the Pacific Gas and Electric Company and associated companies reached 853,300 kw. in July, 1930. In the decade from 1920 to 1930 the load on the system has grown at the average annual rate of 7.25 per cent, which has required the addition of some 40,000 kw. of new generating capacity each year to keep pace with the demand for power. Part of the new capacity has consisted of steam-driven units and part of hydro units, the ratio of the two types of prime movers for the entire system up to 1930 standing at approximately two to five.

Prior to the advent of natural gas into the fuel market of Central California, and the greatly increased steam plant economies recently secured, hydroelectric plants, including their relatively long transmission lines, had been able to deliver power to the consumer at prices which made the development of the hydro resources of the state very attractive from an economic standpoint. The picture has been altered somewhat and natural gas has now placed the cost of power from the two sources approximately on a par. A relatively small change either way in the cost of fuel may have a very large influence on the type of future generating equipment installed.

The power peak growth of the system for the next five years will approximate 50,000 kw. annually. This is favorable to hydroelectric development in that it permits full development of a river over a short period of time, thus spreading the cost of water storage facilities over all of the plants on the stream and reducing the fixed charges per kw. hr. generated. In order to produce hydroelectric power on the most economical basis, hydro plants in California must operate on the highest possible load factor. The average annual load factor for the system of the Pacific Gas and Electric Company is approximately 62 per cent and all studies for the plants on the Mokelumne River, with the exception of the 100 per cent load factor plant at West Point, were on the basis of 85 per cent load factor.

Such a condition of operation will be possible in all years with normal or above normal rainfall. Studies of run-off over a 24-year period have indicated that the average annual output will be 88 per cent of the maximum possible during the wettest year recorded with a 47 per cent output during the extreme dry year. Normal operation of the plants on the Mokelumne River will be on base load with system peaks carried by steam. This condition may be reversed in extreme dry years when the economic use of stored water dictates its use for the carrying of peak loads.

The total cost of the Mokelumne River Development as outlined herein, including transmission and the increased capacity at Newark Substation, is to the order of $39,000,000.00. The entire design and construction of the project have been carried out by the regular personnel of the Pacific Gas and Electric Company.

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Tie-Line Control of Interconnected Networks

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Non-member — Member, A. I. E. E.

Synopsis.—*The paper describes operating experience with tie-line load regulators on the tie connecting the Colfax Power Station of the Duquesne Light Co. and the Springdale Station of the West Penn Power Co. A description is given of the equipment used and also of supplementary equipment, such as program loading equipment installed at Colfax to maintain the most economic division of load between the various generating units. A new hydraulic speed changing device intended to reduce wear on the governor parts to a minimum is also described. Tests made jointly by the Duquesne Light and West Penn Power Companies to determine the effect of operating frequency and tie-line regulators simultaneously are discussed and the curves giving the results are shown.*

INTRODUCTION

THE electrical industry has been successful in continually rendering a more reliable service at a lower cost to its customers. To do this it has been necessary to decrease both capital expenditures and operating cost per unit of system capacity and at the same time provide power systems less susceptible to service interruptions. One thing which has contributed in a great measure to this progress has been the connecting together of power systems in such a manner as to make available to the several systems embracing large areas the economies resulting from the better use factor of both generating and transmission plants, the operation of fewer of the older and less efficient equipments, and the diversity of peak loads. A casual survey is sufficient to reveal that this so-called interconnection is still relatively new and that the immediate years to come will witness a most extraordinary development in this phase of the industry.

Utility officials recognizing the economies resulting from exchange of power from system to system are arranging cooperative or joint operation in a manner to leave no obstacles in the way of the operating man to the end that the greatest number of kilowatt-hours may be produced and delivered to the customer at the lowest cost and with the greatest reliability. A few years ago an interconnected power system embracing more than a few thousand square miles was the exception, whereas today for instance, the power systems between Boston, Chicago, and New Orleans may be formed, when occasion arises for exchange of power, into one continuous interconnected system.

This parallel or interconnected operation of several power systems has brought to the fore a new and most important problem, namely that of controlling the interchange of power between the several systems. This problem first came into prominence under the name of frequency control and many engineers attacked it from that point of view. As a matter of fact it is one of load control and little has been done towards its solution. The division of power in a loop under the control of phase shifting equipment is a related problem but should not be confused with the control of total interchange by the prime mover governors. An attempt to regulate two or more interconnected systems with an automatic frequency regulator on each will promptly develop the fact that the load variations on the systems will be so dissimilar that regardless of frequency control, the power flow over the tie lines cannot be regulated. In fact it has been observed in some cases that when the bulk of the load change falls on one system, the power flow over the tie line exceeds the load change by the amount required to accelerate or retard the systems.

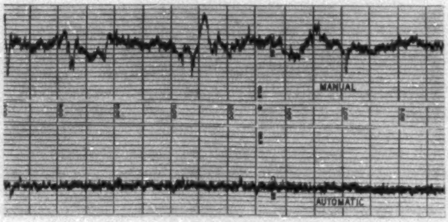

FIG. 1—MANUAL AND AUTOMATIC FREQUENCY CONTROL CHARTS AT COLFAX. (SYSTEMS NOT INTERCONNECTED)

The Duquesne Light Company, having recognized several years ago that the problem was one of load control rather than one of frequency control, enlisted the cooperation of several manufacturers in an experiment with automatic control of the tie line connecting the Duquesne Light Company having a normal capacity of 360,000 kw. with the West Penn Power Company whose interconnections form a network of more than 4,000,000 kw. capacity. The tie which is between the Colfax Station and the Springdale Station of the West Penn Power Company is rated 36,000 kva. at 132 kv.

In addition to the control of power over the tie line it has also been desirable to maintain frequency within a very close range. The control of frequency in itself has been found possible through the use of automatic equipment, and is a comparatively simple problem.

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Presented at the Pacific Coast Convention of the A. I. E. E., Lake Tahoe, California, August 25-28, 1931.

Fig. 1 illustrates system frequency with and without automatic control.

The maximum economy possible as a result of interconnection will not be obtained unless the most efficient loadings of the several generating units on the interconnected system are closely adhered to. This loading must be anticipated and must take account of the distribution of the load, the inherent efficiency of the generating station and units, and numerous other factors. This economy cannot be obtained with manual control, and to obtain the greatest advantage the control must be automatic. Such control must be simple, reliable, and accurate and must be susceptible to being readily set for any division of load between the generating units which may prove advantageous.

Automatic load control over the tie line and automatic control of the output of the generating units in accordance with a predetermined program, both being supervised by or subservient to automatic frequency control, have been in successful operation at the Colfax Power Station for some time. This equipment has been developed as a result of considerable research work both by the Duquesne Light Company and two manufacturers whose equipment is in use. While no doubt many improvements are yet to be made, the operation exceeds expectations held at the time the work was started some two years ago.

Performance of a tie line connecting two large power systems involves the rate of load change, inertia, and sensitivity of the prime mover governors, along with the electrical characteristics, such as amount of reactance in the tie lines connecting the two systems. A load change on one system causes a redistribution of the load between the various prime movers, and also a transient distribution which occurs during the period of adjustment to the new load. This transient condition is not noticeable in a closely tied system such as the average metropolitan utility. However, if there is only one tie line connecting two systems, these load transients are concentrated and assume considerable importance. The majority of load changes occurring on the West Penn tie line connecting the Colfax and Springdale Stations are of a transient nature of rather short duration.

The prime requirements of a successful tie line load regulator are that it must not respond to these small transient load changes, but it must respond to small sustained changes. In addition it must respond to large load changes whether temporary or sustained.

Tie-Line Load Regulators

The Westinghouse Company has developed two types of control equipment. The first type was devised primarily to eliminate corrective action due to transient changes, but to respond rapidly to load changes of reasonable magnitude. This type of load control shown schematically in Fig. 2 consists essentially of two solenoids responding to watts. These coils are connected in *A* and *C* phases and are shown as *C* and *D*. They are arranged to assist each other in balancing the arm *E* which has a movable weight *F* riding on it. The action of the regulator is as follows:

With the weight *F* immediately above the fulcrum, the arm *E* is balanced and requires zero watts in the coils to maintain a balance. Any power flow through the coils will cause the arm *E* to "raise" or "lower" depending upon the direction of the power. For small changes in load the contact *J* engages either the raise or lower stationary contacts *R* or *L*. These contacts send an impulse to the governor motor through time delay relays, which are adjusted to delay the impulse approximately 2½ seconds after the contacts have closed. Experience has shown that on the West Penn tie line the periods of the transient load swings have a shorter period than two seconds. An oscillation occurring on a tie line will cause an unbalance and close the "raise" or "lower" contacts. If the period of contact engagement exceeds the 2½-second time delay a correction is made accordingly. If the period of the contact engagement is less than 2½ seconds no correction results.

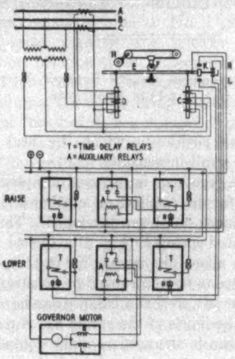


Fig. 2—Westinghouse Solenoid Type Automatic Tie-Line Regulator

A second set of time delay relays is provided to limit the length of impulse delivered to the governors to a predetermined value.

The part of the regulator described above takes care of the first requirement very satisfactorily. To take care of large changes in load, a second set of contacts *K* is provided which operates the speed changer motor directly through the auxiliary relays without time delay. The contacts *K* are set to close for approximately 10,000 kw. change in load, but hunting of the regulator in this zone is prevented by the opening of these contacts at ± 5,000 kw. Below this value the auxiliary contacts *J* are relied upon to return gradually the line load to normal.

This device has shown considerable promise in that it does not cause corrections to be made which must immediately be counteracted by reverse corrections.

A large change of turbine loading to counteract load in a tie line results in considerable energy being supplied to the system to accelerate it while changing the phase angle relation across the tie line. It can be seen that after the phase angle is changed this energy is still available to supply load, which results in an overshooting of the proper value. It is essential that any regulator responding at a rapid rate must have a wide zone about the final position in which the load corrections are made in small increments.

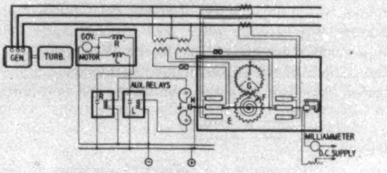


FIG. 3—WESTINGHOUSE KELVIN BALANCE TYPE AUTOMATIC TIE-LINE REGULATOR

The above regulator is not well adapted to remote control, and for this application another design utilizing a Kelvin balance has been developed. This regulator has a calibrated spring *F* (Fig. 3) attached to a geared disk *G* which allows the torque due to the power in the Kelvin balance to be neutralized by rotating the disk *G* to the right or left, so that a balance can be made at any load within the limits of the spring *F*. A motor-driven cam mechanism is arranged to make contact intermittently with the contacts *H* and check the position of the Kelvin balance *E*. Variations in load cause the Kelvin balance to deflect the contact *H* which through suitable mechanism makes contact with the cam *J* thus routing the "raise" and "lower" impulses to the governor motor.

It can be seen that as the unbalance decreases the period of contact engagement with the cams decreases and the duration of the impulse to the governor motor also decreases thus providing an anti-hunting feature. Disk *G* can be rotated by a position transmitter, or a D'Arsonval element can be attached to the lever *E* to counteract the torque in the Kelvin balance, making the device readily adapted to remote control.

No time delay relays are used with this device. It will cause more corrective impulses to be delivered to the governor motors than the device first described and it is somewhat slower in correcting large errors due to rapid changes of load. It will, however, maintain a closer integrated load under average conditions.

The solution of the problem of tie line control offered by the Leeds and Northrup Company consists essentially of a master element comprising a thermal converter and a load setter. The thermal converter is similar to the Lincoln thermal demand meter, except that instead of using a bi-metal strip to indicate the kilowatt load, thermocouples are inserted which produce a potential for transmission to suitable receiving apparatus.

The thermal converter is indicated at *C*, Fig. 4. The load setter is a potentiometer with a calibrated dial, and is adjusted to give a potential equal to the thermocouple potential of the thermal converter for a given tie line load. These two potentials are connected in opposition through a sensitive galvanometer in the load controller. This galvanometer is similar to the Leeds & Northrup Company temperature recorder, and deflects in proportion to the difference in potential between the load setter and the thermocouples of the thermal converter.

The unbalance indicated by the galvanometer is checked by the cam mechanism shown at *F*, which delivers a corrective impulse to the turbine governors in proportion to the load error. This device tends to change the turbine loads until the tie line load flowing through the thermal converter sets up a potential equal to that of the load setter. This control gives very satisfactory operation.

PROGRAM LOAD CONTROL

In the past where frequency control has been adopted, it has been common practise to do all the regulating in one station and usually on one unit. Where the regulating device was put on more than one unit, it was found that the division of load between units was liable to be erratic. Therefore load balancing schemes were devised to force the various units to take their proper share of load increments.

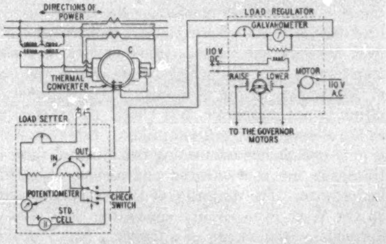


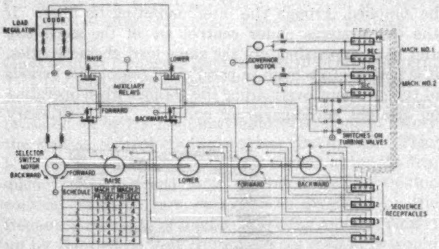
FIG. 4—LEEDS AND NORTHRUP POTENTIOMETER TYPE AUTOMATIC TIE-LINE REGULATOR

These devices are relatively simple and perform satisfactorily, but the principle of dividing up all load increments proportionally between units in a steam station does not necessarily make for maximum efficiency. Most operators have considered the output of the machine and have not taken into account the inefficiency due to operating with steam control valves in a throttling position. In the equipment developed by the Westinghouse Company in accordance with the requirements established by the Duquesne Light operators, load scheduling is based on turbine valve positions, rather than on generator output, thus maintaining each turbine at its most efficient valve opening regardless of the effect of the many variables which enter into the position of the valve with reference to the load.

Fig. 5 shows a schematic diagram of the equipment used to obtain automatic division of load in accordance with the predetermined program loading at the Colfax Power Station. The capacities of the generating units in this station are as follows:

Unit No. 1	60,000 kw.
Unit No. 2	60,000 kw.
Unit No. 3*A*	30,000 kw.
Unit No. 3*B*	30,000 kw.
Unit No. 4*A*	41,250 kw.
Unit No. 4*B*	41,250 kw.

On the larger steam turbines the nozzles are usually grouped in three groups, each fed from the primary, secondary, and tertiary valves. These valves are equipped with auxiliary switches to establish the proper sequence of loading. Referring to Fig. 5, the motor-operated selector switch routes the "raise" or "lower" impulses from the tie-line load regulator through the load scheduling device to the turbine governors. In order to simplify the description and diagram as much as possible only two machines have been shown each with a primary and secondary valve.



SCHEDULE	MACH. 1		MACH. 2	
	PR.	SEC.	PR.	SEC.
1	1	2	3	4
2	1	3	2	4
3	3	4	1	2
4	2	4	1	3
5	1	4	2	3
6	2	3	1	4

Fig. 5—Westinghouse Program Load Controller

There are six loading schedules as shown in the table and in order to set up any one of these schedules, a system of receptacles and plugs is used. There are four machine receptacles and four sequence receptacles. The selector switch has four independent contact arms, two of which are used for routing the raise or lower impulses and two for the control of the selector switch driving motor. The position of the selector switch is determined by the position of the valve auxiliary switches and also by the way in which the machine and sequence receptacles are connected by jumpers. For example consider that machine No. 1 primary receptacle is connected to sequence receptacle No. 1, No. 1 secondary to sequence 2, No. 2 primary to sequence 3 and No. 2 secondary to sequence 4 receptacles. The operation would then be as follows with both machines carrying minimum load and the selector switch in position No. 1: On increasing the load, the load regulator makes the raise contact and the raising impulse is routed through to No. 1 governor motor. After the load has increased to the point where the primary valve is wide open, the primary valve auxiliary switch makes contact which energizes the forward control of the selector switch driving motor on the next raising impulse. The selector switch rotates in the forward direction until cut off by the forward contact breaking the circuit with the first contact. The width of the forward and backward contacts are such that the motor will drift to the next position after the control circuit is broken. The switch is now in position No. 2 and the raising or lowering impulses are still routed to No. 1 governor motor. In the same manner, the selector switch transfers to the next position after the secondary valve on machine No. 1 fully opens. On decreasing load, the selector switch returns, the motor being energized by a circuit through the "backward" selector switch arm and the valve switch contacts which make when the valves are closed. The forward and backward auxiliary relays seal in so that the motor-selector switch travels at least one position and also breaks the control circuit from the regulator so that control is broken during the transfer.

Fig. 6 is a rear view of the program load control panel, showing the 32 receptacles and jacks. One row of receptacles represents the 16 valves on the six turbo-generators and the other row represents sequence positions, these being numbered 1 to 16. By inserting the jacks in the proper receptacles any desired sequence of load scheduling can be obtained. The motor-operated selector switch is at the right. The master raise and lower contactors are at the top of this panel. On the front of the panels are mounted the relays and necessary control and transfer switches.

Hydraulic Speed Changer

Changing the speed or load on a turbine controlled by the usual flyball governor is accomplished by increasing or decreasing the tension on the speed changer spring. The tension on this spring is approximately 1,000 lb. with the governor in full load position and the standard mechanical speed changer has heretofore carried this load under normal operating conditions without appreciable wear.

The introduction of frequency control, however, which called upon the speed changer to respond as often as 30 impulses in a minute, made it necessary to remove the spring load from the speed changer parts in order to prevent excessive wear, and for this purpose a hydraulic speed changer was developed and has been in successful operation for some time.

The hydraulic speed changer shown in Fig. 7 consists of an oil-operated piston, *A*, carrying the load of the spring, *B*, a relay *C* within the piston to control its position, a motor-operated speed changer mechanism *D* for adjusting the oil relay, and mechanism, *E*, to control the governor position by hand in case of failure of the oil pressure.

The relay controls the position of the operating piston by means of four discharge ports. On the up stroke of the relay, oil supplied through an orifice to the upper face of the piston is discharged through the piston to the drain. On the down stroke of the relay the discharge ports are closed, the oil pressure builds up on top of the piston which then follows the movement of the relay. The relay travel in the operating piston is restricted to 1/8 in. in either direction so that in case of oil-pressure failure, the spring load can be taken by the relay itself with only 1/8 in. change in the spring length.

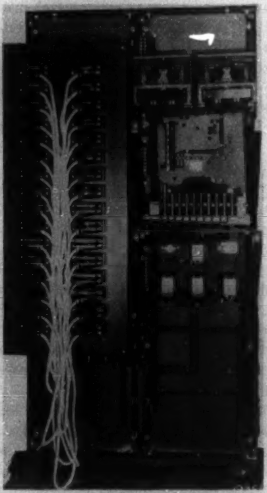


Fig. 6—Rear View of Program Load Controller Showing Plug Receptacles for Arranging Loading Sequence

The relay is moved up and down by a rotating nut actuated either by the speed-changer motor or by the handwheel and is prevented from turning by a key.

The motor-mechanism worm gear is locked to the nut by two long sliding pins, so that the worm wheel housing and nut rotate as a unit. For hand operation the hand wheel is lowered, disengaging the sliding pins, and permitting the handwheel to operate the oil relay independently of the motor mechanism.

The relay controlling mechanism is submerged in oil, which is kept at a constant level by a drain pipe, and is supplied with oil by the discharge from the relay. Since the rotating parts of the mechanism are well lubricated and are not loaded with the spring tension, they are free to respond to a short contact of the tie-line load regulator. To prevent the motor from overtraveling a magnetic brake is provided.

All generating units on the Duquesne Light Company's system on which automatic control, either of the frequency or load type, is used, will be equipped with a speed-changing device of this character.

Operating Experiences with Tie-Line Load Control

The instantaneous speed of any system is a function of the system load and the setting of the turbine governor. Thus, with any given setting the speed of the system will vary as the load changes. If two systems are interconnected through one or more lines, the speed of the entire interconnection becomes a function of the governors and the total load, and the generator load on one system does not bear a direct relation to the demand on that particular system. The generator load being a function of system speed requires that the excess demand be supplied through the interconnecting tie lines. If the two systems under control are of the same size and have approximately the same load characteristics, it is possible to remain in parallel for reasonable periods of time without overloading the tie lines when operating on straight speed-governor control. It is customary when such an interconnection exists for the system controlling the tie line to adjust its governors to maintain 60-cycle frequency if this correction will tend to correct the tie-line loading.

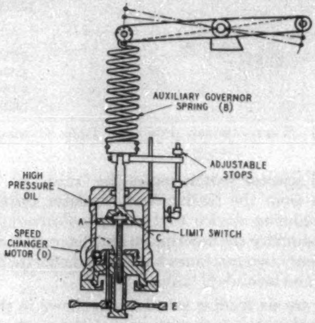


Fig. 7—Westinghouse Hydraulic Speed Changer

With frequency control on one system, it is evident that all of the load changes occurring on the entire interconnection will eventually appear on the units under automatic control, so that all the net load changes of the system without frequency control will appear in the tie line.

Observation on the Colfax-Springdale tie line for a period of two years has shown the above reasoning to be correct. Difficulties were found in maintaining an exchange of power between the systems when it approached the amounts contracted for. The variations in load appearing on the tie line required almost constant supervision of the loading, and the situation was aggravated with the addition of frequency control on the West Penn Power system. Fig. 8 shows typical tie-line load and frequency charts during manual control.

It is generally assumed that tie-line load control tends to make frequency control more difficult to obtain. This is based on the assumption that load changes on one system must all flow over the tie line and be compensated for by the frequency-control equipment before the tie-line control equipment has functioned. That is to say, the load change is first corrected by the frequency regulator and later corrected by the tie-line control, which in turn requires the dropping by the frequency regulator of load equal to the amount picked up by the tie-line load control. This reasoning would be sound if the frequency-control equipment were very much faster in response than the tie-line regulator. As a matter of fact experience has shown that the two types of control work satisfactorily together, although possibly at the expense of increased frequency of operation during periods when corrections must be made by one control to compensate for the operation of the other.

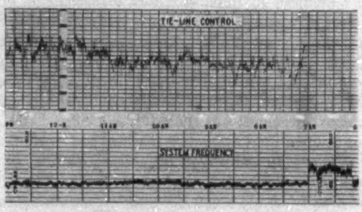


Fig. 8—Tie-Line Load with Manual Control at Colfax and Automatic Frequency Control at Windsor

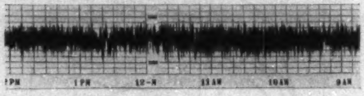


Fig. 9—Typical Chart Showing Tie-Line Control with Regulated Frequency at Windsor

It has also been observed that the use of tie-line regulators affects the permissible neutral zone of the frequency regulators on the interconnected systems. The initial installation of tie-line load control at Colfax was made at a time when the West Penn Power Co. was maintaining frequency within plus and minus 0.05 of a cycle by means of automatic-frequency control devices at the Windsor Power Station. Later in order to reduce the overloading of other tie lines the frequency was allowed to vary as much as plus and minus 0.15 of a cycle. This resulted in a more erratic power flow over the tie line and an increase in corrective operation of the tie-line regulator. Fig. 9 shows a tie-line

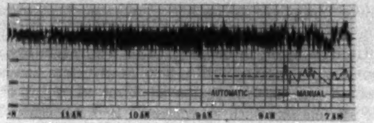


Fig. 10—Manual and Automatic Tie-Line Control

Broken Line Shows the Average of the Load Swings

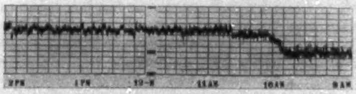


Fig. 11—Automatic Tie-Line Control under Light Load Conditions

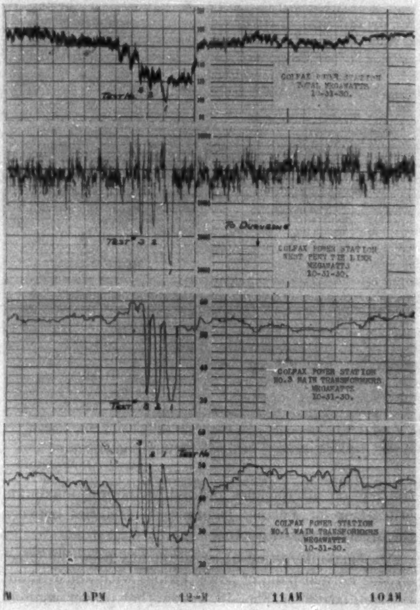


Fig. 12—Instrument Charts at Colfax During Tests

load chart illustrating this effect. The erratic power swings are due to the fact that when there is a load change on one system there is a speed change which affects the governors on both systems. While the West Penn frequency regulator is attempting to readjust frequency, power flows over the tie line and the tie-line regulator functions to change the Colfax governor-speed settings until it is again in balance. The two regulators will continue to operate until a new steady state has been reached and the load change has been taken care of by the system on which it occurred. It is felt that while the solution to tie-line load swings may be temporarily put off by allowing frequency to drift, eventually system speeds will have to be held rigidly at 60 cycles and load control on all tie lines utilized to permit maximum utilization of existing transmission facilities between the interconnected systems.

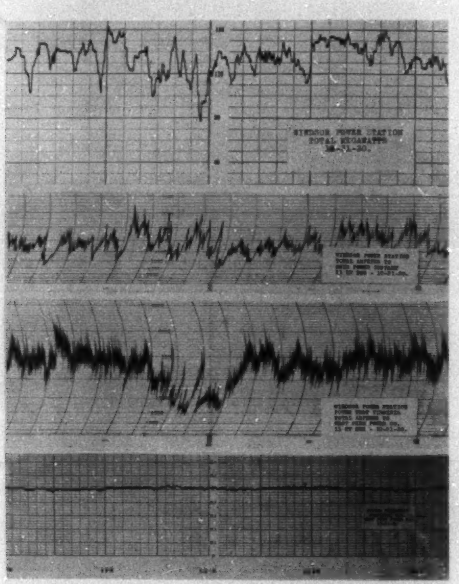


FIG. 13—INSTRUMENT CHARTS AT WINDSOR DURING TESTS

Another interesting observation has been the effect on the normal band of power swings with and without tie-line control. Fig. 10 shows that under manual tie-line load control the average load drifts without any appreciable change in the width of the band of power swings, whereas with automatic tie-line load control the average load is maintained very close to the desired amount, but the band of power swings is greatly enlarged. Fig. 11 is representative of the automatic tie-line control performance during periods of very light load, such as Sundays and holidays. The control gives a very narrow band and maintains the loading without any marked divergence from the normal load. This can be attributed entirely to the reduction in connected capacity and lack of fluctuating loads from industrial plants. During the period in which these charts were recorded the interconnected systems were under automatic-frequency control from the Windsor Power Station of the West Penn Power Co.

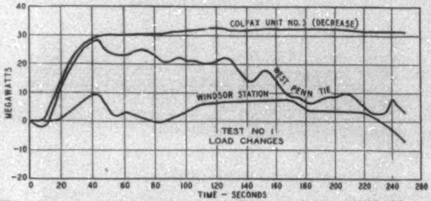


FIG. 14—CURVES SHOWING LOAD CHANGES DURING TEST NO. 1

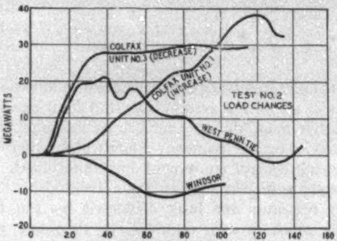


FIG. 15—CURVES SHOWING LOAD CHANGES DURING TEST NO. 2

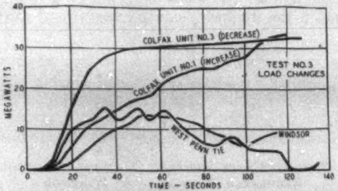


FIG. 16—CURVES SHOWING LOAD CHANGES DURING TEST NO. 3

Tests were conducted by the Duquesne Light and West Penn Power Companies in October 1930, to determine the rate of response of the tie-line control and the frequency control to rapid load changes. This was accomplished by rapidly lowering the load on the Colfax Power Station 30,000 kw. and allowing the load and frequency controllers located at the Colfax and Windsor Power Stations respectively, to correct the load distribution and frequency without manual assistance. Figs. 12 and 13 show the records obtained on the station graphic instruments at Colfax and

Windsor during three consecutive tests. These are of interest primarily because of their time scale and because they are typical of those obtained in ordinary practise. For the purpose of analyzing the instantaneous load shifts, higher speed-test meters were used, and the results are shown in Figs. 14, 15, and 16. It will be noted that in all three tests, when 30,000 kw. was dropped on unit No. 3 at Colfax the power flow over the tie line increased almost simultaneously and at the same rate as the load was dropped. This initial division is a characteristic of interconnected systems when the load change occurs at the end of the tie line opposite from the point of frequency control. Without tie-line control the Windsor Station being under frequency control would have picked up the entire 30,000 kw. dropped. However, the curves show that in all cases the tie-line load control had accomplished considerable correction by operating on Colfax unit No. 1 before the frequency regulator at Windsor had caused that station to acquire much of the load dropped. All three tests were similar except that the speed of response of the tie-line load regulator was increased in tests No. 2 and No. 3. It will be noted that in the second test (Fig. 15) the load was dropped at Colfax generating station but was not picked up at Windsor, due to this station dropping load faster to compensate for load changes on its own system. This is shown on the Windsor charts of Fig. 12, and incidentally is a very good example illustrating the advantages of load diversity through interconnection.

CONCLUSIONS

1. It has been demonstrated that tie-line load regulators and program loading equipment have been developed to a commercial stage.

2. Tie-line load regulators on one system can be operated satisfactorily in conjunction with frequency regulators on a connected system.

3. It has been demonstrated that tie-line control may be used to reduce load fluctuations between interconnected systems.

Discussion

H. S. Fitch: Any discussion of this initial and timely paper on automatic tie-line control should consider first the nature of the interconnection, for upon the type of interconnection depends largely whether the operating problem is the proper loading of tie-lines or the stabilization of frequency or both. As the authors have stated, the important problem of this interconnection is that of controlling the interchange of power between the several systems.

The Pennsylvania-Ohio-West Virginia interconnection is a fairly compact one within an approximate total area of 101,000 square miles and is strongly interconnected by means of the 132-kv. transmission backbone throughout its principal component systems. The total normal running generating capacity is 4,000,000 kw., the Duquesne Light Company generating capacity is 360,000 kw. or approximately 10 per cent and the 132-kv. tie-line between the Duquesne Light Co. and the West Penn Power Co. is limited by transformer and regulator capacities or 36,000 kva. or to approximately 1 per cent of the total interconnected capacity.

Regulation of the load among the interconnected systems of this group and among the individual power stations of each system has been a most difficult problem. As each system joined the interconnected group it would have to scrap, in many instances, much of the operating practise of load and frequency regulation it had built up over a long period of operation. It was recognized early in the history of this interconnected system that in the problems of plant loading, tie-line control and complying with the terms of the many contracts by which each system interconnected with its neighbors, frequency control was a matter of vital consequence. Those companies which were interconnected with more than one other company at different points, were greatly limited in their ability to maintain correct loading on the various interconnecting tie-lines. While it was not difficult for a company on the side adjacent to the major frequency controlling station to maintain its tie-line load at desired value, it was difficult for those companies interconnected to it and farther away from the center of frequency control to regulate the amount of power flowing in its interconnecting tie-line.

The frequency of the system was constantly being changed by the various power stations attempting to regulate the flow on the tie-lines by opening or closing the turbine governors, especially during the abrupt load changing periods and it was usually 9 a.m. before the system settled down to the morning load, which steady condition would last until 11:45 a.m. The Working Committee of operating executives believed that, if the frequency could be held at 60 cycles with but a very slight deviation —less than 1/20 of a cycle—the outlying company would need only to adjust its interconnecting tie-line load, the next company in turn adjust its tie-line load and so on to the last tie-line connecting to the central frequency regulating station. In this manner, the tie-line load equilibrium would not be disturbed.

Tie-line control on this interconnection was, therefore, first attacked from the point of view of frequency control for it was believed then, as the authors of this paper verify now—"that while the solution to tie-line load swings may be temporarily put off by allowing frequency to drift, eventually system speeds will have to be held rigidly at 60 cycles and load control on all tie-lines utilized to permit maximum utilization of the existing transmission facilities between the interconnected systems." It was exactly this problem that the operating engineers tackled in the early part of 1929 by assigning to Windsor Station the task of holding a flat frequency with six 30,000-kva. generators with one operator on each shift assigned practically the sole duty of holding a steady frequency by manual adjustment of the six governors. At that time no commercial type of automatic frequency control was available. The Windsor operators were to be more watchful over frequency, and more agile in adjusting it than the other operators of the system had been and were to concentrate on loading the station but maintaining a close frequency range, whereas all other operators were to concentrate on maintaining a close tie-line load adjustment manually and forget the frequency.

This new scheme of operation worked successfully from the start. Troubles with tie-line regulation seemed to disappear like magic. Whereas it had been usual practise to adjust tie-lines three or four times an hour, now three or four times a morning or afternoon or evening suffices.

The control of close frequency has not been a simple problem as stated by the authors, but has been one in which the manufacturers of the equipment have been actively engaged since 1929 and on which the Working Committee has applied much attention and made many tests gradually working toward a more rigid 60-cycle control. Three times equipment has been installed at Windsor by one manufacturer until now a very close frequency range is attainable and another manufacturer is now installing his equipment for test purposes.

At this writing, frequency control is installed not only at Windsor Station, but at Philo Station of the American Gas &

Electric Company and the Springdale Station of the West Penn Power Company to relieve the Windsor station of the large shifts in load at the load changing periods which were often as high as 80,000 or 90,000 kw. in 8 or 10 minutes. These three power stations by their firm interconnection may be considered to be the nucleus of the system and present a very compact and unified system of frequency control. So far no trouble other than adjustment of speed and amount of response has been necessary to keep these three frequency regulating devices in synchronism. The three of them in unison are doing a very satisfactory job. There has been no serious wear on the governor mechanism at any station due to speed change impulses of nearly 30 times a minute. The dispatching work of putting the control devices in and out of service and of regulating the number of generating units on each device at the three stations is supervised by the Windsor load dispatcher.

The control of frequency has been noticeably assisted by the automatic tie-line control installed on the Duquesne Light Co. interconnection which method is far superior to the manual method formerly used. However, it is very doubtful if automatic tie-line control would be effective with manual frequency control, especially with the kind of frequency illustrated in Fig. 1. Therefore, it appears that a rigid frequency control is the first essential, and it was after this advantage had been secured that tie-line control was installed by the Duquesne Light Co.

Too great emphasis cannot be laid on the statements made by the authors in column 1, page 41 in regard to the use of automatic control to secure the maximum economy possible as a result of the interconnection and to obtain the most efficient loadings of the several generating units. Especially true are the statements that the regulator must not respond to transient load change, but it must respond to small sustained changes. The uses to which the program load controller can be put to obtain these maximum economies will be quickly recognized by every operator. Most operators and several load control devices do not sufficiently take into account the inefficiency due to operating with steam control valves in a throttling position; neither do they allow the economies of very careful operation by having the turbine valve steadily wide open without cracking the next valve. Very close frequency control is necessary for this work.

The tests made by the authors have advanced the subject immeasurably, and they have indicated new methods to operate interconnections more successfully and with greater economy. The job of operating smoothly one large interconnected area with another over a comparatively weak tie-line, each area having a multitude of interconnections, is now providing problems every day for the operating executives of the Chicago, Southern and Pennsylvania-Ohio groups. Whether the answer lies through frequency control or tie-line control, or more probably through the correct application of both methods, is one now sought by tests being continually made by the operators of these systems and the manufacturers of these devices.

Philip Sporn: This paper deals with a problem that is of utmost importance in the operation of interconnected systems' and a problem that is at present receiving a very considerable amount of study. Experience is gradually being obtained with automatic control, and I believe that the final answer depends upon the proper application of automatic control, but not necessarily and certainly not primarily by automatic tie-line control.

In discussing the interconnection problem, the authors refer to operation of the Chicago, Boston and New Orleans systems in parallel when occasion arises for exchange of power. At present there is no economic justification for parallel operation of the various areas mentioned and if they do finally operate in parallel, there will be nothing planned about such operation, but rather, it will be accidental. Power exchange will be a process of relaying and not any actual exchange of power, for example, from Chicago to Boston.

The statement is made that any attempt to regulate two or more interconnected systems with automatic frequency control on each will develop the fact that load variation on the two systems will be so dissimilar that regardless of frequency control, the power flow over tie lines cannot be regulated. This has certainly not been the general experience to date. Experience has shown that automatic frequency control on each of two systems operating in parallel greatly reduces the fluctuation on the tie line, and in one instance was the only practical means of operating the systems in parallel through a relatively weak tie line.

I certainly believe that automatic tie-line control has its place in the operation of interconnected systems and the authors are to be congratulated for the work they have done along the development of such apparatus. However, I do not at all agree with them that the matter of frequency control is comparatively simple and that the entire question practically resolves itself into one of tie-line control.

In my opinion the authors have attacked this problem from the standpoint of a very special case that could hardly be classed as a typical interconnected network since the tie line is of relatively low capacity and further, that there is only *one* tie line between the systems. The case cited in this paper is one in which two areas with generating capacities with a ratio of about 10 to 1 are connected together through a *single* tie (transformer in series) of relatively small capacity. The question is considerably more complicated with two or more large systems connected together through numerous tie lines. In the latter case, it appears from a great deal of work that we have done on the problem to date, that distributed frequency control is the proper solution if we are to keep away from expensive phase shifting equipment in tie lines.

The paper describes tests that were made by the Duquesne Light Company to demonstrate the action of the tie-line controller when generating capacity was dropped on the Duquesne Light Company's system and showed in every case that Windsor, the frequency controlling station, picked up considerable of the load before the tie-line controller was able to make the proper correction. This definitely brings out the fact that the frequency controlling station must always take a certain amount of the "fringe." An interconnecting system in which only one station is designated to hold frequency and in which all other sections have automatic tie-line control will result in the frequency controlling station taking the accumulated fringe of all parts of the systems. On an interconnection of any size, this will amount to considerable capacity, perhaps at times in the order of 100,000 kva. and this would most certainly impose an undue burden on the frequency regulating station. The result is that the system having the station under automatic frequency control must "hold the bag;" this might perhaps be a satisfactory way of doing the job if it were possible to get someone to agree to take the power swings due to frequency regulation. In the ultimate set-up, I do not believe anyone will be willing to sacrifice station economy in order to hold very close frequency, so that each of the interconnected companies may work out its own ideal economy or contract loading program.

Interconnection has been a process of tying together systems that had normally been holding their own frequency, and when these companies started operating in parallel, it was natural that each should attempt to continue to hold frequency. The main difficulty with this method of operation was that the frequency recorders did not have the same degree of accuracy, and naturally resulted in each system attempting to hold a slightly different frequency. Such a condition usually resulted in tie-line loading difficulties.

These difficulties led to the next step which was to designate one station to hold frequency and for other interconnected companies to follow their own loads very closely, so as not to upset tie-line loading. At first the designated station held frequency manually and the improvement over the old method of operation was so marked that automatic frequency control was installed in the one station. While the operation was very much improved over the old method of each station watching frequency, there was nevertheless considerable burden placed on the regulating station. This was mainly due to the fact that manual control on other parts of the system was slow as compared to the automatic control.

This led to the conclusion that for a complicated interconnected system, distributed automatic frequency control would be the ultimate solution. It was realized that this was not a simple problem and that it should not be inferred that the problem has been entirely solved by installing an automatic frequency controller in one station of each of the interconnected companies. The installation of a frequency controller in a given area may smooth out regulation over the interconnecting tie line, but at the same time may create regulation difficulties within the system on which it is installed. The final solution of this difficulty may be additional frequency controllers within the given area, and it is in such cases as this that the automatic tie-line controller may be the ideal solution. But, the fundamental problem will still be one of frequency control. Undoubtedly there will be cases where some limit must be set up as to the amount of power that can flow on tie lines, but the tie-line control should ordinarily be a supplement to frequency control. As a matter of fact, tie-line control itself, such as described by the authors, approaches the question from the standpoint of frequency, because any attempt to adjust governors on one system so that it will supply its own load demands and limit the incoming or outgoing on a tie-line to another system, aims in effect to keep the speed constant.

Experience has borne out the belief that automatic frequency controllers at several stations operate satisfactorily in parallel. For some time the Windsor plant has been holding frequency for a large interconnected system. Undue burden has been placed on this station and relief was sought by installing additional frequency control at other stations. A series of tests in conjunction with two manufacturers is now in progress with frequency control at the Philo plant of The Ohio Power Company, at the Springdale plant of the West Penn Power Company, and at the Windsor plant, which is jointly owned by the West Penn Power Company and The Ohio Power Company. To date operation has demonstrated that available frequency controllers operate satisfactorily in parallel and that they have greatly reduced the swings on Windsor plant. It is of course necessary for system operators to become familiar with automatic frequency control if the results are to be satisfactory. In some cases it may be necessary for them to change their ideas somewhat as to the accustomed methods of operation. One fact that has already been demonstrated is the possibility of having too much capacity under automatic control for a given system load. However, this does not mean that there are too many points of control. It is felt that the surface is only scratched and that the ultimate solution depends on proper distribution and coordination for parallel operation of these automatic controllers.

One of the most important justifications for interconnection is the mutual help that it renders possible during emergency conditions. As at present applied, automatic tie-line control is not entirely satisfactory from this standpoint, that is, if there is a need for help on the system which has automatic tie-line control, this controller will allow power to come in from the interconnection. However, if need for help arises on the other end of the interconnection, the controller will attempt to limit this power flow. Undoubtedly a proper solution of this difficulty can be worked out, but this will probably mean cutting out the tie-line control during such cases of need. On the other hand, if there is frequency control on both sides of the tie line, the situation is somewhat different and it seems to me more satisfactory in that it will allow the maximum amount of help during emergency conditions. There are of course cases where the amount of power that can safely be exchanged between two systems is fixed by line and transformer limitations. In such cases, it may be necessary to use tie-line control.

As stated previously, I believe this problem of operation of interconnecting systems must be attacked from a very broad standpoint and worked out in such a way that all concerned will obtain the maximum benefits to be derived from such interconnections. A solution which imposes undue burden on any system, or acts as a check on the usefulness of interconnection facilities, will ultimately fall down.

John Gosinski: The operation of the tie-line load regulators described in this paper is concerned mainly with normal operation of a tie line, that is, for the transmission of a predetermined amount of power. It will be found that interconnections between neighboring systems are relied upon not only for a normal transfer of power but also for a definite portion of system reserve capacity for use during major system transmission line or generator outages.

Since the value of an interconnecting tie line proves itself by its ability to swing in for large blocks of power in an emergency, any regulating device should preserve this feature of an interconnection. It would be desirable to have the regulating device differentiate between the normal load drifts between systems and a sudden abnormal demand caused by loss of capacity by the neighboring system so that for the latter condition the tie-line regulator would become inoperative.

Quite frequently a generating plant for load regulation may be a considerable distance away from the tie-line terminal, so that the impulses from the tie-line regulator would have to be transmitted over distances as great as 100 miles. The impulses of the regulator should, therefore, have such characteristics as to be suitable for transmission over the less expensive type of telephone or telegraph transmission channels.

R. Brandt: This paper brings forcibly to the front the fact that the main problem to be faced in growing interconnections is that of making the power flow in predetermined amounts and predetermined directions. Frequency control as an end in itself is desirable only from the relatively minor point of view of synchronous clock time. As a step in the solution of the major operating problem, it is, however, of extreme importance since it furnishes a firm, steady foundation without which it would be futile to try to control tie-line loadings.

For two reasons then the development of frequency control has been pushed in the New England region; (1), because it was the logical first step in the general solution of the interconnection problem, and (2), because it could be accomplished fairly readily as the authors of the paper point out, and that its accomplishment did carry with it some desirable results. It can even be rather definitely stated that frequency control has had a steadying effect on tie lines and has, therefore, accomplished at least partially the solution of the main problem.

There are undoubtedly some cases where tie-line control will serve the purpose better than speed control, and the case cited in the paper appears to be one in point. It must not be lost sight of, however, that unless speed were being held steady somewhere else the excellent results obtained with the equipment described would hardly have been found. The case of the Duquesne Light Company when connected with the West Penn Power Company, and through them to an extensive interconnection network by means of one short, strong tie terminating directly at a power station was an almost ideal location for tie-line control, and full advantage has been taken of this fact. Its extended application, however, to the more general case of a system which may be interconnected at the same time with two, three or more adjacent systems over relatively long ties, geographically separated by a hundred or more miles, and terminating at high tension substations rather than controllable sources of power, so complicates the situation as to make the application of any such tie-line controls as are at present available quite impractical.

Frequency control on the other hand does offer at least some measure of help in the complicated general case. Automatic equipment in operation on adjacent systems will have a steadying effect in all of the tie lines between the systems so equipped.

All this does not mean that swings will be eliminated. They may even increase in number, but their average magnitude should be reduced. During the steady load periods of the day it is probable that the ups and downs of the fringe of the load curve on adjacent systems will not absolutely coincide, and that some unnecessary transfer may be brought about which will have to be later corrected. However, any transfer from this cause is minor compared with that which occurs during the fast changing periods of the morning, noon and late afternoon. At these times, the direction of load change on all interconnected systems which do not cross time belts will most certainly be in the same direction and at the same time. In this case the simultaneous operation of frequency controllers having substantially the same sensitivity and response characteristics is of undoubted benefit.

The mechanical devices described in the paper have been well thought out and have served their purpose admirably. In particular, the anti-hunting feature which takes into account the momentum which has to be established and later destroyed in bringing the system from one load condition to another is well explained.

The authors are to be congratulated on the soundness of the conclusions reached, based on the conditions covered by their experience, and on the useful definitions of the requirements of successful tie-line load control.

L. F. Hunt and M. MacFerran: The paper is of especial value in pointing out that the problem of frequency control is intimately related to that of load control, *i. e.*, the division of load among machines according to some predetermined schedule. It is important to note that the desirability of properly correlating load and frequency control is not confined to independent systems connected by a light tie line, but extends also to any system which has several sources of energy which must be coordinated. Especially is this true when two or more units are necessary for regulation.

The use of the hydraulic speed changer is a very interesting development. The question suggests itself as to whether this device does not introduce one more time-consuming step into the already complicated chain of events involved in the action of the governor and the compensating or speed-changer spring. In fact, the very circumstance that makes the hydraulic attachment necessary—the operation of the speed-changer spring 30 times a minute—suggests that the whole governor mechanism is now being put to a use for which it was never intended. Originally the governor and compensating spring were designed to give a drooping speed-load characteristic suitable to the operation in parallel of numerous governed units. Then the necessity for constant frequency gradually arose, and the burden of the regulation was all placed on one unit or plant, with an auxiliary frequency-control device acting constantly to change the tension of the compensating spring. The net result is a perfectly flat speed-load characteristic, obtained not by direct and simple means but by a long chain of adjustments and readjustments. This complicated mechanism, while functioning fairly well under normal load conditions, is unduly sluggish and unresponsive during transient conditions, because there are so many points of slack and lost motion. It would seem that modern conditions dictate a more direct method of control, as for instance an electric frequency device working directly on the pilot-valve of the machine. Such a scheme can readily be extended to include automatic load division control when two or more machines do the regulating.

P. B. Juhnke: This presentation corroborates effectively the opinion held by the writer for some time to the effect that tie-line regulation apart from frequency of regulation is an essential part of system regulation. No matter how fine the regulation and control of frequency may be, drift of power over interconnected ties cannot be entirely avoided without special attention and effort. With frequency of regulation neglected, these drifts will easily reach the limits of the tie-line relay settings and consequently cause trip-outs of interconnecting elements.

This tie-line regulation may be accomplished in different ways. It may be through phase shifting or through action on turbine governors, actuated in either case by the tie-line load. But it must be present and of ample dimensions to accomplish its purpose. Then frequency of regulation can best accomplish its object and the result is a system greatly stabilized and able to function as desired.

T. E. Purcell and C. A. Powel: Most of the experience cited confirms that of the authors but the conclusions drawn are not always identical.

Mr. Fitch has given an excellent summary of the operating experience in his district. It might be interesting to note that the increasing power exchanges between the West Penn system and the Duquesne Light system after automatic frequency control was adopted on the West Penn system led to the installation of automatic tie-line load control on the Duquesne-West Penn interconnection in order to prevent the load changes on the Duquesne system from being transmitted over the tie-line. Experiments had previously been made with automatic frequency control on the two systems but it was found that the tie-line load could not be held within the required limits by this means. Experience since the presentation of the paper indicates that the performance of the tie-line control is not influenced by the quality of the frequency regulation on the interconnected systems. Tie-line control does have a stabilizing effect on the frequency of the interconnection.

Mr. Brandt has given an excellent contribution of his experience in the New England area. It would be interesting to be able to compare an installation of tie-line load control and parallel frequency control on the long tie lines of his system with the results obtained on the Duquesne-West Penn tie. Messrs. Brandt and Gosinski point out that in many cases the tie-line to be controlled is remote from any generating station. Two of the tie-line load regulators shown in Figs. 3 and 4 of the paper are so designed that they can be adapted to remote control.

The use of frequency regulators to accomplish phase-angle control, as referred to by Mr. Sporn, would undoubtedly lead to undesirable distribution of load between generating stations, thereby resulting in decreased efficiency of generation. It is the authors' opinion that this loss might be great enough to more than justify the installation of phase-angle control equipment.

It seems to the authors that the conclusions drawn by Mr. Sporn from the experience at Windsor during the test described in the paper are not fully justified. No doubt the frequency controlling station may take a certain amount of the fringe but the tie-line load regulator is much more sensitive on a large interconnection than any automatic frequency control can be. It seems to the authors that Mr. Sporn has presented a very excellent argument for automatic tie-line control as he points out in his discussion that frequency control in one station is desirable, but up to the present time has not been successful on his system due to the difficulty under such condition of holding load on tie lines within proper limits. This can be accomplished most ideally by the use of the automatic tie-line load control equipment described in the paper. The automatic tie-line control regulator can be so biased with frequency that it can be made to function in the desired manner during emergencies.

With regard to the questions raised on the use of the hydraulic speed changer, we have found the introduction of a program load control eliminates excessive wear on governor parts of any individual machine. The question raised by Mr. Hunt and Miss MacFerran is well taken. Perhaps there are other or better ways of controlling the speed than by the use of the hydraulic speed changer. However, this mechanism not only allows the governor to function well under normal conditions, but also under transient conditions. Full load may be suddenly dropped from this machine without excessive increase in speed.

Tuned Power Lines

BY H. H. SKILLING[*]

Associate A. I. E. E.

***Synopsis.**—A new approach to an important problem in the field of electric power is offered by the author, who limits his discussion to the possibilities of tuned lines rather than their practical application. He presents the results of extensive research work, and on the basis of laboratory experiments points out that through the use of higher transmission frequencies, the efficiency of a given line might be increased as much as 500 per cent.*

* * * * *

LESS than 50 years ago when electrical engineering was young, there was vital and active interest in subjects which today are taken literally for granted. In early electric power work not the least of these was the question of choosing a desirable frequency for a-c. systems; 20 cycles, or 50, or 100? It was only natural that discussion pertaining to the frequency question considered principally the relative merits of different frequencies from the point of view of electric machinery—generators, motors, and transformers—with practically nothing said of the then simple subject of electric power transmission.

Thus it is that the large electric power networks of today find themselves definitely committed to certain frequencies earlier chosen for various expedient reasons. Now there is a definite trend toward the use of more and longer transmission lines for carrying large blocks of power from generation centers to load centers. In this connection the question of transmission stability is supreme; hence it is considered justifiable to reopen the question of desirable frequency, and to study it from the long lines point of view. It is well known of course that transmission characteristics of a given system may be improved by the use of frequencies lower than those now in vogue; but no general study of the influence of frequency has been published.

While the fundamental transmission line equations are:

$$E_g = E_r \cosh \sqrt{Z\,Y}\,l + I_r \sqrt{\frac{Z}{Y}} \sinh \sqrt{Z\,Y}\,l$$

$$I_g = I_r \cosh \sqrt{Z\,Y}\,l + E_r \sqrt{\frac{Y}{Z}} \sinh \sqrt{Z\,Y}\,l$$

where

E is voltage
I is current
g subscript, signifies generator end of line
r subscript, signifies receiving end of line
Z is impedance per unit length of line
Y is admittance per unit length of line
l is length of line

it is most convenient to discuss transmission operation in terms of the general circuit constants:

$$A_o = \cosh \sqrt{Z\,Y}\,l^*$$

$$B_o = \sqrt{\frac{Z}{Y}} \sinh \sqrt{Z\,Y}\,l$$

$$D_o = \cosh \sqrt{Z\,Y}\,l$$

$$C_o = \sqrt{\frac{Y}{Z}} \sinh \sqrt{Z\,Y}\,l.$$

For ordinary transmission lines and for frequencies higher than about 10 cycles per sec., the value of factor $\sqrt{Z/Y}$ is practically constant and need not be considered as a function of frequency in this discussion. Hence the general circuit constants are proportional to the hyperbolic functions. However, if the resistance of the line in question is low compared with the reactance, and if the leakage from line to line is small, factor $\sqrt{Z\,Y}\,l$ becomes purely imaginary and

$$A_o = \cos \sqrt{\mathbf{Z}\mathbf{Y}}\,l^*$$

$$B_o = j\,Z_o \sin \sqrt{\mathbf{Z}\mathbf{Y}}\,l$$

$$D_o = \cos \sqrt{\mathbf{Z}\mathbf{Y}}\,l$$

$$C_o = j\,\frac{1}{Z_o} \sin \sqrt{\mathbf{Z}\mathbf{Y}}\,l$$

with $Z_o = \sqrt{Z/Y}$, the characteristic impedance of the line.

Thus it is made evident why low frequency gives good transmission characteristics. The desirable conditions are low voltage regulation under load, small charging current when lines are unloaded, and high synchronous stability. All three of these factors require a low value for B_o while synchronous stability requires also a value of A_o approximately unity. Both of these conditions are obtainable with low frequency because Z and Y both are proportional to frequency, and as they decrease in value, the sine of $\sqrt{\mathbf{Z}\mathbf{Y}}\,l$ decreases while the cosine of the same angle approaches unity.

Conversely undesirable transmission characteristics are encountered when the value of $\sqrt{\mathbf{Z}\mathbf{Y}}\,l$ approaches $\pi/2$, because then the cosine approaches zero while the sine is near unity. Operation under such conditions is impractical because (1) regulation is so poor that with constant generator voltage the load is required to take practically constant current, (2) line charging current increases *indefinitely* as the receiving voltage rises, and (3) synchronous stability is possible only at extremely low power factor.

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[*] Bold-face symbols (**Z**, **Y**) indicate absolute values (magnitude only) time relationships as indicated by angles omitted.

If the value of $\sqrt{\mathbf{ZY}}\,l$ lies between $\pi/2$ and π, synchronous loading is quite impossible, but a static load may receive power. The static power limit (which is determined by voltage regulation and occurs at the load for which $d\,P_r/d\,E_r = 0$) becomes very large as $\sqrt{\mathbf{ZY}}\,l$ approaches π, because $\sin \pi = 0$. For the same reason charging current to the line is very small; hence, transmission conditions are as good when the frequency makes $\sqrt{\mathbf{ZY}}\,l = \pi$ as when the frequency is very low, except for the possible lack of synchronous stability.

An important point worthy of detailed consideration is that where the frequency used makes the value of $\sqrt{\mathbf{ZY}}\,l$ slightly greater than π, and synchronous stability also is attained. The terminal steady-state conditions are identical when $\sqrt{\mathbf{ZY}}\,l = k$ and when $\sqrt{\mathbf{ZY}}\,l = k + n\,\pi$ with k representing any value of the function and n any integer. The truth of this statement for the idealized case under consideration is evident from the fundamental transmission line equations. The physical meaning is that transmission is good over a line the length of which is equal to a very small fraction of a wavelength of the transmitted voltage, and that it is equally good over a line the length of which is equal to a half wavelength or any multiple thereof. But this consideration includes only steady-state terminal conditions of a line without losses; certain modification is necessary before this statement may be considered to apply generally.

When line losses from resistance and leakage are not negligible it is best to use the hyperbolic-function form of general circuit constants. No change in the general understanding of conditions need occur, but the definiteness of the ideal line is lost; the power limit now is not infinite (as when $\sqrt{ZY}\,l = j\,\pi$) nor is synchronous stability either absolutely perfect or absolutely impossible. No longer is it quite true to say that $j\,\pi$ may be added to $\sqrt{ZY}\,l$ without changing terminal conditions. These things are nearly true, but not quite.

However, a point of far greater importance than line losses is the effect of terminal machinery. Even in the simplest system the general circuit constants will be modified by the generator, the transformers, and the load. If the apparatus at the sending end introduces a series impedance Z_1, and the receiving end apparatus introduces a series impedance Z_2, the general circuit constants become:

$$A_o = \sqrt{\cosh}\,\sqrt{Z\,Y}\,l + \frac{Z_1}{Z_o}\sinh\sqrt{Z\,Y}\,l$$

$$B_o = Z_o \sinh\sqrt{Z\,Y}\,l + (Z_1 + Z_2)\cosh\sqrt{Z\,Y}\,l + \frac{Z_1\,Z_2}{Z_o}\sinh\sqrt{Z\,Y}\,l$$

$$C_o = \frac{1}{Z_o}\sinh\sqrt{Z\,Y}\,l$$

$$D_o = \cosh\sqrt{Z\,Y}\,l + \frac{Z_2}{Z_o}\sinh\sqrt{Z\,Y}\,l.$$

But if the line is long and the machinery is designed to have fairly low synchronous reactance, the only change in the constants that need be contemplated here is that a term must be added to B_o:

$$B_o = Z_o \sinh\sqrt{Z\,Y}\,l + Z'\cosh\sqrt{Z\,Y}\,l$$

where Z' is the total impedance of the machinery. This is an approximation, but it is almost true when the value of $\sqrt{Z\,Y}\,l$ closely approximates π or is any multiple of π. It is not at all applicable if the value of $\sqrt{Z\,Y}\,l$ approximates $\pi/2$, but it may be generally used because it applies whenever transmission characteristics are favorable.

Transient synchronous stability usually is investigated only after line and armature transients have died out; therefore as in the case of steady-state stability, only terminal line conditions are effective. A half-wavelength line would have the same order of transient stability as a low-frequency line except as the terminal machinery might be altered.

TRAVELING WAVES

The nicest description of the operation of a line is in terms of traveling waves. The generator may be considered as imposing on the line a series of electromagnetic waves which travel on the line to be more or less reflected from the receiving end, setting up a wave train traveling back toward the generator. Where the line is short a reflected wave returns to the generator so quickly after its initiation that it still is very closely in phase with the wave leaving the generator. Under this condition the voltages and currents of the two wave trains will add algebraically at the load end (assuming the load impedance to have the same angle as the characteristic impedance of the line) and differ by only a small angle at the generator end. Therefore, if attenuation is low the load voltage will be almost equal to, and in phase with, the generator voltage.

If the line be longer there will be a correspondingly larger difference in phase relation at the generator between the outbound and the returning wave. This involves trigonometric addition of effective voltages and currents at the generator and even with low attenuation, line regulation may be bad. However, if the line be of the proper length the reflected wave will arrive back at the generator just one cycle after its initiation there and consequently in phase with the next wave leaving the generator. Voltages and currents again will add algebraically and line regulation will be very good because the reflected wave will increase both end voltages equally. In this case as with the very short lines the load voltage will closely approximate the generator voltage (if attenuation is low) but 180 deg. out of phase.

The last few paragraphs have spoken of "long" and "short" lines. This refers to the electrical length of the line considering wavelengths rather than miles. However, a half-wavelength line is one for which the imaginary part of $\sqrt{ZY}\,l$ is $j\,\pi$, and hence it may be either a short line operating at high frequency or a very long line operating at normal frequency. Representative dimensions of a half-wavelength line are: 200 mi. long and 450 cycles, or 1,400 mi. long and 60 cycles.

The fact that apparatus at the ends of a line introduces impedance that increases the electrical length of the line already has been mentioned. It was shown that, approximately,

$$B_o = Z_o \sinh \sqrt{ZY}\,l + Z' \cosh \sqrt{ZY}\,l.$$

Since good transmission conditions demand that $B_o = 0$, it is desirable that

$$\sinh \sqrt{ZY}\,l = -\frac{Z'}{Z_o} \cosh \sqrt{ZY}\,l.$$

The hyperbolic cosine may be considered to be unity, either positive or negative. The lowest value of $\sqrt{ZY}\,l$ that approaches a solution of this equation will be between $\pi/2$ and π where the hyperbolic cosine will be negative and hence the values of frequency and length of line to be chosen should as nearly as possible make

$$\sinh \sqrt{ZY}\,l = +\frac{Z'}{Z_o}.$$

Consequently when terminal apparatus is considered, a shorter line or a lower frequency should be used than those given as representative in the preceding paragraph. If it is desired further to shorten the line or lower the frequency, more inductance may be introduced into the line, preferably in combination with static condensers across the line; this will result really in a section of "artificial line."

The presence or absence of synchronous stability for any given line may best be found graphically. Such methods, based on the circle diagram of the transmission line, tell at once the whole story of load limit, stability, regulation, and other factors. Although many statements in this paper now appear somewhat obscure in the light of the equations only, they become perfectly clear when the circle diagrams are studied. The diagrams unfortunately are too voluminous to be included, but Fig. 1 shows results obtained from such charts for a 202-mi. 220-kv. line having three 500,000-cir. mil cables spaced 15 ft. apart in a vertical plane; terminal apparatus not included. Under these conditions the value of sinh $\sqrt{ZY}\,l$ reaches a minimum at 447 cycles. Whether or not there might be synchronous stability at this frequency depends upon the synchronous machinery; it is probable that there would be. It may be seen that the maximum power per phase to be received at 60 cycles by a unity power-factor load is about 50,000 kw. Lagging power factors are much worse, and leading power factors are not much better. With transmission at 447 cycles the maximum power (if supplied to a static load or to an induction motor) obtainable is more than five times the 60-cycle limit.

The reason for the peculiar action of a half-wavelength line is essentially quite simple. The capacitance of the line is so balanced against the inductance of the line that the undesirable effects of both disappear. On an exact half-wavelength line with no losses and without load, with voltage maintained by a generator, the voltage at the receiving end would be equal and opposite to the voltage at the generator; in the middle of the line the voltage would be zero. The momentary charge at any point of the line would be proportional to the voltage; hence charging current would not need to be supplied from the generator, but merely would flow back and forth from one end of the line to the other each half cycle. Although there would be no current flowing at either end of the line, there would be a large current at the middle where the voltage would be low.

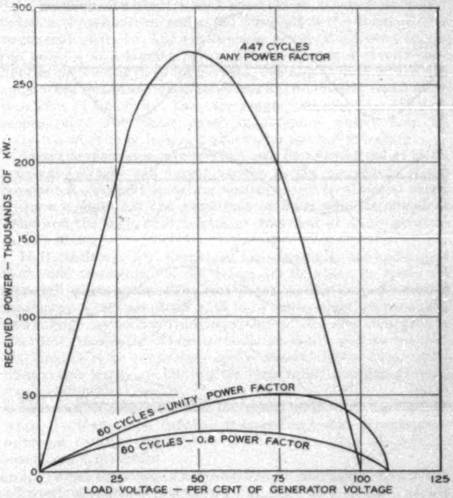

Fig. 1—Power Limits of an Operating 202-Mi. 220-Kv. Line for Normal 60-Cycle Operation and for Suggested 447-Cycle Operation

If load impedance were equal to the characteristic impedance of the line, current and voltage both would be constant along the line, but retrogressing angularly through 180 deg. The charging current would be passed on by each differential section of line to its neighbor as the voltage wave travels, finally to be used by the load.

If load impedance were less than the characteristic impedance, the current would be least at the middle of the line where the voltage would be highest. This resonant voltage rise is valuable because it makes efficiency and regulation of such a line better than could be obtained even from d-c. transmission; by proper insulation, reactance of the terminal machinery, and proper choice of the characteristic impedance of the line, resonant voltage rise may be prevented from becoming troublesome.

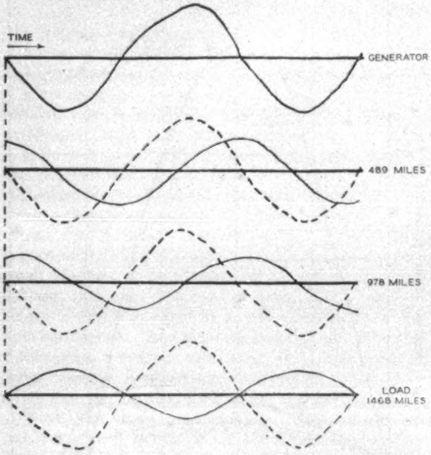

Fig. 2—Tracings from Oscillograms Showing Voltage Phase Relations Along a 1,468-Mi. Tuned Line as Compared with Generator Voltage (Dotted)

It is because of the interaction of the distributed inductance and capacitance that such a line is called a "tuned line." The similarity to a tuned radio circuit is striking.

As evidence that a tuned line will behave as predicted by the fundamental equations, power was transmitted at 60 cycles over a 1,468-mi. artificial line. The characteristics were those of the 220-kv. line previously described (which may be recognized as the Pit River line of the Pacific Gas and Electric Company, Calif.) and the potential used was 127 volts to ground, 0.001 of the real line voltage with a corresponding factor of 0.000001 for the power per phase as compared to the real line. The receiving end was arranged for loading with either a static impedance of any power-factor or with a single-phase synchronous motor. Extensive tests were made and the observed characteristics were found to be in close accord with the computed expectations; synchronous stability of the system was found to be exactly as expected. The half-wavelength line plus the reactance of the synchronous motor gave fair stability. The introduction of reactance into the motor leads was found to improve stability greatly, at the expense, however, of power limit. When the line was shortened to slightly less than a half-wavelength, it was impossible to keep the motor in synchronism; no synchronous power could be obtained although resistance loads continued to operate.

Fig. 2 shows reproductions of oscillographic records of voltages along the artificial line. Load impedance was somewhat less than the characteristic impedance of the line, and there was noticeable resonance. That the receiving voltage actually was lower than the sending voltage was due to the heavy load, the long line, and the unavoidable losses in the artificial line that do not properly represent a true line. Regulation would have been worse had the load been applied at an intermediate point.

While the subject of this paper considers the possibilities of the tuned line rather than its practical application and the details incident thereto, it seems quite reasonable to suggest that it is not at all impossible that 400-mi. transmission at 150 cycles will offer attractive economic possibilities. With the use of such a line of course motor-generator sets (probably induction machines) would be required for frequency-changing, but costly synchronous condensers would be eliminated. Even so considering the cost of building and maintaining 220-kv. lines it seems reasonable that even a relatively large amount of terminal equipment could be justified if it would eliminate one long line.

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Discussion

F. E. Terman: Some years ago I was in rather close contact with a group of engineers who were investigating the possibilities of a 500-mile 60-cycle transmission system. The chief difficulty encountered was in the low power limit of the transmission network at 60 cycles, and this ultimately proved the obstacle which prevented the project from materializing. In order to get around this difficulty many expedients were considered, such as raising the transmission line voltage to about 300 kv., lowering the frequency to 15 or 25 cycles, using intermediate synchronous condensers, etc., but it never occurred to anyone that raising the frequency to perhaps 150 cycles might be the solution to the problem.

The question as to whether tuned power lines will ever be employed in practise is of course dependent upon the economics of the future, but they are unquestionably a possible means of transmitting large blocks of hydroelectric power to distant load centers, and their consideration certainly upsets a number of preconceived ideas that most of us have held.

A. A. Kroneberg: As stated by the author, the subject of this paper is not so much the practicability as the possibility of the tuned line. As such it is quite interesting.

There are several questions that may be asked:

1. In formula $B_o = Z_o \sinh \sqrt{ZY}\, l + Z' \cosh \sqrt{ZY}\, l$ on page 50 of the paper where Z' is the total impedance of the machinery. This value varies with the load, being large at light loads when considerable machinery is taken off the line and reaching a minimum when all the machinery of the system is in service. Does it not mean that the line would have to be operated at different frequencies to take care of variations in Z'? With the load decreasing Z' will be increasing, calling for a shorter line or a lower frequency. Thus the prime movers of the generating station will have to be provided with speed control and the whole system will be operating at a variable frequency. The author suggests that the line length could be controlled by introducing more inductance in series, preferably, with static condensers across the line. This, of course, would enable us to fix the frequency of the system. It would be of interest to know how this could be accomplished in actual operation.

2. A statement on page 52 is open to objection. "Transient synchronous stability usually is investigated only after line and armature transients have died out." The step-by-step, torque-angle, and voltage methods of studying transient stability do not wait for the line and armature transients to die out. After transients die out there is no "transient" stability problem; it becomes a steady state solution.

Edith Clarke: Mr. Skilling has pointed out that a line of wavelength slightly over one-half wave (terminal machine impedances included) has good operating characteristics and power may be transmitted with stability from a synchronous generator at one end to a synchronous motor at the other, while for the line slightly under one-half wavelength no synchronous power can be transmitted.

The same conclusion will be reached by comparing the angles of the general circuit constants A_o and B_o, for lines (plus terminal equipment) of different wavelengths.

With constant excitations on generator and motor, maximum power will be delivered to the motor when the angle between the internal voltages of generator and motor is equal to the impedance angle of the general circuit constant B_o. With E_r as reference vector the angle of E_g must not, therefore, exceed the angle of B_o for the machines to remain in synchronism under steady state.

From vector diagrams of the equation

$$E_g = E_r A_o + I_r B_o$$

it may be seen that when the angle of the constant A_o is smaller than that of the constant B_o the power factor at the receiving end may be lagging, unity, or leading; but when the angle of A_o is larger than the angle of B_o the receiver power factor can only be lagging.

The angle of A is less than the angle of B for lines under one-quarter of a wavelength and also for lines between one-half and three-quarters of a wavelength so that for such lines synchronous power may be delivered at any reasonable power factor; while for lines between one-quarter and one-half of a wavelength and between three-quarters and a full wavelength the angle of A is larger than that of B and synchronous power, if delivered at all, can only be at a lagging power factor. For lines exactly one-quarter or one-half of a wavelength, the angles of A and B are equal when resistance is neglected and approximately equal when resistance is included. They are roughly 90 deg. and 180 deg. respectively. For these cases synchronous power may be delivered at unity or lagging power factor but not at leading.

Since the maximum power which can be transmitted at given terminal voltages will vary inversely as the magnitude of the constant B_o, the best operating conditions will be obtained with a frequency such that the magnitude of B_o is near its minimum value and the angle of B_o is greater than that of A_o by approximately 90 deg.

H. H. Skilling: Mr. Kroneberg has apparently not understood that "line transients" in the paragraph to which he refers are electrical phenomena. The mechanical oscillations of terminal machinery which are dealt with in "step-by-step, torque-angle, and voltage methods of studying stability" are slow compared to electrical transients. These methods generally assume the electric circuits to be continually in the steady state. After transmission line transients die out, the "transient" stability problem has in reality just begun.

Nor does he appreciate that maximum stability is needed only when the load to be supplied is large, and that therefore the optimum value of B_o need be reached only when all terminal machinery is in service.

It is true that large static condensers on high-voltage lines offer serious practical problems.

Intercontinental Radiotelephone Service

From the United States

BY J. J. PILLIOD[*]

Member, A. I. E. E.

***Synopsis.**—Radiotelephone service between the United States and Europe was established January 7, 1927 with one circuit and with service to limited areas. Facilities and service have been greatly improved and extended and rates have been reduced. Present scope of service is described and reference made to consistent increases in transatlantic telephone messages handled. This increase indicates that this service is being found of increasing value by the public.*

Extent of ship-to-shore radiotelephone service from the United States is outlined. Arrangements for service to Buenos Aires and Rio de Janeiro are described, these differing from arrangements used for service to Europe in that operation to these two cities was planned on a part time basis. Proposed short-wave system for operation with Bermuda and proposed new long-wave system to supplement existing facilities to Europe are mentioned.

A description of the new radiotelephone transmitting and receiving stations now being erected at Dixon and Point Reyes, Calif., respectively, is given. These stations will be connected to a terminal office at San Francisco and the system used for the establishment of radiotelephone service to the Hawaiian Islands and later on, to other transpacific points as may be required.

* * * * *

SINCE January 7, 1927, when commercial radiotelephone service was first established between New York and London, there have been continuous and important developments in the application of radiotelephony for intercontinental communications as well as a generally sustained increase in the public demand for this class of service. Today the Bell System, in cooperation with the British Post Office, has four radiotelephone circuits to London, the original long-wave channel supplemented by three short-wave channels. By means of these, telephones in the United States, Canada, Cuba, and Mexico may be connected to practically any telephone in Europe, the city of Ceuta in Northern Africa, part of Australia, and Java in the Dutch East Indies. Service to all European points and Ceuta in Northern Africa is furnished through London where connection is made to the submarine cables and land line facilities making up the European telephone network. As yet Russia, Greece, Turkey, and a few of the Balkan States cannot be reached from the United States over this network. Service to Australia is given by switching the New York-London radio circuits at the latter point to a short-wave radiotelephone channel between London and Sydney, Australia. Service to Java is furnished through London to Amsterdam or Berlin, where connection is made to a short-wave radio channel operated between those points and Java by the Dutch Telephone Administration.

Transatlantic telephone messages between the United States and Europe during the first six months of 1931 were about 20 per cent greater than for the corresponding period for 1930. While the basic rate, which is the rate from New York to London for a three minute period, was reduced from $45 to $30 on May 11, 1930, the increase in traffic is considered very satisfactory in view of the general business conditions which have existed on both sides of the Atlantic, and is an indication that this service continues to be of increasing usefulness to the public.

In December, 1929, a ship-to-shore short-wave radiotelephone service was established on a commercial basis through New York with the *S. S. Leviathan.* Since that date service has been opened with five more of the larger liners plying between North America and various European ports. The additional ships are the *Olympic, Homeric, Majestic,* the *Belgenland,* and the *Empress of Britain.* While this service is normally furnished, at present, with ships only in the Atlantic area, contacts were made last winter with the *Belgenland* during a large part of its cruise around the world and with the *Homeric* then cruising in the Mediterranean.

The Bell System, cooperating with the International Telephone and Telegraph System, established on April 4, 1930 a short-wave radiotelephone channel between New York and Buenos Aires. Through Buenos Aires service may be given by land line connections not only to other parts of the Argentine Republic, but also to the great majority of telephones in Uruguay and Chile.

Construction has been completed recently to the end that the South American Service may be enlarged to include Rio de Janeiro, Brazil. The plans for the radiotelephone system to work with Rio de Janeiro are on a somewhat different basis than for the services established heretofore from the United States and it may be of interest to outline briefly the arrangements made. The previously established services contemplated the full-time use of one or more complete radiotelephone circuits whereas service to Rio de Janeiro is planned on the basis of sharing facilities already provided for service to Buenos Aires. In the United States the transmitter operating with Buenos Aires has been provided with an additional antenna directed toward Rio de Janeiro and at the receiving station there has

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[*] American Tel. & Tel. Co., 15 Dey St., New York, N. Y.

Presented at the Pacific Coast Convention of the A. I. E. E., Lake Tahoe, California, August 25-28, 1931.

31-129

been installed a complete new receiving unit adjusted to receive from Rio de Janeiro. At the transmitting station switches are provided for rapidly changing the transmitter from one antenna to the other while at the receiving station the two receiving units make it possible for the New York control office to keep in constant touch with both distant transmitters. This arrangement allows for the maximum use of equipment and facilities and seems to offer a promising means of providing for the extension of radiotelephone service to a number of points to which it may be desirable to furnish service but where the cost of apparatus for full-time channels cannot at the time be justified. This arrangement also offers an opportunity to reduce the number of frequency assignments required as a number of points may be served from one transmitter using the same transmitting frequencies where the conditions permit. It is expected that this general method will find considerable application in economically extending existing services as by merely providing the necessary antennas and receivers, one set of transmitting equipment, which is the expensive item, can be used for service to a number of points to the extent that the traffic loads permit.

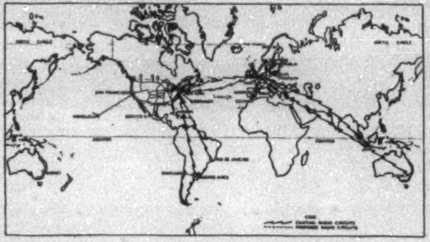

Fig. 1—International Radiotelephone Connections of the Bell System, August 1931

The radiotelephone services previously referred to and which have been established to date now make it possible for telephone users in this country to be connected to more than 91 per cent of the world's telephones. From the map shown in Fig. 1 it may be seen that these existing facilities provide the ground work for furnishing telephone service from North America to the east and south. Preliminary studies have been made of the possibilities, probable demand, and usage of telephone service from the United States to all parts of the world. A world-wide service of this kind is something that is extremely interesting to contemplate but it is also obvious that while considerable progress has been made in establishing such a service, an objective of this magnitude must be approached with due consideration to such factors as equipment and antenna development, costs, probable usage, differences in time and languages, and availability of satisfactory wavelengths.

In line with the desired objective of extending this service where practicable plans have recently been completed and the construction work is now in progress to establish in conjunction with the Imperial and International Communications, Limited, a short-wave radiotelephone channel between New York and Hamilton, Bermuda, there to connect with the land line facilities serving the Bermuda Islands. This radio circuit will, in general, closely resemble the other short-wave radio circuits previously established. The transmitter will be considerably smaller and of lower power since the distance to be covered is only about 700 miles compared to about 3,400 miles between New York and London, 4,800 miles between New York and Rio de Janeiro and 5,300 miles between New York and Buenos Aires. It is expected that the Bermuda system will be ready for operation late this year.

During the past year and a half the Bell System has acquired right-of-way and has constructed two test antennas at Bradley, Maine, near Bangor. These test antennas were constructed for experimental purposes looking forward to supplementing the existing New York-London long-wave circuit now working at 60 kilocycles with an additional long-wave circuit operating at 68 kilocycles. These antennas have proven effective at Bradley and plans are now in progress for the construction of a new long-wave transmitting station at that point. The receiving station for the additional long-wave circuit is planned for Houlton, Maine, the receiving point for the existing long-wave circuit. Owing to the magnitude of this undertaking it is expected that about three years will be required to construct the new long-wave system.

In the Pacific area the points which might logically in the future be furnished direct radiotelephone connection with the United States include the Hawaiian and Phillippine Islands, Japan, China, Australia, New Zealand, and Alaska. A transpacific short-wave radiotelephone system is now under construction, joint arrangements having been made with the Mutual Telephone Company of Hawaii to set up the initial circuit from California to Hawaii. It is a purpose of this paper to furnish some information about this system and particularly the arrangements at the California terminal.

The establishment of an overseas radiotelephone system on the Pacific Coast required, first of all, the selection of station sites for transmitting, receiving, and terminal control stations. A general survey of the west coast of the United States was, therefore, undertaken during the early part of 1930. This survey covered an area extending from a point north of Seattle to a point south of Los Angeles and inland for approximately 100 miles and included in addition to detailed studies of contour and road maps, a physical inspection of much of the area involved. It may be of interest to enumerate here a few of the requirements for transmitting and receiving station sites which were given consideration in this survey. It will be noted from these requirements as well as from the descriptions of buildings and equipment which follow that the establishment of a permanent short-wave radiotelephone system planned to furnish commercial telephone service to a number of points is an undertaking of appreciable magnitude and cost.

The transmitting and receiving station sites for commercial short-wave radiotelephony should preferably be separated by at least 25 miles and by not more than 100 miles, and be so situated that no directional receiving antenna need look toward a transmitter and no directional transmitting antenna need look toward a receiver. Furthermore the transmitting station should not be too close to a receiving station of another agency, nor the receiving station too near a transmitting station although transmitters should operate satisfactorily near other transmitters and receivers near other receivers. This requirement necessitated at the outset, consideration of the locations of existing radio transmitting and receiving stations on the west coast of the United States so that those areas could be avoided which might be expected to be subject to interference from existing stations as well as the areas which would be most susceptible to outside interference.

For a transmitting station site about one square mile of land was desirable to allow for possible future developments. The land should be relatively clear and flat as excessive unevenness will involve difficult and more expensive antenna construction. Except in special cases, low or swampy land is undesirable for radio station purposes. The terrain adjacent to the site in the direction of transmission should be clear and reasonably flat for one or two miles. Distant mountain ranges in front of transmitting antennas should not subtend an angle of more than two or three degrees. In addition a desirable transmitting station site should be reasonably near telephone connecting facilities, reliable power supply, good railroad and shipping facilities and reached by good roads.

For a receiving station site the primary consideration was freedom from outside sources of interference. Experience has shown that considerable interference to short-wave reception may be caused by automobile, airplane, and motor boat ignition systems, industrial electrical machinery, and power lines. These may best be guarded against by the selection of an isolated site. The receiving site should consist of approximately a square mile and be at least a mile from well traveled roads, electric railroads, and commercial establishments in the direction of reception. The land itself should meet about the same requirements as those given for the transmitting station. A receiving station site should, of course, be accessible from the standpoint of telephone facilities. It is also desirable to have good roads, railroad and shipping facilities and power within easy reach. These items are, however, in general, secondary to interference considerations.

From the above mentioned survey there resulted the tentative selection of several tracts of land, the physical characteristics of which fulfilled the general requirements. The next step was to obtain at each of these locations, radio field test data which would indicate their comparative desirability from a radio standpoint. These data indicated the desirability of a site in the Sacramento Valley near Dixon and about 20 miles southwest of Sacramento for a transmitting station and a site at Point Reyes, on the California Coast about 35 miles north of San Francisco for a receiving station. Tracts consisting of approximately 520 acres at Point Reyes and approximately 640 acres at Dixon were subsequently purchased.

The receiving station site has about a mile of ocean water front, with terrain of reasonably satisfactory contours and has outstanding advantages from the standpoint of isolation and radio reception. Adequate power supply is available within a half mile of the property and telephone connecting facilities are being made available by a small amount of construction.

The transmitting station site is about four miles southeast of Dixon. The land is very flat, having a deviation of less than 10 feet in elevation over the entire property. The surrounding country is also essentially flat. This site is so located that power is supplied directly from a 44-kv. substation in Dixon which in turn is tied in with the Vaca-Dixon concentration station about eight miles distant. For telephone connecting facilities the site is also well located as the San Francisco-Sacramento telephone cable crosses the property.

With the selection and acquisition of transmitting and receiving station sites only the terminal control point remained to be fixed. San Francisco was selected as the most logical location since it is a large telephone center and occupies a strategic position in the nationwide telephone network.

At the transmitting station the main building, Fig. 2, is located near the center of the property and is a two story fireproof structure of stucco on reinforced concrete and brick, approximately 40 by 60 ft. It is designed initially to house two transmitters with provision for extension as desired. There is a partial basement for heating facilities and a penthouse on the roof which contains the blowers and radiating system for use in cooling the water for the high power water-cooled tubes. On the ground floor are located the high-voltage transformer vaults, power entrance compartments, rotating machines associated with the transmitters, and battery and battery charging apparatus to furnish power supply for the voice-frequency equipment. Initially only one transmitter is being installed. This is located on the second floor as is also the line terminal or voice-frequency equipment for connecting the wire lines to the transmitter.

The transmitter is of the same general type as those located at Lawrenceville, N. J., and used in the transatlantic service. The transmitter proper consists of a number of individually and carefully shielded units mounted side-by-side. These units include a two-stage audio frequency amplifier, a Piezo electric quartz crystal oscillator with associated amplifier and harmonic generator stages, a modulating amplifier, and a two-stage radio frequency power amplifier. The output of the two-stage audio frequency amplifier is used to modulate the plate voltage of the modulating amplifier which uses two 250-watt tubes in a push-pull circuit. The carrier supply for this modulating amplifier is obtained from the harmonic amplifier stages which are driven by the crystal oscillator located in a temperature controlled oven in order to insure the maintenance of the carrier frequency within narrow limits. The output of the modulating amplifier is applied to the grids of the two-stage radio frequency power amplifier. The first stage of this amplifier contains two 10-kw. water-cooled tubes and the second stage contains six. The tubes are arranged in push-pull circuits and the entire system is carefully balanced to ground. The unmodulated carrier output power of the last stage amplifier is about 15 kw. which under conditions for 100 per cent modulation is equivalent to 60 kw. at the peak of the modulation cycle.

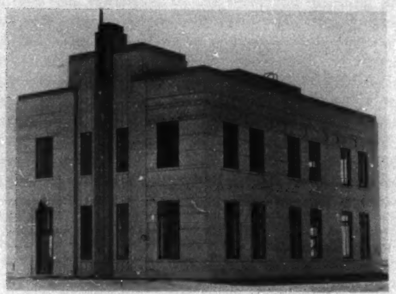

Fig. 2—Transmitter Building at Dixon, California

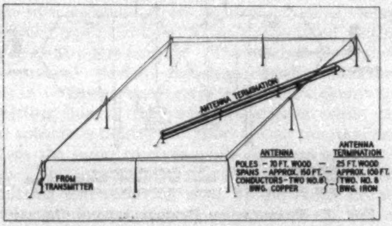
ANTENNA TERMINATION
FROM TRANSMITTER
ANTENNA
ANTENNA TERMINATION
POLES – 70 FT. WOOD – 25 FT. WOOD
SPANS – APPROX. 150 FT. – APPROX. 100 FT.
CONDUCTORS – TWO NO. 8 BWG. COPPER – TWO NO. 8 BWG. IRON

Fig. 3—Schematic of Horizontal Double-V Transmitting Antenna

The power control board consists of nine panels and is equipped with the necessary meters, switches, and relays to remotely control and distribute all power to the transmitter.

Outside of the transmitter building is a power substation which transforms the incoming power at 11 kv. to 2,300 volts, the voltage supplied to the transmitter rectifier transformers. This power substation has sufficient capacity to care for two transmitters with an average power consumption per transmitter, including miscellaneous requirements, in the order of 150 kw. with a power factor of about 85 per cent. In the power substation are also located small transformer banks for lighting and miscellaneous power as well as circuit breakers and disconnect switches so that the building and transmitter power supply may be cut off if required.

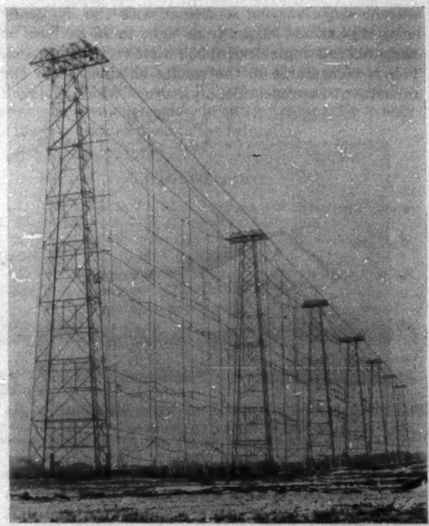

Fig. 4—Antenna Towers and Curtains at Lawrenceville, N. J., Used for Service to Buenos Aires

Three similar antenna systems are used with three radiotelephone channels to England

From the transmitter the radio frequency energy is fed over an open-wire transmission line to the transmitting antenna located about 800 ft. south of the building. The initial transmitting antenna for the Hawaiian service is of the horizontal double-V type, Fig. 3, recently developed by the Bell Telephone Laboratories.[9] This type of antenna which was first installed for use commercially for transmitting purposes between New York and Rio de Janeiro is a diamond shaped structure consisting of one or more horizontal elements supported on wooden poles. The longer axis of the diamond is pointed toward the desired receiving station. The antenna is driven from the rear apex of the diamond and when suitably terminated at the opposite end has a good unidirectional characteristic. This antenna has another very desirable characteristic in that it is effective over a range of frequencies within certain limits. For transmitting to Hawaii the initial antenna is designed to handle two frequencies in the vicinity of 7 and 14 megacycles. The gain of this antenna while varying somewhat with the frequency being transmitted ranges from eight to 16 decibels as compared to a single element half wave vertical antenna. This is comparable to the results obtained from the antennas at Lawrenceville, a picture of which is shown in Fig. 4. When, however, it is considered that each antenna at Lawrenceville occupies two spans between the 185-ft. steel towers shown in the picture, and that each is effective for only the one frequency for which it was designed it may readily be understood that the horizontal double-V antenna which is of very simple construction is much more economical. The development of this relatively inexpensive antenna is of particular significance at this time because if radiotelephone service is to be opened to points which do not justify the provision of individual channels the number of antennas for use with existing equipment may be expected to increase materially in the next few years.

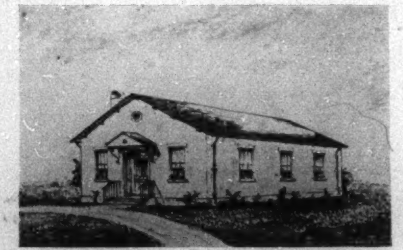

Fig. 5—Architect's Drawing of Receiver Building for Point Reyes, California

At the Point Reyes receiving station, Fig. 5, the building is a one-story semi-fireproof structure of reinforced concrete approximately 26 by 40 ft. so designed as to be capable of housing several radio receivers and their associated power and voice frequency equipment although only one receiver is being installed initially. The receiver building is located near the center of the property and about 2,000 ft. from the road. The receiver which is made up of a number of individually shielded units, mounted on four panels and assembled on four self-supporting racks, employs two stages of screened-grid radio-frequency amplification, a first detector, an intermediate frequency amplifier with input and output filters, a second detector and an audio frequency amplifier. From the output of the intermediate frequency amplifier a part of the received energy is diverted, put through a special two-stage intermediate frequency amplifier, rectified and supplied to the automatic gain control which in turn tends to keep the receiver output volume constant by controlling the bias potential on the radio frequency amplifier and the first detector grid circuits. In this receiver the first two panels which contain the equipment up to and including the first stage of intermediate frequency amplification are duplicates. By means of a switch the equipment in either of the first two panels may be connected to the rest of the receiver. This arrangement makes it possible to switch from one frequency to another with only slight tuning adjustments and, therefore, with the minimum loss of time.

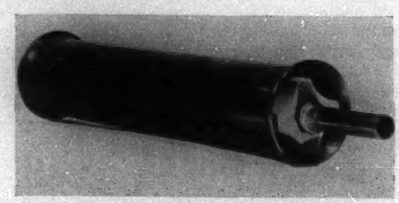

Fig. 6—Section of Concentric Copper Conductor Transmission Line Used Between the Receiving Antenna and the Radio Receiver

Central conductor 3/8 in. outside diameter

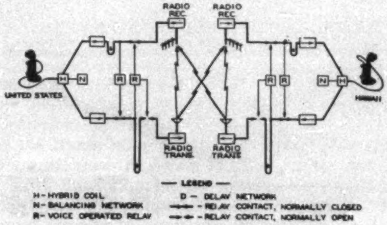

Fig. 7—Transpacific Radiotelephone Circuit

Fundamental arrangement of voice-operated devices

Filament and plate battery is supplied to the receiver from small storage batteries provided in duplicate and so arranged that one set is on charge while the other is supplying the receiver. Automatic regulation is provided in the discharge leads which holds the 24-volt filament battery voltage constant within plus or minus one-quarter volt and the 130-volt plate battery constant within plus or minus five volts.

The receiving antenna is also of the horizontal double-V type similar in construction to the transmitting antenna previously described. It is located about 500 feet northeast of the receiving building and is designed to receive two frequencies 16,195 kilocycles and 7,535 kilocycles. This type of receiving antenna is generally unidirectional and by virtue of this characteristic has a gain in the desired direction of reception about equivalent to the transmitting gain of the corresponding type of transmitting antenna. The receiving antenna is connected to the receiver by means of a concentric conductor transmission line run underground at a depth of about 18 in. This type of receiving transmission line is used because of its low transmission loss at high frequencies and because of the sheilding effect of the outer pipe conductor which is grounded. Fig. 6 shows an illustration of a short section of this type of transmission line. The outer pipe is 1⅝ in. outside diameter copper tubing and the inner pipe is ⅜ in. outside diameter copper tubing. The insulators or spacers are of Isolantite.

The terminal control station at San Francisco is located on the first floor of the Pacific Telephone and Telegraph Company's Grant Avenue toll building. To this location are brought the outgoing radiotelephone circuits to Dixon, the incoming radiotelephone circuits from Point Reyes, and the various telephone and telegraph order wires required for interstation communication. The equipment at the San Francisco terminal control station consists of a monitoring and testing position with volume control apparatus, vodas, delay, and privacy equipments with their associated testing and power supply panels.[5] The vodas equipment (voice-operated device anti-sing) briefly described consists of voice-operated relays which short circuit the transmitting line to the radio transmitter while the distant speaker is talking and short circuit the receiving line from the radio receiver while the local speaker is talking. This arrangement prevents interaction of the transmitting and receiving sides of the radio circuit. The delay equipment is designed to store up the incoming or outgoing voice currents for a fraction of a second until the vodas relay in the side of the circuit involved has had time to function and thus arrange the radio circuit for one way operation.

From the terminal control room the radio circuit is carried to a special position at the San Francisco toll switchboard where the traffic operator handling Hawaiian service is stationed. This operator has available before her a multiple of the toll circuits terminating at San Francisco and here connections may be made from the radio circuit to land line circuits extending to all parts of the country.

At the Hawaiian end the radio equipment of the initial circuit is to be furnished by the Radio Corporation of America and the general layout of the equipment is not essentially different from that at the United States end. Although it is not the purpose of this paper to describe this terminal, the transmitting station is located near Kahuku Point on the island of Oahu about 38 miles from Honolulu. The receiving station is located near Koko Head on the same island and about 12 miles from Honolulu at which point the terminal equipment is located.

It is expected that commercial service to Hawaii will be opened early in 1932. The establishment of this service will offer to the people of Hawaii telephone connection for the first time on a regular basis to the North American continent. In addition the establishment of the transmitting and receiving stations on the west coast will afford opportunities to make tests and obtain data leading to still further extension of radiotelephone communication to other transpacific points as may be decided upon in the future.

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Discussion

F. E. Terman: The double-V non-resonant transmitting antennas described in this paper represent an outstanding development in radio communication in that they provide an inexpensive highly directional antenna that can be used over a wide range of frequencies without readjustment. They also permit the same antenna to be used by different transmitters operating simultaneously on different frequencies.

There is one feature of these antennas in which I am rather interested. Being horizontally polarized, the horizontal double-V antenna is not capable of concentrating the radiation along the horizontal, which is generally considered to be desirable for long distance short-wave communication. I would like to know more about the effect which the angle of radiation has on the ability of the short waves to reach a distant receiving point. In particular, I wonder to what extent the effectiveness of the transmitter is reduced by concentrating the maximum radiation at angles in the order of 10 to 20 degrees instead of along the horizontal as is done by stacking broadside arrays as in the Lawrenceville antennas.

H. N. Kalb: What are the factors which determined the location at Dixon?

L. E. Reukema: Is progress being made in the reduction of the effects of static on long-wave radio telephone circuits, and can not improvement be made in short-wave transmission by suppressing the carrier?

Paul Ost: Why are the short-wave transmission characteristics from the United States to South America better than appears to be the case between the United States and Europe.

George T. Royden: The question has been raised of relaying over two approximately north and south circuits to avoid the difficulties experienced with an east-west circuit such as over the north Atlantic. The International System has such a circuit in operation handling telegraph business between New York and Madrid via Buenos Aires, with greater reliability than equivalent circuits over the north Atlantic.

The technical staff of the Bell System deserves much credit for the manner in which it overcame the many technical difficulties of providing commercially satisfactory transoceanic radiotelephone service. For instance, between San Francisco and Honolulu, the amplitude of the received signal frequently varies over a range twice as great as that perceived by the human ear. This is taken care of by automatic volume control and automatic switching of the circuit to one of the duplicate receiver and antenna systems which may at that moment be receiving the greater energy.

Radio engineers who have followed the cost of long range antenna systems during the past few decades, note with considerable interest the downward trend illustrated by high steel structures during the war period; a group of steel towers several hundred feet high just following the war period, a series of several antenna arrays on high wood poles during the past five years and now the simple double-V antenna supported by common wood poles only 70 ft. high and providing efficient operation over a wide range of frequencies. This development will permit the extension of communication facilities to many localities which could not be economically reached by other means.

Mr. Pilliod: The two horizontally polarized waves to be used at Dixon will be transmitted with an angle of approximately 15 and 30 deg. to the horizontal and will be determined largely by the distance to the Hawaiian station, and the wavelengths to be used. The conclusion that has been reached by the Bell System Engineers as to advantages to be gained by so-called under-modulation is that considering the service as a whole there is nothing to be gained by deliberate under-modulation and it has been concluded that the greatest advantage would be obtained by modulating the carrier wave as near 100 per cent as practicable.

Routing a short-wave radio circuit from New York to Europe via a point south from New York has been considered but taking all things into consideration including cost, the preferable arrangement so far has been to handle the service direct. Relays are of course now used in handling service from the United States to Australia and Java but this method introduces considerable complexity and some real difficulties.

The factors determining the location of the Dixon station are considered to be fairly well outlined in the text of the complete paper but, in general, locations of this kind are determined by economic considerations after the technical features have been satisfied.

Improvements have been made and others are contemplated to reduce the effect of static on long-wave systems. These are directed largely toward transmitting and receiving antenna designs, and it is hoped that the new long-wave system mentioned in the paper will have incorporated in it some of these improvements. While there would be advantages in some directions in suppressing the carrier in short-wave systems there are some disadvantages and so far, the Bell System practise and that of the other companies with which it cooperates, has been to transmit the carrier wave in the services operated from the United States.

As to north and south short-wave radio transmission being better than east and west across the north Atlantic, the times of variable results in the latter case are associated closely with times of magnetic disturbances and manifestations of aurora.

The San Francisco-Los Angeles Section of The Pacific Coast Telephone Cable Network

BY E. M. CALDERWOOD* and D. F. SMITH†

Member, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—In 1926 it became apparent that increasing demands for telephone service on the Pacific Coast would warrant the completion of a telephone toll cable between San Francisco and Los Angeles before the end of 1930. This cable, which is the longest west of the Mississippi and which with buildings and equipment represents an investment of approximately $10,000,000 is strategically located for giving service between points on the Pacific Coast and to points on the North American Continent and the Orient.*

It was completed and placed in service September 2, 1930. It provides approximately 275 telephone circuits or about the same number as would three of the latest type of sixty-wire, six-arm open-wire toll leads fully equipped with three-channel carrier telephone systems, and, in addition, certain broadcast and telegraph facilities. Two hundred and seventy miles of the cable is in underground construction, practically all being conduit which was placed specifically for this cable and other similar cables which it is expected will follow. Equipment associated with the cable is placed in eleven buildings along the route, five of which were newly constructed for this purpose.

This paper describes the fundamental data used and the methods followed in engineering and locating the cable, the problems met in constructing outside plant and buildings, which were unique in the rugged and isolated section across the Tehachapi Mountains, and certain items of interest in regard to its operation and maintenance.

INTRODUCTION

TO COMPREHEND the extensions of long distance plant of The Pacific Telephone and Telegraph Company and Southern California Telephone Company during the past few years, it is necessary for one to have a brief knowledge of the character of the country served by these Companies.

In their territory, there are four major centers of population, namely, the triangle formed by Portland, Seattle, and Spokane on the north, the San Francisco Bay district in the center, the Los Angeles-San Diego Area on the south and the Sacramento-San Joaquin Valley Area. In addition to the above centers, there is a large number of mining and lumbering localities, some active and others entirely or partially abandoned, where there are or have been relatively heavy demands for service.

This geographical situation has led to the provision of "backbone" plant connecting the principal centers of population and the construction of a network of distributing plant to tributary communities. Prior to 1920 all of the plant had been provided over a period of slow growth and on a conservative basis, but beginning about 1920, the Pacific Coast entered a cycle of greater building activity and accompanying this were the related telephone demands which made it necessary to plan for the provision of a number of short toll cables in the vicinity of Los Angeles, namely, to Long Beach, Anaheim and Santa Monica, between San Francisco and San Jose in the central area, and between Seattle and Tacoma in the north.

In 1926 it became apparent that it would be necessary to further consider major toll additions for the main north and south route between Seattle and Los Angeles. Up to this time all relief on the main route had been obtained by the addition of open wire and by the superposition of carrier[1] telephone systems (an arrangement whereby in certain instances three additional talking circuits can be made available per pair of wires by the installation of special terminal equipment) where they could be employed.

Beginning in 1926, an acceptable program was developed for the Seattle to San Francisco route on the basis of open-wire and carrier additions, this plan being warranted because of the distances involved and the relatively small growth. Between San Francisco and Los Angeles, however, the problem was different because of the greater development, and to explain the decisions covering this route, it will be necessary to discuss the plant and facility situation existing at the time major relief measures were considered.

SAN FRANCISCO-LOS ANGELES OPEN WIRE ROUTES

There were two main open-wire routes (see Fig. 1) between San Francisco and Los Angeles; one along the Coast through San Jose, San Luis Obispo and Santa Barbara known as the "Coast Route," and one via Gilroy and the San Joaquin Valley through Fresno and Bakersfield called the "Inland Route." With the exception of two phantom groups (8 wires) on the Coast Route which were of 165 mil (No. 8 BWG) copper wire, all wire on both routes was 104 mil (No. 12 NBS) copper.

The Coast Route consisted of a 12-in. spaced phantomed[2] lead (an arrangement whereby two "side" circuits and a "phantom"[3] circuit are obtained from each group of four wires) bearing three through arms and transposed for carrier[1] operation. In general, the side and phantom circuits were used for the shorter haul circuits such as San Luis Obispo-Santa Barbara, while the longer haul circuits were handled over eight

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[1] **For references see end of paper.**

type "C" carrier telephone systems providing a total of twenty-four channels suitable for longer circuits such as Los Angeles to San Francisco or Seattle. In addition to these regular services, two copper circuits were set aside for connecting radio broadcasting stations with studios originating programs, and one for picture transmission.[4]

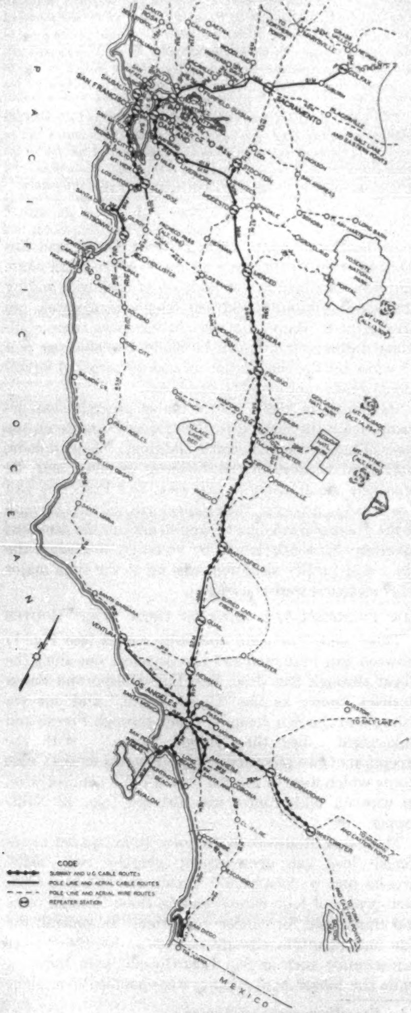

FIG. 1—MAP SHOWING OPEN WIRE AND CABLE ROUTES BETWEEN SAN FRANCISCO AND LOS ANGELES

The Inland Route via San Jose, Gilroy, Fresno and Bakersfield to Los Angeles carried two phantomed crossarms suitable for carrier and short haul service between San Jose and Fresno and a minimum of three arms from the latter point to Los Angeles. Seven type "C" carrier telephone systems, each providing three talking circuits, were operated between San Jose and Los Angeles over this route providing 21 channels for the longer circuits. In addition to these, carrier telephone systems were in use between San Jose and Fresno and between Fresno and Los Angeles for providing shorter haul requirements. One arm of open wire from Sacramento connected with the main inland lead at Fresno and provided for the superposition of two Sacramento-Los Angeles carrier systems furnishing four Salt Lake City-Los Angeles and two Sacramento-Los Angeles circuits. Two program circuits between San Francisco and Los Angeles were routed over the Inland Route, as well as an emergency assignment for the picture circuit.

In addition to these telephone arrangements, a certain amount of telegraph operated simultaneously over these leads, providing private-wire telegraph services to brokers, press associations, and others. The backbone telegraph circuits were supplied by four carrier telegraph systems (two over the Inland Route and two over the Coast Route) each of which provided ten 60-speed (60 words per minute) telegraph circuits suitable for teletypewriter operation between Oakland and Los Angeles. Also a small number of telegraph circuits, used principally for teletypewriter at a speed of 40 words per minute and for manual telegraph, were superposed on the two leads by "compositing," an arrangement enabling a telegraph circuit to be obtained from each of the two wires of a telephone circuit by the addition of special equipment.

The annual growth in telephone circuits, as estimated in 1927 was on the order of ten through or long haul circuits and varying numbers of shorter circuits depending on the location. This growth was such that it was impossible, without large expenditures and unreasonably large carrying charges to take care of the requirements by additions to the open-wire leads beyond the end of 1930.

RELIEF MEASURES

Since the relief required involved appreciable expenditures, a number of suitable plans for providing it was developed. These included:

1. String additional open wire, retaining phantom circuits and superpose additional carrier telephone systems.

2. Change one or both of the present leads to a non-phantomed[2] basis, providing for carrier operation on all arms. Under such a plan, the phantoms on all pairs

except the pole pairs would be sacrificed and the spacing between the wires of each pair would be shortened, but the lead would be so transposed that carrier could be superposed on all non-pole pairs. As an example, a 40-wire lead provides 30 voice-frequency and 24 carrier circuits making a total of 54 circuits where phantoms are retained, whereas it provides 70 on a non-phantom basis with maximum carrier application.

3. Construct a new non-phantomed open-wire lead.
4. Place a toll cable.

The first two plans were discarded because of the fairly high rate of circuit growth and the relatively small amount of relief obtained for the expenditure required. The third plan appeared more attractive but here again the growth was of such magnitude that after all carrying charges were considered, the plan was unattractive and accordingly discarded. As a final measure, the provision of relief by cable was considered and this plan was ultimately adopted.

A cable for a certain period furnishes a surplus of facilities, and accordingly it affords a more flexible arrangement for handling unanticipated circuit demands and as the circuits are not subject to trouble from line swings and other interferences affecting open-wire circuits, a more reliable service results. In general, cable circuits are less subject to interference from power exposures and consequently are quieter than those in open wire, being comparable in this regard to carrier circuits. Also the concentration of facilities on one route makes it easier to avoid high-tension power exposures, economical to make larger underground extensions, and insures minimum maintenance expenses which are in general proportional to the mileage of supporting structure. Balanced against these advantages is the possibility in case of failure that a greater number of circuits may be out of service than is the case of open wire, but with modern maintenance methods, this is not considered to be a serious factor.

In addition to caring for short- and long-haul voice-circuit demands, a cable will provide satisfactory circuits for telegraph, radio program supply, and picture transmission.

As a basis for the cable relief, studies were undertaken covering the telephone development along the desirable routes between San Francisco and Los Angeles, which included the Coast Route (435 mi.); the Valley Route (412 mi.) by way of Oakland, Modesto, Fresno and Bakersfield; and a Combination Route (425 mi.) by way of San Jose, Gilroy, and Fresno, paralleling the Inland open-wire route.

A portion of each route passes through highly cultivated country and in each case high-voltage power line development is considerable. It is perfectly feasible to find routes where private right-of-way costs are reasonable and which are not subject to power interference but these routes are also well removed from the market for telephone service. Private right-of-way costs in the vicinity of areas of extensive telephone development are excessive and the occupancy of highways with aerial plant is undesirable due to power interference and possible maintenance costs arising from changes in highway alinement. There was little advantage from these standpoints in regard to any one of the proposed routes.

It was found that, excluding the telephone stations in San Francisco and Los Angeles and the points immediately tributary to them, the telephone development along the Valley Route was greater than that on the Coast Route or the Combination Route. In addition, along the Valley Route, toll requirements as well as station growth, were greater than for the other routes.

Furthermore, there was a serious facility shortage between the San Francisco Bay area and Fresno which required immediate relief. Between Fresno and Los Angeles the short haul circuit demands were not so pressing, but other factors such as highway widening necessitated major relocation projects.

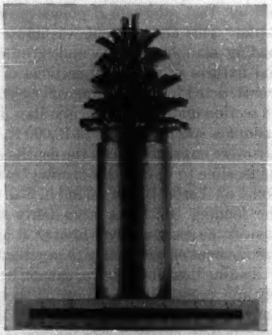

FIG. 2—FULL SIZE TOLL CABLE WITH SHEATH PARTLY REMOVED EXPOSING INTERIOR ARRANGEMENT

In view of these conditions and the fact that the shorter length of the Valley Route would effect a considerable saving, it was decided that the most advantageous relief plan would be to follow the Valley Route.

ENGINEERING FUNDAMENTALS

In the Bell System the cables standardized for toll projects, exclusive of submarine, are, in general, of paper insulated 16- and 19-gage conductors having a sheath composed of a lead-antimony alloy, and available in a number of sizes and combinations of conductors. In the case under discussion, circuit requirements were such that a "full size" cable (2.6 in. external diameter) was warranted. Full size cables (Fig. 2) weigh approximately 7.6 lb. per ft. and the sheath thickness is 0.125 in. Spirally wrapped paper insulation is first applied to each wire and then two wires are twisted together to form a pair. In turn, pairs are twisted together in groups of two to form "quads," the unit dealt with when it is desired to use phantomed[3] operation which enables three-voice paths to be derived from two pairs. In order to decrease the attenuation or transmission loss, "loading"[3,5,6] coils, contained in cast iron or welded steel cases are associated with each pair at regular intervals along the cable. These loading coils consist of series inductances and, since telephone cable circuits are highly capacitative, have the effect of improving the efficiency of the circuit for transmitting voice-frequency currents.

The basic coil spacing adopted for this project was 6,000-ft. or *H* spacing for the Oakland-Los Angeles section. In addition to the *H* spacing, *B* or 3,000-ft. spacing was desired for certain purposes and the loading was so laid out that alternate *B* points coincided with the *H* points. This arrangement was important since special loading manholes in the underground sections and special fixtures in the aerial sections of the cable must be constructed for the loading coil cases. For the submarine section under San Francisco Bay, it was necessary to adopt a special spacing of 12,000 ft. since submarine coils were impracticable in the depths considered and it was feasible to obtain this spacing by locating a loading point on Yerba Buena Island in San Francisco Bay. The loading coil cases were fairly accurately located to meet the spacings given above.

The improvement obtainable by the application of the "lumped" loading described above, cannot be extended indefinitely and practically the limit[6,7] is soon reached. Very exhaustive studies by Bell System engineers in which transmission improvement has been balanced against cost has led to the standardization of a very few types of loading.

Three of these standard loading systems were selected for use on the Oakland-Los Angeles section of the cable. One (known as the H-44-25 loading system) is used for very long haul service and consists of 44 mh. coils on sides and 25 mh. coils on phantoms, these coils being placed on *H* or 6,000-ft. spacing. This loading is placed on 19-gage wire and the facilities so obtained are operated on a "four-wire" basis (*i. e.*, separate paths for the two directions of transmission).[3,7] Having separate paths for transmitting in the two directions enables greater amplification to be employed at repeater offices and greater freedom from difficulties due to electrical reflection or echoes[7,8] results. However, with the longer four-wire circuits, it is still desirable to install equipment which eliminates the echo return path. These "echo suppressors"[8] are installed at Fresno which is approximately midway between San Francisco and Los Angeles. The next loading system, in order of decreasing excellence, consists of the same weight coils placed on the same spacing on 16-gage wire, the actual loading differing only in slight adjustments for crosstalk. These facilities are used on a "two-wire" basis (*i. e.*, the same path for the two directions of transmission). The third system of loading which has been used on either 16-gage or 19-gage wires provides circuits for relatively short haul only. This system of loading (known as the H-172-63) uses 172 and 63 mh. coils on the sides and phantoms, respectively, placed on 6,000-foot spacing. A fourth loading system (B-22) not yet in place is proposed for very high-grade service in transmitting programs to radio stations. This loading[9] is placed on 16-gage pairs and consists of 22-milhenry coils placed at 3,000-ft. intervals.

While loading decreases the attenuation of voice currents it is still necessary to amplify them at certain intervals on longer cable circuits. Amlpification is obtained by using vacuum tube amplifiers or "repeaters"[10] more or less regularly spaced along the cable. Due to the limits on the improvement obtainable by loading and the lower cost of smaller gage wires, it proves economical to place repeaters at about 50-mile intervals. There are three types of these repeaters which have been standardized. The first is the four-wire amplifier and consists of two independent amplifiers for the two directions of transmission. The second is used on two-wire circuits and consists of two one-way amplifiers (one for each direction of transmission) connected together through devices known as hybrid coils which separate the two directions of transmission.

The third type[9] of repeater is for use on the 16-gage B-22 loaded circuits transmitting programs for radio purposes. These are high quality amplifiers passing a very wide range of frequencies and are used for transmitting in one direction only.

The following points were selected for the repeater stations on the cable: Oakland, Livermore, Modesto, Merced, Fresno, Tipton, Bakersfield, Quail, and Newhall, repeaters also being used at the two terminals in San Francisco and Los Angeles. The shortness of the spacing between the repeater points at San Francisco and Oakland was justified for practical and economical reasons in connection with 22-gage cable in the submarine section.

Signaling in the cable is accomplished by the use of alternating current. As a general rule, 20-cycle current is used for signaling on the shorter cable circuits on which telegraph circuits are not superposed. For circuits over two-repeater sections long, 135- or 1,000-cycle signaling is used, depending upon the economies of the particular case. If 1,000-cycle signaling is used, it is interrupted at the rate of 20 cycles per second and has the advantage of passing through and being amplified by the repeaters. It does not necessitate equipment for relaying around the repeaters, which is the case for 20-cycle signaling current and for 135-cycle current on circuits over two-repeater sections in length.

High-grade 60-speed telegraph facilities in a cable are furnished by using the voice-frequency carrier telegraph system.[11] This equipment operates in the voice-frequency range and requires one four-wire circuit for each system supplying twelve telegraph circuits. For shorter haul requirements up to about two repeater sections in length, it is usually desirable to use the metallic telegraph system[12] which either uses pairs, or paths obtained by compositing.

Cable Selection and Description

The individual circuits in the cables are designed to provide a predetermined grade of transmission on the basis of the general toll switching plan[13] which is the fundamental circuit arrangement used in the Bell System for the layout of the toll plant. In many instances more than one design will fulfil the requirements and in these cases the most economical one is selected.

It will be of interest here to note the estimated number of circuits required for one of the periods studied with the type of facilities adopted in each case for a typical repeater section.

Table I shows the results of a study of estimated circuit requirements for 1933 as estimated in October 1928, for the Newhall-Los Angeles section. These are interpreted in terms of quads on the basis of three circuits for each two-wire quad and one and one-half circuits for each four-wire quad.

TABLE I—ESTIMATED 1933 CIRCUIT REQUIREMENTS—NEWHALL-LOS ANGELES SECTION
(Study of October 1928)

		Quads required			
		2-Wire			4-Wire
Circuit group	No. of Circuits	16-Gage H-172-63	19-Gage H-172-63	16-Gage H-44-25	19-Gage H-44-25
San Francisco-Los Angeles	51				34
Los Angeles-Seattle	4				3
Los Angeles-Portland	4				3
Los Angeles-Sacramento	5				4
Los Angeles-Salt Lake City	10				7
Los Angeles-Fresno	12				8
Los Angeles-Visalia	5				4
Los Angeles-Porterville	2				2
Los Angeles-Bakersfield	18			6	
Miscellaneous transcontinental	12				8
Voice-frequency telegraph	6				4
San Francisco-San Diego	3				2
Los Angeles-Sandberg	1		1		
Los Angeles-Mojave	3		1		
Los Angeles-Lancaster	5		2		
Los Angeles-Palmdale	3		1		
Los Angeles-Fillmore	14		5		
Los Angeles-Newhall	7	3			
Eight type C Inland carrier systems	24	8			
Total requirements	189	11	10	6	79
Proposed cable facilities	249	13	21	6	86

Although generally desirable, it was not possible, because of differences in the circuit requirements and the different rates of growth in the various portions of the route, to install a cable with the same make-up in all sections. The make-ups decided upon are given in Table II.

TABLE II—CABLE MAKE-UPS

	Cable section			
	San Francisco-Oakland	Oakland-Stockton Jct.	Stockton Jct.-Fresno	Fresno-Los Angeles
CABLE MAKE-UP				
16 gage quads		33	33	19
19 gage quads		85	85	107
22 gage quads	82	1	1	1
16 gage broadcast pairs		6	6	12
LOADING ARRANGEMENTS				
16 H-44-25, 2-wire quads		26	24	6
16 H-172-63, 2-wire quads		7	9	13
19 H-172-63, 2-wire quads		49	25	21
19 H-44-25, 4-wire quads		36	60	86
16 B-22-N, broadcast pairs		6	6	12
22 2H-44-25, 2-wire quads	42			
22 2H-44-25, 4-wire quads	40			
22 Nonload, tracer quads		1	1	1
AVAILABLE CIRCUITS				
16 H-44-25, 2-wire		78	72	18
16 H-172-63, 2 wire		21	27	39
19 H-172-63, 2 wire		147	75	63
19 H-44-25, 4 wire		54	90	129
16 B-22-N, broadcast	†	6	6	12
22 2H-44-25, 2 wire	126			
22 2H-44-25, 4 wire	60			
22 Nonload, tracer				
Total circuits*	186	300	264	249

*Not including broadcast and tracer.

†Broadcast facilities across San Francisco Bay are taken care of in other cables.

Fig. 3 is a typical cross-section of the cable used between Fresno and Los Angeles showing the location

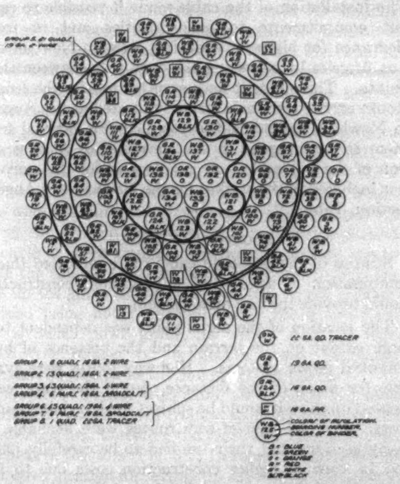
GROUP 5. 21 QUADS/ 19 GA. 2-WIRE
GROUP 1. 6 QUADS/ 16 GA. 2-WIRE
GROUP 2. 13 QUADS/ 16 GA. 2-WIRE
GROUP 3. 43 QUADS/ 19 GA. 4-WIRE
GROUP 4. 6 PAIRS/ 16 GA. BROADCAST
GROUP 6. 43 QUADS/ 19 GA. 4-WIRE
GROUP 7. 6 PAIRS/ 16 GA. BROADCAST
GROUP 8. 1 QUAD. 22 GA. TRACER
22 GA. QD. TRACER
19 GA. QD.
16 GA. QD.
16 GA. PR.
COLORS OF INSULATION
SPACING NUMBER
COLOR OF BINDER
B = BLUE
G = GREEN
O = ORANGE
R = RED
W = WHITE
BLK = BLACK

Fig. 3—Typical Cross Section of Fresno-Los Angeles Section of Cable

of the various quads and pairs contained in it, the coloring of the insulation which enables particular quads to be identified, and the system of numbering. The quads are arranged in layer formation with the 16-gage quads in the center. The twelve 16-gage broadcast pairs and the 22-gage tracer quad are interspersed in the first and third layers (counting from the outside) of 19-gage quads which are placed around the 16 gage. The heavy lines are shown merely to aid in visualizing the segregation of the various groups in the cable and do not represent a shield or separator of any kind. The tracer quad is used for test pairs during the construction period, and for making good any bad quad in a reel length.

The following schedule of completion dates was followed, which provided a reasonable manufacturing load and construction program, and a satisfactory realization of service dates when considered on a broad basis.

Section	Miles	Completion date
San Francisco-Hayward	23	1928
Stockton Jct.-Merced	57	1928
Hayward-Stockton Jct	55	1929
Merced-Fresno	55	1929
Fresno-Los Angeles	222	1930

The installation of the cable did not necessitate the immediate removal of the open-wire leads on the Coast and Inland routes, it still being desirable to temporarily retain all of these leads with a total of 16 carrier telephone systems for emergency and other purposes. The installation of the cable made it possible to carry out commitments to municipalities and to make clearance for highway widening between Newhall and Los Angeles by removing the open wire between these points. This made it necessary to transfer the Inland Route carrier terminals formerly located at Los Angeles to Newhall, extending the voice circuits derived from them to Los Angeles in the Newhall-Los Angeles section of the toll cable. A complement for this purpose was included in the studies for the Newhall-Los Angeles section, Table I.

LOCATION OF CABLE

Studies of the proposed cable route indicated that a combination of aerial and underground construction would provide the best arrangement.

The location of the aerial plant was dependent to a large extent on the terrain and the presence of high potential power systems. Highway routes were available for much of the distance, but if the plant were located on these it would be subject to high maintenance costs. Private right-of-way routes were available in many sections, but their use had to be carefully justified in view of higher construction costs due to the character of the route and the cost of the right-of-way which was an appreciable item. Balanced against these costs was the fact that maintenance expenses could be expected to be lower due to less outside interference and less plant mileage. In locating plant, cost factors are used as a guide, tempered with sound engineering judgment and careful weighting of the factors which it is difficult to evaluate in terms of money.

The route finally selected (Fig. 1) consisted of 138 miles of aerial and 270 miles of underground construction, exclusive of the submarine section under San Francisco Bay. With the exception of 18.6 miles of fiber duct on private right-of-way south of Bakersfield, all of the underground was of clay duct construction. The principal sections of the route on private right-of-way occurred between Hayward and Stockton Junction and between Bakersfield and Newhall. Both sections are hilly and the use of a private right-of-way rather than a highway route could be justified on the basis of the savings in cable mileage.

Between Merced and Madera the cable parallels an 11-kv. circuit of the San Joaquin Light and Power Corporation at a separation of about 150 feet for a distance of some 34 miles. This exposure was considered sufficiently severe to warrant some remedial measure, and a plan was developed jointly with the power company which provided for the breaking of the exposure into a number of sections and the reduction of its length by a reroute of a portion of the power line. The costs of this measure were borne jointly by the two companies.

FIG. 4—VIEW OF CABLE IN THE TEHACHAPI MOUNTAINS

Showing the general type of territory traversed, the construction road, the cleared right-of-way, and, at the upper left, the open wire lead

For 57 miles between Bakersfield and Newhall the cable is carried aerially on a private right-of-way. Thirty-five miles of this is over the Tehachapi and Sierra de La Liebre Mountains (Fig. 4), passing through fairly rough country. Some of the interesting problems encountered are described later in detail.

The right-of-way in this section was purchased from private owners or obtained over government lands and was 100 ft. or more in width. In most instances, the cable was placed in the center of the right-of-way, but on hillsides subject to fire hazards from brush, the poles were set above the center line.

The route was selected to be as short and direct as possible consistent with accessibility and economy. The most desirable location leading through fairly direct canyons and passes had to be avoided due to the presence of a 220-kv. high-tension power line from which a maximum separation was desirable to avoid inductive interference. The Ridge Route Highway, a scenic motor artery, which in general follows the summits of the ridges, was also impracticable due to the length of the circuitous route over the tops of the mountains.

The route finally taken parallels in general the Inland open-wire pole line through the mountains, although no attempt was made to follow this line and numerous departures occur at points where more advantageous locations could be found. In these mountains, elevations of 4,500 feet are encountered, the area is subject to heavy snow and occasional sleet during the winter months and, in accordance with Bell System standards, is classified as a "medium loaded" area.

CONSTRUCTION PROBLEMS IN TEHACHAPI SECTION

The country traversed in this section is generally desolate and uninhabited, there being practically no roads within a reasonable distance of the route. While accessibility had, of course, been considered as far as practicable in selecting the route, special measures remained to be devised over most of the distance, and this phase of the work required considerable study and organization.

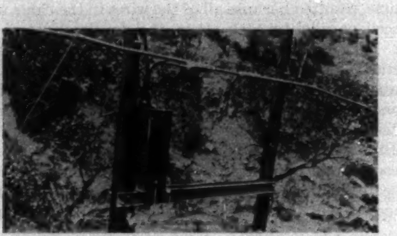

FIG. 5—CLOSE UP OF CABLE AND LOADING COIL CASE

Since all the conductors in the cable at this point were not loaded initially, the conductors were "ballooned out" within the extra splicing sleeve shown, to provide slack for splicing subsequent pots

The Ridge Route Highway following the general route of the cable is of but little value as a transportation route as the line is, in general, inaccessible from it, so it was necessary to build a total of about ten miles of new road and to widen and grade some nine miles of existing narrow dirt roads and paths. These roads were built as economically as possible, bearing in mind that they would be required for line maintenance and future construction.

The nearest railroad station from which the material could be hauled was Lancaster, located on the Mojave desert to the east. From this point a satisfactory road extended for about 40 miles toward the line, from where special transportation equipment was used over the construction roads. From the railroad delivery point at Lancaster, the principal items of material were transported to the job in a series of three steps. Ten-ton, six-wheel trucks and trailers hauled the material over as much of the distance as was accessible to them and then two and one-half ton trucks with drives on all four wheels and, in some instances, tractors and trailers, were used to deliver the material to its point of use.

As the existing open-wire lead carried the maximum wire load for which the poles were designed, namely, four crossarms, it was impossible to consider it for the support of the proposed cable. Considering the growth, it appeared reasonable to build a new pole line capable of supporting three cables and after an analysis of various types of supporting structures, fully-treated Douglas Fir poles normally spaced 110 ft. apart were selected. This particular type of pole was chosen because of its fire resistant qualities, resistance to termite attack and decay, its high fiber strength and reasonable cost.

The lead was built with 25-ft. poles. However, in order to maintain a proper grade and to care for numerous long spans, it was necessary in some instances to use poles as long as 60 ft. While the normal span of the lead was 110 ft., some spans as long as 510 ft. were constructed and required special designs and construction methods. The loading coils were supported aerially on special loading coil fixtures (Fig. 5).

Trucks (four-wheel drive) equipped with earth-boring machines and derricks were used for digging holes, setting poles and pulling cable in accessible locations. A considerable portion of the line was so located that hand digging was required and in many instances rock was encountered making the use of powder necessary.

The pulling of the cable presented interesting problems in many locations where the terrain was particularly irregular or steep. In general, the trucks with a four-wheel drive were used, but in some instances where these were not suitable, tractors especially equipped for pulling cables were employed. In pulling the strand and cable on steep grades, the general procedure was to locate the cable reel at the lower level and to pull the cable up the grade as the cable was then under control, the load being under tension during the entire pull. If it is pulled down grade, its weight must be controlled, the danger of its getting away becoming progressively greater as the pull nears completion.

The reels (weighing with cable about 6,000 lb.) were mounted on two-wheeled trailers specially designed for transporting and unreeling the cable and were delivered by trucks or tractors (Fig. 6). In some instances where the desired location for setting up the reel at the bottom of a steep grade was not accessible, it was necessary to lower the trailer with the reel down the slope on a winch line, with the tractor at the top of the grade checking the descent (Fig. 7).

Some of the grades encountered were much longer then the normal reel length of 750 ft. and in such instances it was necessary first to lower the trailer and reel to the bottom of the grade with a tractor and then to lower a smaller tractor down to the point at which the pull was to be made. The small tractor was held in position while it proceeded to "dig-in" on the side of the grade and establish itself, after which it pulled the cable up to the point at which it was located. When this pull was completed, the small tractor was drawn back up to the top of the grade and a second reel was lowered on a trailer to the point where the small tractor had been previously located and the cable on the latter reel pulled in place.

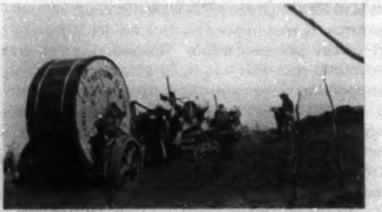

FIG. 6—TRACTORS AND TRAILER FOR HANDLING CABLE REELS

Naturally some of the locations where loading coil cases (weighing as much as 1,675 lb.) and poles were to be installed, came on steep grades and in such instances special rigging and careful handling was often required, it being necessary in some cases to rig temporary spans of winch line across ravines or canyons with special slings devised to carry the cases to their ultimate position.

USE OF GAS PRESSURE IN CABLE SHEATHS

The insulating qualities of the paper strip used for the electrical separation of the conductors of a telephone cable are entirely dependent upon the exclusion of moisture. For this reason it is essential that the cable sheath be made moisture-proof and maintained in that condition.

Methods which have proved valuable during the past few years make use of gas pressure applied within the cable. Dry compressed nitrogen has been selected for this purpose because of its chemical inactivity, low cost, and lack of moisture content. It is supplied in 100-cu. ft. tanks, with a small regulating outfit, consisting of valves, hose, couplings and pressure indicator for controlling the admission of the gas to the cable.

In general, each reel of cable was tested after being pulled into place and before opening the sheath at the ends preparatory to splicing, by injecting the gas into it under pressure. With a pressure of 12 lb. per sq. in. throughout the cable length, any sheath opening was manifested by a drop in pressure and located with the aid of the hissing noise caused by the escaping gas. Then after each splice was completed, it was tested by forcing nitrogen under pressure through a small hole in the sleeve, any opening being indicated by bubbles in the liquid soap with which the sleeve was then painted.

A recent development is the use of continuous pressure in the cable after completion, with low-pressure alarm contactors spaced along the sheath at equal intervals. A drop in pressure in the vicinity of a contactor causes it to operate and close an alarm circuit in the cable which, in turn, sounds an alarm in the testing office. The gas escaping through the hole in the sheath prevents moisture from entering for a sufficient time to permit locating and repairing the hole. This method is in use in the fiber duct section south of Bakersfield.

CABLE LAYUP AND TESTING

Early experimenters with telephone cables were both surprised and disappointed when they discovered that even for a cable only a few hundred feet in length, a telephone conversation taking place over one of the pairs could be heard well on the others. This condition, of course, resulted because all of the wires in the cable were placed parallel to each other, the development of the art not having progressed sufficiently to indicate methods for transposing pairs to avoid crosstalk. Elaborate means are taken in modern toll cables to reduce crosstalk between pairs and the picking up of noise from outside sources. These methods will now be described.

In a toll cable the various groups are kept separate, these groups retaining their identity throughout each repeater section. An example of this is shown in Fig. 3, where it will be seen that the four-wire group transmitting in one direction (Group 6) is kept apart as much as possible from the four-wire group transmitting in the other direction (Group 3) by an intervening layer of two-wire (Group 5) 19 gage intended for H-172-63 loading. The broadcast pairs are located in the four-wire group transmitting in the same direction in which they are used.

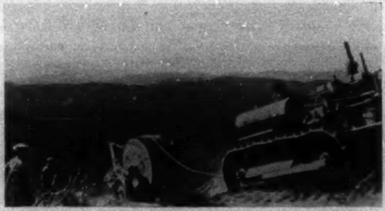

FIG. 7—LOWERING REEL IN DIFFICULT LOCATION

As previously mentioned, the paper-insulated conductors in the cable are twisted together to form pairs, two of these pairs in turn being twisted together to form a quad. This arrangement provides the initial transposing between circuits. In manufacturing the cable, three regular types of quads are made which differ from each other by the length required for the pair twists, and the length used per twist when the pairs are formed into quads. These different types of quads are used to further reduce the crosstalk between the circuits in the cable. To avoid the possibility of two quads in adjacent layers continuing beside each other, alternate layers are stranded in opposite directions during manufacture, the relationship between the layers, as shown in Fig. 3, being true only at exceptional points.

The standard reel length, used wherever possible, is approximately 750 ft. Fig. 8 shows the arrangement for splicing the cable in a normal 6,000-ft. loading section composed of eight reel lengths. When the cable is ready for splicing, the *P* splices are made first. These are known as "planned splices," at which the splicer joins particular quads in one direction to definite quads of the same group in the other. The plan for making the splice is prearranged, and the splicing pattern is so set up that different types of quads are spliced together, satisfactory separation of quads being effected and adjacencies between quads in other reel lengths being minimized. Testers are then located at the *B* splices who measure the difference in capacitance unbalance[14] for the circuits in each quad. Quads in one direction are then spliced to quads of the same group in the opposite direction in such a manner as to neutralize the unbalances to the best advantage. The *C* or final test splice, for the loading section is then made in a similar manner. At the load points the loading coils are spliced into the circuit at random, the splicer connecting the loading coils between quads in the same group in accordance with no definite plan. In case the distance between the splices differs materially from 750 ft. on account of physical conditions, the above procedure is modified so that comparable results are obtained.

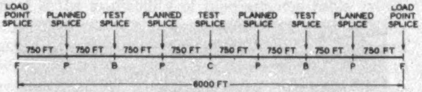

FIG. 8—TYPE AND LOCATION OF SPLICES IN NORMAL LOADING SECTION

As described above, the four-wire group transmitting in one direction is kept separate from the four-wire group used for transmitting in the other. Because one of these groups is always nearer the sheath than the other, the effect of the rotation between the layers is that the length of the wires between two given geographical points is greater for those in the group nearer the sheath. In order to equalize this difference in length, which also results in equalizing the resistance and the transmission loss of the circuits in the four-wire groups which is essential since one group constitutes the return paths for the circuits in the other, a "transposition splice" is located approximately midway between repeater points at a *B* splice where the positions in the cable occupied by the two four-wire groups are interchanged; *i. e.*, groups 3 and 4 of Fig. 3 change places with groups 6 and 7. When the cable is in operation, the direction of transmission on the four-wire group is such that the output of the repeaters at the repeater stations is always applied to the group nearer the sheath, the greater energy thus being carried having the effect of reducing the noise to signal ratio. Its position at the same time shields the quads inside against noise.

In order further to reduce crosstalk between two-wire circuits, at each repeater point the cross-connections are so arranged that while the phantom circuits are connected straight through, the side circuits are systematically rearranged among the quads. A similar procedure is followed for the four-wire circuits, at the repeater offices which are located at the ends of circuit units.

It should be noted that the efforts to reduce the crosstalk described above do not result in its entire elimination, the result being that intelligible crosstalk is minimized, a small amount of unintelligible crosstalk, picked up from many other circuits and known as "babble," being left.

The measurements and tests[14] made on the sides and phantoms (after all the splices for a repeater section are completed, and the pairs with opens, crosses, grounds, or low insulation have been cleared or tagged) are the following: d-c. loop resistance, resistance unbalance, insulation resistance, crosstalk tests, transmission measurements at 1,000 cycles which are checked by calculated values, singing point tests, and impedance-frequency runs on those circuits which have a poor singing point. In general, impedance runs are made on a sample of from 5 to 10 per cent of the circuits in addition to the above, for the purpose of checking the correctness of loading and locating irregularities.

BUILDINGS AND EQUIPMENT

Eleven repeater stations were required for this cable, Livermore, Merced, Tipton, Quail and Newhall being new offices, and building additions being added at Modesto and Bakersfield. The other repeater offices (San Francisco, Oakland, Fresno and Los Angeles) were already in service, and it was only necessary to add the equipment associated with this cable. Each of the new buildings is so constructed as to harmonize with its surroundings, and locations were chosen, with the exception of Quail (which of necessity had to be located in the Tehachapi Mountains), which were readily accessible from established communities. The equipment contained in these offices is required principally for phantoming, ringing, amplifying and regulating purposes, and its extent may be appreciated from the fact that nearly 6,000 three-element vacuum tubes were supplied initially, although, in many offices, sufficient equipment was installed to take care of only a part of the circuits which will ultimately be in service in the cable.

Of all these offices, Quail (Fig. 9), perhaps holds the greatest interest, because of the problems connected with finding a suitable location for it, in obtaining water, gas and power supplies, and housing for the personnel. The location of this station is near the Ridge Route Highway at the western end of the Antelope Valley, an arm of the Mojave Desert, at an elevation of 3,400 ft. In addition to the building for the equipment, which is of steel frame construction with reinforced concrete walls and roof, and the water tower, four modern houses equipped with electric refrigeration and unit heating systems were constructed for the personnel. With the exception of sand, which was obtained locally, all construction materials and telephone equipment were hauled by truck from Lancaster, the nearest railroad point 40 miles away, or from Los Angeles. Before construction could be started it was necessary to develop water, which fortunately was found in satisfactory quantities at depths of from 80 to 150 ft. All power required was generated locally until the middle of 1931, at which time commercial power from a substation, 30 miles away, became available. A high-pressure illuminating gas trunk line, operated by one of the gas utilities, passed nearby, and from this, gas for cooking and heating became available.

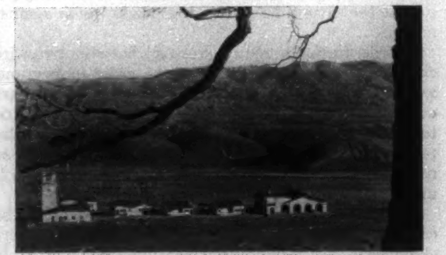

FIG. 9—EXTERIOR VIEW OF QUAIL REPEATER OFFICE AND BUILDINGS

Fig. 10 is an interior view of the Quail repeater station showing four-wire repeater bays, test board, cross-connecting frame and other equipment which is typical of that contained in all the stations. Storage batteries (not shown in Fig. 10), providing a 24-hour power reserve for all of the equipment required for one cable and associated charging generators were installed. These generators are of the commercial type and in order to eliminate noise due to commutator ripple when the 24-volt batteries are being charged while in service, a filter consisting of a choke coil[15] and three electrolytic condensers[15,16] in parallel is installed in the leads discharging from the batteries to the equipment. The choke coil which has an inductance of approximately one mh. weighs about 650 lb. and is rated at 300 amperes. The electrolytic condensers have a capacity of from 800 to 1,350 μf. apiece, depending upon the frequency.

OPERATION AND MAINTENANCE

In operating the cable, it is convenient to establish "circuit units" which consist of circuits lined up for a definite transmission value between the offices designated as "circuit unit terminals" which are San Francisco, Modesto, Fresno, Bakersfield and Los Angeles. These circuit units are then patched together to form longer circuits, it merely being necessary to insert repeaters with the proper gain at the circuit unit terminal offices. In case of trouble on a through circuit, the particular circuit unit which is in difficulty can be replaced by a spare one with only a slight delay. Each circuit unit is under the jurisdiction of a control station (San Francisco, Oakland, Fresno and Los Angeles) which is responsible for keeping it up to the required transmission standard. Test wires are set up in the cable between the various repeater points to enable each station to converse with all others as required in lining up and maintaining the circuits.

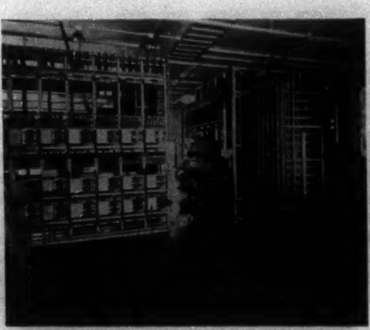

FIG. 10—INTERIOR VIEW OF TYPICAL REPEATER STATION

Due to variations in conductor resistance occasioned by temperature changes occurring between day and night and also those due to seasonal variations, the transmission loss may change for a San Francisco-Los Angeles four-wire circuit (normal loss, 9 db.) as much as 10 or 15 db. comparable variations taking place on all of the other circuits in the cable. In order to compensate for this, a "pilot wire regulating system" is installed in each circuit unit section. This consists essentially of a composited telegraph circuit or pair in the cable known as a "pilot wire," the resistance variations of which cause equipment in certain repeater stations (Livermore, Merced, Tipton and Quail) to function and automatically compensate for variations in all circuit units. The circuits are, of course, lined up for average temperatures and the regulators then provide for temperature variations in either direction.

Considerable testing equipment is required at each repeater office for maintaining the cable circuits at their highest efficiency. This equipment consists of wheatstone bridges, voltmeters, transmission measuring units[14] (for measuring the transmission efficiency of circuits and equipment), impedance bridges[14] (for locating line irregularities) and similar equipment. At Los Angeles a low insulation alarm system has been installed which brings in both a visual and aural alarm as soon as the insulation on certain selected pairs in the cable drops to a dangerous value. When this system indicates that the sheath has been damaged and moisture is slowly entering the cable and short circuiting out the pairs, the location of the trouble is determined by electrical tests and men are sent at once to repair it.

FUTURE EXTENSIONS

It is expected that future demands for toll circuits on the Pacific Coast will result in greater major extensions to the long distance cable network. Several factors tend in this direction. One of these which makes it possible to contemplate greater extensions than in the past is the availability of tape armored cable[2] which may be placed directly in the ground without a subway structure. Another is the use of automotive equipment which makes the extension of heavy cable plant into rougher country possible. It is now feasible to undertake the extension of large-size cable over such rugged country as is encountered in the Sierra Nevada, Cascade, and Siskiyou Mountains. The application of such equipment has resulted in lower costs which, in turn, make greater extensions economically possible. Also the use of gas pressure in cables to indicate potential trouble makes it practicable to maintain cables through sparsely populated areas and provide reliable service.

These items and others, which will undoubtedly be realized in the near future, permit forecasting the extension of the coast cable network to connect the Seattle Area with the Southern California Area and the ultimate use of cable to connect the Pacific Coast cable network with that of the rest of the Bell System.

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Discussion

Glen Ireland: The authors, in connection with their description of the San Francisco-Los Angeles toll cable, also have indicated the general layout of the long haul toll plant on the Pacific Coast. In this connection it is of interest to note how completely this long haul toll plant on the Coast is now connected with that of the rest of the country, including the toll cable network extending over the central, eastern, and southern sections of the country. The major points of connection are at San Francisco, Los Angeles, and Seattle. From San Francisco and Los Angeles there are direct toll circuits to such points as Salt Lake, Denver, Kansas City, St. Louis, Dallas, Chicago, and New York City. San Francisco and Los Angeles together now have 14 direct circuits to Chicago and 13 to New York City. Seattle has direct circuits to such points as Minneapolis, Chicago, and New York City.

These circuits are carried eastward over separate open-wire routes and, allowing for construction now under way, will join the cable network again at Dallas, Oklahoma City, Kansas City, Omaha, and Minneapolis. In the open-wire sections practically all of these circuits are now provided by means of type *C* carrier facilities, the eight-inch non-phantomed open-wire construction being used in many repeater sections. Protection to this transcontinental open-wire network is provided by means of interconnecting toll routes extending, for example, north and south out of Salt Lake and Denver. The San Francisco-Los Angeles cable also provides a valuable tie-in for the transcontinental routes.

The toll cable network east of Minneapolis, Omaha, and Dallas provides innumerable possibilities of routing the circuits so as to insure continuous service. The problems of furnishing transcontinental service through this cable network involves many transmission problems, particularly those brought about by the time of transmission over long distances in cable. In this connection, the use of echo suppressors on the cable facilities, as mentioned by the authors, has been essential in overcoming the more important effects and in attaining the desired transmission results.

The Kindling of Electric Sparkover
Based on Lichtenberg Figures

BY C. EDWARD MAGNUSSON*

Fellow, A. I. E. E.

Synopsis.—*A simple, experimental method is described for studying the kindling mechanism of electric sparkover. Experimental evidence is presented which tends to prove that electrons and not protons or positive ions are the active carriers in the kindling mechanism; that Lichtenberg figures form the initial step in the kindling process; that the sparkover channel develops from the positive Lichtenberg figure and that the sparkover path tends to follow the positive and negative Lichtenberg figures streamers.*

THE mechanism of the electric sparkover process has lately become a much discussed problem. Townsend's theory of the formation of electric sparkover has been challenged and, at least in some respects, found wanting. Investigations by Rogowski,[1] Hippel and Franck,[2] Torok,[3] Marx,[4] Loeb,[5] Uhlmann,[6] Beams,[7] Schumann,[8] Slepian,[9] Peek and others give ample evidence that the sparkover process is very complex; that the speed of formation is in the order of 10^8 cm. per sec. instead of 10^4 or 10^5; that polarity and shape of electrodes, ionization, space charges, voltage gradients, air pressure, temperature and other factors enter into the problem. The results of investigations on the *early stages of electric sparkover* by Steenbeck,[10] Paavola[11] and particularly by Lawrence and Dunnington[12] throw light on certain phases of the initial or kindling stage of electric spark formation, but a comprehensive explanation of the mechanism or process involved in the formation of electric sparks or sparkover is still lacking.

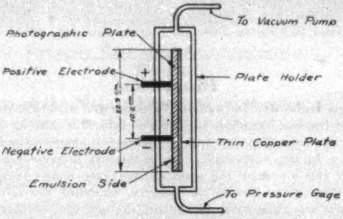

FIG. 1—CROSS-SECTION OF PLATE HOLDER

In this paper a new method of approach is described and experimental evidence presented which tends to show that for *impulse sparkover* the following statements are valid:

a. That electrons are primarily the active elements in the kindling mechanism.

b. That Lichtenberg figures form the initial step in the kindling process.

c. That the sparkover develops from the tips of the positive Lichtenberg figures along the streamers of both the positive and the negative figures.

In connection with an investigation on the *effects of the magnetic field on Lichtenberg figures*[13] it was found that the same method could be used for obtaining evidence on the kindling process of electric sparks; that is, during the first few microseconds of the sparkover process.

The voltage impulse forming the Lichtenberg figures is impressed on the photographic plate located in a strong magnetic field, with the emulsion surface at right angles to the lines of force. Two electrodes, in the holder, spaced 4 in. (10.2 cm.), are in contact with the emulsion side of the plate. The reverse side of the glass is in contact with a thin copper plate. A cross-section of the holder, with photographic plate in position, is shown in Fig. 1. The plate-holder is air-tight and the desired degree of vacuum obtained by a vacuum pump. An illustration of the electromagnet, with plate-holder in position, is shown in Fig. 2. The diameter of the circular pole-face is 10 in. (25.4 cm.); the pole spacing variable, but kept at 2 in. (5.1 cm.) for

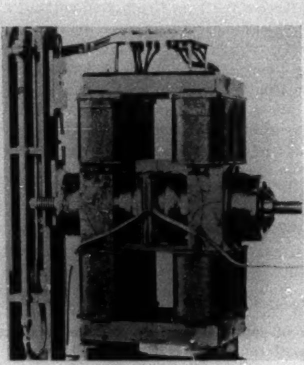

FIG. 2—ELECTROMAGNET WITH PLATE HOLDER IN POSITION

the figures in this paper, except Fig. 7 for which a smaller magnet was used.[13] Eastman: *Speedway* 6½ in. by 8½ in.; Lumiere: *Sigma* 5 in. by 7 in., and Eastman: *Hyper-sensitive pan-chromatic* 5 in. by 7 in., photographic plates were used.

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[1] For references see Bibliography.

Presented at the Pacific Coast Convention of the A. I. E. E., Lake Tahoe, California, August 25-28, 1931.

The voltage impulse was obtained by the discharge of a sphere spark-gap, diameter of spheres 10 in. (25.4 cm.). For Figs. 8 and 9 the impulse energy was increased by placing a glass-tinfoil condenser of $3.6 \cdot 10^{-3}$ μ f. capacitance in parallel with the spark gap. Cathode-ray oscillograms of the voltage impulse showed a very steep wave front, the voltage reaching peak value in approximately two and a half microseconds.

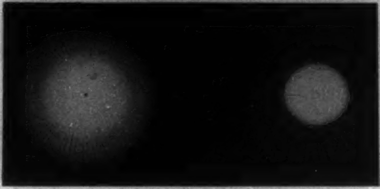

Fig. 3—Lichtenberg Figures

The initial step in the sparkover process
Air pressure: 16 cm. Hg.
Field: *S*, 5,150 lines per cm.²
Electrodes spaced 10.2 cm.

The direction of the magnetic field with respect to the emulsion surface of the photographic plate is indicated by the letters *N* and *S*. If the north pole was in front and the south pole behind the printed figure (reverse of photographic plate) the direction is marked *N*. The letter *S* indicates the reverse direction. The flux density is given lines per cm.² of field-pole surface.

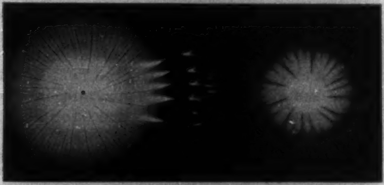

Fig. 4—Lichtenberg Figures

Suppressed sparkover
Air pressure: 13.4 cm. Hg.
Field: *S*, 5,060 lines per cm.²
Electrodes spaced 10.2 m.

In a preceding paper[13] evidence is presented on the formation process of Lichtenberg figures. The deflections of the streamers in the positive as well as in the negative figures, when formed under stress of a strong magnetic field, show that electrons, and not positive ions or protons, are the active elements at *both electrodes*.

That the positive figures are formed by electrons falling into the positive electrode is likewise the conclusion reached by Marx (*loc. cit.* page 68) on the basis of results obtained from an ingeniously devised experiment.

In essence the formation process may be described as follows:

For the negative figures the electrons are projected at high velocity, away from the negative electrode, forming spreading streamers, due to the mutual repulsion existing between the projected electrons in each streamer. The bending of the streamers from the negative electrode in Figs. 3, 4, and 5 is in accord with this statement.

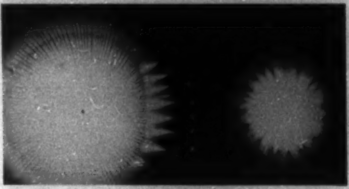

Fig. 5—Lichtenberg Figures

Suppressed sparkover
Air pressure: 12.5 cm. Hg.
Field: *S*, 5,100 lines per cm.²
Electrodes spaced 10.2 cm.

For the positive figures electrons are likewise the active elements but the process is fundamentally different. At the positive electrode the electrons are attracted towards and fall into the positive electrode instead of being repelled and projected away, as at the negative terminal. The locus of the potential gradient wave of the impressed impulse spreads outwards from the positive electrode and, if the gradient be greater than necessary to produce ionization, streams or slides of electrons, produced by secondary ionization, fall into the positive electrode.

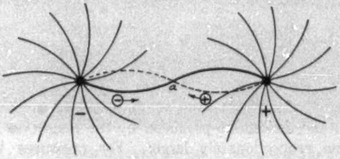

Fig. 6—Deflection of Sparkover Path Produced by Magnetic Field of *S* Direction

The grooves or channels forming the positive figures are assumed to be paths of low resistance during the inrush of the electrons, with the voltage gradient at the tip of the streamers providing the initiating ionization. These sources of the electron streams rapidly progress outwards from the positive electrode until the voltage gradient for any streamer tip becomes less than the ionizing potential. At the instant the initiating ionization at the tip of the groove ceases, the electron stream in that streamer collapses and the cycle for the formation of the positive figure is completed. If the voltage gradient at the electrode continues above the ionizing potential, or recurs, the cycle repeats as in the case of corona, brush discharges, etc. The streamers in each cycle extend outwards with the radial distances approximately proportional to the respective potentials.

At atmospheric pressure the bending of the negative and positive streamers by the magnetic field is very small. By decreasing the air pressure in the plate-holder the deflections produced by the magnetic field become proportionately larger. For pressures less[13] than 3 cm. Hg. the appearance of both the negative and positive Lichtenberg figures becomes strikingly different, when produced under the stress of magnetic fields, from those obtained under atmospheric pressure conditions. While the bending of the streamers in Figs. 3, 4, and 5 is not large, it is ample to show that the moving elements must be negatively charged bodies, that is, electrons and not positive ions for both the positive and the negative figures.

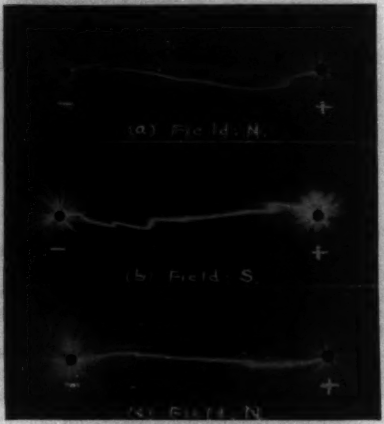

FIG. 7—THREE SPARKOVER PATHS

Air pressure: 15 cm. Hg.
Field: 12,500 lines per cm.^{2}
Electrodes spaced 5 cm.

(a) *The active elements in kindling mechanism.*

A graphical representation of the path followed by an electron, under the combined stress of dielectric and magnetic fields in space quadrature, in passing in a gas from the negative electrode through the middle point *a* to the positive electrode is shown in Fig. 6. The magnetic field direction is *S*, that is, the south pole is above and the north pole under the printed page. Hence the reaction of the magnetic field on electrons projected radially from the negative electrode causes a deflection in the counter-clockwise direction around the negative electrode as center of reference.

Consider an *electron projected from the negative electrode* under the given conditions and passing through the point *a* midway between the two electrodes. The electrostatic force of repulsion from the negative electrode and attraction to the positive electrode combined with the reaction from the magnetic field, at right angles to the direction of motion, will cause the electron to move along the path indicated by the full line in Fig. 6. It is evident that under the action of the magnetic field the paths of all *electrons falling into the positive electrode* will move in paths bending in the counter-clockwise direction, with reference to the positive electrode, as indicated in Fig. 6. Hence, if electrons be the active elements in the sparkover process the path followed by the spark should have a double inflection as illustrated by the full line in Fig. 6. On the other hand, if *positive ions or protons be the active elements* in the sparkover mechanism the path followed by the spark should likewise have a double inflection, but must be curved in the reverse direction, as indicated by the broken line in Fig. 6. *The deflections of the sparkover path therefore can be used as a criterion in determining whether electrons or positive ions or protons are the active elements in the sparkover process.*

On this basis consider the three sparkovers recorded in (a), (b), and (c), Fig. 7. In (b) the field direction was *S*, the same as used for the diagram in Fig. 6. The sparkover path has a double deflection like that of the full line curve in Fig. 6. In Fig. 7 (a) and (c) the direction of the field is *N*, that is, reversed as compared to the direction in (b). The deflection of the sparkover paths in (a) and in (c) are also reversed as compared to (b). It is therefore evident that the photographic records of the sparkovers in Fig. 7 tend to prove that electrons, and not positive ions or protons, were in each case the active elements in the kindling stage of the sparkover process.

By reducing the pressure of the air surrounding the photographic plate the bending effect of the magnetic field is proportionately increased. A series of photographs of sparkovers was taken at air pressures ranging from atmospheric to 0.02 mm. Hg. Typical results illustrating the kindling stage of the sparkover process at reduced air pressures are shown in Figs. 8 to 13 inclusive. Unfortunately it is not possible to reproduce on the printed page by means of half-tone cuts much of what appears on the photographic plate. Even the photographic prints are in many respects inferior to the original plates.

In the series of photographs from which Figs. 8 to 13 were selected the spacing of the electrons was kept constant at 10.2 cm., and the impressed voltage adjusted so as to record on the photographic plate the kindling stage of sparkover formation. The deviation of the sparkover path from a straight line increases with decreasing air pressure. The double inflection of the curve, in each case, follows the path that would be taken by negative charges, that is, electrons, as illustrated by the full line in Fig. 6. It therefore appears from the photographic records, under the given limitations of the experiments, that during the kindling stage and until actual sparkover has occurred, electrons and not positive ions or protons are primarily the active elements in the sparkover mechanism.

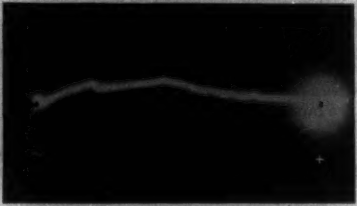

Fig. 8—Sparkover Superimposed on Lichtenberg Figures

Air pressure: 9.9 cm. Hg.
Field: N, 5,050 lines per cm.^{2}
Electrodes spaced 10.2 cm.

Fig. 9—Incipient Sparkover

Air pressure: 1.8 cm.
Field: N, 5,200 lines per cm.^{2}
Electrodes spaced 10.2 cm.

Fig. 10—Sparkover in the Making

Air pressure: 0.15 cm. Hg.
Field: S, 5,200 lines per cm.^{2}
Electrodes spaced 10.2 cm.

Fig. 11—Sparkover in the Kindling Stage

Air pressure: 0.13 cm. Hg.
Field: S, 5,150 lines per cm.^{2}
Electrodes spaced 10.2 cm.

Fig. 12—Sparkover in the Kindling Stage

Air pressure: 0.07 cm. Hg.
Field: S, 5,100 lines per cm.^{2}
Electrodes spaced 10.2 cm.

At low air pressures[13] the negative Lichtenberg figure develops grooves or channels as illustrated in Fig. 13, markedly different from the fan-shaped sectors of uniform texture at higher pressures, as in Figs. 3, 4, and 5 or for those taken under atmospheric pressure. The bending of the rays when the figures are taken under the stress of the magnetic field shows that at all pressures electrons, projected from the negative electrode, are the active elements in the formation of the negative figure. At air pressure less than 3 cm. Hg. the appearance of the positive figure changes into a nebulous form lacking definite structure. The sweep of the nebulous positive figure when formed under the stress of the magnetic field, as in Fig. 13, clearly indicates that the active elements during the kindling stage of the sparkover process are electrons falling into the positive electrode. The double inflection of the sparkover path in Fig. 13, arrested before complete sparkover occurred, is very marked, and in accord with the analysis of the path that would be followed by electrons, indicated by the full line in Fig. 6. There is no evidence in Fig. 13 or on any of the photographic plates of a counter flow of positive ions or protons.

(*b*) *Lichtenberg figures and the sparkover process.*

Although the number and range of experiments made on Lichtenberg figures and sparkovers formed under stress of the magnetic field are limited, the results obtained, combined with the many important relations previously known in regard to Lichtenberg figures and other forms of electric discharges, may warrant the following observations on the interdependence of Lichtenberg figures, electric sparks, and the sparkover process.

1. Lichtenberg figures represent, or are formed by, elementary electric sparks. On this basis the process of formation of the Lichtenberg figures would also represent the kindling mechanism of the simplest form of electric sparks.

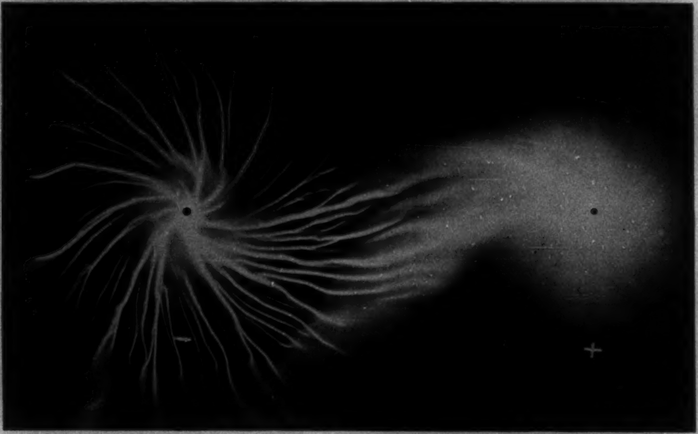

Fig. 13—Arrested Sparkover

Air pressure: 1.3 cm. Hg.
Field: *S*, 5,160 lines per cm.^{2}
Electrodes spaced 10.2 cm.
1 megohm resistance in series with spark-gap

2. Lichtenberg figures, or the electric sparks producing them, form the initial step in the sparkover process. Preceding the sparkover proper, comes "der dunkle entladungs vorström"[11] which, in a way, corresponds to the darts and streamers in Torok's "suppressed discharges,"[3] that is, a rapid succession of Lichtenberg figures forming cycles.

At the *negative electrode*, electrons are projected away at high velocity which with ionization by collision produce the negative figure. As a consequence a negative space charge is formed near the periphery of the negative figure and a positive space charge near the negative electrode.

At the *positive electrode*, as the voltage gradient becomes greater than the ionizing potential, positive streamers or slides suddenly develop and collapse forming the positive figure. For increasing impressed voltage each succeeding set of streamers extends further towards the negative electrode than the one preceding.

3. Sparkover develops from the tips of the positive Lichtenberg figures.

When a positive streamer extends near to the negative figure a rapid increase occurs, as illustrated in Figs. 3, 4, and 5, probably due to the excess of electrons forming the negative space charge at the periphery of the negative figure. If the impressed voltage be sufficiently high, the voltage gradient at the tip of the positive streamers extends the path of low resistance thereby bridging the gap to the negative electrode. This forms the initial conducting sparkover path which rapidly develops into the full sparkover channel, as illustrated by Figs. 7 and 8. The initial sparkover channel, Figs. 7 and 8, apparently follows the positive and negative streamers with sharp turns when changing from one streamer to the next. It seems quite evident that zigzag channels, as shown in Figs. 7 and 8, could not be formed by elements projected from the negative electrode; but could well be developed by electrons falling into the positive electrode; the formation process starting at the positive electrode and extending along the streamers to the negative electrode.

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Discussion

Hugh Skilling: In support of Professor Magnusson's three conclusions I am pleased to mention a study in the Ryan Laboratory last winter. That work led to very similar conclusions. The following is quoted from a report to Dr. Ryan concerning superposed 60-cycle and impulse phenomena between points.

"Contrary to expectation, experiment has shown that the existence of 60-cycle corona between points has very little influence on the phenomena exhibited by the gap when an impulse voltage is applied.

"Arrested discharges of impulses of voltage were observed, measured, and photographed between points; then without changing any other condition of the circuit, a 60-cycle voltage was applied between the points and at the moment of maximum 60-cycle voltage an impulse was thrown on and arrested. Except in slight details, the action of the impulse was unchanged." The 60-cycle voltage applied was 95 per cent of 60-cycle breakdown voltage, and gave profuse corona.

This work, combined with previous knowledge, requires a modification of accepted theory.

"When 60-cycle voltage is applied between points, it rises gradually in each half-cycle until corona commences. At once, space charge becomes the controlling factor in limiting ionization. But at all times of profuse ionization there are many electrons traveling rapidly toward the positive electrode; these are magnetically drawn together into luminous streamers of high conductivity. The cause of the intense ionization that must take place in these streamers is not definitely known, but there are several contributing factors. For instance, energy lost in the streamer will appear as heat, and as the air becomes warmer the mean free paths become greater, ionization more intense, and more heat will appear. As the mean free path becomes quite long, complete ionization of the air in the streamer may be approached. At any rate, ionization is copious enough to maintain the streamer as space charge reduces the voltage gradient from thirty to six kilovolts per centimeter; an intense space charge of positive ions will be formed at the head of the streamer as electrons are carried away, and although this will lower the voltage gradient of the streamer behind it, it will so increase the gradient ahead of the streamer that the streamer will grow longer and by this process extend itself. It will grow out from the positive point until the available voltage is largely consumed by voltage drop along the streamer. When the gradient just beyond the head of the streamer is no longer great enough to supply a large current to the streamer, its consumption of voltage per inch rises, and the current falls even more—dropping suddenly to zero and extinguishing the streamer. But in case the streamer should reach to the negative space charge that has in the meantime been deposited about the negative point, its current will be increased and in most cases it will grow to the negative electrode and so result in complete breakdown of the gap.

"After one streamer has shot out from the positive electrode, the space through which it has penetrated is filled with positive ions, and a dispersion of these is necessary before another streamer can enter the same region. There is a tendency for these positive ions to shield all routes by which streamers might issue from the positive point. Hence, it is only during a small fraction of the total time that a streamer is in actual existence and during the remaining time the electric field between the points is very little different from that which would exist in the absence of streamers. Diffuse corona is the superposition of the thousands or millions of streamers that project themselves each second, while individual filaments bright enough to be distinguished probably have a longer life.

"It is only occasionally that a streamer issues in a similar manner from the negative electrode. Presumably a higher electric field is needed to maintain such a negative streamer, and it is more intense in every way when by chance it is formed. A field-strength greater than 30 kilovolts per centimeter is a commonplace in impulse phenomena, and negative streamers are evident in every arrested discharge, but at 60 cycles it is only by chance that such a streamer may start, and even in the most violent corona they are only seen a few times in a cycle. This is not improbably due to the fact that at the positive electrode, all electrons formed in the corona will converge toward the point, while the only electrons near the negative point are the few produced near it and diverging from it.

"The chief evidence favoring this theory is found in the experiment just described, and in the well known but unexplained relationship of 10 kilovolts per inch for breakdown between electrodes of any shape if corona precedes flashover. The nature of oscillograms of corona current—showing many small positive streamers and few but vigorous negative ones—is explained, and so are many of the characteristics of asymmetrical electrodes such as the case of flashover from point to plane coming always with the point positive."

Moreover, this work agrees very well with Professor Magnusson's results. The additional proposition of the discontinuity of electric discharge seems to make more clear the action in several of Magnusson's excellent figures.

S. S. Mackeown: Some twenty years ago Townsend derived theoretically the conditions necessary to obtain a spark discharge and was able to check his results experimentally over a rather wide range of conditions. He showed that it was essential to have positive ions produce ionization as well as the electrons. Although the positive ions produce a small amount of the ionization compared to the electrons, this ionization is essential for the spark discharge.

The mechanism of the spark discharge seemed to be satisfactorily explained until a few years ago when Rogowski and others found that the time necessary for a spark discharge to occur was of the order of 10^{-8} seconds. Now a spark will occur between plane parallel plates when the field exceeds approximately 30 kilovolts per centimeter. However, it is known that an electron cannot cause ionization in a gas until it has acquired a perfectly definite amount of energy. In air this corresponds to the energy acquired when the electron falls through a potential of about 18 volts. On the average every time an electron hits a gas molecule it loses its energy in the forward direction, and the average energy that an electron will have when it hits a gas molecule is the product of its mean free path and the electric field. The mean free path of an electron in air is approximately 4×10^{-6} cm., so if the electrons are to hit gas molecules with an average energy corresponding to 18 volts, there must be an electric field of some 5×10^{6} volts per cm. However, it is not the average electron which is important in producing a discharge but those few which have acquired high velocities due to either having traveled a distance large compared to the mean free path without making a collision or else due to the fact that they have not lost all of their forward momentum at the last collision. It is difficult to see how there could be an appreciable number of electrons having an energy greater than 10 times the average. If then we assume that the discharge is produced by electrons having energy 10 times as great as the average, we would expect a discharge to occur when the field was approximately 5×10^{5} volts per cm., which is more than 10 times greater than that at which it does occur. The fact that a spark occurs in 10^{-8} seconds indicates that the field must be distorted so that at some place it has a value of 10 times the average and that this distortion occurs in 10^{-8} seconds. Slepian has gotten around this difficulty by assuming thermal ionization, which, however, for several reasons is not acceptable. Hippel and Franck as well as Loeb assume that this distortion is produced by an initial flow of electrons which distort the field, but their theories are not completely satisfactory. If we assume that this distortion of the field does exist, then Townsend's theory is satisfactory to explain the occurrence of a spark. Consequently, the interesting part of the spark discharge is what occurs in the first 10^{-8} seconds.

Professor Magnusson's pictures were produced with a voltage wave which reached a maximum in 2.5×10^{-6} seconds and the pictures are a record of what occurred in probably 10^{-5} seconds. This is 1,000 times the length of time necessary for the sparkover to occur. Unfortunately there does not seem to be any way to determine the sequence of events in this period of time. As far as the pictures go it is perfectly possible that ionization occurred first at the anode and later at the cathode, or *vice versa*, or that they occurred simultaneously. It would, therefore, seem dangerous to draw conclusions regarding the actual formation of a spark from the data present in this paper.

I would like to reiterate again that while the greatest part of the ionization is due to electrons, yet ionization by positive ions is essential and no spark can occur unless both types of ionization are present.

H. G. Cordes: The statements (a) and (c) of Professor Magnusson's paper and the theory deduced from the figures appear to require critical examination. Consideration of either mean free path or space charge invalidates the theory illustrated by Fig. 6. The mean free path of an atom in air at atmospheric pressure is of the order of 10^{-5} cm. and at 0.07 cm. Hg. pressure it is 10^{-2} cm. Although the mean free path of an electron is longer than that of a positive ion, the separation of electrons and ions is too small for observation by inspecting an illustration hence they cannot follow the widely separated paths shown by Fig. 6. Electrons may be the principal charge carriers due to their small size and high velocity but enough positive ions are formed by collisions to reduce the space charge to practically zero.

Reference to Fig. 7 does not seem to sustain the theory because both the positive and the negative streamers are recorded on the emulsion before a spark passes directly between the electrodes. In other words the streamers are ionized paths produced by displacement current flowing through the copper plate and a spark directly between the electrodes is initiated by a relatively short spark from the tip of a negative to the tip of a positive streamer. The short spark took place in Fig. 7A at a point below a straight line between the electrodes whereas in Fig. 8 it passed above the line; this indicates that preliminary ionization of the streamers rather than the blow-out effect of the magnetic field determines the location of the short spark. If this explanation is correct, omission of the copper plate should result in the spark always taking place on the same side of the straight line and in requiring a higher potential to initiate the spark while the pressure and field, of course, remain the same.

The streamers recorded by the emulsion on the photographic plate are probably produced in steps even when a potential of *very steep wave front* is impressed on the electrodes. The presence of grooves at low pressures shows that the record is made by the heat of the discharge instead of by actinic rays; the resulting emulsion vapor is an unknown factor with respect to pressure and composition.

The presence of canal rays in a continuous, but otherwise similar, discharge indicates that positive ions are a component of a spark discharge; pure electron discharges, which take place without this component, have been produced only in very highly rarefied spaces.

C. E. Magnusson: It is indeed gratifying that Dr. Skilling's experiments made in the Ryan Laboratory, "led to very similar conclusions" although the experimental conditions were as widely different as may well be obtained.

In reply to Dr. Mackeown's assertion that "ionization by positive ions is essential and no spark can occur unless both types of ionization are present," let me refer to an important contribution recently made by Rogowski,[*] in which he reaffirms his earlier contention that for impulse sparkover the positive ions do not play the role ascribed to them by Townsend's theory. He shows that for short time impulses, 10^{-5} sec. or less, the positive ions are essentially stationary.[†]

In regard to Mr. Cordes' question let me state that with both electrodes in contact with the emulsion surface (Fig. 1) the bending of the rays and the double inflection of the sparkover path are in the same direction whether a metal plate is in the plate-holder or not. The presence of the metal plate aids in producing more sharply defined figures, especially at low air pressures, but does not affect the direction of the bending of either the positive or the negative streamers or the double inflection of the sparkover path.

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[†] Rogowski, W. *loc. cit.*, p. 593.

Radio Coordination

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INTRODUCTION

THIS paper is written from the authors' experience with radio interference problems in northern and central California. An attempt has been made to present the various phases of the problem and their relation to each other rather than an extensive presentation or solution of any particular phase. The application of mathematical theory for the solution of radio interference problems is in many ways highly complex. Fundamental formulas are used only to present the material more clearly.

Radio interference from natural static and radio sources is not within the scope of this paper and is mentioned only in so far as it has a bearing upon interference caused by electrical sources. Natural static is less severe in the West than in the East, making other types of radio interference more apparent. A larger network of higher voltage lines has been built up in the West because of the extensive hydroelectric development and a more scattered population and this fact also alters the problem to some extent.

The variation of broadcast service area due to power and location and the topography of the surrounding country have an important bearing upon the radio interference problem. The problem will require different treatments depending upon the broadcast coverage in different sections of the country.

HISTORY AND DEVELOPMENT

During the past decade, broadcasting for entertainment and other purposes has grown from a small unimportant industry to one of major importance. The broadcast of scheduled programs for the public really began in the latter part of 1922. In 1923, a few complaints of radio interference were received by the power companies, doubtless, because they are depended upon to a great extent to help their customers in various electrical problems. The power companies with little to draw from by way of a background of experience or research in this field, met the problem with a spirit of cooperation and helpfulness toward the broadcast listener and the radio trade.

The first interference investigations revealed a great number of home-made receivers, many of which were of the single-circuit regenerative type. These home-made receivers had many loose connections and run-down batteries, which accounted for a large percentage of the radio interference complaints at that time. The average broadcast listener did not complain about poor quality of reception or a small amount of noise. He did not protest unless the interference was serious enough to make reception from most broadcast stations impossible. As the radio industry grew, the tone quality of radio transmitters and receiving sets was improved and listeners took more interest in obtaining good reception. Battery receiving sets were used and listeners found they could purchase fairly satisfactory manufactured sets at a reasonable price. Interference due to poorly constructed radio receivers and regenerative receiving sets was reduced. It soon was found that considerable interference originated in industrial and household electrical devices, utilities' lines, and in other miscellaneous electrical and radio sources. Interference was picked up by battery supplied receivers by direct induction in the antenna-ground system and only the more severe cases of interference were found troublesome.

The next important change was from battery-operated receivers to receiving sets supplied directly from the a-c. lighting lines. Battery eliminators coupled receivers conductively to the lines, and noises not previously noticed were brought in and resulted in an increase in the number of complaints received. An increase in the number of interference complaints was also caused because radio was a considerable novelty and listeners attempted to get low powered distant stations and were often very successful. The next change included the radio power supply as part of the receiver and this change further increased the coupling between the power supply lines and the radio receiver itself. The modern radio receiver, made in a single unit, has excellent tone quality, satisfactory selectivity and high sensitivity. Some of the later superheterodyne receivers have a sensitivity of less than one microvolt per meter. With the excellent programs now broadcast and the desire of the radio listeners for clear reception, interference complaints have increased. They have also increased because of the greater number of receivers in use and because keen radio retail competition is pressing radio sales in noisy locations and in places remote from broadcasting stations.

During 1926, after the breakdown of the 1913 radio act, broadcasting stations sprang up almost overnight. Considerable interstation heterodyne interference developed. The present Federal Radio Commission (created by the radio act of 1927) held its first meeting on March 15, 1927. Order in broadcasting was brought about by relicensing all stations, reassigning frequencies and power. On the Pacific Coast, heterodyne interference is a minor factor at the present time. The present trend toward the superheterodyne receivers will allow higher power in broadcasting without

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*Pacific Gas and Electric Co., San Francisco, California.

†San Joaquin Light and Power Corp., Fresno, California.

Presented at the Pacific Coast Convention of the A. I. E. E., Lake Tahoe, California, August 25-28, 1931.

seriously increasing heterodyne interference. Interference from radio sources is not within the scope of this paper and is mentioned only because at this time many interference complaints received by the power companies were found due to this cause.

Fig. 1 shows the total number of receivers sold including consoles, midgets, and radio-phonograph combinations. The number of receivers manufactured has increased from 100,000 in 1922 to 4,438,000 in 1929. The curve of values includes the amount of sales in dollars of all receivers, tubes, parts, and accessories. The retail value of radio sales increased from $60,000,000 in 1922 to $842,548,000 in 1929. The average selling price of a radio receiver has varied from $50.00 in 1922 to $127.00 in 1927. Since the latter year, the price has dropped gradually and in 1930 it was $82.00. The average retail unit price of tubes has decreased from $6.00 in 1922 to $2.30 in 1930. Of the 30,000,000 receivers now in use in the world, approximately 50 per cent or 15,000,000 are operated in the United States.[3]

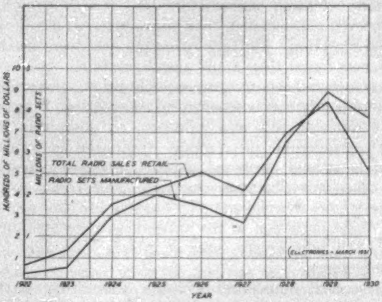

Fig. 1—Number and Value of Receiving Sets Sold in the United States 1922-1930

Advantages of Radio Interference Investigations

The advantages of radio interference investigations to all branches of the radio trade and to the radio listeners are many and obvious. The elimination of radio interference under the present system of broadcasting is of considerable value to the individual broadcast stations in permitting them to reach a greater audience with their programs.

In radio the power company has a very desirable load, as it yields a higher revenue per kilowatt of demand than most other electrical appliances. The connected radio load in the United States is estimated at 1,500,000 kilowatts and on the basis of 4 hours daily use and $0.05 per kilowatt hour, this means an approximate revenue of $100,000,000 per year. Extra heating and lighting also account for an appreciable increase in revenue. Some companies are now making regular use of radio to patrol their lines. Perhaps the most important advantage of interference investigations is the customer good-will building possibilities.

Responsibility

Table I shows the number of complaints investigated yearly by the Pacific Gas and Electric Company from 1924 to 1930. The complaints investigated by agencies other than the company itself are not included. The increase in the percentages chargeable to the power company in 1928, 1929, and 1930 was due to several causes. The first and perhaps the most important was the large number of a-c. receivers sold since 1928. Alternating current receivers being conductively coupled to the supply lines makes them more susceptible to line noises. Radio trade association's investigations also started in 1928 and since they only cover metropolitan areas where the percentages of complaints due to power companies are the least, the omission of these complaints in the table also accounts for the increasing percentages shown since 1928.

TABLE I

Year	Number investigated by P. G. & E.	Per cent due to P. G. & E. Co.	Per cent due to other causes	Per cent cleared themselves
1924	90	30.0	37.5	32.5
1925	469	45.0	33.0	22.0
1926	1,782	30.0	40.5	29.5
1927	2,384	31.8	47.7	20.5
1928	2,973	36.6	40.6	22.8
1929	3,479	50.0	33.1	16.9
1930	3,736	51.5	32.4	16.1
Total	14,913	42.2	37.7	20.1

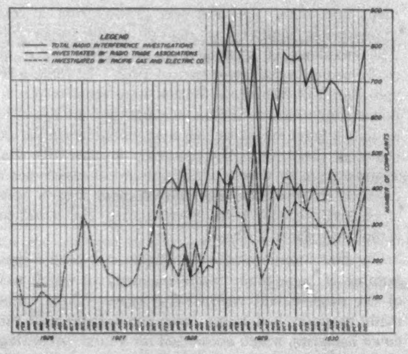

Fig. 2—Radio Interference Complaints Investigated by the Pacific Gas and Electric Company 1926-1930

The increase in the number of complaints shown in Fig. 2 is due to the increased number of radio receivers sold and in use, the increased sensitivity of the receivers, the publicity given radio interference investigations by radio trade associations, and a greater desire on the part of the radio listening public for programs free from interference.

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[3] For references see Bibliography.

The curve shows a definite seasonal trend, the peaks usually occurring during December or January. The increase during the winter months may be accounted for to some extent by the larger number of receivers sold at this time of year, especially at Christmas. People remain at home more during the winter months and spend more time listening to the radio programs. Many noises heard in the summer time are charged to natural summer static and no complaint is made. There is a tendency for the curve to flatten out more each year during the summer months showing people are spending more time than formerly listening to the summer broadcasting.

Any interference producing device connected to the electric power supply network, for which utilities cannot be held responsible and over which the utility has no control, feeds radio frequency disturbances over a considerable portion of the utilities' circuits. This is the responsibility of the appliance manufacturer and not that of the individual owner. In most cases the latter buys and operates such a device without knowledge of its causing interference.

The responsibility for improper or inadequate installations and for defective radio apparatus rests directly with the radio trade. The public's responsibility is doing business with a reputable radio dealer and cooperating with the interference investigator.

COOPERATION

There ought to be close cooperation between parties responsible for radio interference in order that the best possible public service may be given in the most economical manner.

Cooperative radio interference investigations have proved valuable in metropolitan areas where the general noise level is caused by a large number and variety of electrical devices. This is true to a greater degree in congested areas than in the less populated areas. Cooperative investigations usually are conducted by a radio trade association. The radio trade association can take the part of a neutral and unbiased party, locate sources of interference, regardless of its source, and properly place the responsibility. Under such a plan the power company, of course, loses its identity in the matter, which may or may not be to its advantage. Cooperative investigations have been carried on successfully in some parts of California for the past three years (1928-1930).

COVERAGE OF BROADCAST STATIONS

"Fair service broadcasting is roughly estimated by some at one mile for a 5-watt station, ten miles for a 500-watt station and one hundred miles for a 50,000 watt station, but 5 watts may be heard 1,000 miles on a winter night and any amount of power will usually fade at 100 miles, all of which causes confusion and misunderstanding on the part of the uninitiated."

"Reliable service means good reception at all hours of the day and night throughout the year under the most favorable interference conditions. With interference present, the distances will be greatly reduced."[1]

It must be remembered that from an interference standpoint the quality of reception is dependent on the ratio of modulated field strength to noise level. Various writers have suggested values for this ratio varying from 15 to 150. If 80 per cent modulation of the broadcast wave and a noise of 30 microvolts per meter is assumed, the field strength required to overcome the noise would be from 565 to 5,650 microvolts per meter. If the ratio is assumed to be 100, then a field strength of 3,750 microvolts per meter would be required. With these assumptions and the present power and location of broadcast stations in the west, only a small percentage of the area can be receiving fair broadcast reception.

In the San Joaquin Valley there is only one broadcasting station (100 watts) and that station (K. M. J.) is located at Fresno. The larger stations are located at San Francisco and Los Angeles at an average distance of approximately 160 miles and 200 miles respectively from Fresno, and in both cases separated from the valley by mountain ranges. A broadcast field strength survey of this valley was made in the middle of November 1930, readings being taken at seven different cities scattered throughout the entire length a distance of approximately 180 miles. The Pacific Telephone and Telegraph Company had made similar readings at two of the same locations in the month of September. From these readings a curve was made to show what field strength could be expected throughout the valley, if the whole survey had been made in September. Such curves for the largest San Francisco and Los Angeles stations are shown in Fig. 3. It should be noted that the rated power of K. P. O. is 5,000 watts as against 1,000 for K. F. R. C. However, there is not a 5 to 1 ratio between their field strengths, but this ratio continues to become smaller as the distance from the broadcasting stations increases. The average field strength at Fresno of all the stations shown is approximately 130 microvolts per meter. If the power of these stations were increased to ten times the present, this average field strength would then probably be something less than 400 microvolts per meter, which would not give a sufficient ratio between signal and noise to provide coverage of this area. This is not peculiar to the San Joaquin Valley, but applies to all other sections distant from the two broadcasting centers, *i. e.*, San Francisco and Los Angeles.

Therefore, it will be seen that there is a need for better broadcast coverage in California. Table II shows the airline distances from the San Francisco area, where the most powerful northern California broadcast stations are located, to some of the larger cities in northern and central California. A similar condition prevails in southern California, Oregon, Washington and Nevada.

TABLE II

City	Airline distance from San Francisco
San Jose	40 miles
Stockton	65 "
Sacramento	75 "
Salinas	85 "
Auburn	100 "
Marysville	100 "
Chico	140 "
Fresno	160 "
Red Bluff	170 "
Redding	200 "
Eureka	233 "

"The rural listener of the west also has little choice of programs, due to the fact that radio stations in the fifth zone, which embraces two-fifths of the area of the United States, have been allocated only 65,000 watts power, while the stations in the other zones have power aggregating 525,000 watts. Perhaps too much thought has been given to population and not enough to area in the allocation of power and frequencies."[2]

The steady flow of current as in an arc light or rectifier tube under normal operating conditions will not cause serious interference. The air remains completely ionized and becomes a good conductor. A spark causing high-frequency currents may or may not be necessary for the normal operation of the apparatus. The high-frequency currents are either directly radiated by the circuit in which the spark takes place or are transferred to another closely coupled circuit from which radiation takes place. When free to oscillate, the circuit in which the spark takes place and all coupled circuits oscillate at all frequencies in which the inductive reactance is equal or nearly equal to the capacitive reactance.

$$\text{If } R > 2\sqrt{L/C} \text{ No oscillation}$$

$$\text{If } R < 2\sqrt{L/C} \text{ Oscillation}$$

The resistance of power circuits is generally low and therefore usually will oscillate in resonant parts or throughout when excited by a spark discharge.

Interference originating on a 60-cycle primary circuit (2,300 to 25,000 volts) has in most cases a characteristic sound. The gap producing the interference breaks down near the peak of each half of the 60-cycle voltage wave and sets up damped high-frequency wave trains.

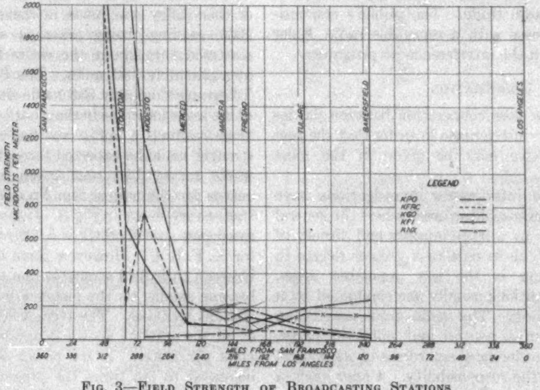


FIG. 3—FIELD STRENGTH OF BROADCASTING STATIONS

SOURCES OF INTERFERENCE

For analytical purposes, any source of radio interference may be referred to as the influence; the ability of the receiving set to pick up and reproduce the interference, the susceptiveness; and the resistance of the path between the influence and the receiving set as the coupling factor.

In general, any spark which causes a rapid change in current flow or the sudden discharge of a stored electrostatic field will generate radio frequency currents.

This excites the circuit 120 times per second on a 60-cycle system and the radiated wave produces 120 vibrations in the telephone headset or loudspeaker. Interference from the higher voltage lines (25,000 volts and up) may best be described as a "roar." This is due to numerous individual sources of interference differing in magnitude and occurring at different times.

Interference from appliances connected to service lines has a different sound than the above mentioned cases, although supplied from a 60-cycle source of power. These sounds may best be described as mechanical. Examples of this type of interference are vibrating devices, thermostatically controlled appliances, brush motors, etc. It is very difficult to describe sound in a satisfactory manner but an interference investigator learns the different types by experience. Interference originating on the low-voltage side of a distribution transformer usually causes trouble only to radio listeners supplied from this transformer or for a short distance on the high-voltage line side.

At one time amateurs were blamed for considerable interference but have proved their innocence to a great extent. Furthermore they have shown a friendly spirit of cooperation in the elimination of interference.

An attempt to list completely all sources of radio interference would result in repetition as far as fundamental causes are concerned. Sources of interference may, however, be divided into the following general groups:

1. Utilities.
    1. Electric power and light companies.
    2. Electric Railway companies.
    3. Communication companies.
2. Industrial and Domestic Electrical Apparatus.

Electric Light and Power Companies

Transmission Lines—30,000 volts and higher.

1. *Brush discharge from conductor.* This is not of major importance at present. It is due principally to roughness of the surface of the conductor.

2. *Suspension type insulators.* This is not of major importance at present. The discharge comes from the gap between the metal cap and porcelain, and around the pin at the point of contact with the cement.

The remedy is to provide sufficient clearance between cap and porcelain or fill the gap with a compound that will remain slightly plastic and will not crack. At the pin the metal should have no sharp edges or points and the cement should be smooth and even with the porcelain surface. Additional units can be used to keep the voltage across any unit below the critical interference voltage. If it is necessary to use corona shields, they should be placed so as not to allow a brush discharge to the porcelain.

3. *Bushings—Switch and Transformer.* Oil-filled or compound-filled bushings should give no trouble except in the possible case of potential transformer bushings where a pin connection is sometimes made. This connection should be made more positive as the oxidation on the pin causes a discharge to take place. Bushings with air columns for conductor space may be lined with metal if a close contact is made between the metal lining and the porcelain. If the conductor can be made a part of the bushing, the hole may be filled with metal filings of non-magnetic material, or with oil. If the conductor is to be removable, a split brass sleeve will answer fairly well. Copper plating, if properly done, provides a very good result. Copper wire screen may be used with good results provided it is well covered with an insulating varnish. In all cases the metal must at some point make good contact with the conductor. Bushings to be used in connection with bushing-type current transformers should have a copper-plated surface from the supporting flange to a point over the cement ring joining the lower part of the outer porcelain tube and the adjacent tube. This acts as a static shield.

4. *Pin Type Insulators.* There are several possible sources of radio interference involving pin type insulators. A great amount of work has been done to develop a new insulator that will not produce interference, by metallic glaze, coating, etc. Inexpensive methods of treating insulators now in service would help the situation.

5. *Bond wires and hardware on wood pole construction.* Bond wires should always be placed between double nuts and never next to the wood. Hardware should be so spaced as not to allow discharges, or it should be well bonded.

Distribution Lines—5,000 Volts to 30,000 Volts.

1. *Hardware discharges.* This can be cared for by changes in construction standards which provide for either greater separation or proper bonding.

2. *Air switches.* Switches now in use can be bonded by the operating companies, and those being manufactured can be provided with proper bonds by the manufacturers. Where iron caps are cemented to the tops of insulators in the making of these switches, sufficient clearance should be allowed between the lower edge of the iron cap and the porcelain.

3. *Transformers.* Interference from distribution transformers is due to several causes and is corrected in most cases by changes in design, such as bushings, windings, terminal blocks, etc., each type of trouble being somewhat peculiar to some location or climate. These troubles are being cared for on present installations by the operating companies and the new products are being put out by the manufacturers according to specifications for the companies operating in the different localities.

4. *Fuses.* Metal parts on fuses for this class of lines should be so placed that there will be no discharge from the metal to the porcelain, or between the different metallic parts. The fuse wire in this class of fuses is subject to what is sometimes called corrosion and when this takes place, discharge to the side of the fuse tube usually occurs. The only remedy for this trouble is to replace the fuse wire. This trouble does not seem to occur where the tube is filled with a nonconducting dust.

Lines Under 5,000 Volts.

1. Interference from this source is comparatively slight and not as important as that from the higher

voltage lines. Poor connections are responsible for many of these cases, and loose fuses account for many complaints. Old type demand meters cause considerable trouble but there are comparatively few of them, they are easy to locate, and are being replaced very rapidly.

General. As an aid to utilities and more especially to electric companies in the elimination of interference the following from an operating company may be a helpful suggestion.

Early investigations brought to light the many points at which radio interference originates on pole line structures. In some cases the remedy was a change in standards to provide more clearance between the metal parts, and in other cases it was necessary to provide for proper bonding. In both instances it was found most advantageous to instruct the field forces not only by letter but by demonstrations and photographs. It is somewhat difficult for the average man to believe that the noise which obliterates "Amos and Andy" comes from a spark smaller than a pinhead, and located between the ends of two crossarm braces. Accordingly a pole top was equipped with dead-end construction, a set of inverted braces, high-voltage sign, space bolts, etc., arranged to cause radio noise. (Fig. 4.) This structure was placed in the transformer warehouse and energized from the high-voltage side of a bank of 5-kw. 11-kv. transformers. This was done at night, in the dark, in order that each man could walk by, see the spark, and hear the noise in a radio set located nearby. This demonstration accompanied with explanations was given before all district foremen, and also before the workmen in many of the district headquarters with excellent results. Along with this demonstration, instructions were given on the proper bonding of 11-kv. air switches. Sample jobs of bonding were placed on the floor, and near the ground on poles just outside of the warehouse. Photographs (Figs. 5A and 5B) of these bonding jobs, together with instructions, were distributed and each district foreman posted these photographs and instructions on the bulletin board for the workmen.

FIG. 4—POLE LINE RADIO INTERFERENCE DEMONSTRATION

Electric Railways

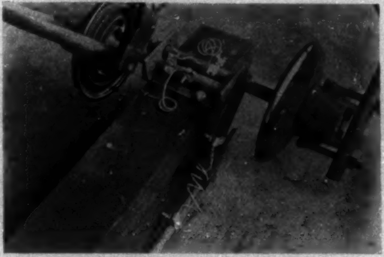
Many power companies either own or are vitally interested in street railways. Their sources of interference are not numerous but very baffling.

1. Commutator sparking on the truck motors can be reduced by increased maintenance.

2. Compressor motor noise can be greatly reduced by making the connections so that the field goes to the trolley and the brushes to the track.

3. Trolley contact. There is not much to be done for this except a more frequent turning down of the trolley wheels and greater maintenance. In some cases, a shoe has been substituted for the wheel and lubrication of the trolley at the same time gives added relief from interference and longer life. This is only in the experimental stage at present. It would seem that the addition of grease would provide a means of partially smothering the small arcs, as the grease is melted, and some further study along this line may prove productive of good results.

4. Rail contacts. The only means of reducing rail contact arcing is to keep the rails and truck wheels clean. This is simply additional maintenance.



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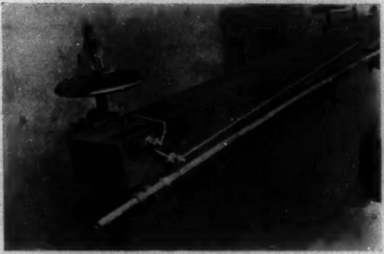

B

FIG. 5—BONDING DETAILS FOR AN 11-KV. AIR SWITCH

Communication Companies

Various units of apparatus associated with communication circuits for giving telephone and telegraph service may be sources of radio interference, among which are ringing machines, pole changers, telephone typewriters and dial telephones. These possible sources of interference are being given considerable attention by the telephone companies and those which would be expected to cause interference are usually treated at the time of installation. Other cases are cared for when and if interference is experienced. The suppression equipment generally used consists of retardation coils and by-pass condensers, sometimes used with resistors, applied singly or in combinations according to the needs of the case.

Industrial and Domestic Apparatus

Radio interference from this class of apparatus may be divided into three types: First, that coming from commutating apparatus,

Second, that coming from make and break contacts, and

Third, that coming from high-frequency apparatus such as violet-ray sets, diathermics and X-rays.

1. Commutating apparatus can be corrected by the use of two series condensers across the brushes, with the center point grounded to the frame of the motor. In larger installations it may be necessary to install air core chokes in the leads.
2. The make and break interference may be corrected by shunting across the contacts a resistor and condenser in series and in severe cases by the use of air core chokes in the leads.
3. To suppress the interference from high-frequency apparatus of this type it is usually necessary to have the room in which it is used thoroughly shielded and a suitable filter inserted in the supply leads.
4. General. It is possible to purchase filters to correct practically all the troubles under this heading. The proper selection may be obtained by either consulting a reputable dealer or writing direct to the manufacturer of these filters.

PATHS OF RADIO INTERFERENCE

In most cases radio interference is conducted and radiated by the power supply system. The distance conducted and radiated depends upon the power generating the spark, the width and resistance of the spark-gap, and the natural periods of oscillation of the power supply system and connected apparatus. The distance also depends upon the high-frequency impedance from line to ground. Transformers furnish a comparatively low impedance path to ground and a line with few transformers will carry radio interference a greater distance.

The interference reaches the receiving set by direct conductive coupling through the power supply wires or by inductive coupling between the power supply wires and the radio antenna-ground system, or by both paths. Some receivers pick up interference by the induction of interfering currents in the chassis of the receiver itself. The sensitivity of the modern radio receiver makes it possible to operate on an inside antenna or on a low short outside antenna. An inside antenna increases the coupling to the power supply wires and in many cases the power supply wires bring in most of the broadcast signal. This close coupling also makes the set more susceptible to interference feeding over the service lines. There may be fading when other loads are switched off and on if the service lines are functioning as an antenna. The fading can often be overcome by putting up a high outside antenna, grounding the power supply neutral wire at the entrance switch and in some cases tuning the house wiring with proper condensers.

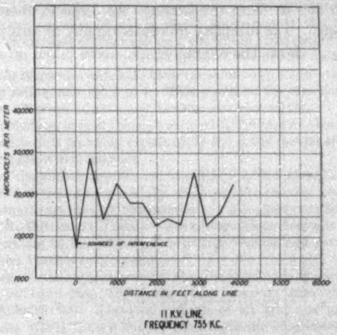

FIG. 6—VARIATION OF FIELD STRENGTH OF INTERFERENCE ALONG AN 11-KV. LINE

TRACING INTERFERENCE

A radio interference test set is operated necessarily from a battery source. It cannot conveniently be connected conductively to the power supply system. This makes a highly sensitive test set necessary in order to pick up the weaker interferences. Some of the cases of interference with an a-c. receiving set cannot be picked up by a battery test set unless very closely coupled to the lines. Contrary to earlier beliefs, sources of interference cannot be located by means of triangulation. The directional property of a loop antenna can be made use of at points of line intersections or at taps. This sometimes enables the operator to determine the circuit causing the interference.

The attenuation of high-frequency currents along wires does not follow the law of radiated electromagnetic field at right angles to the wires. Due to discontinuities of line constants, and reflection of waves, there is a development of unequally spaced nodes and antinodes of intensity. The intensity is sometimes greater a mile from the source of trouble than near its immediate source. Changes in intensity of interference are caused by resonant points, variation of distance from wires, presence of metal structures near lines, reflection phenomena, ground wires, tap lines, right angle corners, changes in line configuration, etc. Due to changing weather conditions and to the changing characteristics of the spark, there is also a daily variation of the intensity of interference. Fig. 6 shows the effect of some of these conditions. The intensity at the source is less than at other points where measurements were taken. There is a rapid increase in intensity in each direction along the line up to 350 ft. from the source. At 3,800 ft. there is a tap line. The intensity increases rapidly at this point.

Fig. 7 shows the characteristic curve taken on a line insulated with pin-type insulators. It is interesting to note the peak, occurring at 25 ft. This is found on all such curves but the reasons are not definitely known.

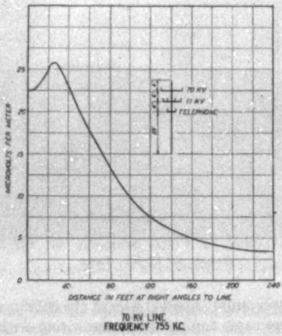

Fig. 7—Variation of Field Strength of Interference at Right Angles to a 70-Kv. Line

In locating interference the usual procedure is to use an intensity method. The effect of noise upon the human ear is not a straight line function and the ear cannot be depended upon accurately. A close inspection of equipment and the use of low-frequency exploring apparatus at suspected points usually shows the source of trouble. An investigator learns to keep on the right track by noting changes taking place at various locations as he drives along the line.

Experience is an important advantage in favor of the interference investigator. He must have a maximum of patience and persistence in actually tracing sources of interference. He should also be a man capable of meeting and intelligently talking to the public.

ELIMINATION OF INTERFERENCE

In general, the most satisfactory method of overcoming interference is stopping it at its source. From the power company standpoint, good radio construction and maintenance of power lines are important in order to avoid creating new sources of interference, thus forestalling future complaints. If the source of interference cannot be corrected, filtering at the source or isolating sections of lines by low-pass filters may be the best possible solution. Preventing the spread of interference by filtering transmission and distribution lines may not be practical from an economical or operating standpoint. Further study of the application and design of filtering devices is needed as each individual case may require special consideration.

The elimination of radio interference from industrial and household electrical devices can best be done at the factory by the manufacturers of such devices. Some manufacturers are and have been for some time aware of the radio interference problem and have taken steps to overcome it.

A method sometimes successfully used to overcome interference from all electrical sources, when the maximum noise pickup is from the service lines, is the use of a filter in the power supply lines at the receiver. In such cases a high outside antenna with a shielded lead-in and shielded ground connection must be installed in addition to the filter. This method of eliminating interference will not be satisfactory when the greatest part of the noise is picked up on the flat top part of the antenna. When no shielded lead-in is used it is very important to keep the lead-in and ground wire as far away from the interior house wiring as possible. Radio ground connections to steam pipes should be avoided and in no case should gas pipes be used. Where possible to install, a good independent ground rod is recommended.

CONCLUSIONS

From the foregoing it is easily understood that there is no simple solution for the interference problem, but it must be dealt with in a spirit of cooperation by the utilities, electric manufacturers, and the public. The interference from electric power supply systems is well in hand except for the troubles originating on high-voltage lines. Although filters on power lines have been suggested their use is yet very questionable. Manufacturers have a great opportunity in the sale of radio noise-proof appliances and apparatus. The public can aid by insisting upon such appliances.

In congested areas, radio trade interference associations have proved valuable to all concerned, while in the less thickly populated areas clearing interference is still left principally to the utilities. Increased broadcast coverage is very desirable to offset the natural noise level in many of the large areas now having low-field strengths as new and additional noises will always be experienced. Continued cooperative effort on the part of all concerned is undoubtedly the best solution to this problem which has come to be paramount to millions of people depending more and more on the radio for entertainment and education. The expression that the United States is becoming "noise conscious" may very appropriately be applied to the radio listening public.

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Discussion

J. J. Smith: The large increase in complaints recorded in Fig. 2 of Messrs. Campbell and Kalb's paper is, I think, typical of the experience of many power companies and shows that the solution of the problem of radio coordination is still in the future. The authors have given a list of sources of interference which is very similar to lists given elsewhere. It is to be regretted that they were not able to amplify their observations on these sources by the addition of field strength data of the type given in Figs. 3, 6 and 7, measured in cases where trouble was experienced and also after remedial measures had been applied.

Our experience to date would seem to show that in one case a complaint may involve a disturbance with a field strength of the order of 10 microvolts per meter, in another 100 microvolts per meter, and in another 1,000 microvolts per meter. Which of these complaints is a legitimate one? This is a question which many are asking and at least one public utility has already taken the position that it will not accept responsibility for complaints unless the broadcast signal strength at the point of reception is of a reasonable level. The criterion used at the present time is that the disturbance must be of sufficient intensity to interfere with the reception of the nearby stations.

About five years ago changes were made in a piece of apparatus so that when it was tested in close proximity to the radio sets being manufactured at that time no noise was heard in the set. In general, such changes cost money and make the apparatus more expensive. With the introduction of more sensitive sets in the last few years complaints have been received that disturbance to radio reception has been caused by some installations of this apparatus. Should it be changed again to conform to present conditions with every likelihood that something further will have to be done in another few years?

These points are mentioned to show that the question of responsibility is more involved than would be assumed from the three last paragraphs in the section entitled "Responsibility" in the paper.

It has been apparent for a considerable time that the problem cannot be solved by the power companies nor the manufacturers of power apparatus nor the manufacturers of radio apparatus acting alone. Cooperation among these interests and with others will be necessary. To further such cooperation a Joint Committee on Coordination of Radio Reception of the N.E.L.A., N.E.M.A. and R.M.A. has recently been organized with a view to studying the problem as a whole so that the public may have the use of power and radio facilities in the most economical manner. Similar joint work on coordination of power and telephone systems which was described in a series of papers at the A.I.E.E. Winter Convention, 1931, has given very gratifying results and it is to be hoped that the work on radio reception will be equally successful.

The work being done by the authors can be of considerable value to this Committee if carried out in a quantitative manner so that the measurements in one locality can be compared with those in another. From such measurements and similar ones made by other investigators reasonable practises to be followed by power companies, manufacturers of power and radio apparatus, etc., will gradually be evolved.

Leonard F. Fuller: Table I of this paper shows a marked increase in the percentage of radio interference cases due to the power company within the last few years, whereas the percentage due to other causes has remained substantially constant, and that of cases clearing themselves has decreased to approximately one-half of the 1924 figure.

I ask the authors for an explanation. Offhand one would expect the percentage due to the power company to decrease with accumulated experience. The reduction in the percentage of cases clearing themselves seems entirely logical.

Referring to the ratio of field strengths of K.P.O. and K.F.R.C., it should be noted that field strength varies as the square root of the *radiated* power, and not in direct proportion to the *rated* power. In the case of these stations, one would expect a $\sqrt{5}$ to 1 ratio between their field strengths provided the powers radiated from their respective antennas have the same ratio as the station kilowatt rating, namely, 5 to 1. This would be the case if their antenna effective heights and resistances were equal. Unexpected reduction in field strength ratio with distance as observed by the authors is probably explained by the many factors affecting attenuation with distance.

One of the authors informs me that a 5 to 1 field strength was mentioned merely because many engineers erroneously believe field strengths are directly proportional to station power rating.

J. O'R. Coleman: There are a few conclusions drawn in this paper relating to the subject of radio interference and revenue with which I do not entirely agree.

On page 82 the authors estimate that the revenue to the central station industry from 15,000,000 radio receivers amounts to about $100,000,000. The total revenue of all light and power companies of the United States, from domestic consumers for the year 1930, amounted to $668,500,000. According to estimates of the National Electric Light Association, based on information collected from a few representative power companies, the revenue to power companies for the whole of the United States, to be credited to radio receiving sets, is now between $30,000,000 and $35,000,000 per annum. The electric refrigerator, electric range and the electric flat iron are appliances which account for much larger annual revenue than the radio receiver. The radio receiver, of course, gives a very substantial and desirable revenue but it cannot be considered as outweighing all other uses of electricity, either in importance from the central station standpoint, or in importance to the ultimate consumer who uses the electricity for the above appliances as well as vacuum cleaners, washing machines, toasters, fans, percolators, etc.

Then there is the question of definition of interference as applied to radio. Apparently the authors of this paper consider that interference is any electrical disturbance which will prevent satisfactory radio reception more or less regardless of any of the many other factors that enter the problem. Consideration of interference is as old as civilization itself. We all recognize that we have to restrain some of our liberties in order that we can live with other human beings. There were questions of interference between railway bridges and steamships in the early days of transportation. Practically all electrical engineers are more or less familiar with the many problems relating to interference which have confronted the power and telephone industries since their very beginning. These problems have not all been solved to this day, but both industries, by recognizing duties as well as rights, have been able to develop their extensive and now indispensable systems without any undue burden placed on either by the requirements of the other system. The telephone company would not even try to render its service in the city of San Francisco with the ground return circuits which at one time were considered not only the last word in communication systems, but practically the only way that a commercial telephone system could be operated. In the study of the coordination of power and telephone systems we have found that the problem naturally divides itself into a study of influence of the power system, a study of the coupling between the two systems, and a study of the susceptiveness of the telephone system. In the principles and practises between the National Electric Light Association and the Bell Telephone System, it is recognized that each industry will limit these factors so far as necessary and practical. It appears to me that the same treatment could be applied in the handline of radio coordination. Of course, there must be a limitation of disturbing fields, but there must also be some limitation of the susceptiveness of the radio receiver and of the coupling between the radio receiver and other electrical systems.

Electrical disturbances which in no way affected the radio sets of five or six years ago, today are frequently complained of as serious sources of interference. Technically, it is possible to increase amplification of signal strength very much further than is practised today. The present limitations on increase in amplification, are the noises produced by the agitation of the molecules of the conducting material. If this is to be the limit of the radio receiver for which other uses of electricity are to be held responsible, we will certainly place a serious burden on power, light, communication and transportation.

The authors state that it is the responsibility of the appliance manufacturers to see that all appliances are completely free of radio interference. The same individuals pay for appliances that pay for radio sets, for communication service, and for power service. The authors have pointed out the particular problem which exists in the San Joaquin Valley, due to low field strengths. However, they advocate that the increased cost of appliances be forced upon all consumers in order that this problem, which is confined to limited localities, may be eliminated. Suppose that the filtering devices which are advocated for vacuum cleaners should add only one or two dollars to the cost of vacuum cleaners. This would add a serious increased total cost of appliances to the public. It is a question as to whether such a cost is justified.

Recognizing that this problem was not to be solved by placing an undue and unnecessary burden upon any one class, the Radio Manufacturers Association approached the National Electric Light Association and the National Electrical Manufacturers Association, to enter with them in a joint study to determine the best overall solution of the difficulty. This joint committee, with seven representatives from each of the three industries concerned, has had several meetings and is now working on this problem. This committee does not hope to find a solution for the problem today or tomorrow, but it hopes to find a solution which will be fair to all parties concerned, and give the best overall economy to the public which we are all striving to serve.

BROADCAST COVERAGE

F. B. Doolittle: The need for better broadcast coverage in California as pointed out by Messrs. Campbell and Kalb, is made more apparent by inspection of a map of the state in which the areas enjoying reasonably good reception are designated and may be compared with the total area of the state. This map (Fig. 1) was prepared by the Railroad Commission of the State of California, which, after an extensive investigation, over a year's time, submitted it as a part of its report to the Governor on "Interference with Radio Broadcast Reception in the State of California," under date of December 1, 1930.

There is little doubt that California has a sufficient allotment of broadcast power and frequencies to serve the area adequately, but the geographical distribution is such that reliable service is not available to large areas and actual inconvenience is worked upon other places by an excessive number of stations in thickly populated centers. There are eighteen broadcast stations in the metropolitan area of Los Angeles, all of them trying to serve the same group of people. One of these stations broadcasts "Amos and Andy," yet many listeners in this area must receive this popular program from a station one hundred thirty miles away because of inability to separate this program from that of other local stations. Would it not be a blessing to all concerned if some of these eighteen stations could be relocated to render service where service is needed?

ELIMINATION OF INTERFERENCE FROM INDUSTRIAL AND DOMESTIC APPARATUS

To cope successfully with the problem of elimination of interference from industrial and domestic apparatus, a definition of radio interference is required. Such a definition is difficult to formulate because, for example, a series motor-driven household appliance may interfere with radio reception in a whole neighborhood in a region of low broadcast signal strength and yet its operation at a location close to broadcast stations may cause no complaint. It is agreed that corrective measures are required on such apparatus in some locations, but is it economically sound to add to the cost of all appliances for the benefit of radio reception in certain areas where they might be used? The ideal answer to this would be to develop non-radio interfering apparatus at no additional cost. If this is not possible, a definite limit of the permissible amount of interference might be established by means of sufficient economic research.

The use of filters with appliances where necessary is an obvious solution of this problem which possesses the advantage of being applicable to appliances now in service as well as to new ones. However, tests made with simple filters or rather radio frequency shunts consisting of two one-tenth microfarad condensers in

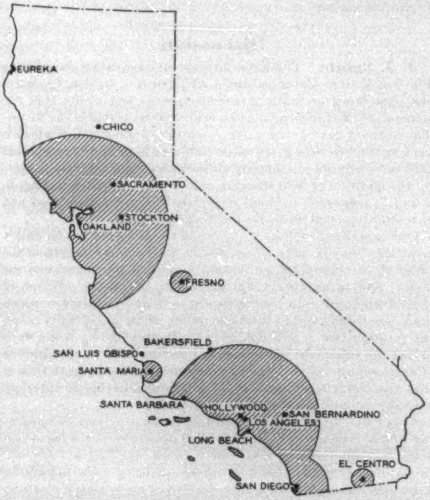

FIG. 1—MAP OF CALIFORNIA SHOWING AREA WITHIN SATISFACTORY RECEPTION RANGE OF BROADCAST STATIONS

Assumed range of 5,000 watt station,—100 miles. Assumed that distance varies as square root of transmitting power. KNX limited power license for 50,000 watts not considered series across the motor brushes with the common point connected to the motor frame proved them to be only partially effective. The average reduction in radio frequency voltage generated by five different appliances with this method was 81 per cent at 1,200 kilocycles and only 67 per cent at 600 kilocycles. A more elaborate filter consisting of series inductances and shunt capacities reduced the radio frequency voltage an average of 98.6 per cent but the retail cost of this filter was 60 per cent of the average price of the appliances and more than three times the price of one of them.

From the foregoing it will be appreciated that this problem of radio interference from industrial apparatus and household appliances is not one which can be simply dismissed by putting it up to the manufacturers to provide such equipment with condensers, but is rather a problem involving many technical and economic aspects which can only be solved satisfactorily by cooperative research involving the public, the electrical manufacturers, the radio industry, and the central stations.

RESPONSIBILITY

The authors have pointed out the increase in interference complaints brought about by the increased sensitivity and susceptiveness of modern radio receivers. Since this is true, a partial solution would consist of limiting those properties of receivers to a certain degree. The susceptiveness of receivers due to conductive coupling to the power supply circuits can be largely eliminated by proper shielding and filtering by the manufacturers of the receivers. As proof of this statement it is only necessary to refer to several popular receivers which are so constructed that without an antenna no interference or broadcast can be tuned in. Limiting the sensitivity to a reasonable degree would compel the installation of an adequate antenna which when well located would react favorably on the signal to noise ratio. Since many of the sources of interference have existed for years and would require very large expenditures to correct, it would seem only reasonable that the radio industry, the new-comer in the field, should shoulder its portion of the burden by cooperating to the extent of seeing that sets are designed and installed with antenna and ground systems of minimum susceptiveness. This calls for education of retail dealers and their installation men.

The authors have truly sounded the keynote in the situation when they call for "close cooperation between parties responsible for radio interference in order that the best possible public service may be given in the most economical manner."

F. E. Terman: The difficulties that are encountered in tracing down and eliminating sources of radio interference produced by high-voltage lines make it of interest to know exactly what this interference consists of. Mr. Overacker who is registered in our Electrical Engineering Department has done considerable work during the past year on interference problems in cooperation with Mr. Kalb. He finds that in the immediate vicinity of the trouble which generates the radio interference the disturbances received by the radio receiver may be caused either by radiation, or by induction through inductive or capacitive coupling between the radio antenna and the transmission line, but that at some distance down the transmission line from the source of the noise the interference is received only by direct coupling with the transmission line. A consideration of the factors involved indicates that this is a highly reasonable result, as the radiated field is inversely proportional to the distance from the radiator and so dies out relatively rapidly with distance, as is indicated by Fig. 7 of the paper, whereas the high frequency currents propagated along the line by ordinary transmission line action for great distances before being appreciably attenuated, but do not produce radiation and so cause interference only by direct coupling.

R. N. Stoddard: In order to coordinate radio interference satisfactorily, it is essential that a method of measurement and a unit of measurement be standardized. This work is now in progress through the Joint Coordination Committee on Radio Reception of N.E.L.A., N.E.M.A., and R.M.A.

It is recognized that the degree of interference may be expressed by the ratio of the useful energy to that which is disturbing or interfering, and that the degree of interference may be modified by a change in this ratio. Interference may be minimized by either an increase in the useful energy or by a decrease in the disturbing energy. Considerable studies have been made of field strength associated with individual broadcasting stations. Some studies of interfering levels have been made by various central stations and utilities. All available data are being collected and coordinated by the Committee and it is hoped a base unit of reference can be arrived at which will be satisfactory to all. This figure will determine the permissible degree of interference based upon a certain minimum serviceable field strength from a broadcasting station.

The authors point out that in general the most satisfactory method is to overcome the interference at the source. It is felt, however, that this should not be taken too literally as prevention of interference at its source may not only be more expensive than other methods, but might be impossible at the present state of the art.

The authors' conclusions state that the use of filters on power lines is yet very questionable. This is in sharp contrast to the sentiment and practises of several utilities in the east. It is felt that the application and use of filters on power circuits have been eminently successful in preventing interference in telephone circuits, carrier current channels, and radio broadcast receivers.

Filters have been applied to the neutrals of generators directly feeding transmission lines to prevent the harmonic frequencies disturbing telephone communication circuits. They have also been quite generally applied to mercury arc rectifiers for the same purpose. High-voltage transmission lines have been filtered to prevent interference between power line carrier communication channels on the same or adjacent interconnected systems. Filters have also been used to simplify high frequency characteristics of complicated transmission line networks.

Remarkable success has been achieved in filtering radio frequency interference from transmission lines particularly where some individual line is coupling the interferences to the secondary or distribution circuits. While it is realized that filters are not the general solution to all interference problems, they have formed a successful and economical solution in many cases of transmission line troubles. It is therefore felt that the application of filters to transmission lines should not be considered as questionable.

C. C. Campbell and H. N. Kalb: Dr. Fuller points out the rapid increase in the percentage of radio interference complaints due to the power company within the past few years, as shown by Table I. This increase was caused by the omission from the table of complaints investigated by Radio Trade Associations in metropolitan areas since 1928. In metropolitan areas the percentage of complaints due to the power company is considerably smaller than in other areas.

Mr. Coleman advises that the estimated annual revenue from radio sets was $35,000,000 instead of $100,000,000, as estimated in the paper. This estimate was made with reports of Pacific Coast Companies as a basis, and may not apply to other parts of the country.

Backfires in Mercury Arc Rectifiers

BY J. SLEPIAN[*]
Fellow, A. I. E. E.
and
L. R. LUDWIG[*]
Associate, A. I. E. E.

Synopsis.—*Mercury arc rectifier design of today was arrived at in an endeavor to reduce the frequency of backfire. Efficiency and cost are sacrificed to obtain sufficient backfire immunity.*

Study of backfires in actual rectifiers and laboratory tubes indicates that this occurrence is a random one. Backfires differ in kind from other breakdown phenomena because they do not occur with certainty after some definite limitation has been reached, and in fact no limit can be chosen above which or below which backfires will or will not take place.

The state of a rectifier in relation to backfiring must be described in terms of a backfire probability, or average rate of backfire. There are no commercial tank rectifiers for which the mean frequency of backfire is absolutely zero, but some probably reach the low value of one per several years.

The significance of the random nature of backfire on the rating and testing of rectifiers is discussed, and suggestions are made as to means for testing which lead to the determination of the true quality of a rectifier.

Data are given showing the variation of the backfire rate with current and voltage. The application of these ideas to research and development is illustrated by several examples.

The theory of the formation of a backfire is described, and backfires are attributed to "causes" which lead to the formation of a cathode spot on an anode, but which do not correspond to any fundamentol limitation of the rectifying process. Several suggestions are made as to the exact nature of these causes.

The time duration of these causes has been found with the cathode-ray oscillograph to be very short, of the order of a few microseconds.

The authors dispute the theory that there is a continuous transition from a discharge of the glow form to that of the arc, at least in the case of the backfiring of mercury arc rectifiers.

* * * * *

Section I

INTRODUCTION

In a normally operating mercury arc rectifier, each anode carries a large current when it is at a positive potential with respect to the cathode and only a small current of the order of a few milliamperes when it is at a considerable negative potential with respect to the cathode. It was found early, however, that with the simplest and most obvious constructions this normal operation was not reliable, and moreover acted in an erratic way. An anode would function properly cycle after cycle over a greater or lesser period of time, and then suddenly fail to hold voltage when negative in some particular half cycle. Such failures of anodes to bear negative voltage and thus cause short circuits are called arc-backs or backfires.

The causes of these backfires are not at all well understood even today, but it was found early that certain constructions greatly reduced the likelihood of their occurrence. Thus it was found that provision of adequate means for cooling the rectifier, moving the anode away from above the cathode, the use of shields around the anodes and grids interposed in the arc path to the anode, and the exercise of extreme care with respect to cleanliness in assembling the rectifier and the removal of all foreign gas to the highest possible degree, all reduced the backfire probability. Thus practically the whole design of present-day rectifiers is based on the endeavor to reduce the frequency of occurrence of backfires and this has caused many of the advantages to be sacrificed which might be expected from the application of the normal characteristics of the mercury arc to the rectifier problem. It seems clear that research for the purpose of advancing the application of mercury arc rectifiers must be largely directed to a study of the nature of backfires so that means may be devised for reducing the frequency of their occurrence without losing the attractive features of mercury arc conversion.

Section II

BACKFIRE—A RANDOM PHENOMENON

The erratic manner of occurrence of backfires thrusts itself upon the attention of all who have to do with rectifiers; not only upon erudite research men in the laboratory, but also upon operators in the field. Yet the great significance of this erratic behavior is not realized; and its implications as to the operation which properly may be expected, as to the correct rating of rectifiers, as to the testing of rectifiers, and as to the appropriate methods for research and development do not seem to have been generally grasped.

It must be recognized that except for backfires due to certain grosser causes, some of which will be mentioned later, backfires are inherently random phenomena.[*] They differ in kind from other breakdown phenomena, or at least they differ so greatly in degree as to be practically different in kind. Let us compare, for example, the backfiring of a rectifier with the breakdown of a spark gap in air at ordinary pressures. As the voltage on a spark gap is gradually raised a point is reached where the spark gap breaks down. If voltage is again applied to the spark gap and is raised up to a point just short of the voltage at which the previous break-

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[*] Research Laboratories, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the South West District Meeting of the A. I. E. E., Kansas City, Mo., October 22-24, 1931.

[*] Since this paper has been written, a paper has been published in the *Revue Generale de L'Electricite* of June 6, 1931 and of June 13, 1931 by M. Leblanc and Demontvignier, which also cites the random nature of rectifier backfire. Furthermore a recent review of patent literature by the authors disclosed a patent of Chas. A. Kraus (U. S. No. 1,086,300) which refers to the random nature of backfire.

down occurred, then the spark gap will not break down. The spark gap will bear this lower voltage indefinitely without breaking down. If, however, the voltage is raised slightly above that at which breakdown occurred in the first test, the spark gap will at once break down. No matter how often this test is repeated the spark gap will be found unable to support the higher voltage for any appreciable length of time.

Now consider the backfire in a rectifier. As the load upon it is increased there will be a point where a backfire occurs. If now the load is reduced below this value, will the rectifier function indefinitely without backfire? Not at all. Backfires also occur at the lower load at random moments. If the load is lowered still further, backfires still occur at random; much less frequently it is true, but they still occur. Conversely if the load is increased above the critical value found in the first test, will backfire occur at once? Not at all. The rate of backfire occurrence is increased, but the rectifier may carry the higher load for one minute or for ten minutes, or maybe only for one second, depending seemingly on chance as to how long the run can be without backfire.

Thus backfires in a mercury arc rectifier seem to occur not under definitely limiting conditions, but under all conditions in their random manner, with an average frequency which depends upon the conditions of operation, and the construction.

Inherently and fundamentally random or erratic phenomena are well known in physics. They usually involve aggregates of a small number of molecules making up small particles of matter. An early example of such phenomenon is the Brownian movement. Brown, an English naturalist, discovered with his microscope that very fine particles of matter suspended in a liquid were in a state of continual motion moving about in a random helter-skelter manner. Such motions are also observed on smoke particles in the air. The explanation of these motions is that the number of molecules of the surrounding liquid or gas which collide with the one side of the particle may not, at a particular moment, agree with the number of molecules which collide with the other side of the particle. Thus there are continual unbalanced forces upon the particle arising from these collisions of the molecules of the medium and because of the smallness of the mass of these particles under these forces, the particle is given a random motion. In the motion of the particle under the Brownian movement, displacements of the particle in various directions are continually taking place. Moderate displacements happen frequently, large displacements less frequently, and very large displacements only very rarely. However, there is no sharp limit as to what displacement may occur. It is merely that the probability of occurrence of a displacement becomes small when the displacement becomes large. If the size of the particles is increased, the magnitude of the displacements occurring as a result of the Brownian movement becomes less. However, large displacements continue to occur, but less frequently than with the small particles. When the particle becomes of ordinary size, say like a billiard ball, then the Brownian movement becomes exceedingly minute, and deviations from the normal motion of the ball under the gravitational field predicted by every day mechanics, become generally indistinguishable. However, a considerable deviation from the regular motion should continue to be expected, but with the frequency of occurrence so low that it should be expected to occur only at a rate of once in tremendous eras of time.

Another more modern example of an inherently random phenomenon is the disintegration of radium. The atoms of radium explode, but apparently in an entirely random way so that in one short interval of time a large number of molecules may explode and in another short time interval only a small number of molecules may explode. The scintillations which are observed on suitable screens in experiments with radium are visible evidence of the random character of the radioactive decomposition.

The backfires are also inherently random occurring phenomena and probably likewise depend for their occurrence upon aggregates involving relatively small numbers of atoms, molecules, ions, or electrons.

Section III

Distribution of Backfires in Time. Mean Frequency of Backfire

The entirely random occurrence of backfires in time may be illustrated by the curve, Fig. 1, which was obtained with a small metal tank rectifier in the laboratory with all controlling conditions maintained as nearly constant as possible and with a constant direct current to one anode and a constant test voltage on the other. This curve cries out that the occurrence of backfire is a matter of probability.

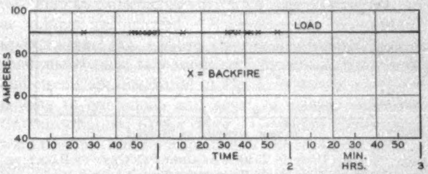

Fig. 1—Random Occurrence of Backfire on a Small Metal Tank Rectifier

In concept at least we may define the probability of backfire of a rectifier in a particular condition, and during a short time interval Δt in this way. Imagine a large number N of tests made with a rectifier under identical conditions, the duration of each test being the short time interval Δt. Suppose that in a certain number of these tests n, backfires occur, but not in the others. Then $\frac{n}{N}$ will be the probability of occurrence of a backfire in the time Δt, and if we write

$$\frac{n}{N} = p \, \Delta t$$

then p is the probability of backfire per unit time for the rectifier in the particular condition.

It is shown in Appendix I that if the probability of backfire of a rectifier operating under constant conditions remains constant, and if the time intervals between successive backfires are observed, then the proportion of these time intervals which is greater than a time interval t will be given by e^{-pt}; that is,

$$\frac{m}{M} = e^{-pt}$$

or

$$\log \frac{m}{M} = -pt$$

where M is the total large number of intervals between successive backfires observed and m is the number of these intervals which are greater than t in length. This relation has been confirmed experimentally by D. E. Marshall using a glass tube with voltage higher than that necessary for maintaining a glow with results shown in Fig. 2, and by F. A. Maxfield for a metal tank rectifier with a constant current arc going to one anode and with test voltage on the other at a value insufficient to maintain a glow by itself, with results shown in Fig. 3.

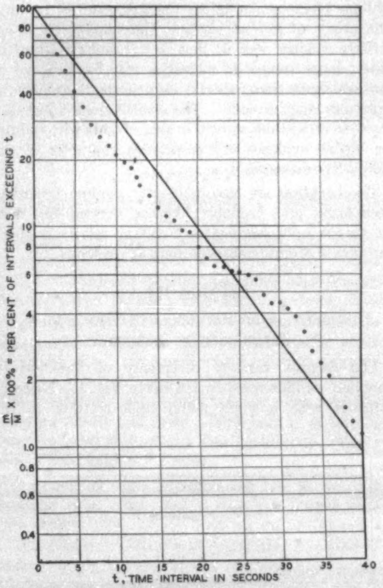

FIG. 2—LOG PLOT OF TIME t, AGAINST PER CENT OF BACKFIRE FREE INTERVALS EXCEEDING TIME LENGTH, t

From the equation above which gives the distribution of time intervals between successive backfires, we find that the average interval is $\frac{1}{p}$. This average interval is not the interval of most frequent occurrence. There is no tendency for backfires to occur at anything like regular intervals. As a matter of fact short intervals occur more frequently than the average, $\frac{1}{p}$. The average frequency of backfire per unit time is p, and $p\,T$ is the average number of backfires in the time T.

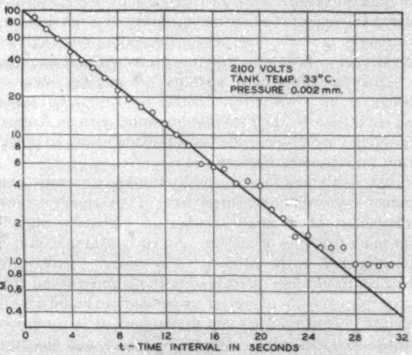

FIG. 3—LOG PLOT SIMILAR TO FIG. 3 EXCEPT THAT DATA WERE OBTAINED WITH A METAL TANK

The state of a rectifier with respect to backfire is then properly described when a mean frequency of occurrence of backfire is given and it cannot be described by saying that the rectifier is operating below or above backfire limits, inasmuch as there are no definite backfire limits. The average frequency of backfire is a function of the construction of the rectifier and the conditions under which it is operating. Thus, under light load the probability of backfire or the mean frequency of backfire is small and under overload the mean frequency of backfire is large. But this does not mean that the rectifier will never backfire on light load or is certain to backfire in a particular finite time interval on overload.

Of course for practical purposes a sufficiently small frequency of backfire is just as good as zero frequency of backfire; for example, a rectifier with mean frequency of backfire of one per ten years will be more reliable than the other electrical apparatus working with it and far more reliable than the human beings operating it. The backfire frequency of successful rectifiers in the field seems to be of the order of several per year.

Section IV

TESTING OF RECTIFIERS

The purpose of commercial tests on rectifiers is to determine their quality and to make sure that they come up to standard; but the quality with respect to backfire propensities must be determined in terms of an average backfire rate. Since the average backfire rate is very low for commercial rectifiers, the determination of this rate by the usual tests under normal conditions is practically impossible.

As an example, suppose that a considerable number of rectifiers were made with such uniformity that the average backfire rate for each rectifier was one per month on normal load; suppose that the manufacturer tested each rectifier on normal load for one month, which is an impractically long test. Then, as shown in Appendix II, 37 per cent of the rectifiers probably would not backfire during the test and 26 per cent would backfire twice or more often. Thus, although all of the rectifiers were of uniform quality and although the test was impractically long, 37 per cent would appear to be above the average quality and 26 per cent below. Testing under normal load for short time is only good for reliably weeding out rectifiers with exceptionally large backfire rate; and at the same time such tests will occasionally reject good rectifiers.

The way out of this dilemma seems to be to test the rectifiers not under normal conditions, but under such abnormal conditions as will make the backfire rate countable in a reasonable time and to have sufficiently well established data to permit extrapolation for determining the backfire rate under normal conditions. For example, if the dependence of backfire rate upon test voltage is well established for any particular type of rectifier, the tests on such a rectifier might be made at such overvoltage as will give a considerable number of backfires in one day. The extrapolation to normal voltage will give backfire rate to be expected under normal conditions.

Section V

SHORT TIME RATING OF RECTIFIERS

In many fields of application of rectifiers, as for example, electric railways, it is desirable that the rectifiers be capable of carrying overload for relatively short periods of time. Dynamo electric machines, transformers, etc., inherently have overload capacity because the limitation to their load carrying capacity is usually determined by the temperature of insulation and because it takes time for the various parts of the machine to acquire their final temperatures under overload.

But with the mercury arc rectifier it is different. It is true that the temperatures in a rectifier may change in time with overload so that the rectifier may get progressively worse from the standpoint of reliability. But apart from such progressive changes, before the temperature and vapor pressure have changed appreciably, the rectifier is immediately less reliable on being overloaded. This fact may not be recognized because of the shortness in time of the usual overload tests and because of false analogy with other types of electrical machines. As an example, increasing the load upon a rectifier to 150 per cent of normal might be expected to multiply the average backfire rate by perhaps ten. If we assume the rectifier is thus overloaded 10 per cent of the time, then if we take the multiplying figure of 10, this overload operation will double the number of backfires per year. If we should take a multiplying figure of 100, it will multiply the number of backfires per year by 11. This reduced reliability under overload will exist in spite of the fact that the rectifier successfully goes through the overload period day after day without any apparent distress.

Section VI

THE VARIATION OF BACKFIRE RATE WITH CURRENT AND VOLTAGE

The preceding discussion has brought out the great practical value of knowing how the average backfire rate may be expected to vary with such control factors as voltage and current. Regarding the dependence upon voltage, Fig. 4 shows the results obtained with a small metal tank, two-anode rectifier whose dimensions and circuit are illustrated in Fig. 5. Current was kept flowing to one anode and negative voltage was continuously applied to the other. Grid glow tube relays were used to record automatically the backfires upon a counter.*

The average backfire rate seemed to vary exponentially with the voltage giving approximately a straight line on the logarithmic scale shown in Fig. 4. The tests were repeated at various times and different straight lines were obtained as the quality of the rectifier changed for unknown reasons. All the lines obtained lay between the dotted lines shown in Fig. 4.

Some data on the variation of backfire rate with current are given in Fig. 6. The two-anode metal tank rectifier illustrated in Fig. 5 was again used. Constant test voltage was applied to one anode and the current to the other anode was varied.

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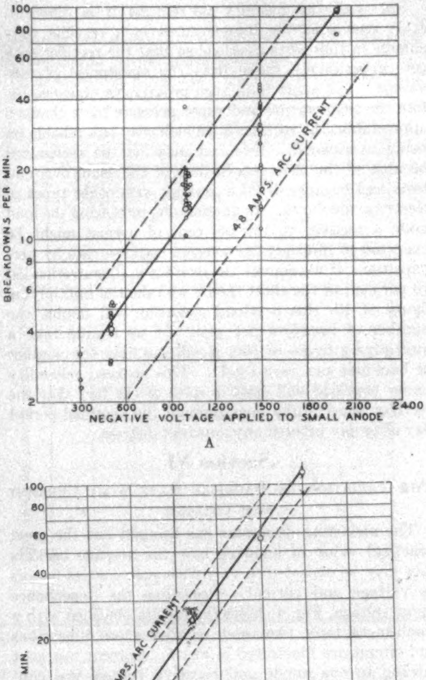
*When a backfire occurred, the charged condenser from which the negative voltage was obtained, discharged through the tank. This discharge current was limited by resistance to a value of a few amperes. After the backfire was completed, the condenser recharged so that negative voltage was then again applied to the anode. The data were obtained by counting the number of backfires in periods of 30 minutes duration. The tank temperature was held constant at 10 deg. cent. during the entire test.

In applying the results of this experiment to rectifiers operating in a-c. circuits, it might seem that sufficient importance is not attached to the moment immediately following the change in direction of the voltage applied to an anode, as it may be thought that backfires occur most frequently then. However, experience shows that this is not the case, so that even if the probability of backfire is high at the moment of voltage reversal, the shortness of this moment more than makes up for the magnitude of the probability with the result that backfires appear to occur more often sometime later than the reversal rather than immediately after it.



A

B

Fig. 4—Average Breakdown Rate Against Voltage for Metal Tank Rectifier

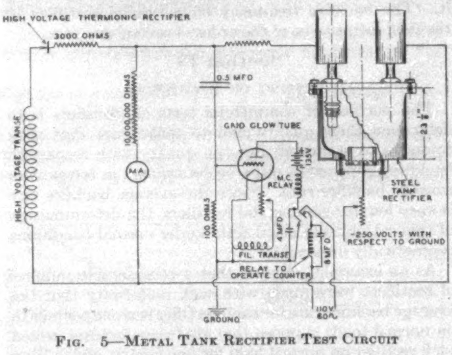

HIGH VOLTAGE THERMIONIC RECTIFIER
3000 OHMS
0.5 MFD
10,000,000 OHMS
MA
HIGH VOLTAGE TRANSF.
GRID GLOW TUBE
135V
M.C. RELAY
100 OHMS
4 MFD
FIL. TRANSF.
8 MFD
RELAY TO OPERATE COUNTER
GROUND
110V 60~
23 1/8"
STEEL TANK RECTIFIER
-250 VOLTS WITH RESPECT TO GROUND

Fig. 5—Metal Tank Rectifier Test Circuit

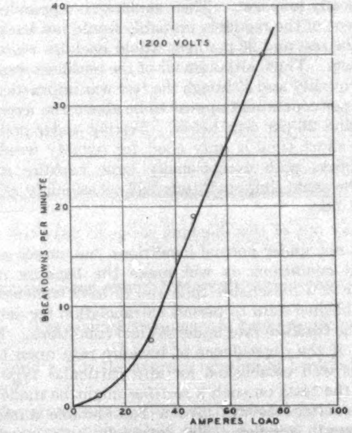


Fig. 6—Average Breakdown Rate Against Current for Metal Tank Rectifier

Section VII

Application to Some Research Problems

The difficulty encountered in determining the proper rating of rectifiers by tests under normal conditions of reasonable duration is also met in researches whose purpose is to determine the relative merits of various constructions or materials entering into rectifiers. For example, consider the question of material for anodes. Graphite anodes are now used by many manufacturers in metal tank rectifiers, but iron anodes are also used extensively. The practise does not seem to be uniform, and may be changed as a result of longer tests. The reason for this uncertainty is the lowness of the backfire rate with either material which made short-time tests under normal conditions inconclusive. Since the inadequacy of the short-time tests was not realized, incorrect conclusions were frequently drawn.

The solution again seems to be to test under such conditions as will give an easily countable backfire rate and then to extrapolate to normal conditions. Hence some tests were made under the authors' direction on iron and graphite anodes with sufficiently high-test voltage as to give a very large backfire rate. The small metal tank rectifier (Fig. 5) was used. One anode was of iron and the other of graphite and both were shielded in a similar manner. An arc was carried to one anode and test voltage applied to the other and the backfire rate counted. Then the arc was carried to the second anode and test voltage applied to the first. The results indicate that fewer causes of backfire appear with graphite, in a given period of time, but the difference is not greater than day to day variations with only one material.

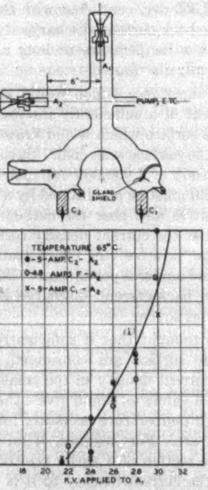


FIG. 7—RESULTS OF TESTS TO COMPARE BREAKDOWN RATE OF TUBE USING POOL *vs.* THERMIONIC CATHODE

From the standpoint of the tendency to backfire alone, the results are very clear. There is no great advantage of either material over the other. If the relative merits of graphite *vs.* iron anode keep this relationship when the voltage is lowered to normal, then an answer is obtained to a question which would require many years of observation on many rectifiers to determine under normal conditions.

Another example of the application of these ideas is in determining the relative merits of cathodes of thermionic type as compared with cathodes of the conventional mercury pool type generally used. In recent years thermionic cathodes have become available with sufficient current capacity and sufficiently long life to make it possible to consider their use in power mercury arc rectifiers. It is certainly highly desirable therefore to determine whether thermionic cathodes have any great advantage over the pool type with respect to backfires. A glass tube was therefore constructed as shown in Fig. 7, containing both a thermionic cathode and a mercury pool cathode. An arc was run between each cathode and the anode A_1 and test voltage applied to A_2. The results obtained are shown in Fig. 7. There is apparently no difference in the backfire rate for the two kinds of cathode for this tube.

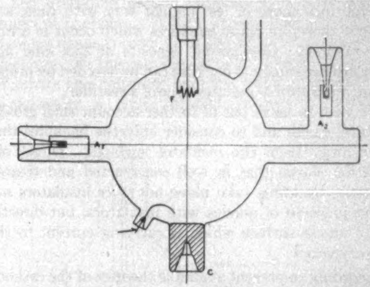


FIG. 8—SKETCH OF TUBE HAVING POOL AND THERMIONIC CATHODE, BUT COMPARATIVELY UNSHIELDED ANODE

With another tube shown diagramatically in Fig. 8, quite different results are obtained as to the relative merits of thermionic *vs.* pool cathodes, as shown in the following table.

Arc current	Volts on A_1	Back current milliamperes	Breakdown rate
7.5 amp. F-A_2	6,000	2	0 in 10 min.
7.5 amp. C-A_2	6,000	1.5	5 in 10 min.
7.5 amp. F-A_2	8,000	4 – 6	29 in 10 min.
7.5 amp. C-A_2	8,000	2 ±	725 in 10 min.

The difference in performance of the tubes is probably due to differences in shielding of the anodes from exposure to the cathode. The thermionic cathode thus seems to possess the advantage that much less shielding is required with it.

Section VIII

Backfires Forbidden by Generally Accepted Theories

It is quite clear that in practical rectifiers there will be certain grosser causes of backfire which will not occur in the random way described in the previous sections. For example, a conducting deposit may form over the insulator between the tank and the anode. Such a conducting deposit may start an arc in the same way as a blowing fuse wire, when negative voltage is applied to the anode. Such deposits may be expected to develop in a more or less regular way with time, and will not therefore cause backfires which occur in a random manner. Conducting deposits of this kind and other grosser causes of backfire can be avoided by proper design and suitable maintenance of a rectifier.

We wish to leave out of further account such grosser backfire causes and to consider only the backfires that occur away from the insulator surfaces. Direct observation shows that in well constructed and treated rectifiers, backfires take place not over insulators nor at the junction of anodes with insulators, but directly at the anode surface which is carrying current to the mercury vapor.

According to present available theories of the cathode of an arc, backfire should never occur without some initiating circumstance which is foreign, not inherent, or not necessary to the kind of discharge which is taking place just before the backfire. An arc is to be distinguished from a glow in that only a low voltage less than 20 volts is required at the cathode for its maintenance. A glow on the other hand is a self-maintaining discharge which requires at least several hundred volts at the cathode.

In mercury arc rectifiers, generally the construction and vapor pressure is such that it is not possible to hold a self-maintaining glow with the available negative voltage which is impressed on the anode. However, with normal negative voltage on one anode and an arc playing to another anode, a current flows to the negative stressed anode in a discharge which does not differ in any essential way from a glow. This discharge differs from the glow only in not being self maintaining; that is, if the arc to the other anode is stopped, then this discharge stops, whereas if conditions are such as to permit a glow, then the glow continues even if the arc to the other anode is stopped. It is possible, as by increasing the vapor pressure, to change the discharge continuously from the non-self-maintaining form to the self-maintaining glow without any great discontinuity in appearance or properties of the discharge. In fact the only way to tell whether a self-maintaining glow has formed is to stop the arc current to the other anode and see whether the discharge stops. In what follows we shall make no distinction between a self-maintaining glow and the non-self-maintaining discharge, but shall refer to them both as the back-current discharge. The current in the back-current discharge is generally quite small, only a few milliamperes for the usual size anode in commercial rectifiers. The current is distributed over the anode surface rather uniformly. Shields, grids and in general any surfaces in proximity to the anode to which ions may discharge greatly lower the back current to the anode.

A backfire consists in the sudden substitution of an arc for the normal back-current discharge. Somehow the conditions for the maintenance of the cathode of an arc must suddenly set themselves up. The conditions required at the cathode of an arc as predicted by present theory are such as should not occur spontaneously so far as the intrinsic characteristics of the back-current discharge go. The conditions required at the cathode of an arc, according to present theories, call for either a very high temperature or a high-current density.[1]

If a temperature at some point of the cathode surface reaches a value sufficiently high for thermionic emission to take place to a considerable degree, then an arc may form. For graphite or iron this temperature is greater than 2,000 deg. cent., but with the low-current density in the back-current discharge it is impossible to obtain such a temperature so long as the current remains uniformly distributed.

Another circumstance which would start an arc is the development of a sufficiently intense electric field at the cathode surface which would draw electrons out even though the surface was cold. Such a field must have a magnitude of 1,000,000 volt per cm. or more. Clearly this field must be maintained by a large concentration of positive ions near the cathode surface and it turns out that the current density corresponding to any such large positive ion concentration must be several thousand amperes per sq. cm.[2] But the current density in the back-current discharge is only a few microamperes per sq. cm.

It might be thought that if sufficiently small areas of the cathode surface are considered, spontaneous variations in current density in the small areas would give rise to such values as would give either the intense field or the high temperature necessary for forming the cathode of an arc. For instance, if an area a little larger than the section of a single positive ion is considered, then the current density to it is zero most of the time and rises to enormously high values during the moment when a positive ion strikes it. However, it is not permissible to consider such small areas. Experiment shows that the cathode of an arc on electrodes of usual size is unstable when the current in the arc is less than 0.05 amperes[3] and that it will change spontaneously to the glow form of discharge. Therefore, we must consider areas sufficient to supply more than 0.05 amperes of arc current. With the current density of 5,000 amperes per sq. cm. such an area would need to exceed 10^{-5} sq. cm. As is shown in Appendix III, fluctuations in the field intensity over such an area, due to fluctuations in the density of positive ions opposite the area, will cause field intensities of a magnitude sufficient for drawing out electrons so rarely as to fail utterly to account for the observed frequency of occurrence of backfires in rectifiers.

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[1] For references see Bibliography.

Scrutiny of the back-current discharge through the spectacles of the present theories of the cathode of an arc fails to reveal the causes of backfire, but in this scrutiny the cathode surface was regarded as being the homogeneous surface of a perfectly homogeneous solid. The possible influences of contaminations on the surface and flaws in the electrode material must then be considered. But before doing this, some interesting experiments carried out by D. E. Marshall are given below.

Section IX

BACKFIRES INITIATED BY PARTICLES MAKING OR BREAKING CONTACT

A tube was constructed as shown in Fig. 9. The electrodes were of graphite. Near the cathode was a glass plunger with a tungsten tip fused into it 0.075 centimeters in diameter, and 2 millimeters long. The plunger could be operated magnetically so as to strike the cathode, remain firmly in contact with the cathode, or be moved out of contact with the cathode. A small amount of liquid mercury was in the tube and the vapor pressure was controlled by placing the tube in an oil bath whose temperature could be varied.

Voltage was applied to the tube from a condenser which was charged from a transformer by means of a thermionic rectifier. When sufficiently high voltage was applied, a glow started in the tube which was quite stable and carried a small current and requiring the continued application of the high voltage. The glow was quite stable if the plunger did not touch the cathode and also when the plunger was firmly in contact with the cathode, but if the plunger was made to strike the cathode, at the moment of contact with the cathode there would be a flash and the condenser would be completely discharged, indicating that an arc had formed with a very large momentary current. Similarly at the moment of breaking contact with the cathode there was a backfire. Thus making or breaking contact of the cathode with a small piece of completely insulated tungsten wire will cause a backfire.

The cause of the backfire is to be sought in the very high-current density at the last contact point. Although the little piece of tungsten collects an exceedingly small back current, this back current must flow to the cathode through the last contact area. As the contact area shrinks to zero the small current collected by the tungsten wire gives a very high-current density at the last contact and a very rapid rise in temperature to a high value must take place there. There is undoubtedly an explosive volatilization of material there with a momentarily very high vapor density and this vapor probably is almost completely ionized. This would permit a high-current density to flow from the gas space to the electrode with enough total current to start a stable arc.

Marshall has also made tests showing that making or breaking of contact of cathode with small mercury drops will cause backfire. Contacting with good insulators, however, did not seem to cause backfire.

Section X

CAUSES OF BACKFIRES

The preceding section suggests a host of possible causes of backfires. A very obvious one is contacting mercury drops thrown from the pool cathode. Proper shielding of the anodes should almost completely eliminate this cause for backfire, but it is possible that occasionally a small droplet subject to Brownian movement may reach the anode in spite of the shield. Small conducting particles of any metal carried around by the mercury vapor streams may contact with the anode and cause backfire, as, for example, a small piece of iron rust or scale. This is an argument for the greatest care in cleanliness in assembly of rectifiers.

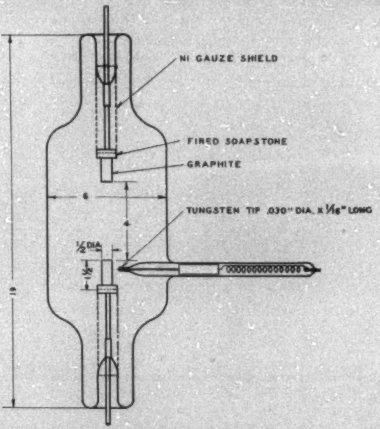

FIG. 9—GLASS TUBE WITH TUNGSTEN POINT TO CONTACT WITH ONE ELECTRODE

Particles may detach themselves from the anode itself. It is well known that under the influence of the back current the anode suffers a very slow disintegration and loss of material known as sputtering. While the sputtered particles may consist in large part of individual molecules, it is believed that larger particles are also detached,[4] although they may be subsequently volatilized in the discharge itself. When a sufficiently large particle detaches itself a backfire may occur. Irregularities in the size of sputtered particles may perhaps be expected to occur spontaneously even in the absence of any impurities on the electrode surface or in the electrode material. However, the presence of such impurities should tend to greatly increase the irregularity of sputtering and make the detachment of large particles more frequent. Thus if there is a small amount of gas in the metal the distribution of this gas should not be expected to be a uniform one, but subject to considerable fluctuations in local density due to the smallness of the actual gas content so that a small area of the electrode may be momentarily rich in gas and in such a state that a large particle may be detached. This suggests that advantage is obtained by removing gas from the electrodes as thoroughly as possible.

It may not be necessary for a large particle to completely detach itself from the electrode. If the particle makes poor thermal contact with the electrodes, then under the influence of the back current it may be raised to a high temperature in spite of the low-energy input density and a backfire may be started.

From this discussion on possible backfire causes it would be expected that the magnitude of the back current is important. If the back current is large, then the particle size necessary to cause backfire will be smaller and particles of such size will occur more frequently. If the back current is large, the sputtering of the electrode will be more rapid and large particles are more likely to be detached or loosened. Any means which reduces the back current to the electrode such as grids, shields, etc., must also reduce the frequency of occurrence of backfire and experience shows this to be the case.

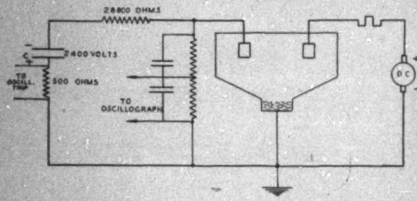

FIG. 10—CATHODE-RAY OSCILLOGRAPH CIRCUIT

Another cause of backfire suggested by Langmuir[8] is the formation of insulating patches on the electrode with dielectric failure of the patch as it gets charged up by back current. These patches are made up of impurities, oxides, or what not and because of the small amount of material in their make-up their development to large sizes will be a random occurrence. Other experimenters also have surmised the existence of such insulating layers.[9]

Section XI

DURATION OF BACKFIRE CAUSES

The random manner of occurrence of backfire as well as considerations of the preceding section make us believe that backfire causes involve only a small amount of matter. Since the amount of matter involved is very small, we should expect also that the time involved by backfire cause would be very small. This is verified by a cathode-ray oscillograph study made by F. A. Maxfield.

The cathode-ray oscillograph was of the type developed for lightning investigation and has been described elsewhere.[6] The circuit used is illustrated in Fig. 10, the Norinder relay functioning to cause the oscillogram to be taken immediately after the starting of a randomly occurring backfire. The small two-anode metal tank was used. An arc of 75 amperes flowed to one anode and 2,400 volts were applied to the other by a charged condenser. In series between this condenser and the anode under test a resistor of 28,800 ohms was placed. The purpose of this resistor was to keep the current too low to maintain an arc even though the backfire cause made the voltage from anode to cathode fall to a low value. Thus it was expected that the voltage on the tested anode which would fall to the low value during the existence of the backfire cause would return to normal when the backfire cause ceased.

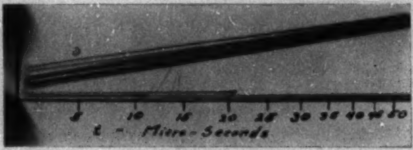

FIG. 11

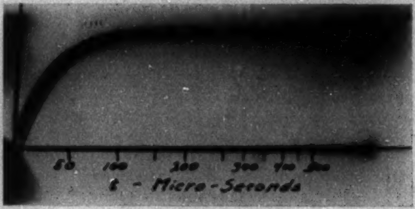

FIG. 12

FIGS. 11 AND 12—CATHODE-RAY OSCILLOGRAMS SHOWING TIME DURATION OF BACKFIRE CAUSES

Oscillograms of the voltage across the tested anode are shown in Figs. 11 and 12. The recovery of normal voltage began to take place in an exceedingly short time, usually less than 5 microseconds from the time the backfire cause started. The finite slope of the recovery curves shown in the figures is due to the electrostatic capacity of the potentiometer by which the oscillograph was connected to the rectifier. The initial drop in voltage upon the occurrence of the backfire cause does not appear upon the oscillograms because of the slight lag in operation of the Norinder relay circuit. A check of this circuit showed that the delay was less than 3 microseconds so that the whole duration of backfire cause was less than 5 microseconds. Out of 500 oscillograms taken with this arrangement, only one backfire cause was found to be of long time duration. It appears in Fig. 12 as a line which is practically horizontal and represents low voltage for the duration of the film. This particular backfire may have been a failure over the insulator surface.

Section XII

TRANSITION FROM GLOW TO ARC

The transition from glow to arc is of great theoretical interest and has been studied by physicists apart from its application to mercury arc rectifiers. Many believe that the transition is continuous; namely, that as the current density is increased in the glow, the voltage will stop increasing and will decrease continuously down to arc voltages. This has been established experimentally for tungsten electrodes under circumstances where the glow could heat the tungsten up to temperatures sufficient for thermionic emission.[7, 10]

But for non-refractory electrodes or electrodes under such circumstances that they cannot reach temperature for thermionic emission under the glow conditions applying up to the moment of experimentally observed transition, it is hard to explain a continuous transition since the transition occurs while the glow current density is very far less than that corresponding to an arc. Furthermore, the transitions appear to occur in an erratic or random manner which is not compatible with the idea of a continuous transition.

A continuous transition was believed to have been observed by Schrum and Wiest[8] with a copper cathode in air at atmospheric pressure. However, since backfire causes may be of such short duration and may occur with very great rapidity, it is possible that the continuous transition believed to have been observed by these investigators was really a rapid alternation from regular glow to a low-voltage discharge maintained by a backfire cause, the frequency of alternation being beyond the capacity of the oscillograph used to follow.

The transition is frequently attributed to a speck of impurity on the cathode, having a lower thermionic work function, that is, electron emission at a lower temperature than the other cathode material. This idea is in better accord with the random occurrence of the transition since these specks are thought of as being of an accidental origin. However, it is just as hard to account for the transition on a cathode of low work function as on one of high, if, as is usually the case, the energy in the glow is insufficient to raise the cathode temperature by an appreciable amount. It seems to the authors that more important than low work function at the speck is poor thermal or electrical contact of the small particle with the cathode and that transition may occur discontinuously and at random even with electrodes of the highest degree of purity and due entirely to irregularities in the manner of sputtering.

Section XIII

CONCLUSION

1. Backfires in mercury arc rectifiers are inherently random phenomena. There are no fundamental limitations of the rectifying process beyond which backfires may be said to occur, and within which they may be said not to occur.

2. The proper appreciation of the nature of backfire requires that the quality of a rectifier be determined and expressed in terms of its probability to backfire, or its average rate of backfire. Clearly the present methods of testing rectifiers at both full load and overload are insufficient to determine its true quality.

3. Research, and the development of mercury arc rectifiers may be greatly aided by the use of means for producing backfires at an abnormally large rate, so that the mean frequency of backfire can be determined in a reasonable length of time.

4. The idea of random backfire agrees very well with the random way in which backfire causes would be expected to take place, based on the best present knowledge of what these causes may be.

5. The duration of these backfire causes is very short, usually less than 3 microseconds. This indicates a very small amount of material involved in the formation of a cathode spot, on an anode.

6. The transition from glow to arc in the mercury arc rectifier is also found to take place in a random manner, and since this fact is incompatible with a continuous transition, the transition must be accepted as discontinuous.

Appendix I

During a long period of testing of a mercury arc rectifier, a number of time intervals between successive backfires will be observed. If the backfires take place in a random manner, what percentage of these intervals will exceed a given time t? Let m be the number of intervals which exceed the time t, and then $\frac{m}{M}$ will be the fraction of the total exceeding the time t.

To find $\frac{m}{M}$, we must know the probability of backfire per unit time, which is p. The probability of backfire in the time t is $p\,t$. Now suppose that this time t is divided into a large number of subintervals Δt. The chance of having a backfire during anyone of these subintervals is $p\,\Delta t$. The chance of not having a backfire during the first subinterval, Δt_1 is therefore $(1 - p\,\Delta t_1)$. The chance of not having a backfire during the first two consecutive subintervals is

$$(1 - p\,\Delta t_1)\ (1 - p\,\Delta t_2)$$

The chance of having no backfire during the first q consecutive subintervals is

$$(1 - p\,\Delta t_1)\ (1 - p\,\Delta t_2)\ .\ .\ .\ (1 - p\,\Delta t\,q)$$

If we make the subintervals equal to each other,

$$(1 - p\,\Delta t)^q$$

is the chance of having no backfire during the first q subintervals. In the time t, there are $\frac{t}{\Delta t}$ subintervals and therefore:

$$\frac{m}{M} = \lim_{\Delta t \to 0} (1 - p\,\Delta t)^{\frac{t}{\Delta t}} \qquad (1)$$

To evaluate this expression as Δt approaches zero, it may be expanded. Then,

$$(1 - p\,\Delta t)^{\frac{t}{\Delta t}} = 1 - \frac{t}{\Delta t}\, p\,\Delta t + \frac{\frac{t}{\Delta t}\left(\frac{t}{\Delta t} - 1\right)}{2\,!}\, p^2\, (\Delta t)^2 - (.\ .\ .\ .) + (.\ .\ .\ .)\ .\ .\ .\ . \qquad (2)$$

As Δt approaches zero, (**2**) becomes

$$\lim_{\Delta t \to 0}.\ (1 - p\,\Delta t)^{\frac{t}{\Delta t}} = 1 - p\,t + \frac{(p\,t)^2}{2\,!} - \frac{(p\,t)^3}{3\,!} + \ .\ .\ . \qquad (3)$$

But

$$e^{-pt} = 1 - p\,t + \frac{(p\,t)^2}{2\,!} - \frac{(p\,t)^3}{3\,!} + \ .\ .\ . \qquad (4)$$

Hence

$$\frac{m}{M} = \epsilon^{-pt} \qquad (5)$$

This expression may also be obtained by solving the differential equation

$$\frac{d\,m}{d\,t} = -\,m\,p \qquad (6)$$

and using the terminal condition that

$$m = M \text{ when } t = 0 \qquad (7)$$

Appendix II

Suppose that a large number of rectifiers N is tested, and that it is known that the probability of backfire of each is p per unit time. If the rectifiers are tested for unit time, what percentage of them will be expected to give no backfires? This percentage may be found by substituting in equation (**5**). For example let p = one per month, and let t = one month. Then,

$$\frac{n}{N} = \frac{1}{e} = \frac{100}{e}\,\% = 36.7\% \qquad (8)$$

During this test, how many rectifiers will be expected to backfire exactly once? Let R be the number of rectifiers which, at a time t, have backfired exactly once. Then,

$$d\,R = n\,p\,d\,t - R\,p\,d\,t \qquad (9)$$

and

$$\frac{d\,R}{d\,t} + R\,p = p\,N\,e^{-pt} \qquad (10)$$

if for n, its value from equation (**5**) is substituted. Solving the equation (**10**)

$$R = p\,N\,t\,e^{-pt} + K\,e^{-pt} \qquad (11)$$

in which K is a constant of integration. The terminal condition is

$$R = 0 \text{ when } t = 0 \qquad (12)$$

Substituting this condition, and using the value found for K;

$$R = p\,N\,t\,e^{-pt} \qquad (13)$$

or the fraction $\frac{R}{N}$ which will backfire exactly once during the test is

$$\frac{R}{N} = p\,t\,e^{-pt} \qquad (14)$$

Again using the numerical values $p = 1$ and $t = 1$;

$$\frac{R}{N} = \frac{1}{e} = \frac{100}{e}\,\% = 36.7\% \qquad (15)$$

Thus if 36.7 per cent do not backfire at all, and 36.7 backfire once, during a test of one month,

$$100\% - 36.7\% - 36.7\% = 26.8\%$$

will backfire twice or more during the test.

Appendix III

FLUCTUATIONS IN THE GRADIENT AT THE SURFACE OF AN ANODE

The average gradient at the surface of an anode during the period of back voltage may be calculated from Poisson's equation, upon the assumption that the positive ion current to the anode is limited by space charge. If the initial velocity of the positive ions at the boundary of the space charge sheath is neglected, if the ions make no collisions within the sheath, and if the anode is considered a plane, the current, voltage, and gradient relations at the anode surface are,

$$i = \frac{5.45 \times 10^{-8}\, V^{3/2}}{\sqrt{M_0}\, d^2} \qquad (1)$$

$$\frac{\partial\, V}{\partial\, x} = \frac{4}{3}\,\frac{V}{d} = 5.7 \times 10^3\, M_0^{\,1/4}\, V^{1/4}\, i^{1/2} \qquad (2)$$

where d is the space charge sheath thickness in centimeters, i is the positive ion current density in amperes per cm.^{2}, V is the anode potential in volts, and M_0 is the molecular weight of the ions.

The average gradient at the anode is also given by

$$\frac{\partial V}{\partial x} = 4 \pi N e \qquad \textbf{(2a)}$$

where N is the total number of positive ions in the space charge sheath opposite one cm.^{2} of the anode surface. Then random fluctuations in the number of ions in the space charge sheath opposite an element of area ΔA will cause fluctuations in the gradient over ΔA. What is the probability that the average gradient will be multiplied by a factor ρ, sufficient for high-field electron emission to occur?

According to Poisson's Law,[11] which will apply in this case with sufficient accuracy, the probability of n ions occurring over an interval ΔA, where the expected number of ions is $N \Delta A$, will be

$$P(n) = \frac{(N \Delta A)^n}{N\,!}\, \epsilon^{-N \Delta A} \qquad (3)$$

In order that a field of the desired magnitude exist, $n = \rho N \Delta A$. For purposes of calculation $N\,!$ may be evaluated by Stirling's formula

$$N\,! = \sqrt{2 \pi n}\, n^n\, \epsilon^{-n} \qquad (4)$$

Making these substitutions, the probability of a field high enough for electron emission occurring over any area ΔA is

$$P = \frac{\epsilon^{(\rho - 1) N \Delta A}}{(2 \pi \rho N \Delta A)^{1/2}\, \rho^{\rho N \Delta A}} \qquad (5)$$

Of course, actually the distribution of positive ions is continually changing; but a good idea of the fluctuations may be obtained if it is considered that any configuration of positive ions endures for a time Δt, then suddenly changes to a different configuration. Δt is made equal to the time of transit of a positive ion across the space charge sheath. Thus, the number of occurrences per second, over the entire anode, favorable for the development of a backfire is

$$\nu = \frac{A}{\Delta A}\, P\, \frac{1}{\Delta t} \qquad (6)$$

Δt may be calculated, on the basis of the assumptions used previously, from

$$M \frac{d^2 x}{d t^2} = e \frac{d V}{d x} \qquad (7)$$

$$\frac{d V}{d x} = \frac{4}{3} \frac{(9 \pi)^{2/3}}{\left(\frac{2 e}{M}\right)^{1/3}}\, i^{2/3} x^{1/3} \qquad (8)$$

So,

$$\frac{d^2 x}{d t^2} = 6\,(2/3\, \pi)^{2/3} \left(\frac{e}{M}\right)^{2/3} i^{2/3} x^{1/3} \qquad (9)$$

Integrating twice

$$x^{1/3} = \left(\frac{2}{3} \pi\right)^{1/3} \left(\frac{e}{M}\right)^{1/3} i^{1/3}\, t \qquad \textbf{(10)}$$

Thus,

$$\Delta t = \frac{d^{1/3}}{(2/3\, \pi)^{1/3} \left(\frac{e}{M}\right)^{1/3} i^{1/3}}$$

or, putting in numerical values, and substituting from (1)

$$\Delta t = 5 \times 10^{-10}\, \frac{M_0^{1/4}\, V^{1/4}}{i^{1/2}} \qquad \textbf{(11)}$$

As typical data, the following may be taken:

Peak Anode voltage = 1,600 volts
Anode current = 10 milliamperes
Anode area = 625 cm.^{2}
Current density = 1.6×10^{-5} amperes per cm.^{2}

For mercury, $M_0 = 200$. So, from equation (2), at the anode surface

$$\frac{\partial V}{\partial x} = 545 \text{ volts per cm.}$$

In order for an appreciable number of electrons to be emitted from cold metals, theory indicates that a field of 5×10^7 volts per cm. is needed. It is hardly conceivable that surface irregularities can multiply the field by more than a factor of 100. So, at least a field of 5×10^5 volts per cm. must result from random fluctuations before backfire can occur from this cause. The value of ρ then is 1,000.

The value of N, from equation (**2a**), is 3×10^8, while from equation (**11**), Δt comes out 3×10^{-6}.

In order to make the probability come out as large as possible, we shall assume that an anode bears the peak inverse voltage continually. From equations (**6**) and (**5**) the average number of backfires to expect per second per anode due to fluctuations in the field, will be

$$\nu = \frac{625}{\Delta A}\, \frac{\epsilon^{999 \times 3 \times 10^8 \Delta A}}{(2 \pi \times 1000 \times 3 \times 10^8 \Delta A)^{1/2}\, 1000^{1000 \times 3 \times 10^8 \Delta A}}\, \frac{1}{3 \times 10^{-6}}$$

If we take $\Delta A = 10^{-5}$, which seems to be a reasonable figure, approximately

$$\nu = 4.78 \times 10^4\, \epsilon^{2.997 \times 10^6}\, 1000^{-3 \times 10^6} = 10^{-7.7 \times 10^6}$$

Thus, the average time between backfires, on this hypothesis, would approach $10^{7{,}700{,}000}$ years.

Suppose ΔA is made 3.3×10^{-9}, so that $N \Delta A = 1$. Then,

$$\nu = 7.87 \times 10^{14}\, \epsilon^{999}\, 1000^{-1000}$$
$$\sim 10^{-2551}$$

The average time between backfires would be about 10^{2554} years.

From these calculations, it seems impossible that

fluctuations in the electric field at the anode surface, due to fluctuations in positive ion density alone, can account for backfires in practical rectifiers.

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Overvoltage on Transmission Systems Caused by Dropping Load

Discussion of Means of Limiting Such Overvoltage with Particular Reference to the Use of Generator Voltage Regulators

BY E. J. BURNHAM*
Associate, A. I. E. E.

Synopsis.*—In a previous article*[1] *the author showed the manner in which under different conditions the voltage on transmission systems will rise due to the dropping of load, and described some means of limiting the voltage rise to avoid overstressing insulation. This paper describes further means of limiting the voltage under such conditions, considering first an improved type of regulator and wheatstone bridge main exciter field rheostat and second, as an alternative, a method of automatically inserting resistance in the alternator field circuit. Tests are described which show the effectiveness of these methods of limiting the voltage.*

Introduction

It is generally understood that total or partial loss of load on waterwheel driven generators or hydro stations frequently results in undesirable voltage rise, and that in such cases it is desirable to limit the voltage rise to avoid overstressing the insulation of generators and connected apparatus.[1]

In the past this has sometimes been accomplished by use of overvoltage, overspeed or overfrequency relays which automatically open the field circuit and trip the generator from the line or bus when the voltage or speed reaches a predetermined value. This is a very effective way of limiting the overvoltage, but it has a disadvantage if several units are in the station because after the machines are separated, considerable time is lost resynchronizing the units and getting them back in service.

It has long been recognized that generator voltage regulators are beneficial in reducing the generator voltage at times when load is lost, but with the older type of excitation systems and regulators which insert a series block of resistance in the main exciter field circuit, the voltage reduction which could be produced often was not adequate either in magnitude or promptness. However, with improved voltage regulators and the use of excitation systems as now developed, it is possible to hold the generator voltage within desired limits when load is dropped.

The purpose of this paper is to show how the voltage on a waterwheel generator or hydro station may be held within reasonable limits at a time when full load is suddenly lost without opening the field circuits of the machines and tripping them from the line or bus.

Tests made on a 47,000-kva. generator at the Spier Falls Plant of the New York Power and Light Corporation are described in which use was made of a newly developed generator voltage regulator and a wheatstone bridge type of main exciter field rheostat, making it possible to reduce quickly the generator field current and thereby limit the generator voltage.

Tests made on the same generator are also described showing the possibilities of limiting generator voltage by inserting resistance automatically in the alternator field.

It is often desirable to determine the overvoltage characteristics of a generator or station under different conditions before the station is built or completed in order that suitable precautions can be taken in the design period. For this reason a method of calculation is included in the appendix, the results of which check very closely with tests.

Test Connections

Fig. 1 shows a simplified diagram of connections used in the tests taken at the Spier Falls Station. The generator is rated 47,000 kva., 81.8 r. p. m., 60 cycles, 13,800 volts, 0.80 power factor, and the direct-connected main exciter, 264 kw., 250 volts. The field of the main exciter is excited from a 6-kw., 250-volt, compound-wound direct-connected pilot exciter. The main exciter is compound wound, but, as tests were desired for a shunt-wound exciter, arrangements were made to insert, by use of a switch, an 0.08-ohm resistor in the alternator field circuit, which would change the time constant of the circuit to a value that would simulate that of a shunt-wound main exciter.

The generator is connected to the 110-kv. bus through a bank of transformers rated 47,000 kva. There being no low-voltage bus or circuit breaker, load was always dropped in the tests, by tripping the high-voltage transformer circuit breaker.

A six-element oscillograph was used on which the following currents and voltages were recorded:

1. 60-cycle timing wave.

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*General Electric Company, Schenectady, N. Y.

[1] See Bibliography.

Presented at the South West District Meeting of the A. I. E. E., Kansas City, Mo., October 22-24, 1931.

2. Alternator armature current.
3. Main exciter field voltage.
4. Alternator armature voltage.
5. Main exciter armature voltage.
6. Alternator field current.

REGULATOR EQUIPMENT

The main control element of the regulator known as the FA-4 is shown in Fig. 2 and the high-speed relays *X* and *Y* in Fig. 3.

The regulator may be used with a wheatstone bridge main exciter field rheostat as shown in the diagram, or with a series type rheostat.

The wheatstone bridge rheostat, Fig. 1, is suitable for use with synchronous condensers or other machines where the excitation range is broad and it is desirable to reduce the main exciter voltage to a very low value or reverse it, or where it is desirable to reduce the exciter voltage very quickly as in the case under consideration.

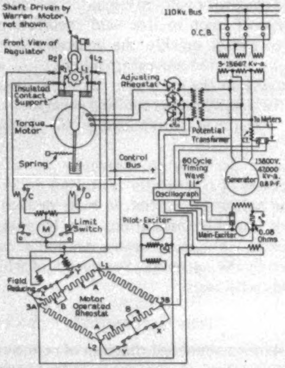
Shaft Driven by Warren Motor not shown
Front View of Regulator
R2
R1
L1
L2
Insulated Contact Support
Torque Motor
Spring
Control Bus
C
D
M
Limit Switch
110Kv. Bus
O.C.B.
3-15667 Kv-a.
Adjusting Rheostat
Potential Transformer
To Meters
60 Cycle Timing Wave
13800V. 47000 Kv-a. 0.8 P-F.
Generator
Oscillograph
Pilot-Exciter
Main-Exciter
0.08 Ohms
Field Reducing
X
Y
A
B
3A
3B
Motor Operated Rheostat
L1
L2

FIG. 1—DIAGRAM OF CONNECTIONS FOR TESTS AT SPIER FALLS STATION

Arms marked *A* are operated simultaneously by motor *M*. The arms marked *B* are operated simultaneously by handwheel and are used for adjustment to give the desired voltage reduction when the high-speed relays *X* open. In the case under consideration a setting was made such that when relays *X* opened the main exciter armature voltage was reduced to a negative value of approximately 10 volts.

The primary control element of the FA-4 regulator consists of a three-phase torque motor to the rotor of which are attached two pairs of contacts. Both front and rear contacts make contact with disks connected to a shaft driven at constant speed by a small a-c. motor. The front disk has cam-shaped teeth while the rear disk is smooth.

Secured to the shaft of the torque motor is an arm to which is attached a spring which balances the torque of the torque motor. The arm also carries a paddle which is immersed in a vessel of oil, producing the required damping action.

OPERATION OF REGULATING EQUIPMENT

This regulator remains inoperative with both pairs of contacts out of engagement as long as no change in excitation is required. For slight changes in voltage only the front contacts *L 1* and *R 1* and the motor-operated field rheostat are affected, the action being a "notching" one varying from a single brief impulse to practically continuous operation of the motor, depending upon the correction required. Only for greater changes in voltage do the back contacts *L 2* and *R 2* and the high-speed relays come into action maintaining the voltage until the motor-operated rheostat has moved to a position corresponding to the required excitation.

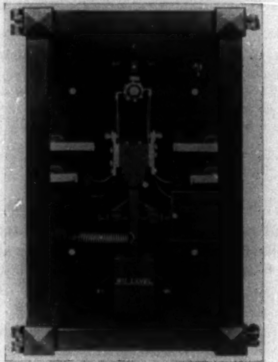

FIG. 2—FRONT VIEW, TYPE FA-4 REGULATOR

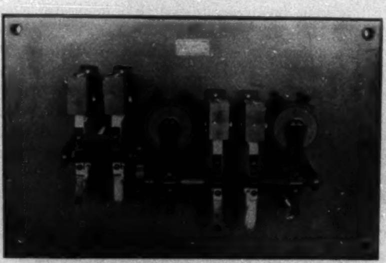

FIG. 3—HIGH-SPEED RELAYS FOR USE WITH FA-4 REGULATOR

The notching action of the front contacts in conjunction with the intermittent operation of the rheostat prevents over-travel of the latter and consequent hunting. On voltage changes of greater magnitude the periods of engagement of the front contacts are longer, tending to move the motor-operated field rheostat more rapidly.

The back contacts go in operation when the a-c. voltage has changed 3 to 15 per cent according to the adjustment made. The frequency range under which the regulator will hold constant voltage is very broad being between the limits of 25 and 85 cycles.

EQUIPMENT TO INSERT RESISTANCE AUTOMATICALLY IN THE ALTERNATOR FIELD CIRCUIT

Fig. 4 shows the diagram of connections and devices used to insert resistance automatically in the alternator field circuit upon the occurrence of overvoltage. The overvoltage relay, which is of the contact making voltmeter type, endeavors to hold alternating voltage in accordance with its setting by opening and closing a contactor connected across a resistor in the alternator field circuit. In the case under consideration the overvoltage relay was set to operate at approximately 16,800 volts or 22 per cent over normal voltage. As soon as the contactor opens, the setting of the overvoltage relay may be changed if desired to hold normal voltage or a voltage of some predetermined value. This change in setting of the overvoltage relay is accomplished by the use of a hand reset relay which upon closing its contacts, changes the value of resistance in the coil circuit of the overvoltage relay.

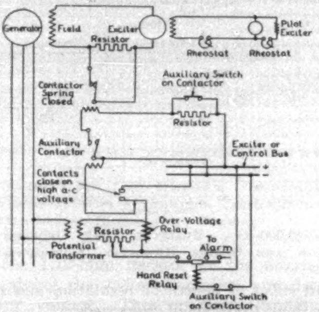

FIG. 4—DIAGRAM SHOWING METHOD OF AUTOMATICALLY INSERTING RESISTANCE IN ALTERNATOR FIELD CIRCUIT UPON OCCURRENCE OF OVERVOLTAGE

TESTS

An outline of the tests which were made is given in Table I.

Fig. 5 shows four tests which were taken at no load to compare the rate of decay of alternator voltage when the excitation was reduced in different ways. In each of the four cases, the alternator voltage was raised under hand control to approximately 16.1 kv. In case 1 the excitation was reduced by inserting a 20-ohm resistor in the main exciter field circuit corresponding to the value of field reducing resistance described later under load tests when regulator and series type main exciter field rheostat connections were used. In cases 2 and 3 the relays X in the wheatstone bridge circuit were opened. In case 4 the contacts of the overvoltage relay were closed, which inserted the 1.12-ohm resistor in the alternator field.

TABLE I

Fig.	Main exciter	Type main exciter field rheostat	Load dropped kw.	Load dropped kva.	Power factor
5	Compound	Series	Tests taken at no load to compare rate of alternating voltage decay.		
5	Compound	Wheatstone Bridge			
5	Shunt	Wheatstone Bridge			
5	Insert resistor in alternator field circuit				
6	Shunt	Wheatstone Bridge	40,000	44,444	0.9
7	Compound	Wheatstone Bridge	40,000	44,444	0.9
8	Compound	Series	0	40,000	0
9	Compound	Wheatstone Bridge	0	40,000	0
10	Insert resistor in alternator field circuit		40,000	44,444	0.9

Figs. 6, 7, 8, 9 and 10 show the results of dropping load on the 47,000-kva. generator under different conditions.

DISCUSSION OF TEST RESULTS

Referring to Fig. 5, the voltage reduction produced with the wheatstone bridge was very rapid as compared with that resulting from the insertion of series resistance in the main exciter field circuit, but not as rapid as the voltage reduction produced by inserting resistance in the alternator field circuit. Comparing cases 2 and 3, a more rapid voltage reduction was given by the shunt than by the compound arrangement of main exciter.

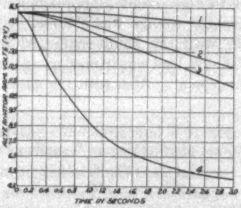

FIG. 5—DECREASE OF ALTERNATOR OPEN-CIRCUIT VOLTAGE

Case 1—By inserting series resistance 20 ohms in main exciter field. Compound exciter

Case 2—By opening relays x in wheatstone bridge circuit. Compound exciter

Case 3—By opening relays x in wheatstone bridge circuit. Shunt exciter

Case 4—By closing overvoltage relay contacts which inserted 1.12-ohm resistor in alternator field circuit

When load was dropped, there were two or three cycles of single-phase operation caused by the three poles of the 110-kv. circuit breaker not opening at exactly the same time. Because of this it will be noticed, that the alternator field current did not decrease instantaneously at zero time, but instead dropped to its new value in about three cycles, which is reasonable when it is considered that it took that time to completely drop the load.

In all cases where 40,000-kw. load was dropped, a maximum speed of 138 per cent of normal was reached in approximately 3.5 seconds. This is about the value expected and falls within general practise as shown by general overspeed curves given in Fig. 12.

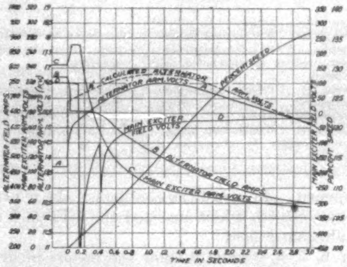

FIG. 6—40,000-KW. 0.9-POWER FACTOR LOAD DROPPED USING WHEATSTONE BRIDGE RHEOSTAT. SHUNT EXCITER

Referring to Figs. 6 and 7, approximately 4 per cent higher alternator voltage was obtained with the compound than with the shunt-connected main exciter when a 40,000-kw. load was tripped using the wheatstone bridge arrangement. Also with the shunt exciter the overvoltage was reduced more quickly than was the case with the compound exciter. The maximum voltage when using the shunt exciter was 16.5 kv. or approximately 20 per cent above normal; and for the compound exciter 17.2 kv. or approximately 25 per cent above normal. This shows very clearly the effectiveness of the regulator with the wheatstone bridge arrangement because with no regulator the alternator voltage would have reached 180 per cent of normal value when tripping the high-voltage transformer circuit breaker and dropping 40,000-kw., 0.9-power-factor load. Without the effect of transformer magnetizing current as would be the case if the load was tripped on the low-voltage side of the transformer, the alternator voltage would reach double value with no regulator or voltage limiting means.

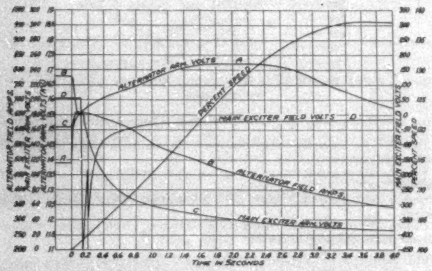

FIG. 7—40,000-KW. 0.9-POWER FACTOR LOAD DROPPED USING WHEATSTONE BRIDGE RHEOSTAT. COMPOUND EXCITER

Figs. 8 and 9 show the results of dropping a 40,000-kva. entirely reactive load. These two tests give the combined effect of generator regulation and regulator action without the effect of overspeed.

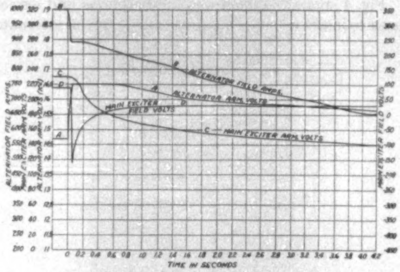

FIG. 8—40,000-KVA. REACTIVE LOAD DROPPED USING REGULATOR AND SERIES RHEOSTAT. COMPOUND EXCITER

Referring again to Figs. 8 and 9, a comparison can be made between the effect of the regulator used with an ordinary series rheostat and used with a wheatstone bridge rheostat in the main exciter field. With the wheatstone bridge arrangement there was, in the first second after load was dropped, a change of alternator voltage of 575 volts, while with the series resistance connection the change was only 150 volts.

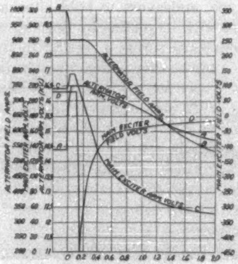

FIG. 9—40,000-KVA. REACTIVE LOAD DROPPED USING REGULATOR AND WHEATSTONE BRIDGE RHEOSTAT. COMPOUND EXCITER

Fig. 10 shows what may be accomplished in limiting alternator voltage by automatically inserting resistance of 1.12 ohms in the alternator field circuit when dropping a 40,000-kw., 0.9-power-factor load. In this test the overvoltage relay shown in Fig. 4 was set to operate at 16,800 kv., and hold this value. It will be noticed that the maximum alternator voltage reached was 17.7 kv. or approximately 128 per cent of normal. In attempting to hold the alternating voltage, the overvoltage relay opened and closed the contactor connected across the 1.12-ohm resistor in the alternator field circuit, giving the sudden increases and decreases in alternator voltage. This operation continued until the regulator could get control and lower the alternating voltage to a point where the overvoltage relay would not operate. This was accomplished at the end of approximately 10 seconds, as shown by the high-speed voltage chart record given in Fig. 11. At this point the regulator which had been doing all it could to reduce the excitation, finally gained control and brought the voltage back to normal, the latter being reached in about 15 seconds. Note that the rather long time which elapsed before the regulator gained control was due to the use of the series instead of the wheatstone bridge arrangement. The voltage chart curve did not follow the oscillograph voltage curve very closely because of the damping action of the voltage chart pen.

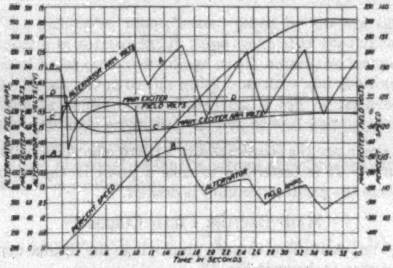

FIG. 10—40,000-KW. LOAD DROPPED. OVERVOLTAGE RELAY INSERTS RESISTANCE IN ALTERNATOR FIELD CIRCUIT. REGULATOR WITH SERIES RHEOSTAT IN SERVICE

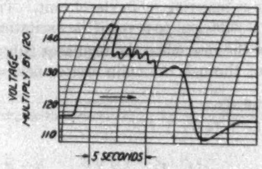

FIG. 11—HIGH-SPEED VOLTAGE CHART RECORD OF TEST SHOWN IN FIG. 10 WHERE RESISTANCE WAS AUTOMATICALLY INSERTED IN ALTERNATOR FIELD CIRCUIT

CONCLUSIONS

The tests described indicate that a simple means of limiting overvoltage caused by dropping load is available, consisting of a suitable regulator and wheatstone bridge exciter field rheostat set to give zero or slightly negative main exciter armature voltage.

Such equipment under ordinary conditions functions as required to hold normal alternating voltage, but at the same time is ready to prevent the voltage from reaching dangerous values which would overstress the insulation of generators and connected apparatus. Furthermore, the use of such equipment has made it possible to design lightning arresters and protective apparatus to give better protection and at the same time not be subject to failure on over-dynamic voltage.

The alternative method of automatically inserting resistance in the alternator field circuit is also very effective in keeping the alternating voltage within desired limits when load is lost. Therefore this method is available for use where overvoltage protection is needed and there are no generator voltage regulators or where regulators are being used, but the excitation system is of the older type and it is not desirable or economical to change the exciter field circuit to include such equipment as the wheatstone bridge.

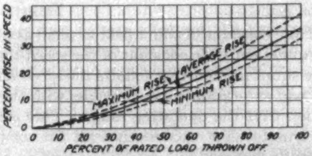

FIG. 12—TYPICAL SPEED CHANGES ON HYDRAULIC TURBINES FOR LOAD THROWN OFF TO ZERO

In the tests described, load was dropped by tripping the high-voltage transformer circuit breaker, therefore the effect of line charging current was not included. The effect of charging current is to give a higher value of voltage, the amount varying with the length and voltage of the line and the number of generators and transformers connected after load is lost.

ACKNOWLEDGMENTS

The author desires to express appreciation for the assistance rendered by the New York Power and Light Corporation in granting permission to make the tests, and to members of the Engineering Department of the General Electric Company for suggestions and valuable assistance in working up the data.

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Appendix

A METHOD OF CALCULATING ALTERNATOR OVERVOLTAGE FOLLOWING SUDDEN LOSS OF LOAD

BY J. W. BUTLER*

In this appendix equations are derived for calculating alternator overvoltage following sudden loss of load. The analysis is made for the bridge-type rheostat, but in general the same method can be used for other types of control.

ASSUMPTIONS

Assumptions used in this article will be summarized below.

(*a*) *Pilot Exciter*

1. Time constant of shunt field so small as compared to the equivalent time constant of the speed-time curve that the shunt field current follows the speed exactly.

2. Load saturation curve coincides with the no load saturation over the operating range. This is approximately true for pilot exciters that are flat compounded and liberally designed.

(*b*) *Main Exciter*

1. Shunt-wound exciters will be considered as they are used in common practise at the present time.

2. Inductance of field circuit remains constant, but is corrected by a factor, depending on the type of field structure.

3. Open circuit instead of load saturation curve is used.

Assumptions 2 and 3 compensate in a measure for the effect of the voltage reduction following a higher curve than the saturation curve, due to eddy currents, when the field current decreases rapidly.

PILOT EXCITER ARMATURE VOLTS

With the above assumptions, the curve of pilot exciter armature volts may be readily obtained as follows:

From the saturation curve of the pilot exciter at normal speed, saturation curves for 110, 120, etc., per cent speed may be drawn. Then drawing in the field circuit resistance line, the voltage for any speed is read at the intersection of the resistance line and the saturation curve for that particular speed. From the values of voltage for different speeds obtained and the hydraulic turbine speed-time curve, a volt-time curve is obtained.

An empirical expression for this curve in operational form using one exponential is

$$e_1 = a + \frac{b - a}{T_1\,p + 1} \tag{1}$$

Where a and b are the initial and final values respectively, and

$$T_1 = \frac{t_o}{\log_\epsilon \dfrac{b - a}{b - e_o}}$$

$$p = \frac{d}{d\,t}$$

e_o and t_o are the coordinates of a particular point on the curve. It is preferable to take a point in the region where the greatest accuracy is desired.

MAIN EXCITER FIELD CURRENT

The excitation circuit, employing the bridge-type rheostat, is shown in Fig. 13.

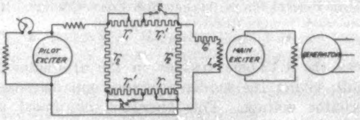

FIG. 13—DIAGRAM OF EXCITATION SYSTEM USING WHEATSTONE BRIDGE RHEOSTAT

The voltage e_1 operates upon the bridge circuit in which r_o and L_o is the shunt field resistance and inductance respectively, and r_1, r_1', and r_2 are the rheostat resistances. Upon indication of overvoltage, the regulator opens x, inserting r_1' in the circuit. The value of r_1' usually is sufficient to reverse the excitation.

Consider the case when the regulator fails to operate. The current through the main exciter field will be

$$i_o = \frac{a\,(r_2 - r_1)}{2\,r_1\,r_2 + r_o\,(r_2 + r_1)}$$

$$+ \frac{r_2 - r_1}{2\,r_1\,r_2 + (r_o + p\,L_o)(r_2 + r_1)}\,\frac{b - a}{T_1\ p\ +\ 1}\,1 \tag{2}$$

The solution of which is—

$$i_o = \frac{a}{m} + \frac{b - a}{n\,T_1}\,(A_o + A_1\,\epsilon^{-a_1 t} + A_2\,\epsilon^{-a_2 t}) \tag{3}$$

$$m = \frac{2\,r_1\,r_2 + r_o\,(r_2 + r_1)}{r_2 - r_1}$$

$$n = \frac{L_o\,(r_2 + r_1)}{r_2 - r_1}$$

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*General Electric Co., Schenectady, N. Y.

$$A_o = \frac{1}{a_1 a_2}$$

$$a_1 = \frac{m}{n}$$

$$A_1 = \frac{1}{-a_1(a_2 - a_1)}$$

$$a_2 = \frac{1}{T_1}$$

$$A_2 = \frac{1}{-a_2(a_1 - a_2)}$$

The high-speed relays x are opened by the voltage regulator in approximately three cycles after the indication of 5 per cent overvoltage which occurs the instant load is lost. Due to this quick operation and the time constants of the circuits involved, it can be assumed with very little error that the loss of load and opening of the relays x occur simultaneously.

Hence, the equation for the current after the regulator acts can be written thus

$$i_o = \frac{a(r_2 - r_1)}{2r_1 r_2 + r_o(r_2 + r_1)} - 2\left[\frac{a(r_2 + r_o)}{2r_2 r_1 + r_o(r_2 + r_1)}\right.$$

$$\left.\text{times}\,\frac{r_2 r_1}{2r_2(r_1 + r_1') + (r_o + pL_o)(r_2 + r_1 + r_1')}\right]1$$

$$+\frac{r_2 - r_1 - r_1'}{2r_2(r_1 + r_1') + (r_o + pL_o)(r_2 + r_1 + r_1')}\,\frac{(b - a)}{T_1 p + 1}1 \tag{4}$$

The solution being

$$i_o = Y_1 + Y_2\epsilon^{-a_2 t} + Y_3\epsilon^{-a_3 t} \tag{5}$$

where

$$Y_1 = \frac{b(r_2 - r_1 - r_1')}{2r_2(r_1 + r_1') + r_o(r_2 + r_1 + r_1')}$$

$$Y_2 = \frac{(r_2 - r_1 - r_1')(b - a)}{L_o(r_2 + r_1 + r_1')(a_2 - a_3)}$$

$$Y_3 = \frac{(r_2 - r_1 - r_1')(b - a)}{[2r_2(r_1 + r_1') + r_o(r_2 + r_1 + r_1')][(T_1 a_3 - 1)]}$$

$$+\frac{2a(r_2 + r_o)r_2 r_1'}{[2r_2 r_1 + r_o(r_2 + r_1)][2r_2(r_1 + r_1') + r_o(r_2 + r_1 + r_1')]}$$

$$a_3 = \frac{2r_2(r_1 + r_1')}{L_o(r_2 + r_1 + r_1')} + \frac{r_o}{L_o}$$

$$a_2 = \frac{1}{T_1}$$

Thus, equation (5) gives an equation of main exciter field current considering regulator action on the bridge rheostat and overspeeding of the pilot exciter.

This current flowing through the main exciter shunt field produces main exciter armature volts and current which will not be considered.

MAIN EXCITER ARMATURE VOLTS AND CURRENT

By plotting equation (5), the curve of main exciter armature volts can be determined considering the increase in speed as follows:

For any particular value of i_o flowing at time t, the voltage is read from the saturation curve. From the speed-time curve of the unit, the speed of the set can be read for the particular time t chosen and the voltage increased in direct ratio. The voltage function thus obtained operates upon the generator field.

The operational equation of such a curve, as shown for (1) is,

$$E = E_o + \frac{E_1}{T_2 p + 1}1. \tag{6}$$

The problem now is to find an equation of generator field current as a function of time. Having this, terminal voltage can be read directly from the saturation curve for any value of field current and corrected for the increased speed at the particular time chosen.

There are two factors affecting the generator field current. Upon sudden loss of load the field current drops immediately to a value necessary to produce flux back of transient reactance. It then builds up to a value that existed before loss of load according to the open-circuit time constant of the generator field circuit providing constant collector ring volts is maintained. But in this case under consideration, due to the increased speed of exciters and regulator action, the collector-ring voltage varies according to equation (6).

With the voltage expressed by equation (6) applied to the collector rings and considering the drop of current at $t = 0$, the following type of equation for generator field current will result:

$$I = I_o - I_o'\frac{p}{T_{do}'p + 1}1 + \frac{E_1}{r}\,\frac{1}{(T_2 p + 1)(T_{do}'p + 1)}1 \tag{7}$$

The solution being

$$I = I_o - I_o'\epsilon^{-c_1 t} + I_1(C_o + C_1\epsilon^{-c_1 t} + C_2\epsilon^{-c_2 t}) \tag{8}$$

Where I_o = field current before loss of load

$$I_1 = \frac{E_1}{r\,T_2\,T_{do}'}$$

T_{do}' = open circuit time constant of generator.

r = resistance of field circuit.

I_o' = the decrease in field current the instant load is lost.

$$C_o = \frac{1}{c_1 c_2} \qquad C_2 = \frac{1}{-c_2(c_1 - c_2)}$$

$$C_1 = \frac{1}{-c_1(c_2 - c_1)} \qquad c_1 = \frac{1}{T_{do}'}$$

$$c_2 = \frac{1}{T_2}$$

Plotting (8), the curve of generator terminal voltage can be determined by using the generator saturation curve and speed-time curve as was explained for main exciter armature volts.

Using this method of calculation, curve A' of Fig. 6 shows the calculated voltage of the 47,000-kva. generator under the test conditions of dropping load. This curve agrees very closely with the test voltage curve A, considering calculated machine constants were used and that there were 2 or 3 cycles of single-phase operation in dropping load.

Discussion

E. J. Burnham: In regard to the question of quick response excitation schemes now in use giving overvoltage protection such as described in the paper: Quick response excitation as ordinarily provided will not necessarily prevent the voltage of a waterwheel-driven generator from rising to an undesirable value upon dropping load. In the past many quick response excitation systems have been designed with reference to voltage build-up to take care of short-circuit conditions. Such systems, especially if series rheostats are used in the main exciter field, may permit the a-c. voltage to rise to a rather high value in case load is dropped. For example the 47,000-kva. waterwheel generator described in this paper will have the same quickness of response as far as build-up of excitation is concerned whether the series rheostat or wheatstone bridge rheostat is used, and in either case may be said to have a quick response excitation system. However, when dropping full load with the wheatstone bridge rheostat in service the a-c. voltage rises to 125 per cent of normal (Fig. 6) and when using a series rheostat under similar conditions, the a-c. voltage rises to a value approximately 145 per cent of normal (calculated).

As to the effect of adequate voltage control on lightning arrester design, the over-dynamic voltage permitted on a transmission system has a direct bearing on the efficiency and cost of the lightning arrester. The use of suitable overvoltage protective equipment prevents the high overvoltage resulting from dropping load on waterwheel generators and permits the use of standard lightning arresters. Unless the overvoltage is controlled, special arresters with increased safety factor are required to prevent possible arrester failure. The protective efficiency of the special arrester will be less than the standard and the increased cost may be comparable to the cost of the overvoltage protective equipment.

Interconnection of the 25- and 60-Cycle System
of the Union Electric Light and Power Company

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HISTORY AND DEVELOPMENT

THE Mississippi River Power Company's hydro plant at Keokuk, Iowa, was developed in the years 1912-1913 as a 25-cycle run-of-river plant with an installed capacity of 135,000 kilowatts in 15 generating units. This plant was projected on the basis of a power contract with St. Louis utilities for 60,000 prime horsepower (45,000 kw.). The smaller city loads at Keokuk, Fort Madison and Burlington, Iowa, were also supplied from the plant. During 1930 the prime load demand reached 110,000 kilowatts with sufficient excess sales to load the plant to capacity over the daily peak.

As a run-of-river plant the capacity of Keokuk fluctuates over the seasons, being divided essentially into three periods:

1. A low-water winter period in which the capacity is reduced to 115,000 kilowatts.
2. A low-water navigation period in which the capacity is reduced to 83,000 kilowatts.
3. A high-water period in which the capacity of the plant is limited to 70,000 kilowatts.

The average duration of these limitations on the plant is 85 days, 102 days, and 150 days per year, respectively. With the average combined limitation for one reason or other of 252 days per year in which the capacity of the plant is less than 100,000 kilowatts, the relay requirements to the water power plant became an important factor. With only 40,000 kilowatts of steam reserve available at St. Louis to augment these periods of deficiency, and with an average rate of growth of 4,000 kilowatts a year on the 25-cycle system, additional relay capacity was needed if the 25-cycle load continued to grow at this rate. Further growth of 25-cycle load in St. Louis has been restricted for several years.

Consideration was given to a number of means of supplying this additional capacity:

1. Additional units at Keokuk. The original construction of the Keokuk Plant provided space for 15 additional units without reconstruction of coffer dams.
2. Installation of a steam plant at Keokuk.
3. Installation of additional 25-cycle steam reserve at St. Louis.
4. Installation of frequency converter at St. Louis.

Coincident with this same problem there arose the requirement for additional 60-cycle generation capacity to supply the rapidly growing metropolitan load at St. Louis. Simultaneous studies revealed the advisability of the development of the Osage Hydro Plant, located on the Osage River about 130 miles southwest of St. Louis, together with the interconnection of this plant with the St. Louis load through the Page Avenue Substation, which is also the terminus of the Keokuk-St. Louis Transmission Lines. The system map is shown in Fig. 1.

The 60-cycle load of 200,000 kilowatts demand growing at the rate of approximately 15 per cent a year required a yearly increase in 60-cycle generating equipment in excess of 30,000 kilowatts; whereas the annual growth in the 25-cycle system of 4,000 kilowatts average indicated that the reserve 60-cycle generating capacity required solely by the 60-cycle system could be reasonably utilized as 25-cycle reserve without additional prime mover investment for the 25-cycle system, providing suitable interconnection were arranged.

Studies of the 25-cycle system load curve as related to the limited schedule of Keokuk Plant capacity showed the loss of approximately 60,000,000 kw-hr. annually with available excess capacity up to 40,000 kilowatts. This was due to the inability of the 25-cycle load to absorb this energy when and if available, particularly during night periods.

The installation of the Osage Plant with its interconnecting transmission lines indicated the requirement at the St. Louis end of approximately 50,000 kva. in synchronous condenser capacity.

GENERAL SPECIFICATION OF FREQUENCY CONVERTER

Consideration of all of these facts from a coordinated point of view gave evidence of the desirability of interconnection between the 25- and 60-cycle systems with a suitable type of frequency changer. The frequency changer should have the following characteristics:

1. It should be reversible in order to provide:
   (a) 25-cycle power to the 25-cycle system to augment the combined water power and steam generation in times of water power deficiency. Comparison of the rates of growth of the 25- and 60-cycle systems indicated that the interconnection through the frequency changers would provide 25-cycle prime reserve without additional investment in 60-cycle generating equipment.
   (b) Access to the 60-cycle system for additional utilization of 60,000,000 kw-hr. annually from the 25-cycle system.
2. The machine should be of such type as would provide interconnection between a fixed capacity 25-cycle generating system of 175,000 kilowatts and an

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31-137

increasing 60-cycle system capacity with a present rating in excess of 450,000 kilowatts.

3. The 60-cycle end of the machine should combine approximately 50,000 kva. of synchronous condenser capacity for use in bringing Osage power into the St. Louis district.

4. Either end of the machine should be constructed to run separately in the event that the diversity between the water sheds of the Keokuk Plant and Osage Plant required such operation. This might also be required in the interest of economy in operation.

5. Two machines of one-half capacity each would be preferable to one since failure of one machine would still permit adequate operation under most cases.

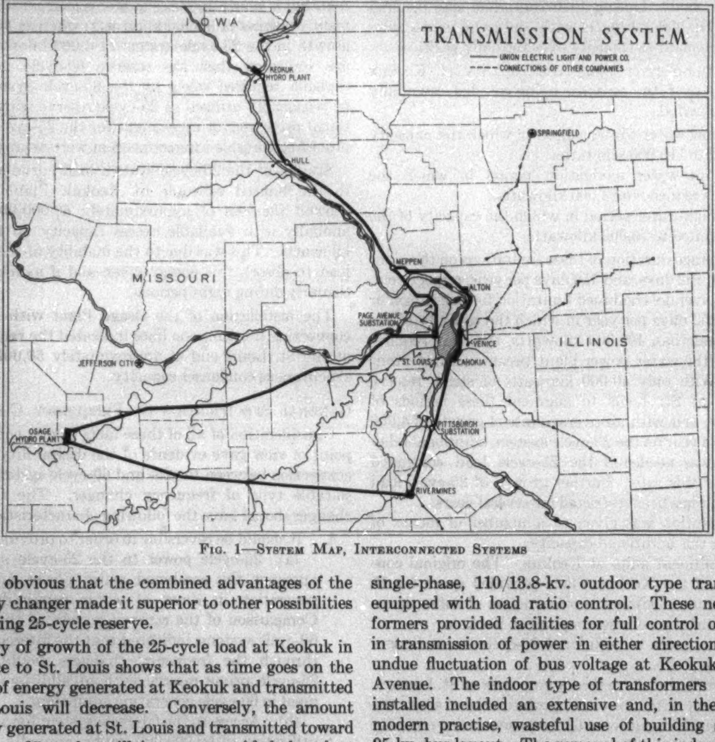


FIG. 1—SYSTEM MAP, INTERCONNECTED SYSTEMS

It was obvious that the combined advantages of the frequency changer made it superior to other possibilities of supplying 25-cycle reserve.

A study of growth of the 25-cycle load at Keokuk in preference to St. Louis shows that as time goes on the amount of energy generated at Keokuk and transmitted to St. Louis will decrease. Conversely, the amount of energy generated at St. Louis and transmitted toward Keokuk at 25 cycles will increase provided the plant capacity at Keokuk remains fixed. The latter amount is now in the order of a few per cent of the former.

SYSTEMS INTERCONNECTION

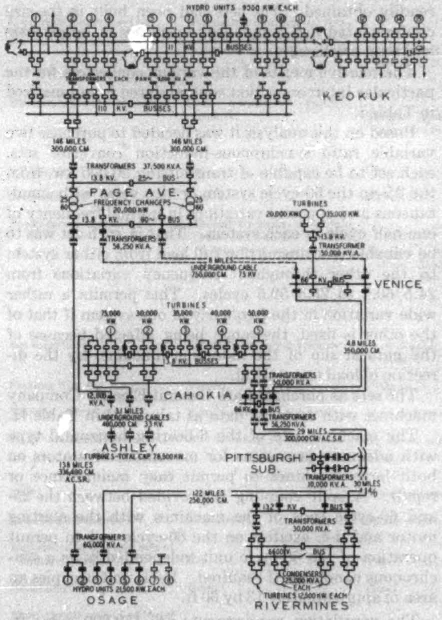
Early development of the Keokuk-St. Louis 25-cycle system anticipated one direction transmission of power, namely, from Keokuk to St. Louis, with voltage drop along the transmission line compensated for by graded voltage ratings of the transformers to be connected at either end. As a result Keokuk transmitted at 112,000 volts and St. Louis received at 95,000 volts. No provision was made for a change of voltage under varying loads. This problem was imminently in need of adjustment and a study of the age of the transformers at Page Avenue, including efficiency, life, and voltage, indicated that with modern design and efficiency to be expected, a new bank of transformers was fully justified. Accordingly the old group of twelve 5,000-kva., single-phase, 95/13.8-kv. indoor type transformers were removed from service and replaced with six 12,500-kva., single-phase, 110/13.8-kv. outdoor type transformers equipped with load ratio control. These new transformers provided facilities for full control of voltage in transmission of power in either direction without undue fluctuation of bus voltage at Keokuk or Page Avenue. The indoor type of transformers originally installed included an extensive and, in the light of modern practise, wasteful use of building space for 95-kv. bus layout. The removal of this indoor bus to a compact outdoor arrangement released sufficient space to provide for a new 13,800-volt, 60-cycle bus required for switching the incoming lines from Osage, and materially simplified the 25-cycle layout.

This coordination of 25- and 60-cycle systems at Page Avenue Substation interconnected to the major source of supply, namely, Venice and Cahokia plants by means of two 66-kv. underground cable circuits providing an effective tie capacity of 100,000 kva. The one line system diagram is shown in Fig. 2.

SELECTION OF TYPE OF SET

The size of frequency converters required was determined by a study of the load requirements. Two sets of 20,000 kw. rating were chosen as combining most suitably the load transmittal requirements, spare capacity, and synchronous condenser requirements.

The type of set to be used was the subject of considerable investigation. It was found that there were five types of machines available, namely:



HYDRO UNITS 9000 KW. EACH
①
②
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④
⑤
⑥
⑦
⑧
⑨
⑩
⑪
⑫
⑬
⑭
⑮
11 KV. BUSSES
TRANSFORMERS EACH BANK 9000 KV.A.
110 KV. BUSSES
KEOKUK
146 MILES 300,000 CM
146 MILES 300,000 CM.
TRANSFORMERS 37,500 KV.A.
13.8 KV. 25~ BUS
PAGE AVE.
25 60
FREQUENCY CHANGERS 20,000 KW
13.8 KV. 60~ BUS
TRANSFORMERS 56,250 KV.A.
TURBINES
20,000 KW.
35,000 KW.
13.8 KV.
TRANSFORMER 50,000 KV.A.
8 MILES UNDERGROUND CABLE 750,000 CM. 73 KV
66 KV. BUS
VENICE
TURBINES
75,000 KW
30,000 KW
35,000 KW
40,000 KW
50,000 KW
13.8 KV. BUSSES
4.8 MILES 350,000 CM.
TRANSFORMERS 50,000 KV.A.
CAHOKIA
12,000 KV.A.
66 KV. BUS
TRANSFORMERS 56,250 KV.A.
3.1 MILES UNDERGROUND CABLES 400,000 CM. 33 KV.
ASHLEY
TURBINES-TOTAL CAP. 78,500 KW.
29 MILES 300,000 CM. A.C.S.R.
PITTSBURGH SUB.
TRANSFORMERS 10,000 KV.A.
30 MILES 4/0
138 MILES 336,400 CM. A.C.S.R.
122 MILES 250,000 CM.
132 KV. BUS
TRANSFORMERS 30,000 KV.A.
6600 V. BUS
TRANSFORMERS 60,000 KV.A.
CONDENSERS 25,000 KV.A. EACH
TURBINES 12,500 KW. EACH
HYDRO UNITS 21,500 KW. EACH
OSAGE
RIVERMINES

FIG. 2—INTERCONNECTED—SYSTEM DIAGRAM

1. The synchronous-converter frequency converter.
2. The synchronous-synchronous converter.
3. The synchronous-induction converter.
4. The fixed-ratio synchronous-induction converter.
5. The variable-ratio synchronous-induction converter.

Each type of machine was analyzed in detail with its adaptability to the particular situation in mind.

Synchronous Converter Set. The synchronous-converter frequency converter consists of two synchronous converters on independent shafts, one operating from each of the systems to be interconnected. All power transferred is received at one frequency, converted to direct current and reconverted to alternating current of the other frequency. The fact that this machine has never been built in large sizes definitely precluded its use in this case.

Synchronous-Synchronous Set. The synchronous-synchronous converter consists of two synchronous machines on a common shaft, one operating on each system. This machine is the one most commonly used. The frequency ratio is fixed and, therefore, the set constitutes a rigid frequency tie between systems within the limits of pull-out torque of the individual machines. The set is completely reversible, the load transmittal being determined solely by prime mover governing and cannot be controlled at the set. Power-factor control of both ends is available through a wider range than on any other type of set. The efficiency is high, preliminary estimates indicating that 95 per cent might be expected on a 20,000-kw. unit. This set is essentially simple in construction and operation and requires few auxiliary machines. The paralleling of machines requires considerable preciseness of synchronizing however. For example, when starting from the 60-cycle end there is only one chance in twenty-four of the machine "coming up" right. Load division between sets is readily accomplished by stator shifting. Machines of this type in the rating proposed had been built so no untried designs would be necessary.

Records indicated that the reliability of this type of set was good provided the machines were of sufficient capacity to withstand shocks incidental to system disturbances. The chief difficulty in applying this machine is the proper determination of the size of unit required. An examination of available data indicated that a capacity equal to from 10 to 25 per cent or more of the capacity of the smaller system would be required. This application would require a set of from 18,000 to 40,000 kw. capacity. Conservatively, it appeared that units of 30,000 to 40,000 kw. each would be required and since load transmittal requirements did not justify units of this size it seemed undesirable to use this type of converter.

Synchronous-Induction Set. The synchronous-induction converter consists of a synchronous machine direct-connected to an induction machine of either the wound rotor or squirrel cage type. The frequency ratio of this set varies according to the load on the set and is not controllable at the set. The set is reversible if sufficient synchronous capacity is available to supply the exciting kva. of the induction machine. There is considerable change in frequency ratio with change in direction of power flow. Load control is determined by prime mover governing. The set is relatively stable, the stability being determined chiefly by the pull-out torque of the synchronous machine. Power-factor control is available on the synchronous end of the set but not on the induction end. The efficiency is relatively low. The set is simple in construction and operation and the reliability is good.

Fixed-Ratio Induction Set. The fixed-ratio synchronous induction converter consists of a synchronous machine and a wound rotor induction machine with the rotor of the induction machine and the stator of the synchronous unit connected to the lower frequency system and the stator of the induction machine connected to the system of higher frequency. This converter is sometimes called the "general transformer" since it can be arranged to transform phase, frequency, and voltage. A peculiar characteristic of this set is that it constitutes a voltage tie between systems of different frequencies in a manner analogous to a transformer on a single-frequency system. In several cases this set has been used to obtain certain advantages which this characteristic gives under particular circumstances. In the case at hand this feature was undesirable.

The frequency ratio is fixed by the design and is rigid. The set is reversible and the load control is obtained by prime mover governing. Stability is good and limited power factor control is available by a combination of transformer tap changing and synchronous machine field control. The efficiency is relatively high and operation is relatively simple although there is only one chance in ten of correct polarity and phase on the second unit. This type of machine has been reliable in service and has been built in large sizes. In the event of a short circuit on the lower frequency system the set overspeeds.

With this type of machine, also, the selection of the proper size of unit is difficult, the determination requiring a consideration of the same factors that govern in the selection of synchronous-synchronous machines.

Variable-Ratio Induction Set. The variable-ratio synchronous-induction converter consists of a synchronous machine direct-connected to a wound rotor induction machine provided with suitable regulating machines, the number and type varying with the particular manufacturer. The distinguishing feature of this set is that the frequency ratio is variable, that is, frequency of one system may vary relatively to that of the other. Voltage and load disturbances are reflected through the set in a lesser degree than with other types. The set is reversible and load control is accomplished at the set itself without dependence on prime mover governing within the design limits of the set. Stability is good, the load transmittal being determined by the pull-out torque of the synchronous machine, the relative speed changes of the two systems and the speed of operation of the regulating equipment. It is interesting to note that if the automatic regulating equipment is not working it requires a relative speed change of twice the slip of the induction machine to reverse the set from full load in one direction to full load in the other whereas with fixed-ratio types this reversal occurs with only a small phase angle change. Power-factor control through a wide range can be obtained on the synchronous machine and through a limited range as determined by economic design considerations on the induction machine. Efficiency is good although not as high as on synchronous-synchronous machines. The efficiency varies with variation in frequency ratio. The set is somewhat complex although its operation is simple.

The main advantage of this set lies in its stability and in the ease with which the sets may be started and the load controlled. Automatic control of load is readily obtained. The sets had been built in the size contemplated and component machines of larger sizes had been produced.

The relative merits of the various converters for the particular interconnection contemplated is summarized in Table I.

Based on this analysis it was decided to purchase two variable ratio synchronous-induction converter sets, each set to be capable of transmitting 20,000 kw. from the 25- to the 60-cycle system or vice versa with simultaneous and opposite variations in normal frequency of one-half cycle in each system. That is, each set was to be capable of transmitting full load from either system to the other throughout frequency variations from 24.5/60.5 to 25.5/59.5 cycles. This permits a rather wide variation in the frequency of one system if that of the other is fixed, the range being affected because of the natural slip of the induction machine, by the direction of load transmittal.

The sets as purchased are General Electric Company machines with essential data as tabulated in Table II.

The machines are of the 6-bearing horizontal type with adequate provisions for movement of stators on both large machines to permit easy maintenance or repair. A split coupling is provided between the 25- and 60-cycle ends of the machines with the starting motor and d-c. exciters on the 60-cycle end to permit operation of the 60-cycle unit independently as a synchronous condenser if required. Each unit occupies an area of approximately 13 by 60 ft.

The ventilation requirements are 150,000 cubic ft. of air per minute per machine. All air into the room is filtered through continuous type filters. The machines are the semi-enclosed type, drawing air from the room and discharging into the basement. Air is forced out of the basement through large chimneys at the building ends which discharge just above the roof of the building. A view of the building housing the machines is shown in Fig. 3, one of the chimneys being visible at the right. The converters installed are shown in Fig. 4.

OPERATION

The frequency converters have, at this writing, been in service for some six months and have fully met every expectation although definite conclusive results of operation cannot yet be evaluated. They operate over a part of the day as 60-cycle generators utilizing the excess 25-cycle water power from Keokuk and the balance of the time as 25-cycle generators supplying 25-cycle relay power. During the first six months' operating period covering initial tests and operation on the normal load cycle outlined above the average over-all efficiency has been approximately 90 per cent.

TABLE I—CHARACTERISTICS OF FREQUENCY CONVERTER SUITABLE FOR INTERCONNECTION OF 25- AND 60-CYCLE SYSTEMS OF UNION ELECTRIC LIGHT AND POWER COMPANY

Characteristic	Variable-ratio Synchronous-induction	Fixed-ratio Synchronous-induction	Synchronous-synchronous	Variable-ratio machine suitability for contemplated interconnection
Load control determined by	Regulating equipment at machine (can be held constant by automatic control)	Prime mover governing	Prime mover governing	Preferable
Load limit determined by	Regulating equipment, relative system speeds and synchronous machine stability	Synchronous machine stability	Synchronous machine stability	Preferable
Machine size determined by	Load transmittal requirements	System synchronizing requirements	System synchronizing requirements	More economical
Voltage tie	No	Yes	No	Preferable
Frequency tie	Flexible	Rigid	Rigid	Suitable
Power-factor correction	Limited to unity on induction end and by machine design on synchronous end	Limited to synchronous machine design and transformer tap range	Limited by synchronous machine design	Satisfactory—requirement for corrective capacity on 25-cycle system is limited
Set complexity	Several auxiliary machines	Considerable auxiliary apparatus	Simple	At a disadvantage
Starting first set	Simple	Difficult	Difficult	Preferable
Starting second set	Simple—same as the first one	Very difficult	Difficult	Preferable
Load balanced between sets	By regulating equipment	By stator shifting	By stator shifting	Preferable because load is always controlled in same manner
Cost	Highest	Intermediate	Lowest	At disadvantage
Expected efficiency	93 per cent	94 per cent	94.2 per cent	Satisfactory
Experience in building sets of required size	Yes	Yes	Yes	On a par with others
Reliability	Good	Good	Good	Equal of others
Short-circuit output of lower frequency machine	Low	High	Intermediate	Preferable in order to eliminate extensive switchgear changes

TABLE II—DATA ON 20,000-KW. FREQUENCY CONVERTERS UNION ELECTRIC LIGHT AND POWER COMPANY

	Synchronous machine	Induction machine	Starting motor	D-c. exciter	Pilot exciter	Regulating machine	A-c. exciter	Exciting transformer
Volts	13,800	13,800	13,800	225/250	240/250	285	35	13,800/37
Kilowatts	20,000	20,000	...	195	5	...	...	...
Kilovolt-ampere or horsepower	28,600 kva.	28,700 hp	1,200 hp	...	...	890 kva	8 kva.	35 kva.
Amperes	1,196	980	66	780	20	1,800	100	...
Power factor	70 per cent	1.0	...	...	...	...	...	...
Speed	300	300	320	300	300	300	300	...
Cycles	60	25	60	...	...	Slip	25/slip	25
Poles	24	10	22	8	4	18	10	...
Phases	3	3	3	...	...	3	6/3	3/6
Corrective kva. zero load	31,200	16,000	...	...	...	...	...	...
Corrective kva. full load	20,000	0	...	...	...	...	...	...

Power from the Bagnell Dam is not yet reaching the St. Louis District and consequently the machines have not been fully utilized for power-factor correction on the 60-cycle system.

It has been found that the inherent ability of the machines to change load gradually with system speed changes has been of marked benefit in stabilizing the systems. For example, with the set feeding the 25-cycle system a voltage dip may cause the loss of considerable 25-cycle load with consequent speeding up of that system. The set reverses and the frequency is held to a small departure from normal. The 60-cycle machines are equipped with high-speed excitation systems so that the 60-cycle voltage is held within narrow limits. The corrective capacity in the 25-cycle induction units varies from zero at full load to 16,000 kva. at no load and exerts a remarkable stabilizing influence on the 25-cycle voltage.

Fig. 3—Page Avenue Substation Building

When a short circuit occurs on the 25-cycle system, obviously the exciting voltage is reduced to a low value and consequently the decrement curve shows a very rapid drop after the first few cycles. This has the effect of holding the interrupting requirements on oil-circuit breakers to a low value.

The machines as installed cannot be operated unless a 25-cycle exciting source is available. Minor modifications would permit direct-current excitation to bring up a dead system. In several cases where the 25-cycle bus voltage has been dropped to a very low value by faults close into the bus the machines have resynchronized themselves inherently without any difficulty. In this connection it is of interest that the voltage of the induction unit is always at the same frequency as the exciting source but its value may vary through wide limits and it may be considerably out of phase with the system voltage. Consequently, the ability to resynchronize depends upon the action of the regulating equipment and the changes in relative system speeds during the interval when the bus voltage is very low.

Fig. 4—Two 20,000-Kw. Variable-Ratio Frequency Converters at Page Avenue Substation

The tested machine efficiencies were considerably higher than anticipated in the original estimates, the actual values including all losses for various operating conditions being given in Table III.

TABLE III—EFFICIENCIES OF 20,000-KW. FREQUENCY CONVERTERS

	Power factor		Load			
Frequency ratio	Induction machine	Synchronous machine	1.0	0.75	0.50	0.25
25/60	1.0	1.0	94.98	94.49	93.94	90.08
25/60	1.0	0.70 overexcited	94.28	93.94	93.94	89.86
25.5/60	1.0	1.0	94.96	94.46	93.90	90.00
24.5/60	1.0	1.0	94.91	94.41	93.82	89.90
24.5/60.5	1.0	1.0	94.88	94.40	93.73	89.80
24.5/60.5	1.0	0.70 overexcited	94.18	93.82	93.38	89.53

Note: Above figures are overall including losses in exciters, regulating machines, and blowers, where required.

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6. "Frequency Converter Tie between Large Power Systems." J. W. Dodge, *Gen. Elec. Rev.*, June 1923. Outlines types available and gives salient features of each.

7. "Parallel Operation and Synchronizing of Frequency Converters." O. E. Shirley, *Gen. Elec. Rev.*, Feb. 1920. Outlines operating problems in connection with synchronous-synchronous sets.

8. "35,000-Kw. Synchronous Frequency Changer Set." M. W. Smith, *Elec. Jl.*, July 1924, p. 343.

9. *A 40,000-Kw. Variable-Ratio Frequency Converter Installation.* Bundy, Van Niekerk, and Rodgers, A. I. E. E. Trans., Vol. 49, p. 245.

10. *Automatic Control for Variable-Ratio Frequency Converters.* Shively and Whitlow. Presented at the South West District Meeting, Kansas City, Mo., Oct. 22-24, 1931.

Discussion

R. H. Park: As brought out by Messrs. Nelson and Hough, there are three principal types of synchronous converters available in large sizes. These are the synchronous-synchronous type, the fixed ratio synchronous-induction type, and the variable-ratio synchronous-induction type.

The first type is the cheapest and most efficient and will frequently be found satisfactory from an operating standpoint. It is believed, however, that many cases arise in which the use of the second or third type is justified in view of the particular operating advantages which they may afford.

For instance, in the case of those metropolitan systems which do not employ generator voltage regulators, there may be a considerable advantage in the use of the fixed ratio synchronous-induction type. Thus when a voltage tie is provided, as is the case with this type of unit, there is less danger of voltage instability in case of the loss of a large unit in either system, since a relatively large amount of wattless kva. can be transferred between systems with only a comparatively slight drop in voltage in the system which is in difficulty.

On the other hand, in many cases, and particularly when large systems, in which relatively large amounts of power are transmitted over overhead wires, are to be tied together, there is a likelihood that the power transfer through the frequency converters connecting the two systems will become excessive during disturbances in either system, if the frequency converter is of the type which establishes a rigid frequency tie. As a consequence, the frequency converter unit or units may pull out of step, and relay out of service. In such cases, the use of a variable-ratio synchronous-induction type frequency converter may be of particular advantage, as, with this type, disturbances may occur in either system which are sufficiently severe to cause a considerable change in frequency, without the overloading or pulling out of step of the frequency converter unit.

While the choice of type of unit will depend greatly on the method of operation and the frequency of occurrence of system disturbances, it would appear that for the interconnection which forms the subject of the paper under discussion, the use of the variable-ratio synchronous-induction type of frequency converter possesses particular advantages.

E. L. Hough: The variable-ratio frequency converter is not a panacea for all interconnections between systems of different frequencies and we have endeavored to point out in the paper that the analysis was based on the particular system under discussion and not on the problem in general. A certain feature of a certain type of set may meet a definite operating problem on a particular system and this one feature may outweigh other considerations. An example of this is the use of large fixed ratio synchronous-induction machines on large metropolitan systems where voltage regulators are not used and a voltage tie between systems is highly desirable. On the other hand, in our particular system it was felt that a voltage tie was not only unnecessary but undesirable as well because of the relative importance of the two systems.

The size of set to be used is determined by different conditions in different situations. As pointed out in the paper, on the St. Louis system the determining factor was load transmittal rather than synchronizing requirements together with a high degree of reliability. Quite obviously a good solution of this engineering problem, as with others of a similar nature, requires the provision of the required facilities at a minimum cost and if a synchronous-synchronous machine of lower cost than a variable-ratio machine can perform satisfactorily, it should be chosen irrespective of whether or not it is of the same size as the variable-ratio machine. In searching for a solution to the Page Avenue interconnection we were unable to obtain guarantees from the manufacturers to the effect that a synchronous-synchronous machine of *equivalent cost* would hold the systems together. A search of the technical press revealed a considerable array of opinions on the size machine required, but no supporting data for the opinions. For example, Mr. Kincaid in the *Electric Journal*, June 1928, recommends machines equivalent to 15 to 20 per cent of the smaller system capacity where the one system is chiefly supplied from hydroelectric plants. Machines of this size would be more expensive than the type chosen and their losses would be higher because they would always operate at partial load.

The ability of the Scherbius type of machine to ride through system disturbance is superior. Due to the natural slip characteristic of the induction machine it requires an actual change of relative system frequencies to cause a change in load transmittal even with the load control equipment blocked or unable to operate with sufficient speed. However, we expect to obtain considerable benefit from the load control equipment because it can run from limit to limit in about 10 seconds and system disturbances require considerable time to materially affect system frequencies. As an example, assuming the induction machine has a natural slip of 2 per cent at full load, in the event the set were loaded to 20,000 kw. capacity, a drop in speed of 1 per cent on the system receiving power from the set would add only 10,000 kw. to the set load, even though the load control were ineffective. With a rigid frequency tie a drop of only a few degrees in phase angle would give a similar result.

While it is true that the stability limit of the Scherbius set is determined by the synchronous machine if voltage is maintained on the induction end it is apparent that the inherent WR^2 effects and reserve capacity of the whole system are brought into play to smooth out the disturbance and allow time for faults to clear without relaying the machine out of service.

As pointed out, the rapid short-circuit decrement reduces the strain on switchgear but may introduce other undesirable consequences. This particular feature must be carefully watched in determining relay settings and may, in some cases, require high-speed relays to take advantage of the initial short-circuit current of the machine which compares quite favorably with an equivalent synchronous machine.

From a reliability standpoint we expect little more difficulty from the a-c. commutating machines than we would from d-c. exciters. Most of the difficulties experienced with these machines in the past have been on those machines operating at a relatively much higher frequency than is the case in this installation. The regulating machine is of the generator type and lends itself readily to compensation. With maximum frequency spread the frequency on the a-c. commutating machine is 0.7 cycle and at normal frequency the commutating machine operates as a continuous current generator.

We expect better performance of the main units of these sets than we would have expected on a rigid frequency tie machine because shocks are cushioned. On a 5,000-kw. machine previously installed a great amount of difficulty was experienced with the amortisseur windings overheating and with winding failures. Similar difficulties have been reported by other operating companies (Trans. A. I. E. E., 1923, Vol. 42, p. 1078).

Up to the present time no commutating difficulties have been experienced and although a number of serious disturbances has occurred on the interconnected systems no difficulties whatever have been experienced with the machines that are not directly attributable to the inexperience of the operators or to incorrect adjustments. In one case, a short circuit on the 25-cycle bus to which the Scherbius unit is connected was cleared properly and the machine resumed its load automatically, although the bus voltage reached a very low value. In another case, during a sleet storm, the lines to the Keokuk Plant developed "dancing conductors" and one line was removed from service. The second line continued to swing together and clear itself for some time. On each occurrence of low voltage the machine output went to a very low value and recovered as soon as voltage came up and the operator being unfamiliar with such operation (a synchronous machine performing in an exactly opposite manner) assumed the machine to be at fault and removed it from service. From all available indications the clearing of the faulty high-tension line would have left the machine carrying its normal load with excitation from the steam system. The transformer method of excitation lends itself peculiarly to satisfactory operation under conditions such as these.

Automatic Control for Variable-Ratio Frequency Converters

BY E. K. SHIVELY* and GEO. S. WHITLOW†

Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.*—An automatic switching equipment for the control of two 20,000-kw. frequency changer sets using the Scherbius method of load and power factor control has been in successful operation at the Page Avenue station of the Union Electric Light and Power Company of St. Louis, for almost a year. This equipment contains several new features in the automatic control of machines of this type. Besides being the first completely automatic switching equipment applied to machines of this type, it is unique in that it controls the largest rotating machines to which automatic switchgear has ever been applied.*

The Page Avenue station is one of the most important links in this Company's system of interconnecting plants. It is the point through which the two 25-cycle generating stations, Ashley Street and Keokuk, are tied. It will also be used as one of the ties for the recently completed Bagnell development, on the Osage River, to the 60-cycle generating system of the metropolitan area. These variable-ratio frequency converters are being used for the transfer of power between the 25-cycle and 60-cycle systems.

* * * * *

GENERAL DESCRIPTION

THE sets consist of seven units on one shaft. These units are, in the order of their physical location, a 250-volt, 5-kw. subexciter; a 250-volt, 192-kw. main exciter; a 13,800-volt, 1,200-hp. (intermittent) 60-cycle starting motor; a 13,800-volt, 28,600-kva. 60-cycle synchronous machine; a 13,800-volt, 28,700-hp. 25-cycle wound rotor induction machine; a 285-volt, 890-kva. regulating machine and an 8-kva. 25-cycle a-c. exciter. There is a removable coupling between the first four and last three units. The speed of the sets is 300 r. p. m. at 60 cycles.

The sets will normally be operated as variable-ratio frequency converters to transfer load in either direction as dictated by system conditions. Both the synchronous and induction machines will supply a limited amount of corrective kva. to the respective systems. The amount of this kva. depends upon the loading of the sets, and can be readily determined for any condition of load by referring to the curves of Fig. 2. If it is so desired, the starting motors, the exciters associated with the 60-cycle units, and the 60-cycle units may be uncoupled from the 25-cycle ends and operated alone as condenser units.

The switching equipment is designed so that the starting, synchronizing, voltage control, load limiting features and protective devices are completely automatic. The control rheostats and switchboards on which the protective devices and automatic relays are mounted, are located on the main machine floor. On the operators' board in the balcony the machine meters and control switches are located. The operator may assume complete control of the equipment at any time by merely turning a control switch from the automatic to the manual position. If the set is in operation, this transfer can be made in either direction without disturbing the operation in any way. Figs. 3, 4, 5 and 6 show general views of the control rheostats and switchboards for one unit.

SYNCHRONOUS UNIT

Starting and Synchronizing. The starting is accomplished with no perceptible disturbance to the system by a wound-rotor induction-type starting motor. This motor has two less poles than the main 60-cycle machine so that when all of the starting resistance is short-circuited, the starting unit will drive the set at a speed slightly above synchronism.

When the initial starting indication is given by means of a control switch, the equipment checks that none of the protective devices has operated, that the starting and synchronizing breakers are in the open position and that normal three-phase voltage is available. If all of these conditions are satisfied, the main 60-cycle oil circuit breaker is permitted to close, which connects the machine bus to the station bus. This operation causes the bearing oil pump motors to start, and after a definite time, sufficient for the oil pressure to build up and lift the machine rotor, the starting motor is energized. Soon after the machine has begun to rotate, the oil pumps are shut down. After the initial starting current has decreased to a predetermined amount, showing that the machine is rotating, successive steps of secondary resistance are short-circuited at definite time intervals until sufficient resistance has been shunted to allow the set to run at slightly below synchronous speed.

During this part of the starting period the subexciter voltage is allowed to build up; this in turn energizes the field of the main exciter. When the machine has reached almost synchronous speed, the field of the synchronous unit is energized and the automatic voltage regulator assumes control of the machine voltage. Normal voltage is then apparent on both sides of the synchronizing breaker and the speed-matching and synchronizing equipment takes control and proceeds to synchronize the machine.

The last step of starting resistance that is shunted

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before field is applied to the synchronous unit, is, as has been stated, sufficient to bring the set up to just below synchronous speed. As this speed is below synchronism the speed matching equipment will cause the last section of resistance to be short-circuited as soon as voltage has built up on the main unit. If the set exceeds synchronous speed and does not synchronize, the speed-matching equipment will insert this last section of resistance to decrease the speed.

The speed control equipment uses the relative vector rotation of the machine and bus voltages to determine whether the machine speed is to be raised or lowered. Fig. 7 shows the vector relationship of the voltages used in this equipment. The voltages *B*-1, *B*-2, *B*-3 are supplied from the station bus potential transformers. Voltage *M*-1, *M*-2 is obtained from *A*-*B* phase of the machine. Contactor coils are connected between the free ends of each of the bus voltages and the machine voltage. The diagram shows the vectors at a position where equal voltages are applied to the contactors, but neither has sufficient voltage to cause it to close its contacts. If the machine voltage rotates clockwise with respect to the bus voltage, indicating that the machine is running too slow, the voltage across 15 *R-C* will increase sufficiently to pick up this contactor, and cause resistance in the starting motor secondary to be shunted. If the vectors rotate in the opposite direction, relatively, 15 *L-C* will pick up and cause resistance to be inserted. It will be seen, because of interlocking, that only one of the contactors can be picked up simultaneously, and that one contactor does not drop out until the voltage across the other has decreased to an amount that is insufficient to pick it up.

When the frequency of the set has been matched sufficiently close to that of the system, the synchronizing relays initiate the closing of the synchronizing breaker when the two voltages are in the proper phase relation. The closing of this breaker is initiated a few degrees before the voltages reach the "in-phase" relationship provided the relative motion of the two vectors is not too fast, which would cause the actual synchronizing to take place after the "in-phase" position was reached, or too slow, which would cause just as severe a disturbance by allowing the synchronizing breaker contacts to make before the proper phase relationship is reached.

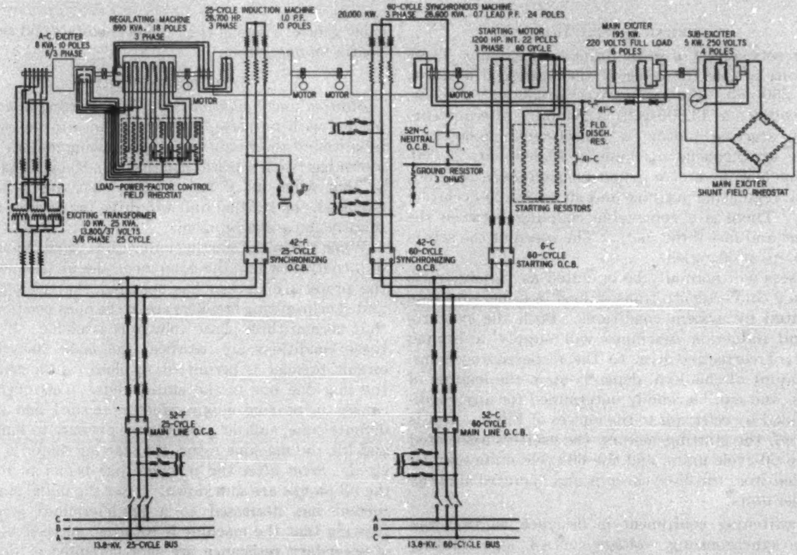
A-C EXCITER
8 KVA. 10 POLES
6/3 PHASE
REGULATING MACHINE
890 KVA. 18 POLES
3 PHASE
25-CYCLE INDUCTION MACHINE
28,700 HP. 1.0 P.F.
3 PHASE 10 POLES
20,000 KW. 60-CYCLE SYNCHRONOUS MACHINE
3 PHASE 28,600 KVA. 0.7 LEAD P.F. 24 POLES
STARTING MOTOR
1200 HP. INT. 22 POLES
3 PHASE 60 CYCLE
MAIN EXCITER
195 KW.
220 VOLTS FULL LOAD
6 POLES
SUB-EXCITER
5 KW. 250 VOLTS
4 POLES
MOTOR
MOTOR
MOTOR
MOTOR
52N-C
NEUTRAL
O.C.B.
41-C
FLD.
DISCH.
RES.
41-C
GROUND RESISTOR
3 OHMS
LOAD-POWER-FACTOR CONTROL
FIELD RHEOSTAT
MAIN EXCITER
SHUNT FIELD RHEOSTAT
STARTING RESISTORS
EXCITING TRANSFORMER
10 KW. 25 KVA.
13,800/37 VOLTS
3/6 PHASE 25 CYCLE
42-F
25-CYCLE
SYNCHRONIZING
O.C.B.
42-C
60-CYCLE
SYNCHRONIZING
O.C.B.
6-C
60-CYCLE
STARTING O.C.B.
52-F
25-CYCLE
MAIN LINE O.C.B.
52-C
60-CYCLE
MAIN LINE O.C.B.
C
B
A
13.8-KV. 25-CYCLE BUS
A
B
C
13.8-KV. 60-CYCLE BUS

Fig. 1—Diagram Showing Main Connections for 20,000-Kw. Variable-Ratio Frequency Converter

Protection. The 60-cycle machine is protected against overload on the starting motor, incomplete start, winding failures, ground faults and undervoltage while starting, by relays that will trip the main oil circuit breaker. In case of a winding fault, incomplete start or overload on the starting motor, a lockout relay is actuated, which makes it necessary to inspect the equipment and reset this relay before further operation can take place. If a bearing overheats, an alarm is sounded for the attendant who has control of the bearing cooling water. He will admit more cooling water to the bearing in distress. In case of an appreciable decrease in excitation, indicating that the machine is delivering an excessive amount of underexcited reactive kva., an alarm is sounded for the switchboard operator in the balcony. This operator, who has control of the load ratio control transformers, changes taps on these transformers to transfer some of this reactive kva. to other synchronous machines on the system.

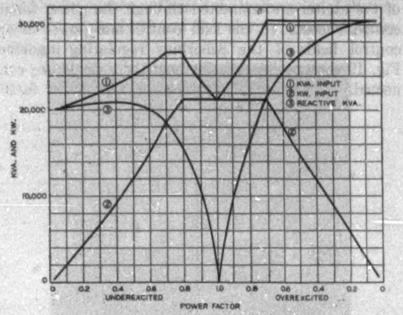

FIG. 2A—MAXIMUM KVA. CURVES FOR 60-CYCLE MACHINE

Voltage Regulation. The generator voltage regulator used with this equipment is one that was developed primarily to eliminate the use of continuously operating vibrating contacts. The regulator performs two functions simultaneously. Upon changed conditions of load or power factor, the regulator immediately meets the changed conditions by rapidly varying the exciter voltage to meet the new excitation requirements. At the same time it operates the motor-driven exciter field rheostat until a corresponding constant exciter voltage is reached. The regulator then remains inoperative until another change requires a different value of excitation.

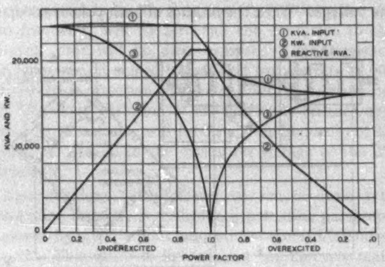

FIG. 2B—MAXIMUM KVA. CURVES FOR 25-CYCLE MACHINE

When operating as a synchronous condenser to correct the power factor of a system, it is sometimes necessary to produce a maximum of underexcited reactive kva. To do this the excitation of the exciter must be reduced to zero or even reversed slightly to overcome the residual, in order that the exciter voltage may be reduced to zero. The wheatstone bridge arrangement of connections of the exciter field rheostat and a separate exciter are used for this purpose. The diagram of this arrangement is shown in Fig. 8. In this arrangement two arms of the bridge circuit are fixed and the other two variable. The variable arms have sufficient resistance to balance the bridge and reduce the exciter voltage to zero.

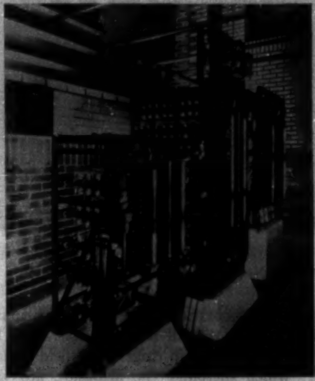

FIG. 3—VOLTAGE AND POWER FACTOR CONTROL RHEOSTATS FOR ONE FREQUENCY CONVERTER SET

FIG. 4—EAST SECTION OF AUTOMATIC CONTROL PANELS FOR NO. 1 FREQUENCY CHANGER SET

INDUCTION UNIT

Excitation System. Excitation is supplied to the armature of the a-c. exciter by a 25-kva. 13,800/37-volt, three-phase/six-phase connected transformer. The high-voltage winding of this transformer is connected in *Y* to the 25-cycle 13,800-volt station bus through the 25-cycle main line oil circuit breaker. The low-voltage winding is connected six-phase diametrical to the six slip rings of the a-c. exciter. The armature of this exciter is like that of a rotary converter, having both commutator and slip rings. The field punchings, however, which surround the armature are without slots or windings and serve simply to complete the magnetic circuit of the armature. The commutator brushes are carried on two brush yokes, the arms of which are spaced 120 elec. deg. apart around the commutator.

Fig. 5—West Section of Automatic Control Panels for No. 1 Frequency Changer Set

Fig. 6—Station Operator's Control Board on Balcony

The brush yokes are shifted by a pilot motor. One yoke, referred to as the geared yoke, is connected to the pilot motor through reduction gearing while the other, referred to as the cam yoke, is shifted from the pinion gear shaft by means of a cam and roller. The two yokes travel at unequal speeds sometimes in the same direction, sometimes in the opposite direction, depending upon the position of the cam. The commutator brushes of the exciter are connected through the power factor control rheostat to the load control and power factor control fields of the Scherbius regulating machine. Fig. 10 shows schematically how these fields are connected. The voltage impressed on the power factor control field circuit is practically constant at all times. The voltage impressed on the load control field circuit, however, depends upon the amount of brush separation on the a-c. exciter, the greater the separation, the higher the voltage. When the brushes of both yokes rest upon the same commutator segment the voltage impressed on the load control field is zero. This is referred to as the neutral brush position.

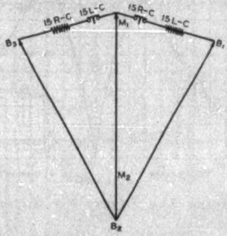

Fig. 7—Fundamental Diagram Showing Operation of the Speed Matching Relays

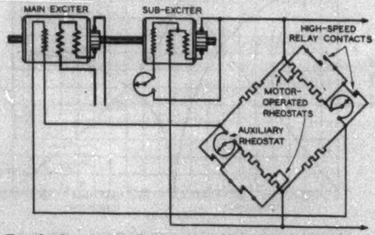

Fig. 8—Schematic Diagram of Excitation System of Synchronous Machine

The basic principle of the Scherbius regulating machine is the same as that of the neutralized d-c. machine with separate excitation. The armature and commutator are of the ordinary type of construction used in d-c. machines and voltage is generated by the rotation of the armature conductors through the field produced by the stator winding. The commutator brushes of the regulating machine are connected to the three rotor slip rings of the 25-cycle induction unit. The excitation thus furnished to the rotor has a frequency which is equal to the difference between the bus frequency and the frequency which the 25-cycle unit would have as a synchronous machine at its driven speed. The difference between these two frequencies will be referred to as slip frequency. Thus if the frequencies of the two systems are exactly 25 and 60 cycles, or, if the frequency of one system is exactly 2.4 times that of the other, the slip frequency will be zero and the excitation to the 25-cycle unit will be direct current. The frequency converters are designed to transmit from zero to 20,000 kw. in either direction with an opposite and simultaneous frequency variation of one-half cycle on each system. This variation will produce a slip frequency of 0.7 cycles per second in the excitation voltage to the 25-cycle unit.

Fig. 9—Variable-Ratio Frequency Converters Showing Regulating Equipment for Induction Unit in the Foreground

Synchronizing. The closing of the 60-cycle synchronizing breaker, described in a preceding paragraph is shown by a white pilot lamp on the operator's control board. This lamp is lighted and the 25-cycle main line breaker can be closed provided the locking out relays for both the 25-cycle and 60-cycle units are in the reset position, the power factor control rheostat is on the position which gives unity power factor at no load and normal voltage, the brushes on the a-c. exciter are on neutral position, the 25-cycle synchronizing breaker is open and the 25-cycle bus voltage is normal. With the appearance of this light the operator closes the 25-cycle main line breaker which energizes the exciting transformer and completes the connection from the 13,800-volt bus to one side of the 25-cycle synchronizing breaker. As soon as the exciting transformer is energized current flows through the a-c. exciter, thence through the fields of the regulating machine and from the commutator of the regulating machine into the rotor of the induction machine.

At this point voltage begins to build up in the stator of the induction machine. Since the excitation of this machine is derived from the station bus the frequency of the voltage generated in the stator is the same as the frequency of the bus voltage. Correct synchronizing, therefore; depends only upon getting the stator voltage approximately equal to and in phase with the bus voltage. The contact arm of the power factor control rheostat remains on its predetermined "run-off" point until after the 25-cycle synchronizing breaker is closed. The contact arm always returns to this point whenever the 25-cycle main line breaker is opened.

The phase angle of the stator voltage with respect to the bus voltage is determined by the position of the two brush yokes on the a-c. exciter. The direction in which the brushes are shifted to obtain the proper phase angle is determined by the operation of the voltage balancing relay. One coil of this relay is connected across the *A* and *B* phases of the bus potential transformers and the other coil of the relay is connected between the *A* phases of the bus and the machine potential transformers. One set of contacts close to run the brushes in the speed lowering direction (*i. e.*, with rotation) when the machine voltage lags the bus voltage, and the other set of contacts close to run the brushes in the speed raising direction (*i. e.*, against rotation) when the machine voltage leads the bus voltage. A phase angle difference between stator and bus voltages of approximately 10 degrees causes this relay to close contacts.

The rapidity with which the brushes are shifted is controlled by the action of two contactors, device 15-*F* and device 15 *X-F*. The *a* contacts of these two contactors are in series with each other and are in series with the contacts of the voltage balancing relay. The connections are arranged so that 15-*F* picks up and drops out with a speed which is proportional to slip frequency. The operation of this contactor causes a shift in the brushes on the a-c. exciter in a direction as determined by the voltage balancing relay.

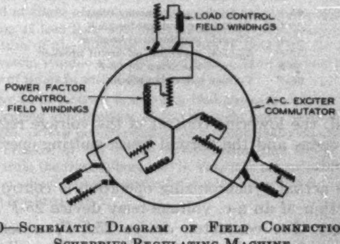
LOAD CONTROL FIELD WINDINGS
POWER FACTOR CONTROL FIELD WINDINGS
A-C. EXCITER COMMUTATOR

Fig. 10—Schematic Diagram of Field Connections for Scherbius Regulating Machine

It may be seen, therefore, that the brushes are shifted with a speed proportional to slip frequency to a position where the stator voltage is approximately in phase with the bus voltage. A slip frequency of zero indicates that the speed of the unit is exactly correct for the system frequency and that there is no phase displacement between machine and bus voltage. The brushes then remain on neutral and the closing of the 25-cycle synchronizing breaker takes place as soon as the machine voltage builds up to a value which is approximately equal to the bus voltage.

The stator voltage is prevented from rising too high during shifting of the brushes by the voltage regulating relay. With the 25-cycle synchronizing breaker open, the lowering contact of this relay picks up a contactor whose *b* contact interrupts the normal brush shifting circuit and whose *a* contact causes the brushes to return toward neutral. This has the effect of reducing the stator voltage. When the voltage has been reduced to normal, the lowering contact of the voltage regulating relay opens and the normal brush shifting operation is resumed.

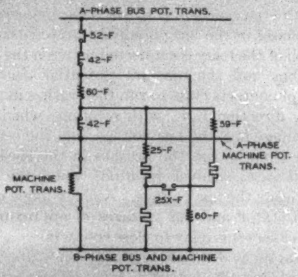

FIG. 11—SCHEMATIC DIAGRAM SHOWING CONNECTIONS FOR No. 25-F SYNCHRONIZING RELAY

25-*F*	25-Cycle synchronizing relay
25*X*-*F*	Auxiliary contactor for 25-*F*
42-*F*	25-cycle synchronous breaker
52-*F*	25-cycle main line oil circuit breaker
59-*F*	Overvoltage relay for 25-*F*
60-*F*	Voltage balancing relay

The actual synchronizing operation is controlled by the action of an a-c. voltage relay device 25-*F* and its auxiliary d-c. contactor device 25*X*-*F*, a schematic diagram of which is shown in Fig. 11. The coil of 25-*F* is connected through two sections of external resistance between the *A* and *B* phases of the machine potential transformer. When the stator voltage rises to 12,000 volts 25-*F* picks up. An *a* contact of 25-*F* picks up 25*X*-*F* which seals itself in. When 25*X*-*F* picks up it short-circuits a part of the resistance in the coil circuit of 25-*F* and causes 25-*F* to be connected between the *A*-phase of the bus potential transformer and the *A*-phase of the machine potential transformer. Its drop out, therefore, is determined by the value of the vector difference voltage between the machine and the bus. This relay is set to drop out at 17 volts secondary or 2,040 volts primary which means that synchronizing takes place at a phase angle difference of approximately 10 degrees or less. The dropping out of 25-*F* causes the 25-cycle synchronizing breaker to close, connecting this unit to the 25-cycle station bus.

Voltage Regulation and Load Control. As soon as the 25-cycle synchronizing breaker is closed the 25-cycle bus voltage is automatically controlled by the voltage regulating relay. This relay, a contact-making voltmeter type, has control of the power-factor control rheostat. The contact circuit of this relay is interlocked with the contacts of two current relays of the contact making ammeter type which limit the current through the unit. One relay is actuated by the stator current and the other by a combination of stator current and a constant current derived from the machine potential transformer to simulate the magnetizing current, the resultant being proportional to the rotor current. This use of a phantom secondary current is made because the frequency of the actual secondary current is too variable for the proper operation of protective devices.

The power factor relay, which is actuated by the stator current and potential, controls the direction in which the power factor control rheostat operates in the event of overcurrent. If the overcurrent is the result of too much overexcited reactive kva. the rheostat is run in the direction which increases the resistance in the field circuit. If the overcurrent is the result of too much underexcited reactive kva. the rheostat is run in the direction which decreases the resistance in the field circuit. The 25-cycle unit can be adjusted to operate at no load between the limits of 24,000 reactive kva. underexcited and 16,000 reactive kva. overexcited and at full load between the limits of 11,000 reactive kva. underexcited and zero reactive kva.

The amount of load transferred through the frequency converters is controlled manually by the switchboard operator. The amount of load transferred under any given condition of frequency spread on the two systems is determined by the amount of brush separation on the commutator of the a-c. exciter. The power transfer is limited to 20,000 kw. in either direction by the action of two power limiting relays whose contacts operate the brush shifting contactors. One relay limits the power transfer from the 25-cycle to the 60-cycle system and the other relay limits the power transfer from the 60-cycle to the 25-cycle system. Both the 25-cycle and 60-cycle buses in the Page Avenue station are supplied through transformers of the load-ratio control type. The 60-cycle transformer banks are equipped with eight 2½ per cent taps in their high-voltage windings and the 25-cycle transformer banks are equipped with nine 2½ per cent taps in their low-voltage windings. The operation of this tap-changing equipment in conjunction with the reactive kva. requirements of the frequency converters gives a very flexible means of controlling the 25-cycle and 60-cycle bus voltages at the station.

The synchronizing and voltage regulation of the 25-cycle unit may be controlled automatically or manually by means of a control switch. This may be turned to either position at any time while the set is operating. The power-limiting and current-limiting relays are operative during both automatic and manual control of the 25-cycle unit.

The unit is protected against winding failure by current differential relays which shut down and lock out the set. The set is also shut down and locked out by the presence of overcurrent on the high-voltage or low-voltage sides of the exciting transformer.

Ventilation. Although the 60-cycle unit is self-cooled the 25-cycle unit requires 75,000 cu. ft. of air per minute to limit the temperature rise to 50 deg. cent. This is supplied by two blowers located directly underneath the air discharge of the induction motor. The first blower is started by the closing of the 25-cycle main line breaker. The second blower is started at 50 deg. cent. by a thermostat located in the discharge air. All ventilating air is supplied through automatic, self-cleaning filters.

CONCLUSIONS

The performance of this equipment during all sorts of system conditions during the past year has demonstrated the feasibility of automatically controlling large frequency changer sets. The stabilizing effect of the 60-cycle high-speed excitation scheme on the voltage during system short circuits and the accuracy and speed of the 25-cycle synchronizing operation are especially noteworthy. Manual synchronizing of the 25-cycle unit is difficult because of the complex character of the excitation system and because of the constantly changing value of the slip frequency. This operation is performed automatically in a very short time with no interchange of power between the two systems, for both zero spread and maximum spread of the two system frequencies.

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Discussion

O. J. Rotty: The authors have given a very clear description of the control scheme employed for the automatic operation of the very complicated requirements of the Scherbius method of frequency changer control. As pointed out by the authors, these machines are located at one of the major interconnecting stations of the company's system. Although operators are required at this station for the various important switching operations, these machines are automatically controlled because experience at other stations has proven the automatic control to be faster, more accurate, and more dependable than manual operation. It is felt by the company's engineers that this is particularly true for machines so inherently complicated of operation.

Although these machines are referred to as being completely automatic they might more properly be described by the popular term, "push-button start and stop." That is, they are started by the operation of a manually operated control switch after which all steps are carried out automatically up to the point where it is desired to bring on the 25-cycle unit when another manual control switch is operated and after which all functions are again automatic. Manual shut-down of an entire set requires only the operation of a single control switch and all apparatus is returned to the normal off-position automatically.

The amount of load transfer through the machines and the reactive kva. carried by each unit are controlled manually by the operator although power limiting and current limiting relays prevent exceeding full load values. The use of load control relays which could be set by the operator for any desired load and which would then definitely hold that load were considered at the time of purchasing the control equipment. However, this scheme was not adopted as it would have prevented the machines from taking full load at times of system trouble (if the relays happened to be operating at less than the full load setting) and it was felt that the feature of the machine automatically going up to full load upon system trouble was very desirable. This has proven out a number of times in actual operation.

It may be of interest to mention that the same scheme as used for indicating synchronism of the 60-cycle unit, as described in this paper, is now being installed for automatically closing the oil circuit breaker of the 132-kv. transmission line supplying this station from the Osage plant, and in time it is intended to employ the same scheme for automatically closing other important transmission line breakers.

Forecasting Population for Engineering Purposes

BY JAMES N. HOLSEN[1]

Non-member

ENGINEERS and persons responsible for the management of engineering projects have a common interest in problems of forecasting population growth and market demands. The soundness of engineering projects depends upon their capacity to fulfil a real demand. Unsoundness, or even failure, may result from planning and designing engineering projects without regard to industrial and population trends. Similarly city planning, housing and home building projects, educational and social programs of various kinds, should ordinarily be based upon the number of people to be served at some future date and their economic and social characteristics.

Practical Value of Population Forecasts

Within the Bell System the careful study of factors influencing the growth, distribution, and character of the population is regarded as essential to the intelligent planning of the telephone business. In planning extensions to the telephone plant, engineers are provided with forecasts of the probable telephone service requirements at periods ranging from five to twenty-five years in the future. These forecasts are based upon the probable growth, distribution, and character of the population. On the basis of these forecasts of probable telephone service requirements and other pertinent factors, engineers determine the initial and ultimate size of buildings, switchboards, underground cables, and the pole lines and aerial cables. Thus, vastly more depends upon the forecasts than the mere provision of telephone instruments, which subscribers are apt to regard as the principal part of the telephone plant. In planning for the future the central aim of the telephone company is to supply the right amount of facilities in the right place. Manifestly, this cannot be done except when based on dependable forecasts of population and its distribution.

Because of the need for reliable forecasts in the telephone business, the American Telephone and Telegraph Company and its various operating companies have given considerable attention to population facts and the interpretation of population changes. It is realized, of course, that population forecasts are subject to the same possible errors as are always present where the human element is an important factor. Unlike the movement of stars which can be predicted by astronomers to within a fraction of a second, the course of human activities does not lend itself to precise foretelling. However, the experience of the telephone company has been that population forecasts can be made with a very helpful degree of accuracy, thereby making for maximum economy in the extension of the telephone plant.

Basic Considerations Involved

Several mathematical formulas have been devised on the assumption that future population can be determined from past trends by the mere application of figures in the formula. It is believed, however, that such formulas place too much emphasis on past trends and ignore the fact that new influences are always appearing and that old influences are frequently ceasing to be of importance. They also tend to give a sense of security to the use of an estimate which is unwarranted by the basic assumptions which have been made. Obviously, it is always well to study the past, but it is never safe to assume that the future will reproduce the past.

Cities and regions grow not from mere wishing or boasting, but usually because of definite geographic and economic advantages which enable them to develop population supporting activities in competition with other localities. Forecasts are apt to miss the mark rather widely if the forecaster has only a local point of view. From a local standpoint, there may appear to be very good reasons why a region should make great advances agriculturally; from a national standpoint, it may appear more reasonable to expect a contraction in its agriculture. Similarly a town may appear to be an ideal location for a particular line of manufacturing; but if a competing town is better situated in respect to raw materials and markets, and if it offers cheaper power and a better labor situation, then the advantages of the first town are likely to be of little avail. The national outlook eliminates the optimism based on the pride of those born or residing in the particular locality for which the forecasts are being made, and the conclusions reached by a broad point of view undoubtedly are much more likely to be realized. Dependable population forecasts cannot be based upon what should or could be done; they must be based upon what most probably will occur.

Population of the United States as a Whole

Because communities are competing with each other for the population growth of the United States as a whole, population forecasts for States and for individual cities should be made with due regard to population trends of the country as a whole. Some of the facts and tendencies that have a definite bearing upon the population growth of the United States briefly may be summarized as follows:

1. The United States is almost certain to continue a

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restrictive immigration policy for many years to come. Until the last decade immigration contributed heavily to our population growth. This is clearly evident from the 1930 census figures, which showed that foreign-born whites and their children constituted 31.1 per cent of the country's total population. In the future, population increases must depend more and more upon the excess of births over deaths.

2. The birth rate in the United States has been declining gradually for several years, due to such factors as generally higher living standards, smaller living quarters, marriages in which both husband and wife are breadwinners, the tightening of immigration restrictions against the prolific races, and finally wider knowledge and utilization of birth control information. Although there have been reductions in infant mortality, the effect has not been sufficient to prevent a decline in the rate of natural increase. For example, in 1910 the birth rate was about 30 per 1,000 population and the death rate was 15, while at present the birth rate is very close to 19 and the death rate about 12. Thus, in twenty years the margin which represents natural increase in population has decreased from 15 per 1,000 to 7.

The number of people in Continental United States in the last forty-year period together with increases in each ten-year period is indicated below:

POPULATION OF CONTINENTAL UNITED STATES

Year	Total	Increase during previous decade	
		Number	Per cent
1890 (June 1)	62,947,714		
1900 (June 1)	75,994,575	13,046,861	20.7
1910 (April 15)	91,972,266	15,977,691	21.0
1920 (January 1)	105,710,620	13,738,354	15.0
1930 (April 1)	122,775,046	17,064,426	16.1

The gain during the 1920-1930 decade exceeds by more than one million the largest previous gain, that occurring in the period 1900-1910, and it is 3.3 million in excess of the growth realized in the decade just preceding, that is, from 1910 to 1920, when the net immigration was 25 per cent greater and the rate of natural increase some 10 per cent higher. A partial explanation of this seeming inconsistency is in the probable greater efficiency with which the 1930 count of inhabitants was made. Due to the better selection, training, and higher compensation of the 1930 enumerators, they probably did a better job of counting, also, the 1930 census was relatively more accurate because of the better rural roads, more extensive telephone service, the policy of permitting recounts, and better organization and supervision of the over all undertaking. Furthermore, the period of counting of the 1910-1920 decade was some six months shorter than that of the 1920-1930 decade. It seems evident that the natural rate of population growth has been steadily slowing down, and that the same general tendencies will prevail over the next twenty years.

Now the slackening in the rate of population growth for the country as a whole will naturally be reflected in the population increase to be divided among the urban and rural communities making up the total population. Unless a community is relatively better situated than in the past, it cannot expect to maintain its past rate of growth. In order to hold its natural rate of increase, the population supporting activities of a community must expand proportionately. In order to increase at a higher rate, its advantages must be such as to attract population at the expense of less favorably situated communities.

There is one factor which lends emphasis to the general statement that a community must be relatively better situated to maintain its past rate of growth, and this consideration is the increasing mobility of the American people. Population in the United States has always been highly mobile; that is to say, the American people have been peculiarly ready to move in response to economic activity, including the opening of new lands for agriculture, and other attractions. But the point is that the development of improved means of communication and transportation and the spread of information concerning the relative advantages or disadvantages of this or that part of the country as a place in which to live and labor have accentuated this mobility. Because of this the real adtanges or disadvantages of communities are likely to play a larger role than heretofore in determining their relative capacities to attract or to hold population.

Urban population has been increasing at a far higher rate than rural population. During the past decade the increase in the rural population was 2.4 millions as against 14.6 millions for the urban centers. Whereas the 1900 census reported only 40 per cent of the population as urban, the corresponding figure in 1930 was 56.2 per cent. The city-ward tendency in population, which is nation wide and regarded by some as a dangerous trend, is intimately related to fundamental economic changes, including (1) the capacity of the farms to maintain a rapidly increasing urban population, aided by the constant development of labor-saving machinery, including farm tractors and scientific agriculture, and (2) the transfer to the cities of many labors that were formerly a part of farm work. Contributing factors lie in the tendency of our export trade to shift from raw materials to manufactured commodities, the increasing per capita demand for manufactures in this country as a result of the rising standard of living (a tendency that is temporarily obscured by the present general business depression), and also in the additional opportunities for untrained workers in the cities, brought about by the decrease in the foreign labor supply after the drastic restrictions on immigration went into effect and by the unusual growth in the service industries.

Further scrutiny of the census figures indicates that the larger cities in the country are growing more rapidly than the smaller cities. Individual exceptions to the general rule may of course, be found. Between 1920 and 1930 the total growth of the ten largest cities in the United States, including their suburban territories, accounted for approximately 40 per cent of the total population increase of the country. The entire group of cities of over 10,000 population obtained 68 per cent of the country's total increase; and if the suburban areas of the cities over 50,000 were included, the proportion is raised to 86 per cent.

Not only has urban population been increasing at a far higher rate than the rural population, but the tendency is one that probably will continue in the future. Looking ahead to 1950, one sees little reason for estimating a materially larger farm population than at present. There will doubtless be substantial farm losses in many sections of the country. Also the group of smaller towns and cities, say those below 5,000 population, probably will increase but slowly. All present indications point to the probability that substantially all of the country's population growth will be absorbed by the group of cities of over 10,000 population and the smaller communities tributary to metropolitan centers.

PURPOSE AND VALUE OF ECONOMIC SURVEYS

The specific population forecasting job of the Southwestern Bell Telephone Company involves individual forecasts for some seven hundred towns and cities in which it operates, scattered throughout the States of Missouri, Kansas, Arkansas, Oklahoma, and Texas. Inasmuch as its long-distance lines serve either directly or through connecting with the lines of other telephone companies virtually every section of these five States, the job includes also forecasts of the probable population of various regions or sections of each State and for each State as a whole.

With a view to providing a more or less definite picture of economic conditions and trends in its territory, the Southwestern Bell Telephone Company inaugurated a series of economic surveys covering the five States in which it operates. The first of these studies was made in 1924 and the last of these completed in 1930 just prior to the 1930 census. It is planned to check the statements and forecasts from time to time against the conditions realized and to correct or supplement them as may appear advisable. The general plan of these studies involved an analysis and presentation of the following subjects:

1. Natural features, including a study of land form, soils, climates, vegetation, and water resources, with respect to their influence on activities.
2. The situation and outlook of the State's basic industries—agriculture, manufacturing, lumbering, and mining—in relation to population.
3. The comparative conditions and prospects in the various natural or economic-geographic regions of the particular State, the regional treatment being essential because of the dissimilarity of conditions prevailing over most States.
4. A detailed analysis of the situation and outlook of the principal cities.
5. The effect of the probable population changes on the demand for telephone service.

Illustrative of the kind of questions for which answers were sought by the State economic surveys are these:

How will the expansion (or perhaps contraction) of agriculture be distributed over the State? Will farmers have a greater or lesser purchasing power? What is the manufacturing outlook in the State? What kind of manufacturing activities may be expected to expand and what about their localization? Will the increase in the population-supporting activities be sufficient to cause the State to attract large numbers of persons from other States?

Since the answers to such questions influence directly the planning and management of the telephone business, the company engineers responsible for the population forecasts necessarily dealt with hard facts and figures; they recognized that reliable forecasts could not be made if unfavorable conditions and undesirable tendencies were ignored and only the bright side were stressed. Always they attempted to maintain an unprejudiced position, recognizing that many parts of the country possess advantages as fields for the employment of labor, capital, and executive ability, and that localities are in competition with one another for shares of the country's total population growth.

The organization for making the surveys and preparing the reports was small. Before issuing the final report the manuscript and the population and telephone forecasts were carefully reviewed in group conference. Personal inspection tours of each state were considered essential. Before making such trips it was considered desirable to collect and review carefully such information as was available on the particular area to be covered. Working diagrams and maps were prepared and the various chapters of the report were tentatively sketched. This procedure usually revealed conflicting statements as to matters of both fact and of opinion, thereby indicating some of the things to be cleared up during the inspection trip. This preliminary study also revealed gaps in the information which needed to be filled. Obviously, such a procedure was necessary in order to obtain maximum results from both the observation and interview work involved in the inspection trip.

Prior to the field trip, working contacts were established with the State University and the State College of Agriculture, with the department heads of the State government, and with the agricultural and industrial departments of the railroads operating in the State. Throughout the conduct of the job, valuable assistance was rendered by various Government Bureaus in Washington.

During the field inspection trips, interviews were had with persons in many lines of work, including State officials, University and College professors, agricultural experiment station directors, bankers, manufacturers and distributors, county agents, representatives of associations of various kinds, and also with local representatives of the telephone company. Of course, all the materials and suggestions gathered in this manner could not be accepted at their face value, because local pride and enthusiasm were not always carefully restrained. However, it was usually possible to detect misinformation and prevent it from coloring the final appraisals and analyses. Apart from furnishing information, a considerable number of persons volunteered or were asked to criticize sections of the report, and their cooperation in this regard helped greatly to reduce the number of erroneous statements and conclusions in the final report.

The formal reports which present the findings and conclusions, vary in length from 150 to 275 pages, depending partly upon the complexity of the problems encountered. It was found desirable to describe the conditions encountered and present the line of reasoning upon which each conclusion was based. Thus the reports can be readily checked and supplemented whenever it is desirable to do so. Readability as well as accuracy was sought. Maps and charts were freely used in the reports, and much of the discussion was centered about them. The subjects covered by the maps and charts of the "Economic Survey of Texas" included the following.

Regional Comparison of Area, Population, and Number of Telephones, 1928 and 1950.
General Land Form Features.
Distribution of Rainfall.
Native Vegetation.
Irrigated Areas and Principal Artesian Belts.
Maps showing details of the various natural regions in Texas.
Population Changes by Regions, 1920 to 1928 (estimated).
Net Interstate Movement of Population in and out of Texas, 1900-1910, 1910-1920.
Population Density by Counties, 1928 (estimated).
Per Cent of Farm Population Colored by Counties, 1925.
Foreign-born White Population as Per Cent of Total by Counties, 1920.
Per Cent Illiterate in Population 10 Years of Age and Over, by Counties, 1920.
Telephones Per One Hundred Population (Texas compared with Missouri, Kansas, Oklahoma, Arkansas, and Total United States) 1905 to 1928.
Telephones per 100 Population, by Exchanges, 1928.
Per Cent of Farms Having Telephones, 1925.
Trend in Acreage of Principal Field Crops, 1900 to 1927.
Trend in Number of Live Stock, 1900 to 1927.
Per Cent of Land Area in Harvested Crops, by Counties, 1924.
Average Value of Farm Lands and Buildings Per Square Mile, by Counties, 1925.
Average Value of All Farm Property Per Farm, by Counties, 1925.
Per Cent of Farms Operated by Tenants, by Counties, 1925.
Change in Number of Farms, by Counties, 1920-25.
Distribution of Cotton Acreage, by Counties, 1925.
Trend in Cotton Production (World, United States, and Texas) 1900-1927.
Series of figures showing distribution of live stock and principal farm crops.
Approximate Per Cent of Land in Forest and Woodland, by Counties (East Texas), 1925.
Sketch Map Showing Location of Principal Minerals (Other than Oil and Gas).
Petroleum Production, 1896-1927.
Trend in Oil Production by Major Districts, by Months, 1926-1928.
Principal Manufacturing Industries (Establishments and Wage Earners) 1925.
Wages Paid Per Wage Earner in Principal Manufacturing Industries, 1925.
Average Number of Wage Earners Engaged in Manufacturing, by Counties, 1925.
Number of Active Cotton Spindles (United States, the South and New England, 1880-1927).
Location of Cotton Mills in Texas, 1928.
Railroad Mileage in Texas, 1870-1926.
Railroad Construction in Texas Lines, 1925.
Map of Texas Illustrating Changes in Freight Rate Making Practises.
Trade Territories as Indicated by Bank Relationships.
Trade Territories as Indicated by Toll Traffic.
Trade Channels Between Texas and Mexico.

It is, of course, too early to determine the accuracy of the 1950 population forecasts made for the States and for the individual cities and towns. Obviously, many uncertainties attend the making of forecasts of this kind. That the population and market guide posts established by the surveys are of considerable value has, however, been thoroughly established. As previously indicated population forecasts were made for future years. It may be of interest, therefore, to compare the 1930 estimates with the 1930 census returns. In the tabulation below the estimates for January 1, 1930 have been projected to April 1, 1930, the census date.

	Census returns 1930 (April 1)	Economic survey estimates 1930 (April 1)	Per cent difference
Missouri	3,629,367	3,626,000	0.09
Arkansas	1,854,482	1,850,000	0.24
Kansas	1,880,999	1,864,000	0.90
Oklahoma	2,396,040	2,378,000	0.75
Texas	5,824,715	5,777,000	0.82

FORECASTING THE POPULATION OF A CITY

By the nature of things, no two cities present situations precisely alike. Indeed, one of the most striking facts is their dissimilarity. However, in analyzing the past growth of a city and in forecasting its future, there are usually five principal influences to be considered:

1. Size and character of its tributary trade territory.
2. Its industrial expansion.
3. Its attraction as a home center.
4. Governmental activities.
5. Institutional activities.

1. Study of the trade territory of a city is essential to an analysis of the population supported or likely to be supported by the wholesale, retail, and general office business of a city. There are various grades of trade centers, grading upward in order from cross-road stores, hamlets, villages, towns, county-seat towns, small cities,

minor cities, major cities, to metropolitan cities. All grades of trade centers function as agencies both for the outward distribution and the inward collection of goods. The typical cross-roads store, for example, may distribute only such articles as are in general demand by a small farming population and serve as a collection agency for, perhaps, only the surplus production of butter and eggs of nearby farms. At the other extreme are the metropolitan cities with varied and far flung distribution, and collection agencies of the type which make them great primary markets for such commodities as grain, cotton, or livestock.

The term "trade territory" is often used rather vaguely. Sometimes it is used to designate the territory tributary to the retail business of a city; and at other times, it may refer to the territory tributary to the wholesale business. For the smaller towns and cities the term is less subject to confusion, since the wholesale business is, in most cases, negligible. The larger cities, however, ordinarily hold a dominant place in the jobbing and wholesale trade of a zone that reaches far beyond the territory served by its retail stores. The metropolitan cities may even have a nation-wide distribution for some commodities. Also, it should be recognized that what is usually designated as the trade territory of a city is seldom, if ever, exclusively its own, for trade territories are over-lapping.

However, for purposes of analyzing the past and probable future, of the wholesale and retail business of a city, its trade territory can usually be defined with sufficient clearness. Aids in this include the analysis of banking relationships, newspaper circulation, and the distribution of sales of representative wholesale establishments. The analysis of the long distance telephone traffic in a number of States has largely confirmed the ideas of the respective trade territories of the principal cities as obtained by the examination of other data.

Many cities have had a rapid growth as trading centers along with the development of a tributary territory which has been settled rapidly. On the other hand, where rival trading centers have developed within the territory formerly tributary to a city, or where the territory has ceased to grow, then the importance of the city itself as a trading center has diminished. The trade territory of practically every city is changing, increasing or decreasing in extent and also undergoing changes in the character of the population. In respect to the latter, it should be remembered that the importance of a trade center depends not only upon the number of people in the tributary area but also upon their standard of living. Low living standards are unfavorable to the development of sizeable trade centers; while high living standards, in housing, furniture, vehicles, clothing, recreation, make for numerous and large trading centers.

In an analysis of the probable future changes in the trade territory of a city, consideration must be given to the trend in agriculture and industry, to such factors as improved highways, motor trucking, and changes in freight rates. For example, it seems clear that improved roads and the use of the automobile will continue to diminish the importance of many small towns.

2. Giving full value to the probable more intensive development of many sections of the country it now appears that the growth of most of our cities during the next twenty years will be due primarily to their relative attractiveness for industrial expansion.

For purposes of population analysis, it is helpful to classify industries. Industries may be grouped according to those which attract population, such as steel mills and automobile factories, and those which result largely from the presence of population, for example, bakeries, laundries, and ice cream factories. Obviously, the growth of industries of the latter type must depend upon the increase in the population of the particular communities which they serve, or upon an increased per capita demand. Again, it is helpful to consider industries from the standpoint as to whether they are conservative or exhaustive in nature, that is, whether their supplies are continuous or temporary. The lumber industry, as it has been carried on in this country, is a striking example of an exhaustive industry; many towns have either become extinct or else greatly declined in population when their lumber industries have moved on following the exhaustion of tributary lumber supplies. In many places, industries based upon the presence of minerals are exhaustive. For example, if a city is supported mainly by oil refining, and if the refining industry depends upon a single oil pool, the time will almost certainly come when the city's population will be greatly reduced unless other population supporting activities are developed.

If a city has diversified manufacturing plants, its growth is likely to be more uniform or stable than for a city where a single industry is dominant. A one industry town may have a rapid growth or a very slow one, depending on whether the industry is prosperous, or the reverse. It is well to note the different industries do not contribute to the population of a city in exact ratio to the number of employees. In a number of industries, including the manufacturing of shoes and clothing, a large proportion of the workers are girls and women, representing second and third members of a family. Further, one must consider the effects of labor saving machinery. In the past virtually all industries have been able to increase their output by the application of labor saving devices, without a proportionate increase in the labor required, and it is certain that this tendency will continue. Thus the industries of a city may increase their output materially, without a corresponding increase in population.

3. There are many people who choose a place to live rather than a place to work. Among these are the independently wealthy, the retired, and the adventurers. These people seek living conditions to meet their desires and adjust their occupations, if necessary, to what is attainable where they choose to live. Thus

large numbers have gone to certain cities because of climate, to other cities because of amusements, and almost every city, large or small, attracts some from the nearby sections because of city conveniences in general.

The classes who seek places to live first and occupations afterwards are likely to increase in the future. As wealth accumulates in the United States, the number who can live on the income from investments will grow. Cities and towns offering special advantages as home centers are likely to attract a larger proportion of the retired element than formerly, owing to the increasing tendency of this class to go longer and longer distances from their original home.

4. The national capital and many of the State capitals owe their growth primarily to the activities of the Federal or State governments. Governmental business requires a certain clerical force which tends to increase with the development of the country or the state, and with the complexity of the governmental machinery. The late war gave an abnormal impetus to the establishment of new departments and to centralization in general. In the past there has been a general tendency to enlarge the functions of governments, but for the immediate future it seems reasonable to expect an increasing pressure for more economical governmental administration expansion.

5. Many cities in the country have military establishments, educational institutions, penitentiaries, etc., which add considerable numbers to the population. Sometimes institutions of this character are relatively so large and are subject to such wide fluctuations in size, that they tend to render past rates of growth of the city misleading and to make forecasting of the future extremely difficult. In analyzing the population growth of such cities, it has been found advisable to isolate the past population related to institutions and treat this influence separately in forecasting the future.

Many cities possess the advantages just described in more or less equal measure, yet their past and prospective rates of population growth vary widely. It is here that well directed efforts on the part of various civic bodies are important. The community which effectively advertises its natural advantages of location is likely to grow, while a neighboring city equally well situated may stand still or even decline. Civic spirit is, therefore, a proper consideration in every population forecast. Through advertising activities a few of our cities have attracted a larger population than they can hold, and the slump which is bound to follow will perhaps permanently injure them. On the other hand, some cities have probably retarded their growth by advertising which so glaringly exaggerated their attractiveness that their real advantages were overlooked.

In the telephone business, and in most other businesses where advance planning is required, it is important not only to know the probable growth of population, but also its character and distribution, and its division into family units. Future population growth is forecasted upon the basis of certain definite factors already referred to, including expansion of industry or commercial activities which support population. In estimating the amount of population that probably will be located in high grade, medium grade, and poor sections of the city, it is essential to know rather definitely the class of population which is supported by the various activities. Obviously, if, in a particular city, population growth is expected to result from expansion of industries which employ principally common labor, both the character and distribution of the population will be quite different than if growth results from, say, an increase in wholesale and general office business. Similarly, character and distribution will be different if the growth comes mainly from an increase in population of the wealthy retired class. Further, it is well to note that each part of a city is in competition with other sections of the city for the limited population growth of the whole community in much the same manner as cities compete with each other.

CONCLUSIONS

Population forecasts, when carefully and intelligently made, serve a valuable purpose in helping to direct the employment of labor and capital to places or projects where they are most needed. What will most probably occur is always the first consideration in population forecasting. Sound forecasts cannot be made to rest upon a local point of view; national factors are always involved, frequently they are international. In boom periods it is necessary to guard against the tendency to overestimate, while the reverse is true of dull periods. Because of the fundamental influence of economic and geographic factors in determining the population growth of cities or regions, the economic survey is invaluable in determining the probable future number of people in a particular city or region, and their social and economic characteristics. Meticulous precision is impossible in making long range population forecasts, but it is both possible and practicable to obtain a valuable working knowledge of conditions in the future.

Discussion

Seymour H. Cronk: After a consideration of Mr. Holsen's paper, I am impressed with the importance of one of his implications. This is, that most engineering cannot be an exact science because of the variable nature of the basic assumptions which it is necessary to make before preparing plans for specific projects. These assumptions usually involve population and marketing in one form or another.

The present depression has emphasized to many minds that a given engineering expenditure would be better apportioned in almost every case if money spent in the preparation of specific plants were limited, if necessary, to the bare essentials and more money were spent in economic research bearing upon the basic assumptions involved.

If there is any measure of truth in the statement of Dr. Wright, President of the A.S.M.E., that the Engineering profession should share in the blame for the present depression because of its part in throwing production out of balance with consumption and distribution, then there is a real need for a closer liaison between economics and engineering.

The purpose of this discussion is to make the point that, according to my observation of engineering trends, the Bell System has for some years been concerned more and more in establishing as reasonably and precisely as possible the basic economic and social assumptions allied with each project for which money is to be spent.

An economic study and estimate which we are completing for Kansas City, Kansas, and which illustrates the economic work done in connection with exchange planning is given in the following.

The Kansas City economic study and estimate will form the basis for the technical plans and will be used to indicate the proper ultimate location and size of new central office buildings, and the best location and size of conduit to be installed under the streets. If plant is to be installed for best service and economy, it must be placed so as to be adequate without being wasteful over a period of at least twenty years.

So the economic problem was to reconstruct Kansas City by blocks on paper as nearly as possible as it will be twenty years in the future. The value of the state economic survey with its population estimates and analysis of industrial trends is obviously great in such work. In Kansas City, all present building development, telephone development, and population or "family" data were recorded by blocks. The amount of vacant and available land was listed by blocks, and these "availability" data were even supplemented by aerial photographs. Past population trends by enumeration districts were studied, and the probable future of each of the controlling industries was studied. All factors were considered which might at this time give any indication of what population and building distribution might be expected in Kansas City in 1951.

To arrive at the final result, which was a detailed growth forecast by blocks, required about 5,000 hours of engineering and economic work.

A less carefully based estimate might be made in a few days' time by other and less thorough methods.

The reason for doing this work in Kansas City, Kansas, and proportionate amounts in other cities, is of course, that the telephone industry has found this, in the long run, to be the cheapest way of "balancing production with consumption and distribution."

D. C. Jackson, Jr.: This paper is a valuable addition to engineering literature, since the matters outlined in it are generally applicable, not merely to any particular type of engineering work, but are generally of use in any field of endeavor having to do with the public.

Of the five points in the general plan of the economic surveys made by the Southwestern Bell Telephone Company, four are of importance to any organization, whether or not of an engineering nature, provided it has a reasonably widespread field for its product. To recapitulate, these points are:

1. Natural features.
2. Situation and outlook of the state's basic industries in relation to population.
3. Comparative conditions and prospects in the natural economic-geographic regions.
4. Detailed analysis of the situation and outlook of the principal cities.
5. Effect of the probable population changes on the demand for telephone service.

I have looked over the report of one of these surveys, that of Kansas, and it is clear that the four parts of this report which correspond to the first four points which I have just named are generally applicable to any large scale project or development of a business or engineering nature.

Tape-Armored Telephone Toll Cable

BY C. W. NYSTROM*
Associate, A. I. E. E.

Synopsis.*—Telephone toll cables buried directly in the earth are coming into increased use. The cables are not installed in the more usual clay conduit but are protected by layers of paper, jute, and steel tapes. The experience of the Southwestern Bell Telephone Company in installing some 500 miles of buried cable is described.*

* * * * *

TOLL cables in this country have in the past been installed either aerially on pole lines or underground in clay conduit depending largely upon the number of cable units needed to meet service requirements over a given period of time. In those parts of the country where relatively few cables will suffice to meet toll needs for a long period of time, it is usually more economical to employ aerial cable rather than underground cable in conduit. While there will continue to be a definite field of use for aerial toll cable particularly where difficult subsurface conditions obtain, there are many toll routes where ground conditions are favorable on which one or two cables will serve all needs for a long period, and it is for this type of case that tape-armored cables were developed as an alternative to aerial cable.

The cable is termed "tape-armored" because of its outer protection. Lead-sheath cable of the type ordinarily used for toll service is first covered with a wrapping of paper followed by two layers of jute roving. Two steel tapes, one over the other, are then applied, each tape being two inches wide and forty-one mils thick. A separation of one-half inch is maintained between adjacent turns of the tape, and the outer layer is so placed that it covers the space left between the windings of the inner layer. Two wrappings of jute cover the tape. The lead sheath, paper, jute, and steel tapes are thoroughly coated with an asphalt compound; the completed cable is given a coat of whitewash to prevent adjacent layers from sticking together on the cable reels. Such a cable of so-called full size has a diameter of 3¼ in. and a weight of 11.3 lb. per ft. as compared with the 2⅝-in. diameter and 8-lb. weight of unarmored cable.

Buried tape-armored cables possess several advantages over cables carried on pole lines or in underground conduit:

1. Easements for buried tape-armored cable on private rights-of-way may sometimes be obtained more cheaply than for pole lines, as the use of the land for ordinary agricultural activities is not disturbed.

2. Induced currents from paralleling power lines are appreciably less than in unarmored aerial or underground cables, due to the shielding effect of the steel tapes.[1]

3. Cable may be laid to follow the contour of rough country and have sharp bends to avoid obstructions while conduit must have a fairly even grade and long radius curves to permit cable being pulled through it.

4. Manholes are required at spacings of from 500 to 700 ft. if conduit is provided, while with tape-armored cables, manholes may be omitted except at loading points which normally occur at 6,000-ft. intervals.

On the other hand tape-armored cables may have some objectionable features. As they are buried directly in the earth, the location and repair of troubles are generally more difficult and expensive than for aerial cables; however, improvements in methods for maintaining buried cables should reduce such costs, and in any event, the amount of trouble per cable-mile may reasonably be expected to be much less than on aerial cables because of their security from damage by storm, fire, bullets, etc.

The first tape-armored toll cable in this country was installed by the Southwestern Bell Telephone Company. Other telephone companies have since placed similar cables but in view of the writer's greater familiarity with work in the Southwestern Bell territory, this paper will deal, in general, only with that Company's experiences.

One of the first considerations following the decision to place a cable is the selection of the route. Between Fort Worth and Cisco, Texas, where the first tape-armored cable was placed, the country for some 15 miles west from Fort Worth is rolling, with hills gradually increasing in height. For the remaining 85 miles the topography becomes extremely rugged with large quantities of rock found in the soil and the country is covered with dense growths of scrub oak and other underbrush. Roads are few and accurate maps were not obtainable. The difficulties and time required in making a detailed ground survey were such that it was deemed advisable to photograph the general route from an aeroplane. From the photographs so obtained a tentative route was marked out and found to be of considerable assistance in expediting the ground survey which is, of course, necessary in any event. With fairly open country, frequent roads and adequate maps, there ordinarily would be no necessity for a preliminary aerial survey.

With regard to right-of-way for tape-armored cables, it is generally felt that private property is more desirable than public highways where road improvements such as straightening, widening, grade changes and reroutes

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*Southwestern Bell Telephone Co., St. Louis, Mo.

[1] *Trends in Telephone and Power Practise as Affecting Coordination*, by W. H. Harrison and A. E. Silver, A. I. E. E. TRANS., June 1931, p. 437.

Presented at the South West District Meeting of the A. I. E. E., Kansas City, Mo., October 22-24, 1931.

might necessitate expensive relocations of cables. On the other hand, if private property easements are high in price or not obtainable or construction costs on privately owned land are too great, it might be best to place cable on the highway and so located as to be subject to the least probability of having to be moved. It is evident that the question of the best possible location for the cable is one requiring careful consideration.

The Fort Worth-Cisco and other tape-armored cables placed later are largely on private rights-of-way, in many instances in cultivated fields. The transportation of five-ton reels of cable over soft and often muddy ground is not usually possible with an ordinary truck or cable reel trailer and it was necessary to design a suitable outfit. Fig. 1 shows the caterpillar type of trailer used as well as a commercial type of tractor necessary to haul the cable to its proper location. There have since been some developments in the line of four-wheel drive truck equipment for handling the deliveries on the right-of-way as well as on the highway.

The equipment in Fig. 1 while ideal for use on soft or uneven ground, is not suitable for hauling cable over long distances because of its limited speed. Some of the hauls on the Fort Worth-Cisco route were as long as twenty miles from the nearest railroad siding and for such cases a rear-wheel drive truck with special winch and body equipment made fairly high speeds possible on highways.

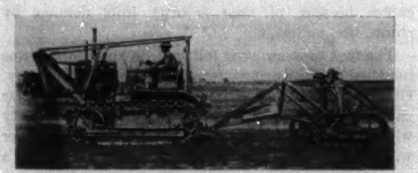
FIG. 1—CABLE REEL TRAILER AND TRACTOR

For trenching in earth a commercial wheel type excavating machine was first used and proved quite satisfactory until soil containing considerable quantities of rock was encountered. This rock lay just below the ground surface and while not in solid ledges, was so hard and in such quantities that the trenching machine could not function. After some experimenting and changes in design the plow shown in Fig. 2 was found to answer the needs quite satisfactorily. For solid rock or where the rock formation was such that the plow could not break through, pneumatic drills and blasting were required.

The cable may be laid in the trench directly from the reel. Where this method is impracticable, as, for example, cables crossing under paved highways, pipe lines or placed on steep slopes, the cable may be pulled over rollers placed on the bottom of the trench. Backfilling was then done by a machine of the type shown in Fig. 3.

The use of the rock-breaking plow focused attention on the further possibility of using a plow for both trenching and laying cable. This led to the development of an outfit which not only dug a trench, but laid the cable in it and backfilled the excavation all in one operation. The result was the plow pictured in Fig. 4. This plow after laying more than 300 miles of cable is still in good condition. The equipment consists essentially of two main parts, a chassis and a vertical central portion which may be raised or lowered to bury the cable at depths up to 28 inches. Fig. 5 shows the lower portion of the mechanism and Fig. 6 the complete cable laying equipment in operation. In connection with the latter figure it will be noted that the plow does not dig a trench, properly speaking, but merely cuts a slot through the ground. Some earth is thrown out and this is drawn to the center of the slot by the backfiller. If desired, a tractor may be run over the disturbed earth to pack it down.

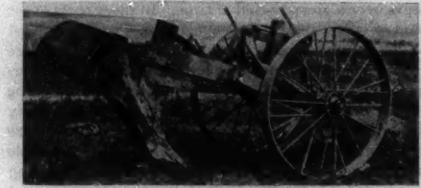
FIG. 2—ROCK-BREAKING PLOW

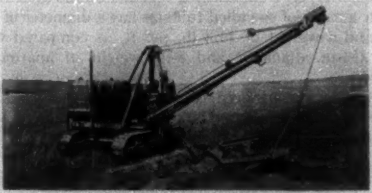
FIG. 3—TRENCH BACKFILLER

Two tractors, sometimes three, are required to pull the cable-reel trailer, plow, and backfiller. As can be seen, the cable passes from the reel through a pipe extending into the ground. Thus as the plow proceeds the cable is deposited at the bottom of the trench and is not dragged through the ground at any time. The essential features of the plow are more clearly seen in Fig. 5. The forward or cutting end is a steel plate having a sharpened edge and a shoe is provided at the bottom to make a larger space for the cable. Side plates are welded to the cutting blade and to the outer sides of the pipe through which the cable passes.

Small boulders and tree roots often made progress very difficult for the cable laying plow. It was found that with a slot first cut through the earth, the plowing in was considerably expedited and cable was laid in locations where successful plow installations seemed, at first, very doubtful. The plow used for this purpose is called a "rooter" plow. The rooter first used was similar to the rock plow of Fig. 2. In fact it was the same plow with the mold boards removed and a curved forward cutting edge added. The first rooter plow was found to lack in requisite strength and following the building of the second cable plow, Fig. 4, the rooter shown in Fig. 7 was designed. This fits in the same chassis as the cable laying plow. Thus either the cable laying or rooter plowing attachments may be used interchangeably.

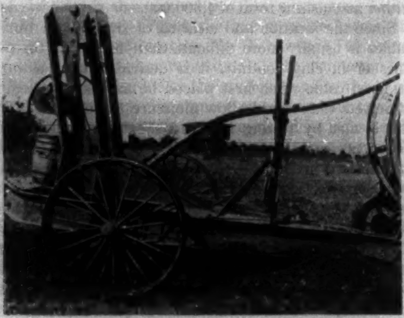

Fig. 4—Cable Laying Plow—Improved Design

Fig. 5—Lower Part of Cable Laying Plow—Improved Design

Road improvement work, pipe-line construction or other excavating activities may result in damage or destruction to the cable if its presence is not definitely indicated. Warning signs are accordingly placed on each side of a road under which a cable is laid, at pipe line and railroad crossings and any other locations where excavating work seems probable. Maps showing the cable location are furnished to highway authorities, railroad, and pipe-line companies and others who might engage in construction or repair work in the vicinity of the cable. The route is also regularly patrolled as an added precaution.

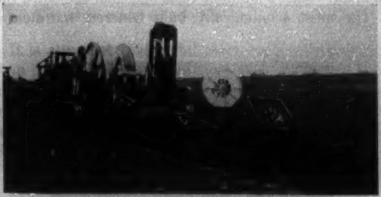

Fig. 6—Complete Cable Laying Equipment

In connection with the splicing, the lead sleeve covering the splice necessary every 750 ft. was at first protected by a cast iron case. When branch or stub cables were required out of the main cable, a modified form of cast iron splice case was used which provided two openings at one end. The splice case was filled with an asphaltum compound to insure protection against soil corrosion for the lead sheath and sleeve, and an asphalt paint was applied to the outer surface of the case.

The cast iron splice covering was found to be rather expensive and it was felt that a cheaper protection would serve equally well. This protection consists of a wood case, made of 2-in. creosoted yellow pine lumber. Space for branch cables is provided by two openings in one of the end pieces. The exposed lead sheath and sleeve within the case are painted with an asphaltum

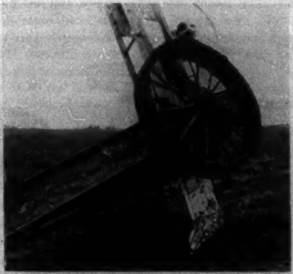

Fig. 7—Rooter Plow—Improved Design

compound and covered with three layers of muslin tape which is also painted with the compound.

In ordinary underground conduit installations the loading manholes are built of concrete or brick. For tape-armored cables, less expensive manholes are built of 2-in. creosoted yellow pine. Fig. 8 shows one containing a cable and loading coil case but with the cover omitted. Such a manhole costs less than one-third that of the most economically built concrete manhole of similar dimensions. It is usually 12 ft. long, 4 ft. wide, and 3 ft. deep. No floor is provided, the loading pots being buried in the earth as shown. After the top is placed, it is covered with earth, the depth from the surface to the top of the manhole being at least 18 inches. This is necessary as it is desirable not to interfere with the use of the land for ordinary agricultural purposes. Since the manholes and cable are not visible above ground, markers are usually provided at each manhole and at points along the cable route to enable ready location in cases of trouble or the placing of additional loading coils or a future cable. The markers are concrete or wood posts to which are attached plates with manhole numbers, distances and direction to the cable or other pertinent information.

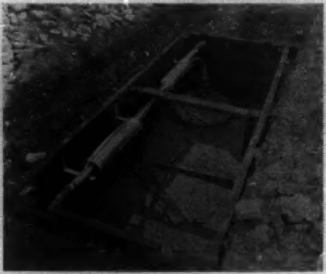

Fig. 8—Creosoted Wood Manhole with Cable

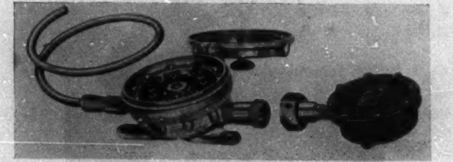

Fig. 9—Combined Contactor—Terminal

Electrical tests on tape-armored cables do not differ appreciably from those used for unarmored cables. It is estimated that 5,150 individual measurements are made per mile of cable and this is the minimum normal amount required. If conditions difficult of correction are encountered—and these are sometimes found—many of the tests must be repeated. For the cables between St. Louis and Dallas, the estimated minimum number of individual test measurements reaches the rather astonishing total of 4,500,000.

Since the location and clearing of troubles in buried cables is usually more difficult than for those on pole lines or in clay conduit, it is desirable that a tape-armored cable when first placed be as free as possible of defects that would allow moisture to enter it. This end is met by testing the lead sheath with nitrogen, under pressure, at the factory, upon arrival at the destination, just before the cable is laid or plowed in, a day or two later, and then before splicing operations begin. As splicing necessitates removal of sheath at each end of a length of cable, most of the gas is lost. Nitrogen is again introduced on completion of splicing in one loading section, 6,000 ft., and allowed to remain for about two days. If no leaks are found the loading coils are spliced in and the loading splices tested with gas. Gage readings are taken and recorded during all these tests. A more extended discussion of the application of gas-pressure testing for the detection of sheath breaks both during installation and afterward while the cable is in service has been given in a previous paper.[2]

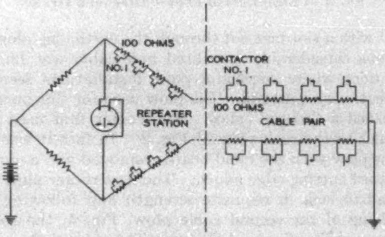

Fig. 10—Gas Pressure Alarm Circuit

Two recent improvements in connection with constant gas-pressure testing on completed cables seem worthy of mention, however. First is the combined contactor terminal shown in Fig. 9. This outfit can be mounted either in a manhole or on a post. It has the advantage over the separate units of being more compact; the contactor, which is the more delicate of the two, can be replaced very easily and the terminal is made gas tight.

Second is an improved alarm circuit that avoids a difficulty possible in the earlier circuits; namely, that the operation of any one contactor will render the alarm circuit insensitive to the subsequent operation of any other contactor in the 50-mile cable section until the first contactor is disconnected or the gas pressure is raised in the cable section it covers. This is due to the fact that a single circuit is used for alarm purposes for an entire repeater section of cable. The plan shown in Fig. 10 avoids this difficulty. The wheatstone bridge principle is used. When the gas pressure lowers, the contactor, say at location No. 1, closes. This short-circuits the 100-ohm resistance at the contactor and unbalances the bridge at the station, causing the alarm there to operate. Resistances equal in value to those at the cable contactors are terminated in jacks at the station and the attendant merely inserts a non-metallic plug in turn in each jack until the correct one is found as evidenced by silencing of the alarm. It will be noted that when the plug is inserted in jack No. 1, a resistance of 100 ohms is added to the circuit. As this is the amount removed by the operation of contactor No. 1, the resistance of the entire circuit is restored to normal and the bridge is automatically rebalanced and remains so until another contactor operates. This system has been tried out and found to operate very satisfactorily.

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[2] *Recent Developments in Telephone Construction Practises*, B. S. Wagner and A. C. Burroway, A. I. E. E. Trans., July 1929, p. 836.

It is evident that there are times when the gas alarm is inoperative. For example, a large sheath break or repair operations may permit practically all the gas to escape from a plug section. To protect the cable during such periods, another maintenance device known as the low-insulation resistance alarm, is often used.

This system operates when moisture entering the cable lowers the insulation resistance between the conductors and the cable sheath. A number of pairs of conductors in the outer layer, *i. e.*, adjacent to the lead sheath, is connected in the repeater station to a relay mechanism which automatically and in regular sequence, switches these conductors to a sensitive insulation resistance measuring device. A decrease in the resistance below a predetermined value causes an alarm to operate. Wheatstone bridge measurements are then made to locate the trouble and men are dispatched to clear it. Because of the minute currents flowing through the moistened insulation, recourse is had to the vacuum tube by which these exceedingly small currents may be amplified sufficiently to cause the operation of an alarm.

So far nothing has been said as to the costs of tape-armored cables and sufficient experience has not been had with them to develop comprehensive and average data. Each of the cables in the Southwestern Company's territory has been placed under conditions which do not admit of direct comparisons. Soil, rock, topography, the time of year during construction, types of machinery and construction methods with continuous improvements on each job, and many other factors have all operated to make cost comparisons difficult. However, some general conclusions seem possible, based on the limited experience so far obtained by the Southwestern Company.

Using the cost of tape-armored cables as unity the following relations are given:

Type of plant	Costs
One tape-armored cable	1.00
One aerial cable on pole line	0.90
Two tape-armored cables	1.00
Two aerial cables on pole line	0.85

It is apparent that aerial plant costs somewhat less than the buried type. However, certain developments now under consideration promise a rather substantial reduction in the cost of buried cable and if these prove successful, it is quite probable that buried plant may cost even less than aerial cable.

There is another factor difficult of evaluation and that is the effect of service interruptions. Continuous and dependable service is one of the principal objectives of a telephone company and service failures are not only annoying to patrons but result in loss of revenue to the company. It would seem that buried cable should be subject to less damage from external sources than aerial construction, and if further experience bears this out the desirability of more extensive use of buried cables may become apparent.

Discussion

H. R. Fritz: In a recent presentation by Harrison and Silver[*] well grounded buried tape-armored cable is shown to provide an approximate shielding of 80 per cent against externally induced voltages, relative to the voltages induced on open wire. This shielding is about 30 per cent larger than that obtained from well-grounded lead-covered cable without armoring.

An extended investigation of the possible shielding benefits of tape-armored cable has been made on an actual field installation. The results were in substantial agreement with the figure given above. The effects of other grounded structures in the neighborhood of the test section were also noted, their contribution being to increase the shielding by a few per cent. It was further revealed that additional shielding may be obtained by grounding the pairs in the cable at each end through the balanced points of repeating coil windings. Under favorable conditions shielding in excess of 90 per cent may be anticipated from the combined effects of the steel tape, other grounded structures and wire drainage.

Of the several parameters governing the shielding afforded by any cable sheath the admittance of the sheath to earth is particularly interesting because of the wide variations that might be expected to occur between different localities. Extensive investigation of tape-armored cables in this territory prior to, during and after installation has revealed a rather singular behavior. Since both the sheath and tape are covered with layers of impregnated jute wrapping it might be anticipated that the distributed admittance of sheath and tape to earth would be large. This was found to be true prior to and just after laying the cable. Subsequently the admittance increased very rapidly. Typical measurements indicated leakances of approximately 0.000004 mho per kilofoot initially, which increased to from 0.01 to 0.3 mho per kilofoot within two weeks after laying. In terms of resistance this means a change from 250,000 to as low as 3.3 ohms per kilofoot. The leakance, once acquired, is retained despite subsequent drying.

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[*] *Trends in Telephone and Power Practise as Affecting Coordination*, by W. H. Harrison and A. E. Silver, A.I.E.E. TRANS., June 1931, p. 437.

The cause of this increase has not been determined nor has it been established whether it may not be affected by changes in the manufacturing process or the composition of the wrapping. For the locations where it does take place the result is of course extremely favorable to a full realization of the shielding from the sheath and tape.

Stanley Skinner: From the standpoint of routine maintenance and operation one of the new problems presented by buried cable has to do with the protection of the sheath from mechanical injury and from soil corrosion.

In the buried cable which was placed in service between Kansas City and Joplin in May of this year, there is a total of a little more than eleven acres of lead sheath which must be kept intact. This sheath is about one-eighth inch in thickness and is easily injured. The tape armor is usually effective in preventing sheath damage such as abrasions and punctures which might be inflicted by sharp stones and the like during the installation period or afterward, but it may be punctured by a direct blow from a pick or digging bar. Consequently considerable thought has been given to the question of protecting the cable from accidental damage during road grading operations, strip coal mining, excavations for pipe lines, etc.

The most effective protection is that afforded by care in the selection of the route. It is usually possible to locate the cable on private property at such a distance from roads and buildings that the probability of its being disturbed during road grading operations and by ordinary farm excavations is remote. It is desirable to follow, as nearly as possible, a direct line from repeater station to repeater station and with proper care such deviations from a straight line as are necessary may be made without materially increasing the length of the cable. Through the use of long tangential sections, a cable may be located as much as a mile away from a direct line between two repeater stations without increasing its total length more than a few hundred feet.

Road crossings are unavoidable and some extra precautions are required at such points. The best protection seems to consist in increasing the depth to which the cable is buried and in carrying this increased depth past the property line to a point where the cable will be clear of probable road widening operations. Similarly it is advisable to increase the depth at points where the cable crosses small streams and gulleys. Certain types of soils are subject to very rapid erosion and this makes it necessary to increase the depth of the cable along hillsides or in descending the banks of streams.

Since experience with tape-armored cable is limited actual field results as to the effectiveness of the jute and asphalt covering as a protection from soil corrosion are not available. Laboratory tests have indicated that the asphalt impregnated coverings should very effectively protect the lead sheath under practically all conditions normally encountered. Some situations may, however, be found which would make it desirable to employ additional protective measures. One or two cases have been encountered in the coal fields where the surface water is unusually acid in reaction; in the same vicinity it was also necessary to use wire-armored cable in crossing streams. In these cases the cable was encased in a wood trough filled with asphalt and the trough buried in a trench under the bed of the stream. While it is doubtful that additional protection was actually needed in these cases, the precautions taken are mentioned as illustrative of the many points which must be watched in a job of this nature.

By establishing favorable contacts between the telephone forces and contractors, pipe line companies, highway officials and others an attempt is made to prevent accidental damage to the cable by enlisting the cooperation of the agencies which may be responsible for this damage.

Due to the limited experience with buried plant, maintenance practises are still in the formative stage. In addition to the buried cable between Kansas City and Joplin, there is an underground cable in clay ducts along the highway between Kansas City and St. Louis. This will enable comparisons of the cost for routine maintenance of these two types of toll plant. From present indications it appears that the cost of maintaining the buried cable will not exceed that of unarmored cable in clay conduit.

The Time Factor in Telephone Transmission

BY O. B. BLACKWELL*

Fellow, A. I. E. E.

UNTIL comparatively recent years the telephone engineer gave little attention to transmission time in his problems. For all practical purposes he could assume that speech was transmitted instantly between the ends of telephone circuits. The rapid extension of the distances over which commercial telephony is given and the introduction of long telephone cables has changed the situation and has introduced time problems in telephone transmission which are of large technical interest and difficulty. As a result, time problems are receiving more consideration in the technical papers published in recent years on transmission. The attached bibliography lists a considerable number of such papers. There seems to be no paper, however, giving a general overall picture of this subject. The present paper gives briefly such a picture.

The time factor introduces five different types of problems in telephone transmission:

1. *A Slowing-Down of Telephone Communication.* In talking over long lengths of certain types of cable, the time interval between the formation of a sound by the speaker and its reception by the listener may become of sufficient magnitude to slow down conversation. This is not a serious matter with the types of circuits now used in the United States, even for the longest distances between points in this country. It does, however, become of considerable importance when we consider the joining together of long lengths of cable in this country and long lengths in Europe with possibly long lengths of intervening submarine cable.

2. *Delay Distortion.* Difference in the speed of transmission over a circuit of the different frequencies which make up speech. This may introduce peculiar distortions in speech which cause considerable interference.

3. *Echo Effects.* These arise from the fact that parts of the energy transmitted over a circuit may be reflected back from points of irregularity in it, particularly at the ends. Small amounts of the energy may wander back and forth over a circuit two or more times. While these echoes may affect both the talker and listener, they generally have the greatest effect on the talker who may have an uneasy feeling that the distant party wishes to break in on the conversation.

4. *Effects of Voice-Operated Devices.* To overcome echoes, and under some conditions to hold circuits stable, it has become the practise of connecting into certain types of circuits, relay devices operated by the transmitted speech currents which render inoperative transmission in the opposite direction. In some cases delay in transmission may be an advantage in the operation of such devices. In other cases it may introduce serious difficulties. Conditions may be set up in which it is difficult for one party to interrupt the other. In other cases, portions of conversations may be locked out. If the voice-operated devices are not properly adjusted or if considerable noise is present, the devices may not function properly and speech mutilation may result.

5. *Fading.* In radio, the well-known phenomenon of fading is due to waves arriving at the receiver over different paths, the transmission times of which are such as to cause alternate strengthening and weakening of the received signal by alternate phase agreement and opposition. While this factor is mentioned here for completeness, it will not be discussed further as it is beyond the scope of this paper to discuss the problems introduced when there is more than a single path between the sending and receiving ends of a circuit. The present paper is limited to the conditions which hold where not more than one path is involved in the transmission in each direction.

SPEED OF TRANSMISSION

Before considering these problems in more detail it would be well to define what is meant by the speed of transmission over a circuit. There are several speeds which may have significance according to the problem involved.

Whenever a change in applied voltage is made at one end of a circuit, some evidence of this is transmitted over the circuit to the receiving end at the speed of light. In general, however, except in radio, no sufficient action to be of use is transmitted at this speed and it is largely of theoretical importance.

The speed which the engineer generally has in mind in thinking of line transmission is the speed at which the crests or the troughs of the waves pass along the line when a single-frequency potential is continuously applied at the sending end. This usually is referred to as the speed of phase transmission in the steady state. While this usually approximates the speed in which we are interested, it may in particular cases differ considerably from it. In fact, in certain types of artificial circuits the crests and troughs of the waves travel toward rather than away from the sending end.

This speed may best be explained as follows:

Since the speed of transmission is generally different for different parts of the frequency range, for simplicity, a particular narrow frequency range, say between the frequencies N_1 and N_2 is considered. It is supposed that

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*Trans. Development Engr., American Tel. & Tel. Co., New York, N. Y

Presented at the South West District Meeting of the A. I. E. E., Kansas City, Mo., October 22-24, 1931.

electrical filters are applied to the circuit limiting the frequencies over it to approximately this range. If then, a voltage having a frequency say at midpoint of this narrow range is applied to the circuit for a short interval and then removed, the speed at which the disturbance thus set up travels down the circuit is the speed in which we are interested. A spurt of energy of this type is evidently similar to that which takes place in carrier telegraph systems when a dot impulse is applied to the circuit. This speed can be looked at, therefore, as that of carrier telegraph signals so formed.

The speeds on this basis of a number of standard constructions which represent good engineering practise today are approximately as follows:

Type of circuit	Approximate speed in miles per second
Cable circuits loaded with 88-mh. coils at 3,000-ft. spacing	10,000
Cable circuits loaded with 44-mh. coils at 6,000-ft. spacing	20,000
Cable pairs of non-loaded 16 B. & S. gage	130,000
Non-loaded open-wire pairs	180,000
Radio	186,000

CAUSES OF TIME LAG IN TRANSMISSION

A pair of wires of zero resistance in free space separated from all other conductors and without leakage would transmit electrical waves over it at the speed of light. It will be noted from the above table that non-loaded open wires transmit at a speed not differing widely from this. What retardation exists comes largely from the glass insulators which cause an increase in capacity and the resistance of the wires, which causes an effective increase in inductance.

In cable circuits there is still further retardation by the increase in the capacity between the wires because of the necessity of using a certain amount of solid dielectric and particularly from the increase in the inductance of the wires when loading coils are inserted in them to decrease attenuation.

In actual circuits there is still some further retardation by the apparatus which is necessarily inserted at the terminals and at intermediate points along the circuit. The figures given in the above table are for the bare circuits. The delays caused by apparatus will, in general, reduce these speeds from 10 to 25 per cent.

SLOWING-DOWN OF TELEPHONE CONVERSATION

Considering the first of the above factors, it is noted that so long as the speaker at one end of a telephone circuit continues to talk, the listener at the other end will hear the speech in proper time relation, independent of how much absolute delay there is in going from one end of the circuit to the other. However, when the speaker asks a question and waits for the answer, the slowing-down effect on his conversation will evidently be the time of transmission of his question to the distant end and the transmission back from the distant end of the answer. Considering the speed shown in the table, however, consideration may be taken of the fact that the non-loaded constructions, both open wire and cable, and the radio, are of such high speed that conversations could be carried on over them for the longest distance between places in the world without appreciable difficulty. This, however, is not the case for the loaded construction. Assume, for example, that a length of 4,000 miles would cover the wire line distance between any two points in this country. For the slowest construction noted, an interval of 0.8 second is required for transmission to the distant end and return. While it is possible to carry on conversation over a circuit with this delay, it is larger than is considered desirable. The faster of the loaded constructions shown would give a delay over circuits of this length which represents somewhere about the limit of what, at the present time, is considered satisfactory. Incidentally, the slowest of the constructions shown, for this and other reasons, is not proposed for use except for comparatively short distances.

Communication engineers must look forward to the time when the longest cable distances in North America are connected, in some cases by submarine cable, to long lengths of cable in Europe. With this as an ultimate objective, this matter of the direct effect of delay on conversation has become of considerable interest.

An appreciation of the transmission time on long telephone circuits may perhaps be gained by considering the distance required to produce an equivalent delay of sound waves traveling in air. For example, it takes about as long for a radio wave to travel half way around the earth at the equator as it does for a sound wave to travel from one speaker to another when the distance separating them is about 75 ft. Incidentally, the time required for a radio wave to travel from the earth to the planet Mars would be from about 3 to 20 minutes, assuming that it got there at all. Evidently if we have neighbors on Mars we can never hope to carry on conversation with them.

Telephone engineers have devoted considerable attention to the effects of delay on the telephone users in an effort to determine how far the electrical waves should be permitted to travel over different types of circuits. Since the present constructions do not offer any particular difficulties, for the distances now in use, a look into the future has been taken by means of artificial delay circuits. One method has been to loop back and forth loaded conductors in a cable until the desired delay was obtained. Another method has been to record the talkers' waves on a phonograph and pick up the impressions with a second needle, displaced the desired amount from the first so as to introduce delay into the conversation.

Considerable use has also been made of pipes or "acoustic" delay circuits. Fig. 1 shows an illustration of a brass pipe delay circuit used in experimental work. Fig. 2 shows the circuit in schematic form. In addition to the pipe, which is looped back and forth to conserve space, the circuit involves telephone receivers at the two ends. The one at the sending end converts electrical energy into sound which is transmitted through the pipe and the one at the receiving end converts the sound waves back into electrical energy. The pipes are quite suitable because they have approximately the same delay at all frequencies. Various devices are required to reduce the reflections which occur at the junction of the pipe and the receiver and to equalize the attenuation.

Using devices of this nature, experimenters have found it possible to talk fairly conveniently over circuits representing time intervals as great as 0.7 second in each direction. So great delays would be considered undesirable for commercial use, however. Delays of about a third of this, in general, are considered about the maximum which is satisfactory.

DELAY DISTORTION

In designing circuits which are electrically long, care must be exercised to insure that the transmission times for all frequencies in the transmission range are sufficiently alike to avoid objectionable transient phenomena. These effects may occur in one-way circuits as well as in two-way circuits and are not related to echo effects.

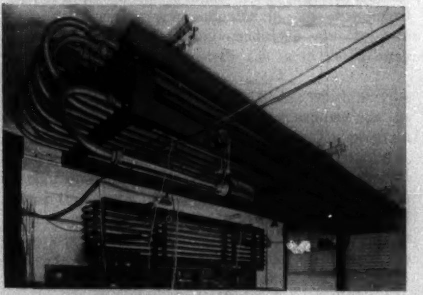

FIG. 1—ACOUSTIC DELAY CIRCUITS

The appearance of these transients to the listener depends on whether the excess delay is at low frequencies or at high frequencies. It is rather difficult to describe the characteristic sound of a circuit with low-frequency delay. A high-frequency delay, if it is in an extreme form, sounds as though a high-pitched reed, such as a harmonica reed, was being plucked whenever there is a sudden transition in the voice sounds being transmitted over the circuit.

The characteristic effects of transients are conveniently described by the aid of oscillograms of spurts of alternating current taken before and after being sent over circuits having various delay characteristics. To begin with, it must be recalled that when any wave shape is applied to a circuit, the transmitted wave in the circuit can be expressed as the sum of the series of sinusoidal waves whose frequencies range from very low to very high values.

In the case where a sinusoidal wave of frequency F is suddenly applied to the sending end of the line, the effect may, therefore, be explained as due to an infinity of sinusoidal waves so proportioned and phased as to add up to zero, up to the instant of application of the wave, and to equal the steady-state value of the wave at that instant. Of these waves, the most important have frequencies close to F. They are propagated over the line individually with the velocity corresponding to the frequency. If the velocity of the line is the same for all frequencies, they will evidently add up to give the same overall wave shape at the receiving end of the line as at the sending end.

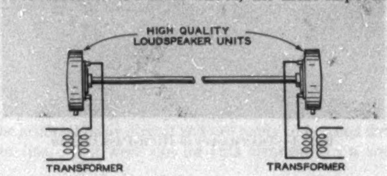

FIG. 2—ACOUSTICAL DELAY CIRCUIT SHOWING TWO HIGH-QUALITY LOUDSPEAKERS WITH ONE CONNECTED TO EACH END OF TUBE FORMING DELAY CIRCUIT

If, on the other hand, the velocity is not the same for all frequencies, there will be more or less distortion and transients in establishing the wave, though ultimately the pure wave of frequency F will be established. Several oscillograms will be shown to indicate transient effects which are experienced under various conditions.

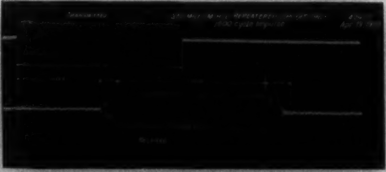

FIG. 3—TRANSIENTS IN MEDIUM HEAVY-LOADED CIRCUIT

Fig. 3 is an oscillogram showing a spurt of 1,600-cycle current as applied to and received from a loaded circuit having fairly large delays in the upper part of the transmitted range compared to the delay at lower frequencies. Remembering the nature of the oscillations at the beginning and end of the applied spurt, it will be observed that the current at the receiving end consists at first of a fairly low frequency which builds up in frequency and magnitude to the steady-state value. At the end of the spurt the same transient is experienced, but in this case the higher frequency currents which have been delayed in the line are at the tail end of the train.

Fig. 4 shows a 200-cycle current with many harmonics of higher degree transmitted over a circuit having large delay at low frequencies. It will be noted that these high or harmonic frequencies are received in advance of the 200-cycle wave. This is because the 200-cycle wave is subject to appreciable delay while the higher frequencies are not. This circuit, while actually made up of artificial networks, had characteristics similar to certain types of long cable circuits for the lower part of the telephone frequency range.

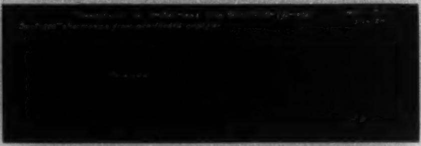

FIG. 4—TRANSIENTS IN HIGH-PASS FILTER

Fig. 5 shows transient effects of a 600-mile composited 19-gage H-174 side circuit. The zero lines of the three curves of this figure show slight effects of crosstalk and other interference. These effects are not, however, of sufficient magnitude to interfere with the general appearance of the signals. The upper line shows the applied 1,000-cycle current. The next line shows the current as it was received at the end of the line. The transients which are produced at the beginning and ending of the signals are evident.

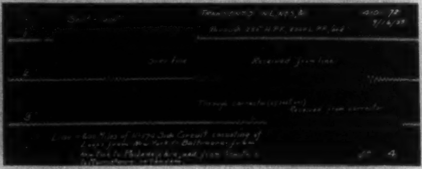

FIG. 5—CORRECTION OF TRANSIENTS IN MEDIUM HEAVY-LOADED CABLE CIRCUIT THROUGH USE OF PHASE COLLECTOR

By the insertion of proper networks it is possible to correct for distortions of this kind. In the last line there is shown the received current when a delay correcting network of this kind is applied in series with the circuit. It should be noted that while the delay in the reception of the signal is somewhat increased (as shown by the displacement of the whole signal further to the right) the transients at the beginning and stopping of the signal are very much reduced.

ECHOES

In designing telephone circuits which are electrically long, an important problem is presented by the necessity of avoiding echo effects. These are caused by reflection of electrical energy at points of discontinuity in the circuit and are very similar to echoes of sound waves in an auditorium. The reflected waves are usually considered as echoes when there is an appreciable delay with respect to direct transmission. Some of the reflected waves return to the receiver of the talker's telephone so that if the effects are severe he may hear an echo of his own words. Other reflected waves enter the receiver of the listener's telephone and, if severe, cause the listener to hear an echo following the directly received transmission.

Reflections of voice waves occur in all practical telephone circuits. It is only in telephone circuits of such length as to require a number of repeaters, however, that echo effects become serious. The fact that the circuits are electrically long makes the time lag of the echoes appreciable. At the same time, the telephone repeaters overcome the high attenuation in these long circuits and consequently make the echoes louder. The seriousness of the effect is a function of both the time lag and the volume of the echo relative to the direct transmission, becoming greater when these are increased.

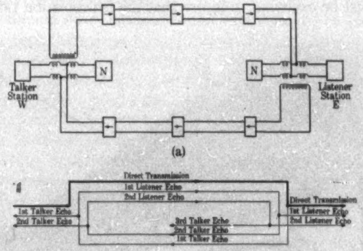

FIG. 6—FOUR-WIRE CIRCUIT

In telephone circuits the most important points of discontinuity are usually the two ends of the circuit. In a four-wire telephone circuit these are the only points of discontinuity.

Fig. 6 shows a schematic diagram of a four-wire telephone circuit and a schematic representation of the direct transmission over the circuit, together with the various talker and listener echoes which are set up. The rectangles at the extreme right and left are intended to represent the telephone sets used by two subscribers at the west and east terminals of the circuit. The rectangles marked *N* represent electrical networks which simulate or balance more or less perfectly the impedance of the telephone sets. In the four-wire circuit the rectangles with arrows represent one-way repeaters or amplifiers. At each terminal the two separate one-way circuits comprising the four-wire circuit are joined together by means of the familiar balanced transformers. When the subscriber at *W* talks, the transmission passes to *E* over the upper path in the four-wire circuit. This is indicated by the heavy line labeled "Direct Transmission" in part *b* of the figure. When subscriber *E* talks, transmission passes over the lower path in a similar manner.

Considering part *b* of the figure, it will be noted that when direct transmission is received at the east end of the circuit, a portion of the current passes to the opposite side of the four-wire circuit and is transmitted to the subscriber at the west end as a talker echo. Similarly, a portion of this talker echo is transmitted over the upper part of the circuit to the listener at the east end of the circuit as a listener echo. Successive talker and listener echoes follow this, as indicated in the diagram. If the networks at the two ends of the circuit can be made to simulate accurately the subscriber circuits, none of these echoes will exist. A high degree of simulation, however, is impracticable in an economical telephone plant under usual conditions. In two-wire circuits with many repeaters the echo paths may become very complicated.

An interesting case of "echoes" is that which may be produced when two radio stations are sending out the same program at the same wavelength. The program as received by one of these stations over wire circuits is, of course, slightly delayed with respect to a station nearer the source of the program. It is possible then for a receiving set properly located to receive both of these stations, in which case if the time difference is sufficient one of them will sound like an echo of the other. In this case there may be the added peculiarity that if the weaker station is the one at which the program is received first it will appear that the "echo" is in advance of rather than following the sound which appears to cause it.

TIME EFFECTS WITH VOICE-OPERATED DEVICES

Switching devices operated by the voice currents themselves are frequently introduced into long telephone circuits. In general, the effect of such devices is to render inoperative transmission in the direction opposite to that of the speech waves which are going over a circuit at the particular instant. The first use of any considerable importance to which such devices were put was in connection with long circuits for the purpose of preventing the building-up of undesirable echoes. More recently, however, long radiotelephone circuits have come into use. These circuits may vary rapidly in transmission effectiveness. If these circuits are arranged to be operative in both directions at a time it would be very difficult to prevent their becoming unstable and possibly setting up oscillations. For this reason such circuits are frequently operated with switching arrangements such that the circuits leading to both transmitting stations are normally disabled and rendered inoperative. When a subscriber speaks at either end, therefore, the voice currents must operate switching devices which restore the circuits leading to his transmitting station. Incidentally, this must render inoperative the receiving circuit at the same time.

A very interesting application of time delay has been made in connection with radio systems so operated. In this arrangement the voice currents when they reach the disabling point are passed through an artificial line in which a desired amount of delay is incorporated. Just before entering this line a fraction of the energy is taken, rectified, and made to operate the switching mechanism for restoring the circuit to operating condition. This switching is so arranged as to be completed by the time that the voice currents have passed through the artificial delay circuit and are ready to proceed down the line. If it were not for this arrangement a small part of the speech currents might be dissipated during the interval while the switching mechanisms were operating.

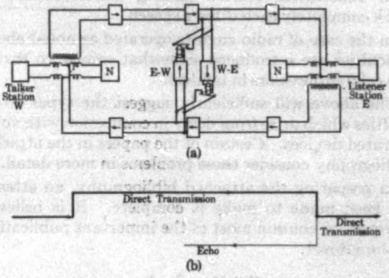

FIG. 7—FOUR-WIRE CIRCUIT WITH ECHO SUPPRESSOR

Fig. 7 is similar to Fig. 6 noted above with the exception that the application is shown of an echo suppressor.

When the subscriber at the left of the drawing begins to talk, the waves set up in his telephone are transmitted over the upper part of the circuit. Upon reaching the input of the echo suppressor, a small part of the energy is diverted to short circuit the lower branch of the circuit, as indicated. Meanwhile, the main transmission passes on to the subscriber at the right. Echoes which return in the lower part of the circuit are blocked as indicated. After the talker has ceased speaking, the device remains operative for a time equal to the delay of the echo as measured from the input of the device to the disabling point plus an additional time to take care of echoes in the circuit between the four-wire terminal and the subscriber.

When the suppressor releases, the circuit is again free to transmit in either direction. When the right-hand subscriber talks, the action is similar except that the other half of the echo suppressor operates.

In practical use the echo suppressors are so carefully controlled that telephone users are generally unable to tell whether a suppressor is on the circuit or not. This is due to the short delays and careful adjustments of the time function of the device. It is possible, however, if the delays are longer and the adjustments are made with less care, to introduce two types of difficulty. If one subscriber talks fairly steadily he may hold the suppressor operative so continuously that it is difficult for the other party to break in on the conversation if he so desires. If one subscriber started to reply almost simultaneously with the termination of the other's speech, part of his reply might be blocked at the echo suppressor along with the last of the echo.

Further difficulties arise if two circuits, each containing an echo suppressor of this kind, are switched together in tandem. In this case it would be possible if both subscribers started talking simultaneously to block completely each other's speech.

In the case of radio circuits operated as noted above, difficulties are introduced somewhat similar to that of two echo suppressors in tandem.

The above will sufficiently suggest the types of difficulties which arise from delay in connection with voice-operated devices. Certain of the papers in the attached bibliography consider these problems in more detail.

In preparing the attached bibliography, no attempt has been made to make it complete. It is believed, however, to contain most of the important publications on the subject.

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Discussion

A. F. Rose: Referring to the table on page 142, it would appear that the speed of propagation of radio circuits is so high that echo effects would not be important as compared with the usual type of long distance cable circuit. For example, the radio portion of the New York-London circuit which is approximately 3,400 miles in length would require 0.018 second for transmission. This time is so short that the chances for objectionable effects, such as lockout, would be small. However, there is some additional delay in the wire line and as discussed in previous articles on the New York-London circuit, it is necessary to add delay artifically to the circuit in order to permit time for the satisfactory operation of voice actuated devices. The additional delay of these circuits and the wire lines increases the one way transmission time to about 0.09 second in the long wave, and 0.05 second in the short-wave circuits. Voice-operated relays which are necessary to preserve stability, also make the echo characteristics of these circuits satisfactory from the standpoint of overall service results. Some of the above delays account for occasional lockouts. Effects which may be mistaken for lockouts sometimes occur due to relays becoming operated by high noise.

As a matter of interest the approximate delays in various parts of the transatlantic circuits are summarized below.

Wire lines	Seconds
New York-Rocky Point (70 mi. H-44)	0.004
Cupar-London (425 mi. H-89)	0.032
London-Rugby (85 mi. H-89)	0.006
Houlton-Bangor (120-mi. N.L.O.W.)	0.001
Bangor-New York (472 mi. H-44)	0.025
New York-Lawrenceville (50 mi. H-44)	0.003
Netcong-New York (50 mi. H-44)	0.003
Delay circuits	
Transmitting	0.021
Receiving	0.004
Filters, etc., about	0.005

Impulse Voltage Tests on a 4,800-Volt
Distribution Substation

BY H. W. COLLINS,* E. E. PIEPHO,* and J. J. TOROK†
Associate, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—Lightning voltage flashovers have been experienced in substations feeding a 4,800-volt ungrounded distribution system. Lines enter the substations through three-phase, underground cables varying in length from a few hundred to about nine hundred feet. Lightning arresters are connected at the substation bus and at the junctions of overhead lines and entrance cables. The arcovers have consistently occurred on lines equipped with voltage regulators and at points between the line terminal of the regulator and the station end of the entrance cable.*

Impulse voltage tests were made on one of the substations which had been taken out of commission, to determine the cause of the flashovers and to find a remedy. Surge voltages from a lightning generator were sent into the station through the entrance cable of a circuit equipped with a voltage regulator and volt-time oscillograms were taken at the principal points of interest.

Oscillograms are presented which show that the impedances of the regulator series windings to surges were high enough to cause reflections of the incident wave and a consequent building up of voltage at the station end of the entrance cable.

It is also shown that the energy drawn from the cable through the regulator series windings is so small for the early part of the wave that the lightning arresters on the bus cannot protect the cable against high-voltage surges which may enter the substation.

Other oscillograms presented indicate that partial protection is provided by the arresters located at the overhead line end of the entrance cable. The effectiveness of this protection becomes greater as the entrance cable is shortened, and as the wave front of the incoming surge is made longer.

It is shown that voltage rises sufficient to break down the entrance cable or cause flashovers at the station end may be prevented by shunting low-voltage arresters across the regulator series winding, or by connecting the usual distribution arresters at the station pothead of the cable. The latter method is more desirable because it insures a lower voltage at the substation end of the cable.

DURING the 1928 lightning season The Detroit Edison Company experienced in five substations a total of six arcovers which were caused by lightning voltages entering the stations by way of a 4,800-volt, ungrounded distribution system. The overhead lines of this system enter the substations through three-conductor, underground cables, and lightning arresters are located at each junction of line to cable and at the substation bus. Some of the distribution lines are equipped with three-phase voltage regulators in the substations. These regulators are Y-connected, but the neutral is not brought out and is insulated from ground. Fig. 1 shows the layout of a typical distribution line with a regulator.

Besides being unusually large in number for a single season, the arcovers were of unexpected severity. In each case the lightning voltage causing flashover entered the substation by way of a distribution line on which there was a voltage regulator. Furthermore, the flashovers occurred at some point between the substation pothead and the "line" side regulator terminals, and their nature indicated that they were caused by voltages in excess of the breakdown voltages of the cable pole and bus lightning arresters which inspection showed to be in good operating condition. This consistency in the location of the flashovers indicated that the impedance of the regulator series winding to surges was probably great enough to cause reflections of surges entering the substation, and so result in the building up of a voltage at the regulator "line" terminals high enough to cause flashovers. Tests were conducted jointly by the Westinghouse Electric and Manufacturing Company and The Detroit Edison Company in an effort to determine the cause of the flashovers and find a remedy. These tests were made in Detroit on Elmdale Substation which had been taken out of commission and was being dismantled.

DESCRIPTION OF EQUIPMENT

Elmdale Substation. This 4,800-volt distribution substation was of the open pipe-frame construction, and the bus copper and all leads were insulated with varnished cambric. One distribution line with voltage regulator was left in the substation, and its 745-ft. entrance cable was looped back so that both ends were in the station. This cable was a three-conductor, 200,000 cir. mils, lead-covered cable insulated with

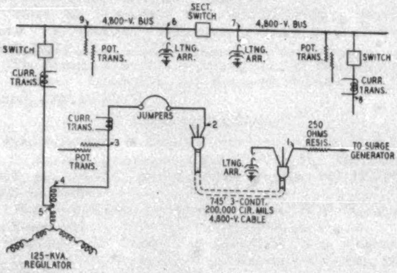

FIG. 1—TYPICAL LAYOUT OF A CIRCUIT IN ELMSDALE SUBSTATION WITH SPECIAL REFERENCE TO THE APPARATUS TESTED WITH SURGE VOLTAGES

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*Detroit Edison Company, Detroit, Michigan.

†Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the South West District Meeting of the A. I. E. E., Kansas City, Mo., October 22-24, 1931.

oil-impregnated paper 5/32 in. thick on the conductors with a 5/32 in. overall belt. A dead, 24,000-volt, three-conductor cable line 8,900 ft. long terminated in the station and was available during the tests.

Lightning Generator. The impulse voltages used during the various tests were obtained from the Westinghouse Electric and Manufacturing Company's 1,000,000-volt lightning generator and auxiliary equipment.[1] Since only 180,000 volts were required in the tests, the generator was reconnected to deliver this voltage with a capacity of 0.075 μf.

Cathode-Ray Oscillographs. Two portable cathode-ray oscillographs were used during the tests, one being an improved Norinder oscillograph,[2] and the other a George oscillograph of the hot cathode type.[3] A concentrating coil was added to the latter to assist in focusing the beam. The apparatus in the measuring system included a resistance potentiometer and a delay cable.

CONNECTIONS OF EQUIPMENT

The substation apparatus tested is shown in Fig. 1. The pothead position (1) on this diagram, was taken to represent the cable pole pothead of the entrance cable. It was to this pothead that the surge generator was connected by a resistance, and through it the voltage surges entered the station. It will be noted that the three phases were joined electrically by potential transformers which were connected open delta, and by the voltage regulator which was *Y*-connected with neutral ungrounded. Each phase had a lightning arrester at the cable pole pothead (1) and at a point on each bus section (6) and (7); and various combinations of these arresters were tried during the tests. The arresters were rated 6,000 volts for use on an ungrounded system. Suitable lightning arresters were also available for connection at the substation pothead, and for shunting the regulator series winding.

DETERMINATION OF RESISTANCE TO SIMULATE LINE CONDITIONS

An analysis of a traveling surge entering the cable from the overhead line shows that, due to the lower surge impedance of the cable, the wave on the line will be reflected with reduced magnitude and reversed polarity. The voltage of the surge entering the cable will be less and the current greater than in the original line surge. The following equations show these effects:

$$e = \frac{2\,Z_c}{Z_c \text{ plus } Z_L}\,E_L$$

$$i = \frac{e}{Z_c} \text{ where}$$

e = voltage of surge in cable
Z_c = surge impedance of cable
Z_L = surge impedance of line
E_L = voltage of surge on line
i = surge current in cable

If the surge generator should be discharged directly into the cable, the current drawn by the arresters would be excessive due to successive reflections of current. Since it was not possible to apply the surge to the cable by means of an overhead line, a resistance was placed in series with the surge generator to limit the current in the cable to a value which would approximate that of a surge entering from an overhead line. The following equation gives the surge in the cable in terms of the surge generator voltage, *E*, and the series resistance, *R*.

FIG. 2

$$e = \frac{Z_c}{Z_c \text{ plus } R}\,E$$

In order to simulate a surge entering from an overhead line

$$\frac{Z_c}{Z_c \text{ plus } R}\,E = \frac{2\,Z_c}{Z_c \text{ plus } Z_L}\,E_L$$

$$R = \frac{Z_L \text{ plus } Z_c}{2} - Z_c$$

Substituting the usual value of Z_L (500 ohms) and Z_c (40 ohms) in the above equation gives 230 ohms for *R*. In the tests a series resistance of about 250 ohms was used.

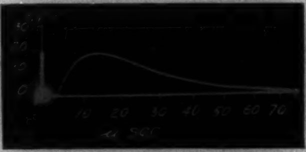

FIG. 3

TEST PROCEDURE

Two general types of impulse voltage waves were used in the tests. One wave, shown in Fig. 2 had a voltage rise from zero to maximum in approximately one microsecond at its front. The other wave, Fig. 3 which was secured by connecting 1,360 millihenrys inductance in series with the surge generator, had a wave front with voltage rise to maximum in about

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1. For references see Bibliography.

ten microseconds. These two different surges were used in the tests to determine generally what effect the steepness of wave front would have upon the reflection of the impulse at the voltage regulator. A very steep front wave, as nearly rectangular as possible, was required to show reflections clearly on the oscillograms. Tests with surges having slower wave fronts more nearly like natural lightning were also desirable in order to gain information more nearly representative of actual lightning conditions.

Various combinations of the lightning arresters shown in Fig. 1 were tried during the tests, as well as other combinations made by adding arresters at the station pothead (2) and by shunting low-voltage arresters across the regulator series winding.

Tests were first made with resistances of various magnitudes connected to the bus to simulate the effects of other circuits connected to the bus. Oscillograms showed that the surge voltage appearing on the bus decreased as the connected resistances were decreased, but that the shapes of the impulse voltage waves impressed at the cable-pole pothead (1) and the station pothead (2) were unaffected. Most of the tests were therefore made with the station bus open-circuited.

Fig. 4

When the two phases which were not connected to the surge generator were grounded at the cable-pole pothead (1) to simulate connection to an overhead line, the oscillograms did not differ from those taken with these phases open-circuited. Consequently, most of the tests were made with the latter arrangement.

In other tests, surges were applied at position (8) Fig. 1, and oscillograms were taken at cardinal points, but the bus lightning arresters limited the impulse voltages to a safe value at all points. Consequently, no further tests of this type were attempted.

During the early part of the testing, volt-time oscillograms were taken at positions (1), (2), (3), (5), (6) and (8), but the effects of the instrument transformers were so slight that the oscillograms taken at positions (2) and (3) were identical, and those taken at positions (5), (6) and (8) were alike. It was therefore decided to make measurements only at positions (1), (2) and (5). The oscillograms taken at these three points are quite different because of the cable-pole lightning arresters, the cable itself and the voltage regulator.

Since all the surges were sent through only one phase, the voltages induced on the other phases are of interest. These two phases have magnetic and capacity coupling with the surged phase, to which they are also connected through the regulator and the potential transformers. These windings, however, have high impedances and it is chiefly by their capacity effect that any surge energy is transmitted through them. Oscillograms were at first taken on the two idle phases concurrently with those taken on the phase connected to the surge generator, but when it became evident that the voltages induced on them were not serious in magnitude (usually about one-fourth the voltage on the surged phase) further measurements on them were omitted.

Fig. 5

Test Results

Effect of the Regular Series Winding. The impulse voltage tests indicated definitely that the energy drawn from the cable through the series winding of the regulator by the lightning arresters and other equipment connected to the bus was very small, especially for the first part of the wave. The high impedance of the regulator series winding is well shown by comparison of the oscillograms taken at the cable-pole pothead (Fig. 4), at the station pothead (Fig. 5) and at the bus (Fig. 6). Fig. 4 shows a surge of about 25-kv. crest entering the cable, and Fig. 5 shows that this wave was increased to a crest value of about 43 kv. by the partial reflection at the voltage regulator. The positive, reflected wave returned to the cable-pole end where it caused sufficient voltage rise to break down the lightning arresters located there. These arresters prevented further voltage rise and returned a negative, reflected wave to the station end of the cable, which reduced the voltage at that point to about 23 kv. Fig. 6 shows

Fig. 6

how much slower the rise of surge voltage is on the bus than at the station pothead, due to the high impedance of the regulator series winding.

Using the slow front wave, oscillograms were taken at the cable-pole pothead (Fig. 7), at the station pothead (Fig. 8) and at the bus (Fig. 9). The reflections at the regulator do not show as distinctly as for the steep front wave since the entering wave does not reach its maximum at the cable-pole pothead before the reflection is beginning to return from the regulator and is adding to the voltage of the entering wave. If the entrance cable had been long enough so that it would have taken an impulse about 10 microseconds to travel forth and back through it, then the reflections would have been just as distinct as for the one microsecond front surge. Comparison of Fig. 7 and Fig. 8 will show that the surge voltage at the regulator rises to a greater maximum value than at the cable-pole pothead. Consequently, although the magnitude of the reflections at the regulator is obscured because the reflections occur on the front of the wave, it is evident that they are appreciable. The slower rise of voltage on the bus shown in Fig. 9 also indicates that the regulator series winding offers considerable impedance even to the slow front wave which was used.

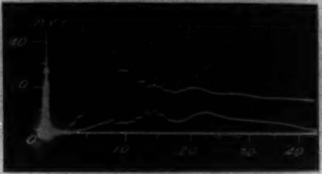

Fig. 7

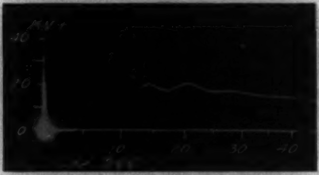

Fig. 8

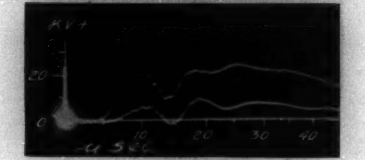

Fig. 9

The slow passage of energy through the regulator series winding is further illustrated by the oscillograms taken at the station pothead (Fig. 10) and at the bus (Fig. 11) with the dead 24,000-volt cable line connected to the bus.

In order to determine if the lightning arresters on the bus provided protection for the cable, the lightning arresters at the cable pole were disconnected. When a voltage surge was discharged at the cable-pole pothead into the cable for this condition, the regulator impedance remained very high for a long enough period to cause a flashover at the station pothead. This is shown in the oscillogram Fig. 12, taken at the station pothead for practically the same impulse voltage as was used when Fig. 5 was taken. It is therefore evident that the regulator series winding makes the lightning arresters on the bus ineffectual in preventing voltage

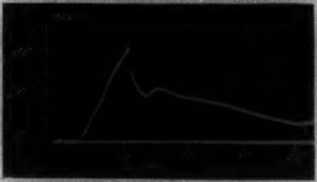

Fig. 10

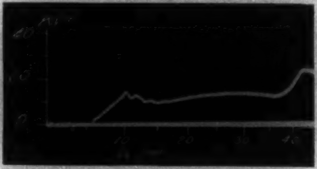

Fig. 11

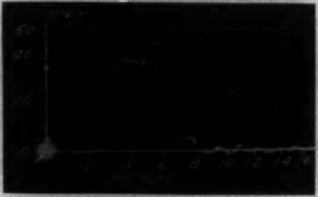

Fig. 12

rise in the cable sufficient to cause breakdown of the cable or flashover at some point between the station pothead and regulator when high-impulse voltages enter the station through the cable.

Action of the Lightning Arresters at the Cable-Pole End of the Entrance Cable. The surge-generator voltage was of such magnitude that the crest value of the impulse entering the cable was less than the breakdown voltage of the lightning arresters at the cable-pole end of the entrance cable. However, when the positive, reflected wave arrived from the substation end of the cable, the voltage on these lightning arresters was suddenly increased and the arresters operated to lower the voltage. This is shown in Fig. 4 and Fig. 5. Operation of the arresters causes a negative reflection of the impulse sent back from the regulator. When this negative reflection arrives at the regulator it causes a sudden drop of potential at that point.

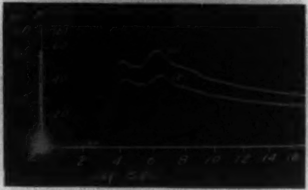

Fig. 13

The time elapsing before the cable-pole arresters relieve the condition of overvoltage at the station end of the cable depends upon the length of the cable, since it is equal to the time required for the reflected wave to travel from the regulator to the cable-pole pothead and back again. This time is about two microseconds for the average length of entrance cable. For such an average case if the wave front of the impulse is greater than two microseconds, crest value will not reach the regulator before the negative reflection from the cable-pole lightning arrester arrives and the voltage at the regulator will not build up as high as for a steep front wave.

Effect of Low-Voltage Lightning Arresters Shunting the Regulator Series Winding. The voltage at the station pothead may be limited to a safe value by shunting low-voltage arresters across it, thus permitting the bus arresters to help lower the voltage at this point. The oscillogram of Fig. 12 was taken at the station pothead with arresters on the bus only, and the flashover shows the lack of protection. When the regulator series windings were shunted by low-voltage arresters, flashover was prevented by the operation of these arresters in series with the bus arresters. This is shown in oscillogram Fig. 13, curve (*a*) having been taken on the bus, and curve (*b*) at the station pothead.

Effect of Lightning Arresters at the Substation End of the Entrance Cable. When lightning arresters were connected at the station pothead, the voltage rise was definitely limited there to the breakdown voltage of the arresters. The flashover in Fig. 12 resulted at the station pothead when the cable-pole arresters were disconnected. This flashover was prevented, as shown in Fig. 14, by connecting lightning arresters at the station pothead. Furthermore, by comparison of curve *b*, Fig. 13 which was taken at the station pothead with low-voltage arresters shunting the regulator series windings, with Fig. 14 it is evident that lightning arresters connected at the station pothead limit the voltage there to a somewhat lower value.

Conclusions

The impulse voltage tests performed on the substation described indicate that:

1. When the surge reaches the regulator, it is reflected by an amount depending upon the surge impedance of the regulator, which in turn depends upon the rate of voltage rise at the wave front and upon the electrical characteristics of the regulator. This reflection adds to the oncoming wave and may cause a rise of voltage at the substation pothead to a value somewhere between 100 per cent and 200 per cent of the wave traveling toward the regulator. The magnitude of this voltage and interval of time it exists at the regulator will depend upon the steepness of the wave front and upon the length of the entrance cable.

2. Because the surge impedances of the regulator series windings remain at high values long enough for a flashover to occur, the bus lightning arresters cannot protect the cable against voltage surges which may enter the substation through it. The lightning arresters at the cable-pole end of the entrance cable limit the voltage there, but they can limit only partially the rise of voltage at the substation end of the cable. If the front of the incident wave is short and the entrance cable is long, approximately double the voltage at the cable-pole end will be impressed at the station end for an appreciable time before the cable-pole arresters relieve this condition.

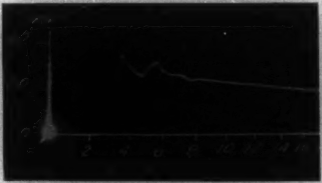

Fig. 14

3. Low-voltage lightning arresters shunted across the regulator series windings operate in series with the bus arresters and limit the rise of voltage on the line side of the regulator to the sum of the breakdown voltages of these two sets of arresters.

4. Lightning arresters at the substation end of the cable limit the voltage on the pothead and the regulator; and, acting in combination with the cable pole and the bus arresters, they will limit the voltage in the cable to the drops across the arresters.

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3. *A New Type of Hot Cathode Oscillograph and Its Application to the Automatic Recording of Lightning and Switching Surges,* by R. H. George, A. I. E. E. TRANS., Vol. 48, July 1929, p. 884.

4. *Lightning Voltages on Transmission Lines,* by R. H. George and J. R. Eaton, A. I. E. E. TRANS., July 1930, p. 877.

Discussion

K. B. McEachron: It is interesting to note how the results obtained by Messrs. Collins, Piepho and Torok check those to be expected, based on theoretical considerations and field tests* which have already been published. Late in the summer of 1929 field tests were made jointly with the Consumers Power Company and the Detroit Edison Company in which various combinations of overhead line and cable were studied, with and without lightning arresters. The last conclusion of that investigation agrees closely with the findings of the present authors. It reads: "The increase in voltage at the other end of the cable from the arrester location was about 15 kv. This increase depends upon the steepness of the wave and the length of the cable and may be estimated for any specific case."

With the voltage limited by arresters at the junction of the overhead line and cable, the voltage at the station end will be dependent on how the reflections add at that end of the cable. If connected to a high impedance as the series coil of a regulator or a reactor the voltage may become double that at the arrester if the wave front of the wave in the cable is steep enough so that a wave of reduction traveling back from the arrester fails to reach the station end of the cable in time to reduce its voltage.

The natural conclusion is therefore to apply an arrester at the station end of the cable or at the cable end of the regulator if a considerable distance exists between the end of the cable and the terminals of the regulator. This arrester is applied largely for the purpose of protecting the regulator and other apparatus between the regulator and cable.

The arrester at the junction of the overhead line and cable is for the purpose of protecting the pothead and also preventing interruption to service as the pothead frequency represents the weakest point in the line insulation. The arrester ground should be connected to the cable sheath for the best results as this eliminates the effect of arrester ground resistance.

On page 150 the authors state that "Tests with surges having slower wave fronts more nearly like natural lightning were also desirable" etc. It seems to me that to be consistent with the studies on high-voltage circuits that waves much faster than those used by the authors are likely to occur. Since the slower waves are less dangerous because the reflections at the station end of the cable are not important it seems that the steepest waves which can occur are the most important. Rates of rise as fast as 1,000 kv. per microsecond have been measured on transmission lines and such a rate of rise would correspond to 20 kv. in 0.02 microseconds which is much faster than any wave shown in the paper. Such short times are difficult from the oscillograph standpoint but their reflections can be calculated. Such a wave would mean practically complete reflection at the station end of the cable if it terminated in a high surge impedance. Since the fast front used by the authors was sufficient to show the reflection, but little would be gained by faster fronts in view of the technical difficulties involved in providing and measuring them. However, it appears very likely that they do occur in practise and must be considered.

D. W. Roper submitted a discussion calling attention to the fact that the Chicago substations were entirely free from the troubles which caused the investigation in Detroit. This discussion was withdrawn because Mr. Roper's paper on *Studies in Lightning Protection on 4,000-Volt Circuits—III,* presented at the Winter Convention, 1932, and printed elsewhere in this issue, explains the immunity of the Chicago stations from such troubles and refers to the paper by Messrs. Collins, Piepho and Torok.

**Study of Effect of Short Lengths of Cable on Traveling Waves,* K. B. McEachron, J. G. Hemstreet and H. P. Seelye, A.I.E.E. TRANS., Vol. 49, 1930, p. 1433.

Corona Energy Loss

Influence of Surface Conditions as Affecting Corona Energy Loss From High-Voltage Transmission Lines

BY WM. D. WEIDLEIN*
Associate, A. I. E. E.

***Synopsis.**—Coatings of various materials, lacquers, oils, oxides or paints were applied to conductors of number 2 B & S gage 257.3 mils diameter, solid aluminum or copper conductors. Measurements of corona energy losses above and below critical corona voltages were made by means of a wattmeter setting and water resistor multiplier.*

INTRODUCTION

DURING the spring of 1930, corona loss measurements on a 220-kv., 60-cycle, three-phase experimental line, 700 ft. in length and of various sizes (0.91 in. to 1 in. in diameter) and types of conductors were made at the Ryan Laboratory, Stanford University. In connection with these measurements some of the test specimens were cleaned with gasoline and then washed, resulting in at least a temporary reduction of corona loss. The question of a permanent reduction of corona loss and of the influence of surface conditions presented itself.

The following report is on research as suggested by this precise work which was undertaken to determine what variation of corona loss results under various conditions of the surface of a conductor.

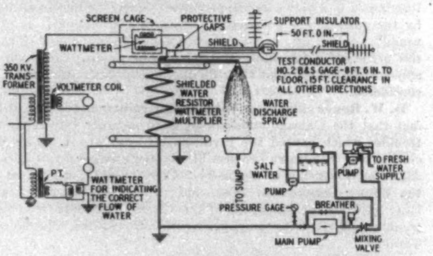
SCREEN CAGE
PROTECTIVE GAPS
SUPPORT INSULATOR
50 FT. 0 IN.
WATTMETER
SHIELD
SHIELD
350 KV. TRANSFORMER
TEST CONDUCTOR NO. 2 B&S GAGE—8 FT. 6 IN. TO FLOOR, 15 FT. CLEARANCE IN ALL OTHER DIRECTIONS
VOLTMETER COIL
SHIELDED WATER RESISTOR WATTMETER MULTIPLIER
WATER DISCHARGE SPRAY
TO FRESH WATER SUPPLY
SALT WATER
PUMP
P.T.
TO SUMP
PUMP
WATTMETER FOR INDICATING THE CORRECT FLOW OF WATER
PRESSURE GAGE
BREATHER
MAIN PUMP
MIXING VALVE

FIG. 1—DIAGRAM OF COMPLETE SET UP OF TESTING EQUIPMENT

EQUIPMENT

The complete set of the test specimens and of the wattmeter setting as diagramatically shown in Fig. 1 is similar to the set-up of a single-phase wattmeter during the tests as noted in the introduction.

The test specimen consisted of 50-ft. lengths of No. 2 B & S gage solid copper or aluminum conductors, supported with 8 ft. 6 in. clearance to ground. The sag at the center of wire was less than 2 in. in all cases. Clearance in all other directions was greater than 15 ft. As to diameters of specimens, when coated, increase in size by 0.001 in. was the maximum, which would produce but very little variation of corona losses.

Insulator loss on the out end is included in all results but is of negligible amount; and for comparison of results this insulator loss does not enter, being present in all tests. Insulator loss of wattmeter end of specimen is completely shielded from wattmeter reading.

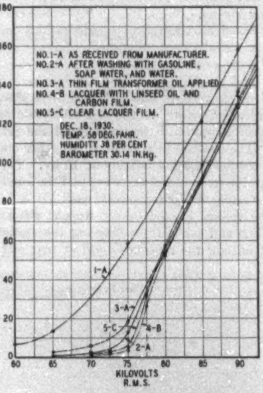

FIG. 2—TESTS ON COPPER SPECIMENS

All tests were made within the laboratory and under as near similar conditions of temperature, humidity, and barometric pressure as possible. Variation of these factors during tests would in no case cause errors that would be appreciable. All results were plotted as read, no corrections being made for such variations. The extreme range of barometric variation was 0.16 in. Hg.

TESTS

The procedure followed in applying voltage was to build up the voltage to 100 kv., reading the loss at this

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*Engineer, Black & Veatch, Kansas City, Mo.

Presented at the South West District Meeting, Kansas City, Mo., Oct. 22-24, 1931.

point and then at lower values of potential, with 5-kv. or 2.5-kv. intervals, depending upon the rate of change in loss.

The accompanying illustrations are presented as showing the variation of corona loss with change in surface structure of conductor or application of thin films.

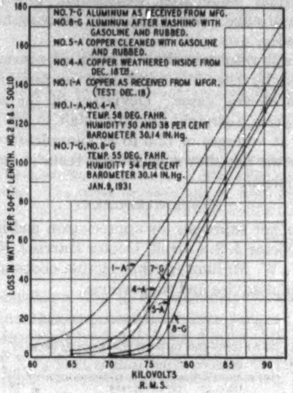

FIG. 3—COMPARISON OF ALUMINUM AND COPPER LOSSES

Results of tests on copper specimens are given in Fig. 2. The shape and shift of the curves between values of 72.5 to 80 kv. should be noted. Joffe's theory that the resistance of a polarized layer decreases when the e. m. f., produced by polarization due to space charge exceeds 2,000 volts, is a very plausible explanation of this action. The high loss from copper conductor as received from the manufacturer is no doubt due to residue left from "pickling" process or from drawing during manufacturing.

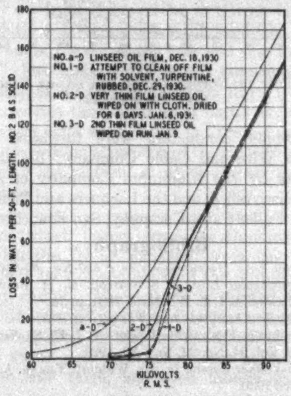

FIG. 4—ILLUSTRATING THE EFFECT OF THICK FILM OF LINSEED OIL

A comparison of losses from aluminum and copper conductors is shown in Fig. 3.

In Fig. 4 the first run No. a-D was of a conductor coated with a thick film of linseed oil, brushed on and dried. This wire was then cleaned and a cloth soaked with linseed oil was wiped once along the wire. A complete film was not formed, accounting for the weathering and higher losses after drying. Curve 3-D gives the loss after wiping a second film onto the wire, resulting in a decrease of losses near 75 kv. applied. Apparently there may be some justification for the fact that with complete covering with a thin film ionizing velocities of electrons cannot be attained within the film, having insufficient ionizing potential fall; then space charge increases to a point where leakage resistance of the space charge starts to decrease, permitting of greater potential fall throughout the adjacent air causing a sharper rise in corona loss.

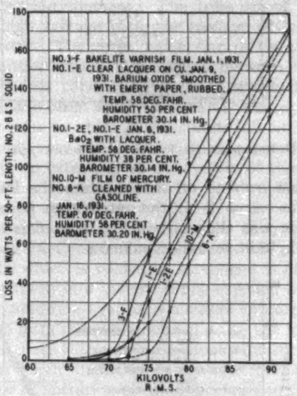

FIG. 5—ILLUSTRATING THE EFFECT OF VARIOUS THIN FILMS

The curves in Fig. 5 illustrate the effect of various thin coatings of materials. The same type of film was applied to copper and aluminum as shown in Fig. 6.

In Fig. 7 copper and aluminum conductors, previously cleaned and then suspended open to weather, show equal losses over the range tested.

Curve 3-F of Fig. 8 is of a test on a copper conductor coated with a smooth surface of copper oxide formed by heating. With corona forming around this wire at 21 kv., there were no steamers formed over the range of voltage tested up to 150 kv., r. m. s. The corona formation appears similar to that forming around a small wire. With this type of discharge there was very little if any ozone smell detected, which was a decided contrast to a discharge with steamers.

Curves 1-N, 2-N and 3-N show the losses from a conductor heated in acetylene gas. On removal from the gas a granular deposit of carbon covered the conductor, and the losses were as plotted curve 1-N. With this carbon rubbed off with a cloth the loss was as plotted in curve 2-N; the interesting point in this connection was that carbon had been deposited to a depth of 1/16 in. into the conductor.

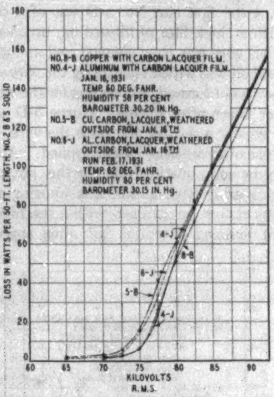

FIG. 6—COMPARISON OF FILMS ON COPPER AND ALUMINUM

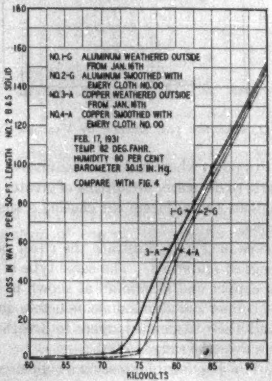

FIG. 7—CLEAN ALUMINUM AND COPPER CONDUCTORS SHOW EQUAL LOSSES IN OPEN AIR

CONCLUSION

Thin film coatings of certain materials are effective in decreasing corona loss from high-voltage conductors, whereas thicker films will increase such loss. Such a film may be the polished surface of a metal itself, or the formation of a film by materials that flow readily and solidify with a glaze or a crystalline surface.

Corona losses from copper and aluminum conductors are of equal value after weathering, providing any detrimental effects remaining from drawing have been eliminated.

Favorable results in the reduction of corona loss are illustrated by the shape and shift in position of the curves in several cases. Deviation of the curves to the right from a straight line from high to low values of loss and at voltages of 72.5 to 80 kilovolts for this size of conductor are shown in several tests in support of this conclusion.

ACKNOWLEDGMENT

The author expresses his appreciation and indebtedness to Doctor J. S. Carroll, Doctor Harris J. Ryan and Doctor J. W. McBain for their active cooperation and very helpful advice and suggestions.

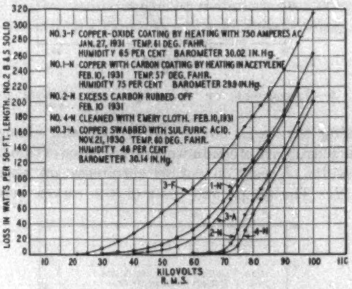

FIG. 8—TEST OF COPPER CONDUCTOR UNDER VARIOUS SURFACE TREATMENTS

The copper conductors were furnished by the Anaconda Wire and Cable Company; the aluminum conductors by the Aluminum Company of America, San Francisco office; and the Lacquers and Pigments by the J. W. Fuller Paint Company, San Francisco.

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The Expulsion Fuse

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***Synopsis.**—The general theory of extinction of a-c. arcs is reviewed. The expulsion fuse is stated to depend upon the gas blast produced by rapid decomposition of fuse tube material under the heat of the arc. Comparison of interrupting capacities of a soapstone fuse tube and a fiber fuse tube supports this view.*

The effectiveness of the gas blast is stated to be due to the high degree of turbulence it introduces into the confined arc space. The theory of the action of such turbulence is discussed.

Data are given as to the composition and volume of gas ejected from a fiber fuse tube.

The voltage interrupting capacity vs. ampere characteristics as obtained experimentally, are given for a fiber tube, and a boric acid lined tube. The curves for boric acid lie much higher than for fiber. The characteristics were found to depen dvery materially on the number of half cycles of arcing. Reasons are given for the shape of the characteristics, and their dependence on length of arcing time.

Data are given showing the effect of variation in size and shape of the tube section. The design problem is discussed of obtaining sufficiently large voltage interrupting capacity at smaller currents without the development of excessive pressures or unmanageably large volumes of flame at large currents.

The use of flame suppressors for deionizing the issuing flame is mentioned. It is shown how, by introducing means for condensing the water vapor, the boric acid lined tube may be completely closed without the development of excessive pressures.

The necessary insulating properties of the gas generating materials used in expulsion fuses are discussed.

I. INTRODUCTION

THE expulsion fuse is an extremely simple device quite extensively used for interrupting a-c. circuits of moderate voltage and power. It consists usually of a straight tube open at one or both ends, made of fiber in which the fusible current carrying element rests. In spite of its apparent simplicity and widespread use, the principles upon which it functions are not at all generally understood. Engineers seem to have been satisfied with extremely loose and inadequate descriptions of the process by which, in it, the arc is caused to be extinguished. For example, a text book on circuit interrupting devices gives as the *modus operandi* "the sudden expansion of the air inside the tube when the fuse blows causes the vaporized metal to be blown out of the end of the tube and thus suppresses the arc." But metal vapor is not necessary for the maintenance of an arc, as engineers learn when long arcs short-circuit their lines upon flashover, and it is difficult to see what would cause the precise synchronization of the sudden expansion of the air with a current zero which is the moment at which the arc is actually extinguished.

The very name "expulsion fuse" itself is a loose description which fosters erroneous thinking in suggesting that the arc is "expelled" from the tube, to perish due to lack of contact with the terminals of the sustaining circuit. Much flame is ejected from the tube prior to current zero, but this does not mean that the arc is expelled for the arc continues to reside in the tube up to that point since large current is continuing to flow there, and even though flame may continue to leave the tube after the last current zero, the arc is not expelled then because the arc no longer exists after that moment. The tube space is withstanding high voltage with the passage of only very small current. The actual extinction of the arc, that is, the transformation of the space between the electrodes from the conducting state, in which it carries considerable current with moderate voltage, to the insulating state in which it withstands full circuit voltage with only very small current, takes place at current zero within an extremely short interval of time, of the order of 10 microseconds. In so short a time, with even the most extreme assumption as to the magnitude of the velocities, the gases within the tube can move only a small fraction of a centimeter and the proportion of the tube which is voided or scavenged is negligible. In the expulsion fuse the arc is not expelled but exists in such a condition that at a moment of current zero the change from the conducting state to the insulating state takes place with extremely great rapidity.

It seems worth while then to consider more carefully the manner of operation of expulsion fuses so that their application may be better understood and their potentialities further developed.

II. EXTINCTION OF A-C. ARCS

It is well to begin with a very brief review of the general principles of the extinction of a-c. arcs. A gas space carrying an arc is conducting because it is ionized; that is, because a considerable number of the normally electrically neutralized molecules are broken up into electrically charged parts called ions. If the arc is to be extinguished, that is, if the gas is to recover its normal insulating state, these ions must disappear. If the ionized gas is left to itself, the ions disappear spontaneously by direct recombination. But this manner of disappearance is relatively so slow as to be of little use in practical a-c. circuit interruption, except for very small powers. In circuit interrupters generally, means are introduced for greatly speeding up the rate of disappearance of the ions.

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To keep a gas space conducting or ionized against the normal recombination or accelerated deionization as occurs in circuit interrupters, the continued activity of certain ionizing agents is necessary. In the electric arc, the combination of electric gradient along the arc and current density in the arc is a very effective producer of ions, and is the only ionizing agent of any consequence in arcs occurring in switching devices generally. While current is flowing, the arc will take such voltage as will cause ions to be produced at a sufficient rate to compensate for the deionization and keep the current going. In an a-c. arc at the instant of current zero, the ionizing activity momentarily ceases but the deionizing activity continues unabated. At such a moment the gas space is recovering insulating properties very rapidly. The voltage necessary to carry the arc into the next half cycle is then much larger than that required by the arc just before the closing of the preceding half cycle, but the external circuit is able to supply this larger voltage of reversed sign to the arc space only after a lapse of time depending on the transient characteristics of the circuit. For practical circuits this time lag is from 10 to 1,000 microseconds. In this time, the arc space may lose its ionization to such an extent that even full circuit voltage is unable to start the arc in the next half cycle. In this case the arc is extinguished. If, however, the deionization has not proceeded far enough, the arc is reignited. Thus extinction of the arc depends upon relative speed of deionization at current zero and the speed of application of reversed voltage to the terminals of the arc by the external circuit.[1]

A very effective means for increasing the rate of deionization is to bring surfaces of solids close to the arc so that ions may collect on these surfaces and recombine there.[2] Thus it is advantageous in circuit interrupters to enclose the arc in chambers or chutes of relatively small dimensions. It is found, however, that there is a great difference in the effect of these surfaces, depending on whether the surfaces are refractory, or whether under the heat of the arc, the surfaces are decomposed giving off large quantities of gas. In the latter case, under proper conditions much higher voltage and higher powered circuits can be interrupted with arcs of given dimensions than in the former case. This increased effectiveness of decomposing walls is believed to be due to the rapid evolution of gas; and the turbulence introduced into the confined arc by the rapid intermixing of freshly generated un-ionized gas is believed to be an extremely effective means for causing rapid deionization at current zero.[2]

III. THEORY OF GAS BLAST CIRCUIT INTERRUPTION DEVICE

For the purposes of this paper, it is enough to know that when an a-c. arc plays in a confined space subject to an intense gas blast, then at each current zero it is in a state where it recovers dielectric strength with exceedingly great rapidity. This may, if preferred, be accepted by the reader as an empirical fact without inquiring further into the details of how this comes about. It is interesting, however, to speculate as to what is the detailed mechanism by which the rapid deionization at current zero takes place.

The authors believe that most important in accounting for the effect of the gas blast is the high degree of turbulence it introduces into the bounded gas space prior to current zero. In fact, some experiments by T. E. Browne, not yet published, in which turbulent motion was produced by suitably disposed magnetic fields and without the introduction of fresh gas, showed a rate of recovery of dielectric strength at current zero exceeding many times that obtained without the turbulent motion. Furthermore, the turbulence prior to current zero is important as well as the turbulence during the extinction period itself. This latter is so short, 10 microseconds, that the amount of deionization due to the actual motion of the gas, carrying ions to the bounding surfaces to be discharged in that time, must be negligible in accounting for the dielectric strength recovered in the very short extinction period.

The great turbulence prior to current zero carries volumes of the gas to the bounding surfaces where they are deionized and thrown violently back into the arc space. The un-ionized, freshly generated gas at decomposing surfaces is also thrown violently into the arc path. Thus, the arc section cannot be continuous with a uniform distribution of ionization, but must consist of small volumes of highly ionized gas in close proximity to un-ionized or slightly ionized volumes of gas.

The arc may be thought of as broken up by the turbulence into a mass of fine threads swirling about in the tumultous motion. These threads are highly ionized and are carrying most of the current and are themselves the regions in which ions are being produced. During the motion threads become broken and cease carrying current, and new threads form as older threads split or as the impressed voltage breaks down new paths through momentarily favorable lining up of highly ionized volumes. Of course under these conditions the arc voltage is much higher than it would be if the turbulence did not exist, but the important thing is what goes on at current zero.

Prior to current zero the highly ionized threads are losing ions rapidly by diffusion into the surrounding un-ionized gas, but the current in the threads and the voltage gradient along them causes ions to be generated in them at as great a rate. At current zero the deionization by diffusion out of the threads continues undiminished, but the production of ions stops. Hence the highly ionized threads disappear almost at once and there is substituted for them a very low density of ionization nearly uniformly distributed over the arc space. A very low density ionization requires a high

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1. For references see Bibliography.

electric gradient for the production of new ions and thus is explained the rapid recovery by the gas space of the ability to withstand volts with the passage of little current.

IV. THE EXPULSION FUSE IS A GAS BLAST DEVICE

Regardless of the details of the theory, we may regard it as empirically established that the expulsion fuse is a gas blast interruption device in which the gas blast arises from the rapid decomposition of the walls of the fuse chamber under the heat of the arc.[2] This may be seen by comparing the arc voltage and the circuit voltage per inch interrupted by an a-c. arc in a tube of relatively refractory material and in a tube of material which is more readily decomposed into a gas. Such a comparison has been made before, and Figs. 1 and 2 give more complete information. Fig. 1 gives data for tests on the circuit voltage interrupting capacity of a 5/8-in. diameter soapstone tube. The numbers next to the points give the number of half cycles of arcing preceding the extinction of the arc. Their significance is discussed in more detail in section VI; they suggest here that the soapstone is not a completely gas-free material, but that under the influence of the arc, a small amount of gas, probably water vapor, is given off. In Fig. 2, the curve for soapstone is compared with that for fiber, this latter curve being taken from Fig. 6. Data and curves for the arc voltage for the soapstone and fiber tubes are also given in Fig. 2. The tests were made in laboratory circuits, air core reactors being used for limiting the current, when the transformers were not directly short-circuited, so that the circuits were highly reactive with a transition period of the order of 10 microseconds.

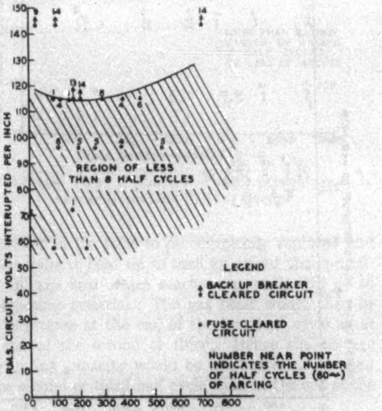

FIG. 1—SOAPSTONE FUSE TUBE 5/8 IN. INTERNAL DIAMETER BY 4 IN. LONG

It is noticed that the circuit voltage interrupting capacity and arc voltage for the fiber tube are much higher than for the relatively more refractory soapstone tube.

V. COMPOSITION AND VOLUME OF GAS IN EXPULSION FUSE

The material most commonly used for expulsion fuse tubes is horn fiber. Under the direction of the authors, Mr. H. H. Evinger collected the gases coming from a fiber tube in which an arc was playing, to determine their composition and volume. The composition was as follows:

	Test A	Test B
H_2	41%	48%
CO	50	50
Hydrocarbons	9	2

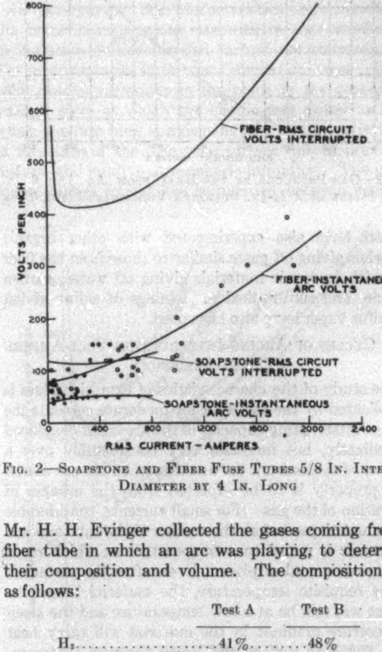

FIG. 2—SOAPSTONE AND FIBER FUSE TUBES 5/8 IN. INTERNAL DIAMETER BY 4 IN. LONG

The water vapor given off escaped collection because it condensed and was not drawn off. Since the fiber contains some 10 per cent of water by weight, it may be assumed that in addition to the above gases there was also about 10 per cent of water by weight.

The volume of gas generated as a function of current for a ¾-in. fiber tube is shown in Fig. 3. The volume is reduced to normal atmosphere conditions. While passing through the tube the volume of gas will be multiplied by 5 to 10 due to the elevated temperature and will be divided by a large factor due to the elevated pressure. Here again the water given off escaped detection because of its condensation.

Materials other than fiber may be used as gas generating materials in expulsion fuses. A material of great promise is boric acid, which may be pressed into solid shapes suitable for lining arc chambers of circuit interrupting devices.[3] The gas given off by boric acid under heat is practically entirely water vapor. The authors have also experimented with other organic materials giving off gases similar to those from the fiber and with inorganic materials giving off water, carbon dioxide, and sulfur dioxide. Linings of sulfur giving off sulfur vapor have also been used.

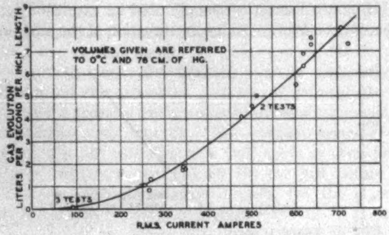

FIG. 3—GAS PRODUCED BY THE HEAT FROM AN ARC 6 IN. LONG INSIDE OF A ¾-IN. DIAMETER VULCANIZED-FIBER TUBE

VI. CYCLES OF ARCING-INTERRUPTION VOLT-AMPERE CHARACTERISTIC

The study of the characteristics of expulsion fuses is complicated by the fact that for moderate currents the voltage interrupting capacity of the arc is not developed immediately, but increases very considerably over a number of cycles from the moment when the arc starts. This property is to be expected from the manner of generation of the gas. For small currents, considerable time must elapse before the surface of the tube exposed to the arc is raised to a temperature at which rapid decomposition takes place. Even after the surface is at the requisite temperature, the material below the surface will still be at a low temperature and the steep temperature gradient in the material will carry heat away from the surface and lessen the rate of decomposition there. After a time the heat will soak into the material, the temperature gradient will be lessened, and the proportion of heat carried away by thermal conductivity will decrease, leaving a greater proportion of heat available for decomposition of the surface. Thus with constant watts input of energy, the rate of gas generation may be expected to increase with time.

The authors have attempted to calculate the influence of this temperature transient upon the gas generation in a fiber tube, using the thermal conductivity taken from tables for fiber, and the specific heat of dry cellulose as a material similar to fiber. With such values for the thermal constants, the time effect should have disappeared before the end of the first half cycle for much smaller currents than experiment shows to be the case. However, the considerable latent heat absorbed by the material due to chemical changes below the temperature for *rapid* decomposition was not taken into account, and as this will be equivalent to multiplying the specific heat by a large factor, the thermal explanation may still be regarded as valid.

It would be expected from this that any means used for reducing the thermal conductivity perpendicular to the surface of the gas forming material would shorten the duration of the thermal transient and lessen the number of cycles of arcing to interrupt a given circuit. The following experiment bears out this expectation. A 40-ampere arc in a 5/8 in. diameter tube, 4 in. long was found to require 9 half cycles before it was extinguished in a 60-cycle circuit of 2,300 volts. A loose spiral of a couple of layers of 0.001 in. paper was placed in the tube and allowed to expand so that it was in loose contact with the tube wall. It was then found that the arc required only 3 half cycles to interrupt the circuit.

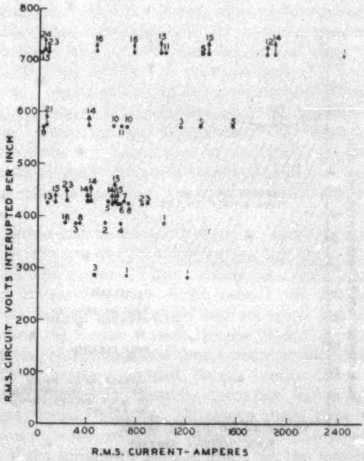

FIG. 4—FIBER FUSE DATA. FUSE TUBE 5/8 IN. INTERNAL DIAMETER BY 4 IN. LONG

Another factor which undoubtedly plays a large part in causing the time delay in the development of the full interrupting capacity was suggested by T. E. Browne and is that for small currents it will take considerable time before the air originally filling the tube is replaced by the gases formed by the decomposition of the gas forming material, which may have greater voltage interrupting capacity. The nature of the gas used in a blast has a great influence upon the voltage interrupting capacity of an a-c. arc subjected to the blast. Hydrogen is particularly effective.[5] Unpublished experiments of Browne show that water vapor is also very effective while nitrogen is much less effective than other gases. Study of the data of Fig. 3 shows that the time for displacement of the original air in the tube may be several half cycles for the smaller currents. For very large currents a single half cycle would be long enough for the air in the tube to be completely replaced and for the walls to heat up to such an extent that practically all the heat which reaches the walls will go to decomposing material. The gas blast would then be just as intense at the end of the first half cycle as at the end of the second or third. Hence the voltage interrupting capacity would be no greater at the end of the second or third half cycle than at the end of the first.

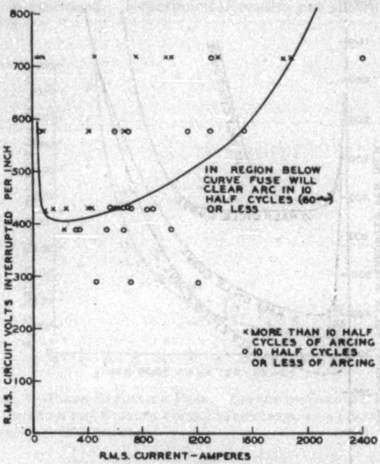

Fig. 5—Fiber Fuse. Circuit Volts Interrupted *vs.* Ampere Curve for 10 Half Cycles

For extremely small currents the gas evolution will become so small that it can play no part in the circuit interruption. The voltage interrupting capacity will therefore be the same as for an arc in the open and again it will be the same for the second, third, or later current zero as for the first. But for intermediate currents the voltage interrupting capacity would be expected to be greater at the second current zero than for the first and greater at the third current zero than the second and so on. Thus it will be necessary to show a family of characteristics, each member of the family corresponding to a different number of half cycles of arcing. The different curves should be expected to coincide for large currents and for very small currents, but should be different for intermediate currents.

Concerning the individual curves, because of the rapid rise in intensity of the gas blast with current as shown in Fig. 3, we should expect large voltage interrupting capacity to be developed by large current arcs. Also on general principles a large voltage interrupting capacity would be expected for sufficiently small current arcs.[2] The individual circuit voltage interrupting capacity *vs.* ampere curves should show a minimum at intermediate currents and rise to large values for very small and for very large currents.

In determining the characteristics experimentally the usual inconsistencies met with in such work due to variation in uncontrolled factors are encountered. The inconsistencies in the tests carried out were principally due to variations in the point of the generated voltage wave at which the circuit was closed and at which the fuse blew starting the arc, and to variations in the state of the fuse tube surface due to previous history and relative atmospheric humidity. The individual curves sought are boundaries between regions where circuit interruption occurred in a certain number of half cycles and regions where the circuit interruption occurred in a large number of half cycles. Since it is practical to obtain only a limited amount of data such boundaries could be determined only very approximately.

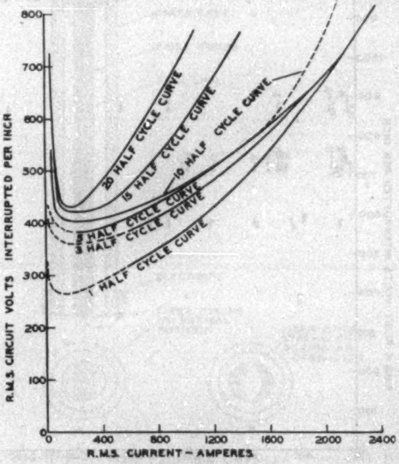

Fig. 6—Fiber Fuse. Circuit Volts Interrupted *vs.* Ampere Curves

Fig. 4 shows the data obtained for a 5/8-in. diameter fiber tube. The circuits used were again 60 cycle reactive, with transient periods of the order of 10 microseconds. The numbers next to the points give the number of half cycles to arc extinction. For those points marked with an arrow pointing upward the backing up breaker interrupted the circuit; the number next to them gives the number of half cycles of arcing up to the point at which the backing up breaker opened.

To obtain the characteristic for a given number of half cycles of arcing, those points of Fig. 4 which represent that number of half cycles of arcing or less were marked with circles and those points which represented more half cycles were marked with crosses. Then with the best judgment of the authors a curve was drawn separating the region of the circles from the region of the crosses. A certain amount of inconsistency in the position of the circles and crosses was permitted in the neighborhood of the region boundary for reasons mentioned above. Fig. 5 shows the curve obtained in this way separating the region of circuit interruption in 10 half cycles or less from the region of more than 10 half cycles of arcing.

A number of curves was drawn in this way corresponding to different numbers of half cycles of arcing and then superposed on the same sheet. The curves were then moved about so as to make them consistent with each other and consistent with the ideas expressed above as to the relations of the characteristics with each other. This could be done up to ten half cycles of arcing without any serious violation of the data as is seen in Fig. 6, but the curves for 15 and 20 half cycles seemed to detach themselves quite definitely from the lower number half cycle curves even for the larger currents.

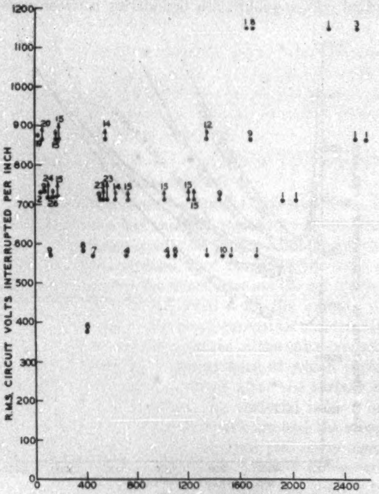

FIG. 7—BORIC ACID FUSE DATA. FUSE TUBE 5/8 IN. DIAMETER BY 4 IN. LONG

This suggests that for 15 half cycles and more there is a new factor coming in which again increases the rate of gas evolution. This may be due to some kind of mechanical disintegration of the surface beginning at this point. In 15 half cycles the heat will have penetrated several thousandths of an inch within the material. Decomposition will take place even below the surface and the high pressure of the confined gas will cause blistering and loosening or even complete detachment of pieces of the fiber. Such loose or detached pieces of fiber will decompose at an accelerated rate, and thus the gas blast will be increased.

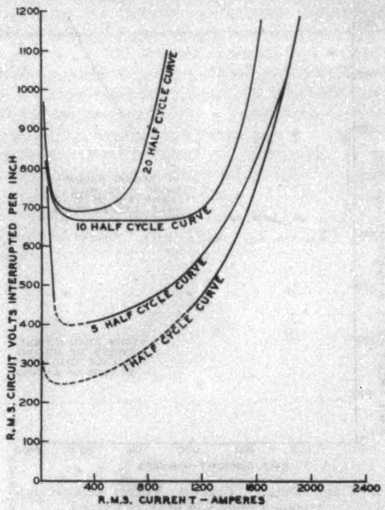

FIG. 8—BORIC ACID FUSE. CIRCUIT VOLTS INTERRUPTED *vs.* AMPERE CURVES

In Fig. 6, the parts of the curves drawn dotted are surmised, and do not have supporting data in Fig. 4.

Data were also obtained and are shown in Fig. 7 for a tube lined with boric acid having 5/8 in. internal diameter. The circuit voltage interrupting capacity *vs.* ampere curves are shown in Fig. 8. These curves lie much higher than those for fiber.

VII. INFLUENCE OF TUBE SECTION SIZE AND SHAPE

From the discussion already given it is quite clear what influence change in size of tube section should have upon the circuit voltage interrupting capacity *vs.* ampere characteristic. With a given rate of gas generation the turbulence and deionization will be far more intense in a small section tube than in one of larger section. Hence for a given current the arc voltage will be much larger in a small tube than in a large one, but with the larger arc voltage, the energy input will be greater and therefore more gas will be generated for a given current in a small tube than in a large one. This will still further intensify the turbulence and deionization. Hence the circuit volts which can be interrupted will go up very rapidly as the tube size is decreased. Experimental results are shown in Fig. 9.

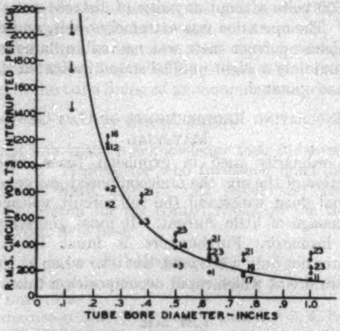

FIG. 9—FIBER EXPULSION FUSE. EFFECT OF VARYING TUBE DIAMETER ON THE CIRCUIT VOLTS INTERRUPTED AT A CONSTANT CURRENT OF 450 AMPERES

The influence of change in shape of tube section may also be anticipated from theory. Comparing a circular section with a narrow rectangular section of the same area it is seen that in the latter all parts of the arc are close to gas generating surface while in the former the center of the arc is more remote from gas generating surface. It should be expected then that the turbulence will penetrate more thoroughly throughout the arc in the latter than in the former and that the circuit voltage interrupting capacity will be larger for the narrow rectangular section than the circular. This is borne out by experiment.

Thus, tests with a slot in fiber 1/8 in. x 1¾ in. and 1 in. long showed that 2,100 volts per in. could be interrupted at 450 amperes. The circular section hole of the same area would have a diameter of 0.53 in., and from Fig. 9 such a hole would be able to interrupt only 400 volts per in. It follows then that while the circular section may be best from the standpoint of convenience of manufacture and for tube strength, it is the worst from the standpoint of circuit voltage interrupting capacity.

VIII. THE PRACTICAL PROBLEM OF EXPULSION FUSES

The curve of Fig. 9 would suggest that in order to make a successful fuse of a length regarded as practical and having a desired circuit voltage interrupting capacity, it is necessary only to use a section of sufficiently small dimensions, but difficulties of a mechanical nature intervene and prevent the solution from being reached so easily. The hole must be small enough to take care of intermediate currents of the order of 300 or 400 amperes, as indicated by Fig. 6, but if so small a hole is used, the gas generation for large currents of the order of a few thousand amperes becomes so intense that enormous pressures are produced which burst any practical and sufficiently inexpensive tube. Also the volume of flame issuing from the tube is so great that it becomes difficult to control, and there is great danger of flashover of nearby live apparatus as well as likelihood of failure of the fuse by flashover outside the tube. If the tube section is made large enough so that for large currents such danger of excessive pressures is avoided, as well as difficulties due to the large volume of flame, then the arc fails to be extinguished for smaller currents. One way of overcoming these difficulties would be to use a construction which would offer a small hole capable of interrupting the smaller currents, but at the same time offering a large hole for the larger currents so as to prevent unduly large gas pressures and large volume of flame. There is room for much ingenuity in devising such a construction. An example of such a construction, with which good results were obtained by Mr. Huffstutter, is shown in Fig. 10. Thin fiber sheets are placed in the tube parallel to a fuse wire, as shown in Fig. 10A.

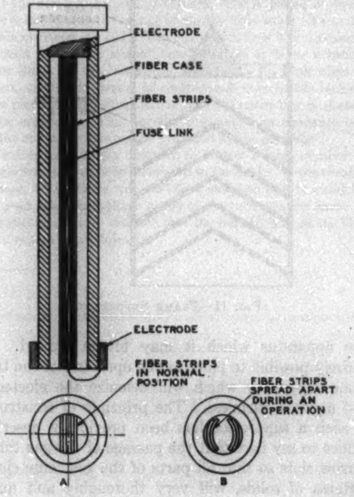
ELECTRODE
FIBER CASE
FIBER STRIPS
FUSE LINK
ELECTRODE
FIBER STRIPS IN NORMAL POSITION
FIBER STRIPS SPREAD APART DURING AN OPERATION
A
B

FIG. 10—HUFFSTUTTER VARIABLE CHAMBER EXPULSION FUSE

For small currents, the section is that of the narrow slot. For larger currents, however, the pressure forces the sheets apart as in Fig. 10B, and the section is that of a large hole with a resulting decrease in the volume and pressure of the gas generated.

IX. Flame Suppression—Complete Closing of Fuse Tube

As mentioned in the preceding section, the flame issuing from an expulsion fuse is a hazard in that it has low dielectric strength and will cause flashover of any live apparatus which it may bridge around. It is entirely possible to place at the open end of the tube a flame suppressor which will deionize the ejected gas and make it harmless. The principle of construction of such a suppressor has been previously described.[4] Suffice to say here that the passage of the gas through narrow slots so that all parts of the gas come close to surfaces of solids, will very thoroughly and quickly deionize it. Of course, the use of such a device will raise the pressure in the fuse tube, and sufficient total cross-section of slots should be used in the flame suppressor to permit exit of the gas without too great rise of pressure in the tube. Fig. 11 shows the construction of a flame suppressor which was used successfully with expulsion fuse tubes.

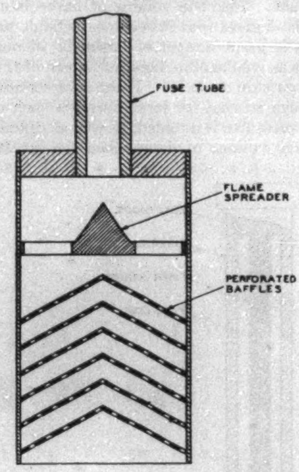

Fig. 11—Flame Suppressor

Boric acid as a fuse tube liner has been found to be better than fiber, in that its circuit voltage interrupting capacity *versus* ampere curves lay higher. In connection with its use there is an additional very remarkable possibility not at all possessed by fiber. The gas given off by the boric acid is water vapor which is easily condensable to a liquid, whereas the principal gases arising from the decomposition of fiber are permanent and condense to a liquid only under very high pressure and at very low temperature. Hence, if means are included within the tube for condensing the water vapor generated, it is possible to close up completely the boric acid lined tube without the development of excessively high pressures. Such a tube was constructed experimentally and is illustrated in Fig. 12. It was able to interrupt all currents up to 3,800 amperes at 6,900 volts, which was all the power available in the test set at that voltage and interrupted 5,000 amperes at 4,600 volts without any sign of distress due to pressure. The operation was extremely quiet, and even at the highest current there was no visible flame whatsoever and only a slight muffled sound indicated that the fuse had operated.

X. Insulating Requirements of Gas Generating Material

As ordinarily used in expulsion fuses, after the extinction of the arc, the undecomposed gas generating material must withstand the full circuit voltage with the passage of little current. It must therefore be a good insulator. Furthermore it must be a good insulator not only when cold, but also when at the high temperature at which rapid decomposition takes place, since the surface of the material will be at this high temperature at the moment arc extinction takes place.

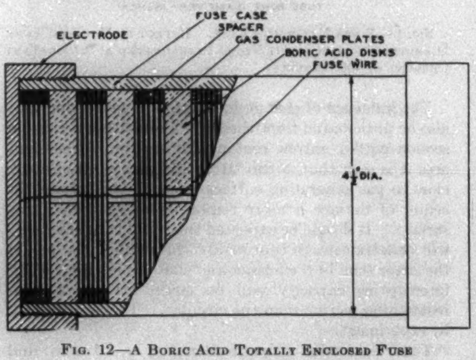

Fig. 12—A Boric Acid Totally Enclosed Fuse

This explains why boric acid is more effective than other inorganic materials which are superior to it from the standpoint of volume of gas generation per unit of arc energy. Boric acid has a high resistivity even at a temperature of several hundred degrees C at which it decomposes rapidly. As an example of a gas generating material which from its constitution would be expected to be superior to boric acid from the standpoint of volume of gas evolution but which when tested in an expulsion fuse appeared to be much inferior as to voltage interrupting capacity,[3] we may take ammonium alum, $Al_2 (NH_4)_2 (SO_4)_4 24H_2O$. Ammonium alum, while a good insulator when cold, melts at 89 deg. cent. and becomes a good conductor. It loses all its water of crystallization at 120 deg. cent., so that this may be taken as the temperature for rapid decomposition. Hence in an expulsion fuse using ammonium alum, the surface left immediately after the arc extinction will be an electrically conducting film of the molten salt. Under voltage this film will break by boiling and restart the arc.

It is possible to separate the gas generating and insulating requirements by suitable constructions. Thus an effective ammonium alum fuse could be made by interspersing short lengths of soapstone, as an insulating material in the tube lining of ammonium alum.

ACKNOWLEDGMENT

The authors wish to acknowledge their indebtedness to Mr. J. W. Huffstutter who furnished much of the experimental data used in this paper and also to Mr. A. P. Strom for his contributions on the totally enclosed boric acid fuse.

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Discussion

L. P. Boll: There is one point which I believe should be elaborated upon especially so when the operator of rural lines and his conditions are considered. The paper treated more or less the action of the expulsion type fuse link under the heavier short-circuit conditions, while the operator of rural lines is primarily concerned with the action of the fuse under abnormal conditions of load which are slightly in excess of the minimum fusing current of the fuse link. The advantage of using tubes of various bores was brought out. The operator in order to minimize his stock and to eliminate several of the difficulties encountered with having various bore cartridges, has chosen to use only two sizes of cartridges. One to accommodate fuse links from one ampere to 30 amperes and the other from 30 amperes to 100 amperes. Naturally satisfactory performance can only be obtained within a small range.

With a small amperage fuse link there are not sufficient gasses generated to create the pressure or cause sufficient turbulence to separate the burned ends of the fuse link sufficiently far to prohibit the reestablishment of the arc, after a period of time, say 2 or 3 hours, due to leakage along the fiber tube. To overcome this shortcoming of the expulsion type fuse it is necessary to surround the link by a small bore cartridge or to use a spring type fuse link. In the case of the spring type fuse link when it is melted, under any degree of short-circuit current the burned ends are mechanically separated, in fact, the entire link is ejected from the cartridge. This permits the maximum clearance between the contact which can be provided by the cartridge.

The writer has had the occasion to inspect a large number of cartridges which have been destroyed in the field. In practically every case, where there was sufficient evidence left, it could be readily seen that the leakage inside of the tube was the cause of the destruction, due to the fact that the burned ends of the link were not separated sufficiently.

The Theory of Oil-Blast Circuit Breakers

BY D. C. PRINCE*
Fellow, A.I.E.E.

***Synopsis.**—Analyzing the oil-blast theory, data are presented to show effect of time and carbon on oil dielectric strength, correlation between voltage recovery rate and required oil velocity, lack of correlation between either current and voltage and necessary oil velocity, correlation between recovery rate and pressure in explosion chambers. Tests on various explosion chamber forms and photographs of the oil blast in action are presented.*

* * * * *

THE oil-blast theory of oil circuit breaker operation has been presented in a previous paper.[1] This theory was rather sharply challenged. The proof presented at that time was somewhat circumstantial, although no more circumstantial than the proof of other theories presented in the past. The oil-blast theory can be substantiated by direct experiment. It is the purpose of this paper to present the evidence at present available in support of this theory; some of this has been published in a fragmentary manner.[2,3,4,5]

The essence of the oil-blast theory is that arc interruption is brought about by interposing between the circuit breaker electrodes, a wall of solid oil which grows at such a rate as to be always more than sufficient to resist puncture by the recovering voltage across the circuit breaker terminals. This growing barrier of oil sweeps before it the arc products so that phenomena of deionization occurring in the gases enveloping the arc do not influence the extinction phenomena. The problem presented in this way may be broken down into its elements, namely:

1. The rate at which the voltage appears across the circuit breaker terminals.
2. Dielectric strength of oil.
3. Necessary velocity of oil.
4. Means of securing that necessary velocity of oil.

The rate at which the voltage appears across the circuit breaker terminals is susceptible of accurate determination. Such rates have been calculated and checked by cathode ray oscillograms.[6,7]

The dielectric strength of oil is, of course, well known, although its variations with different sorts of contamination and with time have not been covered by published data. Tests have been made to determine the effect of short time application of the potential on oil using both fresh oil, filtered and dried, and oil which has been used for a considerable time in an oil circuit breaker so that it contains more or less water and carbon. Fig. 1 shows the variations in the strength of oil with variations in the length of time of impressed potential. These tests were made with a gap consisting of a 1-in. disk and a cone 1 in. in diameter by ½ in. high spaced at 0.025 in. The points at the extreme right are 60-cycle values for this gap. From the figure, carbonized oil sample No. 8 has a dielectric strength at 60 cycles of $\frac{2.4}{17} = 0.14$ compared with 1.00 for clean oil sample No. 1. For a recovery voltage wave reaching its crest in eight microseconds, the ratio is $\frac{27}{42} = 0.64$ that is, from about one seventh the strength at 60 cycles, the poor oil is more than half as strong as good oil for eight microseconds and is 1.6 times better than clean oil under 60-cycle potential tests. This explains why moderate oil deterioration does not have a serious effect on oil-blast circuit breaker performance and also why in the quantitative analysis of the oil-blast theory, the value of 55.0 kv. per 1/10 in. as an oil dielectric strength under oil-blast circuit breaker conditions, which was found, might be expected. Both high strength and small deterioration effects are explained on the assumption that the contaminating elements operate to produce chains of carbon or moisture or both which bridge part of the gap. These chains require time for their formation. This time is not

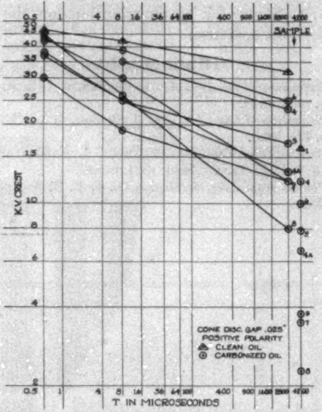

FIG. 1—BREAKDOWN STRENGTH OF OIL UNDER IMPULSE

△ Clean dry oil sample
⊙ Carbonized oil sample

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*General Electric Co., Philadelphia, Pa.

1. For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

available when a sudden application of voltage is made. It also follows that with a high velocity of oil, such as occurs in an oil-blast circuit breaker, the turbulence will tend to prevent the formation of these chains, thus assuring the maximum effectiveness even of contaminated oil in carrying out the actual interruption. These tests are being continued in an effort to ascertain quantitatively the effect of known amounts of moisture and carbon separately and in combination. Sufficient data are not yet available to construct a quantitative paper on this subject but it is hoped that it can be covered more completely at some time in the future.

Having ascertained the rate at which voltage appears across circuit breaker terminals and the dielectric properties of oil under the time conditions represented by such an application of voltage, it should be possible to calculate the rate at which oil must be introduced to interrupt a circuit having a given voltage recovery rate. It should also be possible to ascertain by direct tests whether the interruption occurs as would be predicted by this theory. This has been done.[2] A circuit breaker was constructed in which oil was driven into the gap between the contacts by a piston. The velocity of the piston movement was measured and was varied over a considerable range.

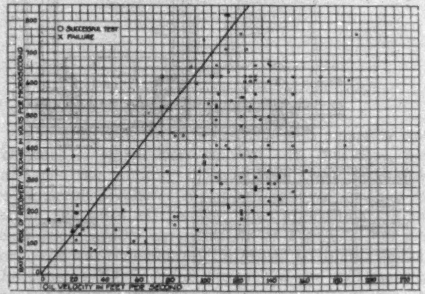

FIG. 2—TESTS SHOWING CORRELATION BETWEEN RATE OF VOLTAGE RECOVERY AND OIL VELOCITY FOR SUCCESSFUL OPERATIONS

A large number of interrupting tests was made. During the series, the test conditions were varied as follows:

1. Change in oil velocity. Range from 5 to 200 ft. per second.
2. Change in test voltage. Range from 3,800 volts to 13,200 volts.
3. Change in current interrupted. Range from 200 amperes to 30,000 amperes.
4. Change in rate of recovery voltage rise. Range from 60 to 820 volts per microsecond.

All of the tests made are plotted in Fig. 2 with rate of recovery voltage rise in volts per microsecond as ordinates and oil velocity in feet per second as abscissas. All other variables are ignored in this plot except that successful tests, namely those on which interruption occurred at the first current zero of arcing, are marked with a dot. By studying the distribution of the points with and without dots, it is possible to draw a line dividing the region of points with dots from that without dots. The slope of this line should be proportional to the dielectric strength of oil under the conditions of circuit breaker interruption and should correspond to the strength of oil measured under the corresponding short time conditions. In this case, the actual slope of the line corresponds to a dielectric strength of 55 kv. per 1/10 of an inch, which is of the proper order for short time voltage applications. To state the case mathematically:

(1) The circuit has a recovery rate of

$$\frac{v}{t} = \text{volts per second}$$

(2) Oil has a dielectric strength of

$$\frac{v}{l} = \text{volts per inch}$$

To ascertain the necessary oil velocity, divide one of these fractions by the other, securing the equation

$$\frac{\frac{v}{l}}{\frac{v}{t}} = \frac{l}{t}$$

where $\frac{l}{t}$ is the required oil velocity. There is thus a positive proof that the oil-blast theory is at least one explanation of the behavior of this circuit breaker. So far, however, the possibility that some other explanation might cover the same set of observations has not been excluded.

To secure evidence on that point, the tests shown in Fig. 2 are further classified in Fig. 3 in order to examine the data for a possible relation of test voltage to the behavior of the breaker. In this chart, test voltages are plotted as ordinates and oil velocity in feet per second as abscissas. Here again successful tests are marked with dots. Those tests not marked with dots are those in which interruption did not occur at the first current zero of arcing. If the test voltage and not rate of recovery voltage rise were predominant in affecting the performance of the breaker, it would be expected that the points indicating successful tests would be so distributed on the chart that some relation to test voltage would be indicated. A study of the chart, however, shows that the points showing successful tests are mixed at random with the unsuccessful tests. It is therefore concluded that there is no relationship between oil velocity and test voltage.

From the analysis so far made, there still exists a possibility that the amount of current interrupted and the oil velocity might bear some definite relationship. To study such a possibility, another chart, Fig. 4, was made. This differs from Fig. 3 in that currents interrupted in amperes are shown as ordinates. The identification of successful tests is the same as that used in Figs. 2 and 3. From this chart, it will be easily seen that the successful and unsuccessful tests are so intermixed that it is not possible to consider that either of them is in any way related to the amount of current interrupted. This discussion, therefore, definitely bears out the conclusion already stated that oil velocity bears a definite relationship to rate of recovery voltage rise and that when the proper oil velocity for a given recovery voltage rise has been determined for successful operation, the breaker performs successfully over a wide range of test voltages and amounts of current interrupted.

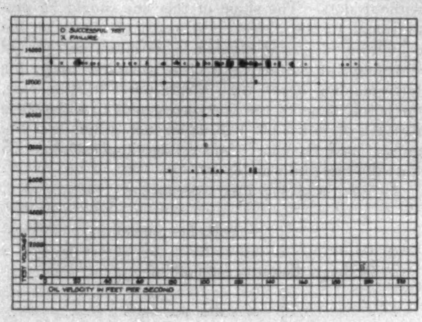

Fig. 3—Tests Showing Lack of Correlation between Voltage and Oil Velocity for Successful Operations

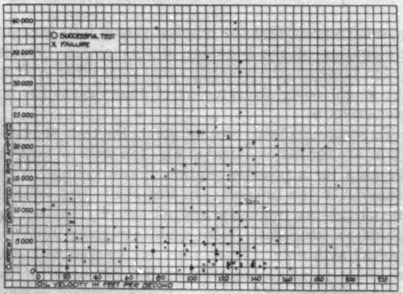

Fig. 4—Tests Showing Lack of Correlation between Current and Oil Velocity for Successful Operations

Indirectly, these data bear upon the subject of deionization. If our phenomenon were one of deionization, a longer time should be required to deionize the dense ionization of a heavy current test than the sparse ionization accompanying a light current interruption. With this design of circuit breaker, there is no element tending to produce more rapid or efficient deionization at a high current where more ionization is present, than at a lower current where little ionization is present and yet no apparent difference appears traceable to this cause.

In the large number of tests made with this circuit breaker, no effort was made to change the oil frequently. The oil thus varied over wide limits as to its 60-cycle puncture value. However, no considerable difference seems to have been produced by this difference in the 60-cycle value of the oil. This result would be expected from the tests already described in which the strength of oil under impulse was measured.

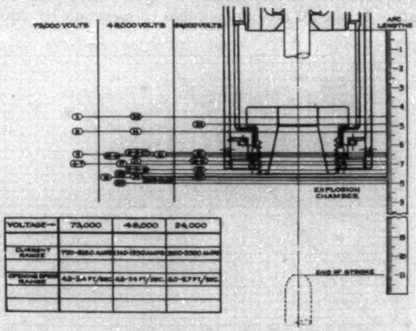

Fig. 5—Sketch Showing Action of Explosion Chamber at Various Voltages and Currents

Once evidence has been secured that the oil-blast theory of operation quantitatively explains the operation of an impulse (oil driven by piston) type of circuit breaker and that other general explanations of circuit breaker behavior are not applicable, it is much more difficult to prove that this same theory of operation explains the behavior of other types of oil circuit breakers. Reasonably direct proofs can, however, be obtained in the case of explosion chamber types of breakers. If these breakers really operate as oil-blast breakers, then steps taken to promote a blast of solid oil will produce an improvement in performance.

Fig. 5 represents a large number of tests with a plain explosion chamber and shows that the arc is uniformly extinguished in the region where the oil-blast action should be most pronounced. At the throat the pressure head has all been turned into velocity head so that there is very little pressure tending to prevent solid oil from displacing the arc products at the current zero.

A special explosion chamber was constructed as shown in Fig. 6 so arranged that there was a series of throats between which there were pockets of oil. This arrangement had the advantage over the simple explosion chamber in providing three points at which the current zero might occur under most favorable conditions of oil-blast. With this arrangement, the arc lengths and arcing time were found to be uniformly less than with the simple explosion chamber.

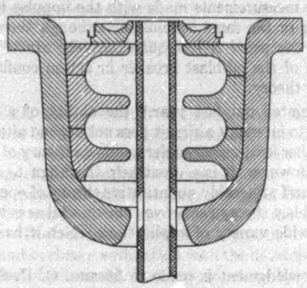

Fig. 6—Multiple Throat Explosion Chamber

In another test, the arc was drawn in a narrow slot, the sides of which were smooth. In this case, performance was notably inferior even to a plain open break circuit breaker. A series of hollows was then cut out in the sides of the slot, thus approximating the explosion chamber arrangement of Fig. 6. The performance was immediately improved to approximately the same as Fig. 6. The porosity of the wall material was not found to influence performance.

If the oil-blast explosion chamber operated exactly in accordance with the quantitative theory proven in the case of the impulse type of circuit breaker, there should be a correlation between pressure shown in the explosion chamber and the rate of recovery voltage rise across the circuit breaker contacts. Some test data have been analyzed and Fig. 7 made up from the data. In this figure, pressure and rate of recovery voltage rise are coordinates. It is not possible to measure the oil velocity directly and due to the wide variation in back pressure, there is not a direct relation between the measured pressures and oil velocity. However, it is apparent that at least some correlation exists between the explosion chamber pressure and the rate of recovery volts at the time of arc interruption. In order to break down this problem into one susceptible of a complete solution, it is necessary to find first the relation between the arc energy and the amount of gas evolved from an arc and second, the conditions governing arc energy. Considerable progress has already been made on the first half of the problem. A large amount of data are available showing that within the error of plus and minus 50 per cent the amount of gas formed is equal to

$$V = 0.07\,W$$

V = volume in liters at normal temperature and pressure

W = arc energy in kilowatt seconds

The amount of arc energy, however, is quite variable in existing types of breakers. This is necessarily the case in a device which interrupts in one or two half cycles of arc inasmuch as the contacts may separate at any point in the cycle. The arc will be interrupted at the first zero after a certain pressure has been reached, but if the current zero does not occur for a considerable time after this pressure has been attained, the pressure will continue to increase to an unnecessary degree. The higher speed of the breaker, the greater will this variation be in per cent, that is, the amount of time represented by a variation of one cycle becomes large by a comparison with the total arcing time. Steps are being taken to make tests with a synchronous closing and opening device which should make it possible to produce and select the type of wave, that is, symmetrical or displaced at the time of contact parting. Tests with this apparatus should make it possible to ascertain what part of the variations in arc energy and therefore pressure are due to the circuit variables and

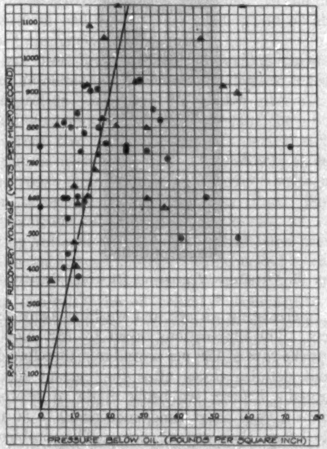

Fig. 7—Tests Showing Correlation between Rate of Voltage Recovery and Explosion Chamber Pressure at Time of Interruption

○ Tests on oil-blast FH-203
△ Tests on oil-blast FK-44
● Pressure when circuit cleared
× Pressure at current zero at which the circuit did not clear

what part still remains to be accounted for as due to the vagaries of the arc.

Statements[8-9] have appeared in the press to the effect that carbon particles are instrumental in arc extinction. Our own tests did not bear this out inasmuch as tests made with liquids which carbonize freely, other things being equal, show less effective arc interrupting properties.

In an attempt to satisfy pardonable skepticism as to the operation of oil-blast breakers by the insertion of walls of solid dielectric, a considerable number of photographic studies has been made. Fig. 8 taken from an article by Mr. E. B. Noel[5] shows the oil in the act of interrupting an arc and washing away the arc products. The illustrations taken by Mr. Noel were taken at the rate of 128 per second and therefore do not represent definite times after the current has been interrupted. Efforts are being made to photograph conditions a very short time, such as 10 to 50 microseconds after the current zero. The problems of technique in securing such photographs have not yet been overcome although there is hope of securing such a record. Even over an interval of 1/128th of a second including the current zero, no marked change appears other than the disappearance of the arc. The arc is clearly subjected to violent washing action in Fig. 8c and 8d and, by Fig. 8e, clear oil is flowing between the electrodes.

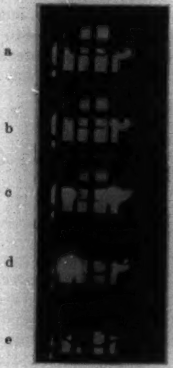

Fig. 8—Oil-Blast Explosion Chamber Operation

In the foregoing, the author has attempted to bring in all relevant information which may bear on the oil-blast theory either to prove or disprove it. It is felt that in the case of impulse breakers, the exact quantitative results obtained by measuring the oil dielectric strength under impulse conditions and the operation of the breaker under known oil velocity and recovery rate conditions prove beyond question that this breaker operates on the oil-blast theory as enunciated and in no other way. All of the factors discovered in connection with the other types of circuit breakers appear to confirm the same law, that is, variations in geometry tending to promote oil-blast action improved performance. Configurations tending merely to hold oil in the vicinity of the arc do not promote operation. Such pressure measurements as can be taken on an explosion chamber of the oil-blast type clearly resemble the velocity measurements made with the impulse breaker pointing to the same conclusion, although the perfection of the proof is less quantitatively exact. Illustrations of the oil-blast breaker in action confirm the oil-blast theory.

As pointed out last year,[10] the ability of a circuit breaker to interrupt a circuit does not depend altogether on whether its designer understands its theory of operation. However, it is a great help in design to have a correct and preferably quantitative theory of operation. The oil-blast theory has proven most useful as evidenced by the wide variety of applications which it has made possible.[11]

Acknowledgment is made to Messrs. O. R. Schurig and B. H. Thompson for the data on gas formation; to Messrs. C. M. Foust and R. F. McAtee for the data on impulse strength of oil; and to Mr. W. F. Skeats and his staff for the test data and cooperation in the preparation of this paper.

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11. *Practical Application of the Oil-Blast Principle of Circuit Interruption*, R. M. Spurck, see page 171 this issue.

Discussion

For discussion of this paper see page 191.

Practical Application of the Oil-Blast Principle of Circuit Interruption

BY R. M. SPURCK*
Associate, A.I.E.E.

***Synopsis.**—The oil-blast principle of circuit interruption may be applied to oil circuit breakers of many types which cover a wide range of service, voltage and interrupting requirements. Although the mechanical construction and configuration of the interrupting elements varies widely for varying requirements, nevertheless the same principle of operation applies.*

It is shown that the oil-blast utilized for efficient arc extinction may be produced electrically by decomposition of oil or mechanically by a spring driven piston in an oil cylinder. Whichever of the two methods is employed, comparable performance results.

Several types of breakers which operate on the oil-blast principle are described and test data given. In all designs of breakers on which the new principle has been used, marked reductions in arc duration and arc length result, even at the higher rates of recovery voltage rise. The principle is therefore applicable to breakers of the high speed type.

THERE were presented to the Institute last year, three papers[1,2,3] which announced the development and practical application of the oil-blast explosion chamber to high-voltage oil circuit breakers. Another paper[4] in this issue also discusses the results of further research on the operation of the oil-blast theory. The research work was performed concurrently and in close coordination with the development of new lines of oil circuit breakers which utilize the oil-blast principle of circuit interruption. It is the purpose of this paper to describe the construction of these new breakers, to give results of interrupting tests made on them, and to discuss their characteristics.

It should be brought out first that the oil-blast principle of circuit interruption is not confined to any particular type of breaker, limited to any voltage, or restricted to any specific design. Developments so far completed apply the oil-blast principle to breakers of various types and designs, generally described as follows:

1. High-voltage outdoor breakers of the downward break type in which the oil-blast explosion chamber is used on each breaking contact.
2. Moderate-voltage indoor and outdoor high-interrupting-rated breakers of the downward break type with two breaks per pole. In this design, an oil-blast explosion chamber is used on each breaking contact.
3. Moderate-voltage moderate-interrupting-rated indoor breakers of the downward break type with three poles in one tank. In this application in a three-phase breaker, one chamber surrounds the contacts of all three poles.
4. Moderate-voltage high-interrupting-rated indoor breakers of the upward break type having two breaks per pole in individual oil vessels. Such breakers are commonly known as the "H" type. In this case, new baffles designed to give an oil-blast are applied.
5. Moderate - voltage moderate - interrupting - rated outdoor breakers of the downward break type with two breaks per pole in individual tanks. One chamber surrounds the pair of contacts in each tank.
6. An entirely new superspeed, 15,000-volt design of single-phase breaker for railway service. In this design, one break in one oil vessel is used. The oil-blast is obtained mechanically.

As has been brought out in previous papers, the oil-blast principle is based on the following conception of its operation. An oil circuit breaker interrupting an alternating current circuit will interrupt the circuit at any current zero after the contacts part if sufficient insulation is inserted between the contacts at current zero to resist puncture by the recovery voltage that appears across the contacts at that time. The rate and magnitude of recovery voltage rise at various operating voltages and conditions has been predicted by mathematical calculations[3,5] and checked by cathode ray oscillograms. Also, by experiment and mathematical analysis,[6] the oil velocity necessary to cause interruption at an early current zero at various rates of recovery voltage rise and oil velocity required for early arc extinction, has put into the hands of designers what may be termed as new tools that enable them to construct apparatus of greater efficiency and consistency of performance than was heretofore possible. Furthermore, with such definite and broadly applicable data available, the designers had opened to them a wide field of application which was limited only by their own ingenuity in devising schemes for applying the new principle. There follows a general description of the various applications that have been made.

APPLICATION TO HIGH-VOLTAGE OUTDOOR BREAKERS

The oil-blast explosion chamber as applied to high-voltage breakers was described in papers[1,2,3] presented last year so no further reference will be made to it here.

APPLICATION TO DOWNWARD-BREAK INDOOR AND OUTDOOR MODERATE-VOLTAGE HIGH-INTERRUPTING-RATED BREAKERS

The explosion chambers used to apply the oil-blast principle to the downward break tank type moderate-

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*General Electric Co., Philadelphia, Pa.,

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

voltage indoor and outdoor breakers having each pole in a single tank are somewhat different in design from the chambers used in the high-voltage outdoor breakers. In these breakers, smaller chambers are used. At such voltages, also, the necessity for high insulation disappears so it is possible to use a metal chamber, properly insulated inside and out, and provided with an insulating throat bushing. Two types of chambers are used, one having high pressure butt contacts, the other segmental sliding contacts and gates. The chamber with the butt contacts is used in breakers where the normal current carrying requirements are not more than 1,200 amperes, in which case no contacts in addition to those on which the arc is broken, are required. Such an explosion chamber is shown in cross section on Fig. 1. As in the high-voltage breakers, an auxiliary gap is provided but due to limitations of space, the mechanical details are somewhat different although the operation is the same.

Because of the similarity of the operation of this chamber to the one used on the high-voltage breakers, no test results will be given. It may be stated, however, that the arc lengths and durations obtained on breakers equipped with it are uniformly very short. As an example, on tests at 23,000 volts, arc durations of approximately 1½ cycles are the rule.

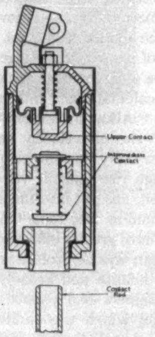

Fig. 1—Oil-Blast Explosion Chamber for Moderate Voltage Downward-Break Indoor and Outdoor Breakers for Current Ratings Up to 1,200 Amperes

Contacts in partially opened position

In breakers of normal current ratings in excess of 1,200 amperes, the arc interrupting contacts must be bridged by additional current carrying contacts and both the main and arcing contacts so designed that in opening the circuit, current will be transferred from the main contacts to the arcing contacts before interruption begins. Such a design is shown in Fig. 2 which shows the contacts in three positions as described.

When the arcing contacts are in the closed position, Fig. 2A, they are bridged by main current carrying contacts. As the breaker opens, the main contacts part before the moving arcing contact separates from its fixed contact, thus transferring all current flow to the arcing contacts. In the position shown in Fig. 2B, the moving arcing contact has separated from the stationary contact a sufficient distance to permit closing of the metal gates. In this position, an arc plays between the stationary contact, metal gates and moving contact. During this phase of the interruption, the arc in the chamber above the gates creates pressure that causes oil to flow rapidly into the passage below the gates in Fig. 2B. Note that Fig. 2C is a section rotated 90 deg. from Fig. 2B. The flow of oil passing through the arc path below the gates causes circuit interruption at an early current zero.

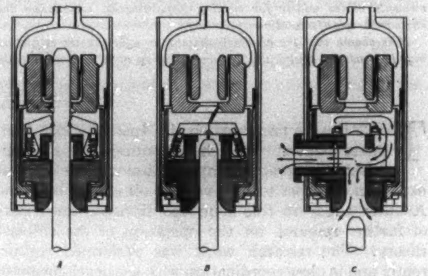

Fig. 2—Oil-Blast Explosion Chamber for Moderate Voltage Downward-Break Indoor and Outdoor Breakers of Current Ratings Above 1,200 Amperes

A. Contacts closed
B. Contacts partially open
C. Contacts open
Note Section *C* is turned 90 deg. from Sections *A* & *B*

Table I shows a few of a series of tests made on such a chamber. All of these tests are circuit interrupting tests. Although the maximum pressure in the explosion chamber was about 80 lb. per sq. in., the pressure above the oil in the tank was very low in all tests. Because of the short arc durations, contact burning and oil deterioration were negligible.

TABLE I

Volts Kv.	Current Interrupted Amperes	Equivalent Three-Phase Kva. Interrupted	Arc Duration Cycle	Arc Length Inches
44	4,100	310,000	1.4	2.8
44	4,200	325,000	1.2	2.5
44	3,600	275,000	1.4	2.8
44	2,500	195,000	1.4	2.8
44	2,500	195,000	2.0	3.8
44	1,300	100,000	1.7	2.8

Single-phase interrupting tests at 60 cycles on gated explosion chamber shown in Fig. 2—arc durations and lengths include both gaps. Recovery voltage rate 1,600 to 1,800 volts per microsecond. Arc durations and lengths include both gaps.

Inasmuch as rate of recovery voltage rise plays an important part in the interrupting performance of breakers, all test data herein will include a statement of recovery voltage rise rate. In some tables of data, it will be noted that a range of recovery voltage rise is given. This range primarily results from changes in circuit reactance made to control the value of short circuit current. Recovery voltage rise rates as high as those listed do not occur in regular practise except in cases where a breaker on a single feeder from a generator bus interrupts a short circuit at a point on the feeder near the stations.

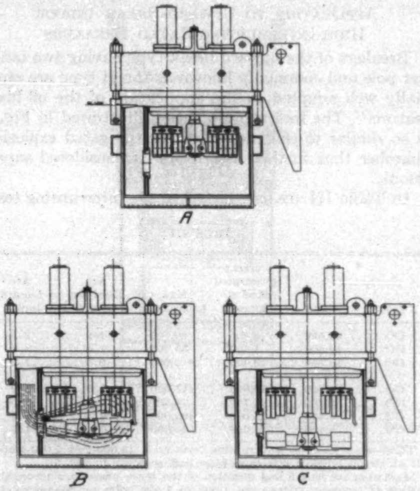

FIG. 3—OIL-BLAST PRINCIPLE APPLIED TO INDOOR 15,000-VOLT DOWNWARD-BREAK BREAKERS HAVING THREE POLES IN ONE TANK

A. Contacts closed
B. Contacts partially open
C. Contacts open

APPLICATION TO DOWNWARD-BREAK INDOOR MODERATE-INTERRUPTING-RATED BREAKERS

So far, the description of applications of the oil-blast principle has included designs in which explosion chambers are used. The application of the principle is, however, not limited to explosion chamber type breakers but is also applicable to the smaller breakers of moderate interrupting ratings in the range of from 50,000 to 250,000 kva. at voltages up to and including 15,000 volts. In applying the principle to such breakers of the downward break tank type in which all three poles of a three phase breaker are in one tank, a box of insulating material inside the tank surrounds the entire group of contacts. The top of the box is just below the oil level. Three of the sides of this box fit the inside of the tank, the fourth side of the box adjacent to one set of contacts is set away from the tank and opposite each contact is an opening in the side of the tank. The arrangement of this box in the tank is shown in Figs. 3 and 4 which should be referred to in the following description of the operation of the breaker.

As the contacts part, the decomposition of the oil by the arc forms gas which creates pressure in the insulating box and causes oil to flow out of the orifices in the side of the box spaced away from the tank. These orifices are so placed that the flow of oil is past the contacts and the arc adjacent to the orifices. Therefore, the arc products in the contact gaps adjacent to the orifices are being continuously washed away and out through the orifices so that at an early current zero, when no decomposition is taking place, the space between the stationary and moving contacts is filled with clean oil, the insulation of which is sufficient to prevent the recovery voltage from breaking down the gap and continue arcing.

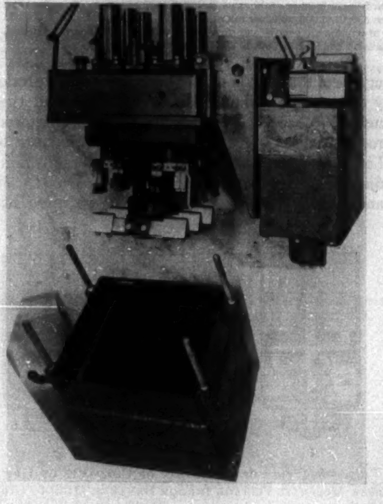

FIG. 4—COMPLETE 15,000-VOLT INDOOR BREAKER WITH OIL-BLAST DESIGN SHOWN IN FIG. 3

In the design of these breakers are a number of interesting features, for example, with the box enclosure, any pressure in the box acts on the three operating rods which pass through the top of the box and tend to reclose the breaker. To compensate for this tendency to close, pistons of an area somewhat greater than the combined area of the operating rods are attached to the cross tie at the bottom of the moving contacts and extend through the bottom of the box. The result is that the pressure in the box exerted on the larger area of the lower pistons gives a definite opening tendency for all pressures inside the box. Another design feature worthy of mention is the arcing horn on the pressure building contacts, that is, the ones adjacent to the box wall with no opening. The purpose of this horn is to curtail box pressure produced by the pressure building gap. To understand this, it must be realized that only a small gap is required to produce sufficient pressure to cause an adequate blast of oil across the actual interrupting contacts which are adjacent to the openings in the box. On the other hand, a somewhat longer gap may be required in the interrupting contacts. As the two gaps without the arcing horn would normally be the same for each increment of travel of the contact blade, the horn is provided to limit the pressure producing gap to a value that is sufficient to provide only proper oil-blasting pressure necessary to accomplish arc extinguishment at an early current zero by the insulation introduced in the interrupting gap.

TABLE II

Volts	Current Interrupted Amperes	Kva. Interrupted	Arc Duration Cycles	Arc Duration Length Inches	Time from Trip Impulse to Interruption Cycles
4,200	30,000	218,000	0.7	1.3	3
4,200	850	6,200	1.0	0.3	6½
4,200	1,700	12,400	0.6	0.3	7
4,200	5,900	42,900	1.0	0.3	7
4,200	15,000	109,000	1.0	0.3	7½
4,200	26,000	189,000	0.7	0.3	6
4,200	49,000	356,000	0.6	0.3	5
4,200	53,000	385,000	0.2	0.3	4½

Three-phase 60-cycle interrupting tests on an indoor 15,000-volt downward break breaker of the type shown in Fig. 4—rate of recovery voltage rise 500 to 11,000 volts per microsecond.

Maximum arc lengths and durations of the three phases recorded. Arc duration and lengths include the interrupting gap only.

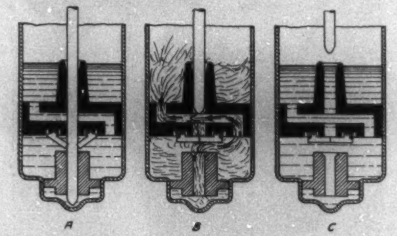

FIG. 5—SEQUENCE OF OPERATION OF CONTACTS OF UPWARD-BREAK MODERATE VOLTAGE INDOOR BREAKER DURING CIRCUIT INTERRUPTION

A. Contacts closed
B. Gates closed—breaker partially open
C. Contacts open

Table II shows some of a series of interrupting tests made at 4,200 volts on a 15,000-volt breaker of this type. If the arc durations shown in this table are compared with arc durations of plain-break breakers previously used in the interrupting and voltage range covered by this line of breakers, it will be found that in general the arc durations of these new breakers are from 75 per cent to 85 per cent shorter. This shortening of the arc duration effects a marked reduction in the oil deterioration and contact burning.

APPLICATION TO UPWARD-BREAK INDOOR HIGH-INTERRUPTING-RATED BREAKERS

Breakers of the upward break type having two tanks per pole and commonly known as the H type are especially well adapted to the application of the oil-blast feature.[7] The interruption action illustrated in Fig. 5 is so similar to that described for the gated explosion chamber that further description is considered superfluous.

In Table III are listed some of the interrupting tests made on one of these breakers. The arc durations in this table are from 50 per cent to 80 per cent less than the arc durations in breakers with the older type of baffles. These reduced arc durations and the correspondingly lessened contact burning permit increases in interrupting ratings in these breaker structures.

TABLE III

Test	Volts	Current Interrupted R.M.S. Amperes	Kva. Interrupted	Arc Duration Cycles	Arc Length Inches
CO	14,500	2,400	60,300	1.2	2.5
CO	14,500	1,400	35,200	1.8	3.7
CO	14,500	16,400	412,000	0.5	1.2
CO	14,500	16,600	417,000	0.6	1.0
CO	8,400	1,740	25,300	0.6	1.0
CO	8,400	27,000	393,000	0.5	1.0
CO	8,400	44,000	640,000	0.6	1.5

Three-phase 60-cycle interrupting tests on a 15,000-volt 2,000-ampere *FH* oil circuit breaker with cross blast baffles shown in Fig. 5.

Maximum arc length and duration of the three phases are recorded.

Rate of recovery voltage rise 1,000 to 1,600 volts per microsecond.

Arc durations and lengths include both gaps.

APPLICATION TO DOWNWARD-BREAK MODERATE-INTERRUPTING-RATING MODERATE-VOLTAGE OUTDOOR BREAKERS

Another line of breakers to which the oil-blast principle has been applied[7] is that comprising 15,000 volt outdoor breakers in interrupting ratings from 50,000 kva. to 175,000 kva. In these breakers, each pole is contained in a separate tank. The general construction and operation are, however, similar to those described for the indoor 15,000 volt moderate interrupting rated breakers previously described. It is therefore considered sufficient to include Fig. 6, a cross section of one tank.

As to tests, those shown in Table IV are typical. Also, as the arc durations are comparable with those already listed for other breakers and the comments regarding performance apply, they will not be repeated.

APPLICATION TO SINGLE-PHASE SUPER-SPEED HIGH-SPEED OUTDOOR BREAKERS

In all of the breakers so far described, the oil-blast resulted from oil decomposition by a pressure producing gap. The same effect can be produced by mechanical means as will be shown in the following description of a 15,000-volt single-phase breaker designed for single-phase 25-cycle, 12,000-volt railway service requiring that 50,000 amperes at 12,000 volts be interrupted in one cycle or 0.04 second.[8]

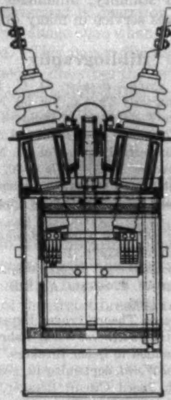

FIG. 6—OIL-BLAST PRINCIPLE APPLIED TO OUTDOOR MODERATE-VOLTAGE DOWNWARD-BREAK BREAKER WITH EACH POLE IN ONE TANK

Contacts partially open

TABLE IV

Volts Kv.	Current Interrupted Amperes	Kva.	Arc Length		Time from Trip Impulse to Inter. Cyc.
			Cy. of Arc	In. per Brk.	
13.2	1,500	34,300	1.7	0.8	9.0
13.2	5,100	117,000	1.5	0.8	8.5
13.2	8,500	194,000	0.6	0.5	6.0
13.2	8,900	203,000	0.6	0.5	6.0
13.2	9,600	219,000	0.6	0.5	6.0

Three-phase 60-cycle interrupting tests on the downward break 15,000-volt outdoor breaker with oil blast feature shown in Fig. 6.

Rate of recovery voltage rise 1,400 volts per microsecond.

Arc durations and lengths included only the interrupting gap.

Maximum arc lengths and durations of the three phases are recorded.

In this case, a piston acting in a cylinder containing oil is used in place of the pressure gap to inject the blast of oil into the gap between the interrupting contacts. Fig. 7 shows the interrupting element of one of these breakers with the contacts in the closed and opening positions. When interrupting a short circuit, the spring charged mechanism pulls the contacts apart at the same time the piston moves downward. The baffle around the moving contact directs the oil flow created by the piston movement over the stationary contact upward through the hollow contact rod into the air chamber. The path of the rapid stream of oil is thus directly through the arc path so that at the first current zero, further flow of current is prevented by the high insulation of the oil that has replaced the arc products.

Quick tripping of this breaker is obtained by the utilization of a very simple mechanism which is released to open by means of a magnetic tripping mechanism of the type used on high-speed air breakers.[9]

TABLE V

Test Volts	Current Peak Values	Current Interrupted Amperes	Arc Duration Cycles	Short-Circuit Duration Cycles
12,000	3,900	2,500	0.15	0.5
12,000	24,000	14,000	0.35	1.0
13,200	30,000	18,000	0.30	1.0
13,200	20,000	13,000	0.45	1.0
13,200	35,000	24,000	0.30	0.5
7,000	65,000	39,000	0.40	1.0
7,000	73,000	47,000	0.30	0.5

Interrupting tests outdoor at 25 cycles single phase on high-speed breaker with oil blast features shown in Fig. 7.

Rate of recovery voltage rise—range 400 to 800 volts per microsecond.

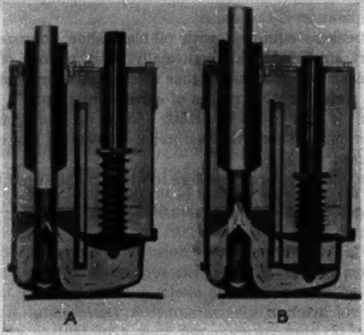

FIG. 7—INTERRUPTING ELEMENT OF A 15,000-VOLT 1,500-AMPERE BREAKER DESIGNED TO INTERRUPT 50,000 AMPERES IN 0.04 SECOND AT 25 CYCLES

Oil-blast obtained mechanically
A. Contacts closed
B. Contacts during arcing period

Table V shows data on a few of a series of 49 tests made on one of the 12,000-volt high-speed breakers. After the first 31 tests at currents interrupted ranging from 2,500 to 18,000 amperes at 12,000 or 13,000 volts, the contacts were examined and found to be only moderately burned.

After the 31st operation, new oil and contacts were put in the breaker. Then after six interruptions, a series of 12 tests was made at 12,000 volts and currents interrupted ranged from 21,000 to 35,000 amperes. The time interval between tests varied from 15 to 40 seconds. After these tests, the contacts were only moderately burned. The oil tested 12.2 kv. in a standard gap.

The points worthy of emphasis in the performance of this breaker are that the mechanically created oil-blast which gives short arcing times can be readily combined with a mechanism and trip which operate so quickly that the short circuit can be interrupted at the first current zero after the short circuit begins. This rapid clearing occurs at both low and high currents interrupted.

CONCLUSIONS

The oil-blast principle of circuit interruption has been applied to oil circuit breakers for high and low voltages and small and large interrupting ratings. Parts can also be supplied to apply this principle to many types of breakers already in service. Although the mechanical construction of the interrupting elements may vary for different requirements as is shown by the ability to obtain the results either mechanically or electrically, nevertheless the fundamental principle of operation is the same.

Breakers equipped with oil-blast show consistently short arc lengths and unusually short periods of arc duration, even at the higher rates of recovery voltage rise and over wide ranges of current interrupted. The result is that contact burning and oil deterioration are so minimized that the maintenance required is greatly reduced.

In many types of breakers to which oil-blast has been applied, it has been possible to increase the interrupting rating of the structure. Many types of breakers already in service may be modified to incorporate the oil-blast feature and obtain high-speed operation. Also, in the newer designs, the interrupting capacity per unit volume is much greater than that previously obtained.

The rapid arc-extinction characteristic of the oil-blast design makes it admirably adaptable to breakers requiring high speed of operation. Such high speed improves system stability, minimizes system voltage dips, and improves service in many other ways.

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Discussion

For discussion of this paper see page 191.

Recent Developments in Arc Rupturing Devices

BY R. C. VAN SICKLE* and W. M. LEEDS*
Non-member Non-member

THE deion grid method of arc rupture has been very successfully applied during the past two years to the design of an extensive line of arc rupturing devices for oil circuit breakers of all voltage classes. The soundness of the fundamental theory underlying this development has been amply demonstrated by the remarkable improvement in operation shown by circuit breakers of every voltage class when designed according to this principle. This paper discusses later developments in this theory, describes the newest types of arc rupturing devices embodying this principle, and presents results obtained with them in special tests and during operating service in the field.

THEORY

The fundamental operating principles and the structural arrangement of the deion grid have been discussed in previous papers before the Institute,[1,2] but recent developments in arc rupturing phenomena make it desirable to review briefly the manner in which this device, Figs. 1 and 2, functions to extinguish an arc. The separation of the circuit breaker contacts draws the arc in a narrow slot open at one end and filled with oil. At intervals through the stack of plates making up the grid, there are interspersed plates of magnetic material which surround the slot on three sides and so modify the magnetic field of the arc as to produce a force which not only keeps the arc within the slot but also moves it back into a series of oil pockets. The intense heat of the arc vaporizes and decomposes the oil, generating gas which can escape only through the open end of the slot. In doing so the gas must pass through the arc stream, subjecting this ionized path to a continuous turbulent blast of fresh un-ionized gas. Furthermore, since this generation of gas is continuous throughout the full length of the arc, the deionization due to the automatic gas blast takes place not alone in some selected portion of the arc but continuously and evenly throughout its entire length. The magnetic blowout is used to advantage in this device by applying the forces so as to control and hold the arc without appreciably lengthening it.

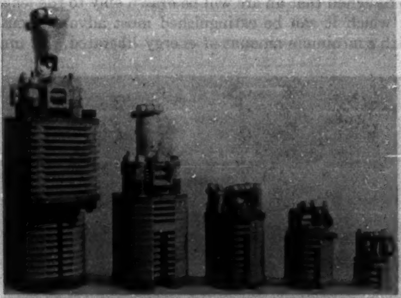

FIG. 1—DEION GRIDS SIZES *A*, *B*, *C*, *D* AND *E* FOR OUTDOOR OIL CIRCUIT BREAKERS

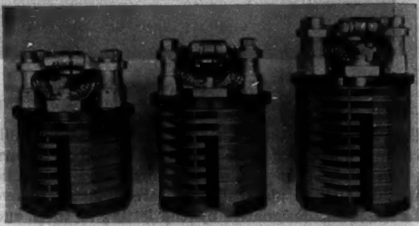

FIG. 2—DEION GRIDS FOR 34.5 KV., 46 KV. AND 69 KV. MADE OF *C* SIZE UNITS

According to the theory recently discussed by Slepian before the Institute[3] and given experimental confirmation by Browne,[4] the turbulent condition of the arc stream causes the arc to be broken up into many parallel arcs of very small cross section, so that the complete stream may be considered as a number of fine threads swirling about in tumultuous motion, these threads being highly ionized and carrying most of the current. At the current zero, diffusion from these slender threads of intense ionization into the fresh un-ionized gas all about them causes the whole arcing space to assume almost immediately a comparatively uniform low ion density so that in a very few microseconds dielectric strength of perhaps twenty times the arc voltage is established between the contacts. Further deionization due to recombination and diffusion continues, aided by the gas blast, but it is the extremely rapid initial rate of recovery of dielectric strength that makes possible the interruption of an arc under unusually difficult circuit conditions. It is evident that a high degree of efficiency is obtained with deion grids since all of the gases generated are caused to pass directly through the arc stream in order to escape. It is thus possible to produce a high rate of deionization in the arc path with a minimum amount of gas generated and a correspondingly small amount of energy liberated within the circuit breaker tank.

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

APPLICATION

Previous discussions before the Institute[2-5] have set forth the constructions by which these principles have been applied in high voltage, outdoor circuit breakers from 34.5 kv. to 230 kv., and this paper describes the later developments applying the same principles to the 15 kv. and 23 kv. outdoor breakers, particularly the low capacity breakers for automatic reclosing service, and to the power house class of oil circuit breakers at 15 kv. For purposes of comparison, representative grids for different voltage applications are shown in Fig. 1. It will be noted that the grids are essentially stacks built up from a number of units, each unit consisting in turn of thin plate elements of insulating and magnetic materials. These plate elements are designed in five different sizes, designated by the letters *A* to *E* and each applicable within a certain range of voltage. The size of the plate varies roughly with the voltage rating in which it is applied, the largest plate being used for the highest voltage. The number of units in each stack is varied according to voltage requirements as may be seen from Fig. 2 in which three grids for different voltages are shown built up from plates of the *C* size.

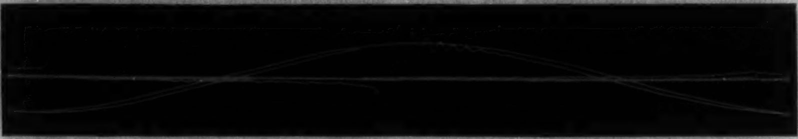

FIG. 3—A CATHODE RAY OSCILLOGRAM OF A SINGLE-PHASE INTERRUPTION BY *A* GRIDS OF 3,300 AMPERES AT 132 KV.

It will be apparent on reflection that in applying the grids over this wide range of service voltages, many diverse conditions are encountered and many different aspects of the problem must be solved. For instance, in the power industry today circuit breakers with rupturing capacities up to 2,500,000 kva. are required both at 15,000 volts and at 230,000 volts, as well as at the intermediate voltages, but the magnitudes of the currents interrupted at this kva. rating differ widely at the two extremes of voltage. At 230 kv. the current interrupted per phase is less than 6,500 amperes while the voltage per contact break is very high. The outstanding problem in operating service at this voltage is one of system stability under short-circuit conditions and the desirability of any circuit breaker for such service is gaged by its ability to interrupt short circuits with a minimum duration of arcing. Therefore, the grid for such service must be inherently a voltage device and all the elements of its design must be shaped to the purpose of building up dielectric strength in the arc path at the highest possible rate in order to prevent fast-rising, high recovery voltages from restriking the arc after a current zero.

At 15,000 volts the conditions are quite different. The voltage per contact break is comparatively low and presents no great difficulties for the modern oil circuit breaker. System disturbances due to voltage surges constitute an operating problem of relatively minor importance and the desirability of any circuit breaker for this application is measured, not by the shortness of its arcing period alone, but by its ability to interrupt heavy short circuit currents without emitting oil or flame and without mechanical injury to its operating parts. The breaker must also be able to withstand a reasonable number of repeated cycles of duty without impairing its adequacy for normal switching service immediately afterward. Since the short-circuit current per phase in this service approaches a maximum of 100,000 amperes, the grid for such an application must be inherently a current rupturing device with its parts so designed that an arc will be drawn only to the length at which it can be extinguished most advantageously with a minimum amount of energy liberated. It must also dissipate the gases which are formed and protect the circuit breaker itself from damage due to the presence of high powered arcs.

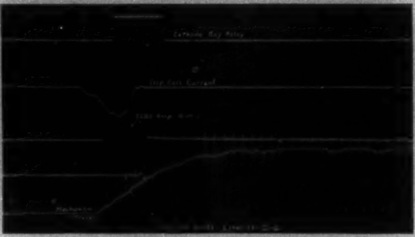

FIG. 4—A MAGNETIC OSCILLOGRAM CORRESPONDING TO FIG. 3

The varied, and in some cases conflicting, requirements for this wide range of service have been met and circuit breakers equipped with deion grids are now available in nearly every breaker class. These grids are all similar and vary gradually from high capacity, high voltage or *A* size to the low capacity, low voltage or *E* size. They are the result not only of the application of a theory, but also of extensive testing and development in both field[6,7] and laboratory. The use of a cathode ray oscillograph is of great assistance in studying the arc voltage since it records the phenomena accurately on a large time axis. One cycle on a cathode-ray oscillogram, Fig. 3, occupies as much distance on the time axis as 75 cycles on a corresponding magnetic oscillogram, Fig. 4.

The length of all of these grids, while graded to the voltage for which they are designed, is such that arc extinction ordinarily takes place before the movable arc-drawing contact is half way down the grid. This large factor of safety is provided for two reasons; first, to insure adequate rupturing ability in the event of a breaker being subjected to unusually difficult circuit conditions involving extraordinarily rapid rates of rise of recovery voltage, or of its being unwittingly subjected to excessive overvoltages; second, to provide for any possible loss of effectiveness of one or more of the uppermost units in the grid due to unusual duty in the form of a prolonged series of repeated heavy current interruptions without an opportunity for inspection. It will be noted that each individual unit in the grid starts to produce its own horizontal gas blast as soon as the arc reaches it, so that the longer the arc, the greater the number of units which are brought into play against it. A great advantage is thus obtained over any device that restricts its deionizing activity to one or at most two points of the arc.

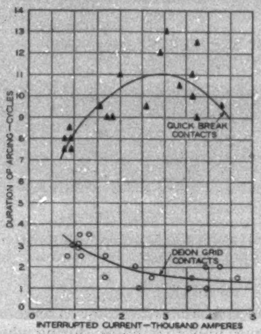

Fig. 5—Curves Showing the Reduction in Arcing Time on a 66-Kv. Circuit by Substituting Deion Grids for Quick-Break Contacts

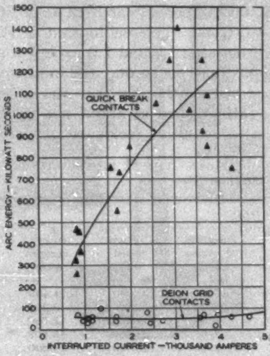

Fig. 6—Curves Showing the Reduction in Arc Energy on a 66-Kv. Circuit by Substituting Deion Grids for Quick-Break Contacts

The remarkable improvement in the interrupting characteristics of the outdoor breakers resulting from the replacing of plain or quick break contacts by deion grids is illustrated by several representative curves taken from laboratory tests. Fig. 5 shows that the duration of arcing on a 66-kv. circuit was reduced to between one-fourth and one-seventh of the former arcing time by the use of deion grids. This result is of great importance in connection with the effect of short-circuit duration on system stability. For the same series of tests, Fig. 6 shows that the arc energy with deion grids was only 5 per cent to 10 per cent of the values associated with conventional quick break contacts. This means a corresponding reduction in contact depreciation and oil deterioration and makes possible long periods of service without maintenance to the breaker.

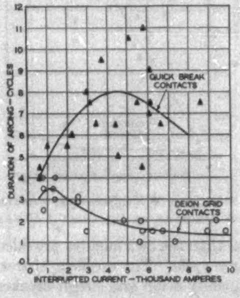

Fig. 7—Duration of Arcing at 38 Kv. Before and After Installing Deion Grids in an Oil Circuit Breaker

Tests on a 38-kv. circuit gave the results shown in Figs. 7 and 8. The deion grids used in these tests were of the *C* size and the improvement over the old style contacts is just as pronounced. The duration of arcing is shortened about 75 per cent while the arc energy is reduced to less than 10 per cent of the value with quick break contacts. It should be remembered that all of these tests were made on a low power factor laboratory circuit involving a much higher rate of rise of recovery voltage than is usually found in actual service in the field. Even better performance of the circuit breakers is to be found under average operating conditions as is confirmed by the data given later in this paper:

Automatic Reclosing Feeder Breakers

In the class of circuit breakers for automatic reclosing service on distribution feeder circuits there is a growing demand for breakers which may be subjected to repeated cycles of interrupting duty without requiring any maintenance. These breakers are also required to reclose after clearing a short circuit with a much smaller time interval than the two minutes provided for in the standard duty cycle. The excellent characteristics of moderate capacity deion grid circuit breakers designed for this type of service have been demonstrated by many series of tests similar to the following.

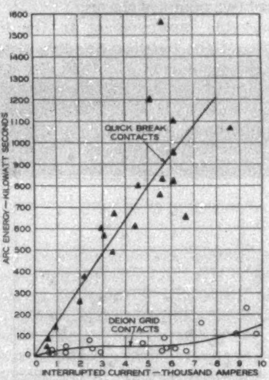

Fig. 8—Arc Energy at 38 Kv. Before and After Installing Deion Grids in an Oil Circuit Breaker

A 15-kv. oil circuit breaker equipped with *E* size deion grids was subjected to a series of 21 short circuits of approximately 14,000 amperes at 3,810 volts, the duty cycle consisting of one *CO* plus two *OCO* tests at zero and 120 second time intervals. The complete series of tests occupied only 45 minutes, during which time the tanks were not even lowered for inspection of the contacts. The data for these tests are given in Table I. The interrupted current varied from 12,200 amperes to 15,300 amperes, while the breaker closed against currents from 15,000 amperes to as high as 28,800 amperes during the *OCO* tests. The oscillogram in Fig. 9 shows an average of between 1.0 and 1.5 cycles of arcing on a frequency of 60 cycles per second and indicates a breaker time from the instant the trip coil was energized until the arc was extinguished of about six cycles. It will be noted that the initial reclosure occurred only 24

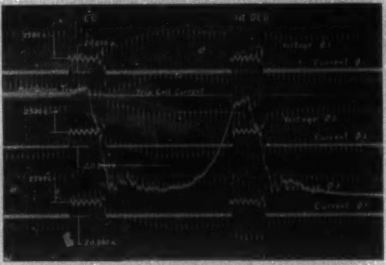

Fig. 9—Oscillogram of a *CO-OCO* Part of a Series of 21 Successive Interruptions Made on a 15-Kv. Deion Grid Oil Circuit Breaker Equipped for Automatic Reclosing Service

TABLE I—15-KV. DEION GRID OIL CIRCUIT BREAKER RECLOSING TESTS AT 3,810 VOLTS—THREE PHASE—UNGROUNDED—60 CYCLES DUTY CYCLE: *CO*—0 SECONDS—*OCO*—120 SECONDS—*OCO*

			CO			1st *OCO*			2nd *OCO*	
Test No.	Time of test p.m.	Phase No.	Interr. current amp.	Duration of arcing cycles	Current closed amp.	Interr. current amp.	Duration of arcing cycles	Current closed amp.	Interr. current amp.	Duration of arcing cycles
29853	5:20	1	13,500	2.0	28,800	14,400	1.0	27,200	14,400	1.0
		2	12,800	2.0	26,800	12,300	1.5	21,300	12,300	1.0
		3	13,300	1.5	16,800	12,700	1.5	18,300	12,700	1.0
29854	5:30	1	14,400	1.5	22,400	13,400	2.0	21,500	15,300	1.5
		2	12,800	1.0	17,100	12,400	2.0	18,800	13,600	1.5
		3	13,000	1.5	17,100	13,000	2.0	27,900	13,000	1.0
29855	5:36	1	14,100	1.0	28,000	13,600	1.5	22,400	13,400	2.0
		2	12,800	1.0	21,700	12,800	1.5	16,200	12,800	1.5
		3	14,600	1.5	16,100	12,700	1.5	23,300	13,300	1.5
29856	5:43	1	13,800	1.0	26,600	12,800	2.0	24,000	13,800	2.0
		2	13,600	1.5	17,000	12,000	2.0	16,200	13,600	1.5
		3	13,700	1.0	18,600	12,400	1.5	23,300	13,700	1.5
29857	5:49	1	13,500	1.0	21,400	13,400	2.0			
		2	12,400	1.5	15,000	12,800	2.5	No record		
		3	14,000	1.0	23,300	13,100	1.5			
29858	5:55	1	13,400	1.0	24,000	12,450	1.5	20,800	14,550	1.5
		2	13,200	1.5	20,850	15,000	1.5	24,300	13,200	1.5
		3	13,950	1.0	15,300	13,000	1.5	18,600	13,950	1.0
29859	6:01	1	14,100	1.5	19,200	12,800	1.5	24,000	12,200	1.0
		2	12,800	1.5	16,200	12,000	1.5	22,500	12,800	1.0
		3	13,600	1.5	17,400	12,400	1.5	17,100	12,700	0.5

cycles or 0.4 second after the first short circuit was cleared, this time interval being about as short as could be obtained with the reclosing relay equipment.

The breaker performed very satisfactorily during these tests. No appreciable drop in oil level was observed, the oil loss being limited to a few drops which seeped out between the operating shaft and bushing. Pressure records were obtained on several of the tests but the values were only about 15 pounds per square inch. Considering the severe service applied to the breaker the oil depreciation was quite moderate, the dielectric strength changing from an average value of 25.8 kv. before the tests to 11.4 kv. after the twenty-first test. When the contacts were examined at the conclusion of the tests, see Fig. 10, they were found to be in condition for further service without maintenance.

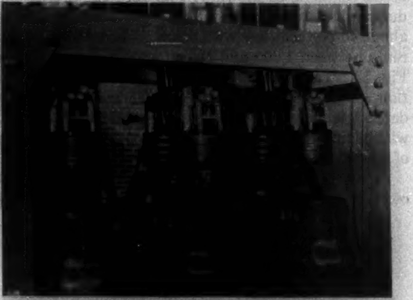

Fig. 10—The 15-Kv. Automatic Reclosing Breaker After Interrupting a Series of 21 Successive Short Circuits

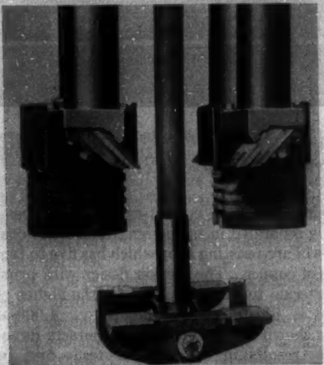

Fig. 11—Heavy Duty Deion Grid Contact Assembly for a 15,000-Volt, 4,000-Ampere Power-House Breaker

Power-House Breakers

As in the case of the various classes of outdoor breakers, the application of deion grids to the power-house class of service presented its own individual problems of design. Since modern powerhouse breakers interrupt heavy short circuits with a duration of arcing close to one cycle at 60-cycle frequency, due to the effectiveness of the inherent magnetic blowout of the current loop, improvement in operation by the application of a more effective form of interrupting device is to be expected more in reduced disturbance and oil deterioration than in shorter arcing time.

In circuit breakers with deion grid contacts the arc is confined in a slot and prevented from moving rapidly by the oil trapped in the pockets. Therefore, the total length of arc is not much greater than the actual contact separation and the arc voltage is kept to a minimum. This reduction in arc voltage combined with the short arcing time decreases the total amount of arc energy, resulting in turn in a smaller volume of gas generated, lower pressures, less depreciation in the dielectric strength of the oil, and less tendency for the breaker to discharge gas and oil during the interrupting process.

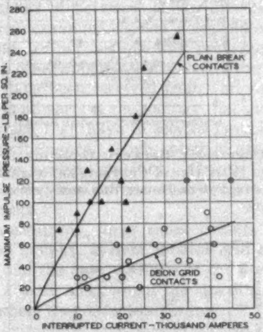

Fig. 12—The Reduction in Pressure During Laboratory Interrupting Tests at 13.2 Kv. Obtained by Replacing Plain Break Contacts by Heavy Duty Deion Grids in Power-House Breakers

The requirements involved in the interruption of heavy currents at low voltage have resulted in the design of a heavy duty form of deion grid for powerhouse service, illustrated in Fig. 11. Certain details of the plate elements are modified so as to provide a more sturdy construction and to give proper means of control of the heavier current arcs. All of the insulating plates are made of fiber and are of sufficient thickness to give ample mechanical strength to the grid. Large oil pockets provide an adequate supply of oil in the grids. A vent located immediately below the top plate of the grid relieves the initial pressure impulse as the arc first appears on the arc-drawing contacts.

It is now generally recognized that the performance of oil circuit breakers is much better when operating on commercial circuits than when connected directly to a laboratory test generator because of the lower rate of rise of recovery voltage. An oscillogram of a field test in which a plain break circuit breaker interrupted 21,000 amperes at 12,700 volts with only one-half cycle of arcing is shown in Fig. 15. On a laboratory circuit a similar breaker interrupting 23,500 amperes with 7,620 volts across a single pole arced for two cycles as shown by the oscillogram in Fig. 16. However, this same breaker when equipped with deion grids and tested on this same circuit interrupted 21,500 amperes at 7,620 volts across a single pole with only one cycle of arcing as is shown by the oscillogram in Fig. 17. Corresponding reductions in the pressure and in the arc energy may be seen on the oscillograms. It is evident that the results of laboratory tests, while not necessarily representative of the speed of breaker operation on the average commercial circuit, give an excellent means for comparing the performance of different types of arc rupturing devices.

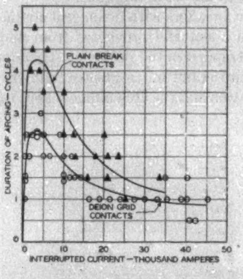

Fig. 13—Comparison Between Duration of Arcing at 13.2 Kv. with Plain Break Contacts and with Heavy Duty Deion Grids in Power-House Breakers from Laboratory Test Results

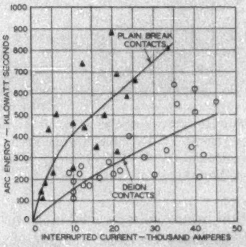

Fig. 14—Comparison Between Arc Energy at 13.2 Kv. with Plain Break Contacts and with Heavy Duty Deion Grids in Power-House Breakers from Laboratory Test Results

By keeping down the rate of liberation of arc energy by control of the arc length and by absorbing some of the impulse in the grid structure, the maximum pressures in powerhouse breakers are actually reduced to about one-third of the values obtained with the same breaker when equipped with the conventional plain break contacts, as shown by Fig. 12. The arcing time during these tests on the difficult laboratory circuit was about 40 per cent less with deion grid contacts for currents between 1,000 and 10,000 amperes, as shown by Fig. 13. The arc energy, upon which the oil depreciation, gas generation, pressure and tendency to throw oil depend, is shown in Fig. 14 to be less than half the value with plain break contacts over the entire current range of a 1,000,000-kva. power-house breaker.

For the lower normal current ratings the arc-drawing contact acts as a main current-carrying member as well, the arcing and main current-carrying surfaces being carefully separated in the design. The arc is drawn on surfaces of arc-resisting alloy which has five to ten times the life of copper. The breaker design with brush contacts external to the grids, used for the higher current ratings, is illustrated by the 15-kv., 4,000-ampere breaker shown in Fig. 11. These contacts have given excellent results in development tests, and brushes rated at 2,000 amperes can carry short circuits up to peak values of 200,000 amperes without indications of burning.

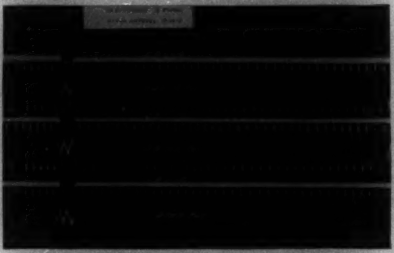

Fig. 15—Oscillogram Showing the Interruption by Plain Break Contacts of a 21,000-Ampere, 12,700-Volt, Three-Phase Short Circuit on a Large System

The principles of the deion grid method of arc rupture are now being applied to single-tank breakers in the 4,500, 7,500 and 15,000 volt classes, and improved operation comparable with that obtained on the larger power-house breakers is expected.

Tests

The results of a number of field tests on high-voltage oil circuit breakers equipped with deion grids have

TABLE II—50-KV. DEION GRID OIL CIRCUIT BREAKER SINGLE-PHASE-TO-GROUND SHORT-CIRCUIT TESTS ON A 44-KV. SYSTEM

Test No.	Type of duty cycle	Current interr. amp.	Kilovolts to ground: Before fault	Kilovolts to ground: After fault	Kilovolts to ground: During fault	Oil test: Before kv.	Oil test: After kv.	Arcing time cycles	Ground
1	CO	3500	25.5	25.6	2.2	29.0	28.4	—	Arcing
2	CO	3900	26.3	27.3	2.2	28.4	28.0	½	Solid
3	CO	4000	25.6	25.7	2.2	29.0	29.7	½	Arcing
4	CO	3900	25.6	25.6	2.5	29.8	28.0	½	Arcing
5,6,7,8	4-OCO 1 minute intervals	3915	25.6	25.6	2.2	28.0	24.8	½	Solid

already been reported.[6-7] Among the lower voltage breaker classes several series of tests of considerable interest have been made recently. The difference in operation of the breakers before and after the installation of deion grids is most striking.

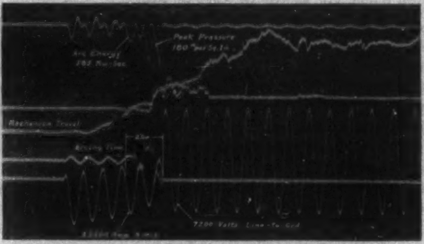

FIG. 16—OSCILLOGRAM SHOWING THE INTERRUPTION BY PLAIN BREAK CONTACTS OF A 23,500-AMPERE LINE-TO-GROUND SHORT CIRCUIT ON A 13,200-VOLT LABORATORY TEST GENERATOR

On one of the large 44-kv. systems, the interrupting duty imposed upon the breaker in a certain location was so heavy due to unusually severe circuit conditions that a single operation was sufficient to blacken the oil. Gas and oil were usually thrown from the breaker and the arc was cleared only by utilizing practically the full contact break distance.

The old style contacts in one of these breakers were replaced by deion grids and the breaker subjected to a series of interrupting tests. An extract from the tabulated data prepared by the operating company is reproduced in Table II. There was no throwing of oil and no sign of distress. The last five tests were made on one pole of the breaker, and the oil tested almost as high after the tests as before.

A smaller circuit breaker rated at 350,000 kva. at 37 kv., was equipped with deion grids and its performance determined by a series of field tests on one of the largest 26.4-kv. systems. The breaker was first subjected to a 2-*OCO* duty cycle at approximately 12,000 amperes, followed after a short interval by one *OCO* test of approximately 16,000 amperes. This last test imposed an interrupting duty of 750,000 kva. on the breaker, which was more than twice its rating with the old style contacts. Although this was rather heavy service for a breaker with 20-in. diameter tanks, there was no external disturbance on any of the tests other than a small discharge of gas from the vent. The oscillograms showed only one cycle of arcing. After the three interruptions the oil was in excellent condition, having depreciated from 28 kv. to an average value for the three tanks of between 23 kv. and 24 kv. The contacts when examined showed a negligible amount of burning and further testing was prevented only by trouble with other apparatus involved in the test set-up.

Reports have been received from various operating companies in all parts of the country that the deion grid breakers which they have in service have been giving excellent performance. High speed of arc rupture is giving better system stability. Greatly reduced oil depreciation and less contact maintenance is lowering operating costs.

After interrupting three short circuits caused by lightning on a 33-kv. system, a deion grid breaker was examined, and the only evidence of the service experienced was a small mark about the size of a pin located at a point where the contacts separated.

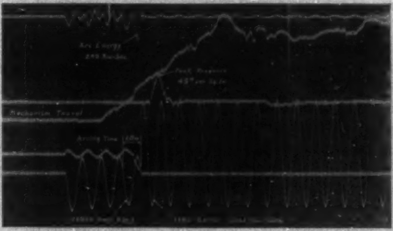

FIG. 17—OSCILLOGRAM SHOWING THE INTERRUPTION BY DEION GRID CONTACTS OF A 21,500-AMPERE LINE-TO-GROUND SHORT CIRCUIT ON A 13,200-VOLT LABORATORY TEST GENERATOR

On another system of the same voltage, eighteen short circuits were cleared by a deion grid circuit breaker before the oil, which then tested 17 kv., was changed. The same company reported that one of its 132-kv. oil circuit breakers was equipped with deion grids and then not examined until after it had operated 21 times due to lightning surges. Although each time it had interrupted from 50 per cent to 75 per cent of its rupturing capacity, the contacts, and oil were found to be in such excellent condition that the breaker was put back into service immediately without any maintenance whatsoever.

CONCLUSIONS

The use of arc rupturing devices employing oil have many inherent advantages, and their disadvantages are being eliminated or minimized by recent developments. With modern methods of controlled arc extinction and a better understanding of the effect of external circuit conditions, uncertain operation and alarming signs of distress during even moderate rupturing duty may be considered almost a thing of the past. Of especial interest to the operating companies is the ease with which old contacts can be replaced by modern deion grids, giving to the breakers greatly improved performance at relatively small expense.

The arc rupturing devices which have resulted from the scientific analysis of the methods of arc extinction have made possible greatly improved interrupting performance and even further progress in the circuit breaker art may be confidently expected.

The authors wish to express their appreciation of the advice and assistance given them by Mr. J. B. MacNeill and Mr. H. M. Wilcox.

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Discussion

For discussion of this paper see page 191.

Extinction of A-C. Arcs in Turbulent Gases

BY T. E. BROWNE, JR.*

Associate, A.I.E.E.

Synopsis.—A recent theory of a-c. arc extinction in gas-blast circuit interrupters is reviewed. It is predicted on the basis of this theory that the interrupting ability of an a-c. arc should increase very considerably with the degree of turbulence to which the arc gases are subjected, and also that the recovery of dielectric strength by a turbulent arc space during a period of zero current should be very rapid for the first few microseconds and then comparatively slow for a considerable time thereafter. Experimental results obtained with an arc in atmospheres of various common gases verify these predictions, and also reveal the importance of the gas medium.

A curve has also been obtained showing recovery of dielectric strength with time during the current zero period by an arc in an expulsion fuse tube. This curve has the characteristic shape predicted by the turbulence theory, thus proving the validity of the theory as applied to the behavior of a practical gas-blast device.

* * * * *

I. INTRODUCTION

Of the published theory relating to a-c. arc extinction, that which explains the behavior of short arcs, depending almost entirely upon phenomena taking place at, or very near, the surfaces of the electrodes, is far more complete and in better agreement with experiment than that relating to long arcs, whose behavior is affected very little by the phenomena occurring very close to the electrodes. The latter, however, are probably of much greater practical importance, since it is with such arcs that engineers must deal in most present-day types of circuit interrupters operating in a-c. circuits of more than a hundred or so volts. Most of the practical circuit interruption difficulties are due to the fact that ordinary unconfined arcs must be drawn out to prohibitive lengths to interrupt power circuits of the higher voltages now in use. Because of this fact, it has been necessary to modify the arc in some way so as to increase the voltage gradient at which it may be extinguished. The various methods by which such modification has been accomplished usually result in a blast of gas being directed into or through the arc stream, which is confined to a restricted space. In circuit breakers employing arcs submerged in liquids, such as oil, carbon tetrachloride, or water, the surrounding liquid surface serves both to restrict the arc space and to furnish the gas blast by its rapid decomposition under the influence of the high-temperature arc. The fiber tube of the common expulsion fuse acts in a similar manner. In liquid circuit breakers, additional surfaces, magnetic fields, gas chambers, hollow electrodes, and so on, may be used to increase the effectiveness of the interaction between arc and liquid. Recently use has also been made of blasts of air,[1-4] carbon dioxide[1a] and hydrogen[1,4,5] directed by mechanical means into suitably confined arc spaces from high-pressure storage tanks.

The great effectiveness of gas blasts in aiding a-c. arc extinction is thus a well established experimental fact, but, though a number of theories has been suggested, there has been presented so far very little definitely conclusive evidence of an experimental nature to indicate the actual mechanism of their action. It is the purpose of this paper to describe some recent experiments that have been and are being made in an effort to obtain such information.

II-A. REVIEW OF THEORY

It is now well established that the extinction of an a-c. arc in any practical circuit interrupter occurs at a moment of zero current, and is the result of a sudden transition of the arc space from a conducting to an insulating state. Since, according to modern theory, electricity is conducted through a gas by the motion of positive and negative ions, it follows that loss of conductivity by a gas must be due to disappearance of the necessary ions. In fact, the various means used in practise to accomplish the extinction of an a-c. arc can generally be shown by sufficiently complete analysis to be means of accelerating the removal of ions from, or deionization of, the gaseous arc space. The time available for this deionization, between current zero and the appearance of full circuit voltage across the arc electrodes, has been shown[6,7,8] to vary from about ten microseconds to several hundred microseconds.

Of the various theories put forward to explain the effectiveness of different types of gas-blast circuit interrupters, that proposed by Slepian,[9] and further developed by Slepian and Denault[10] in connection with the expulsion fuse, seems to the author to be the most general in its applicability and to escape objections that apply to other theories. In it, the major deionizing effect of a gas blast is ascribed to "the high degree of turbulence it introduces into the bounded gas space prior to current zero,"[10] rather than to actual sweeping away of ions,[1] introduction of a non-gaseous insulating medium,[11] "loading" of ions,[3] cooling the arc,[5] and so on, as suggested by proponents of other theories. The action of this turbulence is pictured as causing non-uniformity of ionization of the arc space, so that many comparatively small regions, or "filaments," of highly ionized and conducting gas are interspersed with, or surrounded by, masses of very much less intensely ionized gas, either freshly introduced, or deionized by recent contact with the bounding walls in the course of its turbulent motion. At current zero, when the generation of ions within the conducting regions is momentarily stopped, the dimensions of these regions are such that they very quickly lose their high density of ionization by diffusion into the adjacent non-conducting gas, resulting in a nearly uniform ion distribution having a very much smaller maximum density, and consequently requiring a very much higher breakdown gradient.[9]

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*Research Laboratory, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

1-4. For references see end of paper.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

II-B. PREDICTIONS OF THEORY

On the basis of this theory, then, the dielectric recovery of a turbulent arc space may be thought of as occurring in two stages. The first consists of a very fast rise resulting from the rapid diffusion of ions from the small filaments into the surrounding gas, and continues until a condition of approximately uniform ion density is reached. Following this is the second stage marked by a very much slower further recovery as ions are lost from the arc space by the means usually considered, such as diffusion to surrounding surfaces, recombination, "whisking away," and so forth. This is illustrated by the idealized curve *A B C* of Fig. 1, where *A B* represents the first stage and *B C* the second stage of the dielectric recovery. Since the slope of the line *A B* will depend upon the smallness of the conducting filaments at current zero, it follows that an increase in the turbulence of the arc space preceding current zero will result in an increase in this slope and a decrease in the time required to reach the uniform condition at *B*. For very small conducting regions such as may be expected with a high degree of turbulence, this time may be only a few microseconds. The effect of turbulence alone on the slower recovery *B C* is probably not nearly so great.

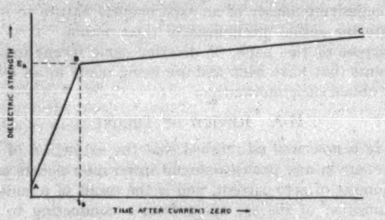

FIG. 1—RECOVERY OF DIELECTRIC STRENGTH OF TURBULENT ARC SPACE, AS PREDICTED BY THEORY

From these considerations, the effect of increasing turbulence on the interrupting ability of an arc in a circuit of given speed may be predicted. As long as the slope of *A B* is such that full circuit voltage is recovered in less than the time t_B, increasing the turbulence will yield a rapid increase in the volts-per-centimeter at which the arc will be extinguished. If t_B should become less than the circuit transient period, however, the improvement resulting from a further increase in turbulence would be only in proportion to the much smaller effect on the height of the line *B C*. This apparent maximum effect of turbulence should appear sooner with slow circuits and with arcs in rapidly diffusing gases, such as hydrogen. Of course, in practical gas-blast devices, the magnitude of the maximum short-time dielectric strength E_B and the time t_B required to reach it will also depend very largely upon conditions other than turbulence existing in the arc space prior to current zero. Such conditions, which govern the average density of ionization throughout the actual arc space at current zero and before, are current magnitude, proximity of confining walls,[6] rate of dilution by fresh gas, presence of solid or liquid bodies, or particles, within the arc space,[10] and composition of the arc gases.[4,5] In practise, of course, the volt-time recovery characteristic of the arc would hardly consist of straight lines and sharp angles, as in Fig. 1, but it may be expected to follow the general direction of these lines.

III-A. EXPERIMENTS WITH ROTATING ARC STRUCTURE

To test the validity of this theory, the arcing structure shown in Fig. 2 was used. In it, factors other than turbulence to which gas-blast effectiveness might be ascribed, such as the cooling, diluting, and scavenging effects of a high-velocity stream of fresh gas, were eliminated by dispensing with the gas blast, as such, and producing turbulence by driving the arc around the circular slot at high velocity by means of a strong radial magnetic field produced by passing direct current through the coils in such a direction that the radial components of their fields were additive in the region of the circular arc space. As illustrated, the apparatus was arranged so that the air in the slot could be replaced by other gases when desired. Ordinary commercial forms of oxygen, nitrogen, hydrogen, and carbon dioxide, as purchased in tanks, were used and a small flow of gas was maintained during the tests to prevent diffusion

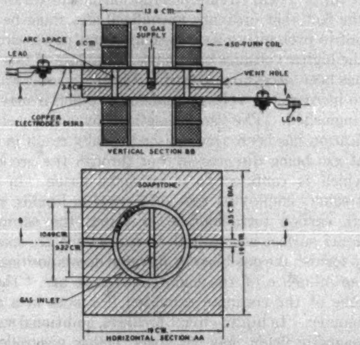

FIG. 2—EXPERIMENTAL ARCING STRUCTURE

of air back into the apparatus. The arc was initiated within the slot by the burning of a fine copper fuse threaded between the two copper-disk electrodes.

Fig. 3 gives the results of measurements, by means of probe electrodes and oscillograph, of the average speed of a 600-ampere, 60-cycle arc at different strengths of the driving magnetic field (produced by direct current) in the 0.635 cm. slot with atmospheres of oxygen, carbon dioxide, air and hydrogen. The speed for a given driving force seems to increase with decreasing density of the gas, if carbon dioxide is presumed to be almost entirely decomposed by the high arc temperature into carbon monoxide and oxygen. Fig. 4 shows similar results for a 1.9 cm. slot, obtained by reducing the diameter of the inner cylinder. The interchange of position, in this case, of the curves for carbon dioxide and for air is difficult to understand, and suggests that the process by which the arc moves through the gas under the influence of the magnetic field is not simple. The large variation shown by the scattering of the points, especially at the higher speeds, also suggests this. However, it is believed that, in general, the body of ionized conducting gas, or arc core, is driven against and through the surrounding non-conducting gas much as if it were an ordinary solid conductor, but having a less definitely marked bounding surface. The action of the moving arc on the rest of the gas in the slot is then similar to that of a leaky piston, driving all of the gas around the slot at a somewhat lower speed than that of the arc itself.

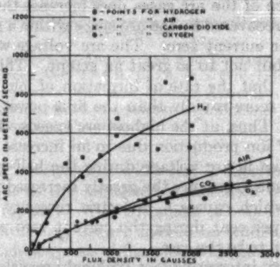

FIG. 3—AVERAGE SPEED OF A 600-AMPERE, 60-CYCLE ARC, 3.8 CM. LONG MOVING IN A CIRCULAR SOAPSTONE SLOT 0.635 CM. WIDE IN VARIOUS GASES AT ATMOSPHERIC PRESSURE, AS A FUNCTION OF AVERAGE RADIAL FLUX DENSITY IN THE SLOT. (D-C. FIELD)

Fig. 5 shows that, with a constant field strength, increasing the arc current increases the speed of the arc according to a curve somewhat similar in shape to that of Fig. 3 with constant current. Detailed comparison of the two curves, however, shows that for the same values of βI, field strength times current, the smaller current arcs move considerably faster than do the higher current arcs. For example, at $\beta I = 222{,}000$ gauss-amperes, the 600-ampere arc moves at 76 meters per second and a 200-ampere arc at 90 meters per second. At 2,000,000 gauss-amperes, the 600-ampere arc is driven at 520 meters per second and an 1,800-ampere arc at only 355 meters per second. This seems to indicate the piston action of the arc, since the larger current arc may be expected to act as a more perfect piston and therefore be slowed up to a greater extent in moving the body of gas at a velocity nearer its own.

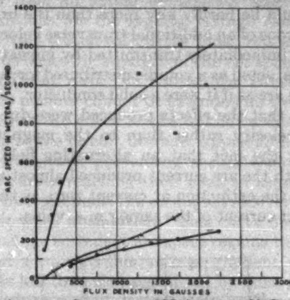

FIG. 4—AVERAGE SPEED OF A 600-AMPERE, 60-CYCLE ARC 3.8 CM. LONG MOVING IN A CIRCULAR SOAPSTONE SLOT 1.9 CM. WIDE IN VARIOUS GASES AT ATMOSPHERIC PRESSURE, AS A FUNCTION OF AVERAGE RADIAL FLUX DENSITY IN THE SLOT. (D-C. FIELD)

⊡ Points for hydrogen
× Points for carbon dioxide
● Points for air

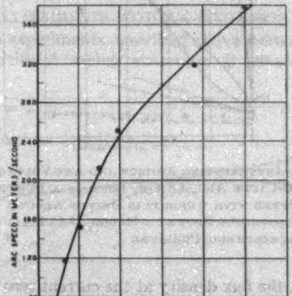

FIG. 5—CURVE SHOWING AVERAGE ARC SPEED IN AIR AT ATMOSPHERIC PRESSURE, AS A FUNCTION OF STEADY-STATE SHORT-CIRCUIT CURRENT (ARC CURRENT, APPROXIMATELY) FOR A 3.8-CM. ARC IN A 0.635-CM. CIRCULAR SOAPSTONE SLOT WITH AN AVERAGE RADIAL FLUX DENSITY OF 1,110 GAUSSES

Of course, the effect, other than producing bodily motion, of the magnetic field upon the arc must be considered. Calculations, which had to be omitted for lack of space, were made on the basis of modern kinetic theory of gaseous conduction to determine the possible effect of the fields used in these experiments upon the mobilities and diffusion coefficients of ions and electrons in the arc space. These calculations show that this effect could be hardly any more than the imparting to the electrons of an additional transverse velocity, which, on being immediately transmitted by impact to the gas molecules, acted as a volume distributed force tending to move the arc as if it were a solid conductor. Additional evidence that the effects produced were due to the resulting velocity rather than to the magnetic field is given by the fact that an alternating field nearly in phase with the arc current produced almost as great an effect on the extinction at current zero, as the field due to a direct current of the same r.m.s. value. In the a-c. field case, the flux density at the current zero of the arc was very slight, so that the change in the arc's extinction characteristics must have been due to a condition caused by the action of the field on the arc prior to current zero. It is believed that this condition was the turbulence of the arc gases resulting from the arc's velocity, which was, in turn, produced by the force due to the transverse magnetic field.

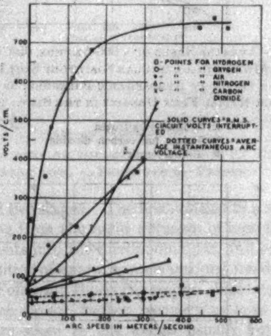

FIG. 6—INTERRUPTING ABILITY AND ARC VOLTAGE OF A 600-AMPERE, 60-CYCLE ARC, 3.8 CM. LONG AS A FUNCTION OF THE AVERAGE SPEED WITH WHICH IT IS DRIVEN AROUND A 0.635-CM. CIRCULAR SOAPSTONE SLOT BY A RADIAL D-C. FIELD IN VARIOUS GASES AT ATMOSPHERIC PRESSURE

The power supply for these experiments consisted of two 50-kva. transformers with 2,300-volt, 60-cycle primaries arranged to provide a considerable number of secondary voltages. The current was limited almost entirely by large air-core station-type reactors, having very low resistance and distributed capacitance. The time required for full circuit voltage to appear across the arc after current zero is estimated to be of the order of ten microseconds for a circuit of this type. For the sake of uniformity of conditions, the reactance of the circuit was adjusted so as to obtain a short-circuit current of approximately 600 r.m.s. amperes in all of the experiments for which results are presented. The circuit was closed by means of a synchronously controlled short-circuiting switch, to obtain symmetrical first cycles. At each of a series of circuit voltages, the current in the field coils was adjusted by repeated trial to the minimum value that would just cause extinction of the arc after not more than two half-cycles. The results of a series of such experiments with arcs in the 0.635 cm. slot in atmospheres of air, nitrogen, carbon dioxide, oxygen, and hydrogen are shown in Fig. 6. The r.m.s. circuit voltage at which the arc was extinguished is plotted against the arc speed which corresponds to the critical field current, as shown by Fig. 3. The arc-speed *vs.* field-strength curve in nitrogen was assumed to be the same as that in air. Values of average arc voltage are also plotted in this figure.

These curves show very clearly that increasing the turbulence of the arc gases does increase the dielectric strength recovered by the arc space within a given short time after current zero. The arc voltage was also increased, but not to so great an extent. This probably indicates that the rate of formation of ions in the arc increases more rapidly than the first power of the arc voltage. Thus, at the highest arc speeds, the increase in rate of ion production due to an increase of 100 per cent or less in arc voltage during the half-cycle would be sufficient to balance the greatly increased rate of loss of ions which causes a dielectric recovery of several hundred per cent during the current zero period; and this is seen to be the case.

One very interesting fact strikingly revealed by these curves is the great difference between the interrupting ability of arcs in different gases for the same arc velocity. The superiority of hydrogen as an arc atmosphere is not surprising, because of the smaller mass and resulting higher rate of diffusion of the hydrogen ions, but the great difference between oxygen and nitrogen is not predictable from their molecular weights. The curve for air, which is largely nitrogen, lies closer to the curve for nitrogen than to that for oxygen, as might be expected. No reason is definitely known for the apparent slow deionization of the turbulent arc in nitrogen. It might be due to the formation of relatively massive, slow diffusing "clusters"[12] of molecules about positive ions, or to some other property or process of similar or different character. The peculiar upward bending of the carbon dioxide curve is difficult to explain, but the general agreement in magnitude of this curve with that for oxygen would seem to follow from the assumption that the gas at arc temperatures is largely a mixture of oxygen and carbon monoxide, which latter has a molecular weight comparable with that of oxygen. The flattening of the hydrogen curve at the higher velocities seems to indicate the approach to a maximum effect of turbulence as predicted in the theory.

In Fig. 7 are plotted a few similar data obtained with a slot three times as wide, for comparison with those of Fig. 6. The line for air, located by two points, has practically the same slope as the corresponding straight line for air in the narrower slot. The difference in the behavior of the arc in carbon dioxide in the two cases is very surprising, however, and seems to defy explanation.

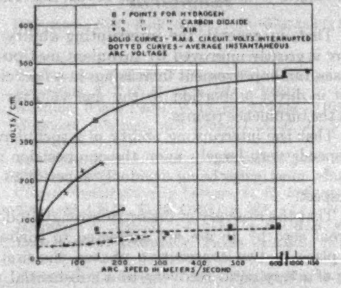

Fig. 7—Interrupting Ability and Arc Voltage of a 600-Ampere, 60-Cycle Arc, 3.8 Cm. Long, as a Function of the Average Speed with which is Driven Around a 1.9-Cm. Circular Soapstone Slot by a Radial D-C. Field in Various Gases at Atmospheric Pressure

At the lower velocities the wider slot is superior to the narrower, but as the arc speed is increased the curve flattens out rapidly. With a gradient of only 350 r.m.s. volts per centimeter, the arc showed no tendency to go out at the highest speeds obtainable with the apparatus. Somewhat similar inconsistencies between the arc speeds in carbon dioxide for the same field in the two slots may be noticed in Figs. 3 and 4. Thus, it seems that there must be processes taking place in the carbon dioxide arc which do not occur with simpler gases. These may be related to the chemical decomposition of carbon dioxide mentioned above. The lower values of volts-per-centimeter interrupted by the arc in hydrogen for the same speeds, and especially the lower value at which flattening apparently occurs, seems to indicate the effect of proximity of the bounding walls on average ionization density at current zero, mentioned in the above discussion of theory.

In Fig. 8 is shown the result of a series of experiments to determine the manner of recovery of dielectric strength with time after current zero for a turbulent arc. The arc, of 600 amperes as before, was driven around the 1.9 cm. slot in carbon dioxide at an average speed of 68 meters per second by a d-c. field of 342 gausses. The rate of recovery of voltage across the arc electrodes was controlled and measured by means of resistance shunts, which were adjusted by trial at each of a series of voltages to the maximum value at which the arc would just be extinguished at the first current zero, which was made to occur at peak generated voltage. The exponential voltage rise curves corresponding to these critical values are plotted in the figure. The solid curves were calculated from the circuit constants, neglecting distributed capacitance and assuming that the voltage started from zero instead of negative arc voltage. The dotted curve was plotted from an actual oscillogram taken at the critical conditions for the highest voltage used. The envelope of this system of curves must represent the instantaneous dielectric strength of the arc space at any time after current zero. Of course, the envelope obtained in this manner cannot be extended all the way to zero time, since the effect of the unknown distributed capacitances of the circuit becomes appreciable for the high-speed transients, preventing accurate plotting of voltage rise curves reaching their peak in times of the order of ten microseconds, the estimated recovery period of the circuit when unshunted. This curve does show, however, that the turbulent arc space regains the ability to withstand a considerable voltage gradient within a time which is less than that permitted by most practical circuits in which circuit interrupters are required to operate. It also shows that further increase of dielectric strength following the initial very rapid recovery is comparatively slow. This behavior is exactly that which would be predicted from Slepian's theory of the deionization at current zero of a long a-c. arc subjected to turbulent gas flow, as illustrated by Fig. 1.

III-B. Experiments with an Expulsion Fuse

Since the experiments described above were all made with an artificial arcing structure, it seemed very desirable to study also the extinction characteristics of a practical gas-blast device in order to prove or disprove, if possible, the validity of the turbulence theory as applied to it. For this study, an expulsion fuse, consisting of a fiber tube 1.6 cm. in inside diameter and 7.6 cm. long, was employed. The tube was closed at one end and vented at the other through a hollow electrode. The power was supplied in these tests through a bank of six

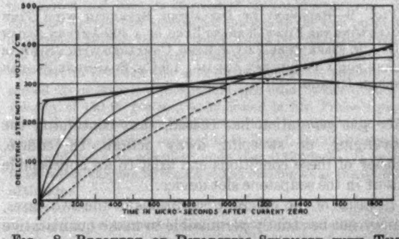

Fig. 8—Recovery of Dielectric Strength with Time Following the First Current Zero of a 600-Ampere Arc in a 1.9-Cm. Circular Soapstone Slot 3.8 Cm. Long, Containing Carbon Dioxide at Atmospheric Pressure, the Arc being Driven at an Average Speed of 68 Meters per Second by a Radial D-C. Field of 340 Gausses

333-kva., 23,000 to 2,300 or 1,150-volt transformers, used in series and parallel combinations to obtain various secondary voltages. As before, the current was limited principally by air-core reactance, 400 amperes (symmetrical short-circuit value) being used in this case. By the same method as that employed in obtaining the data of Fig. 8, an envelope representing the recovery of dielectric strength with time after current zero was obtained for the fiber fuse. This is shown in Fig. 9, with the curve of Fig. 8 repeated for direct comparison. Although the expulsion fuse curve is not quite so flat, its general similarity in shape to the upper curve obtained with the experimental turbulent arc device is evident. The greater slope of the expulsion fuse curve for the first few hundred microseconds after current zero may be due to the diluting effect of a continued injection of fresh gas from the still decomposing fiber walls. The high pressure initially existing within the tube will also result in a continuation of the gas-blast after gas generation has ceased, thus continuing the scavenging, or sweeping away action. Of course, neither of these conditions can exist to an appreciable extent in the soapstone slot device.

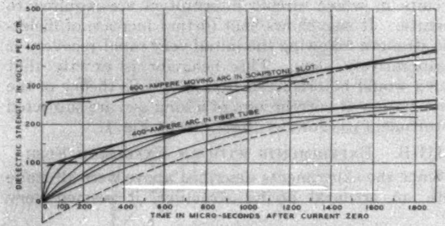

FIG. 9—RECOVERY OF DIELECTRIC STRENGTH WITH TIME FOLLOWING THE FIRST CURRENT ZERO OF A 400-AMPERE ARC IN A 1.6-CM. FIBER TUBE, 7.6 CM. LONG, COMPARED WITH A SIMILAR CURVE FOR A TURBULENT ARC IN A 1.9-CM. SOAPSTONE SLOT IN CARBON DIOXIDE

Because of the different conditions in the two cases, it may not be strictly permissible to make quantitative comparisons based on these curves, but the result of an attempt to do this may be of some interest. Calculations, again omitted for lack of space, indicate that the velocity of the gas in the fuse tube is of the same order of magnitude as that in the soapstone slot. The average arc-voltage gradients during the half-cycles were about the same. In the rotating arc case the gas may be considered to be practically an equal mixture of carbon monoxide and oxygen. The gases generated in the fiber fuse, on the other hand, have been shown[10] to be an approximately equal mixture of carbon monoxide and hydrogen. In the latter case, however, it seems probable that there may have been an appreciable quantity of nitrogen-containing air remaining in the tube at the first current zero to counterbalance the favorable effect of the hydrogen. In justice to the fiber-tube expulsion fuse, it should be explained that the current chosen for these tests is that which gives the lowest voltage interrupting capacity[10] with a tube of this size.

IV. CONCLUSIONS

The results of these experiments reveal the following facts:

1. That, in general, the interrupting ability of an a-c. arc is greatly improved by turbulent motion of the arc gases, this improvement increasing, in a fast circuit, almost in direct proportion to the gas velocity from which the turbulence results.
2. That the interrupting ability of a turbulent a-c. arc depends very largely upon the composition of the arc gases, most gases being decidedly superior to air in this respect.
3. That the recovery of dielectric strength following a current zero by an a-c. arc subjected to turbulence takes place in two consecutive phases, the first consisting of a very rapid recovery to a substantial value in a few microseconds, and the second of a very much slower further recovery for a much longer period, and
4. That at least one practical gas-blast circuit interrupter, the expulsion fuse, exhibits a volt-time interruption characteristic which clearly indicates the role of gas turbulence in its operation.

Conclusions (1), (3) and (4) are entirely in accord with the predictions of the theory proposed by Slepian and Denault,[9,10] and are believed to constitute convincing proof of the essential correctness of this theory. Conclusion (2), which lacks definite theoretical explanation, disagrees with the observations of some experimenters,[4,5] and agrees with those of others,[12] but the experimental fact is well verified, and helps to explain the success of many enclosed-arc circuit interrupters. It is hoped that further results, similar to those from which conclusion (4) was drawn, may be obtained for arcs in other types of gas-blast interrupters. For the oil circuit breaker, which is probably the most important of such devices, some interesting experimental information recorded by Park and Skeats[7]* appears to indicate a volt-time recovery curve which is similar in shape to those given in the present paper. These results were obtained with an ordinary plain-break circuit breaker. Additional results presented by Prince and Skeats[11]† for an explosion pot type of oil breaker, with and without special contacts, seem to indicate similar characteristics, in spite of claims made for an entirely different method of arc extinction in these cases. Further and more systematic investigation of the volt-time dielectric recovery characteristics of arcs in oil circuit breakers, and in other types of gas-blast devices as well, would seem to be very desirable. Also, an accurate determination of such curves for times of the order of ten microseconds and less after current zero would be of considerable interest from the standpoint of theory.

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*Fig. 14 and accompanying text of their paper.

†Curves of Fig. 11 on page 511.

V. Acknowledgment

Much of the credit for this work is due Dr. J. Slepian, who suggested and guided it. The expulsion fuse experiments were conducted by Mr. G. S. Rowley.

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Discussion

THE THEORY OF OIL-BLAST CIRCUIT BREAKERS

(Prince)

PRACTICAL APPLICATION OF THE OIL-BLAST PRINCIPAL OF CIRCUIT INTERRUPTION

(Spurck)

RECENT DEVELOPMENTS IN ARC RUPTURING DEVICES

(Van Sickle and Leeds)

EXTINCTION OF A-C. ARCS IN TURBULENT GASES

(Browne, Jr.)

Philip Sporn: I would like to stress a point that I laid before the Institute a year ago, and that is the question of the standard duty cycle on breakers. The electrical industry is today paying a heavy penalty because of the continuation of the present totally inadequate and in many cases disregarded OCO 2-minute OCO standard cycle. We found, for example, only a month or so ago that one of our own companies was making it a standard practise to open up every breaker on its system above a certain voltage rating after it had interrupted an arc twice under short circuit and this was regardless of whether the guarantee of the breaker was well above the duty that was being imposed upon it. Such practises are bound to exist in many other companies and are costing the industry thousands of dollars every year. In the light of the data as to the successful performance under repeated short-circuit cycles developed during the Philo 1930 tests and also in view of the data developed in the present papers by Messrs. Van Sickle and Leeds and by Mr. Spurck, isn't it high time for the progressive oil switch manufacturers to offer the users of oil circuit breakers a duty cycle that is more consistent with actual and economical power system operation, and that would not at the same time stress the modern designs to anywhere near their limits?

C. L. Fortescue: Some years ago it was felt that circuit breakers using the old method of rupturing arcs had reached the limit of their capacity. We then undertook a fundamental research on the characteristics of long arcs with a view to finding out how they were formed and how they could be quenched. Dr. Slepian was assigned to this research work and the deion circuit breaker and the deion grid oil circuit breaker are part of the results of his investigations.

Some years ago I had the idea that by stabilizing an a-c. arc through a long path it would go out of itself at zero current so I made up two cylindrical choke coils of large diameter and spaced them apart on the same axis leaving a small space between them for the arc chute protecting the choke coil from the arc by thick plates of asbestos board. The choke coils were arranged so that the arc itself formed the series connection between the two choke coils, one terminal of one choke coil was arranged for connection to an 11,000-volt trolley and the other terminal connected to ground and the arc started by connecting together, with fuse wire, the other two terminals which consisted of two small horns and the circuit was closed between the terminal of one choke coil and the trolley. The arc was then established and as expected it stabilized between the two choke coils in the form of a ring but much to my surprise it did not go out at zero current as I expected. The frequency of the alternating current was 25 cycles so that the condition was favorable to deionization of the long arc. As I recall, the mean diameter of the two coils was about three feet so that the length of the arc must have been about nine and one-half feet. This indicated to me that something besides length in the arc was required to quench it and this requirement is, as Messrs. Browne, Van Sickle and Leeds show, a turbulent gas stream across the path of the arc which was not present in the test.

Mr. Prince mentions washing away the products of the arc and in other places he seems to have the idea of interposing a solid layer of oil between the arcing electrodes. He presents test data to support this point of view, but is not this data just as well explained by the fact that the gas formation and its turbulence is a function of the velocity by which the oil is introduced into the ionized path between electrodes? It would seem to me that until the current had approached quite near the zero point any oil directed against the arc stream would be volatilized even at the high pressures produced in the arc chamber. Therefore, the time left to establish the solid oil barrier would probably not exceed 20 or 30 microseconds and this solid barrier even at a velocity of 130 ft. per sec., as indicated in Mr. Prince's paper, would not progress more than 0.04 inch, or taking a value of 100 microseconds, which is believed to be liberal, the solid barrier of oil would not have advanced more than 0.134 inch. It is hard to see where this can be considered as washing away the products of the arc.

In the theory advanced by Dr. Slepian the action suggested appeared to be reasonable and we know that arc quenching, as it takes place in expulsion fuses and the Torok tube, must take place in some such manner as explained in Dr. Slepian's theory. Why, therefore, invent a different answer for a different type of mechanism? Is it not more reasonable to suppose the quenching of the arc takes place in the same way in all cases?

Mr. Browne, I feel, has made tests which support Dr. Slepian's theory very strongly and the successful operation of the deion grid, as pointed out in the paper by Messrs. Van Sickle and Leeds, in which the action could not be due to interposing a layer of cold oil between the electrodes, further supports this theory.

Reverting to lightning phenomena, one reason that I have put forward for the fact that the lightning currents do not reach values high enough to produce serious induced surges is the nature of the cloud atmosphere and the difficulty of producing ionization therein. Now it is known that the cloud atmosphere is in a high degree of turbulence and this, by still further increasing the difficulty of ionization, may be a contributing factor in restricting the rate of discharge of the thundercloud.

In the case of the oil breaker no doubt free electrons are present in large quantities in the arc stream and the idea of washing away the electrons seems to me to be impractical.

J. B. MacNeill: It seems safe to predict that in the future circuit breakers need not be a limitation of system or station development in the way that they were several years ago. Other limitations to the capacity concentrated on short circuit at a given point must be considered. The general problem before the industry of which circuit breaker operation is but one part, is that of overall protective system operation. For some time it has been realized that the accumulation of time increments on large systems to secure selective relay action, provided only inadequate protection. The paper by Goldsborough and Lewis is the result of a scientific study of the factors available for selective action, and outlines ingenious methods of suiting apparatus to the requirements. This paper records a noteworthy contribution to the high speed relay art for all lengths of lines and with dependable high speed operation.

It would appear that the oil blast device given in the papers by Messrs. Prince and Spurck, operating as it does on one end of the arc, would be most effective for low voltages. However, for high voltages in which relatively long arcs are obtained, one questions the ability of a device operating only on the upper arc end to exert sufficient effect on the arc as a whole, to produce effective results. On the other hand, a device such as the deion grid interposing successive increments of arc rupturing ability as the arc lengthens, in which the gas formed passes transversely across the full arc length, impresses one as being better fitted for high-voltage circuits.

If reference is made to Fig. 9 of Mr. T. E. Browne's paper, there will be seen the almost instantaneous restoration of dielectric strength in devices of the turbulent gas type. The speed of oscillation of a high-voltage transformer without shunting lines is usually less than 10,000 cycles per second. Such a circuit obtains full voltage (¼ cycle) in 25 microseconds. Even in this short time the turbulent gas device has reached the knee of the curve. A 132,000-volt transformer circuit connected to an alternator is shown in Fig. 3 of the paper by Messrs. Van Sickle and Leeds, and shows approximately 4,300 cycles per second. Approximately 60 microseconds are required for ¼ cycle of this circuit. The advantage of rupturing devices such as deion grids operating under this principle, is obvious, especially for high-voltage work, over devices which build up dielectric strength only in a straight line relation to time for a considerable period.

T. E. Browne, Jr.: In the paper presented by Messrs. Van Sickle and Leeds, one of the most interesting facts from the standpoint of fundamental theory is that revealed by comparison of the curves showing reduction of arc energy with those showing the corresponding reduction in arcing time. As stated by the authors, the smaller arc energy with deion grids as compared with plain breaks results from an *actual reduction in arc voltage* as well as from the reduction in time duration of the arc. This is brought out most forcefully in the case of the lower-voltage power-house breakers, where, for example, Fig. 14 shows that at 30,000 amperes the application of deion grids effected a reduction in arc energy of approximately 50 per cent, while the corresponding reduction in arcing time, as shown by Fig. 13, is roughly only 20 per cent. The 30 per cent discrepancy obviously must be due to reduction in arc voltage resulting from diminished total length of the arc when controlled by the grids. In spite of this reduction in overall length and voltage, the shorter arc, since it is subjected to a so much more effective turbulent gas blast, is able to interrupt the circuit at current zero just as certainly as is the longer less-efficient unconfined arc.

This is further practical evidence of the fact, brought out by the experiments described in my paper, that the effect of turbulent gas flow on a confined arc is to cause it to approach very close to the ideal a-c. circuit interrupting agent, which, as pointed out by Dr. Slepian and others, should be able to carry the current with negligible power loss during the half-cycle and then change over instantly at a normal current zero from a good conductor to an adequate insulator. It is noteworthy that these desirable characteristics are almost completely attained in the gaseous arc space of the deion grids by gas-blast action alone.

The oil-blast theory of circuit interruption, presented by Mr. Prince, has been of great interest to me since it was first proposed a year ago. After carefully studying the exposition of the theory and the supporting evidence presented at that time and added to at frequent intervals since, as well as the present paper, I find it, however, quite impossible to agree with Mr. Prince's conclusion that the theory is "unquestionably" valid, either for the so-called impulse type, or any other existing kind of breaker. For example, all of the data presented in this paper appear to me to be either inconclusive or actually at variance with the oil-blast theory. In Fig. 1, as a first instance, it is shown that the impulse strength of good and of poor oil approaches equality only for times of the order of one microsecond or less, and that for a surge period of 100 microseconds or more, shown to exist in all practical high-voltage circuits so far investigated, the difference becomes of the order of two and one-half to one, which it would seem ought to make itself apparent in any careful series of tests if the behavior of the breaker actually does depend directly and quantitatively upon the dielectric strength of liquid oil, as stated. If, however, the action of the breaker should depend upon the dielectric strength of gases resulting from decomposition of oil, it is clear that the lack of relation between oil quality and breaker performance actually observed is entirely plausible.

In the case of Fig. 2, showing rate of rise of circuit voltage as a function of necessary oil velocity, it is difficult to understand why, on the basis of the data, that the curve separating the points representing failures from those representing successful operations should have been drawn as a straight line through the origin. Although the wide scattering of the failure points would make the definite location of any curve drawn as a boundary to them doubtful, a curve that is concave upward and passes through the *Y* axis of zero oil velocity at a recovery rate of about 150 volts per microsecond would seem to fit the cross-marked points much more closely. This critical rate of rise of voltage of 150 per microsecond is no doubt about right for the plain-break oil circuit breaker, which Mr. Prince's impulse breaker would be at zero oil velocity. It is exceedingly common knowledge that a plain-break oil breaker is, and has always been, able to interrupt high-voltage circuits having rates of voltage rise which are certainly far in excess of zero. Conceding, however, that there is indicated a certain amount of general correlation between critical circuit speed and oil velocity, it still remains true that such correlation is to be expected on the basis of either a "solid" oil or a gas-blast theory, since the constriction of the arc and the intensity of the gas blast must also increase with the increasing oil velocity, making this figure of little value in comparing the two theories.

In Figs. 3 and 4, where the same data used in Fig. 2 are replotted with voltage and current as respective ordinates, it is also difficult to understand why any existing correlation between these quantities and oil velocity should be expected to show itself, in view of the wide range of circuit speeds represented, after circuit speed had just been shown in Fig. 2 to be a controlling factor. Even so, the difference between these two figures, in which lines are omitted, and Fig. 2 in which the line is drawn, is not impressive. In any case, would it not be necessary, in order to prove or disprove correlation between any one of the three variables (current, voltage and circuit speed) and oil velocity, to hold the other two variables constant?

With regard to Fig. 7, where rate of voltage rise is plotted against oil pressure in an explosion pot, the same general comments made in Fig. 2 may be repeated, including the observation that a certain degree of correlation is to be expected on the basis of either an oil-blast or a gas-blast theory.

The moving-picture photographs reproduced in Fig. 8 from an earlier article, and taken, as Mr. Prince says, "in an attempt to satisfy *pardonable* skepticism," seem to me to have just as much, or as little, to do with the fundamental question of what happens at current zero, as do previous much clearer diagrams of events taking place at these other less important moments. I do not believe that the eventual replacement of arc gases by liquid oil *after* extinction of the arc in any oil circuit breaker has ever been seriously questioned. The question is: "How soon after?"

In summing up this so-called "relevant" information on which his conclusion is based, it seems to me that Mr. Prince has overlooked some data which are far more relevant than any presented in his current paper. These are the data of Fig. 11 in the oil-blast paper by Messrs. Prince and Skeats presented last year. In this figure the results of field tests on a 132,000-volt, 600-ampere explosion-pot breaker, both with and without oil-blast contacts, were presented in terms of inches of arc required to interrupt the circuit as a function of calculated circuit recovery time. In order to reduce these curves to the more fundamental form of interrupted-voltage gradient, or average dielectric strength of the arc, against time, they have been replotted in the accompanying Fig. 1 with ordinates equal to the circuit voltage divided by the original ordinates (inches of arc), giving the effective dielectric strength of the arc space after current zero in volts per inch. The similarity between these curves and corresponding curves for two admittedly gas-blast devices, as shown in Figs. 8 and 9 of my paper, is immediately evident. Study of similar data presented two years ago by Messrs. Park and Skeats reveals the demonstration of generally similar characteristics for another explosion-pot breaker and for a plain-break oil circuit breaker as well. Specifically, I should like to ask Mr. Prince why, if his hypothetical constantly-growing oil film is able to attain a thickness sufficient to produce the dielectric strength indicated for the oil-blast contacts in the extremely short time of 60 microseconds, that it does not continue to grow at at least this rate so as to obtain a further gain in effective dielectric strength of the order of 30,000 per cent after 2,000 microseconds, nearly one-eighth of a cycle later, instead of the mere 75 per cent increase actually observed?

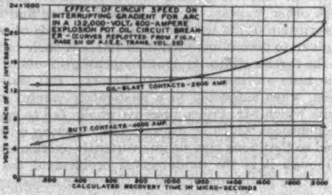

Fig. 1

In conclusion, I should like respectfully to submit three general reasons why it is impossible for me to agree with Mr. Prince's conclusions:

1. As pointed out repeatedly by Dr. Slepian, no detailed picture, capable of surviving critical analysis, of the mechanics by which his oil-film may be formed has yet been presented or discovered.

2. All of the data so far presented in support of the oil-blast theory seem to be preponderantly in favor, rather, of the gas-blast theory.

3. That the oil-film hypothesis is quite unnecessary, since it has been proven by laboratory results and by practical demonstration with deion grids as well, that the properties of a turbulent arc, such as is bound to exist in the oil-blast circuit breaker, are adequate to explain the breaker's behavior.

R. C. Van Sickle and W. M. Leeds: The realization that the gases produced in an oil circuit breaker, formerly considered merely a necessary evil, may actually be utilized to great advantage in the extinction of a-c. arcs, has brought about radical changes in circuit breaker construction. Mr. Browne has shown in his paper that the interrupting ability of a circuit breaking device is dependent among other things on two factors: first, the degree of turbulence of the arc gases, and second, the particular composition of these gases. In the deion grid construction, a very high degree of turbulence is obtained by drawing the arc in a narrow slot and directing the gases as a horizontal blast through the arc stream. If it were necessary to sweep the ionized gas particles a distance of several inches to prevent reignition of the arc at a current zero, a tremendous velocity would be required to accomplish this in the few microseconds available. However, Mr. Browne has brought out the fact that the turbulent condition of the arc just previous to a current zero is responsible for an extremely high initial rate of recovery of dielectric strength. When interrupting circuits with unusually fast-rising recovery voltages, this phase of the extinction process apparently determines whether the arc will be reestablished or not, and the complete removal of ionized particles by the motion of gas or oil volumes becomes a matter of secondary importance.

The gases produced by the decomposition of circuit breaker oil during the arc rupturing process have been analyzed by many investigators and found to contain approximately 70 per cent hydrogen. In view of the marked superiority in interrupting ability shown by hydrogen over air and other gases used in Browne's experiments, it is evident that the use of oil in an arc rupturing device, in addition to advantages of insulation and cooling, provides a supply of gas which as an extinguishing medium is much better than other common gases.

In comparing the results obtained with Mr. Browne's experimental structure and with deion grids certain differences in the functioning of the two devices should be pointed out. In Mr. Browne's device, the arc is driven around a circular path at high velocity by magnetic means so that the arc passes a given point in the gas many times at very short time intervals during which the gas does not have time to become completely deionized. The successive passages of the arc build up a state of moderate ionization in the gas. However, in the deion grid, relative motion is produced by holding the arc more or less stationary and directing a rapid turbulent flow of gas transversely through it. This gas is produced from the decomposition of the oil adjacent the arc and is initially un-ionized. Furthermore, after passing through the arc stream, this gas is discharged from the grid and not used again. It is thus evident that the rate of rise of recovery of dielectric strength is more rapid in both the initial and later stages, due to the lower density of ionization in the space surrounding the arc as the current approaches zero and to the dilution by fresh un-ionized gas after zero.

These differences may explain why the recovery voltage gradients of deion grids are higher than those obtained by Mr. Browne on his experimental device.

J. Slepian: Two years ago, in the closing discussion on my paper, "Extinction of Long A-C. Arcs," Trans. A.I.E.E. V. 49, p. 445, 1930, in commenting upon the suggestion of Mr. Prince that circuits might be interrupted by "whisking the arc products away," I found it necessary to point out that contrary to general impression, the arc in circuit interrupters performs necessary and useful functions and that "whisking it away," if it were really accomplished, would have serious and unpleasant consequences, unless other means were provided to take over the functions normally exercised by the arc. It seems necessary to expound briefly upon this point again, since the significance of this matter does not seem to have been grasped by the proponents of the oil film theory of operation of practical oil circuit breakers.

The useful function of the arc may be seen by considering what would happen if an arc did not form spontaneously upon separating a pair of contacts in a power circuit. If the arc did not form, then at the moment of separation of the contacts the current would necessarily at once drop to zero. Such an instantaneous reduction of the current to zero would induce disastrously high voltages in the circuit. But it may be suggested, why not separate the contacts at a moment of current zero when no high voltages will be induced? This would be an excellent suggestion if the electromechanical problem of synchronizing the separation of the contacts with the current zero was not so extraordinarily difficult to solve. Consider the complicated and variable relations which govern the moment of reaching the current zero in a circuit, and which would make it impossible to start going the contact separating mechanism much ahead of the current zero. Consider the enormous force necessary to accelerate and get going the contact separating mechanism in so short a time. Consider the difficulty of controlling this force so exactly as to synchronize with a normal current zero within a few microseconds. Remember that when dealing with such short time phenomena bars, ordinarily thought of as rigid, are annoyingly flexible, and fluids such as oil are highly compressible.

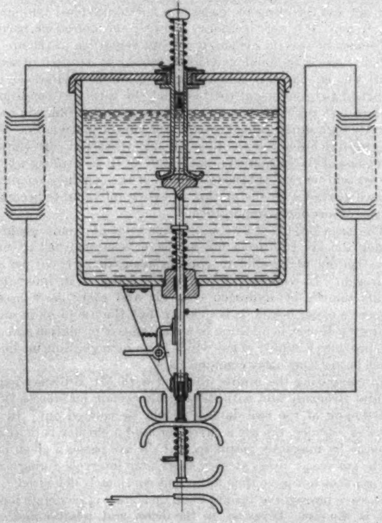

FIG. 2—L. W. CHUBB CIRCUIT INTERRUPTER

(U. S. Patent No. 12.36,674)

The arc relieves us of these difficulties. We separate the contacts when we please and the arc permits the current to continue to flow so that no high voltages are induced. Then, of itself, in a well designed switch of course, the arc seizes upon a moment of current zero for changing from conductor to insulator, thus opening the circuit without the induction of high voltage. Thus no mechanical synchronization with current zero is required. The arc of itself performs this synchronizing function for us so that circuit breakers using arcs are in practical operation everywhere without causing damage to other electrical apparatus.

The problem of improving circuit interrupters is not one of getting rid of, annihilating or "whisking away" arcs, but of using arcs more efficiently and particularly using more effectively the inherent capacity of the arc for interrupting circuits synchronously with a normal current zero. Mr. Prince, who separates his contacts at a random time, admittedly uses an arc up to a moment of current zero, but just at the moment when the arc is ready to work for him and interrupt the circuit, he aspires to drive the arc away and interpose a growing film of oil between the contacts. However, and fortunately, the means he uses are inadequate to form such an oil barrier at any time, let alone synchronously with current zero, so that even in his breakers the arc interrupts the circuit.

It is possible to intensify the mechanical means which Mr. Prince uses so that they will cause a barrier of oil to be interposed between the separating contacts and interrupt the arc but the mere intensification of these means cannot cause them to form a barrier at current zero, and refrain from forming it at any other time. Fig. 2 in this discussion shows a circuit breaker made and tested many years ago by W. L. Chubb, which used means similar to those proposed by Prince. The actuating means, however, which was a sledge hammer blow upon the small piston, was so much more powerful and effective that an oil film really was interposed between the electrodes, squeezing out the arc. But as the lightning arresters indicated in the figure show, high voltages were expected and were obtained since there was no synchronization of the formation of the oil barrier with a normal current zero.

The discussion of Mr. Prince's paper is made difficult because results, even those which he states are the most convincing, are given without supporting data, but references are given to brief articles in other journals. Looking these up, we find that sufficient description of his test methods and methods of calculation are not given to permit any critical estimate of their validity, and there is the further difficulty that frequently reports of results of the same tests are mutually contradictory. But waiving these objections, when subjected to a moderately careful inspection, the results which Mr. Prince presents wholly fail to support any of the conclusions which he draws. The following criticisms are numbered so that none may be inadvertently overlooked.

1. Although it is the fundamental problem of the oil blast theory, neither in this paper, nor in the paper of last year is any detailed attention given to the hydro-mechanical problem of synchronous formation of the growing "solid oil" barrier. Nowhere in the present paper is it stated just where in the oil the velocity occurs which is used as the abscissas of Figs. 2, 3 and 4, although in the next to the last paragraph of the paper it is stated that these curves are the most convincing evidence which the author offers.

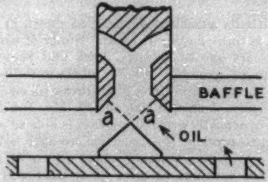

FIG. 3—OIL BOUNDARY AT TIME OF INTERRUPTION

1a. In the reference No. 2 which Mr. Prince cites and from which the data for these figures are taken, a diagram is shown, reproduced here as Fig. 3, in which it is suggested that the velocity in question is the mean velocity over a section of the passage leading up to the arc. The velocity is calculated from the motion of the piston without considering the influence of the compressibility of the oil which must be an important factor when the free surfaces a-a are so violently accelerated.

1b. The obstructing film is said to form by the uniform motion of the free surfaces *a-a* with the calculated velocity used in Figs. 2, 3 and 4. But when actual fluids possessing some viscosity flow through passages, the velocity next to a bounding wall is generally accepted as being zero, taking on larger values away from the wall following a parabolic relationship with distance from the wall. Hence the initial rate of growth of the obstructing film at the cone tip must be zero irrespective of the mean velocity of the oil.

1c. Waiving the objection of (1b), the rate of growth of the obstructing film at the cone tip will be equal to the "oil velocity" divided by the sine of ½ the conical angle made by the free surface *a-a*. Hence when this angle is small the necessary rate of growth of the film should be obtainable with very low oil velocity. No discussion is given in the reference cited by Mr. Prince as to what determines this angle. In his closing discussion last year, TRANS. A.I.E.E. V. 50, p. 529, Mr. Prince, in order to account for the greater interrupting capacity of long arcs in oil blast breakers, suggests that for long arcs the angle is smaller. It would seem equally plausible that for small current arcs this angle should be small. Hence it would seem that for long arcs and small currents in circuits of the same speed, smaller "oil velocities" would be necessary. But this contradicts the conclusions which Mr. Prince draws from his Figs. 2, 3 and 4.

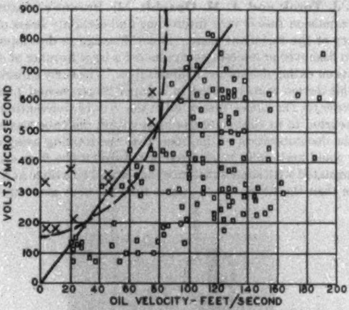

FIG. 4—13,200-VOLT CONNECTION

(D. C. Prince, *Elec. Wld.*, February 28, 1931)

1d. No explanation is given why the free surface *a-a* happens to be at just the right place at current zero to start forming immediately the obstructing film. It may be suggested that long before the current zero, the surface *a-a* exists at the proper position with oil rushing up perpendicularly to the surface with the supposed high velocity, but with the oil being decomposed by the arc as fast as it crosses the surfaces. But with oil coming up with a velocity of 100 ft. per sec., and being decomposed, approximately 20,000 liters of gas at atmospheric pressure and temperature would form per second or about 200 liters per half cycle, for a single square inch of surface *a-a*. This is 10 to 100 times as great as the whole volume of gas usually formed in a complete circuit breaker operation. The electrical energy which the arc would need to receive to decompose oil at this rate would be enormous, so that a high arc voltage would be necessary to keep the arc going. Since the high voltage is not available from the circuit, this is another way of saying that the arc will not be able to keep back the advancing oil surfaces, but that it will be squeezed out with development of high voltages. This same argument applies against the oil having a large perpendicular velocity to any surface abutting on the arc and being kept from closing in on the arc by the decomposition of the oil. If the surfaces are kept from closing in by the gas pressure at the arc, then of course, they cannot have the high velocity which Mr. Prince states is necessary. Hence, if surfaces do exist, closing in on the arc with high velocity, there is no reason why their closing in and final cutting off of the arc should be synchronous with a normal current zero.

1e. In his paper of last year, TRANS. A.I.E.E. V. 50, p. 508, Mr. Prince says of the oil adjacent to the arc, it "exists alongside it without mixing" and thus suggests that the velocity is tangential to the surface abutting the arc. But such a tangential velocity cannot contribute to the growth of an obstructing film and therefore there should be no relation between such a velocity and the interrupting capacity, in contradiction to the conclusions which Mr. Prince draws from his Figs. 2, 3 and 4.

2. We come now to the data shown in his Figs. 2, 3 and 4, which Mr. Prince regards as the most convincing proofs of his ideas. From these curves Mr. Prince deduces that arc extinction depends upon the relation between the circuit recovery voltage rate and oil velocity alone. Since Mr. Prince gives no account here of the experiments from which these data were taken, it is necessary to consult the reference No. 2 cited by him.

2a. In (1a) and (1b) I have pointed out that the calculated velocity used by Mr. Prince in these curves corresponds to no real velocity in the oil.

2b. In (1c) I have pointed out that according to Mr. Prince himself, for a given "oil velocity" the rate of formation of obstructing film should be greater for long arcs and by a similar argument, for small currents, and therefore is not a function of the oil velocity alone. There is no unique relationship between the rate of growth of an oil barrier and the oil velocity.

2c. The rates of voltage rise in the various circuits are given as calculated from high frequency measurements, but no examples of these calculations are given, so that they may be critically examined.

2d. The rate of voltage rise just after current zero in one and the same circuit will vary widely, depending on the point of the voltage wave at which the circuit was closed. No account seems to have been taken of this factor.

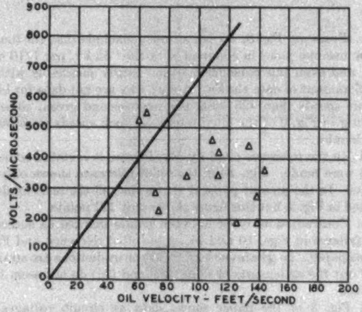

FIG. 5—6,600-VOLT CONNECTION

(D. C. Prince, *Elec. Wld.*, February 28, 1931)

2e. For those points marked as failures, was the arc extinguished at a later zero? If so, were those arc extinctions marked as successful in Figs. 2, 3 and 4? If not, why not?

3. Examining the data of Fig. 10 of the reference cited by Mr. Prince, covering supposedly the same tests as Figs. 2, 3 and 4 of this paper, the contention that the condition for interruption is independent of the voltage of the circuit tested is not supported. This may be seen by separating the data for the 13,200-volt, 6,600-volt and 3,800-volt connections as I have done in my

Figs. 4, 5 and 6 of this discussion. In these figures the straight line of "55 kv. per 1/10 inch" has been drawn in.

3a. In my Fig. 4, a curved line would fit the data better than the straight one shown, but there is not a sufficiently dense region of complete failures to draw any curve with any confidence.

3b. From my Fig. 5, no limiting curve can be drawn whatsoever. The data therefore give no information whatsoever as to whether the oil velocity required for a given circuit speed is the same for 6,600 volts as at 13,200 volts. Why are not data given for oil velocities less than 60 ft. per sec., and so little data for circuit speeds of more than 450 volts per microsecond since the curves of Fig. 9, of the cited reference, show that such speeds were readily obtainable?

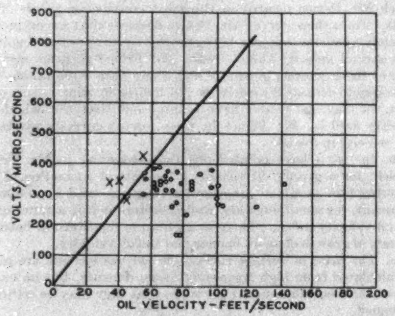

FIG. 6—3,800-VOLT CONNECTION

(D. C. Prince, *Elec. Wld.*, February 28, 1931)

3c. From my Fig. 6, we can at most conclude that the limiting curve has one point in common with the "55 kv. per 1/10 inch" line, and even this conclusion seems hardly justifiable with the small amount of data shown. Again, why are not data for higher circuit speeds than 425 volts per microsecond given, since, according to Fig. 9 of the cited reference, such speeds were readily obtainable.

3d. In the reference cited by Mr. Prince, it is stated some 400 tests were made. Fig. 10 in the cited reference shows only 186 points. In the present paper it states that all the tests made are plotted in Fig. 2, but this figure shows only 121 points.

3e. Contradictions exist between points marked as successful and failures in Figs. 10 and 11, of the cited reference and Fig. 2 of this paper. In whatever way these contradictions are straightened out the statements of (3a), (3b) and (3c) do not seem to be affected.

3f. Fig. 3 of the paper shows data at circuit voltages not mentioned in the paper in the cited reference.

3g. In Fig. 3 the points of the 12,000-volt test do not show lack of correlation. The points of the 10,000-volt test do not show lack of correlation. The points of the 6,600-volt test show as good correlation as the points of Fig. 2. The points of the 3,800-volt test are not shown in Fig. 3. When plotted they show perfect correlation.

3h. The points of Fig. 4 of Mr. Prince's paper determine a curve as well (or as poorly) as the points of Fig. 2.

4. At the bottom of page 168, referring to Fig. 5, it is stated that in the plain explosion chamber the arc is uniformly extinguished at the throat. This is glaringly contradicted by Fig. 9 of the paper by Spurck and Strang, TRANS. A.I.E.E., V. 50, p. 519.

5. At the top of page 169, Fig. 6 and text, it is stated that tests with multiple throat explosion chamber support the oil blast theory of operation. Data are not given which will permit critical examination of this conclusion.

6. Again, on page 169, tests in a narrow slot are said to support the oil blast theory, but data are not given which will permit critical examination of this conclusion.

7. From the bubble size in Fig. 9, it should be possible to compute the velocity of the oil flowing out of the chamber. It would be interesting to know this figure.

8. On the top of page 170, what is meant by "the arc is clearly subjected to violent washing action"? Is it possible that this is another way of saying that there is a "turbulent inmixing of freshly generated gas".

J. J. Torok and A. M. Opsahl: Mr. Browne's treatment of the expulsion fuse is very interesting and certainly bears out the theory of the turbulent gases. Some time ago in developing the deion flashover protectors we conducted a large number of experiments on extinguishing arcs in fiber tubes. The early experiments on this device were reported in a paper *Experimental Lightning Protector for Insulators*, presented at the Winter Convention 1931. It occurred to us that since gas blast within the tube was caused by the decomposition of the fiber, that the wasting away of the fiber could materially be checked or reduced if the fiber could be impregnated with some substance that would volatilize at a lower value than the fiber. At the same time we were also concerned with another theory which was based upon the condensation of moisture within the chamber. This condensation of moisture formed droplets within the tube and thus materially increased the surface on which ions could be collected.

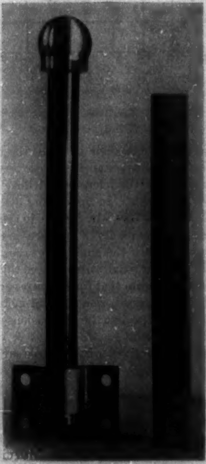

FIG. 7

In some of these tests we wetted the inner wall of the fiber tube with water prior to each test. The results were somewhat startling. We found that the wasting away of the fiber was reduced to about one sixth of its former value. The voltage which could be interrupted on succeeding shots was raised appreciably.

The general conclusions of these tests were that the moisture content of the side walls of the fiber tubes materially improved the operating characteristics of the fiber tube. Thus from the electrical interrupting standpoint it is not harmful for the fiber tube to be left open to the atmosphere especially where moisture might be collected.

In applying this to service conditions it is not desirable to have a stream of water flowing through the bore of the tube during a rainstorm so that it is essential to either close the upper end or properly shield it. The methods of so doing are illustrated in Figs. 7 and 8. In Fig. 9 the top is vented in such a manner that rain cannot go in directly and the gases will be directed so that they will not come in contact with live parts and thus cause secondary flashover. In Fig. 7 the top is completely closed leaving only the bottom open.

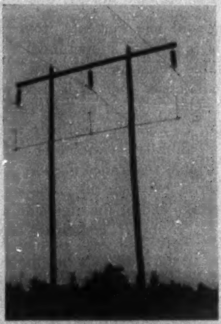

Fig. 8—Typical Westinghouse Expulsion Arrester

Installation, Camden–Magnolia Line

Of about 1,000 such units put in the field during 1931, the operations that have been reported were successful. Further years of service will yield information as to the relative severity of the duty on the tubes in the field and in the laboratory.

W. A. Hillebrand: The curves presented by Mr. Browne confirm earlier experience with the Poulsen radio arc transmitter, which consists of a d-c. arc, generally between carbon and copper electrodes, in an atmosphere of hydrogen, with a magnetic blowout. But for the accidental application by Valdemar Poulsen about 1905 of an atmosphere of hydrogen to the singing arc, the transmitter that bears his name might not have been developed.

The magnetic blow-out functions as a switch to interrupt the arc at radio frequency. Since 1920 there has been in operation an arc transmitter of this type that, hour after hour, regularly interrupts a thousand kilowatts of energy at the arc terminals. This, it should be noted, is a quantity quite different from the rated interrupting capacity of a circuit breaker. The rate of current interruption varies from 10,000 to 20,000 times per second. Unfortunately for the practical application of this device to circuit breaker work, a magnetic field of the requisite intensity requires a magnet frame weighing in the order of 50 tons.

Reference has been made to the fact that hydrogen is a principal constituent of the gases liberated when an arc takes place under oil. In the Poulsen transmitter the hydrogen atmosphere was often produced by the decomposition of a liquid hydrocarbon allowed to fall drop by drop upon the arc flame.

Mr. Browne states that the difference between the curves of Fig. 6 for oxygen and nitrogen cannot be explained by any difference between their molecular weights. I would like to offer as a possible contributing cause the fact that free electrons can exist in an atmosphere of nitrogen but not in an atmosphere of oxygen.

D. C. Prince: Dr. Slepian and Mr. T. E. Browne, Jr. have contributed most of the discussion relative to theory. Mr. Browne is interested in the relative merits of oil-blast and gas-blast whereas Dr. Slepian bases his discussion on the relative accuracy of a deionization theory as compared with the oil-blast theory. In gas circuit interrupting devices such as Mr. Browne has used, the active agent should be a gas blast of sufficient velocity to displace the arc products by an insulating medium, in his case cool un-ionized gas. In an oil circuit breaker advantage is taken of the fact that oil dielectric strength is of the order of ten times that of gas.

Dr. Slepian has quoted correctly the phrase, "whisking arc products away," but then attacked "whisking away arcs." There is no difference of opinion on the undesirability of trying to break a heavy current arc when by waiting less than one hundredth of a second, it can be broken at a current zero.

On account of the volume of matter represented by the "supporting data" requested by Dr. Slepian, its publication is out of the question. In writing our paper pertinent factors have been taken into consideration in making calculations and no data have been withheld which if presented would support a different conclusion (omitted points were mostly surplus successful operations in the region of greatest density of points). Curves plotted have represented the best available engineering judgment.

When a steel structure is to be designed, an engineer calculates all stresses and multiplies by a factor of safety which may be two, five or more. He then selects a structural section for which tests give a sufficient elastic limit or ultimate strength. Engineering judgment may vary the factor of safety but quantitative figures for the stress to be encountered and the physical properties of the steel must be available. The same thing is true of the design and application of oil circuit breakers. Wide changes in duty, as shown by such tests as those at Philo, are not measured in kva. but in recovery rate. The oil-blast theory takes these new factors quantitatively into account. Measurements can be made and calculations based upon them. Factors of safety may be selected as engineering judgment dictates. The theory may ultimately be shown to be in error just as Newton's gravitational theory has been found to be in error, but it will have served its purpose.

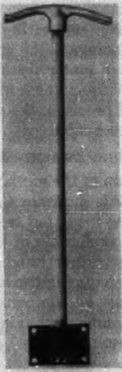

Fig. 9

High-Voltage Bridge for Measurements of Cables with Grounded Sheaths

BY C. L. DAWES* and A. F. DANIEL†
Member, A.I.E.E. Non-member

Synopsis.—This paper describes a bridge that is adapted to making dielectric measurements of cables whose sheaths are in electrical contact with the ground. A balance to adjust the bridge to the system losses to ground is first made with the test cable disconnected; the bridge is then balanced with the test cable connected. The cable properties such as power loss, power factor and capacitance are then determined directly from this bridge balance. Certain precautions, however, are necessary such as the method of substituting the test cable in circuit and the elimination of the ground currents which may come from the low-voltage system.

INTRODUCTION

PRECISION bridge methods are now available for the dielectric properties of cables when the current through the dielectric can be conducted directly from the sheath to the measuring apparatus. These methods however cannot as a rule be applied to cables whose sheaths are in electrical contact with the ground. There is one method‡ in which a bridge measurement of all the system losses is first made with the test cable disconnected; a second bridge measurement is then made with the test cable in circuit. The difference in the power measurements gives presumably the power loss in the cable dielectric. This method does not give directly the electrical properties of the cable (power, power factor, capacitance). Also, unless special precautions are taken, the connecting of the cable to the system may cause a change in the ground currents other than that of the cable under test. The authors have developed a bridge method which measures directly and with a high degree of precision the values of the electrical properties of the test cable.

GROUNDED-BRIDGE METHOD

A simplified diagram of the method is shown in Fig. 1. The cable to be tested is shown as C_5. The transformer case is shown as being insulated from ground. This is not necessary, but is preferable in that all loss currents from windings to case return immediately to the high-voltage winding thus reducing the loss currents through the ground which must be compensated by the bridge. C_2 is a high-voltage condenser having a capacitance of a few hundred micro-microfarads. At any particular voltage setting, its capacitance and losses should remain substantially constant during the time required for making a measurement. In our measurements we used a 10-ft. length of high-voltage cable having a capacitance of the order of 800 $\mu\mu f$. It is desirable that this capacitance should be of such magnitude that its charging current is at least somewhat larger than the stray charging and loss currents from the system to ground. M is a mutual inductance; R_1 and R_2 are ordinary variable resistance decades. C_1 is a fixed standard air condenser such as is used with the usual high-voltage bridge. C_2, M and R_2 constitute branch A of the bridge; C_1 and R_1 constitute branch B of the bridge. L is a calibrated variometer (self-inductance) and R_5 is a variable resistance decade. L and R_5 are connected in series between the cable sheath (ground) and the low-voltage end of the high-side winding of the transformer. C_5, L and R_5 constitute branch C of the bridge.

One terminal of the secondary of M is grounded at b and the detector D is connected between the ground and point a at the junction of R_1 and C_1. The detector D is a tuned magnetic-vane galvanometer and has already been described.[1]

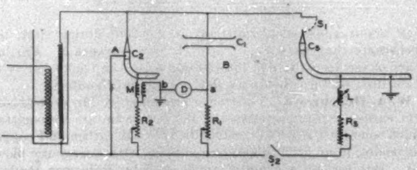

FIG. 1—SIMPLIFIED DIAGRAM OF GROUNDED BRIDGE

Switches S_1 and S_2 are in the high- and low-voltage wires which connect the cable to be measured to the bridge terminals. In making the measurements switches S_1 and S_2 are first opened. This isolates the cable from the rest of the system except through the ground connection and gives a bridge, the two branches of which are A and B. This bridge is of the same type that has been in use in our laboratories for some time[1] past except that the secondary of the mutual inductance M and the detector D are now grounded at b. The bridge is then balanced by varying M and R_1. All the loss currents of the system to ground must flow to point b and return to the transformer through the secondary of the mutual inductance M. Hence this balance does not depend on the dielectric characteristics of C_2 alone

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‡This method is in use at the Commonwealth Edison Company for measuring the characteristics of experimental 1,000-ft. lengths of 76-kv. (to ground) cables under life test.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

[1] For references see Bibliography.

but rather on the characteristics of C_2 in combination with all stray capacitances and leakances to ground. Moreover, since current flows in the secondary of M, its reading cannot be used for computing the power factor of C_2 or C_3 in conjunction with the system impedance to ground.

After this balance has been made, switches S_1 and S_2 are closed, thus connecting the cable C_5 between the high-voltage side of the bridge and ground and also the variable self-inductance L and resistance R_5 in series, between ground and the low-voltage end of the transformer winding. Were S_1 alone closed, all the cable current must flow from the cable sheath to ground and leave the ground at b to return to the transformer winding through the secondary of the mutual inductance M. In so doing this current must disturb the bridge balance already obtained. If the system losses to ground have not been changed by the closing of switch S_1, the original bridge balance can only be restored when *none* of the current through C_5 returns to the transformer through M. Obviously this condition is realized only when *all* the current through C_5 flows through L and R_5. By closing switch S_2 and making proper adjustment of L and R_5, this condition may be realized and is indicated by the detector D again reading zero. The bridge formed by branches B and C will be recognized as the Wien modification of the Maxwell Bridge.[4]

The operation of the bridge may also be regarded from the point of view that since point a has already been brought to ground potential when the detector again reads zero, no difference of potential exists between the ground connection at b and the cable sheath of C_5; hence no current flows between these two points, and the current in L and R_5 must be the same as that in the cable.

Under the foregoing conditions of balance, the current in L and R_5 is the same as that in C_5 and

$$\frac{C_5}{C_1} = \frac{R_1}{R_5} \text{ (very nearly)} \qquad (1)$$

The power factor

$$\text{Power factor} = \frac{L\,\omega}{R_5} \qquad (2)$$

R_5 includes the resistance of the variometer L. A self-inductance may be substituted for M and the ground made at the low-voltage plate of C_2. However, a mutual inductance is particularly well adapted to this ground connection since the stray impedances to ground may require either a positive or a negative phase displacement for balance, and with mutual inductance, both are obtained equally well. It will be noted that arm A of this bridge acts almost in the same manner as the well-known Wagner ground.[3]

LIMITATION OF BRIDGE TYPES

The types of bridge which may be used for this method of cable measurement are limited since for each measurement the branch B of the bridge must remain invariable and the cable sheath of C_5 must be connected directly to the detector. The first limitation precludes the use of the Schering Bridge[2,4] in which the balance is effected by varying R_1 and a condenser shunted about R_1. The second limitation makes it undesirable to use the Heaviside Bridge of the mutual inductance type[1] since in the simplest form of this bridge the primary of the mutual inductance is in series with the cable sheath of C_5.

GUARDING AND SHIELDING

The usual guarding and shielding which are necessary in high-voltage, low power-factor measurements must be used with this bridge. Also, further precautions are necessary to prevent change of system losses to ground when the cable C_5 is switched in circuit.

Although no measurement of cable C_2 is made, guard rings at its ends are desirable to prevent end discharges disturbing the galvanometer balance. These guard rings need not be balanced to sheath potential but may be connected directly to the low-voltage end of the transformer winding as shown in Fig. 2, which gives the complete diagram of connections. As is customary in conventional bridge methods the low-voltage plate of the air condenser C_1 and the leads connecting thereto must be shielded and the shielding brought to the potential of the low-voltage plate. This is done by means of the resistance R_3 and the mutual inductance m' whose primary is connected across the voltmeter coil in series with C_0'. During *this* balance the detector is connected between points a and c.

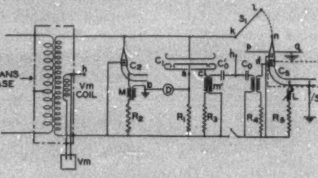

Fig. 2—Detailed Diagram of Grounded Bridge

In a like manner the guards and shielding of the test cable C_5 are brought to the potential of the active sheath by means of R_4 and the mutual inductance m. It is to be noted that with the exception of the guards of C_2 all shielding is at ground potential although not actually grounded. To prevent end discharges entering the guard circuit, and disturbing the galvanometer deflections, double guarding may be used with cable C_5[1].

Some care must be taken in the manner in which switch S_1 is closed. The accuracy of the method depends on the losses to ground of the system remaining unchanged when C_5 is switched in circuit. If the live parts, *kl* of switch S_1 (see Fig. 2) are too near the cable, the system will pick up charge as may be proved by opening S_3 and closing S_2. Hence, in our experiments *kl* was a long lead of No. 1, bare solid wire about 20 ft. (6 m.) long, connected permanently to the high-voltage source at *k*. Connection to C_s was made by *swinging* this lead parallel to the ground until it came in contact with the cable copper at *n*. In both the open and the closed positions care was taken that *kl* had the same relative position to ground so that its loss to ground did not change. Furthermore when point *l* is connected to point *n* some of the loss from *kl* would normally be transferred from ground to the cable shielding, thus causing a variation in ground loss and causing error. To eliminate this source of error a grounded shield *pq* was interposed between the cable shielding and *kl* to intercept this loss current from *kl* and divert it to ground. With a long length of test cable almost entirely buried in the ground, percentage errors from this source naturally would not be large.

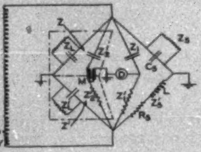

Fig. 3—Conventional Diagram of Grounded Bridge

Equivalent Bridge Circuits

The connections of this bridge in the usual conventional form are shown in Fig. 3. The high-voltage bushing of the transformer and the high-voltage system have an equivalent capacitance and leakance to ground which may be denoted by Z_L. Some intermediate point in the transformer winding such as *o* must be at ground potential. If the case is non-grounded as shown in Figs. 1 and 2, it and the portion of the winding *oo'* form an equivalent capacitance and leakance to ground which may be represented by Z_L'. If the case is grounded Z_L' represents the impedance to case of the portion *oo'* of the winding. Z_2 and Z_2' constitute the arm *A* of the bridge, Fig. 1; Z_1 and Z_1' constitute the arm *B*, consisting of the standard air condenser and R_1, Fig. 1; Z_s and Z_s' constitute the arm *C*, Fig. 1, consisting of the test cable C_s, the self-inductance *L* and the resistance R_s.

In the bridge itself Z_L and Z_2 combine to form an equivalent impedance to ground *Z*; likewise Z_L' and Z_2' combine to form an equivalent impedance to ground *Z'*. The first balance (with switches S_1 and S_2, Figs. 1 and 2 open) causes adjustments among Z_2, Z_2', and Z_1' such that

$$\frac{\overline{Z}}{\overline{Z'}} = \frac{\overline{Z}_1}{\overline{Z}_1'} \tag{3}$$

In the second balance, with switches S_1 and S_2 closed, Z_s' is adjusted until

$$\frac{\overline{Z}_s}{\overline{Z}_s'} = \frac{\overline{Z}_1}{\overline{Z}_1'} \tag{4}$$

The mathematical proofs of (3) and (4) for bridges in general may be found in text books[2,4] and are well known. For simplicity, the guard circuits are omitted in Fig. 3. They would merely constitute two more bridge circuits similar to those shown and having their respective arms in the proportions given in (3) and (4).

Low-Voltage System Disturbances

In our laboratory measurements we were able to verify the accuracy of the method by interposing a switch S_3, Fig. 2, in the ground connection. When S_3 is opened, the branches *B* and *C* constitute a simple Wien-Maxwell bridge[4] with the branch *A* now constituting a true Wagner ground. If the method is correct, little or no change in the balance should occur whether S_3 is closed or opened, provided the bridge system is already in proper balance. Errors in the high-voltage system due to inadequate shielding, and the change in ground loss brought about by the movement of the lead *kl*, were soon appreciated and eliminated in the manner already described.

Another source of error was found to be due to the changes in the ground relations of the low-voltage system caused by the increased capacitive load occurring when C_s is connected to the transformer secondary. When switch S_1 is closed the secondary voltage of the transformer rises due to the increased capacitive load. The high-side voltage is restored to its original value by reducing the voltage of the low-voltage side, including that of the line running from the transformer primary back to the alternator. This causes variations in the capacitive and leakage currents to ground from this entire system. If by chance any of this current enters the low-voltage system at *b* and through the secondary of *M*, Figs. 1 and 2, any change of voltage in the low-voltage system changes this current. This effect may cause large errors. Also to this effect must be added the effect of capacitance and leakance between the high-voltage and the low-voltage winding of the transformer, as is indicated in Fig. 4. Apparently, there is no single connection which will make all such errors zero, particularly if the capacitances are unsymmetrical, but

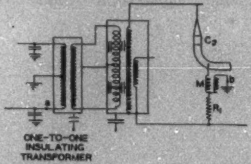

Fig. 4—Stray Capacitance in Grounded Bridge

they can be made very small. It was found that the most accurate results were obtained when the connections shown in Fig. 4 were used. The losses in the low-voltage system back to the generator were eliminated by using a one-to-one insulating transformer and grounding the system back to the alternator at one point. All ground loss currents could thus return to this system through this connection.

The case of this one-to-one transformer was connected to the case of the test transformer and to the middle point of the low-voltage winding of the test transformer. A study of Fig. 4 shows that any change in capacitive and leakage currents of this system do not flow through the ground connection at *b* except the capacitive currents between primary and secondary of the insulating transformer. Since these windings are at low voltage, changes in their capacitive currents are small. If the low-voltage system is grounded at its neutral point a condition of symmetry is produced which again minimizes changes in ground currents caused by change of voltage. When one side of the low-voltage system was grounded at *a*, Fig. 4, rather than at the neutral point little error was found to result. With long lengths of cable such as would ordinarily be measured by this method, the foregoing errors, whose magnitudes are determined almost entirely by the voltage and not by the losses in the test cable, will all be small, proportionately.

EXPERIMENTAL TESTS

In the laboratory the cable was not actually installed in the ground, which made it possible to interpose a switch S_3, Fig. 2, in the lead connecting the sheath to ground. By opening S_3 and switching the detector between point *a* and the sheath C_5 a true Wien-Maxwell impedance bridge is formed, so that it becomes possible to check the accuracy of the measurements which are made when the sheath is grounded. Such check measurements were made on a 31.5 ft. (8.8 m.) length of 00, 9/32 in. (0.715 cm.), wall, impregnated-paper insulated cable. The results of these tests are given in Table I.

TABLE I
Frequency = 60 cycles. C_1 = 120 $\mu\mu f$.

S_3	E (kv.)	R_1	R_3	R_5	L (Mh.)	Power factor	C ($\mu\mu f$. per ft.)	Watts per ft.
Closed	11.42	5780	200	346.5	5.5	0.598	63.5	0.0187
Open	11.42	5780	200	347.8	5.4	0.584	63.3	0.0182
Closed	16.8	5812	200	347.9	5.60	0.606	63.7	0.0411
Open	16.8	5812	200	348.9	5.45	0.588	63.5	0.0497
Closed	22.4	5855	200	348.1	8.75	0.948	64.0	0.0115
Open	22.4	5855	200	349.3	8.60	0.927	63.8	0.0112
Closed	28.0	7213	200	427.7	13.25	1.17	64.2	0.213
Open	28.0	7213	200	428.7	12.80	1.13	64.1	0.222
Closed	28.0	5676	200	334.7	8.4	0.946	64.5	0.180
Open	28.0	5676	200	336.1	8.75	0.984	64.4	0.187
Closed	33.6	5723	200	339.6	14.5	1.49	64.2	0.408
Open	33.6	5723	200	340.7	15.0	1.44	64.0	0.393

The last two sets of measurements were taken three days later than the preceding ones. During this period the cable underwent slight change so that these last data are not continuous with those which precede it. It is to be noted that the maximum difference between corresponding measurements is of the order of 4 per cent which is not large when the character of the measurement is considered.

It may be objected that this method involves too many bridge balances. With the conventional bridge having proper shielding at least three balances should be made. Also when balances for one voltage have been made, subsequent balances are usually made quite readily.

The authors wish to express their appreciation to Dean H. E. Clifford of the Harvard Engineering School for his helpful suggestions in the preparation of this paper.

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A High Sensitivity Power Factor Bridge

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***Synopsis.**—This paper describes an a-c. bridge of high precision, used for power factor measurements in one of the dielectric investigations being carried on at the Johns Hopkins University. The bridge, a modified form of the Schering bridge, possesses several novel features and advantages. The detecting instrument is a moving coil a-c. galvanometer; its field excitation is supplied from a phase shifting transformer, which permits obtaining independent ratio and phase angle balances. A shielded transformer electrostatically isolates the galvanometer from the bridge circuits.*

This bridge is completely shielded and guarded and the analytical theory of the resulting mesh connection is given in the paper. The mathematical treatment is general and may be applied to any similar network. A full discussion is given of the effects upon the balance relation of various sources of error, for example, as failure to balance guard circuits properly and the presence of leakage resistance between measuring and guard circuits. Experimental verification of the equations is presented.

Power factors of specimens of cable material have been measured ranging from 0.00007 to 0.16 with a maximum variation of ± 0.000005.

THE Schering bridge is probably the most widely used a-c. bridge for the measurement of the capacitances and phase defect angles of dielectrics under high-voltage gradients. In a study of the properties of high-voltage cables carried on in the Electrical Laboratories of the School of Engineering of The Johns Hopkins University we have used a completely shielded and guarded Schering bridge[1] for a number of years.

About two years ago, under the auspices of the Utilities Research Commission, an investigation was begun of the properties of cable compounds and papers and of the characteristics that occur after impregnation. In this work it became necessary to modify the Schering bridge in order to obtain the desired range and sensitivity. The bridge that finally resulted is completely shielded and guarded and possesses several novel features.

A bridge has been developed in which it is possible to balance the guard circuits without the use of a second galvanometer or disturbing in any way the main galvanometer connections. It is shown that in this bridge it is not necessary to balance the guard circuits for phase but only for magnitude.

THE BRIDGE

A diagram of the bridge connections is given in Fig. 1, where C_1 is the specimen and C_2 the air capacitor. The latter is adjustable and its high-tension plate is so mounted that any leakage currents from it flow directly to ground and not into its guard circuit.[2] Both the specimen and the air capacitor are provided with guard ring electrodes. The leads from their main or measuring electrodes to the points A and B of the bridge are one-eighth inch (3. mm.) brass rods shielded by one inch (2.54 cm.) diameter tubes. The central conductors are supported at approximately three-foot (1 meter) intervals by small hard rubber washers. Each shielding tube is connected to its proper guard ring as may be seen from the figure.

The capacitance between the main electrode with its lead and the guard ring with its shield is 470 $\mu\mu$f for the specimen and 490 $\mu\mu$f for the air capacitor. It is essential for accurate measurement that the insulation resistance of these two capacitances be maintained *equal to infinity*, as measured with the Megger.

The physical arrangement of the apparatus is such that the specimen and air capacitor may be quickly interchanged. It is also a simple matter to disconnect the specimen and substitute an air capacitor in its place for making check measurements.

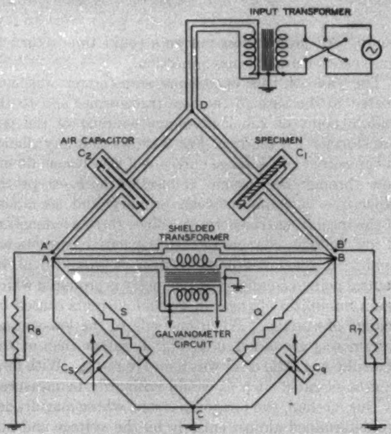

FIG. 1—BRIDGE CIRCUIT—DIAGRAM OF CONNECTIONS

One feature of this bridge, originally used by Hartshorn[3] in low-voltage work, is the use of two variable air capacitors Cs and Cq in parallel with the S and Q arms respectively. The capacitance Cq consists of the variable capacitor shown in the figure plus the capacitance to ground of the detector arm shield, which is connected to the point B of the bridge. In normal operation Cq is kept at a fixed value and balance is obtained

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by varying Cs, a variable calibrated air capacitor. The use of the capacitor Cq permits the neutralization of the fixed capacitance of the bridge and the measurement of very small values of power factor.

As Cq includes certain fixed capacitances it is necessary to determine its value. This is accomplished by substituting an air capacitor for the specimen, and balancing by adjusting Cs, using a 1 to 1 bridge. Then the two high-voltage air capacitors are interchanged and the balance checked. The same value of Cs should hold for both cases to within the sensitivity of the bridge, a phase angle whose cosine is $\pm 5 \times 10^{-6}$. Under these conditions and a ratio of one to one the value of Cq equals that of Cs, the calibrated air capacitor.

The detector is a Leeds and Northrup a-c. moving coil type of galvanometer, which is connected to the bridge through an electrostatically shielded transformer, as shown in Fig. 2. The magnetic field of the galvanometer is excited from the secondary of a phase shifting transformer. The deflection of the galvanometer depends upon the phase relation between its magnetic field and the current in its moving coil. When these two are in phase the deflection is a maximum and when they are in quadrature zero.

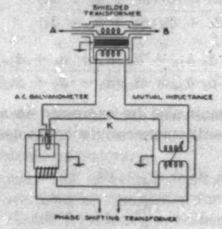

FIG. 2—GALVANOMETER CIRCUIT—DIAGRAM OF CONNECTIONS

In balancing any impedance bridge two conditions must be satisfied, the ratio or magnitude balance and the phase balance. The currents produced by these two factors are in quadrature and it is a simple matter to bring the galvanometer field into phase with either component by adjusting the position of the secondary of the phase shifting transformer. This feature enables the operator not only to balance the bridge for either ratio or phase angle separately and independently, but also to adjust properly the magnitude of the guard resistances, R_7 and R_8, without either disturbing the connections of the main galvanometer or the use of a second detector.

A variable mutual inductance is connected in series with the galvanometer to neutralize the e.m.f. induced in its moving coil by the a-c. field. The value of the mutual inductance is adjusted so as to keep the galvanometer reading on the scale, that is, by making the electrical and mechanical zeros coincide. Balance of the bridge is obtained when there is no change in the galvanometer deflection[4] upon reversal of the input e.m.f.

The simple balance relations of this bridge are: ratio

$$C_1 = C_2\, S/Q \qquad (1)$$

and the phase angle ψ_1, of the specimen

$$\psi_1 = \omega\, Cs\, S - \omega\, Cq\, Q \qquad (2)$$

Where ω equals $2\,\pi$ times the frequency.

The power supply for the bridge and the primary of the phase shifting transformer are taken from a three phase "General Electric Company" sine wave generator driven by a motor supplied by the University storage battery. The sensitivity of the bridge is ample and for phase angle corresponds to an angle whose cosine is $5. \times 10^{-6}$. Values of capacitance ranging from 40 $\mu\mu$f to 1,500 $\mu\mu$f and specimen power factors from 0.007 to 16 per cent have been successfully measured. The bridge is usually operated at 60 cycles and 1,500 volts alternating current.

OPERATION OF THE BRIDGE

In the operation of the bridge the first step is the proper adjustment of the phase shifting transformer. This is attained in the following manner:

Using the specimen, C_1, and air capacitor, C_2, in the high-voltage arms, and having set the ratio arms Q and S to 5,000 ohms each, as tentative values, low-voltage is applied to the bridge. Then the operator proceeds to find by trial the phase shifter setting which will give a minimum deflection when the ratio arms are upset by, say, 1,000 ohms in 5,000. This gives the approximate phase shifter setting for the phase angle balance. The phase shifter is then thrown through 90 deg. to the "ratio" setting and the ratio balance roughly determined. The process is then repeated until it is possible to have an unbalance of several thousand ohms in the ratio arms remain undetected when the phase shifter is in the "phase angle" setting. These adjustments give accurately the phase angle and ratio settings of the secondary of the phase shifting transformer.

In balancing the bridge all four low-voltage arms, R_8, S, Q, and R_7, are set at some tentative values. With the phase shifter on the ratio setting and Q at 5,000 ohms, S is adjusted to the nearest ohm. If Q and S differ materially from each other the spacing of the standard air capacitor is varied to bring the bridge closer to a one to one ratio.

Once a satisfactory ratio of Q and S is obtained, the operator proceeds to balance the guard circuit of the specimen. With the phase shifting transformer in the ratio position the operator connects the points $B - B'$ of the bridge by means of a short lead and then adjusts the guard resistance, R_7, until the upset in balance caused by this procedure disappears. With R_7 at its proper value for magnitude balance, the operator removes the short circuit from across $B - B'$, places it on $A - A'$ and proceeds to adjust R_8 in the same manner as R_7. These adjustments of R_7 and R_8 are for magnitude only and not for phase. The magnitude balance is thus obtained using the main galvanometer as a detector, but without disturbing its connections in any way. The proof of the correctness of this procedure will be discussed later.

After the proper settings of R_7 and R_8 are obtained, the ratio balance is once more checked; usually the balance is not off more than five ohms in 5,000, unless the preliminary settings of R_7 and R_8 were considerably out.

The full test voltage is now applied to the bridge and the operator obtains the final ratio balance, after rechecking the guard resistances for magnitude balance. The phase shifter is then rotated through 90 deg. to the phase angle position and the phase balance obtained by adjusting the variable air capacitor, C_s.

The power factor of the specimen is calculated in accordance with equation (2). It will be shown that this simple relation gives the correct value of the power factor, despite the fact that the guards have been balanced for magnitude only; provided the leakage resistances existing between the guarding systems and their respective measuring circuits are infinity.

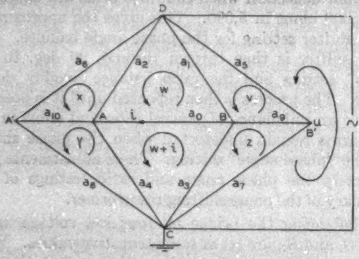

FIG. 3—GENERALIZED MESH DIAGRAM

MESH DESIGN

The mesh diagram of the bridge is given in Fig. 3. This diagram is a general one and may be applied to almost any shielded bridge. The complex impedance operators are represented by $\bar{a}_1$, $\bar{a}_2$, etc. Where $\bar{a}_1$, $\bar{a}_2$, $\bar{a}_3$ and $\bar{a}_4$ represent the measuring circuits of the bridge, and $\bar{a}_5$, $\bar{a}_6$, $\bar{a}_7$, and $\bar{a}_8$ the guard circuits. The operator $\bar{a}_1$ belongs to the measuring circuit of the specimen and $\bar{a}_5$ to its guard. In like manner $\bar{a}_2$ and $\bar{a}_6$ belong to the measuring and guard circuits of the air capacitor, respectively. The operator $\bar{a}_9$ represents the impedance existing between the measuring and guard circuits of the specimen and $\bar{a}_{10}$ performs the same function for the air capacitor circuit. The operator $\bar{a}_0$ expresses the impedance of the galvanometer arm of the bridge.

Using the cyclic currents of Maxwell[6] we obtain a set of six simultaneous equations. Table I below gives the coefficients of the cyclic currents of the generalized mesh.

TABLE I—THE COEFFICIENTS OF THE CYCLIC CURRENTS

Mesh	i	v	w	x	y	z	u
A A′ D	0	0	$-\bar{a}_2$	$(\bar{a}_2 + \bar{a}_6 + \bar{a}_{10})$	$-\bar{a}_{10}$	0	0
A B D	$-\bar{a}_0$	$-\bar{a}_1$	$(\bar{a}_1 + \bar{a}_2)$	$-\bar{a}_2$	0	0	0
B B′ D	0	$(\bar{a}_1 + \bar{a}_5 + \bar{a}_9)$	$-\bar{a}_1$	0	0	$-\bar{a}_9$	$\bar{a}_5$
A A′ C	$-\bar{a}_4$	0	$-\bar{a}_4$	$-\bar{a}_{10}$	$(\bar{a}_4 + \bar{a}_8 + \bar{a}_{10})$	0	0
A B C	$(\bar{a}_0 + \bar{a}_3 + \bar{a}_4)$	0	$(\bar{a}_3 + \bar{a}_4)$	0	$-\bar{a}_4$	$-\bar{a}_3$	0
B B′ C	$-\bar{a}_3$	$-\bar{a}_9$	$-\bar{a}_3$	0	0	$(\bar{a}_3 + \bar{a}_7 + \bar{a}_9)$	$\bar{a}_7$

The current, i, through the galvanometer circuit may be obtained by the solution of the simultaneous equations by means of determinants. For balance the galvanometer current must equal zero and this will only be fulfilled when the determinant for the numerator is zero.

Solving this determinant and equating to zero the complete balance relation is obtained, equation (3), for the generalized network, namely

$$\frac{\bar{a}_1\,\bar{a}_4}{\bar{a}_2\,\bar{a}_3} = \frac{\bar{a}_5\,\bar{a}_9 + \bar{a}_7\,(\bar{a}_1 + \bar{a}_5 + \bar{a}_9)}{\bar{a}_7\,\bar{a}_9 + \bar{a}_5\,(\bar{a}_3 + \bar{a}_7 + \bar{a}_9)} \cdot \frac{\bar{a}_8\,\bar{a}_{10} + \bar{a}_6\,(\bar{a}_4 + \bar{a}_8 + \bar{a}_{10})}{\bar{a}_6\,\bar{a}_{10} + \bar{a}_8\,(\bar{a}_2 + \bar{a}_6 + \bar{a}_{10})} \tag{3}$$

Equation (3) may also be expressed in another form, (4), which is usually more convenient to use.

$$\frac{\bar{a}_1\,\bar{a}_4}{\bar{a}_2\,\bar{a}_3} = \frac{\bar{a}_5 + \bar{a}_7 + \dfrac{\bar{a}_7}{\bar{a}_9}\,(\bar{a}_1 + \bar{a}_5)}{\bar{a}_5 + \bar{a}_7 + \dfrac{\bar{a}_5}{\bar{a}_9}\,(\bar{a}_3 + \bar{a}_7)} \cdot \frac{\bar{a}_6 + \bar{a}_8 + \dfrac{\bar{a}_6}{\bar{a}_{10}}\,(\bar{a}_4 + \bar{a}_8)}{\bar{a}_6 + \bar{a}_8 + \dfrac{\bar{a}_8}{\bar{a}_{10}}\,(\bar{a}_2 + \bar{a}_6)} \tag{4}$$

The terms on the right hand side of equations (3) and (4) are correction terms that express the factors introduced into the simple bridge relation by the presence of the guard circuits. These equations may be expressed in the form of

$$\frac{\bar{a}_1\,\bar{a}_4}{\bar{a}_2\,\bar{a}_3} = \bar{k}_1\,\bar{k}_2 \tag{5}$$

Where $$\bar{k}_1 = \frac{\bar{a}_5 + \bar{a}_7 + \dfrac{\bar{a}_7}{\bar{a}_9}\,(\bar{a}_1 + \bar{a}_5)}{\bar{a}_5 + \bar{a}_7 + \dfrac{\bar{a}_5}{\bar{a}_9}\,(\bar{a}_3 + \bar{a}_7)}$$

and $$\bar{k}_2 = \frac{\bar{a}_6 + \bar{a}_8 + \dfrac{\bar{a}_6}{\bar{a}_{10}}\,(\bar{a}_4 + \bar{a}_8)}{\bar{a}_6 + \bar{a}_8 + \dfrac{\bar{a}_8}{\bar{a}_{10}}\,(\bar{a}_2 + \bar{a}_6)}$$

It is interesting to note that if the guard circuits are balanced with respect to their corresponding bridge circuits, *i. e.*,

$$\frac{\bar{a}_1}{\bar{a}_3} = \frac{\bar{a}_5}{\bar{a}_7} \text{ and } \frac{\bar{a}_2}{\bar{a}_4} = \frac{\bar{a}_6}{\bar{a}_8}$$

that the generalized expression reduces to the simple bridge relation given in

$$\frac{\bar{a}_1\,\bar{a}_4}{\bar{a}_2\,\bar{a}_3} = 1 \tag{6}$$

If no guard circuits are used in the bridge or if the impedances expressed by $\bar{a}_9$ and $\bar{a}_{10}$ are equal to infinity, equation (**4**) again reduces to (**6**) the simple balance relation for a wheatstone network.

Equation (**6**) contains the complex operators of the four arm impedance bridge; each operator having the general form $R \pm jX$. The balance of the bridge requires two adjustments, ratio and phase[6] as stated in the introduction. The ratio and phase balance relations, readily deduced from equation (**6**), are given (**7**) below:

$$\left.\begin{aligned} \frac{Z_1\,Z_4}{Z_2\,Z_3} &= 1 \\ (\phi_1 + \phi_4) - (\phi_2 + \phi_3) &= 0 \end{aligned}\right\} \tag{7}$$

Where Z_1, Z_2, etc., are absolute values and ϕ_1, ϕ_2, etc., are the corresponding phase angles of their respective complex impedance operators.

In like manner the correction terms $\bar{k}_1$ and $\bar{k}_2$ of equation (**5**) are complex quantities for which K_1 and K_2 are their respective absolute values and ϕ_{K1} and ϕ_{K2} their phase angles. For the guarded and shielded bridge the general balance relations are (**8**)

$$\left.\begin{aligned} \frac{Z_1\,Z_4}{Z_2\,Z_3} &= K_1\,K_2 \\ (\phi_1 + \phi_4) - (\phi_2 + \phi_3) &= \phi_{K1} + \phi_{K2} \end{aligned}\right\} \tag{8}$$

CORRECTION TERMS

It is evident from equation (**8**) that the simple ratio and phase angle balance relations mentioned under the description of the bridge must be modified to take care of the correction terms, $\bar{k}_1$ and $\bar{k}_2$ introduced by the presence of the guarding and shielding circuits and that these terms must be evaluated.

It is well to point out here that these correction terms are actually present in every four arm impedance bridge. In unshielded bridges their evaluation is impossible. The use of guards and shields serves to give definite values to these terms and to make their accurate determination feasible.

In order to evaluate the correction terms we must substitute the values of the general operators given below into equation (**5**)

$$\left.\begin{aligned}
\bar{a}_1 &= \rho_1 - \frac{j}{\omega\,C_1} = \frac{1}{\omega\,C_1}\,(\psi_1 - j) \\
\bar{a}_2 &= \frac{-j}{\omega\,C_2} \\
\bar{a}_3 &= \frac{1}{1/Q + j\,\omega\,C_q} = Q\,(1 - j\,\phi_q) \\
\bar{a}_4 &= \frac{1}{1/S + j\,\omega\,C_s} = S\,(1 - j\,\phi_s) \\
\bar{a}_5 &= \frac{1}{\omega\,C_5}\,(\psi_5 - j) = \frac{m_1}{\omega\,C_1}\,(\psi_1 - j) \\
\bar{a}_6 &= -\frac{j}{\omega\,C_6} = -\frac{j\,m_2}{\omega\,C_6} \\
\bar{a}_7 &= R_7\,(1 - j\,\phi_7) = m_1\,Q\,(1 - j\,\phi_7) \\
\bar{a}_8 &= R_8\,(1 - j\,\phi_8) = m_2\,S\,(1 - j\,\phi_8) \\
\bar{a}_9 &= \frac{1}{1/W_9 + j\,\omega\,C_9} \\
\bar{a}_{10} &= \frac{1}{1/W_{10} + j\,\omega\,C_{10}} \\
m_1 &= \frac{C_1}{C_5} \\
m_2 &= \frac{C_2}{C_6}
\end{aligned}\right\} \tag{9}$$

Where ρ_1 is the equivalent series resistance to account for the loss in the specimen capacitor C_1; ψ_1 equals $\omega C_1 \rho_1$, the phase defect angle of the specimen; the angle ϕ_q equals $\omega C_q Q$, the phase angle of the a_3 arm, and ϕ_s equals $\omega C_s S$, the phase angle of the a_4 arm. The angles ϕ_7 and ϕ_8 express the respective phase angles of the R_7 and R_8 arms and are mainly due to capacitance to earth of the shielding. The capacitance of the high-tension electrode of the specimen to its guard is C_5 and ψ_5 its phase defect angle, and the capacitance of the high side of the air capacitor to its guard is C_6. In the operators we have assumed that the phase defect angles of the specimen and its guard are the same, namely that ψ_1 equals ψ_5, and also that both the air capacitor C_2 and its guard C_6 have zero phase defect angles.[3] The capacitance existing between the measuring circuit and the shielding circuit of the specimen is C_9 and the corresponding capacitance for the air capacitor is C_{10}. Both of these capacitances, C_9 and C_{10} have leakage resistances, W_9 and W_{10} respectively. In normal operation these leakage resistances W_9 and W_{10} must equal infinity as already mentioned.

Substituting the values of the general operators into the equations for the correction terms we obtain the corresponding complex values of $\bar{k}_1$ and $\bar{k}_2$ equations (**10**) and (**11**)

$$\bar{k}_1 = \frac{\left\{\Phi\left[1+\frac{C_9}{C_1}(1+m_1)\right]-\frac{Q}{W_9}\phi_7(1+m_1)+\psi_1\left[1+\frac{Q}{W_9}(1+m_1)\right]\right\} - j\left\{1+\frac{Q}{W_9}(1+m_1)+\Phi\,\phi_7\left[1+\frac{C_9}{C_1}(1+m_1)\right]-\frac{C_9}{C_1}\Phi\,\psi_1(1+m_1)\right\}}{\left\{\Phi\left[1+\frac{C_9}{C_1}(1+m_1)\right]-\frac{Q}{W_9}\phi_7(1+m_1)+\psi_1\left[1+\frac{Q}{W_9}(1+m_1)\right]\right\}-\left\{\frac{Q}{W_9}(\phi_q-\phi_7)\right\}-j\left\{1+\frac{Q}{W_9}(1+m_1)+\Phi\,\phi_7\left[1+\frac{C_9}{C_1}(1+m_1)\right]-\frac{C_9}{C_1}\Phi\,\psi_1(1+m_1)\right\}-j\left\{\frac{C_9}{C_1}\Phi(\phi_q-\phi_7)\right\}} \tag{10}$$

$$\bar{k}_2 = \frac{\left\{\Phi\left[1+\frac{C_{10}}{C_2}(1+m_2)\right]-\frac{S}{W_{10}}\phi_8(1+m_2)\right\}-\left\{\frac{S}{W_{10}}(\phi_s-\phi_8)\right\}-j\left\{1+\frac{S}{W_{10}}(1+m_2)+\Phi\,\phi_8\left[1+\frac{C_{10}}{C_2}(1+m_2)\right]\right\}-j\left\{\frac{C_{10}}{C_2}\Phi(\phi_s-\phi_8)\right\}}{\left\{\Phi\left[1+\frac{C_{10}}{C_2}(1+m_2)\right]-\frac{S}{W_{10}}\phi_8(1+m_2)\right\} - j\left\{1+\frac{S}{W_{10}}(1+m_2)+\Phi\,\phi_8\left[1+\frac{C_{10}}{C_2}(1+m_2)\right]\right\}} \tag{11}$$

In equations (10) and (11) the term expressed by Φ is

$$\Phi = \omega C_2 S = \omega C_1 Q$$

A study of the expressions for the correction terms shows that the majority of the terms in the numerators and denominators are alike. They differ, however, only in certain terms in both their real and imaginary parts.

These complex expressions for k_1 and k_2 are, therefore, of the general form

$$\bar{k}_1 = \frac{\alpha_1 - j\beta_1}{(\alpha_1 - \gamma_1) - j(\beta_1 + \delta_1)} \tag{12}$$

and

$$\bar{k}_2 = \frac{(\alpha_2 - \gamma_2) - j(\beta_2 + \delta_2)}{\alpha_2 - j\beta_2} \tag{13}$$

Where the α terms denote those that are alike in their real parts in both the numerators and denominators and the β terms apply to those imaginary terms in the respective numerators and denominators that correspond. The γ and δ terms in (12) and (13) include only those terms by which the respective numerators and denominators differ.

The absolute values and the phase angles of the correction terms introduced by shielding and guarding the bridge may be expressed as follows:

$$K_1 = \sqrt{\frac{\alpha_1^2 + \beta_1^2}{(\alpha_1 - \gamma_1)^2 + (\beta_1 + \delta_1)^2}} \tag{14}$$

$$K_2 = \sqrt{\frac{(\alpha_2 - \gamma_2)^2 + (\beta_2 + \delta_2)^2}{\alpha_2^2 + \beta_2^2}} \tag{15}$$

and

$$\phi_{K1} = +\frac{\alpha_1\delta_1 + \beta_1\gamma_1}{\alpha_1^2 + \beta_1^2 - \alpha_1\gamma_1 + \beta_1\delta_1} \tag{16}$$

$$\phi_{K2} = -\frac{\alpha_2\delta_2 + \beta_2\gamma_2}{\alpha_2^2 + \beta_2^2 - \alpha_2\gamma_2 + \beta_2\delta_2} \tag{17}$$

Taking into account the phase angles of the correction terms, we may write the generalized expression (8) for the power factor of the specimen as follows:

$$\psi_1 = \omega C_s S - \omega C_q Q - (\phi_{K1} + \phi_{K2}) \tag{18}$$

Analysis of the Correction Terms

In the analysis of the correction terms four cases that involve the accuracy and method of operation of the bridge are discussed.

Case 1

Case 1 considers the effect of failure to balance the guard circuits for phase. It belongs to the normal operation of the bridge in which the guard circuits are balanced for magnitude only, and the leakage resistances W_9 and W_{10} are assumed equal to infinity. In this case the normal operators are the general operators given in equation (9) with the exceptions of $\bar{a}_9$ and $\bar{a}_{10}$, which here become:

$$\bar{a}_9 = -\frac{j}{\omega C_9}$$

$$\bar{a}_{10} = -\frac{j}{\omega C_{10}}$$

Case 2

Case 2 considers the phase angle error introduced by the failure to balance the guard circuits for magnitude as well as phase, provided that the leakage resistances W_9 and W_{10} still remain infinite. Let θ_7 represent the ratio of the actual value of guard resistance R_7 to the proper value for correct magnitude balance, that is:

$$\theta_7 = \frac{R_7}{m_1 Q}$$

Similarly $\qquad \theta_8 = \dfrac{R_8}{m_2 S}$

Then, the conditions expressed under this case modify the general operators as follows:

$$\bar{a}_7 = \theta_7 m_1 Q (1 - j \phi_7) \qquad \bar{a}_9 = -\frac{j}{\omega C_9}$$

$$\bar{a}_8 = \theta_8 m_2 S (1 - j \phi_8) \qquad \bar{a}_{10} = -\frac{j}{\omega C_{10}}$$

in which θ_7 and θ_8 obviously become unity when the guard circuits are properly balanced for magnitude.

TABLE II

Case	a_7	a_9	α_1	β_1
1	$m_1 Q (1 - j \phi_7)$	$-\dfrac{j}{\omega C_9}$	$\Phi \left[1 + \dfrac{C_9}{C_1} (1 + m_1) \right] + \Psi_1$	1
2	$\theta_7 m_1 Q (1 - j \phi_7)$	$-\dfrac{j}{\omega C_9}$	$\theta_7 \Phi \left[1 + \dfrac{C_9}{C_1} (1 + m_1) \right] + \Psi_1$	1
3	$m_1 Q (1 - j \phi_7)$	$\dfrac{1}{\dfrac{1}{W_9} + j \omega C_9}$	$\Phi \left[1 + \dfrac{C_9}{C_1} (1 + m_1) \right] - \dfrac{Q}{W_9} \phi_7 (1 + m_1) + \Psi_1 \left[1 + \dfrac{Q}{W_9} (1 + m_1) \right]$	$1 + \dfrac{Q}{W_9} (1 + m_1)$
4	$\theta_7 m_1 Q (1 - j \phi_7)$	0	$- \phi_7$	1

Case	a_8	a_{10}	α_2	β_2
1	$m_2 S (1 - j \phi_8)$	$-\dfrac{j}{\omega C_{10}}$	$\Phi \left[1 + \dfrac{C_{10}}{C_2} (1 + m_2) \right]$	1
2	$\theta_8 m_2 S (1 - j \phi_8)$	$-\dfrac{j}{\omega C_{10}}$	$\theta_8 \Phi \left[1 + \dfrac{C_{10}}{C_2} (1 + m_2) \right]$	1
3	$m_2 S (1 - j \phi_8)$	$\dfrac{1}{\dfrac{1}{W_{10}} + j \omega C_{10}}$	$\Phi \left[1 + \dfrac{C_{10}}{C_2} (1 + m_2) \right] - \dfrac{S}{W_{10}} \phi_8 (1 + m_2)$	$1 + \dfrac{S}{W_{10}} (1 + m_2)$
4	$\theta_8 m_2 S (1 - j \phi_8)$	0	$- \phi_8$	1

Case 3

Case 3 considers the phase angle error introduced by the presence of leakage resistances W_9 and W_{10} in the capacitances C_9 and C_{10}, between the measuring circuits and their guards, provided that the guard resistances have the proper value for magnitude balance. In this case, the general operators of equation (9) hold.

Case 4

Case 4 considers the conditions and ratio errors that exist when the corners of the bridge are connected together, *i. e.*, A to A', as is customary in balancing the guard resistances R_7 and R_8 (See Operation). Under these conditions the operators involved become:

$$\begin{aligned} \bar{a}_7 &= \theta_7 m_1 Q (1 - j \phi_7) \\ \bar{a}_8 &= \theta_8 m_2 S (1 - j \phi_8) \\ \bar{a}_9 &= 0 \\ \bar{a}_{10} &= 0 \end{aligned}$$

To obtain the phase angle and ratio errors produced by the conditions as outlined in these four cases, substitute the corresponding operators in the expressions for $\bar{k}_1$ and $\bar{k}_2$ (5) and determine the respective values of α, β, γ and δ terms (12) to (17) inclusive for each case.

Table II below gives for the four cases the values of the phase angle and ratio errors, together with the modifying operators causing the errors, and the corresponding expressions for the α, β, γ, and δ terms. In working out the values shown in this table the results were simplified by neglecting the product of two phase angles when added to or subtracted from unity.

In most cases this entails an approximation of less than one part in 10,000 of the factor involved, and is, therefore, fully warranted since it means an approximation in the phase angle relation of the bridge of the order of one part in one hundred million.

EXPERIMENTAL VERIFICATION OF EQUATIONS

Case 1

A study of the phase angle and absolute value errors, Table II, shows that the failure to balance the guard circuits for phase causes no error in the ratio balance since K_1 and K_2 here equal unity, and only a negligible phase angle error.

TABLE II

γ_1	δ_1	K_1	ϕ_{k1}
0	$\frac{C_9}{C_1}\Phi(\phi_q-\phi_7)$	1	$+\frac{C_9}{C_1}\left[1+\frac{C_9}{C_1}(1+m_1)\right]\Phi^2(\phi_q-\phi_7)$
$\frac{C_9}{C_1}\Phi(\theta_7-1)$	$\frac{C_9}{C_1}\Phi(\phi_q-\theta_7\phi_7)+\frac{C_9}{C_1}\Phi\Psi_1(\theta_7-1)$	1	$+\frac{C_9}{C_1}\Phi(\theta_7-1)$
$\frac{Q}{W_9}(\phi_q-\phi_7)$	$\frac{C_9}{C_1}\Phi(\phi_q-\phi_7)$	1	$+\frac{(\phi_q-\phi_7)}{\frac{W_9}{Q}+(1+m_1)}$
$\frac{(\phi_q-\theta_7\phi_7)}{\theta_7(1+m_1)}$	$-\frac{(\theta_7-1)}{\theta_7(1+m_1)}$	$\frac{\theta_7(1+m_1)}{(1+2m_1\theta_7+m_1^2\theta_7^2)^{\frac{1}{2}}}$	$+\frac{(\phi_q-\phi_7)}{1+\theta_7 m_1}$

γ_2	δ_2	K_2	ϕ_{k2}
0	$\frac{C_{10}}{C_2}\Phi(\phi_s-\phi_8)$	1	$-\frac{C_{10}}{C_2}\left[1+\frac{C_{10}}{C_2}(1+m_2)\right]\Phi^2(\phi_s-\phi_8)$
$\frac{C_{10}}{C_2}\Phi(\theta_8-1)$	$\frac{C_{10}}{C_2}\Phi(\phi_s-\theta_8\phi_8)$	1	$-\frac{C_{10}}{C_2}\Phi(\theta_8-1)$
$\frac{S}{W_{10}}(\phi_s-\phi_8)$	$\frac{C_{10}}{C_2}\Phi(\phi_s-\phi_8)$	1	$-\frac{(\phi_s-\phi_8)}{\frac{W_{10}}{S}+(1+m_2)}$
$\frac{(\phi_s-\theta_8\phi_8)}{\theta_8(1+m_2)}$	$-\frac{(\theta_8-1)}{\theta_8(1+m_2)}$	$\frac{(1+2m_2\theta_8+m_2^2\theta_8^2)^{\frac{1}{2}}}{\theta_8(1+m_2)}$	$-\frac{(\phi_s-\phi_8)}{1+\theta_8 m_2}$

The latter is shown to be true in the following typical numerical example of the measurement of the power factor at 500 volts 60 cycles of an impregnated paper specimen at 30 deg. cent. Complete data are given below.

Standard Air Capacitor	Specimen
$C_2 = 1110 \times 10^{-12}$ farad	$C_1 = 1159 \times 10^{-12}$ farad
$C_s = 2815 \times 10^{-12}$ farad	$C_q = 889 \times 10^{-12}$ farad
$C_{10} = 489 \times 10^{-12}$ farad	$C_9 = 469 \times 10^{-12}$ farad
$R_8 = 9909$ ohms	$R_7 = 8216$ ohms
$S = 5219$ ohms	$Q = 5000$ ohms
$m_2 = R_8/S = 1.899$	$m_1 = R_7/Q = 1.643$
$\phi_s = \omega C_s S = 0.00554$	$\phi_q = \omega C_q Q = 0.00168$

$$\Phi = \omega C_1 Q = \omega C_2 S = 0.00218$$

$$\psi_1 = \omega C_s S - \omega C_q Q = 0.00386$$

Substituting the proper values in the expressions for ϕ_{K1} and ϕ_{K2} of Case 1, Table II, and assuming as an extreme case that ϕ_7 and ϕ_8 are zero, the following phase angle errors are obtained:

$$\phi_{K1} = +\ 6.7 \times 10^{-9}$$

$$\phi_{K2} = -\ 26.3 \times 10^{-9}$$

These are seen to be negligible when compared with the phase angle sensitivity of our bridge $\pm\ 5 \times 10^{-6}$.

This numerical example is only one of many that could be presented and fully justifies our method of operating the bridge in which no attempt is made to balance the guard circuit for phase. It might also be stated in further justification of our practise that the capacitances to earth of the shields which are in parallel with R_7 and R_8, respectively, serve to reduce this error still further.

Case 2

In Case 2 where the guards are not properly balanced for either phase or magnitude it is evident that the absolute values of K_1 and K_2 are again unity and that ratio errors are absent.

It may be seen from Table II, Case 2, that the phase angle errors ϕ_{K1} and ϕ_{K2} are linear functions of the magnitude unbalance factors θ_7 and θ_8. To verify these equations experimentally the following test was made:

The bridge was balanced normally as in the numerical example given under Case 1, keeping the air capacitor side of the bridge properly balanced for magnitude, we determined the value of ϕ_{K1} corresponding to different values of the guard resistance, R_7. The ratio, θ_7, of the actual value of the guard resistance to its true value was varied from zero to two. The results are given in Fig. 4 and it is evident that a linear relation exists between ϕ_{K1} and θ_7.

As a further check a similar test was performed to determine the value of ϕ_{K2} when θ_8 was varied, keeping θ_7 equal to unity. The results of this run are also plotted in Fig. 4 and we again obtain linear relation between the two quantities.

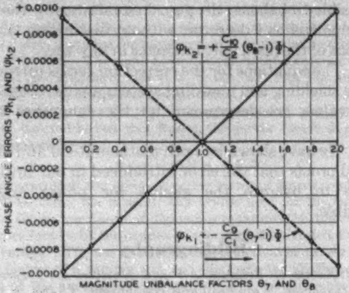

Fig. 4—Experimental Verification of Case 2

Obviously, from such experimentally determined curves it is possible to compute the values of the cross-capacitances C_9 and C_{10}.

As a verification of these relations, the cross-capacitances, C_9 and C_{10}, were measured separately using a 1,000-cycle capacitance bridge. The results given below in Table III show a good agreement and serve further to prove the correctness of the equations.

TABLE III—COMPARISON OF CALCULATED AND MEASURED VALUES FOR THE CROSS-CAPACITANCES C_9 AND C_{10}

Capacitance	Capacitance computed from Fig. 4	Capacitance measured at 1,000 cycles
C_9	.469	.478
C_{10}	.489	.472

It is clearly evident from the equations that the phase angles errors are reduced by keeping C_9 and C_{10} small. The experimental results of Case 2 verify the mathematical relation and also show the importance of accurately balancing the guard resistances for magnitude.

Case 3

This case considers the phase angle errors caused by lack of phase angle balance in the guard circuits acting through the presence of leakage resistances W_9 and W_{10}. As in the two preceding cases, the ratio balance still remains unaffected when the guards are properly balanced for magnitude. The following test was performed to obtain experimental verification of the equations involved; Table II, Case 3:

1. The bridge was balanced in the normal way, with W_9 and W_{10} equal to infinity, the air capacitor was adjusted to one to one ratio with the specimen to within five ohms in 5,000, and the guard capacitance C_6 was varied to make the ratios m_1 and m_2 equal to each other to within 0.1 per cent.

2. The specimen guard circuit was then balanced for phase as well as magnitude using an additional variable capacitor in parallel with R_7.

3. An inductance L_8, of approximately 0.5 henry, was introduced in series with the air capacitor guard resistance R_8, and this resistance was decreased by an amount equal to the resistance of the inductance coil, thus preserving the magnitude balance but greatly increasing the phase difference $(\phi_s - \phi_8)$. It must be noted here that the insertion of L_8 giving the arm R_8 a large inductive phase angle caused no measurable power factor error when the guard of the air capacitor was balanced for magnitude only and not for phase.

4. The actual test was then performed, introducing the resistances W_9 and W_{10} which were kept at all times equal to each other, and which were varied from 100,000 ohms to zero, determining at each step the phase angle error caused by the phase angle difference $(\phi_s - \phi_8)$.

In this test it was necessary to keep W_9 equal to W_{10} to preserve the one to one ratio throughout, thereby eliminating errors due to residual angles in the resistance boxes. It was, therefore, necessary to balance the specimen guard circuit for phase as well as magnitude to confine the error studied to the phase difference $(\phi_s - \phi_8)$ alone, and it was deemed advisable to introduce an inductance in guard resistance R_8 to accentuate the error and make its measurement more accurate.

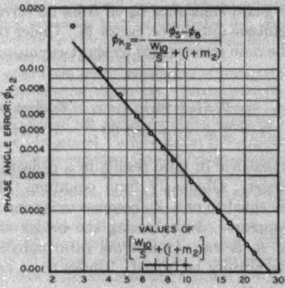

Fig. 5—Experimental Verification of Case 3

The phase angle error under consideration is

$$\phi_{K2} = \frac{\phi_s - \phi_8}{\dfrac{W_{10}}{S} + (1 + m_2)} \qquad (19)$$

which may be written in the form

$$\log\phi_{K2} = \log(\phi_s - \phi_8) - \log\left[\frac{W_{10}}{S} + (1+m_2)\right] \quad (20)$$

It is readily seen from (**20**) that plotting in logarithmic coordinates the phase angle error ϕ_{K2} against the variable

$$\left[\frac{W_{10}}{S} + (1 + m_2)\right]$$

gives a straight line of which the slope is unity for axes of equal moduli. The results of experiment are plotted in Fig. 5 which shows a remarkable agreement between the results of experiment and the above theoretical considerations. It is interesting to note that ϕ_7 and ϕ_8 may be obtained by this test if desired.

Case 4

This case is of importance because it represents the conditions pertaining to the balancing of the guard circuits as described under operation. It considers the ratio error introduced by a given unbalance in the guard circuit acting through a short circuit between the measuring circuit and its guard.

As stated under the operation of the bridge, all four low-voltage arms R_7, Q, S, and R_8 are set at some tentative values, and the operator adjusts Q and S to obtain the preliminary ratio balance. We will confine our attention to the specimen side of the bridge and assume that the preliminary ratio balance is obtained with the tentative guard resistance R_7 off from its correct value by a magnitude unbalance, factor θ_7. Under these conditions the ratio error caused by short-circuiting $B - B'$ is

$$K_1 = \frac{\theta_7\,(1 + m_1)}{\sqrt{1 + 2\,m_1\,\theta_7 + m_1^2\,\theta_7^2}} = \frac{\theta_7\,(1 + m_1)}{1 + m_1\,\theta_7} \quad (21)$$

This unbalance in ratio results in a deflection of the galvanometer. Obviously from equation (**21**), when θ_7 is made equal to one, K_1 becomes unity and the ratio error disappears. In operating the bridge the guard resistance R_7 is varied until the ratio unbalance disappears as already explained. Under these conditions

$$\frac{R_7}{Q} = \frac{C_1}{C_5} = m_1$$

Similarly when the air capacitor side of the bridge is adjusted K_2 equals 1 and

$$\frac{R_8}{S} = \frac{C_2}{C_6} = m_2$$

From the analysis of this case it is evident that our method of adjusting the guard resistance without disturbing in any way the main galvanometer connection is fully justified.

CONCLUSIONS

The conclusions may be summarized as follows:

1. In a completely guarded and shielded bridge it is advisable to keep the cross-capacitances existing between the measuring circuits and the guard circuits low.
2. In such a bridge it is of prime importance that the conductances of the cross-capacitances be maintained at zero.
3. We have developed a method employing a separately excited moving coil a-c. galvanometer that makes it possible to balance the guard circuits using the main galvanometer as a detector, without disturbing the connections of the main galvanometer in any way.
4. The equations for the correction terms introduced by the shielding and guarding system are developed and their values are determined for the constants of our bridge.
5. We have shown by both mathematical and experimental proofs that the error introduced in our bridge by failure to balance the guards for phase angle is negligible.

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Discussion

Leo J. Berberich: For some years, engineers interested in power factor measurements on insulating materials at high voltages and commercial frequencies have felt the need for some suitable means of verifying the accuracy of such measurements. As a result of this, several methods for checking dielectric loss measuring equipment have been proposed in the last two or three years. Some of these are thermal methods utilizing temperature rise associated with power loss, and others involve the use of a standard load usually constructed of glass. These have met with some degree of success, as the literature reveals, but leave something to be desired when it comes to verifying measurements of the small power factors found in good quality insulating oils.

A little over a year ago work was begun on the development of a new "standard of low power factor" at the Johns Hopkins University. This standard has recently been completed and a paper by Dr. Kouwenhoven and myself is now in the hands of the Instruments and Measurements Committee of the Institute. I might say briefly that it consists of a well-shielded and guarded compressed air capacitor and very specially shielded resistors. The standard can be used for voltages as high as 25 kv. and its power factor can be computed accurately. During the course of its development I have had occasion to check the accuracy of the bridge described by Dr. Kouwenhoven as well as that of another bridge similarly constructed. The following table gives an idea of the excellent agreement found between the computed power factor of the standard and the power factor as measured by the bridges:

COMPUTED POWER FACTOR OF STANDARD *vs.* MEASURED POWER FACTOR

Computed power factor 60 cycles	Measured power factor Bridge No. 1	Measured power factor Bridge No. 2
0.000195	0.000196	0.000194
0.000996	0.000992	0.000996
0.001993	0.001983	0.001993

The data listed in the table not only give further evidence as to the accuracy of the bridge described in the paper by Dr. Kouwenhoven and Mr. Baños, but also show that the two bridges used at Johns Hopkins for measurements on insulating oils agree remarkably well. The small deviations from the computed power factor of the standard were due to the fact that the limits of sensitivity of the bridges were approached.

J. C. Balsbaugh: In the bridge described in the paper by Messrs. Kouwenhoven and Baños the power factor ψ_1 of the specimen is given as

$$\psi_1 = \omega C_8 S - \omega C_9 Q$$

This assumes that the absolute power factor of the air condensers is equal to zero. In general, any bridge used for the measurement of the power factor of dielectrics measures the difference in the power factors of the condensers forming the high-tension arms of the bridge. The use of a substitution method in measuring the power factor of a sample will not eliminate the absolute power factors of the air condensers, but will give the difference in the power factors of the sample and the air condenser it replaces. Thus in the measurement of the power factors of dielectrics to a given precision, it is first necessary to investigate the absolute power factors of the condensers to be used in the bridge and determine if these absolute power factors are of the magnitude of the desired power factor precision.

A modified Schering bridge with a precision in power factor measurement of 10^{-6} has been constructed at the Massachusetts Institute of Technology in connection with research carried on in cooperation with the Cable Research Subcommittee of the National Electric Light Association. A paper describing this bridge was presented at the meeting of the Committee on Electrical Insulation of the National Research Council in Cambridge, Mass., November 13, 1931. Progress reports covering the design, construction, and operation of this bridge have also been submitted to the Cable Research Subcommittee of the National Electric Light Association. It is planned to make formal publication of this research when all the factors affecting the power factor precision of the bridge have been fully investigated. This bridge has a sensitivity such that a change in power factor very much lower than 10^{-6}, and a capacitance difference materially lower than 10^{-6} of the high-tension capacitances are detectable. Tests with this bridge have shown that air condensers may have an absolute power factor of the order of 10^{-5}. A study of the probable causes of this power factor has shown that it is not due to conduction currents in the gas, a loss in the solid dielectric between the bridge and shield surfaces, or to extraneous matter on the plates of condensers. A number of tests has shown that the loss depends on the type of metal used in the condenser conducting surfaces, the voltage gradient, surface area, and apparently is due to the gas adsorbed on the condenser surfaces.

In using a substitution method for measuring the power factor of a sample it is essential that the resistances in the low-tension arms be maintained constant in the two balances. The use of a substitution method primarily eliminates the shunt capacitance to ground of the low-tension arms and thereby permits obtaining the power factor of the sample in terms of the difference in capacitances in the two balances of one condenser in the low-tension arms. However, if the low-tension resistances are adjusted in the two balances the shunt capacitances to ground in the resistances are changed. Therefore the measured power factor will not be given in terms of the differences between the settings of one low-tension capacitance. In the bridge at the Massachusetts Institute of Technology previously referred to, variable condensers are used in one of the high-tension arms. Thus a capacitance balance of the bridge may be obtained entirely by an adjustment of the high-tension condensers with the low-tension resistances maintained constant. In this manner it is possible to eliminate completely the shunt capacitances of the bridge to ground from causing an error in the power factor measured by the bridge. The variable capacitances used in this bridge permit obtaining as accurate a capacitance balance as a power-factor balance. These variable condensers are designed with concentric cones that may be moved axially relative to each other. By this method it is possible to obtain and detect changes in capacitances less than 10^{-5} μμf. (10^{-17} farads).

In general, the effect of a phase unbalance in the shield voltage will not cause an error in the power factor measured by the bridge described in this paper. This assumes that the phase shifter is quite accurately set. The effect of a phase unbalance in the shield voltage will be to give an incorrect capacitance balance of the bridge. In the study of dielectrics having very low power factor it is advisable to be able to detect and measure changes in capacitance of the sample of the same order as the power factor. The study of both capacitances and power factor change should permit a clearer and more definite analysis of the results of a test. The magnitude of the phase unbalance of the shield voltage depends principally upon the relative values of capacitances from high-tension to shield and to bridge, and capacitances from shield and bridge to ground. In the usual bridge these values are such that phase balance of the shield is necessary to give a satisfactory bridge capacitance balance where high precision is necessary.

In the bridge described in this paper the shield is balanced by directly connecting the bridge and shield. This method requires two separate shield circuits and thereby requires considerable adjustment to get an accurate shield balance. It would seem preferable to have one shield circuit and balance by connecting the detecting apparatus between the bridge and shield circuit. This method may be accomplished quite easily and will permit a high accuracy in the shield balance.

In this paper there are no calculations or tests giving the sensitivity or precision of the bridge. Furthermore it should be noted that precision in power factor measurement may be very much different from the sensitivity of the bridge to changes in power factor. In general, the precision in power factor measurements is much more important than the bridge sensitivity.

S. K. Waldorf: For Case I it is shown experimentally that the phase angle errors ϕ_{k1} amd ϕ_{k2} in Table II are negligible with specimens having a capacitance of about 1,100 μμf. The authors mention having measured other specimens having capacitances as low as 40 μμf., for which the ratios C_9/C_1 and C_{10}/C_2 become of the order of 30 instead of about ½ for the case given. At the same time there is the corresponding decrease in Φ which enters as a quadratic in expressions for ϕ_{k1} and ϕ_{k2}. The net effect of reduction of specimen size is, then, that the values of these correction terms are not appreciably changed, other factors being equal.

However, to maintain the bridge sensitivity unimpaired, the values of S and Q must be increased, resulting in increases in ϕ_{k1} and ϕ_{k2} which can become appreciable in such circumstances. For the condition to hold that the net correction remain negligible, it then becomes of some importance that the bridge be symmetrical in order for ϕ_{k1} to be very nearly equal numerically to ϕ_{k2}, as the latter quantity is shown to be negative in Table II.

The condition considered in Case III is of practical importance, as it is a very real source of difficulty in humid weather. In such weather there is condensation of moisture on the insulation of the bridge which causes appreciable values of leakage conductance between circuits. The values of W_9 and W_{10}, instead of being nominally equal to infinity as desired, become finite, indeterminate, and variable quantities, making evaluation of ϕ_{k1} and ϕ_{k2} impossible. To a certain extent these troublesome effects can be eliminated, or at least minimized in very bad weather, by coating the insulation with paraffin and by having a minimum of contact surface between the insulation and the conductors. From the very erratic nature of such difficulties due to humidity, it is easily understandable that no method of correction has yet been devised. Indeed, on some summer days of high humidity, conditions become so unsteady that measurements are impossible. This may seem strange to the uninitiated, but is quite common experience for those making refined dielectric measurements of any sort.

Fig. 2 of the galvanometer circuit shows double shielding on the transformer. Such transformers are not in common use nor easily obtained. I believe it would be of value if the authors could tell us briefly the purpose of such shielding and if it is essential, as perfect double shielding with a high value of insulation resistance is not readily made in a compact assembly, and hence is relatively expensive to manufacture.

In conclusion, I should like to add a practical word of precaution for those who may be called upon to build small high-voltage air capacitors for such service as the authors describe. The natural tendency is to make the plates as small as possible for compactness and economy, resulting in small spacings of about 2 mm. or less. Close spacings usually are the source of much trouble and annoyance arising from small fibers bridging the gap between plates and causing either flashover or a power factor in the air capacitor. A much more satisfactory arrangement has been found with plate dimensions large enough to allow a spacing of not less than about 5 mm. for the desired value of capacitance, which is quite effective as a preventative of such bridging.

Perry A. Borden: The method of balancing out induced voltages in the galvanometer coil by the use of a mutual inductance in the field circuit appears to remove one of the objectionable features of the electrodynamic galvanometer. It has been found that by the use of two quadrature reference voltages derived from the secondary of the phase-shifting transformer as shown in Fig. 1, and applied alternately to the field coil of the galvanometer as the length and position respectively of the current vector in the bridge circuit are reduced, much time in balancing such a circuit may be saved.

Another very satisfactory method of detecting a balance condition* consists in exciting the galvanometer field from a frequency differing slightly from that of the system upon which the tests are being made, and proceeding with the balancing process until the "beat" oscillations disappear.

R. P. Siskind: In their conclusions, Messrs. Kouwenhoven and Baños state that it is of prime importance that the conductance of the cross-capacitances between the main electrodes with their leads and their guards be maintained at zero. Under this condition, it is shown analytically and experimentally that failure to balance the guards for phase angle introduces negligible error. In this connection, the authors state that it is essential for accurate measurement that the insulation resistances of these cross-capacitances, as measured by a megger, be maintained at infinity. Since this is an a-c. bridge and these cross-capacitances contain dielectric material, their a-c. leakage resistances cannot be infinite but will have values equal to the equivalent parallel resistances of these cross-capacitances at the test frequency. These values will then depend upon the power factors of the cross-capacitances. However, a rough calculation based upon the values given under the heading, "Experimental Verification of Equations" shows that at 60 cycles, an assumed power factor of 0.100 for C_{10} would give W_{10} a value of 54 megohms and, although finite, this value of W_{10} would lead to an error in measured power factor somewhat less than the accuracy claimed by the authors. According to the equations, the error would be entirely negligible for a more reasonable value of power factor of C_{10}.

Another item worthy of consideration is the possible effect upon the accuracy of the bridge of the presence of harmonics. The authors use a detecting device which operates upon the wattmeter principle. If the potential applied to the test specimen, C_1, is high enough to produce ionization, third and other harmonics will be produced in the current wave through C_1 even with the application to C_1 of a pure sine wave. This effect has been found in measurements with the Dawes-Hoover bridge upon dielectric specimens stressed beyond the ionization point. The bridge is generally out of balance for these harmonics when balanced for the fundamental and this results in the presence of harmonic voltages across the detector. The tuned vibration galvanometer is not affected. With the a-c. galvanometer used by the authors, the presence of third and other harmonic currents in the moving coil would give a deflection if any corresponding harmonic current were present in the field winding. It is hard to believe that the output of a phase-shifting transformer excited with a pure sine wave voltage is entirely free from harmonics. If corresponding harmonic currents of any appreciable magnitude do exist, a torque will be exerted upon the moving coil which would be offset by an equal and opposite torque produced by a slight unbalance of the bridge for the fundamental frequency in bringing the galvanometer deflection to zero. Thus the possibility of some small error does exist when ionization takes place in the test specimen.

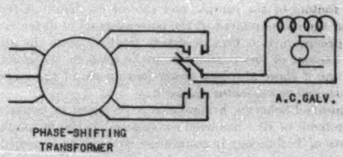

FIG. 1

W. B. Kouwenhoven and Alfredo Baños, Jr.: Mr. Balsbaugh states that we have assumed the absolute power factor of the air capacitor C_2 of the bridge to be zero. That this assumption is justifiable at 60 cycles is clearly brought out by an investigation conducted by one of the authors[1] of this paper and also by the work of Churcher and Dannant.[2]

Mr. Balsbaugh reports that his air capacitors have a power factor of the order of 10^{-5} which he believes is caused by the absorption of gas by the electrode surfaces. If such were the case it is quite possible that this power factor will also be associated with the test cell as well as with the air capacitor, and thus tend to neutralize each other in the bridge.

We have measured power factors of oil samples as low as 0.000025 and we have checked the accuracy of such measurements by a method that does not involve either the standard air capacitor or the Schering bridge. This method is fully described in a paper by one of the authors.[3] Briefly, it consists of the calculation of the power factor of the sample at 60 cycles from the charge and discharge current time relations under continuous potentials. Thus it appears that in measuring power factor of this order an error of 10^{-5} in the power factor of the air capacitor would not permit the excellent agreement that is obtained between the power factor measured in the bridge and that computed from the d-c. characteristics.

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*Balance Methods in A-C. Measurement, P. A. Borden,, TRANS. A.I.E.E., XLII, (1923), p. 395.

[1] *The Phase Defect Angle of an Air Capacitor*, W. B. Kouwenhoven and C. L. Lemmon, TRANS. A.I.E.E., Vol. 49, July 1930, p. 952.

[2] "The Use of Air Condensers as High-Voltage Standards," B. G. Churcher and C. Dannant, *Journal I.E.E.*, Vol. 69, No. A16, Aug. 1931.

[3] *The Predetermination of the A-C. Behavior of Dielectrics*, J. B. Whitehead and A. Baños, Jr., A.I.E.E. Winter Convention, 1932.

The error in the ratio balance that is caused by failure to balance the guard circuits for phase is very small. In our operation of the bridge the ratio arms, Q and S, are only balanced to the nearest ohm in 5,000 and the above mentioned error is much less than this and can be safely neglected.

Our method of balancing the guards by connecting directly the bridge and the shield possesses the distinct advantage of permitting the use of the full sensitivity of the main detector without disturbing in any way its connections. In our bridges using no amplifiers this method has proven very simple and quick and gives accurate results. We recommend it strongly because of the small amount of auxiliary equipment and adjustment that is required.

As regards the precision of the bridge, the data presented by Dr. Berberich in his discussion covers this point fully.

The shielded transformer in the galvanometer circuit is used because it fixes accurately the distributed capacitance that exists between the arms of the bridge and the ground. The connection of this shield to the B corner of the bridge (Fig. 1 of the paper) places this capacitance in parallel with C_2. Unless a doubly shielded transformer is used and connected as shown the effect of this capacitance cannot be evaluated.

The suggestion of Mr. Borden is very interesting and we are at present testing that method of balancing in another research using a similar bridge.

The point raised by Mr. Siskind as to harmonics is of considerable importance. We made an oscillograph study of the currents through the different arms of our Schering bridge when supplied with current from a General Electric sine wave generator. In this work we used the amplifier oscillograph of Dr. Waldorf.[4] We measured the voltage induced by the galvanometer field in its moving coil and found it practically free from harmonics. We found evidence of a small tooth ripple in the currents through the arms of the bridge. These currents, however, were free from low order harmonics. We do not feel that harmonics cause any trouble in our bridge because of the excellent agreement that we have been able to obtain between calculated and measured power factors on the same sample. Furthermore our bridge is operated at potentials that are considerably below the ionization value.

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[4] "The Amplifier Oscillograph Applied to the Study of Dielectrics with Continuous Potentials," S. K. Waldorf. Forth-coming paper through A.I.E.E.

Power Factor Measurement by the Capacitance Bridge

An Analysis of the Parallel Substitution Method as Applied to Two Types of Capacitance Bridges for Capacitance and Power Factor Measurements Upon Dielectric Specimens

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INTRODUCTION

SEVERAL types of capacitance bridges for measurement of the capacitance and the power factor of dielectric specimens are in use at the present time. Some of these bridges make a direct comparison between the capacitance under test and a standard capacitance. The Schering and Dawes-Hoover high-voltage power-frequency bridges are too well-known examples of this type. For accurate power factor measurements upon dielectric specimens at low voltages and audio frequencies a substitution method is generally preferred.

Two substitution methods are commonly employed. Using one method, here referred to as the "direct" substitution method, the test capacitance is first balanced in the bridge circuit against an auxiliary capacitance. The test capacitance is then removed, replaced by a variable standard capacitance, and the bridge rebalanced. In this way a comparison of the test and standard capacitance is obtained, and the power factor and capacitance of the former are determined if the power factor and capacitance of the latter are known at the particular setting used. Using the other method, here referred to as the "parallel" substitution method, the test capacitance and a variable standard capacitance in parallel are first balanced in the bridge circuit against an auxiliary capacitance. The test capacitance is then removed and the bridge rebalanced by variation of the standard capacitance. With this method it is necessary to know the capacitance and manner of variation of resistance with capacitance of the standard condenser used, but it is not necessary to know the actual values of its resistance or phase angle at the settings used.

Both methods are generally applicable to the capacitance bridges commercially available for these measurements upon small capacitances at low voltages and audio frequencies. With each method, the small change in phase angle of the arm of the bridge circuit in which the substitution is made must be counterbalanced in the second balance of the bridge. The compensation for this change in phase angle enters directly into the measurement of power factor of the test capacitance, and it is the manner of making this adjustment in which the bridges in common use show essential differences.

One manner of making this adjustment is the use of a variable resistance in parallel with the test and standard capacitances. The capacitance and conductance bridge of the Bell Telephone Laboratories[1] uses this arrangement. A second arrangement, used in the equal-arm capacitance bridge of the General Radio Company,[2] is a variable resistance in series with either the standard of auxiliary capacitances as the balances may require. A third device, used in the precision audio-frequency bridge of the Leeds & Northrup Company,[3] is a variable capacitance in parallel with one of the fixed resistance ratio arms of the bridge.

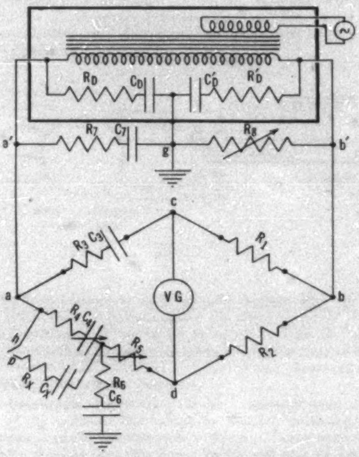

FIG. 1—HIGH-VOLTAGE CAPACITANCE BRIDGE

This paper deals primarily with an analysis of the parallel substitution method applied to capacitance bridges employing the second and third arrangements mentioned in the preceding paragraph.

HIGH-VOLTAGE CAPACITANCE BRIDGE USING THE PARALLEL SUBSTITUTION METHOD

First consider the substitution type of bridge shown in Fig. 1; R_1 and R_2 are equal fixed non-inductive resistances, C_3 is a low loss fixed condenser having an equivalent series resistance R_3, C_4 is a precision variable condenser having an equivalent series resistance R_4 varying with the capacitance C_4, R_5 is a variable non-inductive resistance, and C_x is the capacitance of the dielectric specimen having an equivalent series resistance R_x. In the Wagner ground circuit, C_7 is a large fixed air condenser having an equivalent series resistance R_7, R_8 is a variable non-inductive resistance, and the distributed capacitances of the high-voltage winding of the transformer to the grounded case are represented by the capacitances C_D and C_D' having equivalent series resistances R_D and R_D'. For safety it is necessary to ground the transformer case.

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[1] For references see end of paper.

The bridge is first balanced by adjustment of C_4 and R_5 with the lead h in contact at p thereby connecting the precision condenser and the test specimen in parallel. The lead h is then just removed from contact at p and the bridge rebalanced by adjustment of C_4 and R_5.

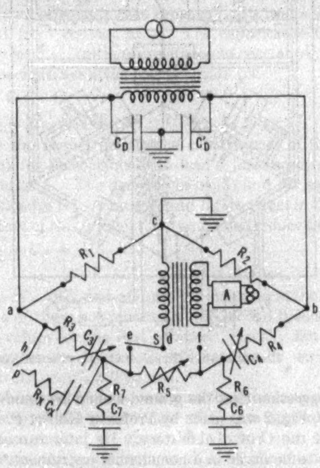

FIG. 2—AUDIO-FREQUENCY CAPACITANCE BRIDGE

A bridge of this type was constructed in 1923 for measurements up to 20 kv.

An early and persistent source of difficulty with this bridge was the impossibility of obtaining an accurate Wagner-ground balance. A study of Fig. 1 will show the reason. The condition for a perfect Wagner-ground balance with the vibration galvanometer connected to points cg is,

$$Z_{7D}/Z_3 = Z_{8D'}/R_1 \qquad (1)$$

where Z_{7D} is the parallel impedance of C_D, C_7, and their equivalent series resistances; Z_3, the series impedance of bridge arm ac; $Z_{8D'}$ the parallel impedance of $C_{D'}$, its equivalent series resistance $R_{D'}$, and the resistance R_8; and R_1 the resistance of the arm cb.

C_3 is a fixed air condenser. Its dielectric material for supporting the high-voltage plate is in positions where the electrostatic field is weak. This makes the phase angle as small as possible. At first, three pyrex glass pillars were used as supports. Later this design was much improved by using a single solid support so placed that the dielectric is located in the electrostatic field between the high-voltage plate and the grounded shielding. This same construction was also used with the precision variable condenser C_4. Therefore, the condenser C_3 had a very low phase angle and R_3 was very small. The transformer was oil-filled, making the angles of defect of the dielectric in the distributed capacitances C_D and $C_{D'}$ very much larger than the angles of defect of the air condenser C_3. Increasing C_7 to an air capacitance of over 1,100 $\mu\mu$f. in an attempt to reduce the phase angle of Z_{7D} sufficiently to give a balance was unsuccessful. R_8 was about 3,000 ohms making the effect of the distributed capacitance $C_{D'}$ upon the phase angle of $Z_{8D'}$ very small. Hence, the net effect of the losses in the distributed capacitance of the transformer was to prevent a satisfactory Wagner-ground balance with this circuit. Subsequent study indicates that the insertion of a suitable variable self-inductance in series with the arm gb' would have made possible a perfect Wagner-ground balance in accordance with equation (**1**).

Also, it was found that the value of R_5 required for a bridge balance showed a surprisingly large variation with a small change in the adjustment of the Wagner-ground balance. In an effort to determine if the presence of the stray capacitance C_6 between the low potential side of the C_4 condenser case and the grounded shielding was responsible for this effect, a mathematical analysis of the bridge circuit was made including this capacitance. Because of its similarity to the analysis of the audio-frequency bridge with Wagner ground given in Appendix II, this derivation is not given.

This analysis yielded an astonishing result. It indicated that the presence of this stray capacitance had an entirely negligible effect upon the capacitance of the test specimen as measured by the bridge but resulted in a very serious error in the measured value of its equivalent series resistance and consequently its power factor. The solution showed that the value of R_x as measured must be multiplied by the factor $[1 + (C_6/C_4')]$ where C_4' is the capacitance of C_4 for the balance with lead h not in contact at p.

With the shielding in use at the time of this discovery, this correction factor had a value exceeding 2. C_6 was somewhat greater than C_4'. This meant that the measured power factors were less than 50 per cent of the true power factors. The shielding was then moved a considerable distance away from the cases of the condensers C_3 and C_4 considerably reducing this correction factor.

In view of the entirely satisfactory development of the Dawes-Hoover high-voltage bridge at that time and the ground balance and correction factor difficulties encountered with this substitution bridge, it was finally abandoned as applied to high-voltage measurements.

A LOW-VOLTAGE AUDIO-FREQUENCY BRIDGE USING THE PARALLEL SUBSTITUTION METHOD

The bridge circuit of Fig. 2 was later used in making power factor measurements at audio frequencies. It was found that a considerable variation in the measured value of R_x was obtained as different settings of the balancing condenser C_4 were used. This indicated the probability of the existence of a stray capacitance error similar to that found in the high-voltage bridge. The only essential difference between the circuits of Figs. 1 and 2 is the interchange of points of application of power input and detector, the circuit of Fig. 1 being preferable for high-voltage work and that of Fig. 2 for low voltages due to its symmetry.

It thus became advisable to derive the bridge equations taking into consideration the effect of the stray capacitance to ground C_6 of the metal case of the condenser C_4. This derivation is given in detail in Appendix II. With switch *s* on point *e*, it is shown that

$$C_X \doteq \Delta C_3 \qquad (2)$$

$$R_X \doteq \Delta R_5 \, (C_3'/C_X)^2 \, (1 + [C_6/C_4]) \qquad (3)$$

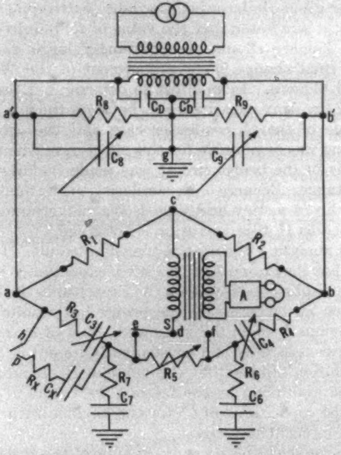

FIG. 3—CAPACITANCE BRIDGE WITH WAGNER GROUND

where ΔC_3 is the change in capacitance of the precision condenser C_3 between the two bridge balances, with and without lead *h* in contact at *p*, ΔR_5 is the change in resistance of R_5 between the two bridge balances, and C_3' is the capacitance of C_3 for the balance with lead *h* not in contact.

In these balances, it is often necessary to have R_5 in arm *db* when balancing with lead *h* in contact and in arm *ad* when balancing with lead *h* not in contact. In this case, equation (7) becomes

$$R_X \doteq [R_5(1+\{C_6/C_4\}) - R_5'(1+\{C_7/C_3'\})](C_3'/C_X)^2 \qquad (4)$$

where R_5 and $R_{5'}$ are values with lead *h* in contact and not in contact respectively. Obviously, with point *e* at ground potential, C_7 will have no effect on the balance, while with point *f* at ground potential the same will be true of C_6. Hence, only one of these stray capacitances can effect a single balance.

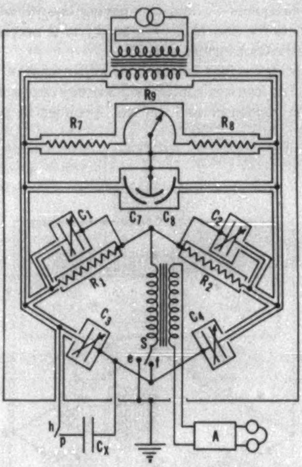

FIG. 4—RECOMMENDED TYPE OF CAPACITANCE BRIDGE

A suggestion that the ground should be removed to point *d*, Fig. 2 was made by Professor Robert F. Field, then of the Cruft Laboratory. He later caused this change to be made in a commercial instrument.[2] The derivation of the bridge equations for this ground position, given in Appendix II, shows that the error due to stray capacitance is practically eliminated.

Fig. 3 shows this capacitance bridge circuit using a Wagner-ground connection instead of a direct ground. In Appendix II, the equations for this bridge are shown to be identical with equations (6), (7), and (8). Therefore, the error and correction factor are not eliminated by the use of a Wagner ground connection.

A RECOMMENDED LOW-VOLTAGE AUDIO-FREQUENCE CAPACITANCE BRIDGE USING THE PARALLEL SUBSTITUTION METHOD

The capacitance bridge shown in Fig. 4 is essentially similar to the precision audio-frequency bridge of the Leeds & Northrup Company[3] with the addition of a Wagner-ground connection and the incorporation of both standard and auxiliary condensers into the bridge unit. Although less flexible, this single unit construction is advantageous when used for this particular type of measurement. No variable resistance is required other than that in the Wagner-ground connection. Since the rotor and shields of the four variable condensers are all brought to ground potential by the bridge and Wagner-ground balances, no stray capacitance error similar to that previously described exists.

In making a measurement, the auxiliary condenser C_4 is set at some convenient value of capacitance depending upon the capacitance C_x of the test specimen; lead h is placed in contact at p; and a balance is obtained with switch S on point f, by variation of the capacitance of the precision condenser C_3 and of the condensers C_1 and C_2. A Wagner-ground balance is then obtained with switch S on point e, after which switch S is returned to point f and a final bridge balance obtained. This three-step process is then repeated with lead h just removed from contact at p. The capacitance C_x and power ratio η_x, (ratio of resistance to reactance), of the test specimen are given by the equations:

$$C_X \doteq C_3' - C_3 \tag{5}$$

$$\eta_X \doteq \omega\,[C_3'/C_X]\,[R_2\,(C_2 - C_2') - R_1\,(C_1 - C_1')] \tag{6}$$

where the primes denote the capacitances of the condensers for the balance with lead h just removed from contact at p. The derivatives of (5) and (6) are given in Appendix III. Since R_1 and R_2 are equal in the construction of this bridge, these equations reduce to

$$C_X \doteq \Delta C_3 \tag{7}$$

$$\eta_X \doteq \omega R_1\,(\Delta C_1 - \Delta C_2)\,(C_3'/C_X) \tag{8}$$

where ΔC_1, ΔC_2, and ΔC_3, are the increases in the capacitances of these condensers for the second balance over their values for the first balance. Usually but one of the condensers C_1 and C_2 will need to be varied and either ΔC_1 or ΔC_2 will be zero.

If the power ratio η_x is low so that the tangent, (η_x), and sine of the phase angle of C_x are approximately equal, equation (8) gives the power factor directly. Otherwise the power factor is given by the equation:

$$(\text{power factor})_X = \sin \tan^{-1} \eta_X \tag{9}$$

CONCLUSIONS

The form of capacitance bridge shown in Fig. 4 has the following advantages:

1. All operating controls are at ground potential reducing the effects of hand capacitance to a minimum.
2. The bridge is entirely self-contained except for oscillator, amplifier, and telephone receivers.
3. All parts are totally shielded except the high-potential terminal for the high-potential lead h to C_x.
4. All controls are continuously variable.
5. The bridge may be constructed entirely of standard parts, the shielding excepted.

ACKNOWLEDGMENT

The author wishes to acknowledge his indebtedness to Dean Harry E. Clifford and to Professor Chester L. Dawes of the Harvard Engineering School for their helpful suggestions in the preparation of this paper.

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Appendix I

DERIVATION OF EQUATIONS FOR THE CAPACITANCE AND POWER FACTOR MEASUREMENT OF DIELECTRIC SPECIMENS USING THE PARALLEL SUBSTITUTION METHOD AND BRIDGE CIRCUIT OF FIG. 2

In Fig. 5 are shown the equivalent series circuit and the equivalent parallel circuit of an imperfect condenser. Since Z_P must equal Z_S, it is easily shown that

$$R_S - j\,(1/\omega C_S) = [R_P/(1 + R_P^2\omega^2 C_P^2)] - j\,[R_P^2\omega C_P/(1 + R_P^2\omega^2 C_P^2)] \tag{10}$$

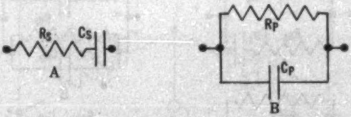

FIG. 5—EQUIVALENT SERIES AND PARALLEL CIRCUITS OF AN IMPERFECT CONDENSER

The angle of defect of the condenser is given by

$$\theta = \tan^{-1}\,(1/R_P\omega C_P) \tag{11}$$

If θ does not exceed 1 deg., $\tan\,\theta \leqq 0.0175$ and $R_P^2\omega^2C_P^2 \geqq 3{,}260$. Therefore, $R_P^2\omega^2C_P^2 >> 1$ and equation (10), to a degree of approximation equal to or better than 0.0306 per cent, gives

$$R_S \doteq 1/R_P\omega^2C_P^2 \tag{12}$$

$$C_S \doteq C_P \tag{13}$$

The ratio of the equivalent series resistance to reactance of an imperfect condenser is defined as the power ratio, η_S.

$$\eta_S = R_S\omega C_S \tag{14}$$

This is obviously equal to $\tan\,\theta$ where θ is the angle of defect of the condenser. If $\eta_S \leqq 0.1$, $\eta_S^2 \leqq 0.01 = 1/R_P^2\omega^2C_P^2$. Then $R_P^2\omega^2C_P^2 = 1/\eta_S^2 \geqq 100$, and the approximations of equations (12) and (13) are within 1 per cent. As will be seen from equation (10), the approximate value of R_S exceeds the true value by this percentage while the true value of C_S exceeds the approximate value by this percentage. The approximation in per cent is about $100\ \eta_S^2$.

With two imperfect condensers in parallel, each having a small angle of defect, the equivalent series resistance and capacitance of the combination may readily be found. In Fig. 6, it is desired to find the constants of the equivalent series circuit shown in (D) to make it equivalent to the given circuit (A). This may be done to a high degree of approximation, by using equations (**12**), (**13**), and (**14**) and taking the two intermediate steps illustrated in (B) and (C).

$$R_{AP} \doteq 1/R_{AS}\omega^2 C_{AS}^2 = 1/\eta_{AS}\omega C_{AS} \tag{15}$$

$$C_{AP} \doteq C_{AS} \tag{16}$$

$$R_{BP} \doteq 1/R_{BS}\omega^2 C_{BS}^2 = 1/\eta_{BS}\omega C_{BS} \tag{17}$$

$$C_{BP} \doteq C_{BS} \tag{18}$$

$$C_{ABS} \doteq C_{ABP} \doteq C_{AP} + C_{BP} \doteq C_{AS} + C_{BS} \tag{19}$$

$$R_{ABS} \doteq 1/R_{ABP}\omega^2 C_{ABP}^2 \doteq (\eta_{AS}\omega C_{AS} + \eta_{BS}\omega C_{BS})/\omega^2 C_{ABS} \tag{20}$$

$$\eta_{ABS}C_{ABS} \doteq \eta_{AS}C_{AS} + \eta_{BS}C_{BS} \tag{21}$$

Equations (**19**) and (**21**) may be carried out for more than two condensers in parallel, and as long as the separate values of η are sufficiently low, the value of ηC for the parallel combination will be the sum of the values of ηC for the separate units, and the total capacitance will be the sum of the separate capacitances, to a high degree of approximation.

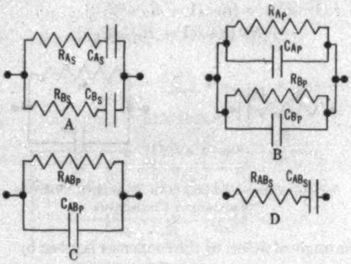

FIG. 6—TWO IMPERFECT CONDENSERS IN PARALLEL

A good variable condenser, such as must be used as a precision standard in this type of measurement, will have a minimum amount of stator supporting dielectric material placed where the electrostatic field is weak and where the movement of the rotor plates causes practically no variation in the flux distribution. This air condenser may then be considered as a variable perfect air condenser in parallel with a small "zero" capacitance containing the equivalent series resistance which represents the loss in the stator supporting dielectric material. Fig. 7A shows the equivalent circuit of the precision variable condenser and (B) and (C), the steps taken in finding its equivalent series resistance and capacitance as shown in (D).

By equations (**12**), (**13**), and (**14**)

$$C_{SS} \doteq C_{SP} \doteq C_{OP} + C \doteq C_{OS} + C \tag{22}$$

$$R_{SS} \doteq 1/R_{OP}\omega^2 C_{SS}^2 \doteq R_{OS}\omega^2 C_{OS}^2/\omega^2 C_{SS}^2 \tag{23}$$

$$\eta_{SS}C_{SS} \doteq \eta_{OS}C_{OS} \tag{24}$$

This last equation indicates that the value of ηC of the precision condenser is approximately a constant regardless of the setting of the rotor plates. This is true to a very high degree of approximation.

The parallel substitution method of capacitance and power factor measurement is illustrated in Fig. 8. In using this method, the bridge is first balanced with the test specimen C_X connected in parallel with the precision condenser C_3. If R_1 and R_2 are equal pure resistances, as is generally the case, the power ratio η_{3X} of the parallel circuit $3X$ must be equal at balance to the power ratio of the balancing arm, $(R_5 + R_4)\omega C_4$; also, the capacitance C_{3X} of the parallel circuit must be equal to C_4.

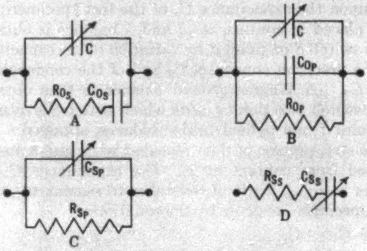

FIG. 7—PRECISION VARIABLE CONDENSER

The value of ηC of the combination $3X$ is found by equation (**21**).

$$\eta_{3X}C_{3X} \doteq \eta_3 C_3 + \eta_X C_X \tag{25}$$

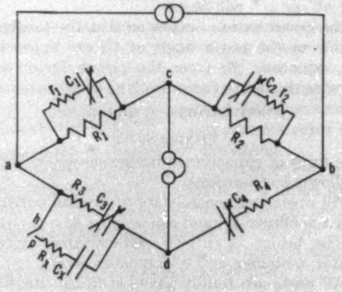

FIG. 8—PARALLEL SUBSTITUTION METHOD

The lead h from the high tension side of the standard condenser, C_3, is then just removed from contact with C_X at p and the bridge rebalanced by adjustment of C_3 and R_5, C_4 being held constant. For this second balance, denoted by primes, the power ratios of the arms 3 and 4 must be equal and also $C_3' = C_4$.

Therefore, $$\eta_3' C_3' \doteq (R_5' + R_4)\omega C_4^2 \tag{26}$$

In the first balance,

$$\eta_3 C_3 + \eta_X C_X \doteqdot (R_5 + R_4)\,\omega C_4^2 \qquad (27)$$

By equation (**24**), ηC for the precision condenser is constant, so

$$\eta_3' C_3' \doteqdot \eta_3 C_3 \qquad (28)$$

Subtracting (**26**) from (**27**), and using (**28**),

$$\eta_X C_X \doteqdot (R_5 - R_5')\,\omega C_4^2 \qquad (29)$$

$$R_X\,\omega C_X^2 \doteqdot \Delta R_5\,\omega C_4^2 \qquad (30)$$

But since $C_4 \doteqdot C_3'$,

$$R_X \doteqdot \Delta R_5 (C_3'/C_X)^2 \qquad (31)$$

$$C_X = \Delta C_3 \qquad (32)$$

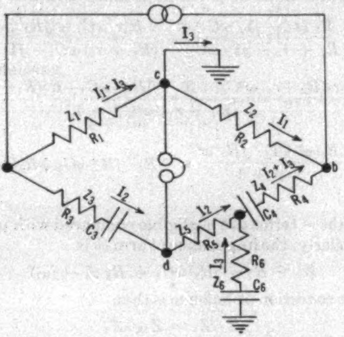

Fig. 9—Capacitance Bridge, Grounded at *c*

where ΔR_5 is the change in resistance of R_5 and ΔC_3 the change in capacitance of C_3 between the two balances, and C_3' is the value for the balance with lead *h* not in contact.

Appendix II

Derivations of Equations for the Bridge Circuits of Figs. 2 and 3 Considering Effect of Stray Capacitance C_6

Referring to Fig. 9 and writing the equations for balance,

$$(I_1 + I_3)\,Z_1 = I_2\,Z_3 \qquad (33)$$

$$I_1 Z_2 = I_2 Z_5 + (I_2 + I_3)\,Z_4 \qquad (34)$$

$$I_2 Z_5 = I_3 Z_6 \qquad (35)$$

Solving these equations simultaneously and eliminating the currents, the equation of balance is found to be

$$(Z_2/Z_1)\,Z_3 = [(Z_2 + Z_4)/(Z_6) + 1]\,Z_5 + Z_4 \qquad (36)$$

If, as is usually the case, Z_1 is made equal to Z_2, the last equation becomes, after substitutions,

$$R_3 - j\,(1/\omega C_3)$$

$$= \left[\frac{R_2 + R_4 - j\,(1/\omega C_4)}{R_6 - j\,(1/\omega\, C_6)} + 1\right] R_5 + R_4 - j\,(1/\omega C_4)$$

$$= \left[\frac{\omega C_6\ \{(R_2 + R_4)\ \omega C_4 - j\,1\}}{\omega C_4\ \{R_6 \omega C_6 - j 1\}} + 1\right] R_5$$

$$+ R_4 - j\,(1/\omega C_4)$$

$$= \left[\frac{C_6\,(\eta_{24} - j\,1)}{C_4\,(\eta_6 - j\,1)} + 1\right] R_5 + R_4 - j\,(1/\omega C_4)$$

$$= \left[\frac{C_6\,(\eta_{24}\eta_6 + 1 + j\eta_{24} - j\eta_6)}{C_4\,(\eta_6^2 + 1)}\right] R_5 + R_4 - j\,(1/\omega C_4) \qquad (37)$$

These second order η terms are negligible compared with unity, since they are of the order of magnitude of the squares of power ratios of air condensers. Therefore

$$R_3 - j\,(1/\omega C_3) \doteqdot [(C_6/C_4)\,(1 + j\eta_{24} - j\eta_6) + 1]\,R_5$$

$$+ R_4 - j\,(1/\omega C_4) \qquad (38)$$

Equating the real parts of (**38**),

$$R_3 \doteqdot [(C_6/C_4) + 1]\,R_5 + R_4 \qquad (39)$$

This last equation shows that correct results can only be obtained for the resistance component of balance when the value of R_5 for each balance is multiplied by the factor $[1 + (C_6/C_4)]$.

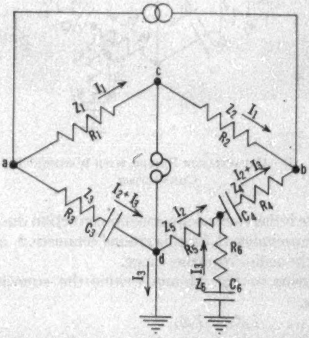

Fig. 10—Capacitance Bridge, Grounded at *d*

Equating the imaginary parts of (**38**),

$$-\,(1/\omega C_3) \doteqdot (C_6/C_4)\,(\eta_{24} - \eta_6)\,R_5 - (1/\omega C_4) \qquad (40)$$

This can be written

$$1/C_3 \doteqdot [\,-\,\eta_{56}\,(\eta_{24} - \eta_6) + 1\,]/C_4 \qquad (41)$$

Since the second order η terms are negligible compared with unity,

$$C_3 \doteqdot C_4 \qquad (42)$$

This last equation shows that no error in the capacitance measurement is caused by the stray capacitance C_6.

Referring to Fig. 10, it is obvious that Z_5 is paralleled by Z_6. This parallel impedance is given by

$$Z_{56} = \frac{Z_5 Z_6}{Z_5 + Z_6} = \frac{R_5 [R_6 - j\ (1/\omega C_6)]}{R_5 + R_6 - j(1/\omega C_6)} = \frac{R_5(\eta_6 - j\,1)}{\eta_{56} - j\,1}$$

$$= R_5 [\eta_6 \eta_{56} + 1 + j\ (\eta_6 - \eta_{56})]/(\eta_{56}^2 + 1)$$

$$\doteq R_5 + jR_5\ (-\ R_5 \omega C_6) \tag{43}$$

Hence, $R_3 \doteq R_5 + R_4$ (44)

$$-\ j\ (1/\omega C_3) \doteq -\ jR_5^2 \omega C_6 - j\ (1/\omega C_4) \tag{45}$$

$$C_4 \doteq C_3\ (1 + \eta^2) \doteq C_3 \tag{46}$$

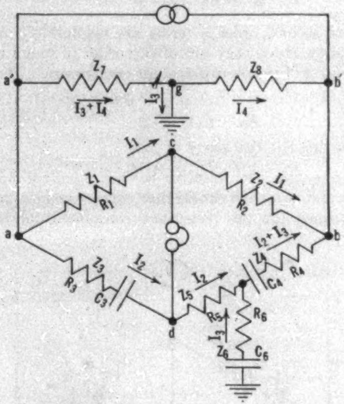

FIG. 11—CAPACITANCE BRIDGE WITH WAGNER-GROUND CONNECTION

There is therefore no appreciable correction due to the stray capacitance C_6 for this ground connection, at least not in the audio-frequency range.

Referring to Fig. 11 and writing the equations of balance,

$$I_1 Z_1 = I_2 Z_3 \tag{47}$$

$$I_1 Z_2 = I_2 Z_5 + (I_2 + I_3)\ Z_4 \tag{48}$$

$$I_2 Z_5 = I_3 Z_6 \tag{49}$$

Solving these equations simultaneously, eliminating the currents, the equation of balance is found to be

$$(Z_2/Z_1)\ Z_3 = [(Z_4/Z_6) + 1]\ Z_5 + Z_4 \tag{50}$$

Setting $Z_1 = Z_2$ and substituting,

$$R_3 - j\ (1/\omega C_3)$$

$$= \left[\frac{R_4 - j\ (1/\omega C_4)}{R_6 - j\ (1/\omega C_6)} + 1 \right] R_5 + R_4 - j\ (1/\omega C_4) \tag{51}$$

Treating this last equation in the same manner as equation (**37**) gives the equation

$$R_3 - j\ (1/\omega C_3) \doteq [(C_6/C_4)\ (1 + j\eta_4 - j\eta_6) + 1]\ R_5 + R_4 - j\ (1/\omega C_4) \tag{52}$$

Equating the real parts of (**52**),

$$R_3 \doteq [(C_6/C_4) + 1]\ R_5 + R_4 \tag{53}$$

Equating the imaginary parts of (**52**),

$$-\ (1/\omega C_3) \doteq [(C_6/C_4)\ (\eta_4 - \eta_6)\ R_5] - (1/\omega C_4) \tag{54}$$

As before, this results in the relation

$$C_3 \doteq C_4 \tag{55}$$

Equations (**53**) and (**55**) show that the correction factor for R_5 still exists when a Wagner-ground connection is used although there is no error in the measured value of capacitance.

Appendix III

DERIVATION OF EQUATIONS FOR RECOMMENDED CAPACITANCE BRIDGE

Referring to Fig. 12 and assuming a balance with the lead *h* in contact at *p*, the impedance of arm *ac* is

$$Z_1 = \frac{R_1\ (r_1 - j1/\omega C_1)}{R_1 + r_1 - j1/\omega C_1} = \frac{R_1 r_1 \omega C_1 - jR_1}{(R_1 + r_1)\,\omega C_1 - j1}$$

$$= \frac{R_1 r_1 (R_1 + r_1)\,\omega^2 C_1^2 + R_1 + j[R_1 r_1 \omega C_1 - R_1 (R_1 + r_1)\,\omega C_1]}{(R_1 + r_1)^2 \omega^2 C_1^2 + 1}$$

$$\doteq \frac{R_1(\eta^2 + 1) - jR_1^2 \omega C_1}{\eta^2 + 1} \doteq R_1 - jR_1^2 \omega C_1 \doteq R_1 (1 - j\eta_1) \tag{56}$$

since the η^2 terms are negligible compared with unity.

Similarly, the impedance of arm *cb* is

$$Z_2 \doteq R_2 - jR_2^2 \omega C_2 \doteq R_2\ (1 - j\eta_2) \tag{57}$$

The condition of balance is that

$$Z_1/Z_2 = Z_{3X}/Z_4 \tag{58}$$

The second balance is obtained by just removing the lead *h* to C_X from contact at *p*, leaving C_4 at its original setting, and rebalancing the bridge by variation of C_3 and either or both C_1 and C_2. Denoting this second set of values by primes, the condition of balance is

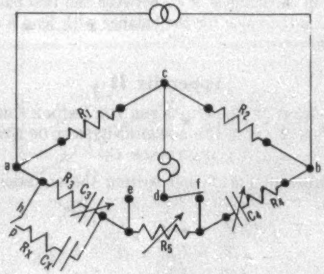

FIG. 12—RECOMMENDED CAPACITANCE BRIDGE

$$Z_1'/Z_2' = Z_3'/Z_4 \tag{59}$$

Dividing (**58**) by (**59**),

$$\frac{Z_1/Z_2}{Z_1'/Z_2'} = \frac{Z_{3X}}{Z_3'} \tag{60}$$

$$\frac{Z_1}{Z_2} \doteq \frac{R_1\,(1 - j\eta_1)}{R_2\,(1 - j\eta_2)} = \frac{R_1\,[1 + \eta_1\eta_2 + j\,(\eta_2 - \eta_1)]}{R_2\,(1 + \eta_2^2)}$$

$$\doteq \frac{R_1}{R_2}\,[1 + j\,(\eta_2 - \eta_1)] \tag{61}$$

since the η^2 terms are negligible compared with unity.

Similarly,

$$Z_1'/Z_2' = (R_1/R_2)\,[1 - j\,(\eta_2' - \eta_1')] \tag{62}$$

$$Z_{3\mathrm{X}} = R_{3\mathrm{X}} - j\,(1/\omega C_{3\mathrm{X}}) = (\eta_{3\mathrm{X}} - j1)/\omega C_{3\mathrm{X}} \tag{63}$$

$$Z_3' = R_3' - j\,(1/\omega C_3') = (\eta_3' - j1)/\omega C_3' \tag{64}$$

Hence, (**60**) may be written

$$\frac{1 + j\,(\eta_2 - \eta_1)}{1 + j\,(\eta_2' - \eta_1')} \doteq \frac{C_3'\,(\eta_{3x} - j1)}{C_{3x}\,(\eta_3' - j1)} \tag{65}$$

Rationalizing,

$$\frac{1 + (\eta_2 - \eta_1)\,(\eta_2' - \eta_1') + j\,[(\eta_2 - \eta_1) - (\eta_2' - \eta_1')]}{1 + (\eta_2' - \eta_1')^2}$$

$$\doteq \frac{C_3'\,[\eta_{3x}\,\eta_3' + 1 + j\,(\eta_{3x} - \eta_3')]}{C_{3x}\,[(\eta_3')^2 + 1]} \tag{66}$$

Since the second order η terms are negligible compared with unity, (**66**) becomes

$$1 + j[(\eta_2 - \eta_1) - (\eta_2' - \eta_1')] \doteq (C_3'/C_x)[1 + j(\eta_{3x} - \eta_3')] \tag{67}$$

Equating the real parts of (**67**),

$$C_{3\mathrm{X}} \doteq C_3' \tag{68}$$

By equation (**19**), Appendix I, $C_{3\mathrm{X}} \doteq C_3 + C_{\mathrm{X}}$, and hence

$$C_{\mathrm{X}} \doteq C_3' - C_3 = \Delta\,C_3 \tag{69}$$

Equating the imaginary parts of (**67**),

$$(\eta_2 - \eta_1) - (\eta_2' - \eta_1') \doteq (C_3'/C_{3x})\,(\eta_{3x} - \eta_3') \tag{70}$$

By equation (**21**), Appendix I, $\eta_{3\mathrm{X}}C_{3\mathrm{X}} \doteq \eta_3 C_3 + \eta_{\mathrm{X}} C_{\mathrm{X}}$; by equation (**24**), Appendix I, $\eta_3 C_3 \doteq \eta_3' C_3'$; and by equation (**67**), $C_{3\mathrm{X}} \doteq C_3'$. Substituting these relations in (**70**) gives the equations

$$C_3'\,[(\eta_2 - \eta_1) - (\eta_2' - \eta_1')] \doteq \eta_{\mathrm{X}} C_{\mathrm{X}}$$

$$\eta_{\mathrm{X}} \doteq \omega\,(C_3'/C_{\mathrm{X}})\,[R_2\,(C_2 - C_2') - R_1\,(C_1 - C_1')] \tag{72}$$

An Automatic Oscillograph

BY C. M. HATHAWAY*
Associate, A.I.E.E.
and
R. C. BUELL*
Associate, A.I.E.E.

INTRODUCTION

ALTHOUGH neither the oscillograph nor the art of oscillography is new, it has only been within the last few years that the oscillograph has been recognized as a thoroughly practical and necessary instrument for analyzing and solving many of the problems of operating engineers. This change in the status of the oscillograph from that of a laboratory device to an instrument of practical field utility has resulted from many recent improvements and simplifications in design, and from an increased demand for that type of instrument in modern engineering practise. Problems have arisen as a result of the new era of extensive interconnection of large power systems and of the high-speed operation of switches and circuit breakers that only an oscillograph can solve, and an oscillograph for such applications must have characteristics that are not found in those designed for more general uses. An oscillograph for such applications must, of course, be fully automatic because short circuits and faults are usually unanticipated; it should be able to take a considerable number of records successively whether the faults follow each other in rapid succession or are days or weeks apart, and it should start recording in as short a time as possible following the beginning of the initiating disturbance. With present-day oil circuit breakers having an operating time of eight cycles or less, and with relays which operate in one-quarter of that time, the oscillograph should certainly start its record in not more than one-half cycle. The automatic oscillograph described in this paper has been designed to meet the demand that has resulted for such an instrument by the advent of high-speed switching and relaying, and by the modern practise of clearing short circuits with extreme speed.

DESCRIPTION

The automatic oscillograph described herein will start recording with full speed within one-half cycle on a 60-cycle system after the current in the operating coil of its starting relay exceeds the critical tripping value. An oscillographic record is then taken at the rate of about 12 in. per second for a predetermined length of time that may be adjusted at will from 2 to 20 seconds, or until the disturbance is over if the fault outlasts this duration setting. The oscillograph will then stop recording, and will wait with no parts in motion for the next fault. The records are taken directly on sensitized paper 4¼ in. wide, and since a 200-ft. roll of this paper may be inserted in the oscillograph at one time, a great number of faults may be recorded in succession without intermediate attention of any kind. The roll of sensitized paper is placed in the right holder on top of the oscillograph, Fig. 1, and as it is exposed it is wound into the left holder. The left holder may be removed from the oscillograph at any time if it is desired to develop and examine the records it contains without waiting for the entire 200-ft. roll of paper to be exhausted.

The oscillograph contains six galvanometers, and each galvanometer may be used to measure either current or potential. The galvanometers are of the permanent magnet type. They employ the familiar bifilar type of moving element, and the records produced are truly oscillographic, showing wave shapes and phase relations as well as absolute magnitudes. The curves are traced on the sensitized record paper by small beams of light reflected from mirrors attached to the galvanometer moving elements. The light is obtained from a standard 6-volt, 32-cp. automobile headlight lamp that burns continuously.

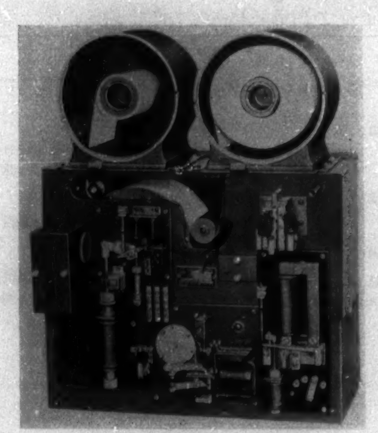

FIG. 1—AUTOMATIC OSCILLOGRAPH SHOWING RECORDING PAPER IN PLACE

Although no parts are in motion except during the time that the oscillograph is actually in operation, it will start its record within one-half cycle after the current in the starting relay exceeds the critical value. The method of obtaining this high starting speed will be made clear by an explanation of the schematic diagram of the optical system shown in Fig. 2. Referring to this figure, a spherical lens *A* focuses an image of the lamp filament *B* upon the front of the galvanometer *E*. The diverging beam from the galvanometer mirror passes through the cylindrical lens *F* and is reflected upward by the plane mirror *G* to the sensitized record-paper surface at *K*. An adjustable slit aperture *D* is placed between the source and the galvanometer, making it possible to vary the width of the light spot at *K*. When the starting relay is tripped, the mirror *G* rotates about the axis *J* and moves the spot of light from *K* to *L* along the sensitized surface, tracing the wave of the quantity actuating the galvanometer. While the spot of light is moving from *K* to *L*, the record paper is itself started in motion in the direction of the arrow, so that after the spot reaches the final position *L* the record is continued by the movement of the paper itself. This device renders it unnecessary to start an appreciable mass in motion in such a short time or to keep it in motion continually in anticipation of a disturbance to be recorded. Attached to the mirror *G* is a shutter vane *H* that intercepts the beam and prevents it from striking the record paper when the oscillograph is not in operation and the mirror *G* is in its extreme counter-clockwise position. A small rotation of *G* about its axis drops this shutter below the path of the light beam.

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*General Electric Company, Schenectady, N. Y.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

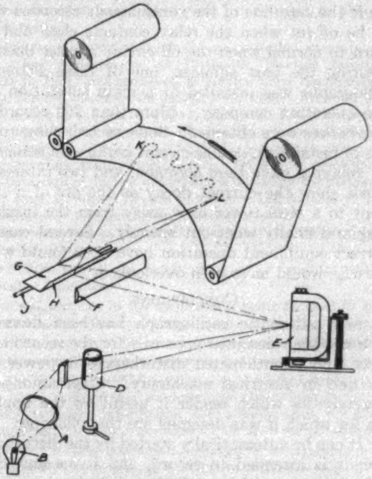

Fig. 2—Schematic Diagram of the Automatic Oscillograph Optical System

Only one galvanometer element is shown

While the oscillograph is not recording, the mirror *G*, Fig. 2, is held in its extreme counter-clockwise position by the attraction of an electromagnet for an armature attached to the mirror shaft. The armature and the holding electromagnet are shown just below the record paper guides in Fig. 1. When the starting relay is tripped, this electromagnet is de-energized, the armature is released, and a spring rotates the mirror clockwise. The movement of the mirror is retarded by an oil dash-pot. Since the tension of the spring decreases as its elongation is decreased, the mirror moves with a retarded motion that combines with the accelerated motion of the record paper to produce an approximately uniform motion of the light beam relative to the record paper.

The starting relay is shown in the lower right-hand corner of the panel in Fig. 1. Its operating coil is intended to be connected directly in the secondary of a current transformer in the neutral connection of a generator or a transformer bank. Its armature is ordinarily held in the downward position by a small permanent magnet against the pull of a spring and the operating coil. Excessive current in the operating coil will pull the armature away from the permanent magnet, and since the attraction between the magnet and the armature is greatly diminished as soon as the contact between them is broken, the armature moves at a high speed when the current in the operating coil only slightly exceeds the tripping value. This device renders the starting speed of the oscillograph more nearly independent of the amount by which the current in the starting relay exceeds the tripping value.

The length of the record produced after each tripping of the starting relay is governed by the cam mechanism shown just to the left of the starting relay in Fig. 1. This mechanism is driven by the motor that drives the record paper, and it may be adjusted to produce oscillograms of definite length of from two to twenty feet. It cannot, however, stop the oscillograph while the fault persists, so it is usually advisable to set it to produce the minimum record length of two feet. The oscillograph will then operate for only two seconds following a switching surge or a fault that is immediately cleared, but will automatically continue operation as long as the fault may persist. A counter is

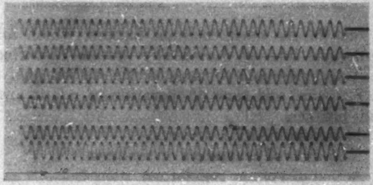

Fig. 3—Section Showing the Beginning of a Record Taken on the Automatic Oscillograph

provided to record each operation so that the inspector may at any time determine the number of records the instrument has taken. A contact is also provided that may be used to close an alarm circuit during operation or to actuate a recording instrument or other type of time-recording device so that the exact time of each fault may be subsequently determined.

The power required for driving the record paper, for operating the lamp, and for energizing the holding magnet may be conveniently obtained from an ordinary automobile storage battery. The battery may be floated across a small charger that supplies continually enough current in excess of that required by the oscillograph while it is not in operation to keep the battery in good condition and always in readiness for service.

APPLICATION

This new automatic oscillograph may be applied to a power system to obtain a continuous check on the operation of the system equipment, or for special investigation of some transient phenomena which occurs during a system disturbance. The following are some of the most common factors, which are of considerable interest during any system disturbance.

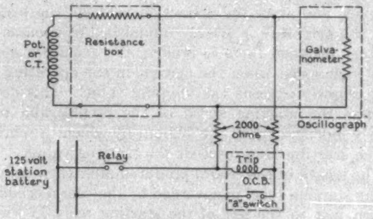

FIG. 4—CIRCUIT FOR INDICATING RELAY AND OIL CIRCUIT BREAKER OPERATION

1. The three-phase voltages on the station bus.
2. Sequence and time of operation of relays and oil circuit breakers.
3. Phase and neutral currents.
4. Kilowatts of generators or tie-lines.
5. Action of voltage regulators.
6. Generator speed and governor operation.
7. Pressure in penstocks, oil circuit breakers, etc.

An over-current relay is furnished to start the oscillograph when a disturbance occurs. This is usually connected to a current transformer in the grounded neutral of a power transformer bank. By connecting such current transformers in parallel the oscillograph may be started by any one of several neutral grounding connections.

In some locations it is impossible to obtain neutral current to start the oscillograph, and undervoltage or overvoltage is the desired indication for the oscillograph to come into operation. Additional relays which will operate under these conditions are available and can be arranged to operate in parallel with the overcurrent relay.

When the oscillograph is used as an aid in the analysis of system operation, it is often installed at a station which has a number of important transmission lines radiating to other sections of the system. The usefulness of the oscillograph can be greatly increased in such a location by the use of circuits similar to that shown in Fig. 4. Such a circuit will not affect the continuously recorded wave and will also show which oil circuit breaker operated, and the length of time for its relay and mechanism to function. With this circuit the zero line of the continuously recorded wave will be off-set when the relay contacts close and will return to normal when the oil circuit breaker opens.

During the past summer, one of these automatic oscillographs was installed at a main substation of a large operating company. More than 200 records of disturbances were obtained. Some records show conditions of instability, double-circuit faults with attendant slow relaying have been common, and two interesting records show the current decay as the arc of a short circuit to a steel tower blew away from the insulator string and finally went out entirely. Several cases of incorrect equipment operation have been found which otherwise would have been overlooked.

CONCLUSIONS

A new automatic oscillograph has been described which has been designed primarily for the recording of chance and unanticipated disturbances in power systems and in electrical machinery. Chief among the characteristics which render it useful for the applications for which it was designed are the following:

1. It can be automatically started by the disturbance which it is intended to record, and it automatically continues in operation until that disturbance is over.
2. Although no parts are in motion while it is not actually recording, it starts recording at full speed within one-half cycle of a 60-cycle system after the current in the starting relay exceeds the tripping value, regardless of the amount by which it may exceed this value.
3. Since a 200-ft. roll of record paper may be inserted at one time, it is capable of taking a large number of records in succession without intermediate attention. The records obtained may be removed at any time without waiting for the entire 200-ft. roll to be exhausted.
4. The records obtained are truly oscillographic, and the speed is great enough to show wave shapes and phase relations as well as absolute magnitudes.
5. The records are taken directly on sensitized paper that is cheaper than film and that produces black lines on a white background immediately after development.
6. The automatic oscillograph may be used to obtain a continuous check on relay operation and oil circuit breaker performance, as well as records of currents, voltages, power, governor operation, penstock pressure, etc., during disturbances.

Discussion

Wm. G. Walker: During the summer of 1930 the development model of the automatic oscillograph described today was installed at the Plymouth Meeting Substation of the Philadelphia Electric Company on the Pennsylvania-New Jersey 220-kv. interconnection lines in order to obtain field experience in the application of such an instrument for recording transient disturbances.

The results of this preliminary installation provided both the manufacturer and the operating company with valuable data showing what might be expected from the application of such an instrument to this purpose. Several of the suggestions advanced by the men responsible for maintaining the instrument during this experimental period have been incorporated in the commercial model.

A commercial model of the device was installed, in conjunction with other oscillographs, at Plymouth Meeting Substation August 6, 1931, and remained in service until temporarily removed for use elsewhere on January 13, 1932. A record of the performance during the 160 days it was installed is of interest.

During this time there were 63 operations from all causes, including testing, as contrasted with the experience cited in the paper covering another installation. Of the 63 operations 21 were the result of system disturbances initiating the instrument. There were 57 correct records obtained. The failures to obtain correct records were due to the following causes:

Three were lost as the result of a burnt out lamp within an interval of 40 minutes.

Two were lost as the result of improper loading within an interval of 4 minutes.

One was lost due to the oscillograph being blocked in the inoperative position for a switching operation.

It will be seen from the above detailed causes that none of the records was lost through failure of the instrument to function.

At the time the records were lost due to a burnt out lamp, data were being accumulated to determine the actual life of these lamps. Sufficient information had already been obtained to indicate that the lamp in question should have been replaced.

The two records which were chargeable to improper loading were directly due to improper maintenance. The case in which the record was lost due to the oscillograph being blocked, was the result of operating instructions issued to reduce the cost of maintenance on other oscillographs on the system, and the unnecessary consumption of film resulting from their initiating on unimportant switching transients.

The earliest experience with the device indicated that the standard Mazda 32-cp. automobile headlight lamp No. 1133 had an average lamp-life of 12.8 days at 6.6 volts battery. Just prior to the temporary removal of the commercial instrument, a new lamp for this purpose was being investigated. At the time the installation was removed the new lamp was still burning satisfactorily after 16 days service. The performance of this new lamp indicates that the lamp-life question has probably been satisfactorily answered although the data are not conclusive.

No particular difficulty was encountered in stabilizing the charge on the supply storage battery to give the desired voltage at the oscillograph terminals. This was accomplished by inserting adjustable resistance in the primary of the tungar charger, observing the gain or loss in battery voltage with the oscillograph connected, and making the necessary adjustments until stability was obtained.

It is our opinion that the ability of this instrument to take 100 consecutive 2-ft. records, without attention, is a considerable advantage where an instrument of this type must be installed at locations which are more or less remote from the headquarters of a specialized maintenance group. Although operating men can be trained to service oscillographs satisfactorily, still at the same time it must be remembered that at times of major system disturbance their primary concern is restoration of normal service. With the instrument under discussion the only duty which develops upon the operator is to log the operation and its corresponding number on the operation counter.

Our experience indicates that the frequency of inspection of this instrument depends entirely upon the life of the lamp and this may result in a two or three week period, depending upon the results obtained with the new lamp. These lamps are made to very close specifications and may be replaced by the regular operating force without disturbing the light focus enough to interfere with the operation of the instrument. There is apparently an adequate supply of light to obtain satisfactory records with a lower voltage than 6.6 volts applied to the oscillograph but it is our opinion that the quality of the records should not be jeopardized in this way.

The instrument is interchangeable for either table or panel mounting. Standard switchboard terminal studs are used when it is panel mounted, thus permitting all wiring to be installed in accordance with local standards.

The Photoelectric Recorder

BY C. W. LA PIERRE[1]

Associate, A.I.E.E.

INTRODUCTION

THE usual direct-acting recorders are the heavier and higher torque counterparts of corresponding indicating instruments. Although such recorders have some highly desirable characteristics, such as a continuous record and a rapid response, they cannot be made to approach indicating instruments in sensitivity.

The photoelectric recorder combines all the advantages of direct-acting recorders with those of the most sensitive indicating instruments. This is accomplished by providing a separately excited recording element to do the actual work of making the record. A sensitive indicating instrument controls the position of the recording element so that the record obtained represents the readings of this basic instrument. The loss in the sensitive basic instrument is the only power required from the measured circuit.

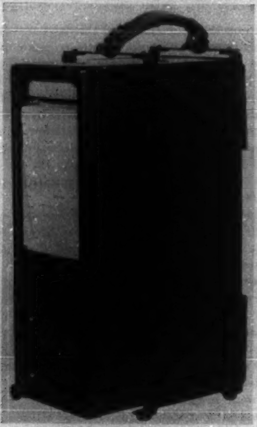

FIG. 1—PHOTOELECTRIC RECORDER WITH PORTABLE FIXTURES

The link between basic and recording elements consists of a combined optical system and photoelectric circuit. The basic element itself rotates a galvanometer mirror through an angle corresponding to its reading, and can be even more sensitive than the usual indicating instruments with pointers. On the other hand, the recording element may readily possess as much torque and be as heavy and sturdy as desired without affecting the basic element or measured circuit in any way.

As a result, the photoelectric recorder possesses all of the desirable characteristics of direct-acting recorders, and at the same time is capable of a sensitivity and a rapidity of response equal to the highest sensitivity indicating instruments. In general, anything which can be indicated can now be continuously recorded by means of the photoelectric recorder.

DESCRIPTION

An external view of one form of the photoelectric recorder is shown in Fig. 1. The instrument is completely self-contained requiring only an auxiliary source of 115 volts, 60 cycles. It is available for either portable or switchboard use.

The internal parts of the photoelectric recorder may be grouped into five units: namely, the basic indicating instrument or galvanometer, optical system, recording element, chart carriage, and power unit. The relative location of the various units is shown in Fig. 2.

The Basic Instrument is the only measuring device within the recorder. Ample space is available for including almost any type of normal size instrument element. For use in the recorder their pointers are removed and replaced by a small galvanometer mirror usually mounted on the axis of rotation, just above the basic instrument proper. A typical mirror and mounting for this purpose weighs approximately 0.07 gram.

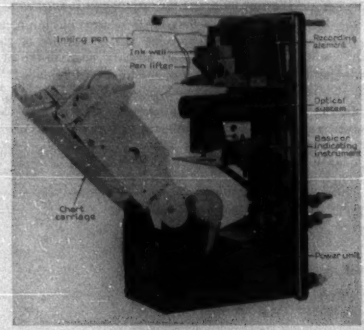

FIG. 2—INTERIOR OF THE PHOTOELECTRIC RECORDER

The Optical System includes mirrors on the basic and recording elements and the fixed reflectors shown in Fig. 3. A schematic diagram of the complete system is illustrated by Fig. 4A. Fig. 3 also shows side reflectors which are for the purpose of directing the light into the photoelectric tubes when it fails to strike the dividing reflector.

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[1] General Engineering Laboratory, General Electric Co., Schenectady.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

In general, the optical system comprises a group of reflectors, so arranged that whenever an angular displacement between the recording and basic elements occurs, there will be a shift in the light distribution between the photoelectric tubes, of the proper direction to react on the recording element and reduce the displacement to zero. Thus the optical system always acts to maintain the recording and basic elements in the same angular position.

The manner in which this is accomplished may be determined by inspection of Fig. 4A. The important reflectors in this group are: the basic element mirror *A*, the curved reflector *B*, the recording element mirror *C*, and finally the dividing reflector *D* which splits the light and reflects each portion into its photoelectric tube. The recording and basic elements are usually located on the same axis, or very nearly so, although this is not necessary for successful operation of the instrument. The lamp and condensing lens converge a source of light upon the basic element mirror *A*, from which it is reflected to the curved mirror *B*. The point at which the light strikes the mirror *B* will depend upon the angular position, or in other words, "the reading" of the basic element. Regardless of where it strikes *B*, however, it is reflected to *C*. The angle at which the light leaves *C* will depend upon the angle at which it arrived from *B* and also upon the position of the recording element. When *C* and *A* are parallel, however, the light will leave *C* and split evenly upon the dividing edge *D*. This holds regardless of the actual position of *A* and *C*; the only requirement is that they must be parallel. If they are not, the light will strike to one side of the edge of *D*; more of it will pass into one photoelectric tube or the other and cause the recording element to turn until they are parallel. The relation between the various angles of incidence and reflection are shown in Fig. 4B. Here, by slightly displacing the mirrors from their true positions, the different paths of light can be readily traced.

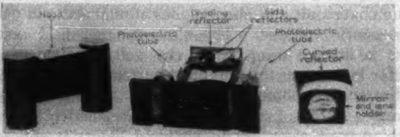
Hood
Photoelectric tube
Dividing reflector
Side reflectors
Photoelectric tube
Curved reflector
Mirror and lens holder

FIG. 3—FIXED REFLECTORS OF THE OPTICAL SYSTEM

The light does not, of course, traverse the optical system as a parallel ray and the optical constants of the system are of major importance. These factors are considered more in detail in the appendix.

The Photoelectric Circuit which supplies the actual energy used by the recording element in making the record is shown in Fig. 5. The output of a secondary winding on the auxiliary supply transformer is rectified and applied to the field coils of the recording element. A capacitor in parallel with the field coils provides additional filtering so that the pulsations in the resulting direct voltage are considerably reduced. The voltage so obtained supplies the pliotron plate and the photoelectric tube circuits. Other windings on the auxiliary supply transformer provide voltages for the lamp, rectifier, and pliotron filaments.

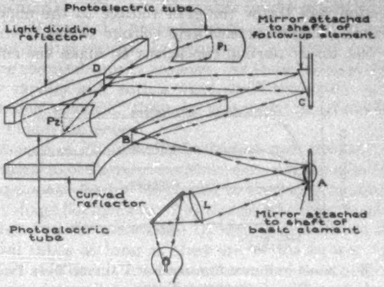
Photoelectric tube
Light dividing reflector
Mirror attached to shaft of follow-up element
P1
D
C
P2
B
A
Curved reflector
L
Mirror attached to shaft of basic element
Photoelectric tube

FIG. 4A—SCHEMATIC DIAGRAM OF THE OPTICAL SYSTEM

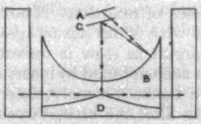

FIG. 4B—VARIABLE ANGLES OF REFLECTION OF THE OPTICAL SYSTEM

Of greatest importance is that part of the circuit beyond the field coils of the recording element. A portion of the direct voltage across the field coils is applied to the three element pliotron tube. The plate current of this tube passes through the recording element armature connected in the filament side of the circuit to permit grounding one armature lead. Another portion of the voltage across the field coils is applied to the two

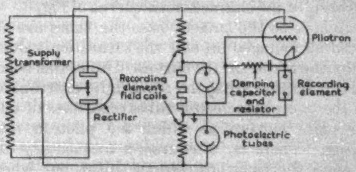
Supply transformer
Rectifier
Recording element field coils
Pliotron
Damping capacitor and resistor
Recording element
Photoelectric tubes

FIG. 5—SCHEMATIC DIAGRAM OF THE PHOTOELECTRIC CIRCUIT

vacuum type photoelectric tubes in series. Their common junction is connected to the grid of the pliotron. This circuit is capable of very high sensitivity to changes in the distribution of illumination between the two tubes. The slightest unbalance of light in a given direction between the tubes will throw the grid voltage to one extreme. A slight unbalance in the opposite direction will throw the grid voltage to the opposite extreme. These extreme grid voltage swings occur far too fast under normal conditions to make the circuit useful for purposes of recording. The high sensitivity may be retained with all its advantages, however, and still have the circuit as slow in operation as may be desired. This is accomplished by inserting between the grid and filament a capacitor and resistor in series. It is now necessary to alter the charge upon the capacitor before the grid voltage can change, and the time lag of this procedure may be adjusted to any degree by changing the value of the capacitance. Without this damping circuit there would be violent hunting between the recording element and the electric circuit, whereas with the damping circuit all tendency to hunt or overshoot can be completely eliminated.

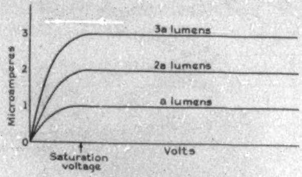

FIG. 6—CHARACTERISTIC CURVES FOR VACUUM TYPE PHOTOELECTRIC TUBES

Besides providing a circuit highly sensitive to changes in light distribution, the vacuum type photoelectric tubes contribute materially toward the high precision obtainable in the transfer from the basic to the recording element. This property can be readily explained on the basis of the characteristic curves of Fig. 6.

As applied to the present case the tubes are always operated at voltages beyond the saturation value indicated by the arrow on the curves. Under this condition the circuit of Fig. 5 has the same light distribution between the two photoelectric tubes regardless of the particular plate current at which the pliotron may be operating. There is, of course, a momentary change every time the basic instrument shifts, but when the recording element takes up its new position the light distribution returns to its former value. This characteristic arises from the fact that above saturation the photoelectric tube current is independent of voltage. In the strictest sense, the tubes are electron valves controlled solely by light. In Fig. 5 it can be shown that the position the optical system always seeks is the condition of equal photoelectric tube currents. If the currents are not equal the grid voltage must be changing as the difference current can only flow into the grid capacitance. Since the photoelectric tube currents are sensitive only to light, it follows that the light distribution must always be the same in order that the tube currents may be equal.

The Recording Element: The particular form assumed by the recording element is not of very great technical importance in the photoelectric recorder whereas in direct-acting types the recording element must be a highly reliable and accurate device. In the present instrument it exists merely for the purpose of actuating the recording pen. Within wide limits its characteristics do not enter at all into the accuracy of the instrument, for its position is controlled by the basic element.

The recording element contained in the instrument of Fig. 2 is illustrated in detail in Fig. 7. This differs from the d'Arsonval type in having an electromagnet instead of a permanent magnet field. The armature consists of a coil moving in the gap formed by the pole pieces and core. The armature current is obtained from the pliotron tube and is under the control of the photoelectric tubes. The field winding derives its current from the output of a rectifier on the secondary side of an auxiliary supply transformer. This transformer is contained in the power unit and also supplies all filament and other necessary voltages. The electromagnet field serves an additional function as the voltage divider and filter inductance for the direct current supplied to the pliotron and photoelectric tubes.

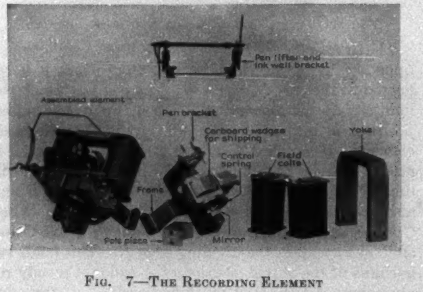

FIG. 7—THE RECORDING ELEMENT

The inking system, comprising a syphon pen and a fixed ink reservoir, and the chart carriage, are of conventional design.

SENSITIVITY AND RAPIDITY OF RESPONSE

The sensitivity of the recorder is dependent only upon the sensitivity of the basic element. Rapidity of response is usually governed by the period of the basic element, although in some isolated cases the period of the recording element may be the limiting factor. Recorders can be constructed utilizing as a basic element almost any indicating instrument or galvanometer that is capable of rotating a mirror as a means of indicating its readings. For recorders that are expected to be carried about and used like any switchboard or portable instrument, the use of extremely sensitive galvanometers with long suspensions is impracticable. However, extraordinarily high sensitivities can be obtained with the more sturdy pivoted instruments. The following discussion gives some idea of what can be readily accomplished.

For d-c. measurements the basic element is usually the same as is found in miniature indicating elements, except that a galvanometer mirror is substituted for the pointer. Such miniature elements are available to provide recorders of as low as 75 microamperes full scale and having a resistance of 350 ohms. Higher currents and other resistance values are also available. A low watt consumption element may be used having a full scale current requirement of 300 microamperes and a resistance of only 10 ohms. A recorder containing this instrument would require less than 1 microwatt from the measured circuit at full scale deflection. With larger elements of slightly more expensive construction, current sensitivities of 20 microamperes full scale can be provided with a watt consumption at that current of approximately 0.3 microwatts.

The figures just given illustrate very forcibly the extremes to which sensitivity has been carried in the photoelectric recorder. Such instruments are not at all delicate. In fact, because of the small weight of their moving systems, they are much more free from difficulties due to rough handling of the recorder than are heavier and less sensitive elements.

Other elements are, of course, available for recording other quantities. For instance, high sensitivity frequency recorders can be constructed with a scale range of 1 cycle at 60 cycles; high sensitivity voltage recording with a scale range of as low as 4 volts on a 115 volt circuit; pressure recorders with full scale pressures down to approximately 1 or 2 inches of water; or, in general, anything for which there is some sort of an indicating instrument, can be continuously recorded with a photoelectric recorder.

The rapidity with which the recorder responds to fluctuations in the measured quantity is of the same order as the response of the basic element. In most cases the instrument responds to a change from zero to full scale within one second, and the usual fluctuations require a much shorter time.

ACCURACY

Many amplifying systems have been devised for using electron tubes in connection with the measurement, recording and control of various quantities. Such amplifying systems are usually effected by variations in the characteristics of the lamps and electron tubes used, and are also subject to variations in the supply voltage. The optical system and photoelectric circuit described above is self-compensating for all normal changes in these variables.

Variation in the Auxiliary Supply Voltage is transmitted to all parts of the photoelectric circuit. However, within wide limits, such variations do not affect the accuracy of the recorder. When the voltage does change, the recording element current will also change momentarily. The resulting shift in the recording element will cause the photo tubes to establish a new grid potential and restore the proper plate current in spite of the changed supply voltage. As these changes occur very rapidly and with a very small movement of the recording element, there is ordinarily no discernible effect upon the record.

Changes in Pliotron Characteristics such as occur due to variation of filament emission, etc., are automatically compensated for by the circuit in the same manner as are voltage fluctuations. No change whatever would be expected in the record due to variations between individual tubes as long as they are of the proper type. In fact this circuit has been constructed with sufficient margin to permit using tubes of widely different characteristics including different filament and plate voltage ratings as well as different values of amplification constant, μ, without any appreciable effect upon the accuracy of the result.

Changes in Lamp Illumination such as occur due to normal variations in supply voltage, deterioration of the filament and blackening of the bulb do not ordinarily produce any effect on the recorder during the life of the lamp. In extreme cases of progressive loss of illumination, the operation of the recorder may be slowed up appreciably without, however, affecting its accuracy.

The Reflectors merely direct the illumination along the desired paths, and variations in their reflecting power, such as those due to accumulations of dust, have the same effect as variations in lamp intensity. Here again serious errors are extremely unlikely, for any trouble is readily detected in the noticeable slowing down of the operation of the instrument long before it develops any appreciable errors.

It is interesting to note, however, that ordinary dust accumulations on the reflectors produce an inappreciable effect on their reflecting power. No matter how dusty the reflectors may appear to the eye, an inspection of the surface through a microscope will show that the particles are widely separated.

Variations in Photoelectric Tube Sensitivity may be added as the final possible source of error of any magnitude. Here again, through proper design of the optical system, the effects of changes in photoelectric tube sensitivity have been reduced to a secondary and negligible order of magnitude. The use of vacuum type tubes is also of considerable advantage as they are much more stable than gas-filled types.

The action of the recorder in respect to these factors may be explained by reference to Figs. 4A and 5. The condition the recording element always seeks is that occurring when the two photoelectric tube currents are equal. If the tubes are not of equal sensitivity, the equal currents can be obtained only by throwing more light on the less sensitive tube. This is actually what happens when tubes having different sensitivities are used. After they are installed and the instrument is set to zero, it will work satisfactorily, and unless the difference in sensitivity is quite large, no one would be able to detect it from the operation of the instrument. In case the difference in sensitivities is large, in the order of two or more to one, for instance, it would be noticed that the recorder followed more rapidly going one way on the scale than the other, and the trouble is readily remedied by replacing the less sensitive tube.

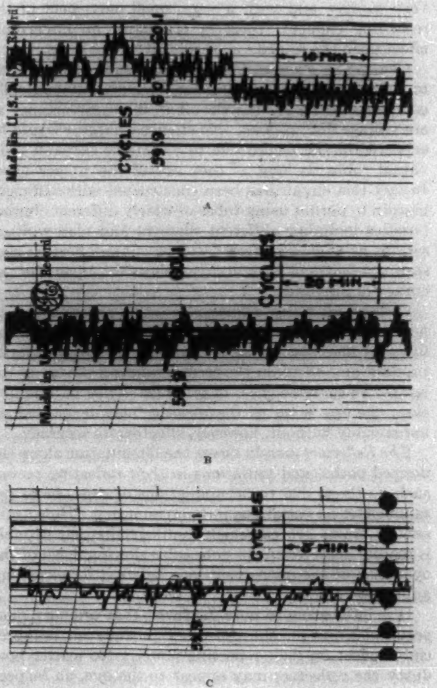

FIG. 8—HIGH SENSITIVITY FREQUENCY RECORDS

Tests show that after photoelectric tubes have been in service a while, they tend to become quite stable and will maintain their sensitivities fairly constant over long periods. There is still some variation, however, and it is desirable to show just what effect this has upon the recorder.

Assuming for the present that the two photo tubes of equal sensitivity are installed, the light *CD* in Fig. 4A will split evenly at *D* into the two tubes for all scale readings. If the sensitivity of one photo tube decreases with respect to the other by 50 per cent (a very unlikely amount) it would mean that the less sensitive tube would require twice the illumination of the other. This would be obtained by a shift in the light ray *CD*. The amount of shift would depend upon the effective width of the ray at *D*. In a specific case, the effective width of the ray is 0.12 inch. Thus for photoelectric tubes of equal sensitivity the split rays would each be 0.06 inch wide at *D*. If one photoelectric tube is to receive twice the illumination of the other, due to a change in relative sensitivity, the two rays would be 0.08 inch and 0.04 inch wide. The actual displacement from the former position is 20 mils. This results in an angular displacement of less than 40 minutes for a length of *CD* equal to 1.75 inches. Since the recording element need rotate through only 20 minutes to produce a 40-minute movement in the ray (the reflection at *C* doubles the angle), the actual error in the position of the recording element is quite small, amounting to only 0.6 per cent of full scale deflection. It is not expected that the vacuum type photo tubes used in the recorder will change relative to one another by any such amount as 50 per cent, in fact, the variations are more nearly 5 per cent. Consequently, the error due to this cause is limited to the order of one-tenth of one per cent at most, and this would occur only over a considerable period of time.

The Other Possible Sources of Error include pliotron grid currents, photo tube dark currents, and leakage currents in the grid circuit. All of these tend to affect the results in the same way as a change in photo tube sensitivity. There is no difficulty, however, in reducing the magnitude of all stray currents to from one-tenth to one-twentieth of the actual control current. When of this order their effect is entirely negligible.

It is believed that in any new instrument of this kind, the possible sources of error should be investigated very carefully. It is for this reason the preceding discussion has been carried out in considerable detail. The mere mentioning of a possible source of error should not in any way be interpreted to mean that it is an important one. In fact they have all been mentioned and discussed only for the purpose of showing how very small are their possible effects.

OPERATION

Several of the photoelectric recorders both in experimental and commercial forms have been under observation under actual operating conditions for some time. The results they have produced are in many cases such as could not be obtained by any other commercial instrument with which the author is familiar.

Figs. 8A, B, and C show frequency records at various chart speeds, made on a well-regulated system. Records such as these show very clearly the variations between the control limits of the regulating devices. They are especially useful in studying system frequency disturbances.

Fig. 9 illustrates sensitive voltage records taken on a constant voltage supply for laboratory use. Recorders of this sensitivity are almost a necessity in running life tests on the filaments of lamps and tubes of various kinds, and in other cases where small variations in voltage produce large variations in the data to be obtained.

Fig. 10 illustrates records of smoke intensity taken in the stack of a steam generating furnace. In this case the recorder was used in connection with a photoelectric smoke detector. Such applications emphasize the usefulness of the photoelectric recorder in connection with other electron tube apparatus. The use of ordinary recorders requires at least one more stage of amplification and the record is subjected to the additional errors of that stage. In photoelectric tube work the photoelectric recorder can often be constructed to record the photo tube currents directly so that the only errors in the record are those of the tube itself. Additional discrepancies due to amplifying equipment are completely eliminated.

Fig. 11 illustrates records taken of variations in the temperature of a refrigerator evaporator under test.

In connection with an electric gage, continuous records have been taken of paper thickness as it passes from one roll to another. Each small division on the

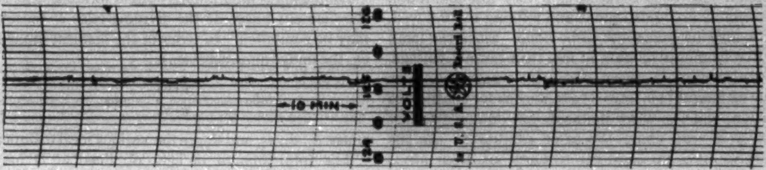

FIG. 9—HIGH SENSITIVITY VOLTAGE RECORD

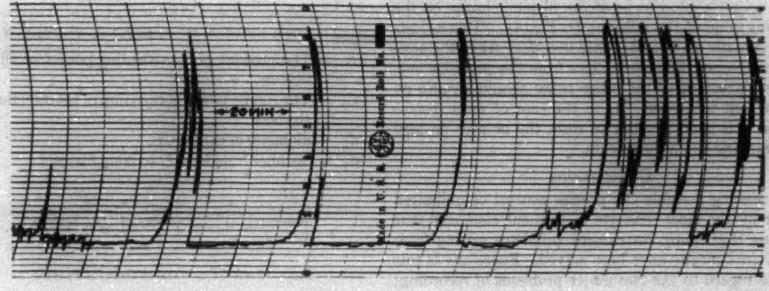

FIG. 10—SMOKE INTENSITY

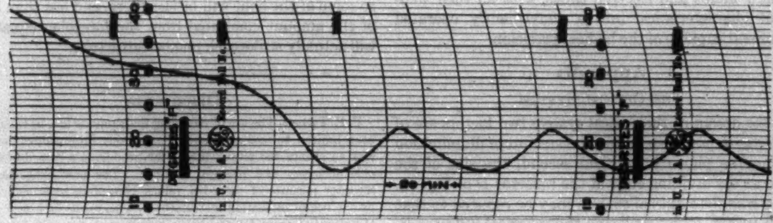

FIG. 11—REFRIGERATOR EVAPORATOR TEMPERATURES DURING TEST

fifty division chart represented a variation in thickness of two one-hundred thousandths of an inch. Other gaging records have been obtained showing the dimensions and distortion of materials under stress.

Still other applications include electrolysis survey records, turbo-generator vibration recording, high sensitivity power recording, small draft pressures and so on.

The Maintenance necessary in the operation of the instrument has been reduced to a very low minimum. Aside from chart replacement and refilling the ink well, there is nothing much else to be done. Since the lamp and tube filaments operate at less than normal voltage, their replacement is a very infrequent occurrence. The estimated lamp life is from 3 to 5 months for continuous operation, and the other filaments are designed to last well over a year under the same conditions. As the phototubes have no filaments to burn out, their life is indeterminate, but they should last at least as long as any of the other tubes.

Lamp replacement may require a slight adjustment in order to get a good focus of the filament image on the dividing edge of the reflector *D*, Fig. 4A. This adjustment is easily made and is only necessary when the position of the new filament differs from the previous one. The replacement of the other tubes should require no adjustment whatever. As in the case of most instruments, it is good practise to reset the pointer on zero after making any adjustments.

CONCLUSIONS

In making possible the precise control of unlimited power by the most sensitive instrument elements, it is obvious that many instrument problems arising from insufficient torque have been solved. Especially is this true in regard to the two major problems of recording; sensitivity and rapidity of response. Of greater fundamental importance, perhaps, is the combination of ordinary lamps and electron tubes, with all their well-known characteristics, into a conveniently operated instrument capable of the highest precision.

Appendix

THE OPTICAL SYSTEM

In the design of the optical system two factors are of dominant importance. First the amount of illumination striking the photo tubes, and second, the size of the image at the dividing edge of the reflector *D* of Fig. 4A.

The complete optical system may be reduced to that shown in Fig. 12 by replacing the spherical reflector *B*, Fig. 4A, with a lens of equivalent focal length. The dimensions of the photoelectric recorder optical system are such that the image of the filament must be focused at a point between the mirror *A* and the lens *B* in order to obtain an image at *D*. Fig. 12 is therefore drawn for this condition.

The plane mirrors *A* and *C* are sufficiently large not to act as light stops and they may therefore be neglected from further consideration as far as the size of image and total amount of light are concerned. However, in order that the light from the mirror *A* will strike the mirror *C* throughout the complete scale range, it is necessary that *A* and *C* be placed at conjugate foci of *B*. Thus in Fig. 4A the two instruments need not be coaxial but may be placed anywhere that will satisfy this condition. The various possibilities and the relative advantages of each, need not be considered here.

By means of well known optical formulas the size of the filament image at the dividing edge of reflector *D* may be computed. In terms of the filament dimensions ab_0 the image dimension ab_D is

$$ab_D = \frac{d_1\,d_3}{d_2\,d_4}\,ab_0$$

where d_1, d_2, d_3, and d_4 are the various focal distances.

The total amount of light obtained at *D* may be computed by:

$$I_D = \frac{K\,C_p\,\pi\,d_L^{\,2}}{4\,d_4^2}\text{ (lumens)}$$

where:

K = The product of the reflecting powers of all the mirrors.

C_p = The candle power of the lamp in the direction of the lens.

d_L = diameter of the lens, *L*.

d_4 = distance from lens, *L*, to lamp filament.

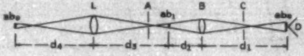

FIG. 12—AN EQUIVALENT OPTICAL SYSTEM

The dividing edge of the mirror *D* may be quite broad so some of the light will be lost there, and less than half of the amount computed from the foregoing equation will actually strike each photoelectric tube.

Discussion

P. E. Twiss: The use of recording instruments has, in the past, been limited to measurements of relatively large values, necessary to obtain the proper torque for producing a record of suitable accuracy and prompt response. The new photoelectric recorder permits of recorded values heretofore measured only by the use of indicating instruments. This opens up a new field of unlimited scope for recording instruments, not only for measurements of low electrical values, but also for many applications of the photoelectric cell in measuring and recording light intensity.

An analysis of the design of this instrument indicates that all component parts are familiar to the trade and have been proven satisfactory by actual service in other applications over a period of years. The photoelectric cell and the amplifying system including the pliotron tube have been in extensive commercial use for several years and their reliability is well known. The indicating and recording apparatus is also of standard design but includes many recent improvements particularly in the chart mechanism.

A recent successful application of the photoelectric recorder to frequency measurement has been demonstrated by a well-known public utility company whose system is regulated by automatic frequency control equipment. The frequency at several points of the system was measured by other recorders of various designs, and operating data over a period of a year have proven conclusively the superiority of the photoelectric recorder.

C. W. Mayott: In May 1930 a photoelectric frequency recorder was installed in our load supervising office at Hartford. This meter being an experimental one had some mechanical defects, all of which have been corrected in the finished product. It was in satisfactory service for approximately a year and one-half and during this period the only replacement was the light source bulb.

The meter gives the much desired instantaneous and continuous record that is so desirable in operation, especially in checking up disturbances. It can give a frequency record in such detail and such magnified scale as to be beyond the requirements of any present operating demands.

H. A. Rolnick: 1. What is the smallest angular motion of the indicating element to which the recorder will respond?

2. Is the above dependent on the electrical circuit or on the optical system or on both?

3. How much difference would a movement of the lamp filament of 0.01 in. to the right or left have on the recorder reading?

C. W. LaPierre: Referring to Mr. Rolnick's questions in the order mentioned:

1. The circuit of the photoelectric recorder is in a position of unstable equilibrium under the balanced condition. Consequently the recording element is responsive to infinitesimal movements of the basic indicating element from balance. This condition would lead to hunting between the recording element and the photoelectric circuit if the tendency were not damped out by the pliotron grid to filament resistance and capacitance (see Fig. 5). The damping capacitance and resistance, however, do not decrease the sensitivity of the circuit—they only make the time of response of the photoelectric circuit the same order of magnitude as the time of response of the recording element.

2. As may be inferred from the answer to question (1), the sensitivity of the response of recording to indicating elements is independent of the optical system and electrical circuit. However, it may be well to repeat that the rapidity with which the recording element responds, is somewhat dependent upon the electrical circuit and optical system. In this answer, I am assuming, of course, that Mr. Rolnick refers to normal variations in the circuit and optical system and not to changes such as would modify the fundamental principles of operation of the circuit.

3. In the design of the photoelectric recorder described in the paper, a sidewise shift in the lamp of 0.01 inch would produce roughly a change in reading of 0.05 per cent of the scale range value. In more recent designs, we have allowed this effect to increase in order to gain other benefits as our experience shows that changes of this order do not occur after a lamp is installed. Any errors at the time of lamp replacement may be completely eliminated by resetting the instrument on zero.

Interconnection of Primary Lightning Arrester Ground and the Grounded Neutral of the Secondary Main

BY C. FRANCIS HARDING[1] and C. S. SPRAGUE[2]
Fellow, A.I.E.E. Associate, A.I.E.E

Synopsis.*—This investigation of the relative merits of various possible transformer connections and protective devices was the result of statistical data secured, over a long period of years, by the Commonwealth Edison Company of Chicago regarding the number and types of distribution transformer failures during lightning storms.* It refers to standard, 60 cycle, overhead, power and lighting distribution systems having 4,000-volt, four-wire primaries and three-wire, 230-volt, three-phase power and 115-230-volt lighting secondary mains.*

Five spans of standard pole line construction were exposed to transient potentials of steep wave front, the latter being induced by the sudden charging of the well insulated artificial cloud suspended over the line. The cloud was charged to potentials, up to 150 kv., by means of a surge generator. Measurements of potentials, ranging as high as 40 kv. in some instances which were induced upon various portions of the distribution system were made by means of ionized sphere gaps and the cathode-ray oscillograph.

Some of the important conclusions reached as the result of this investigation are:

1. The value of an experimental wood pole distribution line with an insulated artificial cloud charged by means of a surge generator was definitely established for lightning protective investigations involving induced as well as direct stroke potentials.

2. The practicability and economy of studying, by means of such laboratory equipment, the operation of various transformer, lightning arrester and ground connections, when exposed to surges approximating those of lightning, were demonstrated.

3. Efficient primary protection upon an overhead distribution system affords a considerable degree of protection to secondaries located below the primaries.

4. A well grounded secondary neutral wire acts to reduce potentials on adjacent wires.

5. With existing transformer design, the insulation of the secondary winding may be considerably over-stressed by steep wave front surges without excessive stress on the primary insulation. Such secondary stresses may be relieved by improvements in secondary insulation.

6. Low ground resistances, although desirable in other respects, do not necessarily reduce the initial potentials which may be induced upon the system.

7. A non-inductive load in the consumer's premises reduces the potentials 60 to 70 per cent at the service entrance.

8. Tests have shown that the interconnection of primary lightning arrester ground to the grounded neutral of the secondary main effects a considerable reduction in voltages at the transformer and imposes no extra hazard upon the consumer's wiring.

INTRODUCTION

UNDER a cooperative arrangement* with the Utilities Research Commission of Chicago, the Engineering Experiment Station at Purdue University has been conducting an investigation of surge protection of distribution circuits. A considerable number of the tests has involved the interconnection of the primary lightning arrester ground and the grounded neutral of the secondary main. It is this phase of the work which will be considered in this paper.

DESCRIPTION OF APPARATUS

After some preliminary work, a five-span distribution system was constructed, over which was supported an insulated artificial cloud for the purpose of inducing surge potentials in the distribution system. The latter was composed of four 4,000-volt primaries on the upper arm and six secondaries on the lower arm comprising a three-phase, 230-volt power secondary and a single-phase 115-230-volt lighting secondary. These are of the standard construction of the Commonwealth Edison Company of Chicago. A bank of three distribution type transformers and a customer's service were included in the distribution system but were not connected to any 60 cycle supply. Circuit grounds were made in the customer's service and on the distribution system as in practise. (See Fig. 1 for circuit diagram.)

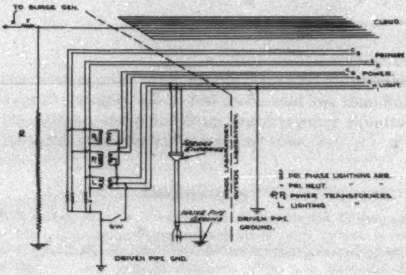

FIG. 1—CIRCUIT DIAGRAM OF CLOUD, LINE, AND TRANSFORMERS

METHOD OF TEST

The cloud was normally held at ground potential by means of a high resistance leak. At definite intervals a surge generator† was made to discharge into the cloud,

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[1] Head of School of Electrical Engineering, Purdue University.

[2] Research Assistant, Engineering Experiment Station, Purdue University.

**Studies in Lightning Protection*, by D. W. Roper, A.I.E.E. TRANS., March 1932, p. 252.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

†"Purdue Builds Lightning Generator," *Electrical World*, January 17, 1931, Vol. 97, No. 3, p. 142.

thereby causing a sudden change in cloud potential with consequent induced voltages in the distribution system. Sphere gap measurements of these induced voltages were taken at the transformers and at the customer's service entrance.

Results of Tests

A series of tests was made in which the induced voltages to ground were measured as the distribution system was built up step by step, that is, starting with the wires only and adding the circuit grounds, arresters, transformer windings, etc., until the complete circuit was obtained.

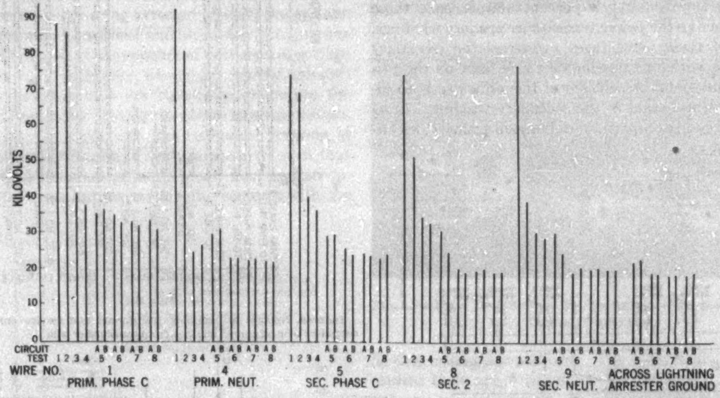

Fig. 2—Induced Potentials to Ground with Increasing Amounts of Apparatus Connected to the System

Induced voltages to ground on distribution wires with the following circuit conditions
Test 1—No apparatus or grounds connected but each wire grounded through high resistance
Test 2—Same as test 1 with secondary neutral grounded one pole from transformer bank
Test 3—Same as test 2 with primary arresters connected
Test 4—Same as test 3 with primary windings connected
Test 5—Same as test 4 with secondary windings connected
Test 6—Same as test 5 with house circuit connected and no load in house circuit
Test 7—Same as test 6 with six 25-watt lamps in house circuit
Test 8—Same as test 7 but with six 100-watt lamps in house circuit
A indicates secondary neutral not connected to primary lightning arrester ground at transformer
B indicates secondary neutral connected to primary lightning arrester ground at transformer

Fig. 2 shows graphically the change in the magnitude of the induced voltages during the above procedure, and also shows the system voltages with and without the interconnection of the secondary neutral to the primary arrester ground. (See Fig. 3 for wire positions on cross arm.)

The graphical presentation of Fig. 2 is, in general, self-explanatory. However, it is desired to call attention to some of the more interesting changes in the induced voltages and their causes. As noted in the legend of Fig. 2, the first line of each group represents the induced voltages on that particular wire with no grounds or connected apparatus. Primary voltages (between wires 1 or 4 and ground) were approximately 90 kv.; secondary voltages (between wires 5, 8, 9 and ground) were approximately 75 kv. Test No. 2 of each group represents the induced voltage when wire No. 9 (sec. neutral) was connected to a driven pipe ground of approximately 15 ohms resistance, one span from the transformer bank. Aside from the reduction in voltage on wire No. 9 itself, it is interesting to note the considerable reduction on wires 4, and 8, adjacent to wire No. 9. (Fig. 3.)

Test No. 3 of each group indicates the voltages with the further addition of primary phase and neutral arresters. As would be expected, there was a marked decrease in all primary voltages. There occurred also a very noticeable reduction in secondary voltages (wires 5, 8, 9) due to the shielding effect of the primary wires after the primary arresters had broken down. The significance and importance of this lies in the fact that it indicates that in the field, a well protected primary system provides a considerable degree of protection for secondaries located below the primaries.

In Test No. 4 the connection of the transformer primary windings caused a very slight reduction in voltage, showing that the primary winding presents a relatively high impedance to the surge.

With transformers, house-wiring circuit, and all grounds connected to the circuit, (Fig. 1) voltages to ground on the power and lighting secondaries and on the primary neutral were of the order of 20-25 kv., while those between the primary phase wires and ground were of the order of 30-35 kv.

With regard to voltages across transformer windings it was determined that for a single-phase lighting transformer with its secondary neutral grounded the voltage across the secondary winding was approximately 5 per cent to 15 per cent of that existing across the primary winding. With three-phase, 230-volt, delta-connected power secondary windings having the mid-point of one winding grounded to the lighting secondary neutral, it was determined that the voltages across the secondary coils were approximately 20 per cent to 80 per cent of those existing across the power transformer primary windings. Either of these percentages overstress the secondary insulation without exceeding the safe limit on the primary insulation. A solution of this difficulty is to increase the insulation of the secondary winding. In so doing, of course, capacitances between primary, secondary, and core will be changed. Consideration should therefore be given to the redistribution of potentials resulting therefrom. These are relatively inexpensive additions to the design and construction of the transformer.

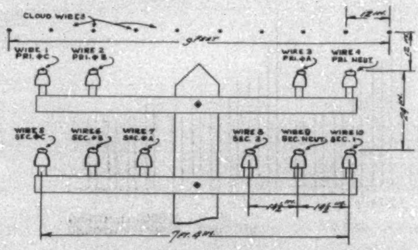

FIG. 3—CROSS SECTION OF CLOUD AND LINE SHOWING WIRE POSITION

In the preceding group of tests the interconnection of the secondary neutral and the lightning arrester ground caused, in general, approximately 5 per cent decrease in the magnitude of the induced voltages to ground.

Throughout most of the work, tests have been made with and without the secondary neutral wire connected to the primary lightning arrester ground lead. The tests indicated that the effect of this connection depends to a large extent upon such factors as the steepness of wave front and the time lag of the primary arresters, hence no definite statement can be made to the effect that this connection is always beneficial. The connection of the lighting secondary neutral to the lightning arrester ground caused 10 per cent reduction in the voltages at the service entrance.

Figs. 4 and 5 portray graphically the magnitudes of the induced voltages at the transformers and at the service entrance under various conditions of ground resistances, and with and without the interconnection of the secondary neutral to the primary lightning arrester ground. In several instances this interconnection caused a material decrease in the voltage between primary phase wire and secondary ground.

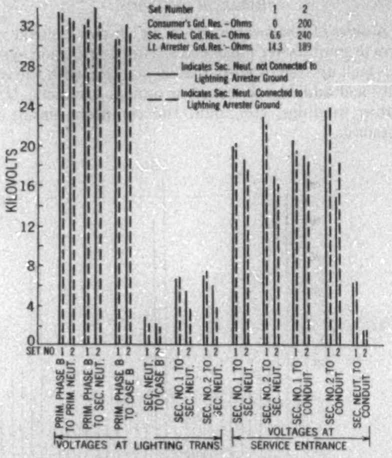

FIG. 4

Induced voltages at lightning transformer and service entrance with secondary neutral ground one span from transformer pole

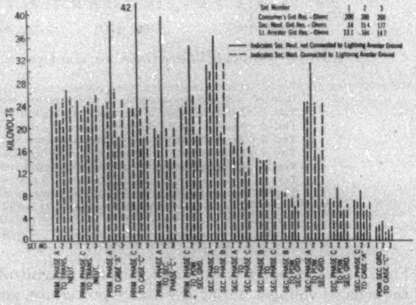

FIG. 5

Induced voltages at power transformers with secondary neutral ground at transformer pole

FIGS. 4, 5—EFFECT OF INTERCONNECTION ON THE POTENTIALS AT THE TRANSFORMERS AND AT THE SERVICE ENTRANCE WITH VARIOUS GROUND RESISTANCES

Field investigations have demonstrated that a very considerable number of transformer failures and cases of fuse-blowing have occurred by flash-over from the primary phase lead to case and thence to secondary neutral. In fact, operating companies have reported that in about 75 per cent of the transformer failures on distribution systems due to lightning, the damage involved both the primary and secondary windings. Measurements of the voltage between primary phase lead and secondary neutral have shown that the *interconnection of the secondary neutral with the lightning arrester ground is, in general, beneficial to the transformer.* In particular, with a low resistance secondary neutral ground and a high resistance lightning arrester ground, the interconnection reduced the above voltage by 30-50 per cent. (See Fig. 5, Set 2.) With no interconnection and a high resistance secondary neutral ground and a low resistance lightning arrester ground, the voltage between primary phase lead and secondary winding was somewhat reduced at the expense of an increased voltage across the high resistance secondary neutral ground. With the addition of a low resistance ground to the secondary neutral by making the above interconnection, thus placing the high and low resistance grounds in parallel, a redistribution of voltage occurred such that the potential between primary phase lead and secondary neutral was approximately the same as with the interconnection in the case of low secondary neutral ground resistance and high lightning arrester ground resistance. (See Fig. 5, Set 3.)

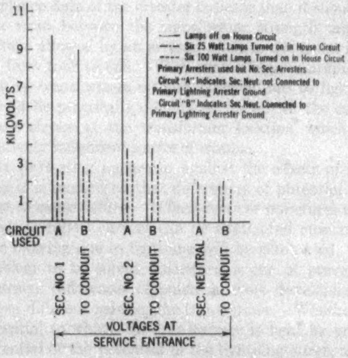

FIG. 6—EFFECT OF LAMP LOAD ON VOLTAGES AT SERVICE ENTRANCE

Induced voltages at service entrance

Fig. 6 shows a 10 per cent reduction in the potentials at the service entrance due to making the interconnection. There is indicated a very considerable reduction in the voltages at the service entrance when the customer's lamps were turned on. This seems to be an argument in favor of turning on the lamps during a thunderstorm.

During the latter part of the work the transformers were mounted on the pole and a testing shelter was constructed as in Fig. 7. The induced potentials were slightly higher but otherwise were in good general agreement with those obtained when the transformers were on the laboratory floor. The majority of the work presented here has been done with the transformers on the poles.

Acknowledgment is gratefully extended to the Utilities Research Commission for the opportunity to work on this interesting problem. The authors also wish to express their appreciation for the encouragement and valuable suggestions and advice received from the Committee, Mr. D. W. Roper, Chairman, supervising the work for the Commission. Also to Messrs. J. S. Malsbary and H. Creviston of the engineering staff and to students who have assisted in the routine testing, we express our thanks.

FIG. 7—VIEW OF OUTDOOR TRANSFORMER INSTALLATION AND TESTING SHELTER

CONCLUSIONS

1. The value of an experimental wood pole distribution line with an insulated artificial cloud charged by means of a surge generator was definitely established for lightning protective investigations involving induced as well as direct stroke potentials.

2. The practicability and economy of studying, by means of such laboratory equipment, the operation of various transformer, lightning arrester and ground connections, when exposed to surges approximating those of lightning, were demonstrated.

3. Efficient primary protection upon an overhead distribution system affords a considerable degree of protection to secondaries located below the primaries.

4. A well grounded secondary neutral wire acts in reducing potentials on adjacent wires.

5. With existing transformer design, the insulation of the secondary winding may be considerably over-

stressed by steep wave front surges without excessive stress on the primary insulation. Such secondary stresses may be relieved by improvements in secondary insulation.

6. Low ground resistances, although desirable in other respects, do not necessarily reduce the initial potentials which may be induced upon the system.

7. A non-inductive load in the consumer's premises reduces the potentials 60 to 70 per cent at the service entrance.

8. Tests have shown that the interconnection of primary lightning arrester ground to the grounded neutral of the secondary main effects a considerable reduction in voltages at the transformer and imposes no extra hazard upon the consumer's wiring. Specific cases of this conclusion are as follows:

a. With the lightning arrester ground resistance and secondary neutral ground resistance below 20 ohms, the interconnection mentioned above has very little effect upon the magnitudes of the potentials at the transformer.

b. This interconnection causes a 30-50 per cent reduction in potentials across the transformer in the case of an arrester ground of 100 to 200 ohms and a secondary neutral ground of less than 20 ohms.

c. With a lightning arrester ground of less than 20 ohms and a secondary neutral ground of from 100 to 200 ohms, the potentials across the transformer were increased about 25 per cent by making this interconnection.

d. With secondary neutral and primary arrester ground resistances both ranging from 100 to 200 ohms, the interconnection had practically no effect upon the magnitudes of the voltages at the transformer.

9. With the usual city conditions consisting of a multiplicity of low resistance grounds on the secondary neutral and the further possibility in some instances of a lightning arrester ground of high resistance, it has been demonstrated herein that potentials at the transformer may be greatly reduced by the interconnection of the primary lightning arrester ground and the grounded neutral of the secondary main.

Discussion

For discussion of this paper see page 271.

Lightning Protection for Distribution Transformers

Field Studies of Lightning Arrester Protection, Including the Effect of Interconnection

BY K. B. McEACHRON* and L. SAXON†

Member, A.I.E.E. Non-member

Synopsis.—*A cooperative field study with portable impulse generator and cathode ray oscillograph was made of the lightning protection of transformers connected to a 4.45-mile 4,600-volt rural distribution circuit. The tests described were made with the pellet arrester. Some of the conclusions are:*

1. Although the arrester discharge voltage is but a fraction of the strength of the transformer, yet for the usual connection to ground now commonly employed, arrester ground resistances may be high enough so that the transformer is not protected, resulting in either blown fuses or a winding failure.

2. Interconnection of the primary arrester ground and secondary neutral gives a high degree of protection to the transformer from surges originating on either primary or secondary regardless of the arrester ground resistance.

3. The interconnection of primary arrester ground and secondary neutral does not increase the magnitude of voltages which appear between various grounded objects on the consumers' premises over that now experienced with exposed secondary conductors and possible bushing flashovers or transformer failure. The effect of the interconnection is to increase the number of impulses reaching the secondary circuits but not their magnitude.

* * * * *

EXPERIENCE has shown that although failures of distribution transformers and fuse blowing during lightning storms are greatly reduced by the application of lightning arresters, yet the record of protection is not as good as the strength of the transformer and performance data of the arrester indicate that it should be. The ratio between the transformer strength and the voltage allowed by an arrester such as the pellet is not far from four to one. With such a large margin, failures of transformers or fuse blowing due to lightning should be practically eliminated, except in the case of direct stroke at the transformer location which is a relatively infrequent source of trouble.

In protecting apparatus against the effects of lightning it is recognized that differences of potential must exist to cause failure. If flashovers or punctures which result in follow current can be eliminated most of the fuse blowing due to lightning will be eliminated. The problem to be solved, therefore, is one of preventing excessive differences of potential from developing between different parts of the transformer. Whether the potential to which the transformer is held be ground potential or the potential of the lightning surge, makes little difference as far as the transformer itself is concerned.

In the past it has been common practise, with a very few exceptions, to connect the primary lightning arresters to a driven ground at the base of the transformer pole, with the idea that the lightning entering over the primary would pass to ground thus holding the primary winding to a potential equal to the arrester discharge voltage. Unhappily this condition is seldom met in practise as arrester ground resistances are not zero and may be as high as several hundred ohms. Lightning frequently enters over the secondary conductors and the arresters connected to the primary are then not in a position to be of service. Indeed the exposure of secondary circuits in urban districts may considerably exceed that of the primary conductors. Thus it is clear that there are several factors which operate to prevent the primary arrester from holding the windings at substantially ground potential.

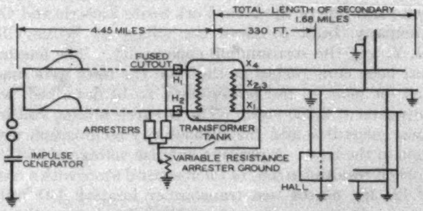

FIG. 1—CIRCUIT CONNECTIONS OF PROTECTION TEST ON TRANSFORMERS

There has been a growing conviction on the part of those closely associated with the problem of protection, that the present method of arrester application is at the mercy of too many uncontrollable factors in practise. Connecting the primary lightning arrester ground to the grounded secondary neutral appears to eliminate many of the uncontrollable factors. With such a connection the voltage between the transformer windings under impulse conditions will not exceed the potential allowed by the arrester, which is so low that flashover of the bushing or failure of the transformer should be

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*General Electric Company, Pittsfield, Mass.

†Utility Management Corporation, New York, N. Y.

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practically eliminated. In proposing such a scheme the question naturally arises: will such a scheme introduce additional hazard on the customer's premises and will it offer the advantage from the protection standpoint which it appears that it should? Experience in those cases where this interconnection has been tried does not indicate any increased hazard, and the protection record seems to be far above that of the present connection.

In order that experimental data might be obtained under controlled test conditions, the General Electric Company has been conducting a field investigation in cooperation with the Associated Gas and Electric Company. The tests were made on a single-phase 4,600-volt rural line of the New York State Electric and Gas Company, built to serve the village of Willseyville, N. Y. and the surrounding community. The line had just been constructed at the time the tests were made and for most of the power tests, to be described, was energized at 2,300 volts. The portable million volt impulse generator and the cathode ray oscillograph were used in the test. A positive impulse voltage of 350 kv. applied resulted in a 6/14 microsecond wave with a crest of 123 kv. at the test transformer location 4.45 miles away. It was necessary to materially increase the insulation at most of the guyed poles to prevent flashover.

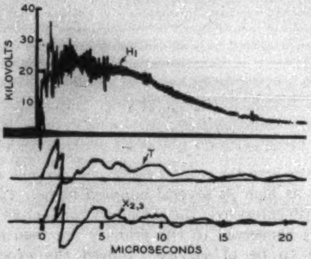

FIG. 2—STANDARD CONNECTION. IMPULSE APPLIED TO PRIMARY. ARRESTER GROUND RESISTANCE 10 OHMS

H_1 Primary to ground
T Tank to ground
$X_{2,3}$ Secondary neutral to ground
All oscillograms have the same calibration

A 10-kva. 2,300-115/230-volt transformer was mounted on pole No. 48 about 30 ft. above ground. The primary conductors terminated on this pole. The three secondary conductors X_1, $X_{2,3}$ and X_4 extended from this pole into the village of Willseyville, having a total circuit length of 3,273 ft. located on four streets. A building which will be designated as the "hall" was connected between X_1 and $X_{2,3}$ at a point about 400 ft. from pole No. 48. This building was the only one connected to the secondary circuits during the tests. In addition to the 65-ohm ground at the hall the secondary neutral was grounded at five points, the nearest being about 330 ft. from pole No. 48. The combined resistance of all secondary grounds was 25 ohms.

The hall was wired with *BX* and had a measured capacitance of 0.04 μf. between one wire and ground. It was provided with six circuits and twenty-six outlets. The hall ground was made to the casing of a driven well. The service meter had not been installed.

All cathode ray oscillograms registered voltages to ground. Voltages between windings are obtained by subtraction, which were checked by direct measurements with sphere gaps. Resistance and capacity dividers were used to reduce the potentials to proper values for the oscillograph deflecting plates.

The tests were made mostly with 3-kv. pellet arresters which have a breakdown potential of about 17 kv. and an *IR* drop of 6 to 7 kv. according to laboratory tests. Some of the tests were made with higher rated pellet arresters.

PROTECTION TESTS

Only results with impulses on both primary conductors will be given as this is the most representative condition of service and the conclusions drawn from the work apply equally well whether the impulse is on one conductor or both. Although nearly 1,000 oscillograms were taken, in the limited space available only a fractional part of the data can be shown.

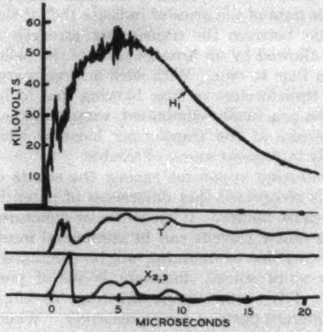

FIG. 3—STANDARD CONNECTION. IMPULSE APPLIED TO PRIMARY. ARRESTER GROUND RESISTANCE 60 OHMS

All oscillograms have the same calibration

IMPULSE APPLIED TO THE PRIMARY

Oscillograms given in Fig. 2 show the potential with respect to ground of the primary H_1 tank T and secondary neutral $X_{2,3}$ with the 3-kv. primary arresters connected to a 10-ohm ground in the usual manner. The secondary neutral potential remains close to ground while the tank takes an intermediate potential, but close to the secondary. As a result most of the potential allowed by the primary arrester and its ground is impressed from the primary to tank and secondary.

Increasing the arrester ground resistance to 60 ohms increases the primary potential to tank and secondary and also increases the tank potential, as shown in Fig. 3. It is clear that further increase in arrester ground resistance is likely to result in either flashover of the primary bushings or a winding failure. Flashover of both primary bushings, or cascade flashover of a primary and secondary bushing if the primary is grounded is likely to result in blown fuses.

Interconnecting the arrester ground and secondary neutral greatly reduces the potentials between windings and from winding to tank, as shown in Fig. 4. With this connection the oscillograms show that the tank and secondary have practically the same potential, while the primary to secondary potential is only the arrester potential, although the arrester ground resistance was 60 ohms.

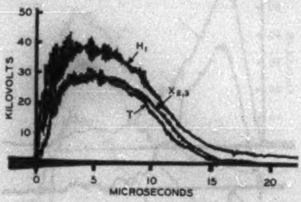

Fig. 4—Interconnection. Impulse Applied to Primary. Arrester Ground Resistance 60 Ohms

In Fig. 5 the oscillograms of Figs. 3 and 4 are subtracted to show the great reduction in potential between primary and secondary when using the interconnection. Fig. 6 shows that there is very little difference of potential between the different windings even with the arrester ground disconnected. Thus, regardless of the arrester ground resistance the transformer is well protected and fuse blowing will be reduced to a minimum by eliminating bushing arcover.

Impulse Applied to Secondary

In making this test the line conductors from the impulse generator were connected to the two secondary wires X_1 and X_4, each of the primary leads of the transformer being connected to ground through a 500-ohm surge impedance. As in the previous protection tests the entire secondary distribution was connected but with no lights on at the hall. The standard arrester with a ground resistance of 19 ohms shows a high voltage between primary and secondary, Fig. 7, which is greatly reduced by the interconnection even though the arrester ground was disconnected, Fig. 8, just as in the case of impulse on the primary. Thus the primary arrester with interconnection will keep the windings at potentials differing only by the arrester potential, whether the impulse originates on primary or secondary. In the case of extremely steep waves originating on the secondary, secondary protection between conductors and neutral may be indicated.

Grounded Tank

Tests made using 6-kv. pellet arresters with and without the tank connected to the arrester ground, which had a resistance of 10 ohms, are shown in Fig. 9. The oscillograms are replotted as they were originally taken with different time scales. The impulse was applied to both primary conductors, the secondary conductors still being connected to the secondary distribution system. The results show that the connection of the tank to the arrester ground lowers the potential between the primary and tank to the arrester voltage, but raises the potential between the tank and secondary to a value equal to the *IR* drop in the ground. Such connection does not lower the voltage between primary and secondary, and if the ground resistance is high, excess potentials will appear between the tank and secondary.

Measurements at the Hall

Oscillograms were taken at the hall during the protection tests and the maximum voltage was found to be 3.9 kv. At this potential sparking took place within fixtures which effectively limited the voltage. With the standard arrester connection on pole No. 48 and an impulse of 123 kv. applied to the primary, the potential between conductors at the hall was 400 and 880 volts when the arrester ground resistance was 10 and 60 ohms respectively. When a gap between primary and secondary set for about 50 kv. r.m.s. sparked, the hall wiring arced over at 3.9 kv. When the interconnection between primary arrester ground and secondary neutral was made the wiring arced over even with arrester ground resistances as low as 10 ohms.

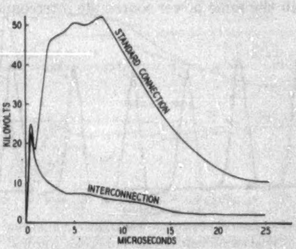

Fig. 5—Standard and Interconnection Compared

Voltages from primary to secondary neutral. 60-ohm arrester ground

With the interconnection, the impulse being applied to the secondary conductors, and the arrester ground disconnected at pole No. 48, the wiring arced over and a potential of 15 kv. was measured to a separate ground at the hall. With a 100-watt lamp connected across the secondary, the potential between wires was reduced to 1.3 kv.

POWER TESTS: ISOLATED-NEUTRAL POWER SUPPLY

Two transformers rated 1½ kva. which had blown fuses quite consistently during lightning storms were placed on pole No. 48 fused with 2 ampere fuses. A 25-ampere non-inductive load was connected to X_1 and X_4. The primary voltage was 4,600 non-grounded, which was obtained from a rural 6,900 single-phase line through the use of two 50-kva. transformers rated 6,900/2,300 and 2,300/4,600, respectively.

Transformer No. 1 which had low oil level sparked over the terminal board with power follow which blew the 2-ampere fuse. With 6-kv. pellet arresters connected and ground resistances up to 170 ohms the terminal board did not flash and of course the fuses did not blow. The oil level was raised, arrester disconnected, and ten impulses applied, which arced to the tank over both the primary and secondary leads. No power follow took place, which was to be expected since the power source was not grounded.

The second transformer did not arc over even with arrester ground resistances as high as 170 ohms. The arresters were then removed and both high-voltage bushings arced over to the tank. Although ten impulses were given, some with and some without secondary 28 ampere load, the primary 2 ampere fuses did not blow. A more intense discharge might have started the power follow. When these tests were made the time was so limited that it was impossible to vary conditions until power follow did take place.

During the tests with the primary ungrounded oscillographic measurements at the hall showed no increase in power voltage either between wires or to ground. Likewise with the same power source the interconnection of arrester ground and secondary neutral did not give any power voltage at the hall during the period of arrester operation.

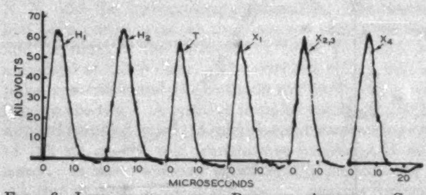

FIG. 6—INTERCONNECTION. PRIMARY ARRESTER GROUND DISCONNECTED. ALL VOLTAGES SHOWN

GROUNDED PRIMARY

The 10-kva. transformer on pole No. 48 was connected to the secondary distribution, its primary receiving power over the experimental line at 2,300 volts from one of the 50-kva. transformers connected to the 6,900-volt rural line. For the tests to be described one conductor was grounded through a 35-ohm ground at the 50-kva. transformer, to simulate one leg of a 2,300/4,000-volt grounded neutral system.

FAULT BETWEEN PRIMARY AND SECONDARY

A ¾-inch gap was connected between the primary phase wire and the secondary neutral. Four impulses out of five blew the 2-ampere fuse on power follow, a voltage of between 600 and 750 volts effective being measured between the conductors and a separate ground for times up to about one-half second until the fuse cleared the circuit. With a lower resistance ground at the power source the voltage measured at the hall would have been higher but the duration shorter.

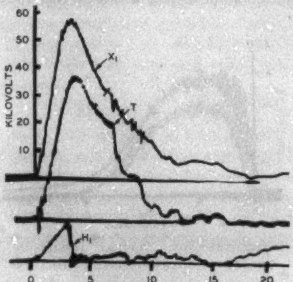

FIG. 7—STANDARD CONNECTION. IMPULSE APPLIED TO SECONDARY. ARRESTER GROUND RESISTANCE 19 OHMS

All oscillograms have the same calibration

TESTS WITH INTERCONNECTION—VOLTAGES AT THE HALL

With the 3-kv. arrester disconnected from ground at pole No. 48, but with the interconnection, a voltage of 101 volts r.m.s. was measured for a fraction of one cycle. The secondary neutral ground was 25 ohms and the primary ground at the 50-kva. transformer, 35 ohms.

When the primary arresters were connected to a 10-ohm ground at pole No. 48 the potential of the conductors at the hall to a separate ground was 33 volts r.m.s. for a fraction of a cycle.

Another test was made in which the sheath of the *BX* conduit was connected to a separate ground from that of the grounded conductor within the conduit. With interconnection, and an arrester ground on pole No. 48 of 10 ohms, and secondary ground of 25 ohms, no 60-cycle potential could be measured to a separate ground, although the sheath ground was of the order of 1,000 ohms. Between the sheath and the grounded conductor a potential of 33 volts would undoubtedly have been measured for this condition.

CONCLUSIONS

1. With the present connection primary arrester ground resistances play an all important role in the protection afforded by the arresters, and a ground resistance which may not be too high for one discharge may be totally inadequate for a higher current discharge.

Since low ground resistance is expensive and frequently cannot be obtained, efficient protection requires the use of some method to eliminate the variable effect of ground resistance.

2. Impulses originating on the secondary network cause the tank potential to rise above the primary and a primary failure may result—although the entrance was secondary. Thus some failures charged to primary entrance may have been due to secondary entrance.

3. Grounding the tank to the arrester ground is not particularly helpful as it reduces the primary to tank stress to the voltage allowed by the arrester, but increases the secondary stress by the *IR* drop in the ground.

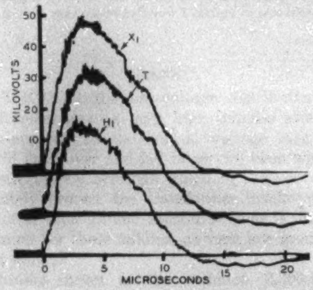

Fig. 8—Interconnection. Impulse Applied to Secondary. Arrester Ground Resistance Infinity

All oscillograms have the same calibration

4. The interconnection of primary arrester ground and secondary neutral will reduce the potential during lightning discharges to the voltage allowed by the arrester, whether impulses come from the secondary or primary circuits. Transformers in reasonably good condition should not fail due to lightning, nor should fuses blow except in case of severe direct stroke very close to the transformer location. Thus accidental connection between primary power supply and customer's premises should be practically eliminated.

5. The potential allowed on the transformer by the arrester with interconnection is independent of the arrester ground resistance so that the transformer is protected whether the arrester ground resistance is high or low.

6. Nothing is to be gained with the interconnection by connecting the transformer tank to the secondary neutral, since with the tank floating, the potentials developed to tank are much less than the strength of insulation involved and the hazard to linemen is decreased.

7. Interconnection of primary arrester ground and secondary neutral allows impulse current to flow into the secondary network, the amount depending on the arrester ground at the transformer. The tests showed that even a 10-ohm ground did not prevent flashing some of the small gaps which exist in the secondary house wiring. Lightning discharges which originate on the secondary do the same thing and interconnection increases the number of such discharges by the amount of exposure added. If a good arrester ground is used the severity of discharge in the secondary circuits will be much reduced. It is not believed that the increases in the number of discharges represent any increased hazard where the secondary neutral is connected to water pipe grounds, as in cities, even if the arrester is not grounded at the transformer. In rural distribution interconnection may represent some increase in hazard if several different grounds exist within consumers premises so that potential differences may be experienced between different grounded objects and the secondary circuits.

8. From the standpoint of hazard due to power voltage, the present situation is that in many cases of fuse blowing which may or may not have resulted from the transformer failure, power voltages of considerable magnitude appear between the secondary ground and separate grounds on the customer's premises until the fuse blows which may take the time of many cycles, depending on the magnitude of the fault current. With good grounds on the secondary this time will be short and the voltage low, with poor grounds the time is longer and the voltage higher.

9. With the interconnection it is believed that contacts between primary and secondary caused by lightning will be largely avoided thus materially reducing any hazard which may have existed before in those cases where the secondary ground resistance was high. In such cases the arrester should probably be connected to a ground at the point of interconnection into the secondary neutral.

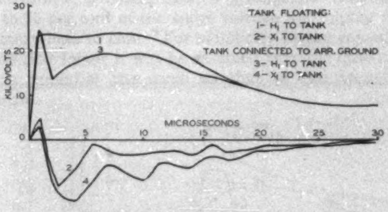

Fig. 9—Standard Connection

Potentials between primary and secondary to tank. Tank floating and tank connected to arrester ground. 6-kv. arrester, 10-ohm ground

10. The tests indicate that the record of arrester protection to distribution transformers, including fuse blowing, can be greatly improved by the use of interconnection, providing proper precautions are taken for those cases where the secondary neutral ground is not a water pipe ground. The tests show that arrester

grounds are not necessary where the secondary ground resistance is low as with waterpipe grounds.

11. The results of these investigations should be brought to the attention of those responsible for the various codes so that the benefits of interconnection may be fully realized.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to Mr. E. J. Wade who conducted the test with the assistance of Messrs. D. D. MacCarthy and M. F. Beavers of the General Electric Company and E. H. Emerson of the Power Company. The authors also wish to thank Messrs. P. L. Chambers and C. B. Signor of the New York State Electric and Gas Corporation for their cooperation in placing the line and the equipment at the disposal of the investigators.

Discussion

For discussion of this paper see page 271.

Lightning Protection for Distribution Transformers

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and
R. N. SOUTHGATE† Associate, A.I.E.E.

***Synopsis.**—Usual methods of connecting arresters for the protection of distribution transformers are often inadequate. In service some transformers still flash over although the arrester, in itself, is capable of protecting the transformer with a large margin of safety.*

Surge current flowing to ground through the ground lead of the arrester gives rise to inductive drop and resistance drop voltages. These voltages added to that permitted by the arrester can be great enough to flash over the transformer bushings to secondary neutral.

By interconnecting the ground lead of the arrester and the transformer secondary neutral, the voltage across the transformer insulation is limited to that permitted by the arrester alone.

Tests have been made to determine the conditions of surge required to duplicate flashovers that occur in practise. Under such conditions measurements were made of the voltages appearing on a representative secondary circuit when bushing flashovers are permitted, and when flashover is prevented by interconnecting the arrester ground and the transformer secondary neutral.

GENERAL

ALTHOUGH the low-voltage distribution transformers have a very high impulse strength as compared to the normal voltage rating, they will still flash over and fail in service even when they are protected by arresters which in themselves could adequately protect the transformer insulation. The effect of arrester ground resistance alone is insufficient to account for these failures, as very low grounds still permit failures.

Lightning surges in the common 2,300-volt, or higher, urban distribution circuits manifest themselves in a somewhat different manner from those on high-voltage transmission lines. In the case of a distribution circuit, the arrester spacing will be usually of such order that the distance traveled by the voltage wave will be short, the crest voltage appearing on the line will be limited, and the surge will manifest itself as current through one or more arresters. The current wave, both as to magnitude and shape, is determined very largely by the lightning stroke, and very little by the characteristics of the distribution circuits.

On rural lines the exposure between arresters is much greater and so in addition to surges originating near the arrester there will be surges arriving over a length of line and these voltage surges will be limited by the insulation of that line.

NATURE OF THE CIRCUIT CONNECTIONS COMMONLY USED

The protection commonly used for a single-phase house lighting transformer provides for phase and neutral arresters on the primary side connected to a common ground lead down the pole. These arresters may be identical or the neutral arrester may be a simple gap. The ground lead is connected to a ground rod that may have a resistance to earth of five to several hundred ohms.

The case of low-voltage class distribution transformers is left free, for the safety of the workman who may touch it while working on the live primary conductors.

The secondary neutral is grounded at each customers service entrance. This ground is made to the water service pipe as it is as good a ground as can be obtained. In addition to the customers ground, there is usually a driven ground at a distance of one span from the transformer.

THE EFFECT OF ARRESTER GROUND CIRCUIT ON THE PROTECTION AFFORDED

Tests show that when a surge is applied to the primary of a distribution transformer that has a secondary ground, the isolated case will take on a low electrostatic potential above ground so that some 80 to 95 per cent of the surge voltage will appear from primary lead to case. For simplicity in the considerations that follow it will be assumed that the case takes on a potential the same as that of the secondary neutral.

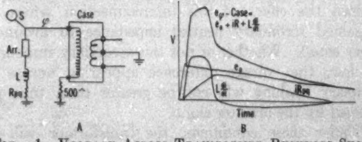

FIG. 1—VOLTAGE ACROSS TRANSFORMER BUSHINGS SHOWN AS THE SUM OF THE ARRESTER VOLTAGE, THE GROUND LEAD INDUCTIVE DROP, AND THE GROUND *IR* DROP

The voltage across the transformer, as shown in Fig. 1, is the total of three components; the *IR* drop across the ground, the inductive drop across the ground lead, and the voltage across the arrester. The drop across the ground is proportional to the amount of current in the surge, and the drop across the lead to the rate of rise of current. For anticipated surge currents, the voltage across the ground alone can exceed the voltage across the arrester, unless the ground resistance is of very low magnitude.

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*Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

†Public Service Electric & Gas Co., Newark, N. J.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Under present methods of construction, the voltages appearing across the primary bushings are often great enough to cause flashovers. The completed path for the surge current to ground is usually over the secondary bushings. Fig. 2 shows the sequence of events during a flashover and the voltages which occur. Primary bushing flashover takes place at a time t_1, and secondary bushing flashover almost simultaneously at a time t_2. With power on the transformer, the flashover path will carry the power arc until the primary fuse opens the circuit. In service this is the path over which the arc burns are usually found.

Under these conditions both the surge current and the power follow current flow through the secondary neutral ground but operating experience seems to show that there is no serious trouble on the customer's property.

INTERCONNECTION OF THE PRIMARY ARRESTER GROUND AND SECONDARY NEUTRAL

By interconnecting the arrester ground and the transformer secondary neutral at the transformer, the potential difference across the primary bushing of the transformer will be limited to that voltage permitted by the arrester alone during a discharge, in accordance with the prior assumption that the secondary neutral and case remain at the same potential. In Fig. 3 is shown the effect of this interconnection where the assumed secondary neutral impedance to ground is very small. Whether or not this secondary impedance is low, the voltage difference appearing across the primary bushing will not be greater than that permitted by the arrester alone.

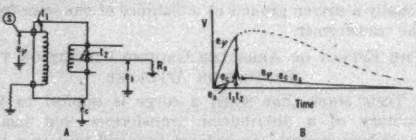

FIG. 2—SUCCESSIVE FLASHOVERS OF PRIMARY AND SECONDARY BUSHINGS, AND RESULTANT POTENTIALS

Under these conditions, the transformer will not flash over and there will be no power current flowing into the secondary neutral from the primary circuit. Whatever impedance drop there is for the surge current flowing in the secondary neutral will appear as voltage in this circuit.

If a solid connection is not permissible between the secondary neutral and the ground terminal of the primary arrester, a gap or valve arrester of a low voltage rating may be placed in the interconnection as shown in Fig. 4A. This arrester will begin taking current when the voltage across the ground lead becomes great enough to discharge the arrester. In this case, the voltage across the primary bushing is the sum of the voltages appearing across the primary and interconnecting arresters as shown in Fig. 4B. This connection will effectively isolate the arrester ground and the secondary neutral as there is no voltage difference between them except during a surge discharge.

A second method of getting improved protection afforded by the interconnection without connecting a solid conductor between the arrester ground and the secondary neutral, is that shown in Fig. 5. Where the effective leakage reactance between the two halves of the secondary winding is small, the surge current in flowing as indicated will result in a very low voltage across the winding as the magnetic effects of the two currents are in opposition. This circuit was proposed by Mr. C. B. Wright of the Duquesne Light Company and very promising laboratory tests were run in January 1931.

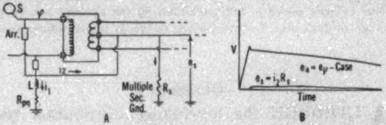

FIG. 3—VOLTAGE ACROSS TRANSFORMER BUSHINGS WITH CONNECTION OF ARRESTER GROUND LEAD TO SECONDARY NEUTRAL

POLICY OF INTERCONNECTING PRIMARY ARRESTER AND SECONDARY NEUTRAL GROUND

At the present time the safety code forbids the interconnection of the primary arrester and the secondary neutral ground. This regulation was made in recognition of the fact that when a surge current flows through a ground a surge voltage appears across it. The driven grounds which were assumed in this case are usually of considerable resistance as compared to the water-pipe grounds which are customary at the present time.

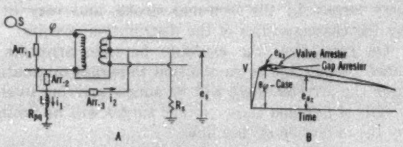

FIG. 4—VOLTAGE ACROSS TRANSFORMER BUSHINGS WITH CONNECTION OF PRIMARY ARRESTER GROUND LEAD TO SECONDARY NEUTRAL THROUGH LOW-VOLTAGE ARRESTER

THE LIMITATIONS OF THE POLICY OF INTERCONNECTING GROUNDS

Where both the primary ground and the secondary ground are driven grounds as in the case of a rural customer, the interconnection can prevent flashover of the transformer insulation but an appreciable surge voltage may appear from secondary to ground. This condition is less serious than permitting a power fault into the secondary circuit, but both grounds should be made as low as possible so that the *IR* drop may be low.

The ungrounded secondary is not the common practise in distribution circuits. Where it is used, a surge voltage on the transformer primary will give the secondary circuit a potential above ground. The smaller the capacity of the secondary to ground, the greater will be its potential. Obviously an interconnection of a poor primary ground with the free secondary will not appreciably lower the voltage across the arrester ground and flashovers from secondary to ground would be expected.

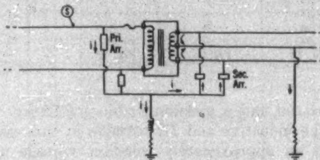

Fig. 5—Path of Current Flow to Ground for Surge Originating on Primary Phase Wire

Tests Made on a Representative Distribution Transformer with Secondary Services

In order to determine the magnitude of the effects required to cause flashovers similar to those which have occurred in service, a program of tests was carried through. A diagram of the test circuit is shown in Fig. 6. A distribution transformer was mounted on a 35-ft. pole. A ground lead was provided down the pole and two ground rods were driven at the foot of the pole in order to get as low a resistance as possible. The secondary wires were racked for about 140 feet. From the transformer pole and from the end of the secondary rack, triplex services were run to a point on the ground. Here the neutral wires were solidly grounded. One wire of each triplex service was left open and is indicated on the figure as "open service." One service wire and the neutral from each service were connected to a 100-foot section of two-conductor *BX* cable, both ends of the sheath being grounded. The two sections of *BX* cable were connected to the same two secondary wires.

In the tests as made, flashovers were necessary in order to simulate the conditions that occur in practise. In order to secure consistent flashovers, gaps were placed around the primary and secondary bushings as shown in Fig. 6.

A 500 ohm resistor was placed between the primary neutral and ground to simulate the effect of an uncharged conductor.

The arresters used in these tests were 3,000-volt auto-valve arresters of the disk and mica spacer type. This type of arrester was selected rather than the porous block type because a larger number is in service and the tests were laid out to duplicate service experience as closely as possible. This type of arrester on test had a maximum voltage of 15 kv. crest across its terminals during a surge discharge of 800 amperes.

Most of the oscillograms have been replotted to the same scale in order to show the relative voltage-time values more readily. Successive test surges were found to be very nearly identical. In Table I the crest voltages measured at certain points are tabulated for the various circuit connections and for each of the two types of impulse wave used.

Voltages at the transformer were measured by means of a cathode ray oscillograph connected to a 1,070-ohm resistance potentiometer through a delay cable. Volt-

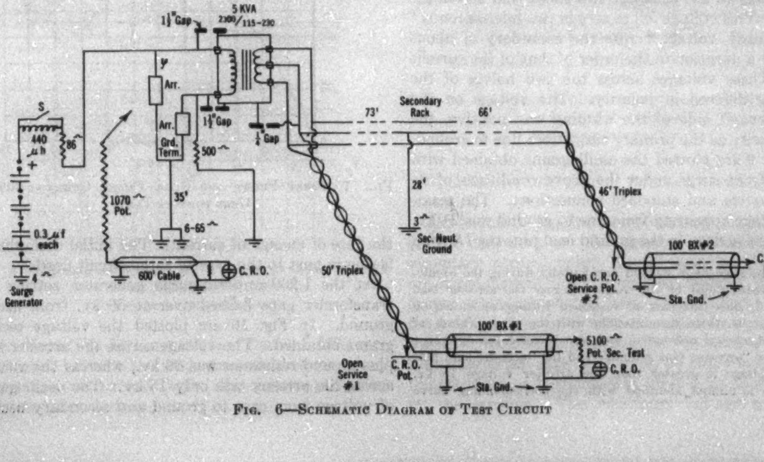

Fig. 6—Schematic Diagram of Test Circuit

TABLE I—CREST VOLTAGE MEASUREMENTS

Figure 7	Steep front surge				Slow front surge		
Column 1	2	3	4	5	6	7	8
	Standard connection						
Point of voltage measurement*	Gaps not flashing	Gaps flashing	Arrester ground and sec. neutral interconnected	Arrester ground through secondary neutral only	Standard connection gaps not flashing	Arrester grounded through secondary neutral only	Arrester grounded through secondary arresters only Fig. 9A
Arrester plus ground	+ 41 kv.	+ 50 kv.	+ 41 kv.	+ 62 kv.	+ 21 kv.	+ 21 kv.	+ 21 kv.
Arrester ground lead	+ 28	+ 36			+ 12		
Trans. sec. neutral		+ 20	+ 28	+ 43			
Open service No. 1	+ 3	+ 25	+ 20	+ 35	+ 3		
BX No. 1	− 3	+ 12	+ 16	+ 23	− 4	+ 8	+ 14
Open service No. 2	+ 3	+ 33	+ 34	+ 56	+ 4		
BX No. 2	− 3	+ 24	+ 18	+ 35	− 5	+ 10	+ 19
Oscillograms plotted	Fig. 8	Fig. 10	Fig. 11	Fig. 12	Fig. 9	Fig. 13	—

*All voltage measurements were made by resistance potentiometers

ages at the secondary service terminals were measured by a 5,100-ohm resistance potentiometer. These potentiometers of course have some effect on the surge voltages.

The Effect of the Ground Lead on Protection

To show the effect of inductance of the arrester ground lead on the voltage appearing across the primary bushing, two rates of current rise were selected. The slow rate of current rise was produced by introducing the 440-microhenry inductance into the circuit of the surge generator in Fig. 6. The steep front surge rose to a maximum of 1,200 amperes in two microseconds while the slow front surge rose to 900 amperes in nine microseconds. These are plotted in Fig. 7.

With the surge generator set to give a current wave of fast front and slightly less than 1,200 amperes crest, in order to prevent flashover of the transformer gaps, the oscillograms in Fig. 8 were taken. The maximum voltage across the ground lead and ground of the arrester was 28 kv. or about two times that across the arrester. This voltage fell to zero in two microseconds.*

The steady voltage across the secondary of about 3 kv. had a duration in the order of that of the current surge. These voltages across the two halves of the secondary differed in polarity. The voltage on the "open service" side of the winding was positive, the same polarity as the primary surge from line to ground.

In Fig. 9 are plotted the oscillograms obtained with the slow front surge under the above conditions of no gap flashovers and standard connections. The maximum voltage appearing from line to ground was 21 kv. The inductive drop in the ground lead plus the *IR* drop in the ground had a maximum value of 12 kv. The sum of the inductive and *IR* voltages in this case resulted in an approximately constant voltage up to maximum current. The secondary voltages were about 5 kv. and of long duration.

The two tests recorded in Figs. 8 and 9 indicate that after an initial disturbance determined by the surge impedance of the ground lead, the voltage across the ground lead is determined by the lead inductance and the rate of change of current. The initial disburbance is due in part to the type of surge circuit used.

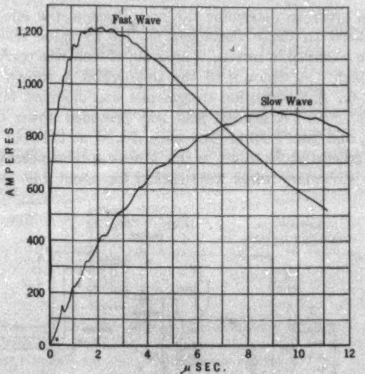

Fig. 7—Steep Front and Slow Front Current Surges Used for the Tests

At the 1,200-amperes surge generator setting the transformer gaps flashed over at 50 kv. from line to ground. In Fig. 10 are plotted the voltage oscillograms obtained. The voltage across the arrester lead plus ground resistance was 36 kv., whereas the voltage across the arrester was only 15 kv. The oscillograms of voltage from case to ground and secondary neutral to ground were almost identical so only one is reproduced. The voltage across the open service No. 2 was 33 kv. as compared to 24 kv. across *BX* No. 2, showing the reductive effect of the *BX* cable on the surge.

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*The gaps may have flashed accidentally during the oscillographic measurement of the voltage across the arrester lead plus ground, since the lack of sustained voltage as shown on the oscillogram seems incompatible with the given values of current and ground resistance, and also because the difference between the "arrester plus ground" and the "arrester lead plus ground" voltage is greater than the arrester voltage 15 kv. This curve is almost identical with the corresponding curve in Fig. 10.

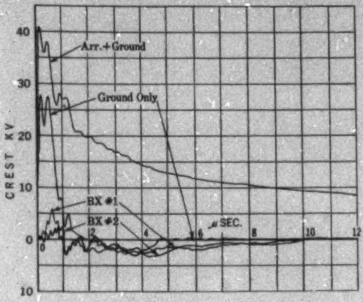

Fig. 8—Voltages Measured in Circuit of Fig. 6 With Steep Front Surge of Reduced Magnitude
No transformer gap flashovers

Interconnection of Arrester Ground and Secondary Neutral

In Fig. 11 are plotted the corresponding voltages when the arrester ground terminal and secondary neutral are interconnected. No gap flashovers took place. From Fig. 11 it can be seen that the difference in potential between the phase lead and the secondary neutral does not exceed 15 kv., the voltage permitted by the arrester alone.

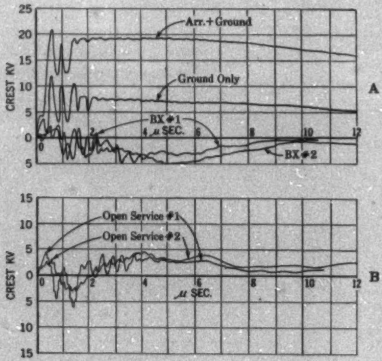

Fig. 9—Voltage Measured in Circuit of Fig. 6 with Slow Front Surge
No transformer gap flashovers

A comparison of columns 3 and 4 of Table I indicates that the crest secondary voltages are about the same with the interconnection as they are during a flashover of the transformer gaps.

Interconnection and a High Arrester Ground Resistance

The extreme case of high ground resistance is that of having no arrester ground at all. Obviously the arrester cannot discharge and any surge above 50 kv. crest would flash over the transformer gaps. Protection under such conditions can be obtained by connecting the arrester ground terminal to the secondary neutral.

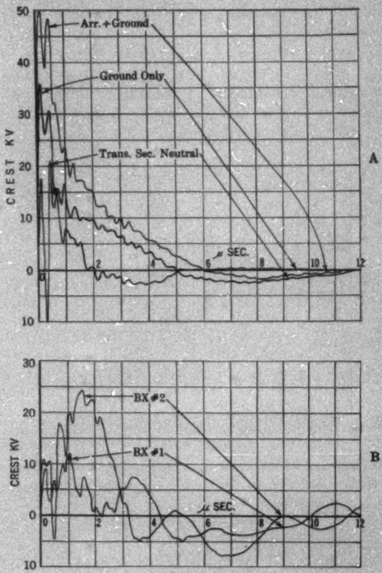

Fig. 10—Voltages Measured in Circuit of Fig. 6 with Steep Front Surge
Transformer gaps flashing over

The circuit of Fig. 6 was modified by connecting the ground terminals of the primary arresters to the secondary neutral only. Under such conditions the full surge current flows to ground through the transformer secondary neutral wire. Figs. 12 and 13 show the voltages measured with the steep front surge and the slow front surge respectively. In neither case did the transformer gaps flash over, as the voltage across the transformer bushings was limited to about 15 kv. by the arrester. The maximum voltage to ground with the fast surge was 60 kv. which was considerably above the gap flashover.

Indirect Interconnection of Secondary Neutral and Arrester Ground

The circuit shown in Fig. 5 was tested using the slow front surge but the separate arrester ground was omitted so that the only path for the arrester surge current was through the secondary arresters, secondary windings, and secondary neutral to ground. It is evident from a comparison of columns 7 and 8 of Table I that the leakage reactance between the two secondary windings of this transformer was high enough to increase the voltage appearing across the service wires.

This same circuit was tested with the steep front surge. The voltage was great enough to cause flashover of the *BX* cable so no measurement was made. No flashover of the transformer gaps occurred however.

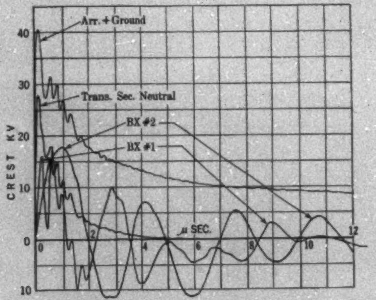

Fig. 11—Voltages Measured with Steep Front Surge in Circuit of Fig. 6 with Arrester Ground and Secondary Neutral Interconnected

No transformer gap flashovers

Surge Voltages Appearing on the Secondary

A surge voltage impressed across the primary of a distribution transformer will induce a voltage across the secondary. This voltage has a duration about the same as that of the primary voltage. A rapidly changing surge current passing to ground over the secondary neutral results in an oscillatory voltage appearing between the outside wires and ground at the service terminals. The maximum voltage is about the same as that of the transformer neutral to ground at the transformer. However, the voltage on the *BX* cable is lower than that on the corresponding open service. The voltage on the more distant service, No. 2, is greater than that on service No. 1.

The voltage appearing at the services when the arrester ground and secondary neutral are interconnected is about the same as that appearing when a steep front surge causes the transformer gaps to flash over due to inductive drop in the arrester ground lead.

The secondary voltages with the arrester ground and secondary neutral interconnected, though of short duration, appear to have a rather high magnitude. While the secondary voltages with this circuit may be compared with the secondary voltages on the other circuits tested, it should be noted that the length of racked secondary was short compared to lengths usually occurring in practise, and that actual voltages will probably be lower. Voltages of the relative magnitude tabulated in column 3 of Table I appear now without serious effect, in the case of transformer flashovers where the ground resistance is low.

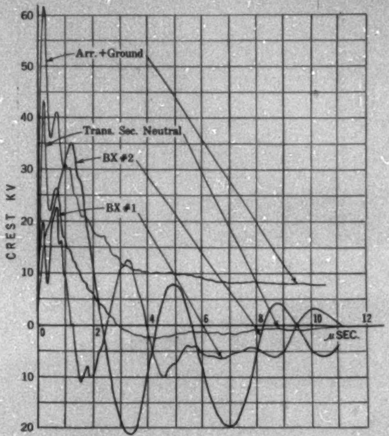

Fig. 12—Voltages Measured with Steep Front Surge in Circuit of Fig. 6 with Arrester Ground Removed and Arrester Connected to Secondary Neutral Only

No transformer gap flashovers

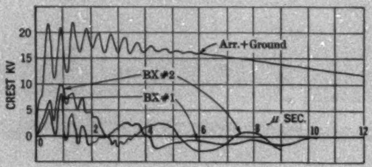

Fig. 13—Voltages Measured with Slow Front Surge in Circuit of Fig. 6 with Arrester Ground Removed and Arrester Connected to Secondary Neutral Only

No transformer gap flashovers

Conclusions

Under present methods of distribution transformer protection, flashovers of bushings or windings due to lightning often occur, causing dynamic current flow into the secondary circuit with resultant transformer outages. Tests show that, when a transformer is subjected to a lightning surge, the case will assume a potential of about 10 per cent of that appearing between the primary leads and secondary neutral. Neglecting the potential on the case, the voltage across the transformer bushings is the sum of the voltages across the arrester, the arrester ground lead and the arrester ground resistance. With a steep front surge, the inductive voltage across the ground lead alone may exceed the arrester voltage. Remedial steps which can be taken are limited to a reduction of the arrester ground resistance and of the inductive drop across the ground lead. Practical application of these measures is limited by available field conditions.

Improved protection of the transformer may be secured by any one of several methods of connecting the arrester ground lead to the secondary circuit. The following methods will limit the surge voltage across the bushings to approximately the characteristic crest voltage of the primary arrester, but will differ in resulting secondary voltages.

1. Direct connection of a low resistance primary arrester ground to a grounded secondary neutral produces no higher voltages at the customer's service than are experienced when flashover occurs with the present protection schemes.

Direct connection of a very high resistance primary arrester ground to a grounded secondary neutral results in higher surge voltages appearing on the customer's leads than when the ground resistance is low.

2. Connection of the primary arrester ground to the secondary neutral through a low-voltage arrester gives substantially the same results as with the direct connection.

3. Connection of the primary arrester ground to the secondary outside wires through two low voltage arresters, results in increased customer voltage.

The authors wish to express their appreciation to E. R. Whitehead for his valuable assistance with these tests.

Discussion

For discussion of this paper see page 271.

Studies in Lightning Protection on 4,000-Volt Circuits—III

BY D. W. ROPER*

Fellow, A.I.E.E.

Synopsis.—*The advancement in knowledge of lightning protection, as developed by the investigations in Chicago since the 1920 paper on the same subject, has been largely due to an intensive study in the past five years made with the active cooperation and assistance of the manufacturers of lightning arresters. The development in recent years of methods of laboratory tests with the lightning generator and the cathode ray oscillograph render unnecessary a continuance of these statistical research investigations.*

The examination of 474 transformers burned out by lightning indicates the desirability of line transformers with non-aging insulation which will be lightning-proof against induced discharges when protected by the modern types of arresters. A comparison of the troubles on the distribution system due to lightning in Chicago with similar results in other cities indicates that considerable advantage is obtained by the use of the four-wire three-phase system with grounded neutral. Induced discharges of 400 kv. crest value have been noted. Direct strokes account for only 5 per cent of transformer troubles. No evidence could be found of shielding effect against induced discharges due to adjacent high trees, buildings, or other structures. Traveling waves are not a factor in the transformer troubles on the distribution system in Chicago. Modern types of lightning arresters excel the best type of 20 years ago in reduced annual charges, but not in efficiency as a protective device. Extra expense to reduce resistance of lightning arrester grounds below 50 ohms is not warranted.

By following the principles developed during these investigations and using transformers of improved design, it appears possible to effect a reduction in Chicago in the rate of transformer troubles due to lightning from 13 cases to 1 case per 1,000 transformers per year.

* * * * *

INTRODUCTION

THIS paper presents the later results of a statistical research investigation that has been in progress for over 20 years. (Fig. 1.)

About the time of the presentation of the 1920 paper[1] on this subject, the manufacturers of lightning arresters were consulted regarding the advisability of discontinuing the investigations on account of the expense. They were opposed to any such plan, as the statistical method described in that paper was the best method then available of securing information regarding the relative protective value of the different types of arresters. It was, therefore, decided to continue the studies, and it was also arranged to install in moderate quantities, for trial purposes, several new types of arresters then being developed.

The lightning machine was developed about 1921 as a device for testing arresters, but the spark gap was the only measuring instrument then available for making comparative observations. About 1924 the cathode ray oscillograph and the necessary accessories were developed in form suitable for use as a measuring instrument[2] on tests made with the lightning machine. In 1926 it was noted that the results of such tests, as then made and interpreted, placed the several types of arresters then on the market in a different order of merit from the order indicated by the field investigations in Chicago. This resulted in a 5-day conference with the manufacturers in January, 1927, to discuss these differences and devise plans for further procedure. As it appeared at this conference that the discussions would be greatly facilitated if the results obtained on all types of arresters could be exposed in the conference to all of the manufacturers, their representatives agreed to this procedure and further agreed that they would not use the information to the disadvantage of any competitor. This agreement was the greatest forward step taken in connection with these investigations. Most of the increased knowledge obtained by the investigations in Chicago since the 1920 paper has resulted from the intensive study begun during this conference and continued in the five succeeding years.

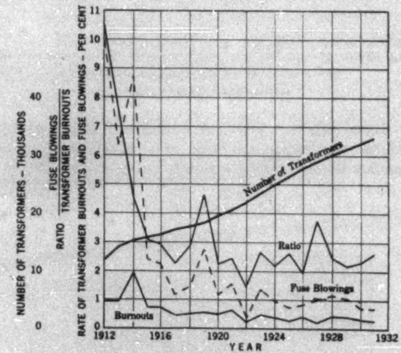

FIG. 1—RECORDS ON TRANSFORMERS AND LIGHTNING TROUBLES FOR 20-YEAR PERIODS

Even though the number of transformers doubled between 1915 and 1931, the actual number of transformer burnouts and fuse blowings remained fairly constant during the period at 105 and 275 per year respectively

About 25 factors which might affect lightning arrester performance or transformer troubles caused by lightning were listed. An investigation, made for each of 119 locations where transformers had failed during the previous year, showed that about one-third of the factors first listed occurred so infrequently that they were of no serious consequence and were quickly dismissed from further investigation.

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*Superintendent, Street Dept., Commonwealth Edison Co., Chicago, Illinois.

1. For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Additional information regarding the factors which appeared important was sent to the manufacturers and discussed in six later conferences. As a result of this procedure, and by utilizing the results of the laboratory tests and field investigations made by the manufacturers during these years, all differences of opinion that were noted in 1926 have been reconciled. The manufacturers have been of invaluable assistance in reaching practically all of the principal conclusions in this paper, and they are now able to make tests with the lightning generator and interpret the results shown by the cathode ray oscillograph, so that the relative merits of various types of lightning arresters can be very closely determined by laboratory tests. The expense of the statistical research investigations in Chicago is, therefore, no longer warranted and it has been materially reduced.

It would be very interesting indeed if the manufacturers could arrange to publish, side by side, the test results and analyses of studies made with the lightning generator and the cathode ray oscillograph on several radically different types of arresters, so as to show how the relative protective value of arresters can be determined.

DESCRIPTION OF SYSTEM

The system of distribution used in Chicago is a four-wire three-phase 2,300/4,000-volt system with the neutral connected to ground through a reactor shunted by a fuse in each of the 57 substations. All of the 606 circuits leave the substations via underground cables, the feeders are entirely underground, and about 20 per cent of the primary mains are also underground. There were 33,093 transformers on August 1, 1931, connected to the overhead mains and in addition 1,593 were installed in manholes or vaults and connected directly to underground primary mains. Over 800,000 customers are served by this system of distribution.

All line transformers are single-phase, with 2,080-volt primary and 115/230-volt secondaries. Lighting transformers are connected between one primary phase wire and the primary neutral; the neutral wire of the secondary is usually grounded on the pole next adjacent to the transformer and if the secondary main is more than one span long the neutral is grounded at two poles. Transformer cases are not grounded. Small lighting customers have two-wire services, but all large lighting customers have three-wire services, and in either case the neutral is grounded to a water pipe on the customer's premises. Secondary lighting mains are generally only one block long. Installations for three-phase power customers consist of three transformers connected in *Y* on the primary and in delta on the secondary. The common point of the transformer primary connections is *not* connected to the primary neutral; the middle point of one transformer secondary is grounded. For the smaller power customers, two transformers are used, each connected between one phase wire and primary neutral with the secondaries in open delta. About 55 per cent of all transformers are single transformer lighting installations, 15 per cent are in 2-transformer banks supplying both light and power, and 30 per cent are in 3-transformer 3-phase banks supplying power only.

About two years ago the plan was adopted of interconnecting the neutrals of all secondary mains wherever this could be done at reasonable expense, and about 80 per cent of these connections have been completed.

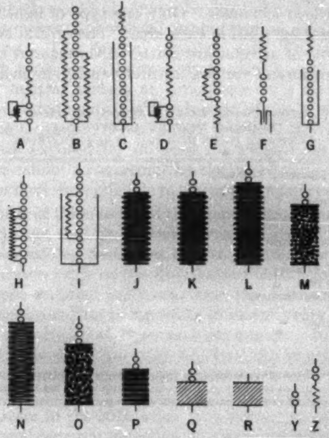

FIG. 2—ELECTRICAL DIAGRAMS OF ARRESTERS USED

The gaps and resistors are conventional. Lower end of diagram is ground side. Types *A* to *R* are 3,000-volt arresters. Types *Y* and *Z* are 300-volt arrestors. In types *C* and *G* narrow grounded bands of metal are carried up the sides, while in type *I*, a grounded metallic cylinder surrounds the lower three-quarters of the arrester. Arresters *J* to *R* are valve types. Arresters are arranged in the order they were installed on the system

On 80 per cent of the circuits, the maximum distance between transformers is less than 1,000 ft. measured along primary mains. At about 370 locations the primary mains are without transformers for distances greater than 1,000 ft.; and at 25 locations, greater than 2,000 ft.

Many of the larger transformers are so located that there is no secondary wire exposed to induced lightning discharges.

The City of Chicago covers about 210 square miles. Eliminating areas in which no overhead lines are installed, the system covers about 120 square miles. The system includes about 135,000 poles, of which about 3,800 are cable poles.

One 3-kv. and one 300-volt arrester (Fig. 2) are installed on the same pole with each lighting transformer, while on 3-transformer installations there are three 3-kv. arresters and one 300-volt arrester, with the further limitation that the maximum number of arresters in any one block is three 3-kv. and one 300-volt arrester. Similar installations of arresters are made on all cable poles and arresters are omitted from transformers within 300 ft. of these poles. There are no lightning arresters or similar devices in any of the substations or on the secondary mains for light or power, and no lightning arresters are installed on transformers connected to underground mains. There are no fuses on the secondary wires outside of customers' premises.

There are now 15 types of 3-kv. arresters on the system, which, for the purpose of this investigation, was divided into 162 areas. Only one type of lightning arrester was installed in each area. The several areas for each type of arrester are scattered throughout the city and in regions of varying arrester density, in an attempt to put all types of arresters on the same basis. Separate performance records have been kept each year for each of these segregated areas.

TABLE I—RESULTS OF EXAMINATION OF TRANSFORMERS BURNED OUT BY LIGHTNING
AVERAGE DATA FOR 1927–1930, INCLUSIVE

Apparent Lightning Entrance	No.	Per cent
Primary only		
Phase wire	133	28.0
Neutral wire	32	6.8
Phase and neutral wire	42	8.9
Total, primary only	207	43.7
Secondary only	100	21.1
Secondary and primary	105	22.2
Doubtful	38	8.0
Indeterminate	24	5.0
Total	474	100.0

Note: In these investigations a transformer is considered to be burned out when it cannot be restored to service without removing it from the pole. About 40 per cent of the transformers were repaired without replacing the coils.

The terrain in Chicago is quite flat as, with one minor exception, the maximum difference in elevation as shown by the Geological Survey maps is less than 50 ft. Satisfactory grounds can usually be secured in most of this area by 10-ft. ground rods except along certain sand ridges where 15-ft. ground rods are used. In a few localities of quite limited area, the rock is within a few feet of the surface, and special methods are used to secure good lightning arrester grounds.

EXAMINATION OF BURNED OUT TRANSFORMERS

The records of the examination of 474 transformers burned out by lightning (Table I) indicate that burnouts which start on the secondary winding (Fig. 3) were nearly one-half of those starting on the primary. This information led to a discussion of the desirability of installing lightning arresters on the secondary mains. Arrangements were made to have this subject investigated at Purdue University, and some of the results of that study are reported in a separate paper.[3]

During the examination of these transformers, considerable information was obtained which confirmed a suggestion in the 1914 paper[4] of the author, reading as follows:

> It is suggested that careful attention by the manufacturers to the primary bushings, the clearance of the primary leads inside the case, the manner of their support, and other minor details, would probably result in a material improvement in these records.

Many bushings, especially in the older transformers, were so small that they were apparently designed to withstand the operating voltage rather than the transient voltages to which they would be subjected when installed on overhead pole lines.

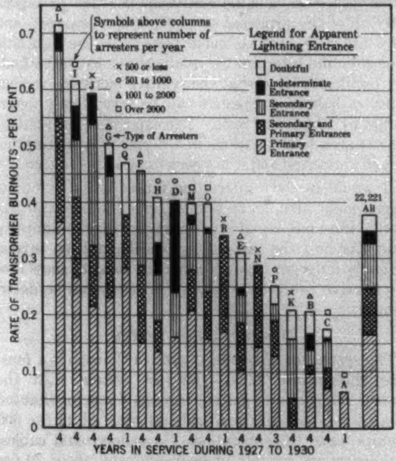

FIG. 3—LIGHTNING ARRESTER PERFORMANCE

The averages are not corrected for size, age, and density of transformers. Probable errors in the individual cases are from 10 to 25 per cent, except for type *A*, for which only one year's performance is shown.

Burnouts are subdivided according to apparent path of entrance of lightning as indicated by examination, and extent of primary and secondary wires is considered

If induced discharges on overhead secondary mains are the cause of transformer burnouts by lightning, it appeared possible that some damage might occur to customers' equipment. A search of the records for three calendar years of the departments that would be involved showed that the total cost of all repairs to customers' wiring, or replacement of lamps or household equipment or devices, averaged less than $100 per year. About 40 failures of watt-hour meters per year appear to be due to lightning.

INDUCED DISCHARGES, DIRECT STROKES, AND TRAVELING WAVES

There is no general accord as to what evidence is considered a conclusive indication of a direct stroke, but if only those cases are included in which there is some shattering of poles or cross-arms, or severe external burning of transformer cases or other equipment on poles, or damage to trees immediately adjacent to the primary wires, then evidence is found of direct strokes on only about 15 poles per year, or about 1 in 9,000. About 5 per cent of the transformer failures due to lightning appear to be caused by direct strokes, and so the other 95 per cent must be due to induced discharges.

Two well defined cases have occurred where, with the assistance of the manufacturers, it was determined that a maximum transient voltage of the order of 400 kv. was indicated. In each instance the maximum dimension across the area, in which signs of damage could be discovered, was less than 1,500 ft.

A large fraction of the transformers on the system are installed at the ends of primary mains remote from the feeding point, and if there are traveling waves on the distribution system, the transformers installed at these open ends should show an increased percentage of failures or fuse blowings due to the piling up of the voltage at such open ends. The records for transformers at such locations show that the percentage of trouble is about 20 per cent below the rate for other transformers on the system. (Table II.) It appears that, as the distance between arresters is generally below 1,000 ft., transient voltage waves can not travel very far before they are dissipated through the lightning arresters and rendered harmless.

TABLE II—EFFECT OF TRAVELING WAVES IN CAUSING BURNOUTS OF TRANSFORMERS IN 1931

	Per cent
Ratio of transformers installed within 100 ft. of open ends of primaries to system total	46
Ratio of burnouts on these transformers, due to lightning, to total on system	41
Ratio of rate of burnouts on transformers near open ends of primary mains to rate for all others	82

UNDERGROUND PRIMARY MAINS

The records of transformer troubles due to lightning, presented in this paper, do not include 1,593 transformers which are installed in manholes or vaults. A separate record of these transformers shows only two failures in the past five years due to lightning, or about 1/30 of 1 per cent per annum.

EFFECT OF VARIOUS FACTORS ON LIGHTNING TROUBLES

Grounding the Primary Neutral. One advantage of a four-wire three-phase system with the neutral grounded in the substation is that it permits the installation of 300-volt arresters on the neutral wire of the overhead primary mains and 3-kv. arresters on the primary phase wires, whereas, with the neutral ungrounded, arresters rated at 3 kv. and 6 kv. are required. The neutral arresters, consisting of a single gap between metal electrodes, sometimes with a low resistance in series, will spark across below 3 kv.; therefore, as the transient voltage rises, these neutral arresters will spark across in about 1/5 of the time required for the transient wave to reach a voltage at which the phase arresters will discharge. If, during induced lightning discharges, the primary neutral wire were grounded at the substations, only, it would not be a factor in reducing troubles caused by lightning; but as a result of this action of the arresters on the primary neutral wire, that wire becomes grounded at each neutral arrester; and it then serves just as efficiently as a ground wire to reduce the maximum voltage, which would otherwise be reached by the primary phase wires, as a special ground wire installed for the purpose. This factor, together with the use of phase-wire arresters of lower voltage rating, gives the grounded neutral system an advantage over systems with ungrounded neutral of a reduction of about 40 per cent in the maximum crest voltage to which the transformers are exposed by induced discharges and this advantage cannot be overcome by the use of any special schemes or devices applicable to the ungrounded system.

As a result of these reduced transient voltages, and in spite of the fact that no lightning arresters are installed in the substations, flashovers due to lightning on the buses or connections in the distribution substations in Chicago are entirely unknown, and the burnouts of voltage regulators due to lightning in recent years have averaged about 1/20 of 1 per cent per annum. In contrast with this situation, may be cited the experience with an ungrounded neutral system having an operating voltage about 25 per cent higher than in Chicago, where the flashovers in the substation had become so serious that a special research investigation[5] was undertaken to determine the cause and the remedy.

A further indication along the same lines is found in the insignificant number of cable failures due to lightning, *viz.*, about 2 per year with 3,800 points of connection between the underground cables and overhead primaries.

Density of Arresters. This subject, quite thoroughly covered in the 1920 paper,[1] is one of the most important factors. Similar calculations, repeated for several succeeding years, confirm the results given in that paper.

Size of Transformers. Recent records confirm the statements and figures in the preceding papers that the percentage of troubles decreases with increasing size of transformers. In Chicago some reduction of troubles has been secured by discontinuing purchases of transformers below 5 kva. capacity.

Age of Transformers. Only 7 per cent of transformer burnouts due to lightning have occurred on transformers less than 5 years old, (Fig. 4) over 75 per cent occurred on transformers more than 10 years old, and the per cent of failures increases with their age.

Resistance of Lightning Arrester and Secondary Neutral Grounds. The records in recent years show no significant change from the results reported in the 1920 paper. The records apparently indicate, however, that any extra expense to reduce the resistance of lightning arrester grounds below 50 ohms is not warranted.

Shielding Effect of Trees, Buildings, Etc. Considerable time and effort has been expended in an attempt to determine to what extent, if any, lightning troubles are reduced by the shielding effect of trees and structures adjacent to the overhead lines. As near as could be determined by the methods used in this investigation, this shielding effect is practically zero. Numerous instances have been noted where transformers in such locations failed during lightning storms. Theoretical considerations do not indicate that any protection should be expected from such structures, as long as 95 per cent of the troubles are due to induced discharges.

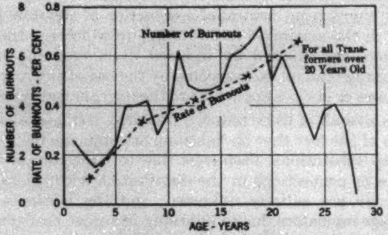

FIG. 4—RELATION OF AGE OF TRANSFORMERS AND BURNOUTS DUE TO LIGHTNING

Figures shown are averages for 1927 to 1931

ACCURACY OF RESULTS

In making research investigations on a subject in which there is a number of variables, it is recognized that, in order to determine the effect of any one variable, all others must be held constant. This, of course, is obviously impossible in field investigations of lightning protection. In spite of all attempts to eliminate the lightning as a variable, the relative performance (Fig. 5) of different types of arresters varies widely from year to year, indicating that the lightning still remains the dominant variable. It seems reasonable to assume that the average results obtained with the different types of arresters over a period of years (Fig. 5) may be in error in individual cases by an amount ranging from 10 per cent to 25 per cent of the values shown. For example, experience in preparation of the 1920 paper[1] shows that the relative order of merit of the different arresters, as shown in Fig. 3, would be altered if density of arresters were taken into consideration.

COMPARISON OF DIFFERENT TYPES OF ARRESTERS

A careful examination of the records obtained from various types of lightning arresters shows that one of the types (*B*, Fig. 3) first placed on the market about 20 years ago and withdrawn about 10 years later, has not been excelled in its protective efficiency by any later arrester. It consisted of a series of metal gaps with two shunting resistors enclosed in a wooden box. This arrester, in common with others of that period, had the disadvantage of requiring annual inspection and maintenance with occasional replacements of some of the parts or the enclosing wooden box or, at times, replacement of the entire arrester. Severe discharges would occasionally melt the metal cylinders and bridge the gaps, so that the arrester would not withstand the application of the working voltage across its terminals and, therefore, it interrupted the service on that circuit until the arrester could be located and disconnected. The manufacturers recognized that lightning protection is an economic problem, and the next step was to produce arresters of the discharge gap type, but enclosed in a porcelain tube instead of a wooden box, so that the annual inspection was eliminated. They still retained in some degree, however, the other defects of type *B*.

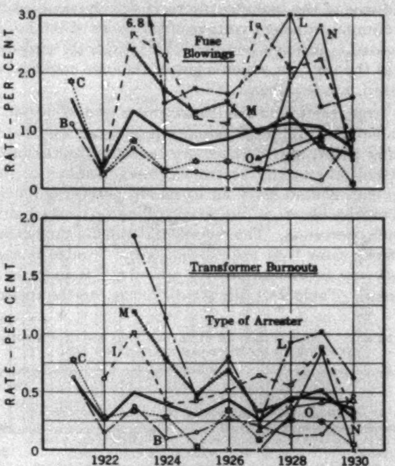

FIG. 5—RECORDS OF TROUBLE ON TRANSFORMERS PROTECTED BY TYPICAL ARRESTERS

Heavy solid line represents average for all arresters on the system

The first arrester of the so-called "valve type" was installed on the Chicago system in 1922. Since that time, all new types of arresters placed on the market by the three principal American manufacturers have been of the valve type. The replacements of the valve type required by failures in service are about one-third of similar replacements of the gap type arresters enclosed in porcelain. The ideal valve type arrester is one without a series gap that will discriminate between induced discharges of high frequency and the 60-cycle follow current, opening wide the valve for the former and closing it promptly against the latter, and repeat the action year after year without reduction in its protective efficiency. Considerable progress has been made in these designs in reducing the annual maintenance and the replacements, and it appears reasonable to expect that a valve type arrester will shortly be produced having a protective efficiency equal to type *B*. One type of arrester, type *M*, appears to improve in efficiency with age as indicated by the records in Fig. 5.

COORDINATION OF EQUIPMENT

In recent years there have been presented to the A.I.E.E. several papers on the subject of coordination of insulation on transmission lines. Apparently the same general principle can be extended to distribution circuits under the term "coordination of equipment," that is, coordination between the performance of protective devices and the equipment which they protect. In this case it means a coordination between lightning arrester performance and the transient voltage limit of distribution transformer windings and bushings. For example, if 50 kv. is the upper limit of the transient voltage between the line terminal of a lightning arrester and ground at the time of induced lightning discharges, then the transformer, when tested by the application of the proposed A.I.E.E. standard impulse waves, should withstand a transient voltage test of at least 100 kv. in order to secure reasonable immunity from lightning troubles. This means that there should be some coordination between the work of the A.I.E.E. Committee on Lightning Arrester Standards and the Committee determining standard tests for distribution transformers. It appears possible to design and construct, at reasonable cost, distribution transformers that will be practically immune to lightning troubles when protected by the modern types of lightning arresters.

SUGGESTIONS FOR SECURING MINIMUM LIGHTNING TROUBLES ON DISTRIBUTION CIRCUITS

1. Use the four-wire three-phase system of distribution with grounded neutral.

2. Where transformers or cables are connected to overhead primary mains, install low-voltage arresters on the neutral wire and valve type arresters suitable for the *Y* voltage on the phase wires; avoid duplication within 500 ft., and reduce the number, for Chicago conditions, by one-third.

3. Use transformers having insulation on the coils and leads that will not deteriorate with age and which will withstand a transient voltage test safely above the voltages to which the transformer will be subjected in service when protected by the arresters.

4. If conditions warrant a second ground rod for reducing the resistance of the lightning arrester ground, install also a second ground wire down the pole and connect it to the second ground rod.

5. Select newest transformers for installation in districts of low arrester density and install the older transformers in high density areas.

6. Require each customer to ground the neutral wire of his house wiring to a water pipe on his premises.

7. Connect the neutral wire of each section of secondary mains to at least two ground rods; on long mains install ground rod connections at intervals not exceeding 600 ft.

8. Interconnect, to a reasonable extent, the neutral wires of adjacent secondary mains.

9. Interconnect the lightning arrester ground wire and the secondary neutral wire when the latter is well grounded.

10. Where transformers are connected to primary mains 1,500 ft. or more distant from the nearest transformers, install on the pole about 600 ft. from the transformers the same lightning arrester equipment as on the transformer pole.

11. Ground the lead sheaths of all underground primary cables.

By following these suggestions and giving the subject continuous engineering supervision, it appears possible to reduce the number of transformer troubles in Chicago caused by lightning, that is fuses blown plus transformers burned out, to less than one case of trouble per year per one thousand transformers installed.

It is regretted that the limitations of space do not permit the presentation of all the data on which the conclusions and suggestions in this paper were based.

ACKNOWLEDGMENT

The author gratefully acknowledges his indebtedness to those of his own staff and the engineering and testing departments who have contributed to these investigations, and especially to H. Halperin who has had general charge of the studies in recent years and materially assisted in the preparation of the paper, and R. R. Schump who has compiled most of the records and prepared the voluminous reports to the manufacturers.

The author is also indebted to V. E. Goodwin and K. B. McEachron of General Electric Company, J. R. McFarlin of Electric Service Supplies Company, and A. L. Atherton and E. Beck of Westinghouse Electric and Manufacturing Company for their careful study of the reports of field observations and their generosity in furnishing results of their laboratory tests, many of them unpublished, during the course of the investigations, and especially for their broadminded policy in setting aside commercial rivalry during seven different conferences lasting seventeen days, so that the investiations might be most efficiently conducted.

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Discussion

For discussion of this paper see page 271.

Lightning Protection for Distribution Transformers

BY T. H. HAINES* and C. A. CORNEY*
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***Synopsis.**—This paper presents a five year history of the results of lightning arrester protection for distribution transformers in New England where the normal ground electrode resistance is much higher than in most other parts of this country.*

The experience shows that the trouble rate of the protected transformers has consistently been materially less than that of those not protected, notwithstanding the seemingly adverse conditions. It also shows that the effectiveness of the arresters is practically independent of the ground resistance when above a value of 100 ohms.

Design features of transformers are shown to be of prime importance in reducing flashovers in and around the equipment.

* * * * *

DESCRIPTION OF SYSTEM

THE Edison Electric Illuminating Company of Boston distributes electrical energy in 40 cities and towns, embracing 650 square miles in and about the city of Boston. The soil is generally sandy and rocky and of such a nature that the obtaining of what are generally considered low resistance grounds is extremely difficult if not impossible. There are many hills and valleys and numerous rivers and small streams wind through the territory. A total of 325,000 retail customers is supplied from 10,200 distribution transformers. All power customers are fed from three-phase transformers of which there are 1,500 overhead and 500 underground. Of the single-phase lighting transformers, 7,300 are overhead and 900 underground.

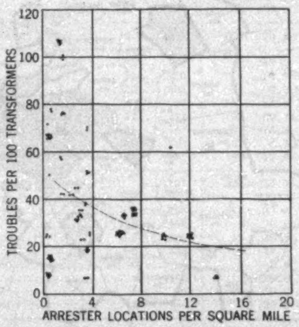

FIG. 1—VARIATION OF TROUBLE RATE WITH LIGHTNING ARRESTER DENSITY 1927

Note: Individual points or groups of points represent the observations in a single town; the number of points in any group being approximately proportional to the number of transformers in the town

In general, the primary distribution system is radial in type and operates at 2,300/4,000 volts, three-phase, four-wire, with the neutral ungrounded. There are approximately 235 such circuits of which 20 per cent of the circuit miles are estimated to be underground.

INTRODUCTION

This investigation was undertaken to determine, if possible, a means of lessening or eliminating the troubles in and about distribution transformers due to lightning.

For years, the occurrence of lightning storms in this territory has been accompanied by numerous blown fuses, damaged and burned out transformers, and other troubles, all of which caused considerable outage to customers with consequent annoyance to them as well as expense to the Company.

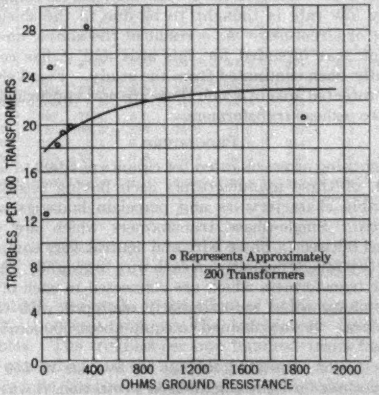

FIG. 2—VARIATION OF TROUBLE RATE WITH GROUND RESISTANCE 1927

Lightning arresters of conventional type had not been used to any extent principally due to the opinion that in order to have satisfactory performance of this equipment it was absolutely essential to have low resistance ground connections. Those familiar with New England soil conditions realize that a low resistance ground at the foot of a pole is a rarity. This condition was emphasized in 1917 when the Bureau of Standards made a general survey of ground resistance conditions and found that they were worse in New England than anywhere else in the country.[1]

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*Both of the Edison Electric Illuminating Company of Boston.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

PRELIMINARY STUDY

With the growth of the system conditions were becoming so acute that early in 1923 it was decided to make a trial installation of a number of standard lightning arresters notwithstanding the fact that there was little hope of getting what was considered a low resistance ground. It was at first necessary to determine where and how a start should be made.

Trouble records as related to the effects of lightning were rather meager at that time, but there were available records of the locations of all transformers which had been changed on account of lightning troubles for a period of years previous to 1923. These locations were spotted on a map of the system, and some of the circuits in the areas where the spots were thickest were chosen as the first to be protected.

From past experience certain areas in and closely adjacent to the City of Boston had shown a trouble rate about one-third that of the rest of the territory, and usually much under 10 per cent. There is now a total of 3,400 overhead transformers in these areas, none of which is protected by lightning arresters. As this section is more thickly settled than the rest, its comparatively low rate is thought to be due to the shielding effect of structures. As a result of the above, no protection was intended for this area and it has consequently been eliminated from the study. Fig. 3 shows the protected area in which there are now approximately 5,400 overhead transformers.

PROCEDURE

Lightning arresters rated for circuits of 4,000 to 5,000 volts, of three manufacturers each having seemingly desirable characteristics and porcelain housings, were selected. Single-phase transformers which are connected between a phase wire and neutral were equipped with two arresters, one from each wire to ground. Three-phase transformers which are connected to each of the three-phase wires were similarly equipped with three arresters. It was planned to equip about 500 locations a year.

As it was estimated to cost an average of $50 per transformer for lightning arrester protection, it was not deemed economically justifiable, in general, to install the equipment on transformers under 7.5 kva. capacity. As the records showed, however, that transformers on dead ends were more susceptible to lightning troubles than the others, it was decided to protect those as small as 5 kva. capacity at these locations.

Due to the previously mentioned opinion as to the necessity for low resistance grounds, it was decided that, for the first trial group, lightning arresters would not be installed unless a ground connection of 250 ohms or less could be obtained. Even this value of ground resistance will appear unreasonably high to many who are accustomed to readily obtaining values as low as one-tenth of this figure or even less.

Accordingly in 1925 the first group of arresters was installed. It was soon found after work had started that even 250-ohm grounds were difficult to obtain and consequently, in order to be able to complete a reasonable number of installations for observation, the limit above which arresters should not be used was raised to 1,000 ohms.

1927 STUDY

No attempt was made to analyze operating records until 1927 as it was thought that the number of installations in service previous to that time was too small to obtain reliable data. The operating forces were kept acquainted with the progress of installations and the object and aims of the proposed study, and were impressed with the desirability of complete and accurate records of troubles.

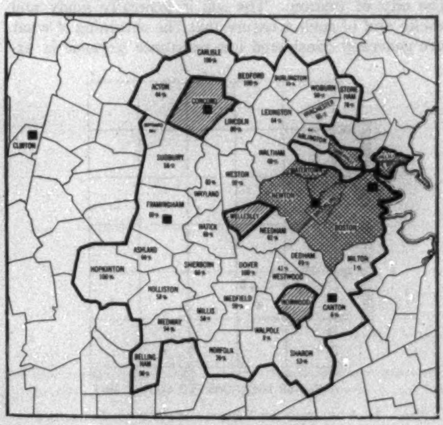

FIG. 3—MAP OF THE SYSTEM

1. Protected and unprotected areas
2. Per cent protection in each town 1930
3. Locations of the Weather Bureau observation stations

Edison territory is shown within heavy line
Single crosshatched areas have municipal distribution
White areas indicate ultimate 100 per cent arrester protection
Double crosshatching indicates unprotected area
Black squares indicate observation stations of the U.S. Weather Bureau
Acton and Maynard were added to the system in 1929

From the procedure established, it will be noted that in any area selected for protection some transformers were protected and some were not. It was thought that in general this practise would give us a means of com-

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[1] Bureau of Standards publication No. 108, "Ground Connections for Electrical Systems," by O. S. Peters, dated June, 1918.

TABLE I—PER CENT TRANSFORMER TROUBLES DUE TO LIGHTNING, 1927

Protected area towns grouped according to % L.A. protection	% Protection	Av. Ground resistance	Transformers with L.A.'s troubles per 100 trans.					Transformers without L.A.'s troubles per 100 trans.					All transformers		
			No. trans. with L.A.'s	Fuses blown	Defects in windings	Miscellaneous defects	Total No. troubles per 100	No. trans. without L.A.'s	Fuses blown	Defects in windings	Miscellaneous defects	Total No. troubles per 100	Total trans.	Troubles in year	Troubles per 100 trans.
1. Ashland	63	167	47	6.4	0.0	10.7	17.1	28	21.4	0.0	25.1	46.5	75	19	25.4
2. Bedford	64	141	45	22.2	0.0	15.6	37.8	25	32.0	0.0	12.0	40.0	70	27	38.6
3. Neeham	73	403	150	3.3	.7	5.3	9.3	54	33.3	0.0	14.8	48.1	204	40	23.5
4. Stoneham	78	129	86	2.3	0.0	0.0	2.3	24	19.2	0.0	3.8	23.0	110	8	7.3
5. Sudbury	64	49	65	7.7	0.0	26.1	33.8	36	13.9	0.0	13.9	27.8	101	32	31.7
6. Weston	74	360	128	13.3	0.0	16.4	29.7	45	42.2	2.2	13.3	57.7	173	64	35.2
Group 60-80		259	521	8.1	0.2	11.1	19.4	212	28.8	0.5	12.7	42.0	733	190	25.9
7. Burlington	42	174	25	12.0	0.0	8.0	20.0	35	34.3	0.0	22.8	57.1	60	25	41.7
8. Dover	52	183	55	40.0	1.8	21.8	63.6	51	29.4	0.0	11.7	41.1	106	54	51.8
9. Holliston	59	166	50	14.0	0.0	26.0	40.0	35	31.4	0.0	17.0	48.4	85	37	43.5
10. Hopkinton	59	128	44	68.2	0.0	15.9	84.1	31	93.5	0.0	38.7	132.2	75	75	100.0
11. Medfield	50	173	44	9.1	0.0	13.7	22.8	43	18.6	0.0	4.6	23.2	87	20	23.0
12. Medway	47	202	42	45.2	2.4	7.1	54.7	48	62.5	0.0	16.6	79.1	90	61	70.0
13. Millis	58	139	37	10.8	0.0	2.7	13.5	27	44.5		14.8	59.3	64	21	35.9
14. Natick	46	127	109	16.5	0.0	6.4	22.9	129	34.9	0.8	11.6	47.3	238	86	36.1
15. Wayland	44	73	44	4.5	2.3	20.4	27.2	57	12.3	1.8	21.1	35.2	101	52	31.7
16. Woburn	53	160	126	6.3	0.0	4.8	11.1	112	30.3	0.9	8.9	40.1	238	39	23.5
Group 40-60		151	576	19.2	0.5	11.7	31.4	568	35.7	0.5	14.6	50.8	1,144	470	41.2
17. Bellingham	35	183	7	42.9	0.0	0.0	42.9	13	38.5	0.0	15.4	53.9	20	10	50.0
18. Carlisle	33	210	6	16.7	0.0	33.3	50.0	12	50.0	0.0	33.3	83.3	18	13	72.2
19. Dedham	22	522	35	0.0	0.0	2.9	2.9	127	11.0	0.8	11.8	23.6	162	31	19.1
20. Framingham	40	99	151	14.6	0.0	5.3	19.9	230	21.3	0.4	8.2	30.4	381	100	26.0
21. Norfolk	31	213	23	26.1	0.0	8.7	34.8	51	35.3	0.0	9.8	45.1	74	31	41.9
22. Sherborn	33	147	24	45.8	0.0	20.8	66.6	48	43.7	0.0	10.4	53.1	72	42	58.3
23. Westwood	39	243	37	5.4	0.0	2.7	8.1	57	1.8	0.0	5.3	7.1	94	7	7.5
24. Winchester	25	181	40	17.5	0.0	15.0	32.5	119	20.2	1.7	11.8	33.7	159	53	33.3
Group 20-40		187	323	16.1	0.0	7.7	23.8	657	21.0	0.6	10.3	31.9	980	287	29.3
25. Arlington	7	196	8	37.5	0.0	25.0	62.5	100	55.0	0.0	28.0	83.0	108	86	76.8
26. Canton	5	158	9	0.0	0.0	11.1	11.1	160	7.5	0.0	6.9	14.4	169	24	14.2
27. Lexington	9	126	19	36.8	0.0	15.8	52.6	202	76.7	2.0	33.2	111.9	221	235	106.8
28. Lincoln	9	233	8	12.5	0.0	0.0	12.5	85	56.5	1.2	25.9	83.6	93	72	77.5
29. Milton	0	—	1	0.0	0.0	0.0	0.0	243	4.1	0.0	3.7	7.8	244	19	7.8
30. Sharon	4	300	5	20.0	0.0	0.0	20.0	115	14.8	0.8	10.4	26.0	120	29	24.2
31. Walpole	5	98	10	0.0	0.0	0.0	0.0	186	10.7	0.0	5.9	16.6	196	31	15.7
32. Waltham	1	—	4	25.0	0.0	50.0	75.0	308	43.2	1.0	18.9	63.1	312	194	65.5
Group 0-20	4	168	64	18.7	0.0	10.9	29.6	1,399	32.0	0.6	15.2	47.8	1,463	690	47.0
Total	34	201	1,484	14.7	0.2	10.6	25.5	2,836	30.0	0.6	13.8	44.4	4,320	1,637	37.9

paring the performance of those transformers protected with those not protected.

The data collected had been tabulated for the areas to be protected. Records of blown fuses, coil, lead, bushing and terminal board troubles comprised part of the data. Previous to the storm season, all lightning arrester grounds were regularly measured for ground resistance and those over 250 ohms were treated with salt and water and/or had additional rods driven in an attempt to lower them.

The resulting tabulation is shown in Table I. From this table it would appear that the installation of lightning arresters on certain transformers had decreased all types of lightning troubles on these transformers by 40 per cent and all troubles except those classified as miscellaneous, by 50 per cent.

A study was made of the effect of lightning arrester density per unit area on the transformer lightning troubles. The troubles per one hundred transformers were plotted against the lightning arrester locations per square mile, the towns on the system being taken as the unit areas. This graph is shown in Fig. 1, and indicated a distinct improvement as the density increased which seems to check the conclusion reached by Mr. D. W. Roper.[2]

The effectiveness of various values of ground resistance was studied by the method shown in Fig. 2. It appears from this that values of ground resistance from 100 ohms to 2,000 ohms have little if any effect on the number of troubles. In the course of discussion of these data the question was raised as to whether the ground resistance offered to 60-cycle secondary current which was used for testing might be different from that offered to lightning waves. The question was taken to the manufacturers and an investigation made by them showed that resistance values offered to lightning waves are appreciably less than those offered to 60-cycle currents.[3] This may account to a certain extent for the relative effectiveness of ground connections having a high 60-cycle resistance.

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[2] *Studies in Lightning Protection on 4,000-Volt Circuits*, by D. W. Roper, A.I.E.E. TRANS., V. XXXV Part 1, p. 655 and V. XXXIX Part 2, p. 1895.

TABLE II—SUMMARY OF LIGHTNING EXPERIENCE IN PROTECTED AREA

Year	No. of Weather Bureau lightning observations	% protection	Av. grd. res.	Trans. with L.A.'s			Trans. without L.A.'s			All transformers			
				No. of trans.	Troubles /100 trans.	Troubles /100 trans. /100W.B.O.	No. of trans.	Troubles /100 trans.	Troubles /100 trans. /100 W.B.O.	No. of trans.	Troubles in year	Troubles /100 trans.	Troubles /100 trans. /100 W.B.O
1927	144	34	201	1,484	25.5	17.7	2,836	44.4	30.8	4,320	1,637	37.9	26.3
1928	82	38	210	1,805	9.9	12.1	2,970	13.9	17.0	4,775	600	12.6	15.4
1929	126	49	196	2,407	12.1	9.6	2,487	38.0	30.2	4,894	1,239	25.3	20.1
1930	118	56	214	2,930	15.7	13.3	2,329	22.6	19.2	5,259	987	18.8	15.9
1931	114	71	234	3,781	4.2	3.7	1,562	7.6	6.7	5,343	278	5.2	4.6

1928 STUDY

The study was continued, and more lightning arresters were installed. This year the lightning storms were much less frequent than in the previous year resulting in only about one-half as many transformer troubles as in 1927. The summary of troubles for this year is presented in Table II which indicated that the troubles with protected transformers were considerably less than the troubles with unprotected transformers. The effect of lightning arrester density and of ground resistance was again studied with results substantially as in 1927.

1929 STUDY

As a result of the experience of the previous two years, lightning arresters were installed at a more rapid rate and ground resistances up to 2,000 ohms for transformers larger than 10 kva. were accepted, although as before salt treatment was given where possible in an endeavor to lower the resistance of such installations.

The same type of tabulation has been made as for the previous years a summary of which is given in Table II. This again shows even more strikingly the effectiveness of protection as in this year there was proportionately more than three times as much trouble on unprotected transformers as on protected ones.

An analysis of the troubles with transformer by sizes showed that while transformers, 5 kva. and smaller, constituted slightly less than 30 per cent of all the transformers, they accounted for more than 50 per cent of the troubles.

1930 STUDY

Three consecutive years had apparently shown the effectiveness of the protection afforded by lightning arresters. The fact that proportionately more troubles occurred on transformers 5 kva. and smaller, and that in general the smaller transformers are more remote from maintenance headquarters, together with the improvements shown with increased arrester density, led to the decision that all transformers in the towns in the protected area should ultimately be protected.

As a further result of the studies and in an effort to apply the new program most effectively all locations which were not protected and where trouble occurred more than twice in 1929, were equipped with arresters. Also a small group of towns which had experienced excessive lightning troubles were selected and arresters were added to give 100 per cent protection before the 1930 lightning season.

The map Fig. 3 shows the proportion of transformers protected by lightning arresters in the various towns as of 1930. The summary of troubles for this year is shown in Table II.

As 100 per cent protection is approached, it is readily seen that the means of comparing the effectiveness of the work tends to disappear and a need for some other measure to show the progress being made becomes evident. The Weather Bureau records observations of lightning storms from several stations in and about our territory. The observations reported have been totaled and called the storm severity factor. This factor is used as a divisor to modify the trouble rate in an attempt to render them comparable.

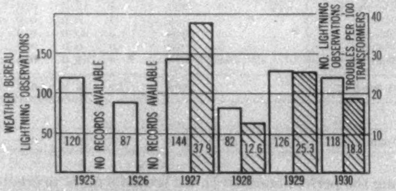

FIG. 4—WEATHER BUREAU LIGHTNING OBSERVATIONS COMPARED TO TRANSFORMER TROUBLES 1925-30

The map, Fig. 3, locates these observation points, and Fig. 4 gives the record each year compared with the troubles in the same year.

The curve indicating the improvement following the application of arresters is shown in Fig. 5, the rate of troubles modified by the storm severity factor is plotted against arrester protection in per cent. Extrapolation of the trend gives an indication of the improvement that may be expected by further additions of lightning arresters. As will be noted, this trend for the area considered is shown as a straight line.

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[3] "Impulse Characteristics of Driven Grounds," by H. M. Towne, *G. E. Review*, November 1928.

The other curves in Fig. 5 showing an individual town and a group of towns are given to indicate the effect of 100 per cent protection in lowering the trouble rate. These towns were selected for complete protection on account of the severity and frequency of storms in their locality.

Using this expectation curve as a basis, it can be shown for this system that the annual saving in cost of maintenance to be expected by the decrease of troubles anticipated by protection of all the remaining transformers in the area will be sufficient to finance the investment required. Additional gain will be realized in decreased customer outage and decreased handling and overhead expenses which were not included in the cost analysis.

The investigation in 1930 furnished additional data for consideration and indicated that the shielding effect of structures and other conductors may have a more pronounced effect in protecting transformers and fuses against lightning outage than had been previously recognized. In support of this, Figs. 6 and 7 show respectively the troubles *vs.* arrester density per square mile, 1927 to 1930, and trouble *vs.* population density per square mile for 1930. Since population density is a rough measure of the shielding afforded by buildings, the shielding may be a factor governing the trend of Fig. 7. This should be interpreted as meaning that the trouble rate in any area is the result of combination of the effects of shielding and arrester density, each of which contributes its share in the final result.

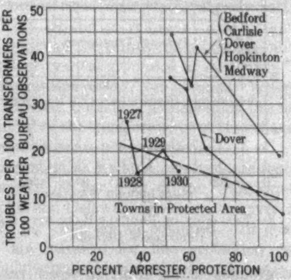

FIG. 5—VARIATION OF TROUBLE RATE (CORRECTED FOR VARIATION IN NUMBER OF WEATHER BUREAU OBSERVATIONS) WITH PER CENT PROTECTION 1927-30

In 1929, it was learned that transformers of small sizes were particularly susceptible to lightning outage. The 1930 analysis has brought out the same condition but more strikingly. Sixty-seven per cent of the total troubles was on transformers of 5 kva. and smaller. This group constitutes 28.5 per cent of the total number of transformers.

In Fig. 8, is presented a comparison of the effect of arrester ground resistance on the efficiency of lightning arresters. As previously mentioned, the average ground resistance is high, being over 200 ohms. This curve covers the four years of the experience records. On resistances below 100 ohms, it is found that decreased resistance makes a definite improvement in the effectiveness of arresters, whereas above that point our experience has indicated that the effectiveness is not materially reduced by high ground resistance. The upper horizontal line on Fig. 8 shows the average trouble intensity for unprotected transformers; compared with the curve it has been concluded that arresters even with very high ground resistances give valuable protection.

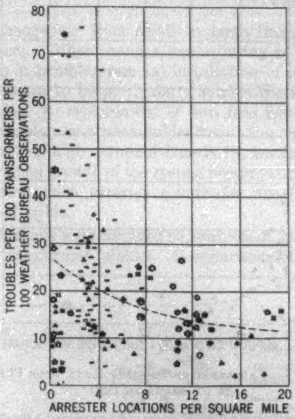

FIG. 6—VARIATION OF TROUBLE RATE WITH LIGHTNING ARRESTER DENSITY 1927-30

Note: Individual points or groups of points represent the observations in a single town; the number of points in any group being approximately proportional to the number of transformers in the town

As stated previously, lightning arresters of three manufacturers were used. Records of troubles have been segregated and tabulated by makers for the years 1927-30 and are shown in Table III. From the table it appears that for our conditions there is little to choose between them.

During 1930 careful inspections were made of nearly all the transformers where fuses were blown during lightning storms. Evidence of arcing was found in nearly every case. The arcing seemed to be concentrated on those transformers which were of more or less ancient design with small primary bushings, insufficient internal clearances, primary terminal boards, etc. Tabulation of the results indicated that transformers made previous to 1910 gave several times as much trouble as those manufactured since then, and inasmuch as previous to this date the use of high loss iron for the transformer cores was the rule, the policy has been adopted of eliminating these old transformers from the system. All other transformers passing through the shop are rebuilt to improve the internal clearances and are equipped with modern bushings.

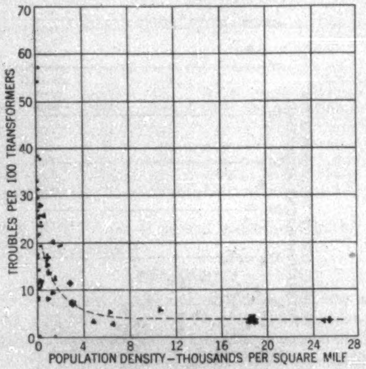

Fig. 7—Variation of Trouble Rate with Population Density 1930

Note: Individual points or groups of points represent the observations in a single town; the number of points in any group being approximately proportional to the number of transformers in the town

1931 Study

While the data for the year 1931 have not been carefully studied on account of lack of time the summary of the record is shown in Table II. A marked decrease in the total number of troubles as well as in the trouble rate will be noted. Again the rate of trouble on protected transformers is less than half that of the unprotected. The marked decrease in troubles is due in part to the rebuilding of about 25 per cent of the transformers which provided them with new bushings and the removal from the system of a large percentage of the small transformers with inadequate clearances and which were built previous to 1910.

TABLE III—TROUBLES FOR 100 INSTALLATIONS OF LIGHTNING ARRESTERS
(By Manufacturer)

Manufacturer	A	B	C	Average
1927	27.1	13.5	20.6	14.1
1928	16.2	4.7	5.6	9.8
1929	12.6	10.6	12.7	11.7
1930	14.5	16.6	15.2	15.7
Average	16.9	12.3	15.3	14.2
Total locations				
1930	1,100	1,452	380	2,941

It should be noted that the improvement in the operation of the protected transformers in general remains the same as in previous years even with nearly all the small trouble making transformers removed from the system.

Conclusions

1. Experience has indicated that the application of lightning arresters to line transformers in areas affected by lightning reduces the trouble rate by about 50 per cent.

2. Variation in ground resistance within the values readily obtainable in this territory has slight effect on the efficiency of the arrester.

3. The value of treating grounds or the installation of multiple grounds in an effort to lower resistance is questionable unless by so doing the resistance can be decreased to a value approximately 100 ohms or less.

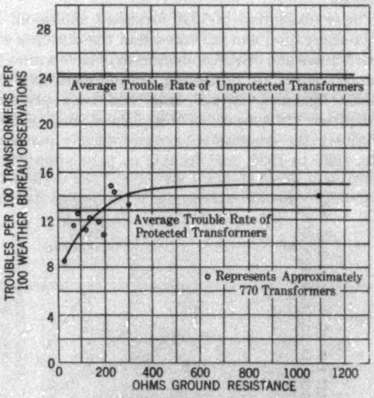

Fig. 8—Variation of Trouble Rate with Ground Resistance 1927-30

4. Insufficient electrical clearances in the smaller and older transformers are an important source of lightning trouble. Larger bushings, and greater clearances are clearly indicated as a necessary feature of correct transformer design.

Discussion

For discussion of this paper see page 271.

Distribution System Lightning Studies
By Philadelphia Electric Company

BY H. A. DAMBLY,* H. N. EKVALL,* and HOWARD S. PHELPS*
Member, A.I.E.E. Associate, A.I.E.E. Associate, A.I.E.E.

***Synopsis.**—In the analysis of distribution lightning problems, many factors must be considered, and long-time experience is necessary to permit reliable conclusions. Philadelphia Electric Company studies indicate that service interruptions due to primary distribution system trouble are in a large part due to lightning. Primary system trouble is found to cause far more customer interruptions than secondary system trouble.*

* * * * *

INTRODUCTION

INVESTIGATIONS of lightning troubles occurring on the distribution systems have been conducted by the Philadelphia Electric Company since 1921. The purpose of these has been to determine:

a. The magnitude and nature of the trouble in various localities.

b. Effectiveness of lightning arresters under field conditions.

c. Ways and means for improving service continuity.

As experience was gained, studies were broadened to cover a large number of the factors which appeared to have an important bearing on the problem.

During the interval 1921-1929 inclusive, investigations were confined to Philadelphia. In 1930 they were extended to all parts of the system, which now includes approximately 23,000 aerial transformers. The territory served covers approximately 1,900 sq. mi., which for operating purposes is divided into six divisions, as indicated in Fig. 1. The density of population ranges from sparsely settled rural districts to densely settled city areas.

The majority of the transformers is single-phase with the secondary neutral grounded at customers' services to the water system or to driven grounds. Where driven grounds are used, outside of Philadelphia the neutral is also grounded at the transformer or at the adjacent pole. Transformer cases are not grounded.

For the system as a whole, arresters are installed on approximately 50 per cent of the transformers. In Philadelphia and Delaware Divisions, however, approximately 25 per cent are so equipped. In general, no arresters are installed for line protection except at aerial-to-underground terminal poles. Arrester grounds in all divisions average 100 to 150 ohms.

LIGHTNING STUDIES—PHILADELPHIA DIVISION 1921-1929

During this period, investigations were made of lightning trouble on transformers in Philadelphia where the distribution system is 2.3 kv. two-phase, ungrounded. A material improvement was accomplished in the first few years of that period probably due in great part to gradual changeover from small to large fuses on all transformers. Other changes, contributing to further reductions in trouble, were (a) installation of arresters in areas known to be particularly susceptible to lightning trouble, (b) replacement of iron fuse boxes, with boxes of porcelain or other moulded insulating materials, (c) removal of high terminal boards in transformers, and (d) gradual growth of the system necessitating larger transformers which experience indicates are less susceptible to trouble.

Investigations were confined to analyses of fuse blowings and transformer failures. Comparisons of trouble on protected and unprotected transformers (reduced to cases of trouble per 100 transformers in service) appeared to indicate that arresters were not providing satisfactory protection. This was attributed to the relatively high resistance of the arrester grounds which averaged well over 100 ohms.

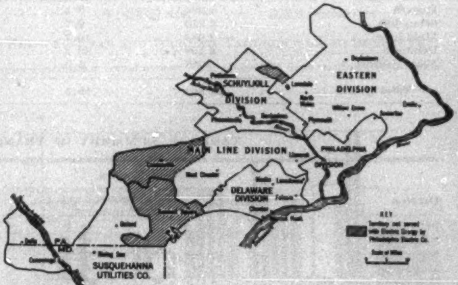

FIG. 1—1930 DIVISIONAL OPERATING BOUNDARY MAP

Philadelphia Electric Co. and Susquehanna Utilities Co.

LIGHTNING STUDIES—ALL DIVISIONS, 1930-1931

In 1930, the studies were broadened to include many factors which from experience, appeared to be of major importance to the problem. Also, these studies were extended to the territory of the former Philadelphia Suburban-Counties Gas and Electric Co. and of the Susquehanna Utilities Co., which had become a part of

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*Philadelphia Electric Company, Philadelphia, Pa.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

the Philadelphia Electric System. This permitted investigations on a variety of systems as follows:

a. 2.3 kv., two-phase, three-wire, ungrounded.

b. 2.3 kv., three-phase, three-wire, ungrounded, delta-connected.

c. 4.1 kv., three-phase, four-wire, grounded, star-connected.

d. 6.6 kv., three-phase, three-wire, ungrounded, delta-connected.

The first is used in the Philadelphia and Delaware Divisions, while the second and third are used in the Eastern, Main Line and Schuylkill Divisions. (See Fig. 1.) The 2.3-kv., three-phase, and the 6.6-kv., three-phase systems are used in the Susquehanna Utilities Co.

From observations thus far it has been impossible to detect any difference in susceptibility to lightning between grounded and ungrounded systems. This will be closely watched in future work.

SUMMARY OF LIGHTNING TROUBLE—ALL DIVISIONS, 1930-1931

During 1930, approximately 5 per cent of the transformers on the combined system experienced lightning trouble. Of these, 68 per cent represents transformer fuse and fuse box trouble, and the remainder transformer (bushing, lead or winding) trouble. In 1931 a similar condition was found. This experience is summarized in Tables I and II.

The higher rates in several of the suburban divisions than in Philadelphia are believed due in part to the use of smaller transformers which influence both fuse and transformer troubles. In Delaware Division the rate was lower than in Philadelphia due probably to less severe storm conditions.

TABLE I—SUMMARY OF TRANSFORMER LIGHTNING TROUBLE—1930

Division	Transformers			Troubles per 100 transformers		
	Number in service	Average size-kva.	Total troubles	Fuse and fuse box	Transformer	Total
Philadelphia	8,467	32.8	230	1.5	1.2	2.7
Eastern	4,204	9.5	250	4.2	1.8	6.0
Schuylkill	3,830	9.7	69	0.6	1.2	1.8
Main Line	3,043	10.7	220	5.1	2.1	7.2
Delaware	2,299	22.3	20	0	0.9	0.9
Susq. Utilities	958	6.0	275	26.2	2.5	28.7
Total system	22,801	19.5	1,064	3.2	1.4	4.6

TABLE II—SUMMARY OF TRANSFORMER LIGHTNING TROUBLE—1931

Division	Transformers			Troubles per 100 transformers		
	Number in service	Average size-kva.	Total troubles	Fuse and fuse box	Transformer	Total
Philadelphia	8,312	32.8	183	1.3	0.9	2.2
Eastern	4,581	10.1	317	3.1	3.8	6.9
Schuylkill	3,863	10.2	190	2.8	2.2	5.0
Main Line	3,197	11.4	221	4.2	2.7	6.9
Delaware	2,475	20.3	49	0.2	1.8	2.0
Susq. Utilities	1,182	6.0	96	7.2	0.9	8.1
Total system	23,619	19.2	1,056	2.5	2.0	4.5

MAJOR FACTORS INVESTIGATED DURING 1930-1931

The following factors which appear to be of major importance to the problem as a whole will be discussed:

1. Size of transformer
2. Age of transformer.
3. Arrester protection of transformer.
4. Fuse protection of transformer.
5. Shielding—exposed and unexposed locations.
6. Lightning storm severity.
7. Effect of ground resistance.
8. Relation of primary (2.3 kv.) to secondary (115/230 volt) trouble.
9. Analysis of customer-hour interruptions.

Discussion of these items will be based largely on experience in the Philadelphia Division, for which more complete information was obtained than for other areas.

SIZE OF TRANSFORMERS

The amount of trouble experienced in all divisions in 1930 decreased with increasing size of transformer, as indicated in Fig. 2. For all transformer sizes, fuse and fuse box trouble was found to be greater than transformer bushing, lead or winding trouble. Similar conditions were also found for all divisions in 1931.

The greater trouble on the small transformers is believed largely due to (a) smaller clearance between leads and case, and (b) smaller bushings, thus permitting more flashovers at these points with resultant short circuits. Lead trouble and winding trouble are compared in Fig. 4.

Therefore reduction in trouble should result from replacement of the smaller transformers with larger ones or with those of like size of recent design.

AGE OF TRANSFORMER

From limited information obtained in 1930, the age of transformers based on date of manufacture (information from serial number, not date installed) appears to be of importance only on sizes of 5 kva. and less. For these, it is believed the newer transformers are less susceptible to lightning than the older.

ARRESTER PROTECTION OF TRANSFORMERS

Analysis of arrester performance, neglecting transformer size, has shown consistently during the past five years that arresters apparently effected little reduction in trouble. However, the same data, when analyzed with respect to transformer size, based on the rate of troubles on various size-groups, indicates that transformers of 15 kva. and less were not materially benefited, but that those above 15 kva. showed a considerable reduction in trouble as indicated in Fig. 5. These observations result from studying all cases of trouble, about 1,000 in number, on approximately 2,500 protected and 5,500 unprotected transformers fairly evenly distributed throughout Philadelphia Division. The significance of the latter is that protected and unprotected transformers were subjected to essentially the same storm conditions.

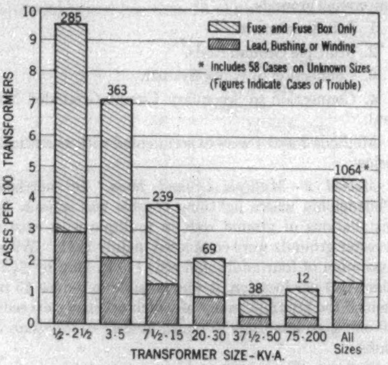

FIG. 2—FUSE AND TRANSFORMER LIGHTNING TROUBLES

Aerial distribution type—Philadelphia Electric Co. System—1930

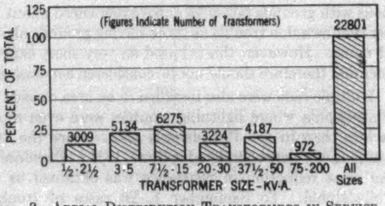

FIG. 3—AERIAL DISTRIBUTION TRANSFORMERS IN SERVICE AS OF AUG. 1, 1931

Philadelphia Electric Co. System—1930

Reductions shown in Fig. 5 were determined by comparing the number of cases of trouble on protected transformers with the number which probably would have occurred had the transformers been unprotected. By this method variations in storm conditions from year to year did not affect the analysis.

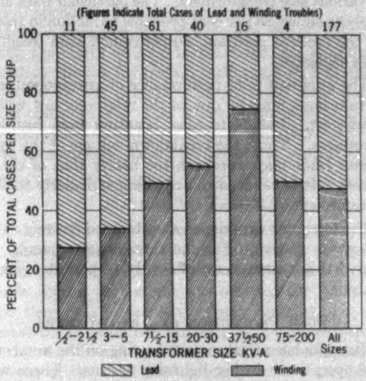

FIG. 4—LEAD AND WINDING TROUBLE DUE TO LIGHTNING

Aerial distribution transformers—Apr. 7, 1930 to Sept. 1, 1931, Philadelphia division

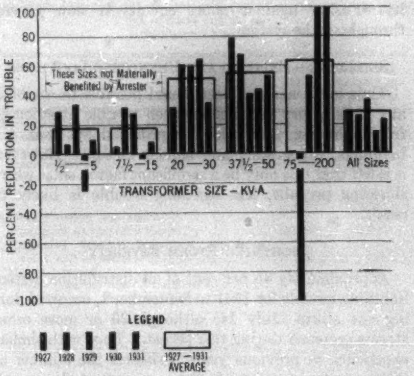

FIG. 5—REDUCTION IN TROUBLE BY LIGHTNING ARRESTERS

Yearly and average 1927—1931 Philadelphia division

Erroneous conclusions concerning overall protection may result by calculating protection from the rate of trouble on protected and unprotected transformers, without regard to the number of transformers or cases of trouble in the individual size groups; the latter method does not properly weight the difference in performance of various size transformers. The particular data under analysis shows 28 per cent probable reduction in trouble by the method used and 10 per cent if such weighting were neglected.

The apparent poor performance of arresters on small transformers may be due to the relatively low voltage breakdown of leads to case, resulting from restricted clearances. Thus, the breakdown voltage of the transformer leads may approach too nearly that of the arresters. It is believed this applies particularly to older transformers.

The foregoing experience was obtained entirely on the two-phase system. For the three-phase systems, the study has not advanced sufficiently to permit a similar analysis.

FUSE PROTECTION OF TRANSFORMERS

Fuse size has an important bearing on the number of fuse operations during lightning storms. These were materially reduced in Philadelphia Division and Susquehanna Utilities Co. by changing from small to large fuses without appreciably increasing transformer failures.

The former practise was to protect against overloads by fusing at one ampere per kva. of transformer capacity. Fusing against short circuits is now general throughout the system.

SHIELDING—EXPOSED AND UNEXPOSED LOCATIONS

Thus far it has not been found possible to establish any satisfactory relation between trouble experienced from lightning, and shielding provided by trees, tall buildings, topography and similar factors. However, it is felt that this may be a significant factor, since where shielding prevails, less lightning trouble is likely to result.

LIGHTNING STORM SEVERITY

Approximately 45 per cent of all distribution system lightning trouble for 1931 to September 1, occurred during one storm (July 14) although 20 or more other storms occurred during this period. This, with similar experience in previous years, indicates the number of storms to be an unsatisfactory measure of the overall lightning severity for a given year.

Although storm severity is believed to have a fairly definite relation to the amount of trouble, personal observations and impressions have not been found satisfactory for estimating lightning severity. A storm severity meter in process of development by the General Electric Co. appears to hold promise as a means for obtaining reliable data.

EFFECT OF GROUND RESISTANCE

Theoretically, the lower the arrester ground resistance the greater will be the reduction in surge voltage upon arrester operation. This, together with the favorable performance of arresters on the Company's 13.2 kv. lines, with grounds reduced to 15 ohms or less, is largely responsible for the effort directed to improving distribution grounds.

Measurements on single driven grounds widely distributed throughout the system indicated an average resistance of the order of 100 ohms. Accordingly, consideration was given in 1931 to the following methods of improving grounds.

1. Salt Treatment.
2. Multiple Ground Rods.
3. Connection to Water System.
4. Connection to Secondary Circuit Grounded Neutral.

Methods 2 and 4 were experimented with as discussed below:

Method 2—Multiple Ground Rods. In northeast Philadelphia where lightning trouble was serious, 345 installations of ground rods in multiple with existing arrester grounds were completed in July 1931. With a maximum of four additional rods (10 ft. long by ¾ in. diameter) per location, it was possible to reduce 45 per cent of the grounds, many of which originally exceeded 50 ohms, to less than 15 ohms, and all to less than 50 ohms.

To September 1, 1931, eight cases of trouble occurred at these locations. Of these eight cases, seven occurred where grounds exceeded 15 ohms. Expressed as troubles per 100 transformers, these data indicated that locations with grounds below 15 ohms experienced about 13 per cent as much trouble as those having grounds above 15 ohms. However, this is based on very short experience and therefore should not be considered conclusive.

Multiple rods were also installed in an area outside of Philadelphia where lightning troubles were even more serious than in the Philadelphia area. Here the 266 installations were completed in June, 1931. Essentially the same reduction in resistance was obtained as in Philadelphia. To September 1, 20 cases of trouble occurred at these locations. However, approximately the same rate of trouble occurred at locations with high as with low resistance.

The foregoing shows a marked difference in experience in the two multiple rod areas. This is believed due to the relatively small transformers in the area outside of Philadelphia, averaging about 4 kva., as compared with 15 kva. in Philadelphia. As previously discussed, studies have shown that little lightning protection can be obtained for the small transformers.

Method 4—Connection to Secondary Circuit Grounded Neutral. Because of its low cost and added advantage of tending to equalize surge potentials on primary and secondary sides of the transformer during arrester operation, connection between the arrester ground wire and secondary circuit grounded neutral has been studied. Trial installations of this connection, hereafter called the "neutral tie" were completed early in June, 1931 at 151 locations, involving 179 transformers. For comparative results these were concentrated in an area adjacent to where multiple rods were installed in Philadelphia, and in which lightning troubles have been equally severe. The scheme is shown in Fig. 6.

The neutral tie was applied only where a prescribed secondary ground grid existed. This was defined as locations where the ground resistance of the secondary neutral measured less than 15 ohms and where the secondary circuit fed 10 or more customers whose service neutrals were grounded to water pipes. Existing arrester grounds were retained, although where of high resistance their protective value is doubtful. The resulting ground resistance was in all cases less than 10 ohms as determined by megger measurement.

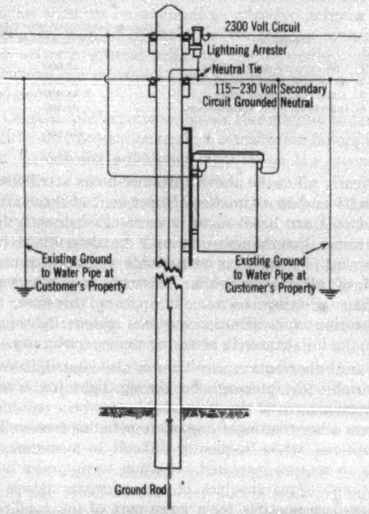
2300 Volt Circuit
Lightning Arrester
Neutral Tie
115–230 Volt Circuit Grounded
Secondary Neutral
Existing Ground to Water Pipe at Customer's Property
Existing Ground to Water Pipe at Customer's Property
Ground Rod

FIG. 6—SCHEMATIC DIAGRAM FOR INTERCONNECTING ARRESTER GROUND WIRE AND SECONDARY CIRCUIT GROUNDED NEUTRAL

Philadelphia division

To September 1, only one case of trouble, a transformer fuse blowing, occurred at neutral tie locations. This was not accompanied by trouble on the secondary circuit or customers' equipment. In this area 94 protected transformers without the neutral tie (because secondary circuit conditions did not meet the requirements) had approximately the same rate of trouble as those with the neutral tie. In the same area, 754 unprotected transformers experienced considerable trouble, thus indicating the area had been exposed to lightning storms of moderate severity.

Although this short experience indicates the feasibility of the neutral tie, observations over a long time and with a larger number of installations must be obtained before dependable conclusions may be drawn.

Analysis of customer meter trouble due to lightning during the storm of July 14 indicated less trouble on secondary circuits fed from transformers with arresters connected to the neutral than on those from unprotected transformers. This indicates the neutral tie did not aggravate secondary troubles and therefore, imposed no additional hazard on persons or property.

RELATION OF PRIMARY (2.3 KV.) TO SECONDARY (115/230 V.) TROUBLE

To determine the relative magnitude of primary and secondary system troubles during lightning storms an analysis of these was made for the storm of July 14. The trouble during that storm was selected for study because there was an unusually large amount of it and because information concerning it was concentrated in the records. It was later found that approximately 70 per cent of the total 1931 primary system trouble in Philadelphia occurred during that storm. The troubles together with the estimated number of customer-interruptions are summarized in Table III.

TABLE III—*PRIMARY AND SECONDARY LIGHTNING TROUBLE AND ESTIMATED CUSTOMER INTERRUPTIONS DUE TO STORM ON 7-14-31

Philadelphia Division

Classification of trouble	Number of cases	Estimated customer interruptions
Primary system (2.3 kv.)		
Transformer fuse and fuse box	69	4,140
Transformer winding	30	1,800
Transformer lead	27	1,620
Primary wires down	27	4,500
	153	12,060
Secondary system (115/230 v.)		
Customers' service fuses blown	1,200	1,200
Customers' defective apparatus, appliances, cords, and wiring	300	300
Meters damaged	138	138
Outside (external to service connections)	90	90
Service pipes	2	2
	1,730	1,730

*Excludes service lost due to circuit interruptions at substations.

The relative magnitude of primary and secondary trouble might also be compared on a customer-hour-interruption basis. However, the duration of the two should be essentially the same as far as nature of repairs is concerned.

Correlating the 138 cases of customers' meter trouble with the 153 cases of primary system trouble shows that only two of the former occurred on secondary circuits fed by transformers experiencing primary trouble. Accordingly, secondary circuit trouble must be due largely to lightning disturbances originating on that circuit.

It is evident from the causes of customers' interruptions that greater overall improvement in service continuity may be accomplished by attacking the primary rather than the secondary situation. However, from the viewpoint of service to the individual customer, interruptions from secondary trouble are just as important as those due to primary trouble. Consequently, efforts should also be directed toward reducing secondary trouble.

ANALYSIS OF CUSTOMER-HOUR INTERRUPTIONS

Primary circuit and individual transformer interruptions exceeding five minutes, throughout Philadelphia and Suburban Divisions in 1931 (to September 1), were analyzed to determine the relative magnitude of (a) those during lightning storms with those during other periods, and (b) those directly due to lightning disturbances with those due to other causes such as wind, trees, etc., during lightning storms.

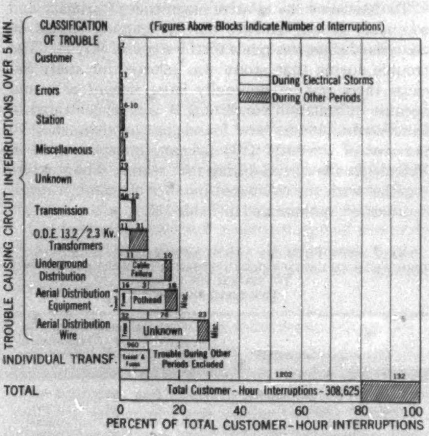

FIG. 7—ESTIMATED CUSTOMER-HOUR INTERRUPTIONS

Due to primary distribution system interruptions Jan. 1 to Sept. 1, 1931
Philadelphia and suburban divisions

Although circuit interruptions of five minutes or less, many of which were only momentary, represented approximately 80 per cent of the total, they were considered to cause only minor inconvenience compared to those of longer duration. Consequently, the analysis was confined to those exceeding five minutes.

As a measure of the inconvenience to the customer, all interruptions were expressed in customer-hours. The duration of circuit interruptions was taken as the time during which the circuit was off at the substation. The duration of interruptions due to individual transformer trouble (not causing primary circuit interruptions) was the time between receipt of trouble report and time "reported cleared." Averages of transformers per circuit, and customers per transformer based on kva. rating, were estimated for each division. The results classified according to the character of trouble are presented in Fig. 7.

This shows that 80 per cent of the total customer-hour interruptions occurred during lightning storms. Of this portion approximately 85 per cent were due to four major causes, as follows:

Trouble on	Estimated customer-hour interruptions during lightning storms
1. Underground distribution	47,900
2. Aerial distribution equipment	48,500
3. Aerial distribution wires	81,600
4. Individual distr. transformers	32,600
	210,600

Nearly all of the above customer-hours attributed to 2 and 4 and an estimate of 50 per cent of those attributed to 1 are believed to have resulted directly from lightning disturbances. It may be determined from this that lightning is responsible for approximately half of all customer-hour interruptions experienced during lightning storms. It appears, therefore, that protection of distribution systems against lightning is of major importance in bettering service continuity.

Aerial distribution wire trouble also contributes substantially to interruptions during lightning storms. However, most of the wire troubles are believed due to causes other than lightning, such as falling trees. Tree conditions, while frequently difficult to overcome, appear to require increased attention to minimize interruptions. Tree troubles of an unknown nature are believed responsible for a large part of the conditions marked "Unknown" on Fig. 7.

SUMMARY AND CONCLUSIONS

From the foregoing experience, the following conclusions are reached:

1. Annually about 5 per cent of the 23,000 aerial distribution transformers experienced lightning trouble. The small transformers have been found more susceptible than large ones. As far as practicable, the larger transformers should be used, or the older units replaced with more recent ones of the same size.

2. Difference in susceptibility between transformers of different sizes must be recognized when analyzing arrester performance.

Observations over five years show little benefit from arresters when considering transformers of all sizes collectively. However, taking size into account, these same data indicate arresters afforded little benefit to those of 15 kva. and less, and substantial benefit to the larger sizes.

3. Many factors influence lightning trouble on distribution systems. It is important to consider all of the major ones if reasonable conclusions are to be obtained.

Also, to permit comparisons with other studies, it is important to have data reduced to a common basis. One method is to compare performance per 100 transformers.

4. Because of the many factors involved, it has not been possible thus far to determine whether there is material difference in susceptibility to lightning trouble on grounded and ungrounded systems.

5. Low ground resistance, which theoretical and field studies show to be essential to effective arrester performance, may be obtained at low cost by connecting the arrester ground wire to the secondary circuit grounded neutral where a well grounded secondary neutral grid exists.

Limited field experience shows this method to furnish satisfactory protection against transformer failure without introducing additional troubles on the secondary system.

6. Customer interruptions from trouble on the primary system far exceed those from trouble on the secondary system. Therefore, major improvement in service continuity can best be accomplished by attacking the primary situation. However, service reliability as measured by the customer is independent of the source of trouble, and consequently improvements on the secondary system cannot be neglected.

7. The number of customer interruptions of less than five minutes greatly exceeds those of more than five minutes. However, when these are expressed as customer-hour interruptions, the latter greatly exceeds the former.

8. The major number of customer-hour interruptions due to all causes occur during electrical storms. Of these, it is estimated that one-half are directly attributable to lightning disturbances.

9. In the interest of improving service continuity, lightning protection of distribution systems has been found to be of major importance. Also, it is significant that it can be justified economically only from this point of view. The important factor is the inconvenience to customers which may thus be minimized. Although intangible, this factor is nevertheless very real and should be carefully considered in establishing policies and practises in meeting the problem.

The authors wish to express their appreciation of the assistance of the Operating Department, especially Messrs. W. Foster, G. S. Van Antwerp and their associates, in accumulating the data obtained in these studies; also, to Mr. V. T. Roth, and associates in the Engineering Department for their untiring efforts in collecting and correlating the information.

Discussion

INTERCONNECTION OF PRIMARY LIGHTNING ARRESTER GROUND

(HARDING AND SPRAGUE)

LIGHTNING PROTECTION FOR DISTRIBUTION TRANSFORMERS

(MCEACHRON AND SAXON)

LIGHTNING PROTECTION FOR DISTRIBUTION TRANSFORMERS

(OPSAHL, BROOKES AND SOUTHGATE)

STUDIES IN LIGHTNING PROTECTION ON 4,000-VOLT CIRCUITS—III

(ROPER)

LIGHTNING PROTECTION FOR DISTRIBUTION TRANSFORMERS

(HAINES AND CORNEY)

DISTRIBUTION SYSTEM LIGHTNING STUDIES

(DAMBLY, EKVALL AND PHELPS)

Harold C. Dean: While the authors of the several papers generally agree that interconnection of the lightning arrester ground with the grounded secondary neutral represents an important improvement in distribution system practise, they rather carefully refrain from showing that it would also be desirable to connect the transformer case to the secondary neutral. Apparently this is due to the prevailing misapprehension that grounded transformer cases would be hazardous to the linemen. Indeed Messrs. Opsahl, Brookes and Southgate dismiss the idea with the statement "The case of low-voltage class distribution transformers is left free, for the safety of the workmen who may touch it while working on the live transformer conductors" while Messrs. McEachron and Saxon conclude "nothing is to be gained with the interconnection by connecting the transformer tank to the secondary neutral, since with the tank floating, the potentials developed to tank are much less than the strength of insulation involved and the hazard to linemen is decreased."

In the New York Edison System we have found that a known hazard is to be preferred to an unknown hazard and we have adopted as a standard the grounding of all transformer cases. Where mounted on poles the transformers are located on a level with the secondary wires and five feet below the primary wires, and we require our linemen to cover up all grounded parts as well as low-voltage and high-voltage wires before doing any work on poles.

In support of our contention that it is bad practise not to ground transformer cases, I would point to the evidence presented in the papers that a large number of the transformer failures do not result in destruction of the windings, but merely result in the ground of a primary lead to the case. Due to the insulation afforded by the poles, these failures between primary leads and cases may not be detected and such cases will of course present very real hazards to linemen who get used to assuming that transformer cases are dead.

Several years ago one of our companies lost a lineman who inadvertently came in contact with a transformer case while working on a secondary wire. As a result of this, the company made a survey of all pole type transformers and found seven cases with full primary potential. Thereafter, pole type transformers were surveyed annually (at some considerable expense) and each year one or two additional installations would be found in which the transformer cases were "hot."

In 1930 The New York Edison System undertook a study looking toward the standardization of transformer installations on poles. Only four of the system companies had overhead lines and four different practises were found. With respect to grounding, we found the following practises:

1. One company connected the lightning arrester ground leads to the secondary neutral, but did not ground the transformer

cases nor install driven grounds at the transformer poles. (This had been the practise for twenty years.)

2. One company connected the lightning arrester ground leads to the secondary neutral and also to driven grounds, but did not ground the transformer cases.

3. One company connected the lightning arrester ground leads to driven grounds only and did not ground the transformer cases.

4. One company connected lightning arrester ground leads to driven grounds only and connected transformer cases to the secondary neutral only. (This had been the practise for sixteen years.)

While the basis for keeping statistical records of transformer failures had not been uniform for all the companies, and while the physical conditions such as exposure to lightning, average size of transformer and transformer fuse, system of primary distribution, etc., were not uniform, nevertheless, all available records of failures due to or suspected of being due to lightning were analyzed and showed the following averages for the years 1928, 1929 and 1930:

AVERAGE RATE PER 100 TRANSFORMERS, 1928 TO 1930 INCLUSIVE

	Transformer failures	Blown fuses	Total
Company with lightning arrester ground leads connected to secondary neutral only. Cases not grounded	0.045	1.03	1.075
Company with lightning arrester ground leads connected to driven grounds only. Cases not grounded	0.88	0.77	1.65
Company with lightning arrester ground leads connected to driven grounds only. Cases grounded to secondary neutral	0.77	4.53	5.30

It is significant that the company which connected its lightning arrester ground leads to the secondary neutral experienced about 1/20th the transformer failure rate of the two companies which depended upon driven grounds. It is also to be noted that the company which grounded its transformer cases to the secondary neutral, but failed to make a connection between the lightning arrester ground leads and the secondary neutral, blew about five times as many fuses as the companies which did not ground cases. As a result of this investigation it seemed obvious that no one company had done a complete job of grounding and that to obtain all the benefits of grounding, the following standards should be adopted for transformer installations on poles:

1. Lightning arrester ground leads shall be connected to the secondary neutral.

2. Lightning arrester ground leads shall be connected to driven grounds at the poles if there are less than two water pipe service grounds connected to the secondary neutral on the pole or poles not more than one section away from the lightning arrester.

3. Lightning arrester ground leads shall be connected to the primary neutral (if any exists).

4. Transformer cases shall be connected to the secondary neutral.

This standard has been in effect since July 15, 1931, and it is expected that all installations on the system will have been made to conform with this standard before the 1932 lightning season.

A. H. Schirmer: When a transformer fails, particularly in areas where there are no water-pipe grounds, the voltage on the secondary circuit may closely approach the primary voltage. Such excessive potentials on the secondary, combined with lightning surges, frequently cause trouble to telephone plants. Any reduction in transformer failures, therefore, will be of considerable benefit to the telephone companies.

The Bell System has been conducting lightning tests for a number of years. While these tests were not made directly on power transformers, the results obtained indicate rather definitely that in order to protect transformers against lightning surges, it is necessary to place the arrester between the primary and secondary circuit. In fact this method of protection has been used for years by the telephone company on open-wire loading coils.

The main question is: "Does such a practise make the situation less safe on the premises of the consumer?" In our opinion, connecting the arrester between the primary and secondary circuit actually makes for an increase in safety on the premises of the consumer, provided a suitable arrester is used. One of the power companies used to a limited extent an arrester consisting of a simple sphere gap connected between the primary and secondary windings. Such an arrester will, of course, protect the transformer but every time it operates, the primary voltage is directly impressed on the secondary circuit. The use of such a sphere gap will also result in excessive fuse operation. Obviously such a practise is not sound. The maximum safety is reached when we use an arrester having a current limiting element so designed that the voltage across the transformer windings is held to a value slightly below the impulse strength of the transformer. With such an arrester the lightning voltage on the secondary from a surge on the primary would be held to the lowest practical value and no power voltages of considerable magnitude would appear between the secondary circuit and ground, after all it is the excessive power voltage rather than lightning surges which make for hazard on the premises of the consumer. Furthermore, the use of an arrester which limits the voltage just enough to prevent failure of the transformer will keep fuse operation to a minimum.

In order to insure that no excessive potentials are imposed on the secondary circuit due to lightning arrester failures, the arrester should be designed so that the leakage current from normal potential will not cause excessive heating in the current limiting element, or if this is not practical, it should be designed so that the arrester will blow up or in some way open the circuit before it can introduce excessive potentials on this secondary circuit.

The use of such an arrester across the primary and secondary circuit will in our opinion be safe and in a large measure eliminate the failure of transformers.

F. W. Peek, Jr.: There is one subject that is not specifically considered in any of the papers, but upon which the successful operation of the various protective schemes depends. The papers show that a strong transformer properly connected to a suitable lightning arrester is necessary. The most important factor is a well insulated transformer. The A.I.E.E. acceptance test should be a measure of the transformer insulation.

At the present time the Institute does not provide for any insulation tests other than that made at or near 60 cycles. In view of the knowledge now available, I believe that the Machinery Committee or other suitable technical committees of the Institute *should review present methods of testing with the object of devising lightning tests* if such tests seem desirable. These tests, if made, should supplement the present tests and not supersede them. *I make the suggestion for a consideration of lightning tests on transformers after years of experience with such tests made for the purpose of devising lightning proof transformers and also in making lightning tests on commercial transformers.* The first commercial tests were, in fact, reported in the *Electrical World* in August 1930. I mention these facts to show that the tests are entirely practicable.

If such tests seem advisable, I recommend that rules be made by a technical committee of the Institute because of the highly technical problems involved, not only in methods of testing, but also in methods of detecting possible damage or deterioration or incipient failures during the test. This follows because it is important that there is assurance that the transformer is as good after the test as it was before the test was made. I hope the Institute can give early consideration to this matter.

A. E. Silver: The paper by Messrs. Dambly, Ekvall and Phelps gives an excellent summary and a measure of the relative importance of types of troubles experienced on an extensive and varied distribution system. It is to be hoped that these investigations will serve as a guide for the further extension of their studies, to including especially the field of the higher distribution voltages.

In this analysis of lightning effects on the Philadelphia Electric system, the authors are to be commended for moving the main objective of their studies from the distribution transformer to the effects on service. The rapid increase of tangible results from these investigations successively for the past two years is significant. This is one of several indications that, I believe, point to a movement into a new order of distribution practises as concerns the minimizing of interruptions.

The Philadelphia Electric investigators found that at least 80 per cent of their troubles, for a given period, came from quickly disappearing causes. I would venture the guess that a large majority of the remaining 20 per cent were from causes that can be made quickly to disappear. Of course, with open-wire lines there will always be some causes that cannot be quickly removed, such as lines put out of service by falling trees and limbs, but I believe these can be made a very small percentage of the total in our eastern areas where lightning is prevalent. While it is not assured that this figure of 80 per cent plus is generally representative, I am going to use it as illustrative of the point I wish to bring out. Most of these interruptions from quickly disappearing causes, by diligent engineering attention, I believe, are capable of being so restricted as either greatly to minimize effect on service or to avoid interruption altogether.

Again, I believe the Philadelphia Electric authors are moving in a constructive direction when they attempt to evaluate causes and effects in terms of customer hour-interruptions or, in other words, to measure with the yardstick of overall economy. In these days of rigid economies, it is encouraging that most of the opportunities that offer a means of cutting down service interruptions do not entail any eventual net increase and, perhaps, a reduction of capital costs, quite aside from the reduction of revenue losses from non-delivery and the decrease of operating and maintenance costs. The directions from which I look for progress in distribution practise include:

1. Improved relaying to realize quick and dependable disconnecting of the circuit in trouble. One of the simpler ways for accomplishing this would be by utilizing residual current which would favor the grounded neutral system.

2. Perfecting and applying the reclosing circuit breaker to general use by bringing the time for the complete switching cycle (including relay action) down to something under 30 cycles. This rapid restoration of service should render the effect of most interruptions essentially negligible except for synchronous load. With more radical developments, it does not seem beyond reason that the switching cycle and equipment may be made such as to protect even synchronous load against falling out of step. Good design and sufficiently quick disconnection should essentially eliminate burn-offs, porcelain breakage and all such damage from fault current that tends to a prolonged interruption.

3. The prevention or quenching of the dynamic fault arc by local means before the circuit has been disconnected or service has been interfered with. This means very rapid deionizing of the arc path,—one possible method being the perfecting of some simple arc interrupting device.

4. Adaptation of the automatic features of secondary a-c. networks, now extensively used in high load density districts, to areas of lighter load density, including some districts served by overhead systems.

5. Self-protecting distribution transformers. This may take the form of suitably related insulation or the simplification of lightning arresters or other protective devices as an integral part of the transformer itself. Efficient grounding and connecting of the protective device to the secondary neutral will play a large part. Messrs. Harding and Sprague's paper and others in this symposium are constructive contributions in this direction.

6. Gradual system simplification through elimination of intermediate voltage transformations and circuits, in service but no longer needed, as a means of reducing the opportunities for interruptions to originate.

7. The economizing in line insulation by setting the levels to meet the duties of surges originating at intervals, dirty surface conditions, etc., but omitting any further increments against lightning spillover. In addition, the intelligent coordination of line, station and equipment insulation.

Even a part of the progress that is promised by these developments will accomplish another substantial economy because of greatly reduced demands for reserve facilities for duplicate service.

C. L. Fortescue: The opinions expressed seem to be unanimously in favor of the by-pass arrangement of lightning arresters and I think the conclusions are correct.

In the old method of protecting transformers the arresters were connected between the high-voltage lines to a ground which was assumed to be of low enough value to reduce the surge to a safe value for the transformer windings but no thought was taken of the effect of the grounding lead which, on account of high surge impedance, would with steep wave fronts produce a high potential between true ground and the line, no matter how effective the arresters might be, nor was any consideration given to the difficulty of getting very low grounds in some territories. Under the conditions imposed by the old rule the transformer case would take up an intermediate potential between that of the high-voltage winding and the low-voltage winding. The low-voltage winding, on account of the fact that the neutral wire was solidly grounded and the secondary line wires had pretty low surge impedance, would remain practically at ground potential. There is a possibility that the difference of potential between the case and the low-voltage windings may be sufficient to cause failure of the low-voltage insulation but usually the failure takes place due to flashover of the high-tension bushing which results in the case being brought up to the potential of the high-tension winding and this in turn causes failure of the low-tension bushing so that the remaining portion of the surge is passed on to the low-tension winding. It was not always the practise to use arresters for the reason that frequently they were rendered ineffective on account of the difficulty of obtaining good grounds. According to my understanding the Safety Code does not make the use of lightning arresters mandatory for the protection of distribution transformers.

In the proposed by-pass connection the surge does not enter the transformer windings but is by-passed through the arrester to the neutral wire of the low-tension winding which is well grounded and where it can do no harm either outside or at the customer's premises because at the customer's premises it has a good ground which is obtained without difficulty. The whole transformer, including the case, is raised momentarily to the surge potential during the passage of the surge but the value of this potential is limited by the ground on the neutral wire at the customer's premises. The action of the arrester absolutely prevents any danger of the high-tension current being carried into the customer's wiring circuit which is not true in the old connection or when no arresters are used and flashover takes place over low-tension bushings which is the most frequent type of failure.

It is necessary to give some consideration to the possible value of the surge that may pass into the customer's wiring circuits. This surge will be limited to a comparatively low value by the customer's ground but consideration must be given to the length of lead between the secondary of the transformer and the customer's circuit because whatever potential appears at the transformer secondary terminals will appear in the customer's circuits and in some cases where these leads are long it may be advisable to use some kind of arrester between lead and neutral at the premises. I believe that in general, in the average case, this will not be necessary but nevertheless it is something that ought to be kept in mind. Where there are a multiplicity of circuits leading to residences the surge impedance of these combined circuits is very low and the multiplicity of grounds also improves conditions and in such cases the arrester at the customer's premises can be dispensed with.

While customers' grounds probably have very low surge impedance, this should not be taken for granted and an investigation should be made of the surge impedance of various types of residence grounds so as to make sure that the grounding arrangements are satisfactory. I think that the surge impedance of a residence ground should be kept as low as possible. I think a reasonable value should be below two ohms. In the case of isolated residences, such as farms and so forth, ready-made grounds may not be available. Driven-grounds should then be used and since the length of line between transformer and customer's circuits is usually longer in such a case I think it advisable to use arresters at the customer's inlet. To sum up, the proposed connection assures very much better protection to the transformer than the old connection and there is no increase in hazard to the customer. As compared with transformers that have no protection it gives very much better protection to the customer because it prevents high voltage from entering into the customer's circuits and the hazard from fire or personal injury is very much greater due to the high-voltage power current than to any surge that is likely to pass through into the customer's circuits.

A. S. Brookes: In reference to the paper by Messrs. Harding and Sprague, I should like to ask what type lightning arresters they used in their tests, what voltages they measured across the arresters and the value of the arrester current.

Their method of testing requires considerable analysis to determine whether and if it simulates the condition of direct strokes. It seems to be the reverse of the induced surge which, according to the accepted theory, is drawn up on the conductors slowly and then released by the discharge of the cloud. I feel that some actual oscillograms would assist greatly in showing just how their method duplicates actual conditions.

In one of their conclusions, they state that a well grounded, secondary neutral wire acts to reduce induced potentials on adjacent wires. A time element enters in here and it should be noticed that this protection is only fully effective when the ground connection is very near the point under consideration. When the ground is an appreciable distance away and the lightning surge has a steep front, flashover of the transformer bushing may occur before a charge can be drawn up on the secondary neutral. In other words, secondary neutral grounds are most effective in protecting the secondary windings from induced surges when placed on the pole adjacent to the transformer pole.

The paper states that high voltages appeared across the secondary windings of the transformer. While we did not measure this potential directly, we did find very little difference was caused in service voltages by removing the low-voltage arresters which were across the secondary of the transformer.

The statement about a non-inductive load in the consumer's premises reducing the service potentials might be amplified. The magnitude of this effect can be computed if the resistance of the load and surge impedance of the service wires are known. I should like to ask the authors if they calculated any checks on the measured values. Actually, it would seem that the branching of the house wiring at the meter board would reduce the voltage at that point very considerably, and particularly so if the wiring was enclosed in *BX*. This is more dependable protection than turning on the lamps during a thunderstorm and considerably safer. The possibility of a surge entering at the same time a consumer turns on a lamp is small but it does exist, and the practise is not to be recommended.

When using the interconnection, a surge on the primary causes almost equal current to flow in each of the three secondary wires. Hence, the voltage difference between the wires is small. The potential difference appears at points where the current in the secondary neutral is drained off by a ground connection. This voltage appears on the consumers' premises with the present construction in the case of a lightning outage and causes no trouble. This voltage can be partially drained off by low-voltage arresters connecting the secondary phase wires to the neutral ground at some pole other than the one the transformer is on. However, the gain probably does not warrant the expense. The interconnection produces essentially the same voltages at the service entrance as are experienced at present with a flashover, and higher potentials than at present with surges which cause arrester breakdown but no flashover.

In the conclusions, the authors give figures on the reduction of voltages at the transformer by the use of the interconnection with various ground resistances. While they do not give directly the effect of the interconnection on the voltages in which we are interested, those across the transformer bushings, we would expect a reduction here irrespective of the primary arrester and secondary neutral ground resistances. This is true because the secondary neutral will drain off part of the current, raising the secondary potential but decreasing the voltage difference across the transformer bushings. In other words, the interconnection will improve voltage conditions at the transformer, whether or not they are so bad that flashovers are anticipated.

The analysis could have been made considerably more valuable by a more detailed consideration of the circuit distances involved, and if more data on the surge current and the circuit details were given. I feel that the interconnection of the primary arrester ground and the secondary neutral has more merits than the results given in this paper indicate.

J. K. Hodnette: The primary requirements for the protection of distribution systems against lightning are:

1. The protection of the connected apparatus against damage from excessive voltage stress.
2. The prevention of circuit outages or interruptions.
3. The protection of secondary service circuits against voltages which might be hazardous to life or secondary apparatus.

The papers of this symposium on distribution circuit protection indicate that the protection to distribution transformers afforded by lightning arresters connected in the usual manner may be inadequate, although arresters at the present time are developed to a high state of perfection. This is due to conditions over which no direct control is possible, such as ground resistance, length of lightning arrester ground connection, etc. Effective protection requires the elimination of these factors, and the authors propose as a solution the interconnection of primary arrester ground and secondary neutral. That this connection produces no additional hazard in the secondary service lines over what exists under the present conditions is confirmed by both experience and laboratory tests.

These facts are of great interest from the standpoint of the design of distribution transformers. The establishment of a definite value for the surge stress permits a more accurate design of the transformer insulation, and suggests the possibility of a self-contained self-protecting transformer. The Westinghouse Company has already developed a distribution transformer which successfully fulfills all of the requirements for a surge-proof transformer. It is self-protecting against surge voltages and requires no external protective apparatus of any kind. It is outage-proof from lightning disturbances. The secondary is fully as safe against abnormal voltages as existing transformers. This transformer is a unit apparatus within which means are provided for protecting it against surge voltages. Surge discharge devices are placed inside the transformer tank and connected between each winding and the tank and core in such a way that the voltage stress that can exist between the high-voltage and low-voltage windings and between these windings and the core and case, is definitely limited in magnitude. In this manner a voltage stress higher than a predetermined safe value cannot exist upon any part of the transformer insulation, and therefore complete and positive protection of the transformer is assured.

The devices used for obtaining these results are deion surge protective gaps. These devices operate on the well-known deion principle. That is, after a surge is discharged, the arc path is deionized and normal voltage restored.

In operation, when a surge potential exists on the primary terminal of sufficient magnitude to cause any damage to the insulation, the device connected to the high-voltage lead limits the stress that can exist upon the high-voltage insulation by flashing over to the tank. Likewise the device connected to the low-voltage winding flashes over and discharges the surge to ground through the secondary neutral. If the neutral is not grounded, a similar low-voltage device conducts the surge from the case directly to ground. By incorporating in the protective gaps a small series resistor, the dynamic voltage is restored without causing the primary fuse to blow or a relay operation to take place with its consequent interruption of the feeder circuit.

Many laboratory tests have been made to prove the value of this surge-proof construction. Conditions as nearly representing service conditions as possible were employed, and surges of various magnitudes up to 1,500,000 volts have been repeatedly impressed upon the excited supply lines. In every case the transformer was protected and normal voltage restored without so much as causing a flicker in secondary lamp leads.

The interconnection of the primary lightning arrester ground and the secondary neutral as recommended by this symposium as the means for securing adequate protection for distribution transformers possibly constitutes an infringement of the National Electrical Code. By providing a circuit for the surge current directly from the case to earth, the surge-proof transformer is self-protecting from surges entering both primary and secondary circuits and does not violate the letter or spirit of the Code.

H. V. Putman: Mr. Roper brought out a point of significance in the fact that the great majority of transformer troubles from lightning occur in the smaller sized transformers where small bushings and close clearances are used. His company has discontinued the use of the smaller sizes on this account, but he strongly recommended larger bushings and more generous clearances. Manufacturers would do well to heed Mr. Roper's criticism, which is well founded on actual operating experience.

Another important point relating to the grounding of distribution tanks was brought out by Mr. Dean. The entire industry will do well to ponder carefully this whole subject of grounding tanks.

With the question of grounding tanks in the picture, the interconnection appears to lose one of its important advantages. The several authors generally agreed that connecting the lightning arrester ground connection to the low-voltage neutral would result in greatly improved protection to transformers. This has been demonstrated in certain particular applications, having isolated tanks and low resistance customer's grounds. While this connection discharges all surges to ground through the customer's ground connection, it was generally agreed that this could be done safely so long as the customer's ground was a water pipe connection of low resistance. It appeared to be the general opinion that if a modification of the National Electrical Code is necessary to permit the use of this interconnection, it should be made.

In considering this matter, the straight interconnection appears to have certain limitations which ought to be given careful consideration. With an isolated tank, the interconnection does give protection entirely independent of the ground resistance, but when the tank is grounded, the protection which the interconnection gives the transformer depends on the customer's ground resistance and the length of the service line or ground connection relative to the steepness of the front of the surge. This has been the source of difficulty with the conventional lightning arrester protection. The same difficulty is present with the interconnection when a grounded tank is used.

If there is no ground on the secondary service, as is usually the case in star delta connected transformers, for supplying three-phase power, then the use of the interconnection is undesirable for two reasons: First, it would pass on to the customer's apparatus the surges coming in over the high-voltage lines; and second, the grounding of tanks under these conditions would destroy the protection afforded by the interconnection completely. The reason for this is easy to understand. Suppose a surge of 100 kv. enters the transformer. The high-voltage winding is raised to this potential. If the lightning arrester discharge voltage is 15 kv., the low-voltage winding will reach a potential of 85 kv., since the surge impedance of the low-voltage service lines and the three-phase connected apparatus is likely to be very high. The potential of the tank and core may start to rise, but will be restored to ground quickly within a fraction of a microsecond, so that a difference of potential of 85 kv. would exist across the insulation between the low-voltage winding and ground, and this would be of sufficient magnitude to cause failure.

It appears, therefore, that the straight interconnection is not well adapted to installations having a low-voltage neutral ground connection of high resistance, nor to installations having no ground connection on the service lines, nor to installations having grounded tanks, nor to three-phase installations.

There seems to be an impression in some quarters that only by the use of the interconnection can effective transformer protection be secured. But this is not the case. The use of a grounded tank with lightning arresters connected between the tank and the high-voltage lines, and a coordinating gap to protect the low-voltage winding against surges entering on the service lines, affords a method of protection which is effective under all conditions, which is universal in its application, which conducts surges directly to ground instead of into the customer's neutral, which appears to be safer to the lineman and the public than the present equipment, with conventional lightning arrester protection, and which raises no question of a possible Code violation.

M. G. Lloyd: In drawing conclusions from these papers, it is necessary to exercise considerable caution. In some particulars these papers present contradictory results, and in one or two cases the conclusions of the authors do not appear to be justified by their observations.

The subject is of particular interest because of the prevalence of transformer failures. Conditions in this respect appear to be improving with better transformer construction and better performance of lightning arresters. It is clearly shown, however, that lightning arresters give very uncertain protection when the impedance to ground is high. Thus, better protection can be secured in a district such as Chicago, where resistances to ground are comparatively low as contrasted with Boston, where resistances to ground are typically high. In the Philadelphia district, good results have been obtained by efforts to keep the ground resistance low.

A proposal to connect the ground lead of a primary lightning arrester to the secondary neutral has had attention from most of the authors of these papers. This practise appears unobjectionable if it does not increase the potential to ground of the secondary conductors. This is a point that requires careful analysis.

Harding and Sprague conclude that no extra hazard is introduced. McEachron and Saxon found that the voltage surge on the secondary was limited to 400 volts when the primary arrester had a resistance to ground of 10 ohms. With interconnection between the primary arrester and the secondary neutral, however, it was found that the secondary arced over (3,900 volts).

Opsahl, Brookes and Southgate found that interconnection produces no higher voltages on the secondary than are obtained without such interconnection if the flashover occurs on the primary. The surge value, however, was definitely affected by the resistance to ground and the secondary voltage was doubled if the ground on the primary arrester was altogether removed.

The Philadelphia Electric Company found no bad effects from the interconnection, but it should be noted that interconnection in that case was definitely limited to locations where there was a low secondary resistance to ground and with connections to ground on the premises of not less than ten customers.

Good results are generally reported to result from the interconnection between the primary arrester and the secondary grounded neutral, and it is to be expected on theoretical grounds that such a connection would limit the voltage difference between the primary and secondary coils of the transformer and thus be effective in preventing flashovers and breakdowns of insulation. It is necessary to examine rather carefully, however, whether the good results reported from the field are due solely to such interconnection. The reports here seem to be somewhat conflicting.

Harding and Sprague found that the potential differences measured at the transformer depended very decidedly upon resistance to ground. When this resistance was low for both the primary arrester and the secondary neutral, interconnection had no effect upon the measured potential. Similarly, when both resistances were high there was no measured effect. When the resistance of the secondary neutral ground was high and the resistance of the ground of the primary arrester was low, interconnection produced an increase in the measured potential. The only case for which they found a reduction in the effect was when the secondary neutral ground had a low resistance and the primary arrester ground a high resistance.

Dambly, Ekvall and Phelps found that interconnection had no effect upon the trouble experienced in the field.

McEachron and Saxon found that the interconnection protected the transformer in all cases, whereas without the interconnection a high resistance of the primary arrester ground may prevent the protection of the transformer. Fig. 4 in their paper shows that the secondary potential was definitely raised by the interconnection when the resistance to ground was 60 ohms. It is regrettable that no similar results are given for a resistance of 10 ohms, which would correspond to the conditions in Fig. 2. The primary potential in both Figs. 3 and 4 was raised by the higher resistance. Similar curves would be of great interest to show what takes place when different values of resistance are interposed in the interconnection.

Interconnection appears to be practically equivalent to a very good connection to ground, on both the secondary neutral and the primary arrester. If the good results can be obtained by such separate grounds, it would seem to have advantages over a direct interconnection. The real problem appears to lie with situations where good grounds are not easily obtained. The authors of three of these papers advocate interconnection when the secondaries are well grounded. The converse proposition would seem to be that interconnection is not desirable where only driven grounds of high resistance are available. Mr. Roper states that the cost necessary to bring resistance to ground below 50 ohms is not warranted by the results obtained.

In studying this problem, it would seem necessary to separate the rural problem from the city problem. In cities, water-pipe grounds are usually available for the secondary circuits entering customers' premises, and the best grounds for the primary arresters can probably be obtained by an interconnection with the secondary neutral. For this practise to be safe, it should be carried out only when there is a multiplicity of grounds on the secondary neutral. Mr. Roper points out the value of interconnecting the secondary neutrals on different transformers even where a secondary network system is not used.

The real problem arises in rural communities where water-pipe grounds are not available and effective grounding is a difficult condition to obtain. Even McEachron and Saxon admit that interconnection under these conditions increases the hazard.

The Committee on Protection Against Lightning of the National Fire Protection Association has had under consideration for some time the advisability of using lightning arresters on overhead secondary circuits where they enter buildings. The fire hazard arising from lightning surges coming in to buildings over such wires is well recognized, and fires have often occurred in this way. Messrs. Opsahl, Brookes and Southgate, in their paper, and Dr. Fortescue, in his discussion, have indicated that such a practise may be desirable. It would seem to be necessary only in rural districts and only where the wires enter the building as separate open wires. Where they are run in conduit the latter would be grounded and would automatically act as a lightning arrester since a surge would probably jump from the conductor to the conduit. For open wires an arrester would seem necessary, but one of cheap construction would suffice for the purpose, since it would be necessary to interrupt follow current at only 120 volts. Experiments with the type of arrester used on radio antennas indicate that a slight modification would make such arresters effective for this purpose.

Edward Beck: The papers indicate rather definitely that in the present state of the art, the principal factors that enter into the protection of a distribution transformer are, outside of the construction of the transformer itself, the primary arrester ground lead and ground resistance. Interconnection of the primary lightning arrester ground terminal with the secondary neutral wire eliminates these variables because it limits the potential differences around the transformer itself to the lightning arrester voltage. Because of the favorable protective results which this interconnection promises, it becomes important to secure some idea of what will occur in the secondary circuits if the interconnection is used, thereby directing surges which may occur on the primary into the secondary neutral.

In order to investigate this, a circuit was set up in the laboratory to simulate an energized 2,300-volt circuit feeding a 110/220-volt house circuit. The house circuit was connected to various common appliances by which the transformer could be loaded to its capacity.

Surges were impressed on the primary from a surge generator, of magnitudes sufficient to discharge 800, 1,600, 3,000 amperes through the primary arresters. The primary arresters were 3,000-volt autovalve arresters of the latest type. Various values of house ground resistances were used, varying from zero to 57 ohms. Tests were made with one side of the primary grounded and ungrounded. Cathode ray oscillograms were made of the voltages occurring between each secondary wire and true ground. Measurements were also made of the potential differences occurring between the secondary phase wires and neutral.

One of the chief things of interest, is the voltages which may occur between the secondary phase wires and neutral, as these may produce flashovers in sockets or appliances. It was found by cathode ray oscillograph measurements that the voltages to ground of the phase wires and the neutral wire are very nearly alike except at the first instant. This potential difference may be due to induction through the transformer before the arrester discharges or it may be due to the release of a steep wave by the arrester discharge. The break down of the arrester releases a very steep surge into the secondary circuit which by reflection produces momentary high differences of potential between the secondary phase and neutral wires. In this test set-up crest voltages of 8 to 10 kv. were measured by means of a sphere gap. No appreciable variation occurred in this voltage between phase and neutral with "house" ground resistances varying from 0—57 ohms. These voltages can be reduced to negligible values by the use of 110-volt lightning arresters or small capacitors between phase wire and neutral at the house entrance. Their use is not considered essential, in the usual house circuit where the neutral is grounded and attachment plugs, etc., provide discharge paths between phase and neutral. Where the secondary circuits are ungrounded or highly insulated throughout, such as may be the case in electric range circuits, protective devices at the house entrance may be of considerable benefit, for instance, in the prevention of damage to ranges. It is to be noted that this is no worse a condition than will exist with the present usual methods of protection when the transformer flashes over. Tests were made with various loads on the secondary circuit varying from zero to full load, that is, 5 kva. The load consisted of the various appliances and lamps mentioned in the figure. It was found that flashovers would occur in lamp sockets and attachment plugs to a considerable degree. These flashovers were rarely followed by a power arc. Consequently few outages resulted. Tests were made with the appliances on the circuit and the radio set in operation and it was found that operation went on normally when the primary was surged although flashovers occurred in the secondary circuit. There was one exception which was a rather old socket which flashed over several times and finally produced a short circuit blowing a low-voltage fuse. A few flashovers occurred on the terminal board of the watthour meter, but the watthour meter was not damaged in any of the tests made.

During the tests well over 1,000 surges were impressed on the same distribution transformer protected by the same lightning arrester feeding the same house circuit. In a particular group of tests, there were three interruptions to the customer's service in about 1,000 operations. These interruptions consisting of blown fuses, one of these three cases necessitated replacement of a lamp socket.

Later, a failure of a toaster occurred. It had been hot for sometime, and had been subjected to over 1,000 surges. The frame of the toaster happened to be grounded to true ground. Several flashovers took place in the attachment plug which was already hot from the heat of the toaster. It developed a short circuit.

The voltages which may exist from all of the secondary wires to the earth will be the *IR* drop across the house ground resistance. It was found that these can be reduced by use of an auxiliary protective gap between the secondary neutral wire and ground at or near the transformer. Tests were made with such a gap in the circuit in series with a resistor to simulate the ground resistance of the driven ground at the transformer pole. This resistance was varied from zero to about 50 ohms and it was found that a considerable reduction in voltage between the house circuits and true earth was secured when the house ground is of some resistance.

During these tests, the lightning arrester was connected behind the primary fused cutouts. This is rather contrary to past practise but presents several advantages. It was found that satisfactory operation was obtained with primary fuse links as small as 5 amperes, and that a 3-ampere fuse link would pass surge currents at intervals of about a minute without blowing, but would rupture on rapidly applied surges.

These data are being presented as information. The improved protection given the transformer by the interconnection is probably admitted by everyone. Regarding the disturbances in the secondary, it is undoubtedly true that the interconnection can make conditions no worse in the secondary than they already are, and apparently they are not bad.

J. J. Torok: The majority of the lightning papers deal principally with the protection of the transformer. In reviewing the subject I was at first somewhat concerned by the hazards that the present systems might have to the consumer or customer. Statistics, some of which are being presented in these papers, show quite conclusively that the hazard to the customer under the present system is very small. A comparison of the two systems shows that the proposed system is far less hazardous. I have arrived at this conclusion in the following manner.

Let us assume that on a pole there is a step-down transformer, the primary of which is 2,200 volts or higher and the secondary is the common three-wire type, the central lead of which is always grounded on or near the customer's property. We will consider then that the consumer during a lightning storm may be in contact with some electrical device and an independent ground of his own. The household equipment usually is of fairly low insulation, consequently, any high voltage may easily flashover to the handle which is in the hands of the customer. With these connections we can now consider the different conditions with respect to the safety of the user.

The small amount of research that has been conducted by the high-voltage laboratories shows quite definitely that impulse voltages are not very dangerous even for high voltages and large currents. Experiments made on rodents and an occasional accidental one on humans show that impulses of such magnitudes as 200,000 volts and several hundred amperes are not as dangerous as 2,200 volts 60 cycles. This is obvious when we consider the time of application of the two voltages, the impulse lasts but a very short time, in fact 30 or 40 millionths of a second, whereas the 60 cycle can last as long as the circuit is complete. It also follows that the shorter the application of the 60 cycle the less dangerous it is. We all know how harmless static gathered from the carpet is, and this may be at potentials of the order of 10,000 volts and that 2,200 volts 60 cycles is used on the electric chair.

Impulse voltages which may appear at the terminals of a transformer may vary in magnitude from a very low value to a very high value. The distribution of these voltage waves with respect to magnitude is of the nature shown in Fig. 1. The shape of this curve for various insulations is very similar. The position with respect to voltage, however, is dependent chiefly upon the insulation of the line. Let us assume on this curve that magnitudes of voltages appearing at this point above the specified value E_1 will cause flashover of the bushings of the transformer. Voltages above this value will flash over the bushings of the primary, raise the potential of the case to a high value, and thus cause flashover to the secondary leads. Since arcs in open air are difficult to extinguish it is very likely that this flashover will establish a 60-cycle arc then power current will flow from the primary circuit into the secondary leads and from there to ground through the customer's service. The potential to which the secondary lead will rise will depend upon two factors; (1) the short-circuit currents in the primary circuit, and (2) the resistance of the secondary ground. Since the customer may be connected to an independent ground he will receive not only the voltage drop of the transformer but also the voltage drop across the ground. From this standpoint one can see that it is possible to obtain 60-cycle potential from 110 volts to 2,200 (assuming of course that the primary circuit voltage is 2,200). This 60-cycle voltage will appear on the secondary circuit as long as the arc is maintained across the transformer terminals, perhaps it may be limited by a

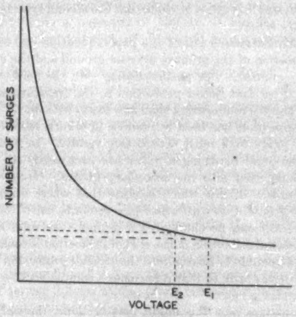

Fig. 1

fuse or by a group of network switches. Generally this period of time may be anywhere from one-half cycle up to several seconds. Such is the present condition.

If, however, we consider the system in which the protective devices are interconnected between the primary and the secondary neutral we will have an entirely different condition wherein these hazards do not appear. Going back to our distribution curve of impulse voltage we find that the action of the arresters comes in at a lower voltage than did the flashover of the bushings but this increase in number of surges that pass on to the secondary is very small. These, as has already been stated, are not very dangerous. On the other hand, due to the action of the arresters there is no 60-cycle power follow or at least if it does, only for a half-cycle, but no longer. The arrester will either cut off at a value above the 60-cycle voltage so that no 60 cycle follows, or at the end of the first half cycle. After that the primary and secondary are again isolated. Under this condition the maximum period of time at which high voltage may appear upon the secondary circuit is one one-hundred-twentieth of a second.

Comparing the two systems again we find that in the present system it is possible to have very high 60-cycle voltages on the customer's leads for several seconds. In the proposed interconnection the maximum period of time that is possible is one one-hundred-twentieth of a second, generally less. From these two conditions it is quite obvious that the second system is much safer than the first.

K. B. McEachron: All of the papers mention the use of the interconnection of the primary arrester ground and the grounded secondary neutral. The papers dealing with the field investigations all agree that better protection to the transformer will be had with the interconnection than now exists without it.

Interconnection has been in practise in several locations for a period of years with most satisfactory results. In these cases the conventional lightning arrester has been used, the arrester ground being tied into the secondary neutral. However, test data are now available which indicate that where multiple low resistance grounds, as with water-pipe grounds, exist, the ground at the arrester can be eliminated if desired. As soon as this is done a material reduction in the cost of protection is realized, and it becomes possible to incorporate the arrester protection inside of the transformer tank so that it becomes a complete self-contained unit.

In considering how this might best be done, the only design of which I am aware, in which this idea of protection within the transformer, coupled with interconnection, was used, was given careful study. I refer to a scheme used in Boston several years ago which was patented in 1919* and with improvements in 1923.† This protection scheme which was developed in Pittsfield consisted of gaps in each primary bushing to the case with an additional gap between the case and the grounded secondary neutral.

A considerable experience was built up with this arrangement in Boston, but it had certain disadvantages, some of which were of sufficient importance to result in its discontinuance. Those disadvantages were not dependent on the idea of interconnection, but were the result of the method of protection used and its application.

This early scheme of using gaps and making the transformer case a part of the circuit without permanently grounding the case gives rise to certain sources of trouble, as follows:

1. The gaps were connected from primary to secondary neutral without a high resistance or its equivalent in the circuit to limit the flow of follow current. When the interconnection is made without the use of the usual form of lightning arrester which has the ability to limit the flow of follow current to small magnitude for one-half cycle, the arc once established will not interrupt itself. This means that the power currents which flow into the secondary system are limited only by the impedance of the secondary ground in series with the impedance of the line. This may constitute a serious hazard in those cases where separate grounds are found in the same building.

It may be argued that the same conditions exist when bushings flash over. When both a primary and a secondary bushing arc over, the same condition does occur, but when protected by the conventional form of arrester properly applied, a bushing flashover seldom occurs, and thus the number of accidental primary-secondary contacts is kept to a small value. If now the arrester is replaced by the interconnected gaps, every operation causes a primary-secondary contact. Where all grounds in a building are not tied together an increased hazard is likely to result from such a use of direct primary-secondary connection.

2. Using the tank as a part of the circuit does not give the degree of safety to a lineman that either solidly grounding the tank or completely insulating it would give. The safety of the lineman may depend upon the integrity of the gap insulation whose strength is much less than that of the bushings. After a period of years such gaps may leak sufficiently so that the tank may take either the primary or the secondary potential. Furthermore, during periods of discharge, dangerous potentials on the tank may become a source of hazard.

3. A necessary part of the use of the gaps is some method of interrupting the flow of follow current following a lightning discharge. The primary fuse served in this capacity, and as a result, the number of fuse blowings was considerably increased over that experienced with the use of the conventional arrester. Today, with increased stress on reliability, any increase in fuse blowing is obviously a step in the wrong direction.

Having had this background of experience with gaps and remembering the good results obtained by those who have used the conventional arrester with interconnection, the best method of obtaining the advantages of interconnection in the self-contained unit appears to be the incorporation of the standard arrester within the transformer.

The experience of those operating companies who have used the interconnection with the conventional arrester has been very good indeed. To apply the scheme generally and to secure the best protection required that the arrester and the transformer be mounted on the same pole, which has not always been past practise. Further, in line with the general effort to reduce costs of installation and reduce the congestion on poles, it appears highly desirable to provide a reliable form of protection which can be placed inside the transformer tank without making the tank a part of the circuit.

In considering what form this arrester should take, certain requirements as to space have to be met, which have not been required of the arrester mounted external to the transformer. Careful study of all of the factors involved indicated the advisability of using the conventional form of arrester which has had much field experience, but so modifying it constructionally that it would fit within the space available without making any change in the existing transformer tank. How this has been accomplished may be seen from Fig. 2 which shows two arresters designed for operation with a 4,600-volt circuit. The transformer shown is rated 1.5 kva. 2,400/4,800/8,320*Y*-120/240 volts.

The arresters shown are of the Thyrite type and completely sealed within a porcelain container. With this construction the arrester operation is in no way different than when applied externally, and has the same factors of safety and the same reliability.

With any modern arrester connected inside the primary fuses the lightning discharge currents will pass through the fuses and may blow fuses below 5 ampere rating in case of extremely severe discharges. Although this tends to increase fuse blowing in the small sizes, its effect is counteracted by a great reduction in the number of bushing flashovers which also result in blown fuses.

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*U. S. Patent, L. R. Brown, No. 1,310,054.

†U. S. Patent, C. H. Kline, No. 1,462,346.

The paper by Harding and Sprague presents valuable data concerning potentials which may be induced on different conductors. In analyzing their results, however, it must be remembered that the voltages were small and consequently the current in the conductors and through the arrester was small, which does not give rise to the significant differences resulting from ground resistance which were obtained in the McEachron-Saxon paper. A statement from the authors of the amount of current would be of value when considering the effects of grounds. On page 236 the statement is made that the voltages across the secondary coils were approximately 20 to 80 per cent of those existing across the primary windings. I wonder if Mr. Roper has any data to show that the power banks are more susceptible in the secondary windings than in the primary. Perhaps secondary protection would be justified in such cases.

On page 236 of the Harding and Sprague paper the statement is made that the effect of the interconnection is dependent on such factors as the steepness of the wave and the time lag of the arresters, and hence no definite statement can be made that the connection is always beneficial. With the interconnection the potential between the secondary neutral and the primary terminal will not exceed the potential allowed by the arrester which, if it has time lag, depends on the rate of potential rise. With the conventional connection the arrester has a ground lead of considerable length and some ground resistance, so the voltage with interconnection will always be less than without it, without regard to wave front. Currents of considerable magnitude are necessary to bring out the effect of ground resistance clearly.

FIG. 2

Concerning conclusion 8 of the Harding and Sprague paper, the potential between the primary winding and secondary winding is not particularly dependent on the resistance between the secondary neutral and the ground, nor the resistance of the arrester ground if the interconnection is used. The arrester operates on difference of potential across its terminals, and its potential drop will not be less with good grounds than with poor grounds. The principle involved in interconnection is the equalization of potential between the primary and the secondary, and this is dependent on the arrester ground or the secondary neutral ground only as these affect the rate of voltage rise across the arrester and the rate of rise and value of current through the arrester. The potentials found on the secondary network are very dependent on the resistances to ground over which the discharge passes, but the maximum potential differences in the transformer are not dependent on the grounds but on the arrester when interconnection is practised.

The data contained in the three papers dealing with operating experience present a wealth of worth while material. I disagree with suggestion 4 of Mr. Roper's paper. While a second ground may be justified, there seems to be no justification for the second ground wire down the pole. It would be interesting if Mr. Roper would give the basis for this suggestion. I would like also to have Mr. Roper give further explanation of just what he means by suggestion 10, and the basis for such a suggestion.

The paper by Messrs. Haines and Corney is interesting because of the comparison of transformer failure with and without arresters. The results of Table I show a considerable reduction in failures even with an average ground resistance of 201 ohms. However, the statement in conclusion 3 seems to indicate that further improvement from the ground resistance standpoint will only come with resistances of 100 ohms or less. While no conclusion can be drawn, it is perhaps worth while to calculate the arrester currents for the induced case and for the direct stroke making certain assumptions. Assuming that the strength of the transformer in service can be taken as 65 kv. and assuming that the arrester potential is 15 kv., then with a ground resistance of 201 ohms the induced voltage on the line would have been 190 kv. (traveling wave 95 kv.) using a surge impedance of 500 ohms and the assumption of instantaneous cloud discharge. If 190 kv. is taken as the maximum traveling wave potential which the line can hold without flashover to ground, it can easily be shown that a ground resistance in excess of 80 ohms would not give protection. Because of varying rates of cloud discharge these two forms of discharge undoubtedly merge into each other, but it is interesting to note a potential of 190 kv. is not far from the probable maximum that such circuits could hold without spilling over.

The experience in Philadelphia, as reported by Dambly, Ekvall and Phelps is particularly interesting in view of the experience with arresters in Boston, as both have high resistance grounds. It is in communities where good grounds are hard to obtain that the benefits of interconnection will be most pronounced. It is gratifying to note the good results thus far obtained in Philadelphia. The data contained in Table III, and in Fig. 7 are of much value in showing the classification troubles and what it means in customer hours of interruption. It is with such data that it becomes possible to evaluate properly the benefits to be gained from lightning protection. The number of transformer troubles alone does not give the complete picture as an economic problem. It would be of value if the authors could also give the number of power follows taking place between conductors and conductors and grounded objects.

A. M. Opsahl: Mr. McEachron mentions the fact that he obtained secondary circuit flashovers when surges were applied to the transformer with various connections. He does not state whether these secondary flashovers resulted in a power arc or a secondary fuse burn-out when power voltage was also applied to the primary of the transformer. I do not think that power would follow and I should like to know whether or not it did in his tests.

In his presentation Mr. Roper showed a slide of the various factors that were discussed in his study of the variables that entered into the failure of distribution transformers. I should like to see a record of these variable factors and the conclusion that he has drawn as to their relative importance in the failure of distribution transformers. Mr. Roper states that about 40 failures of watt-hour meters occur per year due to lightning. It seems to me that there is a probability of several meters failing due to a surge into the secondary with power, whenever a transformer flashes over and burns out or blows a primary fuse. What coordination is there between the meter failures and the transformer failures, or fuse blowings?

Mr. Roper implies that ground resistance has no effect on the protection up to a ground resistance of 50 ohms in the primary arrester ground. Assume a 50-ohm ground and a 1,000-ampere surge current. This would result in 50 kilovolts added to the arrester characteristics for the total voltage appearing across the primary bushings due to arrester and ground resistance alone.

It seems inconsistent that there should be such a small difference in the failures where the ground resistance is low, in the order of a few ohms and where it is in the order of 50 ohms.

I ask Mr. Roper what experience he has had with the parallel ground leads which he mentioned. As I understood Mr. Roper, he has obtained permission from the Illinois Commission to install certain transformers with the interconnections. I assume from this that he has no service experience on this connection to date. From his 20 years investigation of distribution transformer protection, I also ask Mr. Roper what his opinion is as to the hazards to the customer where the proposed interconnection is used.

Mr. Corney does not state what ground resistances can be attained at the customers' premises under the conditions in his territory. He does not state where the transformer flashovers occur but I presume it is over the bushing and into the secondary neutral. Under such conditions, what hazard is there to the customers or what troubles do they experience in the secondary circuit?

R. E. Jones: The interconnection of arrester ground and secondary neutral as described by Mr. McEachron has been common practise in Ontario for many years with the addition that usually primary neutral and transformer case are also tied to the same ground. The results have been very satisfactory under above described conditions.

However in a number of installations on rural lines where the primary neutral was not tied to the secondary neutral the arrester has broken down without destroying itself and allowed power current to flow through the driven rod of the secondary ground without blowing the substation fuse. This ground was gradually dried out and the resulting high resistance caused the current to flow to ground through the consumers' premises damaging radio equipment and motors.

F. W. Gay: It should be possible to obtain additional protection against transformer failure by introducing capacity in shunt between each high-voltage terminal and tank and between each low-voltage terminal and tank. Such capacity having low surge impedance to waves of relatively short duration will tend to maintain all windings and tank at substantially the same potential.

This result can be easily obtained by passing both primary and secondary leads into the transformer through metal clad cables. The metal covering of the cables must be solidly grounded to the transformer tank. Each such cable should include within the metal sheath an oil stop joint so that the continuity of the solid insulation need not be broken from coil to line wire.

Herman Halperin: On page 260 of the paper by Messrs. Haines and Corney it is stated that there is no lightning protection for 3,400 overhead transformers installed in certain areas in and closely adjacent to the City of Boston. What were the rates of burnouts and fuse blowings for these transformers due to lightning?

H. N. Ekvall: The paper by Mr. D. W. Roper covers the results of extensive investigations of many factors associated with the problem of distribution system lightning protection. For many of these factors, the Philadelphia Electric Company through investigations on its systems, has obtained comparable results. There is, however, considerable difference in the methods used in determining arrester performance.

A comparison of the methods of analysis used by the Commonwealth Edison Company in the study of its 4,000-volt grounded three-phase system with those of the Philadelphia Electric Company on its 2,300-volt ungrounded two-phase system follows:

In Chicago, arresters have been installed at practically every transformer. Under these conditions, performance can be determined only from the average rate of trouble over a period of years, before and after installing arresters. In Philadelphia, arresters have been installed on only part of the transformers. Their distribution throughout the entire area makes it possible to study performance by comparing the rate of trouble on protected and unprotected transformers for any one year.

With the Chicago setup, the wide variation in lightning severity for the various lightning seasons may lead to erroneous conclusions. It is believed, however, under conditions such as exist in Philadelphia, the variations in lightning severity are compensated for, since protected and unprotected transformers are exposed to essentially the same storm conditions. Experience indicates this to be very nearly the case since arrester performance or the reduction in trouble by arresters has been fairly uniform over the past five years. During this period the average density of arresters varied from 25 to 50 per cent indicating that there was a continuous change in arrester distribution, thereby permitting performance to be studied on various groups of transformers.

Another factor of importance to arrester performance is transformer size. This is apparent from the large difference in rate of trouble on the various sizes. The Commonwealth Edison Company attempted to eliminate the effect of transformer size by dividing its territory into small areas having approximately the same distribution of transformer sizes in each. Only one type of arrester was installed in a given area, the various types being assigned to areas which would permit the best distribution over the territory. The Philadelphia Electric Company did not try to eliminate the effect of transformer size but accounted for it by obtaining the rate of trouble on various size groups of protected and unprotected transformers. The performance obtained in this way showed very little reduction in trouble by use of arresters on the small transformers and considerable reduction on the large transformers. If the performance of arresters is determined without considering transformer size, the results may not accurately indicate the relative effectiveness of arresters of different types.

F. E. Andrews and E. R. Hendrickson: We believe that the method which Messrs. Dambly, Ekvall and Phelps have used of comparing interruptions by customer hours is a step somewhat in advance of classifications made on the basis of the number of interruptions. It should, however, be pointed out that in many cases customer hours do not give an adequate picture of the relative seriousness of interruptions.

As a means of obtaining a better picture of this factor, it is suggested that some consideration might properly be given to the addition of the kva. of connected transformer capacity which has been interrupted, thus obtaining a factor which takes into consideration the number of customers, the length of interruption, and the amount of connected load interrupted.

From the tabulation in Fig. 7 it is noted that the largest single cause of interruptions is due to aerial distribution wire having caused 30 per cent of the total customer hour interruptions. Of this, 76 per cent is listed as having occurred during electric storms due to causes unknown. Analyses which have been made of such interruptions of the distribution system of the Public Service Company of Northern Illinois indicate that there is some reason to believe that a number of these may be due to flashovers started by lightning and followed up by the 60-cycle power arc. Investigations have been made of a number of cases of such failures. Information which has been developed to date is not sufficiently complete to warrant the drawing of definite conclusions. It does, however, indicate the following tendencies:

1. Most cases of phase-to-phase arcing take place when the clearances between phases have been reduced to a value less than normal line spacing. The path to ground is by way of:
   - *a.* An arrester on adjacent phase.
   - *b.* Ground wire on pole.
   - *c.* Anchor guys on pole.
   - *d.* Transmission pole guys passing between phases.
2. Phase-to-phase arcing has been found to have taken place over a distance as great as 16 inches, at 4,000 volts through air.
3. Interruptions to service do not generally occur when location of disturbance is several miles from the substation.

A study which the Public Service Company has made of transformer and fuse troubles has been segregated by voltage and class of territory (rural or urban), but does not include information with respect to the various sizes of transformers as is covered in this report.

For 1930 investigation of transformer and fuse failures indicates that there were twice as many failures in rural territory than in urban, and that the number was considerably higher on the 4,600 and 6,900-volt lines than on the 2,300-volt lines. A summary of this information expressed in per cent of transformers exposed is shown in the following table.

Voltage	Location in service	Failures			
		Transformer		Fuses	
		Ltg.	All	Ltg.	All
2,300—Urban	19,592	1.1	2.1	3.9	5.2
2,300—Rural	3,605	3.4	4.3	8.0	9.0
4,600—Rural	417	8.6	8.7	31.1	33.0
6,900—Rural	378	5.5	9.8	25.1	27.2
All voltages in per cent of total locations		1.7	2.6	5.2	6.6

Note: The average ground resistance in our territory is somewhat lower than that in the Philadelphia territory, and probably will be in the neighborhood of 50 ohms.

We are particularly interested in the information presented which indicates that arresters were of little or no value on transformers smaller than 15 kva. We believe that this may, to some extent, be due to the higher ground resistance encountered in the Philadelphia territory, thus increasing the voltage to which the transformer is subjected before the arrester becomes operative. This, it appears, would increase the susceptibility of flashovers on the smaller transformers, particularly those of the older type, because of the reduced clearances. A study which has been made of our system in a section of Joliet, Illinois, representing 509 locations over a three-year period, gives results on the value of arresters which are contrary to those drawn in the paper. A summary of these results is given below:

	Arrester ground resistance			
	45 ohms and less	45 ohms and up	No arresters	Total
Per cent of total in each group	52.7	21.0	26.3	100
Per cent transformer failures average per year	0.9	1.8	5.4	2.3
Per cent fuse failures average per year	4.4	3.7	7.4	4.0
Per cent transformer and fuse failures average per year	5.3	5.5	12.8	7.2

Although the transformers have not been segregated by size, over 95 per cent of them were smaller than 15 kva. Since there are over twice as many fuse and transformer failures in the locations having no arresters, it is evident that arresters in this case could be justified on transformers smaller than 15 kva. These transformers are all located in the same vicinity so that they receive the same exposure.

C. F. Harding and C. S. Sprague: In reply to the discussion by Mr. Brookes, the primary phase arresters were of a standard make, rated at 3 to 5 kv. as ordinarily used on 2,300-4,000-volt systems. A 300-volt valve type arrester was used on the primary neutral wire. The voltages across the phase arresters were of the order of from 20 to 30 kv. Arrester currents were estimated at a maximum of a few hundred amperes depending upon the ground resistances. No attempt was made to measure these currents since it is difficult to do this without disturbing the circuit.

The larger part of the investigation was performed using voltages induced by an artificial cloud. It has been shown both theoretically and by oscillograms that the induced transients are the same whether the cloud is suddenly discharged or suddenly charged, providing that in the latter case, the cloud potential is maintained constant until the induced transients have disappeared. The induced voltages thus obtained are less severe than the direct discharge of the surge generator into the primaries, since the energy and surge current are less.

A grounded secondary neutral reduces the induced potentials *to ground* on adjacent wires and is most effective in this respect when the ground connection is close to the point under consideration. Potentials to ground on primary wires on the upper arm were reduced 3 to 10 per cent by the secondary neutral. Potentials to ground on the secondary wires adjacent to and on the same arm with the secondary neutral were reduced approximately 30 per cent. However the ground connection causes a difference of potential to appear between secondary wires although the potential to ground of all the secondary wires is reduced. With regard to bushing flashover a steep front surge is less apt to cause flashover when the secondary grounds are remote from the transformer or of high resistance, assuming no interconnection of secondary neutral and arrester ground. The more effective the secondary grounds are in holding the secondaries close to ground potential, the more voltage will be impressed across the transformer, *i. e.*, between primary phase wire and secondaries.

Induced voltages of from 15 to 30 kv. were measured across the three-phase 230-volt windings of the power transformers, having the power secondary ground made to the midpoint of one winding. On the lighting secondary winding the induced voltages from line wires to neutral were of the order of from 4 to 8 kv. There is evidently some neutralizing action occurring in the lighting transformer secondary winding, which action reduces the impedance of the winding to surge currents. The power secondaries and their ground connection present an unbalanced condition to surge currents flowing to ground and it is here if anywhere that our tests have shown the desirability of secondary arresters.

The statement that a non-inductive load in the consumers' premises reduces the voltages at the service entrance by 60 to 70 per cent is based upon measured values. No attempt was made to calculate this effect. It is quite true that as the surge enters the conduit and branches at the cutout box, the voltage is reduced by the lower surge impedance of the conduit system. However, if all lamps are turned off, positive reflections occur at the sockets and due to the short lengths of the wiring circuit, the voltage at the service entrance and at the sockets builds up very quickly to the value obtaining on the secondary wires outside the service. With a non-inductive load on the circuit, the surge current is continually drained off as it enters the service and due to this larger surge current and the regulation of the external circuit, the voltage is considerably reduced at the service entrance and throughout the service. From the standpoint of personal safety there seems to be no reason why one should not turn on the lamps during a thunderstorm. In fact this is very often done, although for the purpose of obtaining light rather than for protection to the house wiring system.

Number 8 of the conclusions presents the effect of the interconnection on the potentials at the transformer with various values of ground resistances. The potentials described as "across the transformer," *i. e.*, between primary phase lead and secondary winding, are those tending to cause bushing flashover. The conclusions reached are derived from tests using induced potentials with relatively low values of surge current flowing after the arrester had broken down.

Since preparing the paper, tests were made using "direct strokes," that is, with the surge generator discharging into each primary wire in turn. In these tests the discharge currents through the arrester were of the order of 2,000 to 3,000 amperes. With the voltage adjusted to a value not quite sufficient to cause bushing flashover the potentials at the transformers were measured with and without the interconnection and with a variety of ground resistances. The voltages tending to cause flashover were from 50 to 60 kv., without interconnection and from 35 to 38 kv. with interconnection, the latter figure being the maximum voltage across the arrester.

With the voltage raised to a value such that bushing flashover always occurred without interconnection, it was observed that in all cases the interconnection prevented the flashover of the bushing.

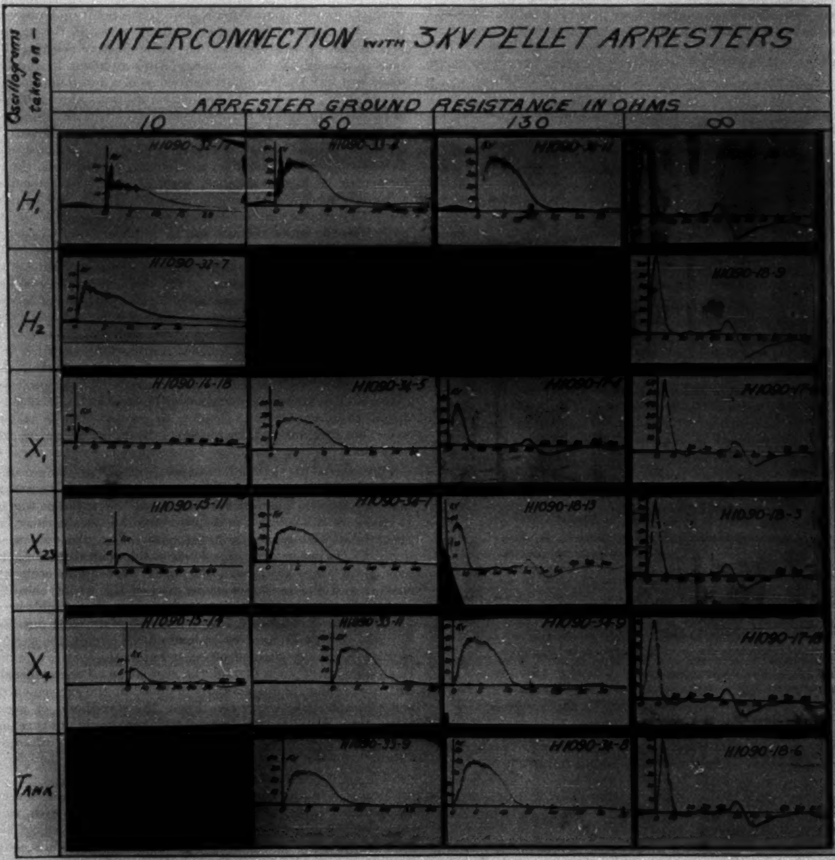

Fig. 3

It is regretted that insufficient time prevented the presentation, in the oral discussion, of these more recent and important tests.

K. B. McEachron: Mr. Dean's discussion giving results from three operating companies with different grounding practises is interesting and ties in very well with the results which were obtained in the Willseyville tests. Grounding the tank with arresters connected to a separate ground allows high stress between primary and tank if ground resistances are high, resulting in primary bushing and lead flashover and consequent fuse blowing. If the windings are stronger than the leads and bushings, substantially all of the interruptions will be due to fuse blowing. With interconnection as shown for the first company tabulated transformer failures were reduced to a very low value. If the tank is connected to the secondary neutral there will be substantially no stress between the secondary winding and the tank and there will be no tendency to flashover secondary bushings. The situation of the primary with interconnection is not materially different whether the tank is grounded to the secondary neutral or not.

From a protection point of view, connecting the primary neutral, if available, to the tank and secondary neutral with interconnected arresters, should give a very high degree of protection.

Mr. Schirmer's comments are particularly helpful from the point of view of the telephone company, especially with regard to the comparative hazard of power and lightning currents reaching the secondary. It is believed that the number of lightning discharges reaching the secondary will be somewhat increased by interconnection, however if conventional arresters are used having suitable valve elements the power current reaching the secondary will be negligible. Apparently Mr. Schirmer feels that even in cases where there are no water pipe grounds the hazard which now exists will be reduced by the use of the interconnection using proper lightning arresters.

Mr. Putman makes a statement that when the tank is grounded interconnection benefit depends on the resistance and length of customers' ground leads. This would be true only in the case where the tank was grounded to a ground independent of the secondary neutral. It is much better practise to connect the tank to the secondary neutral for which condition interconnection gives satisfactory protection.

The scheme proposed by Mr. Putman of a grounded tank with arresters connected between primary and tank with a coordinating gap to protect the low-voltage windings does not prevent lightning surges from reaching the secondary. In fact with the same ground resistance and the same arresters impulse voltages will reach the secondary circuits less frequently with the usual primary-arrester-ground connection than with the scheme proposed by Mr. Putman.

This is true because a smaller discharge is required to elevate the tank sufficiently to arc over the secondary bushings, with tank connected to separate ground, than to raise the primary conductors to high enough potential to arc over the primary bushings which are relatively much stronger than the secondary bushings. However, connection of the arrester between the primary and tank will protect the primary windings and will prevent power current from reaching the secondary. If this connection is used the secondary must withstand the discharge unless the tank is either connected to the grounded secondary neutral or to a separate ground of low enough resistance to prevent the secondary from flashing over. In general, connection of a proper arrester between primary and tank will prevent power current from reaching the secondary but will not prevent lightning from getting through unless the ground resistance is very low.

As Mr. Opsahl suspected, our tests did not show any case where power follow took place across the gaps in the "hall" fixtures, although many oscillograms were taken in which both power and impulse were applied.

The trouble experienced by Mr. Jones would be largely eliminated if the arresters were connected inside of the fuses so that arrester failure would blow the fuses.

Mr. Gay suggests the use of cable to protect the transformer. Calculation and test both show that considerable capacity is required to be of much benefit. A study of the effect of short lengths of cable on incoming waves may be found in a paper* describing an investigation made in Michigan in 1929, which shows that lengths as long as 500 ft. will have a comparatively small effect in reducing crest potentials.

Dr. Lloyd points out apparent contradictions between the results presented by Harding and Sprague and those given in the other two papers dealing with field investigations. The differences can be explained largely by differences in the magnitude of the currents flowing through ground connections. Characteristics of arresters under various conditions of wave front and crest current are well known. When such a device is connected between primary and secondary the only variation in potential which can take place is that resulting from change in wave shape or maximum current. The voltage, with interconnection, between the primary and secondary winding will not be affected to any significant degree by the presence of grounds either primary or secondary. The arrester determines the protection of the transformer while the grounds determine the potentials which are allowed to reach the consumer.

Fig. 3 shows all of the winding voltages with ground resistances of 10 ohms, 60 ohms, 130 ohms and infinite ground resistances. All other conditions were kept constant. Lack of space prevented showing these data in the paper. It should be noted that with the 10-ohm ground the secondary potential to ground was about 6 kv. crest, while it increased to 30 kv., 35 kv., and 60 kv., as the arrester ground resistance was increased from 60 ohms to 130 ohms to infinity. This increase is logical and to be expected. In studying these results one should not forget that the combined secondary neutral resistance was 35 ohms. With water pipe grounds close to the transformer it would not have been possible to elevate the secondary above ground after the negative reflections returned from the water pipe grounds.

D. W. Roper: It is impossible to find any reasonable objection to Mr. Dean's premise that a known hazard is to be preferred to an unknown hazard, but our experience leads us to a contrary conclusion. The Commonwealth Edison Company has had no accidents in connection with ungrounded transformer cases for over ten years, and, on a system with over 30,000 transformers in service, we found only one transformer with a "hot" transformer case in 1931. The Chicago experience, therefore, appears to warrant continuing the present practise of leaving the cases ungrounded.

It is interesting to note that Mr. Peek approves my suggestion that the A.I.E.E. rules for testing transformers should include suitable clauses regarding surge tests on distribution transformers. Considerable weight is added to this suggestion by the announcement of two of the larger manufacturers of distribution transformers during the discussion that they have placed lightning-proof transformers on the market.

In Chicago, the grounds to water pipes on customers' premises have a d-c. resistance of about one-half ohm. Other investigators have shown that for such low values the d-c. resistance is about the same as the surge impedance of the ground, and so we appear to be well within Mr. Fortescue's limit of two ohms.

Mr. Putman states that there is usually no ground on the secondary service of three-phase power transformers connected star delta. In Chicago, the midpoint of the secondary winding of one transformer in the bank is always grounded and hence, his objections to the use of the interconnection for such transformers do not apply here. This ground connection is made to the neutral of the lighting secondary, if available; otherwise a driven ground is used.

Answering Mr. McEachron's question, our data show that the secondary windings on banks of power transformers are about as susceptible to damage by lightning as the secondary windings on lighting transformers and that, in general the apparent entrance of lightning is twice as often on the primary windings as on secondary windings.

The recommendation for a second ground wire down the pole, which Mr. McEachron criticizes, was based on the results of tests made by Professor Harding at Purdue University and by the Westinghouse Company at its own laboratories, the records of which were not published, but are used with its permission. These tests agree in indicating a reduction of about 40 per cent in the impedance of the connection from the arrester to earth. This makes a considerable reduction in the maximum transient voltage imposed on the transformer windings when no interconnection is used. Answering Mr. Opsahl's query, we have had no practical experience with this connection, as we have very few high-resistance grounds.

Suggestion 10 of my paper, which Mr. McEachron criticizes, is based on the theory that when the density of arresters is rather

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***Study of the Effect of Short Lengths of Cable on Traveling Waves*, K. B. McEachron and H. P. Seelye, A.I.E.E. Trans., Vol. 49, 1930, p. 1432.

low, traveling waves on distribution systems become an important factor. The records of a few cases of very high transient voltages noted under the heading of "Traveling Waves" in my paper indicate that the maximum diameter of the area in which these very high-voltage transients appear is limited to something less than 1,500 ft. In Chicago, there are but few locations where the transformers are so far apart. When the transformers are located at intervals greater than 1,500 ft., the protection should partake of the class recommended for transmission lines; and for the terminals of such lines the manufacturers have hitherto recommended the installation of arresters several hundred feet distant from the terminal, in addition to those at the terminal, for the purpose of discharging a portion of the traveling wave before it reached the terminal.

Further discussion on the points Mr. Opsahl raised regarding my paper is as follows:

a. The tabulation showing the factors affecting transformer burnouts due to lightning is given in Table I of this discussion. The importance of the major factors has been shown in my papers except for previous fuse blowings; it was found that transformers which had experienced previous fuse blowings were twice as apt to fail due to lightning, as others.

TABLE I—FACTORS AFFECTING TRANSFORMER BURNOUTS

		Importance	
	Factor	Major	Minor
	A. Data Regarding This Transformer		
1	Size	X	
2	Make and type	X	
3	Age	X	
4	Loading record		X
5	Previous burnouts		X
6	Previous fuse blowings	X	
7	Power or lighting installation		X
8	Secondary grounded or ungrounded		X
	B. Conditions at This Location		
9	Type of arrester	X	
10	Arrester ground resistance		X
11	Height of primary mains		X
12	Length of primary mains		X
13	Number of primary mains		X
14	Number of arresters	X	
15	Length of secondary mains		X
16	Length of phase service wires		X
17	Arrangement of primary and secondary mains		X
18	Grounded telephone cable		X
19	Shielding structures		X
20	Proximity to open ends of primaries		X
	C. Conditions Regarding Neighborhood		
21	Burnouts, this storm		X
22	Burnouts, 5-year period		X
23	Fuse blowings, this storm		X
24	Fuse blowings, 5-year period		X
25	Density of arresters	X	
		7	18

b. Careful attempts on coordination between failures of watt-hour meters due to lightning, and transformer or fuse blowings due to lightning, resulted in our being able to find that such troubles occurred at the same time in only a small percentage of the cases.

c. Our data on the effect of arrester ground resistance on transformer burnouts due to lightning have shown that the resistance is not a predominating variable in our system, where over 90 per cent of the resistances in recent years have been less than 50 ohms. In trying to correlate burnouts with magnitude of resistance, it has been noted that the ground resistance of the arrester on the transformer that burned out might be considerably lower or higher than the arrester ground resistances for adjacent transformers.

The high resistance areas are fairly well defined in Chicago, and they are practically all in regions where the density of arresters is high. A large fraction of the burnouts, however, occur in outlying districts where the density is low and the ground resistance is also low. This situation means that whether or not a transformer is to burn out is not primarily determined in Chicago by the magnitude of the resistance, but by the location of the lightning discharge and by the density of arresters. The existence of the arrester ground does not determine the location or intensity of a lightning discharge, but the resistance may affect the amount of damage to the transformer if it is very high.

d. Our experience and statistics indicate that the use of the interconnection should have little, if any, effect on the hazards to the customer and these hazards at the present time are negligible. A careful study of our records over the past few years show that troubles on customers' premises occur in a few localities each year and the cost of repairing the damaged customers' equipment has been about $200 per year.

Mr. Ekvall thinks that the Philadelphia system of installing arresters on only a part of the transformers in order to get the amount of improvement resulting from the installation of arresters is superior to the system used in Chicago in recent years. The plan favored by Mr. Ekvall was used in earlier years in Chicago, and the results were reported by the writer in a paper before the N.E.L.A. which appears in the Report of the Technical and Hydroelectric Session for 1916, page 86. In those early days we had the same idea as that expressed by Mr. Ekvall, namely, that by some system of records, it would be possible to eliminate the lightning as a variable; but, as expressed in my paper under the heading, "Accuracy of Results," "In spite of all attempts to eliminate the lightning as a variable . . . lightning still remains the dominant variable." If Mr. Ekvall and his associates have been able by their system of records and their analysis to eliminate the lightning as a variable, they are to be congratulated. Perhaps, however, they will change their opinion on this and several other points, as we did in Chicago, with a few more years' experience. It is, of course, very much easier to draw conclusions when only a few facts are available.

Mr. Ekvall criticizes the methods used in Chicago for determining the relative amount of trouble with various sizes of transformers, but his criticism appears to have been answered by Mr. Putman.

T. H. Haines and C. A. Corney: In Mr. K. B. McEachron's discussion he mentions a protective device which was used in Boston for some years previous to the general use of arresters. Mr. McEachron refers to the primary-secondary contact resulting from each operation of the device as being likely to increase the hazard on customers' premises where all grounds in a building are not tied together. It will be of interest to note that although in use for many years and in various locations no serious trouble has ever developed from this cause.

Mr. McEachron calls attention to the result of certain of his calculations which tend to show that complete protection of a distribution transformer connected to an average distribution line cannot be expected with lightning arrester ground resistances in excess of 80 ohms. This need not be interpreted to mean, however, that quite adequate protection cannot be obtained with a ground resistance of 200 ohms or higher, as many induced potentials are less than the maximum which the line can stand and many transformers can and do stand greater potentials than may be assumed as an average value. Experience in Boston where ground resistances are high by comparison with many other districts clearly indicates a marked improvement in the trouble rate due to lightning as a result of the application of lightning arresters. Undoubtedly troubles might be still further reduced with lower ground resistance value, but such protection as is obtained under existing conditions is considered adequate to warrant the application of arresters.

In Mr. A. M. Opsahl's discussion he asks what ground resistances can be attained at the customers' premises under conditions in the Boston territory. In general the customers' grounds are of lower resistance than lightning arrester grounds principally because they are connected to water pipes. This does not apply to outlying territory where water pipes are not available and as a result both types of grounds should be much alike. Secondary network grounds should, of course, be better than lightning arrester grounds due to there usually being more than one ground connection on each secondary network.

In answer to his further questions, we find that our transformer flashovers usually take place over the bushings and at the terminal boards, and that no serious trouble is encountered on the secondary circuits or on customers' premises.

Mr. Herman Halperin requests information on the trouble in the unprotected area where no lightning arresters are installed.

The following table will answer his question.

LIGHTNING TROUBLES IN UNPROTECTED AREA

Year	Total trans-formers	Troubles per 100 transformers			
		Fuses	Miscel-laneous defects	Windings defective	Total troubles
1927	3,070	8.5	4.0	0.2	12.7
1928	3,337	3.3	1.5	0.2	5.0
1929	3,353	4.7	3.5	0.3	8.5
1930	3,450	1.9	2.4	0.3	4.6
1931	3,511	1.6	1.7	0.2	3.5

H. A. Dambly, H. N. Ekvall, and H. S. Phelps: In reply to the questions raised by Mr. A. H. Schirmer and Dr. M. G. Lloyd regarding grounding requirements where the neutral tie is used, we wish to offer the following comments.

As stated in the paper, the neutral tie connection was used in Philadelphia for the first time in 1931. For this trial application, consisting of 151 transformer locations, it was decided to limit the installations to locations where the secondary neutral ground grid consisted of 10 or more water pipe grounds, the ground network to measure not more than 15 ohms resistance. It is realized that these limitations may be unnecessarily conservative.

Mr. Roper expressed preference for the kva-hr. basis over the customer-hr. basis for analyzing customer interruptions. We believe that the kva-hr. basis emphasizes the inconvenience to the large customers, whereas our experience seems to indicate that the customer-hr. basis gives a more complete picture of the inconvenience to customers as a whole. Constant effort to maintain good public relations makes it essential to consider trouble from the standpoint of the largest number of customers.

Messrs. Andrews and Hendrickson believe that high ground resistance in the Philadelphia territory has been partly responsible for arresters being of little or no value on transformers smaller than 15 kva. In their territory, in the section of Joliet, Illinois, where grounds are of lower resistance than in Philadelphia, they found arresters to give considerable protection to the small transformers. Their data show approximately one-half the rate of trouble on protected as on unprotected transformers, 95 per cent of which was below 15 kva. However, if their data were analyzed with respect to transformer size, it would probably be found that the majority of protected transformers are of a larger size than the unprotected. If this is the case then their apparently good arrester performance is most likely due to the fact that transformers of the larger sizes are much less susceptible to lightning trouble than the smaller sizes.

INDEX OF AUTHORS

B

Baños, A., Jr. and Kouwenhoven, W. B., (High Sensitivity Power Factor Bridge) 202
Blackwell, O. B., (The Time Factor in Telephone Transmission) 141
Brookes, A. S., Opsahl, A. M., and Southgate, R. N., (Lightning Protection for Distribution Transformers) 245
Browne, T. E., Jr., (Extinction of A-C. Arcs in Turbulent Gases) 185
Buell, R. C. and Hathaway, C. M., (An Automatic Oscillograph) 222
Burnham, E. J., (Overvoltage on Transmission Systems Caused by Dropping Load) 105

C

Calderwood, E. M. and Smith, D. F., (The San Francisco-Los Angeles Section of the Pacific Coast Telephone Cable Network) 63
Campbell, C. C. and Kalb, H. N., (Radio Coordination) 81
Collins, H. W., Piepho, E. E., and Torok, J. J., (Impulse Voltage Tests on a 4,800-Volt Distribution Substation) 148
Corney, C. A. and Haines, T. H., (Lightning Protection for Distribution Transformers) 259
Crellin, E. A., (The Mokelumne River Development) 28

D

Dambly, H. A., Ekvall, H. N., and Phelps, H. S., (Distribution System Lightning Studies) 265
Daniel, A. F. and Dawes, C. L., (High-Voltage Bridge for Measurements of Cables with Grounded Sheaths) 198
Dawes, C. L. and Daniel, A. F., (High-Voltage Bridge for Measurements of Cables with Grounded Sheaths) 198
Denault, C. L. and Slepian, J., (The Expulsion Fuse) 157
Dexter, B. D. and Wright, E. M., (Power Transmission and Distribution from the Mokelumne River Development) 20
Dunbar, J. R. and Floyd, G. D., (Calorimeter Measurement of Stray-Load Losses of a 55,000-Kva. Generator) 12

E

Ekvall, H. N., Dambly, H. A., and Phelps, H. S., (Distribution System Lightning Studies) 265

F

Floyd, G. D. and Dunbar, J. R., (Calorimeter Measurement of Stray-Load Losses of a 55,000-Kva. Generator) 12

H

Haines, T. H. and Corney, C. A., (Lightning Protection for Distribution Transformers) 259
Harding, C. F. and Sprague, C. S., (Interconnection of Primary Lightning Arrester) 234
Hathaway, C. M. and Buell, R. C., (An Automatic Oscillograph) 222
Hill, H. C. and SeLegue, J. B., (Electrical Equipment for Oil Field Operations) 1
Holsen, J. N., (Forecasting Population for Engineering Purposes) 128
Hough, E. L. and Nelson, L. V., (Interconnection of the 25- and 60-Cycle System of the Union Electric Light and Power Company) 113

K

Kalb, H. N. and Campbell, C. C., (Radio Coordination) 81
Kouwenhoven, W. B. and Baños, A., Jr., (High Sensitivity Power Factor Bridge) 202

L

LaPierre, C. W., (The Photoelectric Recorder) 226
Leeds, W. M. and Van Sickle, R. C., (Recent Developments in Arc Rupturing Devices) 177
Ludwig, L. R. and Slepian, J., (Backfires in Mercury Arc Rectifiers) 92

M

Magnusson, C. Edward, (The Kindling of Electric Sparkover) 74
McEachron, K. B. and Saxon, L., (Lightning Protection for Distribution Transformers) 239

N

Nelson, L. V. and Hough, E. L., (Interconnection of the 25- and 60-Cycle System of the Union Electric Light and Power Company) 113
Nystrom, C. W., (Tape-Armored Telephone Toll Cable) 135

O

Opsahl, A. M., Brookes, A. S., and Southgate, R. N., (Lightning Protection for Distribution Transformers) 245

P

Phelps, H. S., Dambly, H. A., and Ekvall, H. N., (Distribution System Lightning Studies) 265
Piepho, E. E., Collins, H. W., and Torok, J. J., (Impulse Voltage Tests on a 4,800-Volt Distribution Substation) 148
Pilliod, J. J., (Intercontinental Radiotelephone Service from the United States) 56
Powel, C. A. and Purcell, T. E., (Tie-Line Control of Interconnected Networks) 40
Prince, D. C., (The Theory of Oil-Blast Circuit Breakers) 166
Purcell, T. E. and Powel, C. A., (Tie-Line Control of Interconnected Networks) 40

R

Roper, D. W., (Studies in Lightning Protection on 4,000-Volt Circuits—III) 252

S

Saxon, L. and McEachron, K. B., (Lightning Protection for Distribution Transformers) 239
SeLegue, J. B. and Hill, H. C., (Electrical Equipment for Oil Field Operations) 1
Shively, E. K. and Whitlow, G. S., (Automatic Control for Variable-Ratio Frequency Converters) 121
Siskind, R. P., (Power Factor Measurement by the Capacitance Bridge) 214
Skilling, H. H., (Tuned Power Lines) 51
Slepian, J. and Denault, C. L., (The Expulsion Fuse) 157
Slepian, J. and Ludwig, L. R., (Backfires in Mercury Arc Rectifiers) 92
Smith, D. F. and Calderwood, E. M., (The San Francisco-Los Angeles Section of the Pacific Coast Telephone Cable Network) 63
Southgate, R. N., Brookes, A. S., and Opsahl, A. M., (Lightning Protection for Distribution Transformers) 245
Sprague, C. S. and Harding, C. F., (Interconnection of Primary Lightning Arrester) 234
Spurck, R. M., (Practical Application of the Oil-Blast Principle of Circuit Interruption) 171

T

Torok, J. J., Collins, H. W., and Piepho, E. E., (Impulse Voltage Tests on a 4,800-Volt Distribution Substation) 148

V

Van Sickle, R. C. and Leeds, W. M., (Recent Developments in Arc Rupturing Devices) 177

W

Weidlein, W. D., (Corona Energy Loss) 154
Whitlow, G. S. and Shively, E. K., (Automatic Control for Variable-Ratio Frequency Converters) 121
Wright, E. M. and Dexter, B. D., (Power Transmission and Distribution from the Mokelumne River Development) 20

Equivalent Circuits—I

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***Synopsis.**—Equivalent circuits have been found valuable in the solution of certain problems in that they simplify the analysis. Furthermore, equivalent circuits provide a means of replacing magnetic coupling with simple impedance links thereby permitting the use of the calculating board for solution.*

In Part I of this paper, the general equivalent mesh for the n-winding transformer is derived, and other less rigorous transformer circuits are discussed and compared.

In Part II the mesh equivalent for the general network having $(m + l)$ points of entry is derived and its application is explained.

The generalizations of Parts I and II provide a background for the particular problem treated in Part III in which the study of system networks involving groups of parallel transmission lines is considered. The usually difficult problem of accurately determining ground fault currents in such a system is readily solved by applying the special equivalent circuits developed in Part III to a calculating board set-up of the system. These circuits are particularly valuable in the solution of any problem involving an analysis of the zero phase impedance diagram of a system.

The methods and circuits developed in this paper are perfectly general and have many evident applications other than those treated.

* * * * *

INTRODUCTION

VARIOUS authors have used equivalent circuits from time to time in analyzing certain particular problems.[1] The present paper presents general methods of attack suitable for applying equivalent circuits to a great variety of problems. The particular problems treated here concern transformers and networks.

In general, any problem which can be analyzed with an equivalent circuit can be analyzed without it. However, the equivalent circuit may be useful for any or all of the following reasons:

1. Since the equivalent circuit represents only terminal conditions, it is frequently simpler than the actual machine or circuit.
2. It facilitates and reduces calculation in some cases, and even though the amount of calculation has not been reduced, it will have been speeded up and systematized.
3. The equivalent circuit aids in visualizing the problem.
4. Since the equivalent circuit is made up of simple impedance links, it may readily be set up on the calculating board,† thereby lending itself to the solution of complicated networks.

The particular problem which prompted the present study was that of representing a system of coupled circuits by an equivalent composed of simple impedance links. This problem arose primarily through the difficulty experienced in adequately representing on the calculating board groups of parallel transmission lines in which zero phase currents were flowing. In order to represent such a group, it is desirable to replace it with an equivalent circuit in which mutual impedances between the lines are simulated by simple impedance links.

Closely related to the problem of coupled circuits, and probably a broader one, is the general network problem. In the study of transmission and distribution networks, it is often desirable to simplify and reduce them. One method of doing this (the method generalized in this paper) is to replace the actual system with an equivalent network having the minimum number of links required to satisfy terminal conditions.

In order to treat the above mentioned problems adequately, the present paper is divided into *Part I* in which the general mesh linkage to represent *n* electrically isolated but magnetically coupled circuits is developed, and the corresponding star-mesh equivalent is derived; *Part II* in which the equivalent mesh linkage to represent a general network having any number of points of entry is derived; and *Part III* in which, by means of the generalizations developed in Parts I and II, a number of special equivalent circuits representing groups of two, three, and four parallel transmission lines under various conditions are derived. The method of adapting these circuits to a calculating board set-up of the zero phase diagram of a system is fully explained.

The two generalized equivalents discussed in Parts I and II are quite broad and should find many applications other than those cited in this paper.

It should be pointed out here that only linear circuits and networks are dealt with in the present paper. It should also be noted that the circuits developed here are, in general, equivalent only at a single frequency, although certain particular ones are equivalent at all frequencies.‡

Part I

THE TRANSFORMER PROBLEM

In practise, the multi-circuit problem frequently presents itself, that is, the problem of a number of circuits each having a self impedance and a mutual impedance with every other circuit. Notable examples are the

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†The calculating board is referred to in numerous places throughout the paper. It is understood that, in general, an a-c. board will be required since both resistance and reactance are involved in the circuits developed here. If resistance may be neglected, the d-c. board will be adequate.

1. For numbered references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

‡Circuits independent of frequency are shown in Figs. 8, 9, 11, 12, 13, and 14.

multi-winding transformer, a group of parallel transmission lines, the multi-winding generator, etc.[3] The usual problem in such systems is to determine the current in each branch having given the voltage impressed across each circuit.

The multi-circuit problem may always be solved analytically. However, it frequently happens that to do so involves a prohibitive amount of labor, in which case it is desirable to set up the system on the calculating board for solution. If the board is to be used, the system of coupled circuits must be replaced by an equivalent linkage, since magnetic coupling of adequate flexibility usually cannot be simulated on the calculating board.

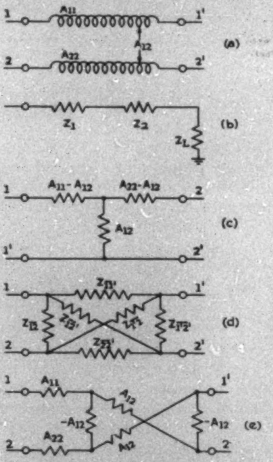

FIG. 1—VARIOUS EQUIVALENT CIRCUITS FOR THE TWO-WINDING TRANSFORMER

a. The two-winding transformer

b. Equivalent circuit used for calculating regulation, magnetizing current being neglected. This circuit gives no voltage transformation

c. Equivalent circuit which gives voltage transformation and does not neglect magnetizing current. This circuit must be viewed from one winding only, since it falls down when viewed as a 4-point network

d. A rigorous equivalent for the two-winding transformer which cannot be distinguished from the actual transformer under any imposed terminal conditions

e. An equivalent equally as rigorous as that of Fig. 1*d*, but much simpler

Before discussing the general equivalent circuit for the *n*-winding transformer, which is derived in Appendix A, it seems pertinent to consider other well known equivalents for coupled circuits, and to point out their limitations as compared with the present development.

In order to study the various types of circuits, the two-winding transformer shown in Fig. 1a will be analyzed. Here, A_{11} is the primary self or magnetizing impedance, A_{22} is the secondary impedance, and A_{12} is the mutual impedance between the windings.

In Fig. 1b is shown a conventional equivalent for the two-winding transformer. This circuit consists of the primary leakage impedance, Z_1 (resistance plus leakage reactance), the secondary leakage impedance, Z_2, and the load impedance, Z_L, all three of which must be expressed on a common voltage base since the same current flows in each. If α is the ratio of primary to secondary turns, then the values of Z_1 and Z_2 referred to the primary are in terms of the characteristic A coefficients,

$$Z_1 = A_{11} - A_{12}\alpha \tag{1}$$

$$Z_2 = A_{22}\alpha^2 - A_{12}\alpha \tag{2}$$

Note in particular that this circuit neglects magnetizing current, and that it does not simulate the voltage transformation of the actual transformer. The circuit of Fig. 1b is quite useful in calculating regulation since magnetizing current in a loaded transformer can usually be neglected. This equivalent is universally used to represent the two-winding transformer in the single line diagram of a polyphase system. Similar circuits have been developed for the three-winding transformer* the double-winding generator, etc.

Now consider the circuit shown in Fig. 1c, which likewise represents a two-winding transformer with loaded secondary. Unlike the circuit of Fig. 1b, this circuit gives actual voltage transformation. Moreover, it does not neglect magnetizing current. The magnetizing impedances of the primary and secondary are respectively A_{11} and A_{22}. The short-circuit impedance of the primary is

$$Z = \frac{A_{11}A_{22} - A_{12}^2}{A_{22}} \tag{3}$$

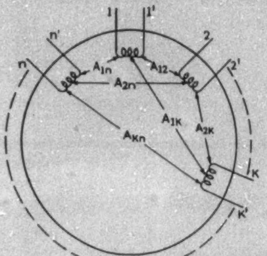

FIG. 2—A SYSTEM OF *n* ELECTRICALLY ISOLATED BUT MAGNETICALLY COUPLED CIRCUITS

Such a system may be replaced by an equivalent mesh of $n(2n - 1)$ links connecting each of the $2n$ points of entry in all possible ways. For example of equivalent mesh when $n = 2$, see Fig. 1d

---

*Mr. A. Boyajian demonstrated the value of this type of circuit in analyzing the three-winding transformer in his paper, *Theory of Three-Circuit Transformers*, TRANS. A.I.E.E., Feb. 1924. In the same paper, he writes the general equations for setting up a similar type of circuit for the *n*-winding transformer.

Thus, it is seen that the circuit of Fig. 1c more nearly represents the actual transformer. However, as a network having four points of entry (*i.e.*, 1, 1′, 2, and 2′), the equivalent of Fig. 1c is still not strictly correct. For example, the impedance between points 1 and 2, $Z_{1\text{-}2}$ is infinite on the actual transformer, whereas on the equivalent circuit, this impedance has a finite value.

The General Equivalent Circuit. A true equivalent circuit for a transformer is one which cannot be distinguished from the actual transformer by any observations which can be made at the terminals. Evidently the circuits (*b*) and (*c*) of Fig. 1 fall down in this respect.

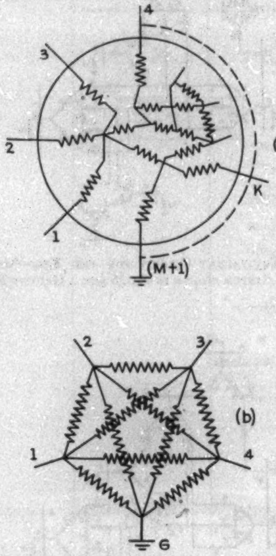

FIG. 3

a. Diagrammatic representation of the general network composed of many erratic links and having $(m + 1)$ points of entry. A general network of this type may be simplified by replacing it with an equivalent mesh whose links are easily evaluated in terms of simple impedance measurements made at the terminals

b. Example of the equivalent mesh for a network having five points of entry, the fifth point being considered as a ground point

In Appendix A, there is derived a general equivalent circuit for the *n*-winding transformer having $2n$ points of entry. This circuit is a mesh composed of $n\,(2n - 1)$ impedance links, one connecting each point of entry with every other point of entry. It is shown that the impedance of the link connecting the two points of entry of some winding *j*, *i. e.*, points *j* and *j*′, is the driving point impedance from point *j*; and the link connecting two points of entry of two different windings, say points *j* and *k*′ of windings *j* and *k*, is equal to the transfer impedance between points *j* and *k*′.

In Appendix A, it is further shown how these link impedances may be measured directly or calculated from known transformer constants, the *A* coefficients.

The general equivalent circuit for the two-winding transformer* is shown in Fig. 1d, and is seen to consist of a six-link mesh, the values of which are[4–8]

$$Z_{\overline{11'}} = \frac{A_{11}A_{22} - A_{12}^2}{A_{22}} \tag{4}$$

$$Z_{\overline{22'}} = \frac{A_{11}A_{22} - A_{12}^2}{A_{11}} \tag{5}$$

$$Z_{\overline{12}} = \frac{A_{11}A_{22} - A_{12}^2}{A_{12}} \tag{6}$$

$$= Z_{\overline{1'2'}} = -\,Z_{\overline{12'}} = -\,Z_{\overline{1'2}}$$

Although there are six links, it is evident from the above equations that only three of them are independent, since four links have values differing in sign only.

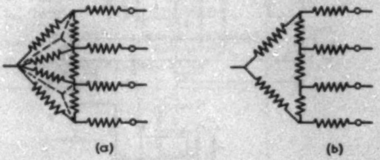

FIG. 4—THESE DIAGRAMS ARE GIVEN TO SHOW THAT AN EQUIVALENT MAY NOT ALWAYS BE UNIQUE

The circuit (*a*) has 11 links only, 9 of which are unique and independent since, by the indicated Y-delta transformations, it may be reduced to circuit (*b*) which has only 9 links. Note that the number of points of entry has not changed

A circuit which is much simpler than that of Fig. 1d, but equally as rigorous and having identical properties, is that shown in Fig. 1e. This circuit is seen to be a combination mesh and star arrangement, and may be arrived at by simple reasoning from the mesh of Fig. 1d. Since the self-impedance of any winding is independent of the other windings, its value need not appear in any of the mutual links if it be lumped as a radial link at the terminals of the circuit. Thus, setting A_{11} and A_{22} equal to zero in equations (**4**), (**5**), and (**6**), and inserting radial links having impedances equal to A_{11} and A_{22} in terminals 1 and 2 respectively, one will obtain the circuit of Fig. 1e. This latter circuit is much more useful than that of Fig. 1d since the mutual links are purely reactive, the resistances appearing only in the terminal links. This arrangement avoids the possibility of negative resistance appearing in any link.

In an entirely similar manner, the mesh equivalent of any number of coupled circuits may be simplified and rendered more useful by reducing to a combination star-

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*The circuit for the two-winding transformer was given by Geo. A. Campbell in his paper, *Cisoidal Oscillations*, PROC. A.I.E.E., 1911. Equations for the *n*-winding transformer, however, have never been published.

mesh arrangement. A rigorous proof for this transformation is given in Appendix A.

Part II

THE NETWORK PROBLEM

In the study of the behavior of a large transmission and distribution system, the transmission lines and connected apparatus are commonly represented by an appropriate single line diagram, all units of connected equipment being replaced with suitable impedance links. In most cases, no trouble is experienced in mapping out the single-line diagram, at least none for the positive and negative phase diagrams,* since standard methods are available for doing this.[9,10]

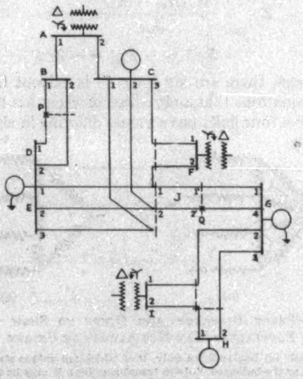


FIG. 5—SINGLE-LINE DIAGRAM OF A SYSTEM OF TRANSMISSION LINES

This system is made up of several groups of parallel lines. In Figs. 8 to 14 is given a number of equivalent circuits to represent such groups when the zero-phase diagram is to be set up on the calculating board

In setting up the single line diagram on the calculating board, for purposes of simplification, it may be desirable to replace either the whole system or some section of it with an equivalent impedance mesh which has the same number of points of entry but a greatly reduced number of links.[12] For example, Fig. 3a shows a complicated network of impedance links having $(m + 1)$ points of entry. It is shown in Appendix B how such a network may be replaced with an equivalent mesh having $\frac{m\,(m+1)}{2}$ links, one connecting each point of entry with every other point of entry. The equivalent mesh for a five-point network is shown in Fig. 3b (note that $m = 4$ and that the $m + 1$ point is considered as a ground point for purposes of analysis).

The values of the links of the equivalent mesh which is to replace the general network may be determined either by direct calculation from the network constants or by measurements made at the terminals of the network. It is shown in Appendix B that the impedance of the link connecting any two points of entry such as j and k is equal to the transfer impedance between the points j and k, *i. e.*,

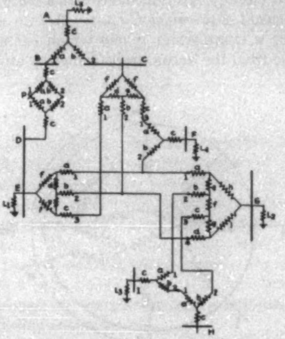


FIG. 6—EQUIVALENT CIRCUIT FOR THE ZERO-PHASE DIAGRAM OF THE SYSTEM SHOWN IN FIG. 5 FOR A GROUND FAULT AT POINT *P*

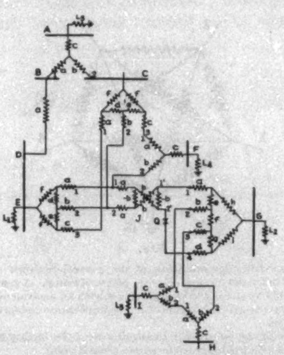


FIG. 7—EQUIVALENT CIRCUIT FOR THE ZERO-PHASE DIAGRAM OF THE SYSTEM SHOWN IN FIG. 5 FOR A GROUND FAULT AT POINT *Q*

$$Z_{\overline{jk}} = Z_{jk} \tag{7}$$

It is further shown that the impedances of the links of the equivalent mesh may be calculated by determinants in terms of the through impedances of the network (by through impedance between any two points such as j and k is meant the impedance between these points with all other points open). In terms of the notation defined in Appendix E

---

*The zero-phase single-line diagram of a system is not so easily set up if the system involves groups of parallel transmission lines. This particular problem is discussed fully in Part III of the paper.

$$Z_{\overline{jk}} = \frac{\Delta_{m-c}}{Q_{c-jk}} \tag{8}$$

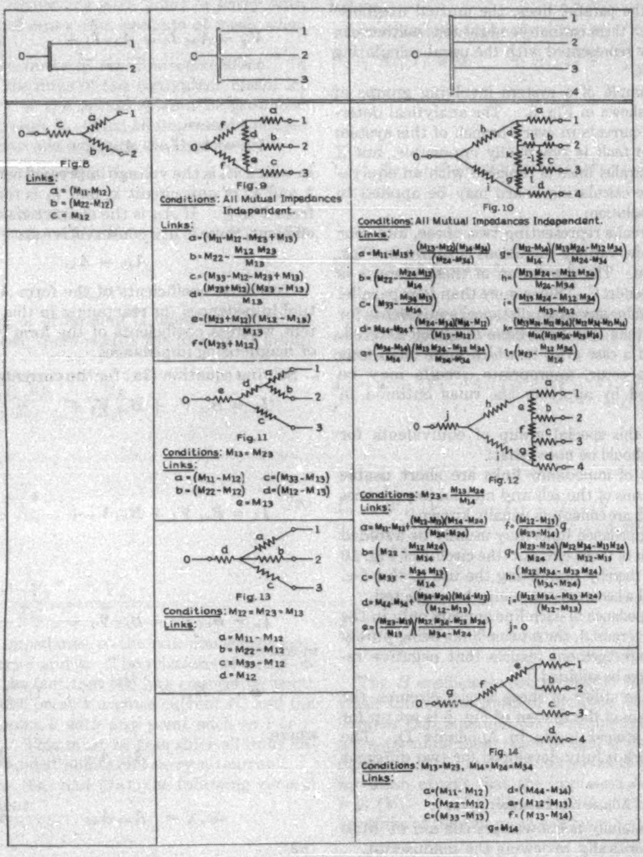
where the characteristic coefficients of the determinants Δ_{m-c} and Q_{c-jk} are

$$C_{jj} = Z_{j-0} \tag{9}$$

$$C_{kk} = Z_{k-0} \tag{10}$$

$$C_{jk} = \frac{Z_{j-0} + Z_{k-0} - Z_{j-k}}{2} \tag{11}$$

The methods developed in Appendix B apply to any general network provided the impedances are linear and provided there are no internal voltages. Note that, although a network is usually composed of simple impedance links, the general network as it is considered here may involve magnetic coupling between any of its elements. In fact, the general network mesh may be applied directly in obtaining an equivalent mesh derived in Appendix A for the n-winding transformer is much simpler and more appropriate to the transformer problem than is the general network mesh.



0 1 2
0 1 2 3
0 1 2 3 4
Fig. 8
Links:
$a = (M_{11}-M_{12})$
$b = (M_{22}-M_{12})$
$c = M_{12}$
Fig. 9
Conditions: All Mutual Impedances Independent.
Links:
$a = (M_{11}-M_{12}-M_{23}+M_{13})$
$b = M_{22} - \frac{M_{12}M_{23}}{M_{13}}$
$c = (M_{33}-M_{12}-M_{23}+M_{13})$
$d = \frac{(M_{23}+M_{12})(M_{23}-M_{13})}{M_{13}}$
$e = \frac{(M_{23}+M_{12})(M_{12}-M_{13})}{M_{13}}$
$f = (M_{23}+M_{12})$
Fig. 10
Conditions: All Mutual Impedances Independent.
Links:
$a = M_{11}-M_{13}+\frac{(M_{13}-M_{12})(M_{14}-M_{34})}{(M_{24}-M_{34})}$
$g = \left(\frac{M_{12}-M_{14}}{M_{14}}\right)\left(\frac{M_{13}M_{24}-M_{12}M_{34}}{M_{24}-M_{34}}\right)$
$b = \left(M_{22}-\frac{M_{24}M_{12}}{M_{14}}\right)$
$h = \left(\frac{M_{13}M_{24}-M_{12}M_{34}}{M_{24}-M_{34}}\right)$
$c = \left(M_{33}-\frac{M_{34}M_{13}}{M_{14}}\right)$
$i = \left(\frac{M_{13}M_{24}-M_{12}M_{34}}{M_{13}-M_{12}}\right)$
$d = M_{44}-M_{24}+\frac{(M_{24}-M_{34})(M_{14}-M_{12})}{(M_{13}-M_{12})}$
$k = \frac{(M_{13}M_{24}-M_{12}M_{34})(M_{12}M_{34}-M_{13}M_{14})}{M_{14}(M_{13}M_{24}-M_{23}M_{14})}$
$e = \left(\frac{M_{34}-M_{14}}{M_{14}}\right)\left(\frac{M_{13}M_{24}-M_{12}M_{34}}{M_{24}-M_{34}}\right)$
$l = \left(M_{23}-\frac{M_{12}M_{34}}{M_{14}}\right)$
Fig. 11
Conditions: $M_{13} = M_{23}$
Links:
$a = (M_{11}-M_{12})$
$c = (M_{33}-M_{13})$
$b = (M_{22}-M_{12})$
$d = (M_{12}-M_{13})$
$e = M_{13}$
Fig. 12
Conditions: $M_{23} = \frac{M_{13}M_{24}}{M_{14}}$
Links:
$a = M_{11}-M_{12}+\frac{(M_{12}-M_{13})(M_{14}-M_{24})}{(M_{34}-M_{24})}$
$f = \frac{(M_{12}-M_{13})}{(M_{13}-M_{14})}g$
$b = \left(M_{22}-\frac{M_{12}M_{24}}{M_{14}}\right)$
$g = \left(\frac{M_{23}-M_{24}}{M_{24}}\right)\left(\frac{M_{12}M_{34}-M_{13}M_{24}}{M_{12}-M_{13}}\right)$
$c = \left(M_{33}-\frac{M_{34}M_{13}}{M_{14}}\right)$
$h = \frac{(M_{12}M_{34}-M_{13}M_{24})}{(M_{34}-M_{24})}$
$d = M_{44}-M_{34}+\frac{(M_{34}-M_{24})(M_{14}-M_{13})}{(M_{12}-M_{13})}$
$i = \frac{(M_{12}M_{34}-M_{13}M_{24})}{(M_{12}-M_{13})}$
$e = \left(\frac{M_{23}-M_{13}}{M_{13}}\right)\left(\frac{M_{12}M_{34}-M_{13}M_{24}}{M_{34}-M_{24}}\right)$
$j = 0$
Fig. 13
Conditions: $M_{12} = M_{23} = M_{13}$
Links:
$a = M_{11}-M_{12}$
$b = M_{22}-M_{12}$
$c = M_{33}-M_{12}$
$d = M_{12}$
Fig. 14
Conditions: $M_{13} = M_{23}$, $M_{14} = M_{24} = M_{34}$
Links:
$a = (M_{11}-M_{12})$
$d = (M_{44}-M_{34})$
$b = (M_{22}-M_{12})$
$e = (M_{12}-M_{13})$
$c = (M_{33}-M_{13})$
$f = (M_{13}-M_{14})$
$g = M_{14}$

FIGS. 8–14

Part III

THE ZERO PHASE SYSTEM DIAGRAM

As pointed out above, the zero phase diagram of a system which involves parallel transmission lines does not lend itself so readily to a calculating board set up. This is owing to the fact that, when zero phase currents are flowing in the parallel lines, the mutual magnetic coupling between them cannot be neglected; neither can it be adequately represented with the usual calculating board equipment.*

A typical example of a system involving groups of parallel lines is shown in Fig. 5. The analytical determination of the currents in every branch of this system due to a ground fault is practically impossible, but if each group of parallel lines is replaced with an equivalent linkage, the calculating board may be applied to obtain an exact solution.

A group of circuits representing two, three, and four parallel lines under various conditions is shown in Figs. 8 to 14 inclusive. The derivation of these circuits is discussed in Appendix C. Since more than four parallel lines seldom exist for any great distance, equivalents for groups of more than four lines have not been derived. However, should a case arise in which more than four parallel lines do occur, appropriate circuits may be readily developed by applying the rules outlined in Appendix C.

In regard to this special group of equivalents for parallel lines, it should be noted that:

1. The values of impedance links are short usable expressions in terms of the self and mutual impedances of the lines, which are constants usually known.
2. Negative impedance links may usually be avoided in the application of these circuits (the circuit of Fig. 10 being excepted) thereby permitting the use of the d-c. calculating board where resistance may be neglected.
3. The self-impedance of each line appears only in the radial link at the terminal, the mutual links being purely reactive. This arrangement insures that negative resistance will always be avoided.

To illustrate the utility of these special circuits, the zero phase diagram of the system of Fig. 5 is set up for the solution of ground faults in Appendix D. The equivalent network is fully developed for two different fault conditions.

ACKNOWLEDGMENT

The author gratefully acknowledges the aid of Miss Edith Clarke in critically reviewing the manuscript.

Appendix A

THE GENERAL EQUIVALENT CIRCUIT FOR THE *N*-WINDING TRANSFORMER

Consider the system of mutually coupled circuits shown in Fig. 2, and having $2n$ points of entry. The canonical equations which completely define such a system are[17]

$$\begin{aligned} V_1 &= A_{11} I_1 + A_{12} I_2 + \ldots\ldots A_{1n} I_n \\ &\vdots \\ V_k &= A_{k1} I_1 + A_{k2} I_2 + \ldots\ldots A_{kn} I_n \qquad \textbf{(1a)} \\ &\vdots \\ V_n &= A_{n1} I_1 + A_{n2} I_2 + \ldots\ldots A_{nn} I_n \end{aligned}$$

in which V_k is the voltage impressed across the terminals k and k' of any circuit k, and I_k is the current flowing from k to k'. If A_{jk} is the characteristic impedance coefficient, there is in a conservative system

$$A_{jk} = A_{kj} \qquad \textbf{(2a)}$$

Note that coefficients of the form A_{jk} represent mutual impedances (or reactances in this case) of the system, whereas coefficients of the form A_{jj} represent self or magnetizing impedances.

Solving equation (**1a**) for the currents

$$\begin{aligned} I_1 &= B_{11} V_1 + B_{12} V_2 + \ldots\ldots B_{1n} V_n \\ &\vdots \\ I_k &= B_{k1} V_1 + B_{k2} V_2 + \ldots\ldots B_{kn} V_n \qquad \textbf{(3a)} \\ &\vdots \\ I_n &= B_{n1} V_1 + B_{n2} V_2 + \ldots\ldots B_{nn} V_n \end{aligned}$$

in which[16]

$$B_{jk} = B_{kj} = \frac{Q_{A-jk}}{\Delta_{n-A}} (-1)^{j+k} \qquad \textbf{(4a)}$$

where

$$\Delta_{n-A} = \begin{vmatrix} A_{11} & A_{12} & \ldots & A_{1n} \\ \vdots & & & \\ A_{k1} & A_{k2} & \ldots & A_{kn} \\ \vdots & & & \\ A_{n1} & A_{n2} & \ldots & A_{nn} \end{vmatrix} \qquad \textbf{(5a)}$$

and Q_{A-jk} is the cofactor of A_{jk} in Δ_{n-A}, (*i. e.*, the minor of Δ_{n-A} with the j'th column and k'th row deleted), the sign of which is $(-1)^{j+k}$.

There are n^2 coefficients in equation (**1a**), $\dfrac{n(n+1)}{2}$ of which are independent. Therefore an equivalent linkage which represents the system must have $2n$ points of entry and at least $\dfrac{n(n+1)}{2}$ independent links. A convenient equivalent linkage which satisfies these conditions is the straight mesh consisting of a system of links, one connecting each point of entry with every other point of entry (for example of mesh when $n = 2$, see Fig. 1d).

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*It is conceivable that actual transformers might be used for this purpose, but the multiplicity of windings with the highly flexible couplings required make such a method impracticable.

Evaluation of the Links of the Equivalent Mesh. In order to evaluate the links of the equivalent mesh, let all points of entry of the actual system be grounded except point k of circuit k. Then all impressed voltages except V_k will be zero and equation (**3a**) becomes

$$
\begin{aligned}
I_1 &= B_{1k} V_k \\
&\;\;\vdots \\
I_k &= B_{kk} V_k \qquad\qquad (\mathbf{6a}) \\
&\;\;\vdots \\
I_n &= B_{nk} V_k
\end{aligned}
$$

If the condition of grounding is likewise applied to the equivalent mesh, there results the analagous group of relations

$$
\begin{aligned}
I_1 &= \frac{-V_k}{Z_{\overline{k1}}} = \frac{V_k}{Z_{\overline{k1'}}} \\
&\;\;\vdots \\
I_k &= \frac{V_k}{Z_{\overline{kk}}} \qquad\qquad (\mathbf{7a}) \\
&\;\;\vdots \\
I_n &= \frac{-V_k}{Z_{\overline{kn}}} = \frac{V_k}{Z_{\overline{kn'}}}
\end{aligned}
$$

where $Z_{\overline{jk}}$ is the impedance of the link connecting any two points of entry j and k. The relations of equation (**7a**) follow from the fact that the link connecting point k with the grounded point k' carries current I_k, and the link connecting point k with any point such as j or j' carries current I_j. This must be true since all links between pairs of grounded points can carry no current.

From equations (**6a**) and (**7a**), the following general relations are evident:

$$Z_{\overline{kk'}} = \frac{1}{B_{kk}} \qquad (\mathbf{8a})$$

$$Z_{\overline{jk'}} = \frac{1}{B_{\overline{jk}}} = Z_{\overline{j'k}} = -Z_{\overline{j'k'}} = -Z_{\overline{jk}} \qquad (\mathbf{9a})$$

It may be observed that links of the equivalent mesh connecting the two points of entry of the same winding are driving point impedances, and links connecting points of entry of different windings are transfer impedances, all of which may be measured directly as indicated in equation (**7a**), or calculated as indicated in equation (**4a**).

The Combination Star-Mesh Equivalent. The general equivalent developed above for the n-winding transformer may be reduced to a much simpler combination star-mesh equivalent, the derivation and proof of which follow.

Let equation (**4a**) be rewritten as

$$
\begin{aligned}
V_1 - A_{11} I_1 &= 0 + A_{12} I_2 + \;.\;.\;.\;.\; + A_{1n} I_n \\
&\;\;\vdots \\
V_k - A_{kk} I_k &= A_{k1} I_1 + A_{k2} I_2 + \;.\;.\;.\;.\; + A_{kn} I_n \\
&\;\;\vdots \\
V_n - A_{nn} I_n &= A_{n1} I_1 + A_{n2} I_2 + \;.\;.\;.\;.\; + 0 \qquad (\mathbf{10a})
\end{aligned}
$$

Considering the left-hand members of equation (**10a**) as the impressed voltages, and solving for currents as before, one obtains

$$
\begin{aligned}
I_1 &= B'_{11}(V_1 - A_{11} I_1) + B'_{12}(V_2 - A_{22} I_2) + \;.\;.\;. \\
&\qquad + B'_{1n}(V_n - A_{nn} I_n) \\
&\;\;\vdots \\
I_k &= B'_{k1}(V_1 - A_{11} I_1) + B'_{k2}(V_2 - A_{22} I_2) + \;.\;.\;. \\
&\qquad + B'_{kn}(V_n - A_{nn} I_n) \\
&\;\;\vdots \\
I_n &= B'_{n1}(V_1 - A_{11} I_1) + B'_{n2}(V_2 - A_{22} I_2) + \;.\;.\;. \\
&\qquad + B'_{nn}(V_n - A_{nn} I_n)
\end{aligned}
$$

The B coefficients are primed in this case to distinguish them from those of equation (**3a**). In the same manner that equation (**3a**) was interpreted to represent a $2\,n$-terminal mesh (such as that of Fig. 4b for $n = 3$), equation (**11a**) may be interpreted to represent a similar mesh except that the impressed voltage on circuit k is $(V_k - A_{kk} I_{kk})$ instead of V_k. The impedances of the links of the new mesh will be

$$Z'_{\overline{jj'}} = \frac{1}{B'_{jj}} \qquad (\mathbf{12a})$$

$$Z'_{\overline{kk'}} = \frac{1}{B'_{kk}} \qquad (\mathbf{13a})$$

$$Z'_{\overline{jk'}} = \frac{1}{B'_{jk}} = Z'_{\overline{j'k}} = -Z'_{\overline{j'k'}} = -Z'_{\overline{jk}} \qquad (\mathbf{14a})$$

Since I_k flows into point k, and since $A_{kk}I_{kk}$ is a voltage drop due to I_k alone, then this drop may be replaced with a series impedance link equal to A_{kk} in the terminal of point k. It is evident that the combination star-mesh is exactly equivalent to the pure mesh since a voltage V_k is impressed at terminal k and a current I_k flows into terminal k in both circuits. Note that the radial impedance link may be placed in either terminal k or k', or may be distributed in both.

Appendix B

THE EQUIVALENT MESH LINKAGE FOR A GENERAL NETWORK HAVING ANY NUMBER OF POINTS OF ENTRY

Consider the general network shown in Fig. 3a, which it is desirable to replace with an equivalent mesh linkage. Assume this network to have $(m + 1)$ points of entry, where m is any number. Let the $(m + 1)$ point of entry be the ground point or point of zero potential. If a point of zero potential does not exist, this terminal may be arbitrarily specified as such in order to provide a reference point for the voltages impressed at the other m terminals.

The canonical equations which define the network at its terminals are[17]

$$\begin{aligned}
V_{10} &= C_{11}I_1 + C_{12}I_2 + \ldots + C_{1m}I_m \\
&\cdot \\
&\cdot \\
V_{k0} &= C_{k1}I_1 + C_{k2}I_2 + \ldots + C_{km}I_m \qquad (1b) \\
&\cdot \\
&\cdot \\
V_{m0} &= C_{m1}I_1 + C_{m2}I_2 + \ldots + C_{mm}I_m
\end{aligned}$$

where V_{k0} is the voltage impressed from any point k to ground, I_k is the current flowing into point k, and C_{jk} is the characteristic impedance coefficient whose value is to be determined.

The solution of equation (**1b**) for currents gives

$$\begin{aligned}
I_1 &= D_{11}V_{10} + D_{12}V_{20} + \ldots + D_{1m}V_{m0} \\
&\cdot \\
&\cdot \\
I_k &= D_{k1}V_{10} + D_{k2}V_{20} + \ldots + D_{km}V_{m0} \qquad (2b) \\
&\cdot \\
&\cdot \\
I_m &= D_{m1}V_{10} + D_{m2}V_{20} + \ldots + D_{mm}V_{m0}
\end{aligned}$$

Expressed by means of determinants as functions of the C coefficients, the D coefficients are

$$D_{jk} = D_{kj} = \frac{Q_{c-jk}}{\Delta_{m-c}}(-1)^{j+k} \qquad (3b)$$

where

$$\Delta_{m-c} = \begin{vmatrix} C_{11} & C_{12} & \ldots & C_{1m} \\ \cdot & & & \\ \cdot & & & \\ C_{k1} & C_{k2} & \ldots & C_{km} \\ \cdot & & & \\ \cdot & & & \\ C_{m1} & C_{m2} & \ldots & C_{mm} \end{vmatrix} \qquad (4b)$$

The relations between the C and D coefficients of equations (**1b**) and (**2b**) are exactly analagous to the relations between the A and B coefficients of equations (**1a**) and (**3a**). In equation (**3b**) above, Q_{C-jk} is the cofactor of C_{jk} in Δ_{m-C}. Since a mesh composed of $\frac{m(m+1)}{2}$ links connecting $(m + 1)$ points of entry in all possible ways may be just as completely defined by equations (**1b**) and (**2b**) as the network itself, it follows that the network may be replaced by an equivalent mesh having $(m + 1)$ points of entry and $\frac{m(m+1)}{2}$ links. The equivalent mesh representing a network having five points of entry is shown in Fig. 3a.

Referring to Figs. 3a and 3b, let a voltage V_k be impressed at point k of the network and let all other points of entry be grounded. Then equation (**2b**) becomes

$$\begin{aligned}
\frac{V_{k0}}{I_1} &= \frac{1}{D_{1k}} \\
&\cdot \\
&\cdot \\
\frac{V_{k0}}{I_k} &= \frac{1}{D_{kk}} \qquad (5b) \\
&\cdot \\
&\cdot \\
\frac{V_{k0}}{I_m} &= \frac{1}{D_{mk}}
\end{aligned}$$

Now let the same conditions obtain with the equivalent mesh (the five-point mesh of Fig. 3b may be used for reference). Current entering ground from any grounded point j will be negative in sign and must flow through the link $Z_{\overline{jk}}$ since links connecting all grounded points can carry no current. In general then

$$\frac{V_{k0}}{I_j} = -Z_{\overline{jk}} = -Z_{jk} = \frac{1}{D_{jk}} \qquad (6b)$$

The link connecting points k and o is not the driving point impedance from k to ground as might be supposed by analogy with equation (**8a**), but rather the transfer impedance between points *k* and *o*. Thus,

$$\frac{V_{k0}}{I_0} = Z_{\overline{k0}} = \frac{1}{\sum\limits_{j=1}^{j=m} D_{jk}} \tag{7b}$$

where

$$I_0 = \sum_{j=1}^{j=m} I_j$$

The method of measuring the *D* coefficients is evident from equation (**5b**).

Evaluation of the C Coefficients. The *C* coefficients of equation (**1b**) may be evaluated by measuring voltages, whereas the *D*'s were evaluated by measuring currents. In equation (**1b**), let all currents except I_k equal zero (*i. e.*, let all points of entry except *k* be opened up). Then,

$$V_{k0} = C_{kk}I_k \tag{8b}$$

$$V_{j0} = C_{jk}I_k \tag{9b}$$

where V_{j0} is the voltage appearing between point *j* and ground when current I_k flows. By impressing a voltage to ground on each of the terminals in succession, and measuring the voltage to ground appearing on each of the open terminals, equations (**8b**) and (**9b**) will suffice to determine all *C* coefficients. If it is not convenient to measure voltages, the following method is available.

Referring to equation (**1b**), let all currents equal zero except I_j and I_k where *j* and *k* are any two points of entry. Also let the condition $I_j + I_k = 0$ be imposed. Then

$$V_{j0} = I_jC_{jj} + I_kC_{jk} \tag{10b}$$

$$V_{k0} = I_jC_{jk} + I_kC_{kk} \tag{11b}$$

which when solved for currents give

$$\begin{aligned} \frac{V_{j0} - V_{k0}}{I} &= C_{jj} + C_{kk} - 2\,C_{jk} \\ &= Z_{j-k} \end{aligned} \tag{12b}$$

where Z_{j-k} is the through impedance between terminals *j* and *k*.

Referring again to equation (**1b**), let V_{k0} be impressed to ground at *k* with all other points open. Then the through impedance from any point *k* to ground is

$$\frac{V_{k0}}{I_k} = Z_{k-0} = C_{kk} \tag{13b}$$

Also $Z_{j-0} = C_{jj}$ (**14b**)

From equations (**12b**), (**13b**), and (**14b**)

$$\frac{Z_{j-0} + Z_{k-0} - Z_{j-k}}{2} = C_{jk} \tag{15b}$$

Equations (**13b**), (**14b**), and (**15b**) give the fundamental relations between the *C* coefficients and the through impedance measurements (or calculations) of the network.

Appendix C

EQUIVALENT CIRCUITS FOR TWO, THREE, AND FOUR PARALLEL TRANSMISSION LINES BUSSED AT ONE END

For purposes of analysis, a group of parallel lines bussed at one end may be considered as a network. Thus, two parallel lines constitute a three-point network, three parallel lines constitute a four-point network, etc. Since the lines are bussed, the through impedance between any two points of entry of such a network will be finite, and therefore equations (**13b**), (**14b**), and (**15b**) may be applied.

As pointed out in Appendix B, a network having $(m + 1)$ points of entry might be represented by the general mesh equivalent. However, a network may be represented by other equivalents having the same number of independent links provided the arrangement of links is unique and non-reducible. Circuits of this type are shown in Figs. 8 to 14 inclusive, and are seen to be combination star-mesh arrangements. The reasons for adopting these particular linkage arrangements will be evident from the following derivations.

TWO PARALLEL LINES BUSSED AT ONE END, FIG. 8

Consider the case of two parallel lines bussed at one end and having self-impedances of M_{11} and M_{22},* and having a mutual impedance between them of M_{12}. The system is a three-point network, and may be represented by two voltage equations. If the bus be considered as the ground or zero point, there is from equations (**13b**), (**14b**), and (**15b**), noting that the *M* coefficients replace the *C* coefficients in this case,

$$Z_{1-0} = M_{11} \tag{1c}$$

$$Z_{2-0} = M_{22} \tag{2c}$$

$$\frac{Z_{1-0} + Z_{2-0} - Z_{1-2}}{2} = M_{12} \tag{3c}$$

Since there are $\dfrac{m\,(m+1)}{2}$ or 3 independent coefficients which describe the system, there must be a minimum of three links in the equivalent. Either a *Y* or a delta satisfies the necessary conditions, but a *Y* is simpler and will be used here. Referring to Fig. 8,† one may write

$$Z_{1-0} = a + c \tag{4c}$$

$$Z_{2-0} = b + c \tag{5c}$$

$$Z_{1-2} = a + b \tag{6c}$$

---

*By self-impedance is meant a term of the form $(r + jx)$ where x is a reactance corresponding to self-inductance. The mutual coefficient M_{jk} is purely reactive. In some cases, resistance may be neglected, in which cases all coefficients are reactive.

†This circuit was derived by Edith Clarke in *Simultaneous Faults*, (see Bibliography 8).

From equations (**1c**) to (**6c**), it follows that

$$a = M_{11} - M_{12} \tag{7c}$$

$$b = M_{22} - M_{12} \tag{8c}$$

$$c = M_{12} \tag{9c}$$

FOUR PARALLEL LINES BUSSED AT ONE END, FIG. 12

Here the method of attack is exactly the same as in the previous case. In this case,

$$m = 4, \quad \frac{m\,(m+1)}{2} = 10,$$

and the minimum number of links is ten. The radial links a, b, c, and d identify the self impedance coefficients, and the links in the loop identify the mutuals.

Link j has been included to illustrate a principle. Note in the simultaneous equations below that j appears in exactly the same manner in each equation. Hence, it will be automatically eliminated in the solution and cannot be considered as an independent link. The circuit of Fig. 12 does not have, then, the requisite ten links but has only nine, one mutual link being deficient. Since six mutual impedances are being expressed in terms of five links, it is necessary that one of the mutuals be related to the others in some manner. In general, one may choose which mutual is to be independent.

From Fig. 12, the ten circuit equations for this case are

$$Z_{1\text{-}0} = a + j + \frac{(e+f+g+i)\,h}{D} \tag{10c}$$

$$Z_{2\text{-}0} = b + j + \frac{(f+g+i)\,(h+e)}{D} \tag{11c}$$

$$Z_{3\text{-}0} = c + j + \frac{(h+e+f)\,(g+i)}{D} \tag{12c}$$

$$Z_{4\text{-}0} = d + j + \frac{(h+e+f+g)\,i}{D} \tag{13c}$$

$$Z_{1\text{-}2} = a + b + \frac{(f+g+i+h)\,e}{D} \tag{14c}$$

$$Z_{1\text{-}3} = a + c + \frac{(g+i+h)\,(e+f)}{D} \tag{15c}$$

$$Z_{1\text{-}4} = a + d + \frac{(e+f+g)\,(h+i)}{D} \tag{16c}$$

$$Z_{2\text{-}3} = b + c + \frac{(g+i+h+e)\,f}{D} \tag{17c}$$

$$Z_{2\text{-}4} = b + d + \frac{(h+e+i)\,(f+g)}{D} \tag{18c}$$

$$Z_{3\text{-}4} = c + d + \frac{(h+e+f+i)\,g}{D} \tag{19c}$$

where

$$D = e + f + g + h + i \tag{20c}$$

Applying the general equations (**13b**), (**14b**), and (**15b**) simultaneously as before, and solving equations (**10c**) to (**19c**) simultaneously, one obtains for the impedances of the ten links,

$$a = M_{11} - M_{12} + \frac{(M_{11} - M_{13})\,(M_{14} - M_{24})}{(M_{34} - M_{24})} \tag{21c}$$

$$b = M_{22} - \frac{M_{12}\,M_{24}}{M_{14}} \tag{22c}$$

$$c = M_{33} - \frac{M_{34}\,M_{13}}{M_{14}} \tag{23c}$$

$$d = M_{44} - M_{34} + \frac{(M_{34} - M_{24})\,(M_{14} - M_{13})}{(M_{12} - M_{13})} \tag{24c}$$

$$e = \frac{(M_{23} - M_{13})}{M_{13}} \left(\frac{M_{12}\,M_{34} - M_{13}\,M_{24}}{M_{34} - M_{24}} \right) \tag{25c}$$

$$f = \frac{(M_{12} - M_{13})}{(M_{13} - M_{14})}\, g \tag{26c}$$

$$g = \left(\frac{M_{23} - M_{24}}{M_{24}} \right) \left(\frac{M_{12}\,M_{34} - M_{13}\,M_{24}}{M_{12} - M_{13}} \right) \tag{27c}$$

$$h = \left(\frac{M_{12}\,M_{24} - M_{13}\,M_{24}}{M_{34} - M_{24}} \right) \tag{28c}$$

$$i = \left(\frac{M_{12}\,M_{34} - M_{13}\,M_{24}}{M_{12} - M_{13}} \right) \tag{29c}$$

$$j = 0 \tag{30c}$$

and the condition that

$$M_{23} = \frac{M_{13}\,M_{24}}{M_{14}}$$

If the parallel lines are grouped at equal spacings in the same horizontal plane, the above relation is closely approximated, and the circuit may be used without great error. However, it is shown in Appendix D that when this circuit is used as a part of a major network, it may be applied without any error by shifting the unbalanced part of M_{23} to another part of the system.

The circuit which provides ten fully independent links for the case of four parallel lines is shown in Fig. 10. Preference is to be given to the circuit of Fig. 12 however, when it can be used, because of its simplicity and because of the absence of negative links.

Rules for Setting Up Equivalent Circuits. From the foregoing analysis and discussion, the following rules for setting up an arbitrary equivalent circuit to represent any network having $(m + 1)$ points of entry and having finite impedances between each pair of terminals, may be inferred:

1. The equivalent circuit must have the same number of points of entry as the network.

2. Assign the $(m+1)$ point of the network as the ground point to which all voltages may be referred.

3. Set up an equivalent linkage having a minimum of $\frac{m(m+1)}{2}$ links, remembering that:

a. $\frac{m(m-1)}{2}$ of these links must be mutual with more than one circuit.

b. It must not be possible to reduce the equivalent, by a Y-delta transformation or similar artifice, to one having fewer links (for example, see Fig. 4).

c. There must be no link common to all of the m circuits in the same way (for example, see link j Fig. 12).

d. More than the required number of links, if used judiciously, often simplify the result (for example, see link c Fig. 9).

e. If less than the required number of links are used, there will be as many coefficients which are not independent as there are deficient links.

4. Evaluate the M coefficients by measuring or calculating the through impedances of the network and applying equations (**13b**), (**14b**), and (**15b**).

5. Set up the values of the through impedances of the equivalent circuit in terms of the unknown links.

6. Solve simultaneously the $m(m+1)$ relations of items 4 and 5 above.

Appendix D

APPLICATION OF THE SPECIAL CIRCUITS OF APPENDIX C TO THE ZERO PHASE DIAGRAM OF A TRANSMISSION SYSTEM

To illustrate the utility of the special circuits shown in Figs. 8 to 14 inclusive for representing groups of parallel transmission lines in the zero phase diagram of a system, the following typical problem and solution are given.

Problem. Map out the zero phase impedance diagram, in a form suitable for setting up on the calculating board, of the transmission system shown in Fig. 5.*

Solution. It may be noted that this system includes eight buses. Four of them are connected to rotating machines two of which are grounded, and three are connected to transformers all of which are grounded. The zero phase diagram of the system of Fig. 5 is shown in Fig. 6 for a fault at point P, and in Fig. 7 for a fault at point Q.

The method of obtaining the equivalent diagrams in Figs. 6 and 7 is evident. The group of magnetically coupled parallel lines emanating from any one bus (each bus is designated with a capital letter) has been replaced with an appropriate equivalent circuit. Note that each equivalent includes line constants up to the dotted lines shown in Fig. 5. Further note in Fig. 5 that D and E are the same bus but are considered as separate circuits since the lines form no mutual with those from D. Circuit J is not associated with a bus, but merely consists of two parallel lines with four points of entry. It is necessary to segregate this portion of lines 1 and 4 emanating from G for the case of a fault at Q. For the case of the fault at P, the constants of circuit J are included with those of circuit G.

Bus G has four lines, all of which are mutually coupled for a certain distance. Space does not permit the inclusion of the circuit constants which were used in the solution of this particular system. However, although the relation $M_{23} = M_{13}M_{24}/M_{14}$ is not quite satisfied by the actual system constants, (the actual value of M_{23} being slightly greater than that obtained from the above relation), the circuit of Fig. 12 was applied. The additional impedance mutual between lines 2 and 3, which could not be included in this circuit, was added to the mutual link between lines 1 and 2 emanating from bus I. Of course this shifting of mutuals is permissible only provided it is not intended to examine voltage conditions at some point in the equivalent circuit between buses G and I.

In setting up the equivalent system shown in Fig. 6 for a fault at point P, it was found that in no case did any of the circuits involve negative links. The significance of this fact is that, in case resistance may be neglected, the lay out may be adapted to a d-c. calculating board. Note that the circuit shown in Fig. 6 may be used without change for a number of fault conditions such as any bus fault for example.

The zero phase impedances in the machines and transformers are included in the L links shown in Figs. 6 and 7. Incidentally, it presumably is evident that any lumped reactance in a line, such as a transformer reactance, may be added directly to the self reactance of the line and the total used as a coefficient in setting up the equivalent circuit.

Appendix E

NOTATION

A_{jj} —Characteristic magnetizing impedance of any winding j in the multi-winding transformer.

A_{jk} —Characteristic mutual impedance between any two windings, j and k, of the multi-winding transformer.

B_{jj} —Characteristic admittance of any winding j of a transformer with all other windings short-circuited (also driving point impedance from point j).

B_{jk} —Characteristic admittance of the multi-winding transformer corresponding to a voltage impressed on winding j and a current in winding k with all windings except j short-circuited, and *vice versa*.

C_{jj} —Characteristic impedance between any point j of a network and ground.

---

*This layout is that of an actual system which was received for solution.

C_{jk} —Characteristic mutual impedance between any two circuits of a network j and k—the voltage appearing between point of entry j and ground when unit current flows from point of entry k to ground, and *vice versa*.

D_{jj} —Characteristic admittance of a network measured from point j to ground with all other points of entry grounded (driving point admittance).

D_{jk} —Characteristic admittance of a network corresponding to a voltage impressed from point of entry j to ground and a current flowing to ground from point of entry k, when all points except j are grounded (transfer admittance).

M_{jj} —Self-impedance of any transmission line, j.

M_{jk} —Mutual reactance between any two transmission lines, j and k, *i. e.*, the voltage induced in one due to unit current in the other.

Δ_{n-A} —N'th order determinant in A.

Δ_{m-0} —M'th order determinant in C.

Q_{A-jk}—Cofactor of A_{jk} in A

$Z_{\overline{jk}}$ —Impedance of a link connecting any two points, j and k, in general mesh.

Z_{j-k} —Through impedance measured between points j and k of a system or network (assuming all other points of entry open).

Z_{jk} —Transfer impedance between points j and k of a network or system.

Z_{jj} —Driving point impedance to ground from any point j of a network or system.

V_k —Voltage impressed across any winding k of a multi-winding transformer.

V_{k0} —Voltage impressed from any point k to ground in a general network.

I_k —Current flowing in any winding k of a multi-winding transformer, or the current entering any point of entry k of the general network.

m —Any integral number referring to the number of points of entry of a general network (excluding the ground point).

n —Any integral number referring to the number of windings of the multi-winding transformer.

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Discussion

For discussion of this paper see page 321.

Transient Oscillations of Mutually Coupled Windings

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***Synopsis.**—The paper is a mathematical study of the oscillations occurring in a pair of mutually coupled windings—the primary and secondary of a transformer—when one of the terminals is subjected to the impact of a traveling wave. Solutions for grounded and open terminal conditions are obtained, but the general solution for any impedances at the terminals is not attempted, except for the initial distribution.*

* * * * *

INTRODUCTION

In the past, the mathematical analysis of transient oscillations in transformer windings has been based upon a single winding having self and mutual inductances between its turns, capacitances along the stack and to ground, and in one instance resistances to represent the losses.[1] Such a circuit ignores the presence of secondary circuits, but strangely enough, proves quite adequate to describe the internal high-frequency transients of the winding under consideration, not only qualitatively, but quantitatively as well. However, cathode ray oscillograms do show some difference in the characteristics of the oscillations, depending upon the terminal connections of the secondary circuits. This is particularly true in the case of an isolated neutral primary winding, where a change from an open-circuited to a short-circuited secondary may bring about considerable changes in the frequencies of oscillations. This shift of frequencies has been loosely attributed to a change in the effective inductance of the circuit, brought about by closing the secondary circuit; just as the natural frequency of a simple lumped inductance-capacitance series circuit is changed if the inductance is reduced by short-circuiting a coupled winding. In addition to the above limitation of the single winding theory of transformer oscillations, there was nothing in the equations relative to the internal oscillations of the secondary circuit.

The idealized circuit of a two-winding transformer is shown in Fig. 1. The primary winding, on which the traveling wave is assumed to impinge, is designated as winding No. 1 and all associated voltages and currents have the corresponding subscript as e_1, i_{c1}, i_{k1}, i_{L1}. The secondary winding is designated as winding No. 2. The terminals are shown connected to impedances Z_1, Z_2, Z_3, and the incoming surge impedance Z. Each winding has capacitance C and K to ground and along the stack respectively, and an effective self-inductance L. The two windings are coupled magnetically by an effective mutual inductance M, and electrostatically by capacitance C_3. The various circuit constants and variables are specifically defined in the List of Symbols. It is pertinent to remark at this point that the effective inductances of the circuit include the partial interlinkages, on certain assumptions, and are not simple uniformly distributed constants. The necessity for these more complicated assumptions is responsible for greatly increasing the complexity of the problem.

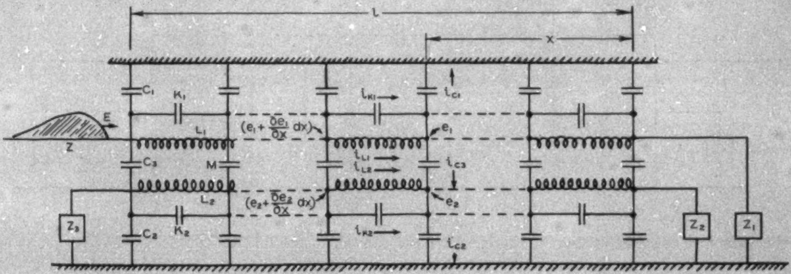

FIG. 1—COMPLETE IDEALIZED CIRCUIT OF A TRANSFORMER TO HIGH-FREQUENCY TRANSIENTS

The application of Kirchhoff's laws to the elements of the circuit shown in Fig. 1, permits the partial differential equations of the circuit to be written. A solution is then found such that the terminal and boundary conditions are satisfied.

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The initial distribution caused by the impact of an abrupt wave front depends only on the capacitance elements of the circuit; because the currents through the capacitive paths are infinite for an infinite rate of change of voltage, whereas the currents in the inductive paths are all zero at that first instant. Thus Fig. 2 shows the circuit which is effective for calculating the initial distribution.

The idealized circuit of Fig. 1 is free of losses, and therefore stabilization does not occur (theoretically) unless the terminal impedances have losses. For zero or infinite values of the terminal impedances the oscillations never do subside but persist forever. However, any actual circuit possesses some losses, and consequently the oscillations eventually die out and the currents reach permanent values, so that there is no change of flux and no induced voltages. The distribution of voltage in the primary winding is then uniform (ignoring the conductance from the winding to ground), and the potential of the secondary is constant throughout the winding. Thus while the realization of these final distributions is contingent upon the presence of losses, either in the circuit itself or in the terminal impedances, nevertheless there are fixed axes of oscillation about which the transient oscillations of the no-loss circuit take place.

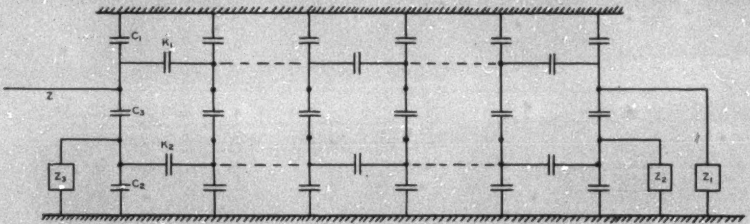

Fig. 2—Circuit Determining the Initial Distribution

The complete solutions are not attempted, except for zero and infinite values of the terminal impedance; that is, for grounded and isolated terminals. The solutions given are also restricted to rectangular applied waves. The response to waves of arbitrary shape can always be found in the usual way by an application of Duhamel's Theorem.[1] The solutions are in the form

$$e = (\text{final distribution}) + \Sigma\ (\text{harmonic space and time oscillations})$$

The time harmonics are always *cosine* functions, and since Duhamel's Theorem is operative only on the *time* functions of the solution, the response to different arbitrary wave shapes may be written down by inspection from the corresponding solutions given in a previous paper,[1] where the time functions were also harmonic *cosine* oscillations. By way of illustration, explicit solutions are worked out in detail for three different terminal conditions.

Conclusions and Findings

As far as a description of the transient in the primary winding is concerned, both qualitatively and quantitatively, it is not necessary to consider the influence of the secondary winding beyond keeping in mind that its terminal conditions will change the relative importance of the natural frequencies of oscillation. But these changes in frequency can be fairly adequately accounted for by changing the effective inductance in the equations.

The transient in the two-winding circuit consists of an infinite series of space and time harmonics, oscillating about the final distribution as an axes of oscillation. The amplitudes of these harmonic oscillations depend upon the initial and final distribution, and are in fact given by the Fourier analysis of the difference between them.

The space harmonics which appear in the solution are decided by the Fourier series chosen to represent the initial distribution throughout the length of the winding. The fundamental wavelength upon which the Fourier analysis is made is dictated by convenience in any specific case, and is therefore more or less arbitrary. Thus for certain terminal conditions, a full range *cosine* series may prove the most appropriate, while in another case a half range *sine* series may lead to the greatest simplification.

To each space harmonic there correspond two distinct time harmonics; that is, each space harmonic consists of two components vibrating at widely different rates. The amplitudes of these two time harmonics are usually quite different, but it is possible, by a change of terminal conditions, to change their relative importance.

The harmonic frequencies of the secondary circuit are the same as those of the primary winding, but the amplitudes bear a ratio to the corresponding amplitudes in the primary which depends upon the order of the space harmonic. In other words, there is no fixed proportionality between the oscillations in the two circuits.

The oscillations corresponding to an infinite rectangular applied wave, start simultaneously throughout the windings, and the time harmonics are all *cosine* functions. The response to applied wave shapes of arbitrary shape can be obtained by an application of Duhamel's Theorem.

List of Symbols

C_1 = capacitance of primary winding to ground.
C_2 = capacitance of secondary winding to ground.
C_3 = capacitance between primary and secondary windings.
K_1 = capacitance between ends of primary.
K_2 = capacitance between ends of secondary.
L_1 = effective self-inductance of primary (see eq. (**12**)).
L_2 = effective self-inductance of secondary (see eq. (**12**)).
M = effective mutual inductance between windings (see eq. (12)).
Z = surge impedance of line at primary.
$Z_1\,(p)$ = grounding impedance of primary neutral.
$Z_2\,(p)$ = terminal impedance of secondary, neutral end.
$Z_3\,(p)$ = terminal impedance of secondary, line end.
E = terminal voltage of primary due to traveling wave.
$i_{c1}, i_{k1}, i_{L1}, i_{c2}, i_{k2}, i_{L2}, i_{c3}$ = currents through C_1, K_1, etc.
e_1, e_2 = instantaneous voltages at any point x of primary and secondary respectively.
x = fraction of total winding from neutral end.
l = 1 = total length of winding.
t = time.
p = $\partial/\partial t$ = partial time derivative.

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Appendix I

The General Differential Equation

The idealized circuit of a transformer with two windings, each having ground and series capacitances and self-inductance, and inductively coupled electrostatically and electromagnetically, is illustrated in Fig. 1. The fundamental circuit equations are (see List of Symbols)

$$i_{k1} = K_1 \frac{\partial^2 e_1}{\partial x \, \partial t} \tag{1}$$

$$i_{k2} = K_2 \frac{\partial^2 e_2}{\partial x \, \partial t} \tag{2}$$

$$i_{c1} = C_1 \frac{\partial e_1}{\partial t} = \frac{\partial i_{k1}}{\partial x} + \frac{\partial i_{L1}}{\partial x} - i_{c3} \tag{3}$$

$$i_{c2} = C_2 \frac{\partial e_2}{\partial t} = \frac{\partial i_{k2}}{\partial x} + \frac{i_{L2}}{\partial x} + i_{c3} \tag{4}$$

$$i_{c3} = C_3 \frac{\partial}{\partial t} (e_1 - e_2) \tag{5}$$

On the assumption of a linearly graded mutual inductance between points of the same winding, and assuming further that the comparative effectiveness of an ampere-turn in one winding in producing flux density at a point in the other winding, is a fixed fraction σ, there is

$$\varphi_1 = \varphi_{m1} + 2\,(m\,l\,t)\,l \int_0^x \int_x^1 \frac{.4\,\pi}{h} (n_1\, i_{L1} + \sigma\, n_2\, i_{L2})\, d\,y \tag{6}$$

$$\varphi_2 = \varphi_{m2} + 2\,(m\,l\,t)\,l \int_0^x \int_x^1 \frac{.4\,\pi}{h} (\sigma\, n_1\, i_{L1} + n_2\, i_{L2})\, d\,y \tag{7}$$

These fluxes induce voltage gradients

$$\frac{\partial e_1}{\partial x} = \frac{n_1}{10^8} \frac{\partial \varphi_1}{\partial t} \tag{8}$$

$$\frac{\partial e_2}{\partial x} = \frac{n_2}{10^8} \frac{\partial \varphi_2}{\partial t} \tag{9}$$

Then by (**6**) with (**8**), and (**7**) with (**9**) there is

$$\frac{\partial^4 e_1}{\partial x^4} = - \frac{0.8\,\pi\,(m\,l\,t)\,n_1\,l}{10^8\,h} \frac{\partial^2}{\partial t\, \partial x} (n_1\, i_{L1} + \sigma\, n_2\, i_{L2})$$

$$= - \frac{\partial^2}{\partial t\, \partial x} (L_1\, i_{L1} + M\, i_{L2}) \tag{10}$$

$$\frac{\partial^4 e_2}{\partial x^4} = - \frac{\partial^2}{\partial t\, \partial x} (M\, i_{L1} + L_2\, i_{L2}) \tag{11}$$

In which

$$\left.\begin{aligned} L_1 &= \frac{0.8\,\pi\, n_1^2\,(m\,l\,t)\,l}{10^8\,h} \\ L_2 &= \frac{0.8\,\pi\, n_2^2\,(m\,l\,t)\,l}{10^8\,h} \\ M &= \frac{0.8\,\pi\, n_1\, n_2\,(m\,l\,t)\,\sigma\, l}{10^8\,h} \end{aligned}\right\} \tag{12}$$

By (1), (2), (3), and (4)

$$\frac{\partial^2}{\partial t\,\partial x}\, i_{L1} = (C_1 + C_3)\frac{\partial^2 e_1}{\partial t^2} - C_3\frac{\partial^2 e_2}{\partial t^2} - K_1\frac{\partial^4 e_1}{\partial x^2\,\partial t^2} \tag{13}$$

$$\frac{\partial^2}{\partial t\,\partial x}\, i_{L2} = (C_2 + C_3)\frac{\partial^2 e_2}{\partial t^2} - C_3\frac{\partial^2 e_1}{\partial t^2} - K_2\frac{\partial^4 e_2}{\partial x^2\,\partial t^2} \tag{14}$$

Substituting (13) and (14) into (10) and (11) respectively, there is

$$\begin{aligned} &\frac{\partial^4 e_1}{\partial x^4} - L_1 K_1 \frac{\partial^4 e_1}{\partial x^2\,\partial t^2} - M K_2 \frac{\partial^4 e_2}{\partial x^2\,\partial t^2} \\ &\quad + (L_1 C_1 + L_1 C_3 - M C_3)\frac{\partial^2 e_1}{\partial t^2} \\ &\quad + (M C_2 + M C_3 - L_1 C_3)\frac{\partial^2 e_2}{\partial t^2} = 0 \end{aligned} \tag{15}$$

$$\begin{aligned} &\frac{\partial^4 e_2}{\partial x^4} - L_2 K_2 \frac{\partial^4 e_2}{\partial x^2\,\partial t^2} - M K_1 \frac{\partial^4 e_1}{\partial x^2\,\partial t^2} \\ &\quad + (L_2 C_2 + L_2 C_3 - M C_3)\frac{\partial^2 e_2}{\partial t^2} \\ &\quad + (M C_1 + M C_3 - L_2 C_3)\frac{\partial^2 e_1}{\partial t^2} = 0 \end{aligned} \tag{16}$$

Solving the simultaneous equations (15) and (16), for either $e = e_1$ or $e = e_2$, there is

$$\begin{aligned} &\frac{\partial^8 e}{\partial x^8} - (L_1 K_1 + L_2 K_2)\frac{\partial^8 e}{\partial^6 x\,\partial t^2} \\ &+ K_1 K_2 (L_1 L_2 - M^2)\frac{\partial^8 e}{\partial^4 x\,\partial t^4} \\ &+ [L_1 C_1 + L_2 C_2 + (L_1 + L_2 - 2\,M)\,C_3]\frac{\partial^6 e}{\partial^4 x\,\partial t^2} \\ &- (L_1 L_2 - M_2)(K_1 C_2 + K_1 C_3 + K_2 C_1 + K_2 C_3)\frac{\partial^6 e}{\partial^2 x\,\partial t^4} \\ &+ (L_1 L_2 - M_2)\,(C_1 C_2 + C_2 C_3 + C_1 C_3)\frac{\partial^4 e}{\partial t^4} = 0 \end{aligned} \tag{17}$$

If $\sigma = 1$ then $(L_1 L_2 - M_2) = 0$ and the above differential equation simplifies to

$$\begin{aligned} &\frac{\partial^4 e}{\partial x^4} - (L_1 K_1 + L_2 K_2)\frac{\partial^4 e}{\partial^2 x\,\partial t^2} \\ &+ [L_1 C_1 + L_2 C_2 + (L_1 + L_2 - 2\,M)\,C_3]\frac{\partial^2 e}{\partial t^2} = 0 \end{aligned} \tag{18}$$

which has the same form as the differential equation for a single circuit, but the coefficients are, of course, different.

The Initial Distribution

The differential equation for the initial distribution can be obtained from the general differential equation (17), by substituting $p = \infty$ in accordance with the method of operational calculus. Then

$$\frac{\partial^4 e}{\partial x^4} - \left(\frac{K_1\,(C_2 + C_3) + K_2\,(C_1 + C_3)}{K_1 K_2}\right)\frac{\partial^2 e}{\partial x^2} + \left(\frac{C_1 C_2 + C_2 C_3 + C_3 C_1}{K_1 K_2}\right) e = 0 \tag{19}$$

Or putting $i_{L1} = i_{L2} = 0$ in (3) and (4), and using (1) and (2), there is

$$K_1 \frac{\partial^3 e_1}{\partial x^2\,\partial t} = i_{c1} + i_{c3} = C_1\frac{\partial e_1}{\partial t} + C_3\frac{\partial}{\partial t}\,(e_1 - e_2) \tag{20}$$

$$K_2 \frac{\partial^3 e_2}{\partial x^2\,\partial t} = i_{c2} - i_{c3} = C_2\frac{\partial e_2}{\partial t} - C_3\frac{\partial}{\partial t}\,(e_1 - e_2)$$

The solution to (19) is

$$e_1 = P\,\epsilon^{\alpha x} + Q\,\epsilon^{-\alpha x} + R\,\epsilon^{\beta x} + S\,\epsilon^{-\beta x} \tag{22}$$

where

$$\begin{aligned} \alpha^2 &= \frac{K_1\,(C_2 + C_3) + K_2\,(C_1 + C_3) + \sqrt{[K_1\,(C_2 + C_3) - K_2\,(C_1 + C_3)]^2 + 4\,K_1 K_2 C_3^2}}{2\,K_1 K_2} \\ \beta^2 &= \frac{K_1\,(C_2 + C_3) + K_2\,(C_1 + C_3) - \sqrt{[K_1\,(C_2 + C_3) - K_2\,(C_1 + C_3)]^2 + 4\,K_1 K_2 C_3^2}}{2\,K_1 K_2} \end{aligned} \tag{23}$$

By (20) and (22)

$$\begin{aligned} e_2 &= P_2\,\epsilon^{\alpha x} + Q_2\,\epsilon^{-\alpha x} + R_2\,\epsilon^{\beta x} + S_2\,\epsilon^{-\beta x} \\ &= \frac{C_1 + C_3}{C_3}\, e_1 - \frac{K_1}{C_3}\frac{\partial^2 e_1}{\partial x^2} \end{aligned} \tag{24}$$

So that the set of integration constants are related by

$$\left.\begin{aligned} P_2 &= P\,(C_1 + C_3 - K_1\,\alpha^2)/C_3 = m\,P \\ Q_2 &= Q\,(C_1 + C_3 - K_1\,\alpha^2)/C_3 = m\,Q \\ R_2 &= R\,(C_1 + C_3 - K_1\,\beta^2)/C_3 = n\,R \\ S_2 &= S\,(C_1 + C_3 - K_1\,\beta^2)/C_3 = n\,S \end{aligned}\right\} \tag{25}$$

Thus the solutions for the initial distributions of e_1 and e_2 have the same functional form, and the integration constants of their separate solutions are related, so that there are only four integration constants that must be determined from the boundary conditions. Referring to Fig. 2, and designating by $Z(\infty)$ the values of the terminal impedances at the first or initial instant, there is:

$$e_1 = E \text{ at } x = 1 \tag{26}$$

$$e_1 = Z_1(p)\, i_{k1} = p\, Z_1(p)\, K_1 \frac{\partial e_1}{\partial x} \text{ at } x = 0 \text{ and } p = \infty \tag{27}$$

$$e_2 = Z_2(p)\, i_{k2} = p\, Z_2(p)\, K_2 \frac{\partial e_2}{\partial x} \text{ at } x = 0 \text{ and } p = \infty \tag{28}$$

$$e_2 = -Z_3(p)\, i_{k2} = -p\, Z_3(p)\, K_2 \frac{\partial e_2}{\partial x} \text{ at } x = 1 \text{ and } p = \infty \tag{29}$$

The employment of these four conditions will yield four simultaneous equations in P, Q, R, S, which will just suffice for their unique determination.

The Final Distribution

The realization of a final steady state distribution at $t = \infty$ for an infinite rectangular applied wave, is contingent upon the presence of losses, either in the transformer or in the terminal impedances, and the characteristics of such a final state will depend upon the nature of the losses. But in the no-loss circuit with zero or infinite terminal impedances, the axes of oscillation are determined by the electrostatic and electromagnetic fields necessary to establish the terminal voltages of the windings. Possible solutions to the general differential equation (17) independent of t and which satisfy the terminal conditions are:

$$\left.\begin{aligned} e_1 &= e_n + (E - e_n)\, x \\ e_2 &= E_2' + x\, E_2 \\ &= \frac{C_3(E + e_n) - (C_2 + C_3 + 2\,C_3')E_2}{2\,(C_2 + C_3 + C_2' + C_3')} + x\, E_2 \end{aligned}\right\} \tag{30}$$

in which E is the primary terminal voltage, e_n is the primary neutral voltage ($e_n = 0$ for grounded neutral, $e_n = E$ for isolated neutral), E_2' is the secondary potential due to its position in the electrostatic field, and C_2' and C_3' are the capacitances of the secondary terminal impedances Z_2 and Z_3 respectively. A comparison of (30) with (87) and (88) of Appendix II identifies $(E - e_n)\, x$ and $x\, E_2$ as the voltage components due to a magnetic field rising without limit at a uniform rate. The value of E_2' is determined from the condition that the total dielectric flux from the primary to the secondary must be equal to that from the secondary to ground. Thus

$$\int_0^1 C_3\,[e_n + (E - e_n)\, x - E_2' - x\, E_2]\, d\,x$$
$$= C_2'\, E_2' + C_2 \int_0^1 (E_2' + x\, E_2)\, d\,x + C_3'\,(E_2 + E_2')$$

The idealized circuit of Fig. 1 does not include any losses, so that the complete solution will not reduce to (30) at $t = \infty$; but will nevertheless represent sustained oscillations about (30) as axes.

Solution of the Differential Equation

The solution must satisfy the initial and final distributions, the terminal conditions, and the differential equation. As a tentative solution suitable for zero and infinite values of the terminal impedances, assume

$$e_1 = e_n + (E - e_n)\, x + \Sigma\, A\, \epsilon^{(ax + bt)} \tag{31}$$

$$e_2 = \Sigma\, A_2\, \epsilon^{(ax + bt)} + E_2\, x + E_2' \tag{32}$$

in which A and A_2 are integration constants. Substituting (31) or (32) into the differential equation (17) and simplifying, there results

$$\begin{aligned}(L_1 L_2 - M^2)[K_1 K_2 a^4 - (K_1 C_2 + K_2 C_1 + K_1 C_3 + K_2 C_3)a^2 \\ + (C_1 C_2 + C_2 C_3 + C_1 C_3)]\, b^4 + [-(L_1 K_1 + L_2 K_2)a^6 \\ + (L_1 C_1 + L_2 C_2 + L_1 C_3 + L_2 C_3 - 2\, M\, C_3)a^4]b^2 + a^8 = 0\end{aligned} \tag{33}$$

This equation is a quadratic in b^2, of which the coefficients are all positive if a is a pure imaginary $\pm j\,\lambda$; and because $L_1 L_2 \geq M^2$ and $(L_1 + L_2) \geq 2\,M$. Moreover, upon expanding the terms under the radical of the solution for b^2, it becomes evident that the radical is a positive real number, and accordingly b^2 has two real negative values. Corresponding to these two negative values, there is

$$b = \begin{cases} \pm j\,\omega \\ \pm j\,\Omega \end{cases} \tag{34}$$

This result is a necessary consequence of the fact that the circuit is a conservative non-dissipative network, and therefore exponential decrement factors cannot appear in its solution. The solutions may now be written in the form

$$\begin{aligned} e_1 = e_n + (E - e_n)x + \Sigma[(A \cos \omega\, t + A' \cos \Omega\, t) \sin \lambda\, x \\ + (B \sin \omega\, t + B' \sin \Omega\, t) \sin \lambda\, x \\ + (C \cos \omega\, t + C' \cos \Omega\, t) \cos \lambda\, x \\ + (D \sin \omega\, t + D' \sin \Omega\, t) \cos \lambda\, x] \end{aligned} \tag{35}$$

$$\begin{aligned} e_2 = \Sigma\, [(A_2 \cos \omega\, t + A_2' \cos \Omega\, t) \sin \lambda\, x \\ + (B_2 \sin \omega\, t + B_2' \sin \Omega\, t) \sin \lambda\, x \\ + (C_2 \cos \omega\, t + C_2' \cos \Omega\, t) \cos \lambda\, x \\ + (D_2 \sin \omega t + D_2' \sin \Omega\, t) \cos \lambda\, x] + E_2 x + E_2' \end{aligned} \tag{36}$$

Thus, there are *two* time harmonics ω and Ω associated with each space harmonic λ. If the coupling between the two windings is perfect, $L_1 L_2 = M^2$ then there will be only one time harmonic corresponding to a given space harmonic. The more nearly that $M^2 \rightarrow L_1 L_2$ the more closely $\Omega \rightarrow 0$. Now in a transformer ω and Ω are usually quite different (say 2:1) but the amplitude factors of the two frequencies are also quite different for given terminal conditions of the secondary. However, a change in the secondary terminal conditions (such as switching from open circuit to short circuit) may cause a complete reversal in the relative importance of the two frequency sets.

In (**35**) and (**36**) there are nineteen constants to be determined. Of these, λ is determined by a comparison with the initial distribution, ω and Ω then follow by (**33**) and (**34**), and the integration constants for the secondary circuit may be expressed in terms of those for the primary circuit through equations (**15**) and (**16**). The remaining constants are determined from the initial distributions and the terminal conditions. Substituting (**35**) and (**36**) into (**15**), and separately equating the coefficients of like trigonometric terms, there results

$$\frac{A_2}{A} = \frac{B_2}{B} = \frac{C_2}{C} = \frac{D_2}{D}$$

$$= \frac{\lambda^4 - \omega^2 (\lambda^2 L_1 K_1 + L_1 C_1 + L_1 C_3 - M C_3)}{\omega^2 (\lambda^2 M K_2 + M C_2 + M C_3 - L_1 C_3)} = r_s \tag{37}$$

$$\frac{A_2'}{A'} = \frac{B_2'}{B'} = \frac{C_2'}{C'} = \frac{D_2'}{D'}$$

$$= \frac{\lambda^4 - \Omega^2 (\lambda^2 L_1 K_1 + L_1 C_1 + L_1 C_3 - M C_3)}{\Omega^2 (\lambda^2 M K_2 + M C_2 + M C_3 - L_1 C_3)} = r_s' \tag{38}$$

Or by (**16**) there is

$$\frac{A_2}{A} = \frac{B_2}{B} = \frac{C_2}{C} = \frac{D_2}{D}$$

$$= \frac{\omega^2 (\lambda^2 M K_1 + M C_1 + M C_3 - L_2 C_3)}{\lambda^4 - \omega^2 (\lambda^2 L_2 K_2 + L_2 C_2 + L_2 C_3 - M C_3)} = r_s \tag{39}$$

$$\frac{A_2'}{A'} = \frac{B_2'}{B'} = \frac{C_2'}{C'} = \frac{D_2'}{D'}$$

$$= \frac{\Omega^2 (\lambda^2 M K_1 + M C_1 + M C_3 - L_2 C_3)}{\lambda^4 - \Omega^2 (\lambda^2 L_2 K_2 + L_2 C_2 + L_2 C_3 - M C_3)} = r_s' \tag{40}$$

That (**37**) is equal to (**39**) and that (**38**) is equal to (**40**) is evident by (**33**). Therefore it is immaterial whether (**37**) and (**38**) or (**39**) and (**40**) be used in subsequent substitutions.

Equating (**35**) at $t = 0$ to (**22**) and (**36**) at $t = 0$ to (**24**), making use of (**25**), (**37**) and (**38**), there is

$$\Sigma \left[(A + A') \sin \lambda x + (C + C') \cos \lambda x\right]$$
$$= (P\, \epsilon^{\alpha x} + Q\, \epsilon^{-\alpha x} + R\, \epsilon^{\beta x} + S\, \epsilon^{-\beta x}) - [e_n + (E - e_n)\, x]$$
$$= \sum_1^\infty \left(X_s \sin \frac{s \pi x}{c} + Y_s \cos \frac{s \pi x}{c} \right) \tag{41}$$

$$\Sigma \left[(r_s A + r_s' A') \sin \lambda x + (r_s C + r_s' C') \cos \lambda x\right]$$
$$= (m P\, \epsilon^{\alpha x} + m Q\, \epsilon^{-\alpha x} + n R\, \epsilon^{\beta x} + n\, S\, \epsilon^{-\beta x}) - E_2 x - E_2'$$
$$= \sum_1^\infty \left(U_s \sin \frac{s \pi x}{c} + V_s \cos \frac{s \pi x}{c} \right) \tag{42}$$

where X_s, Y_s, U_s and V_s are the coefficients in the Fourier analysis of the *difference* between the *initial* and the *final* distributions, and c is the arbitrary half wavelength upon which the Fourier analysis is made. Of course, the choice of c as well as the decision to expand, for instance, as a half-range *sine* or *cosine* series, is dictated by convenience in simplifying any particular special case of boundary conditions.

By direct comparison of the coefficients of like trigonometric terms in (**41**) and (**42**) there is

$$\left.\begin{aligned} A + A' &= X_s \\ C + C' &= Y_s \\ r_s A + r_s' A' &= U_s \\ r_s C + r_s' C' &= V_s \\ \lambda &= \frac{s \pi}{c} \end{aligned}\right\} \tag{43}$$

and solving these equations, there is

$$\left.\begin{aligned} A' &= (r_s X_s - U_s) / (r_s - r_s') \\ A &= (r_s' X_s - U_s)/(r_s' - r_s) \\ C' &= (r_s Y_s - V_s) / (r_s - r_s') \\ C &= (r_s' Y_s - V_s)/(r_s' - r_s) \end{aligned}\right\} \tag{44}$$

The only conditions which remain to be satisfied are

$$\left.\begin{aligned} i_{L1} = i_{L2} &= 0 \text{ at } t = 0 \\ (i_{L1} + i_{K1})\, Z_1 &= e_1 \text{ at } x = 0 \\ (i_{L2} + i_{K2})\, Z_2 &= e_2 \text{ at } x = 0 \\ -(i_{L2} + i_{K2})\, Z_3 &= e_2 \text{ at } x = 1 \end{aligned}\right\} \tag{45}$$

From (**3**) and (**4**) respectively and making use of (**1**), (**4**) and (**5**), there are

$$\frac{\partial i_{L1}}{\partial x} = C_1 \frac{\partial e_1}{\partial t} + i_{c3} - \frac{\partial i_{K1}}{\partial x}$$

$$= \left(C_1 + C_3 - K_1 \frac{\partial^2}{\partial x^2} \right) \frac{\partial e_1}{\partial t} - C_3 \frac{\partial e_1}{\partial x} \tag{46}$$

$$\frac{\partial i_{L2}}{\partial x} = C_2 \frac{\partial e_2}{\partial t} - i_{c3} - \frac{\partial i_{K2}}{\partial x}$$

$$= \left(C_2 + C_3 - K_2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial e_2}{\partial t} - C_3 \frac{\partial e_1}{\partial x} \tag{47}$$

$$(i_{L1} + i_{K1}) = \int \left[(C_1 + C_3) \frac{\partial e_1}{\partial t} - C_3 \frac{\partial e_2}{\partial t} \right] \mathrm{d}\, x \tag{48}$$

$$(i_{L2} + i_{K2}) = \int \left[(C_2 + C_3) \frac{\partial e_2}{\partial t} - C_3 \frac{\partial e_1}{\partial t} \right] \mathrm{d}\, x \tag{49}$$

Now if (**35**) and (**36**) are substituted in the above four equations, and each of the terminal conditions of (**45**) are imposed, it will be evident that for zero and infinite values of the terminal impedance Z_1, Z_2 and Z_3 that conditions (**45**) are satisfied for

$$B = B' = D = D' = 0 \qquad (\mathbf{50})$$

In making such a comparison, a word of warning is advisable. If (**35**) and (**36**) are substituted in (**46**) and $(\partial i_{L1}/\partial x)$ placed equal to zero in conformity with the first condition of (**45**), and the coefficients of like trigonometric terms are separately equated to zero, there results

$$\frac{B'}{B} = \frac{D'}{D} = -\frac{\omega}{\Omega}\left\{\frac{(C_1 + C_3 + K_1\lambda^2) - r\,C_3}{(C_1 + C_3 + K_1\lambda^2) - r'\,C_3}\right\} \qquad (\mathbf{51})$$

Now if the same procedure is followed for (**47**) there results

$$\frac{B'}{B} = \frac{D'}{D} = -\frac{\omega}{\Omega}\left\{\frac{(C_2 + C_3 + K_2\lambda^2)\,r - C_3}{(C_2 + C_3 + K_2\lambda^2)\,r' - C_3}\right\} \qquad (\mathbf{52})$$

But (**51**) and (**52**) are satisfied by (**50**) since $r \neq r'$. The complete solutions may therefore be written (for zero or infinite values of the terminal impedances)

$$e_1 = e_n + (E - e_n)\,x + \sum\left[(A\cos\omega t + A'\cos\Omega t)\sin\lambda x + (C\cos\omega t + C'\cos\Omega t)\cos\lambda x\right] \qquad (\mathbf{53})$$

$$e_2 = \sum\left[(r\,A\cos\omega t + r'\,A'\cos\Omega t)\sin\lambda x + r\,C\cos\omega t + r'\,C'\cos\Omega t)\cos\lambda x\right] + E_2\,x + E_2' \qquad (\mathbf{54})$$

where

$\lambda = s\,\pi/c$ from (**43**)
A, A', C, C' from (**44**)
r, r' from (**37**), (**38**)
$e_n = 0$ for a grounded neutral
$\quad = E$ for an isolated neutral

The principal conclusions which may be drawn from these equations are:

(a) The space harmonics, s, depend upon the Fourier series chosen to represent the initial distributions within the interval $0 \leq x \leq 1$.

(b) To each space harmonic there corresponds two distinct time harmonics ω and Ω whose amplitudes sum up at every instant to give the space harmonic distribution at that instant. In other words, each space harmonic consists of two components vibrating at different rates.

(c) The amplitudes of oscillation depend upon the Fourier analysis of the initial distributions with respect to the final distributions, and therefore depend primarily upon the capacitances and terminal impedances of the circuits.

(d) Any harmonic amplitude of oscillation of the secondary circuit bears a definite ratio to the corresponding oscillation of the primary circuit.

(e) Identically the same frequencies of oscillation appear in both the primary and secondary circuits.

(f) The transient oscillations take place about the final distributions as axes of oscillation.

By way of illustration of the above general equations and procedure, a number of special cases will be worked out in detail.

I. GROUNDED NEUTRAL AND SHORT-CIRCUITED SECONDARY

THE INITIAL DISTRIBUTIONS

Since $Z_1 = Z_2 = Z_3 = 0$ for this case, (**26**) to (**29**) give

$$\left.\begin{aligned} &P\,\epsilon^{\alpha} + Q\,\epsilon^{-\alpha} + R\,\epsilon^{\beta} + S\,\epsilon^{-\beta} = E \\ &P + Q + R + S = 0 \\ &m\,P\,\epsilon^{\alpha} + m\,Q\,\epsilon^{-\alpha} + n\,R\,\epsilon^{\beta} + n\,S\,\epsilon^{-\beta} = 0 \\ &m\,P + m\,Q + n\,R + n\,S = 0 \end{aligned}\right\} \qquad (\mathbf{55})$$

Solving these four simultaneous equations for the integration constants, there are

$$\left.\begin{aligned} P &= n\,E/(n-m)\,2\sinh\alpha \\ Q &= -\,n\,E/(n-m)\,2\sinh\alpha \\ R &= -\,m\,E/(n-m)\,2\sinh\beta \\ S &= m\,E/(n-m)\,2\sinh\beta \end{aligned}\right\} \qquad (\mathbf{56})$$

Then by (**22**) and (**24**), the initial distributions (expressed as half-range sine series) are

$$\underset{t=0}{e_1} = \frac{E}{n-m}\left[\frac{n\sinh\alpha x}{\sinh\alpha} - \frac{m\sinh\beta x}{\sinh\beta}\right]$$

$$= \sum_{s=1}^{\infty}\frac{2\,E}{m-n}\left[\frac{n\,s\,\pi}{\alpha^2 + s^2\,\pi^2} - \frac{m\,s\,\pi}{\beta^2 + s^2\,\pi^2}\right]\cos s\,\pi\,.\,\sin s\,\pi\,x \qquad (\mathbf{57})$$

$$\underset{t=0}{e_2} = \frac{n\,m\,E}{n-m}\left[\frac{\sinh\alpha x}{\sinh\alpha} - \frac{\sinh\beta x}{\sinh\beta}\right]$$

$$= \sum_{s=1}^{\infty}\frac{2\,n\,m\,E}{m-n}\left[\frac{s\,\pi}{\alpha^2 + s^2\,\pi^2} - \frac{s\,\pi}{\beta^2 + s^2\,\pi^2}\right]\cos s\,\pi\,.\,\sin s\,\pi\,x \qquad (\mathbf{58})$$

THE FINAL DISTRIBUTIONS

Since the neutral is grounded, $e_n = 0$, and the final distribution may therefore be expressed as a half-range sine series

$$\underset{t=\infty}{e_1} = x\,E = \sum_{s=1}^{\infty}\frac{(-\,2\,E\cos s\,\pi)}{s\,\pi}\sin s\,\pi\,x \qquad (\mathbf{59})$$

$$\underset{t=\infty}{e_2} = 0 \qquad (\mathbf{60})$$

THE COMPLETE SOLUTION

The difference between the initial and final distributions, expressed as half-range sine series, give—see (**41**) and (**42**)

$$X_s = \frac{2\,E\,s\,\pi}{(m-n)}\left[\frac{n}{\alpha^2+s^2\pi^2}-\frac{m}{\beta^2+s^2\pi^2}+\frac{m-n}{s^2\,\pi^2}\right]\cos s\,\pi \tag{61}$$

$$Y_s = 0 \tag{62}$$

$$U_s = \frac{2\,E\,s\,\pi}{m-n}\left[\frac{n\,m}{\alpha^2+s^2\pi^2}-\frac{n\,m}{\beta^2+s^2\pi^2}\right]\cos s\,\pi \tag{63}$$

$$V_s = 0 \tag{64}$$

$$\lambda = s\,\pi \tag{65}$$

Then by (**44**)

$$A' = \frac{2\,E\,s\,\pi\cos s\,\pi}{(r_s - r_s')\,(m-n)}\left[\frac{n\,(r_s-m)}{\alpha^2+s^2\pi^2}-\frac{m\,(r_s-n)}{\beta^2+s^2\pi^2}+\frac{r_s\,(m-n)}{s^2\,\pi^2}\right] \tag{66}$$

$$A = \frac{2\,E\,s\,\pi\cos s\,\pi}{(m-n)\,(r_s'-r_s)}\left[\frac{n\,(r_s'-m)}{\alpha^2-s^2\pi^2}-\frac{m\,(r_s'-n)}{\beta^2+s^2\pi^2}+\frac{r_s'\,(m-n)}{s^2\,\pi^2}\right] \tag{67}$$

$$C = C' = 0 \tag{68}$$

The solution, by (**53**) and (**54**), then is

$$\begin{aligned} e_1 = x\,E + \sum_{s=1}^{\infty}\frac{2\,E\,s\,\pi\cos s\,\pi}{(r_s-r_s')\,(m-n)}\Bigg\{&\left[\frac{n\,(r_s-m)}{\alpha^2+s^2\pi^2} - \frac{m\,(r_s-n)}{\beta^2+s^2\pi^2}+\frac{r_s\,(m-n)}{s^2\,\pi^2}\right]\cos\Omega_s\,t \\ &-\left[\frac{n\,(r_s'-m)}{\alpha^2+s^2\pi^2}-\frac{m\,(r_s'-n)}{\beta^2+s^2\pi^2} + \frac{r_s'\,(m-n)}{s^2\,\pi^2}\right]\cos\omega_s\,t\Bigg\}\sin s\,\pi\,x \end{aligned} \tag{69}$$

$$\begin{aligned} e_2 = \sum_{s=1}^{\infty}\frac{2\,E\,s\,\pi\cos s\,\pi}{(r_s-r_s')\,(m-n)}\Bigg\{&\left[\frac{n\,(r_s-m)}{\alpha^2+s^2\pi^2} - \frac{m\,(r_s-n)}{\beta^2+s^2\pi^2}+\frac{r_s\,(m-n)}{s^2\,\pi^2}\right]r_s'\cos\Omega_s\,t \\ &-\left[\frac{n\,(r_s'-m)}{\alpha^2+s^2\pi^2}-\frac{m\,(r_s'-n)}{\beta^2+s^2\pi^2} + \frac{r_s'\,(m-n)}{s^2\,\pi^2}\right]r_s\cos\omega_s\,t\Bigg\}\sin s\,\pi\,x \end{aligned} \tag{70}$$

The angular velocities ω_s and Ω_s are given by the solution of (**33**) in terms of $\lambda = s\,\pi$.

II. GROUNDED NEUTRAL AND OPEN-CIRCUITED SECONDARY

THE INITIAL DISTRIBUTIONS

In this case $Z_1 = 0$ and $Z_2 = Z_2 = \infty$. Hence by (**26**) to (**29**) there is

$$\left.\begin{array}{lllll} P\,\epsilon^{\alpha} & +\,Q\,\epsilon^{-\alpha} & +\,R\,\epsilon^{\beta} & +\,S\,\epsilon^{-\beta} & = \mathrm{E} \\ P & +\,Q & +\,R & +\,S & = 0 \\ m\,\alpha\,P & -\,m\,\alpha\,Q & +\,n\,\beta\,R & -\,n\,\beta\,S & = 0 \\ m\,\alpha\,P\,\epsilon^{\alpha} & -\,m\,\alpha\,Q\,\epsilon^{-\alpha} & +\,n\,\beta\,R\,\epsilon^{\beta} & -\,n\,\beta\,S\,\epsilon^{-\beta} & = 0 \end{array}\right\} \tag{71}$$

Solving for the integration constants

$$\begin{aligned} P &= n\,\beta\,E\,[m\,\alpha\,(\cosh\alpha-\sinh\alpha) \\ &\quad + (n\,\beta\sinh\beta - m\,\alpha\cosh\beta)]/T \\ Q &= n\,\beta\,E\,[m\,\alpha\,(\cosh\alpha+\sinh\alpha) \\ &\quad - (n\,\beta\sinh\beta + m\,\alpha\cosh\beta)]/T \\ R &= m\,\alpha\,E\,[n\,\beta\,(\cosh\beta-\sinh\beta) \\ &\quad + (m\,\alpha\sinh\alpha - n\,\beta\cosh\alpha)]/T \\ S &= m\,\alpha\,E\,[n\,\beta\,(\cosh\beta+\sinh\beta) \\ &\quad - (m\,\alpha\sinh\alpha - n\,\beta\cosh\alpha)]/T \\ T &= 4\,m\,n\,\alpha\beta\,(1-\cosh\alpha\,.\,\cosh\beta) \\ &\quad + 2\,(m^2\,\alpha^2 + n^2\,\beta^2)\sinh\alpha\,.\,\sinh\beta \end{aligned} \tag{72}$$

The initial distributions then are

$$\underset{t=0}{e_1} = E\left\{\frac{\begin{array}{l} m\,n\,\alpha\,\beta\,(\cosh\alpha-\cosh\beta)\,(\cosh\alpha\,x-\cosh\beta\,x) \\ +(m\,\alpha\sinh\alpha-n\beta\sinh\beta)(m\,\alpha\sinh\beta\,x-n\beta\sinh\alpha\,x)\end{array}}{2mn\alpha\beta(1-\cosh\alpha.\cosh\beta)+(m^2\alpha^2+n^2\beta^2)\sinh\alpha.\sinh\beta}\right\} \tag{73}$$

$$\underset{t=0}{e_2} = E\left\{\frac{\begin{array}{l} m\,n\,\alpha\,\beta\,(\cosh\alpha-\cosh\beta)\,(m\cosh\alpha\,x-n\cosh\beta\,x) \\ +m\,n(m\alpha\sinh\alpha-n\,\beta\sinh\beta)\,(\alpha\sinh\beta\,x-\beta\sinh\alpha\,x)\end{array}}{2mn\alpha\beta(1-\cosh\alpha.\cosh\beta)+(m^2\alpha^2+n^2\beta^2)\sinh\alpha.\sinh\beta}\right\} \tag{74}$$

and these can, of course, be expressed as a half-range *sine* series.

THE FINAL DISTRIBUTIONS

That of the primary is the same as in (**59**) but the secondary takes on the potential, by (**30b**),

$$\underset{t=\infty}{e_2} = \sum_{1}^{\infty}\left(2\,E_2'\sin^2\frac{s\,\pi}{2} - E_2\cos s\,\pi\right)\frac{2}{s\,\pi}\sin s\,\pi\,x \tag{75}$$

$$E_2' = \left[\frac{C_3\,E}{2\,(C_2+C_3)} - \frac{E_2}{2}\right] \text{and } E_2 = \frac{M'}{L_1'}\,E$$

The Complete Solution then follows by the same procedure as in the previous case, differing only in the values of the coefficients X_s and Y_s because the initial and final distributions have changed. The frequencies of oscillation are the same as in the previous case—but the relative importance of the amplitudes have been changed.

III. ISOLATED NEUTRAL AND OPEN-CIRCUITED SECONDARY

The Initial Distributions

In this case $Z_1 = Z_2 = Z_3 = \infty$, and (**26**) to (**29**) give

$$\left.\begin{array}{lllll} P\epsilon^{\alpha} & +Q\epsilon^{-\alpha} & +R\epsilon^{\beta} & +S\epsilon^{-\beta} & =E \\ \alpha P & -\alpha Q & +\beta R & -\beta S & =0 \\ m\alpha P & -m\alpha Q & +n\beta R & -n\beta S & =0 \\ m\alpha P\epsilon^{\alpha} & -m\alpha Q\epsilon^{-\alpha} & +n\beta R\epsilon^{\beta} & -n\beta S\epsilon^{-\beta} & =0 \end{array}\right\} \tag{76}$$

Solving for the integration constants:

$$\begin{aligned} P &= -(n\beta E\sinh\beta)/T \\ Q &= -(n\beta E\sinh\beta)/T \\ R &= +(m\alpha E\sinh\alpha)/T \\ S &= +(m\alpha E\sinh\alpha)/T \\ T &= 2\,(m\alpha\sinh\alpha\,.\,\cosh\beta - n\beta\,.\,\cosh\alpha\,.\,\sinh\beta) \end{aligned} \tag{77}$$

The initial distributions are

$$\underset{t=0}{e_1} = E\left\{\frac{m\alpha\sinh\alpha\,.\,\cosh\beta x - n\beta\sinh\beta\cosh\alpha x}{m\alpha\sinh\alpha\cosh\beta - n\beta\sinh\beta\cosh\alpha}\right\}$$

$$= \sum_{s=1}^{\infty}\frac{-2Es\pi/2\,.\,\sin s\pi/2\,.\,\cos s\pi x/2}{m\alpha\sinh\alpha\,.\,\cosh\beta - n\beta\sinh\beta\,.\,\cosh\alpha}$$

$$\left[\frac{m\alpha\sinh\alpha}{\beta^2 + s^2\pi^2/4} - \frac{n\beta\sinh\beta}{\alpha^2 + s^2\pi^2/4}\right] \tag{78}$$

$$\underset{t=0}{e_2} = nmE\left\{\frac{\alpha\sinh\alpha\,.\,\cosh\beta x - \beta\sinh\beta\,.\,\cosh\alpha x}{m\alpha\sinh\alpha\,.\,\cosh\beta - n\beta\sinh\beta\,.\,\cosh\alpha}\right\}$$

$$= \sum_{s=1}^{\infty}\frac{-2mnEs\pi/2\,.\,\sin s\pi/2\,.\,\cos s\pi x/2}{m\alpha\sinh\alpha\,.\,\cosh\beta - n\beta\sinh\beta\cosh\alpha}$$

$$\left[\frac{\alpha\sinh\alpha}{\beta^2 + s^2\pi^2/4} - \frac{\beta\sinh\beta}{\alpha^2 + s^2\pi^2/4}\right] \tag{79}$$

The Final Distributions

For an isolated neutral the final distribution expressed as a *cosine* series in the interval $0 \leq x \leq 1$ is

$$\underset{t=\infty}{e_1} = E = E\sum_{s=1}^{\infty}\frac{2\sin s\pi/2\,.\,\cos s\pi x/2}{s\pi/2} \tag{80}$$

and

$$\underset{t=\infty}{e_2} = E_2' = \frac{C_3 E}{C_3 + C_2}$$

$$= \frac{2C_3E}{C_3 + C_2}\sum_{s=1}^{\infty}\frac{\sin s\pi/2\,.\,\cos s\pi x/2}{s\pi/2}. \tag{81}$$

The Complete Solution

The difference between the Fourier series of the initial and final distributions gives, by (**41**)

$$\left.\begin{aligned} X_s &= 0 \\ Y_s &= [\,(\mathbf{78}) - (\mathbf{80})\,] \\ U_s &= 0 \\ V_s &= [\,(\mathbf{79}) - (\mathbf{81})\,] \\ \lambda &= (2s-1)\,\pi/2 \end{aligned}\right\} \tag{82}$$

Appendix II

Exact Equations for the Axes of Oscillation

This appendix offers a more detailed consideration of the axes of oscillation than was given in Appendix I; and incidentally also derives the simplified equations for calculating the current and voltage transmitted to the secondary external circuit.

Substituting (**1**) and (**2**) into (**48**) and (**49**) respectively, and integrating with respect to x there is

$$i_{L1} = I_1 + \int\left[(C_1 + C_3)\frac{\partial e_1}{\partial t} - C_3\frac{\partial e_2}{\partial t}\right.$$

$$\left. - K_1\frac{\partial^3 e_1}{\partial x^2\,\partial t}\right]dx \tag{83}$$

$$i_{L2} = I_2 + \int\left[(C_2 + C_3)\frac{\partial e_2}{\partial t} - C_3\frac{\partial e_1}{\partial t}\right.$$

$$\left. - K_2\frac{\partial^3 e_2}{\partial x^2\,\partial t}\right]dx \tag{84}$$

In these equations I_1 and I_2 are integration constants with respect to x, and are therefore possible functions of t but not of x. They therefore flow through all parts of their windings, and in conjunction with the final electrostatic field, fix the final potentials of the windings. The indefinite integral yields the space and time harmonics of the oscillation.

Substituting I_1 and I_2 in (**6**) and (**7**) there follows by (**8**) and (**9**) the potential distributions caused by these currents:

$$\frac{n_1}{10^8}\int_0^x\frac{\partial\phi_1}{\partial t}\,dx = x\,\frac{n_1}{10^8}\,\frac{\partial}{\partial t}$$

$$\left[\Phi_{m1} + .8\pi\,(mlt)\,\frac{l}{h}\,(n_1 I_1 + \sigma n_2 I_2)\left(\frac{x}{2} - \frac{x^2}{3}\right)\right] \tag{85}$$

$$\frac{n_2}{10^8}\int_0^x\frac{\partial\phi_2}{\partial t}\,dx = x\,\frac{n_2}{10^8}\,\frac{\partial}{\partial t}$$

$$\left[\Phi_{m2} + .8\pi\,(mlt)\,\frac{l}{h}\,(\sigma n_1 I_1 + n_2 I_2)\left(\frac{x}{2} - \frac{x^2}{3}\right)\right] \tag{86}$$

Now the mutual fluxes Φ_{m1} and Φ_{m2} are linear functions of I_1 and I_2 and are large compared with the other terms, so that to an excellent approximation, (85) and (86) are also linear functions of x and may therefore be written in the form,

$$x\,E_1 = \frac{x}{10^8}\,\frac{\partial}{\partial t}\,(n_1\,\Phi_{m1}) = x\,\frac{\partial}{\partial t}\,(L_1'\,I_1 + M'\,I_2) \tag{87}$$

$$x\,E_2 = \frac{x}{10^8}\,\frac{\partial}{\partial t}\,(n_2\,\Phi_{m2}) = x\,\frac{\partial}{\partial t}\,(M'\,I_1 + L_2'\,I_2) \tag{88}$$

Thus comparing with **(30a)** and **(30b)**, it is evident that $x\,E_1 = x\,(E - e_n)$ is the final potential distribution due to the magnetic flux, to which must be added the neutral voltage e_n. Likewise $x\,E_2$ is that part of the secondary final distribution due to the magnetic flux, to which must be added E_2' due to the electrostatic field.

Writing $x = 1$ in **(87)** and **(88)**, there result the equations for calculating the waves transmitted through the transformer to the secondary circuit.

$$\left.\begin{aligned} E_1 &= p\,(L_1'\,I_1 + M'\,I_2) = E - Z_1\,I_1 \\ E_2 &= p\,(M'\,I_1) + L_2'\,I_2) = -\,(Z_2 + Z_3)\,I_2 \end{aligned}\right\} \tag{89}$$

in which L_1', L_2' and M' are the overall self and mutual inductances of the windings, in the conventional sense.

For a short-circuited secondary

$$\left.\begin{aligned} E_2 &= 0 \\ I_1 &= \frac{E_1\,L_2'\,t}{(L_1'\,L_2' - M'^2)} \\ I_2 &= \frac{E_1\,M'\,t}{(L_1'\,L_2' - M'^2)} \end{aligned}\right\} \tag{90}$$

and the currents increase linearly with time at a rate sufficient to consume the primary applied voltage E_1.

For an open-circuited secondary

$$\left.\begin{aligned} I_2 &= 0' \\ I_1 &= E_1\,t/L_1' \\ E_2 &= E_1\,M'/L_1' \end{aligned}\right\} \tag{91}$$

More generally, for any impedances Z_2 and Z_3 in the secondary circuit, the transmitted voltage and current may be calculated from **(89)**; for the currents I_1 and I_2 very soon greatly exceed in magnitude the oscillatory components of **(83)** and **(84)**, and therefore dominate the situation in so far as the secondary external circuit is concerned. Consideration of this aspect of the problem is outside the scope of this paper.

Discussion

For discussion of this paper see page 321.

Geometrical Circuits of Electrical Networks

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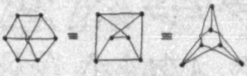
Member, A.I.E.E.

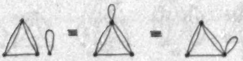
Synopsis.*—If all the electrical properties are abstracted from a given electrical network, there remains a geometrical circuit, completely characterized by the sets of branches terminating at the various vertexes. In this paper, enumerations of geometrical circuits are given, classified according to two different parameters, the nullity (number of branches minus number of vertexes plus number of separate parts) and the rank (number of vertexes minus number of separate parts). There is also a partial list of geometrical circuits which are symmetrical with respect to all the branches and all the vertexes.*

INTRODUCTION

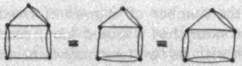
IN many problems in the theory of electrical networks it proves desirable to list all those networks which meet certain electrical conditions. From one point of view this type of problem belongs to the theory of combinations and permutations, but it can be satisfactorily handled on that basis only when the geometrical form of the circuit is specified. One of the first attempts to solve problems of this nature was that of P. A. Macmahon,[2] who, in 1892, investigated the combinations of resistances in series and in parallel, excluding all bridge arrangements such as the wheatstone net. These more complicated circuits, however, are often of greater interest, and it is in enumerating them that difficulties arise. One systematic method of procedure is to start with the geometrical circuits which meet the conditions of the problem, and then assign electrical elements to each branch of the geometrical circuit in all possible ways. At this stage of the investigation we can make use of the well known theory of combinations, taking due account of any symmetries among the branches of the circuit. For a successful application of this method we thus need, to start with, a complete enumeration of all distinct geometrical circuits, classified according to certain specified properties. This method has already been employed in a study of telephone substation and repeater circuits,[3] using as the foundation an enumeration of all distinct geometrical circuits with one, two, three, and four degrees of freedom.



Equivalent by Deformation
(4-1 Used as Example)



Equivalent by Separation
(4-12 Used as Example)



Equivalent by Series Interchange
(5-37 Used as Example)

FIG. 1—EQUIVALENT GEOMETRICAL CIRCUITS

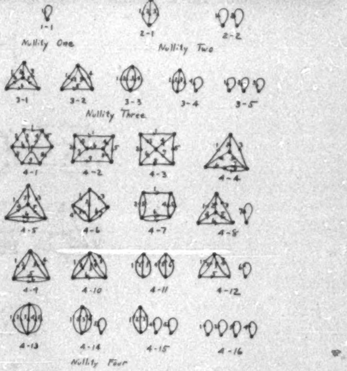


FIG. 2—GEOMETRICAL CIRCUITS OF NULLITY ONE, TWO, THREE, AND FOUR

DISTINCT GEOMETRICAL CIRCUITS

It is assumed that the electrical networks under consideration are composed of two-terminal elements connected together in any fashion. Not all electrical networks are of this type, but many of them may be regarded as such by a suitable designation of elements. The corresponding geometrical circuit consists of branches connected together at branch-points or vertexes, each branch corresponding to a two-terminal electrical element.

Two geometrical circuits are said to be equivalent if they can be brought into geometrical coincidence by one or more of the following procedures:

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[1] American Tel. & Tel. Co., New York, N. Y.

[2] "The Combinations of Resistances," P. A. Macmahon, *Electrician*, Vol. 28, Apr. 8, 1892, pp. 601-602.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

[3] *Maximum Output Networks for Telephone Substation and Repeater Circuits*, G. A. Campbell and R. M. Foster, TRANS. A.I.E.E., Vol. 39, 1920, pp. 231-280.

I. *By deformation:* That is, if wire models of the two circuits have been made, the circuits are equivalent if the models can be made to coincide by (a) translation and rotation of the model, (b) bending and stretching the wires, and (c) tying or untying knots or other linkages in the wires (that is, the wires can be made to cut themselves and each other like beams of light). This type of equivalence is illustrated by the upper row of diagrams in Fig. 1. Although they differ in appearance all three of these circuits are equivalent. In the mathematical theory of linear graphs, here called geometrical circuits, such equivalent structures are usually termed congruent. The essential feature of a geometrical circuit is thus the relative connection of the branches. If the branches are numbered the circuit is completely described by listing the branches terminating at each vertex.

II. *By separation:* That is, two circuits are equivalent if they can be made to coincide by separating two portions of a circuit which have a single vertex in common, and conversely, by connecting two disconnected portions of a circuit at a single vertex. This is illustrated by the middle row of diagrams in Fig. 1. In the mathematical theory of linear graphs the term equivalent is used to cover concepts I and II as defined here.

III. *By series interchange:* Two circuits are equivalent if they can be made to coincide by the interchange of any branches or groups of branches which have a series connection. This is illustrated by the lower row of diagrams of Fig. 1. This extension of the concept of equivalence is needed to meet the conditions of the usual type of electrical network, where in the case of elements in series the order of those elements in the series is inconsequential. There seems to be no corresponding term for this type of equivalence in the mathematical theory, although the concept might well prove useful, especially in the consideration of dual graphs, since series and parallel are dual concepts, and the interchange of branches in parallel is effected by I.

CLASSIFICATION BY NULLITY

Of the possible methods of classifying geometrical circuits, one of the most useful for our purpose is by the nullity.[4] If the number of branches, of vertexes, and of separate parts of the circuit are designated by B, V, and S, respectively, then the nullity N is given by the relation:

$$N = B - V + S.$$

Thus in Fig. 1, the upper circuits are of nullity four, the middle circuits four, and the lower circuits five. The nullity N is sometimes called either the cyclomatic number or the number of degrees of freedom. It is important to note that a closed loop, consisting of a single branch with its two terminals coinciding, has one vertex. In this case $B = V = S = 1$, and so $N = 1$, as it should. Equivalent circuits have the same nullity. This classification by nullity seems a natural method for our purpose since the electrical properties of networks are very often specified in terms of independent meshes, the number of degrees of freedom of the electrical network corresponding to the nullity of the geometrical circuit.

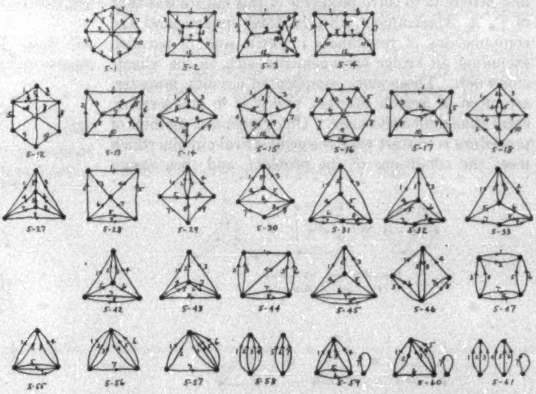

FIG. 3—GEOMETRICAL

It is also important to note that the nullity of a geometrical circuit is not changed by adding or by taking away an isolated, open branch, or by combining two branches in series into a single branch, or by separating a single branch into two branches in series. Accordingly, in enumerating circuits by their nullity, we shall assume that there are no isolated, open branches, and that all branches in series are counted as a single branch. In going over to the physical electrical circuit, a single branch may thus be replaced either by a single electrical element or by two or more such elements in series. It is not likely that electrical elements would be placed in isolated, open branches; but that possibility must be kept in mind.

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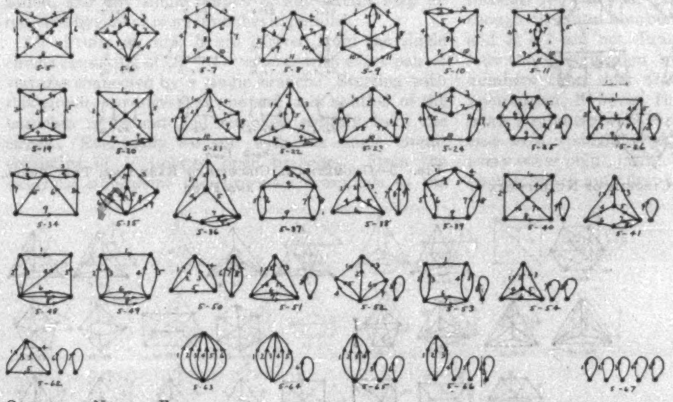
[4] This term is apparently due to Hassler Whitney; see "Non-Separable and Planar Graphs," *Proc. Nat. Academy Sciences*, Vol. 17, Feb. 1931, pp. 125-127.

GEOMETRICAL CIRCUITS OF NULLITY NOT GREATER THAN FIVE

All distinct geometrical circuits of nullity one, two, three, and four are shown in Fig. 2, and those of nullity five in Fig. 3. This list is complete, with the restrictions mentioned in the preceding paragraph, that is, in any one of the circuits shown, any number of isolated, open branches may be added to the circuit, and any single branch of the circuit may be counted as two or more branches in series. For use in applications, all branches are numbered and an arbitrary positive direction marked on each. These drawings may conveniently serve as a reference framework for all electrical networks with not more than five independent meshes, by designating the appropriate circuit number and specifying the particular electrical elements to be inserted in each of the numbered branches.



CIRCUITS OF NULLITY FIVE

DERIVATION OF CIRCUITS

All circuits of nullity N are derived from a smaller group of circuits, called the basic circuits, of the same nullity. In these basic circuits, three branches terminate at each of the vertexes; the circuits are non-separable, that is, each one is connected in a single part which cannot be separated into two parts which have only a single vertex in common; and there are no pairs of branches connected in series. Each of the basic circuits of nullity N thus has $2\ (N-1)$ vertices and $3\ (N-1)$ branches. The number of basic circuits is shown by the following table:

Nullity N	1	2	3	4	5	6
Number of basic circuits	1	1	1	2	4	14
Total number of circuits	1	2	5	16	67	435

In Figs. 2 and 3, the basic circuits are placed first, for each value of N. The 14 basic circuits for $N = 6$ are shown in Fig. 4. In view of the large number involved, the complete set of circuits of nullity six is not shown in this paper.

The basic circuits of nullity N are found from the basic circuits of nullity $(N-1)$ by adding a branch joining the midpoints of a pair of branches, in all possible ways, and then selecting the distinct circuits from the entire set of circuits obtained by this procedure. The fact that this method will yield the entire set of basic circuits of any given nullity follows from theorems proved by H. Whitney.[5]

Then starting with the basic circuits, with $3\ (N-1)$ branches, all circuits with fewer branches are found by allowing first one and then another branch of the basic circuits to shorten and vanish, thus obtaining first a set of circuits each with $(3\,N-4)$ branches, then from these the circuits with $(3\,N-5)$ branches, and so on, until we eventually end with the single circuit with N branches, consisting of N separate closed loops.

From the electrical standpoint this process seems quite natural since allowing a branch to shorten and vanish corresponds to the insertion of an electrical element of zero impedance in the branch, that is, a short circuit. The basic networks for any specified value of N taken alone thus give a complete picture of the entire set of circuits of nullity N, and for some purposes are sufficient, since any electrical network with N independent meshes can then be specified in terms of one or more basic circuits of nullity N by assigning the elements, including possible short circuits, to be distributed among the branches in a specified manner.[6]

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[5] "Basic Graphs," Hassler Whitney,—a paper presented to the American Mathematical Society, New Orleans, Dec. 28, 1931.

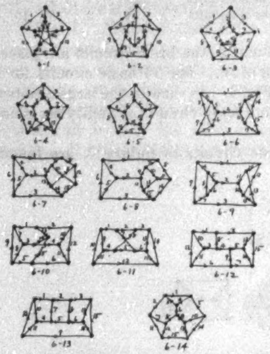
FIG. 4—BASIC GEOMETRICAL CIRCUITS OF NULLITY SIX

CLASSIFICATION BY RANK

Another method of classification of geometrical circuits is by the rank.[7] The rank R of a circuit is defined by the relation:

$$R = V - S.$$

Thus in Fig. 1, the upper circuits are of rank five, the middle circuits two, and the lower circuits four. Equivalent circuits have the same rank.

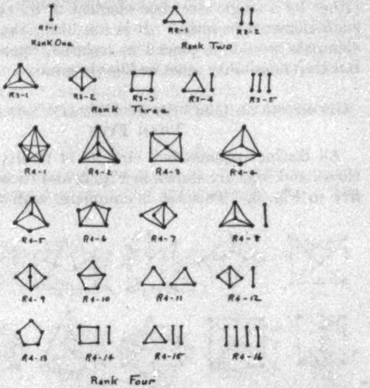

FIG. 5—GEOMETRICAL CIRCUITS OF RANK ONE, TWO, THREE, AND FOUR

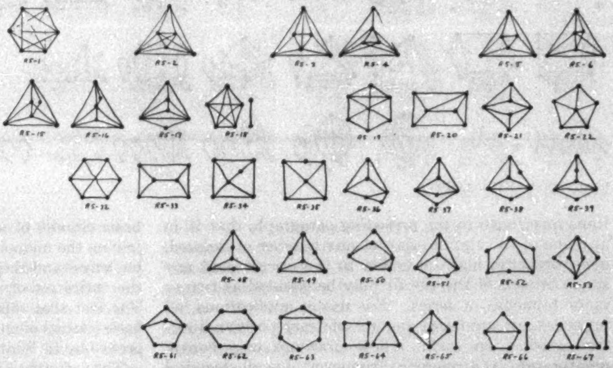
FIG. 6—GEOMETRICAL

It is important to note that the rank of a geometrical circuit is not changed by adding or by taking away an isolated closed loop consisting of a single branch with its two terminals coinciding, or by combining two branches in parallel into a single branch, or by replacing a single branch by two branches in parallel. Accordingly, in enumerating circuits by their rank, it is assumed that there are no isolated closed loops consisting of a single branch, and that all branches in parallel are counted as a single branch. In going over to the physical electrical circuit, a single branch may thus be replaced either by a single electrical element or by two or more such elements in parallel; also, electrical elements may be placed in isolated closed loops, a possibility that may actually arise, such as a short-circuited winding of a transformer.

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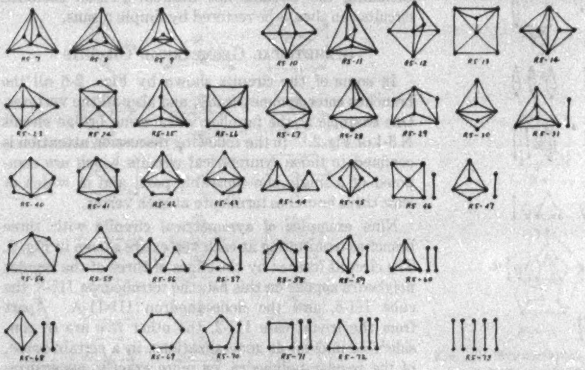
[6] Figs. 2-4 also illustrate the wide variety of forms which, under certain circumstances, equivalent electrical networks may assume.

[7] This term also is apparently due to Hassler Whitney; see "Non-Separable and Planar Graphs," *loc cit.*

GEOMETRICAL CIRCUITS OF RANK NOT GREATER THAN FIVE

All distinct geometrical circuits of rank one, two, three, and four are shown in Fig. 5, and those of rank five in Fig. 6. This enumeration is complete, with the restrictions mentioned in the preceding paragraph, that is, to each of the circuits shown, any number of isolated, closed loops, each consisting of a single branch, may be added, and any single branch of any circuit may be replaced by two or more branches in parallel.

All circuits of rank R are derived from the single circuit consisting of $(R + 1)$ vertexes with every pair of vertexes connected by a single branch. Starting with this circuit, we allow first one and then another of the branches to be cut and removed entirely from the circuit. Eventually we end with the single circuit consisting of N isolated open branches. From the electrical standpoint this process corresponds to the insertion of an electrical element of infinite impedance in the branch which is to be cut, that is, the branch is open-circuited.



CIRCUITS OF RANK FIVE

The number of circuits, classified by the rank, is shown by the following table:

Rank R	1	2	3	4	5	6
Total number of circuits	1	2	5	16	73	538

In view of the large number, the circuits of rank six are not shown in this paper.

There is an inverse or dual relationship between the rank R and the nullity N. The two parameters change places when all series connections are changed to parallel and vice versa, with corresponding transformations in the case of bridge circuits; to every set of branches forming a closed path in one circuit there corresponds a set of branches in the other circuit separating the vertexes into two groups which are not connected by any other branch. H. Whitney[8] has shown that if the geometrical circuit is applicable to the surface of a sphere, there exists a dual or inverse circuit with $N' = R$ and $R' = N$, and that this does not hold for circuits which are not applicable to a sphere, that is, for circuits which cannot be drawn on a sphere or plane with no branches crossing one another. As a result, the enumeration by given values of R is not completely symmetrical with the enumeration by given values of N. There is complete correspondence through $N = R = 3$. For $N = R = 4$, however, although the total numbers are the same, circuits N 4-1 and R 4-1 are not duals. For values greater than four the classification by rank leads to larger total numbers than does that by nullity. It should be pointed out, however, that the enumeration by R is a more straightforward process, since we start in each case with a single circuit, each point connected with every other point, instead of starting with a set of basic circuits which must first be derived.

OTHER METHODS OF CLASSIFICATION

For some purposes neither of the above methods of enumeration may prove quite satisfactory in view of the special restrictions involved. It may be desirable to have a complete enumeration of circuits with B branches, whether or not some of these branches may be arranged in series or parallel or left isolated. Fig. 7 shows a beginning of such an enumeration, the circuits being classified according to values of both N and R. It is to be noted that $N + R = B$. This enumeration may be derived from the classification by nullity, upon inserting branches in series and adding isolated open branches; or it may be derived from the classification by rank, upon inserting branches in parallel and adding isolated closed branches. In either case, given the fundamental enumerations, by nullity or by rank, the problem can then be handled by the regular methods of the theory of combinations.

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[8] "Non-Separable and Planar Graphs," Hassler Whitney, *loc. cit.*,—Theorem 15.

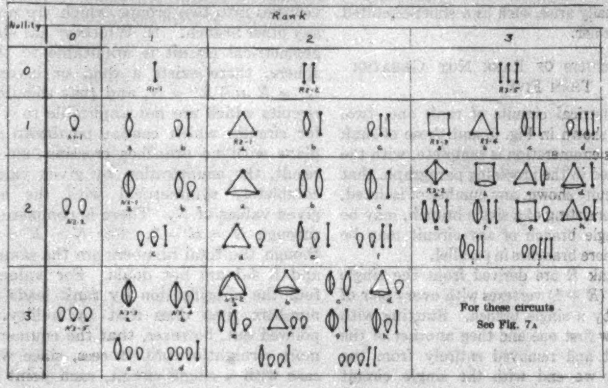

FIG. 7—GEOMETRICAL CIRCUITS CLASSIFIED BY NULLITY AND RANK

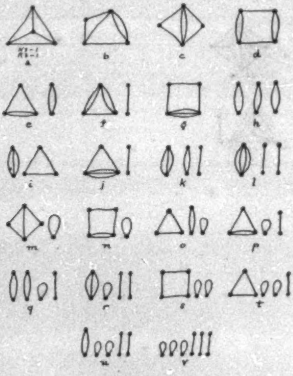

FIG. 7A—GEOMETRICAL CIRCUITS OF NULLITY THREE AND RANK THREE

Such large numbers of circuits are involved in this scheme, however, that it does not seem worth while in this paper to extend it to higher values. A more useful plan, perhaps, is to exclude all series and parallel combinations, as well as all isolated single elements, whether open or closed. Such a classification is shown by Fig. 8, for values of N and R not greater than six. This scheme can be obtained either from the classification by nullity or from the classification by rank, upon excluding the circuits not desired. These excluded circuits can always be restored by simple means.

SYMMETRICAL GEOMETRICAL CIRCUITS

In some of the circuits shown by Figs. 2-6 all the branches enter symmetrically, and also all the vertexes. One example is the familiar wheatstone bridge circuit N 3-1 of Fig. 2. In the following discussion attention is confined to those symmetrical circuits which are composed of a single non-separable part, and in which at least three branches terminate at each vertex.

Nine examples of symmetrical circuits with three branches terminating at each vertex are shown in Fig. 9. The circuits formed by the edges of three of the regular polyhedra appear on this list, the tetrahedron III-3, the cube III-5, and the dodecahedron III-11-A. Apart from the trivial case III-2, the other five are of considerable interest as generalizations, in a certain sense, of the regular polyhedra, or more exactly, generalizations of the geometrical circuits formed by the edges of the regular polyhedra. The circuit III-4 has been used in several places[9] as an illustration of a rigid structure, in the theory of statics, though its symmetrical character has apparently not been pointed out in that connection. The circuit III-6 was used by J.

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[9] See, for example, *Encyklopädie der mathematischen Wissenschaften*, Vol. 4, part 1, p. 403. The symmetrical character of this circuit was pointed out in the 1920 paper by G. A. Campbell and R. M. Foster previously mentioned.

Petersen[10] in showing that Tait's theorem on the coloring of graphs could not be extended to graphs which were not applicable to a sphere, but here again the symmetrical character of the circuit was not mentioned. The three others, III-9, III-11-B, and III-13, are apparently new. It is probable that there exist other symmetrical circuits of nullity greater than six and with three branches at each vertex, but a preliminary investigation has failed to disclose them. On the other hand, it is extremely interesting that there are at least two such circuits of nullity eleven, namely, III-11-A and III-11-B.

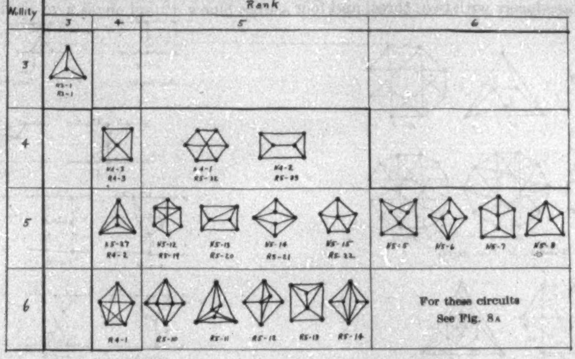
Nullity
Rank
3
4
5
6
3
N3-1
R3-1
4
N4-3
R4-3
N4-1
R5-32
N4-2
R5-33
5
N5-27
R4-2
N5-12
R5-19
N5-13
R5-20
N5-14
R5-21
N5-15
R5-22
N5-5
N5-6
N5-7
N5-8
6
R4-1
R5-10
R5-11
R5-12
R5-13
R5-14
For these circuits
See Fig. 8A

Fig. 8—Geometrical Circuits Classified by Nullity and Rank—Omitting Circuits with Branches in Series or in Parallel or Isolated

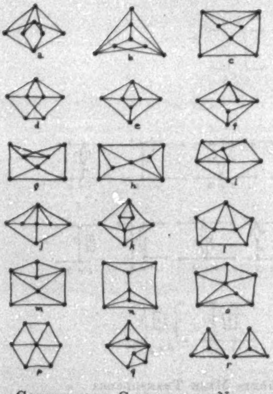
a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p
q
r

Fig. 8a—Geometrical Circuits of Nullity Six and Rank Six—Omitting Circuits with Branches in Series or in Parallel or Isolated

No attempt has yet been made to enumerate completely the symmetrical circuits with four or more branches terminating at each vertex, although a few examples are shown in Fig. 10, including the circuits of the octahedron IV-7 and the icosahedron V-19. Here again it is interesting to note that there are at least two such circuits of nullity eleven, namely, IV-11-A and IV-11-B. A complete list would include many trivial cases, such as, for example, the symmetrical circuits with four branches at each vertex obtained by putting a branch in parallel with each edge of a regular polygon, as is illustrated by circuits N 4-9 and N 5-47 of Figs. 2 and 3. A complete list of symmetrical circuits would

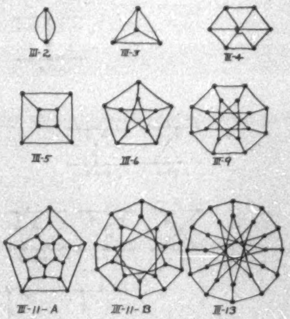
III-2
III-3
III-4
III-5
III-6
III-9
III-11-A
III-11-B
III-13

Fig. 9—Symmetrical Geometrical Circuits with Three Branches at Each Vertex

also include the circuits used as starting points in the classification by rank, that is, the circuits with every pair of vertexes connected by a single branch, for example, III-3, IV-6, and V-10.

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[10] Julius Petersen, *Intermédiaire des Mathématiciens*, Vol. 5, Oct. 1898, pp. 225-227.

GEOMETRICAL FORMS OF TRANSDUCERS

As an illustration of one application of the method described in this paper, Fig. 11 shows the basic geometrical forms of transducers with two, three, and four meshes, obtained from the geometrical circuits of Fig. 2. The transducer has two pairs of terminals; so the problem of enumeration reduces to the problem in simple combinations of picking out a pair of branches from the basic circuits N 2-1, N 3-1, N 4-1, and N 4-2. This can be done in 1, 2, 2, and 6 different ways, respectively, giving a total of 11 different forms. The single basic two-mesh transducer is the familiar T network. The first of the basic three-mesh transducers is the series extended lattice, and the second is the series extended bridged-T. No one of the eight four-mesh transducers appears to have been investigated to any extent. It should be noted that the assigned positive directions on Fig. 2 have been ignored in this enumeration; the number of forms given here could be approximately doubled by twisting one pair of terminals; and the number could be further increased by interchanging the two pairs of terminals in the dissymmetrical cases.

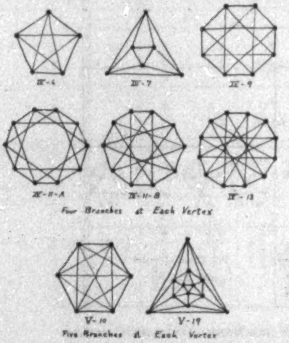
IV-6
IV-7
IV-9
IV-11-A
IV-11-B
IV-13
Four Branches at Each Vertex
V-10
V-19
Five Branches at Each Vertex

FIG. 10—SYMMETRICAL GEOMETRICAL CIRCUITS WITH FOUR OR MORE BRANCHES AT EACH VERTEX

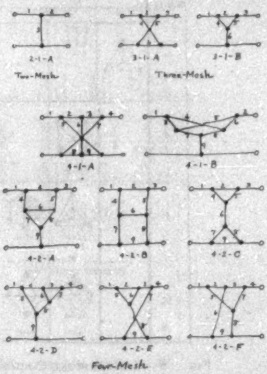
2-1-A
3-1-A
3-1-B
Two-Mesh
Three-Mesh
4-1-A
4-1-B
4-2-A
4-2-B
4-2-C
4-2-D
4-2-E
4-2-F
Four-Mesh

FIG. 11—BASIC GEOMETRICAL FORMS OF TWO-, THREE-, AND FOUR-MESH TRANSDUCERS

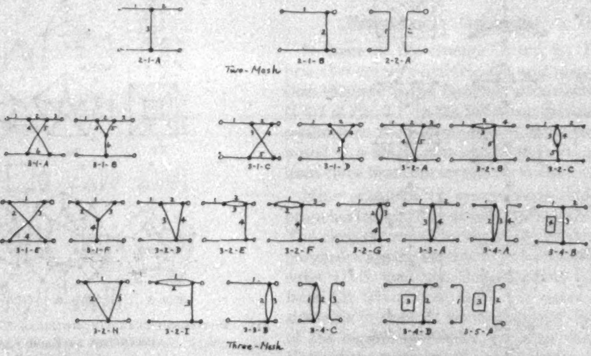
2-1-A
2-1-B
2-2-A
Two-Mesh
3-1-A
3-1-B
3-1-C
3-1-D
3-2-A
3-2-B
3-2-C
3-1-E
3-1-F
3-2-D
3-2-E
3-2-F
3-2-G
3-3-A
3-4-A
3-4-B
3-2-H
3-2-I
3-3-B
3-4-C
3-4-D
3-5-A
Three-Mesh

FIG. 12—GEOMETRICAL FORMS OF TWO- AND THREE- MESH TRANSDUCERS

Fig. 11 shows only the basic forms. Thus the familiar II type of network is not illustrated directly in this figure, although it may be obtained from either 3-1-A or 3-1-B by allowing branches 1, 3, and 6 to shorten and vanish. Since such cases may be of practical interest the complete set of two- and three-mesh transducers is shown in Fig. 12. As before, however, no attempt has been made to show the alternative forms obtained by twisting one pair of terminals or by interchanging the two pairs; nor are the limiting cases shown where the transducer reduces to a mere series or parallel connection. In certain cases, such as 3-2-B, the replacement of a group of branches connecting two terminals by a single branch would reduce the number of meshes from three to two; but this cannot be done, in general, if there is mutual inductance between any branch of the group and a branch outside the group. Accordingly, these cases are included in the enumeration.

Discussion

For discussion of this paper see page 321.

Vector Theory of Circuits Involving Synchronous Machines

BY I. H. SUMMERS*

Synopsis.†—*It has been shown in previous publications that calculations of the performance of a synchronous machine may be simplified by the use of two scalar impedance operators which apply separately to the component of armature current which magnetizes in the axis of the poles, and to the component which magnetizes in the axis of the interpolar space.*

The present paper describes a method of combining these two scalar operators into a single vector operator which applies directly to the total current. With the new operator it becomes possible to write out the complete equations which apply to various numbers of interconnected machines, under various operating conditions, as readily as would be possible if each machine was replaced by a fixed source of voltage operating through a constant value of impedance. The effect is to simplify and thus to extend the scope of synchronous machine analysis. It is believed that the theory developed will be of value to engineers who are working on problems such as stability, hunting, and other problems of this general character which involve detailed analysis of the operation of two or more interconnected synchronous machines under transient conditions.

* * * * *

INTRODUCTION†

THE present paper deals with circuits involving synchronous machines having three armature circuits, salient poles, an arbitrary number of closed circuits on the rotor, and two rotor field circuits, one magnetizing in the axis of the poles, and one magnetizing in the axis of the interpolar space. The performance of such a machine is determined by the history of the three armature terminal voltages, the two field voltages, and the angular position of the rotor with respect to the armature. Let the instantaneous values of these quantities by $e_a, e_b, e_c, e_{fd}, e_{fq}, \theta$, and let the instantaneous values of armature current be i_a, i_b, i_c.

Then in problems which do not involve the flow of neutral current, it can be shown[1] that the equations determining current may be written in the form

$$e_d = p\psi_d - r i_d - \psi_q \, p \, \theta$$
$$e_q = p\psi_q - r i_q + \psi_d \, p \, \theta$$

where

$$p = \frac{d}{dt}$$

$$\psi_d = g_d(p)\, e_{fd} - x_d(p)\, i_d$$
$$\psi_q = g_q(p)\, e_{fq} - x_q(p)\, i_q$$

r = armature resistance

$g_d(p)$, $g_q(p)$, $x_d(p)$, $x_q(p)$ are operators and e_d, e_q, i_d, i_q are particular functions of e_a, e_b, e_c, θ, and i_a, i_b, i_c, θ, and are referred to as the direct and quadrature components of voltage and current respectively.

The operators $g_d(p)$, $g_q(p)$, etc., may be computed from design data, or determined by tests, and may be visualized if desired in terms of appropriate equivalent circuits.

It will be seen that the solution of the equations is complicated by the fact that there is a mutual relationship between the direct and quadrature quantities, that is by the fact that direct component current gives rise to both direct and quadrature components of voltage, and conversely. Nevertheless, when the values of terminal voltage are definitely known, either as a function of time, or of angular displacement, as when only a single machine is being considered, the analysis is comparatively simple.

However, difficulties are encountered when it is desired to study several interconnected machines. These difficulties may be appreciated by considering the case of two machines connected directly together. Let subscripts 1 and 2 be used to designate these machines. Then it is known that $e_{a1} = e_{a2}$, $i_{a1} = -i_{a2}$, and similarly for the other phases. It does not follow, however, that $e_{d1} = e_{d2}$, $i_{d1} = -i_{d2}$, $e_{q1} = e_{q2}$, $i_{q1} = -i_{q2}$ except in the particular case $\theta_1 = \theta_2$. It can be shown, however, that

$$\mathbf{e}_1 = \mathbf{e}_2$$
$$\mathbf{i}_1 = -\mathbf{i}_2$$

where

$$\mathbf{e}_1 = (e_{d1} + j\, e_{q1})\, \underline{|\theta_1}$$
$$\mathbf{e}_2 = (e_{d2} + j\, e_{q2})\, \underline{|\theta_2}$$
$$\mathbf{i}_1 = (i_{d1} + j\, i_{q1})\, \underline{|\theta_1}$$
$$\mathbf{i}_2 = (i_{d2} + j\, i_{q2})\, \underline{|\theta_2}$$

This suggests that the solution for current could be simply effected if suitable symbolic relations could be established between $\mathbf{e}_1$ and $\mathbf{i}_1$, and $\mathbf{e}_2$ and $\mathbf{i}_2$.

Were it not for the salient pole construction it would be possible to write a relation between the current and voltage of any machine in the form

$$\mathbf{e} = g(p)\, \mathbf{e}_f - z(p)\, \mathbf{i}$$

where

$$\mathbf{e}_f = (e_{fd} + j\, e_{fq})\, \underline{|\theta}$$

and $g(p)$ and $z(p)$ are scalar operators which contain speed as a parameter. However, as previously noted, it is a consequence of salient pole construction that the component of $\mathbf{e}$ due to $\mathbf{e}_f$ need not be in phase with $\mathbf{e}_f$ and the component $\mathbf{e}$ due to $\mathbf{i}$ need not be exactly 180 deg. out of phase with $\mathbf{i}$.

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*Deceased. Formerly with the General Electric Co., Schenectady, N. Y.

†Prepared by R. H. Park, Stone & Webster Engg. Corp., Boston, Mass.

[1] *Two Reaction Theory of Synchronous Machines*, R. H. Park, A.I.E.E. TRANS., Vol. 48, July 1929, p. 716-727.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Mr. Summers shows in this paper, however, that a relation can be written between $\mathbf{e}$ and $\mathbf{i}$, in the form

$$\mathbf{e} = \hat{g}\,(p)\,\mathbf{e}_f - \hat{x}\,(p)\,\mathbf{i}$$

where $g\,(p)$ and $x\,(p)$ are linear vector operators which involve speed, armature resistance and the scalar operators $g_d\,(p)$, $g_q\,(p)$, $x_d\,(p)$, and $x_q\,(p)$. These vector operators effect a very great simplification of the differential equations which determine the operation of the machines. In the special case in which the machine speeds are constant, although not necessarily equal, they permit direct solution of the problem in operational form, since the operators are subject to the ordinary rules of the algebra of linear vector operators when they do not contain parameters which are functions of the independent variable. Also, it becomes as simple to write out, in symbolic form, the solution of problems involving the performance of a number of interconnected machines operating at constant or nearly constant speed, under transient or steady load conditions, as it is to write out the solution of the equations which determine the flow of current in a d-c. network. The final explicit form of the result may then be obtained by evaluating the vector operational forms in accordance with established mathematical procedure.

The evaluation of the linear vector operations may be accomplished either by means of dyadics, or by matrix algebra. To quote from Summers' manuscript notes:

"Matrix algebra is of value in linear vector operations for the same reason that ordinary algebra is used in differential operations. Special rules are used in either case when the operations are actually performed on the operand, but transformations are facilitated in the meantime. In the case of the differential operators the expansion theorem is used eventually, and in the case of linear vector operators dyads are supplied and scalar products are formed. Algebra and matrix algebra are used previous to the final operations in the respective cases."

The resultant effect is to simplify and thus to extend the scope of synchronous machine analysis, particularly in cases in which more than one machine is involved.

This work has been taken directly from the author's manuscript notes, except for such minor changes as were considered obviously necessary. The body of the paper will be seen to be incomplete, but it is believed that the development of the thought is adequate to permit any one familiar with the general subject to complete the unfinished portion.

ANALYSIS

It has been shown* that synchronous machine electrical quantities are expressible in terms of linear equations involving direct axis, quadrature axis, and zero phase sequence quantities and that the coefficients of these linear equations are differential operators. Thus the following sets of relations are fundamental:

$$\left.\begin{aligned} \psi_d &= g_d\,(p)\,e_{fd} - x_d\,(p)\,i_d \\ \psi_q &= g_q\,(p)\,e_{fq} - x_q\,(p)\,i_q \\ \psi_o &= -\,x_o\,i_o \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} e_d &= p\,\psi_d - ri_d - \psi_q\,p\,\theta \\ e_q &= p\,\psi_q - ri_q + \psi_d\,p\,\theta \\ e_o &= p\,\psi_o - ri_o \end{aligned}\right\} \qquad (2)$$

$$\left.\begin{aligned} i_{fd} &= y_{fd}\,(p)\,e_{fd} + k_{fd}\,(p)\,i_d \\ i_{fq} &= y_{fq}\,(p)\,e_{fq} + k_{fq}\,(p)\,i_q \end{aligned}\right\} \qquad (3)$$

The differential operators in these equations are deducible by means of auxiliary relations, and other auxiliary relations enable phase quantities to be determined from the direct, quadrature and zero quantities.

For the present purpose we will assume that the differential operators are known, and that the problem is solved when the direct, quadrature and zero quantities are known.

It happens that the presence of zero phase sequence quantities is a fairly rare occurrence in a synchronous machine connected to a large power system so that a large class of problems occurs where the direct and quadrature quantities completely specify the performance. It is with this class of problems that the present note is concerned.

We may define $\mathbf{d}$ and $\mathbf{q}$ as unit vectors in the direct and quadrature axes of the machine respectively. Further define:

$$\left.\begin{aligned} \boldsymbol{\psi} &= \psi_d\,\mathbf{d} + \psi_q\,\mathbf{q} \\ \mathbf{e} &= e_d\,\mathbf{e} + e_q\,\mathbf{q} \\ \mathbf{i} &= i_d\,\mathbf{e} + i_q\,\mathbf{q} \\ \mathbf{e}_f &= e_{fd}\,\mathbf{e} + e_{fq}\,\mathbf{q} \\ \mathbf{i}_f &= i_{fd}\,\mathbf{e} + i_{fq}\,\mathbf{q} \end{aligned}\right\} \qquad (4)$$

Then neglecting zero phase sequence quantities and replacing $p\,\theta$ by $1 - s$, where s = slip with respect to normal speed, there is obtained:

$$\boldsymbol{\psi} = \hat{g}\,(p)\,\mathbf{e}_f - \hat{x}\,(p)\,\mathbf{i} \qquad (5)$$

$$\mathbf{e} = (p + (1 - s)\,j)\,\boldsymbol{\psi} - r\mathbf{i} \qquad (6)$$

$$\mathbf{i}_f = y_f\,(p)\,e_f + k_f\,(p)\,\mathbf{i} \qquad (7)$$

In these expressions a caret over a quantity means that it is a linear vector operator, transforming a vector into another by extensions and rotations. It is equivalent to a dyadic[2] followed by a dot to indicate the dot (scalar) product. Inasmuch as a dyadic is a matrix[3] and operations are wanted between scalars and linear vector operators it will be convenient to designate the operator by its matrix, the vector coordinate and the dot

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*R. H. Park, *loc. cit.*

[2] "Vector Analysis," Gibbs-Wilson, Yale University Press.

[3] "Introduction to Higher Algebra," Bocher, M., McMillan.

being understood. The coordinates may be stated as subscripts when desired. Thus:

$\hat{a}\ r$ is understood to be

$$\begin{pmatrix} a_{11}\,\mathbf{dd} + a_{12}\,\mathbf{dq} \\ + a_{21}\,\mathbf{qd} + a_{22}\,\mathbf{qq} \end{pmatrix} . \mathbf{r}$$

and will be characterized by the convention.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \mathbf{r}$$

In this way the symbolism is freed from superfluous unit vectors and product symbols and matrix algebra is available to carry out operations between scalars and matrices. When vector operations are to be performed the vector dyads and the dot may be written in at will,[4] or may be supplied mentally.

Thus in (**5**), (**6**) and (**7**):

$$\left.\begin{aligned} \hat{g}\,(p) &= \begin{pmatrix} g_d\,(p) & o \\ o & g_q\,(p) \end{pmatrix} \\ \hat{x}\,(p) &= \begin{pmatrix} x_d\,(p) & o \\ o & x_q\,(p) \end{pmatrix} \\ \hat{x} &= \begin{pmatrix} x_d & o \\ o & x_q \end{pmatrix} \\ j &= \begin{pmatrix} o & -1 \\ 1 & o \end{pmatrix} \\ \hat{y}_f\,(p) &= \begin{pmatrix} y_{fd}\,(p) & o \\ o & y_{fq}\,(p) \end{pmatrix} \\ \hat{k}_f\,(p) &= \begin{pmatrix} k_{fd}\,(p) & o \\ o & k_{fq}\,(p) \end{pmatrix} \end{aligned}\right\} \qquad (8)$$

Where any terms in a matrix equation do not contain a matric explicitly as written, unit matrix, $\begin{pmatrix} l & o \\ o & l \end{pmatrix}$, is understood.

Equations (**5**) and (**6**) may be combined to give:

$$\mathbf{e} = \hat{h}\,(p)\,\mathbf{e}_f - \hat{z}\,(p)\,\mathbf{i} \qquad (9)$$

The method of working may be illustrated by evaluating $\hat{h}\,(p)$ and $\hat{z}\,(p)$.

Thus, directly:

$$\hat{z}\,(p) = [p + (1-s)\,j]\,\hat{x}\,(p) + r$$

This gives:

$$\begin{pmatrix} p & s-1 \\ 1-s & p \end{pmatrix}\begin{pmatrix} x_d\,(p) & o \\ o & x_q\,(p) \end{pmatrix} + r$$

Carrying out the multiplication and adding r there is obtained

$$\left.\begin{aligned} &\hat{z}\,(p) = \begin{pmatrix} z_d\,(p) & (s-1)\,x_q\,(p) \\ (1-s)\,x_d\,(p) & z\,q\,(p) \end{pmatrix} \\ &\text{where} \\ &z_d\,(p) = p\,x_d\,(p) + r \\ &z_q\,(p) = p\,x_q\,(p) + r \end{aligned}\right\} \qquad (10)$$

Similarly originally

$$\hat{h}\,(p) = (p + (1-s)\,j)\,\hat{g}\,(p)$$

which gives

$$\hat{h}\,(p) = \begin{pmatrix} pg_d\,(p) & (s-1)\,g_q\,(p) \\ (1-s)\,g_d\,(p) & pg_q\,(p) \end{pmatrix} \qquad (11)$$

As an illustration of the working out of the equations, consider two machines connected together,

$$\left.\begin{aligned} \mathbf{e} &= \hat{h}_1\,(p)\,\mathbf{e}_{f1} - \hat{z}_1\,(p)\,\mathbf{i} \\ &= \hat{h}_2\,(p)\,\mathbf{e}_{f2} + \hat{z}_2\,(p)\,\mathbf{i} \end{aligned}\right\} \qquad (12)$$

Subtracting

$$(\hat{z}_1\,(p) + \hat{z}_2\,(p)\,) = \mathbf{i} = \hat{h}_1\,(p)\,\mathbf{e}_{f1} - \hat{h}_2\,(p)\,\mathbf{e}_{f2}$$

Multiplying through by the inverse operator[5]

$$\mathbf{i} = [\hat{z}_1\,(p) + \hat{z}_2\,(p)\,]^{-1}\,[\hat{h}_1\,(p)\,\mathbf{e}_{f1} - \hat{h}_2\,(p)\,\mathbf{e}_{f2}\,] \qquad (13)$$

Now in writing equations (**12**) it was tacitly assumed that the matrix operators were expressible as vector dyads on the same set of axes.

In order to carry out this requirement consider:

$$a_{ij} = \begin{pmatrix} a_{11}\,\mathbf{ii} & a_{12}\,\mathbf{ij} \\ a_{21}\,\mathbf{ji} & a_{22}\,\mathbf{jj} \end{pmatrix} \qquad (14)$$

Now if

$$\left.\begin{aligned} \mathbf{i} &= l_1\,\mathbf{f} + m_1\,\mathbf{g} \\ \mathbf{j} &= l_2\,\mathbf{f} + m_2\,\mathbf{g} \end{aligned}\right\} \qquad (15)$$

Substitution of (**15**) in (**14**) and collecting terms gives:

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[4] The question of having special symbolism to indicate that a quantity is a scalar, a vector, a differential operator or a linear vector operator, should be given careful thought. It would facilitate writing, typing, printing, and reading if no special symbolism were used, the quantities being completely flexible and capable of taking on the proper significance at will. On the other hand, interpretation is facilitated by having the precise meanings of all symbols indicated at all times. Perhaps a compromise would be best in which the actual equations are written without special symbolism but the interpretations are carefully noted in the text. Special operators, such as synchronous reactance could be indicated, for example, as $x(\infty)$ or x_s and pure vectors could be written as a scalar with the proper unit vector, special letters being reserved for the unit vector to make them readily distinguishable.

[5] This implies that s_1 and s_2 are constant. For uniplanar matrixes the inverse operator $- x^{-1}$ is

$$\frac{\begin{pmatrix} x_{22} & -x_{12} \\ -x_{21} & x_{11} \end{pmatrix}}{\begin{vmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{vmatrix}}$$

$$\left.\begin{aligned} a_{fg} &= \begin{pmatrix} b_{11}\,\mathbf{ff} & b_{12}\,\mathbf{fg} \\ b_{21}\,\mathbf{gf} & b_{22}\,\mathbf{gg} \end{pmatrix} \\ &\text{where} \\ b_{11} &= l_1^2\, a_{11} + (a_{12} + a_{21})\, l_1\, l_2 + l_2^2\, a_{22} \\ b_{12} &= l_1\, m_1\, a_{11} + l_1\, m_2\, a_{12} + l_2\, m_1\, a_{21} + l_2\, m_2\, a_{22} \\ b_{21} &= l_1\, m_1\, a_{11} + l_2\, m_1\, a_{12} = l_1\, m_2\, a_{21} + l_2\, m_2\, a_{22} \\ b_{22} &= m_1^2\, a_{11} + (a_{12} + a_{21})\, m_1\, m_2 + m_2^2\, a_{22} \end{aligned}\right\} \tag{16}$$

For a set of rectangular axes rotated an angle δ from the reference there is

$$\left.\begin{aligned} l_1 &= \cos\delta & m_1 &= \sin\delta \\ l_2 &= -\sin\delta & m_2 &= \cos\delta \end{aligned}\right\} \tag{17}$$

and

$$b_{11} = \frac{a_{11} + a_{22}}{2} + \frac{a_{11} - a_{12}}{2}\,\text{Cos}\,2\,\delta - \frac{a_{12} + a_{21}}{2}\,\text{Sin}\,2\,\delta$$

$$b_{12} = \frac{a_{11} - a_{22}}{2}\,\text{Sin}\,2\,\delta + \frac{a_{12} - a_{21}}{2} + \frac{a_{12} + a_{21}}{2}\,\text{Cos}\,2\,\delta$$

$$b_{21} = \frac{a_{11} - a_{22}}{2}\,\text{Sin}\,2\,\delta + \frac{a_{21} - a_{12}}{2} + \frac{a_{12} + a_{21}}{2}\,\text{Cos}\,2\,\delta$$

$$b_{22} = \frac{a_{11} + a_{22}}{2} - \frac{a_{11} - a_{22}}{2}\,\text{Cos}\,2\,\delta + \frac{a_{12} + a_{21}}{2}\,\text{Sin}\,2\,\delta$$

Discussion

EQUIVALENT CIRCUITS—I

(Starr)

TRANSIENT OSCILLATIONS OF MUTUALLY COUPLED CIRCUITS

(Bewley)

GEOMETRICAL CIRCUITS OF ELECTRICAL NETWORKS

(Foster)

VECTOR THEORY OF CIRCUITS INVOLVING SYNCHRONOUS MACHINES

(Summers)

J. E. Clem: Mr. Starr's paper presents a thorough and comprehensive treatment of the derivation and application of equivalent circuits. Considerable emphasis is given to the greater utilization of the calculating board, by applying equivalent circuits, to the solution of certain problems too difficult to solve analytically. For example, in Part III of the paper, a number of simplified equivalent circuits to represent groups of parallel transmission lines are derived. It is shown in the paper how these circuits may be used to represent the zero-phase diagram of a transmission system on a calculating board in a very simple manner.

It is interesting to note that the group of equivalents shown in Figs. 8 to 15 involve no negative resistance and in most cases involve no negative links of any kind. It is evident that these properties make these circuits of particular value in calculating board work. Anyone interested in system studies should find these circuits, as well as the generalized equivalents developed in Parts I and II of the paper of considerable value, whether or not it is intended to use the calculating board.

More important than the particular applications treated are, perhaps, the general methods and rules which are given to apply equivalent circuits to any type of problem. Thus, the methods developed here might be applied directly to obtain equivalent circuits to represent various types of rotating machinery, and particularly certain multi-circuit motors for which adequate equivalents have not as yet been developed. Representations of the proposed three and four winding generators for use in a single-line diagram also suggest the use of circuits similar to those shown in Figs. 9 and 10.

Many other circuit problems lend themselves to the use of an equivalent circuit as an aid to the solution. For example, they are useful in analyzing any multi-circuit transformer problem where each winding is loaded with a particular type of load and especially if the loads are interconnected electrically. Another suggested application is the use in analyzing the operation of certain types of relays involving combinations of coupled circuits.

Although the equivalent circuit is shown to be quite useful in the analysis of coupled circuits, this is not the only general application. In Part II of the paper there is developed a general equivalent which may be used to replace any part or all of a general network composed of any number of erratically placed links and having any number of points of entry. The value of such an equivalent is evident. The number of links required to represent the network is greatly reduced by using the equivalent.

In Mr. Summers' paper there is presented a new variant of old methods of mathematical manipulation. We have had matrix algebra, vector analysis, and operational mathematic, for a long time. Here we find suggested a method of adapting and combining matrix algebra and vector analysis so that the work may be carried on in the operational form. With the aid of a few rules it is possible to work out the solution symbolically as easily as for an ordinary algebraic problem. The work to obtain this solution is reduced through the simplification and decrease in the number of terms which must be carried along and it is much easier to retain throughout the work a clear conception of the physicial meaning of the equations. This latter concept is of great importance.

The method proposed has been illustrated by an application to an a-c. generator and this immediately suggests that it can be advantageously applied to problems of system operation such as stability, hunting, and synchronizing power. Design of synchronous apparatus can thus be studied in relation to system operation to the end that machines giving better system performance may be built.

This method of analysis should find a broad field of application in all problems involving the use of vector operators. Whenever the reaction coefficient along one axis is different than that along another, as in non-isotropic substances, vector operators can be brought into use. Such problems may occur not only in electrical work, but also in the field of acoustics, mechanics, thermics, and hydraulics.

The paper by Mr. Bewley makes public a mathematical solution of a problem which has been recognized for some time, so that there is now generally available a method by which the internal behavior of a transformer resulting from impact of an impulse may be analyzed. Although the analysis has been worked out on the basis of an applied rectangular wave, the solution for any applied wave shape is immediately available by the use of the well-known principle of super-position.

This paper will be of inestimable value in analyzing the insulation design of transformers. It makes it possible for anyone to evaluate the stress at various points of the windings and correlate the amount of insulation with the requirements. Further, it now becomes possible to insulate the secondary windings of a transformer not only from consideration of the overvoltages coming from the secondary side, but also of the overvoltages coming through the transformer from the primary side. In short, the correlation of the insulation of transformers with experience can now be placed upon a combined rational and empirical basis.

Although written primarily in consideration of transformer needs, this paper will be of distinct value in other fields, particularly for rotating apparatus. Proper insulation of rotating apparatus requires a knowledge of the voltage distribution throughout the windings due to impact of a traveling wave. The method of attack set forth by Mr. Bewley can be applied to this problem. With the modifications requisite for the different types of apparatus, distribution of the internal voltages can be analyzed and the insulation and protective requirements determined.

In Mr. Foster's paper there is given a method of enumerating all possible circuits having certain specified characteristics, defined as "nullity" and "rank." This information is of particular use to communication engineers in the study and design of maximum output networks for substation and repeater circuits.

R. D. Evans: One of the early problems in power system networks was in connection with the supply systems for a-c. railways. Here the layout was determined largely because of transformer loading and voltage regulation which depended upon unusual concentrations that would occur under practical operation. A particular feature of this network problem is the fact that the loads are taken off at varying points along a network branch. Quite elaborate methods were developed to take care of simultaneous loading at different points and to take into account changes in circuit condition arising from the disconnection of a line or of a transformer.

Following this came a very important step in network theory, namely, the introduction of the method of symmetrical components by Dr. Fortescue to make possible the solution of unbalanced power system networks. In the last ten years numerous contributions to the theory have been made which have facilitated its practical application to a large variety of current problems and it is now considered basic for relaying and other applications which are dependent upon unbalanced faults.

In connection with the stability problem it was necessary to develop new forms of circuit constants in order to facilitate the solution of general networks taking into account not only distributed constants but also their combination with lump constants. These constants were also used in extensions of the power circle diagram to take care of the general network. In addition methods for solving networks with synchronous machines in various branches were devised.

The most striking development in the last few years has been the introduction of the a-c. calculating board which has made it possible to solve many problems in a more complete manner and in less time than was possible heretofore. The calculating board has been used extensively in connection with problems of power system stability and relay operation. It also has been used for the purpose of simplifying complicated networks of hundreds of branches to the simplest possible equivalent general network for the smallest number of terminals permissible for the particular case under consideration.

J. W. Butler: The method which Mr. Starr has developed in Part III of his paper for setting up his several simplified equivalent circuits is an interesting and useful one. For example, the circuit shown in Fig. 8 of the paper may be recognized as the equivalent linkage to represent the single-line diagram of the three-winding transformer. For this application, the links a, b and c should be expressed in terms of the Z_{j-k} coefficients rather than the M_{jk} coefficients since the through impedances, that is, the impedances between pairs of windings, are those generally known. We may of course always change any function of M coefficients to one of the Z coefficients by using the general equations (**13b**), (**14b**) and (**15b**) of the paper.

Similarly, Fig. 9 may be interpreted to represent the four-winding transformer, Fig. 10 may be interpreted to represent the five-winding transformer, etc.

As pointed out in the paper, the equivalent circuit used for any particular application may be set up to fit the application in question in the simplest manner.

For example, let it be required to set up an equivalent single-line circuit to represent a four-winding transformer whose six impedances are:

$$Z_{1-3} = Z_{1-2} = 12.0 \text{ per cent}$$
$$Z_{1-4} = 12.53 \text{ per cent}$$
$$Z_{2-3} = 24.0 \text{ per cent}$$
$$Z_{3-4} = Z_{2-4} = 11.9 \text{ per cent}$$

Since there are four independent values of impedance, the equivalent circuit need have only four independent links. It must, of course, have four points of entry and must satisfy the above conditions. The circuit shown below is a simple one which satisfies the particular conditions of this case. For example, one may observe at once that $Z_{1-3} = Z_{1-2}$ and $Z_{3-4} = Z_{2-4}$.

The through impedances are from Fig. 1

$$Z_{1-2} = a + b + \tfrac{3}{4}d$$
$$Z_{1-4} = a + c + d$$
$$Z_{2-3} = 2b + d$$
$$Z_{2-4} = b + c + \tfrac{3}{4}d$$

Solving for a, b, c and d.

$$a = \tfrac{1}{2}(Z_{2-4} - Z_{1-4}) - X_{2-3} + 3/2Z_{1-2}$$
$$b = Z_{1-2} + Z_{2-4} - Z_{1-4} - \tfrac{1}{2}Z_{2-3}$$
$$c = \tfrac{1}{2}(Z_{1-2} - Z_{1-4}) - Z_{2-3} + 3/2Z_{2-4}$$
$$d = 2(Z_{1-4} + Z_{2-3} - Z_{1-2} - Z_{2-4})$$

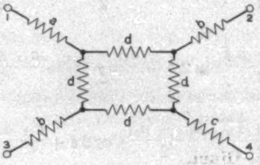


Fig. 1

Substituting the impedance values in the above equations

$$a = -\ 6.32 \text{ per cent}$$
$$b = -\ 0.63 \text{ per cent}$$
$$c = -\ 4.42 \text{ per cent}$$
$$d = 25.26 \text{ per cent}$$

Note that the negative links appear in the terminals and that these may be balanced out by the positive impedance of connected apparatus.

Edith Clarke: Equations corresponding to those given under (**1b**) of Appendix B of Mr. Starr's paper, have been used to advantage in the solutions of the positive sequence system during simultaneous faults when the faults were on different phases and on lines which were not symmetrical with respect to the rest of the system, such as parallel lines not bussed at either end or bussed at one end only. Mr. Starr has expressed the C coefficients of these equations in terms of the "through impedances" between the terminal points taken two at a time, the through impedance being the impedance between the two points under consideration with all other points ungrounded. Since in transmission systems it is usually a simple matter to calculate the through impedance between any two points of the system, this method of determining the numerical value of the C coefficients is a simple and direct one.

Another simplification which will be found of advantage where an a-c. calculating table is not available or when greater accuracy is required than can be secured by its use, is that of placing the self-impedance at the terminals and not in the mesh of the equivalent circuits. The labor of numerical calculations is materially reduced by this procedure since the mesh will be composed of fewer branches all of which will be reactive. For example, two parallel lines in the zero sequence diagram, not bussed at either end, may be represented by equivalent circuits $I(d)$ or $I(e)$. Fig. 1e, which has purely reactive branches in the mesh with the self-impedances at the terminals, is a much more wieldy circuit.

R. G. Lorraine: Mr. Starr has developed equivalent circuits for two, three, and four parallel transmission lines. Mutual effects between lines may be represented by mutual reactors (transformers) instead of by equivalent circuits as pointed out by Mr. Lewis. However, the number of mutuals frequently exceeds the number of mutual reactance units available on the a-c. table and the equivalent circuit must be used. In these circuits all branches representing mutual coupling are pure reactances. For purposes of calculation this is, of course, desirable. However, pure inductive reactances cannot be represented directly on the a-c. calculating table.

The reactance units on a calculating table, to be of reasonable size, must necessarily have an appreciable amount of resistance associated with the reactance. For the accuracy usually required, this resistance cannot be neglected when it occurs in a unit used to represent a pure reactance. To specify arbitrarily this resistance in the mutual branches of the equivalent circuit, is one way of securing proper representation, but at the expense of increasing the number of branches of the equivalent circuit. A better way is to change arbitrarily the impedance reference vector, so that all the mutual impedance branches have a smaller impedance angle than that of the reactance units. This rotation in addition to introducing the required resistance in the mutual branches of the equivalent circuit, produces a different phase angle relation between the currents and voltages throughout the network. Therefore, the resultant currents must be rotated back to obtain the solution. The polar form of the expression for voltage drop through an impedance may be used to find the relations between the impedance and current angles.

The rotation of impedances may produce a negative resistance in any capacity reactance branches which are present. Usually the effect of this resistance may be neglected as the capacity effects are generally small. If they are not the negative resistance may be represented by a potentiometer.

Thus, even though Mr. Starr's equivalent circuits contain pure reactance branches, means are available to represent them adequately on the a-c. calculating table.

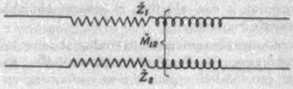
W. A. Lewis: Several of the equivalent circuits presented by Mr. Starr are both new and interesting, and should prove a valuable addition to the work on the subject. In particular the equivalent circuits for the transformer in Fig. 1e and for 0 or more coupled lines in Figs. 9 to 14 should prove useful. The circuit of Fig. 8 for the common case of two lines is rather well known and has been used by others.*

It seems desirable to point out a slight error in the statements that the mutual impedance between lines is purely reactive. The mutual impedance is due not only to inductive coupling but also to the common use by all circuits of the earth and ground wires, if present. This resistance may often be as large a proportion of the mutual impedance as the combined resistances of earth circuit and conductor are of the zero-phase sequence self impedance, and so can seldom be neglected unless the accuracy required permits neglecting all resistances.

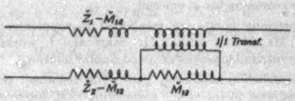
In the footnote on page 292 it is suggested that actual transformers might be used to represent mutual impedance. This has been done with considerable success with one design of a-c. calculating board, where the number of mutual impedances with any one circuit is small. The method, suggested by Mr. C. F. Wagner, is shown in Fig. 2. The transformer has a 1 to 1 ratio and is wound with a bifilar conductor so that the leakage reactance is negligible. It is seen from Fig. 2 that the mutual impedance, M, causes the same drop in both circuits, in one case directly and in the other case through the transformer. To provide the proper value of self-impedance, the mutual impedance must be subtracted from the self-impedance to obtain the values of the external branches, resulting occasionally in negative branches when several mutuals with the same circuit must be represented. The most generally satisfactory method appears to be a judicious combination of equivalent circuits and transformers.

One practical point mitigates against the general use of the rigorous transformer equivalent circuit. Reactance as set on the calculating board inevitably includes with it a certain small proportion of resistance which is usually negligible. When, however, the reactance is neutralized to a small value by means of capacitance, the resistance may no longer be a negligible proportion of the resultant effective reactance.

George W. Hampe: It will be found that the complete or lattice mesh of which Fig. 1d and Fig. 3b of Mr. Starr's paper are examples can be handled more readily if admittances instead of impedances are used to express its links. A simple process for finding the branches of the complete mesh equivalent to the general concealed network was given by Mr. Campbell.* It can be derived without the use of determinants as follows:



Actual Circuit



Equivalent Circuit

FIG. 2

Let Y_j be the admittance from a free entry point j to ground, with all other points of entry grounded. Let each entry point function in turn as the free point.

Let $Y_{j,k}$ be the admittance from the entry points j and k, which are joined but otherwise free, to ground, with all other points of entry grounded. Let each pair of points function in the same manner.

Consider both the concealed network and its equivalent complete mesh to be measured in the above ways. On the equivalent, of which Fig. 3b is an example, some links are thus placed in parallel and the resultant admittance can be equated to a measured quantity from the concealed network.

Thus if $Y_{\overline{jk}}$ is the admittance corresponding to $Z_{\overline{jk}}$ (Appendix E), a set of equations corresponding to each type of measurement can be written. For the first set, the following are instances:

$$Y_{\overline{ja}} + Y_{\overline{jb}} + \ . \ . \ . + Y_{\overline{ji}} + Y_{\overline{jk}} + \ . \ . \ . + Y_{\overline{jn}} = Y_j \qquad (1)$$

$$Y_{\overline{ka}} + Y_{\overline{kb}} + \ . \ . \ . + Y_{\overline{kj}} + Y_{\overline{kl}} + \ . \ . \ . + Y_{\overline{kn}} = Y_k \qquad (2)$$

From the second set:

$$Y_{\overline{ja}} + Y_{\overline{jb}} + \ . \ . \ . \ Y_{\overline{ji}} + Y_{\overline{jl}} + \ . \ . \ . \ Y_{\overline{jn}} + Y_{\overline{ka}} + Y_{\overline{kb}} + \ . \ . \ . \ Y_{\overline{ki}} + Y_{\overline{kl}} + \ . \ . \ . \ Y_{\overline{kn}} = Y_{j,k} \qquad (3)$$

---

*"Theory and Application of Relay Systems," by I. T. Monseth and P. H. Robinson, *Elec. Jl.*, July 1930, p. 411.

Fundamental Basis for Distance Relaying, by W. A. Lewis and L. S. Tippett, A.I.E.E. paper **No. 31-91.**

**Cisoidal Oscillations*, by G. A. Campbell, A.I.E.E. TRANS., Vol. 30, 1911, p. 873.

Substituting from (1) and (2) into (3) yields:

$$Y_j + Y_k - 2Y_{\overline{jk}} \text{ (or } 2Y_{\overline{kj}}) = Y_{j,k} \qquad (4)$$

or

$$Y_{\overline{jk}} = (Y_j + Y_k - Y_{j,k}) \div 2 \qquad (5)$$

For determination of the complete mesh by direct measurement (for instance on the calculating table) this solution is sufficient and leads to easy manipulation. Many other forms of equivalents from which mutual couplings are excluded can be found from the complete mesh by delta-star and star-mesh conversions. For the four-point case there are six in the basic group of which Figs. 1d and e are members. Star-mesh conversions on a general network can be used to show that the complete mesh equivalent of concealed positive resistance will never contain a negative link.

For determination of equivalents analytically, determinants may be useful for the expression of driving point and transfer admittances. The type form equation (5) can be used as the key to the situation. Thus for the n winding transformer it is merely necessary to ascertain which driving point and transfer admittances apply for the measurement conditions specified. The general equations expressing the links of the complete mesh then follow. Parallel lines are coupled in the same manner as the windings of a transformer.

The writer would also like to ask several questions concerning the paper. Why is the term "characteristic impedance" used in defining some of the symbols in Appendix C, but not others? In the introduction it was said that, in general, equivalence held only for a single frequency.

In the transformer equivalent the value of some links is the negative of others. Mr. W. C. Hahn (*General Elec. Rev.*, V. 34, Aug., 1931, pp. 483-489) explained a way of setting up negative resistance on a d-c. calculating board. I wonder if it has been found practical in this connection. Not all coupled lines can be traced to a common bus at one end.

The foregoing points are mentioned to supplement, not to criticize, Mr. Starr's work. Mr. Starr and I, working independently on similar problems, found similar solutions.

Mr. Campbell's remarkable paper had remained practically unused, at least in the power engineering field, for over twenty years. It is to be hoped that since it has been rediscovered the application of other parts of it will lead to the solution of modern problems.

H. L. Rorden: Specific calculations based on Mr. Bewley's derivations illustrate several points of interest. Table I lists the essential terms for three space harmonics as calculated for the given constants.

These calculations show:

1. That of the two time harmonics for each space harmonic, only one of them is important to the primary.
2. The two-winding theory gives practically the same amplitudes of oscillation as the single-winding theory.
3. For practical purposes $A_s' = X_s$.
4. The harmonic frequencies are substantially the same for the single and for the two-winding theory.
5. Both time harmonics are of practically equal importance in the secondary. At the initial instant these time harmonics are of opposite sign for grounded neutral, but due to the great difference in their frequencies they are quite likely to become additive numerically.
6. The first three harmonics are practically of the same importance in the secondary.

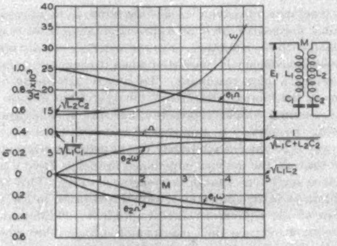


FIG. 3

7. The axes of oscillation in the secondary are quite different, depending upon the terminal connections. For grounded primary neutral these oscillations are about the axes,

$$e_2 = \left[\frac{x}{2} - \frac{1}{12}\right]E$$

for isolated open-circuited secondary and

$$e_2 = 0$$

for short-circuited, grounded secondary.

8. Calculations made with the two-winding theory are extremely sensitive to error, and the computations must be carried out with considerable care to get accurate results. A small error in the calculation of frequencies may cause an enormous error in the amplitudes.

TABLE I—CALCULATED TRANSIENTS

Basic Constants

$K_1 = 1 \times 10^{-11}$ $C_1 = 1 \times 10^{-9}$ $L_1 = 10$
$K_2 = 1 \times 10^{-11}$ $C_2 = 2 \times 10^{-9}$ $L_2 = 2.5$
$C_3 = 1 \times 10^{-9}$ $M = 2.5$

	Short-circuit and grounded secondary			Open-circuited and isolated secondary		
s	1	2	3	1	2	3
ω_s	132,000	500,000	1,140,000	132,000	500,000	1,140,000
Ω_s	74,500	276,000	553,000	74,500	276,000	553,000
r_s	5.65	10.08	81.3	5.65	10.08	81.3
r_s'	0.216	0.308	0.88	0.216	0.308	0.88
X_s	−0.592	0.252	−0.1375	−0.563	0.234	−0.1126
U_s	0.0114	−0.0175	0.0181	−0.2032	0.123	−0.0995
A_s	0.0254	−0.00974	0.00173	−0.015	0.0052	
A_s'	−0.617	0.259	−0.1394	−0.547	0.229	−0.1125
A_{2s}	0.144	−0.0974	0.141	−0.0845	0.0524	
A_{2s}'	−0.133	0.0798	−0.123	−0.118	0.0706	−0.099
$*A_s$	−0.601	0.257	−0.138	−0.601	0.257	−0.138
$*2\pi f$	74,700	278,000	563,000	74,700	278,000	563,000

*Based on single winding theory.

It is interesting, as an analogy, to compare the general distributed circuit with the lumped constant circuit shown in the figure, which exhibits similar characteristics. This consists of two *L-C* circuits coupled by a mutual inductance *M*. The equations for the voltages across the capacitors are:

$$e_1 = \frac{E_1}{(L_1L_2 - M^2)C_1C_2}\left[\frac{1}{\omega\,\Omega^2} + \left[\frac{1 - \omega^2 L_2C_2}{\omega^2(\omega^2 - \Omega^2)}\cos\omega t - \frac{1 - \Omega^2 L_2C_2}{\Omega^2(\omega^2 - \Omega^2)}\cos\Omega t\right]\right]$$

$$e_2 = \frac{E_1 M}{(L_1L_2 - M^2)\,C_2\,(\omega^2 - \Omega^2)}\left[\cos\omega t - \cos\Omega t\right]$$

$$\text{where}\quad \left.\begin{matrix}\omega^2\\ \Omega^2\end{matrix}\right\} = \frac{L_1C_1 + L_2C_2 \ \sqrt{(L_1C_1 - L_2C_2)^2 + 4\,M^2C_1C_2}}{2(L_1L_2 - M^2)\,C_1C_2}$$

The comparison illustrates several points of similarity:

A. The constant term in e_1 of the lumped circuit corresponds to the axis of oscillation of the distributed circuit.

B. The lumped circuit has two distinct frequencies of oscillation, corresponding to two distinct fundamental frequencies in the distributed circuit.

C. The two frequencies are practically of equal importance in the secondary of either the lumped or the distributed circuit.

D. One of the frequencies (that nearest to the natural frequency of the primary alone) is the predominating component in the primary oscillations.

E. As the mutual inductance is decreased in either circuit, the two frequencies approach the natural frequencies of the primary and secondary alone. As the mutual inductance is increased, one of the frequencies increases towards infinity and the other frequency approaches the compromise

$$1/2\pi\ \sqrt{L_1C_1 + L_2C_2}.$$

These conclusions are illustrated in Fig. 3.

Gabriel Kron: For years the customary procedure has been to analyze the transient and steady-state behavior of rotating machinery in terms of currents and voltages appearing at the terminals. Lately a new point of view is applied to the analysis of salient-pole synchronous machines; it is tacitly assumed that at any instant the current-density wave and the actual flux-density wave are sinusoidally distributed in *space* in both field and armature windings, hence they are replaced by two-dimensional space vectors and the behavior of the machine is expressed in terms of these space vectors instead of the terminal currents and voltages. Park in his classical paper "Two-Reaction Theory of Synchronous Machines" assumes two mutually perpendicular, stationary coordinates along the field poles and along the interpolar space and analyzes the *projections* of the space vectors along these coordinates. As a result, for each layer of winding (the field winding being one layer, the armature winding another layer) he sets up two *scalar* equations, altogether four scalar equations, as given in the first column of this paper. Since practically all types of asymmetrical rotating machines, having brushes, or sliprings in any combination with any number of layers of windings on stator and rotor, can be assumed to have sinusoidal current-density and flux-density waves in *space* under any transient or steady-state, balanced or unbalanced conditions, it follows that any rotating machine requires for its analysis *twice* as many *scalar* equations as there are layers of windings on it (neglecting, for the time being, space-harmonics).

The most logical step for simplification lies in the introduction of *two-dimensional vectors* for currents, fluxes, voltages and of *dyadic operators* for impedances and admittances, each operator consisting of the sum of 2 × 2 = 4 dyads, as has been done by Summers for synchronous machines. With the use of these vectors and dyadics (or tensors of rank two) the number of equations is halved and one has to manipulate only as many *tensor* equations as there are layers of windings. In the example used by Summers, the salient-pole alternator, the number of the necessary tensor equations is two [equations (**5**) and (**6**)]; in a repulsion motor with an additional squirrel-cage the number of tensor equations is three, etc.

My studies of the vector currents and vector fluxes existing in different types of rotating machines led me first to a graphical treatment of these vectors. When a limit of their practical use was reached the geometrical relations existing between the different vectors were put in the form of scalar equations. Then quite ignorant of the studies of Summers I also began to use two-dimensional vectors and dyadics with four dyads as being the most obvious tools for a simplified treatment. Soon however, I was led to a more simplified procedure which seemed to be a necessary and *inevitable* step forward.

I assumed that the two stationary unit coordinates of the armature (rotor) $\mathbf{d}_r$ and $\mathbf{q}_r$ are mutually perpendicular to the two unit coordinates of the field (stator) $\mathbf{d}_s$ and $\mathbf{q}_s$ and instead of having a two-dimensional current vector in the field winding and another two-dimensional current vector in the armature winding, I assumed that there is only *one four-dimensional* current in the machine with different components along the four normal coordinates $\mathbf{d}_s$, $\mathbf{q}_s$, $\mathbf{d}_r$ and $\mathbf{q}_r$. Similarly I assumed that the terminal voltage consists of *one* four-dimensional voltage vector

$$\mathbf{e} = e_{ds}\,\mathbf{d} + e_{qs}\,\mathbf{q} + e_{dr}\,\mathbf{d} + e_{qr}\,\mathbf{q}\ .\ .\ .$$

where each scalar coefficient can be any function of time, d-c., a-c., Heaviside unit function, etc., (also zero) or any combination of these. Corresponding to these assumptions I introduced the following *impedance* operator as a dyadic consisting of 4 × 4 = 16 dyads which transforms a four-dimensional current vector into a four-dimensional voltage vector in any machine with two layers of windings

$$\begin{aligned}&(r_{ds} + L_{ds}p)\,\mathbf{q}_s\,\mathbf{q}_s + M_d\,\mathbf{q}_s\,\mathbf{q}_r - M_d p\theta\,\mathbf{q}_s\,\mathbf{d}_r + O\,\mathbf{q}_s\,\mathbf{d}_s\\ &+ M_d p\,\mathbf{q}_r\,\mathbf{q}_s + (r_{dr} + L_{dr}p)\,\mathbf{q}_r\,\mathbf{q}_r - L_{dr}p\theta\,\mathbf{q}_r\,\mathbf{d}_r + O\,\mathbf{q}_r\,\mathbf{d}_s\\ &+ O\,\mathbf{d}_r\,\mathbf{q}_s + L_{qr}p\theta\,\mathbf{d}_r\,\mathbf{q}_r + (r_{qr} + L_{qr}p)\,\mathbf{d}_r\,\mathbf{d}_r + M_q p\,\mathbf{d}_r\,\mathbf{d}_s\\ &+ O\,\mathbf{d}_s\,\mathbf{q}_s + M_q p\theta\,\mathbf{d}_s\,\mathbf{q}_r + M_q p\,\mathbf{d}_s\,\mathbf{d}_r + (r_{qs} + L_{qs}p)\,\mathbf{d}_s\,\mathbf{d}_s\end{aligned}$$

In setting up this impedance operator the "per-unit" concepts have been abandoned, self and mutual inductances were used and the *stator* was assumed to have salient poles, in order to make the tensor available for the transient analysis of asymmetrical commutator machines also. Rotating machines with *three* layers of windings have *six-dimensional* currents and terminal voltages and their impedance operator contains 6 × 6 = 36 dyads, of which the above 16 dyads form the inner square. With greater number of layers of windings proportional relations hold true. This operator, the "magnetic-field tensor" contains all the design constants, is an *invariant*, its value is independent of the presence of brushes, or sliprings and is independent of the terminal conditions.

The presence of brushes, sliprings, taps, and the various possible connections between them is represented by a *transformation of coordinates*. The transformation is made with the aid of another tensor, the "connection-diagram tensor" showing the location of brushes, sliprings, the various connections, the ratio of turns, etc., this tensor being a mathematical representation of the usual connection diagrams of different types of machines. While all types of rotating machines have the same "magnetic-field tensor" containing all the design constants, each has a different "connection-diagram tensor."

With these new assumptions the transient and steady state behavior of all types of rotating machines with any number of layers of windings, under balanced or unbalanced conditions can be expressed (assuming speed as a parameter) in terms of *one* first degree tensor equation

$$\mathbf{i}\ .\ \mathbf{Z} = \mathbf{e} \tag{2}$$

from which the unknown current **i** for a given terminal voltage **e** is found

$$\mathbf{i} = \mathbf{e}\ .\ \mathbf{Z}^{-1} \tag{3}$$

giving the instantaneous values of currents along each coordinate axis by the routine procedure of calculating the *inverse* of the impedance tensor.

The "magnetic-field tensor" with any number of dyads can be expressed as the sum of three dyadics

$$\mathbf{Z} = \mathbf{R} + \mathbf{L}p + \mathbf{R}_v p\theta \qquad (4)$$

where $\mathbf{L}$ represents all dyads containing p and $\mathbf{R}_v$ those containing $p\theta$ showing that any rotating machine consists of a constant (tensor-) resistance and a constant (tensor-) inductance in series with a variable (tensor-) resistance. Also:

$$\mathbf{e} \,.\, \mathbf{i} = \text{power input} \qquad (5)$$

$$\mathbf{i} \,.\, \mathbf{R} \,.\, \mathbf{i} = \text{copper losses inside the machine} \qquad (6)$$

$$\mathbf{i} \,.\, \mathbf{L} \,.\, \mathbf{p} \,.\, \mathbf{i} = \text{rate of increase of stored magnetic energy} \qquad (7)$$

$$\mathbf{i}\, \mathbf{R}_v\, \mathbf{p}\theta \,.\, \mathbf{i} = \text{mechanical output} \qquad (8)$$

Hence

$$\text{Torque} = \mathbf{i} \,.\, \mathbf{R}_v \,.\, \mathbf{i} \qquad (9)$$

Dyadic operators have been set up not only for rotating machines, but also for polyphase networks, vibrating systems, vacuum tubes, etc. One of the most valuable properties of this analysis is that the resultant impedance of any number of interconnected rotating machines and stationary networks (d-c., induction-, synchronous-, commutator-machines, polyphase-networks, vacuum tubes, etc.) is the *sum* of the impedance of each machine of the interconnected system, thereby giving an easier procedure for the transient (and steady state) analysis of systems for speed-control, power-factor control, welding, etc.

In order to have the instantaneous speed of the rotor also as a variable, the concept of current and voltage vectors had to be further extended. I assumed that $\mathbf{e}$ represents not only the multi-dimensional impressed voltage but also the instantaneous *torque* acting along an additional dimension $\mathbf{t}$ in the direction of the axis of the rotor; similarly $\mathbf{i}$ represents not only the multi-dimensional current vector, but in addition the instantaneous angular velocity acting along $\mathbf{t}$ also. If now $\mathbf{R}$ includes in addition the frictional resistance dyad $\mathbf{ptt}$, $\mathbf{L}$ includes the moment of inertia of the rotor $T\mathbf{tt}$ and $\mathbf{L}_v = -\mathbf{R}_v \times \mathbf{t}$, the transient behavior of any machine or combination of machines is described *during acceleration* by the following single, second degree tensor equation

$$\mathbf{i} \,.\, \mathbf{L}_v \times \mathbf{i} + \mathbf{i} \,.\, \mathbf{L}\, p + \mathbf{i} \,.\, \mathbf{R} = \mathbf{e} \qquad (10)$$

By taking infinitesimal changes of vectors in this equation, all problems of hunting and vibration of different types of machines, system stability, starting, etc., can be studied systematically.

It can be proved that this last equation also is a tensor expression of the Maxwell-Lagrangian Equation of Motion for a system containing kinetic (mechanical) and electrokinetic (magnetic) energy.

F. M. Starr: As pointed out by Mr. Lewis, it is stated in the paper that the mutual impedance between parallel lines is purely reactive. I do not agree with Mr. Lewis that there is the slightest inaccuracy in this statement. I do not see any reason why the connotation of the phrase "mutual impedance between the lines" should be construed as including "mutual resistance between ground return circuits."

However, Mr. Lewis has brought up an interesting question, and that is: how may mutual resistances between ground return circuits be represented when the resistance is not negligible? One exact method of doing this would be to represent the ground circuit as an equivalent mesh whose links would be evaluated in terms of resistance measurements made between the various grounding points (with all lines disconnected) in accordance with the method outlined in Part II of the paper. The zero-phase diagram of the system shown in Fig. 5, then, would consist of the equivalent layout shown in Figs. 6 or 7 with the various grounding points connected, not to a common ground, but to the respective terminals of an equivalent resistance mesh representing the ground. Of course, in order to represent the ground circuit by such an equivalent, or in any manner at all, it is necessary that the resistance coefficients be known or possible of measurement, and this may be rather difficult or at least impracticable.

Mr. Lewis further pointed out that the small amount of resistance associated with reactor units on the calculating board might be objectionable when applied to pure reactance links of an equivalent circuit. Although such a condition would rarely occur, if it were important in some particular case, it could be completely corrected by rotating all impedances through the proper angle before setting up on the board, as pointed out by Mr. Lorraine.

Mr. Hampe, in his equations (**4**) and (**5**) has given an interesting equivalent for the general network which is somewhat different than my own. I do not agree with Mr. Hampe in his statement that the general network equivalent can be handled more easily if the links are expressed as admittances rather than impedances. It really makes very little difference whether driving point impedances or driving point admittances are used since they are reciprocals. (See equation (**6b**) in the paper.) Mr. Hampe implies that the links of this equivalent circuit may be calculated without the use of determinants. I think great care should be taken when speaking of measured or calculated quantities. The Y coefficients of Mr. Hampe's equation (**5**) are entirely similar to my B or D coefficients in equation (**9a**) and (**6b**) respectively. In either case these coefficients may be measured directly, but if measurements are not possible, they must be calculated by means of determinants. In this connection, it should be pointed out that to evaluate a link in Mr. Hampe's equivalent requires three impedance (or admittance) measurements or calculations as the case may be, whereas the circuit derived in Appendix B requires only one measurement or calculation. (See equation (**6b**).)

The answers to Mr. Hampe's questions are given briefly in the order in which they were asked:

1. "Characteristic impedance" or "characteristic admittance" is used in defining symbols used in Appendixes A and B (not Appendix C) in accordance with standard nomenclature for determinantal coefficients.

2. Mr. Hahn's method of representing negative resistance is always applicable on a d-c. board, although this method would be unwieldy and impracticable if very many links required it. If an a-c. board is used, there is no reason why negative resistances need ever appear. Negative resistance can always be avoided by applying the reasoning used in Part I of the paper in deriving the circuit of Fig. 1e. That is, resistance may always be lumped in the terminals of a circuit leaving any negative links purely reactive.

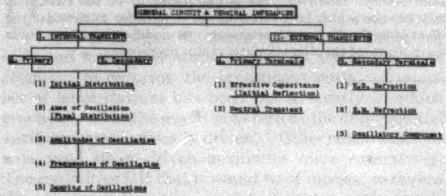
Mr. Butler's discussion shows an interesting application of the method developed in Part III of the paper for setting up simple equivalent circuits. This type of application should prove particularly valuable in mapping out the single-line positive or negative phase diagrams of a system involving 4- or 5-winding transformer banks.

L. V. Bewley: As pointed out by Mr. Rorden, the equations given in the paper are not only complicated, but extremely sensitive to small errors. For these reasons they are not well adapted to numerical calculation. The real practical utility of the analysis is that it suffices to establish rigorously the range of application and limitations of a number of simplified methods pertaining to certain aspects of transformer high frequency transients; such as the primary internal oscillations; terminal and neutral transients; transfer of waves to the secondary external circuit; etc. In this discussion I have compiled and classified the approximate equivalent circuits of the transformer, based upon the solution to the general circuit. It must not be construed, however, that these circuits are new. They have all been employed in the past by various individuals interested in some particular aspect of the problems. In chronological order, the solution to the general circuit was the last one obtained.

The equivalent circuits discussed here are approximate, as contrasted to the mathematically exact equivalent circuits described in Mr. Starr's paper. It often happens in engineering work that approximate equivalent circuits may be substituted which are sufficiently accurate for certain purposes and under specific conditions. Thus a wide variety of circuits may be employed to describe different aspects of a given circuit or piece of apparatus, depending upon the characteristic which it is desired to investigate.

In the accompanying chart a classification of transformer transients due to traveling waves, or high frequency, is given in the form of a family tree; and in Fig. 4 there are tabulated, in the order of their accuracy, various approximate equivalent circuits which suffice, in specific cases and ranges, for engineering calculations. Theoretically, a solution to the general circuit with arbitrary terminal impedances would yield complete information on both the internal and external transients in both windings. Unfortunately, however, the mathematical difficulties are insurmountable, and even could the general solution be obtained it would be too complicated to prove of much utility in practicable engineering calculations. But solutions have been obtained for zero and infinite values of the terminal impedances (short-circuited or open-circuited terminals, either grounded or isolated) and it has been shown that:

Classification of the High Frequency Transients of Transformers


GENERAL CIRCUIT & TERMINAL IMPEDANCES
I. INTERNAL TRANSIENTS
II. EXTERNAL TRANSIENTS
a. Primary
b. Secondary
a. Primary Terminals
b. Secondary Terminals
(1) Initial Distribution
(2) Axes of Oscillation (Final Distribution)
(3) Amplitudes of Oscillation
(4) Frequencies of Oscillation
(5) Damping of Oscillations
(1) Effective Capacitance (Initial Reflection)
(2) Neutral Transient
(1) E.S. Refraction
(2) E.M. Refraction
(3) Oscillatory Component

1. The initial distribution may be calculated for arbitrary terminal impedances.

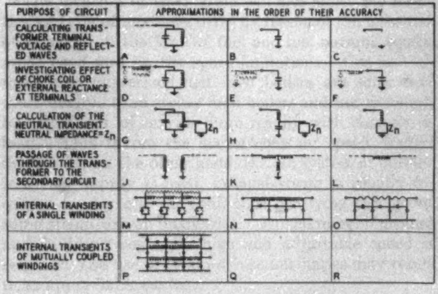
2. The transformer acts as a capacitance network at the instant of impact of steep waves, or when abrupt changes occur in the terminal voltages. The effective capacitance of this network has been derived.

3. Voltages transmitted to the primary neutral and to the secondary terminal impedances through the capacitance network may be calculated. These electrostatic components are exponential impulses of very short duration when the terminals are connected to their surge impedances; but if the terminals are isolated, the electrostatic potentials last as long as the applied voltage.

4. The axes about which the oscillations of the primary and secondary take place are determined by the inductances of the winding (neglecting the losses), and the equations for these axes of oscillation are identical with the simple equations for two mutually coupled windings with arbitrarily terminal impedances. These equations thus suffice for calculating the purely electromagnetic component of voltage and current at the primary neutral and the secondary terminals. This component is very slow and very long compared with the electrostatic component, and is limited in magnitude to a fraction of the primary applied voltage equal to the turn ratio. It is the dominate component of the external transient when the terminals are connected to surge impedances not exceeding a few thousand ohms; but of course it vanishes if the primary neutral is isolated.

5. The internal oscillations consist of an infinite series of space harmonics, and each space harmonic has two components with different frequencies; that is, there are two *time* harmonics for each *space* harmonic. Only one of these two sets of time harmonics is of importance in the primary oscillations, but both sets are of practically equal importance in the secondary oscillations. Moreover, the *amplitudes* of the primary oscillations are almost identical with those given by the single-winding theory of transformer oscillations; and the *ratio* of harmonic frequencies are in the same order, although the *absolute* values of the frequencies may be somewhat different. In any event, if the inductances used in the single-winding theory are empirically changed to correspond to different connections of the secondary winding, the single-winding theory yields very good quantitative as well as qualitative agreement with the double-winding theory and with test. It may also be shown that for the usual values of surge impedances, that the oscillatory components transmitted to the terminal impedances are insignificant compared with the pure electromagnetic component discussed under (4) above. For obviously, the oscillatory components of current can not exceed in value those which flow when the terminals are short-circuited or grounded (values which can be calculated) and therefore the voltages across any terminal impedance can not be as great as that given by the product of that impedance and the current which would flow if that impedance were reduced to zero.

From the above facts, which have been derived from consideration of the general circuit with arbitrary terminal impedances, a table of equivalent circuits for the transformer, under traveling wave conditions and for different purposes, may be compiled as shown in Fig. 4. Starting with the two-winding circuit *P*, which is necessary for calculating the internal transient of the secondary it follows by (5) above that the single-winding circuit *N* may be used, if it is only required to calculate the primary internal transient. Now it may be desirable to include the effect of damping (for example, in investigating the possibility of cumulative oscillations caused by applied waves of resonate frequency), in which case circuit *M* is appropriate. Circuit *O*, which neglects the mutual inductance between parts of the same winding, has been used, but fails to give a correct description of the phenomena of transformer oscillations, because its frequency distribution is erroneous.



PURPOSE OF CIRCUIT	APPROXIMATIONS IN THE ORDER OF THEIR ACCURACY		
CALCULATING TRANSFORMER TERMINAL VOLTAGE AND REFLECTED WAVES	A	B	C
INVESTIGATING EFFECT OF CHOKE COIL OR EXTERNAL REACTANCE AT TERMINALS	D	E	F
CALCULATION OF THE NEUTRAL TRANSIENT. NEUTRAL IMPEDANCE = Z_n	Z_n G	Z_n H	Z_n I
PASSAGE OF WAVES THROUGH THE TRANSFORMER TO THE SECONDARY CIRCUIT	J	K	L
INTERNAL TRANSIENTS OF A SINGLE WINDING	M	N	O
INTERNAL TRANSIENTS OF MUTUALLY COUPLED WINDINGS	P	Q	R

Fig. 4—Equivalent Circuits of Transformers to Traveling Waves and High Frequency

For calculating the external transient of the secondary, when connected to normal impedances and when the primary neutral is grounded, it follows by (4) that circuit *J* may be employed, or its simplifications *K* and *L*.

When the purpose of the calculation is to determine the neutral transient, or the transient of a traveling wave at the transformer terminal, the appropriate circuit is *G*; for at the first instant it acts as a capacitance in conformity with (2); if the neutral is isolated ($Z_n = \infty$) the neutral voltage oscillates at the fundamental natural period of the isolated-neutral transformer; and if the neutral is grounded ($Z_n = O$) the winding becomes an inductance to ground and eventually a short circuit. Now the effect of the terminal capacitance on the neutral current is quite small, so the circuit may be simplified to H, and here again the effect of the fictitious neutral capacitance is dominated by the neutral impedance Z_n, so that circuit I is sufficient for practical calculations.

If, however, the problem is to investigate the effect of an external reactance, the relative importance of the elements of circuit G is completely reversed; for the external reactance (a choke coil, for example) enters into a very high frequency oscillation with the effective capacitance of the transformer, (2), and at this high frequency the inductive winding of the transformer is practically an open circuit. Thus D or E apply.

For calculating the transformer terminal voltage and the reflected waves, circuit G is still the best approximation. If the neutral is grounded, it simplifies to A. For very short waves, or for isolated neutral, circuit B is sufficient. For waves of moderate length, and when slight depression of the wave front by the effective capacitance is not to be considered, the transformer may be regarded as an open-circuit; or as a surge impedance of several thousand ohms if the very small negative reflection is to be included.

R. H. Park: In his discussion of Mr. Summers' paper, Mr. Gabriel Kron suggests a method of analysis which appears to have interesting possibilities. It is to be hoped that Mr. Kron will develop his analysis more completely in the form of a paper for presentation before the A.I.E.E.

Exception must be taken to one point in this discussion. This is the statement that the Summers' vector method of analysis requires "as many tensor equations as there are layers of windings." From this statement, it would be concluded that the form of the equations given by Summer would not be adequate to include synchronous machines equipped with field collars and amortisseurs. However, this is not found to be the case. The only change which is found necessary, in order to consider the effect of field collars or amortisseurs is a modification of the scalar components of the characteristic vector operators.

Auxiliary Drive for Steam Power Stations[1]

A Review of the Characteristics and Economics of Steam and Electric Drive for Steam Power Station Auxiliaries

BY F. H. HOLLISTER[2]

Member, A.I.E.E.

I. *Steam Driven Auxiliaries for Power Plants*, by W. Poole Dryer
II. *Electrically Driven Auxiliaries for Steam Power Stations*, by Lyle W. Smith

DURING the past few years there have been important changes in power station and power system design that have reduced the operating costs and now power station designers are giving more attention to reducing the investment costs. A number of large stations has been built recently in which practically all of the essential as well as the non-essential auxiliaries are electrically driven. Other recent stations have used steam-driven auxiliaries quite extensively. The committee felt that it would be of interest to review the characteristics and merits of steam and electric auxiliary drive in light of present conditions and past operating experience, and has therefore sponsored the papers by Messrs. Dryer and Smith.

The central power station industry as a whole enjoys a well deserved reputation for continuity of service. The public has become accustomed to expect continuous service and the large investment charges of the power station companies and their industrial customers demand it. One of the most important factors in continuity of power service is reliability of the auxiliary equipment required at the power stations. Reliability, therefore, deserves the attention that has been given it in these papers.

The value of adjustable-speed auxiliaries makes an interesting study in the application of auxiliary drive. In the past, the station designers have used adjustable-speed drives for fans, pulverizer mills, boiler feed pumps and circulating pumps quite extensively. Recently there have been developments in the manufacture of auxiliary equipment and station design that have made adjustable speeds less necessary. The one exception to this trend is boiler feed pumps for high-pressure boilers. The paper on steam-driven auxiliaries brings out very clearly the advantages of the steam turbine as a simple and efficient means for obtaining adjustable-speed drive.

It has been shown in the papers that the type of drive best suited for the essential auxiliaries such as fans, boiler feed pumps, and circulating pumps is closely related to and dependent upon the general design of the station. The general station design in turn is influenced by the relation of the station to the balance of the power system with respect to interconnections, size, and location. For instance, it is quite apparent that a power station that forms the chief source of power supply for an area and that does not have reliable interconnections with other power systems warrants more reserve equipment and must have a greater number of steam-driven auxiliaries for starting up the station than a station forming part of a system of interconnected stations.

Furthermore, it is apparent that if a new power station forms part of a large interconnected group of power stations, it may be designed and built to operate at a better load factor and thereby reduce the need for adjustable-speed auxiliary drives.

The location of the station will also affect the selection of type of auxiliary drive by reason of the influence of location on the kind of fuel and fuel burning equipment used and on the space limitations. Gas and oil fired, also pulverized fuel fired boilers are somewhat more flexible than stoker fired boilers and can therefore be taken out of service more readily with changes in load. This favors the use of constant speed electric drive. If, on the other hand, the bin system of pulverized fuel supply is used, exhaust steam is usually required to dry out the coal, thus justifying the use of some steam-driven auxiliaries. Furthermore, if ground and building costs are high and adjustable speed is required, the use of steam-driven auxiliaries may result in an appreciable saving.

It is interesting to note that the total auxiliary power used by a modern station is a relatively small part of the gross generated power, being approximately from 4½ to 6 per cent; hence a radical power saving can not be expected in the selection of auxiliary drive. Likewise, the investment cost of the auxiliary power including drive and control equipments is a relatively small part of the total station cost, being in the case cited for electrically driven auxiliaries approximately 7 per cent of the total station cost. It would appear that the station designer is more or less free to use either type of drive without seriously affecting the cost or the efficiency of the power station as a whole. Electric drive

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[1] Foreword prepared under the Auspices of the Committee on Power Generation.

[2] Sargent & Lundy, Inc., Chicago, Illinois.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

for essential auxiliaries is favored from the standpoint of convenience and cleanliness in operation, also because it is more readily adapted to automatic and remote control particularly in the boiler room.

Maintenance costs would be of interest for a comparison between steam and electric drive; however, representative figures are difficult to obtain chiefly because sufficient time has not elapsed since the modern stations were placed in operation. In addition, the advancement in station design has been so rapid in the last few years it is difficult to find stations of sufficient similarity to make a fair comparison. The subject of these papers is inherently one of generalities because of the many factors that may enter in the selection of auxiliary drive. The discussion of the subject matter should, however, be of interest in bringing out points for consideration in determining the type of auxiliary drive best suited for a specific problem in power station design.

Discussion

For discussion of this paper see page 341.

Steam-Driven Auxiliaries for Power Plants

BY W. POOLE DRYER[1]

Associate, A.I.E.E.

REQUIREMENTS

THE degree of reliability required of a generating station is determined by the kind of community or business served by the station and the relation of the particular station to the other sources of supply for the system. The activities of the downtown section of a large metropolis demand service reliable to a degree not expected in a rural district. So also in the industries, some metallurgical and chemical processes need the continuity of their power supply guaranteed to an extent not essential in other industries. Stations in those strategic situations, metropolitan and industrial, must supply service reliable to the highest degree. Once the city station loses its network load, restarting is extremely difficult; once the industrial process stops, the product, and maybe the machinery, is spoiled and to restart the process takes hours. These are the paramount considerations determining the methods to be used for driving essential auxiliaries in power plants. Secondary considerations are simplicity, initial cost, maintenance expense and economy of power.

RELIABILITY OF AUXILIARY SERVICE

Inasmuch as a station's ability to produce an uninterrupted supply of power depends, among other things, on the ability of the essential auxiliaries to function continuously, the arrangements made for guaranteeing the operation of these auxiliaries should be appropriate to the degree of reliability of service expected from the station. These arrangements involve furnishing a reliable supply of energy, as well as turbines, motors, and accessory equipment which are free from physical defects and which possess suitable operating characteristics.

RELIABILITY OF AUXILIARY POWER SUPPLY

The essential auxiliary driving units must be supplied with energy free from disturbances which would interrupt their operation. To the extent that the source of supply is exposed to those disturbances, arrangements must be made to limit the effect of any single disturbance to as few auxiliaries as possible and to provide a duplicate source immediately available when the primary source fails. Minor disturbances of the energy, such as slight reductions in steam pressure or voltage, are to be expected, and should be provided for in the design of the drives. To what extent is the provision of a duplicate source of supply advisable in stations, when the essential auxiliaries are steam driven, and to what extent when motor driven?

RELIABILITY OF AUXILIARY STEAM POWER

Disturbances threatening interruption of the auxiliary service may originate within the station or without. In properly designed and operated stations, with turbine driven auxiliary equipment, no external circumstances could disturb the characteristics of the steam supply to such an extent as to prevent the auxiliaries from always having available sufficient driving power of suitable characteristics. The one circumstance which, if not anticipated and arranged for, could interfere with the ability of the steam power to sustain properly the operation of the auxiliaries, would be the sudden imposition on the station of an abnormally large block of electrical load. By overdrawing on the evaporating ability of the boilers this would lower the pressure in the steam system possibly to the point where some essential auxiliaries would "lie down." The station then would fail to deliver not only the additional load required of it, but also its original load. With this contingency recognized in the modern station, and provided for by proper schedules for holding in reserve banked boilers, and for accepting load at a rate appropriate to the steaming ability of the boilers, the operation of steam driven auxiliary equipment remains immune to events occurring externally to the station.

Troubles originating within the station and affecting the steam supply to the auxiliaries enough to prevent their operation could be caused only by conditions resulting in an excessive lowering of the steam pressure at the throttles of the auxiliary turbines. Then some of the auxiliaries might reach their breakdown torque and stop running. For instance, certain types of pulverized fuel mills which require a substantially constant torque regardless of output would stop. This might result in the boilers, inadequately supplied with fuel, failing to sustain steam pressure. However, conditions resulting in such an excessive decrease of steam pressure are preventable and not encountered in stations properly designed and operated. These conditions are either under-boilering of the station or else serious damage to the steam system equipment—boilers or piping. Under-boilering is avoided in designing modern utility plants as it would deprive the main turbine generators of their proper capacity for service; however, this defect sometimes exists in small industrial plants. Since the failure of auxiliary power, due to damage to the steam system equipment, is a condition as likely to affect the main steam supply as the auxiliary system, it qualifies equally the reliability of all steam stations, whether the driving power for the auxiliaries comes directly through auxiliary turbines or indirectly through turbine generators and motors. The piping required for auxiliary turbines may be as

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[1] Stone & Webster Engineering Corp., Boston, Mass.

Presented at the Winter Convention of the A.I.E.E., New York, N.Y., January 25-29, 1932.

simple and reliable as the main steam piping on which large turbine generators in all steam stations have to depend in producing the station's electrical output.

Reliability of Auxiliary Electric Power

A station with motor-driven auxiliaries supplied from a single source of energy would not be as reliable, in so far as the supply of energy is concerned, as the station with turbine-driven auxiliaries; for once the steam energy destined for auxiliaries is transmuted into electrical energy for use in the motors, it enters an additional realm wherein its stability is both more sensitive to disturbance and more subject to unexpected attack. Hence, to attain the same degree of reliability of the auxiliary energy supply as in the turbine-driven station, the station with motors must have duplicate sources of energy.

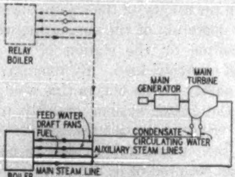

Fig. 1—Station with All Essential Auxiliaries Turbine Driven

—●— Lines and equipment which must function continuously to prevent interruptions

Since the service from the auxiliaries must be of a higher degree of reliability than that of the main power supply, the auxiliaries must be supplied with energy from at least one source not connected to the main generator or to the outgoing lines. This entails furnishing especially for the auxiliaries at least one generator which may be driven either by the main turbine or by a turbine introduced solely for this purpose. In important cases, two special generators are furnished, one driven by each of those turbines. This is frequently done in the case of the downtown stations of metropolitan systems, where so high a degree of reliability of service is expected that it is considered essential to keep the auxiliary system always free from affection by external disturbances.

Not only is it necessary in motor-operated stations to provide special generators and sometimes duplicate distributing systems, but it is customary in the case of such important auxiliaries as boiler feed pumps and exciters, as an ultimate resource, to rely on turbines as the drivers.

In Figs. 1 and 2, one referring to a station with all essential auxiliaries turbine driven, the other to a station with all essential auxiliaries motor driven with power taken from a shaft generator, the solid lines indicate the piping, wiring, and equipment which must always function in order that the supply of auxiliary energy always be maintained. Solid circles indicate auxiliary turbines or motors that similarly must always function; dotted lines and circles indicate duplicate auxiliary energy lines and equipment.

Reliability of Turbines and Motors

The ultimate reliability of the service given by auxiliary equipment to the main units in a station is the compounded reliability of the power supply furnished to the auxiliaries and of the performing abilities inherent in the auxiliary apparatus itself.

Regarding the inherent functional qualities of auxiliary turbines, the main parts in these machines are robust, and, as a concession to low initial cost, simple and inefficient compared with main turbine units. The parts most questioned are the reduction gears, governors, and valve mechanisms. However, the art of gear-cutting has been so developed that gears seldom give trouble without ample warning. Governors and valve mechanisms are less certain in operation unless particular skill is applied to their design.

As a basis for evaluating the degree of functional reliability of turbines, they can be compared with the more usual types of motors; *i. e.*, the squirrel cage, and the adjustable-speed wound rotor types. The simplicity of the construction of squirrel cage motors and their accessories probably place them in approximately the same grade of inherent reliability as turbines. Wound rotor motors and their accessories, however, involve parts and constructions which can be preferred only from considerations of economy of power. It might fairly be said, regarding the qualities of reliability inherent in the drivers and accessories themselves, that steam-driven auxiliaries are as reliable as those electric driven, in the case of constant speed equipment, and are more reliable in the case of adjustable-speed equipment.

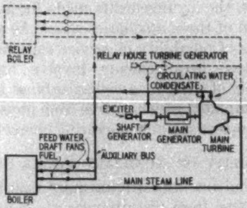

Fig. 2—Station with All Essential Auxiliaries Motor Driven

—●— Lines and equipment which must function continuously to prevent interruptions

A further aspect of the reliability of performance inherent in the driving apparatus itself is the relative ability of turbines and motors to drive adjustable-speed equipment smoothly and without transitional disturbances—particularly draft fans. On such duty, the turbine with its infinite number of speeds performs evenly; the wound rotor motor, with its step action transition in speeds, is more liable to cause disturbance of burner flames, and in the change from the motor of one speed to that of another, further operating complications are encountered.

RELATIVE EFFICIENCY OF TURBINE AND MOTOR DRIVES

Despite their simplicity and reliability, turbine-driven auxiliaries were to a large extent displaced, when it was discovered that a fuel saving could be made if steam for producing auxiliary power was first passed through efficient main turbine units to produce electrical energy and then extracted for water heating, instead of being passed through small inefficient turbines mechanically driving auxiliaries and then into feed water heaters. It was found that the additional electrical energy thereby generated was more than that required for driving the auxiliaries by motors, even after deducting the losses in the generators, transformers, and motors. The fundamental reason for this gain in economy is that the surplus energy thus made available is produced very much more economically than the rest of the station output energy due to the fact that the steam required for generating the surplus energy is used for heating the boiler feed water. As a result, that steam is utilized at a station thermal efficiency between 90 per cent and 100 per cent, whereas the steam used for generating the rest of the output energy is carried into the condenser, where most of its original heat energy is lost, and so is used at an efficiency between 20 and 35 per cent.

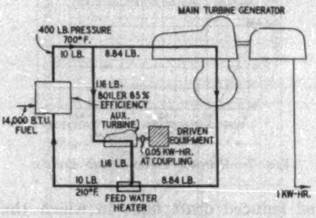

FIG. 3—STATION EFFICIENCY DIAGRAM WITH ALL AUXILIARIES TURBINE DRIVEN

Overall thermal efficiency of station 24.4 per cent

With regard to the economy actually effected by these changes the examples illustrated in Figs. 3 and 4 show that a modern station's net output for a given fuel consumption is increased about one and three-quarters per cent at maximum load by substituting motors for turbines on all essential auxiliaries, and by heating the feed water with extracted steam instead of auxiliary exhaust steam; the over-all thermal efficiency of the turbine-driven station is 24.4 per cent and of the motor-driven station 24.8 per cent. In this comparison other conditions have been kept similar. It should be observed that in Fig. 3 less steam is needed for water heating than in Fig. 4; this is because the auxiliary turbine in Fig. 3 is inefficient, or a poor converter of heat energy into mechanical energy. As a result, hot steam is exhausted, less of which is required for the water heating effect than is needed of the cooler steam extracted from the efficient main turbine. However, as will be inferred from the reasons for the economy explained above, the greater the percentage of the total steam in the main turbine which can be extracted and used for heating, and the less remaining to go into the condenser, the more will be the net station output for the same fuel consumption.

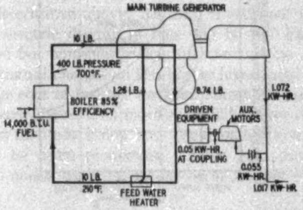

FIG. 4—STATION EFFICIENCY DIAGRAM WITH ALL AUXILIARIES MOTOR DRIVEN

Overall thermal efficiency of station 24.8 per cent

Not as a principle unique to either turbine or motor drives, but as one common to both, it should be realized that the station efficiencies shown in Figs. 3 and 4 could and should be increased by heating the feed water further than shown in these examples by extracting steam from one or two high temperature points of the main turbine so as to raise the water to a higher temperature. Moreover the 1¾ per cent advantage attained in the station shown in Fig. 4 would have been slightly greater if, instead of using one point of extraction, two had been used to heat the water to the same temperature; and due alone to the stray losses encountered in actual operation, the advantage might be as high as 2½ per cent.

The difference in the efficiencies of the two stations compared above would have been very much less if, in the comparison, the high efficiencies now obtainable in auxiliary turbines had been assumed instead of the low efficiencies actually taken as better representing common practise. However, it should be understood at this point that in the two stations compared, their relative efficiencies were estimated for only one condition of station load: both stations were operating at their maximum output and the auxiliary turbines and motors were performing at their maximum and most efficient loads.

Power Consumption

In order to ascertain the total difference in power consumption between steam and electric driven auxiliaries over the whole working range of the boilers and turbine generators, and hence the net annual saving in fuel, it is necessary to determine the power taken at partial loads by both types of drives. This has been done by analyzing separately the power input to each essential auxiliary when driven with turbines, squirrel cage, and adjustable speed wound rotor motors. The results of these analyses are given in Figs. 5 to 9, in each of which the curves are plotted to show, at any gross output of the boiler or main turbine generator, what percentage of that output is consumed by the particular auxiliary with each type of drive. In the case of those curves based on power output of the boiler, that output has been reckoned as the power which the main turbine generator would generate with the quantity and quality of steam delivered by the boiler at its different loads. The power consumed by the turbine-driven auxiliaries in all cases has been calculated as the power which the quantity of steam used in these auxiliaries would have produced if used in the main turbine generators instead.

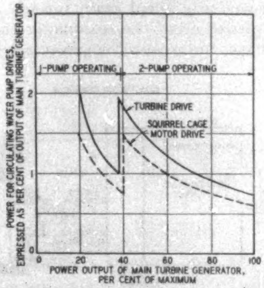

Fig. 5—Circulating Water Pump Drive

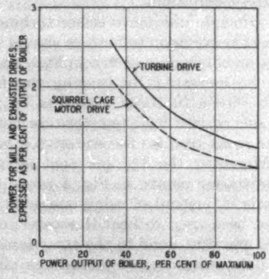

Fig. 6—Unit Pulverizing Mill and Exhauster Drives

These curves represent the performance of equipment in stations having large boilers and turbine generators, where the auxiliary turbines also would be large and slightly more efficient than those often used. However, turbines having higher efficiencies than assumed in this analysis are now in operation on auxiliary service.

It will be seen from the curves that as the boiler and turbine generator loads decrease from a maximum, the relative power consumptions of steam and motor driven auxiliaries vary in the following manner:

1. For such auxiliaries as circulating water pumps and fuel pulverizing equipment, for which the output is substantially constant throughout a wide range of load on the main units, and for which constant speed squirrel cage motor drive is suitable, the electric drive maintains its relative efficiency over the turbine drive, throughout the whole range of load.

2. For such auxiliaries as boiler feed pumps and forced and induced draft fans, in which the output varies with the load on the main units:

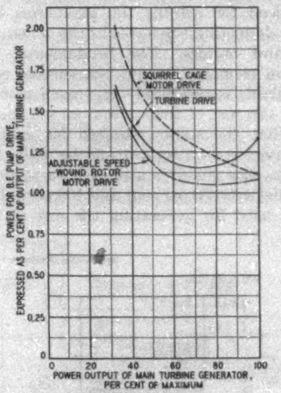

Fig. 7—Boiler Feed Pump Drive

(a) The squirrel cage motor rapidly loses the advantage in power saving which electric drive gives at maximum load, and below about 85 per cent load on the main units it is less efficient than the turbine. At lower loads, the initial relationship between the two drives can be reestablished by transferring the load to a lower speed motor, but on reducing the load still further this motor will in turn become less efficient than the single turbine.

(b) The superior economy over turbine drive which the adjustable speed wound rotor motor gives at maximum load diminishes greatly as the load decreases and would vanish at low loads unless operation is transferred to an additional and lower speed motor.

RELATIVE OPERATING COSTS

These conclusions indicate that from considerations of power consumption, the most favorable application for turbine drive is on equipment like draft fans which, having to traverse a wide range of speed and load, use more than one motor when electrically driven.

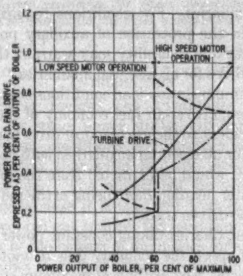

FIG. 8—FORCED DRAFT FAN DRIVE

— — — — — Two squirrel cage motors

— · — · — Two adjustable-speed wound rotor motors

Even on this application the electric drive saves power; however, it involves a larger investment in both driving and generating equipment.

Regarding the investment in driving equipment, it is estimated that for large boilers having a maximum output of the order of one-half a million pounds of steam per hour and with two wound rotor motors per fan, the installed cost of motors, controllers, and wiring for forced and induced draft fans would be about $10.00 per rated fan horsepower more than for turbines with their valves and piping.

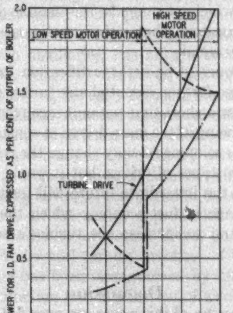

FIG. 9—INDUCED DRAFT FAN DRIVE

— — — — — Two squirrel cage motors

— · — · — Two adjustable-speed wound rotor motors

Regarding the investment in generating equipment assessable against the fan drives, if the fans are electrically driven and assessment is made of all the equipment usually provided for normal and emergency power supply, it will be found that the turbine room cost increases about $49 per installed fan horsepower. If the forced and induced draft fans are steam driven, 0.7 per cent more total evaporating capacity is needed than if motor driven, as is shown in Figs. 8 and 9; as a result the boiler room in this case costs about $12 more per installed horsepower than in the other case. Those investment costs are based on a total station cost of $95 per kw. output.

As determined above, the total investment incident to the electric driving of fans will be $47 higher per installed fan horsepower than for the turbine driving. If the annual charges on this higher investment are less than the cost of fuel per installed horsepower saved by the electric drive, then motors would be more economical than turbines; if not, not. Seldom, however, would motors be more economical. Even with coal at $4 per ton and a capacity use factor of 100 per cent, the combined fuel cost and fixed charges would be less for the turbine drive than for the motor drive.

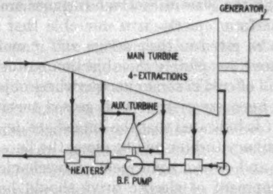

FIG. 10—STEAM AND EXHAUST CONNECTIONS FOR AUXILIARY TURBINE IN HIGH-PRESSURE STATION

In plants designed to deliver large quantities of steam for industrial uses, or for heating, the economic balance is usually in favor of turbine drive regardless of load factor and fuel cost.

FUTURE DEVELOPMENTS

To fully realize the advantages obtainable by using auxiliary turbines in conjunction with large main units, more efficient designs and methods of application become desirable, practicable, and commercially feasible. With the conventional designs and applications of these machines, more stages and larger diameters can be used to raise the efficiency at maximum load; however, care must be taken that the gain so obtained is not at the expense of inefficient partial load operation.

Novel means for securing very high efficiency have been adopted in turbines recently installed in one large central station; there, not only have high rotative speeds and many stages of highly refined design been introduced, but the steam is passed in series through two turbines: first, through the induced draft fan turbine and then through the forced draft fan turbine.

For stations of high steam pressure, about 600 lb. per sq. in. and over, still other means can be invoked to improve the efficiency of auxiliary turbine operation, as indicated in Fig. 10. Here the improvement is derived through using conventional designs of machines, but novel steam connections. In this case the boiler feed pump is shown driven by a turbine receiving steam from one extraction point on the main turbine and exhausting it to a heater connected to a lower extraction point. Thereby, as the pressure drop through the auxiliary turbine is small, this being a factor giving good efficiency, the machine can have relatively few stages and slow wheel speed; moreover, as the throttle steam is of relatively low temperature and pressure, the construction can be simple. All these features contribute to giving a low initial cost of machines and piping, and low maintenance. In adopting systems of this character, much care and some ingenuity of design are required to protect continuity of operation without undue complications of piping, and to achieve the simplicity of operation which is quite as essential as simplicity of design.

Although the gears with which turbines are equipped usually perform quietly, it is desirable that still more attention be given in their design and manufacture so as to remove completely the objection to turbine auxiliaries still offered in some quarters where objectionable noise has been experienced with geared turbines.

While it is indicated that the immediate development of the auxiliary turbine will be along the lines of higher efficiency and greater refinement of mechanical detail, the development of electric drive should be directed toward greater simplicity and lower installation cost. Greater sturdiness of adjustable-speed motors is desirable, as well as wider speed ranges, but the greater field for improvement is in distributing systems and control equipment. It is desirable to retain the efficiency and convenience of motor drives, without sacrifice of reliability, at substantially lower cost than the systems now in common use require. While it is not clear at the moment how this end may be accomplished, the problem is receiving the attention which it deserves. The great variety of systems adopted in recently built stations indicates that power plant designers are far from satisfied with the conventional auxiliary power systems, and that they are striving to produce something better than has been designed before.

CONCLUSIONS

It will be inferred from the foregoing discussion that in the present state of the art the turbine constitutes the simplest and most reliable drive for steam power station auxiliaries. For conditions requiring extreme measures in the protection of auxiliary power supply its simplicity and reliability may warrant the selection of turbine drive for all essential auxiliaries.

For maximum station load conditions there will be an improvement in over-all fuel economy of the order of 2 per cent incident to the use of motor drive instead of turbine drive for all essential auxiliaries: with both types of drive, steam would be extracted from the main turbine for feed water heating, more being extracted in the case of the motor-driven station.

For the auxiliaries taking constant power and driven at constant speed the economy due to motor drive is even greater at partial than at maximum station loads.

For the auxiliaries driven at adjustable speed the advantage in economy with motor drive is slight at lower loads.

Under most conditions the total annual expense for forced and induced draft fans and perhaps for boiler feed pumps, including fuel costs, maintenance, and capital charges, will be less with turbine drive than with motor drive.

There is an element of convenience in starting and stopping, which commends the motor drive to the average operator, when other considerations are approximately equal.

There seems to be little justification for the use of turbine drive for non-essential auxiliaries, (those which may stop for a short time without inconvenience) these auxiliaries being in the main driven at constant speed.

Future developments may change the relative status of these two types of drives, but it would appear that progress in the immediate future will tend to decrease the advantage in fuel economy which the motor now possesses, and to increase the turbine's advantage in reliability.

Discussion

For discussion of this paper see page 341.

Electrically Driven Auxiliaries for Steam Power Stations

BY LYLE W. SMITH[1]

Member, A.I.E.E.

Synopsis.*—There is a definite trend toward the use of all electrically driven auxiliaries for steam power stations. This paper reviews the reasons for this trend, summarizes the advantages of electric drive and discusses in a general way some of the economic factors in the selection of an auxiliary system. A companion paper covers steam-driven auxiliaries.*

INTRODUCTION

THIS paper reviews the advantages of electric drive for steam power station auxiliaries in light of present conditions and outlines some of the economic factors that enter into the selection of an auxiliary system. The subject will be discussed in a general way because it is apparent that each problem has to be worked out to meet the local requirements.

A review of the history of auxiliaries for steam power stations shows a gradual change from complete steam drive to the other extreme of complete electric drive except for a few reserve steam driven auxiliaries such as boiler feed pumps and service pumps. During an intermediate period, dual steam and electric drive was used quite extensively for some of the essential auxiliaries, but the number of dual drives appears to be steadily diminishing.

REASONS FOR TREND TOWARD ELECTRIC DRIVE

The higher efficiency of electrically driven auxiliaries receiving their power from the efficient main turbine or house turbine, as compared with small inefficient turbine-driven auxiliaries, has always been attractive to the station designer. However, lack of confidence in the reliability of the early forms of electric drive for essential auxiliaries retarded its general adoption until more operating experience was obtained and other reasons than efficiency of the auxiliary drive appeared.

The change from steam to electric drive is largely due to an increasing feeling of confidence in the reliability of electric drive, an increasing appreciation of the simplicity and flexibility of electric drive, and important changes in boiler and turbine practise. Confidence in electric drive has been strengthened by improvements in the manufacture and application of electrical equipment, the proved value of station and system interconnections, and a record of successful operating experience with electric drive. The convenience and flexibility of electric drive are generally known; however, they will be discussed more in detail elsewhere in the paper. The important changes in boiler and turbine practise that have influenced the use of electric drive are multi-stage bleeding of turbines for feed water heating, the use of higher steam pressures and temperatures, the development of larger boilers and turbines with their multiplicity of similar auxiliaries, and to some extent, automatic combustion control.

RELIABILITY OF STEAM AND ELECTRIC DRIVE

Reliability has always been considered the chief requirement for essential auxiliaries. Steam is considered more reliable than electric drive chiefly because it is a more direct method of obtaining power. Electric drive introduces additional apparatus such as generators, transformers, switching and control between the boilers and the auxiliary to be driven. If the electrical driven auxiliary is connected directly or indirectly to the main bus, it is also exposed to the effects of line disturbances. The performance of the intermediary equipment required for electric drive will now be considered from the standpoint of reliability.

The main turbines and house turbines are probably more reliable than efficient auxiliary turbines. Furthermore, the main turbines and house turbines receive much closer attention than could be given economically to each auxiliary turbine. Statistics show that generator failures are extremely rare, especially since closed ventilation systems have been used. In the case of a failure of the main generator, there is usually no immediate need for the associated auxiliaries of the generator and boilers. Auxiliary transformers have an equally good record for freedom from trouble. Modern switching and control, when selected and installed with the usual liberal factors of safety, have given relatively little trouble in recent years. A large proportion of the motors used for electric drive is of the squirrel cage induction type, which is unusually sturdy and readily permits full voltage starting with its simplicity of control. The other types of motors when correctly applied and maintained have given good service. Improvements in insulation and bearings have made motor coil failures rare. The extensive use of bus and feeder reactors, and diversity of power feeds to the electrically driven auxiliaries have all tended to minimize the effects of line disturbances. The present trend toward faster operating oil circuit breakers will also improve this situation. It seems apparent that reliable service can be expected from electric drive when correctly applied.

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SIMPLICITY OF ELECTRIC DRIVE

Simplicity is an important requirement for auxiliary drive. It is difficult to conceive a simpler drive for rotating equipment than the squirrel cage motor, or a simpler starting operation than pushing a button. Developments in control have made varying and adjustable speed motors equally simple to operate. A motor can be started in a fraction of a minute whereas the ordinary small auxiliary turbine usually requires a preliminary warming up period. Convenient control of auxiliaries from a few central points often permits a reduction in the size of the operating staff and contributes toward better efficiency in station operation.

FLEXIBILITY OF ELECTRIC DRIVE

Flexibility is likewise an important requirement for auxiliary equipment, chiefly from the standpoint of station design. The flexibility of electric drive is almost unlimited as evidenced by a modern boiler combustion control system. The absence of steam piping to the turbine room and boiler room auxiliaries results in less congested basements, thereby facilitating design, operation and maintenance. Electric drive provides greater flexibility in obtaining the station heat balance.

EFFECT OF STAGE BLEEDING CYCLE

The general adoption of the multi-stage bleeding cycle for boiler feed water heating is one of the most important changes in boiler and turbine practise that has tended to increase the use of electrically driven auxiliaries. The stage bleeding cycle, of course, does not require complete electric auxiliary drive. However, the use of complete electric auxiliary drive does have the following advantages:

1. It decreases the B. t. u. per kw-hr. rate of the station approximately 2 per cent.
2. It simplifies the heat balance system of a station.
3. It simplifies operation because the steam extracted for feed water heating varies automatically with change in load.
4. No provisions are required for transferring auxiliary load to obtain a heat balance with varying station load as is the case when a separate house turbine is used.

EFFECT OF HIGHER PRESSURES AND TEMPERATURES

The increasing use of higher steam pressures and temperatures has made steam driven auxiliaries less desirable than electrically driven auxiliaries. Some of the more important reasons for this condition are as follows:

1. The cost of small high-pressure turbines used for auxiliaries is considerably higher than for small low-pressure turbines. The initial cost of a 1,200-lb., 750 deg. fahr. auxiliary turbine may amount to approximately 40 per cent more than auxiliary turbines for 400-lb., 750 deg. fahr. operation, with a gain in efficiency of but 10 per cent.
2. The efficiency and capacity of small high-pressure turbines fall off more rapidly than small low-pressure turbines with the accumulation of a slight amount of scale in the steam passage, because high pressure requires smaller volumes of steam and therefore smaller clearances. This characteristic also results in higher maintenance cost.

EFFECT OF LARGER BOILERS AND TURBINES

The trend toward larger boilers and turbines has favored the use of electrically driven auxiliaries for the following reasons:

1. Large boilers and turbines permit the economic use of two or more similar auxiliaries per unit which allows diversity of auxiliary power supply, thus increasing the reliability of electric drive.
2. It justifies the use of the more reliable unit system for auxiliary power supply in which all of the essential auxiliaries for a turbine generator and its boilers are supplied from a source connected directly or indirectly to the turbine generator. It also tends to lighten the interrupting duty of the auxiliary switching equipment and therefore reduces the cost.

ECONOMICS OF AUXILIARY DRIVE

The selection of the type of drive for a station auxiliary is not usually made as a result of a direct comparison between the cost and performance of a motor and a turbine alone, but is also determined to a large extent by the general design of the station. Some of the features of station design that influence the type of auxiliary drive have already been mentioned. A comparison between steam and electric drive introduces two factors that are subject to considerable debate. One of these factors is the cost of an electric auxiliary power supply that can be considered equally reliable as steam drive, and the other is the actual water rate of small steam turbines.

Some engineers feel that an electric auxiliary power supply equivalent in reliability to steam driven auxiliaries should make use of auxiliary generators connected to the same shaft as the main generators. Reserve capacity in this case would also be supplied by transformers connected to the station bus and a separate house turbine generator. Other engineers feel that a sufficiently reliable auxiliary power supply can be obtained from transformers connected to the generator leads ahead of the generator oil circuit breaker, or connected to sections of the main bus, if the station forms part of an interconnected system. The use of bus and feeder reactors and the type of system interconnections influence the valuation that can be placed on the reliability of the various methods of auxiliary power supply.

Operating experience with small steam turbines indicates that their efficiency falls off very rapidly unless they receive considerable maintenance. Cases have been reported where the efficiency of small turbines has dropped to half of the guaranteed efficiency, because of lack of maintenance. It is apparent that any comparison between electric and steam drive should take this characteristic of the small turbine into consideration, although it may be difficult to evaluate it.

Some of the factors that affect the economics of an electrically driven auxiliary system will now be considered.

POWER REQUIRED FOR ELECTRICALLY DRIVEN AUXILIARIES

The amount of power required for an electrically driven auxiliary system is relatively small and depends to a large extent on the type of fuel-burning equipment used. The station load factor and nature of condensing water supply also affect the amount of auxiliary power to a lesser extent. A few typical values of auxiliary power with different types of fuel burning equipment are given below:

Fuel burning equipment	Per cent of gross generated auxiliary power
Pulverized coal	5.5—6.5
Stokers	4.0—5.0
Gas and oil	3.5—4.0

The subdivision of auxiliary power for a modern 650-lb. steam generating station of approximately 200,000-kw. capacity, entirely equipped with electrically driven auxiliaries and using the unit system of pulverized fuel equipment as expressed in per cent of the total auxiliary power is approximately as follows:

Coal preparation	22.0	per cent of aux. power
Boiler feed pumps	22.0	" " " " "
Turbine auxiliaries	22.0	" " " " "
Forced and induced draft fans	20.0	" " " " "
Heater drain pumps	3.8	" " " " "
Service pumps	2.6	" " " " "
Station lighting	2.4	" " " " "
Coal handling	1.6	" " " " "
Miscellaneous auxiliary power	1.5	" " " " "
Ash sluice pumps	1.0	" " " " "
Air compressor	0.7	" " " " "
Dust precipitators	0.4	" " " " "
5.5 per cent of gross generated power	=100.0	per cent of aux. power

A study of the above tabulation indicates that the possibilities of further saving in the amount of auxiliary power appears limited to one or two per cent of the gross generated power at the best.

DISTRIBUTION OF COST FOR ELECTRICAL AUXILIARY SYSTEM

The following tabulation shows the major subdivision of investment cost for the electrical part of the electrically driven auxiliary system of the same 200,000-kw. generating station for which the subdivision of auxiliary power has been given. This station utilizes two 4,000-kw. shaft generators, and two 9,000-kva. transformer banks connected to the main bus for auxiliary power. The main switching equipment and the 2,300-volt auxiliary switching equipment are of the vertical lift metal-clad type. Air circuit breakers and magnetic switches are used for 440-volt motors. The boilers are equipped with full automatic control. The investment costs given below are expressed in approximate percentage of the total station cost.

Prorated share of cost for turbo-generator units used for auxiliary power	0.8 per cent
Auxiliary power distribution system, including transformers	1.0 per cent
Motors	1.2 per cent
Control for motors	2.0 per cent
Cable and conduit for motors and control	2.0 per cent
Total for electrical part of station auxiliary system	7.0 per cent

In the above tabulation only the prorated share of the cost of turbo-generator units used for auxiliary power is included, because the boilers and the balance of the station equipment are common to both steam and electrically driven auxiliary systems. No allowance has been made for space occupied by electrical auxiliary equipment because it would be approximately the same for either type of drive. The main auxiliary transformers and high-tension switching in this particular case are installed out of doors. The flexibility of electrical control equipment enables it to use space that might be otherwise wasted.

A study of the tabulation indicates that the greatest possibilities of saving in the cost of an electrical auxiliary system are the control and cables for motors. However, it seems apparent that a radical reduction in the investment cost of the complete station will not be the result of major modifications to the electrical part of an auxiliary system, but will have to come from a series of small savings. The possibility of making some of these small savings in the electrical auxiliary system will now be discussed in a general way.

ELECTRIC AUXILIARY POWER SUPPLY

A review of station designs with respect to source of electric power for auxiliaries shows quite a variety of systems, all of which are modifications of two fundamental systems; power from main bus or power from a separate generator. The advantages and disadvantages of the principal modifications of these fundamental systems are covered very completely in N.E.L.A. Publication 088, April 1930.

It is interesting to note something of a trend back to the original system of supplying auxiliary power from the main bus after passing through a cycle of house turbines and shaft generators. The system of taking auxiliary power from the generator leads ahead of the oil circuit breaker is virtually the same as taking the auxiliary power direct from the main bus, since the generator oil circuit breakers are usually non-automatic except for a fault in the generator. Both of these latter systems are subject to the same criticism of being affected by bus voltage disturbances unless the station has been designed to minimize the effects of line disturbances.

Operating experiences with various types of electric auxiliary power systems appear to justify in many instances the use of the simpler and more efficient system of obtaining the auxiliary power from transformers connected to the generator leads, or from sections of the main bus, for large stations forming part of a large interconnected system. Power from the transformers connected to the generator leads is the least expensive of these two systems because it eliminates the need for auxiliary transformer high-tension oil circuit breakers. The disadvantages of the generator lead system are substantially the same as for an auxiliary shaft generator. Both are subject to interruption if the throttle of the turbine trips on overspeed, and both will cause the auxiliary system to be subjected to a high voltage if the throttle does not trip on overspeed. The shaft generator is, of course, free from main bus voltage disturbances. Improvements in turbine governors and the increasing use of generator voltage regulators for system stability purposes tend to minimize the possibilities of trouble with the auxiliary power system from overspeeding of the turbine.

If an auxiliary system is being selected for an isolated generating station, it is apparent that a more elaborate system of electric auxiliary power supply will be required. This type of station will also require more steam-driven reserve auxiliaries for starting up purposes than are now considered necessary for stations forming part of a large interconnected group.

The economical use of house turbine generators appears to be confined to the stations that used d-c. adjustable-speed motors or where there is use for exhaust steam other than for feed water heating. Many engineers feel that a house turbine is justified for use as a reserve to the other systems of power supply and for starting up the stations. The type of house turbine generator that can be started up almost instantly from a remote point appears well adapted for a reserve power supply when local conditions justify its use.

The unit system of auxiliary power supply, whether it be from house turbines, shaft generators, bus transformers, or transformer on generator leads, appears justified for all large stations. As already mentioned, the chief advantages of the unit system are greater reliability and lower interrupting duty on the auxiliary power control equipment.

ELECTRIC AUXILIARY POWER DISTRIBUTION SYSTEM

The selection of the voltage of an electric auxiliary power distribution system is usually determined by the standard motor voltages available, together with a study of the over-all cost of the motors, cable, and control. In many of the earlier stations, the lengths of cable runs were relatively long and the interrupting duty on the control relatively light, with a result that motors of 50 hp. and above were usually rated 2,200 volts. However, with the present trend toward larger stations with greater concentration of auxiliary power, the cost of control has become a large item of the total cost of the electrical auxiliary system and deserves considerable study. The 1926 report of the A.I.E.E. Power Generation Committee pointed out this condition and suggested that a study be made of the possibilities of using a 440-volt auxiliary power distribution system with carbon air circuit breakers for control. It is interesting to note that a 460-volt auxiliary power system using carbon air circuit breakers was adopted for the Gould Street Generating Station at Baltimore as a result of a thorough study of investment costs and performance of motors and control. The Gould Street Station is designed for four 36,000-kw. units.

In the case of larger stations designed for the use of units with a capacity of 50,000 kw. and upwards, it usually works out that two or more auxiliary distribution voltages are required. The common auxiliary distribution voltages are 2,300 volts and 460 volts; however, 550 volts is being used in preference to 460 volts in some stations. In many instances an appreciable saving can be made in the over-all cost of motors, control, and cable by extending the horsepower range of the low-voltage motors up to 100 hp. or more, and establishing low-voltage power distribution groups central to the group of motors fed by the distribution point. The performance of carbon air circuit breakers for low-voltage control has been very satisfactory.

A further saving can sometimes be made in the auxiliary power switching and control equipment by diversified grouping of duplicate unit auxiliaries. For instance, half of the auxiliaries required for one boiler can be supplied from one bus section and the other half of the auxiliaries from another bus section. This not only makes a more reliable power supply, but eliminates the need for double buses, because it is usually possible to isolate at least one-half of boiler auxiliaries during off peak conditions for maintenance of the switching and control equipment.

AUXILIARY MOTORS AND CONTROL

A review of motor applications for recent large stations indicates a decided trend toward the use of more squirrel cage motors and fewer slip-ring motors. Machinery designers are becoming reconciled to full voltage starting of squirrel cage motors and in many cases, have withdrawn their demands for slip-ring motors merely to obtain smooth starting conditions. Coal pulverizer mills are now available that do not require adjustable-speed motors. Satisfactory results have been obtained with multi-speed squirrel cage motors driving fans equipped with adjustable vanes in the fan inlet. Station designers appear to be more willing to give up small gains in plant efficiency obtained by adjustable-speed motor drives, in order to reduce the investment costs for motors and control. One exception to this trend toward squirrel cage motors appears to be boiler feed pump motors for high-pressure boilers.

It is apparent that if the present high investment cost of auxiliary motors and control is to be reduced, every auxiliary drive application calling for adjustable-speed motors must be investigated and justified by an appreciable saving in over-all cost. The use of adjustable-speed motors increases maintenance and space requirements and to some extent, reduces the reliability of the electric drive. N.E.L.A. Publication No. 088 contains very complete data on the selection of motors and control for station auxiliaries which should be helpful in making decisions of this kind.

Considerable progress has been made by the manufacturers in supplying self-contained motor control groups completely fabricated at the factory. In many cases where the field labor costs are high, a substantial saving can be made by using factory fabricated control groups.

An appreciable saving can often be made in conduit work for motors and control by using armored cable installed in metal troughs or large pipes indoors, or parkway cable outdoors. This method of handling cables has given satisfactory results in the middle west during the past few years.

SUMMARY

The chief advantages of electric drive over steam drive for station auxiliaries are:

1. It facilitates an efficient station design by permitting all of the steam required for feed water heating being taken from the main turbine by stage bleeding.
2. It may be approximately twice as efficient in terms of B. t. u. per kw-hr. as steam drive with small turbines, depending upon the amount of maintenance received by the turbines.
3. It is uniformly efficient, because lack of maintenance does not materially affect the efficiency of electrical equipment, although it may reduce the reliability of the equipment somewhat.
4. It requires less maintenance.
5. It is simpler to operate.
6. It improves the physical design of a station and its operation by reducing the amount of auxiliary piping required, particularly in the basement.

Some of the possible methods of reducing the cost of an electrical auxiliary system are:

1. Obtain auxiliary power supply from transformers connected to the main generator leads ahead of the generator oil circuit breakers, and reserve supply from bus, if reliable interconnections are provided with other stations and station bus is protected by reactors or the equivalent.
2. Establish both high-voltage and low-voltage auxiliary power distribution groups central to location of auxiliaries instead of in electrical bay, thereby reducing length of cable and conduit runs.
3. Increase the horsepower range of low-voltage motors in view of relatively high cost of high-voltage control equipment.
4. Diversify feeds to all duplicate auxiliaries where possible, thereby permitting simpler switching layout for adequate maintenance.
5. Use squirrel cage motors and full-voltage starting equipment wherever possible even at sacrifice of a small percentage of station efficiency.
6. Use factory fabricated control equipment wherever possible.
7. Use armored cable, grouped in metal troughs or large pipes for auxiliary power and control when installed indoors or parkway cable when installed out of doors.

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Discussion

AUXILIARY DRIVE FOR STEAM POWER STATIONS

(HOLLISTER)

STEAM-DRIVEN AUXILIARIES FOR POWER PLANTS

(DRYER)

ELECTRICALLY DRIVEN AUXILIARIES FOR STEAM POWER STATIONS

(SMITH)

A. H. Kehoe: The experience of the New York Edison System with steam and electric drives for power station auxiliaries is as follows:

1. Continuity of Service. Continuity of station output with either steam or electric auxiliary drive has been accomplished in our stations by the following provisions:

All essential steam auxiliaries are supplied from steam headers which are provided with sectionalizing valves and fed from two or more sources.

Each essential auxiliary motor circuit has supply available from several sources.

Auxiliary supply circuits and buses are so sectionalized and diversified that electrical trouble at any one point will not affect more than a portion of the auxiliary service.

2. Flexibility of Control. Flexibility of control in small steps has been found to be necessary only for hoists and for the coal, air, and water supply to boilers.

Satisfactory flexibility of control has been obtained either with steam turbines or with electric motors equipped for speed variations in small steps, or with other types of motors where the driven unit can be properly controlled by throttling the output.

The tendency in modern boiler plant design to provide a minimum of storage for coal, water, and steam within the boiler unit has added importance to the desirability of so controlling the flow of fuel and air to the furnace and of water to the boiler that at all times the correct quantities will be present.

Two-speed and three-speed induced draft motors are operated with throttling control on fans, our practise being to pass the speed-change points on motors by the simultaneous hand control of motor speed and vanes or dampers. Automatic equipment designed to accomplish this purpose is being provided on the units now being erected.

At the fans speed-change points, pulverized fuel fires have been found particularly sensitive to the large increment changes in the flow of fuel or air, due to the difficulty under such conditions of so synchronizing the mechanism causing the flow of fuel and air to the furnace as to maintain the fuel-air ratios at all times consistent with stable combustion.

On stoker fired boilers having a common forced draft duct for a number of boilers, single-speed motors with throttling control on fans have been found to give smooth and efficient operation.

We have experienced serious overloading of some motors driving fans and centrifugal pumps when the efficiency or discharge resistance of the driven unit was materially different from that specified. This was due to a difference in the speed torque characteristics of the motor and the driven auxiliary.

3. Heat Consumption. No comparative heat consumption performance results are available on steam and electric auxiliary drives. The metering of the energy consumed by electric drive is accomplished with comparative ease, but the metering of the energy consumption of steam drive is impractical.

Two recent engineering studies have been made comparing steam drive and electric drive of boiler fans. At our Hell Gate Station, where a boiler deficiency in an existing station was being corrected, the top bleed point in the existing heat balance was so low that it was possible to condense all the steam from the fan drive turbines without exceeding a practical back pressure. For these conditions, the steam and electric drives showed substantially the same net heat rate. From a recent and very thorough comparison of steam and electric drive for induced draft fans only for Hudson Avenue, which case can be considered as much more representative of usual conditions than the circumstances at Hell Gate, it was determined that the net heat consumption of the turbine generator installation under consideration would have been increased approximately ½ per cent had this one set of auxiliaries been driven by steam turbines instead of motors.

4. First Cost of Auxiliaries. We have found some saving in installed cost of auxiliaries with electric drive instead of steam, especially where the simpler types of electric motors are adaptable, such as a-c. squirrel cage motors of either constant speed or two or three-speed pole-changing construction.

5. Effect on Cost per Kw. of Station Output Capacity. The choice between steam and electric drives has been influenced by the effect on net station output capacity where there has been a limitation in available capacity of either the main turbo-generators or the boilers. Such considerations have been highly local in character, and have been of major importance.

6. Policy Considerations. It has been recognized that—other things being equal—there is an advantage to an electric supply company using electricity instead of steam for its own power needs. This advantage is partly in its advertising aspect and partly in the increased familiarity of the company's personnel with problems of utilization of electric service.

I. E. Moultrop: Auxiliary power requirements of the stations built in the future will probably be considerably in excess of the auxiliary power requirements at present and in the past.

Boilers will be operated at much higher ratings than in the past, with greatly increased power requirements for forced and induced draft fan drive.

The use of higher steam pressures has materially increased the power requirements of the boiler feed pumps.

The radical decrease in the surface of condensers per kilowatt of turbine capacity has also increased the power required to pump the circulating water per million pounds of steam condensed. The use of higher steam pressures and the regenerative cycle have fortunately more than offset this increase in power and have actually resulted in a reduction in the power required for circulating water pumps per unit of station capacity.

A comparatively few years ago all essential auxiliaries in our steam-electric generating stations were steam-driven. The development of the small auxiliary turbines was backward because of the false impression that high economy was not necessarily desirable because the exhaust steam was used to heat the boiler feedwater.

The reliability and efficiency of the a-c. motor was improved and within a very few years we had changed to practically all-electric drive for auxiliaries with a very few necessary stand-by turbine-driven auxiliaries.

In the light of recent experiences it now becomes evident that we may have gone too far with a good thing. Both steam-driven and electric-driven auxiliaries have their place in the ideal layout. Now that we have tried both extremes, we will probably find a compromise that will use both steam and electric drives, each in its proper place.

As a starter, we all seem to be agreed that reliability is the primary requisite. This point cannot be too strongly stressed. The reliability of the service from a station can never exceed the reliability of the station auxiliaries.

Unfortunately, we have another requirement that is diametrically opposed to our major requirement. This second requirement is low unit cost. The question of lowering the unit cost of construction is one of the most important facing the station designer today and we cannot forget it in our consideration of the subject under discussion.

The development of the turbines for auxiliary drive in the past few years has been very satisfactory and we can now say that auxiliary turbines of high reliability and very good economy are available.

The feature that still requires improvement in connection with the auxiliary turbine is the valve gear arrangement. The complicated and expensive valve gear designs used for the main generating units are neither justified nor warranted for auxiliary turbines. They should be equipped with an overspeed trip operating a separate valve and a suitable valve chest designed for remote control by the several control devices now in general use. The valves in the valve chest should be designed to give a constant increase in the output of the driven auxiliary for a given impulse from the control system. The failure to meet this basic requirement has been the source of many operating difficulties and the complicated and improperly designed mechanisms often offered for this simple service are very undesirable and unsatisfactory.

Reduction gears have been improved, as pointed out in the paper, to a point where we can safely say that suitable gears are available that will give years of service with reasonable maintenance.

One of the most important auxiliaries in a steam-electric generating station is the boiler feed pump. With the small water storage in modern boilers, 100 per cent reliability of the boiler feed service is a necessity. Turbine-driven boiler feed pumps are generally installed but they are usually used as stand-by units. The boiler feed pump appears to be the auxiliary where turbine drive for normal service seems to be most desirable.

Furthermore, suitable adjustable-speed motors for large size high-pressure boiler feed pumps are not available. To meet this contingency many of these boiler feed pumps have been built to suit the motors available with very unsatisfactory results both from the standpoint of first cost and of maintenance.

The use of turbine drive for high-pressure boiler feed pumps permits the use of the best pump design and the horsepower of the turbines usually required permits of a highly economical design. They can be worked into the heat balance of the station in a number of ways. The turbine drive also permits of proper pressure control which is somewhat unsatisfactory with motor drive.

Electric motor drives for station auxiliaries have many advantages over turbine drives but the one serious drawback is the lack of satisfactory adjustable-speed a-c. motors.

The adjustable-speed a-c. motors and controls available today are abnormally expensive, do not have the desirable range in speed control, and are inefficient when operated at speeds considerably below synchronous speed.

To obtain the necessary range in speed for forced and induced draft fan drive it is often necessary to use two motors on the same shaft.

The lack of suitable adjustable-speed a-c. motors has resulted in the development of a complicated vane control for forced and induced draft fans so that a constant-speed motor can be used.

These makeshifts are a challenge to the motor designer but so far no satisfactory answer has been produced.

It is obvious that more development work must be done on adjustable-speed a-c. motors with regard to wider range of speed variation, particularly for stoker motors, draft fans, and boiler feed pumps. A possibility under consideration is a system based on induction frequency changer equipment in which the stator element is rotated at varying speeds thus controlling the frequency of the auxiliary motor supply.

The application of power tubes is also being investigated. With increased efficiency of rectifying equipments such as phanotrons, rectifiers, etc., it may become practical to provide direct current at the individual motor or group of motors, thereby obtaining the recognized advantages of the adjustable-speed d-c. motor. Power tubes may also be found useful in providing more steps, more evenly graduated, for controlling speed.

The many parts, such as rheostats, contactors, control relays, etc., required to control the adjustable-speed a-c. motor seriously reduces the reliability of the apparatus as a whole.

When the a-c. adjustable-speed motor is brought to approximately the same degree of reliability as the constant-speed motor we shall still have the problem of reliable supply at the motor.

The supply becomes less important as the unit treatment of turbine generator, boilers, and auxiliary equipment is approached. In this connection, it should be recognized that the auxiliaries may be the weak link. More systematic maintenance and the present trend toward increase in size, making necessary the subdivision of auxiliaries into units of half the total capacity, tend to reduce the possibility of complete shut-down of the units. Unit treatment of the auxiliaries further increases reliability where the station contains a number of units.

With regard to the auxiliary buses, it may be well to consider the use of reactors between bus sections, to insure continuity in the source of supply for driving the auxiliaries. When important bus sections are operated together, they should have bus relay protection.

In the matter of protection, surer and faster relaying systems are needed to insure proper selection of circuit or motor at fault. Attention should also be given to time-delay features on low-voltage releases.

The motors available today are really remarkable when compared with the motors available twenty years ago, but it should be distinctly understood that they are far from being developed to a point where they can be regarded as satisfactory. This is especially true of the adjustable-speed a-c. motor.

It may be necessary for the station designers to swing back to all-turbine drive for station auxiliaries in order to spur on the motor designers to produce a motor that will more nearly meet the requirements of the service.

The designers of motors and controls and the designers of station auxiliaries must work together so that the motor and control designers will be more familiar with the present and future requirements, and at the same time the designers of equipment requiring motor drives will design with the problems of the motor and control designer clearly in mind.

Francis Hodgkinson: The importance of reliability of the drives for the essential auxiliaries of a power station cannot be overemphasized. As far as possible, they should be immune from disturbances either within or without the power station. During disturbances to operation, whether they are electrical or otherwise, is a time when the operators are occupied with the difficulty. This is the one time when uninterrupted operation of the auxiliaries is most essential, at least until it has been concluded that the related main generating unit is no longer operable.

These are reasons which, years ago, justified the separate steam drive for auxiliaries, and later the development of separate house turbines with independent electric systems for the operation of motors driving the auxiliaries. It would appear nowadays, however, that a greater degree of reliability of general power station service, in some cases at least, justifies a reversion to the oldest practise of taking the energy from the prime mover either by means of the station bus or by a direct-connected house generator.

There is discussion in the papers of the relative reliability of steam turbines and electric-motor drives. Considering merely the drive itself, there would seem to be little to choose between them. Perhaps the choice depends upon the experience of the individual expressing an opinion. The simple and relatively inefficient types of turbines usually employed for auxiliary drives, together with reduction gears, may be highly reliable. If, however, high efficiency of the steam path is required, the gain in efficiency may, in instances, be secured at some sacrifice of reliability. With the types of small-capacity turbines used for auxiliary drives, efficiency and reliability are sometimes incompatible.

Certain auxiliaries, the boiler house fans and feed pumps, preferably operate at variable speed, as has been pointed out in the papers, when the steam turbine would seem to have the advantage because of the ease with which automatic speed controls may be provided.

Much of the benefits derived from operating the condenser circulating pumps at variable speed may be obtained by the installation of two pumps operating at constant speed, with the added advantage of reliability. One pump only is operated during cold weather, or in hot weather during an emergency, with a loss of about 0.2 to 0.3 in. vacuum from a normal vacuum of 29 in.

The choice of the type of auxiliary drive turns more on the general station layout, as pointed out by Mr. L. W. Smith. The problem is intimately associated with the feed-heating problem, including the feed-heating to be done by the economizer, and preheating of air for combustion, besides the question of the desirability of variable speed in instances previously discussed. Years ago, steam-driven auxiliaries were regarded as imperative in order that their exhausts might preliminarily heat the feed water, before admission to the economizer, to as high a temperature as possible consistent with not too great a loss of exhaust steam to the atmosphere during periods of light load on the main units. Admitting water to the economizer without this preliminary heating introduced difficulties with the economizer because of the deposition of tarry compounds and sulfurous acids on the cold surfaces and the deposition of some of the scale-forming compounds within the economizer instead of in the heater. With the advent of the more economical principle of extracting steam in stages from the prime mover for heating feed water, it became no longer necessary to employ auxiliary exhaust steam for this purpose, and the electric motor operation of auxiliaries came to be generally employed. Just as much a reason, however, for this change in practise was the desire to eliminate the complexity of steam and exhaust piping, the mess incidental to drains, etc., and to reduce the heat incidental to poorly-ventilated basements, where the auxiliaries are generally located.

When air pre-heating is employed, the greater portion of the feed-heating is accomplished by extracted steam from the prime mover, only enough economizer surface being installed to reduce the flue-gas temperature to the desired degree and provide air to the furnaces at the desired temperature. There are instances where the whole of the feedwater heating is accomplished by bleeding the prime mover, no economizer being furnished.

More energy per unit weight of steam is to be had from steam taken in stages at progressive pressures from the efficient main unit for feed heating than from the less efficient auxiliary steam units. A further advantage of the former method lies in the fact that some 20 per cent of the steam admitted to the main unit may be extracted, leaving that much less to go to the condenser. Therefore, the steam turbine may be given a correspondingly higher rating with given dimensions of low-pressure blading. This is important when one considers the high capacities of turbine generators at given speeds that are now available, which results in a problem for the turbine designer to secure sufficient low-pressure blade area with appropriate leaving loss commensurate with available capacities of generators.

House turbines have frequently been employed for furnishing steam for feedwater heating by means of their exhausts. An elaboration of this worthy of note is that employed in the Klingenburg Station of the Berlin Electricity Works, where the house turbines are arranged for stage bleeding. They exhaust into the first feed heater and are bled at two higher-pressure stages. The advantage is that inasmuch as the auxiliary load is nearly constant regardless of the load on any one prime mover, the feedwater temperature will vary much less than in the case of bleeding the main prime mover. In this instance, the house turbine generating set is coupled to an electric motor arranged to give and take energy to and from the main bus. By this means, an equable feedwater temperature may be easily maintained by the switchboard operator, by means of distance control of the house turbine governor.

It would seem that the foregoing considerations, in addition to the statements of the authors, generally lead to a preference for the electric drive for all essential auxiliaries except boiler feed pumps and fans.

H. M. Cushing: The study of the best method for driving power station auxiliaries has been a fascinating one. The trend of the development is clearly depicted in the three papers by Messrs. Hollister, Dryer and Smith. Mr. Dryer, in handling the subject of steam-driven auxiliaries, has made a very favorable case for the electric drive. Our points of view are naturally biased by our own personal experiences. The Huntley Steam Station, with which I am most familiar, started out with a preponderance of steam-driven auxiliaries. The stokers, forced draft fans, boiler feed pumps, hotwell pumps, circulating pumps, hurling water air pumps and exciters, were all equipped with turbine drives. The induced draft fans, a few miscellaneous ventilating fans and pumps, and the coal handling equipment were the only auxiliaries supplied with electric drive.

Although our engineers and operating men had been brought up where steam drive was used almost exclusively, they are now so convinced of the superiority of electric drive, that we have used it exclusively for driving the auxiliaries in the new 60-cycle steam plant, known as Huntley Station No. 2. All boiler feed pumps are motor driven. The only steam-driven auxiliaries in the plant are the small steam-driven lubricating oil pumps, which were furnished by the turbine manufacturer as a part of the equipment.

The general use of electric drive for auxiliaries has been greatly delayed by a number of causes, mostly attributable to the designers themselves:

1. They accepted motor accessories offered by the manufacturer which were of too flimsy design.
2. They accepted standard commercial equipment, using standard insulation where moisture proof insulation should have been used, and used open frame construction where totally enclosed motors would have better met the requirements. Most of the electrical auxiliary equipment in a power house should be drip proof, if not entirely moisture proof, so as not to be damaged by leaks in steam and water lines.
3. They made inadequate provision for protection of bus bars, from the standpoint of insulation and rupturing capacity. We will not permit the use of bare bus bars or bare bus connections for the main power buses, or the auxiliary buses in the station. All must be taped sufficiently to withstand the full operating voltage.
4. They followed the commercial practise of protecting their motors against overload. It is the duty of an operator to see that his equipment is not overloaded. The necessary instruments should be provided for this purpose. It is better to burn out a motor occasionally than to run the risk of a service interruption due to faulty relay operation.
5. They delayed the adoption of the across-the-line starting of motors with its simplified control.
6. They used the holding coil type of contactor for starting and controlling electric motors.

As late as 1928 a manufacturer with engineers specializing on power station controls recommended the holding coil type of contactors in the supply circuits to electric motors. These are utterly unsuitable for a service of this character. Many engineers, realizing the defect of the holding coil type of contactor, have used oil switches on their 220- and 440-volt motor circuits to gain the equivalent of the latched-in contactor. The oil switch is mussy; its contacts are not easily inspected. Small quantities of oil are to be avoided where possible. We believe that the recent development of the latched-in type of contactor is one of the big factors in improving the reliability of electrical equipment for power station auxiliary drives.

I do not understand Mr. Hollister's statement, to the effect that where the bin system of pulverized fuel supply is used, exhaust steam is usually required to dry out the coal, thus justifying the use of some steam-driven auxiliaries. The need for exhaust steam does not necessarily justify the use of steam-driven auxiliaries, for this steam can often be obtained more effectively when bled from the main turbines. It is true that with the bin system, pulverization peaks do not usually coincide with load peaks, and, therefore, the addition of some live steam might be required. Otherwise the question of the bin or the unit system of pulverized coal firing does not enter into the picture. It has been the writer's experience that steam is occasionally used to dry coal in both the bin and unit systems. It is used at Huntley No. 1, on the unit system, as an expedient, on account of the physical layout of the plant. It should not be a deciding factor in the choice between steam and electric drive for auxiliaries.

As Mr. Dryer says, reliability is the most outstanding requirement in the choice of drive for power station auxiliaries. He calls our attention to a point which is often overlooked in considering steam-driven auxiliaries; that is, the sudden dumping of a large block of load on the steam plant may seriously interfere with its ability to sustain proper operation of the steam-driven auxiliaries. This is especially true where the steam plant is on peak load stand-by service. We disagree with his arguments, that a house alternator is necessary to get the proper reliability for the supply of electric power to the auxiliaries. We do agree that two or more separate sources of supply are necessary. These sources, however, may be generators in the same station, or widely separated stations.

In conclusion No. 1 Mr. Dryer states that "The turbine constitutes the simplest and most reliable drive." Our experience at Huntley Stations shows that electric drives are more reliable and easier to operate and maintain than the steam drives. The governors and overspeed devices of small turbines need frequent attention. Many of the original steam drives have been replaced by motors to reduce operating and maintenance expenses. Those adopting electric drives must realize that the class of service required by power house auxiliaries demands that the motors and their controls be built to rigid specifications, and the best of material and workmanship used in their installation.

We follow the practise of using special lock washers on all current, potential and control circuits, and use special insulation on the control wires, instead of the N.E. Code Standard. We thoroughly insulate all buses for the operating voltage.

Conclusion No. 5 advocates turbine drives especially for the induced and forced draft fans. Comparative figures for these two types of drive would be of especial interest if available. We believe that the electric motor lends itself more readily to the adoption of centralized controls, which many operators feel are necessary in our larger plants.

The designers of pulverized fuel plants realize the advantages to be gained by a system of complete interlock between the pulverizers, coal feeders and fans, which will prevent an operator from starting or stopping his auxiliaries in the wrong sequence. This might produce an explosive mixture of the coal and air, which is not present under normal operating conditions.

Conclusion No. 8 predicts that higher efficiency turbines may increase the turbine's advantage over the electric motor drive. We do not think so. The higher efficiencies are obtained by operating with smaller clearances, and we would expect that such a machine would require rather more than less of the operator's attention.

Mr. Smith advises that State Line has provided two 4,000-kw., shaft alternators and two 9,000-kw. banks of transformers (26,000 kw.) for the supply of their electric-driven auxiliaries, for 200,000 kw., of generating capacity. We would like to ask how much additional mechanical power is being supplied by the steam drives of the steam-driven auxiliaries.

At Huntley Station No. 2, there are seven 3,750-kva. transformers (26,250 kva. total) for supplying all the auxiliaries of this 160,000-kw. plant.

I would like to ask whether the percentage figures in Table I of Mr. Smith's paper are in per cent of installed horsepower capacity, or in per cent of their power consumption for a certain station loading, and if the latter, for what loading? Also, whether the figures include the spare equipment or just that which is necessary to carry the load?

The designer who believes in centralized control quite naturally turns to electric drive in preference to steam. With the large units, turbines and boilers which are going into our new power plants, it is not practical for the operator to go around to the different auxiliaries to determine how they are operating, so the designer has to bring the information which is required for successful operation of the equipment to some central point. It is then a very natural development to bring the controls to this same point. Having gone to the expense of bringing the boiler controls to a central point, it becomes a rather simple and inexpensive step to provide automatic control for the boilers. Several features regarding the auxiliary drives at Huntley Station No. 2, may be of interest in this connection. The substation supplying the power house auxiliaries is placed at the center of the load, between the boilers and turbines. The auxiliaries for each unit are split into two sections—one-half being supplied from one bus and the other half from another. Either bus may be interrupted without reducing the output of the unit more than 30 per cent even if the boilers in service are carrying maximum load at the time. The transformers serving these two supplies are connected to the generator leads between the generator and its main oil switch. These supplies are backed up by the auxiliary bus, which is supplied from the main bus bars. These main bars are connected to the outside system by means of two 110-kv. transmission lines and to the adjacent 25-cycle station, by means of a 20,000-kw. frequency changer set.

All auxiliaries in Huntley Station No. 2, are electric-driven except for the two auxiliary oil pumps, which were furnished by the turbine manufacturer as part of his equipment. All auxiliary motors of 15 hp., or larger, are supplied with 550-volt, three-phase 60-cycle current through air break circuit breakers and latched-in contactors.

The adjustable-speed drives for the coal feeders are supplied by Ward-Leonard sets. Miscellaneous small motors are supplied at lower voltages with their stepdown transformers, of the dry type, located near the motors.

F. C. Hanker: The discussion of the relative merits of steam-driven and electrically-driven auxiliaries has continued for a number of years. A review of the conditions as presented in the group of papers discussing the various types of drive is timely and will give the different groups of engineers the opportunity to analyze carefully the characteristics of the available types of apparatus and adopt those most suitable for the conditions existing on the various applications.

The requirements of the different auxiliaries vary from constant-speed operation to adjustable-speed conditions with ratios ranging from 2 to 1 for some applications to 5 to 1 for others and from a service that requires the highest order of reliability to that of the less essential needs.

For the constant speed applications all will agree that suitable designs of motors can be obtained. The development of the squirrel cage induction motor to its present state of efficiency, reliability and simplicity makes it well fitted for the most important applications. The use of full voltage starting has greatly simplified the control problem and made it possible to fulfill any specification that operating conditions impose.

When it comes to adjustable-speed drives the problem is not so simple. The simplest type of a-c. motor is inherently constant speed and a change from this characteristic necessitates the introduction of wound rotors with secondary control, the use of commutator type a-c. motors on some form of mechanical speed changing device where the a-c. power supply is adopted. The use of direct current enables the designer to take advantage of varying speed control through the use of this type of motor. This necessitates the use of commutators with the resultant maintenance of brush rigging and current collecting parts to which some operating engineers are opposed. The d-c. motor, however, has likewise been developed to a high degree of reliability but will usually be found to be higher in cost and difficult to justify economically. Any one of these alternates introduces complications that may be classed as undesirable but nevertheless can be made dependable. Close cooperation between the station designer and the designers of motors and control can greatly simplify the combined equipment over that installed in the past. A corresponding cooperation would be of advantage between the mechanical and electrical equipment designers to secure the best compromise in the inherent characteristics of each type.

I feel that a great deal of the criticism of electric drive is undoubtedly due to the effort to apply motors and control developed for industrial applications to the more rigorous duties imposed by central station conditions. In industrial work there is usually frequent opportunities for minor adjustment of control devices and current collecting parts. The motors are installed where the operating conditions are more suitable and the source of supply usually of smaller capacity. These conditions result in materially lower rupturing duties on the contact devices so that less sturdy equipment is needed to give successful service.

In comparing the industrial requirements with those found in the central station field, it will be found that the motors and control are required to function for weeks or even months at a time without a shutdown. For instance consider the auxiliaries of a large turbine unit; it will be found that such a unit may be in continuous service and obviously it would be undesirable to operate the control equipment at frequent intervals and interfere with the service of, for example, a 60,000-kw. prime mover. This condition is being rapidly improved by developments in contact details. It is also frequently desirable to provide special treatment for the winding insulation to make the equipments suitable for operation in places where coal dust or moisture results in more difficult operating conditions. The specifications for the equipment should be carefully reviewed by engineers familiar with the electrical conditions to insure satisfactory apparatus.

A great deal of attention has been given to the power supply for auxiliary drives. These have frequently been considered in connection with the heat balance of the station and have resulted in the installation of various schemes for controlling the demand on the main unit or some auxiliary source. Many operating engineers have required undue insurance in the question of power supply and this has necessitated complicated control layouts that have increased the cost of electrically-driven auxiliaries as compared with steam. The trend toward the unit system of operation should permit of simplification of both the power supply and the auxiliary drive as it will avoid complication in the sources of supply and thus simplify the control for the auxiliaries as well as minimize the duplication of auxiliaries as is frequently done in practise.

Robert Baker: In the papers by Messrs. Dryer and Smith it is brought out that for the plants on which they base their conclusions, and under maximum load conditions, an improvement in overall fuel economy of the order of 2 per cent will result incidental to the use of motor drive instead of turbine drive for all essential auxiliaries. As plants operate under maximum load conditions for a very small portion of the time, it would seem that comparisons should be made on the basis of between 30 and 40 per cent capacity factor, which would represent more nearly the actual operating conditions met by most generating plants. A comparison on this basis would probably show a reduced saving.

In Mr. Smith's paper there is a tabulation showing the approximate per cent of total station cost for the electric equipment to drive the auxiliaries in a modern station. No figures, however, are given as to the cost of the steam-driven equipment required for a station using steam-driven auxiliaries. Without these figures comparisons can only be made between steam and electric drive on the basis of fuel cost. Comparisons of this nature give only half the picture as an analysis of the overall operating costs of the average plant shows that the cost of fixed charges is approximately equal to the fuel cost. It is also obvious that the capital cost of steam drive for auxiliaries is less than the cost of an equivalent electric drive, when all factors are taken into account. The case then becomes one of balancing the saving in fuel cost obtained with electric drive, against the saving in fixed charges secured with steam drive.

Table II in Mr. Smith's paper gives the cost of electric equipment to drive auxiliaries as 7 per cent of the total station cost. On the basis of a 2 per cent saving in fuel when operating with this type of drive under maximum load conditions, which percentage would probably decrease when corrected to cover average operating conditions, and assuming that the cost of fixed charges on capital investment is about equal to the fuel cost, it can be seen that if the cost of steam equipment to drive auxiliaries is such that it reduces the station cost by 2 per cent, the savings in fuel realized by the more efficient electric drive are eaten up by a like increase in fixed charges. In fact a thorough study of a particular case might show that the saving in fixed charges realized with steam drive would amount to more than the saving in fuel with electric drive. On the basis, however, of a balance in overall operating cost between steam and electric drive the decision should be made in favor of the less costly installation as there would be no increase in economy to justify an additional capital outlay.

In order to compete with steam drive, the first cost of systems of electric drive must be reduced, and this can only be accomplished by a simplification of such systems.

Due to inexperience with electric drive, the earlier systems of auxiliary drive were necessarily complicated, having reserves pyramided on reserves, and still further complicated by automatic throw-overs, interlocking relays, dual drives and what not, all in the name of reliability. Experience has shown, however, that equal reliability could be obtained with a much simpler system of supply. I would point out, however, that the pendulum should not swing too far toward simplification as to affect reliability. In fact an auxiliary system should never be simplified to the point where it becomes less reliable than the station bus, but should be in every case of somewhat greater reliability, the degree to which it approaches this condition being controlled by the importance of that particular station with relation to the system as a whole.

Philip Sporn: To my mind the aim in auxiliary layouts for future power plants should be towards further simplification of the power supply and control equipment. When designers first went to electrical drives, it was natural to feel some hesitancy about their use and to resort to elaborate interconnections and steam stand-by drives. With increased confidence in this type of equipment, fully justified by experience, and with the reliability of supply due to interconnection of large systems, a great deal of this elaboration can be dispensed with. Interconnection particularly has not been taken full advantage of and many station designers are still thinking of individual power stations as if they were the lone forts in a deserted area which have to be capable of being self-sustaining under all conditions of system disturbances. As soon as we get away from that standpoint a great deal of duplication of facilities can be done away with to the ultimate advantage of the system.

There are certain fundamental considerations to be kept in view in making an auxiliary layout, and it seems to me that the requirements in most power plants are sufficiently similar so that only a very limited number of systems need be considered. In looking over various auxiliary layouts, one is struck by the fact that in a dozen different stations there are almost that many methods of providing for the auxiliary supply. Perhaps this has been due to the rapid development of electrical auxiliary systems in the past few years. The merits of the systems are now, however, fairly well recognized so that further standardization and simplification can take place. Designers have perhaps too often been tempted to make their layouts a little different from existing ones. In many instances this has resulted in marked improvement but very often originality was all that could be claimed for this procedure.

A great deal remains to be done toward the development of a simple sturdy adjustable-speed a-c. drive for those few station auxiliaries which require speed variation. Multispeed or double motor drives, adjustable-speed wound rotor motors and special combinations all have disadvantages due to elaborate special switching, poor efficiency, maintenance expense, and entirely too frequent breakdowns due to difficulty with the insulation. For example, when thirty breakdowns are experienced on six motors in a single plant in two years it surely indicates that something is still to be desired in the art of motor insulation. As regards the attainment of a simple and sturdy adjustable-speed a-c. motor, great hope is being placed now in the possibilities of development of vacuum tube devices, but of course it is too soon to tell whether the successful development of what appears a theoretical possibility will actually materialize.

The difficulties with insulation have, without a doubt, been brought about primarily by the fact that not rigid enough attention has been paid by the designers to the problem and by the fact that they have all too frequently been satisfied with making a design without being fully familiar with the conditions of service that the equipment had to encounter. More familiarity with true operating conditions would go a long way toward eliminating many of the difficulties.

It is interesting to note that while the electrical auxiliary drive started originally in low voltages and was gradually pushed up to the point where the great majority of the auxiliaries in large stations were being served at 2,300 or 4,000 volts, there has been a definite swing toward the lower voltages within the last few years in American practise. From studies we have made it appears that the horsepower limit for 440-volt auxiliary motors can be considerably raised over that now considered good practise. We have found that 500-hp. motors, for example, may be economically used at this voltage and only at 750 hp. is the use of 2,200 volts definitely indicated. The reason for that, of course, is primarily the increase in cost of the high-voltage motors. We have recently looked into the question of higher voltage for steam station auxiliary motors, considering voltages as high as 6,600 volts, but the price of motors for this voltage is so much in excess of that for the lower voltage motors that for the present at least it does not appear that any economies can be effected by their use. It is interesting, however, that in Europe 6,000 volts for the larger motors is quite standard practise and the engineers sponsoring that voltage claim that definite economies have been realized.

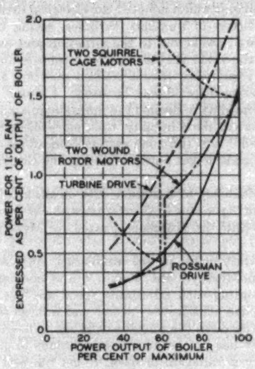

Fig. 1

S. M. Dean: It seems to me that in attacking the problem of developing a low-cost, sufficiently reliable auxiliary system, it would be well to look first to the source of power. If the main generating station switch-house electrical layout were such that each generator section was fairly well protected against voltage disturbances caused by major faults occurring in the other sections, and there were fortunately high-speed relaying and switching, very serious thought should be given to feeding the essential auxiliaries of each individual main turbine unit from transformers directly connected to the generator terminals. In doing this it would be recognized that should faults occur on a feeder or a bus of the section served by the particular generator that the voltage on the generator terminals would probably go so low that the induction motors driving the essential auxiliaries would slow down. But if the switching and relaying were of such character that a fault would be removed from the system in 15 to 20 cycles, the motors would probably come up to speed with sufficient promptness so that the operation of the main turbine unit would not be impaired. Of course, one would have to be very sure that disturbances on a given generator occasioned by troubles on the electrical section connected to its neighbors would not pull the voltage low enough to affect the auxiliaries.

It seems to me that such a method of attack shows distinct possibilities from the point of view of low first cost and acceptable operation. At least, that is the first attack that I should make on the problem.

I should also like to point out that while the papers are confined to steam and a-c. drives the possibilities in d-c. drives should not be overlooked. When account is taken of the extent to which the station rating is reduced (some 5 per cent), the smoothness and ease of operation, the absence of serious short-circuit problems, etc., it does not compare unfavorably with the other methods.

F. H. Hollister: Mr. Dryer has shown some interesting curves comparing the performance of steam turbines and induction motors when driving various types of auxiliaries. He has also pointed out the objection of transitional disturbances which accompany the use of slip-ring motors for driving draft fans.

There are, of course, efficient types of electric drive which do not have this objection, such as the d-c. motors, the a-c. brush shifting motor and the Rossman drive. The use of d-c. motors for fans is largely confined to stations having a d-c. auxiliary power system. Where such is the case the d-c. motor provides a satisfactory method of operating fans. The a-c. brush shifting motor has likewise given satisfactory service in a number of stations, but is limited in size to approximately 25 hp. per pole which would mean 200 hp. at 900 r.p.m.

The Rossman drive, which is a combination of a-c. and d-c. drive,[*] has been in successful operation driving fans at the Powerton Station for the past year. This drive has also recently been placed in service at the Philo and Sheboygan stations. Two 2,500-hp. Rossman drive units are now under construction for boiler feed pumps.

Fig. 1 has been prepared from Fig. 9 of Mr. Dryer's paper on which the performance of the Rossman drive has been superimposed. The performance of d-c. motors and a-c. brush shifting motors would be similar to the Rossman drive.

Fan power data have recently been received from the Operating Department of the Powerton Station, which may be of interest in view of the Rossman drive being relatively new. A comparison has been made between the power input to the fans for two similar boilers, one equipped with two-speed squirrel cage motors, and the other with Rossman drives. For a run of 30 days the boiler equipped with two-speed squirrel cage motors averaged 639 lb. of steam for one kilowatt-hour fan input, and the boiler equipped with Rossman drives for the same period averaged 853 lb. of steam for one kilowatt-hour fan input. This represents a saving of approximately 33 per cent power input to the fans with Rossman drive, and confirms estimated figures given in Mr. Rossman's paper. The cost for each of these types of fan drive and control, when installed at Powerton, is approximately $30.00 per horsepower.

F. M. Stoddard: All auxiliaries at the Columbia Power Station are electrically driven except two boiler feed pumps. The power for the essential auxiliaries is supplied from shaft generators connected to each main turbine unit. Transformers connected to the main bus supply the non-essential auxiliaries and serve as a reserve for the shaft generators. In six years of operation there has been no trouble with the auxiliary system from loss of auxiliary power or fluctuation of bus voltage from external faults. On the whole, the operation of electrically driven auxiliaries has been very satisfactory.

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[*] *A New System of Speed Control for A-C. Motors*, by A. M. Rossman, A.I.E.E. Trans., Vol. 50, March, 1931, p. 162.

The use of electrically driven auxiliaries facilitates operation of a power station in the following manner:

1. It provides convenient control of boilers from one central place.

2. It provides a simple and convenient method of interlocking pulverized fuel equipment for proper starting sequence or other forms of interlocks.

Steam provides a somewhat more reliable power supply particularly for isolated stations. A station using all steam-driven auxiliaries would require a separate turbine-driven lighting generator to be consistent.

Wellen H. Colburn: Probably no part of a steam generating station is so subject to difference of opinion, or open to discussion and variation in design, as the auxiliary drive. Both complete steam and complete electric drive, as well as a wide range of combinations between, and several methods of providing steam and electric supply have their protagonists.

As Mr. Hollister mentions, too little is known about relative maintenance costs of the various schemes but as pointed out these data are difficult to get on a comparable basis. However, maintenance costs are not all and data on overall operating costs are equally desirable and equally difficult to obtain on a comparable basis. More comprehensive costs and the detail of their makeup is desirable.

Mr. Dryer makes an attractive case for "all turbine drive" of auxiliaries but appears to over-emphasize the importance of reliability of auxiliaries, unless his paper is to apply to isolated stations only. However, duplication of generating equipment or interconnection with other stations or systems, as Mr. Smith points out, makes loss of a generating unit from failure either of itself or auxiliaries of lessening importance.

Marginal gains in economy in steam-electric generation are henceforth to rule and these, to be of value, must be gained without a complete offset in increased capital costs.

This factor is reflected in the main unit as well as in the auxiliaries. No analysis is complete which considers the economies of auxiliaries without taking account of the effect of the resulting heat balance diagram upon the initial cost and efficiency of the main turbine. Neither Mr. Dryer nor Mr. Smith appear to have given this consideration.

One of the overlooked items relating to turbine-driven auxiliaries both in design and operation appears to be the steam loss necessary to warm up turbines of larger sizes and better efficiencies required with auxiliaries for modern large generating units. Items of this nature account for discrepancies between practise results and design figures.

Mr. Smith makes a less positive case for electric drive apparently because he recognizes more variables in accomplishing the desired ends.

The point that failure of the main unit makes the auxiliaries unnecessary deserves a corollary. If the main unit can be spared when it fails, we may conclude that it can be spared when its auxiliaries fail. This permits much economy and simplification by non-duplication of auxiliaries and station piping. With such unit design still further economies in electric operation become evident by elimination of house sets and complicated switching equipment. At the same time the growth in size of units, which Mr. Smith points out results in auxiliaries of such size or so located as to require duplicate drives of half the total capacity, tends to minimize the possibility of complete shut-down of a unit by loss of an auxiliary.

That the adjustable-speed a-c. motor leaves much to be desired is too well recognized to be debatable. New designs and methods of adjustable-speed operation with a-c. power supply, possibly with vacuum tube or mercury rectifier applications, are and will be forthcoming which will obviate some, if not all, of the present objections.

It is fairly well recognized that the pendulum swings too far. It is probable that the trend to electric drive of auxiliaries has gone too far and that the economic mean lies between all steam and all electric drive. However, the correct proportion cannot be evaluated without more careful study of the economics of design and operation, taking into account the whole cost of all related factors. Such data are only partially available and undoubtedly will require modification from time to time as equipment designers provide new and more adaptable electric and steam drive and utilization equipment.

John H. Lawrence: There cannot be one general answer to the problem of making a decision between electric and steam drive, any more than we can say that 400, 600 or 1,400 lb. is the correct steam pressure for any station. For instance, a steam plant to act as standby for a hydro plant must be treated differently from a steam plant which operates continuously. Likewise a station to supply an Edison system such as in New York must be treated differently from one supplying the average a-c. system.

There is no doubt about the electrical system gaining the supremacy immediately following the introduction of stage bleeding. This was the logical conclusion as was evidenced by the number of stations which adopted the electrical drive. However, the manufacturers of auxiliary turbines, seeing their markets being cut down, awakened to the situation and as a result the efficiency of the small turbine has been considerably improved, so much so, that today it is a mooted question as to whether steam or electric drive should be used. I would say that the greater the reliability is desired the more steam auxiliaries should be used. The less the reliability, then more electrical auxiliaries should be used. Not that one system is more reliable than the other, but because to get equal reliability the electrical system is more costly.

The heat efficiency of a station with steam auxiliaries can be brought within 1 per cent of the station with electrical auxiliaries, or about one-half the difference quoted by Messrs. Dryer and Smith.

Assuming that many of the auxiliaries, such as circulating pumps, coal pulverizers, etc., should run at constant speed, the advantages coming with adjustable speed are lost. The circulating pumps use considerable power and a large saving can be made by slowing down the pumps as the temperature of the water drops. Also it has been found that variations in characteristics of coals justify a variation of speed to get the best results from some coal pulverizers. If the advantages of adjustable speed are neglected, the efficiency of the electric drive may show up very favorably, but the heat efficiency of a station with properly designed steam drive should show very little loss over electric drive, certainly not over 1 per cent.

I believe the cost of the electric drive with its prorated share of the station cost will be so much in excess of the station with the steam drive, that for the average generating plant steam will be found to be more economical in overall costs.

Assume that the auxiliary system for each system will have the same cost beyond the bus with electric drive or beyond the boiler with steam drive. To be fair to the electrical system assume that the capacity of the boiler plant with the steam drive system must be increased 1 per cent, although it is likely that the units would be overloaded by this amount. This would increase the station cost by about one-half of one per cent. On the other hand if power is taken from the main bus for the auxiliaries, the prorated cost of the turbine room and electrical gallery with equipment should be charged against the electric drive. If this part of the station is worth $50.00 per kilowatt, then if the auxiliaries require 5.5 per cent of the total output, this percentage should be charged against electric drive, for with steam drive this power is available for sending out on the system. In other words, 2¾ per cent of the total station cost, if the total cost is

$100 per kw. is chargeable against increased cost of the turbine room and electrical gallery with equipment.

Deducting the ½ per cent excess cost of boiler plant with steam drive, the excess first cost of an electric drive station is 2¼ per cent more than a steam drive station.

With equally good designs of the two systems, I do not believe the efficiency of the electric drive can be improved to an extent that this extra cost can be justified unless the capacity factor is higher than usually found in a central station.

H. L. Wallau: The Ashtabula Plant of The Cleveland Electric Illuminating Company exemplifies the use of both steam and electric drive for the power supply to station auxiliaries.

Referring at this time, only to those auxiliaries which are directly linked with the operation of the boilers and turbines, the drives used are as follows:

Boiler feed pumps, steam turbine drive.

Primary air fans, squirrel-cage induction motor drive.

Induced draft fans, two-speed induction motor drive.

Fuel feeders, adjustable-speed d-c. motor drive.

House service pumps, two turbine drive and two induction motor drive.

Circulating pumps, two per unit, induction motor drive supplemented by steam turbine (dual) drive on one unit of each pair.

Condensate pumps—two per unit, same types of drives as circulators.

Exciters, (steam) shaft drive from main unit.

Spare exciter, dual turbine—induction motor drive.

Fuel feeder motor supply, two generators, induction motor drive, one supplemented by steam turbine (dual) drive.

All dual-driven apparatus is set to cause the turbine to pick up the load on the equipment at a 3 per cent reduction in speed. Electric power for the motor drives is obtained from bus transformers.

Boiler feed pumps are steam turbine driven to obtain maximum reliability of power supply, regulation being accomplished by excess pressure governors. All of the other essential auxiliaries are normally motor driven. However, as noted, the next most essential auxiliaries have been partially provided with ancillary steam drives, so that failure of electrical supply will not result in total loss of load.

The relatively small amount of steam taken by the feed pumps, neglecting radiation losses, is used at 100 per cent thermal efficiency, the exhaust being used to heat the feed water by means of a jet type barometric heater condenser. Except for the additional heat furnished the condensate by the generator air coolers and steam air ejectors, the balance of the heat supplied (about 75 per cent at full load) is obtained by bleeding at the 13th and 15th turbine stages. This combination of drives is a compromise between Figs. 3 and 4 of Mr. Dryer's paper, more nearly approaching Fig. 4, and has yielded a net annual overall thermal efficiency somewhat in excess of that set up by him. The total use of electricity for all plant purposes was 4.2 per cent of the amount generated.

In our plants the turbine-driven and dual-driven pumps and d-c. generators are all located in the turbine room basement. This simplifies the piping and facilitates operation. If the second set of condensate and circulating pumps connected with each generator had been equipped with steam instead of motor drive, the heat balance would be upset when these pumps are operated on account of the additional exhaust steam supplied to the heater condenser.

Since draft fans cannot be efficiently grouped at one location, piping for these, if steam driven, would have to be carried to many points. This diversity of location is disadvantageous to steam drive as compared with electric drive. Another disadvantage of steam drive, though not a major one, is the necessity of using reduction gearing. Reliability is of less importance for draft fans than for boiler feed pumps, because the loss of a draft fan, while possibly jeopardizing service, does not threaten the destruction of expensive equipment as well.

In our newer plants, the reliability of any given draft fan is of lesser importance than that of a condenser pump, because either of the two boilers associated with a given generator can furnish steam enough to carry 80 per cent of full load on that unit.

The use of turbo-vane draft fans coupled to two-speed pole-changing induction motors furnishes all necessary variations in draft and permits the use of the low-speed low-power rating of the motors for the greatest percentage of the operating time at good efficiencies. The simplicity of full-voltage starting, induction motor drive for the primary air supply is an obvious advantage. Steam drive for the pulverized fuel feeders was at no time considered.

The turbine auxiliaries have been referred to previously. Except at high loading coupled with high summer temperatures of condensing water one set of pumps only is needed, and the dual-driven one employed. Our operating experience over many years has proved that constant-speed equipment is entirely satisfactory for this service.

Of the four house service pumps, two are steam driven for maximum reliability. Had dual-drive been used on all or part of these, the first cost would have been higher, and if only three pumps had been installed, the reserve less.

Mr. Smith has pointed out the relatively small percentage of total investment involved in the cost of electric equipment for auxiliary drive and outlines some of the possible methods for reducing these costs still further.

One of these is the use of house transformers connected to the generator leads ahead of the generator breakers. This does away with breakers on the high side of the transformer and also lends itself admirably to the strictly unit method of operation of main units and their auxiliaries. A supply from another source is, of course, necessary at the time of starting a unit. This method requires one bank per generator and hence more space than if a smaller number of banks of larger individual capacity is used, which latter scheme requires bus transformers and consequently breakers.

At Ashtabula one transformer bank, bus connected, is used to supply the auxiliaries for two turbines and four boilers. Normally each bank feeds an independent section of truck type board. These sections, four in ultimate number, are physically separated one from another and adjacent ones may be electrically joined through two-section tie breakers, one on each board, connected together by a relatively short cable run. Thus the failure of a tie breaker cannot involve more than one section of board. No provision has been made to ring the bus by connections between the first and fourth sections. Spare trucks are provided for all positions so that maintenance or repair work does not interfere with the functioning of any group of auxiliaries.

A transformer bank connected to a given bus section normally supplies the dual-driven condensate and circulating pumps of each of the two turbo-generators associated with it, while the straight motor-driven sets of pumps are supplied from another section. Hence, even when both sets of pumps are in use, a failure of electric supply results in the loss of but one set on each generator.

The four boilers associated with a group of two generators have their primary air fans and induced draft fans fed from a sectionalized 2,300-volt truck type boiler supply board, with two boilers fed from each section. Each section is fed by means of one circuit capable of carrying the load of the four boilers. One of these circuits issues from the section of main supply board associated with the turbines of this group, and the other from another section. Normally both circuits are energized and the tie breaker open. In case of deenergization of a circuit, it is cleared and the tie breaker closed automatically, thus maintaining continuity. Interlocks prevent more breakers from being closed at one time.

Another 2,300-volt truck board, not sectionalized but having duplicate feeds from two independent sections of the main board, supplies the coal handling and preparation equipment. Normally one circuit is open at the receiving end, and loss of the other results in a temporary interruption of power supply to these non-essential auxiliaries until manual throw-over is accomplished. The supply breakers on this board are adjacent and interlocked so that but one may be closed at any time.

Motors below 40 hp. are supplied at 440 volts and distribution centers adjacent to the groups so supplied are established. This is also done with respect to the d-c. fuel feeder motors.

Circuit breaker and stop-valve motor control is obtained from a d-c. 120-volt control bus on which two batteries (468 ampere-hours each at one hour rate) float, trickle-charged through diverter-pole generators.

Normal lighting service is alternating circuit three-wire. In case of failure, a number of key circuits is automatically thrown over to d-c. supply, a neutral for the 230-volt d-c. system being at all times maintained through balancer-sets. A further automatic throw-over to battery supply keeps one-half of these key-circuits operative in case of simultaneous or subsequent failure of the normal d-c. source, (d-c. dual-driven generators).

The above methods of auxiliary supply have been in use for several years at the Avon Plant, Ashtabula including some refinements, providing greater flexibility for inspection and maintenance.

Note the following points:

The sectionalized operation of both the main, 2,300-volt supply and the boiler group boards.

The use of a single bus in all boards.

The use of factory fabricated switching equipment, truck type boards for 2,300 volts and Urelite factory built board for 440 volts. This too is sectionalized.

The availability of spare trucks for any position.

The physical isolation of the various sections of the main 2,300-volt supply board, and of the boiler group supply boards.

The use of two section-tie breakers in series in connection with the main board sections.

The flexibility and reliability obtained for both operation and maintenance by the interlacing of supply circuits and the use of dual-driven equipment at critical locations.

With a few exceptions, use full voltage starting for all a-c. motors.

We believe that the supply scheme used results in a high degree of reliability, operating flexibility, and low installation and maintenance costs.

R. C. Powell: In the matter of reliability, the writer's experience has shown that the reliability of electric drive depends almost entirely upon the reliability of the auxiliary power source. A house generator is impracticable in a station operating at 1,250 lb. steam pressure where all the boilers are reheat boilers, that is, a plant of unit power design, and for such a plant the most reliable auxiliary supply is the shaft-driven auxiliary generator. This would be very reliable were it not that the main turbine is very liable to go out on overspeed following a heavy disturbance. Prevention of tripping on overspeed is a matter of such importance both in the matter of main and auxiliary power supply that manufacturers and operators of steam turbines should make every effort to correct this defect.

The electric drive has considerable advantage in steam plants operated as part of a system which at certain seasons is supplied principally from water power plants, but when even during the plentiful water season most of the steam units are floated on the line to insure service. In such older plants with several boilers per turbine, it is possible to practically shut down the boilers and operate the auxiliaries from water power. In the newer plants with very large boilers and larger turbines, a minimum load of 6 or 7 per cent of maximum must be carried per turbine unit in order to insure safe operation and a fast pick-up of load. At certain times, then, more steam power is generated than is actually required and water that could otherwise be used is wasted. In this case the electric drive is preferable to steam since about one-third of this minimum steam power will be required to operate the auxiliaries, with an appreciable saving in fuel as against steam drive.

As mentioned in the papers, most of the drives may now be at constant speed. This will enable the station designer to use synchronous motors and make some savings in cost with a gain in efficiency.

There is no doubt that the steam drive has attractive features, but the writer agrees with Mr. Smith that the electric drive has on the whole inherent advantages even if there is much to be desired in the matter of cost.

F. W. Gay: The impression left from the papers relating to station auxiliaries is that steam-driven auxiliaries have a more reliable source of power than electric-driven auxiliaries. It would appear that insufficient stress has been laid on the fact that electric-driven auxiliaries may receive power from their own generating station or from electric networks having in themselves great inherent reliability. On the other hand, it is very seldom possible for steam-driven auxiliaries to receive steam from any other station and the transfer of a steam-driven auxiliary from one steam service to another in the same station is not generally so conveniently accomplished as with electric drive.

Supplementing this inherent advantage peculiar to electric drive is the additional advantage with the unit system of supply that upon a prolonged failure of any one generator the auxiliaries for that unit are no longer required. The point has been raised by inference that short circuits on generator buses or feeders, especially in the case of the unit system, may reduce the voltage on the auxiliaries for that unit so as to require their being thrown over to another source of power. No mention has been made of the fact that systems have been devised whereby auxiliary power may be taken from a generator and the voltage of the auxiliary source of supply may be maintained at substantially 100 per cent, while the voltage on the station bus is dragged down to zero by a short circuit. Such a system was thoroughly investigated by Professor Nims and described in the *Electrical World* for April 25, 1931. In the system described by Professor Nims, it is necessary to employ a large amount of transformer capacity and since this transformer capacity is a substantial item for electric auxiliary drive this system is expensive, although probably not as expensive as a shaft end generator. On the other hand, if the unit system of auxiliary drive is employed and the generator supplies its power to a high-voltage bus through a direct-connected transformer it becomes possible to take the auxiliary power from tertiary windings on the main transformer bank.

With substantially no additional expense it is possible to assemble the secondary winding and the tertiary, each in separate inductive relation with respect to the primary, so that the primary has a high equivalent negative impedance with respect to the tertiary for loads on the secondary. Under these conditions the tertiary voltage is over compound, that is, it rises with heavy inductive loads on the secondary and with primary voltage maintained. With a proper design the rise on the tertiary windings may be made to compensate for the drop in generator voltage so that for all loads on the transformer secondary windings up to and including a dead three-phase short circuit on the secondary windings, the voltage on the tertiary winding will be maintained at a serviceable value.

While such a transformer design will not seriously affect the straight through impedance between primary and secondary, it will give a relatively high impedance between primary and tertiary windings and it may be found desirable to equip the tertiary windings with induction voltage regulators and control equipment to maintain a substantially constant voltage on the auxiliary supply bus for all loads on the transformer secondary windings up to and including a dead three-phase short circuit

While induction voltage regulators will not be fast enough to avoid lamp flicker on auxiliary power circuits for sudden changes in generator loading, they should be able to correct for minor changes in the auxiliary supply voltage so fast that the speed of auxiliaries even when driven by slip-ring motors with control resistance in circuit should not vary much over a fraction of one per cent.

A. T. Brown: The writer believes that greater progress could be made if instead of dealing in generalities, careful and detailed studies could be made. After proper reliability has been secured, choice as to the type of drive to be selected for various auxiliaries in a power station, which are intimately connected with the main units, should be determined from the overall costs of operation, including all factors.

It is obvious, if cost is not considered, that from an operating point of view electric drives present many advantages. However, the writer believes that it would be a great advantage to the art of designing power stations if someone would undertake a complete and detailed study covering all-electric, part-steam and part-electric direct steam drive, and great steam drive for auxiliaries for, say, stations of 400 lb., 650 lb., and 1,500 lb. of steam. I have never seen such a detailed study, and it would not be an easy thing for busy engineers to prepare, as it would require practically the designing and plotting performance curves for several entire power stations.

If such data can be secured, I believe points at issue between the advocates of electric and steam-driven auxiliaries can be reduced.

In the last part of Mr. L. W. Smith's paper, it is apparent that he realizes the great cost of motor drives. This is apparent from an inspection of the table on page 393 where it is stated that of the total station cost, the motors are 1.2 per cent of the total, and this appears at the site of the auxiliary drive application. What does not appear at the site is the 5.8 per cent and this apparently does not include the cost of the building space occupied by the control panels, switches, etc., which is often considerable.

If the building cost mentioned above is included, it is safe to say that each motor carries behind the scenes approximately five times its cost in equipment to serve it.

In a large modern power station, which burns, say, $2,000,000 worth of coal annually, a fuel saving of 1 per cent is well worth accomplishing.

If the fixed charges on the capital invested in a power station may be taken as roughly of the order of 90 per cent of the fuel cost, a saving in capital cost of the order of 1.10 per cent will accomplish the same saving as a 1 per cent fuel saving.

If then we take the table referred to above as a fair statement of the cost of electric auxiliary drive, we can see that in order to accomplish this result we must reduce the cost of the behind-the-scenes auxiliary electric equipment in the neighborhood of 18 per cent.

It seems as though this could be done by careful planning and we urge electrical engineers to see what can be done in this direction.

In Mr. W. P. Dryer's paper, the statement is made that if the draft fans are steam driven the boiler evaporating capacity must be increased 0.7 per cent over electric drive. Was this figure obtained from a carefully calculated heat balance, carried all the way through? For unless the loss of heat to the circulating water was increased in the case of steam-driven draft fans, as compared with electric drive, it seems that this figure of 0.7 per cent may give us an erroneous impression.

George A. Orrok: Early practise in this country has always inclined towards the steam auxiliary, while European practise has been always along the lines of bus-driven auxiliaries. The middle ground between the two systems where the auxiliary power is furnished by an auxiliary turbine not connected to the main bus except in emergency was developed to attain the advantages of both systems.

The introduction and development of the regenerative system with its almost universal adoption at the present time has again emphasized the essential differences of the two types of drive. We must also differentiate between types of plant such as the larger industrial installations, with the demand for process steam at low pressures and the central station plant where, due to the regenerative system, no large use of exhaust steam is possible.

The question of reliability is so intimately bound up with the operating policy of the plant, the size and character of the load, the outside interconnections, and the number of units in the plant that it is impossible to give any general solution for the question. The difference in economy between the two types of auxiliary apparatus is not large, possibly of the order of one-half per cent on the overall thermal efficiency of the station, and it has become increasingly evident that all of these factors must be taken into consideration, and the solution for any given plant will not necessarily be a general answer in any sense.

It is of course evident that the smaller number of links between the coal pile and the user represents the most reliable operation, and it is also evident that no solution in which the operation of the auxiliaries is likely to be upset by a short circuit on the outside system is of advantage to a central station.

Modern auxiliary units have been developed to such a state that their reliability in all cases is equally good and the task of the designer has been to use the smallest number of links which with equal cost will give a substantially equal efficiency.

Nevertheless, individual problems must still be settled in an individual way taking into account all of the many factors which make for successful operation and such papers as these, which set forth in such a detailed manner the advantages of competing types of apparatus, serve a most useful purpose.

L. W. Smith: Messrs. Kehoe, Cushing, and Stoddard have brought out two additional advantages of electric drive, namely, it provides a convenient and accurate method of measuring auxiliary power and a simple method of obtaining interlocks for the operation of pulverized fuel equipment.

In answer to Mr. Cushing's questions concerning the station used in the paper as an example of electric drive, it may be noted that there are no steam-driven auxiliaries except boiler feed and fire pumps which are used only as reserves. There is a total connected load of approximately 25,000 hp. in motors including reserves which is equivalent to approximately 9.3 per cent of the station output. The per cent power consumption given for each class of auxiliaries is the average for a period of two weeks during which the station had a maximum net output of 189,000 kw. and average net output of 126,500 kw., which gives a station load factor of 0.669. The apparent excess in auxiliary transformer capacity at the station is largely due to the fact that all of the coal handling equipment is served from one transformer bank. The other transformer bank serves as a reserve for the shaft generators and auxiliary transformers. Subsequent units will not require proportional amounts of transformer capacity. Incidentally, the auxiliary power for units 2 and 3 will be served from transformers connected ahead of the main generator oil circuit breakers instead of by auxiliary shaft generators.

The comments by Messrs. Baker, Lawrence, and Brown are related in that they all touch on the relative investment cost of steam and electric drive. Numerous cost tabulations could have been prepared for steam and electric drive in connection with these papers, but it is doubtful if they would have any value except for the assumed conditions. There are so many variables entering in station design that each case must be worked out on its own merits. The tabulations in the paper were included chiefly for analyzing the distribution of auxiliary power and the cost of the component parts of an electric auxiliary system.

It has been pointed out in the paper that the relative economics of auxiliary drive hinge to a large extent on what constitutes a reliable system of auxiliary power supply and what is the actual water rate of auxiliary turbines. The paper on the East River Generating Station by Messrs. Grady, Lawrence, and Tapscott referred to in the bibliography is of interest in connection with this phase of the problem. The paper on the East River Station states that a comparison of the investment costs between various auxiliary power schemes showed that an a-c. electric supply with shaft generators, reserve house generator, and bus transformers would cost 35 per cent more than steam turbines, and that a d-c. system with storage battery reserve would cost 120 per cent more than turbines. A capitalized credit of 2 per cent is given for fuel saving in the case of a-c. supply and 3 per cent for d-c. supply. It is also interesting to note from this paper that the steam consumption for the auxiliaries at the East River Station is somewhat higher than the original calculations. The conclusions arrived at by the study of the various schemes of auxiliary power for the East River Station are no doubt correct for the particular conditions that confronted the designers, but these conclusions can not be applied to other station designs without a similar analysis. In most cases the number and type of reserve auxiliary power sources could be reduced with an appreciable saving in investment cost. Likewise any major change in the station design might alter the picture as far as the type of auxiliary drive is concerned.

The comparative reliability of electric drive has not been emphasized in this paper because the real question is whether or not electric drive is sufficiently reliable for central station service. The increasing use of electric drive with its increasing simplification of power supply methods would indicate that electric drive does provide a reliable, convenient, and economical method of operating steam power station auxiliaries.

Standard Decrement Curves

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***Synopsis.**—For circuit breaker and relay application, it is frequently necessary to determine the decrement in power system short-circuit currents. For this purpose certain "standard decrement curves" approved by the National Electrical Manufacturers Association have been in general use. The original curves appeared in 1918 and a revised set was prepared in 1923. New decrement curves given in the present paper supersede those formerly in use. The paper also includes a discussion of the need for new curves, the underlying assumptions upon which they are based, the method of application and their limitations.*

INTRODUCTION

For circuit breaker and relay application, it is frequently necessary to determine the magnitude of power system voltages and currents under fault conditions for various intervals of time measured from the instant of the application of a fault. The decay of fault current from initial to sustained values is commonly determined by means of decrement curves. In 1918 a standard set of decrement curves was published for the first time.[1]

The standard decrement curves of 1918 were derived on the basis of the available information on the decrement characteristics of machines and systems. By 1923 considerable additional data had been collected, and accordingly, the curves were revised. These curves, published in the *N.E.L.A. Relay Handbook* (1926) and in various manufacturers' publications are superseded by the new standard decrement curves, Figs. 1 and 2, published for the first time in the present paper. They have been approved by the National Electrical Manufacturers Association. The presentation of these curves is the principal object of this paper, but it also reviews the need for new curves, the underlying assumptions on which they are based, their application, and finally their limitations.

In the past eight years, there has been considerable progress in the analysis of transients in synchronous machines. Furthermore, the method of symmetrical components has come into general use for the calculation of unbalanced faults. These developments in theory, together with the increased knowledge of machine constants, have made desirable the preparation of a new set of decrement curves.

The 1923 decrement curves assumed a single decrement for the a-c. component of fault current. The new decrement curves take into consideration the fact that the a-c. component is made up of two exponential terms and the fact that the transient time constant varies with system reactance. The method of using the curves includes a means to give more accurate results in case the time constants of machines differ appreciably from the assumed average figures upon which the curves are based.

The 1923 curves include three separate sets, one for each type of fault, namely, three-phase, line-to-line, and line-to-ground. The new curves include only a single set which, however, is applicable to the same three types of faults. This is feasible because the new curves are based on the method of symmetrical components. The old curves assume equal initial values of line-to-line and three-phase currents whereas the present curves are plotted for the more usual ratio of 0.866. The old curves for line-to-ground faults were restricted to the particular case of a fault on the terminals of one or more generators, all the neutrals of which were solidly grounded; the new curves are not restricted in this respect.

In the 1923 curves, no distinction was made between machines with and without damper windings. The curves included in the present paper have been designed to apply particularly to salient pole machines with amortisseurs and to turbo-alternators. Therefore, they give inaccurate results for the first 0.2 seconds for salient pole machines without damper windings and are supplemented by a method of calculation which gives the initial value of short-circuit current accurately for these machines.

DERIVATION OF THE STANDARD DECREMENT CURVES

The important differences between the new and the old decrement curves have been outlined above and the derivation of the new curves is given in Appendix I. The basic assumptions, being of more general interest, are given below.

1. The generators are assumed to be operating at rated voltage and kva. at 80 per cent power factor immediately preceding the short circuit.
2. No automatic voltage regulator is used.
3. The actual system subjected to fault may be represented by a single equivalent generator of the same total rating and an external reactance.
4. The load is assumed to be located at the machine terminals, and the machine reactance is taken as 15 per cent unless the total reactance is less than 15 per cent. In this event all the reactance is assumed to be in the machine.

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[1] For references see Bibliography.

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5. The short circuit is assumed to occur on an unloaded feeder.

6. The short circuit is assumed to occur at the point of the voltage wave which corresponds to maximum possible instantaneous current.

7. All resistance in the circuit including the resistance of the fault is neglected.

8. All the machine e.m.f.'s are assumed to be in phase.

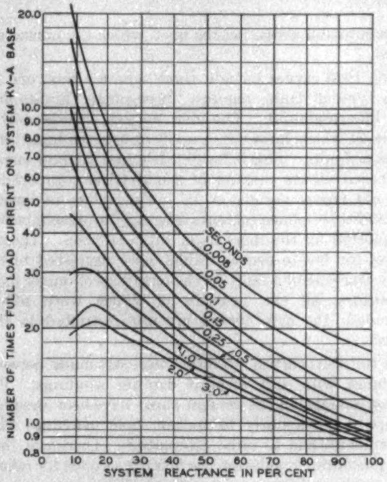

FIG. 1—SHORT-CIRCUIT CURVES FOR SYNCHRONOUS MACHINES

System reactance *must be* based on system kva. and *not* on the particular kva. chosen as a base for calculations.

System kva. is connected synchronous capacity in kva.

Note: These curves are based on $Tdo' = 5$ sec.

For other values of Tdo': multiply actual times by $\frac{5}{Tdo'}$, to get equivalent times to use in curves

How to use curves:

For three-phase short circuit—
  Reactance—Use system reactance to the point of fault
  Times full load (normal) scale—use scale reading
For line-to-line short circuit—
  Reactance—Use two times the system reactance for the three-phase fault
  Times full-load (normal) scale—multiply scale reading by $\sqrt{3}$
For line-to-ground short circuit—
  Reactance—Use $(2x_1 + x_0)$ or

$$\frac{3 \times (\text{initial three-phase short-circuit current})}{(\text{initial line-to-ground short-circuit current})} \times \text{system reactance for three-phase fault}$$

  Times full-load (normal) scale—multiply scale reading by 3

9. The machine reactances and time constants are taken as representative of modern machines. Of particular importance is the relation between transient and subtransient reactance of machines. (See Fig. 3.)

APPLICATION OF STANDARD DECREMENT CURVES

Description of Decrement Curves. The decrement curves of Figs. 1 and 2 give the r.m.s. total current expressed in terms of "times normal current" with total connected synchronous capacity in kva. as a base. In Fig. 1 reactance is plotted as the abscissa with elapsed time from the beginning of the fault as a parameter for the curves. In Fig. 2, the elapsed time is the ordinate and reactance is the parameter.

The reactance to be used with the curves for any kind of a fault may be obtained either by means of analytical calculations, or by use of the calculating board, or indirectly by the substitution in the formulas on Figs. 1 and 2, if the initial values of symmetrical short-circuit currents happen to be known from other sources. In order to choose the proper decrement curve, it is essential that the reactance used to select a curve in Fig. 2 or a line in Fig. 1 be expressed as a per cent of the *total connected synchronous capacity in kva.* rather than as is common practise, an arbitrary value conveniently chosen to expedite system calculations.

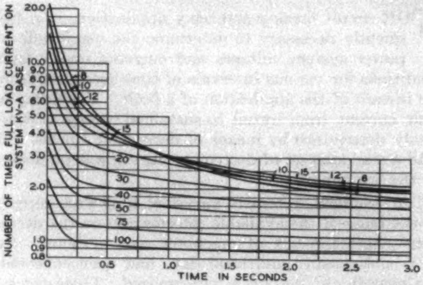

FIG. 2—SHORT-CIRCUIT DECREMENT CURVES FOR SYNCHRONOUS MACHINES (REACTANCE = 8 TO 100 PER CENT)

How to use curves—see Fig. 1

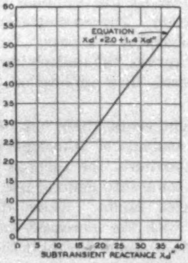

FIG. 3—RELATION BETWEEN Xd' AND Xd'' (SATURATED VALUES) FOR THREE-PHASE SYNCHRONOUS MACHINES

If the system open circuit time constant referred to the point of fault is known to be other than 5.0 sec., the time scale of the curves may be corrected to conform to it by means of the formula given on the curve sheets.[4,5]

The problem in Appendix II illustrates the use of the curves when the initial short-circuit kva. and connected synchronous kva. are known.

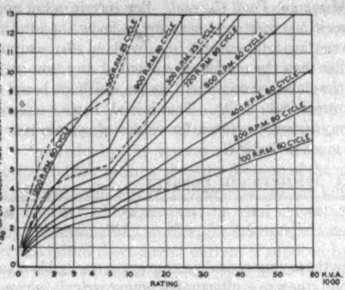

FIG. 4—OPEN-CIRCUIT TIME CONSTANTS OF A-C. GENERATORS AND MOTORS

TABLE I*—TYPICAL THREE-PHASE 60-CYCLE SYNCHRONOUS MACHINE CONSTANTS

	x_d	x_d''	x_2	x_0†
Turbo generator	$\frac{1.10}{0.95-1.45}$	$\frac{0.12}{0.07-0.17}$	$= x_d''$	$\frac{0.03}{0.01-0.14}$
Salient pole motors and generators (with amortisseurs)	$\frac{1.10}{0.60-1.45}$	$\frac{0.22}{0.13-0.35}$	$= x_d''$ (nearly)	$\frac{0.06}{0.02-0.20}$
Waterwheel generators (no amortisseurs)	$\frac{1.10}{0.60-1.45}$	$0.85\ x_d'$	$\frac{0.50}{0.30-0.70}$	$\frac{0.07}{0.04-0.22}$
Condensers	$\frac{1.80}{1.50-2.20}$	$\frac{0.25}{0.18-0.38}$	$\frac{0.24}{0.17-0.37}$	$\frac{0.08}{0.02-0.15}$

NOTE—x_d' for waterwheel generators $= \frac{0.35}{0.20-0.45}$ (saturated)

x_d' for turbo generators and salient pole machines with amortisseur from Fig. 3 (saturated)

*For test data of specific machines, refer to Bibliography item 14.

†x_0 varies so critically with armature winding pitch that the average values given are not very dependable.

Three-Phase Short Circuit. The reactance to be used to select the proper decrement curve for a three-phase fault, is what is commonly spoken of as the "system reactance" referred to or viewed from the point of fault. It is the reactance used with the decrement curves formerly published. When determining this reactance it is important that synchronous machines be represented by their subtransient* reactances, and also that *all loads other than synchronous be neglected.* Proper account has been taken of these loads for the average system in deriving the decrement curves.

Means for obtaining the system reactance referred to the point of fault, and the corresponding short-circuit current have been fully developed in other papers and are not discussed here.[6-11] In case synchronous machine reactances or time constants are not known Table I and Figs. 4 and 5 may be used as a guide in selecting reasonable values.

The system reactance referred to the point of fault enables the proper decrement curves to be selected which gives the three-phase r.m.s. short-circuit current or kva. in "times normal" at any time after the occurrence of the short circuit, subject to the assumptions previously given.

Line-to-Line Short Circuit. For a line-to-line short circuit the fault current is determined by the use of the standard decrement curves of Fig. 1 or Fig. 2 in the same manner as the three-phase short-circuit current by using a reactance equal to twice the system reactance (the value used for three-phase short-circuit calculation) and multiplying the resulting current in times normal read from the curves by $\sqrt{3}$.

The above rule results from the theory of symmetrical components,[7,8,9,11] assuming the negative phase sequence reactance of the system as measured from the point of fault to be equal to the system reactance x'. Where the negative phase sequence of the system is

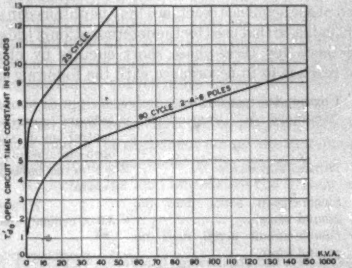

FIG. 5—OPEN-CIRCUIT TIME CONSTANTS OF TURBINE GENERATORS

known, more accurate results may be obtained by using a reactance equal to the sum of the system reactance x_1 and the negative phase sequence system reactance x_2 instead of twice the system reactance x_1. The negative phase sequence reactance of the system is obtained in the same manner as the system reactance x_1 except that the negative phase sequence reactance of the machines is substituted for the subtransient reactance of machines.

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*The "subtransient" reactance is the reactance to use in determining the r.m.s. initial value of the a-c. component of short-circuit current. It includes the effect of induced currents in damper windings, etc., which have a very rapid decrement. The transient reactance on the other hand determines the a-c. component when the rapidly decaying component is neglected while the synchronous reactance is used to determine sustained values of current.

Line-to Ground Fault. For line-to-ground faults, the decrement curves of Fig. 1 and Fig. 2 may still be used, provided that the ratio of the instantaneous values of three-phase to line-to-ground fault currents or the proper value of reactance is known. For this case, the reactance may be taken as three times the system reactance ($3\,x_1$) multiplied by the ratio of the three-phase to the line-to-ground short-circuit currents. Ordinarily the line-to-ground short-circuit current must be calculated by the method of symmetrical components or the reactances for the different sequences used as outlined below. The line-to-ground current at any time will be equal to three times the value of "times normal current" read from the decrement curves using the above values for system reactances.

The value of reactance to be used with the curves for a line-to-ground short circuit may be determined, using the method of symmetrical components, the reactance being taken as the sum of the positive, negative and zero sequence reactances of the system ($x_1 + x_2 + x_0$) as measured from the point of fault. For ordinary calculations it is sufficiently close to use a reactance equal to ($2\,x_1 + x_0$), where x_1 is the system reactance used for three-phase short-circuit calculations and x_0 is the zero phase sequence reactance to the system. The zero phase sequence reactance may be obtained in a manner similar to that used for the determination of the system reactance for three-phase short circuits by changing the reactances in accordance with the following rules.

Ungrounded transformers....... = ∞
*Grounded Δ-*Y* transformers. = ordinary reactance to neutral on *Y* side and open-circuited on Δ side
Neutral impedance............ = 3 × actual impedance in neutral
Single-circuit aerial lines (no ground wires).................. = 3.5 × ordinary reactance
Double-circuit aerial lines (no ground wires).................. = 5.5 × ordinary reactance of double line
Single-circuit aerial lines (copper or aluminum ground wires).. = 2.0 × ordinary reactance
Double-circuit aerial lines (copper or aluminum ground wires).. = 3.0 × ordinary reactance of single line
Cables, three-phase............ = 3 to 5 × ordinary reactance
Cables, single-phase........... = 1.0 × ordinary reactance

For auto-transformers, grounding transformers and other special apparatus, refer to papers and articles which discuss the determination of the zero phase sequence impedance in more detail.[8,12,13]

Minimum Fault Current. For certain relay applications it is sometimes necessary to know the total r.m.s. alternating fault current, *i. e.*, with no d-c. component, rather than the maximum current at any time as given by the standard decrement curves which are based on an asymmetrical fault. The initial r.m.s. alternating (symmetrical) currents for values of system reactance are given in Table II.

TABLE II—INITIAL R.M.S., ALTERNATING (SYMMETRICAL) CURRENT

Reactance on decrement curve	8	10	12	15	20	30	40	50	75	100
Initial symmetrical current	12.3	9.96	8.38	6.88	5.11	3.37	2.52	2.01	1.33	1.0

As the asymmetrical component is negligible after 0.3 second, the r.m.s. alternating (symmetrical) current for any time less than 0.3 second may be obtained by interpolating between the initial value obtained from the above table and the curve values at 0.3 second. Satisfactory results can readily be obtained by sketching in a curve starting from the initial r.m.s. alternating (symmetrical) current and making it tangent to the corresponding standard decrement curve at 0.3 second.

R.M.S. Total Current for Salient Pole Machines without Dampers. Where salient pole machines without damper windings supply the short-circuit current the standard decrement curves give good results after 0.2 second, provided that for these machines the equivalent subtransient reactances are obtained from the known transient reactances by means of the relation given in Fig. 3, but the initial values indicated by standard decrement curves may be 30 per cent too high. If more accurate values of initial current are desired, it is necessary to make a separate calculation of the system reactance using the actual value of the subtransient reactance of the salient pole machines without damper windings instead of the equivalent value obtained from the transient reactance by means of the relation given in Fig. 3. A curve may readily be sketched between the new initial point and the 0.2 second point on the standard decrement curve corresponding to the equivalent subtransient reactance of Fig. 3.

The subtransient reactance of machines without damper windings has generally been taken as equal to the transient reactance. However, there is an appreciable subtransient component in the short-circuit current of such a machine due to the damping action of the pole rivets and other closed circuits and also due to the rapidly disappearing effects of saturation. The test data available indicate that the subtransient reactance of a machine without damper windings should be taken as about 85 per cent of the transient reactance.

Limitations of the Standard Decrement Curves. There

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*Replace by circuit which permits current to flow from *Y* side to ground but not on Δ side.

are certain limitations in the application of standard decrement curves that should be recognized. The decrement curves with the supplementary methods will give safe values for circuit breaker application, except possibly in unusual cases. However, for relay application, it is frequently insufficient to know that the current does not exceed a particular maximum value but instead it is important to know the minimum current values under given fault conditions. In deriving the curves, it was assumed that the system could be replaced by a single equivalent generator. If the initial currents supplied to the fault by the several machines are not in proportion to their ratings, then the division of the sustained currents will not be proportional to the initial current division. Furthermore, the total current will actually be considerably smaller, for example, if a large part of the initial short-circuit current is supplied by a machine whose rating is a small fraction of the total kva. the curve may give a sustained value that is more than 100 per cent too high. It has also been assumed that the system is fully loaded, so that, if the decrement curves are used in the case of a system which is only partially loaded, it is evident that the sustained current will be less than the values given by the curves. It has further been assumed that all of the generated e.m.f.'s are in phase. Departures under operating conditions from this assumption result in smaller values of current.

When voltage regulators are used there will be a tendency to increase the current over that given by the decrement curves. For the first 0.2 second the fault current will ordinarily be unaffected but the sustained current may be increased in proportion to the ratio of the maximum exciter voltage to the nominal exciter voltage corresponding to rated voltage and kva. at 80 per cent power factor. For ordinary excitation systems, *i. e.*, where quick response systems are not used, the sustained currents may be increased as much as 50 per cent by the action of the voltage regulators.

In general, for systems whose characteristics differ radically from the assumptions upon which the curves are based, it may be necessary to use the more accurate methods of calculation such as described by W. C. Hahn, and C. F. Wagner and S. H. Wright.[2,3]

Acknowledgments

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Appendix I

Derivation of Curves

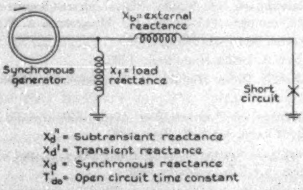
In deriving one set of decrement curves to represent the decrement of a large number of different systems, it it necessary to assume a typical system as a basis for the curves. The typical system assumed is shown in Fig. 6. The value of the load impedance is that which corresponds to full load at 80 per cent power factor on the system.

The initial r.m.s. value of short-circuit current is usually calculated for a system assuming that the system is operating at no-load and that the terminal voltage of each machine on the system is unity. However, the decrement curves apply to a fully loaded system. Thus the decrement curves must include the effect of system load though based on a reactance corresponding to the no-load condition. In order to properly include the effect of load in the decrement curves the amount of internal and external reactance in the system must be known. Neglecting the load impedance the system reactance x_1 is

$$x_1 = x_d'' + x_b \tag{1}$$

For the previously published decrement curves[1] it was assumed that all of the reactance was in the generator for values of x_1 less than 15 per cent. For values of x_1 greater than 15 per cent, 15 per cent was assumed to be in the generator and the rest to be external. This assumption was intended to represent the average system condition and is believed by the authors to be as accurate an assumption as can be made. The same assumption is, therefore, made for the present decrement curves.

The initial r.m.s. a-c. component of short-circuit current may most easily be calculated by employing the superposition theorem. This theorem applies to a network composed of impedances which are independent of the voltage across them and the current through them. It states that the sum of the currents at any point in the network due to two or more sets of terminal conditions applied one at a time, is equal to the current at that point with all the terminal conditions applied together. In the present case, the curves are calculated for a three-phase symmetrical short circuit. By proper conversion factors, (given in the body of the paper, Figs. 1 and 2) the line-to-line and line-to-ground short-circuit currents may be obtained from the three-phase short-circuit curves. The symmetrical three-phase short circuit is assumed to take place on an unloaded feeder which is at normal voltage. In order to satisfy this condition before short circuit and to supply the load on the bus to which the feeder is connected, normal voltage must be applied in series with the short circuit and sufficient voltage applied behind the reactance of the generator to produce normal voltage at the feeder bus. The current which flows under this condition is the load current, no current being in the feeder to be faulted.



X_b = external reactance
Synchronous generator
X_f = load reactance
Short circuit
X_d'' = Subtransient reactance
X_d' = Transient reactance
X_d = Synchronous reactance
T_{do}' = Open circuit time constant

FIG. 6—TYPICAL SYSTEM

All these voltages are now removed and a voltage equal to normal but opposite in direction is applied at the point of fault to determine the short-circuit current flowing in each branch of the network. The sum of the short-circuit current and the load current in each branch is the total current in that branch under short-circuit conditions.

In the derivation of the curves only the current in the faulted feeder is required. Since this feeder is unloaded the subtransient current under short circuit has no-load component and can be calculated in the following way. The load impedance is combined in parallel with the generator subtransient reactance and added to the external reactance to the point of fault. The reciprocal of this total reactance is the current desired since normal (unity) voltage is applied at the point of fault to determine the short-circuit current.

Let i'' = subtransient current in the fault

then

$$i'' = \frac{1}{x_b + \dfrac{x_d'' \, x_f}{x_d'' + x_f}} \qquad (2)$$

The transient a-c. component may be calculated in the same manner by replacing x_d'' by x_d'.

Let i' = transient current in the fault

then

$$i' = \frac{1}{x_b + \dfrac{x_d' \, x_f}{x_d' + x_f}} \qquad (3)$$

The load impedance, x_f, should include both resistance and reactance to represent the load, but in the present case, the inclusion of the resistance affects the result less than 1 per cent, so that x_f is taken as a pure reactance of 1.66 representing the reactance part of an 0.80 power factor load. For the 8, 10, 12 and 15 per cent curves $x_b = 0$. For the rest of the curves, $x_b = x_1 - 15.0$ in per cent. Values of i'' calculated from equations (2) and (3) are given in Table III. Values of x_d' corresponding to assumed values of x_d'' are calculated from the relation.

$$x_d = 1.4 \, x_d'' + 2.0 \text{ in per cent} \qquad (4)$$

which is based upon a large amount of data on turbine generators and salient pole synchronous machines with amortisseurs.

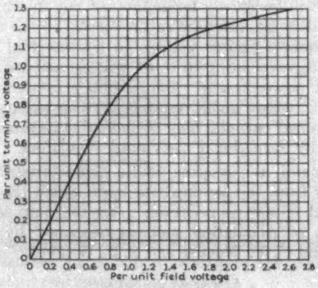


FIG. 7—SATURATION CURVE

The sustained currents (i) are calculated from the Potier diagram, correcting for saturation by adding the difference in excitation taken from the actual no-load saturation curve and that taken from the air-gap line to the excitation obtained neglecting saturation. The saturation curve used is given in Fig. 7. The value of Potier reactance was taken as 1.3 times the subtransient reactance of the machine, as representing an average condition. The load resistance was included in calculating the excitation as it has an appreciable effect. These values of (i) are also given in Table III.

The short-circuit time constant of a machine without saturation is given by Park and Robertson as:

$$T_d' = T_{do}' \frac{x_d' + x_e}{x_d + x_e} \tag{5}$$

In the present case, the external reactance x_e is to be taken as x_a in parallel with x_f.

Oscillograms of short circuits on actual machines show little departure from a true exponential after the effects of amortisseurs and the d-c. component have died away. Therefore, the effects of saturation, if any, are included in the subtransient effect. Inasmuch as the transient reactance is found by extrapolation, back to zero time, from the latter part of the curve, no correction for saturation need be made on the above time constant.

TABLE III

x_e in per cent	8	10	12	15	20	30	40	50	75	100
x_b in per cent	0	0	0	0	5	15	25	35	60	85
x_d'' in per cent	8	10	12	15	15	15	15	15	15	15
x_d' in per cent	13.2	16.0	18.8	23.0	23.0	23.0	23.0	23.0	23.0	23.0
i''	13.1	10.6	8.9	7.3	5.3	3.5	2.6	2.0	1.35	1.01
i'	8.2	6.8	5.9	5.0	4.0	2.84	2.20	1.81	1.24	0.95
i	1.86	1.90	1.92	1.97	1.85	1.61	1.44	1.29	1.01	0.83
T_d'	0.57	0.69	0.82	1.00	1.16	1.43	1.64	1.80	2.11	2.31

TABLE IV

X_e in per cent	8	10	12	15	20	30	40	50	75	100
time in seconds										
0.0833	21.4	17.3	14.5	11.9	8.73	5.79	4.28	3.35	2.23	1.69
0.05	16.6	13.3	11.2	9.37	6.92	4.68	3.49	2.80	1.87	1.40
0.10	12.3	10.2	8.66	7.20	5.56	3.82	2.90	2.34	1.57	1.19
0.20	8.03	6.94	6.07	5.19	4.18	2.98	2.33	1.91	1.32	1.01
0.40	5.18	4.83	4.44	4.02	3.39	2.56	2.06	1.72	1.21	0.935
0.70	3.73	3.72	3.60	3.44	3.00	2.36	1.94	1.64	1.18	0.92
1.00	2.97	3.09	3.08	3.06	2.74	2.22	1.86	1.59	1.15	0.91
1.5	2.32	2.48	2.54	2.63	2.42	2.04	1.75	1.52	1.12	0.89
2.0	2.06	2.18	2.26	2.37	2.22	1.93	1.67	1.46	1.10	0.88
2.5	1.94	2.05	2.10	2.20	2.09	1.83	1.61	1.42	1.08	0.87
3.0	1.90	1.97	2.02	2.11	2.01	1.76	1.56	1.39	1.06	0.86

The open-circuit time constant T_{do}' is assumed as five seconds. The formula for the curve connecting transient and sustained current is then

$$i_t = i + (i' - i)\, e^{-\frac{(x_d + x_e)}{(x_d' + x_e)} \frac{t}{5.0}} \tag{6}$$

The time constant assumed for the subtransient component of current for all the curves, 0.05 second, is an average value corresponding to $T_{do}' = 5$ sec. The formula for the subtransient component of current is

$$i_{st} = (i'' - i')e^{-\frac{t}{0.05}} \tag{7}$$

The sum of the subtransient component (7) and the current from equation (6) is the r.m.s. a-c. component (symmetrical part).

The curves are plotted for the maximum possible offset or d-c. component. The representative time constant assumed for this component is 0.15 sec. Then the d-c. component at any time is

$$i_{dc} = \sqrt{2}\, i'' e^{-\frac{t}{0.15}} \tag{8}$$

The square root of the sum of the squares of this and the r.m.s. a-c. part is the r.m.s. total current, values of which are given in Table IV. It will be noticed that instead of values for zero time, values at $t = 0.0083$ second are given, corresponding approximately to the current at the end of ½ cycle, inasmuch as the value at zero time has no practical significance.

ERROR CAUSED BY ASSUMPTION OF EQUIVALENT SYSTEM

In some cases, the decrement is wanted for a short circuit on a bus to which generators as well as a tie line from a large system are connected. In most cases, the current from the generators can be separated from the current from the large system and the decrement curves applied separately. This gives fairly accurate results. In case a reactance is between the bus and the short circuit, separation of the currents may be difficult. An approximate method which may be used is to apply the decrement curves to the generator and tie line currents separately as before and add them. This results in less error than applying the decrement curves to the total short circuit, but may still be considerably in error. It is relatively difficult to predict the actual error in a large number of cases, such as, the division of short circuit between several machines not on the same bus. Better approximation in cases of this kind will be given in other papers.[2,3]

ASSUMED DISTRIBUTION OF LOAD

The decrement curves are based upon the assumption of a short circuit occurring upon an unloaded feeder as shown in Fig. 6. That this assumption is not productive of a large error is evident from the following:

Since zero external reactance is assumed for the first four curves, for these curves the division of the load between the faulted feeder and the other line feeders is immaterial, as the fault is in effect a generator bus fault with all the load connected directly to this bus. For the rest of the curves, the assumed external reactance is high enough in most cases so that only a small proportion of the total load can be carried through it on account of regulation. More exact methods are given in other papers[2,3] for cases in which it is believed that the effect of load position should be taken into account more accurately.

ALTERNATE FORMS OF DECREMENT CURVES

In Fig. 1, the decrement curves are replotted with system reactance as abscissas, number of times full load current as ordinates, and time in seconds as the parameter. In some cases, the choice of time is somewhat arbitrary while the system reactance is calculated, so that this form of presentation may be more convenient. The formulas for using the curves are the same as in Fig. 2.

Appendix II

SAMPLE PROBLEMS

The following numerical example is given to illustrate the way in which the general decrement curves are to be used. Let it be assumed that the following data have been obtained either analytically or by use of a calculating board and that the system voltage is 66 kv.

The total connected synchronous capacity is 5,500 kva.

The initial r.m.s. alternating component three-phase short-circuit current is found to be 480 amps. or 55,000 kva.

Initial r.m.s. alternating component of line-to-ground short circuit 570 amperes or 65,000 kva.*

The system reactance is then

$$\frac{5{,}500}{55{,}000} \times 100 = 10 \text{ per cent}$$

The three-phase short-circuit current at 0.3 sec. is desired.

Referring to Fig. 2 and following along the 10 per cent reactance curve to 0.3 second time it is found that at this time the short-circuit current or kva. is 5.6 times normal. Thus the short-circuit kva. at 0.3 sec. is

$$5.6 \times 5{,}500 = 30{,}800 \text{ kva.}$$

Since the short circuit occurred on a 66-kv. line, the corresponding short-circuit current is

$$\frac{30{,}800}{66 \times \sqrt{3}} = 269 \text{ amperes}$$

If the equivalent open-circuit time constant of the system is 3.0 seconds instead of 5.0 seconds, then the time to be used with the curves corresponding to 0.3 second is in accordance with the formula given on the curve sheet.

$$0.3 \times \frac{5.0}{3.0} = 0.5 \text{ sec.}$$

For this value of time a value of 4.3 times normal is read from the curve, and the three-phase short-circuit kva. at 0.3 second becomes

$$4.3 \times 5{,}500 = 23{,}600 \text{ kva.}$$

For a line-to-line short circuit the reactance to be used with the curves is twice the system reactance used to find the three-phase short-circuit kva. It is

$$10 \text{ per cent} \times 2 = 20 \text{ per cent}$$

This reactance at 0.5 sec. on the curve of Fig. 2 gives 3.3 times normal, which when multiplied by $\sqrt{3}$ is 5.72 times normal. The magnitude of the line-to-line short circuit at 0.3 second is then

$$5.72 \times 5{,}500 = 31{,}500 \text{ kva.}$$

The line-to-ground short-circuit reactance is found from the formula on the curve sheet as

$$\frac{3 \times 65{,}000}{55{,}000} \times 10 = 35.5 \text{ per cent}$$

The line-to-ground short-circuit kva. is found by interpolation for 0.5 sec. to be 2.2 times normal which multiplied by 3 gives 6.6 times normal as the line-to-ground short-circuit kva. at 0.3 sec., *i. e.*,

$$6.6 \times 5{,}500 = 36{,}300 \text{ kva.}$$

Since this fault occurred on a 66-kv. line the line-to-line ground short-circuit current at 0.3 sec. is

$$\frac{36{,}300}{66 \times \sqrt{3}} = 317 \text{ amperes}$$

As a second illustration, a system having only waterwheel generators without amortisseur windings will be considered. The system has a total synchronous capacity of 8,000 kva., a transient reactance of 38 per cent, of which 15 per cent is external, and an open-circuit time constant of 5 seconds. The three-phase short-circuit current at 0.1 and 2.0 sec. is desired.

The curves of Fig. 2 give the decrement after 0.2 second quite accurately if the proper transient reactance is used to select the curve. Referring to Fig. 3 the subtransient reactance corresponding to a machine transient reactance of 23 per cent is 15 per cent. The system equivalent subtransient reactance is

$$15 \text{ per cent} + 15 \text{ per cent} = 30 \text{ per cent}$$

Using the 30 per cent curve of Fig. 2, the times normal value at 2.0 sec. is found to be 1.9. Thus the three-phase short circuit is

$$1.9 \times 8{,}000 = 15{,}200 \text{ kva.}$$

By referring to Table I it is found that the subtransient reactance of a waterwheel generator without an amortisseur is about 85 per cent of its transient reactance. So this equivalent waterwheel generator machine has a subtransient reactance of

$$23 \times 0.85 = 19.6 \text{ per cent}$$

Adding to this the external reactance the system subtransient reactance which determines the initial short-circuit current is obtained.

$$19.6 + 15 = 34.6 \text{ per cent}$$

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*The short-circuit kva. for a line-to-ground fault is equal to $\sqrt{3}$ times the product of fault current and system voltage.

By interpolation between the curves of Fig. 2, the initial three-phase short circuit is found to be 4.8 times normal. From the curve sketched in connecting this value to the 30 per cent curve at 0.2 sec. it is found that the three-phase short-circuit kva. at 0.1 sec. is 3.7 times normal. It is

$$3.7 \times 8{,}000 = 29{,}600 \text{ kva.}$$

The short-circuit currents may be found as in the preceding problem.

NOMENCLATURE

x_d = direct axis synchronous reactance (unsaturated)
x_d'' = direct axis subtransient reactance (saturated)
x_d' = direct axis transient reactance (saturated)
x_2 = negative sequence reactance
x_0 = zero sequence reactance (saturated)
T_{do}' = open-circuit time constant (unsaturated)

Discussion

R. G. Lorraine: The standard decrement curves presented by Messrs. Hahn and Wagner are based upon the reduction of the actual system subjected to fault to a system consisting of a single equivalent generator and an external reactance. Relations are assumed between the reactances of this equivalent system such that, when the total reactance of the system to the point of fault is known, all of the reactances necessary to determine the decrement curve are specified. In any system to which the decrement curves are applied, the relations between the reactances will probably not be those which were assumed for the standard decrement curves. Therefore, it seemed desirable to investigate the effect of changing these relations.

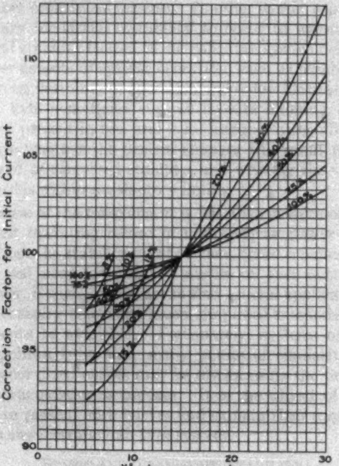

Fig. 1

Based upon the method of derivation given in Appendix I of the paper, three families of curves were developed. One family of curves, Fig. 1, shows the per cent initial current for a specific system based upon the initial current of the corresponding general decrement curve system. The variation in this initial current depends upon the relative magnitudes of the subtransient reactance of the equivalent generator and external reactance which make up the system subtransient reactance. This family of curves is used to determine the departure in the initial current.

Figs. 2 and 3 are of somewhat different character. They show values of transient and synchronous reactance respectively corresponding to a given subtransient reactance, which the equivalent generator must have in order to have approximately the same transient and sustained components of short-circuit current as the related standard decrement curve. An examination of these curves shows that there is a large number of system conditions in addition to those assumed which will produce the same transient and sustained short-circuit current as the standard decrement curves.

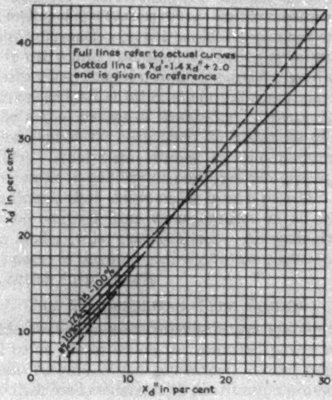

Fig. 2

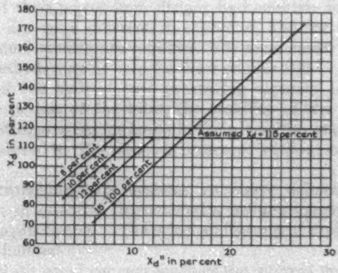

Fig. 3

An example will show how these curves may be used. Assume that the system under consideration has a reactance to the fault of 20 per cent; and that the equivalent generator has a subtransient reactance of 10 per cent, a transient reactance of 20 per cent, and a synchronous reactance of 100 per cent. This gives an external reactance of 10 per cent. The standard decrement curve for 20 per cent system reactance gives an initial current of 5.3 times normal. Referring to Fig. 1 the initial current is 96.5 per cent of this. From the curve of Fig. 2 the transient reactance is found to be 17.6 per cent to fit the standard decrement curve. As 20 per cent is the value for the system, and the external reactance is 10 per cent, the transient component of the standard decrement curve starts about 7 per cent too high. From the synchronous reactance curve of Fig. 3, it is found to be 91 per cent to fit the standard decrement curves. Therefore, the sustained current read from the decrement curve is about 8 per cent too high. Now, it has been found that the decrement curve reading for the specific system chosen is 3.5 per cent high initially, about 7 per cent high at 0.3 seconds and 8 per cent high at the sustained value. Therefore, the currents may be reduced by these percentages to give a more exact result.

A further inspection of the curves will show that while the general decrement curves are developed for a specific system based upon assumed relations between the various reactances, there are many other sets of relations between reactances which will satisfy the standard decrement curves exactly throughout part of their length and with reasonable accuracy throughout the remainder. Thus more confidence can be placed in the curves than one would suppose from an inspection of the assumptions made in their derivation.

Generalized Stability Solution for Metropolitan Type Systems

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Synopsis.—*This paper presents general stability curves by means of which the transient stability during faults may be determined for any metropolitan type system. The results are obtained as the permissible fault duration in terms of certain indexes of the system and fault. These indexes may be found from a short-circuit study of the particular fault made on a d-c. calculating board or its equivalent. For a given fault only three readings are required to obtain the result.*

INTRODUCTION

THERE has been a feeling on the part of a number of engineers that there is no stability problem connected with the metropolitan type of power system. This feeling is not unwarranted because of the excellent operating record of metropolitan type systems.

The fact that the stability of the systems has not been a problem, however, does not necessarily mean that the study of stability would not be profitable. It is possible, for example, that these systems have been laid out very liberally because the factors affecting stability have not been very well known and the design is such that either, (1) greater stability than necessary has been obtained, or (2) the same degree of stability might have been secured in a more economical manner. In other words, it is not necessary for actual instability to occur to warrant a study of stability factors in metropolitan system design.

Other aspects of system operation, such as reliability of sources, flexibility of operation and reduction of short-circuit kva. sometimes suggest layouts differing from those at present in use, and it is desirable to be able to evaluate whether the desired changes will result in an increased or decreased stability, and whether the value obtained is within satisfactory limits.

Because of the foregoing, it is felt that in spite of the excellent operating record of metropolitan power systems from the stability point of view, there is nevertheless a real need for stability studies on such systems. However, if such studies are to find general application, it is necessary that very much simplified methods and short cuts be available. Fortunately, systems of the metropolitan type have many points of similarity so that generalized curves may be drawn using the principal characteristics as indexes. It is the object of this paper to present such curves and explain their application to any metropolitan type system.

STATIC AND TRANSIENT STABILITY

Investigation of many cases of so-called static instability has led the authors to the conclusion that *static instability is never experienced in the normal operation of metropolitan type systems.* In all cases in which sufficient information on the actual happenings was obtained it was found that some operating error or apparatus failure had caused a total or partial loss of excitation, excessive load or other abnormal condition, which were the actual causes of instability. Obviously such examples cannot be considered as instances of static instability. Analytical studies by the authors and others of static instability on metropolitan type systems have indicated that this is so far from a possibility that it may be omitted from consideration on all systems coming under this classification. Accordingly, the present paper is devoted entirely to transient stability, that is, action during faults.

THE METROPOLITAN TYPE SYSTEM

In this discussion a metropolitan type system is considered to be one in which the principal power sources are steam turbine-driven generators, located relatively close to their load centers, with distribution provided by a large number of moderate voltage circuits. The power supply to most metropolitan districts is of this character, but several large districts receive a major part of their power from distant hydroelectric sources which are quite different in character, and for these, individual studies are required. In some cases, short cut methods may be used for these studies also.[1] However, if the hydroelectric generating capacity is one-fourth or less of the total, the stability of the other generating sources for faults in their localities may still be analyzed by the methods presented without serious error.

Metropolitan type systems, as above defined, are similar in essential characteristics making generalized studies practicable. The principal features of similarity are as follows:

1. With equal per cent loadings the internal voltages of all generators on the system are practically in phase.
2. Because of the multiplicity of circuits the reactance of the connecting ties holding a generator or group of generators in step with the system does not change appreciably when the fault is cleared.
3. With steam turbine-driven generators and short transmission distances, the generator characteristics,

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

[1] For references see Bibliography.

no longer being affected by the speed of prime mover, line charging current requirements, etc., tend to become fairly uniform in their essential points, such as reactances, inertia and short circuit ratio.

This similarity makes it possible to calculate the stability of a number of hypothetical system layouts, plot the results of these calculations in the form of curves using the few remaining variable quantities as indexes and apply them to specific layouts by determining the values of these quantities for the specific case.

With a given system layout and considering stability under fault conditions, the measure of the stability is the permissible duration of the fault. Therefore, the curves in this paper are arranged to give as the result the permissible fault duration in seconds. Under the assumed conditions the most important factors affecting permissible fault duration are the initial generator output, synchronizing power and severity of fault. These factors may be expressed in a number of ways and in the curves described in the next section, the authors have chosen the method which appears to be the most useful.

GENERAL STABILITY CURVES FOR METROPOLITAN TYPE SYSTEMS

Metropolitan type systems are usually laid out so that the generating units are not connected directly together at their terminals. The generating units may be individual generators, groups of directly paralleled generators in the same station, or the entire station, if all units are tied solidly in parallel. In what follows, the term "generator" will refer to a generating unit as indicated above.

Since a fault which affects all generators equally has little tendency to make them lose synchronism, the most severe fault from the stability standpoint is one which is electrically close to one of the generators. The stability problem is then one of holding this generator in synchronism with the remainder. The synchronizing power between this "faulted generator" and the remaining generators depends upon the connecting reactance between this generator and the others in parallel. This overall reactance includes the reactance of the system as well as the transient reactance of all machines and will be designated by X. The measure of the severity of the fault used by the authors is the fault current flowing from the faulted generator, expressed in number of times rated current modified if necessary by a factor to take account of fault location. Since the margin of stability is reduced with increased initial load, the curves are given for full load on all machines before the fault occurs, and later a factor will be introduced to take account of other values of initial load.

The fundamental premise underlying the use of the generalized curves for stability determinations is that setups giving the same values of the essential indexes will have approximately the same degree of stability. The problem then becomes one of determining the indexes from the actual system setup, and then obtaining the permissible fault duration of the corresponding hypothetical system setup from the general curves. These indexes were chosen in such a manner that they can be obtained from three readings on a d-c. calculating board.

It was necessary in the development of the general curves that minor influences be eliminated from consideration even though some sacrifice in accuracy was involved. Such influences include resistance in the lines and in the fault, and variations in load characteristics. In obtaining the indexes from the d-c. calculating board, only reactances are used, the results being obtained as reactances or reactive components of fault current. This was taken into account in drawing the curves, so that while the resistance effect of the loads was considered in computing the permissible fault duration, it was not considered in computing the indexes. Thus the difference is common to both the actual and hypothetical systems and is thereby compensated. This is equivalent to assuming that the load characteristics of the actual and hypothetical systems are the same.

Figs. 1 and 2 show the general stability curves for generator short-circuit ratios of 1.0 and 0.8 respectively. The two values of short-circuit ratio are used because of the consideration which has been given this factor in metropolitan system design, although the curves show that it has a minor effect on transient stability. Part A of each figure consists of a family of curves covering the range of overall reactance X, plotted in terms of short circuit current I_{FG} from the faulted generator and the permissible fault duration t. Part B is similar except that the power dropped by the faulted generator is used instead of I_{FG}. The latter curves, as will be shown later, provide a means of correction for resistance in the fault, and for other than rated initial load.

Part A of Figs. 1 and 2 applies directly to faults at locations electrically equivalent to the generator terminals, such as faults on feeders, just beyond the feeder breaker. To provide for faults occurring at other locations, a close approximation is secured by *dividing* the value of I_{FG} by a factor read from Figs. 1C or 2C, before entering Figs. 1A or 2A. This location factor is obtained as a function of the ratio of I_{FG} to I_F, the total fault current. This ratio will be denoted by r_{FG}.

METHOD OF APPLICATION

The application of these curves will be demonstrated by an example. Fig. 3 shows schematically a typical metropolitan type system with reactances as indicated. A fault at point M will be considered. A three-phase fault at this location isolates generator A from the remainder of the system for the duration of the fault, and this generator is therefore the one most likely to fall out of step first. Therefore, machine A becomes the "faulted generator," and all other generators including those in the same group with A become the "remaining generators." Fig. 3 under these conditions reduces to Fig. 4. This reduction may be carried out analytically

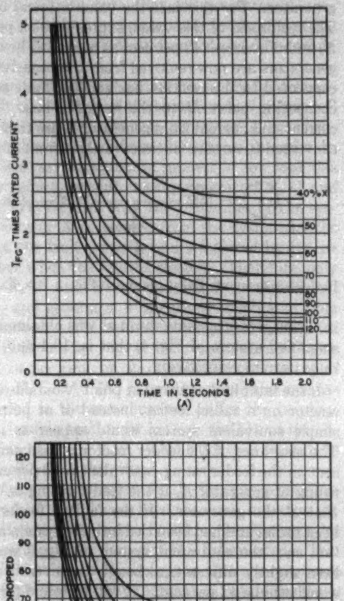

FIG. 1—GENERAL STABILITY CURVES FOR SYSTEMS WITH GENERATORS HAVING SHORT-CIRCUIT RATIO 1.0

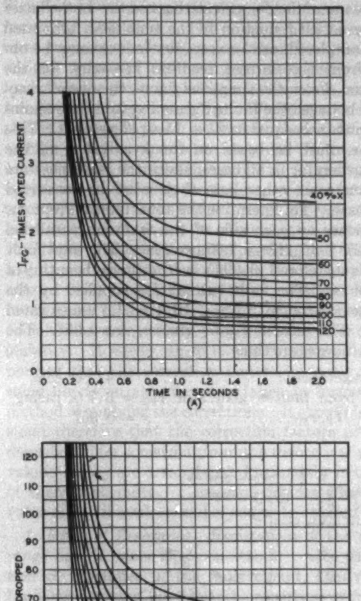

FIG. 2—GENERAL STABILITY CURVES FOR SYSTEMS WITH GENERATORS HAVING SHORT-CIRCUIT RATIO 0.8

A. Principal curve. Short-circuit current I_{FG}, *vs.* permissible fault duration in seconds
B. Auxiliary curve. Per cent power dropped *vs.* permissible fault duration in seconds
C. Location correction factor

$$r_{FG} + I_{FG}/I_F$$

or by the use of the d-c. calculating board. In the latter case, the actual reduction to the simplified form need not be completed, as the necessary indexes may be obtained from the current readings necessary for the reduction. From the simplified figure, the overall reactance X between the faulted generator and the remaining generators is equal to $27 + 15 = 42$ per cent. This reactance must be based on the kva. rating of the faulted generator in all cases, and if the reactances are set up on some other base they must be converted to this base. To determine the total fault current I_F, the branches on each side of the fault are paralleled giving $(27 \times 15)/42 = 9.65$ per cent. The total fault current is therefore $1/0.0965 = 10.4$ times the rating of generator A. The fault current I_{FG} supplied by the faulted generator is $15/42 \times 10.4 = 3.7$ times rated current. The ratio of I_{FG}/I_F gives r_{FG} as 0.357. The essential indexes are thus

Overall reactance X . . . . $= 42$ per cent
Current from faulted generator $I_{FG} = 3.7 \times$ rating
Ratio I_{FG}/I_F, r_{FG} . . . . $= 0.357$

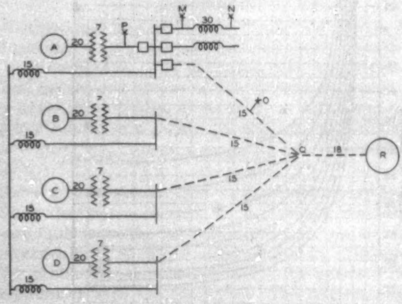

FIG. 3—SCHEMATIC DIAGRAM OF TYPICAL METROPOLITAN TYPE SYSTEM USED TO DEMONSTRATE APPLICATION OF CURVES

Assuming that the short-circuit ratio of generator A is 1.0, the location factor is obtained from Fig. 1C, entering the curve at $r_{FG} = 0.357$ and interpolating for $X = 42$ per cent, giving a value of 0.7. The adjusted value of I_{FG} is then found to be 3.7 divided by $0.7 = 5.3$. The permissible fault duration may now be read from Fig. 1A, for $I_{FG} = 5.3$ (extrapolated) and $X = 42$, and is found to be 0.39 seconds.

If the d-c. calculating board is used, the indexes may be found directly. With the system set up without the fault, generator A is connected to the positive bus, its transient reactance being included as part of the system, and the remaining generators connected to the negative bus in a similar manner. Rated voltage converted to the proper base is now applied between the two buses and the current flowing from generator A is read and converted to number of times rated current for this generator. For this case the value is found to be 2.38. The reciprocal of this value, expressed as a per cent, is X, and of course is 42 per cent as before. The remaining generators are now removed from the negative bus and connected to the positive bus and the fault applied by connecting point M to the negative bus. The fault current and current from generator A are then read and converted to number of times rated current of generator A. This gives the same result as was obtained from the simplified system of Fig. 4, that is, 10.4 and 3.7 times rated current respectively.

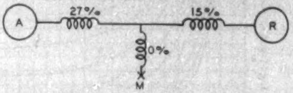

FIG. 4—SIMPLIFIED EQUIVALENT OF FIG. 3 FOR FAULT AT M

If the fault had occurred at point N outside the feeder reactor on a radial feeder, instead of at point M, the simple equivalent system would appear as in Fig. 5. The reactance of the feeder reactor is assumed as 3 per cent on the feeder rating, equivalent to 30 per cent on the rating of generator A, if the feeder rating is one-tenth that of the generator. If the faulted feeder had not been radial and had been tied into the system at the distant end, the equivalent system of Fig. 5 would have been slightly different, although the overall reactance would still be 42 per cent. It will be found for this case that r_{FG} remains the same since it depends only on the reactance branches adjacent to the generators. The total fault current is however greatly reduced owing to the presence of the 30 per cent reactance in series with the fault, and becomes 2.52 times rated current of generator A. I_{FG} is found to be 0.9 times rated current, and when adjusted for location becomes 1.29. When these indexes are referred to Fig. 1A it is found that the point is beyond the range of the curves and the permissible fault duration is over two seconds. Two seconds is about the upper limit of relaying time for most systems and the many variables render it undesirable to carry the curves much beyond this figure.

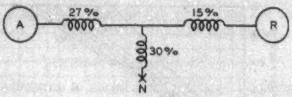

FIG. 5—SIMPLIFIED EQUIVALENT OF FIG. 3 FOR FAULT AT N

If the fault is at some point O so that it will affect all four of the generators in station A more equally, there is a possibility that this entire group may lose synchronism with the remainder of the system. In such a case, the group should be considered as the faulted generator, and a solution should be carried out expressing all indexes in terms of the rating of the group as a base. This solution should be compared with that for generator *A* considered as the faulted generator, and the shorter of the two values of permissible fault duration should be taken as the result.

One more variation will be considered, that of a three-phase fault on the leads of the transformer *A* at point *P* close to its bus. In this case generator *A* must obviously be disconnected from the system in order to clear the fault and the point of interest is how long this fault may persist without causing generators *B*, *C*, and *D*, to lose synchronism with the remainder of the system. Generator *A* is isolated from the system by the fault while the fault persists, and is isolated by its circuit breaker when the fault is cleared. Therefore, only machines *B*, *C*, and *D*, are considered as the faulted generator, and generator *A* is entirely eliminated from the calculations. The equivalent simplified system for this case is shown by Fig. 6, the reactances now being expressed in per cent on the combined kva. rating of generators *B*, *C*, and *D*. From this point the calculations proceed in the same manner as for the previous examples, leading to $X = 94.1$, $r_{FG} = 0.632$ and the adjusted value of $I_{FG} = 1.80$. This gives 0.53 seconds as the permissible fault duration.

If the fault had occurred on the bus the conditions during the fault would have been the same, but after the fault the tie between the synchronizing bus through the bus of generator *A* to the point *Q* is interrupted. This violates the fundamental assumption that the reactance after the fault is essentially the same as before the fault, so that the curves cannot be applied with the same degree of accuracy.

UNBALANCED FAULTS

The preceding analysis applies directly to three-phase faults. In many cases, however, investigations of unbalanced faults are of interest. The method of symmetrical components[2] shows that from the stability standpoint, an unbalanced fault may be represented by an equivalent impedance in series with the fault, the value of this impedance being a function of the negative and zero sequence impedances of the system viewed from the point of fault. These impedances may be found by reducing the negative and zero sequence network either analytically or on a calculating board.

CORRECTION FACTORS

The following influences were not considered in setting up the curves of Figs. 1 and 2:

1. Resistance in the connecting lines and in the fault.
2. Values of initial generator load other than normal rating.
3. Voltage regulators.
4. Systems not capable of reasonably accurate representation by two composite generators.

The correction to take these effects into account in all cases shows that the permissible fault duration is as great or greater than would be indicated if the factors were neglected. It is therefore felt that the curves of Figs. 1 and 2 can be used directly without any correction factor for most work. This view is held because, for most purposes, the limiting cases must form the basis of design. A fault of zero resistance, and operation at full load gives the greatest generator acceleration, and therefore lower indicated stability. Where systems do not reduce to two composite generators, the stability will be as high or higher than when they do. Voltage regulators, of course, increase the stability. Therefore, if the curves are used directly, without correction factors, the error will be on the safe side. It is appreciated, however, that in some cases, the effect of these variations on stability is of interest in system planning and operations, and approximate corrections have been set up to take care of these cases. Each of these variations has been investigated analytically, and in some cases, extensive calculations were necessary to determine their effect on stability. However, since it was considered most important that the correction factors be easily applied, some liberty in accuracy was taken in formulating a method of applying the corrections. It should be understood therefore that the correction factors now to be described differ somewhat from the theoretically correct values. They are arranged approximately in the order of their importance. Correction factors affecting the value of I_{FG}, if used, should of course be applied first.

1. Effect of Resistance. Resistances of the ordinary range have a minor effect on stability, with the exception of resistance in the fault circuit, that is, those branches carrying the greatest proportion of fault current. Due to the large currents in the fault circuit, a relatively small amount of resistance will result in a considerable power loss. This power loss compensates in a large measure the drop in load on the faulted generator due to drop in voltage, so that the tendency to pull out of step is minimized. Both parts *A* and *B* of Figs. 1 and 2 are plotted for faults at the generator terminals. Application of the location factor from part *C* converts a fault at any other location to its equivalent fault at the generator terminals. For each point in part *A* there is a corresponding point in part *B*, from which may be found the amount of power which would have to be dropped by the faulted generator for the equivalent fault at its terminals. If the amount of power dropped is reduced by the presence of resistance in the lines or fault, the effect would be nearly the same regardless of fault location. Hence the result may be found by reducing the equivalent amount of power dropped by the amount of the resistance losses taken by the faulted generator, and reading the corrected value of fault duration from the proper reactance curve of part *B*. In the example previously considered for a fault at *M* with $I_{FG} = 5.3$ and $X = 42$, a clearing time of 0.39 seconds is indicated by part *A*. The same clearing time must be indicated by part *B*, hence, for $X = 42$ and $t = 0.39$, it is found that 100 per cent power would have to be dropped by the generator during the fault if the fault were at its terminals. The curves have been drawn by plotting this power as per cent of the generator kilowatt rating, which is 85 per cent of the kva. rating. Now suppose that generator *A* and its transformer have a total resistance of 1.5 per cent on their rating. With 3.7 times full load fault current flowing, the I^2R loss in the generator and transformer is $3.7^2 \times 1.5$ per cent or 20.5 per cent of the generator kva. rating, or 20.5 per cent divided by $0.85 = 24$ per cent of its kilowatt rating. Then, instead of the 100 per cent load dropped if the generator and transformer had zero resistance, the equivalent load dropped is 100 per cent minus 24 per cent or 76 per cent. The permissible fault duration, considering the effect of resistance, is then found from part *B* for $X = 42$ per cent, and power $= 76$ per cent, and indicates a permissible fault duration of 0.6 seconds.

In the example given above, the resistance in only the generator branch of the equivalent circuit was considered. The power loss was then easily determined, and was all supplied by the faulted generator. If the fault is located at some point on the system where there are a number of paths in multiple for the flow of fault current, the power loss may be determined by adding the I^2R loss in the various branches between the fault and the generator. The currents in the branches may be obtained by measurement of current in corresponding branches of the d-c. calculating board. They may then be squared and multiplied by the per cent resistance of the branches, and summed up to get the total power loss due to the fault.

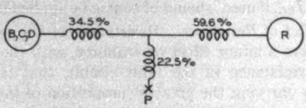

Fig. 6—Simplified Equivalent of Fig. 3 for Fault at P

If resistance in the fault itself is to be considered, the power loss in the fault must be apportioned between the faulted generator and the remaining generators. An approximate method of doing this is to multiply the total loss in the fault by the ratio r_{FG}, and add this value to the loss in the generator branch of the fault circuit, if this is considered. As an example, for a fault at *M* assume ½ per cent resistance in the arc of the fault, in addition to the resistance of the generator and transformer. With 10.4 times normal current, $10.4^2 \times 0.5$ or 54 per cent of generator kva. in power is created in the arc. This is equivalent to 54 per cent divided by 0.85 or 63.5 per cent of generator kilowatt rating. Multiplying by r_{FG}, 63.5 per cent $\times$ 0.357 = 22.5 per cent power in the fault, taken by the faulted generator. The power dropped is 100 per cent for $X = 42$ per cent and $t = 0.39$ seconds and when 24 per cent due to generator and transformer resistance and 22.5 per cent due to fault resistance are subtracted, 53.5 per cent remains as the equivalent power dropped. This value of power dropped is seen from Fig. 1B to give a permissible clearing time in excess of two seconds.

2. Initial Generator Load. For values of initial generator load other than 100 per cent of kilowatt rating, first find the value of kilowatts dropped, corresponding to *X* and *t* for rated initial load. Multiply this by the ratio of initial load to full load, and read the corrected clearing time from part *B*, for the curve corresponding to *X*. Example: with $X = 42$ per cent, $t = 0.39$ seconds, 100 per cent kilowatts is dropped; if the initial load had been 75 per cent instead of 100 per cent, the approximate fault duration may be read as $t = 0.61$ for kilowatts dropped = 100 per cent $\times$ 0.75 = 75 per cent and $X = 42$ per cent.

3. Voltage Regulators and Quick Response Excitation. Voltage regulators with a moderate rate of excitation response will give a certain amount of improvement in stability over the values given by Figs. 1 and 2. The precise amount is difficult to determine, but from calculations made, assuming a rate of response sufficient to maintain constant flux in the generator field, a reasonable idea of possible improvement may be obtained. The results of these calculations when compared with the standard curves without voltage regulators show that, over most of the range, the improvement can be approximated by multiplying the value of I_{FG} by 0.85 before entering the curves of Figs. 1A and 2A.

4. Different Types of Systems. The curves of Figs. 1 and 2 are theoretically applicable only to systems essentially connected in star form, typified by the synchronized at the load method of connection. This is because, as will be discussed later, the simplified systems which were analyzed were of the star form. However, analyses were also made of the ring connection, which represents the maximum possible deviation from the star connection, and the results of these analyses did not differ very greatly from those of the corresponding star systems. Hence, although certain variations will occur in special cases, no correction factor to take account of system connection is required for most work.

Part II

METHODS OF CALCULATION

In previous sections of this paper the general stability curves for metropolitan type systems were presented and the method of their use explained. The method of their derivation will undoubtedly be of interest to many engineers and the following discussion gives the essential features.

In recent years the concentration of large amounts of generating capacity on a single bus required by the larger metropolitan loads produced excessive amounts of short circuit capacity and greatly increased the danger of a system outage from the effects of a single short circuit. This led many utility engineers to consider other arrangements of system connections which would result in greater reliability with reduced short circuit kva., giving rise to the use of "synchronized at the load" and the so-called "loose linked" systems, including the use of a synchronizing bus connected to the individual generators through synchronizing reactors. To determine whether or not the new schemes would be stable, numerous stability studies of particular systems were made.[3,4] The results indicated that certain factors were of prime importance, but others of relatively much less. The similarity of the results indicated that satisfactory accuracy could be obtained with a great saving in labor, if the principal factors were included in a general study and the less important factors neglected.

The "synchronized at the load" scheme, if carried to the limit, would result in a star connection between the various machines of the system, as shown in Fig. 7. From the standpoint of a general study, the star connection is particularly advantageous, because it is the easiest to use in stability calculations. Also, for the same overall reactance, experience and calculations indicated that it would be the least stable, so that any error introduced would be on the side of safety, as the permissible fault duration indicated would be less than that which could actually be permitted. As a result, the star-connected system of Fig. 7, was chosen as the type of system to make the subject of a general investigation. Later other connections were investigated to determine the effect on the stability of the type of connection employed.

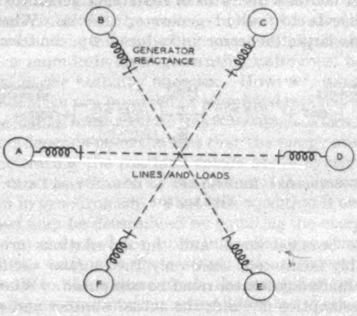

Fig. 7—Schematic Diagram of Hypothetical System, Used for Calculations

An effort was made to choose the constants of the hypothetical system as near to the average of actual operating systems as possible. Typical machine constants and values of system reactance indicated by experience were taken. Because of the consideration which has been given in recent years to the subject of generator short-circuit ratio, two values of this constant were chosen, namely, 1.0 and 0.8. The short-circuit ratio affects the other constants of the machines to some extent, and this was taken into account in choosing the typical values of transient reactance and machine inertia. The following values were used:

Short circuit ratio	1.0	0.8
Transient reactance	20 per cent	25 per cent
Inertia	8.5 kw-sec./kva.	8.25 kw-sec./kva.

The lower value of short-circuit ratio will give a smaller machine, physically, which gives rise to the higher value of transient reactance and the lower value of inertia.

For a loosely coupled system to operate economically, each generating unit must supply its own load and the transmission of power from one generating area to another must be reduced to a minimum. Hence, it is here assumed that each generator carries its own load and that before the occurrence of a fault there is no transfer of power from one generating area to another. Furthermore, the load on a system of this type will generally consist of a large number of relatively small loads, so that the effect of the load in supplying short-circuit current is reduced to a relatively low value, and the load may be approximately represented by means of shunt impedances. Therefore, for the purposes of this analysis, all the load on a generating unit was considered as made up of shunt impedances, consisting of resistance and reactance, of an amount to give full load at the generator terminals at an average system power factor of 0.85. The location of the loads depends to a considerable extent upon the actual arrangement of the system and the distribution voltages, since a system distributing at generator voltage will generally have the load quite close to the generator, whereas a system stepping up to considerably higher voltages will have the load located more distant from the generator terminals. Although the actual location is not a factor of great importance in determining the stability, it is nevertheless desirable to satisfy as well as possible the conditions of all types of systems. What the authors consider to be the best compromise compatible with reasonable facility in calculation is to consider the load to be uniformly distributed along the lines connecting a generating unit to the others. Under the assumptions that each generating unit is operating at full load, 0.85 power factor, and that no load is being transferred from one generating area to another, the distributed admittance which will result in the required load on each generator may be easily determined. The method of treatment is quite similar to that of a line having distributed capacitance, so that each section of line may be represented by an equivalent Π or an equivalent T section, or it may be represented by the corresponding group of general circuit constants. Where the reactance between the terminals of the generators and the common star point is small, it is reasonable to distribute the load over the entire connecting link, but when this reactance becomes quite large, it does not seem reasonable to carry an appreciable portion of the load near the star point. Consequently, when the reactance between the terminals of a generator and the star point exceeded 40 per cent, the load was distributed only over the first 40 per cent of the reactance from the generator terminals.

On the actual system, faults may occur at almost any location. However, except at the generating station and at certain major substations a single short circuit will usually involve only one or perhaps two feeder circuits. Because of the impedance of the feeders, faults at intermediate locations will be much less severe than short circuits occurring at a point which affects all of the feeders to about the same extent. It can be shown, however, that a fault on any single feeder of a group of feeders connecting one generating unit to the remainder of the system is equivalent to a fault on all of the parallel feeders in series with an appropriate value of impedance, or reactance if series resistance has been neglected, as was done here. Thus, to take account of intermediate faults on single feeders, studies were made of faults at different locations on the lines of the hypothetical system, with a fault reactance connected between the point of fault and neutral. Various values of reactance were used, increasing from zero to an amount which would allow the system to remain in synchronism if the fault remained on the system up to two seconds.

With a hypothetical system set up in this manner, calculations were required of many different faults corresponding to variations in three principal factors, namely, the overall reactance between the faulted generator and the remainder of the system, the fault location, and the reactance in series with the fault.

The calculations were carried out in a conventional manner, making several of the simplifying assumptions which would usually be made in the corresponding individual study. Some of the assumptions have already been mentioned, but for completeness they will all be summarized here.

1. Resistances in generators and lines neglected.
2. Load considered as a distributed shunt admittance.
3. Load before fault, rated load on each machine.
4. Generator constants as tabulated above, except as given in 11.
5. Short-circuit losses in generators neglected.
6. All short circuits assumed to involve all three phases.
7. Problem solved as a two-machine problem.
8. Constant prime mover input assumed.
9. Damping factors neglected.
10. Combined inertia of system and "remaining generators" considered infinite.
11. Effect of cross-flux neglected.
12. Effect of voltage regulators neglected.
13. Six equal generating units assumed for purpose of calculation.

Certain of these assumptions may require further amplification. No. 7 indicates that the problem is solved as a two-machine problem. With a star-connected system as shown in Fig. 7 a short circuit on the feeders of any one machine will produce the same effect on all of the remaining machines, so that they will react similarly, and may, therefore, be considered as a single large machine in the calculations. Thus the problem to be solved reduces to the one shown in Fig. 8, comprising two generators connected by the system network which consists of the line with its shunt load and the fault reactance.

No. 8 states that the prime mover input has been assumed constant. This is equivalent to stating that the governors do not operate to change the steam valve position during the critical period of the fault.

No. 9 states that damping factors are neglected. Such damping factors would include the losses produced in the rotor by the asymmetrical direct current component of the short-circuit current, and the damping effect caused by the currents induced in the wedges and solid iron rotor.

No. 10 states that the combined inertia of the system and the remaining generators is considered infinite. In general the remaining generators will total several times the inertia of the faulted generator. Also the inertia effect of the load has been neglected by assuming the load as a shunt admittance, and including it will still further increase the ratio of remaining generator effective inertia to faulted generator inertia. When the ratio is large, the error introduced by considering it infinite becomes small and the calculations are considerably facilitated, since only the angular oscillation of the faulted generator must be computed. Whenever this assumption is valid, the actual number and rating of the machines comprising the remaining generator is of importance only insofar as the overall system reactance is affected.

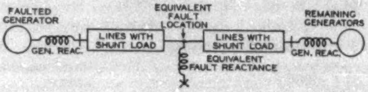

Fig. 8—Simplified Equivalent of Fig. 8 for Fault Near Generator *A*

No. 11 states that cross-flux was neglected. This means that only the flux in the direct axis of the field poles has been included in the calculations.

No. 12 states that the effect of voltage regulators was neglected. Many metropolitan type systems operate without automatic generator voltage regulators so that the short-circuit current causes a decrease in machine voltage and a reduction in stability. To include such systems the effect of decrement was considered, as described below. Later, calculations were made under the assumption of a quick response excitation system of sufficient rate of response and ceiling voltage to maintain constant flux in the fields, and the results made the subject of an approximate correction factor previously described.

No. 13 states that six generating units were assumed for purposes of calculation. As stated above, under the discussion of No. 10, a variation in this number is not of great importance except as it produces a change in the overall reactance.

It has been found by experience that calculations made with the above assumptions yield results which are pessimistic, that is, they indicate that the permissible fault duration is less than that which may actually be experienced without loss of synchronism.

The calculations for the duration of the fault were carried out by a step-by-step method, which assumes that the power output of the generators remains constant for a small interval, from which the acceleration and departure from normal frequency may be calculated, giving a new value of output power which is assumed to remain constant during the second interval, etc. At the same time the field circuit is being demagnetized by the action of the short-circuit current, and this was taken into account, calculating the decrement also step-by-step.

The permissible duration of the fault could have been determined by assuming that the fault was cleared at a certain time, and then making use of the conditions following the fault to determine whether or not the system was stable for the assumed time of fault clearing. This method is the one which must usually be used if three or more individual generating units must be considered in the stability analysis. However, when only two machines are involved, a simplified method known as the equal area method may be used. This latter method depends upon the fact that the energy absorbed in accelerating the machine rotors above synchronous speed must be restored to the system if the system is to remain in synchronism. That this condition is or is not satisfied may be determined by equating the energy absorbed by the generator rotors in accelerating during the period the fault is on the system to the energy which is given up to the system from the generator rotors in returning to synchronous speed after the short circuit has been cleared. This energy for each machine is proportional to the integral of the accelerating or retarding power with respect to the angular displacement of the rotor from its normal position. The representation of this integral as an area gives rise to the name applied to the method. In this method, as usually carried out, the assumption is made that the generator field flux does not decay, so that the generator voltages used remain constant. Under these assumptions the areas involved may be determined readily from the constants of the circuit during and after the fault. In the calculations carried out here, decrement was taken into account, as mentioned above, so that the area representing the energy absorbed during the fault had to be calculated by a step-by-step method and was carried forward for each case simultaneously with the remainder of the step-by-step calculations. It was assumed that when the fault was cleared by the opening of the necessary circuit breakers that the decrement ceased, so that the machine voltages would remain constant for the fraction of a second immediately following at the values which they had reached when the fault was cleared. Under this assumption the energy which could be absorbed by the system if the fault was cleared at a given time was calculated, and by trial a value of time was found which would make this energy equal to that absorbed in accelerating the rotors, as given by the step-by-step process.

The results of these calculations gave the permissible fault duration in terms of the overall reactance between the faulted generator and the remaining generators of the system, the fault reactance in series with the fault, and the fault location. This form was not found to be the most convenient for ready application to actual systems, so that the results were converted to the curves given in Figs. 1 and 2 as previously explained. In making the conversion, the loads were neglected and the indexes determined in terms of quantities which would be obtained if a short circuit study was made on the hypothetical system using a d-c. calculating board. The effect of faults occurring at different locations was found to complicate considerably the curves and render plotting them in convenient form difficult. An effort was therefore made to determine an empirical correction which would give the correct results if the faults were located at some other point than the generator terminals. The method previously described was found to give good results, and to allow convenient plotting of the curves, since only the results for a fault at the generator terminals are given directly.

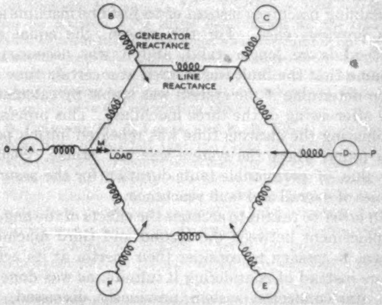

Fig. 9—Schematic Diagram of Hypothetical Ring-Connected System

In the preceding calculations the star connection was used, because it is the most "pessimistic" connection for a given overall reactance. Further calculations were made on a ring-connected system as shown in Fig. 9 to determine the limiting values of correction which might be expected with other forms of system connections, for, as explained previously, a ring connection is about as great a departure from a star-connected system as can be obtained. Six machines were again assumed for convenience in comparison. With this connection the load is assumed to be carried on a group of radial feeders emanating from each generator bus, and therefore for stability purposes may be assumed to be concentrated at that point. For facility in comparison, faults were considered on the radial feeders, for example, at point *M* in Fig. 9. With this setup the system is symmetrical about the center-line *OP*. Hence machines *B* and *F* and machines *C* and *E* will be affected similarly, and therefore may be considered as single generating units of double rating. Hence the problem can be analyzed as a four-machine problem. A calculation was made on this basis for a severe fault, and it was found that the angular displacement between machines *C-E* and *D* was so small that the three machines could be considered as a single unit in subsequent calculations. Therefore, the problem reduces to a three-machine problem, the first machine consisting of the faulted generator *A*, the second of generators *B* and *F*, and the third of generators *C*, *D* and *E*. The analysis followed the same general line as for the two machine problem previously discussed, except that the angular acceleration of the faulted generator now depends upon its relations to the two remaining machines instead of to just one machine as in the previous case. For this reason, the equal area method is no longer valid, and it was necessary to assume that the fault was cleared at a certain time and then determine if the system was stable by calculating the after-swing of the three machines. This process of estimating the clearing time was repeated until a point was found where the system was just stable, yielding a value of permissible fault duration for the assumed values of overall and fault reactance.

In order to take into account the effects of the angular displacement between the second and third machines, it was necessary to consider their inertia at its actual value instead of considering it infinite, as was done for the star-connected system previously discussed. In order to compare with the previous results and bring out clearly the effect of the type of connection, it was necessary to place both on the same basis. This was accomplished by recalculating the results for a few points of the star-connected system, assigning to the remaining generators their actual inertia instead of considering them of infinite inertia. The results were found not to differ materially from those obtained for the corresponding points of the ring connection. Thus with the indexes as set up, the star and ring connections have approximately the same degree of stability, and no correction factor need be employed. It is felt that this same similarity of results will hold when the inertia of the remaining machines is considered infinite in order to approximate the inertia effect of the load, as is done for the general curves in Figs. 1 and 2.

GENERAL COMMENTS

The curves presented in this paper provide a method of solving the transient stability problem on a metropolitan type system without the necessity of investigating the electro-mechanical oscillations as required in a conventional study of this problem. It is to be expected that the results obtained from these curves will have the same order of accuracy as results obtained by a specific analysis of the actual system using the assumptions conventionally made to simplify the work. Experience and test have shown that actual systems have greater stability than the usual calculations indicate. This fact should be borne in mind when using the curves for relay settings or system design, and in general no additional margins of safety need be allowed. The authors feel that the greatest value of these curves lies in the facility with which they allow comparative studies of different layouts to be made. When used in this way the margins between actual and calculated values of stability are largely compensating, and need therefore be of little concern except in extreme cases. Sufficient knowledge concerning load and machine characteristics, particularly those factors affecting damping, is not at present available to determine the margins with precision even as a result of the most careful and involved calculations, so that the curves presented here may be used in all cases where they apply until further data has been obtained on hitherto unevaluated factors.

ACKNOWLEDGMENT

The authors wish to acknowledge the encouragement and assistance of engineers of the operating companies, particularly Mr. S. M. Dean of the Detroit Edison Co., and the assistance of their associates, especially Mr. L. M. Olmsted, in making the numerous calculations.

NOTATION

I_F = total fault current, expressed in times rated current of "faulted generator."

I_{FG} = fault current from "faulted generator," expressed in times rated current.

r_{FG} = ratio I_{FG}/I_F

X = overall reactance between "faulted generator" and remaining generators in parallel, expressed in per cent based on rating of "faulted generator."

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Discussion

R. D. Evans: During the past seven or eight years, attention has been directed to the nature and importance of stability, methods of improving stability, and the calculation of stability. These discussions have been very voluminous and for the most part highly technical. For some time it has been feasible for a specialist to determine, as a result of tedious step-by-step calculations, that a particular system was or was not stable for a particular load condition and a particular system disturbance. From a practical point of view it is necessary to go further than this and to put a dollar value on the difference in the stability characteristics of various power system layouts. To do this has required the development of approximate or short-cut methods as obviously the tedious step-by-step methods are unsuitable. By short-cut methods I do not mean merely shortened stability calculations but instead methods which permit the determination of stability without actually making any stability calculations at all for the cases under consideration. More definitely these short-cut methods involve merely the determination of certain network constants of the particular system and the use of general curves.

The general stability curves given in the paper may be used in a manner similar to that of the "Standard Decrement Curves," see page 353 of this issue. These stability curves are the first that have been presented in connection with the closely connected or metropolitan type system. It is expected that their use together with previous curves developed for long distance straight-way transmission will lead to the development of similar methods for the remaining types of systems so that eventually short-cut methods will be available for all types of systems.

S. H. Wright: When applying the curves presented by Messrs. Griscom, Lewis and Ellis, cases may frequently arise where conditions of the system being studied are met by the conditions assumed in preparing the curves except in one respect, making it desirable to consider the inertia constant as an additional "index factor." This will be illustrated in the following:

Suppose we have a case where the "faulted generator" group consists of a large-capacity unity-short-circuit-ratio generator whose inertia constant is 5.0 kw. sec. per kva. Assume (as under item 10 in the paper), that the combined inertia of the system and "remaining generators" is infinite. The permissible fault duration varies as the square root of the inertia of the faulted generator, hence its value in seconds as read from the curves in the paper would be multiplied by $\sqrt{5.0/8.5}$ or by 0.767, to obtain the actual permissible fault duration; this means a reduction in time of 23 per cent.

As is stated, "The authors feel that the greatest value of these curves lies in the facility with which they allow comparative studies of different layouts to be made." In the spirit of the statement it is obvious that *comparisons* are valid regardless of such a time correction factor as illustrated above, *providing* the faulted generator characteristics are the same in all cases studied.

S. B. Crary: During the past few years, the extensive application of synchronous machine and circuit theory to the study of the transient stability limits of various systems has made it desirable to summarize and classify the results of the investigations already made. In doing this for the metropolitan type of system, the author's have shown some interesting results.

Any generalization or simplification of a problem which comprises as many factors as a transient stability study, must require numerous assumptions. The assumptions which are made, limit the field of application. Care must, therefore, be used in applying the curves of Figs. 1 and 2 of the paper to systems that do not adhere to the assumptions set forth. A few illustrations have been selected as the best means of illustrating this point.

The assumption is made that the reactance of the connecting ties holding a generator or group of generators in step with the system does not change appreciably when the fault is cleared. This assumption is valid when a fault is on a feeder, but is not when the fault is on one of the high-voltage ties between stations. Although the ring-connected system is considered as designed for no power transfer between stations under normal operation, conditions exist when these ties are used to transmit a considerable portion of a generating station load to another area. If a fault occurs on a tie line heavily loaded in this manner, the stability limit may be below that of the hypothetical star system used to determine the curves of Figs. 1 and 2 of the paper.

The assumption is made that the "remaining group" has an infinite inertia. This will probably be valid for most conditions, but the condition may exist, that the faulted group of generators represents more than one-third of the total connected capacity. Under such a condition, the required switching time in order to clear the fault and maintain stability may be reduced or increased an appreciable amount, depending upon the distribution of loads with respect to the "faulted and remaining groups."

The statement is made that "Where systems do not reduce to two composite generators, the stability will be as high or higher than when they do." This can not be taken as a general statement as cases do exist when the stability limit may be lower instead of higher. An example is a system in which "the remaining group" consists of a minor group relatively closely coupled electrically to the "faulted group," and a major group loosely coupled with respect to all of the other machines.

The severity of the fault as it affects stability is determined by the two indexes I_{FO} and I_{FO}/I_F. Could not both of these indexes be replaced by a single index based on the transfer reactance between the two machines when the fault is on? This reactance has been defined in a paper by Summers and McClure.* It is a simple function of the three branches of the simplified equivalent network and can readily be determined from the system set up on a d-c. calculating table.

S. B. Griscom, W. A. Lewis, and W. R. Ellis: In preparing the paper the authors felt that the range of inertia constants normally encountered would be sufficiently narrow that a separate correction or index for this factor would be unnecessary. Mr. Wright mentions an example in which the inertia departs materially from the average assumed, and the deviation is sufficient to warrant a correction. In general, the correction can be made in the manner he suggests. That is, the permissible fault duration as found from the curves and after the application of the other correction factors should be multiplied by $\sqrt{\frac{H}{8.5}}$ for unity short-circuit ratio machines or $\sqrt{\frac{H}{8.25}}$ for 0.8 short-circuit ratio machines, H being the inertia constant of the "faulted generator." It should be pointed out, however, that since decrement was taken into account, the correction obtained in this manner is somewhat too large. With the smaller inertia constant a given angular position is reached sooner and the voltage has a higher percentage of its original value. This higher voltage will give a higher power limit and hence a greater permissible fault duration.

Mr. Crary has emphasized that there are some cases which do not fall within the range of the assumptions made in the paper. From the nature of the problem, this of course is inevitable. However, the authors feel that the application of the curves may be extended to cases outside the range of the assumptions if care and judgment are used in interpreting the results. This will be illustrated by considering the examples selected by Mr. Crary.

If the reactance of the connecting tie holding a generator or group of generators in step with the system changes appreciably when the fault is cleared, upper and lower limits of permissible clearing time may be found by computing the "overall reactance" for both conditions and estimating an intermediate clearing time from the values read from the curves for the two reactance values.

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*"Progress in the Study of System Stability," A.I.E.E. TRANS., Vol. 49, Jan. 1930, p. 132.

If the "faulted generator" is a large proportion of the total system capacity, the assumption that the inertia of the "remaining generators" is infinite is not strictly valid. For the severe faults, which will be electrically close to the faulted generator, the permissible fault duration will be greater than the value given by the curves and the answer is, therefore, on the side of safety. For faults near the electrical center of the system, the deviation may be in either direction, but for such cases the permissible duration will usually be two seconds or more so that the error may be tolerated.

The reduction of the ring connection system of Fig. 9 of the paper for purposes of calculation gives a system which closely approximates the system mentioned by Mr. Crary in which "the remaining group" consists of a minor group relatively closely coupled with the faulted group and a major group loosely coupled with respect to the other machines. That is, machines *B* and *F* comprise the minor group and machines *C*, *D* and *E* comprise the major group. As stated in the paper the permissible fault duration for the case was found to be as great as or greater than the value read from Figs. 1 or 2.

It is appreciated that there may be cases in which accurate results cannot be obtained from the curves. However, it is felt that such cases would be the exceptions and will be recognizable from their special features.

An attempt was made to use as an index the equivalent of the transfer impedance as suggested by Mr. Crary. However, it was found that the driving point impedance was also required to fully represent the conditions, so that the form used was adopted as being more convenient.

The authors realize that as these curves are used certain additions and modifications may be indicated. It is hoped that a standardized set of curves for metropolitan and other types of systems will eventually receive the same recognition as the standard decrement curves referred to by Mr. Evans.

Stability of Conowingo Hydroelectric Station

Philadelphia Electric Company System

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Fellow, A.I.E.E. Associate, A.I.E.E.

***Synopsis.**—This paper gives a brief description of the Philadelphia Electric Company System with particular reference to the features influencing stability of the Conowingo Hydroelectric station, summarizes the various calculations and tests associated with the stability problem, and presents the record of operating experience from a stability standpoint during four years of service.*

Several cases of instability due to line faults have occurred. A thorough study was made of the matter, and as a consequence, changes were made in the equipment and system set-up involving the development and installation of higher speed circuit breakers and relays.

Detailed records are given for all faults on the parts of the system which are intimately associated with the Conowingo development. These records show the improvement of stability obtained by the changes in the system and indicate attainment of a generally satisfactory condition.

* * * * *

INTRODUCTION

THE matter of electric power system stability is becoming of more general interest and concern since the advent of large hydroelectric stations located at considerable distance from their loads and the general interconnection of large capacity power systems. A number of papers has appeared dealing with methods of calculating stability, but comparatively little information has been published showing the experience on actual systems. Such experience is of value as an indication of what may occur on other systems of similar character and is helpful in determining the basis for design of new projects.

The 252,000-kw. Conowingo hydroelectric station which is located 58 miles from the main system of the Philadelphia Electric Company and ties in with the 220-kv. Pennsylvania-New Jersey interconnection by means of two transmission lines, is a typical example of a generating station susceptible to instability under certain conditions of very severe line faults. Studies have been made of the stability of this system, and, during the four years the plant has been in service, considerable operating experience has been obtained including several cases of instability, with the result that several changes have been made in the system set-up and equipment to improve the performance. Automatic oscillographs are permanently installed to record system performance during faults and a large number of records has been obtained.

No operating tests have been made to determine stability, though several short-circuit tests on the 220-kv. system to determine high-speed relay and oil circuit breaker characteristics, intimately associated with stability, were made. A few high speed motion pictures of governor operation were taken, but gave little information.

DESCRIPTION OF SYSTEM

The system involved, shown in Fig. 1, consists of the Conowingo hydroelectric station with seven 36,000-kw. generators connected by two 58-mile, 220-kv. lines to Plymouth Meeting substation, which is a junction point for two lines of the Pennsylvania-New Jersey 220-kv. interconnection, and the location of step-down transformers for connection with the main part of the Philadelphia electric system by means of 66-kv. lines.

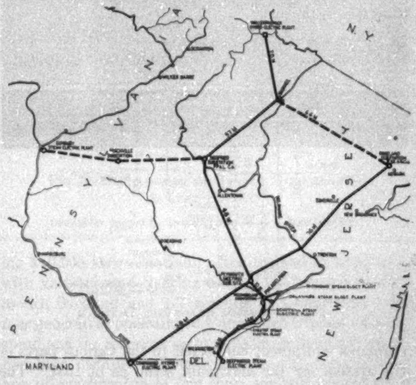
N. Y.
PENNSYLVANIA
NEW JERSEY
ALLENTOWN
READING
TRENTON
RICHMOND STEAM ELECT PLANT
DELAWARE STEAM ELECT PLANT
SCHUYLKILL STEAM ELECTRIC PLANT
DEL.
MARYLAND

FIG. 1—MAP OF SYSTEM

The Pennsylvania-New Jersey interconnection as indicated consists of 220-kv. ties between the Philadelphia Electric Company system at Plymouth Meeting, the Pennsylvania Power and Light Company system at Siegfried, and the Public Service Electric and Gas Company system at Roseland. The 40,000-kw. Wallenpaupack hydroelectric station of the Pennsylvania Power & Light Company is connected to Siegfried with a 220-kv. line which it is planned to extend from Bushkill to Roseland in the ultimate development. The approxi-

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mate lengths of the lines are indicated on Fig. 1. The interconnected systems are all of large capacity, the Philadelphia Electric Company, having a peak load in 1930 of 699,000 kw., the Public Service Electric and Gas Company 540,500 kw., and the Pennsylvania Power & Light Company about 350,000 kw.

The connection between Plymouth Meeting substation and the main part of the Philadelphia Electric system where most of the Conowingo output is used, consists of three 10-mile 66-kv. aerial lines to Westmoreland substation, from which point lines radiate to the various load centers and generating stations as indicated. The Philadelphia Electric Company 66-kv. system is normally sectionalized as shown by Fig. 2 to limit short-circuit duty. This diagram shows the positive sequence reactance of the principal lines and equipment as well as the equivalent reactance of the synchronous equipment connected to the various parts of the system for the maximum setup as of 1931. The set-up for previous years[1] was similar except that the Plymouth Meeting-Roseland line was not in service; there were only two step-down transformer banks at Plymouth Meeting; only two Plymouth Meeting-Westmoreland 66-kv. lines; and the 66-kv. system was not sectionalized at Plymouth Meeting or Westmoreland.

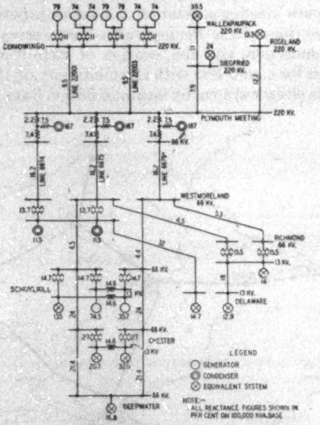

Fig. 2—System Reactance Diagram

PRELIMINARY STABILITY STUDY

Extensive stability calculations and studies were made before the 220-kv. system was built to check stability in case of line faults. The faults were assumed to involve only one phase since information then available indicated this was the general experience on 220-kv. systems. These studies were based on the ultimate system development which involves extension of the 220-kv. system as shown by the dashed lines in Fig. 1. Since the system is rather complicated, one of the studies was made with a mechanical model as described in the paper *Quantitative Mechanical Analysis of Power System Transient Disturbances* by R. C. Bergvall and P. H. Robinson.[2]

The results of the stability studies in general showed that synchronism could be maintained for practically all expected fault conditions, although considerable swinging was indicated for some conditions. On account of this swinging condition together with uncertainty regarding some of the factors used in the study such as clearing time and fault resistance, it was considered advisable to incorporate in the design several special features for increasing stability and reliability.

FEATURES OF INITIAL INSTALLATION HAVING SPECIAL INFLUENCE ON STABILITY

The measures for increasing stability and reliability which were provided with the initial design may be divided into two general classes: preventive and remedial. The first class was aimed at the prevention of faults by means of heavy insulation and protection against lightning with double ground wires for each line, shielding for substation structures and installation of lightning arresters at the substation as described in previous papers.[3,4]

The other class included steps to clear faults quickly should they occur and measures to stiffen the system in case of transient disturbances. A special relay system was installed, designed to clear all faults with minimum delay. A detailed description of the original relay system is given in one of the references.[4] The oil circuit breakers which were the fastest available for such high-voltage service at the time of installation were designed to interrupt fault current in less than one-half second. The generators were built with as low transient reactance as possible consistent with reasonable cost and efficiency. It was rather difficult to obtain low reactance for these machines on account of their slow speed (81.8 r.p.m.), but it was possible to reduce the transient reactance to about 29 per cent and still have a fairly well-balanced design. The transformer reactance was kept as low as possible with safety and this resulted in a value of about 9 per cent. The three windings of the Plymouth Meeting transformers were so arranged that the synchronous condensers connected to the tertiary would be very closely coupled to the 220-kv. system, so as to secure the maximum benefit from them in maintaining stability.

One 30,000-kva. condenser is provided for each of the three transformers at Plymouth Meeting, and these condensers have rather special excitation as described by L. W. Thompson and P. J. Walton.[5] In case of a system disturbance, the exciter voltage is automatically increased at a rate of over 6,000 volts per second, and the load on each condenser increases from a normal value of

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[1] For references see Bibliography.

10,000 kva. to about 58,000 kva. in one-half second. Two additional condensers with automatic high-speed excitation are installed at Westmoreland substation and the 60-cycle motors of a considerable number of frequency converters on other parts of the Philadelphia Electric Company system are used as condensers. These frequency converters and all generators on the Philadelphia electric system have automatic voltage regulators.

The Conowingo generators have special high-speed excitation designed to prevent a drop of more than 2 per cent in the induced voltage of the generators when full-load lagging reactive kva. is suddenly applied to the generator while carrying full rated load at unity power factor.

INSTABILITY EXPERIENCED WITH INITIAL OPERATION

During the first year, 1928, three faults occurred on the Plymouth Meeting-Conowingo lines, each fault involving two phases instead of only one as expected and used as the basis for design. Conowingo became unstable for each of these faults and also on one other occasion when a severe disturbance occurred on the Philadelphia Electric Company 66-kv. system.

The first case of instability was caused by a lightning flashover of two phases of the Plymouth Meeting-Conowingo line about two miles from Plymouth Meeting, tripping the faulty line at Plymouth Meeting and both lines at Conowingo. Five generators were in operation at Conowingo carrying 168,000 kw. and all five machines tripped on over-speed or over-voltage. The Philadelphia Electric System load was 202,000 kw. none of which was lost. The Plymouth Meeting-Siegfried line stayed in and helped materially in supporting the Philadelphia Electric System load after Conowingo was lost.

The second case of instability was caused by an oil fire and explosion near the Chester-Schuylkill two-circuit, 66-kv. line, which enveloped the line in flame causing a two- or three-phase short circuit. This burned down some of the conductors, tripping both circuits. Chester was cut away from the system with a load of about 40,000 kw. One Plymouth Meeting-Westmoreland 66-kv. line tripped at Westmoreland and four out of six machines at Conowingo tripped on over-speed or over-voltage. The lights in Philadelphia were noticed to flicker for several seconds. The Philadelphia Electric System load previous to the trouble was 297,000 kw. of which about 30,000 kw. was lost as a result of the disturbance. The generation at Conowingo dropped from 218,000 kw. to 26,000 kw. The Philadelphia Electric System steam generating capacity in operation at the time was 270,000 kw., the large excess being due to the noon hour drop in load.

The third case of instability was on the 4th of July, 1928, when the system load was small on account of the holiday. This case was caused by a lightning flashover of two phases of the Conowingo-Plymouth Meeting line about five miles from Conowingo while five generators were in operation carrying 112,000 kw. Switches tripped on the faulty line at Plymouth Meeting and on both lines at Conowingo. One Conowingo generator tripped on over-speed. A Hall high-speed recorder which had just been installed at Plymouth Meeting indicated the fault duration as approximately one second and showed that Conowingo became unstable during the fault and remained out of step with the system for about six seconds until separated apparently by opening of the unfaulted Plymouth Meeting-Conowingo line. The remaining generators on the system picked up the load without apparent difficulty.

The final case of instability experienced during the first year's operation was caused by a lightning flashover of two phases of the Plymouth Meeting-Conowingo line about four miles from Plymouth Meeting, while four generators were in operation at Conowingo carrying 140,000 kw. The faulty line tripped at both ends with an estimated clearing time of 0.3 seconds at Plymouth Meeting and 1.7 seconds at Conowingo. All four generators operating at Conowingo tripped on over-speed and the record of an automatic oscillograph which had been installed on the system by this time showed loss of synchronism in less than one-half second after the beginning of the fault. The Philadelphia electric system load at the time of the disturbance was 360,000 kw. and no load was lost. This oscillograph record is shown in Fig. 3.

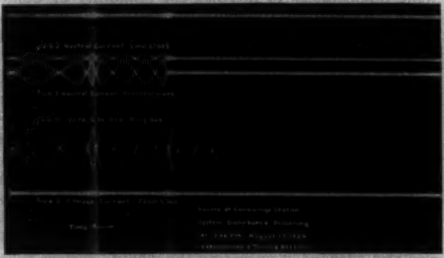

FIG. 3—OSCILLOGRAM OF SYSTEM DISTURBANCE AUGUST 17, 1928

Instability caused by two-phase fault on Conowingo line

Another disturbance experienced during the first year's operation which resulted in the loss of Conowingo was caused by a two-phase fault near the middle of one of the two 66-kv. Plymouth Meeting-Westmoreland lines while Conowingo was carrying 180,000 kw. Both Plymouth Meeting-Westmoreland lines tripped separating Conowingo from the system and all five generators in operation at Conowingo tripped on over-speed or over-voltage. It is believed this interruption was due to too low setting of the relays on the Plymouth Meeting-

Westmoreland lines which tripped the unfaulted line as the result of overload rather than through instability.

This experience during the first year when every fault on the Plymouth Meeting-Conowingo line caused instability was naturally very alarming, and extensive study was given the matter to determine the cause of the trouble and to find out what could be done to correct the condition. The study was handicapped at first by the lack of definite records of the time and sequence of events. Some doubt existed regarding the performance of relays, switch time and operation of the governors. All four cases of instability occurred before the installation of synchronous condensers was complete and before the high-speed excitation system was working satisfactorily.

TEMPORARY MEANS OF IMPROVEMENT

While there was practically no interruption of service to customers, it was realized that something must be done immediately to increase the reliability of Conowingo. Therefore a number of temporary measures was adopted pending the outcome of further study and experience.

It was evident that most trouble was likely to occur during lightning storms and that service to the customer could be maintained by providing sufficient steam reserve. Arrangements were therefore made with the station attendants and line patrolmen to watch for electric storms and to report their appearance to the system load dispatcher so that the load on Conowingo could be reduced and the steam reserve built up whenever an electric storm was on the system. A practise was also adopted of operating the 220-kv. system split during storms if four or more generators were in operation at Conowingo so that the station would operate in two separate sections. While this meant the certain loss of half the station with a fault on either line, it greatly reduced the chance of losing the entire station. The use of this storm set-up was discontinued in 1931 when changes had been made in some of the equipment so that it was no longer required.

SPECIAL TESTS

While relief was obtained by the temporary measures, a series of tests was made to check practically all uncertain factors affecting the performance of the system.

The most comprehensive of the tests was a series of single-phase and two-phase faults made at various locations on the Plymouth Meeting-Conowingo lines while records were obtained by means of observers and special recording equipment on all parts of the system. This test was made with most of the synchronous equipment and lines in service but without load because of danger of disturbing the customers. Twenty-nine oscillograph elements were used in the test and the connections to these elements were changed around to measure various quantities for the different faults. High-speed moving pictures were taken of all the faults and of the governors on several of the generators.

The main conclusions from this test were as follows:

1. Relay operations were proper and all faults were cleared with more than sufficient margin to prevent the disconnection of any equipment other than the faulted line.

2. The overall clearing time for faults including switch and relay operation was somewhat longer than expected, averaging a little over one-half second for the first switch and one second for the second switch.

3. There was no serious swinging of one generator against another at Conowingo.

4. There was no hunting of governors.

5. The faults were apparently of low impedance and did not blow from one conductor to another.

6. The high-speed excitation system was not performing as well as expected.

7. The calculated short-circuit current was in reasonably close agreement with the actual test values.

Another test was made by opening switches on the Conowingo generators while the machines were carrying load to show the speed characteristics and governor response as well as to furnish a service check of the overspeed and over-voltage protection. It showed that the speed increased so rapidly when full load was dropped that some of the units advanced 180 deg. ahead of the system in less than 0.3 seconds. This naturally explains how the plant can get out of synchronism so quickly when the electrical output is seriously reduced by the reduction in voltage as a result of a severe system fault.

Special tests were made of the synchronous condensers under load conditions in order to check the operation of the high-speed excitation system and determine the best adjustments. After some changes had been made, the performance was brought up to expectations. The test of the Plymouth Meeting condensers is described in another paper previously mentioned.[5]

Another test was the measurement of zero sequence reactance of the Conowingo lines. This was one of the uncertain factors used in the calculations and the test showed that the measured value was very close to that used in the calculations.

COLLECTION OF RECORDS OF SYSTEM DISTURBANCES

Complete records and reports of each system disturbance are obtained by the following means:

1. Automatic Oscillographs. As soon as obtainable, automatic oscillographs were permanently installed at Conowingo and at Plymouth Meeting. The installation includes a six-element oscillograph at Conowingo and a seven-element oscillograph at Plymouth Meeting, while most of the time a second oscillograph with six elements has been used at Plymouth Meeting. A four-element Hall recorder is installed at Westmoreland. Through cooperation with the manufacturers, and communication companies, additional oscillographs and recorders have been temporarily installed from time to time to obtain field experience and special information. These oscillographs record residual current in all lines, power in each of the Plymouth Meeting-Conowingo lines, positive sequence voltage and selected line currents and voltages. They are started automatically with station residual current. Arrangements are being made also to start the oscillographs with line current since three-phase faults recently experienced have had insufficient residual current to start the oscillograph.

2. Reports from Station Operators. Most of the relays are equipped with operation indicators and the more important relays are connected to a drop signal system so that it is possible to see which relays operate for each disturbance. From this, it is usually possible to determine the phases which are faulted and the general location of the fault. The operators are instructed to note the time of each disturbance and to make a record of all relay operations. They also report the reading of ammeters in the neutral ground connection of the high-voltage transformer windings. These ammeters are ratcheted so that the pointer remains at the maximum reading until released by the operator. Graphic recording meters make a continuous record of power, voltage and frequency. Of course, station records are kept which show the system set-up and machines in operation at all times.

3. Line Reports. Thorough inspection is made of the line immediately after each fault to determine the damage, location and number of phases involved. If necessary, the patrolmen climb each tower to locate the fault.

All available information is collected for each important disturbance and these records are analyzed to determine the cause, number of phases involved, location of fault, fault current, switching sequence, time of clearing, power swings, etc. The records of all severe disturbances have been referred to the manufacturers of the major equipment to secure their recommendations regarding means of improvement.

ANALYTICAL STUDY OF TWO-PHASE FAULTS

In view of the experience with two-phase faults during the first year, further stability calculations were made based on this type of fault. These calculations showed that stability could not be expected with solid 220-kv. bus operation and heavy load on the plant with a two-phase fault near the end of the line cleared in approximately one-half second, but if the fault was cleared in about 0.15 second, the system would probably remain in synchronism. A similar study was made for two-phase faults on the Plymouth Meeting-Westmoreland lines and this also showed the need of high-speed switching.

DEVELOPMENT OF HIGH-SPEED CIRCUIT BREAKERS

The manufacturers thought it would be possible to increase greatly the speed of clearing faults by means of new design circuit breakers and relays, and with the need for higher speed operation apparent, they immediately undertook the development of suitable equipment. Some experimental models of new relays and oil circuit breakers were tested at Plymouth Meeting by means of short circuits on the 220-kv. system. These tests were very successful showing that interrupting time of the order of 0.14 second was possible. A description of some of the tests was given in the technical press.[6,7]

CHANGES IN EQUIPMENT TO INCREASE STABILITY AND RELIABILITY

As a result of the various studies and tests combined with normal development of the system, it was decided to make the following permanent improvements:

1. New high-speed circuit breakers were purchased and rearrangements made at Plymouth Meeting and Westmoreland so that high-speed breakers were provided for normal operation on all 220-kv. and 66-kv. aerial lines.
2. The 220-kv. line breakers at Conowingo were rebuilt so that high-speed operation was obtained.
3. The relay system was revised and new high-speed relays installed on all lines to reduce the operating time to about one cycle.
4. A third step-down transformer was installed at Plymouth Meeting.
5. A third 66-kv. line was added between Plymouth Meeting and Westmoreland.
6. The 66-kv. system was split into three separate sections at Plymouth Meeting and Westmoreland.
7. The 220-kv. line insulation was increased from 14 to 16 units for suspension insulators and from 16 to 18 units for strain insulators.

OPERATING EXPERIENCE

The general experience with Conowingo since the first year's operation has been very satisfactory. Many disturbances have occurred due to faults on practically all parts of the system, but since the first year when there were four cases of instability, various steps have been taken to improve operation and there has been only one case where Conowingo became unstable.

The disturbances have been classified according to the location of the fault on the system and will be discussed under the following general headings:

PLYMOUTH MEETING-CONOWINGO LINE FAULTS

The principal data for each fault experienced on the Plymouth Meeting-Conowingo lines are shown in Table I. This gives the best known information regarding the type and location of the fault, the clearing time, load conditions and the general effect on the system. Some of the information is only approximate but has been included wherever possible for the sake of completeness.

The residual current is shown in amperes as measured in the faulted line. The positive sequence voltage is expressed in kilovolts equivalent line to neutral on the 220-kv. system at Plymouth Meeting measured during the fault. Comparison of the positive sequence voltage

TABLE I—PLYMOUTH MEETING–CONOWINGO LINE FAULTS

Year	1928			1929				1930				1931			
Date Time	6/9 5:27 p.m.	7/4 4:12 p.m.	8/17 3:06 p.m.	6/14 11:48 p.m.	6/15 1:21 a.m.	7/29 4:12 p.m.	8/21 5:06 a.m.	1/14 12:31 p.m.	6/26 7:48 p.m.	7/6 7:19 p.m.	7/21 7:14 p.m.	5/10 8:52 p.m.	7/9 7:41 p.m.	7/9 9:01 p.m.	7/24 1:47 a.m.
Line No.	22,003	22,003	22,001	22,001	22,001	22,003	22,003	22,003	22,003	22,001	22,003	22,003	22,001	22,001	22,003
Cause	Ltng	Ltng	Ltng	Ltng	Ltng	Ltng	—	Ltng	Ltng	Ltng	Switch failure	Ltng	Ltng	Ltng	Ltng.
No. phases	2	2	2	1	1	2	1	1	1	1	1	2	1	3	1
Miles from Ply. Mtg	2	53	4	—	—	—	—	17	28	—	0	18	5	22	33
Residual current Con.	—	—	—	Open	Open	—	Open	915	500	420	450	1,370	940	—	Open
Residual current Ply.	—	—	—	1,300	1,350	—	—	—	700	3,000	—	2,600	4,500	—	2,600
Pos. sequence voltage Ply.	—	—	—	—	—	—	—	—	115	87	98	76	95	—	115
Clearing time (60-cycle) Con.	—	10	102	Open	Open	69	Open	45	107	71	37	11	12	Est. 40-50	Open
Clearing time (60-cycle) Ply	—	62	20	18	16	20	35	31	35	33	55	8	15	"	8
Load Conowingo mw.	168	112	140	30	0	66	0	263	170	92	150	235	145	165	35
Load P. E. steam gen. mw.	34	46	220	225	211	349	226	159	258	145	323	89	248	241	163
Load P. E. system mw.	202	158	360	252	207	423	250	392	390	223	473	319	345	396	207
220-system set-up	Solid	Solid	Solid	Solid	Solid	Solid	Solid	Solid	Split	Solid	Solid	Solid	Solid	Solid	Solid
Conowingo stability	Unstable	Unstable	Unstable	OK	Off	OK	Off	OK	OK	OK	OK	OK	OK	Unstable	OK
Conowingo power swings	—	—	—	—	—	—	—	Est. 200–325	75–100	70–110	130–155	164–292	114–185	+313–313	31–38

with the normal value of approximately 127 furnishes a general indication of the shock to the system. The clearing time is given in cycles based on the operating frequency of 60 cycles per second. The figures for Conowingo power swings are the first maximum and minimum values of average power output of Conowingo for the surges immediately following the clearing of the fault.

There has been a total of 15 faults on the Conowingo lines including nine single-phase, five two-phase, and one three-phase. Instability resulted from the three-phase fault and from three of the two-phase faults which occurred before the high-speed circuit breakers were in service.

The faults during the first year are discussed in the beginning of the paper. In the second year there were no faults while much load was on the station, so there was no real test of stability. At the beginning of the third year, there was a rather unusual experience of an electric storm in the winter. This caused a lightning flashover of one phase on the Conowingo line while the station was operating with solid 220-kv. bus and loaded to maximum capacity. Very heavy swinging occurred but stability was maintained. Fig. 4 is an oscillograph record of this disturbance as obtained at Conowingo. The fault occurred on *B*-phase of line 22,003. The clearing time at Plymouth Meeting is shown by the length of the first step in the residual current record and the clearing time at Conowingo by the total length of the residual current record. Allowance must be made for the starting time of the oscillograph which in this case is about 8 cycles. The record of voltage on line 22,003 is interrupted sometime before the fault is cleared, due to the operation of an auxiliary switch on the breaker mechanism. The record of *A*-phase current in the unfaulted line (No. 2 vibrator) shows the surging after the fault was cleared.

Another interesting disturbance in 1930 was a fault while using the storm set-up. The 220-kv. system was operating split with two generators carrying 70,000-kw. load connected to the line on which the fault occurred while four generators with about 100,000 kw. were on the other line. Of course, the two generators carrying 70,000-kw. load were lost but the remaining section of the system was disturbed very little.

The last fault of 1930 is rather unusual since it was the equivalent of a 220-kv. bus fault at Plymouth Meeting. An insulation failure occurred inside of one of the Conowingo line circuit breakers which opened but did not clear the fault since it involved the switch terminal connected to the bus. The fault burned clear after a little while and system operation was not very greatly disturbed.

The new equipment for high-speed clearing of faults was in service in 1931 and showed its value for the first fault which was a flashover of two phases while Conowingo was almost fully loaded. The fault was cleared from Plymouth Meeting in about eight cycles or 0.133

seconds and from Conowingo in 11 cycles as shown by the oscillograph record from Conowingo, Fig. 5. The time lost between the beginning of the fault and the first visible record is approximately three cycles. The reduction in oscillograph starting time as compared with previous records such as Fig. 4 was necessary on account of the high-speed switch operation. The improvement in starting time is secured through the use of a new type lamp. Rather heavy surging occurred after the fault but stability was maintained.

Fig. 4—Oscillogram of System Disturbance January 14, 1930

Stability maintained through single-phase fault on Conowingo line with full load on station

Fig. 5—Oscillogram of System Disturbance May 10, 1931, Showing Operation of High-speed Circuit Breakers

Stability maintained through a two-phase fault on the Conowingo line

The worst disturbance and the only one which has caused instability since the first year's operation was the three-phase lightning flashover in 1931. The plant was carrying 165,000 kw. and the fault was not cleared for almost a second because there was practically no ground current, and the only high-speed relays in service were ground relays. Since this experience high-speed balanced protection has been added to take care of such conditions. Due to the relatively small ground current the automatic oscillographs did not start until the circuit breaker had interrupted the fault and no definite record was obtained of the clearing time. The record which was obtained, Fig. 6, is of interest, however, since it shows the power swings of the station out of synchronism with the system. The unfaulted line No. 22,001, finally tripped as shown by cessation of power swings. The crest of the power swing was used as a check on stability calculations by comparison with the crest of the power angle diagram calculated for this condition. These check within a few per cent which seems to indicate that the assumptions used in the stability calculations are reasonably satisfactory.

INTERCONNECTION LINE FAULTS

The faults experienced on the 220-kv. interconnection lines running from Plymouth Meeting to Siegfried and Roseland are listed in Table II. So far as known, all eight faults were single-phase and while most of these were of relatively long duration, none seriously affected stability. The load on these lines is usually rather small, seldom exceeding 50,000 kw. The Roseland line was not placed in service until near the end of 1930 so that comparatively little experience is available on this line. It was equipped initially with high-speed circuit breakers, but the first fault was slow in clearing because it was so near the Roseland end of the line that it was outside the zone of operation of the high-speed relays. The next fault which was probably much closer to Plymouth Meeting was cleared in eight cycles. The last disturbance on the Siegfried line was rather unusual, being caused by the failure of the circuit breaker at Plymouth Meeting when the line was being opened. Some material jammed the oil valve connecting one of the switch tanks to the oil system and most of the oil drained out without being noticed before the breaker was opened. Naturally, the circuit breaker could not open the circuit properly under this condition and flashover to ground occurred inside the tank. This was equivalent to a 220-kv. bus fault at Plymouth Meeting and it was cleared as expected by trip-out of all 220-kv. feeds into Plymouth Meeting. This involved considerable time delay but the system was not seriously dis-

Fig. 6—Oscillogram of System Disturbance July 9, 1931

Instability resulting from three-phase fault on Conowingo line

TABLE II—PLYMOUTH MEETING–SIEGFRIED AND PLYMOUTH MEETING–ROSELAND LINE FAULTS

Year	1928		1929	1930			1931	
Date Time	7/12 5:15 p.m.	7/24 8:02 a.m.	8/13 11:37 a.m.	6/26 7:12 p.m.	7/9 10-17 p.m.	12/10 4:17 p.m.	7/14 6:14 p.m.	9/6 4:28 a.m.
Line	Sieg	Sieg	Sieg	Sieg	Sieg	Rose	Rose	Sieg.
Cause	Ltng	Ltng	Ltng	Ltng	Ltng	Foreign object	Ltng	Switch failure
No. phases	1	1	1	1	1	1	1	1
Miles from Plymouth	40	15	35	17	14	65	—	0
Res. cur.—Plymouth	1,000	1,440	1,070	1,840	1,750	600 +	2,000	—
Pos. seq. volt.—Plymouth	—	—	—	92	98	—	115	—
Clearing time (60-cycles)—Plymouth	65 cyc.	65 cyc.	92 cyc.	30 cyc.	35 cyc.	71 cyc.	8 cyc.	150 cyc.
Load Con. mw	208	218	41	120	25	0	172	28
Load P. E. steam	65	173	341	221	262	434	150	134
Load P. E. system	263	395	349	369	323	453	312	179
220-kv. system	Solid	Solid	Solid	Split	Solid	Solid	Solid	Solid
Conowingo stability	OK	OK	OK	OK	OK	Off	OK	Trip
Power swings	—	—	None	95–142	20–25	Off	166–181	—

TABLE III—P. E. SYSTEM 220-KV. TRANSFORMER FAULTS

Year	1928		1929		1930	1931
Date Time	4/11 11:01 p.m.	12/22 10:33 p.m.	12/28 10:43 a.m.	12/28 10:47 a.m.	5/27 4:53 a.m.	3/28 11:53 a.m.
Station	Con.	Ply	Con.	Con.	Con.	Con.
Cause	Hv bushing flashover	Winding failure	Winding failure	Energize faulty trans.	Winding failure	Winding failure
No. phases	1	1	1	1	1	1
Pos. seq. voltage Ply	—	—	92	—	90	96
Clearing time (60-cycle) 220 kv	—	26	70	36	40	45
Load Con. mw	147	90	262	186	186	258
Load P. E. steam	119	230	198	256	68	147
Load P. E. system	222	300	419	419	232	383
220-kv. system set-up	Solid	Solid	Solid	Solid	Solid	Solid
Con. load dropped	71	0	64	0	33	36
Con. stability	OK	OK	OK	OK	OK	OK
Con. power swings	—	—	180–250	60–300	90–270	120-342

TABLE IV—PLYMOUTH MEETING–WESTMORELAND LINE FAULTS

Year	1928		1931								
Date Time	6/5 10:18 p.m.	7/1 7:28 a.m.	5/16 3:53 p.m.	7/14 7:47 p.m.	7/14 7:49 p.m.	7/14 7:52 p.m.	7/14 7:57 p.m.	7/14 9:03 p.m.	7/14 9:14 p.m.	7/14 10:06 p.m.	8/10 5:58 p.m.
Line No	6,675	6,675	6,674	6,674	6,679	6,674 6,675	6,679	6,674	6,679	6,679	6,679
Cause	Ltng	Short circuit	Short circuit	Ltng	Ltng	Ltng	Ltng	Ltng	Ltng	Ltng	Ltng.
No. phases	2	3	1	3	3	3	1 or 2	2	3	3	1
Miles from Ply. Mtg.	5	0	0	4	2	3	1	3	1	1	1
Positive seq. volt.—Ply	—	—	125	117	92	—	—	—	—	—	110
Residual cur.—Plymouth	—	—	Open	—	750	—	—	—	—	—	7,620
Clearing time (60 cycle)—Plymouth	—	Open	Open	18 +	17 +	—	13	—	—	7	12
Clearing time (60 cycle)—Westmoreland	—	—	16	—	—	—	7	—	—	13	15
Load Con. mw	180	98	255	202	202	202	202	226	226	168	42
Load P. E. steam mw	55	35	47	186	186	186	186	255	255	150	335
Load P. E. system mw	252	116	294	388	388	388	388	473	473	322	379
66-kv. system set-up	Solid	Solid	Solid	Split	Split	Split	Split	Split	Split	Split	Split
Conowingo stability	Trip	OK	OK	OK	OK	OK	OK	OK	OK	OK	OK
Conowingo power swings	—	—	235–290	165–280	140–322	—	—	—	—	—	None

turbed probably because there was very little load on Conowingo.

220-Kv. Transformer Faults

A number of failures has occurred in the 220-kv. transformers at Plymouth Meeting and Conowingo as shown in Table III. Most of these failures have occurred in the windings and each failure has involved only one phase. Synchronism has been maintained through all of these faults, although they were not cleared very quickly and in some cases the station was carrying full load. Fig. 7 shows the oscillograph records obtained at Plymouth Meeting for the last Conowingo transformer failure. Rather heavy power surges occurred when the fault was cleared, but these surges as usual were soon damped out.

Plymouth Meeting-Westmoreland Line Faults

The faults experienced on the Plymouth Meeting-Westmoreland 66-kv. lines are shown by Table IV. Since these lines carry most of the Conowingo output and have considerably higher percentage reactance than the Plymouth Meeting-Conowingo lines the matter of line faults is rather serious. Fortunately, there were very few faults during the first three years when only two lines were installed. The first fault caused both lines to trip apparently due to too low relay settings. The next fault occurred when there was little load on Conowingo and synchronism was maintained. The remaining faults all occurred in 1931 after the third line had been added and after the new high-speed switches and relays were in service. Most of these faults were unusually severe involving two and three phases and in one case two lines, but they were cleared quickly and stability was maintained in every case.

FAULTS ON INTERCONNECTED SYSTEM

Faults occur frequently on the Pennsylvania Power and Light Company system particularly on the Wallenpaupack-Siegfried line as described in the reports of the lightning investigation on the 220-kv. system.[8,9] None of these faults has caused instability of Conowingo even though a three-phase short circuit occurred at Siegfried while Conowingo was carrying full load. This fault lasted about 52 cycles reducing the positive sequence voltage at Plymouth Meeting to 76 kv. or about 60 per cent of normal. The maximum swinging of Conowingo power after the disturbance was about 174 to 334 megawatts. This disturbance was far more severe than generally experienced with faults on the Pennsylvania Power and Light system which usually cause no power surges.

FIG. 7—OSCILLOGRAM OF SYSTEM DISTURBANCE MARCH 28, 1931

Stability maintained through transformer failure at Conowingo with full load on station.

FAULTS ON PHILADELPHIA ELECTRIC COMPANY MAIN SYSTEM

Faults on the Philadelphia Electric Company 66-kv. system excluding the Plymouth Meeting-Westmoreland lines usually cause little disturbance. A large part of the 66-kv. system consists of single conductor underground cable so that most of the faults involve only one phase. These faults sometimes last for more than one second, but there is seldom any trouble besides burning the cable and occasionally tripping some synchronous motors and converters. A system of pilot wire protection is being installed to reduce the clearing time for cable faults.

One case involving a two- or three-phase fault on the Schuylkill-Chester 66-kv. aerial line caused loss of synchronism in some of the machines at Conowingo as described in the section of this paper dealing with instability experienced with initial operation. There have been other cases of two- or three-phase faults on the 66-kv. system since the first year, but none of these has caused instability.

A three-phase short circuit occuring on the low-voltage side of a transformer at Westmoreland while Conowingo was fully loaded, which was cleared in about 30 cycles, caused very heavy power surges, but all generators remained in synchronism even at Delaware Station which is directly connected to the Westmoreland low-voltage (13.2 kv.) bus.

CONCLUSIONS

Based on experience in the Philadelphia Electric Company system, the following conclusions may be stated:

1. Faults on parts of the system other than the main lines from the Conowingo hydroelectric station have little effect on stability.
2. Single-phase faults on the main lines from Conowingo have no very serious effect on stability, but two- or three-phase faults under heavy load conditions are liable to cause instability unless cleared very quickly, and faults of this type are likely on the high-voltage system.
3. Installation of high-speed relays and circuit breakers on the high-voltage system has insured the rapid clearing of faults thereby practically eliminating the danger of instability except possibly for three-phase faults.
4. System stability may be calculated with sufficient accuracy for all practical purposes and such studies are essential for the design of the system.
5. Much credit is due the manufacturers for development of practical methods of studying stability and production of equipment for improving stability of operation.
6. Short-circuit tests on the system and special tests of equipment such as high-speed excitation fully justify the expense incurred.
7. Permanently installed automatic oscillographs are desirable for the following purposes:
   a. To investigate and analyze conditions when instability occurs.
   b. To check calculations of stability limits.
   c. To check relay operations.

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Discussion

J. B. MacNeill: Looking to the future, the more stable operation of large systems under fault conditions will result in large economies in the construction of such systems. At the present time the load that can be carried through a fault is determined by the speed of the protective devices. It has been estimated that in some systems 100 per cent additional load can be carried during storms by the use of sufficiently high-speed circuit breakers and relays over what was possible with corresponding apparatus a few years ago.

On the Conowingo system, I am informed 8-cycle operation of the switches will hold the system together on the majority of faults. This speed is obtainable even at 220,000 volts with the most modern apparatus at all fault currents which affect stability.

When sufficient data become available on high-speed operation of circuit breakers and relays, and enough is known regarding the nature of faults and the speed of deionization thereof, it is not too much to predict that automatic reclosing even of 220,000-volt circuit breakers may be possible. A study of the phenomena occurring both at the fault and in the circuit breaker will be necessary, and considerable operating experience over a period of time will be advisable before speedy reclosure can be considered. In time, however, the necessary data will be secured, and we can have the economies in system operation which automatic reclosing to main power breakers will secure.

An interesting feature of the development work done by Mr. Hentz's organization, was the use of 220,000 volts as a single-phase voltage for testing of oil circuit breakers. Some of the results of these tests have been presented to the Institute in the past. Of especial interest is the light these tests throw on the probable switching speeds for 330,000-volt systems, since 220,000 volts is considerably greater than the line-to-neutral voltage of 190,000 volts, of a 330,000-volt system. With present forms of construction, switching times for 330,000 volts may be approximately 12 cycles, as compared to 8 cycles for 220,000 volts. This will have a considerable effect on the stability of 330,000-volt systems. It is felt, however, that an extension of ideas now available for circuit breaker improvement may result in switching speeds for 330,000 volts comparable with those obtainable on 220,000 volts.

R. M. Spurck: This paper indicates that the stability problems of a large system have been solved. It is hoped that other operating companies who may have stability problems will study this paper and plan on obtaining similar operating data.

It seems worth while to bring out that curing stability problems by high-speed circuit clearing also brings with it many other benefits, such as reduced area of disturbance on the system during faults, reduction in voltage dips, and apparatus that requires less maintenance.

R. A. Hentz: One of the most significant matters in connection with the instability which we experienced is the large percentage (40 per cent) of faults on the Conowingo lines involving more than one wire. From all the information we could glean at the time the system was designed, single wire to ground were to be practically the only faults experienced and that other faults would be extremely rare, but it is evident that, in the region transversed by these lines and with the type of tower construction used, faults involving two wires to ground and even three wires to ground will occur and that this fact should be taken carefully into account in the design of any system.

It is proposed to continue the collection of the data incorporated in the paper and with the passing of years the accumulated experience so obtained will be of further help to those interested in stability problems.

The development of high-speed oil circuit breakers and relays referred to by Mr. MacNeill was the principal factor in eliminating the difficulties which we experienced during the first summer of operation. The fact that this can be accomplished in such very large breakers as those installed on the Conowingo lines, (rated 230 kv. and with 2,500,000 kva. interrupting capacity) gives promise that interruption of faults with rapidity enough to eliminate or at least minimize instability is possible for most other systems.

Radio Interference from Insulator Corona

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Synopsis.—*Extended laboratory tests reveal that the corona formation voltage and the initial radio interference voltage of clean, dry insulators are identical. Oscillograms show many interesting characteristics of insulator corona current and of the radio interference caused by it. Means for reducing the interference are suggested.*

RADIO interference originates on high voltage insulators from two principal causes: (1) from very small ionization or corona discharges due to overstressed air in the dielectric field; and (2) from intermittent leakage currents flowing through conducting deposits of various kinds on the insulator surfaces. Both of these causes of interference are of sufficient importance to warrant investigation. However, because the corona discharge type of interference is universally troublesome, and serious surface leakage is more or less localized in certain geographical areas which are subject to salt fogs or special industrial conditions, this investigation has been devoted to a study of interference due to corona.

The voltage gradient or stress on a dielectric transmitting dielectric flux is directly proportional to the flux density, and inversely proportional to the ease with which the insulating material transmits flux. Therefore when two dielectrics are transmitting the same dielectric flux in series, the dielectric having the lowest permittivity obviously will be subjected to the highest voltage gradient per unit thickness. The insulation of high voltage power lines consists of two dielectrics, air and some other solid insulating material such as porcelain or glass. In their practical application, these gaseous and solid materials are combined in series and parallel arrangements in the dielectric circuit.

Air and solid materials have widely different dielectric characteristics. High voltage insulator porcelain transmits dielectric flux approximately six times as readily as air and has a dielectric strength several times greater. Glass used in modern high voltage insulators has a permittivity four and one-half times that of air, and, like porcelain, also has a dielectric strength several times that of air. Because of the widely different permittivities, exceedingly careful design is necessary to use air and porcelain or air and glass in series combinations with high impressed voltages. Thin air-gaps in series with thick sections of porcelain or glass must be avoided in regions of dense dielectric flux, or failure of the air by ionization will result. This undesirable condition is found in three different places on many of the standard multi-shell pin type insulators as now designed, as follows:

1. At points where the conductor and tie wire are not in intimate contact with insulator head.
2. At the contact between the pin or thimble and the insulating material.
3. Between shells of multi-part insulators.

When insulators so designed are used on high voltage lines, the air in these regions of high voltage gradient fails due to the ionization-by-collision phenomenon. This air failure occurs very suddenly at the time the critical ionizing gradient is reached each half cycle of the alternating voltage impressed on the insulator. Because of the changed energy storage in the capacitance and inductance of the electric circuit the sudden flow of ionizing current resulting from the corona formation sets up oscillations in the electric circuit. The oscillations thus generated cover a wide band of frequencies from the lower range of audibility through the higher radio frequencies. These higher frequency radiations cause radio interference.

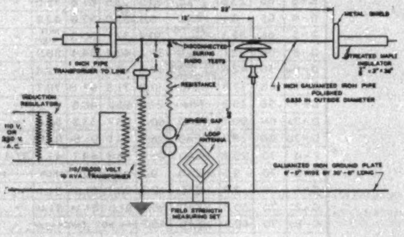

FIG. 1—SCHEMATIC DIAGRAM OF APPARATUS FOR LABORATORY INVESTIGATION OF RADIO INTERFERENCE AND CORONA FORMATION VOLTAGES ON PIN INSULATORS

Laboratory tests show conclusively that radio interference from clean, high-voltage insulators is caused by corona discharges occurring at locations on the insulator where the air is ionized by excessive dielectric flux concentration. Large numbers of tests conducted in a darkened room showed that corona formation and radio interference always are coincident. To obtain this result, great care must be used in performing the tests. The radio receiving set must have high amplification, the loop antenna must be closely coupled with the circuit radiating the interference, and the corona observations made very carefully in a dark room. Obviously

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when the corona formation first occurs under a conductor, behind a tie wire, or in the pin-hole of an insulator, it cannot be observed. Radio noise will occur coincident with this hidden corona and therefore precedes the first observable appearance of corona. The fact that corona formation and radio interference are coincident was determined by a great many tests using insulators so designed and treated that the first corona occurs in locations easily observed in a darkened room. Some of these test data are shown in Table I.

Radio Interference and Visual Corona Formation Voltages

At the time the investigation of radio interference was started, it was known that in general the interference from pin type insulators is more pronounced than that from the suspension type; also, that more interference was radiated from high voltage than from low voltage pin type insulators. For these reasons, practically all tests have been confined to pin and pedestal insulators rated at from 66 to 73 kv. The radio interference and visual corona formation voltage tests were made with apparatus connected as shown in Fig. 1. Great care was exercised to avoid extraneous interference from the testing equipment. To prevent interference from minute discharges between metallic parts in close proximity to each other, all pieces of hardware associated with the experimental apparatus were bonded carefully either to the high voltage conductor or to the ground. A polished ½-in. galvanized iron pipe (0.83 in. outside diameter) shielded at each end, was used as the conductor; this pipe was free from corona for all voltages up to 100 kv.

Table I—Radio Interference and Corona Formation Voltage Data for Pin Insulators

INSULATOR NUMBER	NOMINAL RATING KV	INITIAL RADIO INTERFERENCE DATA		INSULATOR VISIBLE CORONA DATA					REMARKS
		INSULATOR POTENTIAL KV	LOCATION ON INSULATOR	HEAD KV	PIN KV	SHELLS* 1-2 KV	SHELLS* 2-3 KV	SHELLS* 3-4 KV	MEAN ATMOSPHERIC CONDITION BAROMETRIC PRESSURE 757.2 MM. OF HG. TEMPERATURE 21.3 DEGREES CENTIGRADE WET BULB TEMPERATURE 55.3 DEGREES FAHRENHEIT DRY BULB TEMPERATURE 70.4 DEGREES FAHRENHEIT RELATIVE HUMIDITY 36 PER CENT
A-4	66	20.4	PIN	20.4	18.4	30.5	25.7	-	Solder coating in conductor and tie wire grooves.
B-5	66	23.0	HEAD	23.0	32.7	62.2	86.6	-	Silver and Bismuth coating on head. Zinc thimble in pin hole.
B-6	66	23.5	HEAD & PIN	23.5	23.5	67.6	75.0	-	Silver and Bismuth coating on head and in pin hole.
D-4	66	8.1	HEAD	8.5	52.0	53.6	42.9	-	Standard porcelain insulator with ¾ in. malleable thimble.
D-5	66	29.3	CAP TO PORC.	30.2	37.1	50.3	NOT TAKEN	-	Malleable cap on head ¾ in. malleable thimble, discharge cap to porcelain.
D-6	66	32.1	CAP TO PORC.	32.1	40.4	54.4	58.9	-	Malleable cap on head ¾ in. malleable thimble, discharge cap to porcelain.
D-6a	66	42.4	BETWEEN SHELLS 2&3	47.6	45.7	43.4	42.4	-	Crevices head and thimble filled with Portland Cement.
D-6b	66	17.9	HEAD	17.9	71.5	97.1+	70.7	-	Crevices head and thimble filled with Portland Cement, between shells with Pothead Compound.
D-7	66	25.2	PIN	27.0	25.2	46.5	51.9	-	Malleable cap on head ¾ in. malleable thimble, discharge cap to porcelain.
D-7a	66	60.7	HEAD	60.7	86.7	53.8	56.8	-	Crevices head and thimble filled with Pothead Compd. Discharge at head.
D-7b	66	38.7	HEAD	38.7	64.1	91.6+	91.6+	-	Crevices at head, shells, and thimble filled with Pothead Compound. Discharge over compound at head 48.3 KV.
F-4	70	25.2	HEAD	25.5	38.3	44.7	41.1	37.4	Sanded head coated with electrolytic copper ¾ in. malleable thimble.
F-5	70	9.5	HEAD	9.5	← NOT TAKEN →				Same as F-4. Discharge in fault in metal coating.
G-1	70	12.3	HEAD	12.3	38.5	81.1+	81.1+	-	Standard Insulator. Corona on head at tie and in pin hole around metal.
G-2	70	50.3	HEAD	50.3	81.1+	81.1+	81.1+	-	"Nostatic" coating on head and in pin hole.
G-2a	70	40.3	HEAD	40.3	← NOT TAKEN →			-	After 10 A.I.E.E. Std thermal test cycles, 35 impulse flashovers at 450Kv maximum, and five hours heavy conductor and tie wire vibration, corona first formed where impulse arcs destroyed the conducting coating on the head.
G-3	70	59	HEAD	59.0	80.0+	80.0+	80.0+	-	"Nostatic" coating on head and in pin hole.

*Shells numbered from head to pin

Tests on Pin Insulators

A few representative determinations of the radio interference and corona formation voltages for pin insulators are given in Table I. The tabulated values show that radio interference and visible corona occur at the same value of insulator voltage except when the corona starts in locations not in view.

The unshielded insulators started radiating interference and showed visible corona at the surprisingly low voltages of from 8.0 to 12.2 kv. to neutral or approximately 25 per cent of the normal operating voltage. However, all of the shielded insulators except F-5 showed a marked improvement in the radio interference formation point, and only one insulator G-2 was free from radio interference at normal operating voltage.

The lower case subscript letters (Table I) indicate insulators treated or modified as shown under the remarks in the table.

It may be observed that improvements were effected by sealing all crevices with cement or compound; the radio interference voltage was increased thereby from 32 to 123 per cent.

Fundamentals of Preventing Insulator Radio Interference

Because of the nature of the disturbing electrical discharges on insulators there are at least three methods of attack to eliminate them. These are to:

1. Design the insulator in such a manner that no regions of overstressed air exist under normal operating conditions.

2. Provide conducting coatings to act as dielectric

flux distributors at the head, in the pin-hole, and on other necessary parts.

3. Displace the overstressed air with an insulating compound dielectrically strong enough to withstand the potential.

From at least one standpoint, method 1 is the most desirable of the three because it employs no auxiliary material which may be subject to deterioration in service. This method, however, is the most radical departure from present practise and therefore probably will require a longer development period if found practicable at all. If method 2 employing a dielectric flux distributor is used, the conducting material must be in such intimate contact with the solid dielectric that all air between the conducting coating and the porcelain or glass is excluded. The use of method 3 requires that all air in regions subject to being overstressed to the ionizing point be displaced by an insulating compound.

Some difficulties are encountered in employing any of these three methods. Conducting coatings must remain at all times in intimate contact with the insulator material, and must withstand weathering, electrolysis due to leakage currents, temperature changes, and abrasion due to conductor and tie wire motion. Further such coatings must not terminate in positions that will overstress the air in other parts of the dielectric circuit, and the flashover value of the insulator must not be reduced below the required factor of safety. If insulating compound is used, it must withstand weathering, stay in place through the extremes of temperature to which it may be subjected, and not be displaced by the motion of the conductor and tie wire to the extent of impairing its effectiveness.

Although the design of a satisfactory insulator of practical proportions and which will be suitable for mass production apparently is not a simple problem, it is believed to be entirely possible to meet the requirements outlined and build an insulator entirely free from radio interference at a safe margin above its normal operating voltage.

INSULATOR CORONA CURRENT INVESTIGATION

The magnitude and character of the corona current of high voltage insulators is of great interest and importance because these characteristics are direct indications of the radio interference radiation produced. Corona current is not easily studied because (1) it is quite small, attaining maximum values of approximately 40 microamperes for normal voltage on a standard 66-kv. insulator; (2) it is obscured by the insulator charging current which is approximately 0.4 milliampere maximum or 10 times the corona current for an insulator of this size; and (3) it has superimposed oscillations of very high frequency.

An instrument suitable for the investigation of currents of these magnitudes and characteristics must have a high sensitivity and be capable of responding faithfully to high frequencies. The device that meets these requirements most satisfactorily is the low voltage cathode ray oscillograph. Because of the low anode-cathode voltage and resulting low velocity of the electron stream, this instrument has the highest sensitivity of all of the cathode ray oscillographs; and the cathode beam diffusion due to the relative immobility of the positive focusing ions is not excessive for beam speeds of 3×10^6 cm. per sec. on the fluorescent screen.

Although the cathode ray oscillograph is well suited to the study of high frequency discharges, it will not of itself separate the corona current from the charging current. To overcome this difficulty the circuit shown in Fig. 2 was devised to balance out the charging current leaving only the corona current to produce deflection. With this circuit arrangement, it is possible by adjusting R_1 and C_3 to make the vertical deflecting potential on the oscillograph zero for all insulator voltages up to the critical corona formation voltage. At this critical voltage and for higher voltages, the oscillograph vertical deflecting voltage is proportional to the instantaneous value of the insulator corona current in R_2. This condition obtains because C_1 is free from corona. A glass insulator with dielectric flux distributing coatings on the head and in the pin-hole, and found by test to be free from corona up to 50 kv., was used for C_1 when the accompanying cathode ray oscillograms were taken.

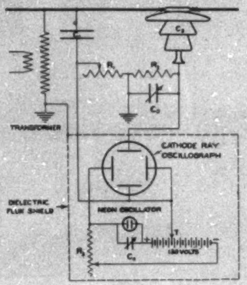

FIG. 2—CIRCUIT FOR OBTAINING CATHODE RAY OSCILLOGRAMS OF INSULATOR CORONA CURRENT. NOTE THAT THE INSULATOR CHARGING CURRENT IS BALANCED OUT

The horizontal deflecting or sweeping voltage for the oscillograph was obtained from a neon oscillator connected as shown in Fig. 2. This type of oscillator produces practically a straight line time axis that can be operated over a wide range of sweeping frequencies by the adjustment of R_3 and C_4. This sweeping oscillator can be synchronized readily with the frequency producing the phenomena under observation so that a stationary image either for observation or recording photographically is obtained. To eliminate stray dielectric fields a grounded wire cage 5x5x6 ft. high was used to shield all apparatus associated with the oscillograph and neon oscillator.

Voltage was applied to the laboratory line and increased to a value just below the critical corona voltage for the insulator under test. At this voltage, resistance R_1 and capacitance C_2 were adjusted to make the vertical deflecting voltage zero. These adjustments were made while the neon oscillator was sweeping the cathode beam over the time axis, conditions under which any slight inaccuracies in balance were detected readily by the deviation of the beam from a straight line. When this balance was established accurately, a series of corona current oscillograms was taken for the insulator at various conductor to pin voltages from 10 to 40 kv., r.m.s., in increments of 5 kv.

In Fig. 4 is shown a series of corona current oscillograms taken on a standard three-shell 66-kv. porcelain insulator with a ¾-in. malleable thimble cemented in the pin-hole. Oscillogram *G* (40 kv.) shows the normal corona current for this insulator operating on a three-phase transmission line at a line potential of 69.3 kv. Atmospheric conditions were representative and normal, air density being 1.006 and the absolute humidity 9.4 g. per cubic meter.

Fig. 3—Apparatus for Obtaining Cathode Ray Oscillograms of Insulator Corona Current

Oscillograms reproduced in Fig. 4 show that when the ionization gradient is reached, the current rises to a maximum value quite abruptly during the positive half cycle and decays rapidly with considerable oscillation, reaching zero practically coincident with the occurrence of maximum voltage on the insulator. During the negative half cycle the corona current does not reach its maximum value in so short a time interval as in the positive half cycle, the oscillations appear to be much more violent, and the corona current is sustained at a high value for a longer time. The higher sustained value of negative current is shown clearly by the fact that for the same line voltage the area under the negative wave is greater than under the positive wave.

Unfortunately the oscillograms do not show the oscillations clearly. This is because that in order to form an image on photographic film it is necessary to trace several successive cycles of corona current on the fluorescent screen. On account of the chance character of corona formation, successive cycles of corona current are not alike, and where the oscillations are the greatest, the variation in successive figures is largest. This variation in the registration of successive cycles is shown on the oscillogram by variations in intensity of exposure and sharpness of the image.

Corona current oscillograms were taken also on a special three-shell 66-kv. porcelain insulator with a silver and bismuth coating on the head and in the pin-hole. This insulator (No. B-6 of Table I) was practically identical with the previous one except for the addition of the metallic coatings. In this case no corona current was observed below 30 kv. The difference in the character of positive and negative corona currents on this insulator was greater than for the standard insulators. The positive current shows much less oscillation than the negative, and the duration of the positive current was observed to be much shorter than that of the negative.

Metallic dielectric flux distributors on this special insulator greatly improved the characteristics from the standpoint of radio interference. The interference starting voltage was increased from slightly above 10 kv. to 23.5 kv., and the maximum corona current at 40 kv. was reduced from 40 to 11 microamperes. However, the voltage gradient was noted to be excessive at the edges of the metallic coatings on the head and in the pin-hole at 23.5 kv., which is only 61.6 per cent of normal operating voltage.

Similar oscillograms were taken for a standard 70-kv. one-piece glass insulator without any treatment to prevent corona formation. These oscillograms show the same general corona current characteristics obtained with the standard porcelain insulator. Cathode ray oscillograms for a similar glass insulator with high resistance iridescent dielectric flux distributors on the head and in the pin-hole revealed no corona current for voltages up to 40 kv. when these coatings were in good condition.

Oscillographic Study of Radio Receiver Output

In order to determine the value of the cathode ray oscillograms of insulator corona currents for indicating the interfering noise that will be produced in a radio receiver, a large number of both Duddell and cathode ray oscillograms was taken of the audio frequency output from a superheterodyne receiver. These were taken with the receiver tuned for a frequency of 1,000 kc. and receiving only the radiation from the laboratory line equipped with one insulator. The circuit conditions and voltages used were the same as those employed when the cathode ray oscillograms of corona current discussed previously were taken.

A typical set of oscillograms for this portion of the investigation may be seen in Fig. 5. In these the polarity and magnitude of the insulator voltage as well as the audio frequency current output of the radio receiver are shown for the standard, three-shell, 66-kv. porcelain insulator for which corona current oscillograms were shown in Fig. 4. It should be observed that the oscillations in the audio output have frequencies reaching values of 5,000 cycles per sec., which is the natural period of oscillation for the oscillograph vibrator element. Therefore the amplitude indications are not very reliable. Notwithstanding this fact, the oscillograms show accurately the beginning of the corona discharge and quite accurately the end of the corona discharge with reference to each half cycle of the insulator voltage. At 14.1 kv. maximum, there is just the slightest trace of interference, occurring at practically the crests of the voltage waves. At 21.2 kv. maximum, the corona starts at a very small electrical angle and at 28.3 kv. maximum, it appears at the zero point of the voltage wave. For higher voltages the corona starts before the voltage wave reverses polarity. For all values of voltage the corona discharge stops at the crest of the voltage wave. This is shown by the fact that at that point the oscillations stop. At the higher voltages the smooth decay of current following the oscillations is due to the recovery of the receiving set from overloading.

The phenomenon of obtaining critical ionizing voltage gradients on the insulator at zero, and even with opposing conductor potentials, is due to the large space charge established in the space and on the insulator surface surrounding the conducting materials where ionization has taken place during the previous half cycle. This is brought out in greater detail in the following discussion.

A THEORY OF THE FORMATION OF INSULATOR CORONA

A theory of the phenomena associated with corona formation on an insulator can be developed by assuming the initial half cycle of conductor voltage to be either positive or negative. For the following discussion the initial polarity is assumed to be negative and the ionization of the air surrounding the conducting parts and in zones of high voltage stress is assumed to be only the normal atmospheric value at the instant the voltage first is applied.

During the initial negative half cycle, the dielectric field of the negatively charged conductor repels the free electrons and negative ions in the regions of high voltage gradient; when the critical ionizing voltage gradient is reached, they bombard the air molecules with such violence that ionization by collision results. The additional electrons and negative ions produced by collision are repelled from the conducting material by the electric field, and form the major part of an outside negative space charge. The heavy immobile positive ions move relatively slowly toward the conductor in the regions of ionization. In this manner corona increases and the positive ion space charge that accumulates due to the relative immobility of the positive ions subtracts from the negative field of the conductor until the crest of the voltage wave is reached. At this point ionization stops because the space occupied by the corona is completely ionized; the voltage gradient at the outside boundary of the corona discharge is just equal to the critical value and must decrease below this value as the conductor voltage decreases. For the negative conductor condition under

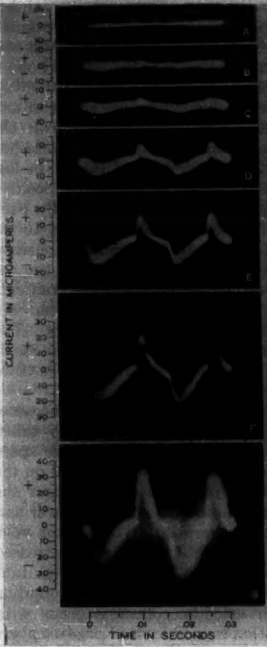

FIG. 4—CATHODE RAY OSCILLOGRAMS OF CORONA CURRENT FOR A STANDARD 66-KV. PORCELAIN PIN TYPE INSULATOR

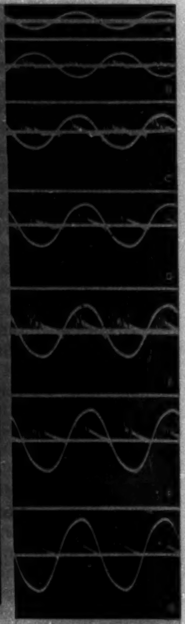

FIG. 5—INSULATOR VOLTAGE (60-CYCLE) AND OUTPUT OF RADIO RECEIVER SHOWING NOISE FROM CORONA ON INSULATOR OF FIG. 4

	Insulator kv. conductor to pin r.m.s.	Insulator kv. conductor to pin maximum
A—	10	14.1
B—	15	21.5
C—	20	28.3
D—	25	35.4
E—	30	42.5
F—	35	49.5
G—	40	56.6

Test conditions (Figs. 4 and 5)	
Barometric pressure	756.0 mm.
Temperature, dry bulb	21.6 deg. cent.
wet bulb	15.3 deg. cent.
Relative humidity	50.0 per cent
Absolute humidity	94.0 g. per cu.m.

consideration the positive space charge limits the corona to a comparatively small discharge distance. Many of the free electrons are swept out of the ionization zone and the loss of positive ions by recombination adjacent to the conductor thereby is reduced to a relatively small number. Large numbers of positive ions always are present therefore adjacent to the conductor; this is such a favorable position that a comparatively small positive space charge will reduce greatly the field of a large negative conductor voltage at the outside boundary of the corona discharge.

After the negative voltage wave passes the crest and starts to decrease, the negative space charge at the outer boundary of the corona no longer is replenished by new ionization; hence it begins to diffuse and be lost by recombination. This action releases the positive space charge and allows it to move more rapidly back toward the conductor, which movement tends to increase the gradient in the space between the positive space charge and the conductor. Because of the decrease in conductor potential, ionizing voltage gradients prior to actual reversal of conductor potential are reached only when the space charge is very large. The latter phenomenon does occur, however, for all maximum applied voltages in excess of approximately twice the maximum visual critical corona voltage.

When the conductor voltage reverses and becomes positive after the described phenomena have taken place, the dielectric flux of the residual positive space charge adds to the field of the positive potential on the conductor. The addition of those fluxes causes the critical ionizing voltage gradient to occur at lower and lower values of instantaneous positive conductor voltages as the maximum conductor voltage is increased above the visual critical corona value. The resultant positive field accelerates the electrons in the surrounding space, and when high velocity collisions occur with molecules of air, ionization is produced. Large numbers of the electrons liberated are swept from the field and removed by the conductor, leaving more immobile positive ions to add to the positive space charge. This increase in space charge increases the dielectric flux density and extends the ionizing voltage gradient to a far greater distance from the conductor than would be possible with the conductor voltage acting alone. When the crest of the positive voltage wave is reached, the voltage gradient is at maximum and the corona discharge stops because the space occupied by the corona is completely ionized; the decrease in the conductor voltage makes ionization at a greater distance from the conductor impossible. As the conductor potential decreases it drops below space charge potential and a field is established between space charge and conductor. The field produces ionizing voltage gradients before the conductor voltage reverses, when the space charge potentials are very high. This phenomenon occurs for maximum applied voltages in excess of approximately twice the maximum visual critical corona voltage.

During the rise in voltage on the positive half cycle a much larger positive space charge is established than that produced during the same period of the negative half cycle. This is because when the conductor is positive, the positive space charge flux adds to the conductor field and extends the ionization gradient, when it is negative, the positive space charge flux subtracts from the negative conductor field at the outside corona boundary.

When the conductor voltage reverses and becomes negative following the establishment of a large positive space charge in the manner just described, a very high voltage gradient is produced between the negative conductor and the residual positive space charge because of the very short distance between them. When this voltage gradient reaches the critical ionizing value (which may occur before the voltage actually becomes negative) breakdown occurs in the form of minute arcs which neutralize the space charge. These corona arcs produce sudden changes in the electric circuit conditions, thereby causing high frequency oscillations of a violent character.

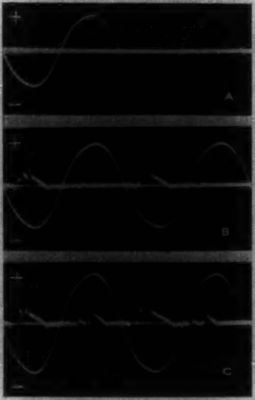

FIG. 6—INSULATOR VOLTAGE (60 CYCLE) AND OUTPUT OF RADIO RECEIVER SHOWING MUCH MORE INTERFERENCE DURING THE NEGATIVE THAN DURING THE POSITIVE HALF CYCLES. PORCELAIN INSULATOR (66 KV.) WITH SILVER AND BISMUTH COATING ON HEAD AND IN THE PIN-HOLE

	Insulator kv. conductor to pin	
	r.m.s.	maximum
A—	30	42.5
B—	35	49.5
C—	40	56.6

Barometric pressure	760.0 mm.
Temperature dry bulb	21.5 deg. cent.
wet bulb	16.0 deg. cent.
Relative humidity	56.0 per cent
Absolute humidity	10.6 g. per cu.m.

RADIO INTERFERENCE FROM POSITIVE AND NEGATIVE CORONA

Near the critical corona formation voltage the electric circuit oscillations and the electromagnetic radiation resulting from them are inherently much worse for 60-cycle a-c. circuits when the conductor is negative than when it is positive. Theoretical considerations outlined in the preceding paragraphs indicate that this condition is to be expected and experimental investigations show a very decided polarity effect under certain conditions. For the special 66-kv. porcelain insulator with silver and bismuth coatings on the head and in the pin-hole, oscillograms taken show a very striking polarity effect in the interference radiation. (See Fig. 6.)

From the experimental investigations it appears that the marked difference in radio interference when the line is positive and when it is negative, respectively, can be observed only when the insulator has a definite critical corona formation voltage. When the voltage is increased far above this critical value, the polarity effect becomes less marked and at still higher voltages probably would disappear almost entirely. This phenomenon accounts for the fact that practically no interference polarity effect was observable in the tests on standard insulators. (See Fig. 5.) On such insulators there is a heterogeneous group of visual critical voltage gradients that cause corona to form in small areas at various locations for every different value of insulator voltages from approximately 25 per cent normal operating voltage up to the flashover value. There the polarity effect is obscured because when sufficient voltage is applied to produce appreciable corona on the insulator, the regions that go into corona at low voltage are subjected to an excessive voltage gradient and therefore produce practically the same interference on both positive and negative half cycles.

SUMMARY

The findings of this study may be summarized as follows:

1. Standard multi-shell pin type insulators of conventional design are susceptible to corona formation at three different points.

2. Radio interference and visible corona formation for clean insulators start at the same voltage.

3. Radio interference from insulator corona may be reduced by (a) designing the insulator so as to eliminate regions of overstressed air; (b) providing conducting coatings on different parts to act as dielectric flux distributors; and (c) displacing the overstressed air with an insulating compound.

4. Insulator corona current can be separated from the charging current and studied independently by means of the cathode ray oscillograph.

5. Duddell oscillograms of the line voltage, and the audio frequency output of a radio set receiving the radiation from an insulator or other source of corona, provide an excellent means for studying certain corona characteristics.

6. The radio interference radiation from an insulator having a definite critical corona voltage shows a decided polarity effect for all corona voltages up to approximately twice the critical disruptive value.

ACKNOWLEDGMENTS

The writer is indebted to his associates in this work, Prof. E. C. Starr and Mr. L. T. Fisher, for their suggestions, contributions, and active participation in the investigation. The work has been made possible through the cooperation of the engineering experiment station (Oregon State College) under the direction of Dean H. S. Rogers and the foreign systems coordination committee of the National Electric Light Association, represented by Mr. H. H. Schoolfield.

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Predetermination of the A-C. Characteristics of Dielectrics

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***Synopsis.**—From the charge and discharge currents of any type of dielectric under continuous potential it is possible to predict accurately the loss, power factor and capacity at 60 cycles. This is done by an empirical determination of the equation for the relaxation function of the dielectric at a given temperature, followed by the application of von Schweidler's method. The method developed is available at any frequency, provided that the continuous potential charge and discharge currents may be measured over initial time intervals comparable with the alternating period.*

It is shown that agreement of the relaxation function with the t^{-n} expression is not general, but on the contrary, it is confined to dielectrics showing negligible or no irreversible conduction. The inadequacy of the t^{-n} expression is proved on experimental and analytical reasons. Chief among them is the failure of the t^{-n} expression to predict accurately the variation of dielectric loss with frequency.

A convenient and sufficient expression for the relaxation function is shown to be a sum of three exponentials. Further, experiment and analysis prove that the method of three exponentials predicts accurately the a-c. behavior of a dielectric at 60 cycles.

The usual forms of irreversible conduction encountered are defined and classified. The case where the initial constant current does not obey Ohm's law has been considered analytically in its contribution to the a-c. behavior.

The manner in which the complete a-c. behavior of a dielectric is accurately predicted from suitable d-c. measurements is illustrated by three typical sets of experimental data; paper, oil, and impregnated paper.

* * * * *

INTRODUCTION

THE purpose of this paper is to give the experimental proof that the a-c. behavior of a dielectric at 60 cycles may be accurately predicted from suitable d-c. measurements. The fact that dielectric loss may be explained in terms of reversible absorption, or residual charge, and anomalous conduction, as observed under continuous potential, has led to several well known extensions to the alternating case of various theories of dielectric absorption. Notable among these are Wagner's[1] extension of Maxwell's stratified dielectric and von Schweidler's[2] extension of Pellat's theory. von Schweidler has deduced the expression for the dielectric loss in an alternating field from the equation of the empirical absorption current-time relation as observed under continuous potential. The work of F. Tank,[3] 1915, apparently represents the first effort made using von Schweidler's method for a direct check between measured and computed losses. Since then other checks on this method have been made by Whitehead,[4] Benedict,[5] and others. This paper discusses further the possibilities and limitations.

The continuous potential measurements, using the amplifier-oscillograph, give records on photographic films of the charge and discharge current-time relations for the interval of time 0.001 to 0.040 sec. after the application of voltage or of short-circuit. The geometric transient, both on charge and discharge, confined by choice of circuit constants to extremely short intervals of time is, therefore, absent from such records. This permits the accurate determination of the reversible absorption current directly from the discharge records and of the irreversible conduction current as the difference between the corresponding ordinates of charge and discharge.[6] The usual measurements of charge and discharge current taken with a galvanometer and beginning about a minute after the application of voltage or of short-circuit, give values which in most cases have no bearing on the a-c. behavior at commercial frequencies. The condition for suitable d-c. measurements for the accurate prediction of the a-c. behavior is that the range of time covered by the d-c. measurements must be of the same order of magnitude as the period of the alternating wave.

VON SCHWEIDLER'S ANALYSIS

The several components of alternating current in a dielectric at a given frequency may conveniently be represented in a vector diagram. Fig. 1 shows the familiar Wagner diagram for a capacitor exhibiting the anomalous properties of reversible absorption and irreversible conduction in addition to the normal property of specific inductive capacity. von Schweidler's method permits the computation from the curves of d-c. charge and discharge currents of each of the separate components, thereby giving dielectric loss, phase difference, and the increase in capacity due to absorption. Under continuous potential the irreversible conduction current is usually characterized by a constant initial conductivity which is independent of the applied stress for low stresses and which is almost invariably larger than the final or normal conductivity. The contribution of the initial irreversible conduction to the alternating case is discussed in a later section. Reversible absorption under continuous potential is characterized by a decaying current function of time

$$i_r(t) = EC_0\phi(t) \qquad (1)$$

where E is the applied voltage, C_0 the geometric ca-

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pacitance, and $\phi(t)$ the relaxation function corresponding to the dielectric at a given temperature; it is measured directly in the d-c. discharge curve. Equation (**1**) embodies Curie's[8] well known laws; namely, that the ordinates of the absorption current-time relation be directly proportional to the applied voltage and to the dimensions of the capacitor. If these laws are obeyed then the Hopkinson[7]-Curie[8] principle of superposition is applicable to the determination of the current under a varying voltage. von Schweidler's fundamental equation for the current, as dependent on any e.m.f. variation $E(\tau)$ in the past, and as based on the principle of superposition, is

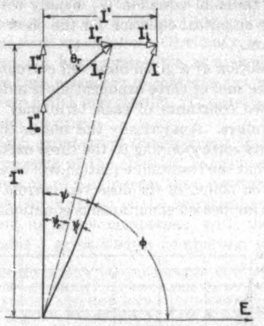

FIG. 1—VECTOR DIAGRAM OF A CAPACITOR SHOWING REVERSIBLE ABSORPTION AND IRREVERSIBLE CONDUCTION IN ADDITION TO SPECIFIC INDUCTIVE CAPACITY

Current Vector	Contribution of
I''_o	Geometric Capacitance
I_r	Reversible Absorption
I'_i	Irreversible Conduction

$$i(t) = E(o)\phi(t) + \int_0^t \frac{d}{d\tau} E(\tau)\phi(t-\tau)\, d\tau \qquad (2)$$

which he applies to determine the steady state for the case of the alternating sinusoidal applied potential. The additional admittance contributed to a capacitor by reversible absorption, in complex notation, is shown to be:

$$Y_r = \omega C_0(B + iA) \qquad (3)$$

where $\omega = 2\pi \times$ frequency and

$$\left.\begin{aligned} A &= \int_0^\infty \cos \omega t \,.\, \phi(t) \,.\, dt \\ B &= \int_0^\infty \sin \omega t \,.\, \phi(t) \,.\, dt \end{aligned}\right\} \qquad (4)$$

The Relaxation Function

Hopkinson, Curie, Tank, and others have reported experimental agreement of the relaxation function with the expression:

$$\phi(t) = \beta t^{-n} \qquad (5)$$

von Schweidler gives in this case the values of the infinite integrals of (**4**), as:

$$\left.\begin{aligned} A &= \omega^{n-1}\beta\Gamma(1-n)\cos\frac{(1-n)\pi}{2} \\ &\text{converges for } o<n<1 \\ B &= \omega^{n-1}\beta\Gamma(1-n)\sin\frac{(1-n)\pi}{2} \\ &\text{converges for } o<n<2 \end{aligned}\right\} \qquad (6)$$

In all the cases mentioned above the values of n given lie between 0 and 1. Benedict[5] shows from experiments with solid dielectrics the same type of agreement up to 100 milliseconds. Whitehead,[4] however, has reported several cases of values of $n>1$, which introduces a serious objection to the use of the type of relaxation function given in equation (**5**). As an example, in tests on commercial abietic acid over a range of temperature, Whitehead[9] has found that the discharge current-time relation is of the t^{-n} type with values of n ranging from 0.1 to 2.5.

In spite of the wide spread experimental agreement with the t^{-n} expression it is found to be inadequate for the accurate prediction of the a-c. behavior for the following reasons:

1. Analytically, when $n>1$, the value of the A integral in (**6**) above does not converge and it becomes impossible to compute the increase in capacity due to absorption.

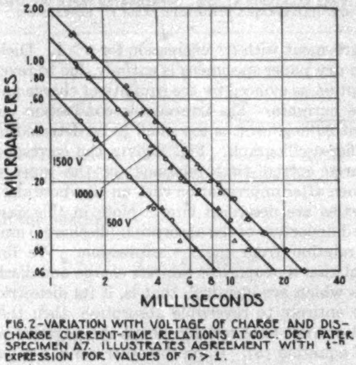

FIG. 2—VARIATION WITH VOLTAGE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 60°C. DRY PAPER SPECIMEN A7. ILLUSTRATES AGREEMENT WITH t^{-n} EXPRESSION FOR VALUES OF $n > 1$.

—○— Charge
—△— Discharge

2. Examination of the expression for the B integral discloses a mode of variation of power factor with frequency which is contrary to both theory and experiment. The computed power factor will decrease, remain constant, or increase with frequency depending on whether the value of n is less than one, equal to one, or greater than one, but in no case will it show a power

factor maximum as required by experience based on well accepted theories.

3. Experimentally, the negative power of time calling as it does for infinite current at zero time, gives rise to computed losses which are usually in excess of the measured losses.

4. Finally, the t^{-n} expression is by no means general, for noted departures are found in many cases.

Fig. 2 gives the current-time relations at three voltages for a specimen of dry, unimpregnated paper showing agreement with t^{-n} expression for $n > 1$. Dielectric loss in dry paper specimens is entirely due to reversible absorption as evinced by the equality of charge and discharge currents. The irreversible conduction current even at 60 deg. cent. is too small to be detected by the amplifier-oscillograph. Fig. 3 gives the corresponding discharge current-time relations for the same paper specimen after impregnation with an oil whose electrical properties are described later. Note in this case the decided departure of the absorption (discharge) current-time relation from the t^{-n} expression. We find in general, that if a capacitor exhibits charge and discharge curves which are identical, that is, if its dielectric loss is due entirely to reversible absorption, then the empirical current-time relation obeys the negative power law of equation (5). If, on the other hand, a capacitor shows measurable irreversible conduction, or if it is made to acquire it, either through elevation in temperature or as in the case of a paper specimen, through impregnation, then the observed discharge current-time relation will differ materially from the simple t^{-n} expression.

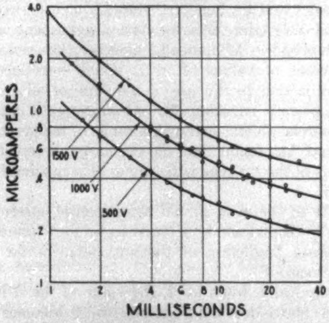

FIG. 3—VARIATION WITH VOLTAGE OF DISCHARGE CURRENT TIME RELATIONS AT 60°C. IMPREGNATED PAPER SPECIMEN C7. ILLUSTRATES DEPARTURE FROM t^{-n} EXPRESSION.

THE METHOD OF THREE EXPONENTIALS

A more general expression for the relaxation function which is not open to the analytical objections of the t^{-n} type is a series of exponentials of the form

$$\phi(t) = \beta_1 e^{-\alpha_1 t} + \beta_2 e^{-\alpha_2 t} + \ldots + \beta_n e^{-\alpha_n t} \tag{7}$$

for which the A and B integrals of (4) become

$$\left.\begin{aligned} A &= \frac{\alpha_1\beta_1}{\omega^2 + \alpha_1^2} + \frac{\alpha_2\beta_2}{\omega^2 + \alpha_2^2} + \ldots + \frac{\alpha_n\beta_n}{\omega^2 + \alpha_n^2} \\ B &= \frac{\omega\beta_1}{\omega^2 + \alpha_1^2} + \frac{\omega\beta_2}{\omega^2 + \alpha_2^2} + \ldots + \frac{\omega\beta_n}{\omega^2 + \alpha_n^2} \end{aligned}\right\} \tag{8}$$

Hopkinson, Steinmetz, Wagner, and others have found that three terms of equation (7) usually will suffice to express the empirical equation for the observed relaxation function.

The resolution of a given observed current-time relation into the sum of three exponentials is indeterminate and the two constants of each term may take wide ranges of values. Analytically this means that a set of six constants corresponding to the three exponentials is so chosen that the resultant equation will be satisfied by any six given points of the observed current-time relation. This implies six simultaneous equations involving transcendental functions, the solution of which can only be obtained by graphical means. To illustrate with a concrete example, consider the 1,500-volt curve of Fig. 2 and resolve it into three exponentials by the method of successive residuals as outlined by J. Lipka.[10]

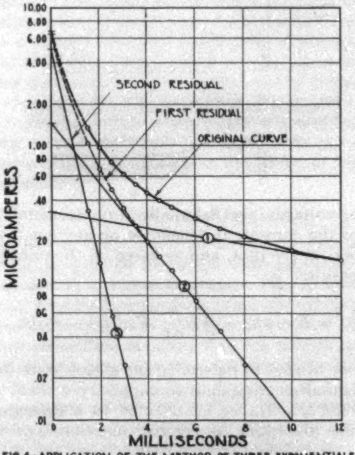

FIG. 4—APPLICATION OF THE METHOD OF THREE EXPONENTIALS TO A CHARGE AND DISCHARGE CURRENT-TIME RELATION AT 1500 VOLTS. DRY PAPER SPECIMEN A7 AT 60°C

1. Redraw the curve in semilogarithmic coordinates; that is, the logarithm of the current against the time in seconds. This is shown by the original curve in Fig. 4.

2. At any arbitrarily chosen value of time, say 12 milliseconds in this case, draw a tangent to the curve terming this straight line the *first exponential*.

3. By subtracting the corresponding ordinates of the first exponential from the original curve obtain the *first residual* (see Fig. 4). Draw a tangent to this curve and note that its position is no longer arbitrary, but is defined by the later part of the curve with points lying in a straight line. This straight line is termed the *second exponential.*

4. By subtracting the corresponding ordinates of the second exponential from the first residual obtain the *second residual,* which this time is completely defined by a series of points lying in a straight line and constituting the last and *third exponential.* Obviously, if the second residual had not been expressible as a straight line it would have been necessary to continue the method to a fourth or even a fifth exponential. In practise, however, this is seldom the case.

5. For each exponential the intercept on the current axis is a measure of the constant multiplier β and its slope a measure of the inverse time constant α. This permits the computation of the empirical current-time relation and its direct comparison with the observed curve. Table I gives such a comparison showing remarkable agreement for the range of time investigated.

TABLE I—COMPARISON OF OBSERVED AND CALCULATED VALUES OF CURRENT. THE METHOD OF THREE EXPONENTIALS. CHARGE AND DISCHARGE CURRENT-TIME RELATION AT 1,500 VOLTS

Dry Paper Specimen A-7 at 60 deg. cent.

Exponential	α	$i\,(o)$
(1)	69	0.33×10^{-6}
(2)	500	1.45×10^{-6}
(3)	1,787	5.00×10^{-6}

(t) Time in seconds	Current: microamperes				
	Calculated				Observed
	$i_1(o)e^{-\alpha_1 t}$	$i_2\,(o)e^{-\alpha_2 t}$	$i_3(o)e^{-\alpha_3 t}$	$i(t)$	$i(t)$
0.002	0.2875	0.5335	0.1402	0.9612	0.960
0.004	0.2504	0.1962	0.0039	0.4505	0.450
0.006	0.2181	0.0722	0.0001	0.2904	0.290
0.008	0.1900	0.0266	—	0.2166	0.215
0.010	0.1655	0.0098	—	0.1753	0.171
0.012	0.1442	0.0036	—	0.1478	0.145

From equation (3) the components of current contributed by reversible absorption at a given frequency and voltage are:

$$\left.\begin{aligned}&\text{Quadrature component} \quad I_r'' = AI_0'' \\ &\text{In phase component} \qquad I_r' = BI_0''\end{aligned}\right\} \qquad (9)$$

where $I_0'' = \omega E C_0$, see Fig. 1, and where the A and B integrals are computed for a given frequency from the constants of the three exponentials by means of equations (8). The above values now permit the computation of the contribution of reversible absorption to the a-c. behavior, since A and B are simple functions of the α's and β's. Thus we have:

$$\left.\begin{aligned}&\text{Phase defect angle } \tan \Psi_r = \frac{B}{1 + A} \\ &\text{Dielectric loss} \qquad W_r = \omega E^2 C_0 B \\ &\text{Apparent capacitance } C' = C_0(1 + A)\end{aligned}\right\} \qquad (10)$$

Equations (10) show the dependence of the computed a-c. behavior on the values of the A and B integrals. These, however, for a given dielectric at a given temperature are functions of the constants α and β and also of the frequency. It therefore becomes important to examine how wide a range of choice of the constants α and β as related to the frequency is permissible for agreement of computed and measured values. Assume therefore for simplicity a current-time relation of a single exponential term and examine the theoretical variation with frequency of the A and B integrals which now become:

$$\left.\begin{aligned}A(\omega) &= \frac{\alpha\beta}{\omega^2 + \alpha^2} \\ B(\omega) &= \frac{\omega\beta}{\omega^2 + \alpha^2}\end{aligned}\right\} \qquad (11)$$

By change of variable

$$e^u = \frac{\omega}{\alpha} \qquad (12)$$

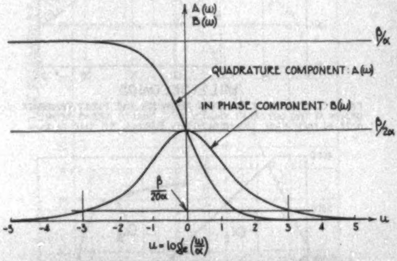

FIG. 5—VARIATION WITH FREQUENCY OF THE IN PHASE AND QUADRATURE COMPONENTS OF A SINGLE EXPONENTIAL.

obtain the relations

$$\left.\begin{aligned}A(u) &= \frac{\beta}{2\,\alpha}\, e^{-u} \operatorname{sech} u \\ B(u) &= \frac{\beta}{2\,\alpha} \operatorname{sech} u\end{aligned}\right\} \qquad (13)$$

which when plotted as shown in Fig. 5 are useful in showing the symmetry of the functions. The contribution to the in-phase component attains a maximum equal to $\beta/2\alpha$ at a value of frequency for which $\omega = \alpha$. The value of $B(\omega)$ falls off rapidly as the ratio ω/α departs either side from unity. For example, an exponential giving its maximum contribution to the in-phase component at $\omega = 100$ would contribute only one-tenth of the maximum value at either $\omega = 5$ or $\omega = 2{,}000$. This is readily deduced from Fig. 5. The variation with frequency of the in-phase component for a relaxation function expressible as the sum of several exponentials is given by the summation of a number of curves of the type $B(\omega)$ in Fig. 5, corresponding to the several exponentials. We have found that the maximum of a fourth exponential that might be introduced occurs at a frequency far from the frequency being studied, and that its contribution to the total in-phase component is negligible. The maximum of each contribution occurs at a value of ω equal to the inverse time constant α of that exponential. Therefore only the exponentials with values of α in the neighborhood of ω at the frequency at which correlation is intended contribute an appreciable amount to the loss component $B(\omega)$. Thus for agreement between measured and computed losses at 60 cycles, where $\omega = 377$, the values of α range approximately from 50 to 2,000. The same reasoning may be applied to examine the time range within which the d-c. measurements will give accurate prediction of the a-c. behavior. It shows that the values of the α's obtained for the three exponentials must range in the neighborhood of the value of ω corresponding to the frequency at which correlation is intended. This means that the range of time covered by the d-c. measurements must be of the same order of magnitude as the period of the alternating wave.

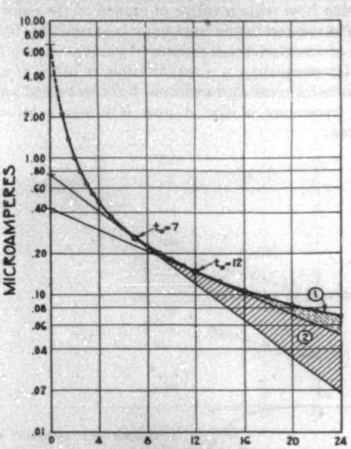

FIG. 6—CURRENT-TIME RELATION SHOWING THE FIRST TANGENT DRAWN AT TWO DIFFERENT VALUES OF "t_0" SHADED AREAS SEGREGATE IN EACH CASE THE NEGLECTED PORTION OF THE CURVE

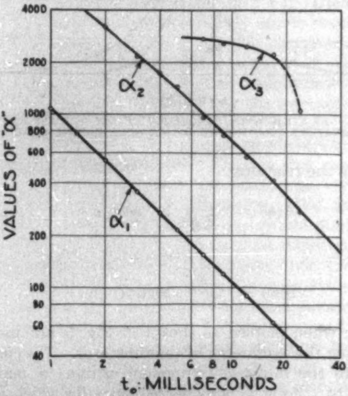

FIG. 7—VARIATION WITH "t_0" OF THE INVERSE TIME CONSTANT "α" OF EACH EXPONENTIAL

Accuracy of the Method

It has been stated that the resolution of a current-time relation into three exponentials is quite indeterminate. The constants of the three exponentials depend entirely on the value of time t_0, of the empirical discharge current-time relation, at which the first tangent is drawn. Fig. 6 illustrates the point for the current-time relation of Fig. 4 showing the first tangent drawn at two different values of t_0 with the shaded areas indicating in each case the neglected portions of the curve.

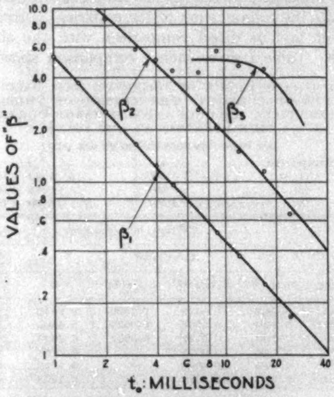

FIG. 8—VARIATION WITH "t_0" OF THE CONSTANT MULTIPLIER "β" OF EACH EXPONENTIAL.

Figs. 7, 8 and 9 give respectively for each exponential the variation with t_0 of the inverse time constant α, the constant multiplier β, and their ratio β/α. This ratio is shown in Fig. 5 to be a measure of the maximum contributions to the in-phase and quadrature components with varying frequency. Figs. 7 and 8 are significant in pointing out the definite dependence of the α and β constants on the chosen value of t_0, while their ratio, as shown by Fig. 9, is found to be essentially independent of t_0 at least for the first and second exponentials. The upper curve of this figure gives the summation of the three ratios, which is seen to be independent of t_0. From Fig. 5 and the equation (10) for the apparent capacitance it follows that this summation gives a measure of the dielectric constant at zero frequency, ϵ_1, in terms of the geometric dielectric constant, ϵ_0, at infinite frequency. Specifically, the relation is

$$\frac{\epsilon_1 - \epsilon_0}{\epsilon_0} = \frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} + \frac{\beta_3}{\alpha_3} \qquad (14)$$

That this summation should prove independent of t_0 further corroborates the sufficiency of three exponentials.

While the constants of the three exponentials are definitely functions of t_0, the total contribution to the a-c. behavior, as given by the in-phase and quadrature components of current at a given voltage and frequency, is essentially independent of t_0 for a limited range of values. This is illustrated in Figs. 10 and 11 which show that for t_0 beyond six milliseconds the in-phase and quadrature components are essentially constant even though the separate contributions of each exponential may vary widely. This is particularly true of the in-phase component, Fig. 10, which is a measure of the power factor, and therefore the one most readily checked by experiment. No physical significance is to be attached to the actual numerical values of the constants of the three exponentials. They serve merely as an empirical means of computing the a-c. behavior. It is apparent from Fig. 10 that the optimum value of t_0 lies in the neighborhood of the period of the alternating wave thus corroborating from another point of view the criterion of what constitutes suitable d-c. measurements.

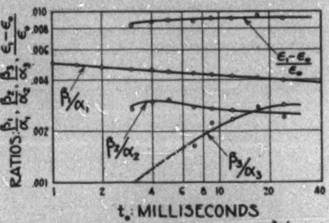

FIG. 9—VARIATION WITH "t_0" OF THE RATIOS β_1/α_1, β_2/α_2, β_3/α_3, AND THEIR SUMMATION, $(\epsilon_1-\epsilon_0)/\epsilon_0$

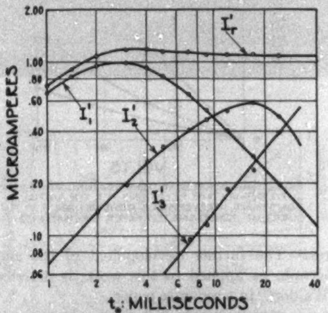

FIG. 10—CONTRIBUTION OF REVERSIBLE ABSORPTION AT 1500 VOLTS, 60 CYCLES. VARIATION WITH "t_0" OF THE IN PHASE COMPONENT OF CURRENT (I'_r) SHOWING THE CONTRIBUTIONS OF EACH EXPONENTIAL.

IRREVERSIBLE CONDUCTION

The irreversible conduction current as observed under continuous potential with the amplifier oscillograph for some of the cable oils tested is characterized by an initial current which remains constant for approximately one second and then decays with time until it merges into the final conductivity. This initial constant current, when it obeys Ohm's law, gives directly the value of the in-phase component of current at the same effective voltage. See Whitehead,[11] "The Conductivity of Insulating Oils, II" for a complete account of this initial conductivity.

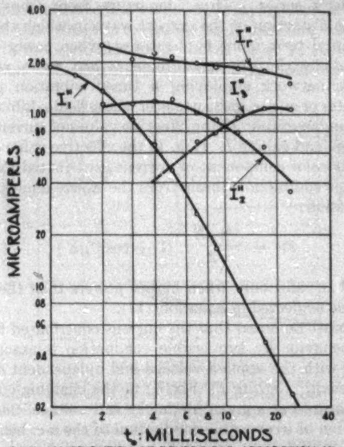

FIG. 11—CONTRIBUTION OF REVERSIBLE ABSORPTION AT 1500 VOLTS, 60 CYCLES. VARIATION WITH "t_0" OF THE QUADRATURE COMPONENT OF CURRENT (I''_r) SHOWING THE CONTRIBUTIONS OF EACH EXPONENTIAL.

In some of the impregnated paper specimens tested the irreversible conduction current does not obey Ohm's law, although it is still characterized by an initial constant conductivity. In such cases the irreversible constant current increases faster than the voltage as illustrated by the typical example shown in Fig. 12. For analytical purposes express the observed current-voltage relation by means of the power series:

$$i(e) = ae + be^3 + ce^5 + \ . \ . \ . \qquad (15)$$

in which the coefficients may be directly determined from the observed current-voltage relation. To facilitate this, however, express the above relation thus:

$$i(e) = \gamma \sinh \mu e \qquad (16)$$

where the γ and μ constants are chosen to give the best agreement with the observed curve within the voltage range required. Obviously μ is a function of the temperature alone while γ is a function of the temperature and the dimensions of the capacitor. Expand equation (**16**) into a series

$$i(e) = \gamma \left[\mu e + \frac{(\mu e)^3}{3!} + \frac{(\mu e)^5}{5!} + \ldots \right] \qquad \textbf{(17)}$$

in which higher powers than the fifth may be neglected.

To obtain the current-time relation for the alternating case substitute for the value of e in equation (**17**) its expression as a periodic sinusoidal function of time. This results in a power series of sin α, where $\alpha = (\omega t + \theta)$, in which all the higher powers of sinα may be broken up into harmonic components by means of Fourier's series. Thus, departure from Ohm's law causes distortion of the current wave in which the fundamental term alone is of interest, when power factor and dielectric loss measurements are made with a bridge network employing a tuned vibration galvanometer or an a-c. galvanometer whose field exhibits only the fundamental. Using effective values of current and voltage and calling I_i' (Fig. 1) the effective value of the *fundamental* component of current contributed by irreversible conduction obtain from the above analysis the expression:

$$I_i' = \frac{\gamma \mu E}{2} \left[1 + \cosh \mu E \right] \qquad \textbf{(18)}$$

which is valid only when higher powers than the fifth may be neglected in equation (**17**).

It must be noted that the current contributed to the a-c. behavior by irreversible conduction is exactly in phase with the applied voltage and independent of the frequency. Letting I'' (Fig. 1) be the charging current as measured at a given frequency and voltage, the contribution of irreversible conduction to the a-c. behavior is as follows:

$$\left. \begin{array}{ll} \text{Phase defect angle } \tan \Psi_i = \dfrac{I_i'}{I''} & \\ & \\ \text{Irreversible loss} \qquad W_i = E I_i' & \end{array} \right\} \qquad \textbf{(19)}$$

where I_i' is given directly by the initial constant current under continuous potential when Ohm's law is obeyed or is computed by (**18**) when the non-linear current-voltage relation is determined according to equation (**17**). Table II gives the computed variation of power factor with voltage and temperature for the typical example shown in Fig. 12.

TABLE II—VARIATION WITH VOLTAGE AND TEMPERATURE OF POWER FACTOR DUE TO IRREVERSIBLE CONDUCTION. NON-LINEAR CURRENT-VOLTAGE CHARACTERISTIC

Impregnated Paper Specimen C-7

Temp. deg. cent.	γ Amp. $\times 10^{-8}$	μ	Power factor		
			500 volts	1,000 volts	1,500 volts
30	9.5	0.00082	0.00016	0.00017	0.00020
45	20.4	0.00076	0.00035	0.00039	0.00047
60	50.8	0.00072	0.00082	0.00090	0.00104

EXPERIMENTAL PROCEDURE

Continuous potential measurements and corresponding 60-cycle a-c. measurements were made on a number of samples of oil, paper, and impregnated paper at voltages of 500, 1,000, and 1,500, and at temperatures of 30, 45, and 60 deg. cent. For a complete description of the constant temperature, drying, impregnating, and measuring tanks and of the test specimens see Whitehead and Kouwenhoven,[12] "Fundamental Properties of Impregnated Paper."

The measurements under continuous voltage include the determination of the charge and discharge current-time relations with the aid of the amplifier-oscillograph. The amplifier increases the normal sensitivity of the electromagnetic oscillograph 10^5 times, giving a deflection on the photographic film of 1 mm. per 4×10^{-8} amperes. The further development of the amplifier oscillograph, as applied to this work, is described by S. K. Waldorf[13] in a separate paper.

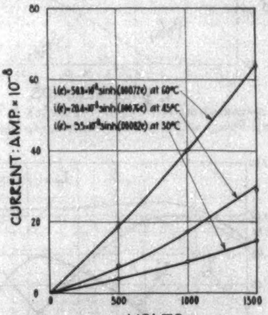

FIG. 12—VARIATION WITH TEMPERATURE OF CURRENT-VOLTAGE RELATIONS FOR THE INITIAL, CONSTANT, IRREVERSIBLE CONDUCTION CURRENT IMPREGNATED PAPER SPECIMEN C7.

The a-c. measurements include the determination of power factor and apparent capacitance using a modified Schering bridge whose power factor sensitivity is $\pm 5 \times 10^{-6}$. The development, theory, and special features of this bridge are described in a separate paper by Kouwenhoven and Baños.[14]

EXPERIMENTAL RESULTS

Paper Specimens. Dry paper specimens reveal a dielectric loss due entirely to reversible absorption; that is, the irreversible conduction current is not measurable as shown by the equality of charge and discharge currents in Fig. 2. Fig. 13 gives a typical pair of oscillograms at a given temperature and voltage. From such oscillograms are computed the current-time relations shown in Figs. 14 and 15 which illustrate the respective variations with voltage and temperature. The insert in Fig. 14 shows how closely Curie's law is obeyed, while the insert in Fig. 15 illustrates the variation of the current ordinates with temperature.

Table III gives the variation with temperature of the α and β constants of the absorption current-time relation for the dry paper specimen before and after impregnation with an oil whose electrical characteristics are described in the next section. Interesting changes of the constants are noted as result of impregnation. These differences are best appreciated by direct comparison of Figs. 2 and 3. Table IV gives the correlation of a-c. and d-c. measurements for the whole range of temperature and voltage explored. Note that in general the agreement between computed and measured losses is very close.

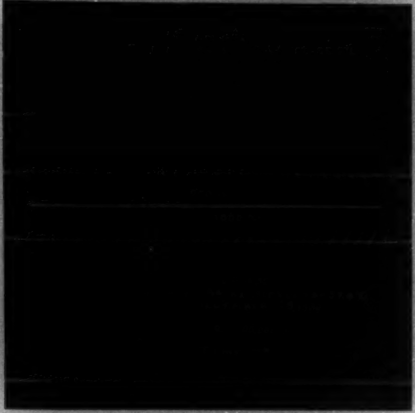

Fig. 13

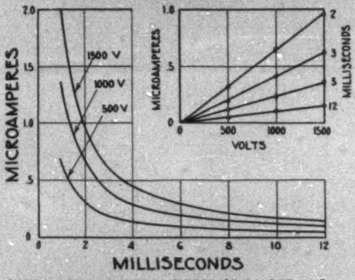

FIG. 14 – VARIATION WITH VOLTAGE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 60°C DRY PAPER SPECIMEN A7.

Oil Specimens. Fig. 16 gives a set of typical charge oscillograms at 1,500 volts and at 30, 45, and 60 deg. cent. for the cable oil used to impregnate the above paper specimen. It is noted that the charge current is characterized by a large initial constant current which accounts for the major part of the loss. Superimposed on this initial current is a smaller decaying current function of time which is reversible and, therefore, attributable to dielectric absorption in oil. Fig. 18 shows the variation with temperature of the charge and discharge current-time relations at 1,500 volts. The equations for discharge show that the small element of dielectric absorption is expressible by a single exponential thus corroborating the results published by Whitehead[11] in "The Conductivity of Insulating Oils, II." Fig. 17 gives the evidence that the initial constant current in the insulating oils tested obeys Ohm's law.

TABLE III—VARIATION WITH TEMPERATURE OF THE AVERAGE "α" AND "β" CONSTANTS IN THE EMPIRICAL CURRENT-TIME RELATION

$$i(t) = EC_0\{\beta_1 e^{-\alpha_1 t} + \beta_2 e^{-\alpha_2 t} + \beta_3 e^{-\alpha_3 t}\}$$

Specimen	Temp. deg.cent.	C_0 $\mu\mu f$	β_1	β_2	β_3	α_1	α_2	α_3
Dry paper A-7.	30	741	0.52	2.52	10.32	112	638	2,160
	45	751	0.40	1.93	6.49	94	614	2,067
	60	760	0.81	1.37	4.09	77	553	1,797
Impregnated paper C-7	30	1,191	0.49	2.33	6.70	95	570	1,827
	45	1,200	0.44	1.92	6.34	59	561	2,030
	60	1,209	0.55	1.32	3.13	30	490	1,635

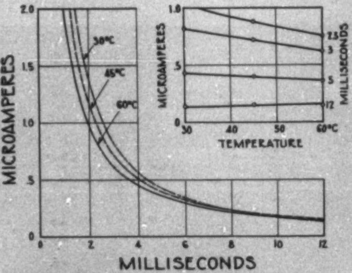

FIG. 15 – VARIATION WITH TEMPERATURE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 1500 VOLTS. DRY PAPER SPECIMEN A7.

Table V gives the comparison between several measured and corresponding computed losses for one oil specimen B-7. The data are typical and the close agreement shown is further evidence of the soundness of the method used in computing the a-c. behavior.

Impregnated Paper Specimens. Some of the changes brought about on a dry paper specimen by impregnation with an insulating oil have already been pointed out in Fig. 3 and Table III. Fig. 3 shows the effect of the conductivity of the insulating oil in increasing the absorption and changing its character. Fig. 19 gives a typical pair of oscillograms in which the presence of the irreversible conduction current is apparent in the charge record. This fact is better appreciated from the equa-

TABLE IV—ANALYSIS OF POWER FACTOR AND DIELECTRIC LOSS AT 60 CYCLES BY COMPUTATION FROM D-C. CHARACTERISTICS

Paper Specimen A-7

		Measured at 60 cycles			Computed from d-c.		
Temp deg. cent.	Volts	Charging current amp. × 10^{-6}	Power factor	Dielectric loss watt × 10^{-6}	In-phase component of current amp. × 10^{-6}	Power factor	Dielectric loss watt × 10^{-6}
T	E	I^{*}	$\tan \Psi$	W	$I_r{}^1$	$\tan \Psi_r$	W_r
60	500	144	0.00237	169	0.340	0.00236	170
	1,000	288	0.00237	676	0.684	0.00237	684
	1,500	432	0.00237	1,518	1.047	0.00242	1,560
	Average	—	0.00237	—	—	0.00238	—
45	500	142	0.00276	197	0.410	0.00288	205
	1,000	285	0.00276	786	0.830	0.00291	830
	1,500	427	0.00276	1,796	1.289	0.00302	1,930
	Average	—	0.00276	—	—	0.00293	—
30	500	141	0.00346	243	0.533	0.00378	266
	1,000	282	0.00346	974	1.024	0.00363	1,024
	1,500	422	0.00346	2,192	1.558	0.00368	2,335
	Average	—	0.00346	—	—	0.00369	—

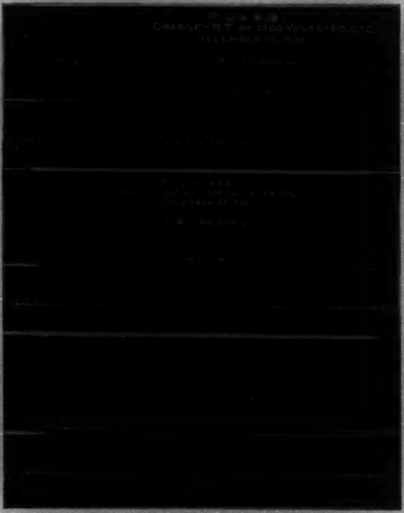

Fig. 16

tions of charge and discharge given in Fig. 20, where the irreversible conduction current is seen to be the *constant* difference between charge and discharge currents. This constant irreversible conduction current does not obey Ohm's law, as shown by Fig. 12, and its contribution to the power factor has already been computed from theoretical considerations and given in Table II.

Summary of Specimens. As an example of the insight afforded by this method of analysis, Fig. 21 gives the power factor as a function of temperature for the oil, paper, and impregnated paper specimens. The shaded areas represent the total increase in power factor with impregnation. Through the analysis of the discharge current-time relations it is possible to resolve this increase into two components:

1. The increase in reversible absorption brought about by impregnation.

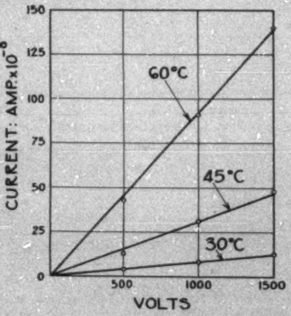

FIG.17—VARIATION OF INITIAL CONSTANT CURRENT WITH VOLTAGE. OIL SPECIMEN B7.

2. The creation of the additional contribution of irreversible conduction totally absent before impregnation and directly attributable to the conductivity of the impregnating oil.

ACKNOWLEDGMENT

Grateful appreciation is expressed to the Utilities Research Commission of Illinois, whose financial sup-

TABLE V—ANALYSIS OF POWER FACTOR AND DIELECTRIC LOSS AT 1,500 VOLTS AND 60 CYCLES BY COMPUTATION FROM D-C. CHARACTERISTICS

Oil Specimen B-7

				Computed from d-c. characteristics							
	Measured at 60 cycles			Reversible absorption			Irreversible conduction			Total	
Temp. deg. cent.	Charging current amp. × 10⁻⁶	Power factor	Dielectric loss watt ×10⁻⁶	In phase component of current amp. ×10⁻⁶	Power factor	Dielectric loss watt ×10⁻⁶	Initial constant current amp. ×10⁻⁶	Power factor	Dielectric loss watt ×10⁻⁶	Power factor	Dielectric loss watt ×10⁻⁶
T	I''	tan ψ	W	I_r'	tan ψ_r	W_r	I_i'	tan ψ_i	W_i	tan ψ_o	W_o
	420	0.00346	2,179	0.065	0.00016	98	1.309	0.00312	1,965	0.00328	2,063
60	421	0.00346	2,184	0.145	0.00035	218	1.251	0.00297	1,880	0.00332	2,098
	422	0.00374	2,357	0.102	0.00024	153	1.349	0.00320	2,025	0.00344	2,178
45	425	0.00116	739	0.029	0.000067	43	0.475	0.001117	713	0.00118	756
	430	0.00037	230	0.027	0.000064	41	0.124	0.000288	186	0.00035	227
	430	0.00034	219	0.018	0.000042	27	0.135	0.000313	202	0.00035	229
30	429	0.00035	225	0.019	0.000043	28	0.120	0.000280	180	0.00032	208
	430	0.00032	206	0.022	0.000051	33	0.128	0.000298	192	0.00035	225
	431	0.00032	210	0.032	0.000074	48	0.105	0.000244	158	0.00032	206
Average	430	0.00034	220	0.024	0.000055	36	0.122	0.000285	184	0.00034	219

port has made possible the active prosecution of a research on the properties of impregnated paper from which the experimental results here reported have been taken. The authors also wish to acknowledge the interest and careful work of Dr. S. K. Waldorf in the development and operation of the amplifier oscillograph.

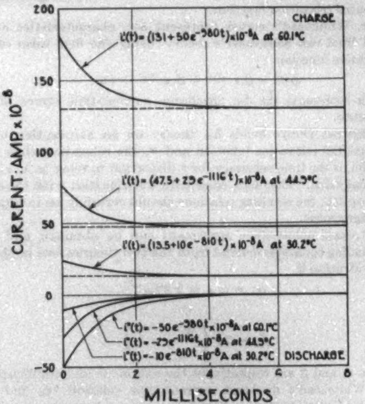

FIG. 18—VARIATION WITH TEMPERATURE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 1500 VOLTS. OIL SPECIMEN B7.

CONCLUSIONS

1. It is shown that for any type of dielectric the loss, power factor and capacity at 60 cycles may be accurately predicted from the charge and discharge currents under continuous potential.

2. The method developed is available at any frequency, provided that the continuous potential charge

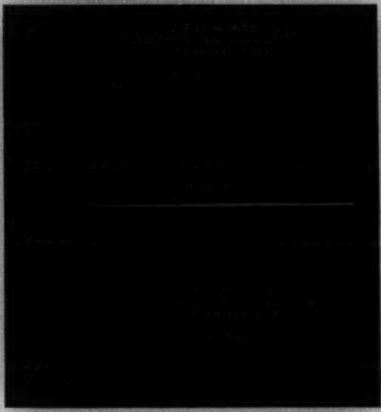

FIG. 19

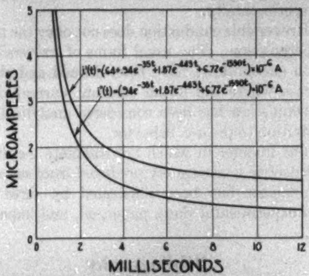

FIG. 20—CHARGE i'(t) AND DISCHARGE i''(t) CURRENT-TIME RELATIONS AT 1500 VOLTS, 60°C. IMPREGNATED PAPER SPECIMEN C7.

and discharge currents may be measured over initial time intervals comparable with the alternating period.

3. The nature of the relaxation function characteristic of reversible absorption in dielectric has been discussed from the experimental and analytical points of view.

4. It has been shown that agreement of the relaxation function with the t^{-n} expression is not general, but on the contrary it is confined to dielectrics showing negligible or no irreversible conduction. The inadequacy of the t^{-n} expression has been proved on experimental and analytical reasons. Chief among them is the failure of the t^{-n} expression to predict accurately the variation of dielectric loss with frequency.

5. The method of three exponentials has been developed as a convenient and sufficient expression for the relaxation function. The sufficiency and accuracy of three exponentials has been proved both analytically and experimentally.

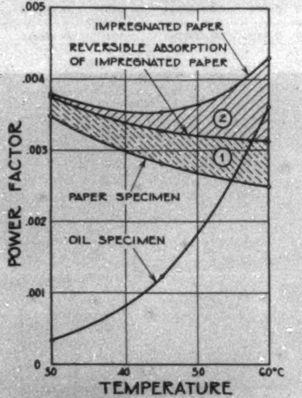

FIG. 21—ANALYSIS OF POWER FACTOR AT 1500 VOLTS. IMPREGNATED PAPER SPECIMEN C7

Increase in Power Factor with Impregnation
(1) Increase in Reversible Absorption
(2) Contribution of Irreversible Conduction

6. Irreversible conduction does not obey the principle of superposition. The usual forms of irreversible conduction encountered have been defined and classified. The case where the initial constant current does not obey Ohm's law has been considered analytically in its contribution to the a-c. behavior.

7. The manner in which the complete a-c. behavior of a dielectric is accurately predicted from suitable d-c. measurements has been illustrated by three typical sets of experimental data, paper, oil, and impregnated paper.

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Discussion

H. H. Race: The authors of this paper have presented some very interesting data but they suggest no physical explanation of their results. Therefore Fig. 5 interested me particularly because the variations of capacitance and loss with frequency are similar to corresponding curves calculated using a very definite physical picture, namely, Debye's theory of polar molecules. A comparison of these two separate lines of mathematical thinking, which lead to qualitatively similar results, shows that a correspondence should be expected since both are based upon an exponential relaxation function.

Dr. Whitehead's curves represent a-c. characteristics calculated from von Schweidler's theory using the first term of the relaxation function

$$\phi(t) = \beta_1 \epsilon^{-\alpha_1 t} + \beta_2 \epsilon^{-\alpha_2 t} + \text{etc.}$$

which represents the d-c. discharge current-time curve of the dielectric.

Likewise, Debye builds his theory on an assumption of an exponential relaxation function and τ, the relaxation time used by him, is the time necessary for a distortion to relax to $1/\epsilon$ of its original value. Therefore, since both theories start with the same assumption, the resulting relations should certainly be in qualitative agreement.

An exact quantitative agreement can be obtained and corresponding equations derived from the two theories can be shown to be identical if

$$\alpha_1 = \omega_m = 2\pi f_m \qquad (1)$$

and

$$\beta_1 = \alpha_1 \left(\frac{\epsilon_0}{\epsilon_\infty} - 1 \right) \qquad (2)$$

where α and β are respectively the exponent and coefficient in Dr. Whitehead's discharge current-time relation. ϵ_0 and ϵ_∞ are the measured effective dielectric constants at zero and infinite frequency and f_m is the frequency at which the maximum a-c. loss occurs.

Equation (1) can also be derived from Dr. Whitehead's work by setting $dB(w)/dw = 0$ in equation (11) in order to determine the frequency at which the maximum a-c. loss will occur. Dr. Whitehead has made this calculation and concludes that "only the exponentials with values of α in the neighborhood of (w) at the frequency at which correlation is intended contribute an appreciable amount to the loss component $B(w)$."

Since equation (1) also relates a-c. dielectric losses resulting from the relaxation of polar orientation to d-c. current-time curves resulting from the same relaxation, it seems that the following more general conclusions can be reached.

1. Any mechanism causing an exponential current-time discharge curve after the removal of a constant potential, will also cause an a-c. dielectric loss-frequency curve having a maximum at $f_m = (\alpha/2\pi)$.

2. Inversely, if a maximum is observed in the a-c. loss-frequency curve, the exponent of a d-c. current time curve taken for the same material will be given by this same relation.

3. Conversely, if in a given range of frequency or relaxation time, one of these effects is not found the other will also be absent.

4. Through these relations, a correlation should be possible between Dr. Whitehead's d-c. and 60-cycle measurements and our high frequency data. However, for the mineral oils in which we have measured the dielectric loss, as a function of frequency, we have found no maxima at frequencies less than 10^4 cycles per second, even in high viscosity oils at temperatures as low as 0 deg. cent. Therefore, the d-c. relaxation curves corresponding to the maxima which we have observed should have become negligible in less than 1×10^{-3} second, which is the time at which Dr. Whitehead's measurements begin.

To check the above conclusions and obtain a better comparison between a-c. and d-c. measurements Dr. Whitehead has very kindly furnished us with a sample of his B-7 oil and we hope to present a paper in the near future containing the reasons leading to equation (1) and data on B-7 oil over a wide range of frequency and temperature.

F. M. Clark: In connection with the paper presented by Dr. Whitehead, it is interesting to note the procedure developed a few years ago in the Pittsfield Works Laboratory of the General Electric Company for the gaging of capacitor characteristics. With 5-kva. oil-filled capacitors designed for a-c. voltage application, ranging from 440 to 2,300, it was found that the relative a-c. characteristics could be successfully determined from the so-called "d-c. leakage." In determining this value, the capacitors were charged under d-c. voltage to the desired value, disconnected from the source of supply and allowed to stand under controlled conditions for a period ranging from one to several days, after which the remaining charge was measured. The difference between the original and final charge was used in gaging the a-c. characteristics of the capacitors. This so-called "d-c. leakage" appeared to be of greater value than insulation resistance measurements, probably because the time used to determine the latter was not of sufficient duration. The work was not carried sufficiently far to predict successfully the power factor from the leakage results obtained. With further investigation, it is believed this could be done.

W. A. Del Mar: The practical control of power factor of impregnated paper now lies with the paper tests. In 1922* the control was essentially a matter of oil resistivity.

The change occurred when we changed our paper from Manila to wood pulp and our oil from petrolatum to cylinder oil. The practical problem has long since been solved, but it would be interesting to have the authors explain these facts in the light of their admirably developed theories.

**Effects of the Composite Structure of Impregnated Paper Insulation on Its Electric Properties*, Hanson and Del Mar, A.I.E.E. TRANS., 1922, Vol. 41, p. 563.

Breakdown of Glass with Alternating Potentials

BY N. D. KENNEY,[1] A. M. LUERY,[1] and J. D. MORIATY[1]
Non-member Non-member Non-member

Synopsis.—*Previous tests on the breakdown of glass with direct potentials indicated the existence of three different regions of breakdown. The present paper describes similar tests on alternating potentials. The following conclusions are reached:*

1. Three distinct regions of breakdown are obtained on alternating current as well as on direct current.

2. In the disruptive and intermediate regions, breakdown occurs at an alternating peak voltage approximately equal to the direct breakdown voltage.

3. In the thermal region, breakdown occurs at an alternating peak voltage somewhat lower than the direct breakdown voltage.

* * * * *

INTRODUCTION

The cause and reason for breakdown of solid dielectrics has resulted in the setting forth of many theories based on both experimental and mathematical research. Research by Kármán, Rogowski and Fock resulted in the establishing of the thermal breakdown theory. Inge and Walther, in their investigations of the breakdown of glass in 1928 found a region of puncture in which the potential gradient is independent of temperature and thickness—the so-called disruptive region. In 1929 Moon and Norcross concluded from their experimental data on glass, that there were three regions of breakdown.

Since the publication of the paper by P. H. Moon and A. S. Norcross[2] describing their experimental work on the breakdown of glass with direct potentials, there has been some question as to whether the same type of results would be obtained with alternating potentials. Our purpose was to determine if three regions of breakdown also existed for the a-c. case. We also hoped to find some relation between the alternating and direct breakdown voltages. Previous work by Hayden and Eddy[3] on the a-c. to d-c. ratio showed widely varying results, due perhaps to lack of knowledge of the types of breakdown and further complicated by edge effect. It was thought that careful tests made with edge effect eliminated, coupled with the present knowledge of different breakdown regions, might lead to a definite a-c. to d-c. ratio in each region.

For these tests it was decided to use glass because of its many advantages.

1. Previous tests have been made on glass, thus allowing a comparison of results.
2. Its characteristics are such that the tubing can be readily blown into spheres, thus eliminating edge effect.
3. Its homogeneous nature makes it desirable for breakdown investigations.

The disadvantages are:

1. Its physical structure is unknown.
2. Its chemical constitution is not exactly known for each batch of glass.

This paper describes the experimental results obtained at the Massachusetts Institute of Technology on the breakdown of glass with alternating potentials, and shows how these results compare with the d-c. tests of Moon and Norcross.

APPARATUS AND METHOD OF TEST

Alternating voltage (60 cycles) was obtained by means of a sine-wave motor-generator set and a 115/230-100,000-volt transformer. A motor-driven rheostat was used to vary the field voltage of the sine-wave set in order to obtain a uniform and easily-controlled rate of voltage rise. The transformer secondary voltage was read by means of a potential transformer and a Weston a-c. voltmeter.

Glass tubes 10 to 15 mm. in diameter and 300 mm. in length were sealed at one end and blown into spheres between 20 and 40 mm. in diameter. This procedure was similar to that used by Moon and Norcross and eliminated edge effect very nicely. These samples were placed in a mercury bath and filled on the inside with mercury. Contact was made to the internal electrode by means of a flexible wire suspended inside the tubing and to the external one by means of a steel container. (Fig. 1.) This was carefully lagged on the sides and top with asbestos and proved to be very effective in keeping the temperature uniform and constant. The voltage was impressed across the sample and increased by means of the motor-driven rheostat until puncture occurred. The sample was then carefully removed from the bath, all pieces being collected, cleaned, and measured. As it was not always possible to measure the thickness of the sample at the exact point of puncture, 5 to 10 micrometer readings were taken on the glass fragments near the puncture and the minimum value used as the approximate thickness at the point of breakdown.

The wave form of the generator was found to be practically sinusoidal; and since the transformer was operated at low saturation and with no series resistance in the primary circuit, it was assumed that the wave form of the voltage across the sample was also sinusoidal. The voltages read from the voltmeter were, of course,

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[1] Massachusetts Institute of Technology.

[2] "Three Mechanisms of Breakdown Obtained on Glass by Elimination of Edge Effect," *Journal of the Franklin Inst.*, 208, 1929, p. 705.

[3] *Dielectric Strength Ratio between Alternating and Direct Voltages*, Hayden and Eddy, TRANS. A.I.E.E., V. 42, 1923, p. 593.

Presented at the Winter Convention of the A.I.Æ.E., New York, N. Y., January 25-29, 1932.

r.m.s. values; but a better comparison with d-c. tests was obtained by using peak instead of r.m.s. values. Thus all readings were multiplied by $\sqrt{2}$, and all results given in this paper are peak kilovolts and not r.m.s. kilovolts.

EXPERIMENTAL RESULTS ON G-1* GLASS

The experimental results are shown in Fig. 2, where each point represents one breakdown test. To show clearly the different regions of breakdown, the logarithm of breakdown voltage (peak kv.) is plotted against the logarithm of thickness in microns. ($1\ \mu = 0.001$ mm.) There is a certain amount of scattering of the points, as can be expected in the results of any breakdown tests. However, it will be noted that this variation is not at all bad, and thus the curves can be located fairly exactly.

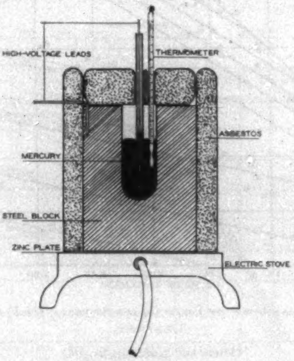

FIG. 1—SCHEME OF TEST APPARATUS

Fig. 2 shows clearly that with alternating potentials *there are three distinct regions of breakdown.* The tests at low temperatures and small thicknesses give a single curve of slope = 1. This represents the *disruptive region.* For somewhat higher temperatures, the curves depart from this straight line; and we have a set of parallel lines of slope = 2/3 breaking off from the disruptive-region line. The clusters of points for the 50 deg. and for 150 deg. curves, for instance, prove that these two are parallel or nearly so and that the slope is distinctly different from that of the 0 deg. curve. This is the *intermediate region.* At still higher temperatures we obtain another set of parallel lines having the slope of ¼. This is the *thermal region.* A comparison of the points at 150 deg. and 250 deg. shows very clearly that these two regions are distinct. The breaks in the 150 and 250 deg. curves may appear purely arbitrary, but they were found necessary in order to make a consistent family of curves when plotted against $(1/T)$. Thus the only questionable aspect of the curves as drawn is whether the transitions should be represented by sharp breaks or be slightly rounded. The accuracy of breakdown data does not warrant any statement as to which is correct. The present scheme, however, is the simpler and will be used in this paper.

In order to show more clearly the relations existing between breakdown voltage and temperature, curves of the logarithm of breakdown voltage (peak kv.) are plotted against the reciprocal of absolute temperature. (Fig. 4.) Here again three distinct slopes are noticed; zero slope for the disruptive region, slope = 430 for the intermediate and slope = 1,940 for the thermal.

How do the results of these a-c. tests compare with d-c. tests? To obtain a comparison, d-c. tests were also made on the same batch of lead glass. The results at room temperature (25 deg. cent.) are shown in Fig. 2 by the solid ▲'s. A very good check will be noted in both disruptive and intermediate regions. Thus we may say that on alternating current and in the disruptive and intermediate regions, *breakdown occurs at a peak voltage approximately equal to the d-c. breakdown voltage*

FURTHER DISCUSSION OF THE CURVES

The three regions of breakdown obtained by Moon and Norcross with G-1 glass and direct potentials are also found when alternating potentials are used. To facilitate comparison of our results with the previous ones, we have added Figs. 3 and 5 giving breakdown voltage for G-1 glass on direct potentials.[4] The a-c. curves (Figs. 2 and 4) are similar in every respect to the d-c. curves (Figs. 3 and 5).

For low temperatures and small thickness a linear relation (slope = 1) exists between breakdown voltage and thickness. The alternating breakdown gradient in this region is constant and has a magnitude of 3,550 kv. per cm., which is the same value obtained with direct current in this research.[5] The potential gradient is not affected by thickness or temperature.

With higher temperatures, an increase in temperature causes a decrease in the breakdown voltage for a given thickness. At intermediate temperatures (between the two previously mentioned regions) the voltage varies with the temperature according to some other relation, giving a third region which varies with the thickness as the 2/3 power.

The general form of the equation found to fit the curves is

$$V_b = K_d{}^n\ (10)^{\frac{b}{T}},$$

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*A lead glass made by the Corning Glass Works.

[4] *Three Regions of Dielectric Breakdown*, Moon and Norcross, TRANS. A.I.E.E., V. 49, 1930, p. 755.

[5] This magnitude differs from the value of 3,100 kv. per cm. obtained by Moon and Norcross (Fig. 3 and Fig. 5) by about 14 per cent. The discrepancy may be due to a slight difference in the chemical or physical properties of the glass used in the two sets of experiments, though both tests were carried out with G-1 glass.

where K is a constant dependent on the region, n is the slope of the *log kv.* vs. *log d* curves and b is the slope of the *log kv.* vs. $1/T$ curves. Table I gives the a-c. peak values for K, n, and b while Table II gives the d-c. values as found by Moon and Norcross.

TABLE I—(A-C. PEAK VALUES)

Region	K	n	b
Disruptive	3,550	1	0
Intermediate	25.5	2/3	430
Thermal	1.635×10^{-3}	1/4	1,943

TABLE II—(D-C. VALUES)

Region	K	n	b
Disruptive	3,100	1	0
Intermediate	21.3	2/3	470
Thermal	0.618×10^{-3}	1/4	1,540

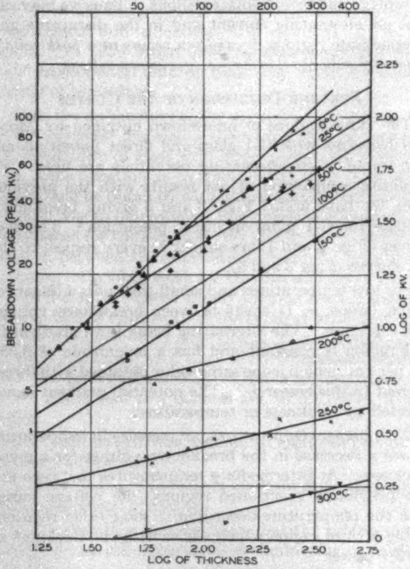

FIG. 2—BREAKDOWN OF LEAD GLASS SPHERES (G-1) ON 60 CYCLE ALTERNATING CURRENT

▲ = d-c. test points at 25 deg. cent.

By comparing Fig. 2 with Fig. 3 it can be seen that in the thermal and intermediate regions the direct breakdown voltage is higher than the a-c. peak value. Comparison of Fig. 4 and Fig. 5 shows that the intermediate region for alternating current is longer than that for direct current. This is shown to better advantage in Fig. 6, where the a-c. and d-c. curves are plotted on the same sheet. For simplicity only the 20μ and 100μ curves are given. It is probable that some of the differences are due to unavoidable experimental variations and that still others are caused by slight differences in the glass used in the two investigations. In the intermediate region, however, the average difference between the a-c. and d-c. curves is only about 6 per cent, which is undoubtedly within the experimental error.

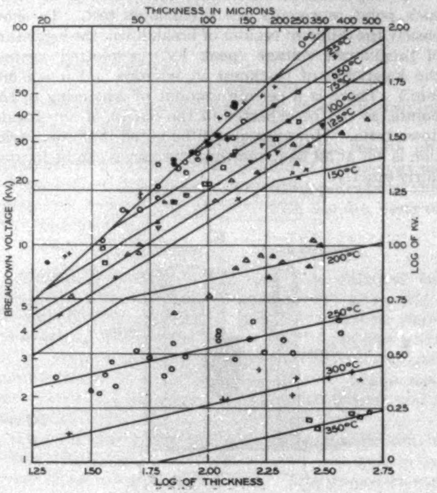

FIG. 3—BREAKDOWN OF LEAD GLASS SPHERES (G-1) ON DIRECT CURRENT

(Moon and Norcross, *loc. cit.*)

At the higher temperatures in the thermal region, the d-c. breakdown voltage appears to be distinctly higher than the peak alternating voltage. If r.m.s. values had been used instead of peak values, this difference would of course be even more marked. In the thermal region there appears to be no simple relation between a-c. and d-c. breakdown voltages or between the slopes of the curves. Such a relation could hardly be expected if we consider that in the a-c. case the heating is due to the total loss whose nature is not understood, and that the results are undoubtedly complicated by the effects of electrolytic polarization which is present in the d-c. case but partially eliminated in the a-c. case.

DISCUSSION OF THE POSSIBLE CAUSES OF BREAKDOWN

Disruptive Region. In the disruptive region the rate of application of voltage has no effect on the breakdown.

Samples were punctured with rates of application varying between 4 kv. per min. and 40 kv. per min. and no change in potential gradient was noticed. Breakdown in this region may be due to submicroscopic cracks in the surface of the glass. This theory was first expounded by Horowitz[6] and seems more logical than the molecular disruption theory, which gives a breakdown value much higher than the experimental values. The equation for breakdown according to the submicroscopic theory is

$$V_b = 1{,}200 \sqrt{\frac{\delta}{\mathcal{K}\,\lambda}} \text{ (volts per cm.)}$$

where

$\mathcal{K}$ = the dielectric constant
δ = the surface tension in dynes per cm.
λ = the depth of the crack in cm.

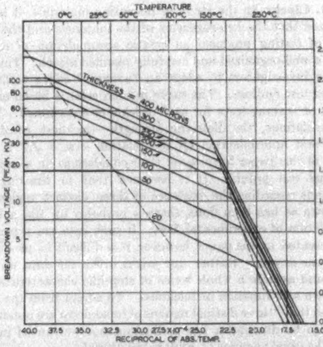

FIG. 4—BREAKDOWN OF LEAD GLASS SPHERES (G-1) ON 60 CYCLE ALTERNATING CURRENT

It will be noted that time should have no effect, according to this equation. Also, as the type of impressed voltage can have no effect on the depth of the crack, the dielectric constant, or the surface tension, the magnitude of V_b will be the same for alternating current and direct current. This was found to be true in our tests.

Intermediate Region. Unlike the disruptive region the intermediate has no constant potential gradient. From the experimental results obtained,

$$V_b = 25.51\, d^{2/3}\, 10^{\frac{430}{T}}.$$

It can be seen from this equation that the breakdown voltage is dependent on the thickness of the sample and the ambient temperature. The time factor appears to enter into this region and a study is to be made of its effect on the magnitude of the breakdown voltage.

Just what governs breakdown in this region is a question for debate. However, the ionization-by-collision theory seems to be a logical explanation of the results. This theory has been accepted for gases and liquids and it may be true also for solids.

Thermal Region. The thermal region, like the intermediate, has no constant breakdown gradient. The equation of this region was found to be

$$V_b = 1.635 \times 10\, d^{1/4}\, 10^{\frac{1940}{T}}.$$

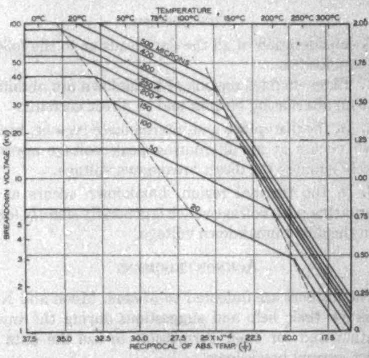

FIG. 5—BREAKDOWN OF LEAD GLASS SPHERES (G-1) ON DIRECT CURRENT

(Moon and Norcross, *loc. cit.*)

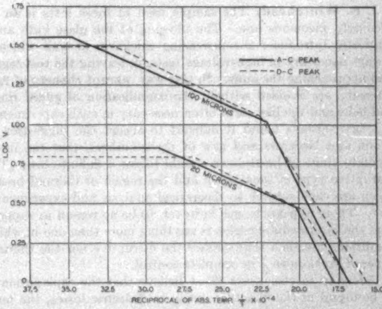

FIG. 6—COMPARISON OF D-C. AND A-C. RESULTS

Thermal breakdown is a function of thickness and temperature.

Heat is generated inside the sample by I^2R and other losses. It was noted during the tests that, by varying the rate of application of voltage, various values of breakdown voltage could be obtained. The rate which was found to be the most satisfactory was 300 volts per min., since further reduction in rate of voltage application had no effect on breakdown voltage.

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[6] Das Griffithsche Prinzip und Dielektrische Durchschlegspestigkeit, *Archiv. für El.*, 18, 1927, p. 535.

When a direct potential is impressed across a sample of glass the sodium ions drift under the action of the outer field and gradually produce a poorly-conducting layer near the anode. With alternating potentials, the effect of electrolytic polarization is very small. This means that the a-c. conduction current will be greater than the direct current and therefore the I^2R losses will be greater with alternating current than with direct current. Hence the r.m.s. alternating breakdown voltage will be less than the direct breakdown voltage. This is substantiated by the tests described above.

CONCLUSIONS

A consideration of all the data leads us to the following conclusions:

1. Three distinct regions of breakdown are obtained, both on alternating current and on direct current.
2. In the disruptive and intermediate regions, breakdown occurs at an alternating peak voltage approximately equal to the direct breakdown voltage.
3. In the thermal region, breakdown occurs at an alternating peak voltage which is generally slightly lower than the direct breakdown voltage.

ACKNOWLEDGMENT

The authors are indebted to Messrs. Moon and Norcross for their help and suggestions during the investigation and for their permission to use the data on direct-current breakdown.

Discussion

J. B. Whitehead: The sample used in these tests is an exceedingly ingenious one. The blowing of the glass bulb automatically introduces the increasing thickness of wall which is always necessary in high-voltage tests, in leaving the test region and approaching the ends. In this way, abrupt changes of field intensity are avoided without the complication of guard rings, end-bells and the like, as is often necessary in such experiments.

I have always found it difficult to accept the suggestion of Moon and Norcross and now of these authors, that the intermediate stage of breakdown should follow a definite law. The disruptive type of breakdown and the region of thermal breakdown are each subject to theoretical analysis and experimental test. There appears to me, however, to be no reason to suppose that the intermediate region is anything more than one in which conditions are such that neither the disruptive nor the thermal types of breakdown is in complete control.

Referring to conclusion (3), I believe that the explanation is to be found in the fact that owing to dielectric losses, the temperature inside the specimen is higher than that which is measured on the thermometer.

W. A. Del Mar: The authors' main thesis is the contention that there exist, with alternating currents, as claimed by Professor Moon for direct currents, three distinct regions of dielectric failure, instead of two, as commonly believed. The evidence of this is that when breakdown is plotted against the reciprocal of the temperature, the graph appears as three lines of different slopes. The authors do not give this evidence, as all the experimental points are omitted from the graphs shown in Figs. 4.

They give experimental points in their graphs of breakdown voltage *versus* thickness, but they are of no significance in this place.

An actual attempt to transcribe the points from Fig. 3 to Fig. 4 resulted in a very erratic looking graph which might have been either what the authors claim or a smooth curve.

The authors, in justice to their thesis, should give Fig. 4 with the experimental points plotted.

W. A. Hillebrand: Doubt has been raised about the existence of an intermediate region of breakdown as postulated by the authors.

This at once brings up the question of the nature of glass, the material used in these tests. Glass is a physical solution, an electrolyte, whose constituents are ionized. It has the high negative temperature coefficient of resistance characteristic of electrolytes. Furthermore, some of its physical properties are indefinite. For example, it will flow at continuously applied stresses much below its short time mechanical ultimate. It therefore seems to me not impossible that in such a material there should be a region of dielectric breakdown intermediate between the disruptive and thermal, partaking of the nature of both and representing a transition stage.

For the same reasons, glass would seem to be a very unsatisfactory material to use in fundamental studies of dielectric breakdown because it is too complex and indefinite in its structure.

F. M. Clark: In the study of insulating materials, it is well recognized that the heterogeneity of the materials and the difficulties of testing produces an erratic accumulation of results despite a well-organized and carefully planned attack. This has led most investigators to adopt an "average" curve to fit the scattered test findings. The curve in such cases is based on the average of several tests for each thickness determined. With most insulations, the dielectric strength—thickness relation—can be best approximated by the relation kva. = AT^n. The authors of this paper have based their conclusions on a series of individual test points. It is assumed that in drawing the conclusions regarding the existence of three distinct regions of breakdown as has been done, there is probably on file a large accumulation of experimental data. If such data are typical of that presented in the paper, however, it is difficult to accept the conclusions drawn. Rather, one might break the data up still further and assume a whole series of step-like characteristics of breakdown as a function of thickness. To accept from the data presented that "three distinct regions of breakdown are obtained" appears to be drawing a fine distinction not warranted in the light of the usual vagaries of insulating material.

In experimental work involving a material such as glass, it must always be borne in mind that one is dealing with a material which may vary from batch to batch and which in itself bears little, if any, relation to other materials. Being a super-cool liquid, largely electrolytic in its electrical conductivity, conclusions drawn cannot safely be extended to apply to pure crystals or ceramic materials on the one hand, or to the usual organic commercial insulation on the other. The conclusions drawn by the authors, even if accepted, should be carefully limited to glass of the type under discussion which in itself might be more clearly described than has been done in the paper.

N. D. Kenney: The question of there being an intermediate region as raised by Dr. J. B. Whitehead and Mr. W. A. Del Mar, can best be answered by an examination of Fig. 2 and Fig. 4. Unless puncture tests on dielectrics are made with edge effect eliminated the field is decidedly non-uniform and the data are such as to make it impossible correctly to interpret the effect of thickness and temperature. The use of glass spheres removes the error introduced by concentration of stresses along an edge and a decided increase in the potential gradient is obtained. An examination of either Fig. 2 or Fig. 3 will show that it would be impossible to draw a line through the intermediate points that would be parallel to either the disruptive or thermal line. In the thermal region, as Dr. J. B. Whitehead has brought out, for thick samples and high temperatures the internal temperature becomes greater than the ambient temperature at the instant of breakdown. In 1931 Professor P. H. Moon presented a very interesting paper on thermal breakdown.*

Mr. Del Mar questioned Fig. 4 because there are no points shown. The data for Fig. 4 were obtained by taking any given thickness such as 200 μ and reading the breakdown voltages for various temperatures from Fig. 2. These values were plotted in Fig. 4 so as to show more clearly the relations existing between breakdown voltage and temperatures for a given thicknes of glass. The points would lie along the curves because of the manner in which they were obtained and there was no need to show them.

It is to be understood that no claim is made that this intermediate region is characteristic of all dielectrics. The physical construction of a dielectric is an important factor of its breakdown strength. This region seems to be characteristic of glass, quartz, and celluloid as consistent tests have shown and its physical form no doubt is closely related to its cause.

The Theory of Thermal Breakdown of Solid Dielectrics, by P. H. Moon, A.I.E.E. TRANS., Sept. 1931, p. 1008.

Magnet Steels and Permanent Magnets

Relationships Among Their Magnetic Properties

BY K. L. SCOTT*
Member, A.I.E.E.

INTRODUCTION

A GENERAL study of the magnetic properties of magnet steels and permanent magnets embraces a number of related problems. Of chief interest to the designer of magnets is the problem of determining the relationship between the open-circuit remanence of a permanent magnet and the various factors which determine its value. These factors include the magnetic characteristics or properties of the steel, as displayed by its hysteresis loop, and the shape and dimensions of the magnet. Of interest to the manufacturer of magnet steel are the questions of how chemical composition, melting practise, and rolling practise affect the magnetic and mechanical properties of the steel. The manufacturer of magnets is interested in the effect of the necessary manufacturing operations, such as heating for hot forming and annealing to increase machinability, upon the flux obtainable in the finished magnet; on the tendency of the steel to warp and crack upon quenching; and on the proper hardening treatment to use.

These interests overlap, of course, and include many special subjects, among which may be listed the manner in which various influences may affect the state of magnetization of a magnet, the correlation of the microstructure and mechanical properties of a magnet steel with its magnetic properties, the determination of a suitable criterion of magnetic quality, the development of accurate and convenient testing equipment, and various other matters of greater or less importance.

It is the purpose of this paper to present data relating to some of the above topics, which have been collected by the writer during the course of several years connection with the manufacture of permanent magnets, both in the laboratory and in the shop.

SYMBOLS AND NOTATIONS

The following symbols and notations will be used. See Fig. 1. All values of B are intrinsic or ferric induction.

B_{max} = The value of magnetic induction corresponding to the tip of a given hysteresis loop, in gauss.

H_{max} = The value of H corresponding to B_{max}, in oersteds.

B_s = The saturation value of flux density corresponding to an infinite magnetizing force.

B_r = Residual induction, the magnetic induction in a ring or infinitely long straight bar after the value of H has been reduced from H_{max} to O. The value of B at the intersection of the hysteresis loop with the B axis, in gauss.

H_c = Coercive force, the value of H required to reduce B from B_r to O in a ring or infinitely long bar. The value of H at the intersection of the hysteresis loop with the H axis, in oersteds.

B_{rem} = Remanence, the magnetic induction at the magnetic equator of a permanent magnet with no external magnetizing or demagnetizing force. Values of B_{rem} given in this paper are without pole pieces on the magnets, in gauss.

L = The actual developed length of a magnet.

A = The area of cross section of a magnet.

D = The equivalent diameter of a magnet $= 2\sqrt{A}/\sqrt{\pi}$.

L/D = The dimension ratio of a permanent magnet.

$(BH)_{max}$ = The maximum value of the product of the coordinates of the demagnetization curve for a given steel. The demagnetization curve is the portion of the hysteresis loop between B_r and H_c.

T = Hardening temperature, absolute scale.

T_0 = Optimum hardening temperature, absolute scale.

FACTORS THAT DETERMINE THE REMANENCE OF A PERMANENT MAGNET

Of the subjects enumerated in the introduction, the first is the one that has been of greatest technical interest to the writer. During the course of several laboratory investigations connected with shop problems, advantage was taken of the opportunity to secure data which might be used for the purpose of making generalizations regarding relations among magnetic properties of magnet steels and permanent magnets.

The specific problem was to find out exactly the way in which the remanence of a magnet is related to its shape and dimensions as well as to the magnetic properties of the material of which it is made. Although it requires the complete family of hysteresis loops for a given piece of magnet steel to portray all of its various magnetic characteristics, it was felt that possibly the

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combined influence upon the remanence of a magnet of the sum of these characteristics could be expressed in terms of the values of B_r and H_c for the material. Such an assumption, if valid, would simplify the problem considerably.

It is true that in the literature on the subject there are several papers[1,5,6] describing more or less accurate methods by means of which a designer may predict the flux of a given magnet provided the demagnetization curve for the steel is known. But these methods are laborious, and unless a complete redesign is worked out for each case, they do not answer such questions, for example, as how much a certain change in B_r or H_c will affect the remanence of a given magnet, other factors remaining constant.

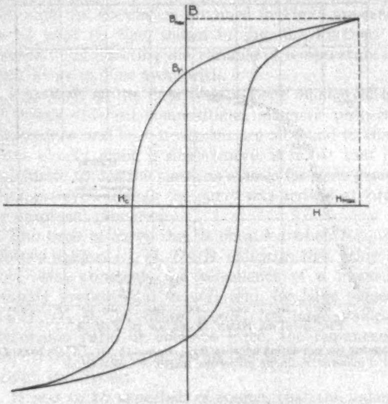

Fig. 1—Hysteresis Loop of Magnet Steel

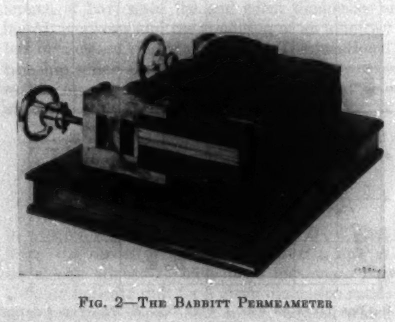

Fig. 2—The Babbitt Permeameter

Used for making magnetic measurements on any kind of magnet steel

Because the flux value of a magnet is affected by shape as well as other factors, and because magnets are made in a bewildering variety of shapes, it was decided to confine the investigations to straight bar magnets and leave the matter of the effect of shape on flux for a later study.

Test Specimens. Accordingly, a large number of straight bars of magnet steel was used for test specimens. Each of the bars was 12 in. in length. In the lot were bars of 1/8 by 1-1/2 in., 1/4 by 1/4 in., 3/16 by 3/4 in., 1/4 by 1-1/4 in., 3/4 by 5/8 in., and 3/8 by 1/2 in. cross sections. All of the types of steel available were used, including 0.9 to 3.5 per cent chromium steel, 0.85 per cent manganese steel, 5 per cent tungsten steel, and 20, 25 and 36 per cent cobalt steels. Along with this range in composition a range in coercive force of from 40 oersteds to 260 oersteds was available, accompanied by a variation in residual induction of from 6,000 gauss to 11,000 gauss.

Experimental Procedure. The test bars were hardened in the manner appropriate to each type of steel, except that some bars were purposely quenched from a high temperature in order to secure low values of B_r. After hardening, the demagnetization curve for each bar was determined by means of permeameter measurements, using the Babbitt[7] permeameter and a Grassot fluxmeter with lamp and scale. The control circuit of this permeameter is the conventional circuit for a ring test, as shown in Fig. 3. The magnetic data are all comparable since accurate measurements may be made with this permeameter on all of the above mentioned types of magnet steel.

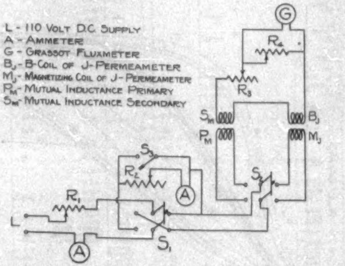
L - 110 Volt D.C. Supply
A - Ammeter
G - Grassot Fluxmeter
B_J - B-Coil of J-Permeameter
M_J - Magnetizing Coil of J-Permeameter
P_M - Mutual Inductance Primary
S_M - Mutual Inductance Secondary
G
R_4
R_3
S_M
B_J
P_M
M_J
S_3
R_2
A
S_2
R_1
L
A
S_1

Fig. 3—Control Circuit of the Babbitt Permeameter

Note that the connections are those of a simple ring test

Following the permeameter measurements, each bar was magnetized as a straight bar magnet in an air core solenoid. Field strengths of over 1,000 oersteds were applied to each bar. Upon removal from the solenoid the flux density at the middle of the bar was measured by means of a search coil and the Grassot fluxmeter with lamp and scale. It is the flux density measured in this way that is referred to in this discussion as the remanence of a magnet, or B_{rem}.

After the measurements at 12-in. lengths were com-

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1. For references see end of paper.

pleted, each bar was cut in succession to 10, 8, 6, 4, and 2-in. lengths by removing 1 in. of material from each end except that in the case of some of the bars of larger cross-section the lengths of 10, 6, and 2 in. were omitted.

Remanence measurements were made at each of the above lengths. The bars were remagnetized for each new measurement of B_{rem}.

The foregoing experiments made available for analysis a wide range of values of the associated variables B_{rem}, B_r, H_c, length L, and cross-sectional area A.

Analysis of Data. The values of remanence *versus* length for a number of the straight bar magnets are plotted in Fig. 4 to show the range of values existing in different bar magnets of the same length, when fully magnetized. The differences in B_{rem} for a particular length of bar are due to the differences in B_r, H_c, and crcss-section of the various samples.

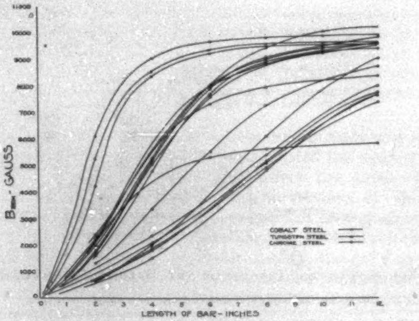

FIG. 4—CURVES OF REMANENCE *vs.* LENGTH FOR STRAIGHT BAR MAGNETS OF DIFFERENT CROSS-SECTIONS AND MATERIALS

The chief characteristic that the curves of Fig. 4 have in common is a resemblance in shape to a normal induction curve and an asymptotic approach to limiting values of B_{rem} as the lengths increase.

In Fig. 5 is shown the result of plotting B_{rem}/B_r *vs.* $(L/D)\sqrt{H_c}(T/T_0)^2$ for the same set of bars. This combination of variables was arrived at purely by a cut and try method, but as shown, a surprisingly good correlation is obtained.

It should be mentioned in connection with Fig. 4 that it was found that the data relating to some of the bars did not fit the curve satisfactorily if B_{rem}/B_r was plotted against $L\sqrt{H_c}/D$, but in each case where this was so the bar had been quenched from above or below the optimum hardening temperature. It was found, however, that the data from these bars could be correlated with those from the properly hardened bars if B_{rem}/B_r were plotted against $(L/D)\sqrt{H_c}(T/T_0)^2$ in which T is the actual, and T_0 is the optimum hardening temperature on the absolute scale. The ratio T/T_0 is, of course, equal to unity for bars properly hardened. The definition of the optimum hardening temperature will be given later.

After this empirical correlation was obtained, it was suggested that B_{rem}/B_r *vs.* $L\sqrt{H_c}/D\sqrt{B_r}$ would be preferable as a choice of variables, from theoretical considerations based upon the assumption of uniform magnetization in the magnets. Although it can be shown that this assumption is not fulfilled in the actual case of straight bar magnets, this method of plotting gives as good a correlation as is shown in Fig. 5. Such a result might be expected, since the values of B_r commonly encountered range from 9,000 gauss to 11,000 gauss, thus differing from 10,000 gauss by 10 per cent at most. The square root of B_r, therefore, differs from 100 by not more than 5 per cent in most cases, whence

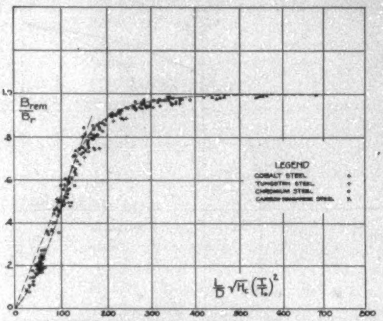

FIG. 5—THE SAME DATA AS IN FIG. 4

Showing the correlation between B_{rem}/B_r and $(L/D)\sqrt{H_c}$ for bars of any kind of material, magnetic properties, length or cross-section

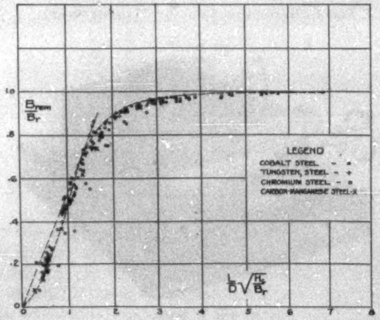

FIG. 6—THE DATA OF FIG. 4 WITH B_{rem}/B_r PLOTTED AGAINST $L\sqrt{H_c}/D\sqrt{B_r}$

it follows that dividing $L\sqrt{H_c}/D$ by $\sqrt{B_r}$ shifts the relative position of the abscissas of the points in Fig. 5 by not more than 5 per cent in most cases. Fig. 6 illustrates the results obtained by the second method of plotting.

Dimensional considerations favor the use of the quantity $L\sqrt{H_c}/D\sqrt{B_r}$, however, since this quantity, like B_{rem}/B_r, is dimensionless, whereas $L\sqrt{H_c}/D$ is not. For this reason it is felt that the method of plotting shown in Fig. 6 is to be preferred to that of Fig. 5 although the correlation obtained in Fig. 5 appears to be as good as that shown in Fig. 6. Further, the necessity for injecting the variable of temperature into the picture is done away with.

Inasmuch as the data from a large number of bars of widely different compositions, magnetic properties, dimensions and heat treatment are all found to cluster quite closely along a single curve, it is felt that it is legitimate to use the curve as a basis for some generalizations, even though the curve was arrived at largely by empirical processes.

The type of curve that is obtained when B_{rem}/B_r is plotted against $L\sqrt{H_c}/D\sqrt{B_r}$ indicates that other factors being constant, the remanence of a magnet is roughly proportional to $\sqrt{B_r}$ and, for large values of $L\sqrt{H_c}/D\sqrt{B_r}$, in other words, for large values of dimension ratio or coercive force, the remanence is practically equal to B_r. This fact is of interest from a design standpoint.

It was to be expected, of course, that the values of B_{rem} would approach B_r as a limit for high values of length of bar, since the end effect diminishes as the length increases and the condition of an infinitely long bar or a closed ring is simulated. The functional relationship which the quantity $L\sqrt{H_c}/D\sqrt{B_r}$ bears to the ration B_{rem}/B_r is not known. A fairly good fit of the observed data is given by the expressions:

$$r = \frac{q^{1.5}}{2.15} \qquad q = O \text{ to } q = 1.25 \text{ and}$$

$$r = e^{-\frac{0.8}{q^{2.75}}} \qquad q = 1.25 \text{ to } q = \infty$$

in which $r = B_{rem}/B_r$ and $q = L\sqrt{H_c}/D\sqrt{B_r}$, but aside from the direct proportionality of B_{rem} and B_r, the equations appear to have little meaning. However, they do indicate that for a given dimension ratio, there is a practical upper limit to H_c, beyond which very large increases in H_c are necessary to produce small increases in the ratio of B_{rem} to B_r. Considerable interest attaches to the fact that the dimension ratio L/D and $\sqrt{H_c}/\sqrt{B_r}$ are of equal weight in affecting the remanence of a magnet. It is also worth noting that the value of B_{rem} is independent of the contour of the cross-sectional area. It is possible that this would not hold for dimension ratios less than one, but it does appear to hold for dimension ratios of practical importance. As shown in Fig. 5, a line drawn through the origin and tangent to the dotted curve of that figure has its point of tangency at values of B_{rem}/B_r and $L\sqrt{H_c}/D\sqrt{B_r}$ of approximately 0.65 and 1.25 respectively. It will be shown later that this is the point of maximum efficiency, *i.e.*, the point at which are obtained the highest values of B_{rem} or external magnetic energy per unit volume of steel.

It follows, if this is true, that magnets should be designed so that $L\sqrt{H_c}/D\sqrt{B_r} = 1.25$, and with this as a basis, the nomogram of Fig. 7 was laid out. The use of this chart to design a magnet for maximum efficiency, is illustrated by the dotted lines of the figure. In the case shown a total flux in the magnet of 3,000 maxwells is desired, and a steel with a B_r of 10,000 gauss and a coercive force of 54 oersteds is assumed. The correct values of A and L are found to be 0.461 cm.² and 13 cm., respectively, and the dimension ratio is 17.

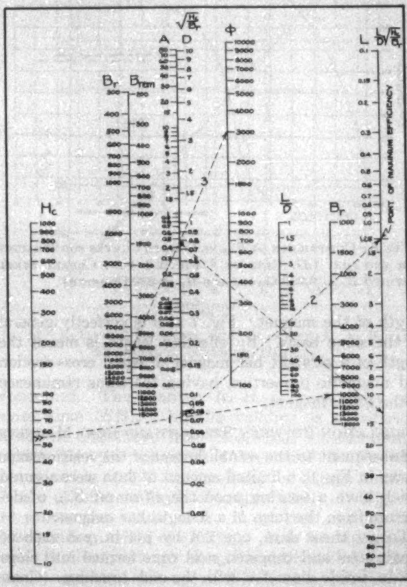

Fig. 7—Nomogram Chart for Permanent Magnet Design for Maximum Efficiency. Based on the Curves of Figs. 6 and 10

It should be borne in mind throughout this discussion that Figs. 4, 5, 6 and 7 apply directly only to straight bar magnets. Formed magnets with short air-gaps with or without pole pieces, will have higher remanence values than are shown by the curve of Fig. 6. This is because, compared with straight magnets, their effective lengths are greater than their actual lengths. The difference between effective and actual lengths is negligible, however, except for magnets whose poles are brought quite close together, either actually or by means of pole pieces of soft iron. Watt-hour meter damping magnets are good examples of magnets whose poles are brought thus closely together without the aid of pole pieces.

On the other hand, the general conclusions drawn from the curve of Fig. 6 are applicable to magnets of any shape if L is assumed to stand for the effective length of the magnet. Fig. 7 also is perfectly general on the same basis. By effective length is meant the length of a straight bar magnet of equal cross-section and magnetic properties, having the same remanence as the given magnet.

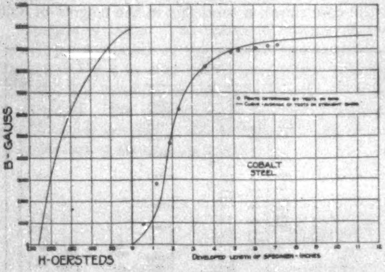

FIG. 8—COMPARISON OF B_{rem} *vs.* LENGTH CURVES FOR STRAIGHT BAR AND 2-IN. I.D. RING OF 1/4-IN. x 1/4-IN. COBALT STEEL IN WHICH $B_r = 9{,}980$ GAUSS AND $H_c = 230$ OERSTEDS

CORRELATION BETWEEN STRAIGHT AND BENT MAGNETS

Subsequent to the establishment of the relationships shown in Fig. 6, a limited amount of data were secured which have a bearing upon the effect on B_{rem} of departure from the form of a straight bar magnet.

To get these data, one 1/4 by 1/4 in. rod each of cobalt steel and tungsten steel were formed into rings of 2-in. inside diameter, with the ends touching. Companion test bars were cut adjacent to each end of the rods used in forming the rings. All three pieces of each kind of steel received as nearly identical heat treatments as possible, even to heating and cooling the straight test bars at the time of forming the rings. After hardening, demagnetization curves were obtained on each of the two test bars of tungsten steel and of cobalt steel. Then data for B_{rem} *vs.* length curves were obtained from both the test bars and rings by the process of cutting off the ends and remeasuring B_{rem} already described.

The demagnetization curves and the B_{rem} *vs.* length curves, for the two straight cobalt steel bars are practically identical. Inasmuch as these bars were cut adjacent to the ends of the bar from which the ring was formed, and heat treated with it, it is safe to assume that the material in all three samples is alike magnetically, and that the demagnetization curve for the ring is the same as for the straight bars. The same conclusions hold for the tungsten steel, as shown in Fig. 9.

It follows then that it is valid to compare the B_{rem} *vs.* length curves for the straight bars with those for the rings, and conclude that all differences in the curves are due to differences in shape. It is interesting to note that the ring for cobalt steel yields the same curve for B_{rem} *vs.* length as the straight bar, but that, in the case of the tungsten steel, the proximity of the ends of the ring has an appreciable effect on the value of B_{rem}, causing the curve for B_{rem} *vs.* length to turn upwards for those lengths corresponding to short air gaps.

Additional data along these lines would be very useful, and should include measurements on magnets with pole structures having well defined air gaps.

CRITERIA OF MAGNETIC QUALITY OF PERMANENT MAGNET STEELS

A number of quantities determinable by magnetic measurement has been proposed at various times by different investigators for use as criteria of magnetic quality of magnet steels. Among these quantities may be listed $(BH)_{max}$⁽¹⁾, the maximum product of the coordinates of the demagnetization curve; and the quantities $B_r\, H_c$⁽²⁾, B_r/B_{max}, $B_r\, H_c/B_{max}$ and B_r/H_c⁽³⁾. High values of all of the proposed criteria except the last were considered desirable.

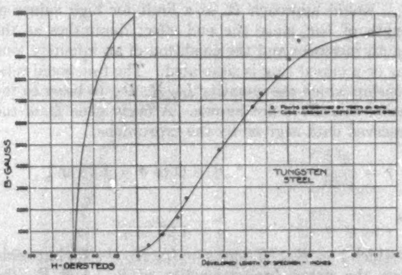

FIG. 9—COMPARISON OF B_{rem} *vs.* LENGTH CURVES FOR STRAIGHT BAR AND 2-IN. I.D. RING OF 1/4-IN. x 1/4-IN. TUNGSTEN STEEL IN WHICH $B_r = 10{,}830$ GAUSS AND $H_c = 59$ OERSTEDS

The form B_r/H_c has decided disadvantages, since a low value of the criterion is obtained when B_r is small as well as when H_c is large, and low values of B_r are distinctly undesirable in view of the fact that B_{rem} is directly proportional to B_r. The ratio B_r/B_{max} was found to be practically constant for some types of steel, regardless of the value of H_c, and thus was of no value as a criterion. The quantity $B_r\, H_c/B_{max}$ would accordingly amount to a constant times H_c and would give no actual weight to the value of B_r. These three criteria would, therefore, fail either to indicate a choice between different types of magnet steel or to aid in the selection of an optimum heat treatment for a given kind of steel.

This leaves the quantities $(BH)_{max}$ and $B_r H_c$ as the only ones not obviously faulty. The first was proposed by S. Evershed in a noteworthy paper in the *Journal* of the Institution of Electrical Engineers for September 1920, in which he showed that the quantity $(BH)_{max}$ is a measure of the maximum amount of external magnetic energy which can be supported per unit volume of a given magnet steel, and that this in turn defines the term "magnetic quality" as applied to magnet steel.

Evershed's derivation of his criterion of magnetic quality, given in the article noted above, is complete and convincing, but it was thought desirable to obtain experimental verification. The data from the bars of Fig. 4 were used to this end. In Fig. 10 is shown a number of curves each corresponding to a different type of magnet steel. The demagnetization curves for the bars are shown plotted in the usual way, and to the right of these are plotted the curves of (BH) *vs.* B. To the right of these are plotted further the experimentally determined curves of B_{rem} *vs.* length for the bars in question.

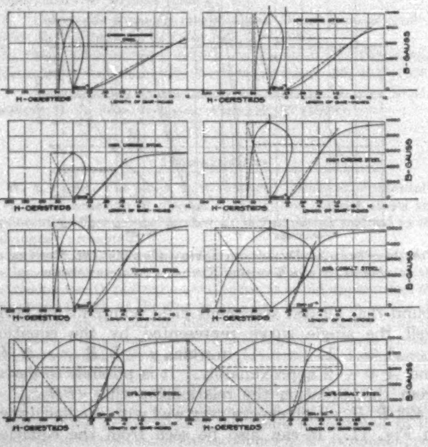

Fig. 10—Curves Showing Experimental Verification of Validity of Evershed's Criterion for Magnetic Quality of Permanent Magnet Steels

Fig. 10 shows that if tangents to the curves of B_{rem} *vs.* length are drawn through the origin the point of tangency in each case indicates the value of B_{rem} and length at which the ratio of B_{rem} per unit length or volume of steel is a maximum, and it is clearly demonstrated that in each case these values of B_{rem} coincide closely with the values of B for which the product (BH) is a maximum. The writer feels that this constitutes a sufficiently convincing verification of the correctness of Evershed's criterion.

The chief objection to the use of Evershed's criterion is that a number of points on the demagnetization curve for a piece of magnet steel must be known before the numerical value of the product $(BH)_{max}$ can be determined. The quantity $B_r H_c$ is more easily obtained than $(BH)_{max}$ but its validity as a criterion cannot be established from theoretical considerations. Accordingly, the values of $(BH)_{max}$ and $B_r H_c$ were determined by actual measurement for a large number of bars of different kinds of magnet steel. In Fig. 11 are shown the values of $B_r H_c$ plotted against the corresponding values of $(BH)_{max}$. It is felt that these points lie closely enough along the straight line in that figure to warrant the use of $B_r H_c$ as a criterion of magnetic quality of magnet steel for all practical purposes. In cases where very accurate comparisons are required, it may be necessary to go to the greater trouble of determining $(BH)_{max}$.

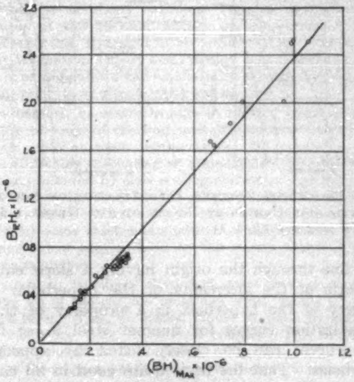

Fig. 11—Relation Between the Quantity $B_r \times H_c$ and Evershed's Criterion, $(BH)_{max}$

Referring to a previous mention of optimum hardening temperatures, it will be understandable now to state that the optimum hardening temperature for a given type of steel is that temperature at which a maximum value of $(BH)_{max}$ is obtained, or practically speaking, a maximum value of $B_r H_c$.

Fit of Demagnetization Curve by Hyperbola

It will be noted that the points on the demagnetization curves for which the product (BH) is a maximum are given quite accurately in each case by the intersection of the demagnetization curve with a line through the origin having the slope B_r/H_c. That this should be so follows from the fact that any demagnetization curve for magnet steel may be closely approximated by a rectangular hyperbola whose equation is $B = a - k/(H + b)$ in which a, b, and k are parameters of each particular curve. It is a mathematical property of the foregoing rectangular hyperbola that the coordinates whose product is a maximum are located by a line through the origin having a slope equal to the ratio of the intercepts of the hyperbola. This property of the hyperbola is a property of the demagnetization curves for magnet steel in so far as these curves can be closely fitted by appropriate hyperbolas. That the fit is quite good in all cases is shown by Fig. 12 in which hyperbolas calculated for each demagnetization curve are shown by dots and the observed points by a full line. The graphical method of Fig. 10 for determining the point on each demagnetization curve the product of whose coordinates is a maximum is more accurate than the method of plotting the curve of (BH) *vs.* B, because the latter curve is usually quite flat-topped and its maximum is hard to locate exactly. The fact that the curve is flat-topped explains why many methods of magnet design give good results. It is because about equal efficiencies are obtained in any case over a fairly wide range of values of $B_{rem.}$

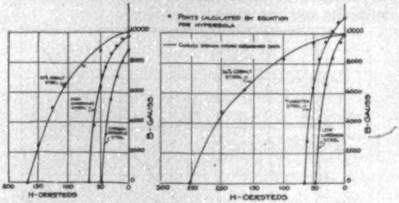

FIG. 12—CURVES SHOWING APPROXIMATION OF ACTUAL DEMAGNETIZATION CURVES BY POINTS ON THE GENERAL HYPERBOLA, $B = a - k/(H + b)$

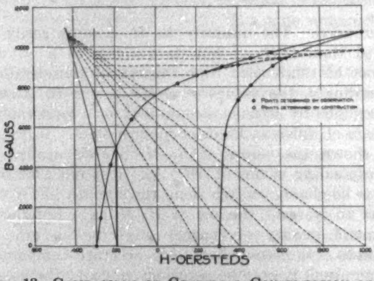

FIG. 13—COMPARISON OF GRAPHICAL CONSTRUCTION OF THE DESCENDING PORTION OF THE HYSTERESIS LOOP WITH ACTUAL MEASURED DATA

Note the lack of agreement for positive values of H

A further application of the idea that the demagnetization curve can be represented by a hyperbola is made by Mr. E. A. Watson.[4] Watson applies a graphical construction for a hyperbola to the demagnetization curve and extends the construction into the region of positive values of H, thus providing a means of extrapolating the hysteresis loop to the saturation value of B. It has been the writer's experience that Watson's graphical construction cannot be legitimately extended into the first quadrant because the curve thus predetermined almost always lies considerably under the true curve, and yields too low a value for the saturation flux density. Fig. 13 illustrates the case in point.

If, instead of potting B *vs.* H for the points of the demagnetization curve, we plot B/B_r *vs.* H/H_c, it is found that all test specimens yield points fitting fairly well the single curve represented by the equation $y = A\,(x + 1)/(x + A)$, in which $y = B/B_r$, $x = H/H_c$ and $A = \sqrt{2}$ approximately. It is possible to derive from this equation the relationship that $(BH)_{max} = 0.423\ B_r\,H_c$ which checks very well with the line drawn in Fig. 11. It can also be seen from the equation $y = A\,(x + 1)/(x + A)$ that the representation of the demagnetization curve by a hyperbola cannot be extrapolated to give the correct value for B_s since for values of H approaching infinity, the equation yields the result that $y = B_s/B_r = A = \sqrt{2}$ whence $B_s = \sqrt{2}\,B_r$, while values of B in excess of $\sqrt{2}\ B_r$ are commonly encountered at ordinary values of H.

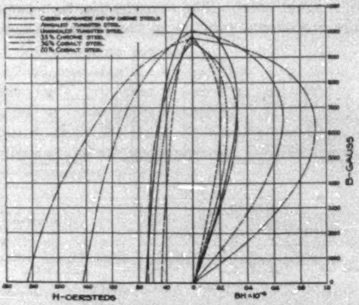

FIG. 14—TYPICAL DEMAGNETIZATION CURVES AND CURVES OF BH *vs.* B FOR VARIOUS KINDS OF MAGNET STEEL

CHARACTERISTIC CURVES FOR MAGNET STEEL

In Fig. 14 are shown demagnetization curves and (BH) curves for the various types of magnet steel with which the writer has had experience. It should be remembered that while these curves are more or less typical of the steels in question, there is no one curve which can be given as the curve for the material. There will be as many curves as there are samples tested, because magnetic characteristics of a piece of magnet steel, or any other magnetic material depend upon the whole previous thermal history of the sample from the time of melting to the time of testing, the amount of hot and cold working the sample has received, and the conditions of stress in the material at the time of testing. The relation of all of these variables to the remanence of a permanent magnet, fortunately, is focussed, as it were, in the values of B_r and H_c for the material, and the relation of these variables to B_{rem} is as shown by Fig. 6.

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4. In discussion, *loc. cit.*(1), *Journ. Inst. Elect. Eng.* **58**, 829 (1920), also *ibid.* **61**, 641 (1923).

5. "Calcul des appareils magnétiques, Diagramme génerale,' Picou, R. V., *Rev. gén. dé l'élect.*, **22**, 259 (1928).

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Discussion

W. A. Codd: The paper quite evidently checks experimentally Evershed's criterion of magnetic quality for magnet steels and apparently offers a considerably simplified design method in the nomogram presented.

In Fig. 9, from the data given on the closed ring sample, 2 in. inside diameter and ¼ in. by ¼ in. section, together with the maximum length plotted on the curve, it would appear that the developed length of the ring sample has been taken as the circumference on the outside diameter of the ring rather than the mean length which appears to be more logical. Is there a reason for this or is the 2 in. inside diameter of the ring merely an approximate figure.

Also in connection with Fig. 9, what is the reason for the difference of the effect of proximity of the ends of the magnet for Tungsten steel as indicated in this figure as compared to this effect for the Cobalt sample as shown in Fig. 8. It would appear that the reason for this lies in the facts brought out in the paper in regard to the relatively small effect of increases of H_c, or equivalent changes, as B_{rem}/B_r approaches unity. Is this correct?

There is one other question in regard to the data taken on the ring samples. How was the measurement of the B_{rem} flux value made on the ring sample for maximum length with the ends touching?

It is understood that the measurements of B_{rem} were made, after the sample was magnetized, by use of a search coil. In this case to measure the total flux of the magnet at this neutral section it would be necessary to draw the coil from the neutral section off the end of the magnet. Otherwise only the leakage flux would be measured, of course.

In regard to the nomogram shown in Fig. 7, for use in permanent magnet design, this apparently offers a decided simplification over Evershed's step-by-step method of calculation. However, for use in connection with bent magnets, as the author points out, it is necessary to use the effective length rather than the actual length of the magnet. Figs. 8 and 9 evidently offer a method of obtaining this effective length for a given shape of bent magnet. Is it necessary to make such an experimental determination for each particular shape which is to be used, or is there a general method for obtaining the effective length? If such a method of readily calculating this effective length were available, the design nomogram would be extremely valuable.

The author mentions variations between samples due to differences of the thermal history of the sample. In connection with this, Evershed in his article of Aug. 1925 on "Permanent Magnets in Theory and Practise" discusses these thermal effects in considerable detail emphasizing in particular two effects which he refers to as ultra-heating and spoiling.

Ultra-heating, he states, results in reduced values of B_r due to failure of some of the iron molecules to return to the α or magnetic state at room temperatures. He suggests a curing process consisting of heating to a moderate temperature (1,380 deg. fahr.) followed by slow cooling and states that this process should always be used by the magnet maker before forming magnets. This does not seem to agree with the process recommended by some steel manufacturers which indicates that the total amount of heating used by the magnet maker should be kept at a minimum.

In regard to "spoiling," Evershed states that this consists of breaking up of the carbide molecules which results in reduced coercive force. This, he states, should be cured by heating to about 2,370 deg. fahr. for several minutes. This he indicates should be done at the mills before rolling into bars. If this is not done the magnet maker will receive steel low in coercive force.

Evershed gives the above particularly in regard to Tungsten steel. Has the author any experience in regard to these points?

The author has made numerous measurements on a large number of samples of each of the various kinds of magnet steel available. Does he find that any of these steels show any better uniformity than others when exercising equal care in control of heat treatment?

K. L. Scott: The ring magnets, about which Mr. Codd has asked were formed hot about a mandrel roughly two inches in diameter. The developed length, as given in the paper, represents the actual mean length of the ring. Mr. Codd's interpretation of Figs. 8 and 9 is correct. A gap was ground between the ends of the ring magnets, large enough to admit a small search coil, before any flux measurements were undertaken, although an initial value of B_{rem} could have been obtained before this was done by following the procedure for a standard Rowland ring test.

A general method of determining the effective length of a magnet, with or without pole pieces, has not been worked out to the writer's knowledge. Such a general method would probably involve the calculation of internal and external reluctances for a given design of magnet and pole pieces, and the calculation of effective length from these reluctances either by reference to an empirical curve or formula.

The writer has had no first hand experience with steel mill practise in the manufacture of tungsten magnet steel. His own experience does bear out the recommendation that the total amount of heating by the magnet maker should be kept as low as possible. The writer's experience does not enable him to differentiate between the various kinds of magnet steel as regards uniformity of magnetic quality when exercising equal care in control of heat treatment. It appears to the writer that exact control of the heat treatment of magnetic materials is yet to be attained.

An Induction Motor with Paralleled Rotor and Stator

BY A. G. CONRAD[1]
Associate, A.I.E.E.

and

R. G. WARNER[1]
Member, A.I.E.E.

***Synopsis.**—A wound rotor induction motor may be operated with the stator and the rotor independently magnetized, (neither supply frequency corresponding to the slip frequency). If the stator and rotor are paralleled the pull-out torque as a motor is approximately doubled and the continuous horsepower output may be increased, and as an induction generator, the maximum kva. output is increased. It may be operated simultaneously as a motor and as a generator.*

This method of operation was new to the authors and to several eminent induction motor designers and as no reference has been found to others employing this principle, the theory has been developed and is presented with the experimental results of tests.

INTRODUCTION

WITH machines wired as in Fig. 1 it is possible to transfer power from rotor to stator or from stator to rotor at high frequency and while this is taking place, the machine can either receive power or deliver power at its shaft. Using more general terms, if stator and rotor are independently excited then power can be delivered to the machine through three channels namely, the stator, the rotor or the shaft. When energy is supplied through two of these channels it can be taken from the third and when it is supplied to one it can be taken from one or both of the other two.

The rotor and stator voltage and frequency can be adjusted to the same values; thus it is possible to parallel stator and rotor and make the machine operate either as a double motor or double generator.

PRINCIPLE OF OPERATION

Fig. 1 shows a wiring diagram of a wound rotor induction motor with the rings of the motor connected to a synchronous machine. The armature of the synchronous machine serves to provide a path for the secondary current and consequently the induction motor can be started. The induction motor will behave much the same as it would with its rings short-circuited. If the phase rotation of the stator was in the order of 1-2-3 the voltages induced in the rotor would be a low-frequency voltage with a phase rotation of *a-b-c*. This will cause three currents to flow in the synchronous machine armature that are 120 deg. out of phase and in the order of *A-B-C*. This means, of course, that if the windings are placed on the machine as shown in the diagram that the induction motor will rotate in a clockwise direction.

With the stator disconnected from the supply and short-circuited directly or through transformers, and power supplied to its rotor from the alternator in Fig. 1, the induction machine may be started again. If the phase rotation of the alternator is *C-B-A* the induction machine will rotate in the same direction as above and the low-frequency currents in the stator will be in the direction 3-2-1.

If the stator is now supplied as usual and at the same time the rotor is supplied from the alternator with phase rotation *C-B-A* the supply transformers will act as a low impedance path for the slip-frequency currents in the stator and the alternator will complete the path for the slip-frequency currents of the rotor. The magnitude of the voltage impressed on the rotor is dependent on the field excitation of the synchronous machine. The frequency of the voltage depends upon the speed of the synchronous machine. By adjustment of the input of the d-c. machine the frequency in the rotor circuit can be made equal to the stator frequency. By the field adjustment of the synchronous machine the rotor voltage can be made equal to the stator voltage. When the rotor and stator have the same voltage and frequency they can be synchronized and electrically connected through the switch S_1 of Fig. 1. The circuit to the synchronous machine can now be opened at S_2. The induction motor will continue to operate with its rotor and stator electrically and magnetically connected; both the stator and rotor will receive power from the line. The machine will rotate at a speed less than synchronous speed. The high-frequency current in the stator will be in the order of 1-2-3 and the low-frequency currents in the order of 3-2-1. The high-frequency currents in the rotor will be in the order of *c-b-a* and the low frequencies in the order of *a-b-c*. The machine when

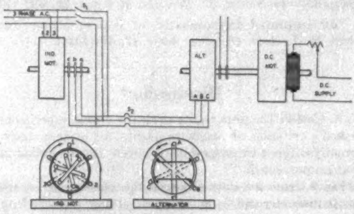

FIG. 1

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Abridgment of a thesis submitted by A. G. Conrad to the Graduate School of Yale University, June 1931.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

so connected has characteristics similar to those of the normal induction motor. When loaded the speed decreases slightly.

When driven above synchronous speed the machine acts as an induction generator. The low-frequency currents in the stator and rotor are opposite in phase rotation to what they are as a motor and are in the order of 1-2-3 in the stator and *c-b-a* in the rotor.

The characteristics of the motor with its rotor and stator paralleled can be determined by applying ordinary induction motor theory. Using slip as parameter it is possible to determine the torque, stator, copper loss and rotor loss, etc., by the power fed into the stator and for the same slip determine value for the same items which are produced by the power fed into the rotor.

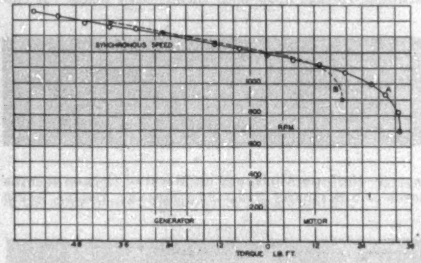


FIG. 2

Curve *A*—Speed torque curves of machine with rotor and stator paralleled
Curve *B*—Speed torque curves of machine with rotor short-circuited

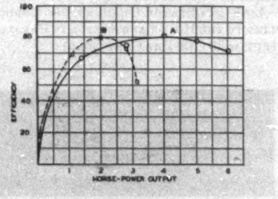


FIG. 3

Curve *A*—Efficiency of the motor with its rotor and stator paralleled
Curve *B*—Efficiency of the motor with its rotor short-circuited

EXPERIMENTAL INVESTIGATIONS

Calculations and analysis of operation were checked experimentally. The machine selected for tests was a Westinghouse wound rotor induction motor rated as follows:

Horsepower...... = 10 Stator volts....... = 220
Cycles........... = 60 Rotor volts....... = 206
Three-phase...... 1,140 r.p.m.
Serial No. 4191333

The motor has nearly a 1 to 1 ratio of stator to rotor turns.

In order to keep within the limitations of the laboratory circuits, the voltage on each phase was dropped to 110 volts, 60 cycles and the characteristics were determined with stator and rotor paralleled. The machine was operated both above and below synchronous speed to determine the characteristics as a motor and as a generator. In this machine the voltage supplied to each phase is a fundamental voltage while the current will contain not only a fundamental component but also a low-frequency component.

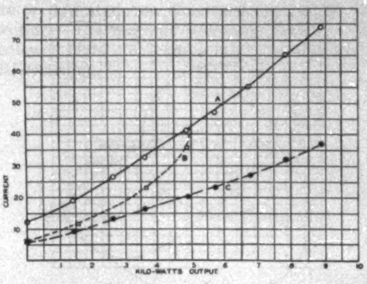


FIG. 4

Curve *A*—Line current delivered by generator with rotor and stator paralleled
Curve *B*—Current in stator phase of the generator with rotor short-circuited
Curve *C*—Current in stator phase of the generator with its rotor and stator paralleled

The results of these tests are shown as curve *A* in Figs. 2 to 5. For comparison curve *B* in each figure shows the characteristics of the motor with normal connections.

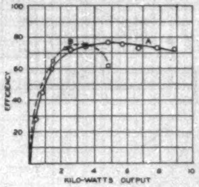


FIG. 5

Curve *A*—Generator efficiency with rotor and stator paralleled
Curve *B*—Generator efficiency with rotor short-circuited

Fig. 2 shows that the breakdown torque of the motor can be approximately doubled by using the stator and rotor in parallel; likewise that the generator capacity and maximum output can be increased by the parallel connection.

The relative efficiencies for the two methods of operation are shown in Figs. 3 and 5. With the parallel connection high efficiencies are obtained for large loads on account of reduced copper losses per unit output but the efficiencies are low for light loads because of larger iron losses and copper losses produced by the additional exciting current. The maximum efficiency for the two cases seemed to be approximately the same for both the condition of motor and generator action.

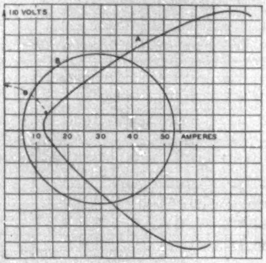


FIG. 6

Curve *A*—Locus of line current with rotor and stator paralleled
Curve *B*—Locus of line current with rotor short-circuited

The heating in the machine due to the copper losses will always be less for heavy loads when parallel connected than when the rotor is short-circuited. In Fig. 4 is shown the line current for different loads on the generator for each of the methods of operation. It can be seen that the phase current with the parallel connection will be less than the phase current when the generator is supplying the same load with its rotor short-circuited.

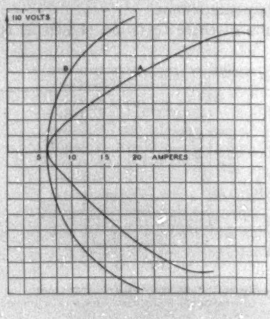


FIG. 7

Curve *A*—Locus of current in stator phase with rotor and stator paralleled
Curve *B*—Locus of current in same state phase with rotor short-circuited

In Fig. 6 is shown the circle diagram of the motor when the rings are short-circuited. The no-load losses for this machine are very small but it can be seen that when the rotor and stator are paralleled that the no-load line current contains a larger power component. This is because of the increased iron losses due to the high-frequency rotor flux and also the super-position of the two fluxes traveling in the iron at different speeds. In addition to this the exciting current in the rotor causes copper losses that increase the no-load input.

It will also be noted that the maximum power component of the input current is a little less than twice the maximum power component of the motor when its rings are short-circuited. This is caused by a slight terminal voltage drop with these heavy loads. It can be assumed that the heart-shaped diagram would have had a slightly higher maximum power component if the voltage had remained constant. The arithmetical sum of stator and rotor current was almost equal to the line current. If the rotor and stator constants were the same each current would be one-half the line current and when compared with the current with the rings short-circuited we would have current loci per stator phase as shown in Fig. 7.

FIG. 8—NO-LOAD CURRENTS OF MACHINE OPERATING WITH ROTOR AND STATOR IN PARALLEL. VOLTAGE APPLIED = 245

FIG. 9—LINE AND STATOR CURRENT FOR CONDITION OF 220 VOLTS APPLIED TO THE ROTOR AND STATOR IN PARALLEL. HORSEPOWER OUTPUT = 10 (100 PER CENT LOAD)

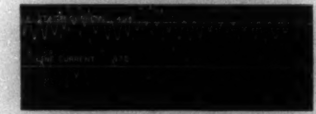


FIG. 10—STATOR AND LINE CURRENTS TAKEN BY MOTOR WITH ITS ROTOR AND STATOR PARALLELED SUPPLY VOLTS = 220; SPEED = 1040; HORSEPOWER OUTPUT = 20 (200 PER CENT LOAD)

The exciting current taken by the motor is indicated in the oscillogram of Fig. 8. It can be seen in Fig. 9 that some of the high-frequency waves have pointed peaks and the other maximum value of the same wave has a rounded point. This indicates a high saturation during part of the cycle.

When the machine is loaded, the high-frequency currents are slightly out of phase and the low-frequency currents are slightly out of phase. This is caused by different constants in the rotor from those in the stator, *i. e.*, the ratio of stator inductance to stator resistance is different than the ratio of rotor inductance to rotor resistance. It can also be seen that the low-frequency current is slightly less than the high-frequency current in the same phase as is shown in Figs. 9 and 10.

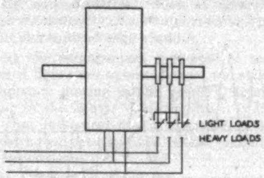
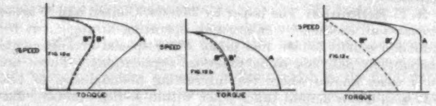


Fig. 11—A Suggested Connection by Which Operating Characteristics Might be Improved

Conclusions

When the practical value of the connection is considered it can be seen that this connection increases the break-down torque approximately 100 per cent. The efficiency for heavy loads is higher than that of the same motor operated with its rings short-circuited. (See Figs. 3 and 5.) For light loads the efficiency of the motor with the rotor short-circuited is higher and the total excitation current is one-half or less depending upon the degree of saturation. When the machine is to be subjected to a wide range of loads it is possible to use a switching arrangement in the rotor that will short-circuit the rings when the load is light and connect the rotor and stator in parallel when the load is heavy. (See Fig. 11.)



A B C

Fig. 12

Curve *A*—Torque developed by motor when rotor and stator are paralleled
Curve *B*—Torque developed by motor with rotor short-circuited and power supplied to stator
Curve *C*—Torque developed by the motor with stator short-circuited and power supplied to the rotor

The advantages of this parallel connection are:

1. The rating of a given motor can be increased over what it would be if the rings were short-circuited.
2. Initial cost of motor required to drive a given load will be less and with the suggested switching arrangement smaller motors may be utilized giving the additional advantages.
3. The no-load losses may be made smaller.
4. The all day efficiency of the motor when subjected to a wide range of loads will be higher than that of a larger motor operating with its rings short-circuited.

The tests made on the machine as a motor and as a generator indicated two disadvantages of the parallel connection. The machine was noisy and the exciting current was high. This noise might be eliminated by special designing.

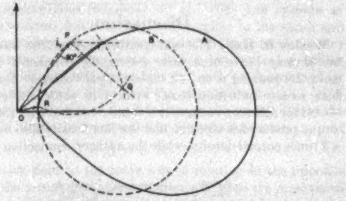


Fig. 13—Construction of Current Locus Diagram

Due to the super-imposed fluxes rotating in opposite directions the iron of the motor was saturated as is indicated in Fig. 14. This difficulty could be minimized by a special design.

It is possible to operate a wound rotor induction motor with its rotor and stator paralleled when ratio is not 1 to 1, by connecting the low-voltage rotor to the high-voltage line through an auto-transformer. The behavior of the machine under these conditions should be better than when no transformer is used, since the transformer will by-pass part of the low-frequency current that ordinarily would flow through the supply transformers.

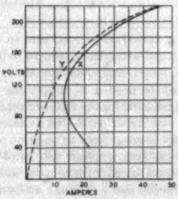


Fig. 14

Curve X—No-load current taken by the machine at different voltages with its rotor and stator paralleled
Curve Y—Estimated saturation curve

By designing the machine so that the rotor and stator phases have the same constants it is possible to obtain speed torque curves as shown in Fig. 12A, the total torque being the sum of the action and reaction. By putting resistance in each phase a characteristic can be obtained as in Fig. 12B. If either the rotor or stator are designed with a high resistance and those of the other element with a low resistance it is possible to obtain a total characteristic as shown in Fig. 12C, obtaining both high starting torque and also good regulation.

The effects of the low-frequency currents on the behavior of the line will depend to a certain extent on the ratio of the motor rating to the rating of the transformers supplying the load. It is obvious that if this ratio is large the efficiency might be reduced.

Discussion

Wesley B. Hall: The same performance as the authors obtained may be secured from a normally connected induction motor by running it on $\sqrt{2}$ times normal voltage. This can be done to any induction motor even if its rotor and stator are wound for different voltages or if it has a squirrel-cage rotor. The torque produced is uniform, and the flux density also uniform at $\sqrt{2}$ times normal density, while the authors' connection produces a torque pulsating periodically from zero to twice load torque, and a flux density pulsating from zero to twice normal density. The performance, losses, and efficiency are in general the same. A transformer may be needed to supply the increased voltage, which may not be required for the authors' scheme.

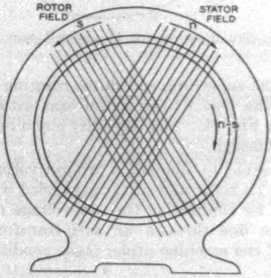


Fig. 1

With the rotor and stator paralleled as described in the paper, the air gap flux may be considered as made up of two components; a field set up by the stator, which will be of normal value and rotate at synchronous speed with respect to the stator, cutting the rotor at slip speed; and a field set up by the rotor, of equal value, which will rotate at synchronous speed with respect to the rotor and cut the stator at slip speed. Each of these fields will be of uniform constant value. This is shown in Fig. 1, where *n* indicates synchronous speed, *s* indicates slip speed, and *n-s* indicates rotor speed.

If the load on the motor is twice normal torque, it may be considered that the field produced by the stator carries normal torque and the field produced by the rotor carries an additional normal torque (although it will be shown later that this involves only the average value of a pulsating torque). Then the stator carries normal current at line frequency to maintain its own field and carries an additional normal current at slip frequency caused by the rotor's field. The true stator load current is the sum of these components and is shown in Fig. 2. The r.m.s. value of this sum is, of course, $\sqrt{2}$ times normal current in the stator winding. Similarly the current in the rotor winding is of exactly the same character and has an r.m.s. value of $\sqrt{2}$ times normal. This is the same value as would be present if the motor were normally connected and carrying twice normal torque on $\sqrt{2}$ times normal voltage. The similarity exists at all loads, so that for the two methods of connection the I^2R losses are equal, the efficiencies are the same, and the slips alike for any given load, neglecting differences in exciting currents.

The torque obtained by the authors' connection, however, is not the uniform sum of two uniform torques obtained from these two uniform rotating fields. The fields are periodically aiding or opposed, so that the resultant air gap flux is periodically zero or twice normal, at a frequency equal to line plus slip frequency, as may be seen by an inspection of Fig. 1. This result is shown in Fig. 3. When the fields are opposed the torque is zero, when aiding, it is twice load torque (four times normal in the specific case assumed). The periodic occurrence of twice normal flux density calls for very large magnetizing current, and the torque pulsation causes very evident vibration.

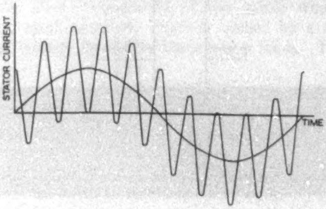


Fig. 2

The average value of torque equals the sum of the individual torques of the two component fluxes (twice normal in the case assumed).

R. E. Hellmund: The paper by Messrs. Conrad and Warner discusses an exceedingly interesting operating arrangement for induction motors, which like many other special arrangements undoubtedly will find some practical applications. There are other cases known where the interesting phenomenon of two induction motor actions takes place within a single motor. Examples of this are the various internal cascade arrangements and the operation of an induction motor with a single-phase secondary, which is equivalent to an internal cascade arrangement. To my knowledge, however, the particular arrangement described in the paper has not been covered previously in literature.

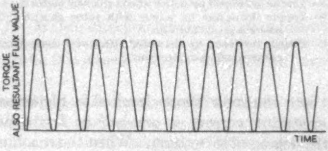


Fig. 3

Like many other special arrangements, the one covered in the paper probably will not find application for general purpose work because the same results can usually be obtained with the ordinary induction motor. This is evident from the following simple considerations. In the proposed scheme, double full-load torque, for instance, can be obtained by having a full-load current of slip-ring frequency superimposed upon full-load current of line frequency in each member. This means twice the full-load losses, because with two currents of different frequency superimposed, the losses are equal to the sum of the losses caused by the individual currents. On account of the superimposition of the two fields in the core portions, the maximum magnetic densities, however, are doubled. This, of course, will mean correspondingly high core losses and perhaps difficulties with noise, etc. If, on the other hand, the same motor were operated with 41 per cent higher voltage and 141 per cent load current, it would also give twice the output with twice the copper losses and only 41 per cent increase for the core densities. Such an arrangement would further obviate the necessity of a wound rotor and permit the use of the much simpler squirrel-cage winding.

In employing the arrangement suggested in the paper care must be taken if lights are operated from the same circuit, because low frequency flowing in the primary circuit is likely to cause flickering of the lights. This condition is similar to that obtained with an induction motor operated with single-phase secondary, which, except for this reason, might have found more general application.

Although for the reason stated the general application of the proposed arrangement is unlikely, it may nevertheless be useful in special cases, as, for instance, where speed torque characteristics as shown in Fig. 12c are desirable, and therefore the publication of the results obtained by the authors should be considered a welcome contribution to the art.

C. L. Fortescue: In dealing with rotating apparatus, by the method of symmetrical coordinates, the usual convention is to consider all phase sequences with reference to the stator. With this convention a symmetrically-wound induction motor running at normal slip will have only positive sequence currents in both stator and rotor. The same motor when driven above synchronous speed will have positive phase sequence currents in the stator and negative phase sequence currents in the rotor. Analytically this is taken care of by the fact that the negative phase sequence currents and e.m.fs. in the rotor are conjugates of the positive phase sequence quantities that appear in the analysis and which have negative indexes. The analytic method has no discontinuities but the solution is obtained, partly in terms of positive sequence quantities having the conventional positive indexes and partly in terms of positive sequence quantities having negative sequence indexes which in the final statement of the solution may be replaced by their conjugate negative sequence equivalents with positive indexes.

In dealing with problems in which the rotor is coupled with the primary circuit inductively or conductively the phase sequence relations of the motor with respect to the primary circuit must be considered with respect to these conventions, in order to obtain the correct analytical solution.

In the present problem from the standpoint of the stator and the circuit connections thereto the circuit connections to the rotor must be made negative sequence in order to give the same direction of rotation. Therefore, in the rotor the primary currents will be negative sequence as far as the motor reactions are concerned and the resulting secondary currents will likewise be negative sequence. But whereas the primary current in the circuit will be positive sequence for both stator and rotor, the secondary currents will both be negative sequence with respect to the primary circuit.

$S' \check{I}_{a1}$ is the positive sequence system of currents in the stator
$S^2 \check{I}_{u2}$ is the negative sequence system of currents in the rotor
$S^2 \check{I}_{a2}$ is the negative sequence system of currents in the stator
$S' \check{I}_{u1}$ is the positive sequence system of currents in the rotor

The equations of the induction motor operated with both primary and secondary connected to the same source of power are:

$$S' \check{E}_{a1} = S' \left\{ (R_a + j\omega_0 L_a)\, \check{I}_{a1} + j\omega_0 M \check{I}_{u1} e^{\,j(\omega_0 - \omega_1)t} \right\}$$

$$\begin{aligned} O &= S^2 \left\{ (R_a + j\omega_1 L_a)\, \check{I}_{a2} + j\omega_1 M \check{I}_{u2} e^{-j(\omega_0 - \omega_1)t} \right\} \\ O &= S' \left\{ (R_u + j\omega_1 L_u)\, \check{I}_{u1} + j\omega_1 M \check{I}_{a1} e^{-j(\omega_0 - \omega_1)t} \right\} \\ S^2 \check{E}_{a1} &= S^2 \left\{ (R_u + j\omega_0 L_u)\, \check{I}_{u2} + j\omega_0 M \check{I}_{a2} e^{\,j(\omega_0 - \omega_1)t} \right\} \end{aligned}$$

To simplify the problem assume $R_u = R_a$, $L_u = L_a$, then $I_{a1} = I_{u2}$ and $I_{a2} = I_{u1}$. The slip frequency is given by ω_1 and the synchronous frequency by ω_0. The two systems of primary currents, that is to say those supplied to the stator and rotor respectively, are entirely unrelated in the induction motor circuits as they are throughout without any mutual coupling. The two fields will be independent and each be determined by the impressed e.m.f., $S'\check{E}_{a1}$ as if the other did not exist. If B be the maximum value of the induction in the air gap due to the stator field, it may be represented by the rotating vector

$$S' \check{B} = S' B\, e^{\,j\omega_0 t}$$

The rotating field to the rotor will be rotating in the opposite sense to the actual mechanical rotation and since the mechanical rotation is $e^{j(\omega_0 - \omega_1)}$ the actual rotation of this induction in the air gap will be $e^{-j\{\omega_0 - (\omega_0 - \omega_1)\}}$ or $e^{-j\omega_1}$. Its value will also be B the same as for the stator so that the total induction will be

$$B\,(e^{j\omega_0 t} + e^{-j\omega_1 t})$$

So that in less than every complete cycle there will be a doubling up of the induction.

For the same output the heating due to copper loss is very much reduced. To get at this let us take I_a as the normal full load current when the machine is operating in a normal induction motor. We may assume the I_u is approximately equal to I. Then we shall have

$$I^2R \text{ loss} = (I_a^2 + I_u^2)\, R_a = 2I_a^2 R_a$$

Assume the two-winding scheme, and let the two currents supplied by the primary be mI_a, mI_u, etc. Then

$$I^2R \text{ loss} = 2\,(m^2 I_a^2 + m^2 I_u^2)\, R_a = 4m^2 I_a^2 R_a$$

For the same loss we have

$$4m^2 I_a^2 R_a = 2I_a^2 R_a$$

and therefore

$$m^2 = \frac{1}{2}$$

$$m = \frac{\sqrt{2}}{2} = 0.707$$

The current that may be supplied by the primary circuit to give the same I^2R loss will therefore be 41.4 per cent higher than for the straight induction motor. That is to say, its output for the same copper loss will be increased 41 and a fraction per cent.

To offset this the induction is doubled though the actual iron loss will not be so greatly increased in each element by the doubling of the induction but the rotor iron loss and the stator iron loss will have approximately the same magnitude, so that we may say that the iron loss for the same applied voltage will be approximately double that in the regular induction motor.

Some Considerations in the Design of Damper Windings for Synchronous Motors

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Associate, A. I. E. E

***Synopsis.**—As the application of synchronous motors is extended to include more specialized fields, the design of damper windings for special starting performance becomes more of an exact art. Methods have been presented before the Institute by which the starting performance can be determined with good accuracy. The purpose of this paper is to present certain possibilities and limitations of the performance which may be obtained by variation of the damper winding design.*

For purposes of consideration, damper windings are arbitrarily classified as low reactance and high reactance types. It is pointed out that low reactance damper windings are well suited to give a high ratio of torque to kva., especially during the initial part of the starting period. Many other types of performance, including a fairly uniform torque over the starting period or, a low starting torque with high pull-in torque—all with a relatively low kva. can be obtained with high reactance windings.

INTRODUCTION

THE constant effort of the designers of synchronous motors to improve the starting characteristics of their machines has led to marked advances in the art of predetermining their performance. Putman[1] and Linville[2] have given methods of calculating the performance of the motor throughout the entire starting period. By their use the designer may calculate the starting performance with reasonable accuracy provided, first, that he can calculate the machine constants correctly, and second, that he has an appreciation of the possibilities and limitations of the various combinations of these constants. It is the purpose of this paper to point out certain aspects of this second condition of successful design.

MACHINE CONSTANTS

For purposes of comparison, the machine constants may be classified as stator reactance (x_1) and resistance (r_1), damper winding reactance (x_b) and resistance (r_b), total rotor reactance (x_2) and resistance (r_2) and the magnetizing admittance (H). Often the individual damper bars do not all have the same reactance and resistance. Putman used an equivalent reactance and resistance for the damper circuit, in which the constants of the individual bars are combined into the constants of a damper winding made up of equivalent bars. Linville reduced the motor circuits to an equivalent network, placing each bar with its individual constants in a separate branch.

In this paper all constants are referred to the stator and expressed in per cent.

CHARACTERISTICS OF DAMPER WINDINGS

A good criterion for the best starting performance for a synchronous motor appears to be that the motor shall develop the required torque throughout the starting period with the lowest kva. that can be obtained without impairing the operating characteristics at synchronous speed.

The design of special damper windings for induction motors has been studied for years and probably most of the practical combinations have been used in them. In studying the problem of starting windings for synchronous motors, many of the steps of the induction motor designers doubtless are retraced.

Types of Windings. For the purpose of discussion, it is convenient to divide damper windings arbitrarily into two classes. The first is called "low-reactance windings." This includes all windings which are made up of round or rectangular bars of such size that skin effect has only a negligible effect on the distribution of the current in the bar. Such windings are so arranged in the pole head that the ratio of specific permeance to effective specific resistance of the winding as a whole is only slightly affected as the motor comes up to speed. The second class is called "high-reactance windings." This includes all forms of double deck windings, such as those having two distinct bars placed one over the other and all windings made entirely or in part of L bars or inverted T bars as well as plain rectangular bars of such a shape and material that they are subject to an appreciable skin effect. Moreover, the second class includes all single deck windings which are so arranged that the characteristics of the winding change noticeably as the motor comes up to speed.

Low-Reactance Windings. Most of the ordinary single deck damper windings are of the low-reactance type, although it will be shown in Appendix III that certain of them actually have a considerable change in characteristics as the motor comes up to speed.

In order to obtain the highest ratio of torque to kva. throughout the starting period, it is necessary to have the rotor circuits of high resistance and low reactance. Very high-resistance damper windings have the serious practical objection that they produce low torque at the higher speeds unless the motor has a low stator reactance. This difficulty becomes quite marked in low-speed motors. However, low-reactance

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1. For references see Bibliography.

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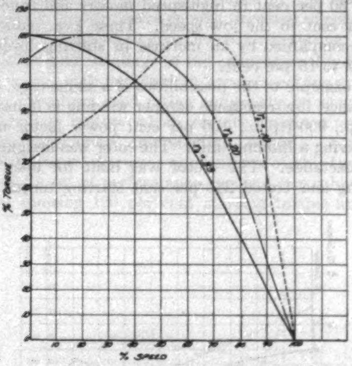
damper windings are well suited for low-speed motors with 40 to 50 per cent pull-in torque accompanied with a starting torque as high as 75 per cent. Higher torques with approximately the same ratio of pull-in to starting torque can be built with an increase in frame size. Most high-speed motors will produce 100 per cent pull-in torque with 150 to 175 per cent starting torque with low-reactance dampers.



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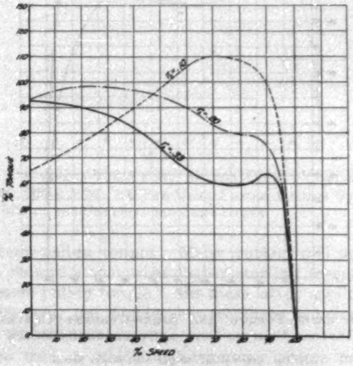

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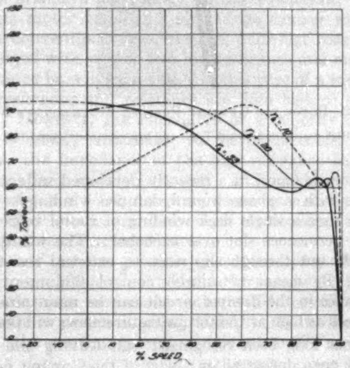

C

FIG. 1—EFFECT OF CHANGE IN DAMPER WINDING RESISTANCE ON TORQUE 2,000-HP., 150-R.P.M., 100 PER CENT POWER FACTOR SYNCHRONOUS MOTOR

A. Field open
B. Field closed through normal starting resistor
C. Starting resistor varied in same ratio as damper winding resistance

The effect of changing the damper winding resistance is shown in Fig. 1. Fig. 1A shows the case with the field open. The effects are the same as with an induction motor. Fig. 1B shows the same conditions except with the field winding closed through a starting resistor. It will be noted that the curve with the lowest resistance has the highest maximum value. This is because the maximum of the torque from the damper winding for this condition coincides most nearly with the maximum of the torque from the field winding. The effect of the field winding has been covered in a previous paper.[1] Fig. 1C shows as a matter of interest the results obtained when the starting resistor is varied in the same proportion as the damper winding resistance.

The per cent starting current for these cases is shown in the following table:

FIG.	1A	1B	1C
$r_b = 0.10$	360	400	400
$r_b = 0.20$	320	370	370
$r_b = 0.30$	260	320	320

An increase of the damper winding reactance decreases the torque at all slips. Probably most designers of synchronous motors have noted that increasing the damper winding reactance for a given value of resistance in some cases results in an actual increase in starting kva. The reason for this becomes apparent upon examining an approximate equation for starting current which was derived originally for the induction motor. The equation is:

$$I = \frac{\sqrt{1 + 2Hx_2}}{Z_0}$$

where Z_0 is the total impedance of the motor. Differentiating this expression with respect to x_2, it is found that the current has its maximum value when the following relation is fulfilled:

$$X_2 = \frac{-\frac{1}{H} + \sqrt{\left(\frac{1}{H}\right)^2 - 4\left(\frac{x_1}{H} - x_1^2 - r_0^2\right)}}{2}$$

where r_0 = sum of stator and rotor resistance.

An excellent example of a low-reactance winding in which a high ratio of torque to kva. is maintained at all times is found in a recently developed salient-pole motor with a phase wound damper winding.[4] This motor uses a single deck winding of round bars with a low permeance slot over each bar. The winding is brought out through slip rings to external resistance grids. By means of suitable control equipment, the resistance in the damper circuit can be maintained at all times as high as the torque requirements will permit.

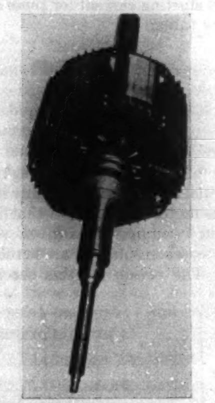

FIG. 2—ROTOR FOR GENERAL PURPOSE SYNCHRONOUS MOTOR

High-reactance inverted T bar and low-reactance round bar

It has long been recognized that leaving the field circuit open during all or a part of the starting period is an effective means of increasing the rotor circuit resistance and decreasing its reactance. Recently a number of applications calling for a particularly high ratio of torque to kva. has been filled with motors which had the field circuit open during the greater part of the starting period. Large and medium size slow-speed motors can be built readily for open field starting. They may be excited from standard 125 or 250-volt circuits by the use of a field sectionalizing switch mounted on the rotor.[4] Such switches have been successfully applied to low-speed motors both with a normal single cage winding and with the phase-wound damper. In high-speed motors of small and medium size, it has been found more convenient to use reduced voltage excitation, so that the voltage across the rings at starting will be kept within the limits of a reasonable amount of insulation on the field coils, the collectors and the control equipment. The reduction of kva. by the use of open field varies from 20 to 30 per cent in high-speed motors and from 8 to 15 per cent in the low speed. These kva. reductions are accompanied by an increase in starting torque of from 3 to 15 per cent.

An example of the possibilities of a single-cage, high-resistance, low-reactance damper winding is found in a 250-hp., 900-r.p.m., 100 per cent power factor motor for driving a hammer mill. The rotor was designed for low reactance. The motor was built for open field starting and tested 245 per cent starting torque with 110 per cent pull-in torque and 425 per cent starting kva.

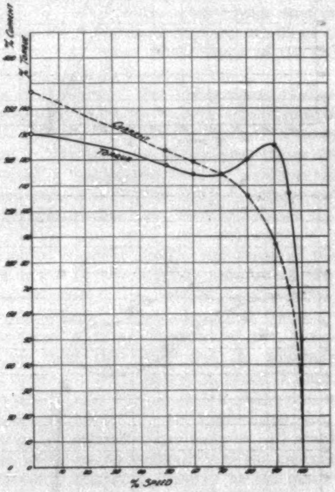

FIG. 3—SPEED-TORQUE AND SPEED-CURRENT CURVES FOR GENERAL PURPOSE TYPE OF SYNCHRONOUS MOTOR

250 hp., 900 r.p.m., 80 per cent power factor

Where a high ratio of torque to kva. is required, especially during the early part of the starting period, and where the starting torque itself must be high with a relatively low kva., low-reactance damper windings are well suited.

High-Reactance Windings. It is well recognized that to get any appreciable change in the effective resistance of any type of a double deck damper bar it is necessary for at least a portion of the material of the bar to be of low resistance. In high-speed, high-torque motors, where the damper winding must furnish a major portion of the impedance to limit the starting current, a straight double deck winding defeats its own purpose. If its reactance is high enough to limit the starting current, it is apt to be low in starting torque. And if the resistance of the winding is high enough to contribute materially to the impedance of the machine, it probably will not have enough decrease in resistance at the higher speeds to give the required torque.

Double deck windings of the conventional induction motor type, with two distinct bars placed one over the other, have been used rather commonly on slow-speed synchronous motors which require nearly a uniform torque throughout the starting period. A recent A.I.E.E. paper[6] discusses applications of this type and presents a set of performance curves for a rubber mill motor showing 120 per cent starting torque and 100 per cent pull-in torque. Many rubber mill applications require a somewhat higher starting torque with the same pull-in torque.[5] On these latter applications motors with the simpler low-reactance winding give good performance.

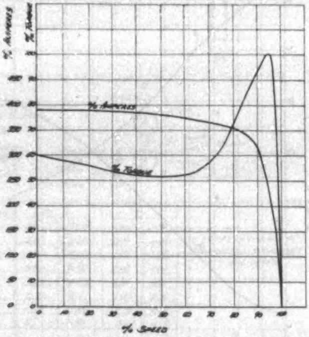

FIG. 4—SPEED-TORQUE AND SPEED-CURRENT CURVES FOR A 700-HP., 900-R.P.M., 100 PER CENT POWER FACTOR SYNCHRONOUS MOTOR FOR CENTRIFUGAL PUMP DRIVE

One field in which high-reactance rotors may be used to advantage is the general purpose class of motors and those of somewhat higher horsepower rating, which usually have general purpose characteristics. These motors require 100 per cent pull-in torque and only a little higher starting torque. Due to the usual small physical size of these motors, it has been found that bars of an L or inverted T section are well suited. The use of a few high-reactance bars, combined with low-reactance bars in the same pole head, presents no serious manufacturing difficulties and gives sufficient impedance at the start to limit the current, yet low enough resistance at low slips to give the required pull-in torque. The constants of such a winding are given in Appendix II. Fig. 2 shows an 8-pole rotor for a general purpose motor of this type.

Curves of the tested starting performance of a 250-hp., 900-r.p.m., 80 per cent power factor motor are shown in Fig. 3.

An interesting example of the possibilities of special forms of high-reactance rotors is found in the design of high-speed motors for centrifugal pump and blower drive. This is a good example of an application where a low torque per kva. is quite acceptable. The present day requirements are at least 100 per cent pull-in torque with 50 to 60 per cent starting torque and the starting current between 400 and 450 per cent.

One solution of the problem which has been quite successful is the two winding motor described in a recent A.I.E.E. paper.[6] This method often gives the required starting performance from motors of practically standard design. However, there are cases, especially with small high-voltage motors, where it is difficult to obtain the required number of parallels. And the control equipment must include an auxiliary contact or breaker for closing the second half of the winding.

For this reason, a high-speed motor with a simple type of high-reactance rotor and a normal, high efficiency stator has been developed, which permits the use of standard full voltage starting equipment.

This motor employs a single-cage damper winding of the conventional type, except of quite high resistance, combined with one or two low resistance and medium reactance bars in each pole. In this type of a winding, the resistance and reactance are both high at standstill. However, the impedance of a majority of the bars is mostly resistance. As the motor comes up to speed, the impedance of the high-resistance bars remains approximately constant. The impedance of the low-resistance bars, however, decreases materially at the higher speeds, and thus effects the necessary reduction in total rotor impedance.

A set of test curves on such a motor rated at 700-hp., 900-r.p.m., 100 per cent power factor is shown in Fig. 4.

These tests were made with the field circuit closed through a proper starting resistor. With the field open, the starting current is reduced to 275 per cent, with a slight increase in starting torque.

As the applications of synchronous motors become more specialized, there will doubtless be a marked improvement in the results obtained from special, yet simple, forms of high-reactance damper windings.

ACKNOWLEDGMENT

The author acknowledges the aid of Mr. M. R. Lory in calculating several of the examples given in this paper.

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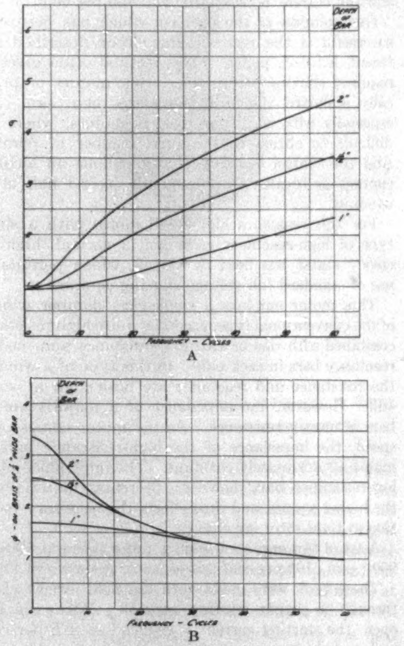


A

B

FIG. 5—CHARACTERISTICS OF RECTANGULAR BARS

A. Resistance σ = ohms per cu. in. × 10^6
B. Reactance ϕ = specific permeance per cm length

Appendix I

CHARACTERISTICS OF VARIOUS TYPES OF DAMPER BARS

The constants of a number of types of damper bars in common use are presented here for reference and comparison. Much of the material has been presented before. See Fig. 7 for reference to dimension letters.

1. Round Bar. The resistance of the round bar is ordinarily determined by its cross section and the specific resistance of the material without reference to skin effect. Mr. Fechheimer[7] showed that the permeance is

$$1.26 \left[0.62 + \frac{d}{W} \right]$$

The figures were obtained by integration in polar coordinates based on flux lines going straight across the round slot. Careful flux plots have later shown the value of the constant inside the bracket to be 0.66 instead of 0.62.

2. Square Bar. In this class are included all bars of square or approximately square cross section. Here, also, the resistance usually may be calculated without regard to skin effect. The permeance is given by the well known expression

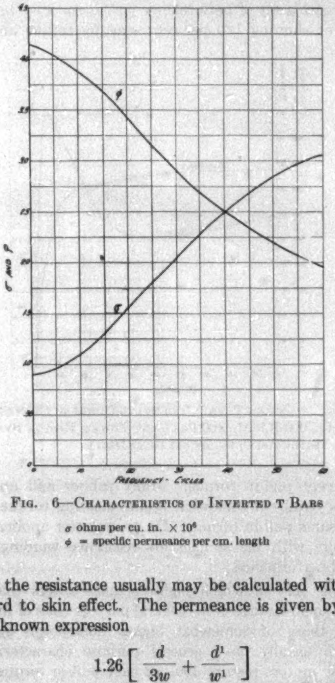


FIG. 6—CHARACTERISTICS OF INVERTED T BARS

σ ohms per cu. in. × 10^6
ϕ = specific permeance per cm. length

$$1.26 \left[\frac{d}{3w} + \frac{d^1}{w^1} \right]$$

It is apparent at once that a square bar has a better power factor than a round one for the same resistance.

3. Rectangular Bar. In determining the constants of a rectangular bar the displacement of the current in the bar itself at the higher frequencies must be taken into account. The expressions may be conveniently derived by the use of integral equations. A more general formula has already been presented[1] which reduces for a rectangular bar to

$$Z = \frac{r(A + jA)}{ab}\left[\frac{\cosh 2A \cos 2A + j \sinh 2A \sin 2A}{\sinh 2A \cos 2A + j \cosh 2A \sin 2A}\right]$$

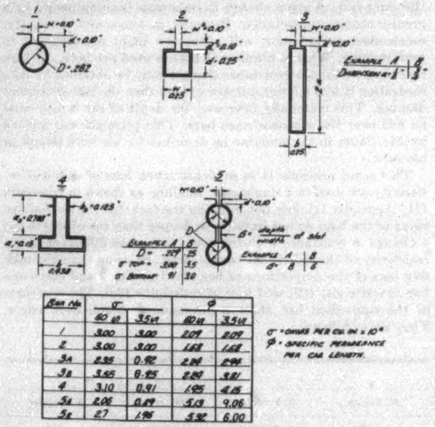
where

$$A = 2\pi a \sqrt{\frac{f}{r}}$$

r = specific resistance—absolute units

f = frequency

The impedance Z will be given in absolute units. The expression does not include the permeance of the slot above the bar. A set of curves covering various depths of 1/4 inch wide bar is shown in Fig. 5A and 5B.



Bar No	σ		φ	
	60 ∿	35 ∿	60 ∿	35 ∿
1	3.00	3.00	2.09	2.09
2	3.00	3.00	1.68	1.68
3A	2.35	0.92	2.84	2.94
3B	3.55	0.95	2.89	3.81
4	3.10	0.91	1.95	4.15
5A	2.06	0.89	5.13	9.06
5B	2.7	1.96	5.92	6.00

FIG. 7—CHARACTERISTICS OF SEVERAL COMMON BARS

4. Inverted T Bars. The theory of the calculations for the inverted T bar has been presented in a previous paper.[1] The impedance formula in hyperbolic notation is

$$Z = \frac{r(A + jA)}{b_2}\left[\frac{R \cosh (S+jS) - \cosh (D+jD)}{R \sinh (S+jS) - \sinh (D+jD)}\right]$$

where $R = \dfrac{b_1 + b_2}{b_1 - b_2}$, $s = A\,(a_2 + a_1)$, $D = A\,(a_2 - a_1)$.

Fig. 6 gives a set of curves for a copper bar of the dimensions shown in Fig. 7.

5. Double Deck Bar. Perhaps the best known form of double deck bar for induction motors is that consisting of two bars, one above the other. They have been used to a certain extent on synchronous motors, although the limited depth of pole head presents a handicap to their application.

In general, bars of this type are of such a shape and size that the skin effect in the conductor itself is negligible. The impedance of the type shown in Fig. 7 is given by

$$Z = \frac{r_a\, r_b - 105\ w^2(1+G) + jw\ [r_a(24.3+12.6G) + r_b\ 8.32]}{r_0 + jw\ 4\pi\ (1.025 + G)}$$

where

r_a is resistance of top bar in absolute units per centimeter length

r_b is resistance of bottom bar in absolute units per centimeter length.

$r_0 = r_a + r_b$

$w = 2\pi f$

G = ratio of depth to width of slot between bars.

The impedance Z is given in absolute units. It does not include the permeance of the slot above the bars.

The above list is not inclusive of all types of bars in use, but fairly covers the range of common practise.

Fig. 7 shows sketches of the various types of bars together with a comparison of their characteristics.

Appendix II

CHARACTERISTICS OF A COMBINATION OF INVERTED T BARS AND ROUND BARS FOR A GENERAL PURPOSE MOTOR

In this appendix examples are given of the constants which can be obtained by the use of high and low reactance bars in the same pole. The desirable characteristics are a high impedance with fairly high power factor at the start and an appreciable decrease in resistance as the motor comes up to speed. A sketch of a pole using 2 high-reactance bars of the inverted T type combined with 4 low-reactance bars is shown in Fig. 8.

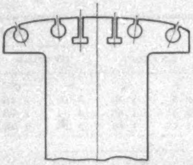


FIG. 8—POLE PUNCHING FOR A GENERAL PURPOSE MOTOR

Instead of handling each bar separately in calculating the starting performance, it is convenient to find the resistance and permeance constants of an equivalent bar of such characteristics that the winding may be considered to consist of 6 (in the case of Fig. 8) such equivalent bars per pole. The impedance of the equivalent bar may be found by multiplying the actual impedance of all bars in parallel by the number of bars.

The following table gives the specific resistance and specific permeance of the equivalent bar for the pole shown in Fig. 8 with various resistance materials in the round bars. The inverted T bar is copper in all cases. The equivalent bar will have the same area as one of the inverted T bars.

		⊥ bar				Equivalent bar			
Round bar		60 cycles		3.5 cycles		60 cycles		3.5 cycles	
σ	φ	σ	φ	σ	φ	σ	φ	σ	φ
1.87	2.51	2.07	2.89	0.831	4.09	2.10	2.62	1.45	4.23
2.84	2.51	2.07	2.89	0.831	4.09	2.84	2.69	1.74	5.34
5.50	2.51	2.07	2.89	0.831	4.09	4.20	3.08	2.08	7.23
12.82	2.51	2.07	2.89	0.831	4.09	6.21	4.68	2.44	9.70

σ = resistance per cu. in. × 10^6
φ = specific permeance per cm. length.

It will be noted that the highest resistance round bars give the greatest decrease in resistance as the motor comes up to speed. A 250-hp., 720-r.p.m. motor using the last combination listed calculated 14.8 per cent damper circuit reactance and 17.3 per cent resistance at standstill.

Appendix III

CONSTANTS OF SINGLE-CAGE WINDING ON ROUND BARS

In some cases where the winding is made of round or square bars, all the same size, the slots above the bars are deeper at the center of the pole than near the tips.

Consider a pole with five 3/8 inch round bars. The slots over the middle 3 bars are 3/16 inch deep by 1/8 inch wide and over the two end bars they are 1/16 inch deep by 1/8 inch wide. In case it is necessary to use bars of different materials to meet the necessary resistance, an appreciable change of constants may occur as the motor comes up to speed. For bars all of the same material, the change is quite small. Three examples are illustrated in the following table:

				Equivalent bar			
Center bars		End bars		60 cycles		3.5 cycles	
σ	φ	σ	φ	σ	φ	σ	φ
5.5	2.72	5.5	1.46	5.56	2.16	5.51	2.21
1.98	2.72	5.5	1.46	3.04	2.93	2.66	3.05
5.5	2.72	1.98	1.46	3.30	2.08	3.18	2.12

Discussion

M. R. Lory: Synchronous motors are being applied to a great variety of applications which differ widely in starting requirements. The popularity of full-voltage starting with its cheaper and simpler control has increased the importance of limiting the kva. drawn by the motor during the starting period. The problem of the designer is to meet the torque requirements of the load and still keep the current drawn from the line to a minimum. In his paper, Mr. Shutt has given a comprehensive summary of the various means by which the designer can accomplish this result.

In Fig. 7 and Appendix I, Mr. Shutt has given the constants for several types of damper bars including two double-deck arrangements. Mr. Shutt pointed out the difficulty of using double-deck windings where the depth of pole head is limited. As a matter of interest, I have calculated these two examples with the two bars in parallel but in separate slots, making the permeance of the slot above each the same as the permeance which it has in the arrangement given in Fig. 7. Calculated on the same basis, the values of the constants for the two bars are as follows:

	σ		φ	
	60 cycles	3.5 cycles	60 cycles	3.5 cycles
5-A	1.83	0.713	3.62	8.16
5-B	2.66	1.95	3.83	4.93

In many cases the above characteristics are not as desirable as those given by the conventional arrangement with both bars in the same slot. A given change in resistance is accompanied by a greater change in reactance. However, in some cases the simpler mechanical construction will offset the slight disadvantage in performance. When a number of bars is used per pole, the high reactance of the low-resistance damper may be obtained by concentrating it into a fewer number of bars than the high-resistance damper. This materially decreases the depth of slot which must be put over the low-resistance bars. This principle was utilized by Mr. Shutt in the machine he describes for use with pumps or blowers.

This same principle is in evidence when bars of different resistivity are used in a single cage winding, as shown in Appendix III. Appendix III does not mention the fact that the slots above some of the bars do not have to be deeper than the others to get a change in resistance with frequency when the difference of the resistivity of the individual bars is large. So as an example, take five bars of the proportions of bar No. 1 of Fig. 7 and make one bar of resistivity 0.91 and four of resistivity 13.6. The constants of the equivalent bar show a pronounced double-deck effect. They are as follows:

σ		φ	
60 cycles	3.5 cycles	60 cycles	3.5 cycles
4.46	3.59	6.08	6.6

I believe that the formula for the value of X_2 for maximum inrush is more of theoretical interest than of practical value. I find that with the constants normally found in design this formula gives either negative values or positive values which are so small that they are below what would be obtained in a machine.

R. E. Hellmund: Mr. Shutt's paper is further evidence of the excellent progress which has continuously been made by American designers of synchronous motors in the development of motors suitable for the many commercial applications. As a result of such work, the American market has been kept free of the numerous more complicated and therefore less desirable excitation and phase-advancing schemes for induction motors which we find so extensively discussed and produced in Europe. In this country these various arrangements have been investigated and built experimentally, but it was found that economically the synchronous motor designs were always just one step ahead of anything which could be accomplished with the various possibilities of excitation of induction motors. By the use of the simplest design features and through painstaking design work, many desirable characteristics have been obtained with synchronous motors, and at the same time design proportions have been worked out which have resulted in very favorable cost comparisons with domestic induction motors and which for many years have been away ahead of any European synchronous motor designs which have come to my attention.

Part of the credit for keeping the more complicated induction motor arrangements off the American market is also due to the central station engineers through their permitting starting-current and kva. values in excess of those usually permitted in other countries.

These remarks may answer the question frequently raised as to why various exciter and phase-advancer schemes for induction motors have not been commercially exploited in this country.

C. C. Shutt: The intensive study of the starting performance of the synchronous motor has been carried on for a comparatively short time when compared to its predecessor in the alternating current field—the induction motor. The designer of synchronous motors can obtain a great amount of benefit from a study of what has been tried and adopted—or discarded—in the starting windings of induction motors.

In commercial design work the designer is somewhat handicapped by the fact that present methods of calculation, which accurately take into account the effects of the field winding, are rather laborious. It is expected that improvements will be made in the near future in the methods of calculating the starting performance.

In regard to Mr. Lory's comment on the practical value of the formula for the value of X_2 which gives the maximum inrush, an inspection of the equation as given in the paper will show that for high values of resistance X_2 may have an appreciable magnitude. In actual design work this "critical" value of rotor reactance will be noticed particularly on high-speed motors where the starting current must be limited largely by the impedance of the rotor. In these cases the damper winding resistance is often made quite high.

Mr. Hellmund's remarks are particularly applicable to the low-speed field. The operating advantages of the low-speed synchronous motors permit the use of many low-speed direct-connected motors in industrial applications which would otherwise require high-speed induction motors, which have relatively high full load power factor and efficiency, and reduction gears or belts.

Film Characteristics of Electrolytic Condensers

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Associate, A.I.E.E.

***Synopsis.**—Electrolytic condensers of the film-forming type have some interesting characteristics which have never been satisfactorily explained. Extremely thin oxide or hydroxide films formed on the surface of the metal anodes seem insufficient in themselves to explain the complete action. Likewise, an electromotive equilibrium is insufficient to explain the action.*

A résumé of former research reveals a great deal of information of large practical value in building electrolytic condensers but little of value from a theoretical standpoint. Reviewing the prominent papers, it appears that aluminum has been the chief subject of discussion for condenser anodes. Such anodes show small leakage currents in selected electrolytes at low temperatures. Increasing voltage, temperature, or electrolyte concentration cause increases in leakage; the addition of impurities also has a deleterious effect.

The oxide-gas theory of rectification as applied to electrolytic condensers by Guntherschulze predicts changes in capacity for variations in pressure. Experiments do not substantiate the presence of gas. The oxide layer alone is unable to fulfil the necessary conditions found.

Electrolytic condensers with extremely low applied voltages show large variations in measured capacity from high-voltage measurements. The results indicate a film in which ion penetration plays a large part.

The presence of a film saturated with water as the dielectric overcomes a number of former obstacles, and lessens the necessity for the very thin films formerly assumed, due to the high dielectric constant of water.

* * * * *

ELECTROLYTIC condensers have been of general interest to electrochemists since 1855 when Wheatstone[1] presented a paper on the polarization of aluminum in electrolytes.

A number of metals possesses the property of forming on their surfaces protective coats when immersed in electrolytes. The skin formed is extremely thin normally; but if the metal is made electrically positive with respect to the electrolyte the thickness will increase until the layer is readily apparent. This film has a high electrical resistance, rising in some instances to several thousand megohms per square cm. of surface, the resistance remaining only as long as the metal is the anode in the electrolyte.

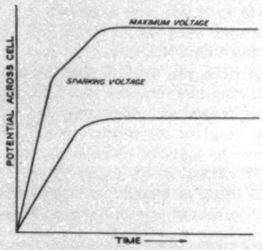

FIG. 1—THE FORMATION OF AN ELECTROLYTIC CONDENSER ANODE WITH CONSTANT CURRENT THROUGH THE ELECTROLYTE

Upper curve high current density, lower curve low current density

This film assumes the role of the dielectric in a condenser,[2] the plates of which are the metal on the one side and the electrolyte on the other.

The metals which form protective films in electrolytes include aluminum, antimony, bismuth, magnesium, zinc, niobium, vanadium, colombium, zirconium, tantalum, their alloys, and a number of other metals. These metals all can be used in electrolytic condensers. Copper and silver exhibit ionic valve action as opposed to the electronic valve action of film forming metals.[3]

The choice of electrolytes is narrow when aluminum is used as the electrode, but tantalum exhibits valve action in all electrolytes except hydrofluoric acid. The usual practise in selecting an electrolyte for aluminum is to choose the salt of a weak acid and a strong base which will nearest meet all specifications. A sufficient amount of free acid is usually added to the solution to insure an acid reaction to retard the precipitation of aluminum hydroxide.

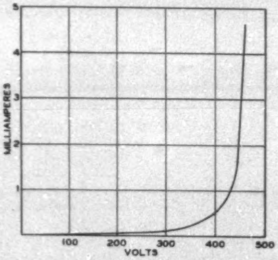

FIG. 2—LEAKAGE CURRENT OF 8-MICROFARAD CONDENSER FOR RADIO FILTER CIRCUITS, FORMED TO 450 VOLTS, 25 DEG. CENT.

If a strip of pure aluminum is made the anode in a suitable electrolyte and a constant current is caused to flow, the voltage across the cell will rise with increasing time as indicated by the curve of Fig. 1. The initial portion of the curve is straight, but as the voltage is raised, the rate decreases and abrupt bends appear in

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

the curves. Close examination will disclose a faint luminescence at the surface of the anode after the first bend; and upon further increases small sparks appear at the surface of the aluminum and finally heavy sparking takes place between the anode and the electrolyte and large current increases result from small increases in voltage. This maximum voltage varies widely under different conditions and it is known as the breakdown voltage; but the useful operating range of the condenser is below the first bend in the curve. The lower curve of Fig. 1 is for a lower forming current; in this case the maximum voltage is not approached.

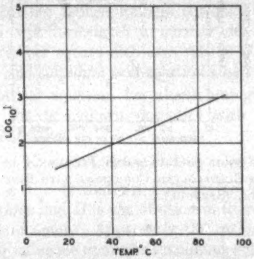

Fig. 3—Leakage Current of Aluminum Anode Plotted as a Log Function against Electrolyte Temperature. (Bairsto and Mercer)

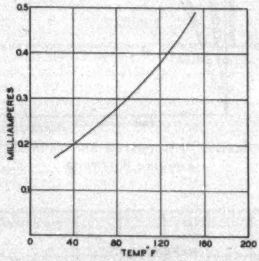

Fig. 4—Leakage Current of 8-Microfarad Aluminum Condenser with 350 Volts Impressed, for Different Operating Temperatures

Fig. 2 is the characteristic leakage current of an eight-microfarad aluminum condenser formed to 450 volts. The area of the anode is 72 square inches, and the solution is saturated sodium borate with the addition of sufficient boric acid to insure an acid reaction from the electrolyte. The leakage current of a condenser depends not only upon the voltage, but also the materials used and the operating temperature. The effect of temperature on the leakage current of aluminum is expressed by the empirical equation.[4]

$$I = A\epsilon^{\alpha T}, \qquad (\log_e I = \log_e A + \alpha T$$

where A is a constant, α is a constant dependent only on the equivalent weight of the anion in solution, and T is the temperature. When the current density is high the rise of current with temperature is more rapid than predicted. Fig. 3 shows the leakage of an aluminum anode plotted as a log function against temperature. The variation of film resistance with impressed voltage[5] is illustrated in Fig. 5 which shows clearly the asymmetrical characteristic of the cell.

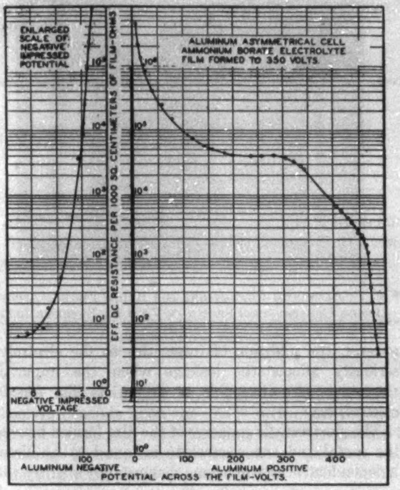

Fig. 5—Variations of Film Resistance with Impressed Voltage in Aluminum Electrolytic Condenser. (Siegmund)

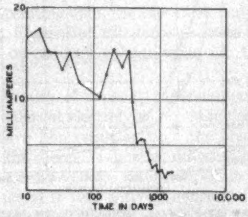

Fig. 6—Decrease in Leakage Current of Electrolytic Condenser with Time. (Siegmund)

After an anode has had high voltages applied the leakage currents are found to be slightly higher for normal voltage values upon reduction of the voltage. After a relatively short time has elapsed, practically the same values of leakage current are obtained. Over a long period of time condenser characteristics change appreciably,[5] particularly when the condenser is formed initially to a voltage different from the operating voltage, Fig. 6.

With metals that are soluble or slightly soluble in the electrolyte, leakage current may increase rapidly with concentration of the solution. Using aluminum as the anode, the addition of very slight amounts of chlorine will increase leakage current many fold. In general, the addition of impurities to the electrolyte seriously affects the leakage when the impurities themselves are incapable of acting as film forming electrolytes.

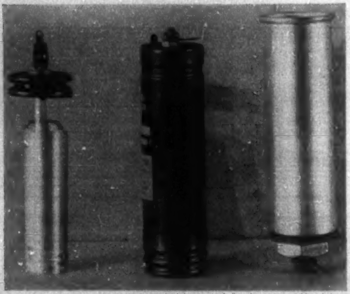

Fig. 7—Aluminum Condensers for Radio Filter Circuits. Anode Construction Shown in View on Extreme Left

The electrostatic capacity of the film formed between the metal and the electrolyte is found to be almost independent of the solution, changes only slightly with variations in temperature, and is inversely proportional to the voltage of formation of the anode, Fig. 8.

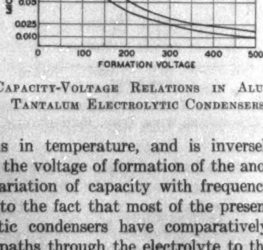

Fig. 8—Capacity-Voltage Relations in Aluminum and Tantalum Electrolytic Condensers

The variation of capacity with frequency is slight but due to the fact that most of the present forms of electrolytic condensers have comparatively long resistance paths through the electrolyte to the film, the effective capacity decreases with increases in frequency, Fig. 9.

The breakdown voltage of a condenser depends upon the electrolyte and the anode metal. An aluminum anode has been formed to 1890 volts.[6] The usual commercial aluminum condenser is formed to 500 volts. Tantalum condensers are formed commercially to 150 volts, with possibilities of units to withstand 600 volts. The other metals and alloys are not in general use and only a small number are suitable for commercial applications.

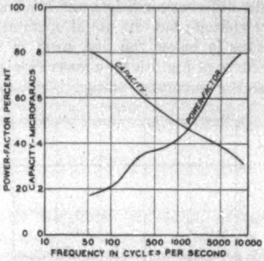

Fig. 9—Effect of Increasing Frequency on Effective Capacity of Electrolytic Condenser with High Resistance Electrolyte. (Sprague)

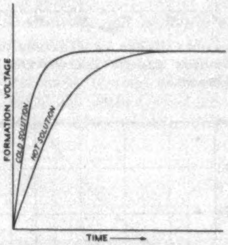

Fig. 10—Formation of Aluminum Anodes in Hot and Cold Forming Solutions

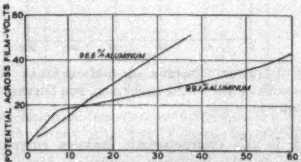

Fig. 11—Influence of Purity of Aluminum on Rate of Formation. Forming Current 0.078 Milliamperes per Sq. Cm. (Siegmund)

The rate of formation of a film varies widely under different conditions. In general the gradient of the curve in Fig. 10 is independent of the concentration of the electrolyte, and decreases considerably with increasing temperatures as indicated by the lower curve. The rate of formation in volts per second is almost directly proportional to the current density at high current densities, so that equal formation voltages are produced by equal quantities of electricity through the cell. Pure aluminum forms at a lower rate than impure aluminum, Fig. 11.

Commercial aluminum condensers formed to 450 volts approximately are formed in hot electrolytes; current densities range from 5 to 15 milliamperes per square inch. After the formation period, the condensers are given an aging period of two to eighteen hours to further reduce leakage current. Tantalum condensers will form with a current density of 15 milliamperes per square inch at a rate of 60 volts per minute in cold sulphuric acid solution.

An empirical equation for aluminum[6] gives a decrease in the breakdown potential with increasing ionic concentration,

$$V = (514.51 \log D\text{-}330) \text{ volts}$$

where V is the breakdown voltage for small ionic concentrations and D is the dilution in litres per gram equivalent of anion. Apparently the cation has very little or no influence upon the maximum voltage.

Table I compares the breakdown voltage of tantalum with that of aluminum in 0.1 N. solutions after 24 hours of formation.

TABLE I

Solution	Aluminum	Tantalum
Sodium chloride	— volts	390 volts
Oxalic acid	—	445
Potassium cyanide	275	520
Potassium permanganate	112	105
Ammonium chromate	122	147
Sodium silicate	445	540
Sodium sulphate	40	490
Ammonium citrate	470	550
Borax	480	540
Diamonium phosphate	460	575
Ammonium bicarbonate	425	525
Citric acid	536	605

Some metals will "form" in molten, water-free salts.[7] Aluminum will form to approximately 80 volts in molten potassium nitrate. However, the effective film thickness is found to be as high as forty times as much as in aqueous solutions for aluminum, and up to five times as thick for tantalum.

The maximum voltage is affected also by the nature of the anion. In case of anions containing metals, a specific maximum voltage is given by that anion for any stated concentration and the maximum is dependent upon the metal of the anion. Table II for tantalum in 0.05 N. solutions of chromium gives practically the same breakdown voltage in each case.[8]

TABLE II

Metal	Cation	Anion	Maximum voltage
Chromium	K	Cr_2O_7	131
		CrO_4	141
		$Cr(C_2O_4)_3$	136
		$Cr(CN)_6$	126
			133 Mean.

The following list of the specific maximum voltages applies for 0.05 N. solutions of the electrolytes,[8] Table III.

TABLE III

Pd	19.5	Cr	133	H	448
Pt	20	Ni	181	Bi	450
Rh	23	Sn	190	Hg	453
Au	24	Co	243	Cd	478
Ir	32	Cu	335	Al	660
Mn	46	W	363	Sb	435
Fe	84	Ag	422		

Only the metals seem to have a definite voltage but a few of the other elements seem to give fairly consistent results,[8] Table IV.

TABLE IV

Nitrogen	470	Silicon	490
Arsenic	465	Fluorine	505

After voltage is removed, the rate at which the film dissolves depends upon the anode metal and the electrolyte. Antimony and tantalum are very sensitive to current interruptions, but bismuth, magnesium, and aluminum films dissolve very slowly. With a reduction in thickness of the film there is also an increase in capacity. A condenser originally formed to a voltage higher than the operating voltage will gain capacity until it stabilizes at the new value of voltage, the time depending upon the condenser and the voltage difference. Capacities will increase more rapidly for condensers on reduced voltage when the temperature is high.

The uniformity of capacity measurements for films formed in different electrolytes preclude the possibility of an effective film resulting from the reaction of the metal and the anion. Likewise, different metals have different capacity-voltage ratios, which indicates that the anode metal is very actively concerned in the film structure with film thicknesses ranging from 10^{-6} to 5×10^{-4} cm.

The existence of a film on the surface has been questioned,[9] and due to the fact that chlor-ion concentrations destroy valve action in aluminum and also due to the fact that an amalgamated electrode of aluminum will not form a film, it has been proposed that an electromotive equilibrium exists.[10]

Interference measurements[11] of the thickness of films have given results of 5×10^{-6} to 5×10^{-4} cm. for aluminum. A method of removing the aluminum and leaving the film intact has been tried;[12] the thickness of these films as measured with a direct vision spectroscope was approximately 7×10^{-4} cm. Film thicknesses were also measured by computing the weight of anion deposited in the formation period by measuring the forming current in coulombs; and also by the increase in weight,[12] Fig. 12. By assuming a dielectric constant for the film equal to unity, the effective film thickness can be computed from capacity measurements and comparisons have been made for a number of anode metals and electrolytes,[13] Fig. 13.

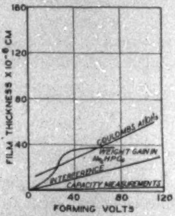

FIG. 12—FILM THICKNESS ON ALUMINUM ANODES AS MEASURED BY CAPACITY MEASUREMENTS, INTERFERENCE METHODS, INCREASE IN WEIGHT OF FORMED ANODE OVER UNFORMED ANODE, AND BY THE FORMATION CURRENT. (MOTT)

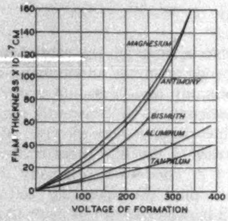

FIG. 13—RELATIVE FILM THICKNESS OF DIFFERENT METALS; DIELECTRIC CONSTANT UNITY. (GUNTHERSCHULZE)

Relative thickness of films for aluminum and tantalum in aqueous solutions and in fused, water-free salts are given in Table V.

TABLE V—RELATIVE THICKNESS AT 50 VOLTS AFTER 30 MINUTES FORMATION. (THICKNESS × 10^{-7} CM. FOR DIELECTRIC CONSTANT UNITY)

Electrolyte	Tantalum	Aluminum
Dilute aqueous	4.10	6.36
Pure sulphuric acid	6.69	10.65
Fused water-free		
NH_4NO_3	6.26	—
$KHSO_4$	6.98	13.10
$AgNO_3$	7.12	220.
$TlNO_3$	10.10	—
$LiNO_3$	7.93	—
$NaNO_3$	12.50	133
KNO_2	11.10	142
$K_2Cr_2O_7$	18.20	30

The electromotive equilibrium theory has been advanced as a complete explanation of the action at an anode surface;[10–14] evidence has also been presented for the presence of a film which forms when the aluminum is the anode and dissolves when the aluminum is the cathode.[15] However, oscillograms do not show such action during rectification, current passing immediately when the voltage reverses across the film. On high voltage rectifiers, the weight of the supposed aluminum hydroxide film which would have to be formed, dissolved, and reformed many times a second is prohibitive to such a theory. One investigator[16] was convinced by a microscopic examination of the luminous points, that they constitute an immense number of Wehnelt interrupters in parallel.

The first satisfactory theory accounting for alternating current rectification was advanced in 1902 by Guthe.[17] This was the oxide-gas film theory which proposed that a solid oxide or hydroxide film is formed, proportional in thickness to the formation voltage. Accompanying the oxide layer a thin film of oxygen formed on the solid film increases the leakage resistance. Rectification was attributed to the high mobility of free electrons when penetrating the films as compared to the relatively heavy and slow moving anions of the electrolyte. The electrons are able to easily penetrate the films even under very low accelerating fields, but the ions are not able to penetrate the oxide film deeply enough to reach the anode metal. Since there is an abundance of free electrons in the metal and none in the electrolyte, rectification will take place.

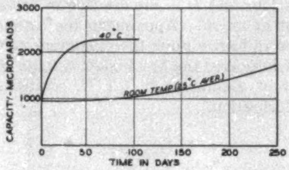

FIG. 14—ALUMINUM CONDENSERS OPERATED AT HALF OF THE FORMATION VOLTAGE. EFFECT OF TEMPERATURE ON THE RATE OF CAPACITY CHANGE. (SIEGMUND)

This theory has undergone considerable expansion. The gas layer was thought to carry the principal burden so far as voltage drop was concerned due to the low specific inductive capacity of oxygen; and the oxygen film was supposed to constitute the real capacity of the anode. Fig. 15 shows the relative thickness of the gas and oxide layers on an aluminum anode. This convenient mechanism was employed to explain many of the former discrepancies observed for electrolytic rectifiers and condensers. Guntherschulze[18] calculated potential gradients as high as 8,000 volts per micron in such very thin films.

In an experiment to detect the presence of gas,[18] an aluminum anode was formed to 400 volts, and the voltage was suddenly reduced to 4 volts. The calculated reduction in pressure was from 2,100 to 0.21 kg. per cm.^{2}; but instead of an expansion of gas and reduction in capacity, the capacity increased slightly. Fig. 16 is illustrative of the supposed arrangement of oxide layer and gas film adsorbed in the pores of the oxide film. Hydrostatic pressure applied to the electrolyte of rectifiers caused decreases in rectification.[19] Pressures up to 20 atmospheres were applied, and results indicated that the gas-oxide theory was correct.

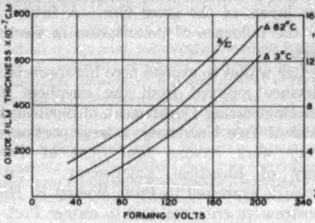

Fig. 15—Relative Thickness of Gas Layer Compared to Oxide Film on Aluminum Anode. (Guntherschulze)

An attempt was made by the writer to correlate the discrepancies found, and a condenser was constructed with provisions for applying pressure up to one hundred atmospheres. Both electrodes were of aluminum and of the same area of 350 sq. cm. The condenser was formed to 100 volts on a 60 cycle circuit, and oscillograms were made of the applied voltage and current at 80 volts 60 cycles for pressures ranging from atmospheric to 1,500 lb. per sq. in. The r.m.s. value of current was found to be 0.297 ampere in every case, and the oscillograms showed no change in current wave shape when sine wave voltage was applied.

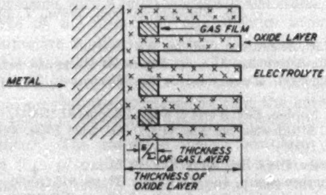

Fig. 16—Roughly Equivalent Structure of Film on Anode Showing Gas Layer in the Gross Film, and the Continuous Oxide Layer

If the experiments of Carman and Balzar[19] on the effects of increased pressure on rectification are discounted, the reduced voltage experiment of Guntherschulze and the writer's work with condensers under pressure both indicate that there is no gas film present. There may be oxygen present; but if so, it is undoubtedly present as an adsorbed gas in the oxide film, relatively free from external disturbances and not obeying the gas laws. Oscillograms from an electrolytic condenser under pressure are shown in Fig. 17.

The results of some measurements by the writer on electrolytic condensers operating in alternating current circuits at very low voltages are shown in Fig. 18. For both aluminum and tantalum the effective capacity decreased rapidly when the applied voltage fell below five millivolts. Both condensers were symmetrical condensers for a-c. operation formed to 60 volts a-c. at 60 cycles, and all measurements were made at 60 cycles.

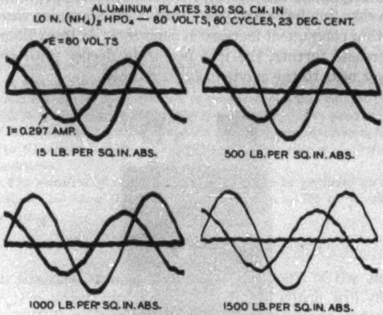

Fig. 17—Aluminum Electrolytic Condenser under Hydrostatic Pressure

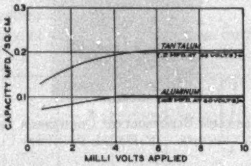

Fig. 18—Electrolytic Condensers with Very Low Applied Voltages on 60 Cycles A-C.

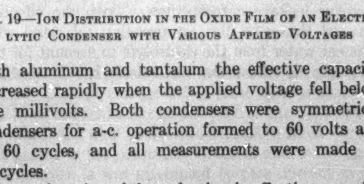

Fig. 19—Ion Distribution in the Oxide Film of an Electrolytic Condenser with Various Applied Voltages

An explanation of the reduction in effective capacity can be made on the basis of diffusion of ions throughout the oxide film under accelerating fields. Referring to Fig. 19, there are shown in an illustrative manner cross-sections of an oxide film under three conditions of impressed voltage. For the case of zero applied voltage very few ions from the electrolyte penetrate the oxide film, and the effective thickness of the film approaches the gross thickness. When low voltages are impressed more ions penetrate the film under the accelerating fields, and the effective thickness decreases with a consequent increase in capacity. As the voltage increases further, the ions penetrate deeper until the layer next to the anode metal is reached.

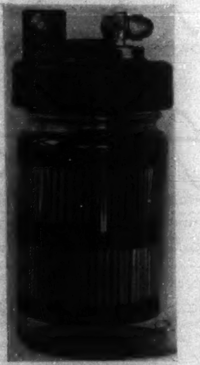

FIG. 20—TANTALUM ELECTROLYTIC CONDENSER FOR BATTERY CHARGING CIRCUITS. (FANSTEEL PRODUCTS CO., INC.)

Any ions penetrating this film constitute leakage current; the film therefore preserves a fairly constant thickness of effective film by natural growth resulting from leakage current. Thus after a certain critical voltage is reached for a given film the effective capacity of the film can change only slightly with variations in voltage.

Summarizing the foregoing, it appears that the normal film in electrolytic condensers is an oxide or hydroxide of the anode metal which is of a greater gross thickness than the effective film, this effective film maintaining itself relatively impermeable to ion penetration by the expedient of natural growth of film wherever leakage currents flow. Both the effective film and the gross film are probably saturated with sufficient water from the electrolyte to account for the extremely high capacities encountered by virtue of the high dielectric constant of water, approximately 80. A dielectric constant approaching this value for the effective film makes possible films of sufficient thickness that the voltage gradients encountered formerly may be materially reduced.

Recent work of the writer has shown that polarization capacity may account for a large part of the measured capacity, this polarization probably occurring in the outer layers of the gross film. A future paper discusses the influence of polarization in electrolytic condensers.

The writer wishes to express here his appreciation of the assistance received from the members of the Electrical Engineering Department of Sheffield Scientific School of Yale University, a large portion of the preparatory work having been done at Dunham Laboratory of Electrical Engineering. Particular appreciation is expressed to Prof. Wesley B. Hall for much constructive criticism, and to former Prof. A. E. Knowlton for valuable assistance in the preparation of this paper.

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Discussion

For discussion of this paper see page 449.

Power Losses in Electrolytic Condensers

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Associate, A.I.E.E.

Synopsis.*—Electrolytic condensers on a-c. circuits have higher power losses than would be expected from the leakage currents and electrolyte resistances. Likewise, losses change with the applied voltage, with temperature of the cell, with the frequency of the applied voltage, and with external circuit conditions governing current wave shape in such ways that it is almost impossible to predict results in advance.*

The current wave usually contains large harmonics when a sine wave of voltage is impressed, indicating a non-linear function in the impedance of electrolytic condensers. Increases in voltage, frequency, electrolyte temperature, and concentration of the electrolyte cause increases in the harmonics in the current wave.

Dielectric losses in an oxide film or a gas-oxide film are too low to account for the total losses in electrolytic condensers; and in general the losses in a polarization cell are higher than those found in electrolytic condensers. Polarized cells show some characteristics which are similar to the peculiarities of electrolytic condensers; a number of low-voltage condensers very closely approach the action of polarization cells.

Electrolyte potentials of symmetrical condensers on a-c. circuits are lower than the theoretical values. These discrepancies can be accounted for only by a voltage distribution in the electrolyte. Controlling the potential from outside sources has small influence on power losses except for extreme values of potential.

An equivalent circuit can be drawn for a condenser from the losses, the applied voltage, and the electrolyte potential. On the basis of dielectric theory, the principal portion of the losses is attributed to the voltage drop above the rectifying region. The losses now approach those for a polarization cell.

A film on the anode surfaces saturated with the electrolyte solvent would explain the characteristics as measured, and also make it possible to predict characteristics with some accuracy. Ion penetration of the outer regions of the film and a comparatively continuous film near the anode explain the non-linear characteristics of electrolytic condensers.

The variation of the electrolyte potential can be predicted on the basis of the above theory, and power losses are also more easily predictable.

ELECTROLYTIC condensers consist of electrodes immersed in proper electrolytes with means for obtaining electrical contact to the electrodes and the electrolyte. The action of electrolytic condensers is due to films formed on the surfaces of the anodes. One class of condenser known as the dry type utilizes a hygroscopic, viscous electrolyte which will not flow from between two parallel electrodes separated by a suitable spacer,[1] such condensers have comparatively short electrolyte paths, and consequently much lower electrolyte resistance. They possess the further advantage that it is very simple to roll the electrodes and the electrolyte into a compact cartridge similar to the impregnated paper condensers. One such condenser is now available having 2.5 μf. of capacity per cubic inch of space, with a rated voltage of 450 volts.

Three of the important film theories include the electromotive equilibrium theory, the solid oxide film, and the gas-oxide film theory. The writer's adsorbed liquid theory supplements the older theories.[2] The solid oxide film theory considers that the entire electrical stress is distributed across the oxide film as the dielectric.[3] The gas-oxide theory postulates an inactive film of oxide and hydroxide which serves to support an extremely minute layer of gas carrying the major portion of the electrical stress.[4] The electromotive equilibrium theory states that the oxide film is only supplementary to the principal action of the condenser which is entirely electrochemical in nature.[5] The writer's adsorbed liquid theory[2] more recently advanced proposes that the oxide film is the carrier of an adsorbed volume of liquid, the solvent of the electrolyte, which carries ions the signs of which will vary with their positions with respect to the anode and the applied voltage. This theory takes advantage of the fact that the solvent, in most cases water, has a dielectric constant over 80.

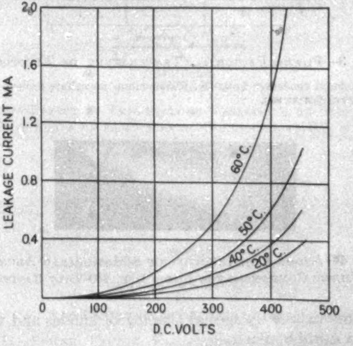

FIG. 1—LEAKAGE CURRENT OF 8-MICROFARAD FILTER CONDENSER FORMED TO 500 VOLTS D.C.

Aluminum anode in saturated borax soln

As a source of power loss, the leakage current of the majority of the electrolytic condensers is negligible, Fig. 1. Leakage current will in most cases account for less than one-tenth of one per cent of the total losses. A redissolving film[6] which would dissolve when current was passed in the permeable direction and would again reform when it was attempted to pass current in the

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*Development Engineer, New Haven, Conn.

1. For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

impermeable direction would account for high losses, but this theory is open to many objections.

Resistance of the electrolyte can be controlled by the temperature of the condenser, and by the composition of the electrolyte; the majority of suitable electrolytes have high electrical resistance. However, the power factor of an electrolytic condenser increases with rising voltage, Fig. 2. Increases in temperature should reduce power losses, but measurements show increased losses with increasing temperatures, Fig. 3. The anode construction of Fig. 4 gives rise to a temperature power factor curve such as shown in Fig. 5 due to the long current conduction paths to a large portion of the anode. This defect has been overcome to some extent by special shaping of anodes and venting in suitable places.[7]

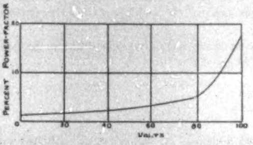

FIG. 2—DEPENDENCE OF POWER FACTOR ON APPLIED VOLTAGE

Aluminum condenser in 1.0 N. diammonium phosphate formed at 100 volts 60 cycles per sec., 23 deg. cent.

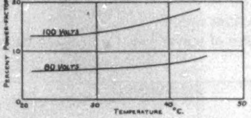

FIG. 3—POWER FACTOR *vs.* TEMPERATURE OF ELECTROLYTE

Aluminum condenser in 0.9 N. diammonium phosphate formed at 100 volts, 60 cycles per sec.

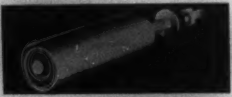

FIG. 4—ANODE CONSTRUCTION OF 8-MICROFARAD ALUMINUM FILTER CONDENSER FOR RADIO USE, 450-VOLT RATING

Typical electrolyte resistances range from 400 ohms to 100 ohms per cm. cube for aluminum condensers, and may go as low as 3 or 4 ohms per cm. cube for low voltage tantalum condensers. The total resistance of the electrolyte of the condenser of Fig. 2 and Fig. 3 is approximately 0.8 ohms, decreasing possibly 50 per cent for the higher temperatures of Fig. 3. The maximum power loss in the electrolyte of this condenser accounts for a power factor slightly under one-half of 1 per cent, much lower than the over-all values which range from 1.25 to 20 per cent. Tantalum in sulfuric acid emphasizes the inadequacy of the resistance of the electrolyte to explain the power losses in electrolytic condensers, Fig. 6; the total resistance of the electrolyte is only a few hundredths of an ohm, and the power factor would be less than one-tenth of one per cent were the losses found in the electrolyte.

When a sine wave of alternating voltage is applied to an electrolytic condenser, large variations in current wave shape are found (Fig. 7); the oscillograms show the current waves 180 deg. out of phase. The wave shapes for aluminum anodes are different in detail from tantalum, but of the same general form, Fig. 8.

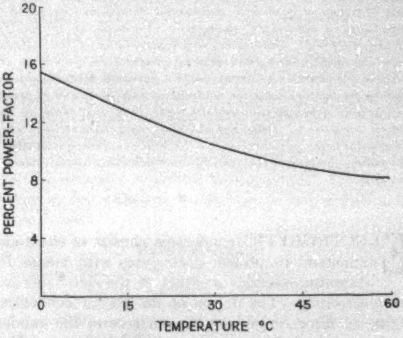

FIG. 5—POWER FACTOR *vs.* TEMPERATURE IN ALUMINUM ELECTROLYTIC CONDENSER FOR RADIO SERVICE

Electrolyte borax and boric acid

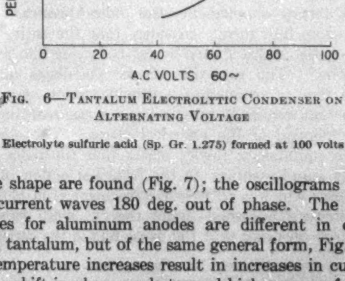

FIG. 6—TANTALUM ELECTROLYTIC CONDENSER ON ALTERNATING VOLTAGE

Electrolyte sulfuric acid (Sp. Gr. 1.275) formed at 100 volts

Temperature increases result in increases in current and a shift in phase angle toward higher power factors as shown in Fig. 9. These changes are small in so far as the wave shape is concerned until comparatively high temperatures are reached. Changing the concentration of the electrolyte does not seem to alter the effect of temperature changes to any great extent except by a further shift toward an increase in power factor when comparatively high concentrations are reached. (Figs. 10 and 11.)

Increases in the frequency of the voltage applied to the condenser are evident in the current wave shape in the same manner that all influences tending to increase the current loading per unit of area of the anodes become evident, namely by increases in the harmonics and a general if not rapid decrease in phase angle. However, the out-of-phase component of current is as usual almost unchanged as seen in Fig. 12.

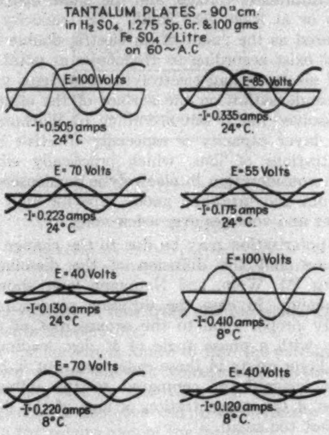

FIG. 7—CURRENT WAVE SHAPE AND APPLIED VOLTAGE

Tantalum condenser

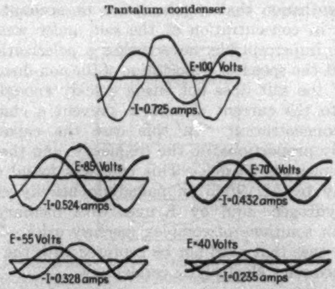

FIG. 8—CURRENT WAVE SHAPE AND APPLIED VOLTAGE

350-sq. cm. aluminum plates on 60 cycles a.c. in 3.5 N. diammonium phosphate, formed at 100 volts, temperature 23 deg. cent.

In general, the current wave shapes of electrolytic condensers in circuits where sine waves of alternating voltage are impressed may possess a large number of harmonics of varying amplitudes. Any changes in operating conditions which tend to increase current density at the anode surface will increase the amplitudes of the harmonics. The harmonics are usually distributed in such a way that a distinct peak in current is evident aside from the out-of-phase condenser component of current which is plainly evident and changes very little, this peak occurring in phase with the impressed voltage.

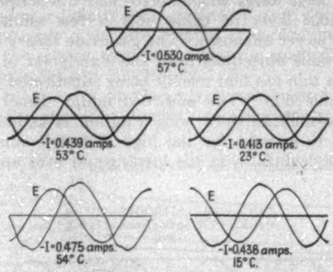

FIG. 9—CURRENT WAVE SHAPE AND TEMPERATURE OF ELECTROLYTE

350-sq. cm. aluminum plates on 60 cycles a.c. in 1.75 N. diammonium phosphate, 80 volts impressed and formed at 100 volts

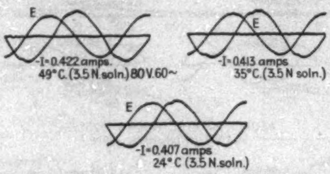

FIG. 10—EFFECT OF TEMPERATURE VARIATIONS ON CURRENT WAVE SHAPE OF ELECTROLYTIC CONDENSER OF FIG. 9

3.5 N. solution

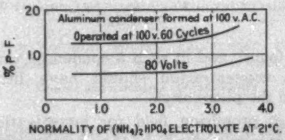

FIG. 11—POWER FACTOR VARIATIONS WITH CHANGES IN ELECTROLYTE NORMALITY IN AN ALUMINUM CONDENSER

Power factors of electrolytic condensers are high for reasons other than leakage and resistance. To ascribe an additional loss to a poor phase angle for the effective dielectric film will account for another part of the total loss. Adhering to three of the original film theories presented, the solid oxide film, the gas-oxide film, and the adsorbed liquid film, the possibilities of the three can be investigated. The solid oxide film theory calls for an extremely thin film of oxide of the anode metal, which may however be shielded to some extent by a gross thickness of film in which there is considerable penetration of the electrolyte. The phase angle of a crystalline film would be much too small to account for the large losses encountered; and the action of an amorphous layer of oxide only a few millimicrons thick is as yet unknown. The gas-oxide theory is open to the same objections from the standpoint of power losses, a thin gas film merely being substituted for the thin oxide film. The adsorbed liquid theory offers more possibilities since polar rotation is undoubtedly present to account for the high dielectric constants, yet it is doubtful that the losses could ever approach the losses found; and in such a condenser variations in electrolyte concentration should have little or no effect upon the phase angle.

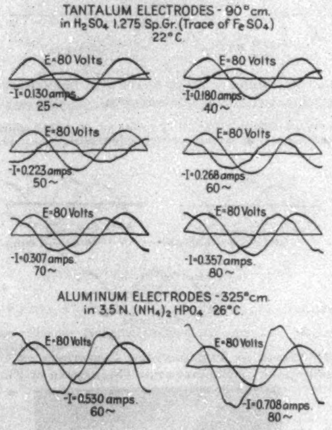

FIG. 12—INFLUENCE OF FREQUENCY ON CURRENT WAVE SHAPE

The most promising field for investigation of the power losses now seems to be the electromotive equilibrium theory. As mentioned above, the electromotive equilibrium theory considered alone is insufficient to explain all the action occurring in electrolytic condensers, but the extent to which polarization capacity enters the question has been investigated in some detail.

An electrolytic cell capable of polarization can be subjected to a flow of current and then discharged through a resistance, acting in some respects as a condenser which has been charged and which has a certain capacity; this is known as the polarization capacity of the cell. Its place is the point of contact between the electrode and the electrolyte, and is quite distinct from the case of a non-conducting layer, for instance an oxide layer as on an aluminum condenser. Polarization capacities have been found to be dependent upon the frequency of the impressed voltage and the magnitude of the voltage or current intensity; and the phase displacement varies from 0 deg. to 90 deg. leading. There are in general three characteristic groups into which electrolytic cells may be divided.

The capacity may be independent of the frequency of the impressed voltage, and the phase displacement is zero or at least very small. The capacity can be considered as the result of an electric double layer as it must exist according to Helmholtz at point of contact of electrode and electrolyte, or perhaps produced by a condensation on the surface of the electrode of the dissolved metal salt according to Warburg. The double layer capacity is especially effective in small concentrations of ions, which practically eliminates it from consideration in electrolytic condensers of the aluminum and tantalum anode type for high current densities and voltages over a few volts.

The polarization may be due to the change in concentration and the diffusion of the dissolved salt. Warburg, M. Wein, and Neumann have shown that the capacity, in case the diffusion predominates, is inversely proportional to the square root of the frequency, with a phase angle of 45 deg. leading. The double layer exists in these cases also, but it becomes entirely insignificant compared to the influence of diffusion if the concentration of ions of the dissolved salt is not too small.

In the case of a salt so little dissociated and of such a concentration that a polarization on account of the change in concentration of the salt under weak currents is imperceptible, nevertheless a polarization can occur, if the speed of dissociation of the non-dissociated part of the salt does not ensue quickly enough compared to the current change to prevent a change in ionic concentration. In this case the capacity is inversely proportional to the frequency, and the phase displacement is 90 deg. This case was found experimentally by M. Wein on palladium heavily charged with hydrogen—and by Kruger with mercury electrodes in solutions of complex mercury salts. This is another case that need not be confused with the action in the ordinary electrolytic condensers.

It therefore appears that the only case of polarization capacity which must be considered is the case of concentration or diffusion capacity. Warburg[8] has developed the theory of concentration cells when a sine wave of alternating current is forced through the cell. Selecting a symmetrical cell with parallel plate electrodes

$$E = \frac{\alpha}{K\omega} \sin\left[\,\omega - \frac{\pi}{2} + \psi\,\right]$$

where K is the polarization capacity. The value of the constant ψ is $\frac{\pi}{4}$ for this case, and

$$K = \frac{c_0\sqrt{kqe^2y}}{2RT\sqrt{\omega f}}$$

In the above equation R is the gas constant, f is the number of ions given by one molecule of the salt, y the total valency of ions from one molecule, and T is the absolute temperature.

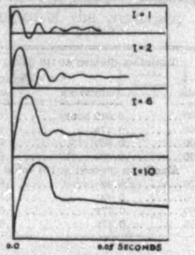

Fig. 13—Degeneration of Oscillatory Charge of Polarization Cell Due to Increase in Current Density

Thus, the current is found to lead the voltage by 45 deg. when a sine wave of voltage is impressed on a concentration cell for low values of current density. The capacity is also observed to be inversely proportional to the square root of the frequency.

When the current density is high the above relation is no longer true, and harmonics are introduced into the current wave when a sine wave of voltage is impressed. However, it is very interesting to note that the effective capacity will not change greatly, and the phase angle of 45 deg. is particularly interesting from the standpoint of power losses in electrolytic condensers.

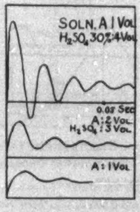

Fig. 14—Degeneration of Oscillatory Charge of Polarization Cell Due to Increased Ion Concentration

Kruger[9] developed equations for the charge of polarized cells in inductive circuits, and obtained experimentally the necessary results to confirm them. Very low voltages were applied to the cells to avoid gas polarization, and it was found that increases in current density would change a charge from an oscillatory charge to an aperiodic charge, Fig. 13. Likewise, increases in the ion concentration of the solution show the same effect, Fig. 14; and one very interesting development is the fact that an oscillatory circuit can be converted very easily to an aperiodic circuit by increasing the inductance in the circuit. It is also observed as a definite characteristic of most electrolytic cells that only a definite number of oscillations occur before the charge degenerates into an aperiodic form which may tail-out indefinitely.

The behavior of an electrolytic condenser made up with nickel anodes in a caustic solution for use in low-voltage filters was very similar to the action found by Kruger for concentration cells. The charging characteristics were strongly suggestive of a condenser in which concentration capacity plays a large part, Fig. 15: the discharge characteristics in inductive circuits also indicate the predominance of concentration capacity with very high initial concentration in the electrolyte. A low-voltage electrolytic condenser of the semi-dry type intended for the same low-voltage filter use showed better condenser characteristics, Fig. 16. The charging characteristics of the semi-dry condenser were found to approach those of a perfect condenser, although the decrease in charging current

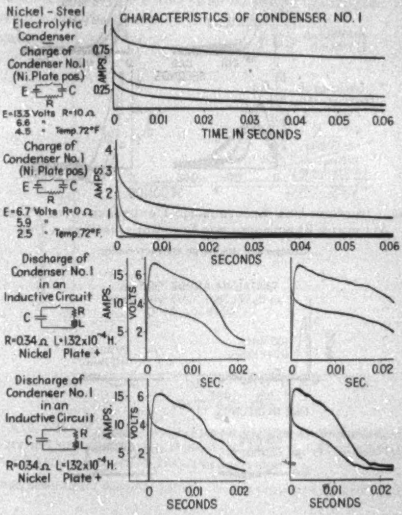

Fig. 15—Low-Voltage Filter Condenser with Large Amount of Concentration Capacity

Charge and discharge characteristics

with time was not rapid enough to satisfy a logarithmic function.

When a high-resistance film is present on the surface of an electrode immersed in an electrolyte, somewhat different results may be expected than from the case of diffusion capacity alone. However, diffusion capacity undoubtedly exists at the point of contact of the film and the electrolyte.

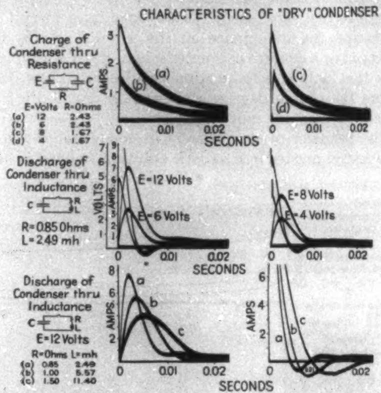

Fig. 16—Semi-Dry Electrolytic Condenser with Pasted Aluminum Electrodes for Low-Voltage Filter Use

Charge and discharge characteristics

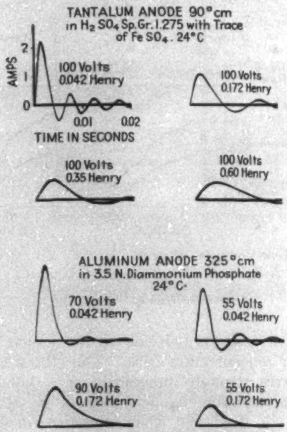

Fig. 17—Oscillatory Discharge of Electrolytic Condensers in Induction Circuits of Very Low Resistance

Ordinary aluminum and tantalum condensers were discharged through inductive circuits and oscillograms of current were obtained, Fig. 17, with strong indications of the presence of diffusion capacity. Tantalum showed the characteristic increase in attenuation of the oscillatory component with increase in inductance, and also was noteworthy in that the period of the oscillatory component became shorter as the cell discharged and the concentration decreased. Aluminum indicated an even greater diffusion capacity than tantalum, and the decrease in attenuation is strikingly evident as the initial concentration is decreased for the same circuit conditions by lowering the initial voltage. Increasing the inductance of the discharge circuit for the aluminum condenser caused a rapid approach to the aperiodic discharge predicted by Kruger[9] for concentration cells.

TABLE I

Volts	Inductance	Capacity μf. per sq. cm.
Tantalum (formed at 110 volts)		
100	0.042 henry	0.201
100	0.172	0.190
100	0.350	0.170
100	0.600	0.161
Aluminum (formed at 110 volts)		
70	0.42	0.085
55	0.42	0.091
90	0.172	0.093
55	0.172	0.087

The effects of diffusion or concentration capacity in ordinary electrolytic condensers are somewhat similar to the effects predicted for simple concentration cells; but there is the fundamental difference in that in most electrolytic condensers the diffusion capacity is in series with a more or less effective dielectric layer.

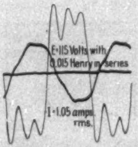

Fig. 18—Aluminum Electrolytic Condenser Formed at 120 Volts in 1.75 N. Diammonium Phosphate in Series with Inductance of 0.015 Henry

Area of aluminum electrodes 350 sq. cm. each, temperature 23 deg. cent., total applied voltage 115-sine wave 60-cycle.

The following capacities calculated from the oscillograms of Fig. 17 by integrating over the first quarter cycle of the discharge to find the coulombs of electricity stored in the condenser for a given voltage, show a decrease in effective capacity for decreasing frequency, Table I.

These variations in effective capacity have been noticed in electrolytic condensers only in freely oscillating circuits and exist to a negligible extent in circuits where alternating voltages are impressed from a source of power supply. Condensers with non-linear current voltage characteristics are difficult to handle as simple capacities without introducing errors.

Harmonics in the current wave can be emphasized in a-c. circuits by placing inductance in series with the condenser, Fig. 18.

In a symmetrical electrolytic condenser on a-c. circuits, in order to prevent heavy leakage currents, it is necessary that the aluminum or tantalum electrodes always have a positive potential with respect to the electrolyte of the condenser. Assuming an equal voltage division between electrodes, the maximum alternating voltage applied to one electrode is equal to half of the total applied voltage. The maximum instantaneous voltage is therefore equal to 0.707 by the total applied voltage, and the d-c. potential of the electrolyte which must be negative with respect to the electrodes at all times is therefore equal to 0.707 by the a-c. voltage on the condenser. In order to insure a minimum of leakage, it is possible to raise the potential of the electrodes with respect to the electrolyte by means of a separate source of direct current, Fig. 19. Referring to Fig. 2, the power factor is improved greatly by operating the condenser on an a-c. voltage well below the voltage of formation; by maintaining the voltage of formation higher than the operating voltage by means of some external source of direct current it should be possible to greatly improve the power factor, providing that other effects do not overweigh the advantages gained. However, when the voltage of the electrolyte was controlled from an external source, the expected results were not obtained, very little if any improvement in power factor being noted. (Figs. 20 and 21.)

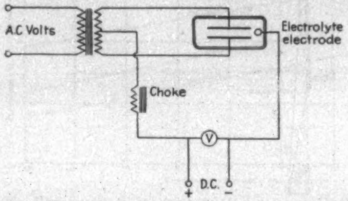

Fig. 19—Circuit for Applying and Measuring Electrolyte Potential in Electrolytic Condensers on A-C. Circuit

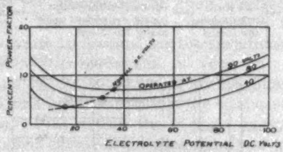

Fig. 20—Power Factor Control by Means of Control of Electrolyte Potential of Electrolytic Condenser

Since there was no improvement in power factor when the electrolyte of a symmetrical condenser was maintained at the proper potential with respect to the electrodes by auxiliary means, the actual potential of the electrolyte was measured with an electro-static voltmeter connected to the midpoint of the transformer supplying the condenser and to the electrolyte. The potentials found for aluminum and for tantalum electrolytic condensers were 50 to 75 per cent of the expected values, Fig. 22. These departures from the theoretical values on the basis of complete rectifier action at the film and a dielectric capacity type of condenser are much too large to admit of experimental error, and furthermore are very consistent in their values.

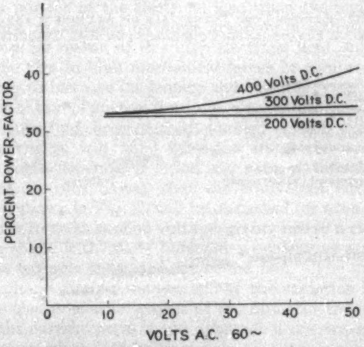

Fig. 21—Variations of Power Factor of 8-Microfarad Filter Condenser with Amplitude of the A-C. Component for Various Values of D-C. Polarization

High power factor due principally to the long electrolyte conduction paths

It was thought at first that these low values might be due to a pulsating electrolyte potential which would reach a maximum value of the theoretical voltage twice every cycle, but subsequent oscillograms of the variation in electrolyte potential disclosed a ripple in the d-c. voltage of approximately one-quarter of one per cent or less, Fig. 23. This is further proof that losses in electrolytic condensers on a-c. circuits should not be ascribed to leakage current when operated at normal voltages.

The maximum value of alternating voltage applied to the rectifying film on the electrodes of electrolytic condensers is therefore 50 to 75 per cent of the theoretical value.

The phase angle of the part of the electrolytic condenser across which this voltage appears is an unknown quantity; but if it is assumed that this effective film is any one of the three mentioned above, a solid dielectric, a gas filled solid, or a liquid filled solid, the phase angle is extremely close to 90 deg. leading.

On the assumption that the power loss in the effective film is negligible, it is possible to locate the voltage vector of the effective film with respect to the current and voltage vectors for the entire condenser, Fig. 24. The condenser current I_c is shown leading the total applied voltage by the phase angle α, and the effective film voltage E_{eff} is approximately in quadrature with the condenser current. In order to complete the voltage triangle, the vector E_{e1} must be added with a phase angle of β with respect to the condenser current. This vector E_{e1} is of interest now, since the power losses are practically all accounted for by its phase angle; it is the voltage drop which appears in the electrolyte of the condenser, and it has just been shown that the voltage drop in the electrolyte is much too small as calculated to account for the magnitude of this vector on the basis of a resistance drop, furthermore the phase angle shows that it is not a pure resistance.

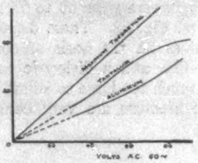

FIG. 22—ELECTROLYTE POTENTIALS OF ALUMINUM AND TANTALUM ELECTROLYTIC CONDENSERS ON A-C. CIRCUITS

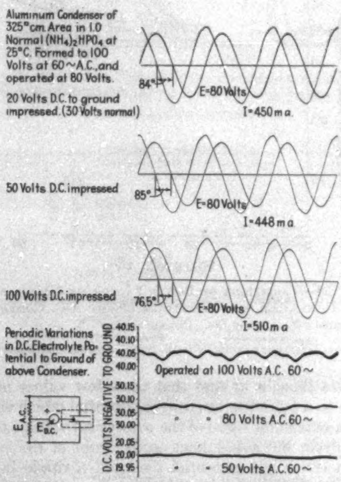

FIG. 23—EFFECT OF WAVE SHAPE ON IMPRESSED ELECTROLYTE POTENTIAL, AND THE NORMAL FLUCTUATIONS IN ELECTROLYTE POTENTIAL

It becomes very quickly apparent upon consideration of the properties of this residual voltage vector after all of the measurable quantities are accounted for, that the electromotive equilibrium theory is justified to some degree. The phenomenon of polarization capacity will account for the leading phase angle of the vector E_{e1}, and when a larger portion of the loss is assigned to the voltage appearing across the effective film, the phase angle of the electrolyte voltage vector increases. Thus, it appears that in the great majority of electrolytic condensers of the formed film type there are two distinct kinds of capacity present, dielectric capacity and polarization capacity.

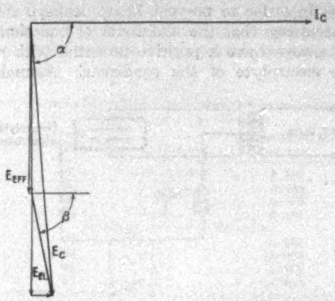

FIG. 24—VOLTAGE DISTRIBUTION IN ELECTROLYTIC CONDENSER ON ALTERNATING CURRENT CIRCUIT

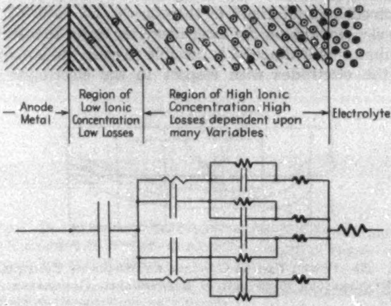
Anode Metal
Region of Low Ionic Concentration Low Losses
Region of High Ionic Concentration. High Losses dependent upon many Variables.
Electrolyte

FIG. 25—SCHEMATIC DRAWING OF CROSS-SECTION OF FILM ON ELECTROLYTIC CONDENSER ANODE

Showing regions of high and low ionic penetration and the equivalent circuit diagram

Selecting a typical example of an aluminum condenser, with a total impressed voltage of 80 volts at 60 cycles, the power factor was found to be 9 per cent. The d-c. potential of the electrolyte was measured at 32 volts, leaving a distribution of 44.8 volts on both electrodes of the symmetrical condenser in series. The remainder of the out-of-phase voltage 34.8 volts, appears as a voltage drop in the electrolyte. Added to this quadrature drop is the in-phase drop of 7.2 volts, giving a total drop in the electrolyte of 36 volts at 20 per cent power factor, or a phase angle of approximately 78 deg. leading. The resistance of the electrolyte will account for less than one volt drop, and that in phase with the current; apparently polarization is occurring in the gross film covering the effective film, or very near to the gross film.

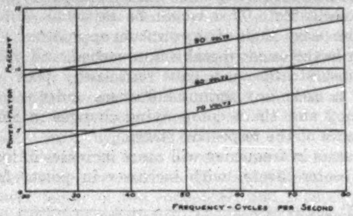

Fig. 26—Effect of Variations in Frequency Upon the Power Factor of Electrolytic Condensers

Aluminum plates 350 sq. cm. in diammonium phosphate 1.0 N. at 23 deg. cent. (formed at 100 volts)

However, polarization capacity of the kind found in this type of electrolytic condenser was found to have a phase angle of 45 deg. leading, which does not agree with the 78 deg. for the above case. Obviously, there are modifying factors and it will appear that polarization capacity as such must be handled differently than from the standpoint of clean electrodes in an electrolyte. The writer has advanced a theory of film penetration to account for the variations in capacity of electrolytic condensers at very low voltages.[2] A diagrammatic cross-section of such a film is shown in Fig. 25, and below it is shown the equivalent circuit. In the equivalent circuit, the in-phase components of the equivalent circuit are shaded lightly or heavily, to indicate low conductivity or high conductivity of the film at that point as the case may be.

Calculations from the measured capacities of such anodes show that the gross film as measured by optical methods may reach thicknesses as great as fifty times that necessary for the requisite capacity. Thus, it is necessary that the film be conducting through at least a part of its thickness, and it can best be described as a semi-permeable membrane which increases in density as the anode is approached.

In such a film structure, the penetration of the film by the electrolyte will be retarded in the case of cations by reason of the direction of the electric field, and in the case of anions the field will accelerate them to even further penetration than under the case of normal diffusion. By way of explanation, the terminology used for the anion and the cation assigns to that ion which is attracted to the cathode the name cation, and vice versa, there being some ambiguity in the terms as used by physicists and electrochemists.

Thus referring again to Fig. 25, a film of insulating material of a thickness comparable with the wave-length of visible light, varying in composition from an extremely dense layer against an anode to a relatively porous outer surface and presumably saturated with water, acquires a space charge which varies in density with the location in the film. The distribution of the electrical field through the film will depend upon the configuration of the space charge and movements in the film, assuming that the film material itself is a relatively good insulator. Thus, the potential gradient will increase as the anode is approached, placing a larger portion of the electrical load upon the region of the film unoccupied by large numbers of anions; and likewise by reason of this high electrical field, subjecting the film to high mechanical forces in compression, tending to increase its density and continuity.

The action of such a film in an electrolytic condenser with pulsating direct current applied is such that the space charge will vary with the impressed voltage, with some dependence upon the rate of diffusion of anions, in such a way that the equivalent electrical circuit shown in Fig. 25 will be disturbed for even small changes in total applied voltage, giving rise of a current wave shape with many harmonics evident as a result of the variable characteristic of the film. The smaller the voltage change, the less will be the variation in the space charge characteristics of the film and the nearer will the current wave shape approach a sine wave for a sine wave of applied voltage.

The region of the film nearest to the anode was described as being relatively free of ions; whether or not it is saturated with the solvent of the electrolyte is open to question, but it is more likely that it is saturated than not since the dielectric constants of the aluminum compounds are relatively low compared to water, and the volt-ampere loading of this region would be excessive in the case of low dielectric constants. This portion of the film is shown in the equivalent diagram as pure capacity with little or no resistance loss, and it is believed that across this region the rectifying action of electrolytic rectifiers occurs. The

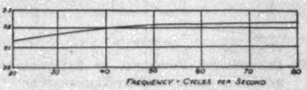

Fig. 27—Effect of Frequency Upon Electrolyte Potential of Symmetrical Electrolytic Condenser

Aluminum

boundary can be fixed definitely on the side adjacent to the anode metal, but on the other side of this rectifying region of low dielectric loss and high electrical resistance, the boundary cannot be fixed so definitely. It moves about with the applied voltage, the thickness of the formed film, the mobilites of the anions, and the temperature of the film and electrolyte. Thus, the leakage current and breakdown voltage of a particular film are dependent upon the above mentioned variables. No definite plane of demarcation between the rectifying region and the adjacent region saturated with ions can be located, the one shading gradually into the other. A definition of the boundary of this rectifying region can be stated as the point at which the maximum voltage drop from the anode is equal to the electrolyte potential when the film is made half of a symmetrical electrolytic condenser in an a-c. circuit.

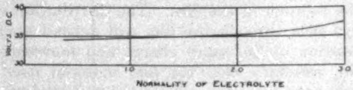

FIG. 28—EFFECT OF ELECTROLYTE CONCENTRATION UPON ELECTROLYTE POTENTIAL OF SYMMETRICAL CONDENSER

Aluminum

The entire film is then seen to form a dielectric sheet similar to the stratified dielectric of Maxwell showing absorption as do all stratified films; and possessing the further interesting feature that the leakage of any one strata is controlled by many factors, and will change periodically with an applied periodic voltage variation, giving rise to a dielectric sheet with a non-linear voltage-current characteristic.

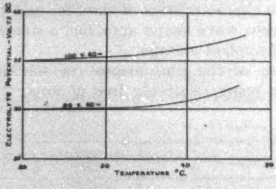

FIG. 29—ELECTROLYTE POTENTIAL OF ELECTROLYTIC CONDENSER AS INFLUENCED BY TEMPERATURE

Aluminum

For the current flow in the resistance paths, the ions in the film constitute the space charge in the film and must also supply the electrical charge to take care of the excess voltage imposed on the rectifying film by reason of the resistance paths. Thus, the ions in the film fulfil two requirements, the one providing electrical conductivity through regions of the film and the other providing an electrical charge of real thickness to supplement the dielectric strain in the parallel strata. The thickness of the ionic space charge influences the characteristics of the condenser, as well as does the mobility of the ions in the film. The increase in effective capacity of the condensers in the above described freely oscillating circuit with increases in natural frequency of the circuit, Table I, can then be attributed to the predominance of the leakage resistance paths over the space charge effect in the cases cited. However, this is the only case where such action has been noted by the writer, the effect of variations in frequency upon effective capacity usually being negligible. This would be expected ordinarily since the phase angle of any stratum approaches 90 deg. leading as the anode metal is approached, and only the outermost strata must have sufficiently poor phase angles to exert any serious influences under increased frequency and the accompanying changes in relative impedance of the respective strata.

Increases in frequency will cause increases in loading of the outer strata, with increases in power factor. The effect of frequency upon the power factor of an aluminum electrolytic condenser is shown in Fig. 26, and the variation in d-c. potential of the electrolyte for the same condenser is shown in Fig. 27.

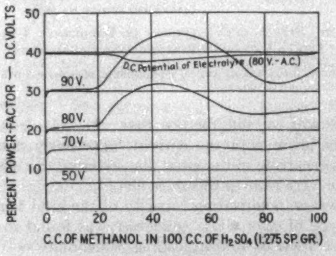

FIG. 30—ELECTROLYTIC CONDENSER POWER FACTORS AS CONTROLLED BY SOLVENT OF THE ELECTROLYTE

Tantalum condenser

Increasing the concentration of the electrolyte should increase the space charge density in the film, with a resultant increase in d-c. potential of the electrolyte, Fig. 28. The condenser illustrated in Fig. 28 is the same condenser as in Fig. 11.

Variations in impressed voltage have a similar effect to both frequency changes and increased concentration for high voltages; and the results are shown in Figs. 2, 6, and 22. Increases in the space charge density are general for increases in voltage, with particularly high increases in the outer strata. Temperature rises produce a similar effect to voltage rises through increased mobilites of anions, Fig. 29.

The temperature drop through the film can be calculated from the known losses and the assumption that the transfer of heat from the film to the electrolyte follows the general rules of heat transfer between liquid and solid surfaces in contact, a value of approximately 7.3 g-cal. per sq. cm. per hour per deg. cent., being acceptable for the case of electrolytic condensers. For the condenser of Fig. 2, with a total area of both electrodes of 700 sq. cm. and a current flow of 400 milliamperes at 100 volts, the power factor is 17 per cent. The loss per hour is 6.8 watt-hr. or 5,860 g-cal. per hour, with a temperature difference between the film and the electrolyte of approximately 1.15 deg. cent.

An attempt was made to change the capacity of a tantalum electrolytic condenser by changing the nature of the solvent for the electrolyte. Methyl alcohol was added to sulfuric acid electrolyte, with some expectation that penetration of the film by the alcohol might occur with a consequent loss in capacity due to the lower dielectric constant of methyl alcohol as compared with water, approximately 30 to 80. Capacity variations were negligible, and the small errors in observation were as large as the observed variations. However, some very curious effects were noticed upon the power factor of the condenser, and a small effect upon the d-c. potential of the electrolyte for the higher operating voltages. The peculiar humps in the curves of Fig. 30 are attributed to the increased loading of the outer strata of the film due to penetration by the alcohol, thereby increasing the voltage drop in the outer strata and thus raising the power factor. The curve of d-c. potential of the electrolyte against alcohol content indicates this action also. Work on electrolyte condensers was discontinued before any further investigations could be made along similar lines.

The author wishes to express his appreciation to Professor Wesley B. Hall of Sheffield Scientific School of Yale University and to Mr. A. E. Knowlton, formerly of Yale for their valuable assistance rendered in the preparation of the material for this paper both while at Yale and subsequently.

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Discussion

FILM CHARACTERISTICS OF ELECTROLYTIC CONDENSERS

(Godsey)

POWER LOSSES IN ELECTROLYTIC CONDENSERS

(Godsey)

H. H. Race: Figs. 25 and 26 of Mr. Godsey's paper interested me particularly because of some recent work in our laboratory which may help to explain power factor—frequency characteristics of electrolytic capacitors even though it was undertaken for an entirely different purpose. In studying the causes of dielectric loss in insulating materials we have made artificial imperfect dielectrics by sealing conducting liquids in fused quartz tubes. We have found that the measured effective capacitance and dielectric loss as functions of frequency of such a composite dielectric can be completely explained in terms of the equivalent circuit shown in Fig. 1 which may be reduced to the simpler circuit shown in Fig. 2.*

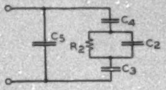

Fig. 1—Equivalent Circuit of Fused Quartz Tubes Containing a Conducting Fluid

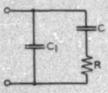

Fig. 2—Simpler-Circuit Which Can be Made Equivalent to Fig. 1 by the Use of Definite Algebraic Relations Between Coefficients

If a dielectric can be represented by the equivalent circuit of Fig. 2 in which the coefficients are assumed to be independent of frequency, then the general form of the effective capacitance and loss curves will be as shown in Fig. 3.

If the coefficients of capacitance and resistance in Mr. Godsey's Fig. 25 can be assumed to be independent of frequency then this complicated combination of series and parallel arrangements of resistance and capacitance can be reduced to an equivalent circuit similar to Fig. 2 above and the changes in effective capacitance and loss with frequency in an electrolytic capacitor should follow Fig. 3. Thus Fig. 26 may be only a special case of this more general form.

Probably the coefficients in Fig. 26 cannot be assumed to be independent of frequency since the ionic concentration at a given point in the oxide film will change with time. Nevertheless, I feel that a study of the dielectric properties of electrolytic capacitors over wide ranges of frequency and temperature should add considerably to existing knowledge of the mechanism involved.

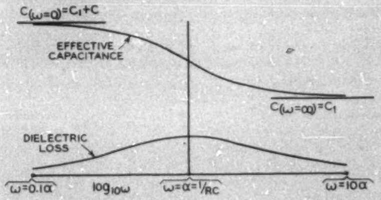

Fig. 3—Variations with Frequency of Effective Capacitance and Dielectric Loss for the Equivalent Circuit Shown in Fig. 2 (Plotted for Case Where $C = C_1$)

J. Slepian: The principal obstacle to the use of electrolytic condensers in the power field has been the high power factor usually obtained in such cells. An attempt to locate and explain the losses occurring in these cells, such as Mr. Godsey has made in this paper, is therefore extremely praiseworthy.

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*The results of this study will be published in the near future.

In work which I carried on over a period of years some twelve years ago, I quickly found that the properties of aluminum cells were extremely complicated and that such cells frequently behave in most unexpected ways. For example, it was generally believed that the halogens were extremely deleterious. I found that the addition of a fractional per cent by weight of a soluble fluoride had an extremely beneficial effect upon the power factor, reducing the losses generally many fold. With the addition of such traces of fluoride I was able to make cells with power factors under 2 per cent.

However, I was never able to make cells with such low power factors which would stay efficient indefinitely. In all cases there was a gradual rise in power factor with continuous operation over a period of years. Starting at 2 per cent the power factor would become 5 or 6 per cent in the course of a year and continue to rise slowly for years afterwards. This was with a superposed d-c. excitation upon the cells. Without the superposed d-c. excitation the rise in power factor was very much more rapid. Investigation showed that apparently changes in the electrolyte played a part in the rise in power factor since, if the plates of an old cell were placed in fresh electrolyte the power factor returned to a low value, although not quite as low as in the original cell. The power factor rose much more rapidly, however, with time in the new electrolyte than in the old, indicating that there had also been a change in the electrode. The change in the electrolyte I assumed to have been the formation of colloidal aluminum hydroxide and I ask Mr. Godsey if he can suggest any reason why the presence of such substance should effect the losses in the cell.

In general I found that in operation on alternating current some of the generalizations made from d-c. studies were not applicable. For example, on direct current the capacity of an aluminum cell seems to be independent of the nature of the electrolyte and to vary inversely with the d-c. voltage, but on alternating current the capacity was different in different electrolytes and varies with the time. Generally also, the capacity was lowest in those cases where the power factor was low. This may seem to be inconsistent with the picture of Fig. 25 which Mr. Godsey has drawn explaining the losses and it would be interesting to have him comment upon that point.

F. W. Godsey, Jr.: The study of artificial imperfect dielectrics referred to by Mr. Race is interesting from the point of view of electrolytic condensers in that the action in a composite dielectric such as Mr. Race describes would closely approach the action in an electrolytic condenser under certain conditions.

However, the circuit of Fig. 25 is only very roughly equivalent and the coefficient of capacitance and resistance can under no circumstances be assumed to be independent of frequency. This is due to the fact that the conducting particles and ions in the outer layers of the film not only constitute leakage resistance paths, but also constitute a considerable portion of the electrical charge of the inner layers of the film. Thus, the capacitance and resistance coefficient are not only variable for different frequencies, but vary through wide limits for different current densities and applied potentials. It is the author's conclusion that it is a hopeless task to attempt to set up a simple equivalent circuit for an electrolytic condenser.

As Mr. Race suggests, it would add considerably to existing knowledge of electrolytic condensers if a study of the dielectric properties were made over wide ranges of frequency and temperature, provided that sufficient precautions were taken to overcome the inherent difficulties due to electrolyte resistance and cell constants.

Mr. Slepian's comments in connection with the addition of halogens to the electrolyte of electrolytic condensers are very interesting. Since most of the previous work on electrolytic condensers was done on direct current, it was soon found that the addition of halogens greatly increased leakage current for a small amount of the halogen ion. However, as I have shown, the leakage current in electrolytic condensers is in general a negligible factor in computing power losses.

Referring to Fig. 25, it will be seen that the addition of any ion to the electrolyte which will serve to penetrate the outer film layers will reduce the value of leakage resistance in the outer layers.

By a proper control of the leakage resistance of these outer film layers, it is possible to either increase the power factor or to reduce the power factor of the condenser. Theoretically, the power factors encountered will be lower when the resistance is zero and when the resistance is infinite. By the addition of halogens, this resistance can be controlled to some extent and it would appear from Fig. 25, that with increased conductivity of the outer film layers, there should also appear an increase in effective capacitance.

The observation that the capacity in alternating current circuits was lowest in cases where the power factor was low can very probably be assigned to the case of newly formed condensers upon which the oxide films are relatively thin and continuous in the outer layers so that there is less penetration of the outer layers by ions. Upon aging, the film grows in thickness and the continuity of the outer layers is in all probability so broken up that ion penetration is much more readily effected and the leakage currents in the outer layers are therefore excessive. It is apparent from an inspection of Fig. 25 that if the leakage resistance in the outer layers is high, the effective capacity will be low.

Mr. Slepian's suggestion that the formation of colloidal aluminun hydroxide in the electrolyte accounts for a large part of the rise in power factor with time of the electrolytic condensers is I believe, substantiated by the references to the importance of the condition of the outer film layers made in the papers under discussion. The placing of an anode with a formed film upon it in an electrolyte in which there is colloidal aluminum hydroxide would result, upon the passage of current, in the collection of this colloidal hydroxide upon the plate. Since the thickness of the oxide film upon the anode is only about one micron for a 300-volt film, a very small deposit of the colloidal hydroxide would very effectively increase the thickness of the outer film layers and would in all probability increase the series resistance of the outer film layers with an attendant increase in power loss. (Two additional papers on electrolytic condensers were presented at the Sixty-First General Meeting of the Electrochemical Society in Baltimore on April 23, 1932. These papers have some bearing upon the comments of Mr. Race and Mr. Slepian, and are identified as preprint 61-27 and preprint 61-28, pages 331 to 357 inclusive.)

Electrical Precipitation

Some Fundamental Theory and Experiments

BY A. W. SIMON*
Non-member
and
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Non-member

WHILE numerous articles have been published on electrical precipitation (the Cottrell process), they have been concerned largely with the engineering features of Cottrell treaters or the special features of the application of the process to particular industries. The fundamental theory of the process or the quantitative laws underlying it, on the other hand, have received but scant attention. From the theoretical side we have but two papers which treat the subject with any degree of thoroughness: one, based on the kinetic theory but limited to tube treaters, by Deutsch,[1] and the other, based on the law of mass action, by one of the present authors.[2] From the quantitative experimental side also but little has come to light; we need mention only that in a paper by Schmidt[3] it is set forth that the precipitation efficiency (percentage removal) is an exponential function of the gas volume treated (although it is not stated what kind of an exponential) and also of the "length of discharge electrode;" while in a paper by Horne[4] the relation between precipitation efficiency (percentage removal) and secondary voltage is exhibited in the form of a curve; further that in another paper by Deutsch it is stated that Horne and Anderson have arrived at an empirical formula, connecting the precipitation efficiency η with the length of pass L and the velocity V of the gas, of the form:

$$\eta = 1 - K^{L/V}$$

and, finally, that Anderson[6] gives the equivalent formula:

$$1 - \eta = K^{t}$$

where t is the time the gas remains in the precipitator.

As a matter of fact, outside of the few points just mentioned, *quantitatively* very little is generally available regarding the fundamental principles involved in electrical precipitation; and many operators today are still entirely in darkness regarding the effect of such fundamental factors as voltage, current, velocity, size of discharge electrode, etc., on the percentage removal, *i. e.*, the "precipitation efficiency".[7] In the present work a number of these relations are, we believe, for the first time, clarified and exhibited.

I. QUANTITATIVE THEORY OF THE COTTRELL PROCESS

From the theoretical point of view it is logical to infer that the Cottrell process can be considered as a reaction between the dust particles on the one hand and the electrically charged gas particles (gas ions) on the other. Such a reaction would presumably follow the law of mass action familiar in chemistry, since after all the chance of a dust particle being precipitated depends on its change of encountering an ion, and therefore should be proportional to the number of ions present, whereas the actual number of dust particles encountering ions should also be proportional to the number of dust particles present. On this basis, if we denote by dq/dt the rate of removal of the dust particles, we have:

$$-\,dq/dt = kqi \qquad (1)$$

where q is the concentration of dust particles at any instant, i is the concentration of the ions, and k a constant, which depends, however, on the particular gas and fume treated.

Integrating this equation directly we have:

$$q/q_o = e^{-kit} \qquad (2)$$

where q_o represents the original concentration and t the treatment time.

This equation can be interpreted in two ways; either statically or dynamically. The static case would correspond to the experiment where the whole volume of the precipitator is filled with a quiescent gas containing suspended material of concentration q_o and the precipitation process is continued for a length of time t, at the end of which period q is the concentration remaining in suspension; the dynamic case, on the other hand, is where the dust-laden gas is flowing through the precipitator, in which case q_o is the entering and q the emergent concentration, and t the time the gas remains in the precipitator.

The left member of (2) is obviously the fractional part of the entering concentration passing through *unprecipitated*, *i. e.*, emerging from the precipitator, whereas in many cases the fractional part *precipitated* is of interest. If we denote this (fractional) part by η, we have that the left member of (2) is equal to $(1 - \eta)$, whence we can rewrite (2) as:

$$\eta = 1 - e^{-kit} \qquad (3)$$

In a particular case we may assume i to be constant, in which case we can put ki identically equal to a new constant α, whence we have as a final result:

$$\eta = 1 - e^{-\alpha t} \qquad (4)$$

Equation (4) expresses two fundamental laws of electrical precipitation, which we shall now consider in detail.

First, from the definition of η, it follows: *The fractional part of the incoming concentration precipitated is independent of this concentration.* This really justifies the use of the quantity η, that is to say, establishes the

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*Tennessee Coal, Iron & Railroad Company.

1. For references see end of paper.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

"precipitation efficiency" (percentage removal) as a uniquely determinable quantity (independent of the incoming concentration).

If interest centers not so much on the amount of material precipitated, as on the *cleanliness* of the emerging gas, it is the quantity $(1 - \eta)$, which we will denote by ξ, which is of importance. In this case we can rewrite (**4**) as

$$\xi = e^{-\alpha t} \tag{5}$$

A second fundamental law which follows directly from eq. (**4**) is that *the fractional part of the incoming concentration precipitated depends on the length of time the gas remains in the precipitator.*

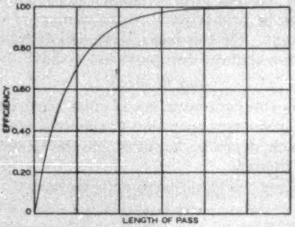

FIG. 1—THEORETICAL PRECIPITATION EFFICIENCY (PERCENTAGE REMOVAL) OF A COTTRELL PRECIPITATOR AS A FUNCTION OF THE LENGTH OF THE PRECIPITATOR

From this law we can deduce directly also how the precipitation efficiency varies with the gas volume V (treated per minute) and the distance D traversed by the gas in the precipitator, *i. e.*, the "length of pass." Obviously, if A represents the effective cross-section of the treater, these quantities are related to each other, and also to the velocity v of the gas in the precipitator by the equation:

$$t = D/v = DA/V \tag{6}$$

whence we can rewrite (**4**) in the several forms:

$$\eta = 1 - e^{-\alpha D/v} \tag{7}$$

$$\eta = 1 - e^{-\alpha AD/V} \tag{8}$$

$$\eta = 1 - e^{-KD/V} \tag{9}$$

where K is a constant of the precipitator depending, however, on the particular gas and fume treated and on treater conditions (temperature, etc.), but, of course, not on D and V. Eq. (**9**) expresses some important facts regarding Cottrell treaters now to be considered in greater detail.

In order to exhibit the relations expressed by eq. (**9**) more clearly, the curve giving the precipitation efficiency as a function of the effective length of the treater is reproduced in Fig. 1; and as a function of the *volume treated per minute* (or its equivalent the velocity of the gas in the treater) in Fig. 2. Anticipating somewhat, it might be stated that most of the quantitative laws involved in electrical precipitation satisfy or approximate either one or the other of these curves, so that they may be regarded as the two fundamental types of characteristic curves of the Cottrell process.

Without, for the moment, going into the mathematical relations, it will be seen from Fig. 1 that the cleaner the gas becomes, that is to say, the more dust is precipitated, the more difficult it becomes to precipitate the remaining dust or to make the gas cleaner, a fact which in practise is found pretty much universally true. In fact the curve of Fig. 1 really expresses a law which holds generally for gas cleaning; for the assumptions made in deducing eq. (**9**) for Cottrell cleaning hold also *mutatis mutandis* for other types of cleaner. To be specific, it will be noted that the foregoing theory is based on the chance of an encounter between a gas ion and a dust particle; however, it might equally well be, in the case of a wet washer, the chance of a water droplet encountering a dust particle, or in the case of a bag filter, the chance of a fiber ensnaring a dust particle. Therefore, with the proper interpretation of D, this formula can be applied to these cases also.

The above more or less qualitative statement, that the cleaner the gas is the harder it is to make it cleaner, can be restated quantitatively as follows:

Consider the total pass D split up into equal intervals of length l and consider the relation between the dust concentration entering and emerging from the jth of these intervals. In practice each interval might represent a separate Cottrell unit, the gas passing through the separate units in succession (*i. e.*, the units being connected in series). For the purpose in hand it is more convenient to rewrite eq. (**9**) in the form:

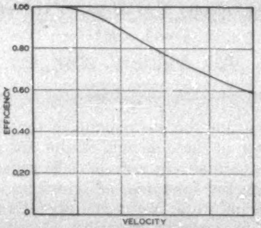

FIG. 2—THEORETICAL PRECIPITATION EFFICIENCY (PERCENTAGE REMOVAL) OF A COTTRELL PRECIPITATOR AS A FUNCTION OF THE GAS VOLUME TREATED PER MINUTE OR THE VELOCITY

$$\eta = 1 - e^{-\beta jl} \tag{10}$$

and then to substitute for $(1 - \eta)$ its value ξ, whence we have finally:

$$\xi = e^{-\beta jl} \tag{11}$$

as the formula to be used. Next if we denote by q_j the concentration of the dust leaving the jth segment of length l, and by f_j the fractional part which q_j is of the concentration q_{j-1} *entering* the interval, we have:

$$f_j = q_j/q_{j-1} = e^{-\beta jl}/e^{-\beta(j-1)l} = e^{-\beta l} = \text{const}_1 \tag{12}$$

In other words: *the fractional part of the dust passing (unprecipitated) through any pass of a series of identical Cottrell passes is the same for every pass.*

If we denote by p_i the fractional part of the dust precipitated in any pass, we have also, since $p_i = 1 - f_i$:

$$p_i = (1 - f_i) = 1 - \text{const}_1 = \text{const}_2 \qquad (13)$$

that is to say *the fractional part of the dust precipitated in any pass of a series of identical Cottrell passes is the same for every pass.*

In practise this means that if the first pass of a series of passes takes out 70 per cent of the dust entering it, the second pass will take out also 70 per cent of the dust *entering it*, but 70 per cent of the 30 per cent of the original dust leaving the first pass and entering the second; the third pass will take out also 70 per cent of the dust entering it but 70 per cent of only the 9 per cent of the original dust leaving the second pass and entering the third; etc.

View of Set-Up Used in Experimental Study of Electrical Precipitation

From the point of view of the dust concentration *leaving* successive passes, the relations become still simpler, the first pass *transmits*, for example, in the above case 30 per cent of the dust entering it; the second pass 30 per cent of the dust entering it but 30 per cent of 30 per cent of the original dust; the third pass transmits 30 per cent of 30 per cent of 30 per cent of the original (entering) dust, etc.

Gathering these results in a table we have:

Pass	In	Down	Through
1	100.0%	70% of 100.0% = 70.0%	30% of 100.0% = 30.0%
2	30.0%	70% of 30.0% = 21.0%	30% of 30.0% = 9.0%
3	9.0%	70% of 9.0% = 6.3%	30% of 9.0% = 2.7%
4	2.7%	70% of 2.7% = 1.89%	30% of 2.7% = .81%

From the point of view of costs, after the first 70 per cent has been removed, it costs just as much to take out the next 21 per cent as it did the first 70 per cent, and then as much again to take out the next 9 per cent as it did the first 70 per cent, and so on. Of course to most engineers this is a familiar case, since there are numerous other devices which obey the same economic law.

Let us now turn to an examination of Fig. 2. As already stated, this curve exhibits the relation between the precipitation efficiency and the gas volume treated (per unit time). Since the latter is proportional to the velocity of the gas in the treater Fig. 2 also exhibits the relation between the precipitation efficiency and the *velocity* of the gas in the treater.

From this curve it is readily seen that the rating of a Cottrell treater in terms of the gas volume alone is meaningless, since the rating depends on the desired efficiency, which in turn depends on the dust content of the original gas and the desired cleanliness of the treated gas. That is to say, if a customer is satisfied with dirtier gas, the rating of the treater can be increased, and *vice versa*.

A further series of deductions which can be made from the above fundamental formulas, namely eqs. (**7**), (**8**), and (**9**), concerns the effect of the *manner of connection* of Cottrell units on the resulting precipitation efficiency. In practise, of course, for well-known reasons, a Cottrell treater is not made in the form of a single large unit, but of a number of smaller units, and in installing these the question naturally arises whether there is any advantage (as regards precipitation efficiency) in connecting them in series over connecting them in parallel; or whether perhaps a series-parallel connection is better, etc. In particular, for example, if six units are available they can be connected symmetrically in four different ways as shown in Fig. 3 and designated *A*, *B*, *C*, *D*.

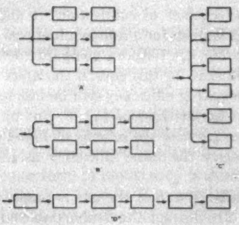

Fig. 3—Four Methods of Connecting Symmetrically Six Cottrell Passes as Regards Gas Flow

In general, if there are available a total number N of passes, such that $N = mn$, where m and n are any two integral factors of N, then it is obvious that the installation can be connected so that it consists of m units in parallel where each unit consists of n passes in series, or n units in parallel, where each unit consists of m passes in series.

In order to answer the question raised above, we have only to note that the fundamental variable which determines the cleaning efficiency is the treatment time t, *i. e.*, the length of time the gas remains in the precipitator. Since this quantity is equal to D/v, where D is the total distance traversed by the gas in the electric field and v is the velocity, which in turn is given by V/A, where V is the total volume treated per minute and A the total effective area of the installation, the cleaning efficiency for any two arrangements is the same provided D/v is the same for the two cases.

For the case then that we have m units in parallel each consisting of n passes in series, we have, provided we let l represent the length of each pass and a its effective area:

$$t = D/v = nl/(V/ma) = N\ (al/V), \tag{14}$$

that is to say, the resulting cleaning efficiency of the installation depends only on the total number of passes (and the total volume treated) and not on their manner of connection. For the alternative case where we have n units in parallel each consisting of m passes in series we have:

$$t = D/v = ml/(V/na) = N\ (al/V), \tag{15}$$

as before.

On the basis of cleaning efficiency alone then the four schemes of connection shown in Fig. 3 would be exactly equivalent.

It will be seen that the schemes of Fig. 3 all represent *symmetrical* schemes of connection; it might be asked what would happen for a non-symmetrical scheme such as shown in Fig. 4. Without going into details, it might be answered that for this case, if the units are left wide open, the resulting efficiency will be less for the case of Fig. 4 than for the case of Fig. 3; but by distributing properly the gas flow the scheme of Fig. 4 can be made to give exactly the same efficiency as the scheme of Fig. 3.

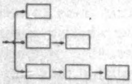

FIG. 4—NON-SYMMETRICAL CONNECTION OF SIX COTTRELL PASSES

In particular, if 1/6 of the gas is passed through the single pass, 2/6 through the double pass and 3/6 through the triple pass (by properly adjusting suitable valves), then the resulting efficiency theoretically will be the same for Fig. 4 as for Fig. 3.

Here again what we have said for Cottrell cleaners ought to hold also for wet washers or bag filter installations, or any other type of cleaner which satisfies our fundamental assumptions.

II. EXPERIMENTAL STUDY OF THE COTTRELL PROCESS

Unfortunately the theory developed above does not give us the precipitation efficiency in terms of all the fundamental variables involved in the Cottrell process; that is to say, we have had to predicate always that certain conditions remained constant; for example, the curve of Fig. 1 would hold for a particular gas and fume in a particular treater, whereas if the nature of the gas, nature of the fume, spacing of discharge electrodes, operating voltage, etc., were changed, our formulas give us no inkling of how the precipitation "constant," that is, the quantity α in eq. (4) would change. Strictly speaking, the theory developed above indicates how the precipitation efficiency varies with the treatment time, *all other conditions being constant.* For this reason, in the experimental portion of this work, it is necessary to follow a slightly different procedure than would correspond to a direct verification of the theoretical formula.

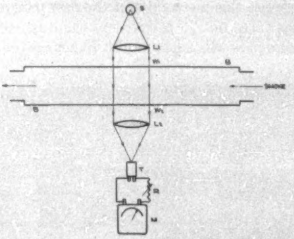

FIG. 5—SCHEMATIC DIAGRAM OF LABORATORY SMOKE CONCENTRATION METER

To begin with we shall have to list those variables which are known by experience to enter in, and determine experimentally how they affect the precipitation efficiency. We shall also verify the validity of our theoretical formulas for the conditions under which they are assumed to hold.

The selection of these variables requires some knowledge of the Cottrell process. We might list: the (secondary) operating voltage, the (secondary) current, the gas velocity, the length of the treater, the size of the discharge electrode, the spacing of the discharge electrodes, spacing of the discharge electrode to collecting electrode, the nature of the fume treated, the nature of the gas carrying the fume (last including temperature and pressure).

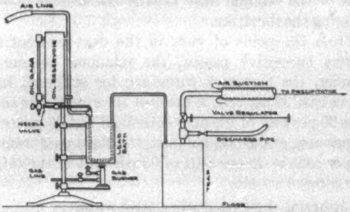

FIG. 6—SCHEMATIC DIAGRAM OF SMOKE GENERATOR USED IN THE EXPERIMENTS

Of course it is not necessarily true that all these variables are independent or fundamental; that is to say, the size of the discharge electrode or the voltage might enter in only as they affect the current flow; and it might be possible to express the whole process in terms of a few fundamental variables such as the space current, the electric field, the velocity, and the mobility of the fume particles in the particular gas.

However, since the effect of the fundamental variables might be difficult of direct verification, the procedure in the present study has been to determine the effect of the secondary variables (voltage, current, etc.) directly on the precipitation efficiency and also to determine the interrelation between the secondary variables; for example, the effect of the voltage on the precipitation efficiency is determined directly, and, also, for example, the relation between the voltage and current is determined independently. With this prelude we can proceed to the experiments.

Development of the Method of Measuring the Concentration of Suspended Material in Smokes, Fumes and Mists

Possibly the reason so little has been done heretofore in the experimental study of the Cottrell process was the lack of a suitable means of measuring quickly and accurately the concentration of the suspended material entering and leaving the experimental precipitators.

Close-Up of Experimental Precipitation and Smoke Recorder

Heretofore, the only method available for this purpose was the rather tedious and inaccurate filter method, in which a measured volume of the gas is drawn off through a filter and the resulting deposit weighed. Theoretically this process sounds simple enough, but in practise it is attended with many difficulties tending to produce inaccuracy, furthermore, the method is very tedious, and finally, the result is only an *average* value over a considerable time interval. Where many experiments have to be carried out, as was the goal in our present study, the filter method is too slow even if it were sufficiently accurate.

We set ourselves the problem of developing a rapid (in fact, instantaneous) and accurate method of determining the *concentration* of suspended material in gases. In the following we shall refer to the resulting instrument as the *smoke concentration meter*, now to be described.

The development of the instrument and a study of the principles underlying it was a considerable task in itself, and since the fundamental principles involved and a form of the instrument have been described elsewhere,[8] in the present work we shall confine ourselves to a brief description of the method and apparatus used.

Briefly the instrument depends on the fact that when a beam of light is passed a fixed distance through a gas containing suspended material, the fractional part of the light transmitted depends on the concentration of the suspended material. In particular it can be shown that if L_o is the intensity of the light *entering* the smoke-laden column of gas, L the intensity *emerging*, x the length of the column, and m the concentration of suspended material, (expressed, of course, in weight of material per unit volume) then we have the relation:

$$\log (L_o/L) = kmx$$

where k is a constant which depends, however, on the nature of the material (only on the grain size in the case of *opaque* material, but also on the wave length of light used and on the optical properties of the suspended material in the case of semi-opaque material such as acid and oil mists).

In the case of electric precipitation studies one is interested usually not so much in the actual concentrations (in grams per cubic meter or grains per cubic foot) as in the *ratio* of concentrations, as appear, for example, in our fundamental formula (2). It is then not necessary to make an absolute calibration of the meters, but more convenient to choose an arbitrary unit, which in our particular case we chose as that concentration for which over a two-inch path, 100 log (L_o/L) is equal to unity. It will be seen that this is a very convenient unit to operate with where this type of meter is used, since in order to get the concentration (in terms of this arbitrary unit), it is necessary only to subtract log L from log L_o and multiply the difference by 100. In fact, in our particular case we kept L_o at a constant value and prepared a chart which gave us the concentration in terms of the observed L directly.

The particular form of meter used is shown diagrammatically in Fig. 5. In this, B represents a long rectangular box, with two glass sides W_1, W_2, through which passes lengthwise the smoke-laden gas to be measured. The box is made long so that the gas (and smoke) will distribute itself uniformly between the windows W_1, W_2. Perpendicularly to the axis of this box and on an optical bench is set up an optical system consisting of a light source S, a lens L_1 which renders the light from the light source parallel, after which it passes as a parallel beam through the window W_1, the smoke-laden gas, and the exit window W_2, whence a second lens L_2 reconcentrates it on a light sensitive element T (a thermopile) which in turn sends a current I (proportional to the amount of light L falling on it) through the milliammeter M.

Careful experimental investigation showed that the current developed by the thermopile T was rigorously proportional to the amount of light L falling on it, so that we can put:

$$\log (I_o/I) = \log (L_o/L) = kmx \qquad (17)$$

The setting corresponding to L_o, namely I_o, is made (with no smoke in the apparatus) by adjusting a series resistance R in the thermopile circuit. As above stated I_o is adjusted to a constant value and to facilitate the work the concentration (in arbitrary units) is read directly from a previously prepared chart.

Two such meters are used; one on the inlet and the other on the outlet side of the precipitator.

The "Smoke" Generator

Prerequisite for experiments of this type is a constant and controllable source of smoke of constant characteristics. The development of this element of our set-up was also no small task. Furthermore, it was desired to produce a smoke which would not deposit on the windows of the smoke meters and thus coat them over, since otherwise the meters would read inaccurately. As might be inferred, the production of the required smoke generator was successful only after considerable experimentation with various substances and design of generators.

Ironically, while it is fairly easy to produce smoke, it is by no means an easy task to produce a *constant* source of smoke of *constant* characteristics, *i. e.*, to produce a smoke both the quantity and quality of which are controlled, and furthermore one which will not coat the windows of the measuring apparatus.

After much experimentation the scheme finally adopted was a modified form of the one originally developed by Tolman[9] and his colleagues in their experimental study of smokes. The "smoke" strictly speaking, mist-generating apparatus is shown in Fig. 6. The material "smokefied" was lubricating oil just as in Tolman's work. The principle of operation is that oil is dropped at a regulated rate into a copper U-tube immersed in a lead bath maintained at a temperature of 480 deg. cent. by means of a Bunsen burner and thus vaporized, while a stream of air is blown simultaneously through the U-tube, this air stream recondenses the oil in the form of a fine mist and serves to carry it out of the apparatus.

The amount of oil and air fed into the apparatus is carefully controlled and duplicated by means of needle valves fitted with graduated dials. The function of the various parts of the smoke generator will be clear from the accompanying figure.

Inserted between the generator and the electric precipitator, however, is a galvanized iron tank which serves the double purpose of an equalizer or mixer and a trap for the larger droplets. This tank is provided with two valves by means of which the amount of smoke fed into the precipitator is regulated, the unused portion being allowed to escape to the outer air.

By duplicating the air pressure (read on a manometer) and the oil feed, as well as the temperature of the bath, smoke of uniform quality and quantity is produced, the exact concentration required being regulated by means of the gate valves on the mixer tank.

Furthermore, our last requirement of a smoke which will not coat over the windows is realized, because in practise the windows take on a transparent film of oil of constant thickness, the excess simply draining off. The absorption by this film, of course, is corrected for by making the I_o setting with the oil film present.

The Experimental Electric Precipitator

The precipitator used in the experiments to be described below is shown diagrammatically in Fig. 7.

It consists of a horizontal iron pipe, through which the smoke-laden gas flows as shown by the arrows, with a discharge electrode in the form of a wire extending along the axis of the pipe and supported by means of insulators (made of Pyrex cane) bent in the form shown. The wire is adjusted to center by sighting through four sets of peep sights and adjusting the insulators. The high tension current is supplied by the lead-in, which enters through a Pyrex bushing (made of Pyrex capillary tubing). The effective treater length is from the center of the inlet pipe to the center of the outlet pipe. If desired, the length of discharge electrode can be fixed exactly by slipping a tube over the wire, thus limiting the distance over which corona forms.

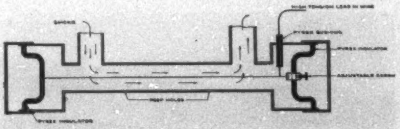

FIG. 7—SCHEMATIC DIAGRAM OF THE ELECTRIC (COTTRELL) PRECIPITATOR USED IN THE EXPERIMENTS

The precipitator is set on a slight incline so that the precipitated oil runs out of the precipitator pipe and into the end bells from which it is removed from time to time.

The High-Tension Generator

It would be desirable to use high-tension direct current for the experiments; however, since this is difficult to generate, we chose pulsating alternating current rectified by means of a kenotron. While in commercial installations mechanical rectification is employed, the mechanical rectifier with its attendant sparking complicates the phenomena by introducing considerable high frequency into the circuit. However, a more serious drawback is that with mechanical rectification a considerable and variable fraction as high as half of the secondary voltage is across the mechanical rectifier, and this fact complicates the voltage measurement. In the case of the kenotron it may be assumed that there is a negligible drop across the kenotron during the transmitting cycle, so that the secondary voltage can be calculated from the applied primary voltage or in most cases the phenomena can be expressed directly in terms of the primary voltage, to which the secondary voltage is simply proportional.

The secondary current was measured by means of a direct current milliammeter. It would have been better, of course, to use double-wave rectification, however, it is not likely that the phenomena would be essentially different, and we chose single wave rectification for the sake of economy.

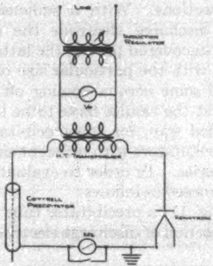

FIG. 8—DIAGRAM OF HIGH-TENSION CIRCUIT EMPLOYED IN THE ELECTRICAL PRECIPITATION EXPERIMENTS

A wiring diagram of the high tension circuit is shown in Fig. 8. The kenotron was rated at 30,000 volts, 100 milliamperes, although only about 1 milliampere was used in the experiments. The voltage applied to the primary of the high-tension transformer was regulated with an induction regulator.

The amount of gas passing through the precipitator was determined by means of an orifice meter, the differential across the orifice being measured by means of a draft gage.

The set-up consisted of (1) the smoke generator, (2) the equalizer trap, (3) the orifice meter, (4) the inlet concentration meter, (5) the precipitator, (6) the outlet concentration meter, (7) an exhauster to draw the air through the system, (8) an outlet to the stack for the waste gas; all in series.

In setting up the apparatus extended use was made of the laboratory supports and supplies furnished by the Gaertner Scientific Corporation of Chicago, whom we wish to thank for the excellency and usefulness of this apparatus. In particular, for example, the concentration meters were made by mounting light source, lenses, etc., on Gaertner optical benches; the boxes of the meters, the precipitator, the orifice meters, piping, etc., were mounted on Gaertner universal laboratory supports, which were found extremely useful and convenient.

III. EXPERIMENTS ON ELECTRIC PRECIPITATION PROPER

The use of an oil mist in these experiments in place of a dry fume has three great advantages which need to be emphasized:

First, a practical difficulty encountered in precipitating dry dust, metallurgical fume, etc., is that after the material reaches the collecting electrode it does not necessarily stay there but may be blown off by the gas stream or thrown off by the electric (coulomb) force exerted on it by the charge on the collecting electrode (it is of course a well-known fact in electrostatics that an element of area of a charged surface is subjected to a repulsive force numerically equal to $4\pi\sigma$, where σ is the surface density of charge). In such cases the precipitation proper is masked by this phenomenon. By the use of a liquid such as oil, it is assured that all the material which reaches the collecting electrode is held fast there, that is to say, it is assured that one is dealing *purely* with electric precipitation.

Secondly, in the precipitation of certain dry (dielectric) materials, as soon as these deposit on the collecting electrode, the latter exhibits corona also due to the local overstressing of the field by the dielectric material. This phenomenon, known in precipitation parlance as "back corona," is obviated in our case by the use of a liquid, in particular by the use of an oil.

THE SMOKE GENERATOR

Thirdly, when dry fume is precipitated, as the process goes on, a layer of material is built up on the collecting and also on the discharge electrode, these layers, even if they do not exhibit back corona, progressively change the electrical conditions. This is obviated also by the use of oil, for the latter drains continuously away and leaves a certain equilibrium thickness of film on both electrodes, which film then remains constant. The high degree to which results could be duplicated in the experiments described below attest to the effectiveness of this step. Also, of course, the use of oil obviates having to clean out the apparatus mechanically, since the accumulated oil drains out by itself.

1. *Relation between the outlet and the inlet concentrator.* According to our theory above, in particular eq. (**2**), the ratio of the outlet to the inlet concentration, other factors being constant, should be constant; that is to say, if the inlet concentration is varied, the outlet concentration should vary in proportion.

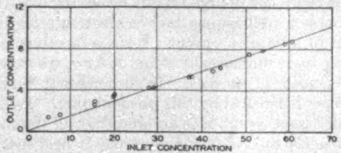

Fig. 9—Relation of Outlet Concentration to Inlet Concentration in the Experimental Precipitator

In Fig. 9 the experimental results obtained in this connection have been plotted. They show that over a considerable range proportionality indeed exists between the outlet and the inlet concentration. This fact greatly simplifies further experimentation, since except in special investigations the subsequent experiments can all be carried out at a single conveniently chosen concentration.

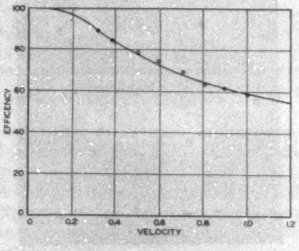

Fig. 10—Experimental Efficiency-Velocity Curve. Compare with the Theoretical Curve of Fig. 2

In particular, also, this experiment establishes the uniqueness of the quantities ξ and $(1 - \xi)$ or η; and justifies the use of the term "precipitation efficiency," since this is now experimentally proved to be independent of the concentration.

2. *Relation between the precipitation efficiency and the treatment time.* Since the treatment time is difficult of direct measurement and in practise is directly proportional to the length of pass and inversely proportional to the velocity, we shall discuss the phenomenon in terms of the latter two quantities, (see below).

3. *Relation between the precipitation efficiency and the length of pass.* While strictly speaking, in order to determine the relation between the precipitation efficiency and the length of pass, the length of the precipitator tube itself would have to be varied, in a *tube* treater the effective length of the treater can be varied also by varying the effective length of the discharge electrode. This can be done with the apparatus of Fig. 7 by slipping a tube over a definite section of the discharge electrode, thus cutting down the corona. For convenience, however, in place of a tube, slotted aluminum cylinders (large size aluminum wire) is slipped over the discharge electrode in sections. With a sufficiently large wire covering the discharge electrode the corona can be limited to the uncovered part of the latter. We found, however, that with the particular size of wire we used, there was still some corona coming off the aluminum sections, so that the results have to be interpreted in a slightly modified way, for the precipitator really consists of two precipitators with different sizes of discharge electrodes in series. In order to evaluate this case it is necessary to proceed as follows:

Consider (Fig. 11) a precipitator tube of total length D, of which a section of discharge electrode of length x is of one diameter, while the remainder of length $(D - x)$ is of another. Let q_o denote the concentration entering the first section; q_1, that leaving the first and entering the second; and q_2 that leaving the second. Then we should have the relations:

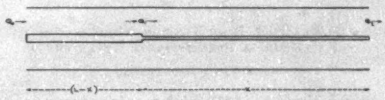

Fig. 11—Method of Limiting Effective Length of Discharge Electrode

$$q_1/q_o = e^{-\alpha_1(D-x)} \quad \text{and} \quad q_2/q_1 = e^{-\alpha_2 x} \tag{18}$$

from which by direct multiplication we obtain:

$$q_2/q_o = e^{-\alpha_1(D-x)-\alpha_2 x} \tag{19}$$

or taking logarithms:

$$\log (q_2/q_o) = -\alpha_1 D - (\alpha_2 - \alpha_1)\, x \tag{20}$$

Since all quantities except x on the right hand side of (**20**) are constant we can put:

$$\log (q_2/q_o) = c_1 - c_2\, x$$

In other words for a precipitator of this type, provided the inlet concentration is held constant, the logarithms of the observed $(1 - \eta)$'s when plotted against the corresponding exposed lengths of discharge electrode should fall on a straight line. The slope of this line furthermore represents logarithm of the ratio of the concentration leaving a section of unit length of the precipitator to that entering it.

The experiment is then to hold the inlet concentration constant and vary the exposed length of discharge electrode by slipping sections of (slotted) larger wire over it. In particular, 4-in. lengths of aluminum wire 0.125 in. in diameter were slipped over a discharge electrode 0.023 in. in diameter.

The experimental results obtained are exhibited in Figs. 12 and 13. In the first of these the logarithm of $(1 - \eta)$ is plotted against the exposed length of discharge electrode. It is seen that except for very short lengths of discharge electrode, the logarithm of $(1 - \eta)$ under these conditions is indeed a linear function of the length of discharge electrode as demanded by eq. (**21**), which it will be noted is a direct consequence of the fundamental law expressed by eq. (**2**). In order to exhibit these results in a way more in accordance with Fig. 1, in Fig. 13 the observed efficiencies have been plotted directly against the exposed length of discharge electrode, the circles representing the experimental results and the full line the theoretical exponential curve which best fits the data. It will be seen that the agreement is very close except for very short lengths of discharge electrode, hence we can conclude that the *precipitation efficiency is an exponential function of the length of pass* as demanded by eqs. (**7**), (**8**) and (**9**).

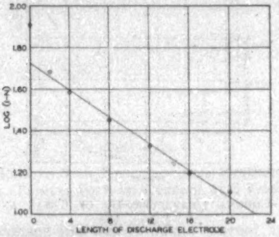

FIG. 12—TEST OF WHETHER THE EXPERIMENTAL EFFICIENCY-LENGTH OF PASS CURVE IS AN EXPONENTIAL

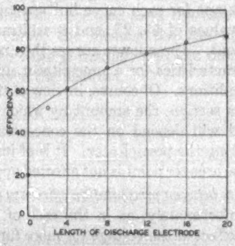

FIG. 13—PRECIPITATION EFFICIENCY AS A FUNCTION OF LENGTH OF PASS (LENGTH OF DISCHARGE ELECTRODE) IN EXPERIMENTAL PRECIPITATOR

Circles denote experimental points; full line the theoretical exponential curve

4. Relation between precipitation efficiency and the velocity of the gas in the treater. Since the exact velocity is not of great interest but merely the law of variation of the efficiency with the velocity, for convenience in the curves in place of the velocity the square roots of the pressure differentials across an orifice plate inserted ahead of the experimental precipitator have been plotted. The square root of this differential is of course proportional to the velocity. According to theory the efficiency-velocity curve should have the same general form as the curve exhibited in Fig. 2. In Fig. 10 the experimental values obtained have been entered as the small circles, while the full drawn curve represents the theoretical exponential curve which best fits the data. It is seen that the agreement here between theory and experiment is very perfect.

5. Relation between precipitation efficiency and the secondary current. It will be noted that our fundamental relation, namely eq. (**2**), involves the quantity i, which was defined as the concentration of the ions. This quantity can be considered under a given set of conditions (ionic velocity) as proportional to the space current in the precipitator. If the latter does not vary from point to point *lengthwise* of the precipitator we may take i proportional to the (total) secondary current, in which case the precipitation efficiency would be also an exponential function of the secondary current. Unfortunately, however, it is not possible directly to vary the secondary current in a precipitator without simultaneously varying the voltage, hence we must be satisfied with the determination of the relation between the precipitation efficiency and the secondary current *with the voltage also varying.* The exponential curve obtained in this connection is exhibited in Fig. 14. It will be seen that it does resemble an exponential (compare Fig. 1), although careful analysis shows that it is not a pure curve of this type.

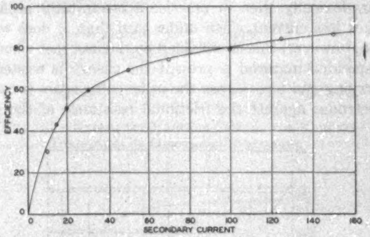

FIG. 14—PRECIPITATION EFFICIENCY AS A FUNCTION OF THE SECONDARY CURRENT (CURRENT VARIED BY CHANGING APPLIED VOLTAGE)

It was noted, however, in taking these points that the secondary current itself, and to some extent the secondary voltage, also, was affected by the *concentration* of the smoke entering the precipitator; it was noted, for example, that the admission of smoke to the precipitator always resulted in a drop in the secondary current, and that this drop increased with the concentration of the smoke admitted. This led to a special study of the secondary *current* (smoke) *concentration* relation now to be described.

6. Relation between the secondary current and the (inlet) concentration of smoke. The phenomenon mentioned just above was studied quantitatively. For this purpose the inlet concentration of the smoke was varied in definite steps and the corresponding secondary currents (primary voltage being constant) were noted.

The experimental results are exhibited in Fig. 15. Here again the experimental points are entered as small circles while the theoretical exponential curve which best fits the data is drawn as the full line. It is seen that the fit is fairly good, but still not perfectly exact, pointing to some disturbing factor.

The explanation of this phenomenon is simple. The admission of smoke results in the formation in place of the very mobile gas ions present in the clear air, of the much less mobile and slower moving charged oil drops with a corresponding decrease in the speed of transfer of charge between electrodes, *i. e.*, reduction in current. Paradoxically then in practise a precipitator actually takes less current when under load than it does when idle; however, this is explained by the fact that when no suspended material is present the energy is wasted in carrying gas ions across the intervening space between electrodes against the frictional resistance of the gas.

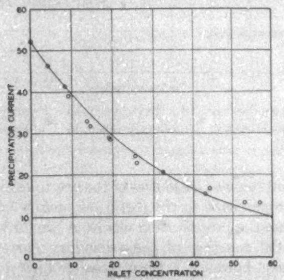

FIG. 15—SECONDARY CURRENT AS A FUNCTION OF THE CONCENTRATION OF THE SMOKE ENTERING THE PRECIPITATOR

Small circles represent experimental points; the full line a theoretical exponential curve

Incidentally, of course, the question also arises whether the secondary voltage is modified by this phenomenon. We tested also this point by connecting a Braun electrostatic voltmeter across the precipitator, and it was noted that there was an *increase* in secondary voltage simultaneously with the *decrease* in the secondary current as might be expected.

7. Relation between secondary current and primary voltage (corona characteristic). The relation between the current and voltage for the case of concentric cylinders for air is of course well known. However, in view of the results of the previous sections it becomes of interest to examine how the corona characteristic of such a system is modified by the presence of a dispersoid. Accordingly, characteristic curves of the particular precipitator were obtained with clean air and smoke-laden air in the precipitator.

The results are exhibited in Fig. 16. In this figure the upper curve 1 represents the corona characteristic for clean air or gas; the other three the same for various definite concentrations of smoke- or mist-laden air. The latter were obtained by keeping the inlet concentration constant for each curve but assigning to it the successive values of 55, 25, and 8 arbitrary units (of concentration). These lower curves then represent the corona characteristics for a precipitator under various working conditions. Of course, in view of the results of the previous section, the amount by which the current is depressed will depend on the concentration of the smoke entering the precipitator. It is of importance to note that the arcover point is not affected appreciably.

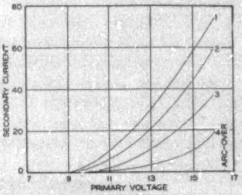

FIG. 16—CORONA CHARACTERISTIC OF CLEAR AND SMOKE-LADEN AIR

Curve 1 for clear air, lower curves for successively increasing concentrations of smoke

8. Relation between precipitation efficiency and primary voltage. Unfortunately, as in the case of the current curves, the effect of the voltage (or more fundamentally the electric field within the precipitator) cannot be determined directly, since a change of the voltage involves a change of the current also as already noted in a previous section. It is of interest nevertheless to determine how the precipitation efficiency varies directly with the applied voltage.

The results of the experiment are exhibited in Fig. 17. It will be noted that the curve also partakes of the general nature of an exponential. It will be seen that no precipitation occurs until the voltage reaches about 9.3 volts. This is explained by the fact that for the particular wire used the corona current does not begin (Fig. 16) until this voltage is reached. The curve then rises almost linearly, but eventually bends over much like the curve of Fig. 1, and ends abruptly at the arcover voltage. This curve is very similar to that obtained by Horne. However, it must be emphasized in connection with this curve that the relation is not a fundamental relation because the current is also allowed to vary (compare section 11).

9. Relation between precipitation efficiency and power input. Since it is not possible to vary current and voltage independently in a given precipitator, it is of interest to study the precipitation efficiency as a function of the power input. For this purpose, while this procedure is not rigorously correct, we formed the product of the primary voltage and the secondary current and took the result as proportional to the power input.

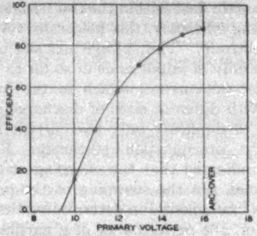

Fig. 17—Precipitation Efficiency as a Function of the Applied Primary Voltage (Current Also Varying)

The experimental results are plotted in Fig. 18. The curve is very similar to the efficiency-current curve. It will be seen that here also as in the case of the efficiency-pass curve, the cleaner the gas becomes the harder it is to clean it further, *i. e.*, the more energy it takes to remove the residual.

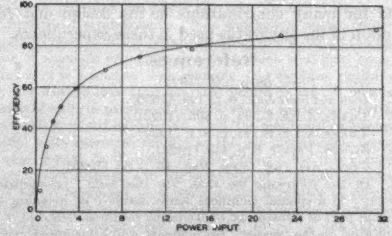

Fig. 18—Precipitation Efficiency as a Function of Power Input into the Precipitator

10. Effect of diameter of discharge electrode on the efficiency-voltage relation. Of great interest in connection with the design of a Cottrell treater is the effect of the diameter of the discharge electrode on the precipitation efficiency. Firstly, of course, it is well-known how the wire diameter affects the corona characteristic, in particular, that with larger wire sizes the same current is obtained at a higher voltage than with a smaller wire. The first effect is then to shift the corona characteristic, that is to change the operating voltage. Secondly, it is also well known that the size of the discharge electrode affects the magnitude of the electric field at any point within the treater tube; in particular for the same applied voltage the field in the vicinity of the collecting electrode and in the greater portion of the treater tube will be increased by using larger wires. Both effects combined tend to increase the electric field within the treater (ignoring, of course, the zone immediately around the discharge electrode). Hence by changing the wire size we have a method of changing the electric field within the treater; in particular of answering the question whether a high field and low space current or the reverse is desirable. In this and the following sections this point is studied in detail.

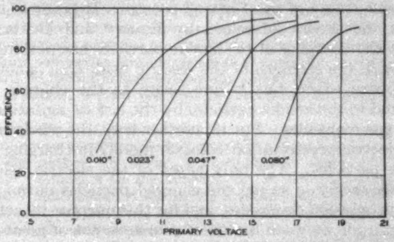

Fig. 19—Efficiency-Voltage Curves for Various Diameters of Discharge Electrode

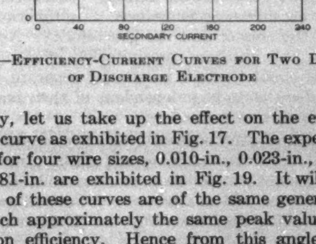

Fig. 20—Efficiency-Current Curves for Two Diameters of Discharge Electrode

Firstly, let us take up the effect on the efficiency-voltage curve as exhibited in Fig. 17. The experimental results for four wire sizes, 0.010-in., 0.023-in., 0.047-in. and 0.081-in. are exhibited in Fig. 19. It will be seen that all of these curves are of the same general shape and reach approximately the same peak value of precipitation efficiency. Hence from this angle *there is apparently little to be gained by changing the wire size, the result is only to shift the phenomenon to a different voltage range.* This indicates that the voltage *insofar as it determines the electric field within the treater volume is of little importance*—a highly interesting result—all the more so when it is considered that the voltage did not enter into our fundamental deductions. It would seem then that the voltage enters in only as it determines the current flow, and we shall next study the phenomenon from this angle.

11. Effect of size of discharge electrode on the efficiency-current relation. The rather remarkable result obtained in the previous section suggests plotting the efficiency for various wire diameters in terms of the (secondary) current flowing in the treater. The experimental results for two wire sizes, namely 0.010-in. and 0.017-in. are plotted in Fig. 20. They show that *the current alone is the determining factor;* that is to say, if the current flow is duplicated (with different sizes of discharge electrode) the efficiency is duplicated—a very remarkable and important result, particularly as regards the theoretical interpretation of the Cottrell process. It tends to support the theory advanced by Hauer[10] and Deutsch.[11] that the charging of the smoke particles has nothing to do with the strength of the electric field, *i. e.,* is not an influence effect, but is due solely to the charges imparted to the smoke particles by the kinetic agitation of the gas molecules. Put in another way, the whole trick of electric precipitation consists merely in charging the dust particles. The only effect of the electric field is subsequently to sweep the charged particles out of the space between electrodes, and for this purpose its actual magnitude between limits, of course, is not of great importance. The result also supports our theoretical deductions, since our fundamental assumption was that the whole process could be treated on the basis of chance encounters between ions and smoke particles, and that this chance of encounter was not influenced by the electric field.

IV. SUMMARY

On the basis that the chance of a dust particle being precipitated depends on its chance of encountering an ion, a formula is deduced giving the precipitation efficiency (percentage removal) as a function of the length of time the gas remains in the treater. As a corollary the precipitation efficiency is deduced as a function of the length of the treater and the gas velocity. As further corollaries the resultant precipitation efficiency of a number of equal Cottrell passes (or separate treating units) is deduced to be independent of their manner of connection as regards gas flow.

An optical method of measuring the concentration of suspended material in smokes and mists, a method of generating smoke of constant and controllable quality and quantity as well as a laboratory Cottrell precipitator for studying experimentally the fundamental laws involved in electrical precipitation are described.

The relation between the concentration leaving a Cottrell precipitator, other conditions being constant, is shown experimentally to be a constant fraction of the concentration entering the precipitator. The precipitation efficiency (percentage removal) as a function of the length of pass, the gas velocity, the electric current, the voltage, and the power input is determined experimentally. It is demonstrated that these curves partake of the nature of exponentials. The current flowing in the precipitator is shown to be a function of the incoming concentration, decreasing with it according to an exponential law. Corona characteristics (current-voltage curves) of a smoke-laden gas are determined and shown to be different from that obtained in clean air or gas. It is shown that the voltage insofar as it determines the electric field within the treater has little effect on the cleaning efficiency, this holds also for the size of discharge electrode. The voltage and size of discharge electrode are only of importance in so far as they determine the electric current, which is the fundamental variable. With different sizes of discharge electrode it is necessary to duplicate only the current in order to duplicate the precipitation efficiency. From these results it is indicated that the charging of the dust or smoke particles, not the sweeping of the particles out of the field is fundamental in determining the efficacy of the process, or, the performance of a particular treater under a given set of conditions.

V. CONCLUSION

While the present theoretical and experimental study is more extensive perhaps than any that has been made before, it is still far from complete; and it is hoped that the experiments described and experimental method developed will stimulate further research in this field.

In conclusion we wish to thank Mr. G. G. Crawford, at whose suggestion this work was undertaken, for his interest and inspiration, and Mr. H. C. Ryding for the generous support he has given us in this work. We also wish to acknowledge our indebtedness to Mr. H. Raymond for many contributions in the design and construction of the apparatus used in these experiments.

References

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2. *Iron and Steel Eng.*, **6**, p. 143, (1929).
3. TRANS. A.I.E.E., **41**, p. 805, (1922).
4. TRANS. A.I.E.E., **41**, p. 811, (1922).
5. *Zt. tech. Physik*, **6**, p. 433, (1925).
6. *Trans. Am. Inst. Chem. Eng.*, **15**, p. 85, (1925).
7. In the following we shall use the term "precipitation efficiency" in a special technical sense, namely as meaning the fractional part or percentage of incoming material removed, *i. e.*, precipitated, in the treater. Both theory and experiment justify the use of this term, since the percentage removed or "precipitation efficiency" is independent of the incoming concentration.
8. See paper read before the American Iron & Steel Institute Meeting of October 24, 1930; also "Blast Furnace & Steel Plant," November 15, 1930.
9. *Jl. Am. Chem. Soc.*, **41**, p. 307, (1919).
10. *Ann. d. Phys.* **61**, p. 303, (1920).
11. *Loc. cit.*[1]

Discussion

J. C. Hale: The collected data covering work done under the auspices of Research Corporation, Western Precipitation Company, Lurgi Apparatebau-Gesellschaft, Siemens-Schuckertwerke and Lodge-Cottrell, Ltd. have been freely interchanged and it is from all these that I am drawing the following information.

As the authors have pointed out, there has not been a great amount of data presented through the engineering societies on the quantitative laws governing precipitation. One reason is that few outside of those in the development and engineering departments of the companies engaged in the application of the processes in the various industries are interested in this phase of the problems of gas cleaning. Of the large volume of data which has been collected and exchanged by the various investigators, usually only that portion has been published which has a direct bearing on some particular application.

Referring to the authors' mathematical analysis, equations (4), (5), (7), (8) and (9) are, as the authors state, various forms of the same expression. Consider equation (4); ϵ is the base of the naperian logarithm and α is a constant. I may, therefore, select a new constant K equal $\epsilon^{-\alpha}$ and rewrite this equation as

$$n = 1 - K^t \text{ or } 1 - n = K^t$$

With this simple substitution, we have the identical equation developed and reported by Anderson and his co-workers and presented in his 1919 report of tests made during that year at an experimental plant at Santa Cruz.

Equivalent formulas have been arrived at and published by European investigators, notably Heinrick and Deutsch, Engineers of the Precipitation Companies in Germany. This equation has been a most valuable contribution to the art of precipitation and it has been very extensively used. It must not, however, be regarded as inexorable. The fact that the "constant" of this equation is not always constant has served first, to stimulate the attention given by engineers in an attempt properly to evaluate it and second, to retard the publication of the results so far obtained until these results are more complete. It might be mentioned that K has been shown to vary considerably with the gas velocity. Maximum and minimum determinations of K under different conditions have shown well established variations of the order of 2 to 1. In the hands of one thoroughly familiar with its limitations, the formula is nevertheless a valuable working tool.

The mathematical analysis leading to equation (15) shows that identical efficiencies will obtain whether precipitator units be connected in series or in parallel or in series parallel with respect to gas flow provided the gas is properly divided in the cases of unsymmetrical arrangements. This conclusion serves, however, as a good example of the pitfalls to which one is led by the application of scientific principles to a field which is at least as much an art as it is a science. For example, in a problem requiring a very low friction loss, equation (15) points to the use of a large number of units in parallel. It can be shown mathematically by a somewhat involved calculation, that the combined efficiency of parallel units decreases rapidly as the gas distribution becomes unequal. Equal distribution of gas through too many parallel units is most difficult and the selection of a proper number of units to be paralleled is, therefore, not a matter of efficiency calculated on the basis of any formula, but a matter of experience with regard to distribution. Similarly, to go to the other extreme, we might be led to a design with a large number of units connected in series, to eliminate the difficulties of equalizing gas distribution. This, however, would involve a high gas velocity and material precipitated in the first of a series of units might be mechanically swept along the surface of the collecting electrode of that unit, picked up by the gas stream and carried to the following units and ultimately to the outlet main. This contingency is not considered in any formula for efficiency, but serves to indicate one of the many dangers to which scientific formulas may lead unless they are made subservient to the many requirements of different conditions met with in practise,—again pointing to precipitation as an art more than a science.

As the authors point out, the constant which appears in the formula applying to smoke concentration meters is dependent upon the grain size of the smoke particles. In spite of this statement, however, efficiency is determined in all of the tests without regard to this factor. That it is not an insignificant factor is indicated in the paper by Tolman and others, referred to by the authors. It is there stated that the light beam strength varies with the concentration of dust between certain limiting formulas in which, for small particles, the particle diameter appears directly as the cube and for large particles inversely as the first power. The Tyndall beam instrument gives, fundamentally, a measurement of visibility which is proportional to concentration only if the size of particles is a constant. A user of a precipitation installation will seldom be satisfied with an efficiency determination based upon visibility, as he requires the removal of the suspended matter from the gas stream and not merely its conversion into larger particles.

This characteristic of the concentration meter used will account for some of the errors which I believe exist in the authors' results as given in sections 8, 9, 10 and 11.

Admittedly, the filter method of measurement of concentrations gives integrated values over a period of time and is a slow, tedious process. It is our experience that it is the most accurate method and the only one acceptable as a true measurement not requiring calibration and independent of variables, such as size of particles, which influence other apparatus. Even this method has, of course, certain practical limitations in its commercial application, especially where there may be very sudden changes in gas velocity.

The smoke generator used is certainly a most ingenious and carefully worked out device. I most heartily agree with the authors' comments on the difficulty of producing a controlled quantity and quality of smoke for experimental use. That the smoke produced may vary as to size of particle is pointed out by Tolman in the paper previously mentioned.

In the absence of conclusive information as to the relative merits of half wave and full wave rectification, of locating the vacuum tube in the grounded side of the circuit or in the high potential side, and of using a mechanical rectifier, I see no reasonable objection to the use of any of these methods. The important point is that the same equipment and the same methods of measurement of electrical values be used throughout.

There are three of the authors' conclusions which lead to a difficulty if they are expressed mathematically and then compared.

Equation (3) may be written $\eta = f(i)$.

The statements in section 6 and the curve Fig. 15 may be expressed $i = f_1(c)$.

By combining these we may write $\eta = f_2(c)$.

The authors' statement in italics at the bottom of page 451 may be expressed mathematically as "η is not any function of c."

The incompatibility of these equations indicates the necessity of some explanation or qualifications of the authors' conclusions upon which they are based.

I do not believe that any general conclusion should be drawn or simple formulas of relationship established between efficiency and current voltage or power input as given by the authors in Figs. 14, 17 and 18. While these are characteristic curves applicable to some problems, there are marked exceptions which are both too numerous and too conclusive to be ignored. Many well confirmed tests on commercial installations prove that under some conditions increasing the current and power does not give any increase in efficiency.

The conclusions based on the data represented by Figs. 19 and 20 are misleading in that two important factors entering into the results exert an entirely different effect on the results obtained by the experimental equipment used and results obtainable by equipment of commercial dimensions. These factors are the limitations of the smoke concentration meter already referred to and the relatively small diameter of the pipe used.

The authors state that "the whole trick of electrical precipitation consists in merely charging the dust particles." This reduces the whole problem to the simple question of selecting an electrode assembly to give the maximum corona discharge, which is certainly a question capable of accurate scientific solution. We know, however, based upon a very wide experience in dealing with a variety of problems, that it is out of the question to draw any such an ideally simple conclusion. We have learned the futility of attempting to disregard the wide variations in other conditions. It is the existence of an almost infinite number of combinations of these variable conditions which so far removes the problem from the range of an exact science as to lead those most intimately connected with it to prefer the designation "art of electrical precipitation."

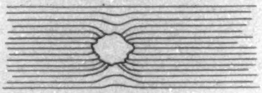

FIG. 1

Electrical precipitation has been under study and development both here and abroad for over twenty years, and from my nineteen years experience I feel that in justice to those scientists and engineers, who have during that time devoted so much research and effort to the art, the part of the authors' conclusion that refers to the possible scope of the work which has been done heretofore should be sincerely questioned.

J. J. Torok: In making some studies on precipitation some time ago I reviewed a treatise on this subject by Ladenburg which was published in the *Annalen der Physik*, 1930, Vol. 4, page 863. Herein Ladenburg presents a very interesting picture of the process of electrostatic precipitation. I believe that it would be worth while to review this at this time for it attacks the problem from a different standpoint than that taken by Messrs. Simon and Kron.

Ladenburg considers the smoke or dust as a minute particle floating in the atmosphere. This particle he assumes to have various diameters. The irregular shapes of such particles can effectively be represented as spherical. In an electrostatic field through which a large number of ions are moving the behavior of this particle will be as follows:

First assume that the particle has a dielectric constant greater than one. Under this condition the electrostatic field around the particle will be distorted. Lines of force now tend to concentrate into the particle as shown in Fig. 1 of this discussion. The ions which are moving in this field will follow very closely the direction of the lines of force of the electrostatic field, consequently they will strike the dust particles where they will be retained. Thus the dust particle acquires a charge. This charge increases until the field around the particle is distorted by the charge so that instead of the lines of force converging on the particle they now swing around it and the other ions in passing through the field will not strike the particle. In other words, the charges acquired by the particle in a particular field will increase until they repel the oncoming charges and thus it will come to an equilibrium. It can readily be seen from Fig. 2 that the larger the particle the greater the distortion it will cause in the field and consequently the larger charge it will acquire. The forces acting upon the particle, however, are proportional to the charges it acquires. Consequently, the larger particles which have acquired the greater charge will be removed from the field first. The smaller particles which have acquired less charge will require more time to be forced across the field and as a result their time of precipitation is somewhat longer.

If we assume that the dust particle has a dielectric constant equal to air there is no concentration of the lines of force into the particle, thus only the ions in whose path the dust particle lies will be entrapped upon the dust particle. This perhaps will require a little longer period of time for the particles to acquire a charge which will distort the field so that no further ions will be entrapped upon it. It is to be remembered also that under this condition the charges entrapped on the particle are less than if the particle had a higher dielectric constant.

As the size of the particle diminishes, the charge acquired by it becomes so small that the time of precipitation becomes very long. However, these smaller particles are precipitated in a period of time much less than required from a theoretical standpoint. This discrepancy is accounted for by the electric wind effect. The turbulent motion set up within the precipitator by the corona causes the air to move rapidly from one electrode to another. In this turbulent action the small particles are from time to time brought very close to the electrode and with the small motion it can obtain through its electrostatic forces it will be pulled to the plate and be deposited there.

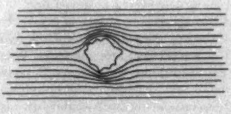

FIG. 2

From this analysis we find that in precipitators the larger particles are removed first and as time progresses the smaller particles will be taken out.

Commutation Considered as a Switching Phenomenon

BY R. E. HELLMUND* and L. R. LUDWIG†
Fellow, A.I.E.E. Associate, A.I.E.E.

THE usual commutation theories deal with the reversal of the current in the commutating coil and with means for bringing about such reversal either along a straight line or a curve. The principal object in such studies is to accomplish commutation without excessive current densities under the brush. It is recognized that frequently it is impossible to realize this, but little attention has been paid to the fact that unless such ideal conditions are obtained we actually have to deal with a switching phenomenon and that the problem should be analyzed with this in mind. The necessity for introducing the concept of switching is important inasmuch as ideal conditions can be obtained in but very few practical cases. This paper gives a brief analysis of the various considerations entering into the switching phenomena at both the leading and the trailing edges of the brush and indicates how commutator and brush burning is affected by aiding or hindering the switching operations.

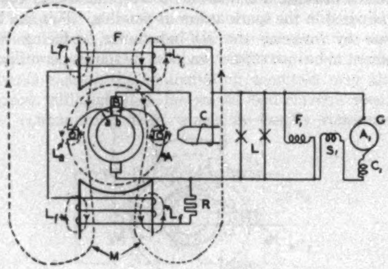

FIG. 1

Let us assume that the circuit of armature coil 1 in Fig. 1 is about to be short-circuited by brush B and that at this moment a voltage e is induced in the coil by its rotation in the commutating field, or possibly by the stator field of an a-c. machine. It is evident that sparking in the minute point or edge of the brush B which first touches the segment a will be largely influenced by the speed with which the circulating current in the coil can build up as compared with the rate of increase of the area common to the brush and the segment a. This in turn means that the speed of the commutator will have an influence as well as the rate of increase of the circulating current. The latter is determined by the voltage e and the self-inductance and resistance of the circuit of coil 1, if the coil carries no flux common to other closed circuits. This condition would exist if brush B does not short-circuit any other armature coil and if the field winding F is open-circuited, and under the further assumption that there are no secondary damping currents in the stator structure. Under these conditions, the entire main flux F and the leakage fluxes L_a cause a rather high self-inductance in the armature coil 1, and consequently a slow building up of the circulating current. Fig. 2 represents a circuit equivalent to the armature coil 1 with the assumptions made so far.

The conditions change materially if the shunt winding F is connected across the armature circuit as shown in Fig. 1, as well as across certain load circuits L and various circuits of the generator G. We now have within the armature coil a leakage reactance flux L_a and a mutual inductance M between the armature coil and the field circuit. The latter in turn has a leakage flux L_f, and it is closed partly through the armature circuit having resistance and inductance, and partly through several other circuits also having a resistance or inductance, or both; in other words, we have equivalent circuits somewhat as illustrated in Fig. 3. The rate of change in the circulating current will now depend upon all these factors, and therefore any change of constants in the field circuit or any of the other circuits may have a certain influence upon the commutating conditions. Similarly, any damping currents which can be induced in the solid portion of the stator yoke, or in coil shield or damper windings surrounding the poles, may have an influence upon commutation. The various resistances and reactances shown in the secondary circuit of Fig. 3 are replaced by a single inductance X_2 and a single resistance R_2 in Fig. 4, which while not fully equivalent to Fig. 1 somewhat simplifies the problem.

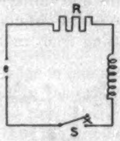

FIG. 2

All the various constants previously mentioned will also have an influence when the circuit of the coil 1 is

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opened while a circulating current is still flowing in it. There will be one difference, however, in that the high leakage and self-inductances, which are helpful in the case of closing the circuit, will be harmful in the opening because they considerably increase the energy to be dissipated in the spark which forms at the trailing edge of the brush. Mutual inductance, which aids in opening the circuit as will be explained, is disadvantageous when it is closed because it allows a rapid building up of current. The effect of inductance on the opening operation may be made clear by following the switching operation in detail first for a machine with open stator circuits, wound with one coil per slot, and a brush spanning one commutator bar, so that no account need be taken of mutual inductance between a commutating coil and any other closed winding. A circulating current is assumed to be flowing when coil 1 passes from under the brush.

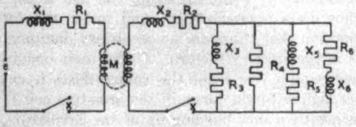

FIG. 3

When a circuit containing resistance and self-inductance such as shown in Fig. 2 is opened by the separation of contacts (the brush and the bar in this case), an arc or spark will be formed between the electrodes, because at the last point of contact the temperature may become very high and "explosions" take place at this point, giving rise to sufficient energy density to establish the cathode spot of an arc on the negative electrode.[1] As the contacts separate further the arc becomes longer and finally becomes unstable and vanishes. Any calculation of the decrease to zero of the circulating current due to commutator bars passing from under the brush must consider this short arc or spark formed. The arc characteristic, *i. e.*, the arc voltage as a function of the current through it, depends on the arc length. The latter in turn is varying and depends on the way in which the arc is lengthened as a function of time, or, in other words, on the speed of the commutator.

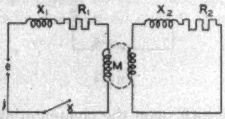

FIG. 4

If the transient arc characteristic is accurately known for a particular condition, the energy dissipated in the arc can be calculated.[2] This energy may be divided into two parts; the energy stored electromagnetically in the circuit, and the energy supplied to the circuit during the time duration of the arc. In the case of the commutating coil, the latter energy may be supplied by the voltage induced in the coil by its rotation in the commutating field, or in an a-c. machine it can also be induced by transformer action between the stator field and the coil. Usually the total arc energy will lie between the values of once and twice the energy stored in the magnetic circuit. This stored energy varies directly with the self-inductance coefficient L and the square of the current to be broken, and hence the arc energy also depends upon these factors. The length of time during which the arc or spark is present also increases with an increased coefficient of self-inductance. Since a trailing edge spark is harmful in so far as it burns the commutator and brushes, it is desirable to keep the energy to be dissipated in the spark as low as possible. This can be done by lowering the self-inductance, reducing the current to be interrupted, and also by the proper utilization of mutual inductance, which will be discussed next.

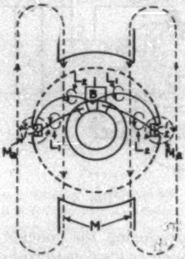

FIG. 5

As previously indicated, circuits having mutual inductance with a commutating coil are established by the field or stator circuit being closed through the armature and other circuits, or by damping currents in the stator structure. Other circuits having mutual inductance with the commutating coils usually exist if the winding is chorded, if there is more than one coil per slot, or if a wide brush is used; or, in fact, whenever the sides of at least two coils which are being commutated during a common time interval lie in the same slot. In this case the mutual inductance between coils must be considered. For example, if the brush is wide as shown in Fig. 5 and short-circuits the two armature coils 1 and 2, coil 1, which is about to be open-circuited, still has leakage fluxes L_1, but it also has mutual fluxes M with the field winding as previously described, and mutual fluxes M_a with the armature coil 2, the latter in turn having leakage fluxes L_2 of its own. This may better be

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[1] Slepian, *Temperature of a Contact*, JOUR. of A.I.E.E., 1926, p. 930.

[2] Rudenberg, "Elektrische Schaltvorgange," p. 188. (Julius Springer, 1923.)

shown by the simplified although not fully equivalent circuit indicated in Fig. 6. If such a circuit containing mutual inductance in addition to self-inductance and resistance is opened with a switch, the decrease of the current in the primary circuit may again be calculated if the transient arc characteristic curve is known. It is found that in this case all of the stored energy need not be dissipated in the switch. The decrease of current in the primary circuit will induce voltages in the secondary circuits such as to cause their currents to change. This means that part of the energy stored in the primary circuit may be transferred to the secondary circuits and dissipated as I^2R loss in them or held in storage in the secondary circuits. Consequently, the energy dissipated in the spark may be, and usually is, reduced by introducing mutual inductions. The magnitude of the reduction depends on the degree of mutual coupling between the circuits, and upon the circuit constants, such as resistance and self-inductance, of all the circuits having mutual inductance with the commutating coil. It follows, therefore, that as conditions in the stator circuits change, the commutating condition may also change, although such factors as load current, speed, and commutating fields, which are usually considered as the only ones determining the commutating conditions, remain unchanged. For similar reasons, different conditions of mutual inductance between the various armature coils will have an appreciable influence. The conditions of the mutual flux M_o and leakage fluxes L_1 and L_2 in Fig. 6 will of course be appreciably different when the coils are located in separate slots and when the coils are located in the same slot. This difference leads to the frequently observed condition that certain commutator segments show more burning than others. These considerations also explain why frequently a split-throw of the armature coils improves commutation as a result of its effect upon the relation between leakage and mutual inductances of the coils. Without the split-throw a good transfer of energy from coil to coil in the same slot may take place, but the last coil in the slot is not well coupled to the first coil in the next slot and hence considerable energy must be dissipated when the circuit of the last coil in a slot is broken. The split-throw reduces this maximum dissipation of energy in the spark because at least one coil side of the coil under commutation is always closely coupled with another coil side, and, assuming that the brush is wide enough to short-circuit the adjacent coil, better transfer of energy can take place. The beneficial effect thus obtained consists not only in decreasing the maximum sparking but also in bringing about more uniform sparking on the different commutator bars, which in turn reduces irregular wear and differences in the heating of the bars; irregular heating of the bars if occurring to any marked extent is likely to cause differences in expansion of the bars and therefore mechanical irregularities in the commutator surface, with consequent further accumulative harmful effects on commutation.

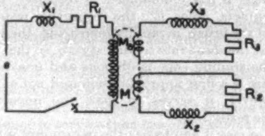

Fig. 6

In all of the previous discussions on the constants of the circuits which may influence commutation, only resistance and reactance have been mentioned. However, with the high frequencies of commutating currents, it is quite possible that at times certain capacity effects in the various circuits may exert an influence upon commutation conditions.

Having established the new concept of switching and the fact that the various machine and external circuits may have an influence upon commutation, we have found a means for explaining many other practical experiences which cannot be explained by the conventional commutation theorites. It can be readily seen, for instance, that there may be a difference in commutation between shunt, series, or separately excited machines, between laminated or solid field structures, etc., although the speed, current, and commutating field are the same. It follows that in the case of a-c. commutator machines the commutation may be affected by the size and type of transformer to which the machine is connected and by various other factors. Interesting results may be obtained with two series-connected motors on a motor car, especially in a-c. motors having no damping effects in their field structure. If in such a case the commutating impulses transformed into the fields of the two motors are exactly in opposition, no damping current can flow in the fields. Such opposition of voltages will occur if the relative position of the two commutators is such that the brush edge in one motor lines up with the mica insulation while in the other motor it happens to be in the center of the bar. If the relative commutator position changes so that the brushes in both motors will have the same position with regard to the mica insulation, the commutating impulses reflected in the fields will be in phase and damping current current can flow, thus causing conditions entirely different from those previously mentioned. With two such motors on a motor car having slightly different speeds caused by a difference in the wheel diameters, periodic variations of commutation will take place. With the two motors connected in parallel across the same transformer, similar variations may take place. Even a change in design from series to parallel connected fields in the motor may influence commutation. With such machines as phase-advancers or regulating machines connected into the secondary of induction motors, the switching theory readily explains differences in commutation which have been observed by the authors for different connections of such machines to various external circuits.

The introduction of the switching concept into the commutating problem is, of course, not necessary when ideal commutating conditions exist. There are also cases which approach the ideal sufficiently close to make the consideration of the switching concept unnecessary in practise. Again, where the commutating conditions are not ideal but where the brush short-circuits several armature coils, their mutual effect may be so predominating that the characteristics of the field and external circuits may have little influence. However, in general the commutating phenomenon cannot be considered as completely analyzed unless the possibility of the switching phenomenon as influenced by the various circuits has been given due consideration.

Discussion

Ernst Weber: I cannot quite agree with the authors in that the commutation problem has been given little attention as a switching phenomenon. On the contrary, the complete fundamental differential equations of commutation in a general case were established as early as 1908 especially by R. Rüdenberg (*E.T.Z.* 1908, page 65) following previous discussions on the problem which go back to 1905 when G. Mie and E. Arnold derived a mathematical condition for good commutation which since then often has been the center of much discussion. The complete differential equations take into account all the inductive couplings of the armature coils at the point to be short-circuited, and under short-circuit conditions. In addition the inducing effects of the external fields as produced by the main poles and by the commutating poles are taken care of. The one reason why no further progress has been made in this mathematical investigation is that it is not possible in general to integrate the underlying systems of differential equations.

An excellent brief summary of the theory of commutation may be found in German Manual *Starkstromtechnik*, Vol. I, U. 494 (published by W. Ernst u. Sohn, Berlin 1930) treated by C. Trettin. The system of differential equations, when the brush covers for example four segments and thus short-circuits three armature coils, is given by:

$$\begin{aligned} i_1 r_1 - i_2 r_2 - L \frac{du}{dt} - M_{12} \frac{dv}{dt} - M_{13} \frac{dw}{dt} &= e' \\ i_2 r_2 - i_3 r_3 - M_{12} \frac{du}{dt} - L \frac{dv}{dt} - M_{23} \frac{dw}{dt} &= e'' \qquad (1) \\ i_3 r_3 - i_1 r_1 - M_{13} \frac{du}{dt} - M_{23} \frac{dv}{dt} - L \frac{dw}{dt} &= e''', \end{aligned}$$

where i_ν denotes the currents in the segments; u, v, w the respective currents in the armature coils under short-circuit condition, r_ν the resistances of the closed coil-circuits, L the self, and $M_{\nu u}$ the mutual inductances of the coils, and e', e'', e'' the electromotive forces induced by the fields in the commutation zone. If all the parameters involved were constants it would be easy to solve for the currents and to deduce theoretically their shape under any condition of external uncompensated fields. However, all the parameters are variable, such as the inductances due to saturation, eddy currents and the changing positions of the coils relative to the field system; the resistances due to the varying contact area of brush and segment, the dependence of brush resistance on temperature, contact pressure, and current density. The equivalent circuits given in Figs. 3, 4 and 6 of the paper appear, from this recognized point of view, as very rough approximations. The variation of resistance for example extends over a range from several thousands of ohms to infinity.

In view of this extreme complexity we may finally state that commutation is one of the most intricate problems of scientific engineering. To arrive at some result, it was but natural, that either restrictions in great number were imposed on the illustrative system (1) in order to fit it to the mathematical means available for a solution, or estimations were made as to the influence of the various terms when straight line commutation was set as the aim. Several of the conclusions arrived at by authors from energy relations can thus be found in the literature. In a recent book published by J. Springer, Berlin, 1929, L. Dreyfus treats the whole problem of commutation on a mathematical, rigorous, but not very convenient way. The Siemens Schuckert-Werke, (Mr. C. Trettin), Berlin, as well as L. Dreyfus, hold several patents to use damper windings in slots or as wedges in order to influence the inductance at the instants of short-circuiting and breaking the short circuit of the coils, by these means causing the electromagnetic energy to be dissipated in heat in several parallel circuits.

The variations of the inductances with time are very small contrasted with those of the resistances. Assuming that over small time intervals both parameters may be taken as constants, the transient will be of the type of an exponential curve, the time constant of which is given by $T = L'/R'$, if L' is the effective inductance for the considered interval of time. At the first instant, R' is very large, the time constant very small, the rate of change of the current large and therefore a tendency to spark. However, since the time constant increases rapidly to a finite value, the danger is not pronounced. Conversely for the breaking of the contacts, since R'', the respective resistance at the trailing edge of the brush, is rapidly increasing, the time constant will decrease at the same rate to almost zero, so that the tendency to spark at the trailing edge is obvious and unavoidable. This physical picture is a first approximation and not at all the single criterion; however, it serves to show the importance of the variation of the resistance.

In conclusion may I emphasize the paramount influence of the brush material. Practise shows again and again that cases of bad commutation, considered as hopeless by theoreticians, are satisfactorily solved by engineers who patiently try various kinds of brushes until they succeed in finding the "adequate brush" for that special case. Investigations are under way to throw some light on the action of contact resistance and its dependence on the brush material, but we are still lacking complete data on this subject.

Thomas T. Hambleton: The consideration of commutation from the standpoint of switching may not be entirely new as most designers of commutating machines must recognize commutation as a high-speed switching of armature coils. The consideration of the possible influence of external circuits is entirely new to me. Of course it is desirable to view any complex problem from as many angles as possible, and it is in this respect that the paper should prove to be of value.

I fail to see where the paper presents any new knowledge that will enable designers to produce better commutating d-c. machines. I would therefore ask the following questions:

1. Has the consideration of commutation as a switching operation resulted in a method of more accurately pre-determining the commutation performance of d-c. machines?
2. Has it indicated any change in design proportions which has produced better commutation?
3. Have any specific cases been observed where commutation in d-c. machines has proved to be affected by external circuits; field circuit, damping circuits or external load circuits?
4. Is it not a fact that the considerations set forth in this paper apply almost entirely to a-c. commutating machines?
5. Has it been possible to compensate in the design of machine for the effect of external circuits on commutation?

G. W. Penney: The usual theory of commutation is based on the assumption that the brush contact drop is the same for all bars and that every other part can be built exactly as designed, so that the current reversal can be a straight line function or some other function which would be uniform for every coil. We are all familiar with the fact that the theory developed from these simple assumptions does not explain many of the troubles encountered in practise. For example, in the case of a large 250-volt generator running at constant speed and load there is commonly one coil per slot, so that all coils are theoretically alike and therefore for any given load and speed there should be a value of interpole flux which would exactly reverse the coil current in the period when a bar is under the brush, thereby giving black commutation. And yet we know that there are many cases where even for steady load conditions there is no value of commutating pole flux which will give black commutation. If the sparking is observed with a stroboscope it will often be found that a given value of interpole flux will give sparkless commutation on certain bars, but sparking at other bars. If the interpole flux is changed slightly, the sparking will disappear at the given bars, but sparks will appear at other bars, so that when observed with the unaided eye, the sparking will appear to be constant for a considerable range of commutating pole flux.

When we consider that a relative movement of one-ten-thousandth of an inch between one bar and the adjacent bars may seriously disturb the uniformity of contact drop, or that a slight deposit of carbon on one bar may have an equally serious result, it seems unreasonable to expect to maintain a uniform contact drop at all times.

The method of analysis described in this paper when combined with the characteristics of the brush under quickly changing current density in a portion of the brush face gives a means of analyzing the performance of the coil during commutation when ideal conditions do not exist.

The ability of a machine to commutate satisfactorily when slight disturbances occur either in the contact, at the commutator or in the rest of the circuit, should be a much more valuable index of its performance in service than the conventional analysis based on perfect conditions.

C. G. Veinott: The general principles, given in the paper, of course, apply equally well to lap or wave windings but the statement that with "a brush spanning one commutator bar . . . no account need be taken of mutual inductance" does not. This is true with a lap winding but is not true for a two-circuit wave winding of four poles or more because the number of commutator bars per pole is not an integral number. The discussion to follow does not refute any of the facts presented in this paper but only supplements the latter with information on wave windings not covered in the paper.

Some time ago I studied the commutating conditions in a wave winding and obtained, by graphical integration, the curve of the reversal of current in the commutating coil against time. I came to the conclusion that, in a wave winding of 4 or more poles, when only two brush arms are used, the commutating conditions are different under the two brush arms. This is due to differences in the coupling conditions of the coils undergoing commutation. As the authors have brought out that differences in the coupling conditions affect commutation, I only need to show how the coupling conditions under the two brush arms differ.

To be specific, consider a 4-pole wave-wound, retrogressive armature having 33 slots and 33 commutator bars with the armature winding chorded ¼ of a slot. Assume two brushes spaced 90 deg. apart, each brush spanning ¾ bars. Calling the brushes positive and negative solely for the purposes of identification and not to indicate actual d-c. polarity, consider the period when the positive brush just starts to span bars No. 8 and No. 9. This condition is represented in Fig. 1. Let us ignore, in this analysis, the mutual inductive effects of the field circuit and only consider other armature coils since effects due to each other circuits would only be superimposed upon the effects we are considering. (In Fig. 1 the coils represent single coil sides, the numbers the slot in which the coil side lies, and the letter *b* or *t*, whether the coil side is in the bottom or in the top of the slot.) We observe this significant fact: the coils short-circuited by the positive brush *begin* commutation without being coupled with any other coils undergoing commutation. At this point the negative brush is merely riding upon bar No. 33.

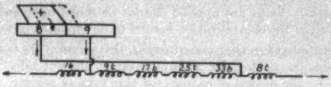

FIG. 1

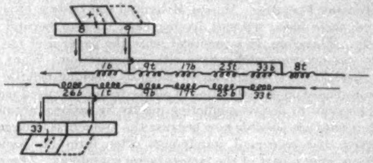

FIG. 2

In Fig. 2, we see the positive brush finishing its commutation. Observe that during the last ¾ths, including the *finish* of the period of commutation, three of the four coil sides short-circuited by the positive brush are mutually coupled with three of the four coil sides short-circuited by the negative brush at that time. It can be seen further that the coils short-circuited by the negative brush *start* commutation with three coil sides coupled.

In Fig. 3 we see the coils short-circuited by the negative brush *finishing* commutation with no coupling. At this time the positive brush is riding upon bar No. 9.

This brings out very clearly the fact that the commutation may be quite different under one brush than under the other.

If the brushes span one bar, somewhat the same conditions obtain except that the coils under the positive brush start commutation loosely coupled (one coil side coupled) and finish with close coupling (3 coil sides coupled). The coils under the negative brush start commutation closely coupled and finish commutation loosely coupled.

Considering commutation as a switching phenomenon, it follows that, when the coils under the positive brush finish commutation, if an arc is formed, all of the stored energy need not be dissipated in the arc to burn the commutator since much of this energy can be transferred through mutual inductive effects. When the coils under the negative brush finish commutation, if an arc is formed, they can transfer but little, if any, energy to other coils undergoing the process of commutation. Of course, there is always the possibility of transferring some of this energy to the armature working circuit, but this transfer of energy must be relatively small.

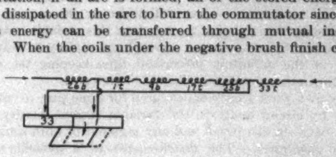

FIG. 3

F. W. McCloskey: It has become increasingly difficult of late years to predict the commutation characteristics of d-c. machines in which the loads applied are of a rapidly fluctuating nature.

Undoubtedly this is due in part, at least, to the fact which is brought out in this paper, that the customary method of figuring commutation does not embrace all the factors involved.

Street-car motors are subjected to this type of load, particularly in notching up the controller. Observations of such motors have shown that with peak currents of, say, twice the one-hour rating obtained in notching, the sparking may be in the neighborhood of twice as severe as when the same load is applied under steady conditions. It is therefore evident that stand tests in which steady loads are applied cannot be relied upon entirely as a criterion of the commutation results to be expected in service.

As pointed out in the present paper, this is one of the conditions which the usual theories of commutation do not take into account. In the design of motors for this type of service, particularly in these days of rapid acceleration of street cars, careful consideration must be given to this angle of commutation.

Thomas Linville: Messrs. Hellmund and Ludwig have presented their ideas without having carried them beyond pure theory. Therefore, in a practical sense the paper does not help solve the designers' difficulties with commutation. What must be hoped for is a commutation analysis reaching definite conclusions based on consideration of all the factors involved making it impossible to derive conflicting results by rational argument. Such results are possible now because the problem is so large and complex that restricted standpoints often form the basis for analyses, only a part of the factors being considered. The result is that many different opinions are held making confusion and disagreement.

Rising from these observations to the plane of theoretical thought in order to discuss the paper on its own level it seems that commutation has always been considered (in most of our minds and broadly speaking) as a switching phenomenon. The object has been to accomplish the switching; *i. e.*, to make and break the contact between the brush and each segment, with no current flowing. It makes little difference whether we start, knowing that the current will flow and appear as sparks if certain conditions exist, or with the premise that the current is already flowing. The objective in each case, is to prevent sparking and the analysis is likely to be the same. The possible difference is that we cannot definitely treat a sparking case mathematically as such.

In the commutation of a machine, the brush in making and breaking the contact with each segment increases and decreases the contact area at a definite rate. If in decreasing the area, for instance, the circuit conditions tend to diminish the current at a slower rate, the current density in the contact increases. This is equivalent to saying that the job of "forcing" the current change is thrown on the brush. Carbon brushes have a fairly definite limit to their ability in this connection. As the area diminishes the circuit resistance increases diminishing the current faster. If too much is demanded of the brush (if this variable component of the circuit resistance must increase too rapidly to reduce the current in the definitely prescribed time keeping the density within the limits of stability) then the brush arcs.

In my own work I have endeavored for some time to calculate, entirely by circuit analysis, the variable current density in the contact between the brush and any segment of both simple and complex machines. The development of a general method giving definite and practical conclusions now seems possible.

It is based, of course, upon the assumption of stable brush characteristics. Hence, sparking is not considered. Interesting results are promised since, for poorly commutating machines, the solution shows high current density occurring in some cases on the leading edge of the brush, more often on the trailing edge, and sometimes on both. These high current densities indicate that sparking is likely,—that it actually will occur if the density is sufficiently high. When it is determined at what density (for average conditions) the contact drop characteristic of a brush becomes unstable, or glowing and arcing begin, I expect we shall be able to say, by the calculation, when a machine will spark and when it will not. Furthermore since the circuit conditions depend upon the design dimensions it looks possible to answer by calculation many of the questions now perplexing the designers.

In one sense it does not seem that commutation should be called a "switching phenomenon" or by any name suggestive of a restricted point of view. Rather, it seems preferable to consider it always broadly as a circuit problem in which boundary conditions must be met in a fixed interval of time. Truly in the contents of the paper Messrs. Hellmund and Ludwig have considered it thus.

One point should be called to attention. In practical machines where several coils are commutating simultaneously the mutual inductances between the armature coils and the main pole windings, when taken together, have the effect of canceling one another with the result that no current is induced in the main pole windings. Hence, the external load circuits have little effect upon commutation except in particular cases. The fact that they do is a true idea but, possibly, overemphasized. The most important disturbers of good commutation are still those fluxes locally in and about the commutating slots; fluxes established by each armature coil which are not mutual with any other circuit. As suggested in the paper reducing these to improve the contact breaking conditions makes the contact making conditions worse but the balance is now, and surely always will be, in favor of the former.

R. M. Baker: The authors have exposed in this paper the very interesting observed fact that the commutation of a machine is indirectly affected by the nature of the connected load. As described in the paper, the effect is due to the change in the possible damping of the main pole windings. The results of some measurements of armature coil inductance taken with and without the field coils short-circuited; and also the maximum possible inductance of the armature coil with no damping follow:

Inductance with field coils short-circuited separately 7.28×10^{-6} henrys.

Inductance with field coils open 10.6×10^{-6} henrys.

Maximum possible inductance (calculated) 31.0×10^{-6} henrys. The machine on which these measurements were made was a 20-hp. d-c. shunt generator with frame made up from heavy laminations (0.05 inches thick), and the measurements were made at 700 cycles.

From the above data, it can be seen that for this particular machine, a change from perfect field damping to no field damping can increase the inductance of an armature coil by about 30 per cent. In a machine having a very perfectly laminated field structure, the increase would be greater, whereas if a machine has a solid yoke, the increase would surely be less. We should expect, therefore, to find the phenomena described by the authors principally in those machines having a perfectly laminated magnetic circuit.

R. E. Hellmund: Referring to the discussion by Mr. Weber, I cannot agree that the subject matter of our paper is anticipated by the work which he mentioned and the mathematical treatment given by him. It is true that the commutating phenomenon has been referred to as a switching phenomenon for a great many years. It is also true that the effect of mutual inductance between various short-circuited coils has been taken into account in a great many previous theories, including the one referred to by Mr. Weber. However, as far as I am aware, all of this previous work deals exclusively with the reversal of the current in the armature coil while it passes under the brush. All of such work is referred to in our paper as the conventional commutation theory. In contrast to these conventional theories dealing with phenomena under the brush, our paper is intended to deal only with the phenomenon at the two edges of the brush, and, more particularly, with the arcing and its effect at the trailing edge under commutating conditions which result in arcing. The theories referred to by Mr. Weber relate to our paper only in so far as they have an influence upon conditions which may be found to exist at the edges of the brush as a result of what happens during the passage of the coil under the brush.

Referring now to both Mr. Weber's and Mr. Hambleton's discussion, I can only again emphasize what is stated at the end of the paper; namely, that the material given in the paper is of practical importance only in those cases in which ideal conditions by means of the proper commutating fields can not be obtained. Since this is more frequently the case under the various operating conditions of a-c. machines than of d-c. machines, it follows directly that the considerations are of greater importance in connection with a-c. problems.

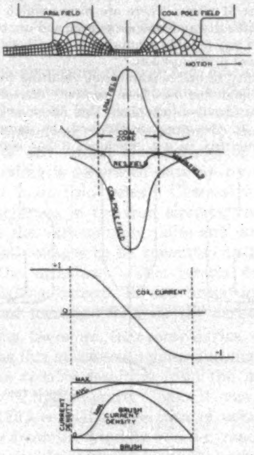

FIG. 4

Some discussion relating to Mr. Hambleton's first and second questions will be given later in connection with some figures. With regard to his third question, some cases have been observed where the commutation of a d-c. machine under actual operating conditions was different from that obtained with the test method used at times, in which the armature is short-circuited and the field excited through a high resistance just sufficient to give the desired current in the armature. In this case the damping currents in the field are of course lower than under normal operating conditions, and this may partly or entirely account for the difference noted in commutation. On the other hand, this case is further complicated by the fact that the commutating field is somewhat different under the two conditions of operation. The considerations given in the paper may also be of importance in certain machines having a third-brush or other special d-c. arrangements. In these and other cases the considerations in the paper may indicate the desirability of using the split throw in the armature, closely interlinked damper windings, etc. Aside from the fact that in most normal conditions of d-c. commutation the commutating fields can be adjusted reasonably well, there are other factors favoring the usual d-c. machines. In this connection the tests described by Mr. Baker in his discussion, which were made after this paper was written, are most interesting and somewhat surprising. They show that even in a machine with heavy laminations the damping effects of the stator may be appreciable, which in turn indicates that in machines having solid frame portions the normal damping effects may have a very favorable and predominating influence. Mr. Hambleton's fourth and fifth questions are answered by the previous remarks, as far as d-c. machines are concerned.

Referring now to a-c. machines, various examples can be cited in which the considerations of the paper have been of utmost practical importance and where a definite influence of the machine and external circuits have been noted. In several instances the change to a split throw has made sufficient difference to change practical failure to successful operation. This was especially important when for reasons other than the motor design proper the motors were built with more flux per pole than is desirable from a motor design point of view. Another practical example is that of the periodic sparking described in the paper. With certain low-frequency exciter machines for induction motors, the results when such machines were operated in connection with the induction motors were entirely different from results obtained on the test floor with resistance load. Here again the difference was very marked. These examples are also interesting because greater difficulties were experienced with 25-cycle motors than with 60-cycle motors, which is the direct opposite of what designers of a-c. commutating machinery are accustomed to. The reason for this was that in these cases the leakage reactance between the short-circuited armature coil and other circuits is made up largely of differential leakage, which in turn is proportional to the total main flux per pole. The latter was larger in the 25-cycle than in the 60-cycle machines and as a result led to less favorable commutating conditions in the 25-cycle machines.

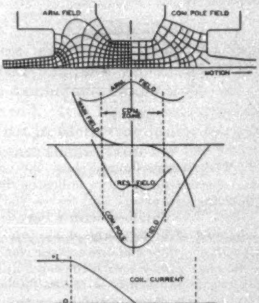

FIG. 5

The influence of external circuits in a-c. machines is also of greater importance not only because due to the highly laminated stator structure there are no damping effects in the stator core, but also because there frequently is, especially in motors for reversing operation and with consequent tilting of the carbons in their holders, only one armature coil short-circuited by the brush and consequently none of the other armature coils is able to supply any damping effect.

With reference to the question as to whether the switching operation at the edges of the brush in d-c. machines can be avoided, a brief discussion of the following figures may be of interest and may cover certain points raised in the discussions by

Messrs. Weber, Hambleton, and Penney. Ideal conditions are obtained if at the moment an armature coil is short-circuited its voltage is zero, and consequently zero current in the edge of the brush at that instant. Similarly, sparking and switching conditions at the trailing edge are impossible if the current in the edge of the brush is zero when the circuit is opened. This condition is fulfilled in Fig. 4. The armature and commutating pole fields have been carefully charted,* and it will be seen that the resultant field is zero both at the entering and trailing edge. The current in the commutating coils is shown below and has a characteristic which is sometimes referred to as "sinusoidal commutation." The bottom figures show the current density in the brush, and the minimum curve indicates that zero values apply at the time the coil enters and leaves the commutating zone. These results were obtained with a very small gap after much careful scheming with the shape of the interpole and with an armature without chording and one coil per slot.

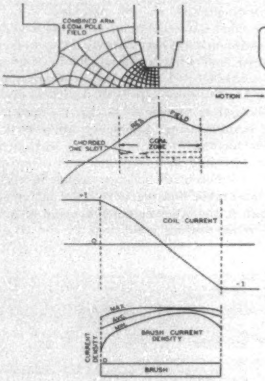

Fig. 6

Fig. 5 shows the same machine with the conventional large air-gap. The conditions are not nearly so favorable inasmuch as the resultant field induces a voltage in the commutating coil at the ends of the zone and there are certain current densities at both the entering and trailing edges. Fig. 6 shows a machine with chording of one slot and results are still less favorable; in fact, it was found impossible with chorded armature to obtain by changes in the interpole the ideal conditions shown in Fig. 4. In cases where two or more coils per slot are used, switching conditions are altogether unavoidable because, due to the influence of the main poles, the commutating field as shown in Fig. 7 is usually unsymmetrical. This means that the field cannot be correct for both coils. It is too weak for the commutation of the first coil and therefore commutation is incomplete at *b* when the first coil leaves the brush. However, on account of the close coupling with the other coil in the same slot, the energy can be readily transferred into the other coil, as indicated by the upswing of the current in the second coil. The curve for the second coil shows that because the commutating field in the range of its commutation is too strong, a reverse current is built up as indicated at *d* which has to be interrupted when the circuit opens. These figures, except Fig. 4, indicate clearly that there actually is a switching operation at the edges of the brush in most practical d-c. cases. This, however, is not of very great practical consequence in most d-c. cases for the various reasons previously indicated, although it may at times be largely responsible for the sparking which cannot be entirely eliminated.

L. R. Ludwig: In one of the discussions a question was raised regarding the feasibility of some special commutating arrangements. In answer to this, it may be stated that many such arrangements lead to irregular wear of the commutator parts, which would result in sparking regardless of any merits from an electrical point of view. There are means which would not be open to this difficulty and they have been and are being carefully considered. If any notable results are obtained, this work will be made the subject of later papers.

The possibility of bad commutation resulting from pulsations in the main field flux has been carefully investigated by us several times. In d-c. machines this effect is of minor importance due to the large air-gap and the damping effect of the stator structure. In a-c. machines it has been found that such pulsations may introduce high short-circuit voltages into the commutating coil, and consequently this point should always receive careful attention. It is the usual practise to construct a-c. machines, particularly those having relatively weak fields, with skewed slots in the rotor. Under this condition careful investigation has failed to reveal any commutating difficulties due to pulsations of the main field fluxes.

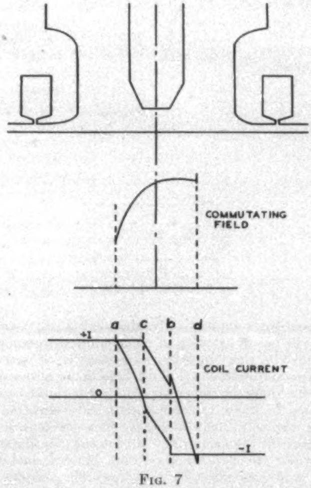

Fig. 7

The discussion given by Mr. Veinott is a valuable study of the special problems involved in a wave winding, and it again makes clear the necessity for a complete analysis of the electrical conditions existing within a commutating coil.

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*We are indebted to C. S. Newell for carrying out this work.

Equalizing Currents in the Armature of a D-C. Machine

BY ROBERT M. BAKER*

Associate, A.I.E.E.

Synopsis.*—This paper contains an analysis which shows how the equalizing currents in the armature coils of a d-c. machine act to correct the unbalance causing these currents to flow. It is shown that even with very small inequalities of main pole air gaps (plus or minus 7 per cent of an average main pole gap), the equalizing current in an armature coil may reach, several times each revolution of the armature, a peak value as high as 25 per cent of the full load coil current value. The analysis is unique in that it is based upon experimentally determined curves of equalizing current variation in an actual machine.*

INTRODUCTION

WHEN the parallel type winding was first used on the armatures of large d-c. machines it was almost impossible to obtain sufficiently good magnetic balance of the main field circuits to permit satisfactory operation. This was due to the fact that the voltage of any circuit of a parallel or lap-wound armature winding is generated entirely by the flux of two adjacent main field poles. Unequal air gaps, or unequal reluctances in the iron circuits, result in unequal flux in the various main poles and consequently cause unequal voltages to be generated in the parallel circuits of the armature. This results not only in heavy circulating current in the armature winding, but also in an increased load on the carbon brushes.

It is obvious, therefore, that some device was needed to balance the flux of the main poles automatically and overcome the trouble resulting from the unavoidable irregularities of manufacture. Thus it was that B. G. Lamme in 1896 conceived the idea of connecting together in the armature winding of a d-c. machine, those points which should normally be at the same potential. He reasoned that with these polyphase connections, currents would circulate between the various parts of the armature winding to equalize the flux of the different main poles, just as a number of synchronous machines operating in parallel tend to equalize their fluxes by causing exciting currents to flow in the armature circuits. This was the beginning of the equalizing connections or cross connections as they are used today.

Equalizing connections have proved themselves capable of taking care of considerable unbalance in the main fields, and have made it possible to build large d-c. machines to operate satisfactorily with lap-wound armatures of twenty or more circuits. Their performance has in fact been so satisfactory that very little time has been given to a study of the details of this performance. Purely theoretical analyses are valuable in giving one a clear mental picture of the process by which the equalization is accomplished; but they are always necessarily based upon simplifying and often inaccurate assumptions and are therefore of little quantitative interest. The analysis given in this paper is based upon the experimentally determined variation of equalizing currents in the armature coils and shows how these known currents act to overcome the magnetic unbalance causing them. The value of such a treatment lies in the fact that it will give the reader some idea of the magnitude and nature of equalizing currents arising from a certain unbalance in the main field circuits and will make the whole subject more tangible by showing records of the equalizing currents in the armature winding of an actual machine. It will also suggest a method for studying the subject in more detail.

THE MACHINE

The machine used in this study is a 25-hp., 4-pole, 220-volt, 1,500-r.p.m., d-c. generator. The armature is full pitch lap wound and has 40 slots, one coil per slot, ten turns per coil. It is 100 per cent cross connected on the front. That is, each commutator bar is in turn connected to another commutator bar just two pole-pitches ahead. Since the armature has 40 slots and a full pitch winding, it is entirely symmetrical. The average main pole air gap for this machine is only 0.074 inches.

ANALYSIS OF TEST RESULTS

To obtain oscillographic records of equalizing currents in armature coils it was necessary to insert a small resistance in series with the coil or coils to be studied, and to record the voltage drop across this resistance. The value of resistance chosen was only 25 per cent of the resistance of a single armature coil, and since equalizing currents are limited almost entirely by armature coil inductance, the insertion of this small resistance caused no distortion. All records were taken with the commutator brushes lifted, so that the only current in the armature coils was the equalizing current.

The first record taken was of the equalizing currents in two diametrically opposite armature coils with all four field coils normally excited in series. The only unbalance in the magnetic circuit was due to the air gaps under the two lower poles being slightly less than those under the two upper poles. The record of the coil currents resulting from this unbalance is shown in Fig. 1. It is seen that the equalizing connections paralleling these two coils are sufficiently low in resistance

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that the two coils carry at each instant, exactly the same current. Although it is not shown on the oscillogram, the currents in these two coils are in opposite directions. Thus it is equivalent to a circulating current between the two coils set up by the difference of their generated voltages. Their generated voltages result not only from the cutting of flux, but also from their mutual interlinkages with other coils which are carrying current. An inspection of Fig. 2 will show that from symmetry, the equalizing current variation in any armature coil should repeat, except for a reversal of sign, twice each revolution. The fact that this condition was not obtained in Fig. 1 is explained by a dissymmetry of rotor iron. That is, the air-gap under any main field pole varies slightly as the armature is rotated.

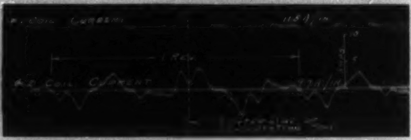

Fig. 1—Equalizing Current in Two Diametrically Opposite Armature Coils. (See Fig. 2)

Using the experimentally determined curve of equalizing current given in Fig. 1, an analysis will be made to show how equalizing currents correct the unbalance causing them to flow.

To eliminate the effect of dissymmetry of rotor iron, the curve of equalizing current is averaged so as to repeat, with signs changed, every two pole-pitches, and is plotted as the solid line curve of Fig. 3. It can be assumed that this is the current variation in the conductors of the top coil side of slot *A*, for after this assumption is made, the relation between the direction of the current and the direction of the resulting m.m.f. can be determined by the substitution of a known magnetic unbalance with a known polarity such as the partial excitation of a single field coil. From an inspection of Fig. 3, it is obvious that the bottom coil side of slot *A* is, at any instant, carrying a current of the same magnitude, but opposite in direction to that carried by the upper coil side just one pole-pitch earlier. Thus it is possible at once to plot the dotted curve of Fig. 3 showing the variation of conductor current in the lower side of slot *A*. The average conductor current in a slot multiplied by the number of conductors gives the total slot current as shown in Fig. 4.

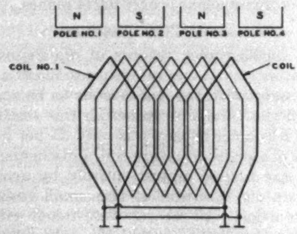

Fig. 2—Development of Armature Winding Showing Coils Nos. 1 and 2

For simplicity, only one pair of equalizing connections is shown

Since the value of current in a slot is dependent only upon the position of the slot relative to the main field poles, the curve of slot current will just as well represent a current sheet around the armature, fixed in space and therefore setting up a constant magnetic field relative to the main field poles. The m.m.f. resulting from this current sheet is represented by the heavy line curve of Fig. 4, which is simply the integral of the slot current curve. The scale of the m.m.f. curve was determined from the current scale and the number of slots per unit length of armature periphery, and is plotted in per cent of the normal excitation of a single main field pole.

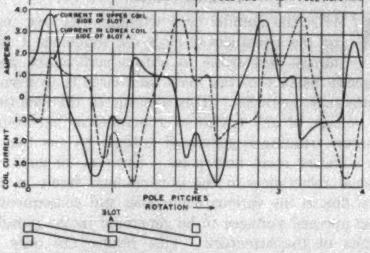

Fig. 3—Conductor Current in Upper and Lower Coil Side of Slot *A* Plotted Against the Position of Slot *A* Relative to the Main Field Poles

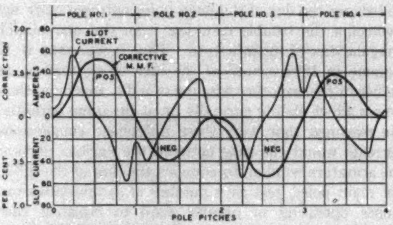

Fig. 4—Slot Current and Corrective M.M.F.

Referring to Fig. 5, it can be seen that the correction is proper. The corrective m.m.f. is in such a direction as to aid the flux of poles Nos. 3 and 4 and to oppose the flux of poles Nos. 1 and 2. An oscillogram of generated coil voltage showed that the equalization was practically perfect.

In a way the four-pole machine is unique. For instance, it has already been observed that the curve of equalizing current in an armature coil must repeat with the signs changed every two pole-pitches or twice each revolution. From the foregoing analysis it follows that the curve of corrective m.m.f. must also repeat twice each revolution. That is, the correction of two opposite poles must be equal and opposite. This is not at all surprising, for the unbalance must occur between opposite poles before it is effective in setting up equalizing currents. The unbalance of adjacent poles may be very great, and yet if opposite poles are equal in strength, the flux in all four poles will be equal.

The oscillogram of Fig. 6 was taken with only pole No. 2 of the machine excited, and shows records of generated coil voltage and coil current of one armature coil. It is interesting to notice the completeness of the equalization. Although only one field coil was excited, the flux in all poles is of the same amount and in the proper direction. The excitation of the pole was about 30 per cent normal. An analysis of the coil current curve showed that the equalization was accomplished by a corrective m.m.f. of 15 per cent sending flux from pole No. 4 into the armature, and a corrective m.m.f. of 15 per cent opposing the original 30 per cent excitation of pole No. 2. The correction under poles Nos. 1 and 3 was practically nil.

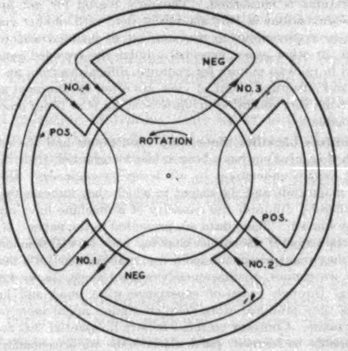

Fig. 5—Schematic Diagram of Magnetic Circuit Showing Normal Flux Distribution

A similar record taken on the machine with poles Nos. 1 and 2 excited to 30 per cent normal value is shown in Fig. 7. In this case the equalization required a corrective m.m.f. under each of the poles. Under pole No. 1 it was 15 per cent from pole to armature; under pole No. 2 it was 15 per cent from armature to pole; under pole No. 3 it was 15 per cent from armature to pole; and under pole No. 4 it was 15 per cent from pole to armature. With two poles originally excited, the final flux density under the poles was just twice that obtained from the original excitation of one pole.

It might be said that the analysis set forth in this paper was started almost with the answer. No attempt was made to solve the very difficult problem of theoretically determining the equalizing current set up in the armature coils by a certain unbalance in the main field circuit. The main purpose of the writer was to show how the oscillograph might be used in a study of the behavior of equalizing connections.

The process by which equalizing currents are set up in an armature is fairly well known, but the complications arising from the mutual inductance between armature coils and the existence of many overlapping circuits are so great that an accurate analytical solution of the problem seems practically impossible. It is firmly believed that a complete analysis similar to the one given in this paper, but one made on a machine with a large number of poles, would furnish enough empirical data that an accurate quantitative analytical solution would not be necessary. The number and size of the equalizing connections could be changed and if necessary, the analysis could be extended to two or three sizes or types of machines, so that the results could be interpolated for intermediate points. If later it does appear desirable to make a complete theoretical analysis of the problem, the analyzed data obtained on the actual machines would be of great value as a check on proposed theories.

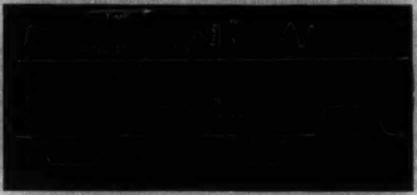

Fig. 6—Current and Generated Voltage in Armature Coil, with Pole No. 2 Excited 30 Per Cent Normal

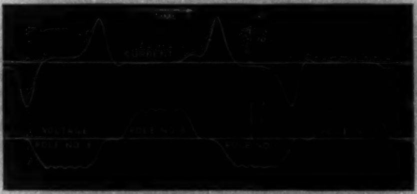

Fig. 7—Current and Generated Voltage in Armature Coil with Poles Nos. 1 and 2 Excited 30 Per Cent Normal

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1. "The Development of the D-C. Generator in America," B. G. Lamme, *Elec. Jl.*, Vol. XII, p. 117.

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Discussion

R. E. Hellmund: Such a study as that covered by Mr. Baker relating to equalizing connections on commutating machinery is very desirable because, in spite of the fact that equalizing connections have been used for a great many years, the design engineer is still continuously confronted with questions relating to their application. While equalizing connections are of course always used with parallel-wound armatures, it is frequently difficult to decide to what extent they should be used and which of the various possible arrangements at the front or rear end of the armature leads to the most economical and reliable design.

Equalizing connections fulfil two functions: that of equalizing the fields of the various poles, as covered in Mr. Baker's paper, and, in addition, a function which is not as generally recognized. This second function relates to the transfer of current from one brush arm to another of the same polarity whenever the brush, on account of mechanical inaccuracies of the commutator, temporarily jumps and fails to make contact with the commutator. This is exceedingly important in practise because, as is probably recognized by all designers of large and high-speed commutating machinery, by far the largest part of their commutating difficulties are due to mechanical inaccuracies or so-called "high bars" of the commutator. It can readily be seen that when a brush leaves a commutator bar which is directly connected by an equalizing connection to another bar of the same polarity, the current can temporarily transfer through this equalizing connection to such other bar, and unless such transfer is opposed by high self-induction of the equalizing connection, no sparking will result at the brush leaving the commutator. If, on the other hand, there is no equalizing connection to such bar, the transfer of current has to take place through a part of the armature winding. This may apply either if the cross-connections are at the rear of the armature or at the commutator end but limited to a small number of bars. Any transfer of current taking place would then be opposed by the self-inductive effect of part of the armature winding, and the self-inductive voltage thus obtained is likely to maintain a spark between the commutator and the jumping brush. It is for this reason that best results are usually obtained with 100 per cent cross-connections at the commutator, or, still better, with the so-called "involute winding."

Unfortunately the merits of the more expensive and often difficult arrangements for cross-connections cannot be readily determined from ordinary comparative tests because both functions previously mentioned are affected by inaccuracies of manufacture which are difficult to duplicate. The function relating to the equalizing of the field is affected by inaccuracies in various portions of the air-gap which cannot be measured easily. The function relating to the transfer of brush current is affected by inaccuracies of the commutator as small as one ten-thousandth of an inch, which are also difficult to determine, and which moreover are subject to change during the tests due to different expansion and contraction with changes in temperature. It is for this reason that the only hope of clarifying the subject rests in systematic studies in which a number of variables is eliminated, such as carried on by Mr. Baker.

Thomas T. Hambleton: The introduction to the paper recognizes the fact that armature equalizer connections are a matter of ancient history and that their action is thoroughly understood by all designers.

It seems appropriate from the historical standpoint to mention a coincidence which is not infrequently encountered in the development of an art or science. It occasionally happens that two or more individuals independently and concurrently arrive at the same solution of a problem. Such a coincidence appears to have occurred in connection with armature equalization. The old records of the General Electric Company contain drawings made in 1895 showing involute formed equalizers mounted at the end of commutators and connected directly to the segments, thus providing complete equalization at the most advantageous point as regards another function of the equalizers which Mr. Baker has not mentioned.

Besides providing paths for the circulation of magnetizing currents which equalize field fluxes, equalizer connections also carry between parallel brush groups interchange currents caused by irregularities in the mechanical condition or the contact surface condition of the commutator.

An equalizer located at the commutator end of the machine has minimum impedance for these currents; when the equalizer is located at the back end of the armature these currents must at least twice traverse the armature slots in addition to the equalizer, thereby encountering the high impedance of the iron-embedded conductors.

It is evident that a type of equalization considered perfect today was used on some very early machines.

R. E. Hellmund: I agree with Mr. Hambleton in his statement that 100 per cent cross-connections at the front end are desirable as far as obtaining the best result from the equalizing connections is concerned. The only reason for not applying cross-connections in this manner is that with smaller machines and less severe conditions they may not be justified economically. Again, in other cases, especially under the crowded conditions found in railway motors for instance, difficulties from an insulation and structural point of view are often encountered and the use of the very desirable involute winding is at times practically impossible.

Thomas Linville: Equalizing connections and the currents which flow in them have been taken for granted, their function being clearly understood by all designing engineers. However, their magnitude and the extent to which they increase the losses and thereby diminish the capacity of a machine have not been widely known. These data are presented in the paper. It seems that the value of the connections has been so well demonstrated that they must be considered a fundamental part of a machine. Since we cannot divorce ourselves from them, as we know in general through years of experience what good and harmful effects they produce, more detailed studies might seem of little importance. Contrary to this I believe it essential that as much as possible be learned, particularly since we occasionally want to build a machine with purposely unbalanced magnetic fields and want to know how effectively the equalizing currents will function. Again because of the existence of these currents in commutating coils, and because the brushes always form parallel equalizing paths, it is important to know exactly how they affect commutation. Also a great deal of information concerning the action of equalizing currents in some types of a-c. machines is needed. Finally for the currents to be effective in the desired way they must be largely reactive currents. Hence, we want to know how low the resistance of the connections must be and how complete the equalization will be with a given resistance. The fact that the currents are determined practically by reactance alone is what makes their calculation difficult. Much experience, test data, and well-established convictions answer these wants but the need for still more definite data is great. Therefore, I think Mr. Baker's work is worth while.

It is encouraging to see such excellent results obtained with the oscillograph because its use is essential in making experimental studies and in proving theories. In some cases, with big machines, real difficulties are met due to stray fields, etc., causing distorted oscillograms when extreme care is not used.

Calculation of No-Load Damper Winding Loss in Synchronous Machines

BY E. I. POLLARD*

Non-member

Synopsis.—Pulsations in the air-gap induction due to the slotted stator construction produce pole-face loss. When a damper winding is present the pulsations link the damper bars producing circulating currents causing an additional loss, which is designated by the term "no-load damper winding loss." The magnitude of this loss is found to vary from practically zero to as much as eight times the pole-face loss, or to as much as half of the total no-load core loss. A method of calculating the no-load damper loss is developed based on theoretical considerations. As far as the writer is aware no theoretical work has been previously published on this subject.

DEVELOPMENT OF THE FORMULA

IN synchronous machines, the non-uniform nature of the air-gap due to the presence of alternate teeth and slots on the stator results in a non-uniform distribution of flux density at the surface of the rotor, the maxima of density being opposite the teeth and the minima opposite the slots. Flux maps of the air-gap field for a slotted stator have been published by several writers.[1,2] The relation between the actual and average densities from point to point over a tooth pitch along the air-gap, derived from flux maps, is shown by Wieseman.[1] The induction at the rotor side of the gap may be resolved into an average induction and a superimposed pulsation or ripple. The latter is the object of interest from the standpoint of damper loss. In producing the ripple, the teeth may be regarded as north poles, the flux returning to consequent or imaginary south poles occupying positions in the slots as indicated in Fig. 1.

In most machines, the damper slots are but partially closed, so that the reluctance of the opening above the slot is large compared with the reluctance of the path through the iron around the slot. The pulsation flux from a north to a south pole will, therefore, link the damper slots which interrupt the otherwise smooth surface of the pole-face. Any two bars form a closed circuit linking the enclosed pulsation flux. The position of maximum interlinkage occurs when the central point between the bars is opposite the center of either a stator tooth or slot. The frequency of the flux pulsation with respect to a point on the rotor surface is equal to the number of stator slots per pair of poles multiplied by fundamental or line frequency. Therefore, the rate of change of interlinkage of pulsation flux with a pair of damper bars is quite large, inducing a considerable voltage around the closed circuit formed by the two bars. Assuming that the circuit is open, it is possible to calculate the voltage induced around the circuit, the total inductive reactance, resistance, and impedance of the circuit, and from these calculate the current which flows when the circuit is closed. However, since the effect of the resistance upon the impedance is quite small, it is preferable to assume that the reactance and impedance are equal and calculate the current directly on the basis that the magnitude of the current is determined by zero interlinkage of flux with the circuit. In other words, the flux produced by the flow of current is equal to and completely cancels the flux enclosed by the circuit before it is closed. This damping action leads to the suggestion that the circuit formed by each pair of adjacent damper bars be defined as a "damping circuit." On this basis each damper bar becomes a part of two damping circuits, the return paths of these circuits being the two bars adjacent to the one considered, as indicated in Fig. 1. The total current in the bar is, therefore, the resultant of the currents flowing in the two damping circuits of which the bar is a part. The current in the damping circuit is obtained by use of the relation:

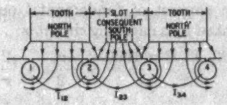

FIG. 1

damping circuit m.m.f.

$$= \frac{\text{total enclosed pulsation flux}}{3.19 \times \text{total permeance of damping circuit}}$$

or

$$I_c = \Phi/\sqrt{2}\ 3.19\ \lambda \qquad (1)$$

The total enclosed pulsation flux must be construed to include the mutual flux set up in the damping circuit considered by currents in all other damping circuits, as well as the flux set up in it by the non-uniform air-gap permeance. That is:

$$\Phi = \Phi_p + \Phi_m \qquad (2)$$

The total permeance of the damping circuit may be separated into the air-gap permeance and the permeance of the slots forming the circuit:

$$\lambda = \lambda_g + 2\ \lambda_s \qquad (3)$$

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1. For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Let K_p represent the difference between the maximum and the minimum values of air-gap induction measured at the rotor side of the gap opposite the centers of a tooth and of a slot respectively, and expressed as a fraction of the maximum induction. Then the average amplitude of the pulsation component of induction is

$$\frac{K_p\, K_g\, B \times 1000}{2}$$

The factor 1000 is required since B is expressed in kilolines per square inch.

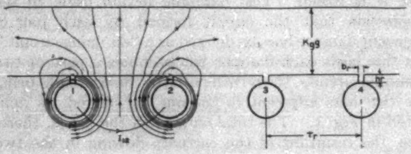

FIG. 2

The dissimilarity between the positive and negative halves of the tooth pulsation field form reveals the presence of a prominent second harmonic in addition to the fundamental, both of which will be considered. Accuracy does not seem to warrant extension of the Fourier's analysis beyond the second harmonic. Denoting the amplitude of the fundamental of field form as K_{w1} × average amplitude of induction pulsation, then the maximum amount of fundamental pulsation flux that can be linked by a damping circuit is:

$$\Phi_p = \frac{K_{w1}\, K_g\, K_p\, B \times 1000}{2} \times \frac{\tau_s}{2} \times \frac{2}{\pi} \times \sin \phi/2$$

or

$$\Phi_p = 159\, \tau_s\, B\, K_{w1}\, K_p\, K_g \sin \phi/2 \qquad (4)$$

The term $\sin \phi/2$ is the chord factor of the damping circuit, ϕ being the "throw" of the damping circuit, or damper bar pitch, expressed in electrical degrees. Since τ_s = 360 electrical degrees for the pulsation,

$$\phi = \tau_r/\tau_s \times 360^\circ$$

The angle ϕ is also the time phase angle between the currents in adjacent damping circuits.

Referring to the circuit consisting of bars numbered 1 and 2 in Fig. 2 whose current I_{12} at some instant is outward in bar 1 and inward in bar 2. The various paths taken by the flux produced by I_{12} are indicated by arrowed lines. The flux crossing the gap may be regarded as returning through some other damping circuit in which the current is in phase opposition to I_{12}. The net effect of the m.m.f. of the current I_{12} on the permeance of the damping circuit consisting of bars 3 and 4 is practically zero, and does not alter the amount of flux linked by bars 3 and 4. However, the m.m.f. of the current I_{12} acting on the slot permeance of bar 2 produces flux linked by the circuit consisting of bars 2 and 3, which is the flux Φ_m mentioned above. The magnitude of Φ_m is:

$$\Phi_m = 3.19 \sqrt{2}\, I_c \cos \phi \times 2\, \lambda_s \qquad (5)$$

Substituting equations (2), (3), (4), and (5) in equation (1) gives

$$I_c = \frac{159\, \tau_s\, B\, K_{w1}\, K_p\, K_g \sin \phi/2 + 2 \times 3.19 \sqrt{2}\, \lambda_s\, I_c \cos \phi}{3.19 \sqrt{2}\, (\lambda_g + 2\, \lambda_s)}$$

Solving for I_c:

$$I_c = \frac{35.3\, \tau_s\, B\, K_{w1}\, K_p\, K_g \sin \phi/2}{\lambda_g + 2\, \lambda_s\, (1 - \cos \phi)} \qquad (6)$$

The resultant current in the bar is the vector difference of the two currents I_c in the two damping circuits of which the bar is a part, displaced in time by the angle ϕ. Hence

$$I_r = I_c \sqrt{2\, (1 - \cos \phi)}$$

or

$$I_r = \frac{35.3\, \tau_s\, B\, K_{w1}\, K_p\, K_g \sin \phi/2 \times \sqrt{2\, (1 - \cos \phi)}}{\lambda_g + 2\, \lambda_s\, (1 - \cos \phi)}$$

Since

$$\sin \phi/2 = \sqrt{\frac{1 - \cos \phi}{2}},$$

$$I_r = \frac{35.3\, \tau_s\, B\, K_{w1}\, K_p\, K_g\, (1 - \cos \phi)}{\lambda_g + 2\, \lambda_s\, (1 - \cos \phi)} \qquad (7)$$

The effective bar resistance is

$$K_f\, R = \frac{\rho\, K_f\, l}{10^6\, a} \qquad (8)$$

Assuming that all damping circuits are physically identical, since n is the total number of bars on the machine, the total loss resulting from the fundamental of the pulsation field form is:

$$P_1 = n\, K_{f1}\, R_1\, I_{r1}^2 \times 10^{-3} \qquad (9)$$

The principal error introduced by the above assumption is that the damping circuits consisting of the outer bars in the pole tips of adjacent main poles are quite dissimilar from the remaining circuits so that the loss in the outer bars is actually different from the assumed value. Correction will not be made for this effect in the present paper.

Substituting equations (7) and (8) in (9) gives:

$$P_1 = \frac{1.246\, n\, l\, \rho\, K_{f1}}{10^6\, a} \left[\frac{\tau_s\, B\, K_{w1}\, K_p\, K_g\, (1 - \cos \phi)}{\lambda_g + 2\, \lambda_s\, (1 - \cos \phi)} \right]^2.$$

The same reasoning applies throughout to the second harmonic of pulsation field form, giving a similar formula. The total loss for both fundamental and second harmonic is:

$$P = \frac{1.246\, n\, l\, \rho}{10^6\, a}\, (\tau_s\, B\, K_p\, K_g)^2$$

$$\times \left[K_{f1} \left(\frac{K_{w1} (1 - \cos \phi)}{\lambda_g + 2 \lambda_s (1 - \cos \phi)} \right)^2 + K_{f2} \left(\frac{K_{w2} (1 - \cos 2 \phi)}{\lambda_g + 2 \lambda_s (1 - \cos 2 \phi)} \right)^2 \right] \qquad (10)$$

EVALUATION OF UNDETERMINED FACTORS IN EQUATION (10)

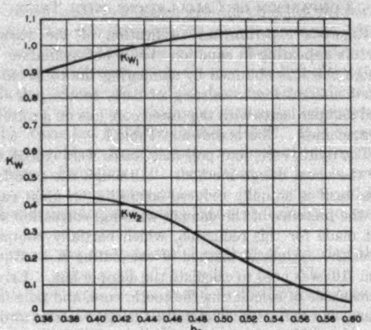

FIG. 3

1. *The effect of proportions of air-gap and stator slot—K_p.* The length of air-gap and stator slot width determine the factor K_p. The relation between these quantities is given by Mr. F. W. Carter[2] as follows:

$$K_p = 1 - \frac{1}{\sqrt{1 - (b_s/2\,g)^2}}$$

This has been checked graphically by Mr. R. W. Wieseman.[1]

2. *The effect of proportions of stator slot and tooth—K_{w1} and K_{w2}.* From flux maps of the air-gap field in the region of a stator slot, it is evident that the field is practically uniform over a sizeable portion of the tooth.[1] This, of course, results in a flat-topped shape for that portion of the tooth pulsation field form, which is opposite the stator tooth. Obviously, the effect of increasing the width of the tooth is to widen the flat-topped portion of the field form. Thus the relative widths of slot and tooth have a very considerable effect upon the field form and consequently upon the magnitudes of the harmonics into which the field form is resolved.

Several other flux maps similar to those of Mr. Wieseman show that the field form is also affected to some extent by the length of air-gap. Other proportions remaining unchanged, an increase in the air-gap shortens the flat-topped portion of the field form. However, within the range of proportions usually found in salient pole synchronous machines, this effect is relatively much less important that that of the proportions of slot and tooth, so that the magnitudes of the harmonics of the field form may be assumed to be functions only of the ratio of stator slot width to tooth pitch, b_s/τ_s. The values of K_{w1} and K_{w2} are plotted against b_s/τ_s in Fig. 3, and were obtained from Fourier's analyses of pulsation field forms.

3. *Calculation of the permeances λ_g and λ_s.* The slot permeance λ_s may be regarded as the sum of three component permeances, the permeance of that part of the slot occupied by the bar, λ_b, the permeance of the slot above the bar, λ_a, and a tooth-tip permeance, λ_t, representing the flux fringing out into the air-gap. Then

$$\lambda_s = \lambda_a + \lambda_b + \lambda_t.$$

For uniform current density in the bar, $\lambda_b = 0.75$, approximately.[3] Since the frequency of the damping currents is quite high, the current density will not be uniform but will be high at the top of the bar, decreasing toward the bottom. Thus for the same total current in the bar the flux crossing it will be reduced since h_b is in effect reduced, and λ_b will be less than for uniform current distribution. Since the increase in resistance K_f is a measure of this skin effect, the value of λ_b will be reduced, approximately, by K_f. Since λ_b is a small portion of λ, this rough approximation is acceptable. Then, for round or square bars

$$\lambda_b = \frac{0.75}{K_f}$$

The magnitude of the slot permeance above the bar is:

$$\lambda_a = \frac{h_r}{b_r}$$

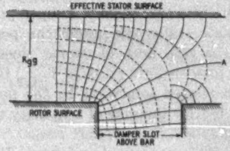

FIG. 4—MAP OF FIELD PRODUCED BY CURRENT FLOWING IN BAR IN SLOT SHOWN AND RETURNING IN ADJACENT BAR AT THE LEFT FOR $\frac{b\,r}{K_g\,g} = 1.0$

The values of λ_g and λ_t are obtained from a series of flux maps similar to Fig. 4. The flux line marked *A* in Fig. 4 is the dividing line between the gap permeance λ_g and the tooth-tip permeance λ_t. Since the field of Fig. 4 is practically uniform a short distance to the left of the damper slot, it is evident that for a given effective gap, λ_t is independent of τ_r.

Maps similar to Fig. 4 show that λ_t is a function of $b_r/K_g\,g$. Calculated from these maps, the magnitude of λ_t is submitted in curve form in Fig. 5. The set of maps for various values of $b_r/K_g\,g$ shows that λ_g is independent of $b_r/K_g\,g$ while Fig. 4 shows that λ_g does depend upon $\tau_r/K_g g$ and in fact may be taken as approximately equal to $\tau_r/K_g g$.

$$\lambda_g = \tau_r/K_g\,g.$$

By calculating λ_g in this manner, the non-uniformity of the air-gap permeance due to the stator teeth and slots is neglected. More complete analysis would show that for a given combination of slot and gap dimensions and bar pitch, λ_g is equal to $\tau_r/K_g g$ plus a superimposed periodic function of time. Referring to Fig. 1, λ_g for bars 1 and 2 is larger than for bars 2 and 3, so that the current I_{12} is less than I_{23}. But since we are interested chiefly in the resultant current in bar 2, which is the same as if the average value of λ_g had been used for both circuits, the final result is unaffected. In Fig. 1 by omitting bars 1 and 3 the condition of $\tau_r/\tau_s = 1.0$ is obtained. In this case, since a damper pitch extends over a complete stator slot pitch, the periodic component of λ_g is zero and, of course, does not affect results. For intermediate ratios of τ_r/τ_s, the resultant bar current is altered somewhat by the periodic component of λ_g, but for the sake of simplicity this will not be considered.

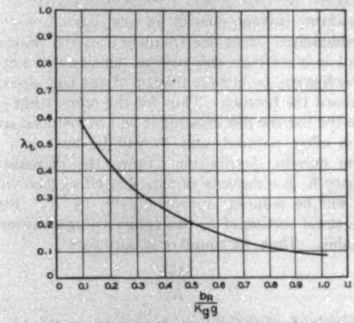

FIG. 5—DAMPER TOOTH-TIP PERMEANCE λ_t

4. *Calculation of increase of resistance*, K_f. Various investigators[4] have calculated the ratio of the a-c. to d-c. loss in a solid conductor placed in an open rectangular slot, based on the assumption that the flux produced by current in the conductor crosses the slot in straight parallel lines. Since partially closed rather than open damper slots are almost universally used in modern synchronous machinery the effect on K_f of partially closing the slot is to increase the ratio of a-c. to d-c. loss, K_f. In the absence of a rigorous treatment of this subject, K_f may be calculated by the open-slot formula, remembering that it is optimistic.

COMPARISON OF CALCULATIONS WITH TESTS

Rigorous experimental verification of the various factors appearing in equation (**10**) is not available. A rough check is obtained by comparing the total calculated no-load loss consisting of core, tooth, pole-face and damper losses with the tested core loss on a number of machines. This is shown in Table I.

The tooth, core, and pole-face losses were calculated by the usual design methods. Although the pole-face loss itself is actually reduced somewhat in most cases by the presence of the damper winding, correction was not made for this reduction, which partially compensates the optimistic method of calculating K_f. Equation (**10**) was used to calculate the damper loss. Errors in methods of calculating the tooth, core, and pole-face losses, and variations in manufacturing processes and in the materials themselves tend to prevent consistency of the results of such a check. In choosing the examples for Table I, machines having relatively large pole-face loss were preferred in order to minimize as much as possible the effect of errors in the calculation of the core and tooth losses These machines were all designed and built before the factors controlling damper loss were known. As a result, some very unfavorable combinations were accidentally used. Example No. 12 is outstanding in this respect, the damper loss being over eight times the pole-face loss. The designer of this machine estimated the total core loss to be about thirty kilowatts, and the test results could not be explained. These remarks also apply to examples Nos. 6, 7, and 10. In each case the high damper loss was due to a combination of unfavorable conditions: τ_r/τ_s = about 0.5, bar resistance is high and the total number of bars is large. In examples Nos. 1, 3, 8, 9,

TABLE I

	Poles	Kva.	Calculated core and tooth loss	Calculated pole face loss	Calculated damper loss	Total calculated loss	Test loss
1	8	3,000	17.1	5.1	0.38	22.6	18
2	8	2,920	17.3	5.3	4.5	27.1	31
3	14	5,270	28.3	10.0	0.40	38.7	40
4	16	692	3.32	1.86	3.04	8.22	7.5
5	24	3,750	30.5	15.8	13.5	59.8	57.5
6	26	1,500	7.8	5.4	12.7	25.9	26.0
7	26	1,500	6.39	1.9	3.74	12.03	15
8	40	1,125	8.8	5.5	2.4	16.7	17.7
9	48	1,750	10.5	3.5	0.33	14.3	13.6
10	56	1,250	14.6	9.8	17.2	41.6	36.7
11	58	2,000	11.9	9.4	0.49	21.8	23.6
12	88	5,000	24.1	3.3	27.6	55.0	54.0

and 11, favorable combinations were accidentally chosen when these machines were designed, and the test core losses were considered unaccountably low. Table I emphasizes the extremely variable nature of the no-load damper loss, and indicates the value of some method by which the magnitude of this loss may be estimated.

CONCLUSIONS

Several ways by which the damper loss may be minimized are suggested by equation (**10**). In general, any change made in a design with the object of reducing pole-face loss also decreases the magnitude of the damper loss, unless the ratio τ_r/τ_s is thereby made less favorable. However, such changes usually affect the fundamental proportions of the machine; air-gap length, number of slots, etc., which are frequently determined not by losses but by other characteristics. It is, therefore, preferable to look for better methods of controlling this loss. Equation (**10**) shows that the loss is proportional to the number of bars on the machine and to the bar resistance, suggesting that it is desirable to keep these quantities small.

Perhaps the most powerful method of reducing the loss is by adjustment of the ratio τ_r/τ_s. When $\tau_r/\tau_s = 1.0$ the terms $1 - \cos\phi$ and $1 - \cos 2\phi$ in equation (**10**) are equal to zero, causing the damper loss to vanish. The loss is quite small down to $\tau_r/\tau_s = 0.85$. However, when $\tau_r/\tau_s =$ about 0.5 the loss is usually excessive, and this condition should be avoided if possible. In cases where a low value of telephone interference factor is imperative, the use of nearly equal rotor and stator slot pitches should be applied with caution since some unpublished work of Mr. S. L. Henderson* indicates that this condition frequently results in a higher telephone interference factor than where τ_r/τ_s is about 0.65. With this exception, this method of reducing damper loss may be applied without impairing other characteristics.

Increasing the rotor slot pitch increases λ_g, and assists in reducing loss, being a secondary effect accompanying reduction in the number of bars. Increasing the term λ_t by deeply embedding the damper bars in the pole-face also assists in reducing loss.

In conclusion the author wishes to express his indebtedness for helpful criticism and suggestions to Messrs. J. F. Calvert, L. A. Kilgore and S. Beckwith,* with whom he conferred from time to time during his investigation of this problem, and to Messrs. M. W. Smith and C. M. Laffoon* for criticism of the manuscript.

NOMENCLATURE

a = area of damper bar (sq. in.)
B = average air-gap induction (kilolines per sq. in.)
h_b = depth of damper bar (in.)
h_r = depth of rotor slot tip above bar
f = stator tooth frequency = number of slots per pair of poles × machine frequency
g = air-gap length (in.)
I_c = current in damping circuit (amperes)
I_r = resultant current in damper bar
K_g = air-gap coefficient
K_f = ratio of a-c. to d-c. loss in damper bar
K_p = double amplitude of air-gap induction pulsation in terms of maximum induction, at rotor surface
K_w = magnitude of field form harmonic in terms of K_p (Fig. 3)
l = length of damper bar (in.)
n = total number of damper bars on machine
P = no-load damper loss (kw.)
τ_r = rotor slot pitch (in.)
τ_s = stator slot pitch (in.)
R = d-c. resistance of damper bar (ohms)
b_s = stator slot width (in.)
b_r = width of rotor slot opening above bar (in.)
b_b = width of damper bar (in.)
λ_a = permeance of rotor slot above bar
λ_b = permeance of that portion of the rotor slot which is occupied by the bar
λ_g = air-gap portion of damping circuit permeance
λ_t = rotor tooth-tip permeance (Fig. 5)
Φ_p = maximum amount of pulsation flux due to air-gap permeance variation, which is linked by a damping circuit
Φ_m = mutual pulsation flux linked by a damping circuit due to currents in other circuits
ϕ = phase angle between currents in adjacent damping circuits $= \tau_r/\tau_s \times 360$ deg.
ρ = resistivity of bar material (microhms per cu. in.)

Subscripts 1 and 2 refer to fundamental and second harmonic respectively.

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1. *Graphical Determination of Magnetic Fields*, R. W. Wieseman, TRANS. A.I.E.E., 1927, p. 141.

2. "Pole Face Losses," F. W. Carter, *Journal I.E.E.*, 1916.

3. *Discussion of Papers on Graphical Determination of Magnetic Fields*, J. F. Calvert, Figs. 13 and 14, TRANS. A.I.E.E., 1927, p. 151.

4. *Eddy Current Losses in Large Slot Wound Conductors*, A. B. Field, TRANS. A.I.E.E., 1905.

Discussion

C. C. Shutt: Mr. Pollard's paper gives the designer of synchronous machines another useful tool for the predetermination of their performance. The test results of example 12, Table I, demonstrated clearly the necessity for the investigation which he undertook. The particular machine of example 12 in Table I happened to be a motor. The damper bar material had a resistance 15 times that of copper and the slot ratio was conclusive to high damper winding losses.

In synchronous motor design the use of relatively high resistance damper bars is becoming increasingly common as a means of reducing the starting kva. and improving the starting power factor. From equation (**10**) of the paper, it is apparent that the loss increases directly with the effective resistance of the damper bars. This makes it particularly necessary to check the design of synchronous motors from the standpoint of ratio of rotor to stator slot pitches.

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*All of the Power Engg. Dept., Westinghouse Elec. & Mfg. Co.

On a new line of synchronous machines, which was recently developed, this feature was checked before making the drawings. The number of stator and rotor slots per pole was determined by other considerations. The pitch of the slots in the pole punchings was set so as to avoid improper ratios from the standpoint of no-load damper loss. This necessitated unequal pitches of the pole face slots in several of the poles. The tests of no-load loss on this line of machine compare favorably with the calculated values.

R. E. Hellmund: The practical importance of damper winding losses has, of course, been appreciated by designers for some time. I recall cases where what seemed to be a rather insignificant increase in the ratio of the secondary bars of a squirrel-cage motor to the number of primary slots increased the no-load losses of the motor from 50 to 100 per cent. Furthermore, it is easily seen that if the top of the teeth of the rotor are smaller than the slot openings of the stator, it will result either in large flux fluctuations in the rotor teeth and consequent core losses, or else in large losses from the corrective current in the squirrel-cage bars. As a matter of fact, in the example of the induction motor previously referred to, the losses caused by the flux changes in the teeth, if allowed to exist by removing the winding, were of about the same order as the losses caused by the corrective current with the winding in place. Although there was previously a qualitative appreciation of the factors touched upon in Mr. Pollard's paper, his quantitative analysis of the problem is of great value.

As in most other cases of design, there are in this case several reasons why the use of the proportions giving ideal results in one respect, such as the minimum damper winding losses covered in the paper, cannot be applied. This in turn makes it desirable to determine just how great a departure from the ideal is advisable in order to favor other factors of importance. A large number of secondary bars, for instance, is usually desirable on account of the favorable effect upon the power factor and other performance features of induction motors. Again, it is pointed out in the paper that while a slot ratio of 1:1 gives minimum damper winding losses, it is likely to lead to telephone interference. This is due to the fact that higher harmonic currents different from those calculated in Mr. Pollard's paper are set up in the primary and secondary of synchronous and induction motors for the following reason:

With a definite voltage impressed upon these motors, the various currents have to adjust themselves so as to set up fields inducing a counter e.m.f. equal and opposite to the impressed e.m.f. Inasmuch as the field forms of the machines are usually not sufficiently ideal to accomplish this with sinusoidal magnetizing current, a flow of certain higher harmonic currents is necessary. These higher harmonic currents are usually of large amplitude with a slot ratio of 1:1 or equal slot pitch, and therefore are likely to cause telephone interference. The losses caused by these harmonics and not covered in the paper are usually of no practical importance. (In the usual open-circuit, no-load test of synchronous machines, such losses are of course not present at all.) This problem can be attacked along the lines suggested in a paper which I presented before the A.I.E.E.* and an article published in the *Electrical World* of 1906 on the subject "Magnetizing Currents in Polyphase Induction Motors."

The method given by Mr. Pollard should prove an excellent guide in choosing a secondary slot pitch appreciably different from the primary without reaching the point where the secondary damper winding losses assume undesirable proportions.

Ernst Weber: The formula (1) of the paper for the total damper winding loss contains two factors kw_1 and kw_2 which are very important since they represent the relative magnitudes of the fundamental and second harmonic of the pulsation wave and are involved in equation (10) as the square. It is important, therefore, to know their values with some accuracy. The author determined kw_1 and kw_2 from flux maps taking into account the effect of the relative widths of slot and tooth upon the field form. Obviously this method can be very inaccurate.

In 1928, I published a paper on the air-gap coefficient in machines* in which I gave a mathematical expression for the pulsation curve as

$$B_p = \overline{B}_p \cdot \left(\sin \frac{x\pi}{t_s} \right)^{2n} \tag{1}$$

with

$$n = \frac{t_s - b_s}{b_s}.$$

This expression represents for the integers n, discrete wave forms of just the type desired to take care of the ratio of slot width b_s to slot pitch t_s. The mean value follows by integration as

$$(B_p)_m = \overline{B}_p \cdot \frac{1.3.5. \ldots . (2n-1)}{2.4.6. \ldots . 2n}, \tag{2}$$

and by Fourier's analysis the amplitudes of the first and second harmonics may be determined. Their ratios to $(B_p)_m$ in (2) give the amplitude factors

$$K_{w1} = \frac{4}{\pi} \cdot \frac{2.4.6. \ldots (2n)}{1.3.5. \ldots (2n-1)(2n+1)}, \quad K_{w2} = \frac{2n}{2n+2} \cdot K_{w1}. \tag{3}$$

Contrasted with those values obtained by Mr. Pollard it is to be seen that formula (3) gives lower values for kw_1 and higher ones for kw_2 than Fig. 3 of the paper; but the variation with $\frac{b_s}{t_s}$ is of the same character. This may be taken as a satisfactory check and yet may provide a means for further correction and approximation, since it is quite easy to compute even the higher harmonics.

The effect of increase of resistance of the damper loss due to alternating current has been treated with close approximation by R. Rogowski, *Archiv. f. Elektrotech.* 1913, Vol. 2, p. 81. A formula for the ratio of a-c. loss to d-c. loss is given there so that the statement regarding the absence of a rigorous treatment of this subject does not seem correct.

As to the effect of the damper slots on telephone interference may I summarize some experiences I had in 1926 when I was associated with the Siemens-Schuckert-Werke in Vienna. In an unpublished paper I proved that minimum loss and minimum interference harmonics are two nearly opposing tendencies. For slot harmonics to be a minimum the voltage wave of the generator, I could set up the condition $0.62 < \frac{t_s}{t_r} < 0.70$ and experiments as well as careful investigation of many oscillograms showed satisfactory agreement. The damper losses, however, are not at the minimum within this range of the above ratio. Although it does not make any difference whether we write $\frac{t_s}{t_r}$ or the inverse value $\frac{t_r}{t_s}$, it seems to me that generally the stator slot pitch is made smaller than the damper slot pitch and I should like to know if Mr. Pollard purposely used $t_r < t_s$.

E. I. Pollard: Mr. Weber questions the accuracy of the

No-Load Conditions of Single-Phase Induction Motors and Phase Converters, A.I.E.E. Trans., 1918, Vol. 37, p. 539.

*E. Weber, *E.T.Z.*, 1928, Vol. 49, p. 858.

curves for Kw_1 and Kw_2 in Fig. 3. As stated, these curves were calculated from flux maps which Mr. Weber states "can be very inaccurate." Concerning the accuracy of flux maps, I refer Mr. Weber to reference 1 of the bibliography. Although I am not familiar with the derivation of Mr. Weber's equations, his relatively simple solution of a complex mathematical problem indicates that various simplifying assumptions must have been made. For instance, he neglects the effect of the ratio of slot width to gap length, as I did in Fig. 3. The assumption of a considerably different ratio of bs/g would in part account for the discrepancy, although the effect is secondary importance, except in extreme cases.

The article by Von W. Rogowski referred to by Mr. Weber considers the calculation of eddy current factors for various arrangements of coils and bars in open slots, as has been done by various other investigations, notably A. B. Field, B. Hague and Gilman. But does Rogowski take into account the effect upon field distribution and eddy factor of partially closing the slot above the bar? He does not consider the case of the round bar.

In general the rotor slot pitch is smaller than the stator slot pitch, except possibly in the case of small low-voltage machines. This is because a large number of bars is desirable from the standpoint of reduced rotor reactance and better power factor for induction motor performance of synchronous motors as pointed out by Mr. Hellmund. Also, if a smaller number of larger bars were used, they would occupy a greater depth in the pole head and pole tips, requiring a deeper pole head and resulting in increased pole tip leakage.

Sine-Wave Generators

Cylindrical Rotor Winding Connections and Analysis of Rotor M.M.F. Waves

BY LLOYD P. SHILDNECK*
Associate, A.I.E.E.

Synopsis.—*This paper describes and compares various types of double layer rotor winding connections for non-salient pole machines, with a particular application to sine-wave generators. Design coefficients are obtained by a mathematical analysis of the derived equations for the rotor m.m.f. Five specific types of connections, the V, Y, X, U, and W types, are discussed, as well as the general case. The new types advocated are seen to be superior to the old, giving a greater output for a given maximum temperature.*

GENERAL

CERTAIN applications, as iron testing, meter testing,[1] cable testing, etc., require a sine wave of generator voltage under all load conditions. The voltage wave form of an ordinary salient-pole generator is usually very good at no load, but becomes distorted under load. To overcome this inherent defect, the round-rotor type of sine-wave generator was developed. The stator of this type is very similar to that of a salient-pole generator. For the field, however, a round punched rotor is used with a double layer winding distributed in evenly spaced slots and connected to give nearly a sine wave of m.m.f.

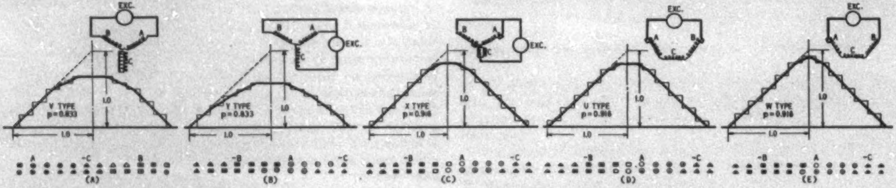

FIG. 1—ROTOR M.M.F. WAVES

Because of the equal permeances in both axes of this type, a sinusoidal air gap flux density wave is set up by any combination of sinusoidal rotor and stator m.m.f. waves. Thus a sine-wave generator has fundamentally three requirements: a round rotor, a sine wave of stator m.m.f., and a sine wave of rotor m.m.f. Of these, only the third requirement is discussed here, since the construction of the rotor is similar to that of a wound rotor induction motor, and the methods of producing a sinusoidal stator m.m.f. are well understood.[2,3,4]

The very useful, but rather tedious, *graphical* method is used to plot the flux density fields of salient-pole machines.[5] In round rotor machines, however, the permeance variable disappears, and the field m.m.f. and field flux density wave shapes are identical. Consequently, to determine the field flux density wave shape, it is but necessary to derive *analytically* the shape of the field m.m.f. wave. (The assumptions are: infinite permeability of the iron and infinite number of rotor teeth.) This method allows a direct determination for the maximum or minimum value of any coefficient, by equating the derivative to zero. The analytical method is general and applicable to all round rotor machines, but the analyses made in this paper are directly applicable only to round rotor machines with double-layer rotor coil windings. This includes the synchronous-induction motor and the sine-wave generator, but not the turbine generator.

DESCRIPTION OF TYPES OF ROTOR WINDING CONNECTIONS

A study of the two types of connections used heretofore (V and Y) revealed certain inherent undesirable features. Three new types (X, U and W) were developed as a direct result of eliminating these deficiencies.

V Type. This old type is obtained by winding the rotor with an ordinary three-phase double-layer winding, and connecting phases A and B in series with the exciter, as shown in Fig. 1A. Thus only two-thirds of the coils are used to produce the field m.m.f. The remaining coils are short-circuited individually to assist in damping out the backward rotating component of single-phase armature reaction. So

$$s = 0.333$$

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[1] For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

is the defining equation for this type, where s = the ratio of short-circuited to total coils. This type is inferior because an insufficient ratio of coils is used in the d-c. circuit.

Y Type. In the other old type, the three phases are connected to the exciter as shown in Fig. 1B. This type has good damping features (since polyphase currents can flow to damp out the backward rotating flux), but it has serious heating limitations. The third leg C, carrying full field current, has a heating four times as great as that of either leg A or B. The heating in this "bottle-neck" leg then definitely limits the output of the rotor. The Y type has another undesirable feature in that opposing currents in the top and bottom of the same slot are used to obtain the flat top of the m.m.f. wave. (See Fig. 1B.) This, of course, increases the I^2R loss.

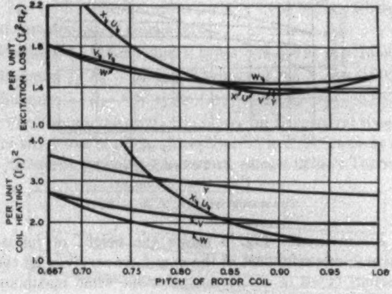

FIG. 2—FIELD EXCITATION LOSS ($I_f^2R_f$) AND HEATING (I_f^2) FOR UNIT FUNDAMENTAL

Five specific types (V, Y, X, U, W)

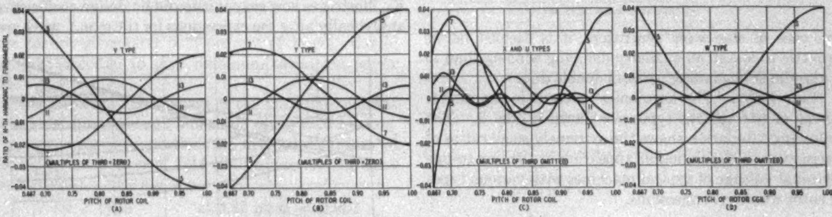

FIG. 3—HARMONICS IN ROTOR M.M.F. WAVES

Five specific types (V, Y, X, U, W)

X Type. This new type is a modification of the Y type with the following changes as shown in Fig. 1C.

Phase C is split into two parallel circuits, *thereby eliminating the bottle-neck* feature of the Y type; also the coils that carry opposing currents in the Y type are removed from the d-c. circuit, *thereby eliminating the unnecessary power loss.* From the beginning of each pole group in phase A (1-pitch) × (coils per pole) are taken and a like number from the end of each pole group in phase B, so that the two groups of short-circuited coils are adjacent. Thus

$$s = 2\,(1 - p)$$

is the defining equation for the X type, where p = rotor coil pitch.

U Type. This new type is a modification of the X type, obtained from it by joining the circuits in series, instead of in parallel, as shown in Fig. 1D. In practise this makes a very simple winding. All the coils in each pole pitch are placed in series; then those end coils which would oppose each other, are short-circuited individually. The defining equation for the U type is also

$$s = 2\,(1 - p)$$

W Type. This new type is the same as the U type, except just half as many coils are short-circuited. (See Fig. 1E.) The defining equation for this type is then

$$s = (1 - p)$$

The W type is a limiting case; any larger value of s would cause "opposing" coils, as in the Y type.

General Case. In the five specific types previously described, s is either a function of p or else is constant, enabling all the coefficients to be plotted in terms of the one variable p. In a study of the general case, however, both p and s are allowed to vary independently. Fig. 2 shows very strikingly the definite superiority of the V type (in which a series connection is used) over the Y type (in which one leg is placed in series with two paralleled legs). This superiority of the series type has also been proved by tests made by Messrs. Ganapati and Parikh.[6] Since a part series, part parallel type is inherently inferior to a series type, only the latter will be discussed in the general case. The general type of winding connections then results from placing all the coils under each pole in series, short-circuiting individually any fractional part s of them, and using any coil pitch p. It necessarily includes the V, U, and W types, but not the Y type.

It is always possible to obtain values

$$s = n\,(1 - p)$$

where n is any integer or zero. Under certain conditions of pitch, n may be fractional: $n = 1$ gives the W type; $n = 2$ gives the U or X type; all possible values of n are included in the general type.

DISCUSSION

The mathematical expressions for the m.m.f. waves of the various types of winding connections are analyzed by the method of Fourier series to obtain A_n, the value of the nth harmonic. The substitution $n = 1$ gives the fundamental (A_1) or useful part of the wave, which is proportional to the output of the rotor. The reciprocal of A_1 gives the field current (I_f) necessary to produce unit fundamental. From this value (I_f), the heating (I_f^2) and the excitation power loss ($I_f^2\ R_f$) are easily obtained. These values are listed in Table I.

Damping. The damping provided by the series types of winding connections (V, U, and W) is not very effective, because currents cannot flow in such windings so as to set up a revolving magnetic field of constant amplitude. The Y type and the X type are better fitted for damping out any backward rotating flux. However, in the case of sine-wave generators, it is deemed better practise to install an auxiliary damping winding than to make the field winding perform double duty.

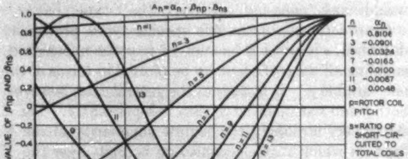

FIG. 4—MAXIMUM HEIGHT OF NTH HARMONIC (A_n)

(General type)

Wave Form. The harmonics* present in the m.m.f. waves of the various types are shown in Fig. 3. The following tabulation shows the extremely small ratios of harmonics to fundamental appearing in the line-to-line terminal voltage of typical machines with various types of rotor winding connections.

	Y type Stator pitch = 0.833 Stator slots = 72 Rotor pitch = 0.800	X or U type Stator pitch = 0.800 Stator slots = 60 Rotor pitch = 0.928	W type Stator pitch = 0.81 Stator slots = 84 Rotor pitch = 0.889
A_3/A_1	0	0	0
A_5/A_1	0	0	0.00027
A_7/A_1	0.00047	0.00003	−0.00016
A_9/A_1	0	0	0
A_{11}/A_1	0.00081	0.00026	−0.00009
A_{13}/A_1	0.00030	0.00004	−0.00014
A_{15}/A_1	0	0	0

The curves in Fig. 4 are rather unique in that they give A_n, the value of the nth harmonic, for the general type winding connections, for any value of (n) from 1 to 13, any value of (p) from 0.65 to 1.00, and any value of (s) from 0 to 0.35, *without interpolation.* This is obtained by using a double abscissa, as shown.

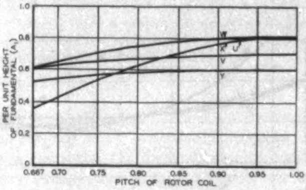

FIG. 5—HEIGHT OF FUNDAMENTAL (A_1) FOR UNIT COIL CURRENT

Five specific types (V, Y, X, U, W)

Fundamental. Fig. 5 shows the height of fundamental, or useful part of the m.m.f. wave, given by the various types of connections, for the same maximum current flowing in any part of the winding. A comparison at the pitches necessarily used to give good wave forms shows a very decided advantage in favor of the new types of connections (X, U, W).

A_1 for	Y type	at 0.82 pitch	=	0.584
A_1 "	V type	" 0.82 "	=	0.674
A_1 "	X and U type	" 0.93 "	=	0.786
A_1 "	W type	" 0.90 "	=	0.790

Thus the new types have a greater output for the same maximum temperature.

K_m and Air-Gap Ampere-Turns. The derivation of K_m illustrates how easy it is to obtain design coefficients analytically after the expressions for the m.m.f. wave are set up. K_m is simply the ratio of the maximum to the average height of the m.m.f. wave. This coefficient is very useful in determining the maximum air-gap flux density, which is proportional to the air-gap ampere-turns. The values of K_m for the general type are given in Fig. 6.

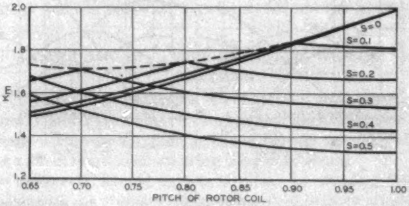

FIG. 6—$K_m = \dfrac{\text{MAXIMUM FLUX DENSITY}}{\text{AVERAGE FLUX DENSITY}}$

(General type)

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*The third and multiples of the third are omitted in the figures for the X, U, and W types, since the Y connection of the stator eliminates them from the line-to-line terminal voltage.

Excitation Power Loss ($I_f^2 R_f$). Fig. 2 shows the power required for unit fundamental with the various specific types of connections. A comparison at the pitches to give good wave form shows that the new types require slightly less power.

$I_f^2 R_f$ for Y and V types at 0.82 pitch = 1.47
$I_f^2 R_f$ " X and U types " 0.93 " = 1.39
$I_f^2 R_f$ " W type " 0.90 " = 1.44

The minimum excitation for any type occurs where the first derivative equals zero.

Minimum $I_f^2 R_f$ for Y and V types = 1.356 at $p = 1.000$
Minimum $I_f^2 R_f$ " X and U types = 1.385 " $p = 0.902$
Minimum $I_f^2 R_f$ " W type = 1.445 " $p = 0.887$

Fig. 7 shows the excitation power for the general type. For any value of pitch, the excitation loss decreases as s is increased, reaching a minimum at $s = 0.26$. Therefore, for any given pitch p, 26 per cent of the coils should be short-circuited to give minimum excitation power loss.

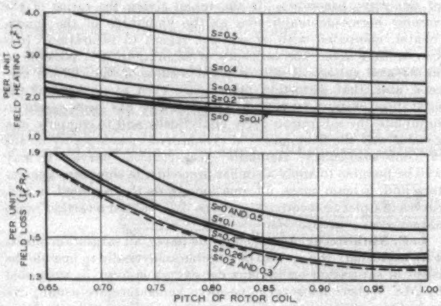

FIG. 7—FIELD HEATING (I_f^2) AND EXCITATION LOSS ($I_f^2R_f$) FOR UNIT FUNDAMENTAL

(General type)

Field Heating. The square of the maximum current in any slot gives a rough indication of the field heating. The new types of connections are decidedly superior to the old in the matter of field coil heating for the same fundamental. (See Fig. 2.)

I_f^2 for Y type at 0.82 pitch = 2.932
I_f^2 " V type " 0.82 " = 2.200
I_f^2 " X and U types " 0.93 " = 1.620
I_f^2 " W type " 0.90 " = 1.601

To obtain low coil temperature, it is seen from Fig. 7, general type, to be necessary to use a very low value of (s). The high values of pitch (and consequently, low values of s) permissible with the new types of winding account for the cool winding. Since field temperature usually limits the output of a machine, this low field heating constitutes the major superiority of the new types over the old.

CONCLUSIONS

1. Three new types of connections, X, U, and W, have been developed and shown to be superior to the old types, V and Y. By their use 15 per cent or more increase in output can be secured from the rotor without increasing the maximum temperature rise, or impairing the voltage wave shape. The Y type is decidedly inferior to the other types, because of its high maximum temperature rise.

2. The analysis made in this paper is quite general and is directly applicable to all round-rotor synchronous machines with double-layer rotor coil windings. Sine-wave generators and synchronous-induction motors are good examples of this type.

TABLE I

	A_n(For Unit I_f)	I_f(For Unit A_1)
(V)	$(8/n^2\pi^2)(\cos n\pi/6)(\sin n\pi p/2)$	$1.425/(\sin \pi p/2)$
(Y)	$(8/n^2\pi^2)(\cos^2 n\pi/6)(\sin n\pi p/2)$	$1.648/(\sin \pi p/2)$
(X)	$(8/n^2\pi^2)(-\cos n\pi p)(\sin n\pi p/2)$	$2.470/(-\cos \pi p)(\sin \pi p/2)$
(U)	$(8/n^2\pi^2)(-\cos n\pi p)(\sin n\pi p/2)$	$1.235/(-\cos \pi p)(\sin \pi p/2)$
(W)	$(8/n^2\pi^2)(\sin n\pi/2)(\sin^2 n\pi p/2)$	$1.235/(\sin^2 \pi p/2)$
(G)	$(8/n^2\pi^2)(\cos n\pi s/2)(\sin n\pi p/2)$	$1.235/(\cos \pi s/2)(\sin \pi p/2)$

	$I_f^2 R_f$ (For Unit A_1)
(V) and (Y)	$1.356/(\sin^2 \pi p/2)$
(X) and (U)	$1.525\ (2p - 1)/(\cos^2 \pi p)\ (\sin^2 \pi p/2)$
(W)	$1.525\ (p)/(\sin^4 \pi p/2)$
(G)	$1.525\ (1 - s)/(\cos^2 \pi s/2)\ (\sin^2 \pi p/2)$

	K_a	K_m
(V)	$1.21/(\sin \pi p/2)$	$12/[8 - 9(1 - p)^2]$
(Y)	$(3p+1)/(9 \sin \pi p/2)$	$3(3p+1)/[7 - 9(1-p)^2]$
(X)	$(\pi/2)(2p-1)/(-\cos \pi p)(\sin \pi p/2)$	$2(2p-1)/[1 - 5(1-p)^2]$
(U)	$(\pi/2)(2p-1)/(-\cos \pi p)(\sin \pi p/2)$	$2(2p-1)/[1 - 5(1-p)^2]$
(W)	$(\pi/2)(p)/(\sin^2 \pi p/2)$	$2(p)/[1 - 2(1-p)^2]$
(G-1)	$(\pi/2)(1-s)/(\cos \pi s/2)(\sin \pi p/2)$	$2(1-s)/[(1-s^2) - (1-p)^2]$
(G-2)	$(\pi/2)(p)/(\cos \pi s/2)(\sin \pi p/2)$	$2(p)/[(1-s^2) - (1-p)^2]$

	(M.M.F. wave (For unit I_f)
(V)	$[x]_0^{x_1}$; $[x/2 + (p/2 - 1/6)]_{x_1}^{x_2}$; $[2/3]_{x_2}^{1}$
(Y)	$[x]_0^{x_1}$; $[3x/4 + (p/4 - 1/6)]_{x_1}^{x_2}$; $[x/2 + 1/6]_{x_2}^{x_3}$; $[p/2 + 1/6]_{x_3}^{1}$
(X)	$[x]_0^{x_1}$; $[x/2 + (3p/2 - 1)]_{x_1}^{x_2}$; $[2p - 1]_{x_2}^{1}$
(U)	$[x]_0^{x_1}$; $[x/2 + (3p/2 - 1)]_{x_1}^{x_2}$; $[2p - 1]_{x_2}^{1}$
(W)	$[x]_0^{x_1}$; $[x/2 + (2p - 1)/2]_{x_1}^{1}$
(G-1)	$[x]_0^{x_1}$; $[x/2 + (p - s)/2]_{x_1}^{x_2}$; $[1 - s]_{x_2}^{1}$
(G-2)	$[x]_0^{x_1}$; $[x/2 + (p - s)/2]_{x_1}^{x_2}$; $[p]_{x_2}^{1}$

	x_1	x_2	x_3	y_1	y_2	y_3
(V)	$p - 1/3$	$5/3 - p$	–	$p - 1/3$	$2/3$	–
(Y)	$p - 2/3$	$4/3 - p$	p	$p - 2/3$	$5/6 - p/2$	$1/6 + p/2$
(X)	$(3p - 2)$	p	–	$3p - 2$	$2p - 1$	–
(U)	$(3p - 2)$	p	–	$3p - 2$	$2p - 1$	–
(W)	$2p - 1$	1	–	$2p - 1$	p	–
(G-1)	$p - s$	$2 - p - s$	–	$p - s$	$1 - s$	–
(G-2)	$p - s$	$p + s$	–	$p - s$	p	–

(G-1)—General type, region I, where $1 \geqq p \geqq (1 - s)$
(G-2)—General type, region II, where $(1 - s) \geqq p \geqq s$.

3. All practical methods of making rotor coil connections have been considered, and the final results of the study listed in Table I. This comprehensive list of analytical expressions for design coefficients has been partially expanded into curve form for greater ease in making comparisons. By virtue of the information given in this table, it is now possible to choose the proper values of p and s to fit any particular case.

SYMBOLS

A_1 = maximum height of fundamental of rotor m.m.f. wave.
A_n = maximum height of nth harmonic of rotor m.m.f. wave.
I_f = field current (= coil current in series type).
K_a = ratio of fundamental armature flux to fundamental field flux for same peak values of m.m.f.
K_m = ratio of maximum to average air gap flux density.
p = rotor coil pitch.
R_f = resistance of the field winding.
s = ratio of short-circuited rotor coils to total coils.
x_1, x_2, x_3 = x-coordinates of the first, second, and third points of discontinuity of the slope of the m.m.f. wave.
y_1, y_2, y_3 = y-coordinates of the above three points.

UNIT QUANTITIES

Unit ampere-turns = 1 ampere-turn
Unit field current = 1/(turns per pole) amperes.
Unit number of coils = coils per pole.
Unit resistance = resistance of all coils in series.
Unit distance along X-axis = (pole pitch)/2.

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Discussion

Ernst Weber: Although the author emphasizes in the first sentence of his paper the need for a sinusoidal wave form of the generated voltage under all load conditions, he does not mention the influence of saturation on the voltage curve in the new types of sine-wave generators. Is the table, giving the ratios of the various harmonic amplitudes to the amplitude of the fundamental, computed with or without regard to saturation? It is known that under the influence of saturation, it is possible to secure sine voltage at only a definite value of the exciting current, and, that distortion takes place even at no load, if the excitation current is varied. The attempt has been made to formulate the saturation curve analytically and to compute in a general way the field curves of synchronous machines by means of using a so-called "standard" magnetization curve.* It may well be possible to apply a similar procedure to sine-wave generators and to gain more information as to the required m.m.f. curves in order to secure sine voltage over at least a certain range of excitation current.

L. P. Shildneck: As stated in the paper, all saturation effects were neglected. It may well be possible analytically to include the effect of saturation on the flux density wave form, as suggested by Mr. Weber. However, sine-wave generators are usually required to operate over a wide range of conditions, and saturation effects must be reduced to a minimum. This is done by making the air-gap length relatively large, and the flux density in the teeth relatively low. Thus for sine-wave generators of current design it was considered unnecessary to include the effect of saturation.

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*"The Magnetic Field Forms of Synchronous Machines Under No-Load Condition," E. Weber, *Archiv. f. Elektrotech.*, 1927, Vol. 19, p. 193.

Engineering Features of Three-Power Locomotives

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Synopsis.—*Three-power locomotives capable of operating from an external distribution system or on internal power are in use by the New York Central and the Delaware, Lackawanna and Western Railroads. In one case the supply voltage is 600 volts direct current and in the other 3,000 volts direct current.*

The authors discuss the characteristics of this new type of locomotive and its equipment, as well as the methods of calculating the capacity and characteristics required for the service. The coordination of the diesel engine, generator, battery, and motor equipment has been carefully worked out to insure successful operation of the complete unit.

* * * * *

GENERAL

THE three-power electric locomotive operates from any of the following energy sources:

1. External power; that is, trolley or third rail.
2. Internal power from a storage battery.
3. Internal power from a storage battery and engine-driven generator connected in parallel.

Ability to operate from these power supplies gives the three-power locomotive an outstanding degree of flexibility.

Regardless of the system voltage upon which the three-power locomotive is designed to work, the general scheme of operation is the same. When using external power, the unit performs exactly as a straight electric locomotive, although the engine-generator set may be run to charge the battery. On non-electrified tracks power may be taken from the battery alone. However, if the engine is running, traction and auxiliary power are taken entirely from the engine-generator set up to its capacity above which the battery automatically assumes the excess load. On light loads, the engine-generator may run the locomotive and charge the battery at the same time.

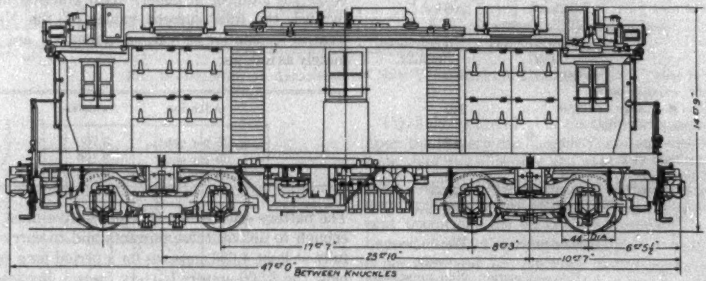

FIG. 1—OUTLINE OF NEW YORK CENTRAL LOCOMOTIVE

The New York Central Railroad tried out the first three-power locomotive in 1928, and in 1930 put 35 units into transfer and switching service on the West Side in connection with the complete elimination of steam service from Manhattan Island. The Delaware, Lackawanna & Western Railroad put two three-power locomotives into similar service in 1930 to help eliminate steam from the Bergen Tunnels with the advent of electrified suburban service from Hoboken.

Fig. 1 shows outline dimensions of the New York Central locomotive and Fig. 2 the location of apparatus in the cab. The Delaware, Lackawanna & Western locomotive is of similar mechanical and electrical design. Table I gives the pertinent data on the two locomotives and shows wherein they differ, due largely to the difference in voltages.

GENERAL REQUIREMENTS FOR INTERNAL POWER PLANT

The average power required for switching service is small while the peak demands are relatively large. The theory upon which the internal power operation of the three-power locomotive is based is that the battery will provide the excess power required for the peak demands, allowing the engine size to be cut to a minimum. Extensive tests on internal power were run in 1928 on the first New York Central three-power locomotive to determine the actual extent of these power

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Presented at the Winter Convention of the A.I.E.E., New York, N.Y., January 25-29, 1932.

TABLE I—DATA

	New York Central	Delaware, Lackawanna & Western
Weight of locomotive in running order	257,000 lb.	248,000 lb.
Rated voltage on external power	600 (3rd rail)	3,000 (trolley)
Continuous horsepower on external power	1,300	1,450
Hourly horsepower on external power	1,665	1,600
Continuous tractive effort	24,600 lb.	22,200 lb.
Hourly tractive effort	34,100 lb.	25,200 lb.
Speed on external power at continuous tractive effort	19.8 mi. per hr.	24.5 mi. per hr.
Speed on external power at hourly tractive effort	18.3 mi. per hr.	23.8 mi. per hr.
Maximum tractive effort (25 per cent adhesion)	64,250 lb.	62,000 lb.
Traction motors:		
Number	4	4
Type	Axle hung	Axle hung
Voltage	600	1,500/3,000
Gearing	72/17	72/17
Ventilation	Forced	Forced
Oil Engine:		
Type	I-R solid injection	I-R solid injection
Horsepower	300	300
Traction generator	200 kw.	200 kw.
Battery:		
Type	Exide-Ironclad TL-27	Exide-Ironclad MVA-21
Number of cells	240	360
Ampere-hr., capacity at 6 hr. rate of discharge	650	340
Kw-hr. capacity at 6 hr. rate of discharge	301	242
Average voltage at 6 hr. rate of discharge	464	712
Connection of cells	Series	Series
Grounded at	Negative	Negative
Control:		
Type	Electro-pneumatic Non-automatic	Electro-pneumatic Non-automatic
Motor combinations on external power	3	2
Motor combinations on internal power	3	3

requirements in heavy switching service. The average for the maximum observed day for three-shift service was approximately 500 kw-hr. per eight-hour shift, or 62.5 kilowatts average load. The average work for 65 shifts of 3-shift service was 447 kw-hr. per shift. Frequently peak loads of 800 kilowatts were recorded, or more than twelve times the maximum observed average load per eight-hour shift.

For long battery life and reasonable maintenance the battery discharge should be limited to 125 per cent of the total capacity at the six-hour rate for each twenty-four hours the locomotive is operating in three-shift service. To limit the battery discharge to this value successfully, it is essential that the correct generator characteristic be used. This characteristic should be high enough and of the proper shape to insure that the engine carries the entire load up to the engine-generator capacity, but not high enough to allow the battery to recharge so rapidly that the engine must be shut down frequently to prevent overcharging. Without the engine, the battery must supply the entire load, which results in the discharge over a twenty-four hour period being considerably greater when the engine is shut down often than with the engine running most of the time. The initial locomotive tests showed that the engine should be run at least 70 per cent of the time to meet these discharge limitations.

A generator characteristic low enough to insure engine operation 70 per cent of the time is too low to give the battery an equalizing charge. Therefore, two generator characteristics are necessary; first, a normal characteristic for regular operation; and second, a higher or equalizing characteristic for use periodically for giving the battery a gassing charge.

The minimum size of the battery is fixed by traction motor characteristics and weight of the locomotive on one hand and speed requirements on the other. The former determine the maximum current which may be drawn from the battery as limited by adhesion, and the latter the number of cells. With traction motors in parallel, the total current values on the New York Central locomotive at various adhesions are approximately as follows:

Adhesion	Amperes
25 per cent	3,700
20 per cent	3,100
15 per cent	2,500

The battery selected for this locomotive must be large enough to deliver these currents and to carry successfully at least 4,000 amperes for a period long enough to blow the 2,500-ampere battery fuses. Because of the decreased available capacity of the battery at high discharge rates, it is not practical to operate this type of locomotive on internal power at high adhesion and high speed for any great length of time. However, the extreme load conditions given above are possible and must be successfully met for short periods in practical operation.

In case of necessity it is possible to operate the three-power locomotive at high adhesion over considerable distances by running at reduced speeds with the motors in series or series-parallel. In such cases the current to the motors is reduced to ¼ or ½ of the value which would be required with the motors in parallel for the same total tractive effort. As the engine-generator set will continue to deliver its full power, all of this reduction in current is taken from the battery discharge rate, thus greatly increasing its available capacity. In addition to this, the increase in available kw-hr. capacity of the battery increases the duration of the movement and to this extent increases the kilowatt hours delivered by the engine, thus further increasing available mileage.

For fast and responsive operation of the locomotive it is advantageous to have the battery voltage near the rated voltage of the traction motors.

Battery short time current rating—2 minutes—3,000 amperes.
Engine horsepower—300.
Assumed time engine running—85 per cent.
Maximum amount battery may be discharged in 24-hour period, 125 per cent of capacity at 6-hour rate which is equivalent to the following:
  812 ampere hours in 24 hours.
  271 ampere hours per 8-hour shift.
  114 kilowatt hours per 8-hour shift, (average discharge voltage 420).

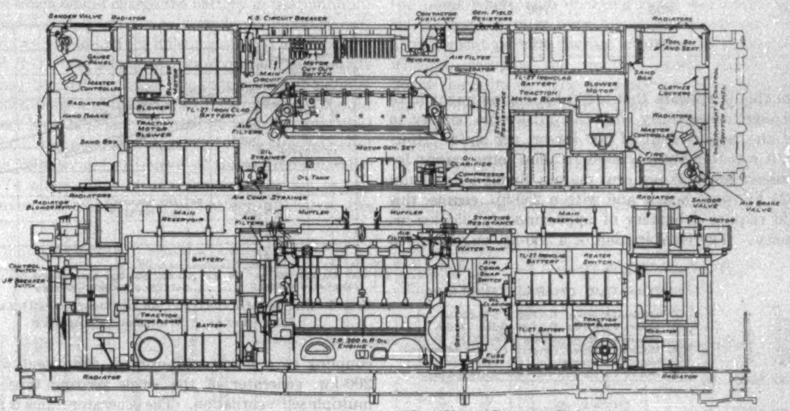
Sander Valve
Radiator
K.S. Circuit Breaker
Contactor Auxiliary
Gen. Field Resistors
Radiators
Gauge Panel
Master Controller
Radiators
Hand Brake
Radiators
Sand Box
Blower Motor
Blower
Traction Motor Blower
TL-27 Iron Clad Battery
Main Circuit Contactors
Motor Cut Out Switch
Reverser
Air Filter
Generator
Air Filters
Starting Resistance
TL-27 Ironclad Battery
Traction Motor Blower
Blower Motor
Tool Box And Seat
Sand Box
Clothes Lockers
Radiators
Master Controller
Instrument & Control Switch Panel
Fire Extinguishers
Oil Strainer
Oil Tank
Motor Gen. Set
Oil Clarifier
Compressor Governor
Air Comp Strainer
Radiator
Sander Valve
Air Brake Valve
Radiators
Radiator Blower Motor
Main Reservoir
Air Filters
Muffler
Muffler
Starting Resistance
Air Filter
Main Reservoir
Motor
Water Tank
Control Switch
J.R. Breaker Switch
Battery
Traction Motor Blower
Battery
Air Comp Snap Switch
TL-27 Ironclad Battery
Heater Switch
I.R. 300 H.P. Oil Engine
Generator
Oil Clarifier Sw.
Fuse Boxes
TL-27 Battery
Traction Motor Blower
Radiator
Radiator
Radiator

Fig. 2—Location of Apparatus on New York Central Locomotive

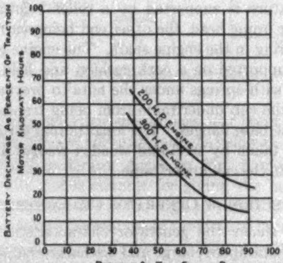

Fig. 3—Relation of Battery Discharge to Time of Engine Operation

Curves showing time oil engine is operated plotted against battery discharge as per cent of motor kilowatt-hours for initial New York Central locomotive in heavy switching service

SELECTION OF INTERNAL POWER PLANT FOR THE NEW YORK CENTRAL LOCOMOTIVE

As a result of the initial New York Central locomotive tests and the weight and speed considerations just outlined, a design for the New York Central locomotive was adopted to which the following data apply:
(See Table I also.)

Fig. 3 shows the effect of the time the engine operates upon the battery discharge. These curves are plotted from data taken during the tests on the initial New York Central locomotive. For the 300-hp. curve a number of points was secured between 45 and 75 per cent operation, the curve being extended somewhat beyond these values upon reasonable assumptions for 0 per cent and 100 per cent engine operation. The 200-hp. engine curve is based largely on assumptions since only one point (93 per cent) was available from the test data, this test being made by setting the fuel finger of the oil engine governor to limit the engine output to 200 hp. It is apparent, however, that this curve will intersect the ordinate with the engine running 0 per cent of the time at the same point as the curve for the 300-hp. engine, and, hence, should draw nearer to the latter curve as the per cent of the time the engine runs is decreased. Therefore, it is believed that the curve approximately represents the condition that would exist for a 200-hp. engine.

Fig. 3 shows that with a 300-hp. engine running 85 per cent of the time, the battery discharge will be equivalent to approximately 15 per cent of the power delivered to the traction motors. Therefore, the maximum average work which the locomotive can do per 8-hour shift when operating 3 shifts per day on internal power is: 114/0.15 = 760 kilowatt hours. Using Fig. 3 and interpolating for the 250-hp. engine, the following maximum kilowatt hours per shift may be calculated for different engine sizes using the present 301-kw-hr. battery:

Engine hp.	Maximum kw-hr. per shift
300	760
250	531
200	403

From this it appears that a 250-hp. engine might have been used successfully with the 301-kw-hr. battery to meet the requirement of 500 kw-hr. per shift. However, it was thought advisable to allow some margin for contingencies, a 25 per cent margin giving a maximum of 625 kilowatt hours, and with a 250-hp. engine, this would require a battery of approximately 355-kw-hr. capacity. The combination of a 300-hp. engine and a 301-kw-hr. battery was finally chosen because of the greater margin of capacity per shift as well as the ability to use an oil engine which had been thoroughly proved in railway service.

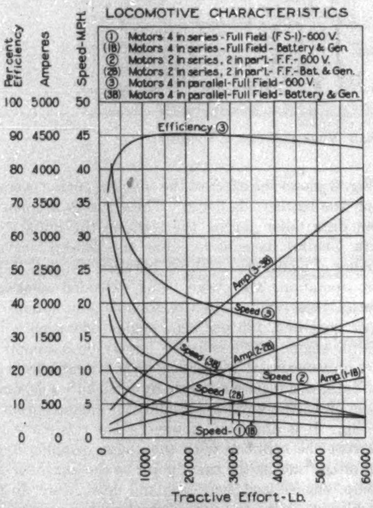

Fig. 4—Speed-Tractive Effort Characteristics of New York Central Locomotive

Fig. 4 shows locomotive characteristics on both external and internal power.

Description of Internal Power Plant of New York Central and Delaware, Lackawanna & Western Locomotives

The oil engine is the Ingersoll-Rand 300-hp., vertical, 6-cylinder, 4-stroke cycle, trunk piston, single acting type with direct fuel injection. The engine is controlled by a constant speed governor which regulates between 550 r.p.m. full load and 575 r.p.m. no load. The engine is directly connected through a flexible coupling to a 6-pole, self-excited, compound wound, 200-kw. generator of the single bearing type, with multiple self-ventilation. The generator frame is bolted to a bracket on the engine and is also supported by springs resting on the floor. The commutator end of the armature is supported by a roller bearing in the generator frame head, the other end being carried by a pilot bearing in the engine shaft. The engine-generator unit is supported on a cork cushion and held in place by bolts with springs under the nuts to prevent deflections of the cab underframe from being transmitted to the engine base flanges. The longitudinal thrust of the engine is taken by steel and wooden wedges and shear lugs cast in the cab underframe.

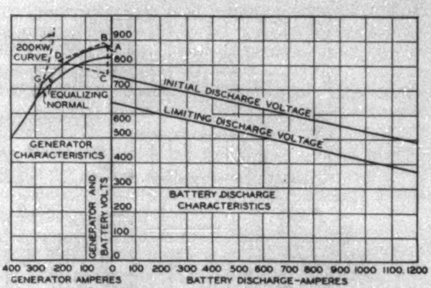

Fig. 5—Battery-Generator Characteristics of Delaware, Lackawanna & Western Locomotive

Selection of Generator Characteristic

Fig. 5 shows the generator characteristic and the battery voltampere charge and discharge characteristics of the internal power plant of the Delaware, Lackawanna & Western locomotive. These characteristics are representative of three-power locomotive practise and will be explained. When the battery is fully charged, any rate of charge however small will produce gassing, and if the charging rate be high enough, gassing will occur with the battery in any state of charge, although the rate may be reduced during the latter part of charge to a value which, unless abnormally sustained or occurring too frequently, will produce so small an amount of gassing as to be practically harmless. This value is called the "finishing rate" and is point *B* in Fig. 5. The charging rate at the end of charge should be confined between the finishing rate and one-half of the finishing rate. Consequently, line *A B* represents the voltampere charge characteristic of the battery at finish of charge condition. The ability of the battery to absorb high charging rates at the start of charge is indicated by point *D* which is the maximum charging rate recommended, and hence line *C D* is the voltampere charge characteristic at start of charge condition. Line *D B* represents the theoretical generator characteristic which would charge the battery in the minimum time and *A C* that which would produce a constant current charge at one-half the finishing rate. Therefore, the area *A B D C* includes all values of current and voltage that may be used to charge this battery under permissible conditions and the generator characteristic should intersect lines *A B* and *C D* and lie between lines *B D* and *A C*. The generator characteristic is also restricted by the 200-kw. output curve of the engine-generator set. To allow delivery of approximately full engine load over the whole range of battery discharge, the generator characteristic should pass near point *G* and lie close to the 200-kw. curve at voltages below G.

The higher generator curve in Fig. 5 meets these requirements and is suitable for giving the battery an equalizing charge, but is too high to permit the engine to run a sufficient percentage of the time to keep battery discharge within the daily limit of 125 per cent of rated capacity without overcharging the battery. Therefore, the lower curve is used for normal charging and the higher characteristic for periodic equalizing charges. The charging rate at the end of charge, when using the normal characteristic, is less than one-half the finishing rate, but with a properly shaped characteristic and the engine running almost continuously, the battery will be maintained between equalizing charges in a satisfactory state somewhat below full charge. Use of the low generator curve for normal charging has the further advantage of leaving margin for variation of characteristic between inspections without running the risk of overcharging the battery seriously, such as would happen, for example, if the engine speed increased and the equalizing characteristic were being used in daily operation.

The control is arranged so that whenever the locomotive is operating on internal power and *the traction motors are connected to the battery generator bus*, the higher characteristic is automatically brought into operation. This is permissible because usually the traction motor load is sufficient to hold the generator voltage below the point where high charging rates will occur unless the battery is in a very low state of charge. This arrangement permits the use of a more drooping curve without dropping the traction power characteristic below the point *G*. The necessity for a drooping curve may be better appreciated when it is considered that the battery is intended to deliver only 42 per cent of its capacity per 8-hour shift, whereas a generator characteristic lying along the line *B D* would give the battery a complete charge in 4 hours if continuously applied from full discharge and neglecting the small auxiliary load.

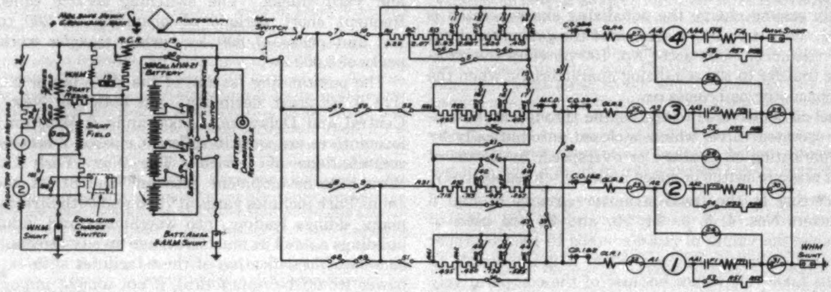

FIG. 6—POWER CIRCUIT CONNECTIONS OF DELAWARE, LACKAWANNA & WESTERN LOCOMOTIVE

CONTROL OF THE INTERNAL POWER PLANT

Both the New York Central and Delaware, Lackawanna & Western locomotives have a three-speed control on internal power. The series or lowest speed connection develops maximum tractive effort with minimum current from the battery-generator unit, allowing most slow switching movements to be made almost entirely on the engine-generator set without using battery current.

Fig. 6 shows the power circuit connections of the Delaware, Lackawanna & Western locomotive. Contactors Nos. 7, 8, 9 and 10 connect the traction motors to external power, and Nos. 47, 48, 49 and 50 to internal power. On both the New York Central and Delaware, Lackawanna & Western locomotives, transfer from external to internal power is controlled by a potential relay and is automatic upon loss of external power, although the automatic feature is selective on the New York Central locomotive because of interruption of power on third rail gaps when transfer to internal power is not desired. On both locomotives, transfer from internal to external power is accomplished by turning the controller handle to the off position and then notching out, necessitated because of going from a lower to a higher voltage power source. Indicating lights show whether operation is from internal or external power and whether collector shoes or pantograph are in contact with the third rail or trolley wire. The battery may be broken up into four sections to limit the maximum voltage to ground when doing any work in the battery compartments. The engine is started by motoring the generator from the battery through contactor No. 20 and the starting resistance; maximum field for starting is secured by short-circuiting the generator shunt field resistance with contactor No. 101. The reverse current relay controls contactor No. 19 and connects the generator to the battery when the proper generator voltage has built up; it likewise disconnects the generator should the generator voltage drop below that of the battery due to loss of engine speed, generator field, etc. To give the battery its weekly gassing charge the equalizing charge switch is closed, short-circuiting a section of the generator shunt field resistance. Contactor No. 100 provides the automatic transfer to the equalizing characteristic when the traction motor load comes on.

Fuel oil is delivered to the engine through an electrically operated valve, which is closed automatically by low lubricating oil pressure or overspeed, by means of an oil pressure switch or speed limit switch respectively.

Referring to the traction motor circuits in Fig. 6 contactors Nos. 4, 5, 6, 34, 35, and 36 are used to rearrange the values of resistor steps to secure proper operation on internal power, when it is impossible to use the 3,000-volt values because of the comparatively low battery voltage. The same rheostat values are used on both internal and external power on the New York Central locomotive because the voltages are near enough together. The control of the battery-generator combination is almost identical with that on the Delaware, Lackawanna & Western locomotive.

AUXILIARIES

Auxiliaries on the New York Central locomotive operate automatically from the third rail when available, but on the Delaware, Lackawanna & Western locomotive these run at all times on internal power as this scheme is simpler and less expensive than furnishing auxiliaries for 3,000-volt operation.

Operating cabs are heated by hot water from the oil engine cooling system. Electric immersion heaters operating from third rail are used on the New York Central locomotive to heat the cooling system water for cab heating as well as prevention of freezing when working on external power. The Delaware, Lackawanna & Western locomotive has 3,000-volt heaters for trolley operation.

SERVICE CONDITIONS

Tests on the first New York Central locomotive were confined to internal power operation and the results of these tests have been covered under *General Requirements of Internal Power Plant.*

Tests were made on the Delaware, Lackawanna & Western locomotives in a combination of transfer and switching service. The transfer work consists of hauling heavy trains at relatively high speed on external power between Secaucus and Jersey City Yards, a distance of 3½ miles. The switching, done entirely on internal power, consists of hauling these trains into the yards, partially breaking them up, making up outgoing trains, charging the train line, and hauling them to the electrified tracks. The ratio of the average power required in the switching work to that in the transfer work proved to be approximately 50 kw. to 450 kw. or 1 to 9. It is believed that this is the first case where actual data have been obtained which show the relative amount of work done on the main and yard tracks. The switching service called for frequent short periods of operation at 200 to 400 kw. and peaks of 800 kw., while transfer work had peaks of 3,000 kw.

The outstanding feature of the three-power locomotive is its great flexibility. On both the New York Central and Delaware, Lackawanna & Western these locomotives are used in different classes of service and combinations of service. The New York Central West Side development between 72nd Street and St. Johns Park includes yards at 72nd and 30th Streets with many sidings leading into warehouses and industrial buildings as well as much trackage on city streets. The complete electrification of these facilities with external power would be impractical, if not almost impossible; yet the service demanded a self-propelled locomotive capable of handling the heaviest type of switching and short transfer work, as well as road freight and transfer service on third rail. The three-power locomotive was developed to meet these requirements.

This locomotive has a field of its own and is not intended generally to supplant the straight oil or gas electric unit. The work of the locomotive when operating on internal power must be sufficiently intermittent to allow the battery to be maintained in a satisfactory state of charge. Heavy switching and short heavy transfer work on internal power or a combination of both, together with external power operation, offer conditions for which this type of locomotive is best suited. The internal power feature makes this type approach a "universal" locomotive for road freight, transfer, and switching service on sections of road which, for economic or practical reasons, could not be completely electrified.

ACKNOWLEDGMENT

The authors gratefully acknowledge the assistance rendered by Mr. W. S. H. Hamilton of the New York

Central Railroad and Mr. D. P. Orcutt of the Electric Storage Battery Company in the preparation of this paper. The original analysis of the tests on the initial New York Central locomotive by Messrs. Hamilton and Orcutt has been largely drawn upon. The discussion of the selection of generator characteristics is based on an analysis of this subject by Mr. J. H. Tracy of the Electric Storage Battery Company, to whom thanks are extended for the use of this material as well as valuable comments.

Discussion

W. S. H. Hamilton: This discussion is confined to the three-power locomotives used by the New York Central around New York City.

Some question has been raised as to the size of internal power plant required on these locomotives and also whether with the present power plant they are capable of meeting emergency conditions.

The size of the internal power plant was based on the requirements in our 60th Street yard where there is heavy switching and float work. It was estimated that to do the work quickly, internal power equivalent to 900 engine hp. should be available. This makes the locomotives "snappy" even in heavy work and lets them kick and drop cars without difficulty.

It should be borne in mind that while switching service averages only about 60 kw-hr. per hour input to the traction motors, peak inputs of 600-800 kw. are frequently encountered. They are of short duration and do not represent many kw-hr. in the course of a trick. While the lack of this amount of power would only make a small difference in time in each movement, where it was required, at the same time these small differences are cumulative and have an effect on the total work done in the trick. Furthermore, in congested yards slowing down one movement even a little usually affects movements of two or three other locomotives.

There are other locations on the West Side where this amount of power is not required and where a lighter locomotive could be used but it was desired to have all locomotives alike to increase the utilization, decrease the number of spares, and relieve the yard forces of having to worry about which locomotive to get ready for each job.

As to the adequacy of the internal power plant to stand overloads or conditions not anticipated, I wish to point out that from August 1930 until June 1931, these locomotives handled 2,000-ton trains in regular road service on internal power from 72nd Street to Spuyten Duyvil a distance of about 8 miles. This should be ample demonstration of their ability, as they are not intended regularly to handle such service and with the installation of third rail down to 72nd Street, this service has been discontinued.

The assignment of these locomotives is approximately as follows:

30th Street Yard	11
60th Street Yard	11
Hudson Side-Electric Division	8
Harlem Side-Electric Division	3
In Shop	3
Total	36

The locomotives at 30th Street operate entirely on internal power, those at 60th Street mostly on internal power and the others mostly on third rail. Those assigned to the Hudson and Harlem Sides of the Electric Division are used for express trains, way freight trains, and as traveling switchers. These latter do the industrial switching, picking up and dropping cars at various stations on internal power and then run to the next station on external power. There is a number of industrial plants where it would be very difficult to install either third rail or trolley and yet a locomotive switching them must be capable of moving rapidly between stations in order to keep out of the way of other traffic.

For such service the three-power type of locomotive is ideally suited and is a very economical solution of the problem. It should be pointed out that at the continuous rating of the traction motors, these locomotives develop 1,300 hp. at the wheel rim on external power, whereas on internal power they develop only 682 hp. To duplicate the amount of power developed on external power, a 1,600-hp. engine would be required.

For the year 1931, the whole fleet of 36 locomotives was in service with crews on 61 per cent of the time, and available for service 85.5 per cent of the time. This performance with new locomotives and new crews, we consider very creditable, but look forward to bettering it as everyone becomes more used to them.

D. P. Orcutt: Information has been asked regarding the expected life of the battery on three-power locomotives and in this connection the fact was mentioned that in an investigation of a proposal for the use of a large straight storage battery locomotive for switching service, it was found that the battery life had been stated to be 4 years and that this life had resulted in an excessive battery maintenance cost for the work under consideration.

It should be remembered that with a three-power locomotive operating on internal power, a very large part of the energy used by the motors flows directly from the generator to them and does not pass through the battery.

Data collected from the operation of the first three-power locomotives on the New York Central, No. 1525, showed that in switching service about 85 per cent of the energy used by the traction motors flowed directly from the generator to the motors and 15 per cent from the battery to the motors. It is therefore evident that a very much smaller battery can be used with a three-power unit, thus greatly reducing battery maintenance cost from what it would be with a straight battery locomotive in switching service.

Incidentally it may be stated that this same data obtained from No. 1525 showed that the losses in the battery amounted to only about 5 per cent of the total energy generated while in a straight battery unit the battery losses would probably exceed 30 per cent of the energy required to keep the locomotive in service.

A straight battery locomotive may be the most economical unit to use under conditions where the size of the battery necessary to do the total amount of work does not require more battery weight than can be placed on a locomotive of the total weight required and where time is available for recharging the battery. In neither of the installations discussed by Messrs. Brehob and Craton would a straight battery locomotive give the required service which includes 24-hour operation as well as heavy transfer service.

The economies resulting from the use of any form of internally powered locomotive for use with an electrification project can be determined only by a comparison of the quality of service made available and of the cost of the entire service under consideration. A study based only on the cost of locomotive operation per hour does not give a true picture of the economies effected on the whole project.

As the paper and the discussion are confined to locomotives capable of operating on both external and internal power, we have combined data obtained from both the D. L. & W. three-power locomotives and New York Central three-power locomotives and in this way obtain the following as being approximately representative of this type of locomotive operating in connection with a railroad electrification project:

In an 8-hour shift the locomotive would operate on non-electrified tracks approximately 6 hours at an average input to the motors of 60 kw. per hour which would represent a total of 360 kw-hr.

On electrified tracks it would operate approximately two hours at an average input to the motors of 450 kw-hr. per hour which would represent a total of 900 kw-hr.

The power required for auxiliaries and all electrical losses during this 8-hour period would represent about 220 kw-hr. making the total power required for the locomotive for the 8-hour shift approximately 1,480 kw-hr.

Approximately 15 per cent of the 360 kw-hr. delivered to the motors on internal power while the locomotive is on non-electrified tracks would come from the battery or 54 kw-hr. This represents only about 4 per cent of the total power required by the locomotive during the 8-hour shift. As compared to this a straight battery locomotive would have had to deliver all the power required by the locomotive if the locomotive were to do the same work that the combination locomotive did during this 8-hour shift.

It can easily be seen, therefore, that when only 4 per cent of the work which this combination locomotive does in an 8-hour shift comes from the battery, that battery costs will be a relatively small proportion of the total operating costs of the locomotive as compared with a type of locomotive in which all the work is done by the battery.

Even when called upon to operate entirely on internal power the input from the battery to the traction motors is only 15 per cent of the total input to the motors whereas it would have to be 100 per cent (plus input to auxiliaries) in a straight battery locomotive.

It has been pointed out that this locomotive has decided advantages in connection with an electrification project:

It can be operated in warehouses and other places where it may be objectionable to run an internal combustion engine or install third rail or trolley.

It is capable of accelerating rapidly.

It is the most flexible type of locomotive tried out or suggested for this class of service.

It has high fuel economy because the engine operates with a favorable load factor.

It is our belief that the battery costs on this locomotive have been and will be entirely justified by the advantages obtained.

Hermann Lemp: To design an oil-engine required only to operate at nearly constant load and speed is a much simpler problem than to design one for adjustable speed and load, as anyone conversant with oil-engine design will readily appreciate. Similarly a storage battery, floating on a system, receiving and delivering energy continuously, on a hand to mouth proposition operates under a far better condition than when first fully charged then full discharged sometimes way down to the very lowest figure permissible. It is therefore quite conceivable, as Mr. Orcutt pointed out, that the storage battery manufacturer's guarantee of four years life can be met. This combination, when used in switching service, permits the time load factor of 15 per cent to be exchanged for 62 per cent and hence requires a smaller oil engine to perform the work of a bigger unit.

Mr. Brehob laid particular stress on the necessity of providing heavier axles and frames owing to the addition of the internal combustion engine plant, while he made no mention of the weight of the storage battery. If my records are correct the weights of the first three-power locomotives were as follows:

Total weight—257,000 lb., in which the battery weighs 34,300 lb. and oil engine generator and accessories weigh 31,500 lb.

In other words another engine equipment of 300 hp. would not cost and weigh any more than the battery, good only for four years, while the engine equipment, if properly maintained, would be available at least for ten years. To me the question is therefore one of original investment.

Railroads I think are particularly anxious to obtain an equipment which can maintain the rated output substantially continuously and which is not limited by a "time power decrease," such as a battery under a continuous full discharge rate would constitute. A similar operating condition is presented by the Hoboken Manufacturers Railroad. Most of the switching can be performed by a 300-hp. unit. But sometimes during the day, particularly when the tide is low and loaded cars have to be pulled up a stiff grade from the float, a second 300-hp. engine in reserve is started up and gives the additional power and acceleration needed. The engine units *A* and *B* are shifted every day, so as to equalize the time service of each. Such a locomotive, when climatic or service conditions call for nearly continuous full load operation, can give such a service at a lower first cost and maintenance than the battery combination.

Mr. Hamilton has expressed a fear that the oil-engine part of the equipment might prove unreliable. He stated that the actual experience with the three-power locomotive had demonstrated an availability for service of 85.5 per cent. It was brought out in a paper read by the writer before the Metropolitan Section of the A.S.M.E., April 17, 1928, that the average availability of straight oil-electric locomotives using the same oil-engine unit as the three-power locomotive had been established to be 91.5 per cent. This figure was obtained from returns to a questionnaire sent to all railroads and industrial plants using these locomotives for about three years in various locations and service.

F. H. Brehob: Due to the fact that the service requirements to be met were severe and the type of locomotive to be built was of unusual design, a number of difficult problems in mechanical design had to be solved.

The locomotive as developed represents the heaviest four-axle swivel truck locomotive in service today, and utilizes the heaviest axle loading using single-geared axle-hung motors.

Naturally, the railroad desires no more weight in a locomotive than is absolutely necessary to obtain the required adhesion for the maximum load to be handled. Generally, the basic design of the ordinary straight electric freight locomotive results in approximately the desired weight. By basic weight I mean using normal stresses and using the kind and grade of material ordinarily used for the various parts. For example, on the New York Central R-2, six-motor freight locomotive utilizing the same traction motor as on the three-power locomotive, approximately 2½ per cent of the total weight of 266,000 lb. is ballast.

The three-power locomotive as furnished the New York Central is not only a straight electric locomotive converting electrical power into mechanical power, but carries in addition a power plant with its accessories such as water tank, fuel tank, radiators and piping. The weight is thus considerably increased due to this apparatus and also to the increased weight of the mechanical portion requiring heavier supporting members to carry this load and because of the necessity of building a larger locomotive to house this apparatus. The railroad specified a minimum weight of 125 tons. By utilizing material to best advantage, reducing sections of material where feasible, and in one or two cases using a higher grade of material, it was possible to limit the weight in working order to 257,000 lb. For example, the axles were made of vanadium steel, although the selection of this material was also influenced by a limitation in the diameter of the axle to 8 in. so that the GE-286 motor used on other locomotives on the New York Central could be utilized without change on the three-power locomotive.

One of the most effective ways of keeping the weight to a minimum is to design the locomotive as compactly as possible which involves a suitable arrangement of apparatus using every bit of space to best advantage. The apparatus must be arranged to be readily accessible for inspection and repair and for easy removal from the locomotive.

The arrangement finally adopted was the one as illustrated in the paper. It will be noted that the blowers for ventilating the traction motors are placed directly over the hollow center plate into a small space recessed into the battery compartment although the upper tier of batteries is not affected. The control apparatus is mounted along one side of the locomotive opposite the engine, access being obtained from one side by removable covers on the outside of the cab. In this manner the space between the engine and the control equipment serves a double purpose; on the one hand giving accessibility to the engine for maintenance and on the other hand providing the necessary arcing clearances for the contactors. The space between the trucks is practically all taken up with the accelerating grids and the air compressor. The space not required on the roof for the hatch is occupied with engine radiators, and air reservoirs. Utilizing all the available space to advantage permits building the locomotive as short as possible with a minimum distance between center plates resulting in a minimum cross section of material in the supporting underframe, and consequently a minimum weight of material.

Owing to the fact that collection of power on the D. L. & W. locomotive is from a 3,000-volt overhead trolley, the arrangement of apparatus is somewhat different. It was desirable to enclose the 3,000-volt control apparatus somewhat more than the 600-volt apparatus on the New York Central three-power locomotive, for the sake of safety and cleanliness. This resulted in an arrangement whereby the engine was placed in one side of the cab leaving a greater space for the high-voltage equipment opposite the engine. This also permitted the use of enclosing covers on the inside as well as on the outside. Instead of locating the rheostats beneath the locomotive they were placed inside the locomotive immediately above the floor, and chimneys for heat dissipation were provided. The space underneath was used for air reservoirs instead of the roof, thereby providing space for mounting the overhead trolley. The battery trays were arranged in three tiers due to the fact that this battery was of a higher voltage and consisted of a larger number of smaller cells.

Another reason for arranging the apparatus as compactly as possible was to obtain a design which would negotiate the minimum radius of curvature of 100 ft. required by the railroad. This severe curvature is the principal reason for the swivel truck design as it is doubtful whether an articulated type could have been built which would prove successful.

Certain definite height and width limitations had to be met by the railroad, and these had to be considered especially on a curve when a displacement from normal position is encountered. This was not an easy matter to accomplish because a straight electric locomotive required to negotiate the same curve would be shorter. The design required that the distance between center plates be kept to a minimum and of even greater importance that the overhang from the center plate to the end of the coupler be kept at a minimum.

The draft rigging is, in effect, radial in that the coupler is free to swivel in a coupler extension which also swivels.

The three-power locomotives for the New York Central and D. L. & W. railroads are a modification of the original three-power locomotive jointly designed by the American Locomotive Company and the General Electric Company, and the operating experience gained by the New York Central on this locomotive and straight electric locomotive has been most helpful. It is gratifying that the combined efforts of the railroad and builder produced a new type of motive power which in a relatively short length of time has already proven successful.

F. H. Craton: Space limitations in the paper did not allow much detail regarding the interesting background behind the development of the three-power locomotive. There are 36 of these locomotives in service on the New York Central in New York, 2 on the Lackawanna and 2 on the Illinois Terminal Railroad. As the so-called West Side Improvement of the New York Central was largely responsible for the development of the three-power locomotive, it is briefly described in the following.

This project is the last step in eliminating steam locomotive operation from Manhattan Island. The New York Central's freight tracks enter Manhattan at Spuyten Duyvil which is at the northwest tip of the island at the junction of the Hudson and Harlem rivers, the line running from Spuyten Duyvil south along the east shore of the Hudson River for 12.4 miles as far as St. Johns Park. The electrification of these tracks constitutes a part of the so-called West Side Improvement. From Spuyten Duyvil to 72nd Street, which is at the entrance to the 60th Street yard, 600-volt third rail has been installed as an extension of the present third rail electric zone. The heavy freight service from Harmon, the northern terminus of the electric zone, to 72nd Street, approximately 30 miles, is handled by Class R-2 straight electric 600-volt road locomotives, 42 of which were purchased during 1930 and 1931. However, south of 72nd Street, owing to unusual conditions involving much trackage following city streets, a number of sidings entering industrial buildings and warehouses, much yard trackage and special conditions connected with float work, it was impractical to electrify with either third rail or overhead distribution systems. Therefore, the railroad management was faced with the necessity for choosing a suitable type of internally powered locomotive for the service south of 72nd Street as well as a locomotive which would operate also on third rail in order to increase its flexibility. After a detailed study of the question involving consideration of all the conditions to be met, the three-power type of locomotive was finally chosen.

Discussion of the three-power locomotive usually leads to its comparison with a two-power design, which is a unit with a straight engine-generator power plant and external power facilities. The justification for using one type or the other hinges entirely on the service requirements to be fulfilled and can be determined only after a thorough study of all the conditions involved. A power plant of the engine-generator-battery type is best fitted for operation in service in which the peak demands are high and the load factor low. The operating conditions on the New York Central West Side as well as the Lackawanna are well suited to take advantage of this type of internal power plant. The three-power type possesses the advantage of being able to operate on battery power into buildings or restricted areas where there might be an objection to an internal combustion engine because of sparks or odor, but the value of this feature would be dictated by the service requirements.

The question has been raised as to whether the internal power plant should be regarded as an auxiliary to the external power operating facilities or vice versa. This appears to be dependent upon the service conditions under which the locomotive operates on each particular application. Because of the unusual flexibility of this type, it is possible that the predominance of one type of operation over the other would shift from one to the other in operation on a single diversified application such as the New York Central where the locomotives are used with different combinations of external and internal power operating conditions but all units are interchangeable in the various classes of work.

F. H. Brehob and F. H. Craton: If the term "power plant" as referred to in the discussion has been misunderstood, it should be stated that this term includes the battery inasmuch as the battery and engine generator in multiple combine to furnish power to the traction motors. It was realized that the battery presented a design problem on account of its weight, which is one reason the battery was divided, one-half of it being placed almost directly over each center plate, where its weight would require a minimum amount of supporting structure.

Mr. Hamilton has pointed out that a locomotive without a battery would require additional engine-generator capacity together with additional accessories. This additional weight of power plant in the form of engine-generator, radiators, etc., would offset the weight of the battery.

The two-power locomotive undoubtedly has its place in the field of transportation. By two-power is meant a locomotive which will first operate from external power and secondly from an engine-generator set, but without battery for traction purposes. We believe the selection of either type can only be satisfactorily arrived at when all the service requirements to be performed are known and studied.

Motors for 3,000-Volt D-C. Multiple Unit Cars

BY JOHN C. AYDELOTT*
Member, A.I.E.E.

A NEW multiple unit car motor, incorporating many novel features of design, was developed for the electrification of the suburban service of the D. L. & W. R. R. The absence of trouble, in commencing service and in maintaining it, marked the whole electrification program. This was the first installation of multiple unit suburban service in this country employing a trolley potential of 3,000 volts direct current.

Locomotive motors to operate from a 3,000-volt trolley were first built for the C. M. & St. P. R. R. in 1914 and subsequent installations include the Spanish Northern Railway in Spain, the Paulista Railway in Brazil, the Mexican Railway in Mexico and the Cleveland Union Terminals Company. Experience gained from several years of operation with the earlier designs of 3,000-volt locomotive motors has been invaluable in perfecting the design of the Lackawanna multiple unit car motor which rounds out the line available for 3,000-volt d-c. electrification.

There are now in service on the Lackawanna Railroad 564 of these motors. The breaking in of equipments started in August, 1930. Revenue service started September 3, 1930, when the first passenger-carrying train ran from Hoboken to Montclair. With the final extension to Dover on January 25, 1931, the last of 141 units was placed in service. Traffic has so increased that units originally intended as spare cars are all needed regularly and there are times during the day when all but three units are on the road.

By June 1, 1931, 4,500,000 car-miles had been operated with only two train delays, one of 19 minutes and one of 4 minutes, chargeable to motor failure. One train was held at Hoboken Terminal 19 minutes due to a burned out armature bearing and the other was 4 minutes late arriving at Montclair.

During the period of initial operation these motors have been remarkably free from flashovers, insulation failures, and mechanical troubles.

The minimizing of motor flashing is an achievement embracing many features of motor design. Flashing may be caused by interruptions or disturbances in the voltage supplied to the motors, and many things within the motor itself may contribute to motor flashing, such as poor commutator condition, commutator roughness, oil and dirt between commutator bars, flat spots, bouncing brushes, and low motor stability.

The Lackawanna motor has 281 commutator segments. With 1,500 volts applied to the motor terminals there is an average of 21.3 volts per bar. The successful operation of this motor demonstrates the feasibility of high volts-per-bar, higher than previously used in any 3,000-volt railway motor, and depends upon commutating characteristics that will keep the commutator always in first class condition, and upon the ability of the motor to withstand frequent wide and sudden variations of trolley voltage on full or weak field with a minimum of spit at the brushes.

Good commutation hinges first upon keeping the reactance of the armature conductors at a minimum so that the energy involved in the reversal of the current in a conductor is low. Then the compensation, depending upon the quantity of commutating pole flux, must have the required value at all loads. The design of this motor employs a one-turn armature winding. Careful attention was given to the number of slots and the proportions of the slots, air-gap and pole faces. The armature bars are clipped at the back permitting a "breaking up of the slots" which has long been a feature of good railway motor design. Non-magnetic steel binding wire is used to reduce slot reactance. At light loads the effect of non-magnetic binding wire as compared with steel binding wire is particularly noteworthy. With non-magnetic binding wire the slot reactance flux and slot reactance are not only low in value but are essentially proportional to load current. .With steel wire the slot reactance flux is proportionately very high at light loads until the binding wire saturates and is increased by the amount of flux carried by the wire at all loads. When the slot reactance voltage is not proportional to load it cannot be correctly compensated at all loads.

It is well known that good commutation can usually be obtained at some one motor load according to the adjustment of the commutating field strength and it has been common practise to adjust compensation for minimum sparking at some load around the one-hour rating. If the commutating poles were somewhat saturated at this point the motor would be over compensated for all lighter loads. The design of this motor includes ample commutating pole cross section. Commutation is good from the lightest load to four times the continuous rating. Light loads are mentioned particularly because no sacrifice of commutation at light loads has been made to obtain correct compensation at heavy loads.

This motor has ability to withstand flashover from interruption to an unusual degree. This may be attributed largely to the use of highly responsive commutating poles. Extra turns are wound on the commutating coils and the extra magneto-motive force is consumed under steady load condition in non-magnetic shims at the back of the commutating pole pieces. Thus the commutating pole flux follows very quickly any change in the armature current and tends to maintain correct compensation even under extreme transient conditions.

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

A strong exciting field winding adds to the stability of the motor. The use of an inductive shunt rather than field tapping to obtain weak field gives any motor a greater ability to withstand flashover from interruption during reduced field operation. In the case of the Lackawanna motor the inductance of the field shunt has been so proportioned that the ability of the motor to withstand flashover from interruption on reduced field is superior even to its performance on full field.

Motor flashing that might be caused by dirt is avoided by the ventilating scheme developed for this motor. Dirt may lodge between commutator segments causing short circuits which, especially with high volts per bar, will lead to flashovers. Dirt that causes brushes to stick in the holders inevitably leads to flashing. It is also highly desirable that all other parts of the motor as well as the commutator and brushholders be kept as free from dirt and moisture as possible.

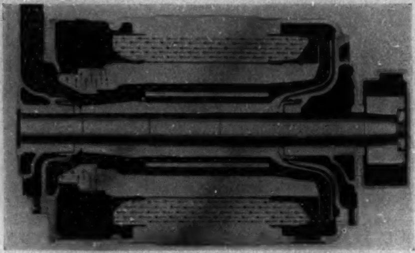

Fig. 1—Ventilating Scheme for D. L. & W. R. R. Motor Showing Air Circulation Inside the Frame

The most successful 3,000-volt locomotive motors have been blown, clean air being taken from the cab; and a blast of air has been directed straight against the motor commutator serving to prevent any accumulation of dust from the brushes in the mica grooves. The increased pressure in a blown motor tends to prevent the entrance of brake shoe dust or other dirt, or snow from the lower levels of the air surrounding the running gear of a locomotive in motion.

No external blowers are used on the D. L. & W. multiple unit cars but with the system of ventilation used the motor frame operates at all times either at or slightly above atmospheric pressure. The motors are self-ventilated as shown in Fig. 1. The cooperation of the car builder permitted the ventilating air to be taken from settling chambers over the vestibules to which it is admitted through louvers at each corner of the car roof. A separate duct to each motor goes down through the side and along under the floor of the car to a point over the commutator end bearing as shown in Fig. 2. A flexible air connection made of cloth sewed over a coiled wire leads the air from the duct on the car to the motor inlet in the commutator end framehead. The flow of air is maintained by a fan on the motor shaft. The air enters the motor under the commutator and passes first through the ducts in the armature. It is then returned by the fan through the field structure of the motor and exhausted at the commutator end. The air outlet opening is covered with an expanded metal screen and a hood so that foreign material cannot fall or be thrown into the motor. This method maintains pressure inside all the exposed surfaces of a self-ventilated motor, which means that track dirt, fine snow, or brakeshoe dust cannot enter the motor. It should also be noted that before the ventilating air reaches the commutator chamber of the motor it is filtered, so to speak, through both the armature and field coils. Furthermore, any carbon dust that originates in the commutator chamber is carried directly to the exhaust openings and does not enter any other parts of the motor. Routine inspections have shown these motors to be unusually clean. Surplus oil from the bearings is led through ducts to the outside of the motor and none finds its way inside.

Fig. 2—Ventilating Scheme for Motor Car Showing Air Intake at the Roof and Path to Motors

Smooth commutators depend not only on good commutating characteristics but also on good mechanical design and construction. The use of four brushholders and the low current necessary because of the high voltage permit the commutator to be compact and strong. The commutator diameter is 16¼ in. and there is a brush surface of 2¾ in. Six bolts hold the commutator cap and shell together. The cars operate at 70 mi. per hr. at least once during every round trip. With 35-in.

wheels and 59/22 gearing the motor reaches 1,800 r.p.m. at 70 mi. per hr. This gives a peripheral speed at the commutator of 7,620 ft. per minute.

When the motors were first put into service there was some brush breakage at high speeds but in a short time the commutators seasoned and brush breakage is now practically eliminated. The brushholders play their part in maintaining good commutating conditions. Pressure fingers are of the steel clock spring type, of low inertia, maintaining their tension permanently, and are centrally located by recessed adjusting sleeves clamped by the first turn of the springs. A non-resilient brush is used to maintain contact with the commutator at high speed. No abrasive action is necessary to keep the commutator smooth. Thus the brush may be comparatively soft, of low friction coefficient and high contact drop.

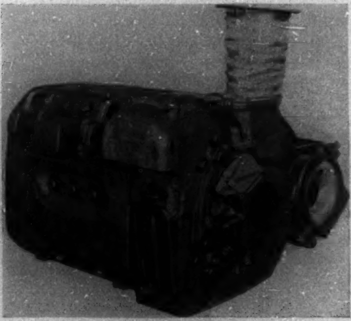

FIG. 3—GE-700-A 1,500/3,000-VOLT RAILWAY MOTOR SHOWING AIR INTAKE AND OUTLET

The insulation of the motor throughout is in accordance with recognized standards for 3,000-volt railway motors. While the absence of insulation failures in the motors thus far covers a comparatively short period, experience has shown that the same kind of insulation in other 3,000-volt motors lasts indefinitely, failures seldom occurring from any other cause than mechanical damage.

The ratings of the motors, when ventilated as in service, are as follows:

	Volts	Field strength	Shaft hp.	Amps.	R.p.m.	Rise deg. cent.
Continuous	1,500	50 per cent	181	97.0	1055	105
One hour	1,500	50 per cent	255	137.0	913	120

Although during the time that it is connected, the inductive shunt carries 50 per cent of the current passing through the motor, it has a continuous rating of only 34 amperes. The field shunt is designed with a temperature rise characteristic which at the r.m.s. current obtained in continuous suburban service will give a temperature rise approximately equal to the corresponding temperature rise of the traction motor exciting fields. Cold equipments coupled in train with hot equipments will share the load equally.

On certain length runs the r.m.s. current of the shunt slightly exceeds the rating, reaching a peak value at runs of approximately 3,000 ft. in length, where the calculated r.m.s. is approximately 37 amperes. However, under these conditions the r.m.s. current of the motor exceeds the continuous rating for 105 deg. cent. rise by resistance. Not only was the present electrification of the D. L. & W. R. R. considered in studying shunt capacity but also the possibility of extensions into more mountainous territory involving long uphill runs. These studies indicate that these runs do not require a shunt rating more than one-third the continuous motor rating for 50 per cent field strength operation. The reduction in shunt rating also permits a saving in weight.

Views of the complete motor are shown in Figs. 3 and 4. Four motors are used on a motor car weighing 81.5 tons loaded, which together with a trail car weighing 56.5 tons forms a two-car unit of 138 tons. The motor weighs complete with gear, gear case and axle linings 6,850 lb., giving a weight per horsepower of 26.9 lb. The four motors give a capacity for the two car units of 7.4 hp. per ton. Trains for use in short run commuter service may be made of 2, 4, 6, 8, 10 or 12 cars, half of which in each case are motor cars. A typical six-car train has a free running speed of 67 mi. per hr. on level track (Davis Friction) and a maximum permissible speed of 70 mi. per hr. Some of the trains shown on the time tables make the following stops and schedule speeds.

Stations	Miles	Stops	Avg. run	Schedule speed
Hoboken to South Orange	13.85	10	1.38 mi.	28.7 mi. per hr.
South Orange to Hoboken	13.85	10	1.38 mi.	29.7 mi. per hr.
Hoboken to Summit	20.10	13	1.55 mi.	30.9 mi. per hr.
Hoboken to Morristown	29.77	18	1.65 mi.	31.3 mi. per hr.
Morristown to Hoboken	29.77	17	1.75 mi.	31.9 mi. per hr.
Dover to Hoboken	40.49	22	1.84 mi.	32.8 mi. per hr.
Hoboken to Montclair	13.12	6	2.19 mi.	34.2 mi. per hr.
Dover to Hoboken	40.49	14	2.89 mi.	40.0 mi. per hr.
Hoboken to Dover	40.49	7	5.78 mi.	41.2 mi. per hr.
Montclair to Hoboken	13.12	1	13.12 mi.	41.4 mi. per hr.

The motors will permit the use of as high an accelerating rate as desired, limited only by the weight on drivers and the slipping point of the wheels. Normally an automatic acceleration of 1.5 mi. per hr. per sec. is maintained corresponding to an adhesion of 12.8 per cent. Acceleration tests have shown an equipment to be capable of reaching 30 mi. per hr. in 20 seconds and 47 mi. per hr. in 60 seconds.

In frequent stop service a four-motor equipment is well loaded by a motor car and trailer unit, and the motor heating approaches the permissible limit for class B insulation. Experience has shown that for railway motors, class B insulation will have a longer life if the motors are run hot enough to keep them free from moisture. In long run service the motors will have sufficient excess capacity to permit the handling of additional trailers. Such train make-ups as two standard units and one trailer, or three standard units and three trailers are contemplated. With such combinations the accelerating rate will be reduced but in infrequent stop service these trains will be capable of fast schedules.

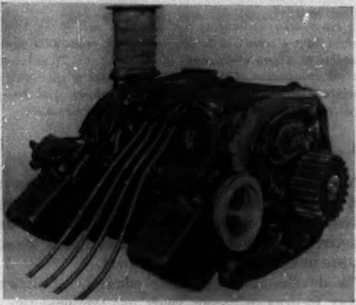

Fig. 4—GE-700-A 1,500/3,000-Volt Railway Motor Showing Pinion and Axle Preparation

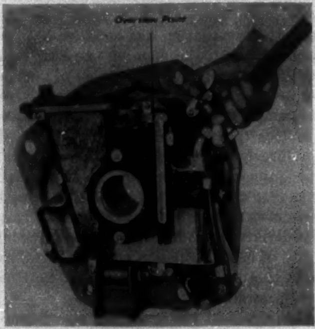

Fig. 5—Model Showing Principle of Constant-Oil-Level Bearing for Railway Motors. Oil in Reservoir at Overflow Point During Filling Operation

Conservative interpretation of experience characterizes every feature of the mechanical design, contributing to low maintenance. Stresses in all working parts are low. Both armature and axle bearings are of the sleeve type and embody the constant oil level feature permitting oiling periods to be extended to several times the usual period. There is a layer of babbitt on the wearing surfaces of the bore and flanges of all the bearings. The almost complete absence of hot bearings during the initial stages of the electrification was due not only to design, but to the fact that all of the linings were line reamed after assembly in their respective final positions and due to the careful packing of the bearings, oiling and breaking in.

In the bearings of the motor the oil level is maintained practically constant in the waste chamber as long as there is any oil in the reservoir. There are separate openings for waste and oil, hence there is no chance for dirt to fall into the waste chamber when replenishing the oil supply. The principal of operation of the constant oil level bearing is illustrated in Fig. 5 showing a view of a sectional model.

This type of bearing has been designed to operate between inspections without addition of oil. Should it become necessary to add oil on the road, the oil can be poured into the bearing through the filler opening and the bearing will operate as an ordinary bearing.

The operation of the gearing on the motors is quiet, promising long life. Gearing is of special heat treated steel, General Electric Company grade F, 2¼ pitch, with long and short addendum tooth form. The gear teeth were ground after treatment to insure correct profile. The possibility of giving off ringing notes from the gear has been overcome by a non-resonant feature shown in Fig. 6. This is obtained by snapping rings of small cross section into annular grooves machined in the under side of the rim. These rings are of a size having a different period of vibration from that of the gear section thereby counteracting or destroying the vibrations in the gear. The manner in which the rings are assembled provides an annular pocket for the retention of gear grease. This also has a decided damping effect. After assembly the rings are welded at the joint to prevent any possibility of their coming loose.

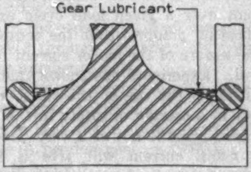

Fig. 6—Section of Non-Resonant Gear

Gear cases are of malleable iron with three point support.

Six months to a year of operation gives only an indication of what the maintenance expense will be. A longer time will be necessary before sufficient data are accumulated and ready to publish. The absence of trouble thus far, however, promises unusually low maintenance for this first multiple unit suburban electrification employing a trolley potential of 3,000 volts direct current.

Temperature Rise of Ventilated Railway Motor Armatures†

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Synopsis.—*The calculation of temperature rise and of time-temperature curves of a railway motor for which only preliminary design data are available, usually presents considerable difficulty. Modern railway practise demands increasingly large power units in a limited space, so that accurate methods are necessary for the predetermination of temperature rise as a basis for guaranteed ratings and of time-temperature curves for the application of a motor to a complicated service cycle.*

For calculating the temperature rise of the armature in a new design of motor for any time interval and load condition, the author presents a method which is shown to give consistent results when applied to motors of widely varying size. A brief outline is given of the method by which the equations are derived from the usual theory of heat flow between related bodies. It is shown that by making certain assumptions the armature can be reduced to a simple equivalent model consisting of two conducting bodies separated by an insulating wall, and that the equations resulting from an analysis of this thermal system form a practical method of calculating time-temperature curves of an armature from preliminary design data. A detailed mathematical treatment is presented in an appendix.

By substitution of existing test data into part of the formula it is shown how a curve can be obtained which renders the equations applicable to a very wide range of motor sizes. To illustrate the practical application of the formula, a number of calculated time-temperature curves for existing motors is shown and compared with data from actual temperature tests of these same motors.

* * * * *

THE accurate predetermination of temperature rise for loads of short duration in a new design of traction motor is of the utmost importance since the application of all such motors involves service of an intermittent nature. The usual method of using a r.m.s. current over a given load cycle as a basis of motor application assumes that the temperature rise varies as the square of the motor current over a given time interval. Experience has shown that this is not the case, although the use of the r.m.s. current will always result in a margin of safety in the final result. Where the load cycle is not of a very widely varying nature the r.m.s. current will give a fairly close approximation of the actual conditions. In such service as mine haulage or yard switching where a widely varying load cycle has been definitely specified by the purchaser, greater economy of space and cost can be accomplished if an accurate method is available for calculating the time-temperature curves for any load condition. Such a method can then be applied successively to each load condition of the cycle and a true picture of the motor heating will be obtained.

In many cases a motor designer is called upon to guarantee ratings which allow very little margin for guesswork in estimating temperature rise and consequently feels the need of a reliable means for its calculation. Railway motors are usually worked much closer to the limits allowed by existing insulating materials than machines used in other applications. Also, the exacting space limitations imposed upon motors placed under locomotives or rail cars, together with the present demand for still greater capacity within these limitations calls for closer designing than is generally required for other types of machine. Consequently, there is an urgent need for more accurate predetermination of temperature rise if a constant advance in the art is to be maintained.

The development of highly efficient ventilation of railway motors in recent years has rendered the estimating of short-time temperature rise more difficult than was the case with the older type of motor of larger thermal capacity. In the newer designs, the value of the nominal one-hour rating as a true indication of thermal capacity is not very great since the much more effective ventilation has considerably decreased the margin between the nominal one-hour rating and the continuous rating. The rate of temperature increase for short intervals cannot be safely regarded as the function of an unvarying time constant in a motor of this type since the rate of heat dissipation, which is largely dependent on the speed, has a considerable influence on the so-called thermal time-constant. In the following treatment a method of predetermining the thermal characteristics of ventilated motors is derived which takes into account the influence of the rate of heat dissipation on the thermal capacity as well as the dissimilar nature of the materials contained in the structure of the armature.

In a well-designed motor having an efficient ventilating system, the temperature of the armature windings will increase more rapidly than that of the field windings so that the armature is the more sensitive to changes in load. On this account it becomes essential to know the thermal characteristics of the more complicated rotor windings if a close estimate of the behavior of the motor under a varying load cycle is desired. This treatment, therefore, is devoted entirely to the more difficult problem of arriving at suitable equations from which the temperature rise of the armature can be determined for any desired load and speed.

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In order to arrive at anything approaching a simple result, it is necessary to reduce the armature to the simplest possible equivalent model. Nearly all railway armatures are designed so that a portion of the cooling air is drawn through ventilating ducts running axially through the armature punchings underneath the slots. Since the axial length of the armature core is usually long in comparison to the radial depth of the punchings, the heat flow across the plane of the laminations will be very small compared to the flow parallel to the plane of the laminations. Also, the temperature of the end windings in modern railway motors is usually fairly close to that of the embedded windings over most of the working load conditions and, on account of the comparative length of the embedded part, the temperature variation along the whole length of armature winding is comparatively small. From these considerations, the following assumptions are made:

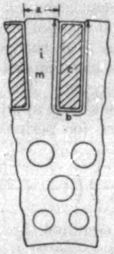

FIG. 1

A. The temperature of the copper in the slot is assumed to be uniform in all three dimensions.

B. The temperature throughout the iron of the armature punchings is assumed to be uniform in all three dimensions.

C. The rate of heat transfer to the ventilating air in the axial ducts of the core is taken as being one-half that at the outer surface. This is a purely empirical assumption and is found to give the most consistent results in the calculation of the surface heat transfer constant from temperature test data. It is also based on the supposition that the air in the ducts moves relatively to the surface of the ducts with a velocity which is about one-quarter to one-third the velocity of the air relative to the armature surface. The rate of heat transfer in axial ducts for various conditions of air velocity and nature of surface has been obtained experimentally by Luke.

D. The flow of heat through the insulation at the top of the slot is neglected.

The first two of these assumptions reduce the problem to a two dimensional one so that it is only necessary to consider the flow of heat in the armature in the plane of the laminations, that is, perpendicular to the axis of the armature. There are two main component parts of the armature, the iron of the core, represented by i in Fig. 1, and the copper c embedded in the slot. The iron and the copper, whose thermal characteristics differ from each other quite considerably are separated by a comparatively thick insulating wall m. It is the interchange of heat between the iron and the copper through this insulation that is usually neglected in most methods of predetermining temperature rise in railway motor armatures. The arrangement of the component parts of the armature as shown by Fig. 1 can be represented by the simple thermal model shown in Fig. 2. The body c represents the copper separated by the insulation m from the body i which represents the iron of the core. The equations given below are derived from the mathematical consideration of this system presented in the Appendix.

The following symbols are employed:

a = width of tooth at the armature surface in inches.
b = periphery of the slot in inches.
K_1 = rate of heat conduction through the slot insulation in watts per sq. in. per deg. cent. $= \frac{\kappa}{\tau}$
K_2 = coefficient of heat transfer from the armature surface to the ventilating air in watts per square in. per deg. cent.
κ = conductivity of the insulation in watts per in. per deg. cent.
p = sum of the peripheries of all the axial ventilating ducts through the armature core divided by the number of slots.
S_c = specific heat of copper in watt-hours per lb. per deg. cent. = 0.0485.
S_i = specific heat of the armature iron in watt-hours per lb. per deg. cent. = 0.0605.
t = time in hours.
T_c = temperature reached by the copper in time t.
T_i = temperature reached by the iron in time t.

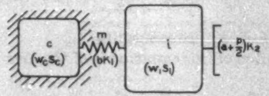

FIG. 2

T_a = mean temperature of the ventilating air adjacent to the armature surface. (It is assumed that this temperature will be the same in the vent ducts as at the armature surface.)
T_k = temperature rise of the copper above the average temperature of the ventilating air after infinite time.
τ = thickness of insulation between copper in the slot and iron at sides and bottom of slot.
θ = temperature rise of the copper above the ambient air temperature at time t.
θ_a = average temperature rise of the ventilating air above the ambient temperature at time t.
θ_c = temperature rise of the armature copper above the average temperature of the ventilating air at time t.

θ_k = temperature rise of the armature copper above the ambient temperature when t is infinite.

V = rate of air flow through the motor in cu. ft. per min.

v_s = peripheral speed of armature surface in ft. per min.

W_c = loss in the embedded conductors in one inch length of one slot in watts

W_i = core loss in watts per inch length of core per slot.

w_c = weight of copper in one inch length of one slot + 50 per cent of weight of insulating material in one inch length of slot. (In a motor insulated for 750 volts with single-turn coils this will be equivalent to 1.3 times weight of copper in one inch length of one slot.)

w_i = weight of armature punchings in lb. per in. length of core per slot.

Mathematical consideration of the interchange of heat between the two component bodies c and i leads to two simultaneous linear equations of the first order:

$$A\frac{d\,T_i}{d\,t} + (B + C)\,T_i - B\,T_c - C\,T_a - W_i = 0 \qquad (1)$$

$$D\frac{d\,T_c}{d\,t} + B\,T_c - B\,T_i - W_c = 0 \qquad (2)$$

where A, B, C, D are constants (see Appendix).

Since railway motors are now usually rated on the basis of the temperature rise of the windings as measured by increase in resistance we are only interested in the copper temperature T_c. Eliminating T_i from the above equations we obtain a second order equation in T_c:

$$L\frac{d^2\,T_c}{d\,t^2} + M\frac{d\,T_c}{d\,t} + N\,T_c = K' \qquad (3)$$

where L, M, N, K' are constants.

The solution of this equation is of the form

$$T_c = C_1\,e^{-\lambda_1 t} + C_2\,e^{-\lambda_2 t} + \frac{K'}{N} \qquad (4)$$

where C_1 and C_2 are constants of integration and $-\lambda_1$ and $-\lambda_2$ are roots of the equation $L\,D^2 + M\,D + N = 0$. Introducing the original constants into equation (**4**) and making certain simplifying approximations, the temperature rise of the armature copper at any time t is given by the following set of equations:

$$\theta_c = T_k\,(1 - e^{-\lambda_1 t}) + C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t}) \qquad (5)$$

where:

$$C_2 = \frac{w_c\,S_c\,W_i - w_i\,S_i\,W_c}{(2\,w_c\,S_c + w_i\,S_i)\,b\,K_1} \qquad (6)$$

$$\lambda_1 = \frac{\left(a + \frac{p}{2}\right)K_2}{\left[\left(a + \frac{p}{2}\right)\frac{K_2}{b\,K_1} + 1\right]w_c\,S_c + w_i\,S_i} \qquad (7)$$

$$\lambda_2 = \frac{b\,K_1}{w_c\,S_c} + \frac{\left(a + \frac{p}{2}\right)K_2 + b\,K_1}{w_i\,S_i} \qquad (8)$$

$$T_k = \frac{\left[\left(a + \frac{p}{2}\right)\frac{K_2}{b\,K_1} + 1\right]W_c + W_i}{\left(a + \frac{p}{2}\right)K_2} \qquad (9)$$

θ_c is the temperature rise of the armature copper above the mean temperature of the ventilating air at time t. Since the ventilating air temperature varies quite considerably, it cannot be conveniently considered as a constant over the period of time under consideration, so that some means is necessary to take its variation into account. The following empirical relation is found to be a close approximation in modern self-ventilated motors for peripheral speeds not less than 1,000 ft. per min.:

$$\theta_a = T_k\left(\frac{700}{v_s + 1000}\right)\left(1 - e^{-\frac{v_s^3 t}{2\times10^{10}}}\right) \qquad (10)$$

Introducing this into equation (**5**)

$$\theta = T_k\,(1 - e^{-\lambda_1 t}) + C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t}) + T_k\left(\frac{700}{v_s + 1000}\right)\left(1 - e^{-\frac{v_s^3 t}{2\times10^{10}}}\right) \qquad (5a)$$

The correction for rise in ventilating air represented by equation (**10**) is only approximate and can, of course, be made more exactly by the introduction of a more complicated expression. The rise in ventilating air temperature, however, is relatively small in comparison with the copper rise so that a correction of the above type is close enough for most applications. In all the foregoing, the mean temperature of the ventilating air is taken as the arithmetic mean between the temperature of the ingoing and outgoing air so that θ_a actually will be one-half the temperature rise of the ventilating air through the motor.

In equation (**9**), T_k is the temperature rise of the copper above the mean temperature of the ventilating air when t is infinite, that is, after a continuous run under the load condition represented by the losses W_c and W_i. Equation (**9**), therefore, gives us a means of arriving at values of K_2, by substitution of data from actual temperature tests into the equation, for various conditions of speed and ventilation. By taking a number of temperature runs on each of a wide range of motors of varying size it was possible, from equation (**9**), to find a relation between K_2 and the peripheral speed of the armature or other condition affecting the rate of heat dissipation. Fig. 3 shows the values of K_2 so obtained for a range of self-ventilated motors varying in size from 50 hp. to 310 hp., plotted against the armature peripheral speed. In these calculations and in all subsequent calculations, the quantity W_c was taken as the $I^2\,R$ loss calculated from conductor resistance plus the eddy current loss in the conductors as calculated by Field's method. As the tests from which these points are taken were ordinary test floor runs in which no special precautions for absolute accuracy were taken as would be the case in laboratory tests, the points are very favorably consistent for a curve of this type. The temperature rise by resistance was measured by taking a four point cooling curve immediately after shutting down the motor and projecting the curve back to the shutdown point. By measuring V, the rate of air flow through the motor in forced ventilated machines and, in the case of self-ventilated motors, estimating it from the losses and air rise for each test, and assuming that the axial velocity of the air over the armature surface is approximately $\frac{V^*}{0.3\,D^2}$, the velocity of the air relative to the armature surface will be, approximately, $\sqrt{v_a^2 + \left(\frac{V}{0.3\,D^2}\right)^2}$. Fig. 4 shows K_2 plotted against this relative velocity for a range of motors including both self and forced ventilated machines varying in size from 50 hp. to 800 hp. and in armature diameter from 11 to 30 in. With the aid of the curves of Figs. 3 and 4, the temperature rise above the mean ventilating air temperature after a continuous run can be predetermined from equation (**9**). However, equation (**9**) can be modified to correct for the rise in ventilating air temperature. For the line of self-ventilated motors mentioned above, this is done by adding the right hand member of equation (**10**) when t is infinite:

$$\theta_k = \left(\frac{\left[\left(a+\frac{p}{2}\right)\frac{K_2}{b\,K_1}+1\right]W_c + W_i}{\left(a+\frac{p}{2}\right)K_2} \right) \times \left(1+\frac{700}{v_a+1000}\right) \qquad \textbf{(9a)}$$

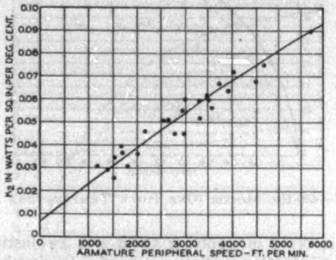

FIG. 3—K_2 FOR SELF-VENTILATED RAILWAY MOTORS

In determining the curves shown in Figs. 3 and 4 the value of κ, the conductivity of the insulation between the copper and the walls of the slot, was taken as 0.0025 watts per in. per deg. cent. This corresponds to an average value for the combination of mica, paper and varnish which is used for armature coil insulation. The temperature rise of the copper as will be seen by an inspection of equation (**9**) is considerably influenced by the value of K_1, the rate of heat transfer through this insulation; so that, considering the possibility of variations in the conductivity due to air pockets and incomplete drying out of compound, it is probable that the variations from the curve of the points of Figs. 3 and 4 can, to some extent, be ascribed to variations in the condition of the insulation.

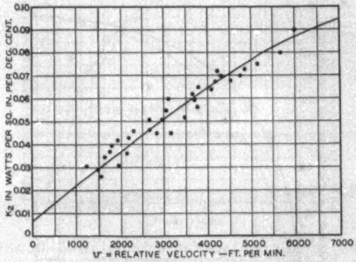

FIG. 4—K_2 FOR BOTH SELF- AND FORCED-VENTILATED RAILWAY MOTORS

The right-hand side of equation (**5**), from which time-temperature curves of a motor armature can be calculated, consists of two parts, $T_k\,(1 - e^{-\lambda_1 t})$ and $C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t})$. The equation was arranged in this form because it illustrates clearly how the time-temperature characteristics vary from the usually accepted simple logarithmic function which the first of these two expressions represents. The second expression, $C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t})$, is very small compared with the first when t is greater than $\frac{1}{\lambda_1}$ and disappears when t becomes infinite so that for finding the temperature rise over intervals of time exceeding the usually accepted thermal time constant, the second expression can be neglected. The equation then assumes the familiar form:

$$\theta_c = T_k\,(1 - e^{-\lambda_1 t}) \qquad \textbf{(11)}$$

For short time intervals, however, this second expression becomes very important and cannot be neglected. Figs. 5 to 10 show several time-temperature curves which were calculated by means of equation (**5**) for three motors, one a forced-ventilated motor of 450 hp. and the others, self-ventilated motors rated at 100 hp. and 50 hp. respectively. These curves were calculated for the same load and speed at which some actual temperature test giving consistent results was taken. The points plotted on the diagrams show the actual temperature rise above the mean ventilating air temperature obtained in the corresponding temperature run.

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*Where D is armature diameter in inches.

It is recognized that a simple logarithmic curve of the form $\theta = T_k\,(1 - e^{-\lambda_1 t})$, involving a single thermal time constant, could be found which would pass through any such set of two, or even more, test points. But it has been found very difficult to discover a sufficiently simple law which would connect the constants obtained experimentally of such an expression with the main design dimensions of a wide range of armature sizes. It has been shown that the equations (**5**) to (**9**) can be made, with the aid of a curve or set of curves of the type shown in Figs. 3 and 4, to represent within a reasonable degree of accuracy, the thermal characteristics of a very wide range of machines. The only feature which such a group of motors need have in common is that the path of the cooling air through the motor should be essentially similar in all machines of the group. The chief purpose of this method is the predetermination of the thermal characteristics of a projected motor design when only preliminary data are available. The method may appear unwieldy at first sight but with a little practise these equations can be handled quite easily and the extra time and labor involved in applying them will be justified in most cases by the smaller margin of safety with which it will be found that a new design of motor can be applied.

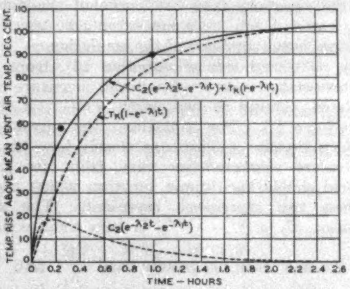

FIG. 5—450-HP. MOTOR (ONE HOUR TEMPERATURE RUN)

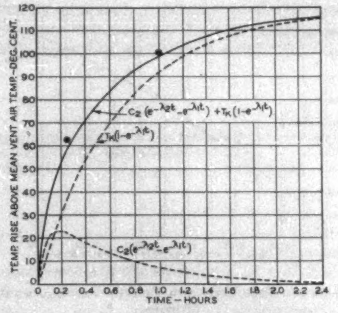

FIG. 6—450-HP. MOTOR (ONE HOUR TEMPERATURE RUN)

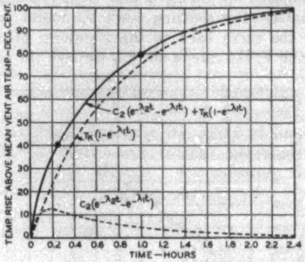

FIG. 7—450-HP. MOTOR (ONE HOUR TEMPERATURE RUN)

APPENDIX

Since the flow of heat through the insulation at the top of the slot is neglected, all the heat escaping from the copper will be transferred to the iron. Considering only one inch length of core, the rate of heat flow through the insulating wall will be $b\,K_1$ watts per deg. cent. Referring now to Fig. 2, the heat increment in time $d\,t$ in the body i will be:

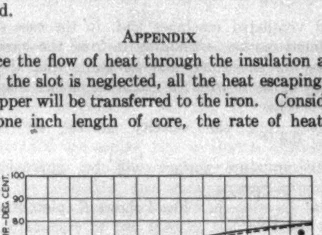

FIG. 8—450-HP. MOTOR (CONTINUOUS TEMPERATURE RUN)

$$W_i\,d\,t + (T_c - T_i)\,b\,K_1\,d\,t \qquad \textbf{(12)}$$

The heat stored in time $d\,t$ will be

$$w_i\,S_i\,d\,T_i \qquad \textbf{(13)}$$

The heat imparted to the ventilating air will be

$$\left[a\, K_2 (T_i - T_a) + p \frac{K_2}{2} (T_i - T_a) \right] d\,t \qquad (14)$$

Equating these values and rearranging:

$$w_i\, S_i \frac{d\,T_i}{d\,t} + \left[b\, K_1 + \left(a + \frac{p}{2} \right) K_2 \right] T_i$$

$$- b\, K_1\, T_c - \left(a + \frac{p}{2} \right) K_2\, T_a - W_i = 0 \qquad (15)$$

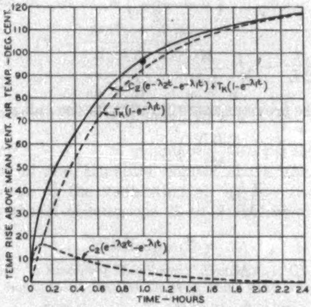

Fig. 9—100-Hp. Motor (One Hour Temperature Run)

In the body *c*, the heat increment will be:

$$W_c\, d\,t$$

The heat transferred to the body *i* will be:

$$b\, K_1 (T_c - T_i)\, d\,t \qquad (16)$$

The heat stored will be:

$$w_c\, S_c\, d\, T_c \qquad (17)$$

Equating these quantities and rearranging

$$w_c\, S_c \frac{d\,T_c}{d\,t} + b\, K_1\, T_c - b\, K_1\, T_i - W_c = 0 \qquad (18)$$

Let

$$w_i\, S_i = A$$

$$b\, K_1 = B$$

$$\left(a + \frac{p}{2} \right) K_2 = C$$

$$w_c\, S_c = D$$

Then we have:

$$A \frac{d\,T_i}{d\,t} + (B + C)\, T_i - B\, T_c - C\, T_a - W_i = 0 \qquad (15a)$$

$$D \frac{d\,T_c}{d\,t} + B\, T_c - B\, T_i - W_c = 0 \qquad (18a)$$

From (18a) we have:

$$T_i = \frac{D}{B} \frac{d\,T_c}{d\,t} + T_c - \frac{W_c}{B} \qquad (19)$$

Differentiating (18a) with respect to *t*:

$$D \frac{d^2\,T_c}{d\,t^2} + B \frac{d\,T_c}{d\,t} - B \frac{d\,T_i}{d\,t} = 0$$

or

$$\frac{d\,T_i}{d\,t} = \frac{D}{B} \frac{d^2\,T_c}{d\,t^2} + \frac{d\,T_c}{d\,t} \qquad (20)$$

Substituting these values for T_i and $\frac{d\,T_i}{d\,t}$ in equation (15a) and rearranging

$$A\, D \frac{d^2\,T_c}{d\,t^2} + [A\, B + (B + C)\, D] \frac{d\,T_c}{d\,t}$$

$$+ B\, C\, T_c - B\, C\, T_a - (B + C)\, W_c - B\, W_i = 0 \qquad (21)$$

Let

$$A\, D = L$$

$$A\, B + (B + C)\, D = M$$

$$B\, C = N$$

$$B\, C\, T_a + (B + C)\, W_c + B\, W_i = K'$$

Equation (21) becomes:

$$L \frac{d^2\,T_c}{d\,t^2} + M \frac{d\,T_c}{d\,t} + N\, T_c = K' \qquad (21a)$$

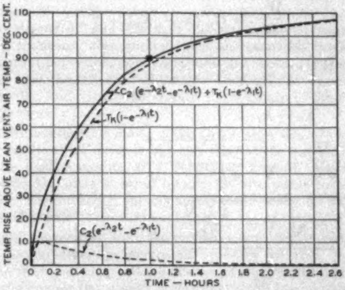

Fig. 10—50-Hp. Motor (One Hour Temperature Run)

The solution of this equation is:

$$T_c = C_1\, e^{-\lambda_1 t} + C_2\, e^{-\lambda_2 t} + \frac{K'}{N} \qquad (22)$$

Where:

$$\lambda_1 = \frac{M}{2\,L} - \sqrt{\frac{M^2}{4\,L^2} - \frac{N}{L}} \qquad (23)$$

$$\lambda_2 = \frac{M}{2\,L} + \sqrt{\frac{M^2}{4\,L^2} - \frac{N}{L}} \qquad (24)$$

$$\frac{K'}{N} = T_a + \frac{(B + C)\, W_c + B\, W_i}{B\, C}$$

Substituting this in equation (**22**) and calling:

$$\frac{(B+C)\,W_c + B\,W_i}{B\,C},\ T_k:$$

$$T_c - T_a = C_1\,e^{-\lambda_1 t} + C_2\,e^{-\lambda_2 t} + T_k \qquad (\mathbf{25})$$

When $t = 0,\ T_c - T_a = 0$

$$C_1 + C_2 + T_k = 0$$

or

$$C_1 = -C_2 - T_k$$

so that

$$T_c - T_a = C_2\,(e^{-\lambda_2 t} - e^{-\lambda_1 t}) + T_k\,(1 - e^{-\lambda_1 t}) \qquad (\mathbf{26})$$

From equation (**19**), knowing that when $t = 0$,

$$T_c = T_i = T_a:$$

$$T_a = \frac{D}{B}\,\frac{d\,T_c}{d\,t} + T_a - \frac{W_c}{B}$$

or

$$\frac{D}{B}\,\frac{d\,T_c}{d\,t} = \frac{W_c}{B} \qquad (\mathbf{27})$$

Differentiating equation (**26**) with respect to t:

$$\frac{d\,T_c}{d\,t} = C_2\,(\lambda_1\,e^{-\lambda_1 t} - \lambda_2\,e^{-\lambda_2 t}) + T_k\,\lambda_1\,e^{-\lambda_1 t}$$

Putting $t = 0$ and $\frac{d\,T_c}{d\,t} = \frac{W_c}{D}$ (from equation (**27**):

$$\frac{W_c}{D} = C_2\,(\lambda_1 - \lambda_2) + T_k\,\lambda_1$$

or

$$C_2 = \frac{\frac{W_c}{D} - T_k\,\lambda_1}{\lambda_1 - \lambda_2} \qquad (\mathbf{28})$$

$$\lambda_1 = \frac{M}{2\,L} - \sqrt{\frac{M^2}{4\,L^2} - \frac{N}{L}}$$

$$= \frac{1 - \sqrt{1 - \frac{4\,L\,N}{M^2}}}{\frac{2\,L}{M}}$$

$$\frac{1 - \left(1 - \frac{2\,L\,N}{M^2}\right)}{\frac{2\,L}{M}}$$ (approx.) since $\frac{4\,L\,N}{M^2}$ is of the order of 0.2

$$= \frac{N}{M}$$

$$= \frac{B\,C}{A\,B + D\,(B+C)}$$

$$= \frac{b\,K_1\left(a + \frac{p}{2}\right)K_2}{w_i\,S_i\,b\,K_1 + \left[\,b\,K_1 + \left(a + \frac{p}{2}\right)K_2\right]\,w_c\,S_c}$$

$$= \frac{\left(a + \frac{p}{2}\right)K_2}{\left[1 + \left(a + \frac{p}{2}\right)\frac{K_2}{b\,K_1}\right]w_c\,S_c + w_i\,S_i} \qquad (\mathbf{7})$$

$$T_k = \frac{(B+C)\,W_c + B\,W_i}{B\,C}$$

$$= \frac{\left[1 + \left(a + \frac{p}{2}\right)\frac{K_2}{b\,K_1}\right]W_c + W_i}{\left(a + \frac{p}{2}\right)K_2} \qquad (\mathbf{9})$$

$$\lambda_2 = \frac{M}{2\,L} + \sqrt{\frac{M^2}{4\,L^2} - \frac{N}{L}}$$

Using the previous approximation for the radical:

$$\lambda_2 = \frac{1 + \left(1 - \frac{2\,L\,N}{M^2}\right)}{\frac{2\,L}{M}}$$

$$= \frac{M}{L} - \frac{N}{M}$$

$$= \frac{A\,B + D\,(B+C)}{A\,D} - \frac{B\,C}{A\,B + D\,(B+C)}$$

Neglecting the last term which is very small in comparison with the first:

$$\lambda_2 = \frac{B}{D} + \frac{B+C}{A}$$

$$= \frac{b\,K_1}{w_c\,S_c} + \frac{b\,K_1 + \left(a + \frac{p}{2}\right)K_2}{w_i\,S_i}$$

$$C_2 = \frac{\frac{W_c}{D} - T_k\,\lambda_1}{\lambda_1 - \lambda_2}$$

$$\lambda_1 - \lambda_2 = -2\sqrt{\frac{M^2}{4\,L^2} - \frac{N}{L}}$$

$$= -\frac{M}{L}\sqrt{1 - \frac{4\,L\,N}{M^2}} = -\frac{M}{L}\left(1 - \frac{2\,L\,N}{M^2}\right) \text{ approx.}$$

$$= \frac{2\,B\,C}{A\,B + D\,(B+C)} - \frac{A\,B + D\,(B+C)}{A\,D}$$

$$T_k\,\lambda_1 = \frac{N}{M}\;\frac{(B+C)\,W_c + B\,W_i}{N}$$

$$= \frac{(B+C)\,W_c + B\,W_i}{M}$$

$$\therefore C_2 = \frac{\dfrac{W_c}{D} - \dfrac{(B+C)\,W_c + B\,W_i}{M}}{\dfrac{2\,B\,C}{A\,B + D\,(B+C)} - \dfrac{A\,B + D\,(B+C)}{A\,D}}$$

$$= -\frac{\dfrac{A\,B}{D}\,W_c - B\,W_i}{\dfrac{A\,B^2}{D} + \dfrac{D}{A}\,(B+C)^2 + 2\,B^2}$$

Putting $(B + C)^2 = 2\,B^2$, dividing through by B^2 and neglecting D^2

$$C_2 = \frac{D\,W_i - A\,W_c}{B\,(A + 2\,D)}$$

$$= \frac{w_c\,S_c\,W_i - w_i\,S_i\,W_c}{(2\,w_c\,S_c + w_i\,S_i)\;b\,K_1} \qquad \textbf{(6)}$$

Putting $T_c - T_a = \theta_c$ in equation (**26**), the temperature rise of the armature copper above the mean temperature of the ventilating air at any time t is given by the set of equations (**5**) to (**9**) inclusive.

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Discussion

John Basta: We know how difficult it is to obtain a simple method of calculation of heating curves on assumptions following closely the experience. With those machines where the exchange of heat from winding to air cannot be neglected, the heating curve differs widely from a simple exponential curve. However, Mr. Lightband found it justified to neglect the exchange mentioned with armatures of railway motors and succeeded in showing that the heating curve for them is composed from two exponential curves. The actual heat runs used for verifying the calculated curves, however, do not give more than two, and in some cases even one point only. That is why we would suggest to Mr. Lightband that he undertake another series of tests in which complete curves would be obtained. This, of course, should not be done by measuring the resistances of armature windings, as in the tests presented, but rather by placing temperature detectors into the winding, which would be connected through slip rings to meters. With such an arrangement, the distribution of temperatures along the winding could be ascertained at the same time.

Carl J. Fechheimer: It has been recognized for several years that the usual transient equation for temperature in an armature in which there is one time constant is inadequate, particularly during the first part of the period after the load on the machine is changed. The thermal time constant is given by the ratio of the energy stored to the power dissipated, and that ratio is usually considerably different for the copper and the iron. The errors that may arise for short time ratings, if only one time constant is considered, may be of considerable magnitude. Mr. Lightband has brought out these points clearly.

The general forms of the transient equations when two bodies are considered are analogous to those of the two electric circuits which are magnetically coupled, and when self and mutual inductances are considered. In either case, it is essential to solve two differential equations simultaneously. The solutions for the thermal circuit may be obtained by making use of the principles of superposition and reciprocity. The solutions obtained thereby are perhaps somewhat more general than the forms obtained by Mr. Lightband.

Mr. Lightband's solutions are for the heating state, but they might easily be extended to the cooling state. When the cooling curve is plotted, it is found that the initial part is similar to the "sub-transient" for the decay of current in a suddenly short-circuited alternator, and like it, it dies out relatively soon, and only the "transient" state then obtains, and then only the second time constant is needed. Mr. Lightband has shown for the heating curve that the influence of the first time constant diminishes fairly rapidly, and subsequently only the influence of the second is of importance.

The third assumption made in the paper, that the rate of heat transfer in the axial ducts is one-half that at the outer surface, limits the final equations to a particular type of ventilation. This should be borne in mind by anyone who plans to use the equations. Undoubtedly, the equations could easily be modified so as to make them of more general form.

It should be recalled that the problems involved in heat flow become too involved to secure an accurate solution. For example, in many machines the influence of the longitudinal heat flow in the copper has a very large effect upon the copper temperature. While the differential equations for this case can be written, the solutions for the transient state become too involved to admit of placing them in practical form. It is essential to make assumptions such as Mr. Lightband has done, although his assumptions may not be the ones to adopt for some other class of apparatus. Thus, his results are all based upon the average copper temperature as measured by resistance. For machines having long cores, the copper at the middle is usually at considerably higher temperature than the average, and consequently his assumption *B* would introduce too large an error.

P. H. Powell: A point which is not quite clear in Mr. Lightband's paper is the definition of K_2 and the surface of the armature regarded as radiating. His argument seems to be based on the assumption that all the heat is dissipated by the surfaces of the tops of the teeth and by the surface of the ventilating ducts; on the other hand, K_2 is defined with respect to the armature surface again somewhat indefinitely. If one regards the outside barrel surface of the armature (circumference multiplied by overall length) as being the only radiating surface, the value of K_2 for a speed of 3,000 feet per minute is about 0.1 for open type machines. If one adds up the external barrel surface, the internal barrel surface, and the ducts, the figure for K_2 appears to be about 0.04. Mr. Lightband's result seems to indicate a figure of about 0.05.

There are, of course, differences in types and in ventilating arrangements that make a comparison almost impossible, but it would seem that if only the tops of the teeth and the iron surface of the ventilating ducts are allowed as radiating surface, the figure for K_2 should be considerably higher than 0.05. However, it is very interesting to find a method involved which will enable the short-time rating to be better defined.

D. A. Lightband: It is pointed out that in machines where direct flow of heat from the winding to the air cannot be neglected, the heating curve differs widely from a simple exponential curve. I have shown that in railway motors, even though this heat flow is neglected, there will still be considerable deviation from the simple exponential curve. Attention has been drawn by Mr. Basta to the use of only two test points as an illustration of the closeness of the method. It should be pointed out that the two points are really three, since the origin must be included. Any kind of exponential curve cannot be drawn through three such points, which would represent a reasonable characteristic heating curve of a motor. Even a simple exponential curve drawn through these three points has been found to differ very widely from an exponential heating curve estimated from the one hour and continuous temperature tests.

The suggestion by Mr. Basta that test curves be taken by temperature detectors in the winding rather than by measuring the temperature rise by the resistance method would be rather defeating the object of this method since it is on temperature rises measured by resistance that the ratings of all modern railway motors are based. Measurement of distribution of temperature along the winding longitudinally is beyond the scope of this paper.

Mr. Fechheimer mentions the fact that the solutions are for the heating state only. In the consideration of cooling, it should be pointed out that if the different time constants in the copper and iron are to be taken into account, a knowledge is necessary of the exact nature of the heating immediately prior to the commencement of the cooling curve. This is necessary because the nature of the cooling curve will depend on the initial relation between the temperature rise of the iron and that of the copper. The following equations will give the cooling curve when both the initial temperature rise of the copper and of the iron are known.

Equation for cooling with no power on:

$$\theta_c = \theta_o e^{-\lambda_1 t} + C_4 \left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right) \qquad \text{(c)}$$

where θ_o = initial copper temperature

λ_1 and λ_2 defined by equations (7) and (8)

$$C_4 = \frac{(w_i S_i + w_c S_c)(\theta_o - \theta_i) - w_c S_c \left(a + \frac{p}{2}\right) \frac{K_2}{b K_1} \theta_i}{w_c S_c + 2 w_i S_i} \qquad \text{(e)}$$

θ_i = initial iron temperature

For cooling with power on:

$$\begin{aligned} \theta_c &= T_k \left(1 - e^{-\lambda_1 t}\right) + C_3 \left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right) + \theta_4\, e^{-\lambda_1 t} + C_4 \left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right) \\ &= T_k + (\theta_o - T_k)\, e^{-\lambda_1 t} + (C_3 + C_4) \left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right) \qquad \text{(f)} \end{aligned}$$

Mr. Powell has brought up the point that my definition of K_2 is somewhat indefinite since I do not specifically state whether it is based on the heat radiating from the whole external and internal radiating surface of the armature. He is of the opinion that if this constant refers to the radiating surface obtained by adding up the external surface of the armature core and winding, the internal surface of the armature coils, and the surface of the ducts, it should have a value higher than that given in the paper.

I have made the definition of K_2 as the coefficient of heat transfer from the armature surface to the ventilating air. By "armature surface" is meant the external surface of the armature core and the internal surface of the ventilating ducts. Since I have assumed that no heat flow has taken place from the armature core to the windings in a longitudinal direction, it is quite unnecessary to include the outer surface of the windings as part of the radiating surface. The amount of heat which flows through the insulation at the mouth of the slots is a very small proportion of the total heat flow from the armature and has been neglected, and therefore the only radiating surfaces which need be considered are the tops of the teeth and the ventilating ducts.

The figures for K_2 which are calculated from temperature tests by means of the equations given in the paper and shown in Figs. 3 and 4, correspond very closely to values of rate of heat transfer for various kinds of rotors given in Luke's paper referred to in the bibliography. Luke's results were obtained by means of an exhaustive series of experiments and offer a very sound basis of comparison for heat transfer constants in rotating armatures.

High Capacity Rectifier Efficiency Improves by Sectionalizing

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Member, A.I.E.E.

Synopsis.—The reasons for adopting the sectional construction are given and discussed at some length. The new features of design and construction which make the sectional arrangement feasible are described. One arrangement of sections into a unit is described and alternative arrangements are discussed briefly.

Performance characteristics are described in broad terms and the extent of the improvement over previous type is pointed out.

A forecast of future development is indicated.

THE idea of the sectional type mercury arc rectifier is based on the obvious, but apparently heretofore unrecognized, fact that the best possibilities inherent in the principle are realized only in the smaller sizes. Efficiency, reliability, economy, and flexibility of both application and use are all far better in small than in large units. The advantages appear to be fundamental and, therefore, permanent.

The rectifier is not alone in this. Many familiar things are inherently at their best in small sizes. The established and common "unit type" things are so well known that we have come to overlook the limitations and inefficiencies introduced by the subdivision and to accept the scheme as natural and right. In suspension insulator units, a large part of the insulation material is shunted by metal, assembly operations are duplicated many times in a single unit of use, and the distribution of potential along the string is wrong unless corrected; but the mechanical strength, flexibility, adaptability to all voltages, and the economy of concentrating manufacture and stocks in a single size of unit are so much more important that the disadvantages almost vanish. Because the benefits outweighed the disadvantages, the introduction of the idea of sectionalization in line insulators created a new utility, and because the increase in utility was great, the whole of the associated art of long distance transmission received a major stimulus.

So with the metal tank mercury arc rectifier. Subdivision into unit sections introduces both advantages and disadvantages, but the advantages so far outweigh the disadvantages that, once the conception is seriously considered and the sectional design actually worked out, doubts disappear and it seems the right and natural way to proceed.

Development through the past twenty-nine years has resulted in rectifiers of small capacity, 500 kw. for example, with a high degree of reliability, a reasonable cost, and an efficiency which makes available for the higher voltage conversion applications the unique quietness and freedom from mechanical wear which has provided the major incentive throughout the development. When size is increased to get larger output capacity, however, the quality decreases markedly. Reliability decreases not only in proportion to size as might be expected, but more rapidly, because the complexity of the problem of control of the hurricane flow of vapor from the cathode is increased many fold when size is even doubled. The "outage time" per year with 3,000-kw. rectifiers is several times as great as with 500-kw. units. Cost per unit of output does not decrease with increase of size, as it does with some apparatus, because manufacturing problems increase faster than in proportion to size, and because the total requirement for this kind of equipment, with capabilities and limitations as they are even in the smaller sizes, will not support organized "line manufacture" for the larger capacities. Efficiency falls off with increase in size because of the longer arc and greater exposure to deionizing surfaces, and because the fundamentally less reliability makes necessary an increase in loss producing devices to get acceptable quality. With present designs, the internal losses for a 500-kw. unit are approximately 25 per cent less than for a 3,000-kw. unit, both based on 600-volt operation.

The major disadvantages which go with size appear to be fundamental and permanent, whatever advances may be made in the art. Larger units will always be less efficient, and the reliability and cost disadvantages will probably persist.

If we think of multiple installations of conventional small capacity units of the sort which have been available in the past, we realize that the project is impracticable. The space required is too great and the multiplicity of control and protective devices appears questionable. But when we think of subdivision of a large capacity, taking advantage of space economies resulting from recent advances in the art, building the structure in sections but installing, controlling, protecting and using it as a whole, the project becomes feasible and the desirable qualities of the small rectifier are made available for the larger capacities while the disadvantages of sectionalizing are so reduced as to become negligible.

Our work with rectifiers of conventional size and characteristics has resulted in the conviction that the limitation in capacity for a given design, as for example

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the 500-kw. unit shown in Figs. 1 and 2, lies in local limitations of the various parts, for example, the limits of conductivity and heating at the anode, and not in the perhaps more fundamental and difficult tendency to arc back at the higher currents. Such a unit carries a nominal current of 833 amperes continuously, 1,250 amperes for two hours, and 2,500 amperes for one minute. In all our testing, which includes use at currents up to 17,000 amperes on short circuit, and which includes considerable experimental experience of loads of 2,000 to 2,500 amperes for periods of 5 to 30 minutes, dependent on temperature rise, we have not experienced arcing back as a limitation. Extended service experience with a limited number of installations gives the same result. Evidently, change in the details, leaving the general arrangement unaltered, would make it possible materially to reduce the size of the containing tank without affecting the ability of the unit to carry the loads required by its rating. Experiments along these lines have verified this conclusion.

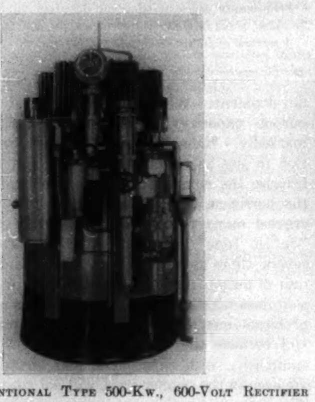


FIG. 1—CONVENTIONAL TYPE 500-KW., 600-VOLT RECTIFIER

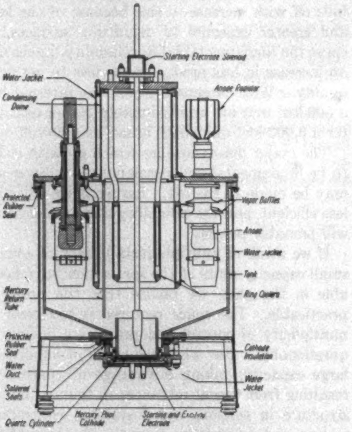

Starting Electrode Solenoid
Water Jacket
Condensing Dome
Anode Radiator
Protected Rubber Seal
Vapor Baffles
Anode
Water Jacket
Tank
Ring Coolers
Mercury Return Tube
Protected Rubber Seal
Cathode Insulation
Water Duct
Water Jacket
Soldered Seals
Quartz Cylinder
Mercury Pool Cathode
Starting and Exciting Electrode

FIG. 2—SECTION SHOWING ARRANGEMENT OF PARTS IN CONVENTIONAL 500-KW., 600-VOLT RECTIFIER

The diameter is 54 inches and the overall height is 104 inches

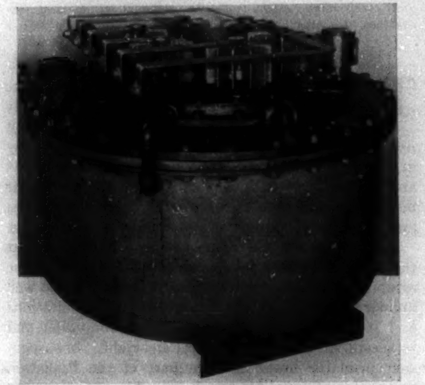
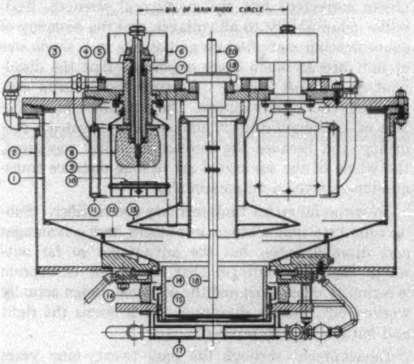


FIG. 3—RECTIFIER SECTION, 750-KW., 600-VOLT RATING


DIA. OF MAIN ANODE CIRCLE
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18
19
20

FIG. 4—SECTION SHOWING ARRANGEMENT OF PARTS OF UNIT SHOWN IN FIG. 3

The diameter is 54 inches and the overall height is 41½ inches. The kw. rating per unit volume is thus over 3 times as great as for the conventional unit

As a further step, a rearrangement of parts made possible a still further reduction in size for a given rating, and at the same time brought about some reduction in the internal losses. The rearrangement was based on the knowledge gained in our experimental experience as to the effect of the way the vapor flows from the cathode to the condensing wall and the position of the anode structures in relation to this stream of ionized vapor on the limiting current of which the rectifier unit is capable, without excessive arcing back. Although the whole mechanism of vapor flow and its effect on arcing back are obscure as yet, several factors which influence performance have been established. The design shown in Figs. 3 and 4 is an attempt to take these as yet rather intangible and, it must be admitted, somewhat uncertain factors into account in the most satisfactory way possible at the present time.

The anode support and shielding arrangement also offered an opportunity for progress and a rather extended series of experiments resulted finally in the anode structure shown. Processing to eliminate foreign material and keep the gas evolution under operation at as low a value as possible, have progressed along with the design development.

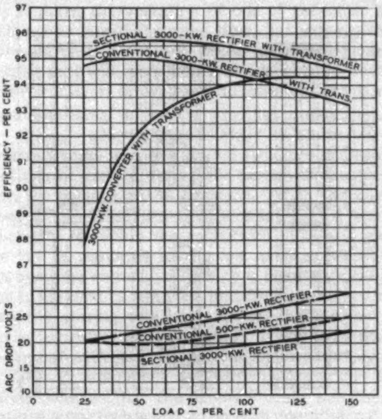

Fig. 5—Representative Arc Drop and Efficiency Curves for Conventional and Sectional Rectifiers

As a result of these improvements, the rectifier section shown in Figs. 3 and 4 is capable, with a reasonable margin, of a standard nominal rating of 750 kw. at 600 volts, and even of higher short-time overloads as sometimes required.

This section, at 750-kw., 600-volt rating, is required to carry a normal current of 1,250 amperes, with 50 per cent overload, or 1,875 amperes for two hours. Owing to the fact that final temperatures are reached in times of the order of two hours, it is felt that the two-hour capacity can best be indicated by continuous operation at the 50 per cent overload value and our experimental tests have been made in this way, thus including in a single test some margin of safety along with a degree of acceleration of the test. Double load, or 2,500 amperes is required by standard nominal rating for a period of one minute following full load. Experimental tests have been made at this current up to 5 minutes duration.

Fig. 6—Sectional Rectifier, 3,000-Kw., 600-Volt Rating

This unit consists of four sections each of 750-kw. rating, the supporting structure and the necessary auxiliary equipment for maintaining the vacuum and dividing the load. Some control and protective devices are also conveniently mounted on the structure

Fig. 7—View of Rear of 3,000-Kw., 600-Volt Sectional Rectifier Showing the Arrangement of the System for Maintaining the Vacuum

In all these operations, arcing back is very infrequent if it occurs at all and it is a requirement that the rectifier can be put into operation again immediately following such an occurrence.

The internal losses in the rectifier are determined by the arc drop, or voltage from anode to cathode during the conducting period. This value varies with load current and can be indicated by a curve of voltage drop against load current. The value that determines efficiency is the average for the various anodes. Fig. 5 shows average arc drop curve for the unit indicated by Figs. 3 and 4. The dotted curve shows the value for the older, conventional, unit shown by Figs. 1 and 2. As the size of a rectifier is increased, the arc path becomes longer and losses per kw. output become greater. Experimental rectifiers of 3,000 kw. with a single tank have had arc drop values as shown by the dot and dash curve of Fig. 5. It is seen that at full loads the difference between these experimental single tank 3,000-kw. rectifiers and the section shown in Figs. 3 and 4 amounts at full load to approximately 6 volts or 1 per cent, on the basis of 600 volts output. With ordinary load factors and rates for power, this difference in 3,000-kw. rating saves from $600 to $1,500 per year power cost, which represents an added value of from $4,000 to $10,000.

The proportions of the new units are such as to make it feasible to mount them in service one above another. Thus, a 3,000-kw. unit can be made of four 750-kw. sections mounted two side by side and two deep. Such an arrangement is shown in Fig. 6. The floor space required for such an assembly is approximately 118 inches wide, by 68 inches deep over all parts, including auxiliary equipment. The over-all height is 113 inches, and the ceiling height of the room necessary to accommodate this equipment need not exceed this value except for electrical clearance of a few inches and whatever may be necessary to provide for adequate working space certainly not more than two feet. It is evident that there is a considerable space economy.

Such an arrangement is likely to prove the most convenient for usual applications. There is no reason, however, why other arrangements cannot be used where available space places a limitation on one or more dimensions. For example, the auxiliary equipment and wiring can be arranged between the units at the center, at some penalty from the standpoint of accessibility, making the floor dimensions approximately 150 by 55 inches. Or the four units may be mounted in a single column with floor dimensions of about 65 by 70 and a height of 210 inches. Or the parts may be separated in two or groups, possibly at some expense in auxiliary equipment.

The problem of paralleling is the same as in the case of a single tank rectifier using the same number of anodes and, except in detail, is the same as for any rectifier using twelve or more anodes. Load balancing is inherently approximately correct owing to the fact that the voltage across the arc rises with increasing current after a minimum at about 40 per cent load is passed. To get a closer balance, anode balancing coils are used and these are so arranged as to introduce no appreciable impedance into the circuit under normal operation. In the event that it is desired to take a section out of service, for maintenance or any other reason, the corresponding balancing coil sections are short circuited by connection links provided for the purpose.

One arrangement of vacuum pumping is indicated in Fig. 7. Use is made of two independent pumping systems, connected through manifolding to the various

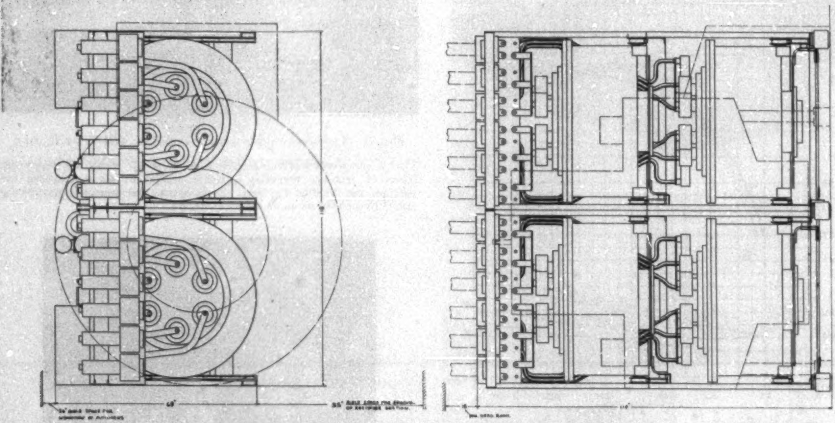

FIG. 8—OUTLINE OF 3,000-KW., 600-VOLT SECTIONAL RECTIFIER

Approximate dimensions of an experimental single tank unit are indicated by the dot and dash outline sections and with cross connections so arranged as to permit the use of either pumping system on any or all of the tanks. The various sections of the rectifier are connected to the vacuum manifold through two valves of unique design, provided with a sealing plate as an extra precaution against outside air. Even if such a valve should develop a leak, this safeguard would prevent any leakage into the tanks except at a very slow rate and for the minute or less while the valve position is being changed. The use of two valves permits the separation of a single section from the balance of the structure without admitting air to either part. In the event that maintenance work is required, it is thus possible to put the repaired section in condition by pretreatment, then install it and by proper manipulation of the valves, exhaust it completely without admitting air to either that unit or any part of the operating system. With the pumping arrangement shown, this requires the operation of one of the other units with valves closed for some time, but experience has indicated that if properly made and treated out the rectifier unit will carry normal loads without pumping for several hours or even several days without damage.

Sectionalizing inherently provides the utility that any damage which occurs is likely to be limited to a relatively small part of the total structure and that the load can be carried during the period of maintenance with the balance of the structure. With the arrangement in Fig. 7, a faulty section is segregated from the others by removing the connection links and closing the valves, requiring only a few minutes, before load can again be carried. Although a maintenance operation is required to remove a unit from the structure for repairs requiring two hours or so, this can be done during a light or no load period. It would be possible to arrange with jack connections and separate systems of auxiliary equipment or flexible pumping and water connections, so that the operator could pull the unit forward a few inches and go on with operation with the balance of the equipment after an interruption of one or two minutes. The choice between these two plans depends on the frequency with which interruptions are likely to occur. The inherent quality in the design, together with the experience thus far indicate that the interruptions requiring maintenance will be so infrequent as to make the simpler plan preferable.

The project for sectionalizing, together with the development in working out sections of suitable dimensions and proportion, thus makes available an increased degree of reliability, increased flexibility in application and use, improvement in manufacturing requirements, and an efficiency advantage of 1 per cent or more. Beyond this is the belief that as time goes on and as we acquire a clearer understanding of the fundamental knowledge gained from our research, but not yet fully used in design, further improvement will be made in respect to efficiency and size. It is to be expected that, as this progress occurs, the development will be along the lines of sectional structures rather than along the lines of single unit arrangements.

It is obvious that a development such as this is one to which essential contributions are made by many individuals. The successful completion of such a project requires a variety of knowledge, experience and talents, no part of which can safely be omitted. The author wishes to acknowledge essential contributions to this development by the research, substation, switchboard, transformer, and manufacturing engineers of the Westinghouse Electric & Manufacturing Company as well as by the design engineers whose work has made possible the design improvements. Special acknowledgment is also made of the enthusiastic and courageous support and encouragement of our executives, especially Messrs. Rugg and Kintner.

Discussion

A. W. Hull: This discussion relates to the fundamental limitation of rectifiers as given in the paper: namely, the prophecy that "the major disadvantages which go with size appear to be fundamental and permanent. Larger units will always be less efficient and probably less reliable."

The theory of rectifiers and our experience up to the present does not seem to me to justify this conclusion. The statement that "efficiency falls off with increase in size because of the longer arc" does not appear to me to apply to large rectifiers. Efficiency does not depend fundamentally on arc length but upon the ratio of length to width, or diameter, of arc path. When the dimensions of a rectifier are doubled, the width of the arc path is doubled as well as the length, and the total arc drop remains unchanged. This is in agreement with the theory of the nature of arc drop and with all experimental evidence, so far as I am aware. If large rectifiers are at the present moment less efficient than small ones, it is, I believe, a mere chance which is due to the relative state of development, and not a fundamental limitation of large rectifiers.

The same is true in regard to reliability. The only theoretical prediction regarding reliability, as a function of size, is that the arc does not tend to concentrate on a single point of a large anode but, after reaching a definite density, which is the same as the density in small rectifiers, it spreads uniformly until it covers the whole anode at full load. Hence, size does not introduce danger from concentration. There is no evidence, with which I am familiar, either theoretical or experimental that would indicate any fundamental reduction in reliability with increase in size, or the need of increased baffling in order to obtain reliability.

C. A. Butcher: It is perhaps quite generally known that I have not been an enthusiast for mercury arc power rectifiers. During eighteen years of experience having much to do with power supply for electric railways, I have witnessed many improvements in the design of substations. In that time, high reluctance commutating poles and high-speed circuit breakers solved the flashover problem in the operation of 60-cycle synchronous converters. The perfection of automatic control including supervisory, effected economies and improvement in operation.

Shortly after the first display of enthusiasm for the rectifier in America, following a lapse of a few years during which development on the part of American manufacturers ceased, I made a careful review of the available literature on the subject. This included the early development experience of my own company as well as that of one European manufacturer—also reports covering the operation and performance of a number of European rectifier installations. In a paper* I compared the rectifier with the synchronous converter. It did not appear that the conventional design of rectifier which has changed but little since then could on economic grounds replace the converter, in the 600-volt class. The demand for conversion apparatus for higher voltages is at present not of itself such as to be conducive to commercial development. The inefficiency of the rectifier at lower voltages has limited its application with few exceptions to the voltages of 600 and above. Even in the 600-volt class, it has not had the advantage of equality from the standpoint of service availability or first cost. Only at light loads is there any margin in efficiency favoring the rectifier. This advantage is very much discounted in metropolitan areas where conversion units are added to and taken out of service as demands require, which effects an average load per unit of from 75 to 100 per cent of rating.

It has not been demonstrated that the maintenance of metal tank mercury arc rectifiers will be any less than for other types of conversion apparatus.

The auxiliary equipment required with rectifiers is much greater than with synchronous converters.

On the whole, large rectifiers of the conventional type are perhaps in a class with 600-volt, 60-cycle synchronous converters before the introduction of high reluctance commutating poles and high-speed circuit breakers.

It is certain that if the rectifier is to stand as a distinct and economic advance in the art, something must be done to assure fewer service interruptions. The record established by modern synchronous converters in this respect will be difficult to equal. Also definite improvement in full load efficiency is fundamental to successful exploitation; otherwise the synchronous converter will be continued as a competitor with consequent economic loss.

Mr. Atherton has described a new development—a sectional rectifier. Tank volume per unit of capacity has been so reduced that an assembly of four 750-kw. units requires considerably less space than a single 3,000-kw. unit of conventional design. Brief as has been experience in actual service following extensive research laboratory work and testing, indications are strong that a high order of availability is to be expected. Efficiency is definitely one per cent better than that of a single tank unit of equal rating. One or more sections may be removed from service without impairing service from the remaining sections. Only a few minutes are required to disconnect a section. Division of load between sections is practically perfect. A sectionalized assembly minimizes the necessity for spare conversion equipment which is no small advantage in its favor considering capital investment in real estate, buildings, and switching equipment.

The multiplication of auxiliaries might seem a formidable objection, but surely not more so than the number of devices that go into an automatic substation. Having been intimately connected with this latter development, the success of which is unquestionable, I feel certain that there is very little cause for concern.

Truly, for the first time, I am enthusiastic about a rectifier.

W. E. Gutzwiller: The idea of the sectional type rectifier is not new. Previous to the year 1926, when 1,000 amperes was about the maximum rating per rectifier tank, sectional arrangements were used by all manufacturers wherever larger capacities were involved. Practically all rectifier installations in this country between the years 1923 and 1926 had two rectifier tanks connected to a common transformer. In Europe at about the same period as many as six tanks were operated from one transformer. Then rectifiers were usually assembled side by side on a common frame and were exhausted by a common vacuum pump set. As soon, however, as rectifiers of larger capacity had been developed sufficiently, it was found that their manufacturing cost, floor space requirements, and attendance were considerably less than for a sectional arrangement of smaller units and their reliability greater. The result was that the sectional idea was almost entirely abandoned.

The smaller sizes of rectifiers have been undergoing a process of development for a great many years, whereby certain of their features have been sufficiently perfected resulting in a slight gain in efficiency when compared with the larger sizes more recently produced. Without doubt, however, further development of the larger types will follow along similar lines so that the present slight difference in efficiency will probably disappear. Where this slight gain in efficiency is a factor of considerable commercial importance as for instance in electrolytic plants, then the sectional idea can and has been adopted.

The new feature of the rectifier described in the foregoing paper is the reduced overall height which permits at least for lower voltages, the compact assembly of a number of tanks side by side or one above the other in a cubicle form. This reduction in height is made possible apparently by placing the condensing dome into the arc chamber whereas in the conventional type rectifier, it is mounted on top of the arc chamber. The diameter and height of the arc chamber are the same as for our conventional 750-kw. 600-volt rectifier. But in my opinion this advantage of compactness has been purchased at too high a price for it has necessitated the placing of additional water cooling cylinders at points within the chamber where they are much exposed to the heat of the arc—an innovation that is perhaps dangerous.

The claims of this new sectional type rectifier are based on comparison of the conventional 500 and 3,000-kw. rectifier with a 3,000-kw. sectional type unit all at 600 volts direct current. No comparison is made between this rectifier and the conventional type in case of say 1,500 and 2,000-kw. 600-volt units or large rectifier units operated at 1,500 or 3,000 volts direct current. In other words, the author picked out the most favorable condition for his comparison and omitted all reference to medium size 600-volt units and units at higher voltages.

The arc drop curve for the sectional rectifier shown in Fig. 5, is substantially the same as for the conventional 750-kw. 600-volt rectifier built by other manufacturers. It is stated by the author that with increased tank capacity, the quality and reliability decreases and further that the outage time per year with 3,000-kw. rectifiers is several times as great as with 500-kw. units. My experiences do not check with this; in fact, I find the reverse to be true. This is independently confirmed by the 1931 Report of the A.E.R.A. which shows a 2 per cent higher availability factor for the 2,500 and 3,000-kw. units of which a total of ten units is listed as against twelve 500-kw. units. The sectional type 3,000-kw. unit has more electrodes, more seals, more weldings and several times as many auxiliaries and temperature control devices than the conventional type rectifier. One would expect, therefore, that its reliability would be inferior to the single tank arrangement.

As far as space requirements are concerned, there is a slight gain in floor space for a 3,000-kw. 600-volt sectional unit. Since, however, space for transformer equipment, switchgear, and control remains the same as for a single tank unit and the rectifier seldom occupies more than 35 per cent of the total substation floor area, the total gain in space is not of much account. The headroom requirements of the above mentioned unit are somewhat less than for a single tank type. Here again the space advantage is less pronounced in the case of 1,500 and 2,000-kw. 600-volt units and does not exist for 1,500 and 3,000-volt rectifiers which the author omitted to mention.

Contrary to statements made by the author, it has been found that the cost per unit of output decreases with larger capacity rectifiers. I find that for a 3,000-kw. 600-volt single tank unit, it is from 30 to 40 per cent lower than for a 500-kw. unit. Furthermore, in case of small units the auxiliary apparatus represents a large percentage of the total unit cost. This, however, is not the case with large rectifiers as they require about the same amount of auxiliaries as small tanks.

**Application of Mercury Arc Power Rectifiers*, by C. A. Butcher, A.I.E.E. JOURNAL, May 1927, p. 446.

The demand for rectifiers so far is not in sufficient volume that much could be gained from mass production. Also in any event it would seem that even this small volume will require more than one tank size to take care of all requirements; *i. e.*, low-voltage and high-voltage units. Mass production, therefore, is hardly feasible; nevertheless, individual parts, such as, anodes, etc., can and are produced in quantities even with conventional type rectifiers.

It appears to me that the claims made for this rectifier have been somewhat over emphasized. Experience has shown that small sectional types have usually been later supplanted by large single tank types for reason of simplicity and reliability except in very special cases where other considerations were dominant. This is particularly true on fluctuating loads, such as heavy traction service where a real gain in reliability is of more importance than a slight gain in efficiency.

R. E. Hellmund: Some of the discussion refers to the comparative reliability and availability of sectionalized designs versus large unit designs. A somewhat similar example taken from the transportation field is given as follows.

In the early days of a-c. main-line electrification, a great deal of discussion took place on the relative merits of locomotives having one or two large motors and those having many small motors, such as the New York & New Haven locomotives having the multiple twin-motor arrangement. Practical experience showed very quickly that the reliability and availability with the large motors were very low because such unavoidable accidents as the breaking of a single carbon might damage the large commutator and keep the locomotive out of service a great part of the time. In contrast to this, a similar occurrence in the locomotives having many small motors, that is, "sectionalized power," permitted the quick replacement of the damaged motor and resulted in much greater availability of the locomotives. This was not a matter of theory but rather one of absolute fact and experience, and naturally it has practically eliminated any desire on the part of designers or operators for locomotives having large single motors.

The same tendency would not be quite so marked in apparatus having but very few parts to get out of order, such, for example, as synchronous machinery, and in such cases the tendency is toward a few large units rather than toward sectionalization. The mercury arc rectifier, however, in its present stage of development is not in this latter class. It involves the fulfilment of various severe and difficult requirements, such as tightness of a great many welded and other joints, as well as absolute cleanliness both during the construction of the rectifier and during repairs in the field. The smallest defect in either direction results in troubles and in failure. I have no doubt that under such conditions as these the sectionalized design will in the near future demonstrate its superiority over the large unit design from the standpoint of both maintenance and availability.

A. L. Atherton: Mr. Hull indicates disagreement with the reasons on which the sectional arrangement of large capacity rectifiers has been based. In view of his standing as a scientist, we may accept his fundamental statements, especially since they deal with a point which is relatively well established and generally known. It seems, however, doubtful to me if we may so readily accept the application of these fundamentals. That part of the arc path within the shield around the anode is not increased in cross section with increase in size of the rectifier. If a single set of six anodes was used for the same total output in the large and small capacity rectifiers, we might assume this relation to be as he has stated but this is not the case in today's practise. The 3,000-kw., 600-volt rectifiers are made usually with either 12 or 18 anodes which gives a rating of 1,000 to 1,500 kw. per set as against 500 or 750 for the sectional arrangement and for the smaller conventional units. Similarly, throughout the arc the effective cross section is not increased with length as the tank size is increased. The barriers, constructional or otherwise, to retaining the same design parameters in the larger units as were used in the more fully developed smaller units, are likely to prove sufficiently permanent to prevent this kind of working out of large single-tank units.

As Mr. Hull has indicated, reliability is probably affected by the same factors as efficiency, since it depends on conditions in the gas path. I do not think that we have anything so simple to deal with as the obvious but rather negative thought that the arc does not tend to concentrate on a single point. The amount and condition of vapor available for conductivity, its state of rest or motion as a body, the interaction between arcs which are carrying current and paths which are expected to withstand back voltage, and many other complicating and complicated factors enter the question of reliability.

As is usually the case in a development or investigation in which all of us know a relatively small share of what is to be known, we shall each of us have to be guided by our available knowledge and beliefs at the time. The course of events will determine which is the correct procedure. This much seems to me to be quite definite; in its present state of development, the sectional rectifier does have a material and valuable advantage in efficiency over the single-tank units of large capacity. I think that the same relation holds for reliability factors. I think that the convenience of concentrating the manufacturing problem in a reduced number of sizes will offset the fundamental disadvantage of the requirement for a greater number of pieces and a greater amount of material. I may be wrong in believing that these advantages will be retained as the developments progress. Time alone can tell.

Mr. Gutzwiller's reference to the experience in making up larger capacities of parallel installation of smaller capacity units, adds to our overall picture and it demonstrates anew the fact that valuable procedures are not always recognized at first.

He refers repeatedly to the gain in efficiency as slight. This gain at 3,000 kw., 600 volts amounts to approximately 1 per cent. For average load conditions in metropolitan traction service and with usual power rates, this gain in efficiency is worth from $750 to $1,500 per year, or capitalized at 15 per cent, from $5,000 to $10,000 in first cost. This is from 10 to 25 per cent of the normal cost of a rectifier of this capacity. The magnitude of the gain is evidently a question of point of view.

Here again, we shall have to depend on time to give the final answer. In the meantime, we who think of sectional designs as desirable can take considerable comfort in the example Mr. Hellmund has given of a well known and definitely established practise of sectionalizing for the sake of just these same gains that we feel are secured by sectionalizing the large capacity rectifiers.

Preparing Substations for Electric Operation
Philadelphia to Trenton Electrification, P.R.R.

BY L. B. CURTIS[1]

Member, A.I.E.E.

Introduction

INVOLVED in the extensive electrification program recently announced by the management of the Pennsylvania Railroad between New York and Washington are approximately forty substations. Eleven of these, including those covered by this paper, are already in operation. A definite program for testing these substations before placing them in operation had evolved, and it was thought advisable to combine these tests in a single set of instructions. This has been elaborated somewhat in this paper.

Since Edgely substation is typical of the majority of the forty substations, it is used to exemplify the testing. The other substations included in the Trenton electrification are Frankford Junction, Cornwells, and Morrisville. The standard apparatus consists mainly of 4,500-kva. step-down transformers, high-speed trolley breakers, necessary 132-kv. and 11-kv. switches and breakers, and control equipment.

The testing may be separated into the following three groups:

A. Preliminary tests without d-c. control and a-c. power.
B. Tests with d-c. control but no a-c. power.
C. Tests with d-c. control and a-c. power.

Group A—Preliminary Tests without D-C. Control and A-C. Power

Current Transformer Polarity Test

Due to the fact that differential protection is employed in some instances it is imperative that the polarities of current transformers installed in the bushings of all power transformers and circuit breakers be correct. All protective relays receive their actuating current from these transformers. The end of the current transformer corresponding to the top of the bushing is lettered *a*, and is understood to have the same instantaneous polarity as the top of the bushing terminal.

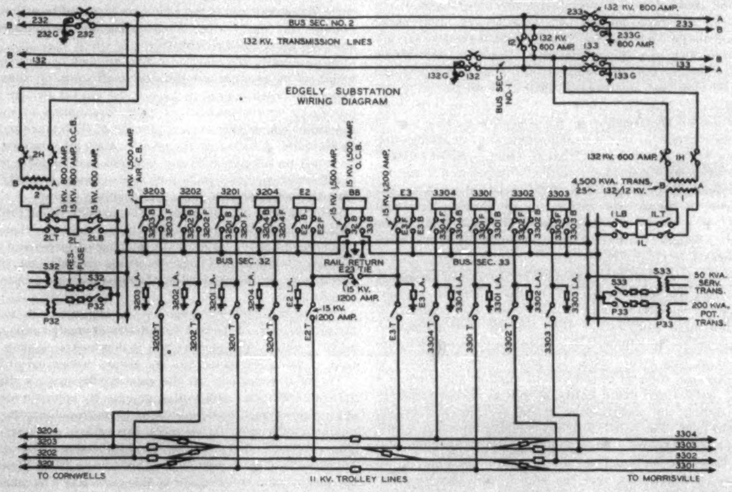
EDGELY SUBSTATION WIRING DIAGRAM
132 KV. TRANSMISSION LINES
BUS SEC. NO. 2
BUS SEC. NO. 1
132 KV. 800 AMP.
132 KV. 600 AMP.
132 KV. 600 AMP.
232
232G
233
233G
132
132G
133
133G
12
A
B
2H
1H
2
1
4,500 KVA. TRANS. 25~ 132/12 KV.
15 KV. 800 AMP.
15 KV. 800 AMP. O.C.B.
15 KV. 600 AMP.
15 KV. 1,500 AMP. AIR C.B.
15 KV. 1,500 AMP.
15 KV. 1,500 AMP. O.C.B.
15 KV. 1,200 AMP.
2LT
2L
2LB
1LB
1LT
1L
3203
3202
3201
3204
E2
BB
E3
3304
3301
3302
3303
BUS SEC. 32
BUS SEC. 33
RAIL RETURN
E23 TIE
15 KV. 1200 AMP.
15 KV. 1200 AMP.
RES. FUSE
S32
P32
S33
P33
50 KVA. SERV. TRANS.
200 KVA. POT. TRANS.
3203 LA.
3202 LA.
3201 LA.
3204 LA.
E2 LA.
E3 LA.
3304 LA.
3301 LA.
3302 LA.
3303 LA.
3203 T
3202 T
3201 T
3204 T
E2 T
E3 T
3304 T
3301 T
3302 T
3303 T
3204
3203
3202
3201
3304
3303
3302
3301
TO CORNWELLS
11 KV. TROLLEY LINES
TO MORRISVILLE

Fig. 1—Edgely Substation Wiring Diagram

Fig. 1 shows the connections for checking polarity. A millivoltmeter is connected across the secondary of the current transformer about to be tested with the plus terminal attached to the *a* side. Two or three dry cells connected in series have their negative terminal connected to the bottom of the bushing terminal. When the positive terminal is touched to the top of the bushing, or on the "make" there should be a positive deflection on the millivoltmeter. A negative deflection takes place on the "break." When the current transformer is connected through to the control and relay board, the above test is repeated with the millivoltmeter connected across the test switch behind the board. Every current transformer is tested in a similar manner.

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[1] Supervisor of Substations, P.R.R.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Circuit Breaker and Transformer Oil Test

Samples of oil from all oil-filled equipment must be tested for electrical breakdown strength before the equipment is energized. At least six samples are taken in each case. With the electrodes in the test set spaced 0.1 inch apart the average breakdown value must not be less than 25,000 volts.

Transformer Megger Test

All transformers are meggered to test their insulation. Readings are taken from high winding to low winding, from high winding to ground, and from low winding to ground. All readings must be sufficiently high to insure the absence of moisture, short circuits, and grounds.

Power Transformer Tap Changing Device Test

The power transformers are equipped with a tap changing device on the top cover in order to vary the voltage ratio between the high-voltage and low-voltage windings. A definite ratio is decided upon and all step-down transformers on the system are set on this tap.

Manual Operation of All Switching Equipment

All manually and electrically operated switches must be carefully adjusted for alignment and smooth operation by hand. No electrical operation of motor operated switches should be attempted before perfect operation is performed manually.

Electrical Interlock Test

In order to secure safety for the operators, and protection for the equipment, some of the apparatus are interlocked. The physical features of the interlocking parts should be carefully inspected so that all moving parts move freely.

Physical Check of All Relays

All protective relays are given a predetermined setting after an examination as to their condition. The relays are operated by hand to determine the contact adjustment and if any friction exists between movable parts.

Group B—Tests with D-C. Control but No A-C. Power

D-C. Energizing of Relay and Control Board

The combined relay and control board is now energized with the 125-volt control battery, one section at a time. Two lamps connected between the negative and positive buses and ground indicate whether any grounds exist.

Electrical Control and Indication Test

In the switch house on the control board there is a double pole switch that throws the control either on "station control," that is, control from the switch house, or "tower control" which is control from the nearest railroad tower. Every electrically operated piece of equipment in the substation is operated by means of the drum control handles on the control board both in the substation and in the tower. A green light indicates "open" and a red light "closed." Every operation and indication must be perfect.

Electrical Interlock Test

The apparatus that is interlocked together, some to open a load circuit in a breaker instead of in a disconnecting switch, and some to prevent a ground switch from being closed on a live circuit is now operated from the control board. The interlocking features must work every time.

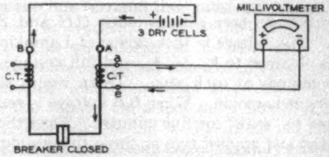

Fig. 2—Connections for Checking Current Transformer Polarity

Manual Relay Operation Test

The last test of this group is to check the relay operation of all equipment by causing the relays to function by hand. Inasmuch as the operation of these relays will be described under the a-c. tests it will be omitted here.

Group C—Tests with D-C. Control and A-C. Power

Preliminary

It is necessary to take the following precautions and preparations before these tests begin:

a. The entire catenary and transmission lines to be tested must be inspected for visible grounds and broken insulators.

b. A direct telephone connection must be had with the power director.

c. All temporary grounds at the substation must be removed except those on trolley feeders for tests one to eight inclusive.

d. Sufficient number of ground sticks for applying grounds must be on hand.

e. All necessary instruments for testing must be on hand.

f. A program of procedure must be set up and all involved in the test must have a copy so that it is thoroughly understood by all concerned what is to take place.

g. All interested in the test must be previously notified and be present when and where required.

Energizing Test of Transmission Line

The first of the a-c. tests is to build up the potential on the transmission lines. All 132-kv. line switches (132, 133, 232, and 233 in Fig. 1) are closed. The 132-kv. transformer horn-gap switches (1*H* and 2*H*) are opened. The voltage is then raised at Lamokin where power is obtained to ¼, ½, ¾, and full voltage, checking for grounds at each step. These would show on ammeters at Lamokin. When full voltage is reached it is allowed to "soak" for five minutes. Then the power is removed and applied two or three times suddenly at full voltage.

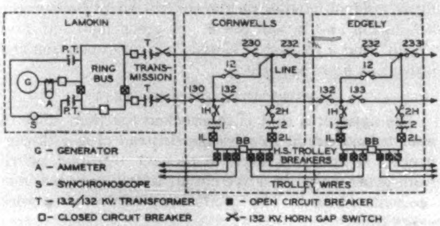

Fig. 3—Schematic Diagram for Phasing Tests

Energizing Test of Power Transformers

Both transformer horn-gap switches (1*H* and 2*H* in Fig. 1) are now closed, in addition to those in test 1. The voltage is built up as before, including the step-down transformers in the energizing.

Phase Test between 132-Kv. Lines and 11-Kv. Buses

a. The first step is to phase out the 132-kv. transmission lines. The 132-kv. line and tie switches (132, 133, 232, 233, and 12 in Fig. 3) are closed and all other switches and breakers opened. The voltage is built up and fed into one bus section at Lamokin from the generator, over one transmission line through the 132-kv. tie switch, and back to Lamokin over the other transmission line. If the pointer of the synchronoscope stops in the proper position the lines are in phase.

b. The next step is to phase through the 11-kv. bus instead of the 132-kv. tie switch. The tie switch 12 is opened and the transformer horn-gap switches 1*H* and 2*H*, the transformer low side breakers 1*L* and 2*L*, and the bus tie breaker *BB* are closed. The voltage is built up at Lamokin as in the preceding test.

Energizing Test of Entire Substation

All switches and breakers in the substation are now closed except the T-disconnect switches where the feeders leave the substation. The substation is energized with full voltage to allow the equipment and buses to "soak" for several hours in order to show up any defective insulation.

Calibration of Potential Device

While the substation is energized the potential devices connected to the 132-kv. bushings of the step-down transformers are calibrated. These devices furnish potential to the transmission line ground protective relays and must be calibrated for the particular relay used. A phase meter is used to read the phase relation between the system or reference voltage and voltage of the potential device. The watt component of the phase meter's burden should closely approximate in value that of the relay burden. The relay is blocked so it cannot operate and is disconnected during the test. The test connections are shown in Fig. 4. A condenser *C* is used across the phase meter in order to make the burden of the latter approximately that of the relay. A high resistance voltmeter is used to read the voltage across the potential device. The reference voltage is obtained from a potential transformer which is connected to the 11 kv. bus and should be constant during the test. A 100-watt lamp is used to obtain approximately one ampere 100 per cent power factor load. The values of reactors, transformer ratio, and condensers in potential device are varied so as to get a combination that will give approximately half reference voltage across the potential device, and the lowest possible phase difference. When this condition has been obtained the phase meter is disconnected and the relay reconnected.

The potential device on the other side of the transformer is then checked as above, and the same results obtained. The blocking may then be removed from the relay.

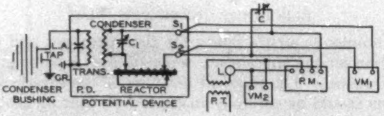

Fig. 4—Potential Device Test Connections

Reduced Voltage Ground Test on High-Speed Breakers

At Lamokin, jumpers are placed across the low- and high-voltage bushings of a step-up transformer so that generator voltage is impressed directly on the transmission line. The generator voltage is built up to approximately 11,000 volts which is reduced to 1,000 volts at the substation. With the 11-kv. bus dead, a high-speed breaker is grounded on the feeder side to the substation steel with a ground stick. This is indicated by × *U* in Fig. 5. Then all switches and breakers in the substation are closed except the disconnect switches on the out-going feeders, and the high-speed trolley breakers. The grounded breaker is then closed from the control board in the switch house.

The oil breakers are controlled from a set of three relays; impedance, load compensating, and instantaneous and time delay which are fed from current transformers located on the bus side of the trolley breaker. The load compensating relay can be considered as a part of the impedance relay, by changing the resistance value in series with the restraining coil of the latter. The impedance setting of the impedance relay is equal to 9/10 of the impedance of the trolley section to which the corresponding breaker is connected in order to avoid getting any overlapping of protection caused by inaccurate calculation. There is a directional element in series with the impedance element so that the relay will operate only when the fault is on the trolley or feeder side of the breaker.

The instantaneous and time delay relay has two overload elements, one high-speed with a current setting, and the other time-delay with a lower current setting. When a ground occurs close to one substation such as the one imposed above, the impedance relay will usually open the breaker feeding the fault. At the next substation, the breaker at the other end of the line will be opened by the high-speed element of the instantaneous and time delay relay. The impedance relay at this point being outside the setting zone will not operate. When the ground occurs midway between two substations the breakers feeding the faulty line will be opened by the impedance relay. The time delay element of the instantaneous and time delay relay acts as a back-up protection and also functions to clear stub end faults at minimum generating capacity.

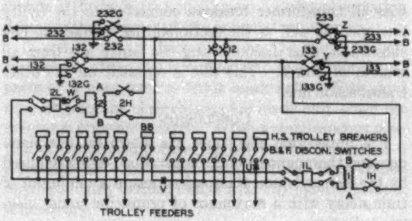

Fig. 5—Location of Grounds—Edgely Substation

The high speed air trolley breaker operates from either of two independent devices serving different purposes, a high speed device which is a part of the breaker, and a directional overcurrent, undervoltage relay.

The high speed device operates on a sudden current increase which is characteristic of a ground. It does not operate in the case of a gradually increasing current. The directional overcurrent, undervoltage relay takes care of this condition. This relay is fed from a current transformer on the trolley feeder side of the breaker. As the current demand increases, the voltage drops, and if this condition reaches a predetermined value the relay will operate to open the breaker.

If the oil or air breaker as the case may be operates satisfactorily, the bus is killed, and the ground stick applied to the next breaker. Every high speed trolley breaker is tested in this manner.

Reduced Voltage Ground Test on 11-Kv. Trolley Buses

A ground is now imposed on one of the 11-kv. trolley buses as shown in Fig. 5 at *V*. All breakers and switches in the substation are closed except the switches on the outgoing feeders, and the two transformer breakers 1*L* and 2*L*. A bus ground is caused by closing these two breakers simultaneously.

The bus differential protective relay is connected to current transformers located on the side away from the section to be protected of the bus tie breaker, the corresponding transformer breaker, and the trolley feeder breakers. The sum of the currents leaving the bus through the trolley breakers connected to one bus section under normal conditions is equal to the current entering the bus through the transformer breaker plus or minus the current flowing through the bus-tie breaker. The secondaries of all the above current transformers are connected together in such a way that under normal conditions there is no current flowing through the relay. On a bus short circuit or ground, however, this equilibrium is disturbed and the relay operates. This energizes a set of auxiliary relays which opens all breakers connected to the faulty bus section, and also a lock-out relay which locks open all closing circuits of these breakers so that they cannot be closed from the control board until the lock-out relay is reset by hand. The other bus section is tested as above.

Reduced Voltage Test for Transformer Internal Faults

A ratio differential relay is used to protect the 4,500-kva. step-down transformers against internal faults. A ground stick is attached near the trolley side of the low voltage bushing as shown at *W* in Fig. 5. If the test is made on No. 2 transformer the transmission line horn-gap switches 232 and 233 (Fig. 5) of the transmission line feeding this transformer should be opened. The other transmission line is energized with the reduced voltage, and 1*H*, 2*H*, 1*L* and *BB* are closed. The ground is caused by closing breaker 2*L*.

The ratio differential relay is energized by two sets of current transformers, one set consisting of two cross-connected current transformers on the high voltage side of the step-down transformer, and the other of a similar combination on the bus side of the transformer breaker. The relay has an operating coil and a restraining coil, there being no current flowing in the former under normal conditions. When a fault occurs in the transformer the balance between the two sets of current transformers is disturbed and the unbalanced current flows through the operating coil causing the relay to operate. This causes the transformer breaker 2*L* to open and after a sequence of operations which allows the fault current to decrease to safe value, the transformer horn-gap switch 2*H* opens thus isolating the faulty transformer. The test is then repeated on No. 1 transformer.

PHASING AND INSULATION TEST OF CATENARY SYSTEM

The reduced voltage tests having been completed the catenary system is now tested for insulation and phasing using the same system set-up as for test 3, except that the ring-bus at Lamokin is not sectionalized. The temporary grounds on the out-going trolley feeders are removed and all breakers and switches are closed. The voltage is raised to ¼, ½, ¾, and full voltage, thus energizing the entire catenary system as far as the test is to go. If the generator ammeter at Lamokin registers zero the trolley system is properly phased and the insulation is good. If there is a current flowing the trolley feeder breakers should be opened one at a time until the faulty section is located.

FULL VOLTAGE GROUND TEST ON TRANSMISSION LINES

After full voltage ground tests are made on high-speed breakers similar to test 6, the transmission lines are tested at full voltage. Reduced voltage tests are also made but as the operation is the same for both tests one description will suffice.

The transmission line ground protection as stated in test 5 is obtained with a differential relay which receives its potential from condenser potential devices. The ground is obtained by closing one leg of the 132-kv. ground switch, the other leg being previously disconnected from the motor-operated mechanism when the system is dead. The location of this ground is shown at *Y* in Fig. 5.

When a ground occurs on one side of the transmission line the voltage on that side of the potential device approaches zero depending upon the nature of the ground, and the voltage on the other side approaches 110 volts. The balance in the relay being disturbed, it operates, causing the corresponding step-down transformer breaker to open. This same operation takes place at each substation feeding into the faulty line. In Philadelphia at D-1 substation, the line is sectionalized by a 132-kv. breaker which also opens, thus disconnecting the faulty transmission line from all sources of power. The same test is performed on the other transmission line.

FULL-VOLTAGE SHORT-CIRCUIT TEST ON TRANSMISSION LINES

The 132-kv. transmission lines are protected from short circuits by means of directional control relays. The short circuit is caused by closing both sides of the 132-kv. ground switch. The trolley feeder network supplies power to the short circuit through all step-down transformers connected to the faulty line. The power flow is thus in the reverse direction from normal and is used as an indication of fault conditions.

The directional control relay has an overcurrent element which receives its current from a current transformer located on the low-voltage side of the 4,500-kva. step-down transformer, and a power directional element receiving potential from a 11,000/110-volt potential transformer connected to the corresponding trolley bus section. The directional element contacts must be closed before the overcurrent element can start to operate. The operation of this relay causes the corresponding transformer circuit breaker to open. Likewise all transformer breakers connected to the faulty line open as well as the sectionalizing breaker at D-1 substation, thus disconnecting the faulty line from all sources of power. This test is repeated on the other transmission line.

CONCLUSION

It is hoped that this paper has shown the completeness and thoroughness of the substation testing required by the Pennsylvania Railroad to insure a minimum of train delay with a maximum of protection under electric operation.

ACKNOWLEDGMENTS

Acknowledgment is hereby made to Mr. J. Westby for his care and interest in reading over this paper in its preliminary form and in offering valuable contributions.

Instructions prepared by Gibbs & Hill, Consulting Engineers, for use in the operation of the above electrification were of assistance in preparing this paper, particularly in describing the relay operation.

Acknowledgment is also made to two manufacturers who furnished most of the equipment in the above substations for reading over this paper and in offering many helpful comments.

Time and Its Measurement

BY ERNEST W. BROWN*

Non-member

THE nature of time and its relation to space in the theory of relativity, is not discussed here but it is assumed that, for our purposes, the Newtonian definitions give a sufficiently accurate picture.

DEFINITION OF A TIME-KEEPER

A time-keeper, whether terrestrial or celestial, may be considered as consisting essentially of a fixed dial and a moving pointer. A complete observation can be regarded as a comparison of the angles described by the pointers of two time-keepers between two signals, at each of which the positions of the pointers are simultaneously observed. This comparison of angles is then translated into a comparison of intervals of time by means of certain assumptions made as to the nature of time. In our daily life one of the time-keepers is used as a standard for judgment of the other which may, for example, be the interval between the departure and arrival of a train or that between the successive demands of the body for nourishment. Every observation demands the presence of two or more time-keepers.

In order that an observation may be made, the circumference of the dial is divided into two or more equal parts or it may have a single mark only. With some time-keepers the observations are made only when the pointer is opposite a mark. When a simultaneous position of the second pointer cannot be well estimated, a simultaneous observation with a third time-keeper, having a highly divided dial or a rapidly moving pointer, is necessary.

The pointer is actuated by a mechanism which is governed by the laws of motion. In some mechanisms the law of gravitation also plays a leading part: in others, physical laws of a different nature are mainly responsible for its motion. Most of the terrestrial mechanisms are constructed by human agency and are therefore under a certain degree of control. Celestial mechanisms can not be controlled.

ERRORS

Since our ability to deduce results from these mechanisms is limited and is subject to error, the discussion resolves itself from the outset into a consideration of the errors which may exist when an observation is made.

It will be convenient to divide these errors into three classes:

1. Lack of precision in the statements of the laws under which the mechanism is supposed to move.

2. Errors in the applications of these laws to the mechanism.

3. Errors in the reading of the pointer, usually called errors of observation.

From the practical point of view, Newton's three laws of motion on which our chief time-keeper, the earth, rotating on its axis, is mainly dependent, may be regarded as possessing an accuracy far beyond our powers of detection. The same may be said of the Newtonian law of gravitation in the practical applications to time measurement. Less accurately known are the laws governing the kinetic reactions of matter usually stated under the terms, elasticity, rigidity, hysteresis, etc. Perhaps in the same class can be placed the phenomena classified under the terms electricity and light. With the earlier time-keepers these have been used only for the transmission of signals where the accuracy for time-keeping purposes is very high; in the latest time-keeper, the oscillating crystal, electricity has become part of the mechanism

Errors in the application of the laws are usually numerical or mathematical; these can be eliminated. A more frequent source of error is the omission of the effect of known or unknown reactions of matter, such as friction, elasticity, etc. Errors of observation of terrestrial time-keepers are largely under control and can be eliminated to almost any extent by the development of suitable devices. Those of celestial observations are numerous: some follow known laws and can be allowed for to a greater or less extent; others are unknown and their effects can be discovered by indirect methods only; still others which appear to have no systematic character are supposed to follow certain laws of error and are reduced by multiplying the number of observations.

STANDARD TIME-KEEPERS. DEFINITIONS OF ERROR AND CORRECTION

The use of the word error implies the existence of a perfect time-keeper. No such mechanism exists. We can only make comparisons between different time-keepers. Any definition to be useful in practise must take this fact into account.

The error of a time-keeper at any instant is the difference between the reading of its pointer and that of another time-keeper which has a similar dial and whose pointer is constructed to have the same angular velocity, the latter being used as a standard. The angular velocity of the pointer of the standard may be different from that of the first time-keeper provided its dial is proportionately divided. The "accumulated error" in any interval is the difference of its readings at the beginning and the end of the interval, less the corresponding difference of the standard. The error is taken to be positive when the pointer is in advance of that of the standard. Thus, the error of a time-keeper includes

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that of the standard. They may be partially separated only by the use of the theory of errors applied to numerous time-keepers observed on many occasions. With an actual mechanism as a standard, the position of its pointer must be corrected for any errors which are known.

Fundamentally, all our observations can be considered as those of angles, but it is convenient to express them in time. The angular error of a time-keeper is expressed in time by its ratio to the angle described by the pointer of the standard in a given interval, multiplied by this time interval. Since the errors with which we deal are usually small, whether of the time-keeper or of the standard, the error of the standard has little effect in this translation of an error of a time-keeper from angle to time and can ordinarily be neglected. In future it will be supposed that this translation has been made.

The *correction* of a time-keeper is the amount which must be applied to it in order that it may agree with the standard. It has, therefore, the opposite sign to the *error*.

DEFINITION OF RATE AND CHANGE OF RATE

The *rate* of a time-keeper is less easy to define accurately. The average rate over a given interval is perhaps best defined as the ratio of the difference of the readings at the beginning and end of the interval to the corresponding difference of the standard in the same interval. The real rate at a given instant is the limit of this ratio as the interval approaches zero, but as this is not measurable, average rates must be used. Unfortunately, the word "rate" is used sometimes in this sense and sometimes as meaning the difference of this ratio from unity, expressed in terms of the standard. Thus if we talk of a time-keeper having a daily rate of $+1^s$ we mean that at the end of a day the reading is behind that of the standard by an angle which is 1/86400 that described in a day by the standard. The rate of a time-keeper thus defined is positive when it is losing on the standard.

A daily change of rate is the difference of the rates thus measured on successive days.

TIME-KEEPERS WHICH DIVIDE INTERVALS OF TIME

In the description of the dial and pointer illustration it was mentioned that with some time-keepers the position of the pointer could be observed only when it was opposite a mark. Different kinds of time-keepers are needed, some of which can be observed accurately at more frequent intervals than others. The dial may be divided into more parts or the pointer may revolve more rapidly. We thus have a second function of the time-keeper, that of dividing an interval into equal parts.

Our fundamental standard is, in theory, observed once a day and the chief function of the terrestrial clocks has been to divide this into 86,400 equal parts called seconds, the equality of the parts being a fundamental aim of the mechanism. For smaller divisions, various devices known as chronographs, are employed, the accuracy with which these can be constructed to divide the second into any number of equal parts being far in excess of practical needs.

THE FUNDAMENTAL STANDARD

As stated, the fundamental standard for time measurement is the rotation of the earth about its axis. According to the laws of motion, certain ideal bodies will rotate about their axis with constant angular velocities. Most of the apparent deviations of the actual earth from such an ideal are calculable or measurable or both, and can be applied to the observations. One, which will be mentioned later, has been made definite only within the last few years and is of considerable importance.

With this standard, the pointer is the telescope of the transit instrument and, in theory, the dial is marked with a single star only, so that the observations are made once only in each revolution, that is, when the pointer is on the star. In practise, however, many stars are used, the time from one to the next being carried by the astronomical clock whose rate is afterward determined by the observations.

ERRORS OF THE FUNDAMENTAL STANDARD

The errors of the dial of this time-keeper are those of the angular positions of the stars relatively to one another and of their relative motions; these have been reduced to very small quantities by numerous observations and thus the relative position of and motions of these "clock stars" are known with high accuracy. Next comes the personal error of the observer in recording the time on the clock when the pointer is on the star. This error again, partly by mechanical devices, partly by experience, and partly by measurement, can be nearly eliminated. Next the errors due to movements of the instrument and of its parts relative to the ground on which it is placed, are under a certain degree of control and measurement. Finally, there are anomalous refraction errors due to the movements of the atmosphere and others due to the local movements of the ground relative to the whole earth which are probably very small but which are not under control.

Under the best conditions none of these circumstances should produce serious accumulated errors over periods of a few days or longer. Meridian observatories of the first class expect to be able to give the daily time within one or two-hundredths of a second. But considerable doubts concerning the actual accuracy must be entertained as long as these observatories show differences amongst themselves running up to one or two-tenths of a second and persisting for weeks or months at a time. There is too little known about the nature or causes of these differences for any one observatory to be able justly to claim substantial superiority over all the others in the accuracy of its time determination. The day-to-day accuracy permits of tests; the week-to-week or month-to-month accuracy is doubtful.

CHANGES IN THE RATE OF THE FUNDAMENTAL STANDARD

The important exception in our knowledge of the deviations of the rate of rotation of the earth from constancy has been demonstrated by observations of the moon, sun, planets and satellites. In the comparison of gravitational theory with observation of these bodies, they agree substantially with one another in their measure of the time and disagree with the earth. The nature of these variations is such that errors in theory or observation as their source are highly improbable. Quantitative calculations also indicate that they arise from forces which act below the earth's crust. Calculation and dynamical theory then indicate that one result of their action must be changes in the external radius of the crust.

These changes of rate of our principal time-keeper are irregular. Ordinarily they are very small and can be detected as having occurred only in the discussion of observations of the moon. But there have been two occasions when no doubt concerning their orders of magnitude can be raised. During the interval 1893 to 1900 the rate can not have diminished by less than one part in 30,000,000 and was probably greater. Again within two or three years of 1917 it increased again by about one part in 40,000,000. The chief doubt is the interval during which each change was completed; it may have been anything less than seven years in the first case and less than five in the second.

The complete technique by means of which these changes are demonstrated is so highly specialized and so laborious to carry out that few living investigators have been able to examine it in detail. The results depend for their reliability on several investigations which have been published within the last eighty years and which have been carried out by different methods. The observational effects are so large, speaking relatively, that no doubt can be entertained regarding them. The rejection of them implies that an astronomically determined unit rate may sometimes differ from its average value by as much as one part in 30,000,000: their acceptance permits the error to be later reduced to about one-tenth of this amount.

COMPARISONS BETWEEN TIME-KEEPERS

It will be seen that there are considerable differences in practise between the accuracy which is attainable in the comparison of two or more time-keepers in the past, at the present moment and in the future. High accuracy in many astronomical determinations depends on the multiplication of the observations. Slow systematic changes of rate can only be determined by observations spread over a considerable length of time. Other changes of rate cannot at present be predicted. No method of judging the accuracy of a time-keeper is of general application unless it specifies the nature of the errors under consideration, the length of the interval over which it is to be judged, and the standard with which it is to be compared. While mention of an ideal time-keeper may give mental satisfaction, it has no place in actual measurements. Indeed, the tendency of modern physics appears to lean towards the abolition of all consideration of hypotheses which assume the existence of phenomena which cannot be observed.

MEAN SOLAR TIME

The direct use of the fundamental standard is confined to the astronomical observatory: it is more convenient for the layman to use an intermediate standard which is related to the position of the sun. For purposes of definition we can regard this time-keeper as a pointer referred to a mark on a dial which is made to move with constant angular velocity, this velocity relative to the fixed stellar dial having been previously determined by observation and gravitational theory.

In the early stages of human history, the mark on the moving dial was the actual sun. It was later found that such a mark judged by the stars was in advance or behind its average position by as much as one per cent in the course of a year. Later still gravitational theory enabled the astronomer to calculate the deviation very accurately and finally, by observations extending over a long interval of time, to obtain the average velocity also with high accuracy. The moving dial makes a complete circuit in the same time that the pointer takes to make some 365 circuits.

In practise, the astronomer compares the time shown by transit observations of the stars with his "sidereal clock" constructed to have the same rate. For the layman he translates this into civil time by calculation. The instant at which the day shall begin is a matter of agreement and definition.

The mean solar day is the interval between successive coincidences of the pointer with the moving dial. The mean solar second is 1/86400 of this.

GRAVITATIONAL EFFECTS ON PENDULUM TIME-KEEPERS

This paper concludes with a few remarks concerning the astronomical and geophysical forces acting on certain types of terrestrial time-keepers. All of the latter which are in practical use for accurate time-keeping are oscillators. These are of two types, those which oscillate principally under the action of the laws of gravitation and those whose oscillations are controlled chiefly by that property of matter which we call elasticity. As far as we know, the latter are practically independent of gravitational action or can be made so. They need not, therefore, be further considered in this connection.

The pendulum clock depends mainly on gravitational action for its motion and any change in this action may alter its time-keeping properties. As far as the gravitational actions of outside bodies are concerned, no practical effect can be detected. Just lately, for the first time, the action of the moon, which is the greatest external effect, was measured by the Loomis chronograph and shown to give accumulated errors which were always less than two ten-thousands of a second as indicated by theory. This also included those indirect effects caused by the moon such as tidal action both in the oceans and in the change of shape of the earth, etc.

A much more serious effect can be produced by a change in the distance of the pendulum from the center of the earth. If the clock be raised one foot, its rate will be changed by one part in twenty million and the accumulated loss as a time-keeper will be one and a half seconds in a year. The changes in the rate of rotation of the earth have been mentioned previously. Under two extreme hypotheses, the outer radius was changed near 1897 by 5 in. or 12 ft. The former change would have meant an accumulated error of the clock of $0^{s}6$ in a year; the latter 18^{s}. In either of these two cases the star transits would have shown 1^{s} as against the moon. Thus the comparison of the clock with star transits would have shown an apparent gain of $0^{s}4$ in the one case and an apparent loss of 17^{s} in the other. If any such change as the latter takes place in the future within an interval of a year, it can certainly be detected by the averaging of results of the Shortt clocks which are now in operation under the best conditions. Even if the change took three or four years to complete, it probably could still be detected. It must be remembered, however, that it is an extreme hypothesis constructed to account for a certain phenomenon, without any collateral evidence as yet to support it. The hypothesis may be good for a smaller value or it may have to be rejected altogether. At present, it appears to be necessary qualitatively, whatever the quantitative value may be.

Discussion

For discussion of this paper see page 550.

Modern Developments in Precision Clocks

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AT THE beginning of the present century it was believed by many that the limit of accuracy attainable by mechanical clocks had been reached. This presumption was based on the performance of specific mechanisms which had indeed been brought to a high state of perfection, but this presumption did not allow for the possibility of other *types* of mechanism for measuring time nor for certain refinements that have since then more than doubled the accuracy and reliability of the best clocks then available.

In 1911 Lord Grimthorpe wrote[1] "It seems to follow that the extent to which astronomical clocks can be made accurate, *viz.*, 1/30 of a second average variation from their mean daily rate . . . is a degree of accuracy sufficient for present purposes, and it seems rather doubtful whether mechanical science will, in the case of clocks, be likely to reach a much higher figure."

As the words clock, chronometer, etc., have come to have definite technical meanings in the art, we shall use the more general term "Timekeeper." In the broadest sense a timekeeper may be defined as any physical means that can be used for the subdivision of time into intervals known in terms of a standard time interval. Thus any periodic phenomenon can be adopted in a timekeeper, provided its period can be recorded in terms of another. In this sense the earth and the moon are timekeepers, as well as vibrating pendulums, tuning forks, and other sustained oscillators. As all man-made timekeepers are essentially "oscillators" we will consider first the equation of motion of a simple oscillator which can be expressed as

$$M\ddot{x} + R\dot{x} + Sx = A \sin (\omega t + \phi)$$

where M corresponds to the mass, R to the resistance or damping and S to restoring force proportional to displacement x. The right hand term represents the sustaining force required on account of the dissipation of energy in the oscillator.

For an oscillator to be a perfect timekeeper, all of the quantities, M, R, S, A and ϕ must remain invariable. While M and S are the chief rate governing quantities, the others are, from a design point of view, the most difficult to control.

The ultra precise practical timekeepers of today can all be contained within two classes, depending on whether S (the restoring force) is

(a) Gravity (as in pendulum clocks)
(b) Elasticity (as in quartz crystal oscillators)

The second class is a development of the last decade and was made possible by the development of the vacuum tube and associated electrical circuits. Intercomparisons between timekeepers of the two classes provide a most valuable means for the study of changes in gravity and related phenomena. These two classes are discussed separately.

GRAVITY TIMEKEEPERS (PENDULUM CLOCKS)

Unfortunately the motion of a pendulum is not quite simple harmonic motion since the restoring force varies as the sine of the angular displacement (instead of directly as the displacement) and therefore the period is not truly independent of the amplitude. This can be seen from the equation of motion of the pendulum which in its simplest form, neglecting damping and sustaining forces, is

$$Ml\theta + Mg \sin \theta = 0$$

the free period T for any given angular displacement can be expressed by the series:

$$T = 2\,\pi \sqrt{\frac{l}{g}}\,(1 + 0.0625\,\alpha^2 + 0.00358\,\alpha^4 + \,.\,.\,.)$$

Where α is the maximum deflection from mid swing expressed in radians. The corresponding change in rate expressed in seconds per day is shown in the curve Fig. 1.

Various attempts have been made to construct pendulums having a period independent of arc. Huygens[2] demonstrated that a pendulum constrained to move in the arc of a cycloid under the influence of gravity would have a rate independent of arc, but mechanisms to accomplish this result have always introduced larger irregularities than those which they attempted to cure. Suggestions have also been made that an elastic restoring force could be used to supplement gravity in such a way that the sum of the two would make the restoring force directly proportional to the displacement. Loseby first made this suggestion in 1851.[3] But such an additional control robs the gravity pendulum of one of its greatest advantages, as the rate is markedly affected by changes in the elastic restoring force.

Practical design has been confined to keeping the amplitude small and to providing a governing action so that any slight increase in amplitude causes an opposing tendency to decrease the amplitude, and *vice versa*.

The old anchor recoil escapement[4] accomplished this by increasing the friction on the pendulum with increase of amplitude. The inertia escapement[5] in the Shortt clock attains a similar result without the serious disadvantage of introducing friction. This inertia escapement is based on the principle that a weight rolling down an inclined plane applies a greater horizontal force

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*Loomis Laboratories, Tuxedo Park, N. Y.

†Bell Telephone Laboratories.

[1] For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

32-52

to a slowly moving plane than to a faster moving one. A suitable inclined plane attached to a pendulum receives an amount of energy roughly inversely proportional to the amplitude, or to the velocity at midswing. As the amplitude increases, the energy delivered into the system becomes less. But in spite of these controls the amplitude does vary in fact, which at the present time probably accounts for more irregularities than any other factor. Dr. Jackson has shown[6] that if the daily amplitude is regularly recorded and the clock rate corrected accordingly, a number of the irregularities in the Shortt clocks at Greenwich can be explained.

If a direct amplitude control could be applied to a clock such corrections ought not to be necessary. Such a method is now being tried on a Shortt clock in the Loomis Laboratory. When the amplitude of the pendulum exceeds a predetermined amount by even 0.0001 inch, a signal is sent, via a pointed electrode which hangs in the evacuated pendulum casing, to a vacuum tube which operates a relay so that the next 30-second driving impulse is omitted. The electrode is so adjusted that about every fifth impulse is omitted in this way. It is found in practise that the device operates without the bob ever coming in contact with the electrode, so that no energy is taken from the pendulum by physical contact. The difference in amplitude between the operating and non-operating conditions is only about 10^{-6} cm. which corresponds to about 0.3 second of arc. Thus the mean amplitude can be controlled to within approximately 0.1 second of arc. In order to insure that changes in level of the whole clock mechanism will not be the direct cause of changes in amplitude through this control mechanism, the mounting for the control electrode is suspended so as to be free to swing in the same plane as the pendulum, though of course having a different period and being highly damped in order to avoid sympathetic oscillations. With this control the constancy of rate appears to be improved, but at least a year's record is needed for conclusive proof, as the variations which this device seeks to control usually only occur at intervals of several months.

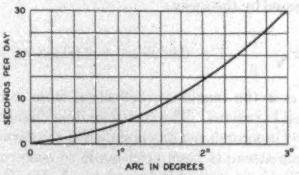


FIG. 1—CHANGE OF RATE OF PENDULUM WITH SEMI-ARC EXPRESSED IN SECONDS LOST PER DAY

The complete solution for the period of a pendulum swinging in a resisting medium, such as air, involves a factor $\sqrt{1 + kR^2}$ where R is the resistance to motion and k is a constant depending on the structure of the pendulum and the nature of the drive. Although it would appear from this that all pendulums should be operated in a high vacuum, practically it has been found[7] that the most satisfactory pressure for the Shortt clock, from the standpoint of stability of period with pressure variation, is about 15 to 20 mm. of mercury. When the pressure is very low or zero the arc, when not definitely controlled, is very unstable and causes large changes in rate, but in the neighborhood of 15 mm. of pressure there is a stabilizing action due to the air friction, and a neutralization of the following four effects of pressure variation:

1. Change in the restoring force due to buoyancy of pendulum in air.
2. Change of center of gravity due to buoyancy when pendulum is made of dissimilar materials.
3. Change of effective mass due to air carried along by the pendulum.
4. Change of friction and consequent change in arc.

The first and fourth are probably of greatest importance. With decrease in pressure the first decreases the period and the fourth increases it. At the critical pressure the two effects are about equal and opposite. In the determination of gravity,[8] where very precise pendulums are used, all of these factors must be considered very carefully.

When the amplitude control device described above is used, the pressure can be reduced to a small fraction of a millimeter, which thereby reduces the effect of friction to a minimum. The term $\sqrt{1 + kR^2}$ then becomes practically unity and the required sustaining energy can be very much reduced.

So far we have considered variations in the restoring force. Variations in the inertia term are now considered.

The expansion of the pendulum with temperature must either be reduced or compensated for. At the present time practically all clocks of high precision employ invar pendulum rods with bobs of type metal or invar. The bob rests on an expansion collar, the dimensions of which are calculated to compensate for the lengthening of the rod. Some recent studies[9] in Japan have produced a "super-invar," to use the expression given, which may have a zero or even a slightly negative coefficient of expansion. If these new alloys are as stable as invar, they may have an important use in precision clock making, as the entire pendulum could be made from a single piece of material.

With the new technique recently developed in the laboratories of the General Electric Company it is now possible to produce pieces of clear fused quartz large enough to construct the entire pendulum for a clock. Some experiments are now being conducted with such clocks in the Loomis Laboratory. Fused quartz has a temperature coefficient of expansion of about 0.5 parts in 10^6 per deg. cent. A temperature variation of one degree would therefore cause a change in rate of only 0.02 second a day due to this effect alone. In a constant temperature vault with a fused quartz pendulum a variation of 0.01 degree would only affect the rate 0.0002 second a day. Means have been developed whereby temperature can be controlled to within 0.01 degree over long periods.

There are two other major effects that should be considered in connection with pendulum length. These are (1) variations in the position of the point of oscillation, and (2) the instability of materials with time, in other words, aging. The effect of knife edge wear on the period of an ordinary pendulum is appreciable. A wearing of one thousandth of a millimeter will change the rate of a seconds pendulum nearly a tenth of a second a day. Largely for this reason, a flexible spring suspension of elinvar[10] is used in most precision clocks. But even the spring is suspected of changes in length and, what is just as bad, also of variation in stiffness which would change the point of flexure and the restoring force. Aging effects in the pendulum rod can be reduced only by finding better materials. It may be that fused quartz, or possibly some large single crystal, will be found finally to be the most suitable material to use.

The effect of knife edge wear on the period of a pendulum can in theory be almost entirely avoided by suspending the pendulum at a distance from its center of mass equal to the radius of gyration about that point. This simple principle of mechanics, has of course long been recognized,[11] and experiments are being conducted in Germany[12] and elsewhere to determine its value in a precision timekeeper. In the case of a uniform straight rod this distance should be $\frac{1}{2\sqrt{3}}$ of the total length from the center. With this condition fulfilled a change in this length due to knife edge wear of one part in a thousand would affect the period less than one part in a million.

Returning to the fundamental equation at the beginning of the paper

$$M\ddot{x} + R\dot{x} + Sx = A \sin (t + \phi)$$

we have considered briefly the terms on the left-hand side.

We will now consider the right-hand term—the driving force—and especially the quantity ϕ the phase angle, which presents perhaps the most difficult problem in the design of all timekeepers.

The impulse should be delivered to the pendulum at the time when the velocity is a maximum, that is, at the center of the swing, in order to reduce to the minimum the effect of variations in the driving force on the rate. If the impulse is applied *before* the instant of maximum velocity, the rate is momentarily increased. If the impulse is applied *after* the instant of maximum velocity the reverse is true. Of course the impulse actually is of finite duration and the desired effect is approximated by supplying the pulse symmetrically with respect to the instant of maximum velocity thereby tending to neutralize the above mentioned acceleration and retardation effects.

The necessity for this method of applying sustaining power pulses to a pendulum has been realized almost from the beginning of precision clock history but cannot be over-emphasized.

In the Shortt clock[13] the impulse to the master pendulum is controlled by means of an auxiliary or "slave" pendulum which in turn is automatically synchronized with the master pendulum to within ± 0.003 second at all times. The means by which this is accomplished can be seen from the schematic drawing Fig. 2 which shows the master and slave pendulums with all of the essential control elements, and the counting or time indicating dials. Although the master pendulum determines the rate of the system, the slave pendulum and associated mechanism does the work of releasing the driving pulses at the proper time. The slave pendulum, through a jeweled ratchet and pawl-operated count wheel, releases mechanically a weight arm once each 30 seconds at just the right time so that a small wheel on the gravity lever, rolling down a curved inclined track associated with the slave pendulum rod, imparts the sustaining impulse to it. As soon as the mechanical impulse is delivered, this arm closes an electrical contact which resets the weight arm, before the return swing of the pendulum, in readiness for the next 30 second impulse. The current which resets the slave clock drive also releases the impulse mechanism for the master clock thereby avoiding the use of *any* mechanical connection with the master pendulum other than that of the impulse wheel. As in the case of the slave drive mechanism, the impulse lever makes an electrical contact after delivering its impulse to the master pendulum, which resets the gravity arm and in addition operates the "hit and miss" synchronizer on the slave clock. The slave pendulum normally runs a little slower than the master and is speeded up when necessary by means of a spring attached to the pendu-

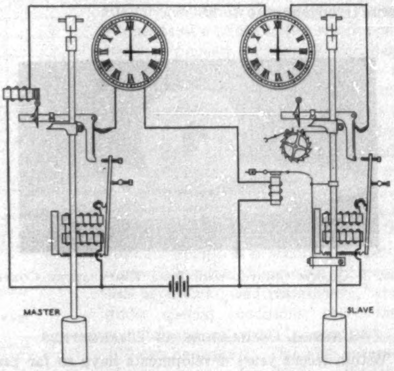


FIG. 2—SCHEMATIC DRAWING OF SHORTT CLOCK MECHANISM AND CIRCUIT

lum rod which is engaged by the synchronizer whenever the slave lags by more than 0.002 second behind a definite phase relation to the master.

There are reasons to believe, however, that small changes in phase do occur especially with variations in amplitude. Probably future improvements in gravity timekeepers will include improved methods for keeping the phase of the driving force more nearly constant.

It would be impossible even to survey adequately the developments leading up to the final design of any given type of clock in the limited space for this paper. It must not be assumed that the excellent performance of any timekeeping device is due entirely to any one person or company or period, but represents the combined efforts of a large number of people over an extensive period.[14,15,16] The three clocks most often recognized as precision timekeepers are the Leroy,[17] Riefler[18] and Shortt[13]. One or more of these is to be found in most of the important time laboratories and astronomical observatories throughout the world.

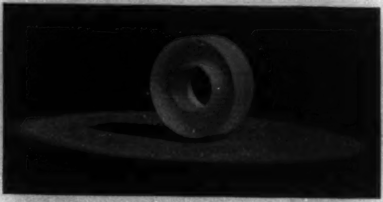


FIG. 3—QUARTZ CRYSTAL WITH ZERO TEMPERATURE COEFFICIENT USED IN CRYSTAL CLOCK

CRYSTAL OSCILLATORS AS TIMEKEEPERS

Within recent years developments have so far progressed that the constancy of rate of certain mechanical resonators, maintained in vibration by vacuum tubes, may be comparable with that of the best pendulum clocks. Such oscillators have the chief attributes of a good clock and can be used to operate or control a wide variety of time mechanisms with even greater versatility than the conventional pendulum.

The most accurate oscillators of this type known to the authors are controlled by plates of quartz crystal vibrating at a high frequency, that is, high in comparison with the frequency of a pendulum. Probably the most accurate of these at the present time are a set of four 100,000-cycle oscillators[19] that have been built by Bell Telephone Laboratories primarily for use as a primary frequency standard. These laboratories have also supplied a similar set of oscillators to the U. S. Bureau of Standards for the same purpose.

In this type of oscillator the frequency is controlled by a ring of quartz crystal about three inches in outside diameter as shown in Fig. 3. The frequency of the oscillator is the same as that of the crystal, that is, 100,000 cycles per second. One complete oscillator, including temperature and pressure controlled crystal, and shielded vacuum tube circuit, is shown in Fig. 4.

In order to operate clock mechanisms in the most direct way it is necessary to control from this high frequency another which is low enough to operate a synchronous motor. This is done by an electrical circuit known as a sub-multiple generator which controls one frequency at a definite fraction of another in absolute synchronism. Several types of circuit have been used for this, all of which are as positive in action as a set of gears. Usually the low frequency used to operate the motor is 1,000 cycles. Each pole on the motor in motion therefore corresponds to one milli-second in time and hence any variation in the motor mechanism or operation can amount at most to a fraction of a milli-second. Such a motor can be used to operate time indicating or measuring apparatus in a wide variety of forms.

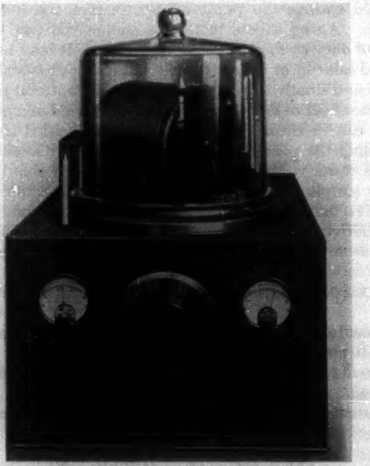


FIG. 4—COMPLETE 100,000 CYCLE QUARTZ OSCILLATOR

Including temperature and pressure controlled crystal and shielded vacuum tube circuit

As a rate controlling element a crystal of quartz used in this way has a number of outstanding useful properties. As previously discussed, the rate of vibration is controlled chiefly by the effective mass and stiffness of the resonator. On account of the chemical stability of the substance Si O_2 and the physical stability of the crystal structure of quartz these may be expected to be constant in very high degree. At the present time, variations introduced by the vacuum tube circuit and the crystal mounting are such that no conclusive evidence of aging in the quartz has been obtained.

Since the elasticity is a constant for such displacements as are required (amounting to less than one part in 100,000 change of dimensions) the rate is substantially independent of amplitude. The amplitude can be controlled at nearly a constant value, however, by proper adjustment of the vacuum tube circuit.

The temperature coefficient of frequency can be made as near to zero as required at any given operating temperature by properly adjusting the shape of the resonator. Crystals that have been in continuous operation for three years have not shown any appreciable change in this adjustment.

Crystals are not affected by gravity or magnetic fields and can be shielded readily from electrostatic fields. They can be mounted so as to be relatively immune to vibrations which always seem to be present in the earth, especially near traveled roads and manufacturing centers. This may be found to be an important factor in the location of accurate clocks in cities or near earthquake zones. The crystals referred to above are mounted on the seventh floor near a much traveled thoroughfare in New York City.

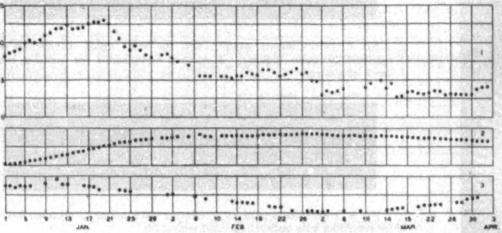


FIG. 5—PERFORMANCE DATA FOR CRYSTAL CLOCKS

Curve 1. Rate comparison between crystals 1 and 2 expressed as parts in ten million

Curve 2. Integral of curve 1, showing running comparison between crystals expressed in seconds

Curve 3. Indicated time of one crystal clock vs. corrected time signals from U. S. Naval Observatory

The electro mechanical coupling to a crystal for imparting the sustaining energy to it makes use of its piezo activity. As far as has been determined there is no increase in decrement directly due to this means of coupling, although, of course, properties of the electrical circuit, such as damping, may be manifested in the crystal through this coupling. The logarithmic decrement of a 100,000-cycle low coefficient crystal mounted in air at atmospheric pressure and coupled into the electrical driving circuit is about 0.00012.

Probably the largest rate disturbing factors in the crystal clock at the present time are due to effects of the phase ϕ of the applied driving force. This factor varies slightly with circuit variables, aging of vacuum tubes and power voltage variations. Developments now in progress indicate, however, that the effects of these variables can be greatly reduced, which should make a marked improvement in performance.

The performance, as clocks, of a pair of crystals is shown for a three-month period in the graph Fig. 5. The comparison rate curve shows the variations in relative rates between the crystals while the time comparison curve shows the variation in indicated time (integrated rates). The third curve gives the rating of one of the crystals against corrected Naval Observatory time signals for the same three-month period.

While the crystal clock assembly is more costly than a pendulum clock and requires more elaborate associated electrical equipment, we believe that for many purposes it is decidedly superior because of its great versatility. For example, its rate can be adjusted at will over a sufficient range to allow for such aging effects as occur. Also, the phase, or indicated time, can be adjusted with extreme accuracy. This can be done most advantageously in the electrical circuit by means of a phase shifter which operates continuously through 360 elec. deg., or any fraction or multiple thereof. If used in the 100,000-cycle circuit, one complete turn of the dial corresponds to a final adjustment of one hundred thousandth of a second, which obviously is greater accuracy than needed in any ordinary time mechanism.

The accuracy of inter-comparison of crystal clocks may be very high. An absolute comparison accurate to better than one-hundred thousandth of a second can be maintained continuously, while under special conditions, short time comparisons accurate to one part in 10^{10} can be made.[19] This high accuracy of comparison is due chiefly to the large number of vibrations per second, 200,000 times greater than with a "seconds" pendulum. Even greater accuracy of comparison could be obtained by the use of higher frequency crystals, but for reasons of convenience and greater freedom from the effect of external circuits and the mounting, the lower frequency is preferred.

One application of considerable interest is that two clock mechanisms can be operated from the same crystal control in this way so that one keeps true mean solar time when the other keeps true sidereal time.[20] It can be shown readily that any ratio of rates can be obtained for such a purpose accurate to at least one part in 10^{10} without the use of an unduly complicated mechanism. Lord Grimthorpe has shown why a pendulum cannot be used for this purpose,[21] but his reasons do not apply to the crystal control method.

By means of a shutter of the type shown in Fig. 6, and a photoelectric cell, signals of extreme accuracy can be controlled which could be used for radio transmission of time signals or for any purpose requiring great accuracy. When this disk is rotated at ten revolutions per second by a crystal controlled motor the current from the photo cell is 1,000 cycles modulated, in the manner indicated in oscillogram No. 1, at 100 cycles per second. By marking one 0.01 second element, as for example by partially blocking one sector, a positive 0.1 second indication is obtained. The oscillograms shown in Fig. 7 were made with the disk shown and indicate how, with the use of a suitable oscillograph, a continuous time signal can be recorded that can be read to a fraction of a milli-second. This resolution would be very useful for accurate time measurement work, for studies in radio propagation times, and for measurements of gravity. Observers requiring less resolution than this could obtain 0.01 second accuracy by the use of a rectifier and a recorder accurate to only 0.01 second. The same is true for 0.1 and 1.0 second accuracy by a suitable choice of transmitting disk and receiving equipment.

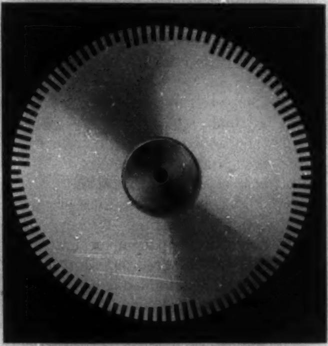
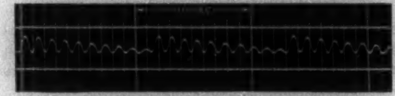


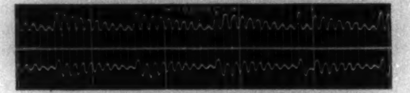
Fig. 6—Special Light Interrupter for the Production of Accurate Timing Signals by Photoelectric Cell

The second oscillogram shows the signal directly from the disk and after it had gone through a complete radio transmitter and receiver consisting of a total of eleven tandem vacuum tube stages and the associated equipment. As indicated, the total delay encountered was about 0.1 milli-second, and the distortion is not appreciable. Of course any combination of make and break signals can be controlled in this way using either a continuous or a modulated tone. If desired a series of half second dashes could be controlled, interspaced by a dot-dash code to designate each dash.

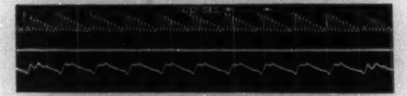
A complete time signal control mechanism based on this idea is shown diagrammatically in Fig. 8. In the method shown, light from a straight filament lamp is imaged on the slotted edge of a disk driven at 10 revolutions per second by a 1,000-cycle motor. The light entering the photo cell is modulated as indicated, at 1,000 times and 500 times per second alternately, and controls a corresponding electrical signal. The slow speed shutter, rotating at one revolution per second, causes the effective light path to alternate between the two disks once each second, and gives a corresponding variation of electrical signal. This signal can be used for any resolution from one second down to one milli-second and can be received through static and fading. It can be reduced to the dash-space type by the use of a simple voice-frequency filter to suppress the undesired half signal, or used as a continuous signal for oscillograph recording.



No. 1. Direct signal obtained from photo-cell



No. 2. Direct signal (above), and the same signal transmitted and received by radio (below). A method is indicated for marking every tenth of a second



No. 3. Direct signal as in No. 2 (above) and same signal rectified (below) for recording with only 0.01 second resolution

Fig. 7—Oscillogram of Time Signals Produced by Photo-Cell Method

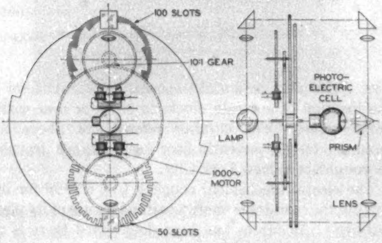


Fig. 8—Proposed Method for Production of Accurate Time Signals by Photoelectric Cell

Among the advantages of the crystal clock it should be mentioned that in order to control any number of clock mechanisms with the same precision it is only necessary to distribute the constant frequency current derived from the crystal to the separate mechanisms. This can be done on any scale economically feasible.

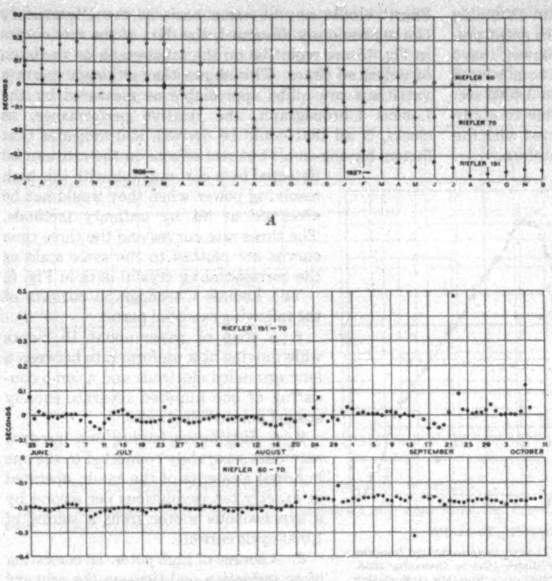


Fig. 9—U. S. Naval Observatory, Washington, D. C.

A. Observed mean daily rates of Riefler Clocks No. 60, No. 70 and No. 151
B. Relative rates of Riefler 151 *vs.* 70 and Riefler 60 *vs.* 70. (Data supplied by U. S. Naval Observatory)

Performance of Clock Installations

In order to indicate the performance that may be expected of precision clocks, data have been obtained from several actual clock installations and presented herewith in the form of curves. These do not show in every case, perhaps, the performance that could be obtained under ideal conditions but they do indicate the type of performance to be expected under present normal working conditions. In studying these curves, care should be taken to examine the coordinates because it was impossible to present all the data plotted to the same scale on account of the different times and the different errors involved. Also some of the data are plotted as daily rates and others as indicated time. The effect of this is that over a short period a rate curve shows greater apparent variations than a time curve and *vice versa*. In some cases the mean gaining or losing rate was removed from the data, as this has no bearing on the *constancy* of rate which alone determines the value of a clock as a timekeeper.

The essential descriptive information concerning the clocks represented accompanies each figure. Some data relative to the performance of Shortt Clocks 20, 21 and 22 in the Loomis Laboratory and the Crystal Clocks in Bell Telephone Laboratories are discussed in the following section.

Experiments in Time at Loomis Laboratory

The clock installation at Tuxedo Park and some of the experimental results obtained there have been described elsewhere.[7] In brief it consists of three Shortt clocks mounted in an especially favorable location practically free from traffic and electrical disturbances, and carefully temperature controlled. The excellence of the location is enhanced by the fact that the clock vault is excavated in the solid rock of the mountain on which the laboratory stands, and the three massive masonry piers for mounting the clocks are effectively a part of this rock. One of the three Shortt master pendulums in its casing is shown in Fig. 14 mounted on its pier within the temperature controlled vault.

By means of the Loomis Chronograph[7]

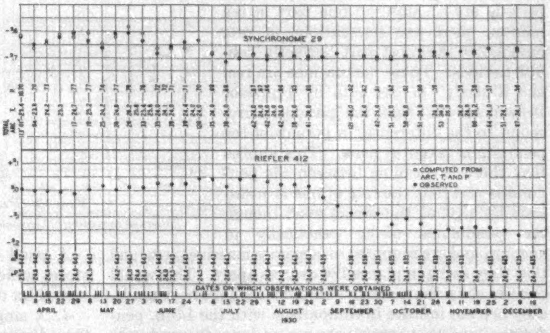


Fig. 10—Dominion Observatory, Ottawa, Canada

Observed daily rates of Riefler 412 and Shortt 29. (Data supplied by Dominion Observatory) it has been possible for the first time to obtain a running phase comparison of a number of clocks with great precision. Prof. E. W. Brown and Dr. Brouwer have shown[22] that the probable error of the mean hourly rates as measured by this chronograph is less than 0.0001 sec. With a resolution of this order, it is possible to study some effects that would be forever unsuspected with the use of any comparison equipment used heretofore.

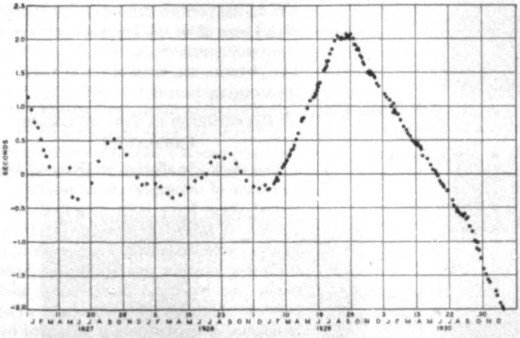


FIG. 11—ROYAL OBSERVATORY, GREENWICH, ENGLAND

Comparison of observed correction for Shortt clock No. 11 with those computed from the following formula which was based on observations from March 1927 to December 1928. Formula: $98^{s}905 + 28^{s}2002t + 1^{s}7542t^{2} + 0^{s}2524 \int (T\text{-}62^{o}8)\, dt$ + Nutation (Information from Monthly Notices of R.A.S. 1928-1931 inc.)

The rate curves on Fig. 15 will illustrate a case in point. These three curves show the differences in rates between the three Shortt clocks taken in cyclic order, plotted on a very open scale over a period of about three months. Although the *mean* rates over this and much longer periods are constant to one part in ten million, there are fluctuations in rate having for the most part perfectly definite periods. These fluctuations are real and a study[22] of the data by Professor Brown showed that the periods correspond to the differences in periods of the pendulums taken in pairs. This implies that there is coupling between the pendulums and that the rate of one is *modulated* by that of the others even though the pendulums swing practically in vacuum and are mounted on separate massive piers, and that they swing in planes 120 degrees apart. This would seem to show that, massive as the piers have been made, they are not infinite in comparison with the 14-lb. pendulums, and that strains are set up by each pendulum that are felt in some degree by the others through the piers and solid bed rock.

In order to indicate the comparison of these three Shortt clocks on the same basis as the Observatory clocks previously discussed, the data of the rate curves in Fig. 15 are replotted on the same graph on the basis of indicated time. This shows that although the rate variations are quite appreciable as measured by the Loomis Chronograph, the relative performance, as clocks, is all that could be desired, and indicates that factors having small but real effects on the rate can be detected by a chronograph with this high resolving power when they would not be observed at all by ordinary methods. The three rate curves and the three time curves are plotted to the same scale as the corresponding crystal data in Fig. 5.

The Loomis Chronograph consists of the following essential parts:

1. A strip of paper about 10 inches wide moving at a uniform rate between a long grounded electrode and a comb consisting of one hundred separate equally spaced pointed electrodes.

2. A distributor having one hundred segments separately connected to the one hundred elements of the comb, operated at exactly ten revolutions per second by a synchronous motor from a source of 1,000-cycle current.

3. A source of high potential consisting of an induction coil through the primary of which a large condenser is discharged when a record is to be made.

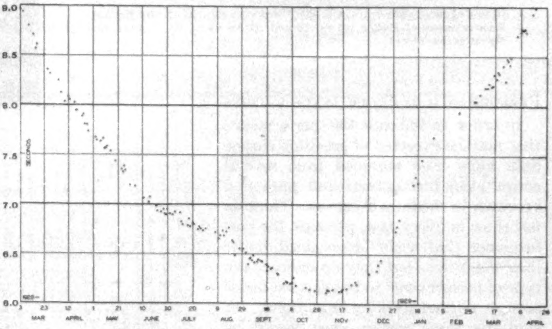


FIG. 12—PARIS OBSERVATORY, PARIS, FRANCE

Observed correction of Leroy clock No. 1185 (Information from Bulletin Horaire 1928-1930)

A small condenser on the induction coil secondary stores sufficient energy to produce an intense spark of very short duration; and

4. A single relay in the common circuit of all the time devices to be recorded on the moving paper strip.

Fig. 16 shows this chronograph in operation.

If the relay is operated every time the distributor makes some exact whole number of revolutions a series of perforations will be made in a line parallel with the edge of the paper. If in the time between sparks, the motor revolves a whole number of times plus or minus one hundredth, successive perforations will be displaced an amount corresponding to one milli-second in time. One second in time is represented by ten complete transits of the chart, so that, in effect, the chart is 100 inches wide and the effective recording element moves at the rate of 500 ft. a minute. Thus from the slope of the line of perforations with respect to the edge of the paper the difference between the rate of the distributor and the rate of the sparks can be determined with high precision. Similarly any number of separate sets of sparks can be recorded simultaneously on the same chart, using the same actuating relay and distributor. The relative rates may be determined by measuring the difference of slopes of the rows of perforations.

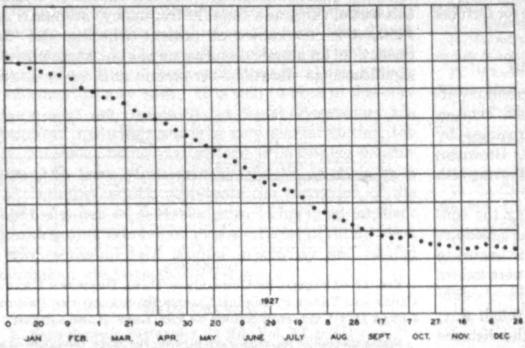


FIG. 13—ROYAL OBSERVATORY OF EDINBURGH, SCOTLAND

Observed correction of Shortt clock No. O. (Information from Proc. Royal Soc. Edinburgh Vol. XLVIII, page 161)

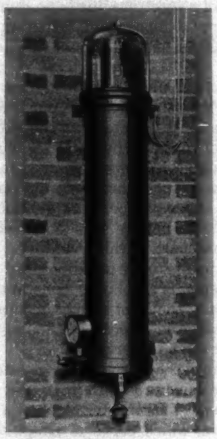


FIG. 14—MASTER PENDULUM (IN CASING) FOR ONE OF THREE SHORTT CLOCKS IN THE LOOMIS LABORATORY, TUXEDO PARK N. Y.

This is the method used for intercomparing the three Shortt clocks and for comparing them with the crystal oscillators in New York. To accomplish this, 1,000-cycle current controlled by the Bell Laboratories' crystal is sent over a private wire to Tuxedo and there used to drive the distributor as outlined above. The resulting records are continuous comparisons of the three Shortt clocks and the crystal accurate to better than one milli-second in time.

Since the crystal does not respond to variations in gravity while the pendulum does, the difference in rates contains a term having the period of the lunar day, corresponding to the direct gravitation effect of the moon on the pendulums. Several months' record of the Loomis Chronograph have been analyzed,[22] by Professor Brown and Dr. Brouwer and a lunar term observed having the proper period and magnitude. A graph of the lunar effect derived in this way is reproduced from

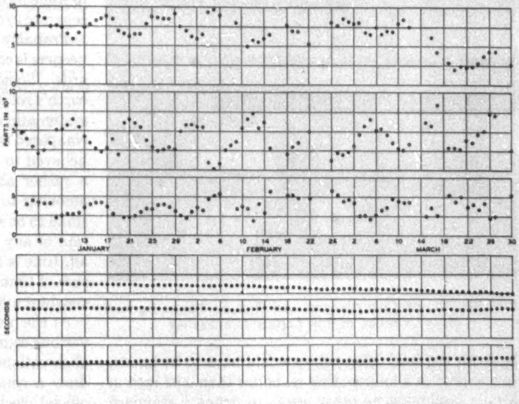


FIG. 15—INTERCOMPARISON OF THREE SHORTT CLOCKS IN LOOMIS LABORATORY

A. Rates, expressed in parts in ten million
B. Integrated rates expressed in seconds

their paper in Fig. 17. The periodic gain and loss in indicated time is about 0.0002 second per lunar day.

The results of time studies obtained at the Loomis Laboratory have been very encouraging and it is planned to continue the researches in clocks and time measurement methods. Some aspects of this work were discussed by Professor C. V. Boys in *Nature* for October 17, 1931.

POSSIBLE FUTURE INVESTIGATIONS

No doubt there is a practical limit to the accuracy attainable in a timekeeper but as long as random changes in rate are observed which cannot be explained by uncontrollable properties of matter (such as Brownian movement) that limit has not been reached, even with the types of mechanism at present in use.

The suggestion has been made of utilizing the constant velocity of light between fixed points as a measure of period. At present the velocity of light is known to less than one part in a million.[23] Even if it were known with greater accuracy, the practical difficulties of establishing such a standard, except in a very rough way, would be insuperable. In the first place the distance between the measuring points would have to be known to one part in a million, that is, better than 0.06 inch in a mile, assuming no other error, to define a standard good to only 0.1 second a day. Considering the difficulties in measurement, an accuracy of a little better than a part in a million is about all that could be expected of such a method by any means now known.

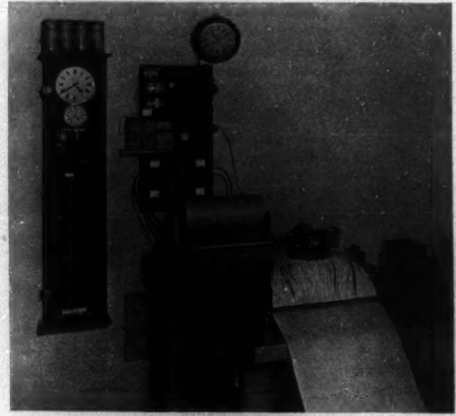


FIG. 16—LOOMIS CHRONOGRAPH AND ONE OF THREE SHORTT SLAVE CLOCKS IN THE LOOMIS LABORATORY

The extreme definition of some spectral lines indicates a very slightly damped oscillation of some sort in the atom, and some thought has been directed to means of making a frequency comparison between these vibrations and others of lower frequency that could be used to measure time as previously outlined. The effective logarithmic decrement of the "oscillator" behind the red line of cadmium can be estimated as about one tenth of that of a quartz resonator. The difficulties of utilizing this sort of thing as a time (or frequency) standard are: First, it is not known at present whether the "frequency" of an atomic disturbance has the same physical significance as vibrations-per-second and, second, even if the significance were the same, the orders of the two frequency ranges are so far separated that no means known at present could be used to effect the comparison. Besides, the energy of a spectrum line is no doubt made up of a large number of damped wave trains superposed in some fortuitous fashion as would cover the identity of separate cycles.

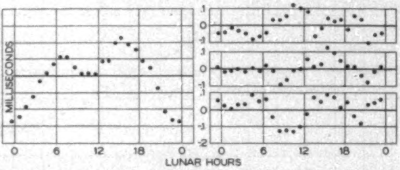


FIG. 17—ANALYSIS OF COMPARISON DATA BETWEEN CRYSTAL CLOCK AND THREE PENDULUM CLOCKS REVEALED THE PERIODIC LUNAR DAY VARIATION SHOWN IN THE LEFT HAND CURVE

Similar analyses of the comparison between the three pendulum clocks taken in pairs showed no corresponding effect

Probably the real final limit in the accuracy of timekeepers is concerned with the measure of our time standards. Some observations indicate that the rate of the earth's rotation is not constant, but that there are both systematic and apparently random changes in rate as compared with the motion of other astronomical bodies believed to be more stable.[24] For example; about 1918 a rather abrupt change in rate was observed which amounts to about one part in thirty million. This is small to be sure, but points to the improbability that the rate of any phenomenon is absolutely constant. After all, time is relative and the most nearly accurate measure of time must always be in terms of the most nearly constant motion that can be observed.

At the present time in the official observatories of the various countries the usual procedure is to plot the star observations in terms of their master clock and then to draw a smooth curve, which curve is adopted as the official clock correction. To some extent the drawing of this curve is a matter of judgment, based partly on the number and certainty of the star observations and partly on the estimated performance of the master clock in terms of their other clocks. These observatories at the present time also transmit and record radio time signals and publish periodically corrections to these signals in terms of their "standard clock error." From these data alone it is not possible for any one observatory to compare its clocks directly with the clocks in another observatory. If the observatories would publish one additional number for each of their important clocks, namely, the comparison of each clock with their official time, it would be possible to compare the rates of the clocks in such observatory directly with clocks anywhere else in the world where the radio time signals can be received. This would involve only a very small amount of additional work as the numerical data have to be obtained in any case. It is well to note in this connection that the United States Naval Observatory has adopted a procedure which is very similar to this, the only difference being that instead of publishing the corrections to each of its principal clocks, it publishes a single number which represents the correction of the weighted mean of several clocks. This is an excellent beginning and it is hoped in the interest of future clock developments and of studies in gravity and related phenomena, that others will follow.

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Discussion

For discussion of this paper see page 550.

Time Service of the U. S. Naval Observatory

BY J. FREDERICK HELLWEG[1]
Non-member

INTRODUCTION

THE U. S. Naval Observatory determines the standard time for the United States with scientific precision and broadcasts the results so as to make them available throughout a large part of the world. Since 1830 the Navy has had charge of this very important task. In the early days the demands for time service were confined principally to the mariners of our country. Scientific pursuits, increased demands of public utilities, greater business efficiency, increased speed in all activities of daily life, new inventions, all have demanded greater accuracy, and repeated expansion of the time service. At present the time service is broadcast six times a day and each broadcast is sent over a spread of about fourteen different frequencies to insure universal reception.

DESCRIPTION OF THE OBSERVATIONS WHICH ESTABLISH STANDARD TIME

The period of the earth's rotation on its axis is accepted as the standard of time measurement. The stars appear to rise in the east and after swinging across the heavens to set in the west. If a person stood on the earth's equator and measured the interval of time between two successive transits of a distant star he would have the most accurate measure of the rotation of the earth. If the same measurement was then made using the sun instead of a star the interval would be about four minutes longer due to the earth's orbital motion around the sun which continuously changes the apparent place of the sun among the stars. Therefore, as the earth rotates on its axis, this apparent motion of the sun necessitates the earth's rotating through more than 360 degrees to bring the sun again in the observer's meridian.

The stars are not within the earth's orbit, they are so far distant that their apparent positions are only very slightly affected by the earth's orbital motion. The positions of the stars are measured from a point in the sky known as the vernal equinox, and the period of the earth's rotation measured with respect to the vernal equinox is called a sidereal day. The same period measured with respect to the sun is called an apparent solar day. Owing to the variable length of the apparent solar day, astronomers had to devise a uniform measure of time and called it a mean solar day, so now we have apparent time, mean time, and sidereal time, and each time can be converted into the other by the application of the proper corrections. Because of the greater accuracy of sidereal time, that time has been universally used by astronomers.

In order to simplify the important question of time throughout the world, standard time zones have been established. All points within each zone use the same standard mean time although this may differ from their local time by as much as one-half hour. These zones represent a belt just one hour wide or fifteen degrees of longitude wide. In continental United States there are four zones:

Eastern Standard Time or 75th meridian time which is exactly five hours behind Greenwich standard mean time.

Central Standard Time or 90th meridian time, six hours behind Greenwich time.

Mountain Standard Time or 105th meridian time, seven hours behind Greenwich time.

Pacific Standard Time, or 120th meridian time, eight hours behind Greenwich time.

The Naval Observatory is thus able to furnish one time signal which provides time for all zones, not only in continental United States but for its island possessions and ships far out across the Pacific.

THE DETERMINATION OF THE STANDARD TIME

The Naval Observatory uses small quickly handled telescopes known as transit instruments in observing celestial bodies for the determination of our standard time.

These transit instruments as their name indicates are rigidly mounted so that there can be no motion either side of the meridian. The instruments are pivoted however so as to rotate from north to south. Astronomical bodies are therefore only visible through the transit instrument when on or very near the celestial meridian.

The sidereal time at which each star will cross the meridian is very accurately calculated. Stars are selected which cross the celestial meridian near the zenith of the Observatory and also whose positions are the most accurately known.

The observer operates a micrometer attachment which causes interruptions in an electric circuit as the star passes certain points in the field of the instrument. These interruptions are recorded graphically by a chronograph on the same sheet which records the standard sidereal clock ticks. By measurements of this sheet and the application of known corrections, the exact time of meridian transit is determined, according to that particular clock. The difference between this time and the previously calculated time represents the error of that clock on sidereal time. A continuous record of the standard clock corrections is maintained, determined

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Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

32-38

every clear night by observing a list of stars with one or more telescopes at the Observatory.

THE PROCEDURE USED IN BROADCASTING STANDARD TIME

Sidereal time is more convenient and more accurate for the use of the Observatory in rating the standard clocks, but it is not at all suitable for the general public.

The transmission of the time service is therefore in mean solar time and the transmitting clocks are rated to mean solar time.

Immediately before each time signal is broadcast, the error of the transmission apparatus is determined by a chronograph comparison with the standard clocks. The record sheet of this chronograph permits very accurate measurements to the fraction of a second. The continuously changing differences between sidereal and solar time and the error of the sidereal clock are also applied resulting in our mean solar time broadcast being accurate within one one-hundredth of a second.

This comparison and correction is made immediately prior to each broadcast and requires several minutes to complete.

Five minutes before the zero hour and after the transmitting clock has been corrected to mean solar time with the greatest possible accuracy, the operator throws the controlling switch which gives the Naval Observatory direct control over the Navy's high power radio stations at Arlington and at Annapolis. From then on, the operation is entirely automatic.

Every second is broadcast except certain ones omitted for identification signals, and others which act as fingers pointing definitely to the next audible second following the period of silence.

The twenty-ninth second of each minute is omitted and therefore definitely points to the 30th second of each minute. The 56th, 57th, 58th, and 59th seconds are omitted therefore definitely indicating that the next succeeding second, the 60th, is the beginning of another minute. This occurs on the 55th, 56th, 57th, and 58th minutes before the zero hour. On the 59th minute there are no audible "ticks" from the 50th to the 60th second, indicating that after a silence of nine seconds, the next audible tick is the zero hour either midnight, 3:00 a.m., noon, 4:00 p.m., 7:00 p.m., or 10:00 p.m., seventy-fifth meridian time, or Eastern Standard Time.

In order that those listening for the broadcast can distinguish the minute when they tune in for reception, the fifty-first to the fifty-fourth seconds are used as identification signals. If the 51st beat is omitted, the listener will count four more beats before the end of the minute silence occurs, the 52d, 53d, 54th, and 55th beats. This indicates that there are four more minutes to go. If the 52d beat is omitted, the listener can only count three more beats before the silent period, the 53d, 54th, and 55th beats indicating three more minutes to go. If the 53d beat is omitted, the listener can count only two more beats, indicating two more minutes to go, and if the 54th beat is omitted, the listener can count only one more beat and knows that he has only one more minute to go before the zero hour.

This standard broadcast is known all over the world and is religiously listened for in the most remote parts of the globe.

If for any reason, the broadcast is interfered with, it is repeated the following hour.

In addition to our direct control of the two high power stations at Arlington and Annapolis, the Observatory also controls by radio the retransmitting station at San Francisco, and eventually it is hoped that this service will be extended to the east.

During the time broadcasts, automatic chronographic records are made at the Observatory which show the time of the sidereal clock, the time of the transmitting clock, (mean solar time) and the actual time of broadcast by Arlington and by Annapolis.

Immediately after the completion of the time signal broadcast, this record is measured and the difference noted. This gives the time of actual emission by each station. The corrections to the standard clock are then recalculated using star sights taken both before and after the broadcast. Errors are carefully computed and a correction sheet is sent weekly to all those requiring extreme accuracy noting the error on 75th mean solar time of each broadcast of each day. In addition, immediate corrections are sent out to those requiring more rapid service.

DESCRIPTION OF THE TIME CLOCK VAULT—SIDEREAL CLOCKS—COMPARISON OF THE PERFORMANCES OF VARIOUS CLOCKS

The standard sidereal clocks are rated very accurately to sidereal time. They are maintained in a vault under constant temperature and air pressure in order to minimize the effect of changes in either temperature or pressure on the rates of the various clocks.

A new vault is now under construction. It will accommodate seven clocks each on a separate pier and so arranged that no two adjacent pendulums will swing in the same plane. The vault consists of an inner and an outer vault separated by a three-foot air space. The inner vault will be illuminated by flood lights let in through the walls from light boxes in the three-foot air space, exactly as magazines are illuminated on board naval vessels. This will permit cleaning and repairing the lighting apparatus without entering the inner vault.

The temperature control will also be installed in this air space. A periscope to be lowered through the roof of the inner and the outer vaults from the office space above ground will permit visual inspection of the clocks in the vault.

The walls of the outer vault will be continued above ground and the enclosed space will be built as an office for the time service, record rooms, etc. There will also be a sleeping room for the use of astronomers on night watches.

At present there are three clocks made by Riefler in

Germany which are about thirty years old. Also one new Shortt clock made by the Synchronome Company of England and an order has been placed for a second Shortt clock.

A study of the Riefler clock No. 60 during a period of twelve years, 1916 to 1928, showed that its performance could be predicted 10 days in advance with an average error of 0.09 second. One day predictions showed an error of about 0.009 second. The other two Riefler clocks did not perform so well.

Shortt clock No. 38 has been under observation only for a relatively short period. Based on its performance from March 15 to August 12, 1931, its performance could be predicted ten days in advance with an average error of 0.05 second and daily predictions with 0.005 second error. A study of the Shortt clocks at Greenwich Observatory over an extended period confirms these figures. Taking all three Riefler clocks together, it is not possible to predict as well as by the Shortt clock, in other words, the Shortt compares favorably with the three Rieflers combined.

For longer periods, the Shortt clock is considerably better. The Rieflers are all subject to slow progressive changes in rates amounting to a tenth or even a quarter of a second per day in a year. No such change has been evident in the running of the Shortt clock. The Greenwich Shortt clocks show evidence of such a change but to a much smaller degree than the Rieflers.

NEW DEVELOPMENTS—NEW CLOCKS—PROJECTS FOR EXTENSION OF THE TIME SERVICE

The Observatory now has under construction three new clocks, an additional Shortt clock, a crystal controlled clock, and an experimental clock. The crystal controlled clock is being constructed by the Naval Research Laboratory under the supervision of Doctor L. P. Wheeler and Mr. W. E. Bower in accordance with their design which differs from those heretofore in use. The crystal is bar shaped and about four inches long. It has a fundamental frequency of 1,000 cycles per second, and is suspended at its nodes. Condenser plates placed in proximity to its ends, are connected to the input of a vacuum tube amplifier. The output of the amplifier is used to operate a telephone unit, the acoustic output of which is directed at the middle of the crystal, thus sustaining the oscillations. It is claimed for this method that the coefficient of the coupling between the crystal and the vacuum tubes is much less than in the conventional type. The frequency of the crystal is consequently less affected by variations in the tubes.

The crystal oscillator, in addition to serving as a new standard time keeper, will be used for controlling a new automatic time signal transmission apparatus which is being developed at the Naval Observatory. This is being designed for greater accuracy and reduced labor for maintenance.

The experimental clock is being planned by the time service personnel and embodies several new ideas.

In addition the Observatory is planning a more efficient apparatus for retransmission of the Observatory's time signal via San Francisco. If this proves successful, a similar installation will be tried at Honolulu and it may be carried on to a similar one in Guam and later in Manila.

Discussion

For discussion of this paper see page 550.

Time Services of the Telegraph Companies

BY G. W. JANSON[1]

Member, A.I.E.E.

GENERAL

IN the solar time system which forms the basis of the coordination of most human activities, a specified time at a given point is understood to mean a definite position of that point in respect to the sun. Thus, the true solar time at any two places upon the earth's surface not on the same meridian is different at any given instant. In order to avoid the confusion resulting from a large number of time standards, the earth's surface has been considered as divided into time zones and a standard of time arbitrarily chosen for each such zone. The North American Continent is included in seven such zones, the width of each being approximately fifteen degrees longitude or one twenty-fourth of the earth's circumference. In each of these, a standard time is used which differs by one exact hour from that of the adjoining zones and which represents the mean solar time of a meridian passing through it.

It is essential that the indication of time by clocks and other devices within a time zone be kept alike and at a proper relationship to those of other zones and to time as indicated by the sun. This might be done at each point by frequent comparisons of the time indicating devices with determinations of true time made from observation of the sun or other heavenly bodies. Such a procedure would be quite involved and hence for general practise the observations of a selected few places are relied upon as the basis of accurate time for many other points. The results of the determinations of the selected observatories must therefore be accurately and widely communicated to the dependent places and for this a system of electrical time signals is utilized.

The distribution of time signals as described in this paper may be divided into three classes of service. The first of these, commonly known as the "noon beats," comprises the transmission during a three minute period, ending at noon Eastern Standard Time, of a series of electrical impulses at intervals of exactly one second or multiples thereof. These signals originate from the Naval Observatory at Washington and are transmitted to the Western Union and to the Postal Telegraph-Cable Company who further distribute them to telegraph and railroad offices where they are used as a comparison standard by means of which these offices may determine the error in their time indicating or recording devices.

The second service commonly known as the "jewelers beats" comprises a similar transmission throughout the day of a coded series of impulses by which the time of day may be determined to a small fraction of a second. These beats are used principally by jewelers and watchmakers for the purpose of checking and regulating time pieces. Some of this work, for example the regulation of ships' chronometers, requires a high degree of accuracy and hence a continuously available standard of correct time.

The third and largest of these services is the clock or synchronizing service. This comprises the automatic correction or resetting of clocks or other devices which independently maintain an approximately uniform motion. The Western Union Telegraph Company's synchronizing system corrects about 120,000 clocks of which 95 per cent are rented to subscribers and the remainder are in its own service or owned by individuals or firms who subscribe for the synchronizing service which hourly resets the clocks to the correct time.

NOON BEAT SERVICE

The noon beat service which forms the basis of comparison for the accuracy of all other parts of the systems described originates with the Naval Observatory at Washington. Here astronomical observations are made each night when conditions will permit and from these, the error of the three principal naval master clocks is determined. These clocks are of great accuracy, and specially installed so that their rate is not affected by temperature and other variations.

The observatory equipment includes in addition to the three master clocks, two secondary master or transmitting clocks. The rate of these clocks may be altered by magnetic means shortly before it is expected to send signals from the observatory. Adjustments are made to bring these clocks to the correct time as indicated by the master clocks and the correction for their known error.

Signals are transmitted from the Observatory several times a day for five minute periods to various radio stations and communication companies who redistribute them. The signals are always sent during the last five minutes before the exact hour and are made by closing a circuit for a brief period immediately following the end of each second except the following:

29th, 51st, 56th, 57th, 58th and 59th seconds of the 56th min.
29th, 52nd, 56th, 57th, 58th and 59th seconds of the 57th min.
29th, 53rd, 56th, 57th, 58th and 59th seconds of the 58th min.
29th, 54th, 56th, 57th, 58th and 59th seconds of the 59th min.
29th, 51st, 52nd, 53rd, 54th, 55th, 56th, 57th, 58th, and 59th seconds of the 60th min.

The signals from the observatory control a special form of telegraph repeater in the Washington office of The Western Union Telegraph Company and in the

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[1] Western Union Telegraph Company, 60 Hudson St., New York, N. Y.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Washington office of the Postal Telegraph-Cable Company, so that daily, commencing at 11:57 a.m. Eastern Standard Time, the beats are repeated and transmitted to all of the large telegraph centers. At each of these, the incoming lines carrying the beats are similarly connected to time repeaters which repeat the signals into a number of telegraph circuits covering the territory served by that central office. In this distribution, the beat signals are connected to more than 200,000 miles of telegraph line wires.

The signals indicating the time determined as correct by the Naval Observatory are thus automatically repeated throughout the United States and made available at all points having local master clocks controlling time functions so that these devices may be corrected daily for any error which may have accrued during the preceding twenty-four hour period. All the other distributions of time described in this paper are on a local basis, that is, controlled by a local master clock serving a city or other territory not exceeding a few miles radius.

JEWELERS' BEAT SERVICE

The jewelers' beat service mentioned at the beginning of this paper is quite similar to the noon beats. Each city or local territory in which this service is given is provided with a master clock equipped with the necessary contacts and with auxiliary equipment for transmitting time beats. These signals are transmitted to circuits upon which are connected in series a number of telegraph sounders installed in the offices of the jewelers, watchmakers, and others subscribing for the service.

In general, the code followed for this service differs from that described above for the noon beat signals. The local master clocks are equipped with contacts for sending beats in accordance with a code which divides each hour into five minute periods, the first of which commences at the exact hour, with the exception of a silent minute following the exact hour. Beats or impulses are transmitted after each second except the 29th second and the last five seconds of each minute. During the last minute of each five minute period, the beats indicating the end of the 50th to 54th seconds are also omitted, making the final silent period of ten seconds duration.

CLOCK SYNCHRONIZING SERVICE

The most extensive distribution of correct time carried on by the Telegraph Company is the automatic correction of clocks. Each of these clocks is provided with an electro-magnetic device by means of which the hands may be set to a predetermined time. This mechanism is independent of the motive power moving the clock hands and is thus applicable to practically any of the general design and types of clock mechanism. A large majority of the clocks actually in service in the system described have mechanisms of the spring-driven pendulum-controlled type, provided with a local battery and an electric motor which automatically keeps the driving spring properly wound.

A number of the clock magnets is connected in series on a line wire and all such line wires serving the local territory are carried to a central or control office. Once each hour, there is sent from the Central Office a "synchronizing" impulse which operates the magnets and sets the clock hands to indicate correct time. There is in use a number of methods of distributing the synchronizing impulse so as to make all clocks conform to the correct time, and of insuring against interferencee with the service.

This paper describes the latest of these arrangements, designed for the larger cities, in which contacts on a local master clock control a machine equipped with a large number of contacts, each pair of which connects a line circuit to a source of power at the precise hour. This synchronizing machine is motor driven and as constructed at present, is capable of handling 120 line wires, each of which may connect together groups of approximately 60 clocks. The synchronizing machine is provided with many special features to safeguard the clock service and with adjusting arrangements by means of which the time as transmitted by the machine may be made different from that indicated by the master clock controlling it. This latter feature permits correction for any slight master clock error with no readjustments of that device. It is only necessary to regulate the master clock by means of weights on the pendulum when the accumulated error exceeds four seconds. In this way, frequent regulations and consequent irregularity in the operation of the master clock are eliminated.

Included with the synchronizing arrangements at the Central Office are automatic means for testing the clock circuits. These are embodied in a motor-driven test machine which is started by auxiliary clock contacts at predetermined intervals. When the machine is thus started, it connects successively to each line certain electrical testing circuits which compare that line with a simple artificial line of correct characteristics. The mechanism is so arranged that when a line under test varies considerably from the artificial line, a printed record is made on a paper tape of the designation identifying that line.

The two machines are mounted upon a 22 in. by 26 in. shelf attached to a metal cabinet 26 in. by 18 in. by 81 in. high. This cabinet houses the auxiliary apparatus required in the operation and control of the machines and is provided with a jack panel at which each synchronizing clock circuit is made available for such tests and trouble analysis as may be desired. The switchboard is provided with a volt milammeter and other equipment to facilitate such work. The completed switchboard is shown in Fig. 1.

While the above describes in general the synchronizing clock service, a more detailed description of the clock mechanism, and of the electrical circuits of the synchronizing and testing machines may be of interest.

Clock Mechanism. Fig. 2 shows the mechanism applied to a clock for resetting its hands if the clock is in error when the synchronizing pulse is received.

Regular clock mechanism connected to the shaft *G* causes that shaft to make as nearly one complete revolution per hour as practicable. The shaft is connected through a friction coupling to the double disk *Q* and hence the latter is driven by the clock mechanism but may be moved independently. To this disk is mounted the minute hand of the clock and the gear train driving the hour hand one revolution for each twelve revolutions of *Q*. A lever *D* is so arranged that when the electromagnet *A* is energized its armature *B* causes *D* to strike projections on *Q* and thus move *Q* and the clock hands controlled by it to a definite position; *i. e.*, that corresponding to the exact hour. Normally, the synchronizing lever *D* is locked in the position shown by means of the locking lever *H*, which carries a pin *S* that engages the catch *T* on *D*. This arrangement prevents any current which might be used for testing during other portions of the hour from operating the synchronizing mechanism. At three minutes to the hour, the pin *U* on the disk *Q* engages the lever *H* and moves the locking pin *S* from under the catch *T*. The mechanism is now free to operate when the synchronizing current passes through the magnet *A*. The pin *U* remains engaged with the locking lever until three minutes past the hour, at which time it allows the lever *H* to drop back in position and lock the synchronizing mechanism until the next hour.

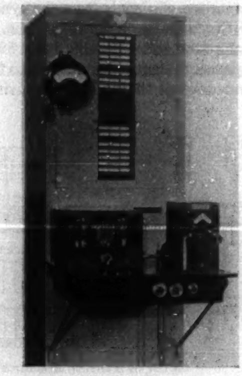

FIG. 1

A somewhat different mechanism is generally used to set the second hand of a clock and that is also illustrated in Fig. 2. The second hand is regularly attached to the heart-shaped cam *R* which is driven through a friction coupling by the clock mechanism. When the synchronizing magnet is operated, the lever *C* is pressed against this cam to advance or retard the second hand to a position indicating the beginning of a minute.

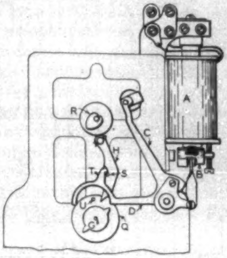

FIG. 2—CLOCK SYNCHRONIZING MECHANISM

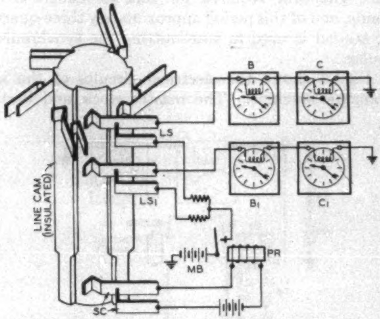

FIG. 3—ELEMENTARY CLOCK CIRCUIT

Synchronizing Machine. The clock magnets are grouped in series on line circuits which are connected to contact springs on the synchronizing machine. Each machine is provided with two cams for closing these line springs and for controlling the application of power for the transmission of the synchronizing impulse. These elements are illustrated in Fig. 3 in which *B*, *C* and B_1, C_1 represent clocks on two lines connected to the springs *LS* and LS_1 arranged for operation by the insulated cam. Around each cam are disposed ten layers each containing six sets of contact springs, the cam thus controlling contacts for a total of 60 lines. One of the cams is also arranged to operate a pair of "synchronizing contacts" shown in Fig. 3 at *SC*. Once each hour, the master clock causes the cams to revolve through one-third of a revolution. As they do so, each set of line springs is closed and then restored to its normal condition except that they rest upon a similar portion of the cam 120 deg. farther advanced. The movement of the cam also closes the spring *SC* which through the operation of the power relay *PR* connects the line wires to a source of operating current *MB*. It should be noted that the raised portion of the cam which passes under the springs *SC* is shorter than that under the line springs. This insures the line circuits being closed before power is applied and that they remain closed until the power relay has been deenergized and has removed the power connection. In practise, a number of power relays is used, each controlling the current to a group of lines varying in number from 12 to 30, depending upon the potential used.

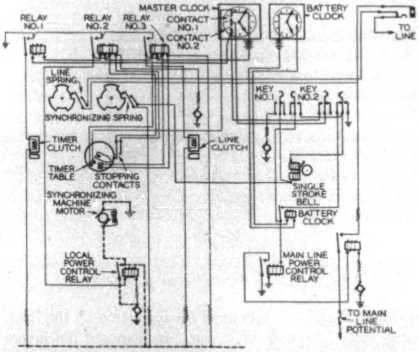
RELAY NO. 1
RELAY NO. 2
RELAY NO. 3
MASTER CLOCK
CONTACT NO. 1
CONTACT NO. 2
BATTERY CLOCK
TO LINE
LINE SPRING
SYNCHRONIZING SPRING
KEY NO. 1
KEY NO. 2
TIMER CLUTCH
LINE CLUTCH
TIMER TABLE
STOPPING CONTACTS
SYNCHRONIZING MACHINE MOTOR
SINGLE STROKE BELL
BATTERY CLOCK
LOCAL POWER CONTROL RELAY
MAIN LINE POWER CONTROL RELAY
TO MAIN LINE POTENTIAL

FIG. 4—ELECTRICAL CIRCUIT OF SYNCHRONIZING MACHINE

The cams are driven by an electric motor through gear trains so controlled by a master clock that once each hour the cams turn through one-third of a revolution. The time required for this movement is four seconds, and of this period approximately three-quarters of a second is used in transmitting the synchronizing impulse.

Fig. 4 shows the electrical circuits of the synchronizing machine. The master clock and battery clock shown control the machine in transmitting the impulses. At five seconds before the hour, the battery clock contacts close and remain closed until five seconds after the exact hour. These contacts through suitable relays connect the machine to the source of operating current for the clock line circuits. The master clock contact shown as Contact 2 closes at approximately 15 seconds before the exact hour and opens at five seconds before the exact hour. The closing of these contacts starts the machine in operation and sets the various control parts to a correct initial position. Through suitable clutch mechanisms, the opening of Contact 2 causes a timing arm to revolve and connect to contacts on a timer table. The timer table contacts control a "line clutch" mechanism so that during the interval between the closing of the two pairs of these contacts, the cams are turned and transmit the synchronizing signal. By manual adjustment of the position of the timer table contacts, the time relation between the connections to them and the opening of Contact 1 may be varied to compensate for errors in the master clock.

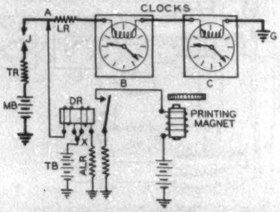
A
LR
CLOCKS
G
TR
MB
DR
B
C
PRINTING MAGNET
X
TB
ALR

FIG. 5—ELEMENTARY TESTING MACHINE CIRCUIT

Two keys are also shown in Fig. 4. Key No. 1 when depressed operates the main power control relay, thus permitting an attendant to make tests requiring line battery. Key No. 2 and an extra set of master clock contacts (shown as Contact No. 1) are provided to facilitate comparison of signals from the synchronizing equipment with standard observatory time. Pressing Key No. 2 before the end of the 58th and 59th minute of each hour causes the entire mechanism to function in a normal manner except that the synchronizing impulse is not sent to any line but is sent to a bell, the stroke of which may be compared audibly with the corresponding signal from the Naval Observatory.

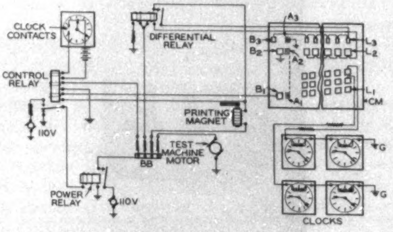
CLOCK CONTACTS
DIFFERENTIAL RELAY
A3
B3
B2
A2
L3
L2
CONTROL RELAY
B1
A1
L1
CM
PRINTING MAGNET
110V
TEST MACHINE MOTOR
BB
G
POWER RELAY
110V
G
CLOCKS

FIG. 6—CIRCUIT OF TESTING MACHINE

Testing Machine. In Fig. 5 is shown in heavy lines a schematic representation of a clock circuit and in light lines the electrical elements of the testing arrangement used in checking the quality of the clock circuit. The tap resistance *TR* is fixed at approximately one ohm per volt of operating potential of the main battery *MB*, while resistance *LR* is selected so that the current during the synchronizing impulse will be 250 milliamperes. The resistance from *A* to *G* will therefore be the same for all circuits operating from a given potential. A fault on the clock circuit would either decrease or increase this resistance value. The testing arrangement of Fig. 6 is designed to indicate when such a change from the normal resistance exists.

The center point *X* of a differentially wound relay is connected to the testing battery *TB*. One winding of the relay is connected to the line at *A* and the other winding is connected to the artificial line resistance *ALR*. Resistance *ALR* is adjusted to equal the nominal value which should exist for all circuits operating at the potential of the main battery *MB*. If the resistances in series with the two windings of the differential relay are equal, the current flowing from the battery *TB* divides equally and the relay will not be operated. Should the line resistance vary from that of the standard *ALR* as a result of a ground, open or other fault, the currents in the two relay windings will be unequal and the relay will operate the printing magnet.

Fig. 6 shows schematically the electrical circuits of the testing machine. Once each hour, the motor controlling this machine is started by the closing of the clock contacts. A brush shown at A_1 moves spirally and contacts with the line studs L_1 while brushes A_2 and A_3 contact with studs L_2 and L_3. In making these connections, each line is connected to one of the differential relay windings and thus compared with the artificial line or standard. If, as a result of an improper line condition, the relay is operated, the printing magnet causes an impression to be made upon a paper tape to indicate the position of the Brush A_1 at the time the printing magnet has operated. This identifies the line in trouble to an attendant who may then make the necessary tests to determine the nature of the fault.

SUMMARY

The proper distribution of time information requires a continuity of accurate service. This the systems described aim to accomplish by the subdivision of the distribution into local areas each entirely independent of all others. The equipment in each area is capable of rendering an accurate time service to subscribers in that area for extended periods without assistance from other sources and by the nationwide dissemination of standard signals may be held to a negligible variation from true time. The equipment and operating methods are such that troubles occurring within the area will produce a minimum of interference with the accuracy of the distribution within that area and will be promptly indicated so that they may be remedied.

Discussion

For discussion of this paper see page 550.

Synchronous Electric Time Service

BY H. E. WARREN[1]

Associate, A.I.E.E.

BEFORE the beginning of the present century, there were a few individuals who realized the possibility that commercial alternating current might be used in some manner, by means of its recurring pulsations, to measure time; but dreams of this nature did not materialize in practical performance.

Up to 1916 there was no commercial a-c. system in existence in which generator speed was regulated with sufficient accuracy so that the current could be used for the purpose of indicating time by means of synchronous clocks. At that time two devices were brought out which in the succeeding years made possible commercial a-c. time service.

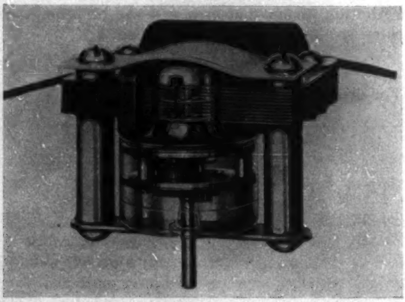

Fig. 1—Type B Telechron Motor and Gear Train

The first of these devices was a self-starting synchronous motor which utilized residual magnetism to enable a rotor to start and run at synchronous speed. This type of motor differs from all the common forms of a-c. motors in that eddy currents, induced either in the mass of the rotor or in some form of winding upon the rotor, are not utilized to start the rotor from rest. In place thereof, the rotor is made of such permanent magnetic material and is so proportioned with respect to the field that remanent magnetic poles are set up in the rotor material by the a-c. magnetism of the field in such a manner as to cause the desired motion. Such a motor is sometimes known as a hysteresis motor. Fig. 1 shows a phantom view of a recent model of this motor.

The hands of a clock were connected by a gear train with the first of these self-starting synchronous motors and this clock, supplied by current from the local electric light system, was compared with standard time, at intervals, over a period of several weeks. The new motor-driven clock proved to be a very inferior timekeeper, developing daily errors as great as five or ten minutes, thus proving the need of greater accuracy in the average frequency of the power supply if it were to be used for timekeeping.

By observing these daily errors, it was easy to compute the average value of the system frequency. There are 1,440 minutes in a day and the gearing of the clock was such that, if the frequency had been exactly 60 cycles, there would have been no time error. A 1 per cent error in average frequency (0.6 cycles at 60 cycles) would mean 1 per cent error in the time of the clock, which amounts to 14.4 minutes in a day. The actual computed error in the average frequency, as determined by these observations, is shown in the upper curve of Fig. 2 which was originally published in the article "Clocks in the Field of Electric Light Appliances" in the *Proceedings* of the N.E.L.A. for 1917.

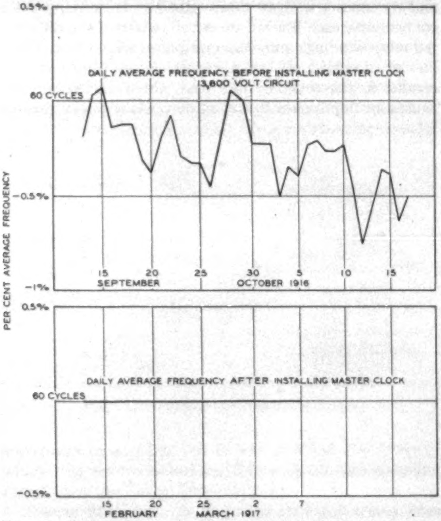

Fig. 2—Daily Average Frequency Before and After Installing Warren Master Clock

The second device consisted of an indicating instrument that would show to a generating station operator, directly in terms of time, deviations of average frequency from standard. Guided by this instrument an operator could so regulate the speed of his generators

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1. Warren Telechron Company, Ashland, Mass.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

as to maintain such small deviations from standard average frequency that the alternations could be used to measure time for all ordinary commercial purposes.

Fig. 3 shows the dials of a type of master clock which is in common use. The large dial in the center has two hands, one colored black and the other gold. The black hand makes a revolution on the dial every five minutes; its rate of motion being regulated by an accurately adjusted pendulum beating seconds. The gold hand is driven by a gear train directly from one of the Telechron self-starting motors. The gear ratio is such that, when the average frequency is correct, this hand will also make one revolution in exactly five minutes.

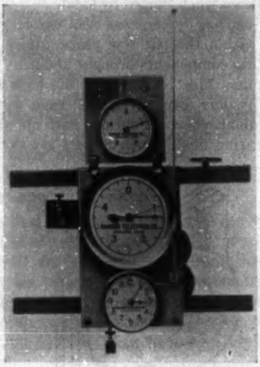


Fig. 3—Operating Dials of Type A Master Clock

The two hands are started off together. So long as the average frequency remains correct, they will keep together and there will be no visible angle between them. An error in the average frequency will cause the gold hand to move slightly faster or slower than the black hand and they will become separated. The angle of separation will be a direct visible measure of the error in the integrated alternations, which may be conveniently called the system time. If the gold hand is ahead, this will show that the system time is fast; if behind, that it is slow.

An error in the average frequency as small as 1/60th of a cycle will cause a spread of one second between the two hands of the master clock, in one hour. If such an error continues, the spread will continue to increase as time passes. The accuracy of the indication is limited only by the time keeping precision of the pendulum itself. When carefully regulated this is better than one second per day, or an absolute precision of about one part in one hundred thousand.

When the operator, at the generating station which is regulating frequency, adjusts the turbine governors from time to time so as to keep the black and gold hands of the master clock together, he automatically causes all other synchronous motor clocks on the same system to keep correct time. The lower curve of Fig. 2 shows the great improvement in the average frequency value which was obtained after the first master clock installation.

The standardization of average frequency is of great value to the power companies in the following ways:

1. By improving the service furnished to manufacturing establishments, ensuring more uniform speed of motor-driven machinery and as a general result assuring the quality of the product.

2. By facilitating interconnection with other power companies which have correspondingly standardized their frequency.

3. By enabling power companies to use synchronous motor movements in their maximum demand meters and graphic recorders, thereby obtaining more accurate and synchronized records at lower cost of maintenance.

Before master clocks were available, the inherent errors in frequency indicators were sufficient to handicap operators on interconnected systems in their efforts to equalize the frequency preparatory to synchronizing the two systems, and to lead to controversy as to the true value of the frequency after the two systems were synchronized.

Master clocks have been generally adopted by the power companies of the United States for the purpose of regulating frequency. At the present time, it is estimated that the regulation of the current to over 90 per cent of the consumers' meters in this country is guided by these master clocks.

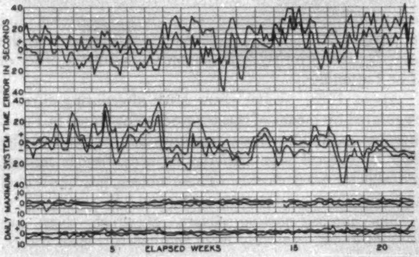


Fig. 4—Maximum Daily System Time Errors of Four Large Power Systems

The accuracy which may be expected at present from a synchronous electric clock is indicated by the records shown on Fig. 4. The records shown on this chart are taken from four typical very large interconnected systems east of the Mississippi River. The ordinates of the curve in each case are seconds deviation from standard time, and the abscissas weeks over a period of five months. The solid line through the center of each curve represents standard time, and the two points plotted for each day at the top and bottom of the band represent the maximum range of the deviations from standard for that day. The widest deviations shown are 44 seconds fast or slow, which means that a synchronous clock on these systems set absolutely accurate with standard time did not, through the period plotted, indicate incorrect time more than 44 seconds fast or slow. The records of the two systems which show the smallest departure of standard time indicate that a clock operating on these systems would never be more than 9 seconds fast or slow. There are few uses of time which justify higher accuracy, and even the worst of these records is far superior to that of previous types of commercial clocks.

Some power companies throughout the country may be giving better than the best time service indicated on this chart. Probably there are some permitting wider variations. Practically all power companies, however, are giving sufficiently accurate frequency regulation to provide far better time on synchronous clocks than on commercial clocks previously available.

All factors in power system development are tending to require continued improvements in frequency regulation. As a matter of fact the very large interconnected systems have so much momentum stored in rotating equipment (generators, motors, etc.,) that they will give, when properly adjusted at the beginning of a flat load period, almost perfect time with no adjustment of governors for speed. Governor adjustment is necessary, however, over the irregular portions of the load curve and much work is being done at present to divide the burden of this regulation on large interconnected systems between the various power companies by the use of automatic frequency control to supplement the master clocks. When perfected and applied these instruments will undoubtedly reduce the system time error to insignificant values of the order of 1 or 2 seconds.

About 1928, twelve years after alternating current regulated by means of master clocks illustrated in Fig. 3 became available, there appeared on the market the first of several new varieties of synchronous motors suitable for use in clocks. These motors may be divided into two classes; non-self-starting and self-starting.

The *non-self-starting motors,* of which there are several kinds, generally have toothed soft-steel rotors. The usual stator construction is bi-polar with teeth on each pole corresponding to the spacing of the rotor poles. In all multipolar motors of the common varieties the speed is found by dividing the number of current alternations per minute by the number of rotor or stator poles. In nearly all cases no attempt is made to introduce a rotating component into the field. A common arrangement of rotor and stator is shown in Fig. 5. On account of the difference in magnetic reluctance between rotor positions, where the poles do or do not correspond with the stator poles, a strong reactance torque exists when the rotor is at rest. This tends to prevent the rotor from starting; but when running at synchronous speed, tends to keep it at that speed.

It is somewhat difficult to spin such a rotor by hand so as to cause it to lock into synchronism with the alternating field, unless some means are provided which will permit a sudden phase shift of the rotor poles as the rotor speed passes through its synchronous value. Among the satisfactory schemes for accomplishing this result are the use of loosely coupled fly-wheels on the rotor shaft or spirally arranged rotor poles with means for the axial shifting of the rotor in the field. Mechanical launching devices are also sometimes used automatically to bring the rotor up to approximately synchronous speed when started by hand.

The *self-starting synchronous clock motors,* which have appeared within the last few years, utilize a-c. fields with rotating components brought about by the use of shading coils. Most of these motors have multipolar rotors with some form of squirrel cage winding or a copper element wherein Foucault currents are induced by the rotating field. Such motors get their starting torque, like ordinary induction motors, through the reaction of the Foucault currents in the field. Synchronous driving torque, in these varieties of self-starting motors, is produced by polar projections of the steel rotors or by a separate polar element loosely coupled to the rotor shaft. One variety of the recent self-starting synchronous motors utilizes remanent magnetism for starting and for synchronous operation.

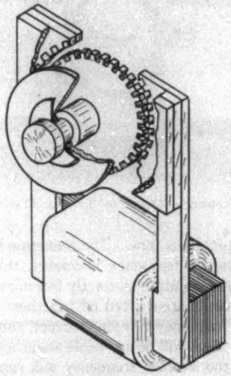


FIG. 5—NON-SELF-STARTING SYNCHRONOUS CLOCK MOTOR

Any of the rotors heretofore described may be used to drive the hands of a clock through a simple train of gears. Two forms of such gear trains are in common use, one composed entirely of spur gears and the other utilizing worm gears and worms. On account of the greater efficiency of transmission, a spur gear train is better for large clocks but for small clocks other mechanism is satisfactory. Frequently, the high-speed gears of a spur gear train are made of non-metallic material so as to run more quietly and the motor or the whole clock movement is usually mounted on cushioning material so as to reduce the a-c. hum. Some clocks are much quieter than others depending upon the rigidity of the field laminations and the design of the moving parts.

Any one of the motors mentioned is adequate to drive the hands of a clock dial 12 in. or less in diameter. The actual output required for this purpose is exceedingly small. If the clock hands are balanced and the gear train is fairly efficient, not more than one-twenty millionth of a horsepower is needed for a 12 in. dial. The power output of the various motors ranges from less than one-millionth of a horsepower to more than one-ten thousandth of a horsepower. The larger amount has been found adequate to drive tower clocks ten or twelve feet in diameter. The power output of these motors seems exceedingly small as compared with commercial motors which are used for other purposes but it is, nevertheless, fully adequate for this time keeping duty and is many times greater than the power of ordinary spring clock motors.

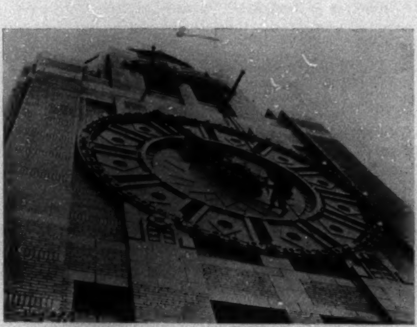
The electrical input to these motors is generally in the neighborhood of two watts, but as the power factor is low, the volt amperes will vary from three to six. Obviously, the efficiency of these motors is very low indeed, amounting to a fraction of 1 per cent. However, the power used is so little (17 kw-hr. per year or approximately $1.00 worth) that this feature is of minor importance.

Most synchronous motor clocks are provided with some form of indicator visible through a small hole in the dial. In clocks which are not self-starting, this indicator shows by its continuous motion that the clock is in operation. After an interruption in the current, the motion of the indicator stops permanently. In self-starting motor clocks of the Telechron brand, the indicator, back of the dial, shows a steady white target as long as the current flows. This target changes to red and so remains after an interruption in the current supply, although the clock, of course, starts and continues to run as soon as the power is restored.

Inasmuch as most kinds of synchronous motor clocks will either stop permanently or show errors after current interruptions, it is reassuring to know that system interruptions are rare in our thickly settled districts. Often, these clocks continue to show correct time within a very few seconds for many months, even for years. In order to provide against the rare stopping of the hands because of interruptions, clocks have been available for many years which have auxiliary spring movements. These movements appeared more necessary in the beginning before alternating current service had become as reliable as it is today.

The successful operation of synchronous motor clocks is quite independent of certain influences which are detrimental to other clocks such as level position, vibration, and ordinary temperature variations. Extreme temperatures near zero fahrenheit will stop some kinds of clock motors, but other varieties will operate well below zero.

Application of these small clock type synchronous motors to many devices and instruments has been going on ever since these motors became available. The first users were the power companies themselves who had many instruments and meters, especially those of the maximum demand type, which required time keeping elements. The spring clocks, which had been used for these purposes, were inaccurate, troublesome, and costly to maintain. This was especially true because they needed frequent winding and were often located at distant points. The instrument load was sometimes too great for a spring clock to handle with accuracy. As soon as the motors were substituted for the spring clocks there was a great gain in accuracy and reliability and the costly winding was eliminated. It then became possible to redesign the instruments so as to give still better performance.



Pacific & Atlantic Photo, Inc.

FIG. 6—DIAL OF LARGE SYNCHRONOUS TOWER CLOCK

Gradually, nearly all the makers of instruments began using synchronous motors in place of spring movements until the list of such motor-driven instruments is a nearly complete catalog of devices in which the measurement of time is involved. The largest class of these motor-driven instruments is perhaps the graphic recording type. The load of driving a paper chart, especially of the strip type, is very great for a spring clock, but comparatively trifling for a synchronous motor. Moreover, the motor can handle a roll of paper so as to make a continuous record for weeks or months without regard to winding. There are also many kinds of instruments which do not make a record but serve to control or actuate, and synchronous motors have advantageously been incorporated into certain forms such as time switches, temperature controllers, traffic signals, etc.

There is a large and important class of timing devices exclusive of ordinary clocks. Among them are numerous kinds of time and cost recorders, program machines, time stamps, etc., wherein synchronous motors have gradually been replacing spring clock movements. This change has made it possible, in many cases, to eliminate local master clocks, batteries, and expensive wiring.

In building installations, where Telechron clocks, or timing devices driven by Telechron motors, are numerous it is customary for the customer to provide a so-called "resetting device" or else a duplicate source of alternating current energy to guard against the effect of interruptions in the supply of current. In all such installations, a separate wiring system is used and this is connected with the public service system through a suitable switchboard. The resetting device automatically measures the duration of an interruption and, after the current is restored, applies higher frequency or otherwise speeds up the clocks until they are again correct.

The problem of driving very large tower clocks by means of self-starting synchronous motors is much less difficult than that formerly involved in the use of conventional pendulum tower clock movements. It is necessary, of course, for the gears and shafts to be very strong because the stresses due to the great weight of the hands and the pressure of the wind are rather high. Fig. 6 shows a tower clock movement installed on the Williamsburgh Savings Bank at Brooklyn, N. Y. where each of the four dials is 26 ft. in diameter and the hands alone weigh 700 lb. per pair. The hour hand sleeves of this clock are of steel and their outside diameter is 4 inches. The power required to drive these hands during a heavy wind is less than 0.001 hp. The motors used have a very large surplus power margin.

The use of synchronous motor clocks and timing devices has increased since its conception in 1916 with extraordinary rapidity, especially during the last few years. According to estimates more than one million two hundred thousand synchronous motor clocks were sold during 1930.

There seems little doubt that, during the coming years, spring and weight driven clocks will be replaced in large measure by the new a-c. timekeepers, which have proven greatly superior in every important respect.

The author wishes to express in this paper sincere appreciation of the generous aid which has been given by numerous executives and engineers to promote the attainment of commercial time service by means of alternating current—especially those men with far sighted vision who realized the possibilities before there were any actual results.

He would also express thanks to Mr. George M. Keenan, Superintendent of the Pennsylvania-New Jersey Interconnection, who has cooperated in the preparation of this paper.

Discussion

TIME AND ITS MEASUREMENT

(Brown)

MODERN DEVELOPMENTS IN PRECISION CLOCKS

(Loomis and Marrison)

TIME SERVICE OF THE U. S. NAVAL OBSERVATORY

(Hellweg)

TIME SERVICES OF THE TELEGRAPH COMPANIES

(Janson)

SYNCHRONOUS ELECTRIC TIME SERVICE

(Warren)

L. S. Harrison: This discussion presents an important development in the particular field of institutional electric time systems—where a multiplicity of clocks is designed to influence the coordination of activities in schools, hospitals, office buildings, railroad terminals, industries, etc. It is a master clock system which hourly supervises the uniform agreement of electric secondary units.

This system which has had widespread application during the past seven years combines certain basic principles of practical timekeeping in a unique manner. The most reliable means of obtaining commercial accuracy is with a high grade 60-beat pendulum regulator. A well designed unit, properly mounted and free from vibration is capable of rating within two to three seconds per month of standard time without resetting, or an error of approximately 1 in 900,000.

The practise of all of the best known systems of electric clocks to employ, somewhere in the system layout, a 60-beat master regulator, confirms this basic necessity of time system engineering. However, the high accuracy of a master clock does not by itself, assure the successful transmission of system time to the various individual secondary time indicators because to complete its function as a time governor, a master regulator must do two things:

1. It must provide an electrical operating medium to propel secondary clocks at an accurate constant rate of advance, and
2. It must transmit to each secondary clock, periodically, a point origin of time on which to establish such constant rate.

It should be borne in mind that secondary time units are not of themselves, clocks; they cannot keep time by themselves but must depend upon an uninterrupted supply of accurately timed pulses and they, hence, inherently reflect as any other meter reflects, conditions in everything which may contribute to or affect their operation; yet these secondary clocks are the real points of utility in the time system and the medium through which the master clock delivers its service.

Further, any clock indicator which depends initially for its timekeeping quality on original setting by hand, obviously does not receive its accuracy from the master clock and cannot be said to do so.

The master clock, which is automatically self winding, transmits its own accuracy over a wiring system wholly devoted to time service transmission to the secondary timepieces both as to rate of advance and as to a point origin of time by means of a periodic time check.

The secondary units are designed electrically to sense their own agreement with system time once an hour and automatically to accept only the exact required number of driving impulses during the hour. These secondaries may be in the form of devices for signaling and recording of time as well as for indicating.

Since its introduction in 1924, this system has been placed in use by large numbers of schools, colleges, banks and office buildings, and industries throughout the United States and Canada. Three outstanding recent installations are the Pennsylvania Railroad Philadelphia Terminal Development, Cincinnati Union Terminal and the Cleveland Union Terminal Group.

A short description of this system is placed on record for reference as follows:

A description of the Hourly Supervised Electric Time System is quite readily grasped when two of its operating conditions are laid down, namely:

1. Each secondary clock (Fig. 1) individually connects itself to the supervising circuit at the 59th minute on its own indicating dial, permitting it to sense its own position with the correct system time.

2. The master clock shown in Fig. 2 provides supervising also during the last minute of the hour by a simple manipulation of the circuit. This supervision, being supplied for variable differences in error, must be handled over two of the three wires in the system, as follows:

(a) On one wire a suspension of impulses for a period of ten minutes before, so that any fast secondaries may be retarded which may have arrived at the last minute of the hour point ahead of the master clock.

(b) On the other wire extra impulses are supplied, during the last minute of the hour, in order that slow secondaries may be stepped ahead.

All units which are on time, are of course, in perfect step with the master clock and therefore do not partake of any of the effects produced by the above supervision. The third wire is a common return.

Philip Sporn: A reading of Mr. Warren's very interesting paper is likely to give the impression, particularly to one who has but a passing acquaintance with the subject, that the question of frequency control on the part of the operators of large power systems is a rather simple affair, involving, as it does not, very little more than the use of a master clock, and that such practise is very beneficial in the operation of a power system. Such, of course, is far from the case; I would, therefore, emphasize some of the dark shadows in the picture in order to give a representation more nearly in accordance with the facts. In connection with Fig. 2 of the paper there is shown a graph of daily average frequency before and after the installation of a master clock. It ought to be pointed out, however, in this connection that while a master clock permits the speeding up or slowing down of system frequency after a certain period has elapsed and the error had been allowed to accumulate, that the straight line curve does not truly represent system frequency.

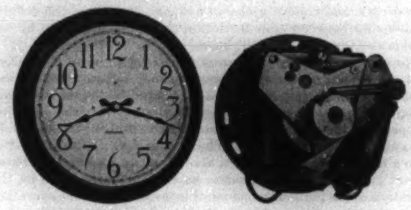


Fig. 1—International Secondary Clock and Movement

Nor is the very jagged and irregular line typical of what can be done without the use of a master clock. The fact of the matter is that during the period covered by the upper curve in Fig. 2, the frequency meters available were of such a crude nature that the results shown are about all that could be expected. On the other hand, the use of a modern frequency meter with its high degree of accuracy without any master clock whatsoever could be made to give substantially as good an average time curve as is indicated by the lower curve in Fig. 2 of the paper. I think it is worth while stressing this because otherwise we may get the old familiar *post hoc, ergo propter hoc* train of reasoning started in connection with the very complicated problem of frequency control and we all know the subject is complicated enough as it is without having to encounter that.

In Fig. 3 is shown a frequency chart taken over approximately 12 hours on our middle western interconnected system in June 1925. A rough examination of the chart will indicate that the *instantaneous* frequency is pretty ragged. At the same time it will be noted that at no time during the 12 hours has it been worse than ½ cycle plus or minus the normal of 60 cycles. It will also be seen that the average frequency is substantially perfect, or if it is not, over this 12-hour period, there would be no difficulty by governor manipulation, say over the 24-hour period, to bring it to perfection, as far as *average frequency* is concerned without in any way altering very much the characteristic of the frequency curve.

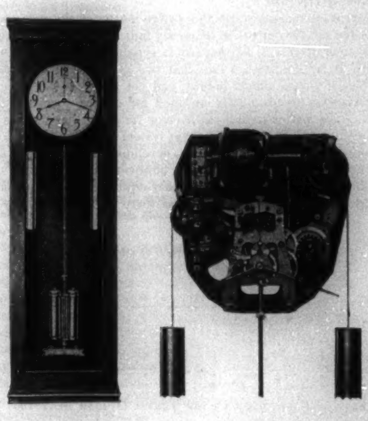


Fig. 2—International Master Clock and Movement

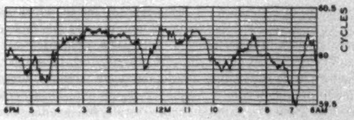


Fig. 3—Frequency Chart Taken in June 1925

Showing excellent average frequency but wide variations of instantaneous frequency

This again is shown because of the claims made in the paper by Mr. Warren that the standardization of average frequency is of great value to the power companies in the ways enumerated:

1. "By improving the service furnished to manufacturing establishments, ensuring more uniform speed of motor-driven machinery and as a general result assuring the quality of the product."

It will be seen from Fig. 3 that so far as giving good service to processes requiring accurate speed, the frequency represented by Fig. 3, although having excellent average characteristics, is decidedly unsuitable and offers no improvement over some other frequency that might have the same characteristics, but which might be floating above or below the 59-cycle line as a base.

2. "By facilitating interconnection with other power companies which have correspondingly standardized their frequency."

Again referring to Fig. 3, it will be noted that a frequency of the type shown there is not adequate in a large interconnected system although again it may be perfect from an average frequency standpoint. The attempt to regulate frequency to make satisfactory interconnection is much more complicated and brings with it a train of many other problems, some of which will be mentioned later.

3. "By enabling power companies to use synchronous motor movements in their maximum demand meters and graphic recorders, thereby obtaining more accurate and synchronized records at lower cost of maintenance."

An examination of Fig. 3 will again show that a considerable error can be introduced in the timing of devices utilizing the frequency of the type indicated there, even though the average frequency is perfect.

Mr. Warren again states that before master clocks were available the inherent errors in frequency indicators were a handicap in system operation, particularly in synchronizing different systems. It is, of course, a fact that two systems can have their *average frequency* almost perfect, and yet at the instant of synchronization be absolutely unsuited for coming together. The fact of the matter is that the average frequency has nothing to do with system operation, but that system operation was greatly facilitated and improved by the development of accurate frequency indicators. It is admitted, however, that it is highly desirable to maintain average frequency equivalent to the nominal frequency of the system as long as there is the slightest claim with regard to constancy of frequency. A master clock along with a time deviation recorder is, of course, very desirable as a means of checking the accuracy of the frequency recorders.

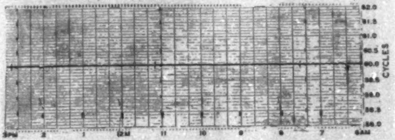


Fig. 4—Frequency Chart Taken During May 1931

Illustrating to what extent instantaneous frequency variations can be reduced

Mr. Warren is, of course, on solid ground when he states that the factors in power system development are tending to require continued improvements in frequency regulation. These factors, however, it should be pointed out, are totally independent of and have no relation to the problem of supplying time and it is doubtful whether the advantages of being able to supply correct time would pay for the difficulties that are involved in maintaining such close frequency. He is not right, however, in saying that very large interconnected systems, because of the heavy momentum, etc., give almost perfect time with no attachment of governors for speed. In Fig. 4 there is shown a frequency chart taken on the same system which furnished the data in Fig. 3, the difference being that it was taken approximately 5 years later. It will be noted that the instantaneous frequency here is almost perfect, having a maximum deviation of approximately plus or minus 0.03 cycles. However, it was not obtained by merely bringing together a large power system and letting it run in parallel after the governors had been adjusted.

In Fig. 5, for example, there is shown a typical loading on one of the tie lines of the particular system tying together two large portions of it during a day when frequency of the order given in Fig. 4 was obtained. It will be noticed that over this particular line within a period of less than 2 hours this tie-line had its load increased from zero to about 95,000 kw., back to zero again with an actual reversal of flow, then up again to approximately 110,000 kw., and down to 20,000 kw. It is obvious that this particular line, in so far as it was intended to function as a steady deliverer of power somewhere close to its theoretical limits, was vitally handicapped by what it was doing in the way of maintaining system regulation so that a frequency shown in Fig. 4 could be maintained.

A more definite relationship between tie-line loading and time deviation on an interconnected system is shown in Fig. 6. It will be noticed that during the 2-hour period, the time deviation from true time was at no time greater than approximately 2 seconds, yet that involved carrying the tie-line through several very erratic and violent cycles of loading. Note, too, the definite relationship between time deviation and load picked up by the tie-line and furnished to the system:

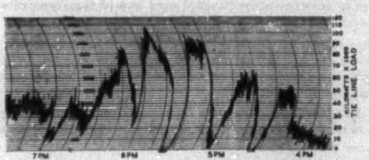


Fig. 5—Typical Example Showing the Effect of Close Frequency Regulation on the Loading of Tie-Line Connecting the Regulating Station to the System

For example, during the period from 5:00 p. m. to 5:20 p. m. load was coming on the system rapidly and the frequency control station was picking up this load as indicated by the tie-line chart in order to maintain frequency. At about 5:20 the upswing stopped and the regulating station backed off about 70,000 kw. in a matter of some 10 minutes. The process was again reversed at even a more rapid rate between about 5:40 and 5:50 p. m. These violent swings in load, of course, have their repercussion on the operation of the plant from which this tie-line is fed and operation under these conditions is not at all a simple matter. For that matter the entire problem of frequency regulation on an interconnected system is highly involved both from a technical and from an economic standpoint. It is important, therefore, that the impression be not allowed to remain that frequency regulation is rather simple; nor that it becomes more simple as more systems are thrown together in parallel; and that the mere installation of a master clock of the type shown in Fig. 3 of the paper will do the trick.

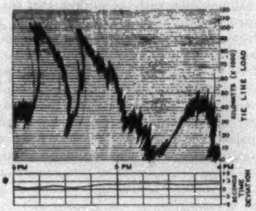


Fig. 6—Typical Case of Power Transfer Across a Tie-Line as a Result of Holding Accurate System Time

Time service should be a logical by-product of system operation; at the present time it appears safe to say that the ultimate method of operating large interconnected systems will be facilitated by extensive use of automatic frequency control devices and that it will be possible to obtain very accurate time service from such systems.

Considerable use has been made of synchronous motor clocks in indicating type and in round chart type demand meters, as brought out in Mr. Warren's paper and this has effected considerable economy and increased reliability in the use of these meters. There is, however, a need at the present time for a synchronous motor clock of sufficient power to drive the timing device as well as the reroll device in strip chart instruments. Some use has been made of synchronous motors in strip chart meters to date, but in practically all cases either an additional motor or a spring is used for the reroll. It is obvious that to replace these two units with single synchronous motors would simplify the construction and considerably lower maintenance. Such a device would be applicable to strip chart demand meters, wattmeters, ammeters, voltmeters, etc. In the case of voltmeters, it would probably be advisable to incorporate some type of carry-over device in order that interruption and duration of interruptions would be properly indicated on the chart.

R. J. Wensley: It is interesting to note that the first interest displayed by the central station industry in synchronous timekeeping was in the use of such methods for the integration of demand against definite time intervals. Such devices began to come into use some years before the distribution of time as a service was instituted by the power companies. As each demand interval was short, usually 15 or 30 minutes, and as the errors were not cumulative the variance from true time over the 24-hour period found in the frequency of all power systems did not affect the commercial application of such devices.

As Mr. Warren's paper is historical in character, it is perhaps in order to add a few comments to complete the historical review of the devices used in synchronous timekeeping. As the motor is the main problem in the secondary clocks, the various classes of motor and the origin of each are of some interest.

Self-starting synchronous motors suitable for clock application are of several varieties:

1. Hysteresis motors, fully synchronous.
    - (a) Smooth rotor hardened material.
    - (b) Toothed rotor softer material.
2. Reaction type, fully synchronous.
    - (a) Phase split by resistance.
    - (b) Phase split by capacitance.
3. Sub-synchronous motors.
    - (a) Two magnetic circuits and rotors.
    - (b) One magnetic circuit, two rotors.
    - (c) One magnetic circuit, one rotor.

Class 1 is the oldest as the smooth core hysteresis motor depending on the remanence of a hardened rotor was quite completely described by Nikola Tesla in 1887. The toothed core hysteresis motor depending on the reluctance of the slots for its synchronous characteristics first shows in the patent granted to Durfee, of Rochester, New York, about 1910.

Class 2 is so well known as to hardly need comment. The use of phase splitting capacitors and resistances to accelerate induction motors with salient poles to synchronous speed dates from about 1900. This type of motor has not been used to any great extent for timekeeping, though it is not at all impossible to design a motor of this type that would be suitable for some varieties of time service.

Class 3 dates back to 1890 when Elihu Thomson described a synchronous motor with an eddy current disk tending to run at high speed, while toothed magnetic structures on the rotor, acting as a magnetic escapement prevented operation above synchronous speed. Sub-synchronous motors are all of this type, differing only in the details of construction. Each of the three types listed consists of an induction element of higher speed characteristics and a magnetic escapement to determine synchronous speed. All of these types have the advantage of slower speed of rotation for a given size of motor. To counterbalance this advantage is the rather critical nature of the design due to the necessity for accurate balance between the induction and synchronous elements.

Mr. Warren mentions the difficulty he experienced in persuading the central stations to agree to frequency regulation. It is very interesting to note that the first inventor to file an application in this country on a complete timekeeping system was Mr. A. F. Poole, who, in 1914, filed on a system comprising a master clock regulator and secondary clocks with synchronous motors. Mr. Poole tried to bribe the power companies by showing a resistance connected in parallel with the secondary clocks to increase the energy consumption. It is very fortunate that the power companies did not need this added inducement even though the problem is not simple as has been quite well covered by Mr. Sporn in his discussion.

It might be well to mention at this time that the usual power system is not regulated with sufficient accuracy to permit of the use of synchronous timing of extremely short intervals. The momentary variation may amount to as much as a tenth of a per cent or more. This error would be too great to permit of the calibration of fine watches by stroboscopic methods using central station frequency as a standard. Many such problems are presented to the manufacturers and of course must be refused.

W. A. Dudley: Mr. Janson has described various time services which the telegraph companies furnish to their subscribers, and in this connection it may be well to mention another important time service which these companies provide for their own purposes.

Synchronized clocks identical with those furnished to subscribers are provided throughout the several operating rooms and offices of the telegraph companies and usually one or more of the clocks in each office is provided with cam-operated contacts which, through suitable relays, are used for the control of timing and dating stamps. The stamps are magnetically operated in equal steps to position type wheels for printing the year, month, day, meridian, hour and minute, and in some cases fractions of minutes. The principal use of the stamps is for timing of messages but they are also used for stamping employees' time cards and other supervisory purposes.

Since all of the time stamps throughout an entire telegraph system are controlled from clocks which are part of a synchronized time service as described by Mr. Janson, it may be seen that the timing of messages at all offices of the system will be on the same basis, and will thereby provide an accurate means for controlling and checking speed of service.

J. C. Wilcox: Mr. Janson's paper very ably outlines the technical characteristics of the equipment used to transmit Naval Observatory time beats to master clocks and master clock correcting signals to subscribers' clocks. As all of these subscribers' clocks are equipped with a special type of self-winding movement particularly adapted to the maintenance of a continuous correct time service, I would enlarge on that part of Mr. Janson's paper which refers to the movements, and give some further idea of their construction, use, and application.

First and foremost stands the fact that these clocks are independent operating units. They do not depend on any outside energy for operation. They are spring-driven and pendulum-operated. A motor automatically winds the main spring once each hour and maintains a constant tension on the main spring to give accurate pendulum control. Clocks running independently can be regulated within three or four seconds in twenty-four hours. To check for any variations, however, they are synchronized every hour by the Telegraph Company's master clock. If connecting line should be open for any reason and the synchronizing signal not received the clocks continue to operate continuously and within good commercial limits until the next signal brings them to time. Their outstanding feature is their ability to operate continuously and accurately regardless of outside line interruptions.

This characteristic makes them particularly desirable for use in commercial and transportation fields. In New York we find every railroad and subway system using self-winding clocks in their stations and dispatching headquarters. The largest installation is in Grand Central Terminal where over 1,000 units operate under the control of a central master clock which is checked daily with the noon time beats. The synchronizing line not only covers the terminal proper but extends as far out as Harmon, a distance of thirty miles.

The Pennsylvania Terminal has a similar system with about 250 clocks in the terminal proper at 33rd Street, and a separate synchronizing circuit running eastward to the Sunnyside freight yards at Long Island City and in the opposite direction as far west as Bristol, Pa., a distance of 67 miles.

Local subway lines are similarly equipped. The Interborough Rapid Transit System uses about 600 self-winding clocks synchronized from a master at its 59th Street power house. The Brooklyn & Manhattan Transit has approximately 100. The Hudson & Manhattan under-river tubes has 50 synchronized by a Christopher Street master clock. Approximately 100 miles of new subway system is now under construction, and a master clock with suitable relay and battery equipment has been installed in the 53rd Street power house and is available for inspection. No estimate can be made of the number of clocks the new subway will use but probably between 200 and 300.

I refer to these private installations only to point out the fields of application of this self-winding synchronized type of movement which is essentially the same as that used throughout the Telegraph Company's time service.

Herman Lemp: A symposium of time and time services would not be complete without mentioning the work of Mathias Hipp the pioneer in this art in Europe. Those who have traveled recently in Switzerland have undoubtedly seen the electric clocks on street corners, church towers, and hotels in the principal cities. These clocks have been in continuous service since about 1868 furnishing accurate time to a critical public, makers of Swiss watches and chronometers.

At the Centennial exposition in Philadelphia in 1876 Mr. Hipp received Gold Medals for his exhibit. His electric clocks are known all over Europe. The underlying fundamentals of the first central stations established to furnish accurate electric time are found in Mr. Hipp's conception of using feeble impulses but of some duration following each other every minute with reversed polarity, to actuate polarized receiving dials. These impulses have the advantage of never causing the receiver armature to advance more than *one step at a time*, even if a bad contact such as might be likened to a wire passing over a file, should give many impulses in lieu of one. To cause the minute hand to advance another step, the current will have first to travel *in opposite direction*, and as stated above is independent of the number of impulses in the same direction. Hence if atmospheric interference should cause the hand on one dial to advance between legitimate signals the current from the master clock would find its work already done and at the next minute the dial would be in step again.

Mr. Hipp was the first to invent the tuning fork escapement for a chronoscope, used in technical schools to measure the time a ball takes to fall—say three decimeters. He then invented the Hipp escapement, a toggle joint electric contact making device, which would, upon lowering of the amplitude of a free swinging pendulum, send an impulse through an electromagnet acting upon an armature attached to the pendulum and thereby restore its amplitude. This device was unaffected by strength of current and permitted the master clock to be kept going indefinitely as long as electric energy was supplied and with an accuracy of a few hundredths of a second variation per day.

It was my duty in 1878 to correct the master clock of the city of Neuchâtel every noon, by signals received from the Observatory of the same place. To ascertain deviation from observatory time Mr. Hipp invented the "coincidence clock," doing for time what a vernier scale does for linear dimensions. A pendulum next to the master clock pendulum had a period of oscillation of 60 beats in 59 seconds. It was released by current from the observatory and started to swing immediately. If the master clock pendulum was fast the coincidence pendulum lagged behind, each beat approaching nearer that of the master clock. As soon as the two beats coincided, the number of beats was read on the dial, indicating the number 1/60th of a second the master pendulum was fast. On either side of the master pendulum two others were provided which, when connected to the master pendulum, would either accelerate or retard the latter 1/60th for each beat they were connected. As a check the coincidence pendulum was reset again and released a second time from the observatory when, if the master pendulum had been properly corrected, the beats of the two pendulums would coincide right at the first swing.

Mr. Hipp was also the first to construct an astronomical pendulum of steel with mercury compensation (Invar was unknown at that time) located in a glass tube hermetically sealed and kept under constant (vacuum) pressure and temperature. This pendulum was located in a vault in the observatory while its dial was placed next to the observer at the transit instrument. This astronomical pendulum which had a maximum variation of 1/200th of a second per day was exhibited at the first electric exhibition in Paris in 1881 and I think another was sent to the Naval Observatory in Washington.

Mr. Hipp's successor Mr. Albert Favarger greatly simplified the polarized synchronized dials in 1905 and his book on electric time measurements is today considered authoritative.

John D. Crawford: Time (integrated frequency) and frequency (time rate) are similar aspects of the same physical idea. Yet these two ways of viewing the problems before us can cause very real differences of opinion. The discussion between Mr. Warren, who measures time, and Mr. Sporn, who measures frequency, illustrates this point.

In the field of precision measurement, on the other hand, it is significant that Messrs. Loomis and Marrison have pooled their experiences with clocks and with frequency standards, respectively, to the very great benefit of both. This paper is the first summary of the field to appear (in recent years at least) and the authors are to be congratulated. I know that I can speak for one organization interested in frequency measurements when I say that we are glad indeed to know how the other half lives.

In connection with Mr. Marrison's reference to the use of distance as a measure of frequency in terms of the velocity of light, a mention of the recently completed measurements of the velocity of light *in vacua* may be of interest. This series of experiments was undertaken by the late Dr. A. A. Michelson on a ranch near Santa Barbara, California. The measurement involved the determination of the time it took light to travel ten times between two sets of mirrors placed at the ends of an evacuated tube about a mile long. The distance, as measured by the United States Coast and Geodetic Survey, had a probable error of something like one part in three and one-half million. A crystal clock, similar in operating principle to the one described by Mr. Marrison, was used as the reference standard for time. Data that we have available on some 15 units of this type now in commercial service, indicate that, when working under the best conditions, it is capable of determining the average frequency to within three parts in ten million. The experimental details and the results of the measurements have not yet been formally published, but it seems as though the velocity of light will soon be known with a somewhat greater accuracy than heretofore.

If this increased accuracy is realized, it will be due as much to the high accuracy of comparison that is possible with a crystal clock as it is to its frequency stability. Due to the fact that the crystal is vibrating at 100,000 cycles per second, it is inherently better suited to precise, short-interval timing than is a pendulum clock whose bob vibrates once every second. Because of this greater accuracy of comparison, it is a relatively easy matter to determine with high precision the rotational frequency of the rotating mirror used in the experiment. This point has already been mentioned by Mr. Marrison.

So it may be that the Coast and Geodetic Survey will find it possible to follow out a suggestion made some time ago by Major William Bowie, Chief of its Division of Geodesy, and lay out precise base lines in archipelagoes, mountains, and other inaccessible places by means of a crystal clock and a beam of light. Distance would then be measured in terms of time and a standard base line, using the constant velocity of light as a transfer medium.

R. C. Thayer: The noon beat signals as sent out by the two telegraph companies serve a very valuable purpose to the railroads of this country. These time signals as sent on the telegraph companies' wires are picked up by the railroad companies and re-distributed throughout their own network of communication circuits, thus each of the railroad's stations, and the train dispatchers receive the correct time daily. At terminals and the more important offices clocks designated as "standard clocks" are installed, whose accuracy is checked daily and recorded, with which the trainmen can compare their watches. The ability to obtain accurate time is of prime importance to the railroads, and without which it would be impossible to operate the fast train schedules now in effect in this country.

H. E. Warren: In answer to the comments of Mr. Philip Sporn I would not minimize the difficulties that stand in the way of securing satisfactory frequency regulation. I would agree with him that the requirements today are much more severe than was the case in 1916 when master clocks as an aid to regulating frequency first became available.

At that time the need for extreme accuracy in frequency regulation was much less and the instruments which could be used were far inferior to those which are now available. Standardization of frequency was then considered of rather minor importance. Power companies were operated regularly, for what seemed sufficiently good reasons, at frequencies as much as two or three cycles above and below 60, although rated as giving 60-cycle service. Many companies which were attempting to operate at 60 cycles were actually delivering current as much as 1 cycle above or below that value on account of instrument errors.

This state of affairs was not of course conducive to extensive interconnection. The use of the master clocks by the power companies, whatever the reason may have been for adoption, did actually tend to standardize the average frequency at an exact 60-cycle value. Moreover the use of the master clocks with their very high inherent precision probably served both as a spur and an aid to the manufacturers and to the users of frequency meters. This is for the reason that errors in the meters could be very easily discovered by comparison with the master clock so that all uncertainty regarding the relative precision of different meters for use on the same system could be readily removed.

A condition has now been brought about, as pointed out by Mr. Sporn, where it is possible by commercial instruments to maintain frequency regulation that was utterly impossible fifteen years ago. Whether or not this result would have come about if the use of the master clocks as basic standards for the average value of the frequency had not been generally adopted remains at least open to question. Certainly the adoption by many power companies of methods of regulation which call for the maintenance of exact average frequency, primarily or secondarily for the purpose of giving time service to their customers, has been a rather important factor in bringing about the present situation, where practically all power companies have the same exact standard of average frequency.

I heartily agree with what Mr. Sporn has said about the extremely difficult problems of regulating station output on large interconnected systems so as to maintain tie-line loads within satisfactory limits and I certainly would not claim that the adoption of master clocks alone, has or could completely solve these problems.

One idea which I desire to convey in my paper is that the adoption of the master clocks by the power companies has, apart from the time service that has been made possible, been a factor and aid in helping to solve the operating problems of some of the very large power companies.

In answer to the comments of Mr. R. J. Wensley I would recognize the background that has been furnished by numerous inventors in this field, as in practically all fields where something worth while has suddenly been accomplished. It seems to me that in most cases, however, the merit of the work that has been done can be measured by the accomplishment.

Forces of Electric Origin in the Iron Arc

An Explanation of Overhead Welding

BY F. CREEDY[1] Member, A.I.E.E.
R. O. LERCH[2] Non-member
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IT will be readily admitted that it is desirable, if possible, to reduce the study of the phenomena of high-current metal arcs, particularly iron arcs, owing to their importance in welding, to a state in which they can be exactly described and measured by a suitable number of parameters, so that the effect of any change of conditions may be specified by the changes in these parameters. Owing to its intense brilliance the arc cannot be studied directly and the most convenient way of studying it is to project the magnified image on a screen by means of a lens in the manner shown for instance in Fig. 3. When this is done a series of remarkable phenomena at once becomes apparent. Considering first a low current arc of four or five amperes between electrodes of about the same size as welding wire, say about an eighth of an inch in diameter, it is seen that the two ends of the electrodes each become fused into a globule, the positive electrode being materially hotter than the negative. In some cases the globule may be exclusively on the positive electrode while the negative electrode remains black containing only a small incandescent area corresponding to the cathode spot. On each electrode there is a brilliant incandescent spot and frequently a reflection of the cathode spot may be seen in the fused surface of the drop on the end of the positive electrode which presents a characteristic glazed appearance.

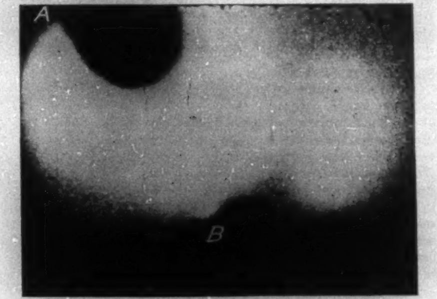

Fig. 1

The arc itself consists of a column of blue vapor surrounded by a yellow envelope of burning iron which is suppressed if the arc is struck in nitrogen or other inert gas. All this has been the subject of much study, but if an arc between electrodes of similar area but carrying a welding current of say 150 amperes is examined in the same way, entirely different phenomena become apparent. Suppose that the positive electrode is a piece of steel plate of considerable area while the negative electrode as before is a welding rod, the drop on the end of the welding rod can be seen as before but now it appears to be the seat of violent forces. It oscillates to and fro and quivers like a jelly while on its surface are to be seen what seem to be fine wave formations. Moreover it does not drop off in the same manner as in a small current of say 5 amperes when the drop leaves the electrode very much as a drop of water leaves a hose. When a powerful current is used the drop can be seen to grow to a certain diameter, usually somewhat larger than that of the rod and then there is a sharp explosion or crackling noise and the drop disappears, its motion being so fast that it is impossible to see it cross the gap between the electrodes either by the eye or by means of movie cameras of ordinary speed.

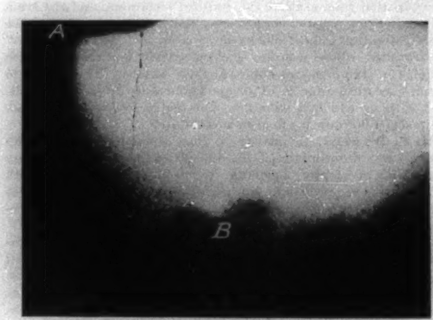

Fig. 2

In Figs. 1 and 2 are shown two succeeding pictures on a film which was given 32 exposures per second. In the first the drop can be seen clearly while in the second it has totally disappeared, (the end of the electrode being seen at *A* and the cathode spot at *B*) having therefore crossed the gap in less than this space of time (probably a great deal less). These phenomena urgently need explanation since the origin of these forces is not at all well known.

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Other curious phenomena are that when welding overhead, the drop can travel in the direction opposite to that of gravity; and again, the crater or hollow space which appears to form under the welding rod requires explanation. This explanation resolves itself into two portions.

1. What is the force that causes the drop to travel across the arc space?

2. Why is it that the drop is detached with explosive violence rather than gradually? Light was thrown on the first of these questions in the following way:

1. Studying low current (five amperes) iron arcs by means of an apparatus which involved burning the arc inside a glass bell jar containing gas at atmospheric pressure it was found that, starting with a clean jar, within a few seconds of the arc being first started, a deposit began to form on the jar which gave sharp shadows of the electrode holders. This was apparently due to particles projected in straight lines which might be thought to give rise to mechanical forces on the electrodes. Within a few minutes this sharp pattern was blurred by the oxides, nitrides, etc., given off by the arc in vapor form. Investigation of these forces, whose magnitude must be far less than a gram, is difficult because the heavy arcs which it is necessary to use rapidly melt away any apparatus delicate enough to measure these small forces. Apparently the only way in which this difficulty can be overcome is to reduce the time of application of the heavy arc to a very short interval.

In order to make use of this principle the following apparatus was designed based on the principle of the ballistic galvanometer. This is shown in Fig. 3 and consists of a brass rod about 5/16-in. diameter and 8 in. long terminated at either end by an iron electrode about 3/8-in. diameter and 2 in. long. This forms the moving system which therefore consists of a double headed *T* which is suspended at the center of gravity by a bifilar suspension whose dimensions are as follows:

Length 26.6 cm. Distance apart at the bottom 2.93 cm. Distance apart at top 1.82 cm. Diameter 0076 cm. Molybdenum wire.

A galvanometer mirror is also fixed to the suspended system and reflects a spot of light on a screen in the usual manner and a pin projects from it below, dipping into a mercury cup. This forms the moving electrode. There is in addition a fixed electrode which consists of a piece of welding wire held in a clamp as shown so that it may either be brought in contact with the end of the moving electrode or moved back to any desired distance by depressing the key. The current therefore passes through the fixed electrode, the arc between it and the moving electrode, then through the moving electrode and out through the mercury cup.

To strike the arc the fixed and moving electrodes must be momentarily touched together and then drawn apart by depressing the key. This at first sight gives rise to a difficulty owing to the sensitivity of the suspension. No matter how light the contact between the fixed and moving electrode, it is sure to set the spot of light on the screen into violent oscillation which may take a long time to die away since the apparatus has no damping except that due to the pin immersed in the mercury cup. To overcome this difficulty, vanes are attached to the moving system on either side of its axis of rotation and arranged to dip into mercury troughs which could be raised or lowered by moving a lever. When the vanes are in the mercury troughs, the damping is extremely great so that contact can be made without agitating the spot of light. Having struck the arc it is possible to lower the mercury troughs and free the moving system without either shock or agitation of the spot. The method of operation of the instrument therefore consisted in striking the arc with a small current, say 5 amperes, the forces due to which are very small, then throwing on the heavy current for a short period varying from 0.1 second to 0.6 second and observing the first deflection of the spot of light. From this and the moment of inertia the impulse may be readily calculated on the familiar principles of the ballistic galvanometer. The impulse of course is defined as the force multiplied by time of application.

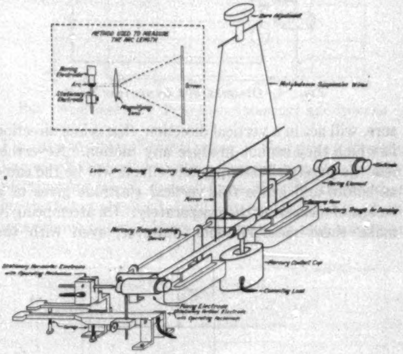
Zero Adjustment
Method used to measure the arc length
Moving Electrode
Arc
Stationary Electrode
Magnifying Lens
Screen
Molybdenum Suspension Wires
Lever for Raising Mercury Troughs
Electrode
Mirror
Mercury Trough for Damping
Mercury Trough Lowering Device
Mercury Outlet Cup
Connecting Lead
Stationary Horizontal Electrode with Operating Mechanism
Moving Electrode
Stationary Vertical Electrode with Operating Mechanism
Spring

Fig. 3

It was also desired to study the relation between these forces and the arc length, hence the moving and fixed electrodes and the arc between them were projected on a screen as shown in the separate sketch included in Fig. 3 with a magnification of approximately 23.8 diameters which rendered the arc length readily measurable.

Examining this apparatus it is obvious that currents passing into the moving element by means of the arc and out by the mercury cup will produce certain forces in conjunction with the earth's field and the fields of other current carrying wires in the neighborhood. It is necessary therefore to find some means of estimating the magnitude of these forces. For this purpose a second electrode is provided and arranged so that an arc may be struck onto the moving electrode from below, this second electrode being vertical. This is clearly shown in Fig. 3. When an arc is struck in this position the arc forces, which the apparatus is designed to measure, will act in a vertical direction, that is in a direction in which they cannot produce any motion. Nevertheless the purely electromagnetic effects will be the same as before and hence this vertical electrode gives us a means of measuring them separately. On attempting to make these measurements, however, even with the heaviest currents, it was found that the electromagnetic effects were so small as to be quite incapable of measurement by the instrument used. By this means it was proved that the deflection obtained when the horizontal electrode was used was entirely due to the arc forces and not to these electromagnetic forces. Two different types of circuit were used to throw on the heavy arc for a short period. These are shown in Figs. 4 and 5.

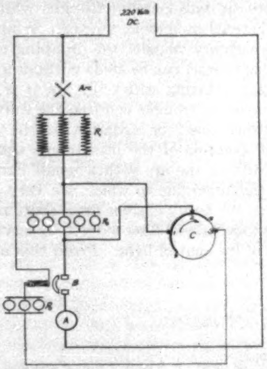

Fig. 4—Diagram of Connection No. 1

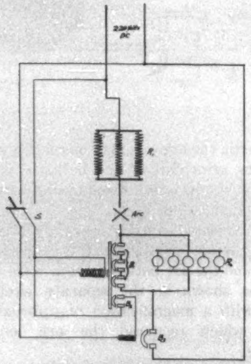

Fig. 5—Diagram of Connection No. 2

In Fig. 4 resistances R_1 and R_2 are connected in series with the arc, R_1 being of such a value as to limit the current to the value which it is desired to study, say 150 amperes while R_2 further limits it to a value of 5 amperes. A revolving contact maker C is employed which short circuits resistance R_2 between brushes 1 and 2 for a certain period depending on its speed of rotation. Between brushes 2 and 3 is connected the trip coil of a circuit breaker so that the arc circuit will be broken as soon as brushes 2 and 3 are joined. This insures that the operation of short-circuiting R_2 shall take place once only and not periodically with the rotation of the contact maker.

For very powerful currents this apparatus as constructed was not found satisfactory owing to insufficient brush area and therefore another circuit shown in Fig. 5 was employed. Here R_1 and R_2 have the same meanings as before while B_1 is a three-phase contactor, all three blades of which are placed in series, and which is fitted with an extra pair of contacts B_2 which close the trip coil circuit of the circuit breaker B_3. With this circuit the heavy arc is thrown on only for a period depending on the time lag of the circuit breaker which can be adjusted to any figure desired. This was found to give very steady and reliable results, particularly for very short time intervals.

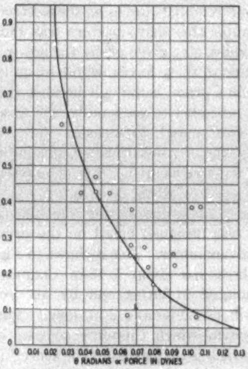

Fig. 6—Arc Length *vs.* Force ($\alpha\theta$) for 120 Amperes Inductance in Circuit Time is 0.1 Second

In using this instrument it was not found possible to regulate the arc length beforehand to the desired value before measuring the impulse. It was necessary to take a considerable number of observations with constant current and random arc lengths after which it was found that a curve could be plotted without much difficulty.

One of these curves is shown in Fig. 6 which shows the relation between the angle of deflection of the instrument in radians and the arc length. It will be seen that the points are rather widely scattered but nevertheless not so much so that a fairly satisfactory curve cannot be drawn through them. All of these instruments are rather delicate and it may be noted that the points on the curves obtained by Duffield, Burnham, and Davis referred to below are quite as widely scattered. At least one cause of these scatterings has been discovered (referred to below) and work is in progress by which it is believed it may be reduced.

A set of curves similar to Fig. 6 was observed for currents of 38, 68, 90, 120, and 154 amperes.

As may be shown easily from the theory of the ballistic instrument, the angular deflection in radians is proportional to the impulse or force multiplied by time of application. The times of application of the force corresponding to these different currents were carefully measured in the manner described below and the forces calculated from the theory of the ballistic instrument as may easily be done. (It may be noted that the moment of inertia of the moving system is 6,760 gram-(cm.)^2.) The result is shown in Fig. 7 which of course shows the smooth curves drawn through the rather widely scattered points such as those shown in Fig. 6. The most striking feature of Fig. 7 is the rapid increase of the force with decreasing arc length. Apart from this the curve for 120 amperes does not seem to be parallel to the others, at any rate for long arcs. This probably is due to the fact that arcs longer than 3 millimeters tend to take a curved form which does not lend itself to accurate measurement especially when it is remembered that this measurement has to be made within a period of 1/10 of a second.

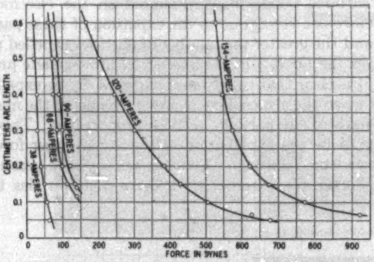

FIG. 7—ARC LENGTH *vs.* FORCE FOR VARIOUS CURRENTS BASED ON UNIT TIME

From Fig. 7 values may be taken of the force corresponding to fixed arc lengths, say 1 millimeter, and this will give 5 points for the curve of force plotted against amperes. A set of such curves is shown in Fig. 8 and it will be seen that they are approximately at least parabolas. In Fig. 9 the curves of Fig. 7 are replotted on logarithmic paper which shows that those for less than 3 millimeters are straight lines while for greater lengths some disturbing cause appears to enter which gives rise to curvature. For arcs of less than 3 millimeters therefore it is possible to obtain a simple equation. For instance for 1 millimeter $\log i = \log 17 + 0.5 \log F$ is obtained which when transposed gives

$$F = \frac{i^2}{290}.$$

For different arc lengths the constants of this equation will vary and the different values are plotted in Fig. 10 also on logarithmic paper. This gives a straight line which is parallel to other straight lines obtained by plotting arc length against force directly, these other curves being obtained simply by replotting Fig. 8 on logarithmic paper. From this curve we may ascertain how the constant in the above equation depends on the arc length and thus derive our final formula namely,

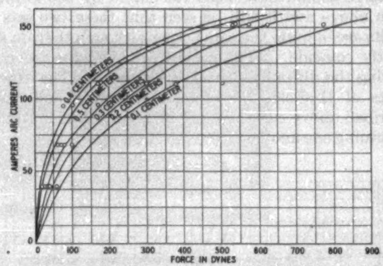

FIG. 8—CURRENT *vs.* FORCE FOR CONSTANT ARC LENGTH

$$F = \frac{0.035\, i^2}{\sqrt{l}} \quad (F \text{ in dynes, } i \text{ in amperes, } l \text{ in millimeters}).$$

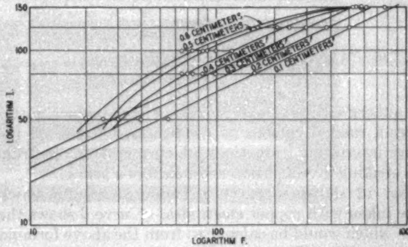

FIG. 9—LOG *I* *vs.* LOG *F* FOR CONSTANT ARC LENGTH

A study of the literature reveals that very little work indeed seems to have been done on this subject. The only papers which the writers have been able to find are a paper by Duffield, Burnham, and Davis, in the *Proceedings of the Royal Society* for 1920, v. 97, and a paper by R. Tanberg, in the *Physical Review* for 1930, v. 35.

Messrs. Duffield, Burnham, and Davis studied various materials including iron and measured the force for currents lying between 3 and 10 amperes by means of a torsion balance which must have been a very delicate experiment. The arc length corresponding to their curves is not very clearly stated but there are several references in the paper to arcs of 3 millimeters and the writers assume that this is the length of arc they employed. Their curves are reproduced in Fig. 11. Curve 1 showing the force with the moving system positive and curve 2 with it negative. Curve 3 shows what the force should have been calculated by the formula given above and it will be seen that these two sets of results are of quite the same order although they do not exactly coincide. This is hardly to be expected considering the difficulty of the experiments. The experiments of Duffield, Burnham, and Davis, and those described above can therefore be regarded as mutually confirming one another.

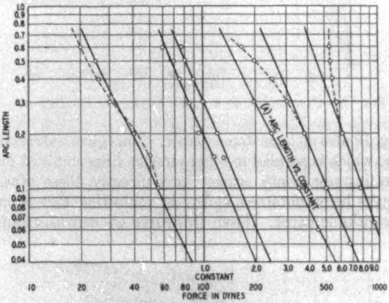

FIG. 10—(*A*) ARC LENGTH *vs.* CONSTANT, OTHER FIVE CURVES ARE ARC LENGTH *vs.* FORCE

Tanberg's results were taken *in vacuo* which introduces a radical change of conditions. A few of the many interesting deductions which may be made from this striking investigation are mentioned below.

Fig. 12 shows in curve 4 Tanberg's results which were taken with copper electrodes. Curve 6 shows the force which would be calculated from the above formula for arcs of 3 millimeters. This, of course, is for iron electrodes. But Duffield, Burnham, and Davis experimented both on copper and iron and found that the force in the case of the copper electrode is roughly double that obtained with an iron electrode under the same conditions. Curve 5 therefore has ordinates twice as great as curve 6 and might be supposed to be the curve which might be obtained by a series of experiments similar to the above, taken with copper electrodes at atmospheric pressure.

Nothing can be more striking than the difference between the forces obtained by Tanberg and those calculated in the manner just described and this difference can only be ascribed to the absence of atmospheric pressure in Tanberg's case. This therefore enables us to give a probable explanation as to why the force is so much greater with short arc lengths. The most reasonable hypothesis as to the origin of this force ascribes it to particles projected from the electrodes, particularly the cathode. The force measured on the electrodes therefore is a recoil due to the expulsion of these particles and the greater the number of particles expelled the greater the force. In the absence of atmospheric pressure it is clear that many more particles can be expelled than when the atmosphere is present reducing the mean free path of the particle to small proportions. This is likely to be the explanation of the much larger forces observed by Tanberg. Again at atmospheric pressure the shorter the external path between electrodes the greater the number of particles which can be expelled and therefore the greater the force which is in accordance with our observations.

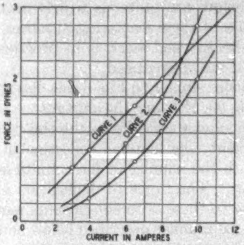

FIG. 11—CURVE SHEET NO. 1

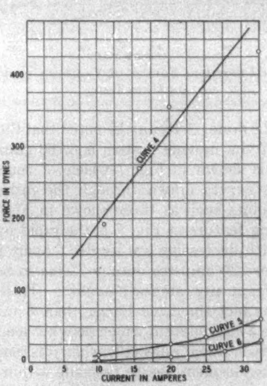

FIG. 12—CURVE SHEET NO. 2

It will be observed that the quantity measured by the instrument described above is the impulse due to the arc forces, that is the product of force multiplied by time of application. It is essential that the time of application should be accurately measured. This is normally done by measuring the speed of rotation of the revolving contact maker of Fig. 4, but in order to check this a number of oscillograms was taken some of which are reproduced in Fig. 13.

Where the circuit of Fig. 5 is employed there is no other convenient method of measuring time except by the oscillograph. Fig. 13, (No. 1) shows the current impulse when the circuit of Fig. 5 is used, the arc being excited with rotary converter current. The effect of the slots in the rotary converter on the current wave should be noted. In order to eliminate this, battery current was substituted. The result of this is shown in Fig. 13, (No. 2) from which it will be seen that the oscillations have by no means disappeared. For the reasons mentioned below, a heavy self-induction was connected in series during this test and it will be seen that instead of the current rising instantly as it does in No. 1 it rises in the well-known exponential form.

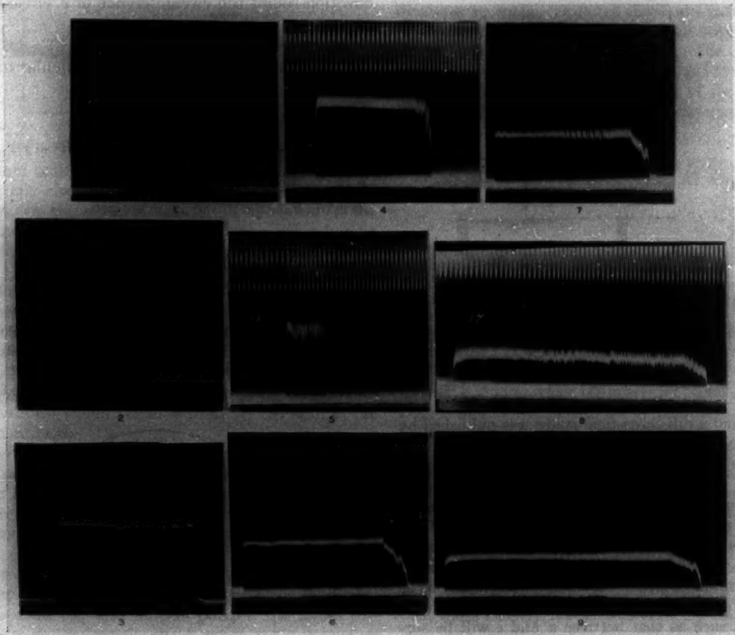

Fig. 13

When the contact maker of Fig. 4 is employed some remarkable phenomena become apparent as will be seen from numbers 3, 4, and 5. These show current impulses which correspond to the same time of closing of the contact maker. It will be seen that the times during which the current is on are entirely different in these three cases which reveal some of the reasons why the points on the curves deviate from the average. It will be seen that the current contains extremely pronounced oscillations of very high frequency. Apparently if one of these oscillations becomes large enough to reduce the current to zero, it cannot restart, so that the length of the impulse depends entirely on these oscillations, and not on the rate of rotation of the contact maker. In order to reduce these oscillations a large self-induction was placed in series with the arc circuit. The effect is shown in numbers 6 and 7 and it will be seen that the oscillations are much reduced though not entirely suppressed. Numbers 8 and 9 show a longer time period. In number 9 the oscillations are almost entirely suppressed by the self-induction but in number 8 taken under the same conditions as far as can be judged the oscillations are still pronounced.

Considering the whole series of oscillograms it will be seen that the oscillations do not start immediately the current is switched on but only after an interval which would seem to correlate them with the heating of the electrodes. This is about the only comment that can be made in general since the cause of these oscillations has not yet been discovered.

THE FORCE DETACHING THE GLOBULE

Coming to the question of the nature of the force which detaches the globule from the welding rod; about 25 years ago Carl Hering drew attention to what he called the "pinch effect" which is found in electric furnaces and a full theoretical study was published by Dr. E. F. Northrup, *Phys. Rev.*, v. 24, 1907, p. 474.

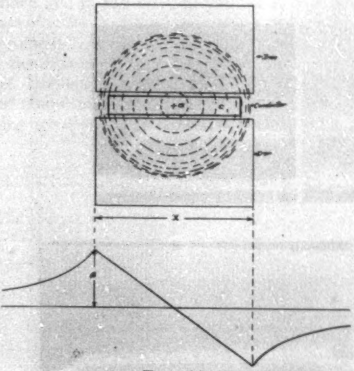

FIG. 14

In order to understand the pinch effect more clearly consider a flat conductor (Fig. 14) carrying a current of uniform current density flowing say at right angles to the paper and surrounded by magnetic material of the form shown. If we consider the central filament of current at a it will be surrounded by a field proportional to its magnitude which is very small, vanishingly so as the width of the filament decreases. A wider filament of current such as that between a and c will be surrounded by a field again proportional to its magnitude which is much greater. Hence the magnetic density will be zero in the middle of the strip and will rise in either direction towards the edges as shown in the curve below, whose ordinates represent B the magnetic density. Given this field distribution and assuming that the current is flowing upwards in the diagram, then it is clear applying the elementary rule, that the outer portions of the conductor will experience a force urging them towards the center and if the conductor is a fluid this will result in a hydrostatic pressure which becomes a maximum at the center reducing to zero at the circumference. The pressure in dynes per sq. cm. at radius r (Fig. 15) is given by the formula

$$g = \frac{I^2}{100\,\pi\,R^4}\,(R^2 - r^2)$$

for the case of a circular conductor of external radius R cm. carrying current I amperes. Clearly becoming a maximum when $r = 0$.

If the values corresponding to a welding rod 4 mm. in diameter carrying 300 amperes are inserted in the formula, $g = 7{,}000$ dynes per sq. cm. at the center of the rod. The value of g in the above formula gives us for a fluid conductor the hydrostatic pressure, which is equal in all directions. The total pressure in an axial direction round the circumference of a circle of radius r is

$$2\,\pi\,r\,g$$

and the total force on a plane perpendicular to the axis is

$$\int_0^R 2\,\pi\,r\,g\,d\,r = \int_0^R \left[\frac{2\,r\,I^2}{100\,R^4}\,(R^2 - r^2)\right] d\,r$$

Carrying out this integration we get

$$F = \text{total force} = \frac{I^2}{200}$$

The total force is therefore independent of the radius of the cylindrical conductor. If $I = 300$ amperes F = 450 dynes. This is approximately equal to the weight (at room temperature) of a spherical globule of iron 4.9 mm. (0.193 inches, a little over 3/16 in.) in diameter.

Hence for dimensions usually employed the force due to the pinch effect tending to separate the globule from the rod is at least equal to its weight particularly if it is remembered that due to the high temperature and the occluded gases the weight of the hot globule is probably a good deal less than that of the cold one.

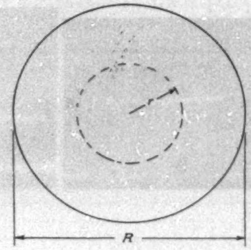

FIG. 15

Thus it is shown in general that the passage of a current through a fluid conductor, such as mercury or the molten contents of the electric furnace, causes a force of the nature of a hydrostatic pressure whose intensity is proportional to the square of the current density. Consequently if there is a constriction in the fluid path at any point so that the current density is greater there than elsewhere, the fluid will be forced out at that point and will be heaped up on either side of the constriction as shown in Fig. 16. There is then a depression in the fluid surface where the constriction occurs and a corresponding elevation at other portions, the difference of level between the two being sufficient to balance the hydrostatic pressure due to the high current density in the constricted portion. But of course a fluid cannot maintain the form shown in Fig. 16 without constant circulation hence the fluid is constantly flowing down the sides of the notch as shown in the dotted lines and being forced out of the portion having the minimum area thus setting up a circulation as shown.

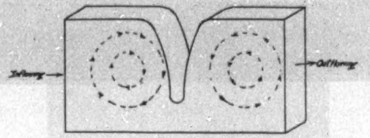

Fig. 16

It is shown above that the force starting from zero at the outer circumference of the circular conductor becomes a maximum at the center, and if a circular liquid column is arranged vertically the conducting liquid will be forced out of the center and will fall back along the outer circumference thus producing a continual fountain which remains in action as long as the current continues.

Now consider a welding rod which is arranged in a vertical position with a fluid drop on the top (see Fig. 17a) and a powerful current flowing through it leaving by an arc at the top of the fluid bead which has a diameter somewhat greater than the rod. It is not certain to what extent this bead is liquid or merely viscous. Since the diameter of the bead is larger than that of the rod, there is a constricted portion where the fluid bead joins the solid rod and at this portion the forces corresponding to the pinch effect exert their maximum power.

It was pointed out above that in the case shown in Fig. 16, the fluid in the narrowest cross section is constantly replenished by that running down the sides of the notch and that it is due to this alone that the phenomenon is a stable one, that is, that the depth of the notch has a definite value for each value of the current and may become shallower or deeper by decreasing or increasing the value of the current. But if there is a viscous fluid in which this replenishment cannot take place, then as soon as the electric forces become sufficiently powerful to overcome the viscosity of the fluid and cause the notch to begin to appear, it seems to the writers that this notch must get deeper and deeper (owing to the forcing out of the material in the smallest cross-section and to the fact that there is no means of replenishing it) until the conductor is broken in two. In other words the stability of the phenomenon depends on the free circulation of the fluid, that is on the absence of viscosity.

Now apply this to the case of the bead on the end of the welding rod. These electric forces must cause a constriction (see Fig. 17b) to form at the narrowest portion of the fluid mass, that is where it joins the solid material. This constriction forces out the material in the center of the bead the axial pressure being 7,000 dynes per sq. cm. as calculated above, and as soon as the cross section becomes smaller the pressures are increased as the square of the current density. The total force tends to sever the parts still attached, and remains constant no matter how small the area of the attached portions. Hence as soon as the constriction starts it deepens at an ever increasing rate until the bead is cut off from the rod with explosive violence in a way which can be readily observed. It may be that some of the fine wave motion which is apparently observable in the bead is due to attempts to set up a circulation such as that described in connection with Fig. 16. As soon as the globule is detached the forces measured above are ample to carry it over. For instance with a current of 300 amperes and an arc length of 0.1 millimeter the force is approximately 10,000 dynes or about 10 grams.

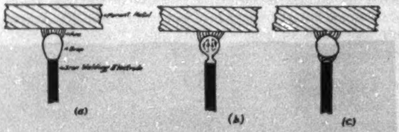

Fig. 17

Discussion

J. Slepian: Is force itself the proper measure of the capacity of the pinch effect in throwing particles from the electrode with considerable velocity? It would seem to me that impulse or work being more directly related to momentum and energy would be more appropriate units for describing the particle-throwing capacity. Hence, it should be necessary to consider either the time factor or the distance factor, that is, for how long a time can the pinch effect force act so as to give the particle momentum, or through what distance does the pinch effect force act so as to give the particle kinetic energy?

L. R. Ludwig: Mr. Creedy has attacked a rather interesting problem in his paper in attempting to describe the means by which drops are caused to travel across the arc space during welding. He has assumed that the drop first breaks contact with the welding electrode, travels across the space, and fastens itself to the work which is overhead. This, however, is not generally the case. Fig. 1, for example, is an oscillogram of the voltage and current taken during an overhead welding operation. It will be seen that the voltage drops from arc voltage to a value of about 2 volts very frequently, and it is believed that these voltage decreases correspond to a period during which the drop actually short-circuits the welding arc. In this event, it seems probable that when the drop of molten metal touches the work it freezes at the upper contact, and the surface tension will probably be sufficient to draw the drop up onto the work. It seems that Mr. Creedy has neglected to mention this possibility.

With regard to small drops, one must agree with Mr. Creedy that the vapor blast from the cathode of the arc may be an important element in the transfer of the welding material if the electrode spacing is short. If this is to serve as an explanation, however, it is essential that the welding electrode be negative and the work positive. Overhead welding is frequently done with polarities opposite to those mentioned, and this fact again seems to mean that Mr. Creedy's explanation of the transfer of material is not sufficient.

In the latter part of his paper Mr. Creedy has discussed the effect of the "pinch effect" on loosening molten drops from the welding electrode. He has calculated a force of 450 dynes, which hardly seems sufficient to explain the explosive action which he has observed when the drops leave the electrode. In order to be certain of the results, either the impulse or the momentum given to the drop should be calculated. It seems possible that the explosive action is really due to the very high temperature of the last point of contact between the drop and the electrode. The formula for the calculation of this temperature was given by Mr. Slepian before the A.I.E.E. in 1926. Thus, if it is assumed that due to the pinch effect the area common to the drop and the welding electrode is reduced to a very small value, the voltage between the welding rod and work would concentrate across the last contact point. This would give rise to a voltage drop of 2 volts across the contact (taken from Fig. 1), which according to Slepian's formula would result in a temperature of 13,000 deg. cent. at the last contact point. Although there are few data on the vapor pressure of iron at this temperature, an extrapolation would give a value of about 1,000 atmospheres. This pressure is of course exerted on a very small area, but some idea of this area may be had by calculating the amount of material vaporized as a function of the pressure, temperature, and area; finding from this and the latent heat of vaporization for iron the watts required; equating this latter result to the actual watts input and solving for the area. The result is an area of about 10^{-3} square centimeters. If the pressure given above acts on this area, the force would be about 10^6 dynes. This is much greater than the force calculated by Mr. Creedy.

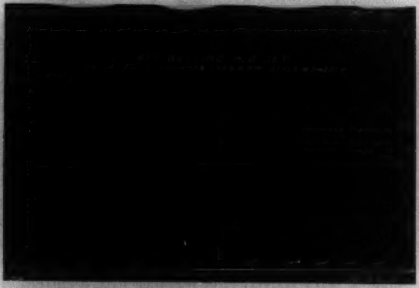

FIG. 1

Such calculations are conceded to be very rough. The calculated temperature of 13,000 deg. cent. is probably much too high, but it does seem as though the force due to this cause is of major importance in causing the transfer of welding material.

F. B. Lucas and E. C. Easton: Mr. Ludwig of the Westinghouse Company states in his discussion that the transfer of metal in arc welding is due to a globule building up on the end of the electrode until it comes into contact with the plate being welded then pinching off, leaving part of the molten metal on the plate and part on the electrode. Mr. Ludwig shows in Fig. 1 that the voltage across the arc dropped to approximately two volts at regular intervals, supposedly when metal was transferred.

FIG. 2

We have been experimenting with electric arcs and our results show that Mr. Ludwig's statements are not necessarily true in all cases.

We have constructed a contact maker which, when connected to a moving picture camera, makes electrical contact each time an exposure is made by the movie camera. This contact maker was connected in a circuit with one of the oscillograph vibrators thus recording the moving picture exposures on the oscillograph film. Using the above apparatus we have been able to see just what is happening in the arc at each instant on the oscillogram.

In Fig. 2 the moving picture film has been printed above the oscillograph film. The upper curve, No. 1, is the curve made by the contact maker. Curve 2 is the voltage across the arc, curve 3 is the current, and curve 4 is a 60-cycle timing wave. The moving pictures clearly show that a globule of metal was transferred at exposure No. 21 but there is no change in the value of voltage as shown by the voltage curve.

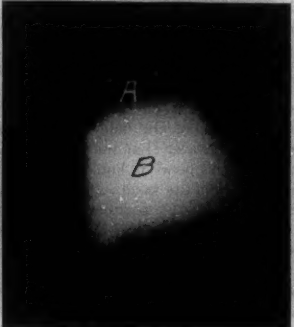

FIG. 3

Fifteen pictures such as the one shown in Fig. 2 were taken and they all gave the same characteristics.

Figs. 3, 4, and 5 are enlargements of exposures 20, 21, and 22 respectively. Fig. 3 shows the arc just before the globule passed. The globule is suspended from the upper electrode as shown at *A* and the arc is shown by the bright spot *B*.

In Fig. 4 the globule can be seen just as it starts across the arc. The globule shows up as a dark gray space at *A*. The arc in this print has become quite short.

Fig. 5 shows the arc after the globule has passed. The arc now is much longer than the arc shown in Fig. 3 due to the globule of iron passing across to the larger electrode below. It can be noticed that the arc length in Fig. 5 is just about the same as the length of the arc plus the length of the globule in Fig. 4.

These pictures show that metal can be transferred across the arc by means other than contact between the two electrodes. They also show that the transfer of metal is not necessarily accompanied by a decrease in the voltage across the arc. The voltage and current both remain approximately the same.

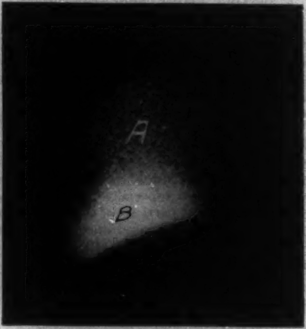

Fig. 4

G. E. Doan: So little is known about the welding arc that any data resulting from a carefully planned experiment, such as the present one, are sure to be important. For some years I have inclined toward the opinion that while the force called gravitation is the principal one in transporting arc metal in horizontal welding, that the force of first importance in overhead welding is the explosion of gases dissolved in the hot metal. If Mr. Creedy's electrodes contained gases from the beginning, I think their explosion or sudden expansion as they are heated by the arc might be responsible for at least a part of the back pressure which he has measured. These results would hold, therefore, only for electrodes of this certain gas content. The transfer by action of the "pinch effect" clearly explained by Mr. Creedy seems warranted except for one observation where it should operate but does not do so. Messrs. S. L. Hoyt and Gerald Brophy of the General Electric Laboratories observed several years ago that metal cannot be transferred upward, as in overhead welding, if the electrode is pure gas free iron, even in the hands of an expert operator. The metal merely melts and runs down the electrode somewhat as wax from a dripping candle. If the electrode contains gases, overhead welding is possible. For these reasons I consider the gas content of the electrode metal as important in determining the back pressure at the electrode and of greater moment than the pinch effect in overhead welding.

J. Leland Myer: *Pressure not necessarily due alone to particle emission from the electrodes.* The author has stated that the most reasonable hypothesis as to the origin of the force (on the electrodes) ascribes it to particles projected from the electrodes, particularly the cathode. The assumption of this same hypothesis led Tanberg[1] to the erroneous conclusion that the temperature of the cathode of a copper arc was 500,000° K. Compton[2] has shown that it is unnecessary to assign this high temperature to the cathode, as the force on the cathode can be easily accounted for by the "accommodation coefficient" of the incoming positive ions.

In addition to the original work of Duffield[3,4] referred to by the author, there is a series of interesting papers by Ratner,[5] Tyndall,[6] and Duffield,[7] wherein the exact origin of the particles and the nature of their forces are discussed. The conclusions are by no means definite. More recent work by Schmick and Seeliger[8] on the transport of material in the iron arc shows that the loss of iron at the cathode is not a linear function of time and current. This is in disagreement with the energy balance theories of Compton, Gunther-Schulze and Seeliger. It is evident that our knowledge in this general field is very meagre, and reliable data are very much needed. At any rate, it does not seem justifiable to assume that all the pressure on the electrode is caused by particle emission.

Polarity of the moving electrode not given. The author does not state whether the forces were measured on the anode or cathode of the iron arc. In view of the widely different forces measured by Beer and Tyndall[9] for the anode and cathode of a carbon arc, it would seem desirable to mention the polarity of the moving electrode in the present investigation.

Pinch effect not of sufficient magnitude to account for forces measured. The pinch effect has been applied to the arc as early as 1923 by Barlow.[10] Using Duffield's data, he found that the pinch effect accounts for less than 30 per cent of the observed force. Likewise an examination of Fig. 8 in Mr. Creedy's paper shows that the pinch effect contributes only a fraction of the forces actually measured.

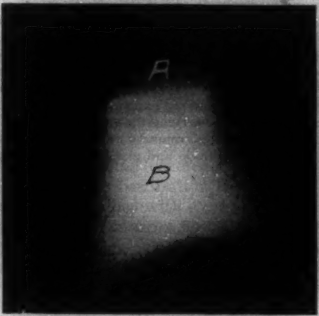

Fig. 5

For example, the pinch effect for a 100-ampere arc should exert a pressure of 50 dynes on each electrode, while the actual measurements indicate forces of 100 to 300 dynes. For 150 amperes the force of the pinch effect should be 112 dynes, while the measured forces lie between 440 and 800 dynes. The author's calculations show that the pinch effect of a 300-ampere arc gives just enough force on the average globule to counteract the action of gravity—allowing none to accelerate the globule upward. The discrepancy cited above is outstanding, and strongly suggests that there must be an additional force other than the pinch effect in overhead welding.

Gas content of the electrodes possibly an important factor. It might prove useful to know something of the quality of metal in the electrodes, particularly as concerns their gas content. The force on the electrodes might be expected to vary widely with different qualities of iron. For example, with a carbon arc, Sellerio[11] (using a torsion method also) noted that the carbon quality had a great deal more influence on the electrode repulsion forces than the arc length itself. Likewise Beer and Tyndall[9] made tests on the gas content of their carbons. On plotting time—pressure curves (maintaining constant arc length and current) they found actual pressure variations of 100:1 over a period of 12 minutes. After 12 minutes heating, the pressure reached a constant minimum value, and consistent data were then obtainable. Their final tests were carried out with degasified carbons which showed no pressure variations with time.

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[1] See References.

If gas be present in the metal electrodes, the intense heat of the arc will certainly cause gas explosions (*i. e.*, very rapid expansion of included gas upon sudden heating). It is believed that such gas explosions afford a reasonable explanation for—

1. The unusually sudden detachment of the globule, as shown in Fig. 1 and 2.
2. The sidewise spattering of metal from the molten tip of the metal electrode.
3. The excess force which must be added to the pinch effect to account for the actual forces observed.
4. The random distribution of points on the force—arc length curve of Fig. 6, as well as on similar curves of other observers.

An obvious test suggests itself, namely the repetition of these experiments with degasified iron.

F. Creedy: The discussion shows that the paper must have been badly phrased in one respect at least since not only Mr. Slepian but Mr. Myer have fallen into a serious misapprehension as to what was meant by it. It was not intended to maintain that the "pinch effect" had anything whatever to do with the passage of the globule from the electrode to the work. According to our view the function of the pinch effect is solely to separate the globule from the electrode and if there were no other forces in existence when the globule had been so separated it would fall to the ground under the action of gravitation. Our view is that the pinch effect separates the globule which is then taken across by the forces measured in the first part of the paper, and these forces, which have nothing whatever to do with the pinch effect and which are measured but not explained in the paper, are those which are responsible for carrying the globule over.

Referring to Mr. Ludwig's remarks we agree that as soon as the connection between electrode and globule is reduced to a small area a great amount of heat would be developed which would assist the explosive effect noted, but we also think that the explosive effect is partly due to the viscosity of the molten part of the electrode. Study of the mechanics of the pinch effect, as for instance described by Dr. Northrup and referred to in the paper, will show that its stability depends on the circulation of the liquid conductor. Where no circulation is possible, the reduction of area once started must go forward with greater and greater rapidity and finally with explosive violence until the conductor is completely severed. Thus the explosiveness is explained even without resorting to high temperatures.

All those who have taken oscillograms of welding must have noted that the arc is frequently short-circuited by a globule in the manner referred to by Mr. Ludwig, but the work done in our laboratories by Mr. E. C. Easton and Mr. F. B. Lucas as mentioned by them in the present discussion, has shown that it is quite possible for the globule to pass without any indication whatever in the oscillograms. Thus not all the globules short-circuit the arc and we have to explain the passage of those that do not as well as of those that do. I am obliged to Mr. Myer for the interesting references which he has given. At the same time I can not help feeling very skeptical as to the explanation of these forces as due to the explosion of occluded gases. Since the discussion took place, tests have been made with especially purified gas free iron in vacuum and forces as high as 4 grams observed. Some of the references that Mr. Myer gives refer to work done with carbon, in some cases especially degasified. It is hardly conceivable in any case that particles of carbon in the solid form should fly off in the manner supposed since the products of combustion of carbon are gaseous. This is still more true, of course, for the degasified carbons.

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11. A. Sellerio, *Phil. Mag.* 44, Oct. 1922.

An Improved A-C. Arc Welder

BY A. M. CANDY[1]

Member, A.I.E.E.

THE use of fusion welding processes has increased tremendously during the past decade having largely superseded the forge and hammer method of welding, due to the greater flexibility and continuity of the fusion processes. The two principal fusion processes are the gas (oxyacetylene) and the electric arc. This paper however will deal only with the arc process.

In lighting service both direct current and alternating current arcs have been used in the past with only detailed advantages being advanced for either because, with the electrodes used, either direct current or alternating current produces a reasonably satisfactory arc. In the welding field however the d-c. arc in the past has been used much more extensively than the a-c. arc, in spite of the fact that for a given rating the cost of the apparatus is less for an a-c. arc.

There are several reasons for this former feeling among which are:

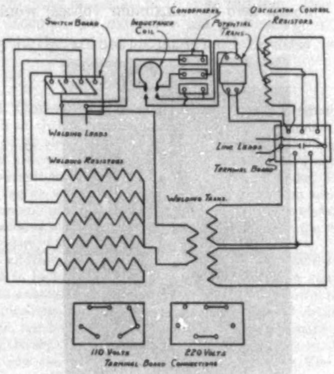

FIG. 1—100-AMPERE IMPROVED A-C. ARC WELDER

1. The open-circuit voltage of an a-c. welder is about double that of a d-c. welder thereby increasing life hazard.
2. An a-c. arc is much more difficult to strike and hold especially with bare welding wire and for vertical and overhead welding.
3. Low power factor.

The improved a-c. welder however eliminates these objections. The open-circuit (secondary) voltage is as low or lower than that of a d-c. machine. The arc is as easy to start and hold in any position under normal conditions as is a d-c. arc. In fact under certain conditions where an interfering blast (termed magnetic blow) will seriously affect the d-c. arc, the troublesome blow will be materially less when the new a-c. welder is used. The power factor at fractional loads is above that of a commercial induction motor used to drive a correspondingly rated d-c. welding generator.

The welding transformer proper shown as a part of the diagram Fig. 1 is perfectly standard in all of its design characteristics and features and is identical with standard commercial distribution transformers in construction. It is provided with a primary which can be connected in parallel or series for 220 volts or 440 volts, and with a secondary which delivers a practically constant potential of 73 volts which is used on the welding system. In this respect from the apparatus standpoint the equipment is very similar to d-c. multiple operator systems where the d-c. generator operates at a constant potential of approximately 60 volts and the current is supplied to the welding circuit through a series of control resistors for stabilizing the arc and for permitting the operator to adjust the current delivered at the arc to the value required for the work at hand. It is therefore apparent that the most interesting and novel portion of this a-c. welding equipment is that of the oscillator circuit.

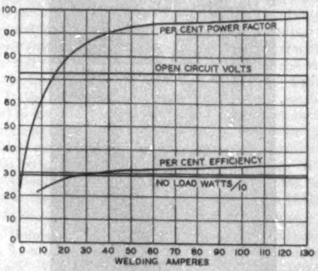

FIG. 2—PERFORMANCE CURVES OF AN IMPROVED A-C. ARC WELDER

100-ampere capacity, 110-220 volts, single phase, 50-60 cycles, 7½ kva.

Most of these and other desirable features are obtained by superimposing high-frequency oscillations on the welding circuit. The general basic idea of such use of an oscillating circuit is not new since it was

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[1] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

(according to records) proposed in 1879. However we do not know of any previous commercial use of the idea in this country.

The scheme of connections for the welding circuits, oscillator circuits, and power input circuit is as shown by Fig. 1. The oscillator circuit comprises a small standard transformer, a condenser across the high side or secondary, two spark gaps, and an air core choke coil. The transformer is connected on the primary side to the power input for the welder, and supplies through its secondary the necessary voltage to charge the condenser and jump the spark gaps. This produces oscillation between the condenser and choke coil of a frequency determined by the relation between the capacity of the condenser and the inductance of the choke coil. This particular equipment is designed so that the frequency is below the radio broadcast band to prevent interference and as an additional assurance a condenser is connected across the power input terminals of the equipment. On the other hand the frequency selected is sufficiently high to preclude the possibility of any injurious physiological effects upon the person using the apparatus.

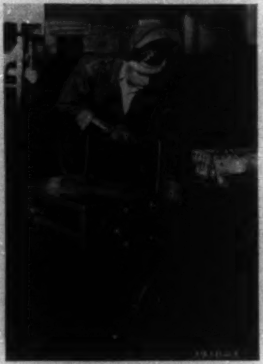

Fig. 3—Improved A-C. Arc Welder

To perform the desired functions it is necessary to have the oscillator circuit discharge between the welding electrode and the work but it is also necessary to prevent the welding circuit from supplying any energy to the oscillator, therefore two small condensers are placed in the circuit between the oscillator and the welding electrodes. These condensers block the low-frequency welding current but allow free passage of the high-frequency discharge through the welding electrode and the work. Similarly a condenser is connected across the input terminals to prevent the oscillator from feeding into the power circuit which might cause radio interference with sets in the immediate territory.

The voltage developed by the oscillator circuit is sufficiently high so that the discharge will actually start just before the electrode contacts the work. As soon as the discharge starts the ionized gases between the electrode and work produce a low resistance path for the welding current so that the arc starts very easily, in fact just as easily as a d-c. arc. Some operators actually prefer this circuit to some d-c. circuits because the arc can be started without actually making contact, thereby eliminating the sticking (freezing) of the electrode to the work.

So long as the welding arc length is maintained, the oscillator continues to discharge between the electrode and the work and eliminates the tendency for the arc to snuff out each time the welding current passes through zero. In previous designs of welding transformers these characteristics were approached by designing for an open-circuit secondary voltage of about 125 volts (compared with 73 volts for the improved transformer) and for high leakage reactance to throw the voltage and current out of phase so that when the current passed through zero the maximum voltage would be available to prevent the current from being interrupted. The old design naturally produces a very low power factor (25 to 35 per cent) compared with approximately 90 per cent for the improved transformer. (Fig. 2.)

Fig. 4—Provision for Handling

The welding operator will not experience any bothersome shock from the welding circuit nor from the oscillator circuit because of the very high frequency of the oscillator circuit producing the well-known "skin-effect," and because of the low capacity of the oscillator circuit.

The mechanical construction of the new welder is both simple and sturdy, the general appearance being as indicated by Fig. 3. The welding transformer, oscillator, and current regulating resistors are assembled in an arc-welded sheet metal case, which is provided with wheels and a suitable handle for conveniently moving the unit as shown in Fig. 4.

The current regulating resistors are connected to five switches and are designed so that currents of 4, 8, 16, 32, 64 amperes, and any combination from 4 to 120 amperes in steps of 4 amperes may be obtained. It is therefore possible to adjust the current value to within 2 amperes of any desired value. The equipment is especially desirable for welding sheet steel and aluminum in the thinner gages below No. 16 in which field the d-c. arc has not been particularly satisfactory up to the present time.

The quality of the welds made with this improved a-c. welder is in every respect equal to d-c. welds. Many tests of such welds have been made and in one series of ten tests using hot rolled steel plate the average tensile strength was 52,000 lb. per sq. inch.

Because of the low no-load losses, low investment charges and low maintenance (no rotating parts) the a-c. equipment is very economical to operate especially when welding materials of 16 gage and less. It is probably destined for extensive use particularly in fields where the d-c. arc has not made much headway.

Discussion

J. B. Gibbs: For a long time inventors and designers have recognized that an a-c. welder would be very desirable, on account of its simplicity, low cost, and absence of moving parts. Several a-c. welders have been built, but they have not found wide acceptance, for the reasons given in the paper. The low power factor might perhaps have been tolerated on account of the economy of the apparatus, but the combination of danger to the operator and difficulty of operation has made the field for a-c. welders of the old type rather small.

The addition of the oscillator removes the difficulties which seemed to be inherent in a-c. welding. It is another example of the combination of well-known ideas to produce a useful result—a combination which having been perfected is of obvious advantage, but however, requiring a breath of vision and a knowledge of the art.

The new a-c. welder is essentially a transformer. The details which are added to secure portability, current control, and stability of arc are of importance but they are so simple that the whole apparatus can be built in a factory designed exclusively for the manufacturing of transformers.

The quality of the welds made with this a-c. welder is fully up to the high standard which is expected of d-c. welds. In the series of tests mentioned in the paper, the hot rolled steel plates were welded together, machined to a standard dimension, and tested to destruction. The final failure occurred sometimes in the parent metal and sometimes in the deposited metal, indicating approximately equal strength.

The interest displayed in this welder since its announcement a few months ago indicated that it meets an existing need and that it is now feasible and economical to apply electric arc welding to many uses for which, heretofore, it has been found unsuitable.

R. E. Hellmund: During the last eight or ten years a great deal of discussion has taken place regarding the relative merits of a-c. and d-c. arc welding and the possibility of using a-c. welding to a greater extent. However, up to this time the a-c. welder has not been commercially exploited to any extent in this country. This is rather surprising in view of the fact that such welders have been commercially available in Europe for almost ten years. Quite a number of them was exhibited in England about 1924 or 1925, and on the Continent both single-phase and three-phase a-c. welding were carried on commercially about that time. Shortly after this, about 1925 or 1926, one of the first a-c. welders with high-frequency "keep alive" was commercially available in France and it gave reasonably good results as far as the weld was concerned. Unfortunately it was possible for the operator to receive severe and disagreeable shocks under certain conditions, a factor which of course would not have made this welder very popular in this country and which carried with it a certain hazard, not so much from the danger of the shock itself as from the fact that the reaction from the unexpected shock might result in other accidents, such as falling, etc.

Possibly it is to be regretted that a-c. arc welders were not available earlier in this country, as they might have hastened a more general introduction of arc welding because of their somewhat lower cost, greater portability, and the absence of a rotating member and consequent reduction in attendance and maintenance. On the other hand, if we consider that arc welding in general in the absence of reliable test methods for the finished weld had to overcome many prejudices, it may be fortunate that the inferior welds likely to be obtained with the earlier types of a-c. welders were not added to the many other difficulties which had to be overcome in the introduction of arc welding. Now that an a-c. welder which is not only equal but in some respects superior to the d-c. welder is available, it will most likely open up new fields and broaden the application of arc welding.

C. J. Holslag: As we were the first ones to manufacture a-c. welders we are naturally interested in the article presented by Mr. Candy. A description of our welders was published by the American Institute of Electrical Engineers, 1919.

The open-circuit voltage of an a-c. machine is not generally double that of a d-c. welder as stated by Mr. Candy. At the present time, the use of a "shielded" arc makes it necessary to have about 70 volts open-circuit voltage in d-c. welding. This is the open-circuit voltage of the a-c. arc welders we are now manufacturing and is the same as the a-c. welders now manufactured in Europe.

Power factor and efficiency with either alternating current or direct current are closely related. In fact, the power factor depends on how much loss there is in the welding circuit. The best conditions for welding, either alternating current or direct current, require some dead resistance. With an a-c. transformer we can weld without resistance—this results in an efficient apparatus but a poor power factor. With 75 volts open circuit and 35 volts across the arc the power factor is around 50 per cent. If resistance is introduced in the circuit, the power factor will be raised. This is also true in the d-c. circuit except that the power factor is better because of the higher loss in the motor and generator. In general, with an a-c. welding transformer we have an efficiency of 80 per cent and the power factor of 50 per cent; whereas, in the d-c. circuit the total efficiency is around 50 per cent and the power factor 80 per cent. In other words, there is about the same current demand on the main line.

Mr. Candy's second point is decidedly controversial. We would also like to have Mr. Candy comment on the safety of his circuit from the operator's standpoint especially for transformers of the larger size.

A. M. Candy: Output capacity of the welding transformer has no bearing on the safety to the operator inasmuch as the voltage of the welding circuit remains the same.

Recent Developments in Design of Arc Welding Generators

BY K. L. HANSEN[1]

Associate, A.I.E.E.

THE rapid expansion of arc welding in recent years has served in an ever increasing degree to focus attention on the design and operating characteristics of arc welding generating equipments. Up until a few years ago the d-c. arc welding circuit invariably constituted a welding system rather than a welding generator. This applies even to the variable voltage individual operator sets as well as to the constant potential multiple operator equipments. Generally speaking, the so-called arc welding generator does not possess all the characteristics required for arc welding, and to compensate for this, external auxiliary devices of one kind or another are introduced. The generator to be discussed in this paper is designed so as to have inherently all the characteristics required for arc welding without the assistance of any external auxiliary apparatus whatever, and is, therefore, in every respect an "arc welding generator."

Resistance Characteristics of Arc Welding Circuit

It is well known that the negative resistance characteristic of the arc makes it inherently unstable when operating from a constant potential circuit. To overcome this instability, the circuit must have a drooping volt-ampere characteristic; that is, the voltage must decrease with increasing current. Individual operator welding generators are, therefore, designed to have a drooping volt-ampere curve.

When there is no actual transfer of material across the arc so that its inherent resistance characteristic is the only cause for instability, the requirements are not very exacting. For example, very little more than a drooping volt-ampere characteristic is required to maintain a carbon arc. However, when there is metal transferred across the arc, as in metallic arc welding, and abrupt changes in the arc resistance are caused thereby, the conditions become much more complex and exacting.

When welding with a mild steel, bare, or lightly flux coated electrode, oscillograms show that the voltage across the arc drops to zero at more or less regular intervals. This drop of voltage takes place whenever a drop of molten metal goes across the arc and thus momentarily short-circuits the arc, at which instant the resistance of the arc is reduced to zero.

The time during which the arc remains short-circuited is exceedingly short, being on the order of one to two hundredths of a second. As compared with the length of time on short circuit, the average interval of time between drops is approximately four to five times the length of time on short circuit. The resistance of the metallic arc, therefore, varies rapidly between the value zero and such a value of resistance as to make the drop across the arc at normal welding circuit equal to the normal arc voltage.

The variations in arc resistance resulting from metal transfer across the arc are much wider and much more rapid than variations in the arc resistance caused by its inherent negative resistance characteristic. Because of the predominating character of these resistance fluctuations, the circuit to which an arc welding generator is delivering energy consists, from the standpoint of the generating equipment, of a resistance which is periodically being short-circuited at a rate of from ten to twenty times per second.

The equivalence of the metallic arc to a circuit, the resistance of which varies continuously and rapidly from a definite value to zero and back to its original value, is of great importance in the mathematical investigation of the instantaneous values of current fluctuation in welding generators. In a mathematical paper,[2] the writer made the following statement:

> It must, of course, be understood that it is not attempted to calculate the current fluctuations in the arc circuit when welding is being performed, but merely the momentary increase or decrease in current that would take place if the resistance in the main circuit is suddenly changed from one value to another; as, for example, from open circuit to short circuit. It is, however, fair to assume that any special features in design of the machine which will reduce the fluctuations when calculated in this manner will also reduce the fluctuations during the welding operation and thus help to stabilize the arc.

It has been argued against this method of treating the problem that the arc does not obey Ohm's law in so far as the voltage across the arc is not proportional to the current, but remains approximately constant over wide range of current, and that the arc should not, therefore, be considered a resistance, but an independent electromotive force. However, this objection to the method used in the writer's earlier paper does not appear to be well taken.

Regardless of whether the arc obeys Ohm's law or not, it is in its nature a resistance inasmuch as it always consumes electrical energy and voltage and is not a source of voltage. Furthermore, the reaction of the machine to abrupt and wide changes in the external resistance is, of course, precisely what it is desired to

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[1] K. L. Hansen Engg. Co., Inc., Milwaukee, Wisconsin.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

[2] *Design of Constant Current Generator for Arc Welding,* TRANS. A.I.E.E., Vol. XXXIX, Part 2, 1920.

know. Indeed, all tests that have so far been suggested for measuring the ability of welding generators to meet properly the exacting conditions of metallic arc welding consist of methods for determining the amount and duration of the momentary increase or decrease in current when the external resistance is suddenly changed from one value to another.

TREND IN DESIGN

The importance of minimizing the momentary current fluctuations, or transients as they are frequently called, is clearly discerned in the most recent modifications made in design of arc welding generating equipments.

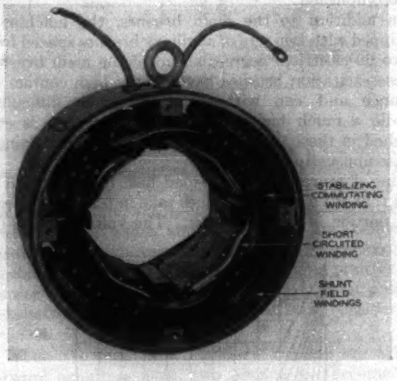

FIG. 1—COMPLETE STATOR OF 200-AMPERE HANSEN ARC WELDER

Publications which have appeared lately on the subject indicate that considerable effort has been directed toward that end. For example, in some makes of apparatus the well-known reactor, which has been used extensively to smooth out the current fluctuations, has been modified to utilize impulses from it to speed up the flux changes in the field. One such modification, known as the "flexactor," has been described by Blankenbuehler[3] and another, the "transformer reactor" by Bergman.[4] These modifications of the reactor have apparently resulted in marked improvement in operation.

In the writer's paper referred to, the desirability of eliminating the external stabilizer was strongly emphasized. Furthermore, it may be of interest to quote from this paper comments made therein on the self-excited machine in use at that time, that is, about a dozen years ago. These comments referred to the well-known system of self-excitation using a two-pole armature in a four-pole field structure and a third brush. The quotation follows:

> Since there are only two sets of main brushes, the commutator must be longer than in the four-pole machine to keep the same current density in the brushes. On the whole the material is not so economically employed as in the four-pole generator with separate excitation.
>
> Furthermore, since the series field turns generally have to be changed for different current values, either by shunting or by taps, the current adjustment is not comparable to the fine adjustment that can easily be obtained over a wide range in the separately excited machine.
>
> On the whole, the writer feels that the advantage obtained by eliminating the separate excitation is not sufficient to compensate for the disadvantages mentioned. Undoubtedly efforts will be continued in this direction, however, and if a generator can be developed possessing all the desirable features of the present separately excited machine without the need of separate excitation, the advantage of such a machine is obvious.

It probably will be conceded that an ideal welding generator is one in which:

(1) The external reactor has been eliminated, provided it has been accomplished without introducing undue complications.

(2) Current impulses in the main circuit are reflected into the field to quicken flux changes in the field and thereby make the machine more responsive to changes in the external resistance.

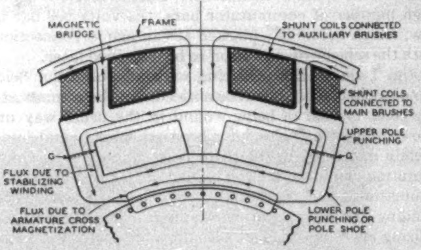
MAGNETIC BRIDGE
FRAME
SHUNT COILS CONNECTED TO AUXILIARY BRUSHES
SHUNT COILS CONNECTED TO MAIN BRUSHES
UPPER POLE PUNCHING
G
G
FLUX DUE TO STABILIZING WINDING
FLUX DUE TO ARMATURE CROSS MAGNETIZATION
LOWER POLE PUNCHING OR POLE SHOE

FIG. 2—CROSS SECTIONAL VIEW OF MAIN POLES AND SHUNT COILS OF 200-AMPERE WELDER

Stabilizing and short-circuit windings omitted

(3) The exciter has been eliminated without resorting to the two-pole armature four-pole field structure.

(4) All regulating rheostats and switches have been eliminated and still a wide range of welding currents can be obtained in fine steps which will have desirable arc characteristics over the whole range.

A generator which, in the writer's opinion, fulfils these requirements is briefly described in the following section.

COMPLETELY SELF-CONTAINED ARC WELDING GENERATOR

The first two of the objects enumerated above have been attained by a novel design of the commutating winding, shunt windings and main poles, and by adding a special winding consisting of one turn short-circuited upon itself.

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[3] *An Improved Arc Welding Generator*, TRANS. A.I.E.E., Vol. 50, June 1931, p. 657.

[4] *A New System for D-C. Arc Welding*, TRANS. A.I.E.E., Vol. 50, June 1931, p. 678.

The armature is similar to a standard d-c. armature, except that it is wound with a relatively large number of turns and correspondingly large number of commutator bars. In arc welding generators, this is desirable for several reasons.

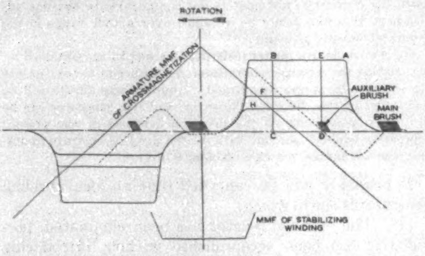
ROTATION
ARMATURE MMF OF CROSSMAGNETIZATION
B
E
A
AUXILIARY BRUSH
MAIN BRUSH
F
H
C
D
MMF OF STABILIZING WINDING

Fig. 3

It is generally recognized that with a high number of turns on the armature a comparatively great change in the terminal voltage requires only a small change in the field flux, which makes the machine more responsive to external resistance variations. Furthermore, with a high number of commutator bars, the volts per bar is low, which becomes a marked advantage in connection with the scheme of excitation to be described later.

Fig. 1 shows the field ring with all poles and field windings in place. It will be noticed that the commutating coils, instead of being wound in the usual way on the commutating poles, have been extended to and completely imbedded in the main pole flanges. The commutating winding thus extended and surrounded by laminations, functions both in capacity of stabilizing winding and commutating winding. The desired stabilizing effect is obtained without adding any windings to those ordinarily employed in commutating pole machines.

The method of assembling the pole flanges around the stabilizing winding can readily be observed from Fig. 1, assisted perhaps by a reference to Fig. 2. The pole, it will be noticed, is made up of two parts, the upper and the lower part or pole shoe. The upper part is bolted to the frame. The stabilizing coil, which has been wound and formed to the proper curvature on a press, is then laid in slots in the upper part of the poles. The pole shoe is then put in place and bolted to the upper part.

From Fig. 2 it will also be noticed that the generator has two shunt coils separated by a magnetic bridge. The function of this arrangement and of the short-circuited winding shown in Fig. 1 will be described later. The pole shoe, it will be seen, serves as a common magnetic path for the stabilizing flux and the flux due to armature cross-magnetization.

It may be pointed out here that the axial length of the back of the pole, that is, the part bolted to the frame, is considerably shorter than the axial length of the part of the pole constituting the magnetic circuit of the stabilizing winding.

Excitation and Current Control

The well-known reentrant volt-ampere curve of the shunt generator is not at all suited for arc welding. It is essential that the permanent short-circuit current be equal to or greater than the welding current. For that reason some source of excitation must be available even on short circuit when the voltage across the main brushes is zero. Reference to Fig. 3 will show how this is accomplished.

In addition to the main brushes, the machine is equipped with two sets of auxiliary brushes spaced from 50 to 60 electrical degrees ahead of the main brushes. These excitation brushes have a very high contact resistance and can without commutation difficulties handle a much higher voltage per bar than is ever reached in these generators, which, as already pointed out, is unusually low.

Line *A* represents the no-load field form. It will be seen that the auxiliary brushes take in part of the negative portion of the field form. The voltage across the auxiliary brushes on open circuit is therefore proportional to the rectangle *BCDE*. What will be the condition on some load, for example, on short circuit? Assume that saturation is negligible on short circuit so that fluxes are proportional to m.m.fs. The voltage on the shunt coil excited from the main brushes falls to zero on short circuit and the m.m.f. of the field is reduced to line *F*. In addition, there is direct demagnetizing force from the armature which reduces the

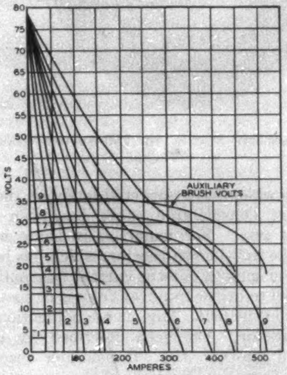

Fig. 4—Volt-Ampere Curves of 300-Ampere Hansen Arc Welder

Nos. 1, 2, 3, etc., indicate different brush positions m.m.f. on the field still further to line *H*. The remaining m.m.fs. are those due to the armature cross-magnetization and the stabilizing winding.

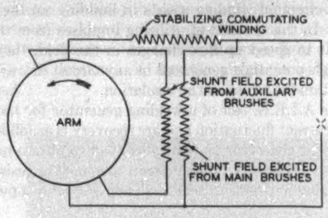
STABILIZING COMMUTATING WINDING
ARM
SHUNT FIELD EXCITED FROM AUXILIARY BRUSHES
SHUNT FIELD EXCITED FROM MAIN BRUSHES

Fig. 5

The distribution of flux in the air gap under the main poles is not directly affected by the m.m.f. of the stabilizing winding. It is indirectly affected by this m.m.f. when the pole shoe becomes saturated, but neglecting that for the moment, the flux distribution is the resultant of the net m.m.f. of the field and the m.m.f. of armature cross-magnetization. This is shown in Fig. 3 as a dotted line. The area included between this line and the base line, and lying above the base line, represents positive flux, and similarly the area below the base line represents negative flux. It will be noticed that on short circuit there is still a considerable positive flux between the auxiliary brushes, whereas the net positive flux between the main brushes has been considerably reduced.

When the main and auxiliary brushes are shifted forward simultaneously, two phenomena become pronounced. The armature demagnetization is increased and the voltage on the auxiliary brushes is decreased. Both of these tend to reduce the current so that a wide variation in current can be obtained with a relatively small shift of the brushes. The brushes are shifted by means of a micrometer screw arrangement, making possible minute graduations of welding current.

Fig. 7—Oscillogram of 300-Ampere Welder When External Resistance is Changed from Zero to Give 25 Volts Drop at 300 Amperes

Fig. 4 shows a set of volt-ampere curves of a 300-ampere welder. It will be noticed that for larger current values the voltage on the auxiliary brushes remains about constant up to about normal current for that setting and then drops off some at short circuit. However, even at short circuit there is plenty left to maintain a sufficiently large value of short-circuit current. For lower current values, the tendency for the auxiliary brush voltage to remain substantially constant down to short circuit becomes more pronounced.

It has been mentioned that when the pole shoe which carries the flux of both the stabilizing and the armature cross-magnetization becomes saturated, the air gap distribution, and consequently the compounding of the auxiliary brushes, are affected thereby. This saturation can, therefore, be modified by introducing air gaps in the path of the stabilizing flux, as, for example, at points marked *G* in Fig. 2.

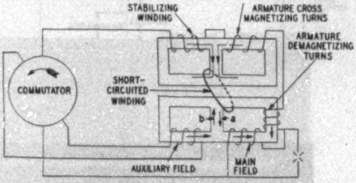

Fig. 6

Momentary Current Fluctuation and Arc Recovery

Fig. 5 is a diagram of the actual circuit of the machine, except that the one turn short-circuited winding is not shown. However, this diagram is of no assistance in forming a mental picture of the interactions between the various circuits during transient conditions, and an equivalent diagram to facilitate this has been shown in Fig. 6. It should be understood that this is not an actual diagram of the circuits, the separation of the armature cross-magnetizing and demagnetizing turns, for example, being for convenience only.

The stabilizing winding and armature cross-magnetizing turns are shown opposing each other with a common magnetic circuit, the pole shoe, between them. The armature demagnetizing turns are shown opposed to the two shunt fields and it should be observed that it is the flux within these turns which is responsible for the voltage at the main brushes. The magnetic circuit, encircled by the auxiliary field winding, is the main pole body, and the magnetic bridge is interposed between the auxiliary field and main field windings. In the absence of saturation there is no mutual induction between the stabilizing winding and the two shunt windings, so they have been shown as having separate magnetic circuits.

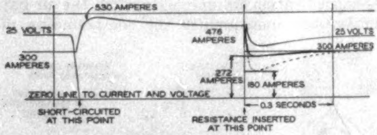

FIG. 8

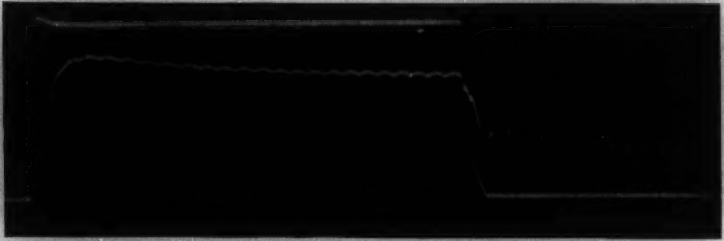

FIG. 9—OSCILLOGRAM OF CURRENT AND VOLTAGE OF 300-AMPERE WELDER IN PASSING FROM OPEN CIRCUIT TO SHORT CIRCUIT AND BACK TO OPEN CIRCUIT

The two shunt coils are excited in the same direction, and unless the main pole body is saturated, little flux passes through the magnetic bridge on open circuit. What flux there is passes through in the direction indicated by arrow *a*. On short circuit the m.m.f. of the main field winding disappears and an opposing m.m.f. of armature demagnetization appears. Now a considerable amount of flux passes across the bridge in the direction of arrow *b*; that is, instead of reducing the flux in the main pole body, which flux interlinks with the auxiliary field winding, most of it is on short circuit diverted through the bridge. The result is that the energy stored in the auxiliary field changes but very little as the terminal voltage varies up and down. The advantage of eliminating the necessity of changing the energy stored in this field when the external resistance varies is obvious.

To make the machine still faster in response to variations in the external resistance, the short-circuited winding is made to form a link between the stabilizing winding and the field windings. When the current in the stabilizing winding increases rapidly, a current will flow in the short-circuited winding, assisting in reducing the field flux. When the current in the stabilizing winding is decreasing rapidly, the transient current in the short-circuited winding assists in building up the main flux. In this method of utilizing impulses from the arc circuit to speed up flux changes in the field, there are no high potentials generated in any circuit at any time to put undue strain on the insulation.

The A.I.E.E. test of a welding generator for momentary current fluctuation and arc recovery is as follows:

Adjust generator on resistance load to obtain normal current at 25 volts. Short circuit one-half the resistance and allow current to settle. Then suddenly open the short circuit and reinsert the resistance in circuit. The current momentarily drops below normal and the time of recovery is the time required for the current to return to within 5 per cent of its original value.

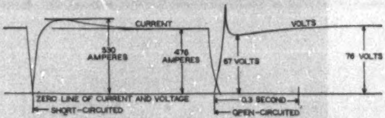

FIG. 10—OSCILLOGRAPH RECORD OF CURRENT AND VOLTAGE OF 300-AMPERE WELDER IN PASSING FROM OPEN CIRCUIT TO SHORT CIRCUIT AND BACK TO OPEN CIRCUIT

This test is of little value because the resistance variation is not wide enough to cause any noticeable depression of current below normal upon reinserting the resistance in the circuit; that is, if the machine is worth being termed an arc welding generator. The Navy specification calls for the same test for arc recovery with the exception that all of the resistance must be short-circuited and then reinserted. This test means something.

An oscillograph record of a test made according to Navy specifications at normal load on a 300-ampere welder is shown in Fig. 7. The timing is a standard 60-cycle wave. As the current and voltage scales are in opposite directions on the oscillogram, it has for convenience been replotted to scale in Fig. 8. The dotted line indicates the drop in current and the time of recovery permitted by the Navy specifications. It will be noticed that whereas the Navy permits a drop of current to 180 amperes, it actually drops to only 272 amperes.

Fig. 9 shows the record of current and voltage of the same machine in passing from open circuit to short circuit and back to open circuit. As in the preceding case, the voltage and current scales are in opposite directions, and for that reason voltage and current have been replotted to scale in Fig. 10.

Fig. 11 is a record of welding current and arc voltage when welding with a bare, mild steel electrode; average welding current, 200 amperes; average arc volts, 20.

Discussion

L. R. Ludwig: Mr. Hansen makes the point in his paper that the chief cause of the welding arc instability is the frequent short-circuiting of the arc by molten drops. Instability apparently is taken as meaning that the arc will extinguish, and must be manually restruck. The paper states in particular that when there is no transfer of material across the arc its inherent resistance characteristic is the only cause of instability and further, the carbon arc is cited as being very stable because there is no transfer of material. This stability of the carbon arc is more probably due to another reason and that is the thermionic nature of its cathode, which has been carefully discussed by K. T. Compton. The cathode of the usual welding arc is probably not thermionic as evidenced by its rapid motions.

The paper also states that variations in arc resistance resulting from metal transfer, are wider and more rapid than variations in arc resistance caused by its inherent negative resistance characteristic. It is difficult to see why the short-circuiting of the arc by drops will cause it to be unstable since the arc certainly restrikes with ease when the drop breaks away from the electrode. The only requirement placed on the generator by this bridging action is that its current must not rise to a high value during the short circuit. On the other hand the welding arc is subject to many rapid variations of voltage and current which the generator is called on to follow if the arc is to be stable.

One of the principal causes of instability may be variation in gas dissociation about the arc as suggested by Alexander in 1928. At any rate, there does not seem to be sufficient evidence just now for believing that short-circuiting of the arc by drops is the chief cause of instability.

The generator described by Mr. Hansen is a clever design which certainly combines into one machine all of the major requirements of a d-c. welding apparatus. The question of the desirability of such a combination is an economic one; the use of two associated pieces of apparatus would certainly have no disadvantage from a technical standpoint, but such design may have advantage from a standpoint of cost and simplicity.

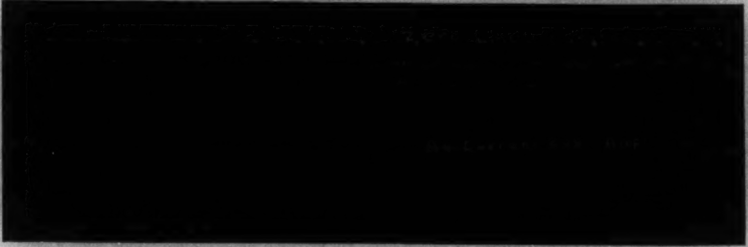

Fig. 11

The characteristic curves of Mr. Hansen's generator shown in his Fig. 4, can hardly be accepted as ideal. The stability of an arc is increased if the change in terminal voltage is made as great as possible for a given change of current. It is for this reason that an arc is more stable on a high-voltage circuit than on a low-voltage one. In order to avoid high open-circuit voltage the ideal welding generator characteristic would be quite steep in the region corresponding to about 25 volts, and flat in the vicinity of the open-circuit voltage, which is almost the opposite of the curves given by Mr. Hansen. Transient characteristic curves of course are of utmost importance, but they cannot be steeper than the so-called static curves; that is, those taken point by point.

R. E. Hellmund: Mr. Hansen has discussed in his paper an exceedingly interesting arrangement of an arc welding generator, which by skillful design can most likely be made to operate satisfactorily. I doubt, however, that the definition of an "ideal welding generator" as given at the beginning of Mr. Hansen's paper is correct for present-day manufacturing conditions. The latter dictate that in order to obtain the best economic results, all apparatus and arrangements, although they may accomplish special results, should be an assembly of standard parts manufactured in quantity. This is essential both for the sake of minimum manufacturing costs and of general availability of spare parts for nation-wide distribution at a minimum investment. While I can conceive that the solution offered by Mr. Hansen may work out satisfactorily for a small concern, it certainly is not the ideal solution for the large manufacturer. As far as the user of the apparatus is concerned, it is immaterial to him whether the desired results are obtained by a theoretical combination in a single machine or by a well-designed physical combination of standard parts; possibly he would prefer the latter in order to facilitate his renewal part problem.

Conditions as just described are often most discouraging to the engineers originating such arrangements as covered in the paper, but inasmuch as we are dealing with practical engineering questions we cannot overlook the economical factors mentioned in this connection. They are uncompromising and have prevented the commercial exploitation of many otherwise very ingenious arrangements. Of the many hundred schemes which have been proposed for combining the functions of several pieces of apparatus into one, very few have commercially survived; of those which have, the most outstanding in this country are the synchronous rotary converter, which is extremely simple, and the third-brush generator used in automobiles, which is also very simple and does not present great difficulties on account of the low voltage of the commutator and which, furthermore, is manufactured in large quantities for its own specific application. All this merely means that conditions have to be particularly favorable if specialized combination arrangements are to win out over combinations of standard parts.

D. W. Ver Planck: From a theoretical standpoint, at least, the Hansen generator is very interesting.

In all welding generators so far described it is necessary, in order to obtain the desired volt-ampere characteristic, to have a change in the amount of field flux linking the armature, as the terminal conditions vary from open circuit to short circuit or vice versa. The method of accomplishing this in separately excited machines is to cause the total flux to decrease toward zero by cancelling out the exciter m.m.f. with a m.m.f. derived from the welding current. In the self-excited machine described by Mr. Bergman* the flux from one pair of poles remains constant while that from the other set is reversed by means of a series field carrying welding current. Both of these methods require that the net number of flux linkages of the entire field circuit change by an amount equal to a little less than their no-load value. The Hansen generator is unique in that its main field flux is not changed in magnitude, but is diverted from the armature into the magnetic bridges between the coils. This involves little change in the auxiliary field linkages but the main field linkages are decreased to nearly zero. The total net change in flux linkages for both fields is somewhat less than *one-half* of the normal no-load linkages. This is an excellent point in the Hansen machine and indicates that equally as good transient characteristics as those of the Bergman type could be obtained by the use of a small corrective device.

The oscillogram of Fig. 9 when compared with the one given by Mr. Bergman seems to indicate that the two machines have about the same transient performance. However, closer examination shows that the short-circuit on the Hansen machine was interrupted quite slowly, six times more slowly than on the Bergman machine. Now it can be shown by analysis, and experience bears this out, that "instantaneous recovery voltage," or the minimum of the recovery voltage, is higher the more slowly the short circuit is interrupted. This emphasizes the need for making tests of this kind on a standardized basis so that a comparison of results will not tend to be misleading.

Referring to the volt-ampere characteristics of Fig. 4, the question arises as to whether or not the nearly perpendicular approach of the curves to the zero axis is desirable. It would seem that this is approaching dangerously near to an unstable condition.

The use of a third brush as it is done in the Hansen generator appears to be a somewhat questionable practise because this brush must commutate coils which are well under the poles where the generated voltages are high. The use of a high resistance brush may limit the short-circuit current in these coils, but even so one would suspect that considerable sparking with the consequent commutator blackening would result when the machine began to get dusty. It should be noted that the use of the excitation brush in the Bergman generator is quite different from this as it commutates coils which are in the neutral zone between the cross and main poles.

K. L. Hansen: Mr. Ludwig is undoubtedly correct in his statement that the stability of the carbon arc is conditioned by the thermionic nature of its cathode. The point which I desired to bring out was that if very wide and abrupt variations in arc resistance are superimposed on the resistance variations due to the inherent instability, the conditions, from the standpoint of the generating equipment, become much more severe and exacting than if the inherent instability alone has to be taken care of.

For example, let us assume that a carbon arc is so arranged that its negative electrode periodically touches the positive at a rate of 15 to 20 times per second, separating the electrodes to a distance of about one inch between strikes, and each time, at the moment of striking, a portion of the negative electrode is left deposited on the positive. Under these conditions, the resistance of the arc will vary between wide limits, and the conditions for thermionic emission from the cathode will change with each stroke. Also assume, as frequently happens in metallic arc welding, that there are forces present, such as magnetic fields, tending to disrupt the arc. The reestablishment of the arc after each extinction on short circuit would then probably not be so readily accomplished as Mr. Ludwig seems to think. At any rate, the demand upon the welding generator to maintain a uniform current under these conditions is without the slighest doubt much more severe than if it simply had to maintain a carbon arc with approximately constant distance between the points of the electrodes.

However, the only purpose for bringing this discussion into my paper was to justify my contention that the mathematical analysis of the arc welding generator can best be approached by investigating its reaction to wide and abrupt changes in the resistance of the external circuit. Mr. Ludwig, I believe, agrees with me on this point.

Before introducing the latest innovation, the magnetic bridge, in the Hansen arc welder, it had a volt-ampere characteristic substantially like the one described by Mr. Ludwig as being the ideal one. The introduction of the bridge, modifying the volt-ampere characteristic to the one shown in the paper, immeasurably improved the machine for welding. Whether or not the modification in the volt-ampere curve had anything to do with this improvement in operation, or whether it was entirely due to improvement in the transient conditions, such as stability and arc recovery, is problematical.

Mr. Hellmund states that the arrangement discussed in the paper "*can* most *likely* be made to operate satisfactorily." This and other statements in his discussion lead me to believe that he labors under the impression that the Hansen arc welder is just being introduced as an untried novelty. He may be interested to learn that the Hansen arc welder, in substantially its present form, has been on the market since 1927.

The number of machines now in operation is well on the way to the second thousand, one customer having had for the past three years 327 machines in operation in factory and field. Mr. Hellmund may have been influenced by the title of the paper. The more recent developments which the title has specific reference to are the short-circuited winding and the magnetic bridge, the latter being the latest innovation. However, as the distinctive features of excitation and stabilization had not been previously described in any technical publication, the machine was discussed as a whole.

It may be remarked that the introduction of the bridge has very markedly improved the welding characteristics of the generator, entirely overcoming occasional criticism in the operation of the earlier models. There is, therefore, every reason to believe that the machine will be even more favorably received in the future than it has been in the past. I wish to emphasize this because the justification for developing a distinct generator to

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**A New System for D-C. Arc Welding*, by S. R. Bergman, A.I.E.E. Trans., June 1931, p. 678.

meet a specific application naturally depends on the number of units that eventually go into operation. When this number is large enough the machine obviously becomes a "standard" machine for that application, and the parts are regularly carried in stock.

In developing the Hansen arc welder, every effort was exerted to reduce the number of parts to a minimum. The reduction of the number of electric circuits or windings to less than one-half the number required when the usual auxiliary devices, such as stabilizer and exciter, are used, is an indication of how well that effort has succeeded. Furthermore, attention was concentrated on the elimination of those parts which experience has shown to be most vulnerable in operation.

The result has been to reduce the demand for spare parts, aside from brushes, and on rare occasions bearings, both of which are standard, to an astonishingly small amount. This was considered a much more satisfactory solution of the renewal problem from the user's standpoint than furnishing an excess number of standard parts which can readily be supplied from convenient distributing points, although the latter may have some attractive features from the manufacturer's standpoint.

As compared with another standard make of welding equipment, the elimination of unnecessary parts by correct design of the generator itself has made possible a reduction in weight from 35 to 47 per cent. Or, putting it the other way, the excess baggage required for the external auxiliary devices ranges from 54 to 90 per cent. Conservatively estimated, the average amount of finished material saved per machine is 500 pounds. The amount saved on the machines in operation so far is, therefore, moving towards the million pound mark.

If a large manufacturer, because of rigid adherence to standards, lacks flexibility to take advantage of a scheme which effects such savings while a small concern can do it, the reflection appears to be on the large manufacturer rather than on the scheme.

Mr. Hellmund is obviously under the impression that the experience of developing and assisting in putting on the market the generator in question must have been a discouraging one to the writer as the originator of the arrangements covered in the paper. I am sure that he will be pleased to know that quite the opposite has been the case. The experience has been a most interesting one and well worth while. The fact that economic forces are uncompromising is a help rather than a hindrance to those who wish to exploit new schemes of real merit. They are indeed the forces which assist the inventor in overcoming the inherent tendency in human beings to oppose change, or what might be termed human inertia. However, to succeed it is not sufficient that the proposed scheme be ingenious. It must have merit, that is, have a distinct advantage over the schemes already in use, and that advantage must in some form or other be an economic one.

Mr. Ver Planck evidently sees the full significance of the magnetic bridge between field coils, and I wish to express my appreciation of his comments on this point. The rather slow interruption of current on oscillogram, Fig. 9, at the instant of opening the short circuit, is evidently caused by the tenacity of the arc which is formed to hang on when the circuit is broken. On the earlier machines, with slower arc recovery, this phenomenon was not observed, and the voltage would have an instantaneous sharp rise due to the inductance of the circuit. Increased tenacity of the arc to hang on is obviously the result of improvement in design with reference to arc recovery. I do not believe that the minimum voltage following the first rapid rise would have been any lower if means had been employed to break the arc more rapidly. That is a point, however, which can be definitely settled by some additional tests.

Regarding the auxiliary brushes operating in the field, it was pointed out in the paper that the combination of low voltage between bars and high resistance brush has insured entirely satisfactory operation. In fact, five years of experience with this system of excitation have shown that the commutator will polish just slightly better where the auxiliary brushes are bearing on it in addition to the main brushes.

INDEX OF AUTHORS

A

Atherton, A. L. (High Capacity Rectifier Efficiency Improves by Sectionalizing) 511
Aydelott, John C., (Motors for 3,000-Volt D-C. Multiple Unit Cars) 498

B

Baker, Robert M., (Equalizing Currents in the Armature of a D-C. Machine) 473
Baños, Alfredo, Jr. and Whitehead, J. B., (Predetermination of the A-C. Characteristics of Dielectrics) 392
Bewley, L. V., (Transient Oscillations of Mutually Coupled Windings) 299
Brehob, F. H. and Craton, F. H., (Engineering Features of Three-Power Locomotives) 489
Brown, Ernest W., (Time and Its Measurement) 523

C

Candy, A. M., (An Improved A-C. Arc Welder) 567
Conrad, A. G. and Warner, R. G., (An Induction Motor with Paralleled Rotor and Stator) 418
Craton, F. H. and Brehob, F. H., (Engineering Features of Three-Power Locomotives) 489
Creedy, F., Lerch, R. O., Seal, P. W., and Sordon, E. P., (Forces of Electric Origin in the Iron Arc) 556
Curtis, L. B., (Preparing Substations for Electric Operation) 518

D

Dryer, W. Poole, (Steam-Driven Auxiliaries for Power Plants) 331

E

Ellis, W. R., Lewis, W. A., and Griscom, S. B., (Generalized Stability Solution for Metropolitan Type Systems) 363

F

Foster, Ronald M., (Geometrical Circuits of Electrical Networks) 309

G

Godsey, Jr., F. W., (Film Characteristics of Electrolytic Condensers) 432
Godsey, Jr., F. W., (Power Losses in Electrolytic Condensers) 439
Griscom, S. B., Lewis, W. A., and Ellis, W. R., (Generalized Stability Solution for Metropolitan Type Systems) 363

H

Hahn, W. C. and Wagner, C. F., (Standard Decrement Curves) 353
Hansen, K. L., (Recent Developments in Design of Arc Welding Generators) 570
Hellmund, R. E. and Ludwig, L. R., (Commutation Considered as a Switching Phenomenon) 465
Hellweg, J. Frederick, (Time Service of the U. S. Naval Observatory) 538
Hentz, R. A. and Jones, J. W., (Stability of Conowingo Hydroelectric Station) 375
Hollister, F. H., (Auxiliary Drive for Steam Power Stations) 392

J

Janson, G. W., (Time Services of the Telegraph Companies) 541
Jones, J. W. and Hentz, R. A., (Stability of Conowingo Hydroelectric Station) 375

K

Kenney, N. D., Luery, A. M., and Moriaty, J. D., (Breakdown of Glass with Alternating Potentials) 404
Kron, L. C. and Simon, A. W., (Electrical Precipitation) 451

L

Lerch, R. O., Creedy, F., Seal, P. W., and Sordon, E. P., (Forces of Electric Origin in the Iron Arc) 556
Lewis, W. A., Griscom, S. B., and Ellis, W. R., (Generalized Stability Solution for Metropolitan Type Systems) 363
Lightband, D. A., (Temperature of Ventilated Railway Motor Armatures) 502
Loomis, Alfred L. and Marrison, W. A., (Modern Developments in Precision Clocks) 527
Ludwig, L. R. and Hellmund, R. E., (Commutation Considered as a Switching Phenomenon) 465
Luery, A. M., Kenney, N. D., and Moriaty, J. D., (Breakdown of Glass with Alternating Potentials) 404

M

Marrison, W. A. and Loomis, Alfred L., (Modern Developments in Precision Clocks) 527
McMillan, F. O., (Radio Interference from Insulator Corona) 385
Moriaty, J. D., Kenney, N. D., and Luery, A. M., (Breakdown of Glass with Alternating Potentials) 404

P

Pollard, E. I., (Calculation of No-Load Damper Winding Loss of Synchronous Machines) 477

S

Scott, K. L., (Magnet Steels and Permanent Magnets) 410
Seal, P. W., Sordon, E. P., Creedy, F., and Lerch, R. O., (Forces of Electric Origin in the Iron Arc) 556
Shildneck, Lloyd P., (Sine-Wave Generators) 484
Shutt, C. C., (Some Considerations in the Design of Damper Windings for Synchronous Motors) 424
Simon, A. W. and Kron, L. C., (Electrical Precipitation) 451
Smith, Lyle W., (Electrically Driven Auxiliaries for Steam Power Stations) 337
Sordon, E. P., Creedy, F., Lerch, R. O., and Seal, P. W., (Forces of Electric Origin in the Iron Arc) 556
Starr, Frank M., (Equivalent Circuits—I) 287
Summers, I. H., (Vector Theory of Circuits Involving Synchronous Machines) 318

W

Wagner, C. F., and Hahn, W. C., (Standard Decrement Curves) 353
Warner, R. G. and Conrad, A. G., (An Induction Motor with Paralleled Rotor and Stator) 418
Warren, H. E., (Synchronous Electric Time Service) 546
Whitehead, J. B. and Baños, Alfredo, Jr., (Predetermination of the A-C. Characteristics of Dielectrics) 392

Surge-Proof Transformers

BY H. V. PUTMAN*

Member, A.I.E.E.

A short report of artificial lightning tests made on a 42,000-kva., 220-kv. shell type transformer was published in the October issue of ELECTRICAL ENGINEERING.[1] A complete report of these tests was published in the *Electric Journal.*[2] In these reports three points of particular interest in connection with these tests are mentioned: (1) the transformers tested were the largest on which lightning tests have been made; (2) the tests made were more severe than previous tests;† and (3) the transformers tested were of a new surge-proof construction involving fundamental improvements in the design of the insulation. An illustration is shown in Fig. 1 of a lightning test during which the transformer bushing was flashed over at approximately one and one-half million volts.

It is the purpose of this paper to describe the novel features of this improved transformer construction.

Two features are of special interest:

1. Without the use of shields to neutralize ground capacity, it has been possible to build shell type transformers in which the distribution of surge voltages is substantially uniform, and which are practically non-oscillating even for the longest surges.

2. A major improvement in the impulse strength of the insulation has been made through the elimination of creepage surfaces from the entire insulation structure.

Appendix I discusses the merits and limitations of uniform voltage distribution. It is pointed out that uniform distribution of voltage within transformer windings has for its object the uniform distribution of the voltage stress throughout the insulation. Even perfect distribution of the surge voltages throughout the windings equalizes the voltage stress primarily on only part of the insulation structure, which is the insulation between turns and coils. There still remain at the ends of the coil columns or groups, high stress concentrations in the major insulation between the winding and ground, which are in no way mitigated by uniform distribution within the winding. Furthermore, in shielded transformers the introduction of the large metal surfaces at line potential in close proximity to the entire high-voltage winding requires the introduction of an additional insulation structure between the shields and winding in which high stress concentrations exist. In the new surge-proof transformers the initial distribution is not only substantially uniform within the winding, but the voltage stress is fairly uniformly distributed throughout the entire insulation structure.

These results have been obtained by simply employing proper design proportions and coil arrangements—a surprisingly simple explanation of such an important achievement. A static plate has been used across the face of the first coil to assist the surge in entering the winding and to eliminate the necessity of padding the line coil. No shields have been employed.

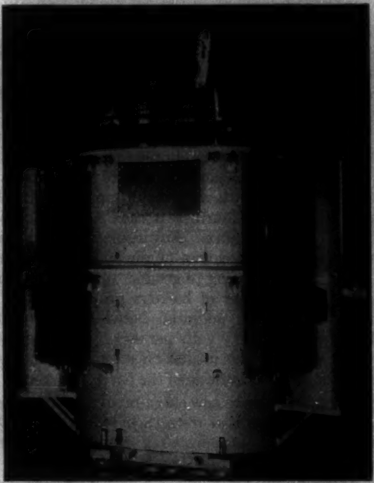

FIG. 1—LIGHTNING TEST ON 42,000-KVA., 220-KV. TRANSFORMER, SHOWING BUSHING FLASHOVER AT APPROXIMATELY 1,500,000 VOLTS AT A TIME LAG OF 2½ MICROSECONDS

Two distinct arcs are seen, because of the metal flange at the middle of the bushing

As pointed out on previous occasions, the initial distribution of surges within individual shell type groups is practically uniform because of the relatively high ratio of coil-to-coil capacity compared to the capacity to ground.[3] It is this inherent characteristic of shell type groups which has been utilized to produce an initial distribution that is substantially uniform, and to eliminate internal oscillations.

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†Both long and short wave surges, representative of lightning surges coming in over the line, were applied, and, in addition, a final surge having a wave front of 11,000,000 volts per microsecond was applied to simulate a direct stroke of lightning to the line close to the apparatus. This final surge was probably much more severe than any likely to be produced by real natural lightning.

[1] For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Appendix II discusses the research work done on transformer insulation which pointed the way to the elimination of creepage surfaces.

It was reported last year by the Transformer Subcommittee that there was at least an approximate relation between the ultimate impulse strength of insulation structures as used in transformers and their low-frequency dielectric strength, and therefore the ordinary low-frequency induced voltage tests could be considered a measure of impulse strength.[4] The results of researches now indicate that this conclusion is questionable; that there is no necessary relation between impulse strength and 60-cycle strength; that the impulse ratio of a complete transformer depends on how much of the low-frequency dielectric strength is obtained by the use of creepage surfaces.

Experiments indicate that creepage surfaces have low impulse ratios compared to puncture barriers, and hence to build an insulation structure with the highest impulse ratio and therefore with the highest ultimate impulse strength requires the complete elimination of creepage surfaces.

It was found possible to apply this principle to practical designs with only very simple changes in the insulation structure. In the new surge-proof transformers the insulation barriers are arranged substantially coincident with the equipotential surfaces of the electrostatic field as explained in the appendix. Any insulation failure either between coils, or lengthwise of the coil group, or from any point in the winding to ground, must result from a puncture of a number of barriers proportional to the normal difference in potential between those points. In these transformers, "volts per inch," the design engineer's measure of the insulation strength of creepage surfaces, has become a thing of the past.

In appearance, these new surge-proof shell type transformers are not different from their predecessors. The casual observer would probably notice no difference between the core and coil structure of the old and the new. Yet the simple changes which have been made have set a new standard of performance in withstanding lightning surges.

The improvements in insulation design which have made this possible are the final achievement of several years of fundamental research. That these important results have been obtained in such simple fashion is a tribute to the skill of those engineers who participated in this development.

Appendix I

Merits and Limitations of Uniform Voltage Distribution

A fundamental problem to which consideration must be given in designing surge-proof transformers is that of reducing the dielectric stress, in the insulation, resulting from surge voltages penetrating the winding.

It is clear that nothing within the transformer winding itself can in any way affect the magnitude of surge voltages to which the high-voltage end of the winding is subjected. Whatever voltage is impressed upon the terminal of the transformer bushing is also impressed on the high-voltage end of the winding. Therefore, it is necessary to insulate between the high-voltage end of the winding and ground for the very maximum surge voltage which may be impressed on the transformer bushing by the most violent lightning surge. This voltage is, of course, limited by the flashover value of the transformer bushing, but this may be very high for surges having extremely steep wave fronts.

However, even if nothing can be done in the design to reduce the surge voltage at the high-voltage end of the winding, it is desirable to have that voltage uniformly distributed throughout the winding in order to equalize the voltage stress on the insulation between the turns and coils.* It is clear that if the surge voltage piled upon the end of the winding more insulation would be required between turns and coils at the end of the winding, and this extra insulation would be unnecessary and ineffective for normal voltage. If, however,

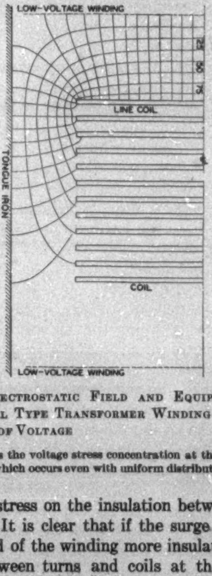

Fig. 2—Electrostatic Field and Equipotential Surfaces in Shell Type Transformer Winding with Uniform Distribution of Voltage

This plot shows the voltage stress concentration at the high-voltage end of the coil group which occurs even with uniform distribution surge voltages are uniformly distributed like the normal voltage, then the insulation which would be placed uniformly between turns and coils for the normal voltage is properly located to withstand the surge voltages also. Therefore, uniform distribution does make it possible to make the most effective use of a given amount of turn-to-turn and coil-to-coil insulation.

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*Engineers have long recognized the merits of uniform voltage distribution and given consideration to methods of obtaining it. The condenser bushing is perhaps the best known example of an insulation structure in which means are provided to give a uniform distribution of voltage stress throughout the insulation. As early as 1912, Fortescue invented and patented (see U.S. patent 1,129,463) means of shielding core type transformers to obtain uniform distribution. The development of the cathode ray oscillograph stimulated investigations of transient phenomena in transformer windings generally, and in the case of core type transformers, led to the work of Palueff in developing means of shielding.[5]

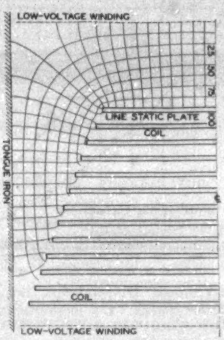

Fig. 3—Electrostatic Field and Equipotential Surfaces in Surge-Proof Shell Type Transformer Winding of the Single Group Type with Uniform Distribution of Voltage

It will be noted that the voltage stress throughout the insulation between the winding and tongue iron has been made substantially uniform and the stress concentration at the corner of the group has been materially reduced

But uniform distribution, even though perfect, does not equalize the voltage stress in any marked degree in the major insulation between the high-voltage winding and the low-voltage winding or ground. At the ends of the coil columns or groups there still remain points of high stress concentration which are practically unaffected by the distribution of voltage within the winding. In Fig. 2 is shown a plot of the electrostatic field with uniform distribution of voltage showing these points of concentration at the end of a shell type group. Similar points of concentration occur in core type transformers at the ends of the coil columns. Figs. 3 and 4 show similar plots for two of the winding arrangements used in the new surge-proof transformers. It will be noted in Figs. 3 and 4 that points of stress concentration have been greatly reduced throughout the entire insulation structure and the insulation is stressed substantially uniformly at all points. This is a very important result. Uniform distribution, desirable as it is from the standpoint of the turn and coil insulation, is only a partial solution to this major problem of equalizing the voltage stress throughout the entire insulation structure. But this major problem has been solved directly in these new surge-proof transformers and by the simple expedient of employing proper design proportions. In solving it, a distribution of surge voltages which is substantially uniform within the winding follows as a secondary result.

There is nothing in this simple method of solution which in any way limits its application to practical transformer designs. Fig. 5 shows a distribution curve plotted from test results on a sample transformer having a single shell type group. Fig. 6 shows a comparison between calculated and tested results on another sample transformer. Fig. 7 shows the same comparison for the 42,000-kva., 220-kv. transformer mentioned in the paper. Fig. 8 shows a calculated initial distribution curve for a 20,000-kva., single-phase, fully insulated transformer to be used for single-phase railway distribution.

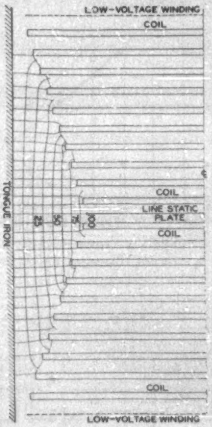

Fig. 4—Electrostatic Field and Equipotential Surfaces in Surge-Proof Shell Type Transformer Winding of the Twin Group Type with Uniform Distribution of Voltage

Since the high-voltage lead enters the middle of the group, voltage stress concentrations at the corners are inherently eliminated, and due to the construction, a uniform stress distribution throughout the entire insulation structure is obtained

No discussion of this subject would be complete without touching on the subject of internal oscillations produced by surges. Engineers are agreed on the following facts pertaining to internal oscillations:

1. Surges cannot produce severe internal oscillations unless the duration of the surges is longer than the natural period of the transformer.

2. Surges cannot produce severe internal oscillations regardless of their duration if they are uniformly distributed throughout the winding.

3. In certain types of transformers where natural periods are such that violent internal oscillations occur due to ordinary long wave lightning surges, such

oscillations may overstress and cause a failure in the major insulation between winding and ground, unless these stresses have been taken into consideration in the design of the insulation.

The shell type transformers with graded insulation previously manufactured by the company, with which the writer is connected, have not been subject to violent internal oscillations because their natural periods have been much longer than the wavelength of surges of any appreciable magnitude produced by lightning. On this account, no failure of major insulation has ever occurred in these transformers due to lightning surges. While the new transformers have much shorter natural periods than their predecessors, they are practically nonoscillating because surges are distributed substantially uniformly throughout the winding. Fig. 7 shows both the initial and maximum surge voltage distributions. In all these tests surges having a steep front and a very long tail (60 microseconds to half value) were used. Test points taken both by sphere gap measurements and by cathode ray oscillograph are shown. These indicate close agreement between calculated and tested results. As would be expected, the results show no appreciable internal oscillations.

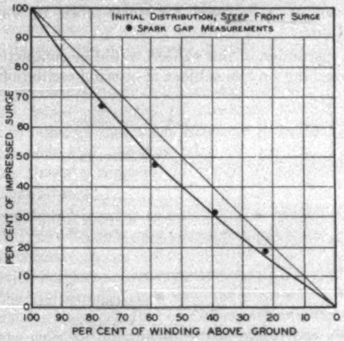

Fig. 5—Initial Surge-Voltage Distribution in a Sample Shell Type Transformer Having One High-Voltage Group

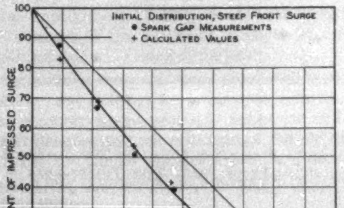

Fig. 6—Initial Surge-Voltage Distribution in a Sample Surge-Proof Shell Type Transformer

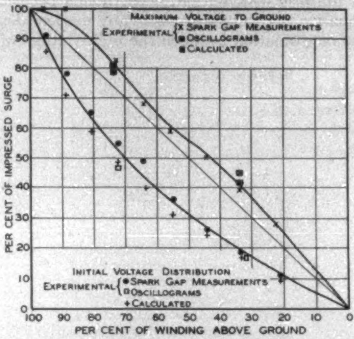

Fig. 7—Surge-Voltage Distribution in a 42,000-Kva., 220-Kv. Shell Type Transformer

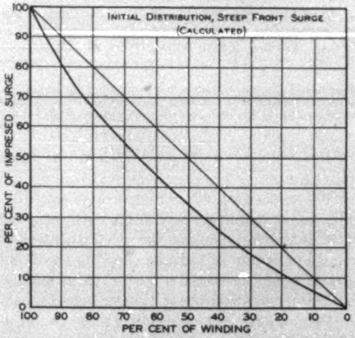

Fig. 8—Calculated Initial Surge-Voltage Distribution in a 20,000-Kva., 132-Kv. Surge-Proof Shell Type Transformer with Full Insulation for Single-Phase Railroad Transmission

Appendix II

Improvements in Transformer Insulation

It is well known that insulator strings, needle gaps and similar structures have a much higher dielectric strength when subjected to impulse voltages than when subjected to 60-cycle voltages. The ratio of the impulse strength to the 60-cycle strength is called the impulse ratio. For a particular structure, the impulse ratio is constant for a given wave, but for very short waves the impulse ratio increases in value. Tests on insulation samples indicate similar characteristics. The report of the Transformer Subcommittee published in 1930 states that different types of transformers having the same low-frequency dielectric strength have about the same impulse strength, and therefore about the same impulse ratio.

A very simple experiment can be performed which might cause one to question this conclusion. In Fig. 9 is shown an electrode separated from a ground plate by an insulation barrier. The whole is immersed in oil and tests made to determine the dielectric strength of the barrier as function of its extension beyond the electrode for both low-frequency voltages and impulse voltages. Fig. 10 shows the results. The low-frequency dielectric strength increases rapidly with the extension, but there is no measurable increase in the impulse strength as the extension is increased. If the impulse ratio is plotted as function of extension, it is seen that the greater the extension the lower the impulse ratio. Here at least is a humble example of a type of insulation structure whose impulse ratio may vary over a wide range.

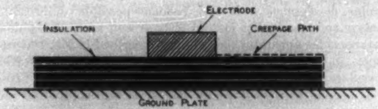

FIG. 9—EXPERIMENT TO DETERMINE CHARACTERISTICS OF CREEPAGE PATH

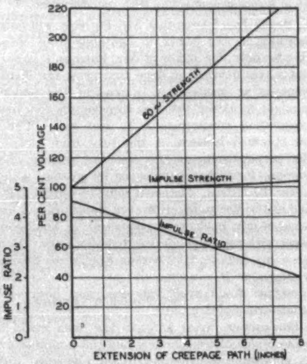

FIG. 10—DIELECTRIC STRENGTH OF A CREEPAGE SURFACE FOR 60-CYCLE AND IMPULSIVE VOLTAGES

This simple experiment seems to indicate that the creepage surface over the barrier gives practically no impulse strength. In Fig. 11 is shown the electrostatic field through the barrier of Fig. 9. The equipotential surfaces cut obliquely through the surface of the barrier. Similar creepage surfaces located in essentially the same manner with respect to the electrostatic field are found in both core or shell type transformers extending lengthwise of the coil groups or columns. See Fig. 2, Appendix I. It is also known from field experience and from tests on sample transformers that failures due to surge voltages very often take place along these surfaces.

Accordingly, an investigation was made to see what effect the elimination of these creepage surfaces from the transformer insulation might have on its impulse strength. Two sample transformers were built with the same insulation clearances. One was insulated in the conventional manner; in the other, creepage surfaces were eliminated. Both were tested to destruction with impulse voltages. The one without creepage surfaces withstood nearly twice as much voltage as the other.

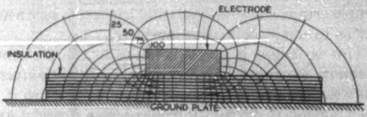

FIG. 11—ELECTROSTATIC FIELD DISTRIBUTION BETWEEN ELECTRODES OF TEST PIECE

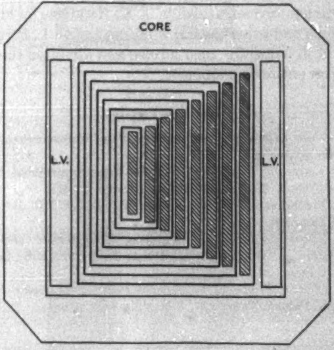

FIG. 12—ARRANGEMENT OF INSULATION BARRIERS IN THE HIGH-VOLTAGE WINDING OF THE SURGE-PROOF TRANSFORMER

These and similar investigations led to the conclusion that the impulse ratio of a complete transformer depends on how much of the low-frequency dielectric strength is obtained from the use of creepage surfaces.

It follows that for the maximum impulse ratio, creepage surfaces should be eliminated entirely from the insulation structure. This means that the insulation barriers should be coincident with the equipotential surfaces of the electrostatic field. While this cannot be accomplished exactly in a practical design, it has been very closely approximated. In fact, any failure in the insulation structure from one coil to another coil or to ground must result in the puncture of a number of barriers proportional to the normal potential difference between those points.

Fig. 12 shows how this is accomplished by the unique arrangement of the insulation barriers between coils and the core. By referring to Fig. 3, which shows the electrostatic field, it will be seen that the surfaces of the barriers lie essentially along the equipotential surfaces.

It will be noted that the same proportions and arrangements of coils which have made it possible to equalize the dielectric stress so well throughout the entire insulation structure have also made possible this simple arrangement of insulation barriers in such a manner as to eliminate creepage surfaces.

This is indeed a simple solution to the problem of building surge-proof transformers. Its effectiveness has been proved by the most drastic tests which it has been possible to devise. It resulted from a careful study of the basic fundamentals of the problem.

In conclusion, the writer desires to acknowledge the helpful assistance of P. L. Bellaschi for his invaluable theoretical investigations, of J. K. Hodnette for his suggestions and experimental work, and of C. L. Fortescue and F. J. Vogel for their advice and direction throughout this entire development.

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Discussion

R. E. Hellmund: While in Europe in the summer of 1925, I was very much surprised to find that it was quite general practise with most manufacturing concerns on the Continent to carry out both experimental and commercial surge tests on insulators and transformers. Apparently such practise had been initiated shortly after the war, and by 1925 had been quite generally adopted. Since this was in marked contrast to American practise at that time, I was naturally interested in learning why such a difference existed. Although it was of course impossible to obtain very definite reasons, answers to my inquiries gave me the impression that the practise of surge testing in Europe had been brought about by a greater number of failures of transformers in service, which in turn most likely should be considered as an aftermath of the war in so far as it had undoubtedly interfered with normal design progress. In other words, the European engineers evidently were forced to the expediency of surge testing at a time when, due to the lack of sufficient data on surges occurring in service and the lack of means for analyzing their laboratory and commercial test data, the entire procedure contained a great many elements of uncertainty. Nevertheless, this may have contributed to the improvement of their designs, which seems to be somewhat borne out by the fact that they have now obtained a great degree of reliability in service.

In contrast to this condition prevailing in Europe, the service record of high-voltage transformers in America always had been very satisfactory and it was therefore quite natural that the American engineers should have hesitated to introduce the practise of surge testing until sufficient data had been accumulated with regard to surges actually occurring in the various systems and until, by the use of the cathode-ray oscillograph and similar developments, it had become possible definitely to analyze and standardize factory tests as well as to study design features of the apparatus such as those described in Mr. Putman's paper. In the absence of definite operating troubles, such orderly procedure was undoubtedly justified until now, when after a great amount of work has been carried on and completed along this line, surge testing not only can be carried on but also can be interpreted as a measure of what might be expected of a piece of apparatus in actual service. The practise now arrived at in this country is, as a result of the more deliberate method of developing it, in marked contrast to the European methods previously mentioned. In nearly all cases, the latter consisted of the discharge of the stored energy in the transformer winding itself, usually at relatively low voltages, while, with the testing methods referred to in Mr. Putman's paper, a surge involving several times the rated voltage, considerable amounts of energy, and very steep wave fronts is impressed upon the transformer winding from a surge generator and recorded by means of the cathode-ray oscillograph. Although the method of testing with surge generators was used in Europe in 1925 in connection with lightning arresters, insulators, and switches, I did not find any instances of its use in connection with transformers.

F. W. Peek, Jr.: I hope that Mr. Putman will continue the research which he has begun on lightning-proof transformers. I will discuss only one phase of Mr. Putman's paper. Fig. 9 illustrates a test devised to show that creepage distance is of little value as an insulation. In reading the paper the impression might be gained that this corresponds to a usual transformer insulation arrangement. It does not. The poor results are obtained because of the very high stress between electrodes perpendicular to the creepage surface. This stress starts corona, for either impulse or 60 cycles, which causes leakage over the surface. The effect could be temporarily stopped by boxing in the electrodes. However, deterioration and failure would soon result. Such an arrangement is analogous to the transformer with very high stresses between coils. Such dangerous stresses are indicated by corona discharges during the induced voltage test if measures are taken to detect them. The skilled designer would avoid this condition because it would mean a transformer of short life.

C. L. Fortescue: Great progress has been made in recent years in the design of transformers and the manufacturers have made an important contribution to the electrical industry in bringing out these surge-proof non-oscillating transformers. The principles used in obtaining these results have been known for a good many years, perhaps the first germ of the idea was incorporated in a transformer constructed at Cornell in 1899 by Professor Harris J. Ryan with the aid of Mr. E. F. Scattergood using what he has called the "sectored guard surface." About 1908 Mr. Reynders conceived the idea of using alternate layers of insulation material and conductors in transformers but, while he had all the elements for obtaining the surge-proof transformer, he failed to coordinate the field distribution of the coils with that of the insulating structure. In March 1913 in a paper before the A.I.E.E., I showed that it was possible to group a system of conductors of different potentials so that they will assist in insulating one another. In the discussion there was a great deal said about the principles of shielding insulating structures, illustrated by the theory of the condenser terminal and methods for obtaining uniform distribution of voltage over insulator assemblies, and so forth. The discussions illustrated rather forcibly the confusion of mind that existed at that time with regard to surges in general, surges being often referred to as high-frequency voltage and sometimes as high-frequency surges and this confusion of mind still exists at the present time as illustrated by the many requests by engineers as to the frequency of the lightning surge and references to artificial surges as having a certain frequency. In the discussion I pointed out that transformers designed on the electrostatic principle expounded in this paper could be made highly resistant to surges. In applying this principle to produce non-resonant transformers the so-called condenser type insulation could be used to insulate the core and case from the windings. In this way the winding space is isolated from the external space and through condenser action between the high-and low-voltage cylinders and plates produces a uniform electric field in the winding space which will therefore have the same potential distribution as the windings themselves as determined by electromagnetic induction. The use of condenser action to obtain desirable field distribution in isolated space is plainly stated in the paper and discussion, and in the text of my patent No. 1,129,463 and reissued patent No. 14,473. The essential requirement to obtain strength against impulses is that the dielectric circuit shall conform at every point with the electromagnetic circuit. This is the principle on which the present surge-proof transformers are based.

In 1915 Weed presented before the A.I.E.E. a paper on surges in transformers and in 1922 he was the author of a paper describing surge-resistant winding structures. More recently important contributions were made by Blume, Boyajian, Palueff, Hodnette, Bellaschi, and others which were directed towards simplifying the structures. Unfortunately there was not much incentive to designing transformers to withstand a lightning surge, the reason, I think, being that lightning was regarded as an act of God and it was assumed that nothing much could be done about it. It was realized that in most cases the lightning stroke was too far away to cause much damage and it was assumed that if it struck the line it was all up and nothing could be done about it. However, various protective devices were used and engineers were satisfied that nothing more could be done to insure the transformer windings against lightning.

The lightning failure at Wallenpaupack had the effect of rousing transformer designers from their false feeling of security and while I feel that danger to transformers has been somewhat exaggerated in some quarters, the jolt was just the stimulant required to start designers thinking about the importance of the dielectric design of transformers. I have always emphasized the importance of the dielectric circuit in transformers but exigencies of manufacture and the lack of knowledge of lightning surges acted as a deterrent in development progress. However, the work of recent years must be highly commended for the reason that the dielectric problem under surge conditions has been solved by recourse to relatively simple expedients which produce transformers presenting no serious difficulties in manufacture. In general, the simpler the insulating structure the better it is from a manufacturing and operating standpoint and in this respect the surge-proof shell type transformer must be acknowledged by everyone to be ideal.

A. B. Hendricks, Jr: The construction described appears to be logical and successful, and good results have been obtained under test.

Of "the three points of particular interest" mentioned, the third states; "transformers tested were of a new surge-proof construction involving fundamental improvements in the design of the insulation."

These "fundamental improvements" are not so very new. Fig. 1 is from my own drawing of a transformer which I designed and built and put in service in 1906, which exemplifies the fundamental principles involved. The winding is of the concentric form with circular coils, but the insulation is varied more or less in proportion with the voltages, and by the use of interleaved cylinders, flanged rings and formed insulating casing presents an absolute barrier between the high-voltage winding, low-voltage winding and core. I well remember my own statement at the time this transformer was built, that the insulation formed "an absolute and complete barrier and little or no dependence was placed on mere surface distance."

Since the middle point of the high-voltage winding is connected to the core, the voltmeter coil is located at the same point.

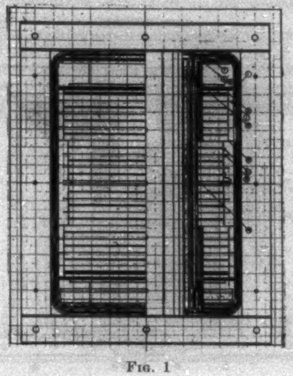

Fig. 1

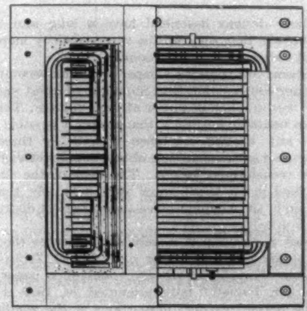

Fig. 2

The two coils at each end of the series are strip-wound, one turn per layer, the insulation between turns and coils being greatly reinforced. These were called "buffer coils," and I think this was the first design to which this term was applied, and it has persisted ever since.

Fig. 2 shows a 300,000-volt transformer of similar design built in 1913, the insulation in this case being in three steps.

Fig. 3 shows a 500-kilowatt, 578,000-volt transformer, being one unit of our original three-phase million volt outfit, as described in the A.I.E.E. JOURNAL October and November 1922. In this design, the insulation of the coil groups is varied in many steps, each group being also heavily taped separately and insulated from the adjoining group by collars. A heavily insulated large radius static plate is placed at the top of the coil stack.

All of these three designs are testing transformers with concentric windings, in which the boxing covers three sides only of the coil group, the fourth side being insulated by the taping and collars.

The principle is identical with the scheme described in Mr. Putman's paper, in that the barriers are placed along the equipotential surfaces. Such an arrangement is common practise in transformer design. Naturally, our later designs show an improvement in arrangement of the insulation over the earlier ones.

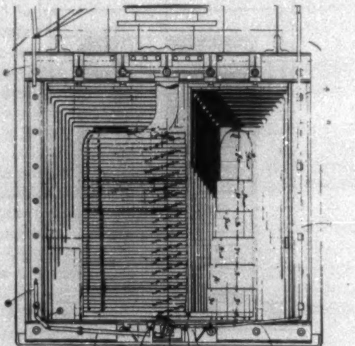

Fig. 3

While these designs described have a long and honorable history, I believe that they will be superseded by more modern and efficient designs of the non-resonating type.

J. F. Peters: Mr. Putman's paper covers an accomplishment in transformer construction that should be of great value in the economical transmission of power at high voltage. This type of construction contains flexibility that makes it possible to obtain practically any desired reactance in two- or three-winding transformers and at the same time obtain impulse strengths never obtained in transformers before. The tests of the 42,000-kva. units described in his paper show very definitely that proper proportioning of the windings gives almost perfect distribution of all impulse voltages.

This is not a surprise; it merely demonstrates theory. In a discussion of Mr. Palueff's paper on *Effect of Transient Voltages on Power Transformer Design*,* and of a paper by Mr. Brand and Mr. Palueff,* both presented in 1929. I pointed out that the proper physical proportions of the windings have a great deal to do with getting a good distribution of impulse voltage along the coil group and in keeping the amplitude of oscillations between the initial and the final distribution relatively small.

Mr. Putman's paper proves the correctness of those statements both as to voltage distribution and practical elimination of internal oscillations. In addition, he shows how to obtain impulse strengths of extraordinary value.

J. E. Clem: Near the end of the first page of this paper there is a statement to the effect that the "initial distribution of surges within individual shell type groups is practically uniform" and that this is an "inherent characteristic of shell type groups." It is rather difficult for me to visualize the basis of these statements since my transformer experience and understanding of transients in transformers would lead me to draw an entirely different conclusion. In support of my belief that the initial distribution in a non-shielded shell type transformer is not uniform, I refer to Mr. Palueff's discussion of J. K. Hodnette's paper, A.I.E.E. Trans., Vol. 49, 1930, No. 1, p. 75, Figs. 4 and 5.

Reference to the coil sketch in Fig. 5 of the above discussion shows that the inside turn of the first coil was connected to the inside turn of the second coil and the outside turn of the second coil to the outside turn of the third coil (adjacent connection) and so on down the stack. It has been claimed that the difference between the voltages along the inside of the coil stock and those along the outside of the coil stack could be removed by changing the connections so that the inside of coil No. 1 was connected to the outside of coil No. 2 (cross connection) and so on down the stack. Although some shell type transformers may have been built in the past with this type of connection, I believe that the practise has been abandoned on account of the hazard. That a hazard will be introduced by this type of connection is substantiated by the following quotation from a discussion by Mr. J. F. Peters, which appears on page 701 of the 1929 Transactions, "but it is then necessary to carry the series connections through the oil ducts between coils where the space is necessarily limited and, therefore, produce somewhat of a hazard."

The initial distribution in a shell type transformer is not uniform even though the coils are cross connected. This is made clear by reference to Fig. 4, which shows the initial distribution over a group of shell type coils connected first one way and then the other. The solid curve shows initial distribution when the coils are adjacent connected. The dotted curve shows the initial distribution when the coils are cross connected. The sawtooth appearance so pronounced when the coils are adjacent connected is not entirely eliminated by the use of the cross connection. The low points when the coils are cross connected occur at a point along the face of the coil nearer the inside than the outside end. Actually, the voltage between the first pair of coils along the outside is higher when the coils are cross connected than when they are connected in the other manner, the figures being 60 per cent and 51 per cent of the applied voltage respectively. It is very apparent from these curves that the initial distribution of a non-shielded shell type transformer can not

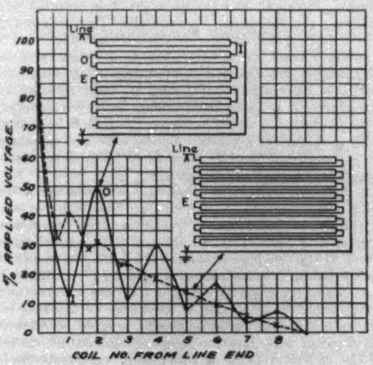

Fig. 4—Initial Voltage Distribution Non-Shielded Shell Type Coils (Clem)

be made uniform by using the cross connection method of connecting the coils.

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*A.I.E.E. Trans., 1929, Vol. 48, Part 3.

The author claims that a short stack of wide shell type coils inherently has a better initial distribution than a long stack of narrow (core type) coils. Comparison of Fig. 5 and the solid curve of Fig. 4 of this discussion indicates otherwise. The coil stack in Fig. 5 was obtained from the coil stack in Fig. 4 by making the coils narrower and the stack correspondingly longer, the same values of winding being used, however, without changing the insulation. Non-modification of the insulation makes the comparison more favorable for the shell type group, since the wider coils would of necessity have a higher voltage between them and thus require more insulation. Comparison of the curve in Fig. 5 with the solid curve in Fig. 4 shows that the distribution for the shell type winding is actually less uniform than that for the equivalent core type winding. It follows, naturally, that the initial distribution in a non-shielded shell type transformer is not inherently uniform.

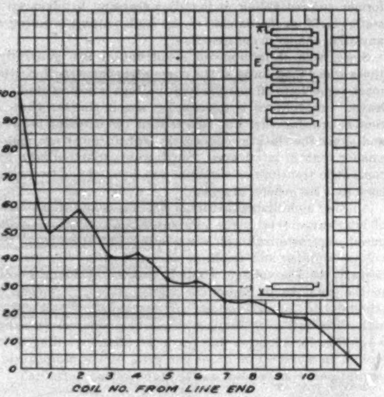

FIG. 5—INITIAL VOLTAGE DISTRIBUTION NON-SHIELDED CORE TYPE COILS (CLEM)

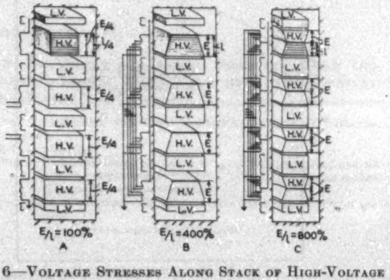

FIG. 6—VOLTAGE STRESSES ALONG STACK OF HIGH-VOLTAGE WINDINGS UNDER ASSUMPTION OF UNIFORM VOLTAGE DISTRIBUTION

A. Conventional design of shell type transformer. (Total voltage is distributed between all groups)

B. New shell type design (full voltage is applied across each group)

C. New shell type design, twin groups. (Full voltage is applied across each half of each group. Maximum voltage per unit length of stack, maximum number of interconnections and leads)

K. K. Paluef: The outstanding features of the core-type transformer as ordinarily built for grounded neutral service, are: first, the winding consists of two multiple paths from the middle of the stack to the ends (see Fig. 4 of Putman's paper); second, the winding is made non-resonating by means of an electrostatic shield connected to the line end and properly spaced from the high-voltage winding.

The first feature was adopted by us for high-voltage transformers about 1919. It eliminates the concentration of dielectric field at the end of the stack shown on Figs. 2 and 3 of the paper and therefore stresses more uniformly the insulation between high- and low-voltage windings.

The second feature was put into practise in 1926. It permits the most effective use of insulation throughout the winding since it allows accurate predetermination of voltage distribution throughout the entire structure. It also reduces the transient voltage stresses very materially by distributing them uniformly and preventing oscillation.

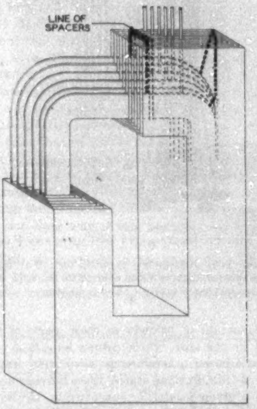

FIG. 7—CREEPAGE PATH IS SHOWN BY HEAVY DOTTED LINES

It is most gratifying to find that these essential features of transformer design gradually are receiving more general recognition, as unqualified support given to them by Mr. Putman indicates.

The practical importance attached by Mr. Putman to these two features is evident from his willingness to adopt a distribution of windings radically different from that previously employed in shell type transformers in spite of the fact that this change resulted in as much as eightfold increase in voltage stress between coils and along the high-voltage stack, and also in an increase in the number of high-voltage leads (line and taps) in the same ratio as can be seen from comparison of sketch *A* and *C* of Fig. 6 of this discussion.

Mr. Putman believes that he succeeded in the introduction of a third major improvement in transformer design "through the elimination of creepage surface from the entire insulation structure."

It is obvious from Fig. 7 of this discussion that creepage path in the new design exists on the top and bottom of the insulation in essentially the same manner as it existed in the old design. It seems to me that such a creepage path is unavoidable because if the windings were "boxed in" on the top and bottom in the manner shown in Fig. 12 of the paper, no oil could possibly circulate between the windings and tank which is an impossible condition for a power transformer.

The comparison of arrangement of windings and of the points of application of voltage in the old and new shell type design (Fig. 6 of this discussion) indicates that the full line-to-ground voltage is crowded in the new design into from 1/3 to 1/8 of the space used for it in the old design. This results in a correspondingly increased voltage between coils and density of dielectric field.

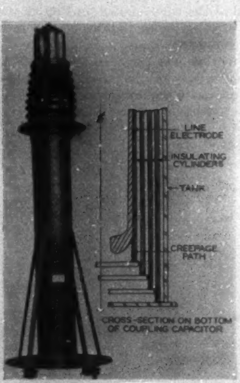

Fig. 8—Reliability of Creepage Path

Oil insulated (0.001 μf.) coupling capacitor for 110-kv. transmission system, tested in 1925 with 225 impulses of artificial lightning with amplitudes of applied waves just below or above bushing arcover

The introduction of barriers in those parts of the structure surrounded by the core, where proper length of creepage path can not be secured, therefore is absolutely imperative. Unfortunately, as Mr. Putman states, these barriers only double the strength of a given insulating space while, as shown here, the stress is increased as many times as there are groups (from two to eight times or so).

In view of all this, the stress along the creepage surfaces at top and bottom in the new design is necessarily appreciably greater than in the old. Since the vertical extension of the insulating barriers in the old design could be made just as long as in the new, it is not clear why the author expressed such a definite preference for the new arrangement of the windings.

Dielectric Strength of Creepage Path. The dielectric strength of a creepage path (whether for 60 cycles or impulse voltage) follows very definite laws, which are well known; where creepage path is used in accordance with these laws it is perfectly dependable.

One of the fundamental principles of design, where creepage path is used, is the proper proportioning of the thickness of barriers or other solid insulation to the length of the creepage path.

Another important characteristic of creepage path is illustrated in Fig. 9. It shows that the arcover voltage for a given creepage distance falls rapidly with decrease in distance between coils (or other electrodes).

This happens because the strength of a creepage path falls suddenly as soon as oil corona is formed on the edges of the coils. On the other hand, the voltage necessary to start corona is the lower the smaller the distance *d*.

Impulse Ratio. It is stated in the paper that the impulse ratio of creepage path is not a constant value, but depends on the length of the creepage path. I fail to see the practical significance of such a statement. The insulation allowances used between turns, between coils and between winding and ground, are selected by the designer without any reference to impulse ratio, but in accordance with the impulse strength determined by test for these various elements.

Non-Resonating Windings. To make a concentric type transformer non-resonating, it is only necessary to place an electrostatic shield on the outside of the high-voltage winding without any other change in construction.

Since coils of interleaved construction are generally many times wider than those of the concentric construction (they may reach some 25 to 30 in.) the high-voltage winding stack in interleaved transformers is necessarily much shorter than in concentric transformers. For this reason the stresses between coils and along the stack in concentric transformers are many times smaller than in interleaved transformers, particularly when the concentric transformer windings are located on two legs of the core, as is the general practise.

Another undesirable feature of the non-resonating transformer of interleaved type is the extremely high lightning voltage induced electrostatically in low-voltage windings from the high-voltage winding and shield as shown in Fig. 10. This happens because the low-voltage winding instead of being placed next to the core, as in concentric transformers, is placed in close proximity to the high-voltage coil and shield. From the approximate distribution of electrostatic field produced by a lightning wave, Fig. 10, it follows that the low-voltage winding rises above ground to some 60 or more per cent of the amplitude of the lightning wave impressed on the high-voltage terminal.

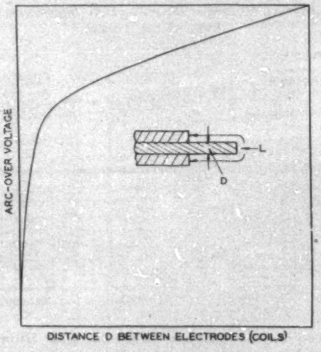

Fig. 9—Reduction of Arcover Voltage of a Creepage Path

With reduction of distance *D* between electrodes when total length *L* of path is kept constant

Incidentally, it is interesting that a single shield placed on the top of the line coil as shown in Fig. 10 can not produce a uniform voltage distribution throughout the winding. This follows from the consideration of the electrostatic field shown, which results from the fact that only a few turns of the high-voltage winding, in the immediate vicinity of the grounded end of the winding, remain practically at the ground potential after the impact of lightning wave, while the rest of the ground coil, as well as all other coils, "float" in the electrostatic field as shown.

Short Stack of Wide Coils. Calculations and tests of electrostatic (or initial) distribution of voltage produced by steep wave front are so simple that there should be no difference of opinion on the subject. In spite of it, the difference evidently exists.

For a number of years I have maintained that without a shield, initial distribution can not be made uniform or even made to approach uniformity by varying the proportions of the width of coils to the length of the winding stack. Furthermore, I know that in a short stack of wide coils without shielding, the initial distribution is extremely non-uniform. This knowledge is based on numerous tests and calculations, some of which were published by me before. Fig. 13 shows the initial distribution in an absurdly short stack of only four wide coils.

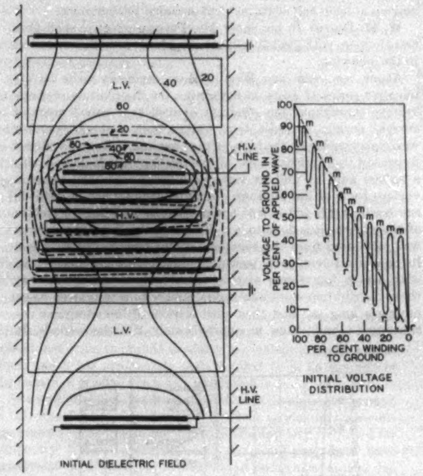

Fig. 10—Character of Initial Dielectric Field and Corresponding Initial Voltage Distribution Produced by Lightning Waves

Interleaved construction with high-voltage winding arranged similar to Fig. 3 of the paper

Solid lines—actual field

Dotted lines—Field necessary to make winding non-resonating and to prevent electrostatic induction of abnormally high potentials in low-voltage windings

l and *r* indicate the left and right edges respectively of the coils

m indicates the middle of the coils

I also previously explained that the increase in the width of the coils which gives, as stated in the paper, "a relatively high coil-to-coil capacity compared to the capacity to ground" can not have the beneficial effect, expected by some, on the initial distribution or on the subsequent oscillations.

It follows that an ordinary winding, not equipped with a shield, can not be made non-resonating by mere selection of proper width of coils and length of stack.

In spite of the simplicity of tests, certain precautions are necessary to secure correct results. In the case of wide coils, for example, it is imperative to measure voltages in different parts of each coil rather than limit these measurements only to turns located on the edges of the coils, as is frequently done. For instance, if on Fig. 10, a line were drawn over points marked *r* and *l*, which indicate the voltages on the outside and inside edges of coils, it would give quite a smooth curve which approaches the uniform distribution line. However, the actual distribution, shown by the solid wavy curve, is extremely non-uniform with the *middle* of each coil at much higher potential than its edges and with as much as 30 to 40 per cent of transformer terminal voltage appearing across half of a coil near the ground end of the stack.

In case measurements are made only on the outside edges, the difference between conditions indicated by such tests and the true conditions can be seen from a comparison of curves 1 and 1*a* of Fig. 13.

With the above in view, I ask Mr. Putman whether the curves shown in his Figs. 5, 6, 7 and 8 are the results of measurements made at the *middle* and at *both edges* of each coil.

Mr. Putman states that he uses a "static plate" but not a "shield." I should appreciate it if Mr. Putman were to give his definition of the word "shield" since, in the light of the above, it is obvious that, in case his tests indicate values measured on both edges and in the middle of each coil, such results could not be obtained without the use of some special means to neutralize, to a marked degree, the effect of ground capacitance.

Could Mr. Putman also tell us how essential is the presence of his "static plate" for securing distribution shown by the above curves? What would the distribution be without "static plates"?

Reduction in Major Insulation Stresses. The author of the paper points out that in the winding of Fig. 3 of his paper the dielectric field concentration at the edge of the line coil is not as high as in the old construction shown in Fig. 2. It is obvious that if the distances between the core and the line coil were made the same in the two constructions, the overall stress and the concentration of field on the winding of Fig. 2 (old design) would be less than that of Fig. 3, while the field concentration on the line coil will not change appreciably. The reduction shown in Fig. 3 is caused by a 30 per cent increase in distance between coil and core rather than by any improvement in construction. Since the voltage from line coil to core in both constructions is identical it is not clear why it was necessary to make such an increase.

Lightning Tests. It is stated in the paper that the lightning tests described therein were the most severe ever performed on assembled power transformers. I question the correctness of this statement. The tests we made about a year and a half ago (and reported at the time in the A.I.E.E. Transactions, January 1930) on one of the 13,000-kva., 230-kv. transformers built for the New England Power Co. consisted of over 200 impulses. Of these, over 40 were equal to or in excess of that necessary to arc over the 64-inch coordinating gap. Many were sufficiently high to arc over the transformer bushing. In Fig. 11 of this discussion are shown copies of the oscillograms of the most important waves used during these tests. Waves shown by dotted lines are taken from Dann's paper referred to by Mr. Putman

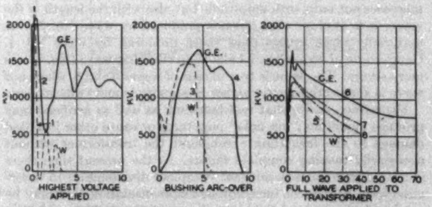

Fig. 11—Lightning Test on Core and Shell Type Transformers (230-Kv. Operating Voltage)

——— G. E. test August 1930 (core type)

— — — W test August 1931 (shell type) waves shown are applied to high-voltage terminal

(*Electric Journal*, Dec. 1931). Wave No. 5 was taken from Mr. Putman's lecture given at Hazleton last fall. Our waves are shown by solid lines. No. 2 was not previously published. No. 4 was published in the *G. E. Review*. A wave practically identical to No. 6 was also published in the same periodical.

Mr. Putman considers that the wave No. 1 was the most severe because it rose at the rate of 11,000,000 volts per microsecond. It would be easier to agree with him if he gave some experimental data showing the effect on insulation of rate of voltage rise. I believe that this impulse was not particularly severe because of its extraordinary shortness. It lasted only two tenths of a microsecond.

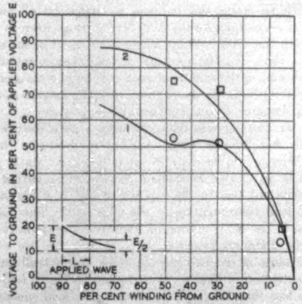

FIG. 12—MAXIMUM VOLTAGE TO GROUND PRODUCED BY WAVES WITH EXPONENTIAL TAILS OF DIFFERENT LENGTH (TEST RESULTS)

Length of wave in per cent of natural period of transformer 1—21 per cent (Palueff's test on core type transformer)
○ —22 per cent (Hodnette's test on shell type transformer)
2 —107 per cent (Palueff's test on core type transformer)
□ —113 per cent (Hodnette's test on shell type transformer)

Since it is recognized that the destructiveness of impulse waves increases not only with amplitude but also with the length of the wave, it is obvious that stresses produced by wave No. 2 were materially more severe than those produced by wave No. 1. Comparison of waves 3 and 4 also shows that our wave (No. 4) was more severe. The same is true in case of waves 5 and 6, where our wave is about 38 per cent higher in amplitude and twice as long.

Experience shows that real lightning as well as artificial may produce numerous "pin-hole" punctures or cause other localized damage to the insulation throughout the transformer without necessarily causing complete failure. At the present time there is no means available, without complete disassembly, to determine local damage of turn, coil or major insulation that may be produced by artificial lightning. I would like to know whether the 42,000-kva. transformers subjected to lightning tests by Mr. Putman were disassembled for minute examination of each coil and insulation parts.

For a number of years we have felt that the standard induced voltage test specified for grounded transformers was not an adequate test alone because the voltage stresses produced by such a test do not correspond either in magnitude or distribution to stresses produced by transient voltages in transformers of ordinary construction. This, in case of a grounded transformer for example, would permit building a transformer of given operating characteristics (and meeting all A.I.E.E. rules and tests) at a cost appreciably less than that of transformers designed to withstand transient voltage stresses. It is obvious that should lightning tests be made a part of A.I.E.E. acceptance test it would help to bring the standards of transformer design to a more uniform level.

For the above reason I heartily endorse Mr. Peek's suggestion made at the Convention, that the proper A.I.E.E. committee should consider the possibility of standardizing artificial lightning tests as a part of the acceptance tests on transformers, supplementary to the present induced voltage test of 3.46 times normal.

Effect of Length of Wave on Amplitude of Oscillations. Mr. Putman states "surges can not produce *severe* internal oscillations unless the duration of the surges is longer than the natural period of the transformer." A more accurate statement of this relation is as follows:

The voltage of any particular harmonic of transformer oscillation will not reach its *crest* value unless the length of the applied wave is at least half of the natural period of the harmonic.

W. M. Dann: In his paper Mr. Putman refers to shell type transformers with graded insulation, and this subject is discussed in the following.

About ten years ago Westinghouse engineers made an exhaustive series of tests to determine the distribution of surge voltage throughout the windings of shell type transformers of current design. In these tests, surges from a surge generator were impressed upon the high-voltage winding, and surge voltages to ground from various parts of the winding were measured with a sphere gap, as well as voltages between coils and between various parts of the windings. In those days we had no cathode ray oscillograph and it was not possible to reproduce a picture of the wave as it is done today. However, the shape of the wave was calculated from the constants of the circuit, and of course its magnitude could be measured directly with a sphere gap with a very fair degree of precision. It was on this basis that these distribution tests were made, first with a very steep wave having a long tail and then with a wave of less steepness decreasing to half value in approximately 20 microseconds. It took almost a year to complete them, and an idea of the thoroughness and care with which they were made may be gained from the fact that all results were checked by three of four different observers in repeated tests.

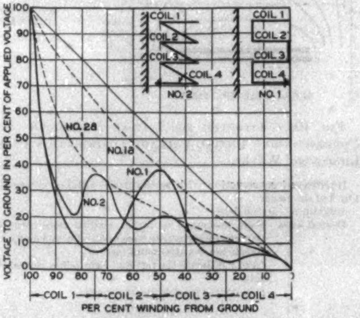

FIG. 13—INITIAL DISTRIBUTION IN A STACK OF FOUR VERY WIDE COILS

1 and 2—Actual distribution (voltages measured throughout the coils)
1*a* and 2*a*—Apparent distribution (voltages are not measured throughout the coils)

These tests showed very clearly that in the shell type transformer there were no dangerous voltages of the kind which one would expect if violent internal oscillations were developed by the entering surge. They furnished direct data on surge voltage stresses between winding and ground for the proper design of graded major insulation.

These actual tests supplemented in an important way the fundamental data on which Westinghouse design practises have been based. At the time they were made they were considered of prime importance; since then, they have become of great interest because of the repeated statements of Mr. Palueff and others to the effect that the practise of grading major insulation is hazardous.

That this practise is far from hazardous, I think, is clearly shown by the following facts:

1. There are more than three million kva. of 220-kv. Westinghouse transformers in successful operation with graded insulation, and the practise of grading the insulation for 220-kv. transformers is practically universal in this country.

2. There are approximately five million kva. of Westinghouse power transformers for 110-kv. and above in successful operation with graded insulation.

3. With all these transformers in service, to the best of our knowledge not a single failure of major insulation to ground due to lightning has been experienced in a Westinghouse shell type transformer having graded insulation; this record could hardly have been attained if dangerous voltages between winding and ground were actually set up by internal oscillations due to lightning surges.

References have been made at various meetings of the Institute by Mr. Palueff to the failure of a 220-kv. transformer at the Leaside station of the Hydro-Electric Commission at Toronto during a lightning storm. This transformer happens to be a Westinghouse shell type unit having graded insulation. These references appear on page 79 of the January 1930 TRANSACTIONS, and again on page 1195 of the July 1930 TRANSACTIONS, and once more on page 69 of the March 1931 TRANSACTIONS. In these references, this failure has been explained as a failure of the graded insulation to ground produced by violent internal oscillations, and it has erroneously been pointed to as an example of the hazard of grading the major insulation.

As a matter of fact, there was no failure of insulation to ground in this transformer. It was purely a failure across the high-voltage group of coils due to the presence of a tap lead, required for temporary 154-kv. operation, and this lead proved to have insufficient surge strength. It was not an inherent weakness due to the principle of grading the insulation, because the grading had nothing whatever to do with the failure.

The new surge-proof transformer, which Mr. Putman describes, represents a distinct forward step in methods of insulating, and in the practise of grading major insulation. Heretofore the insulation of a transformer having graded insulation has been graded by groups; that is, the high-voltage group nearest the line has been given the full amount of major insulation, and the group nearest the grounded end has been given a reduced amount of major insulation to correspond to the voltage stresses developed in the group, and intermediate groups, if any, have had an intermediate amount of insulation. I think Mr. Putman has made it plain in Figs. 3 and 12 and in the text of his paper, that in the new surge-proof transformer the coils themselves have been graded in width and the insulation around the individual coils has been graded in amount. The result is that concentrations of stress have been eliminated and the amount of solid puncture insulation around each coil has been proportioned to match the stresses developed in that coil.

It seems to me that the remarkable surge strength which has been demonstrated by the actual application of surges to these transformers shows the soundness of the principles on which the new design is based.

V. M. Montsinger: I discuss Mr. Putman's paper from the standpoint of 60-cycle and impulse arcover of barriers as affected by creepage. My experience does not check the data given in Fig. 10 showing no increase in the impulse arcover voltage for long collar extensions.

In the first place if the impulse strength shown as a practically horizontal line in Fig. 10 is correct, this shows that creepage strength across the edge of the barrier must be approximately the same as the puncture strength. This follows because the creepage certainly can be made great enough to cause puncture. Since the impulse strength curve is practically horizontal the two strengths; *i. e.*, creepage over edge and puncture, must be approximately equal. This does not seem reasonable.

Numerous tests have shown that for a practical barrier as used in design work puncture occurs at approximately twice the voltage at which arcover occurs when the electrodes are even with the edge. This is true for both 60-cycle and ½/5 microsecond impulse waves. It follows, therefore, that under these limiting conditions the impulse ratio is approximately the same.

Fig. 14 shows the results of tests made on a barrier with various creepage distances.

My experience in breaking down complete transformers shows that approximately the same impulse ratio is found whether failure is entirely by creepage or by a combination of puncture of solid insulation and creepage when using the ½/5 microsecond wave. This refers to cases where oscillations are not involved.

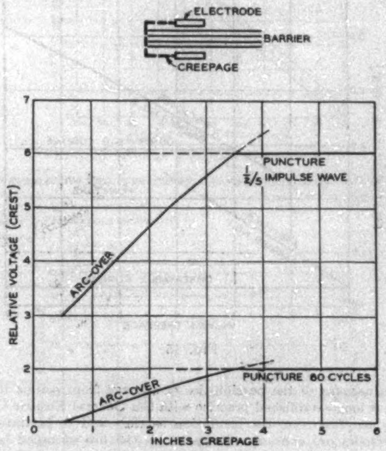

Fig. 14

I feel, therefore, that it would not be advisable to make any radical change in design practise as a result of the data given in Fig. 10 of Mr. Putman's paper.

So far my remarks have been confined entirely to breakdown, no reference being made to the appearance of corona in oil.

Fig. 15 illustrates the very wide range in 60-cycle voltage between the beginning of corona and the breakdown of two barriers of different thicknesses. These data show the importance of properly proportioning the barrier extension.

It is evident that the observable corona depends only on the thickness of the barrier and shape of the terminals; also that it is not of any use so far as suppressing corona is concerned to make the extension very much in excess of twice the thickness of the barrier.

If the barrier extension is many times its thickness and if the voltage is raised above the corona point, heavy local arcing may result in serious damage to the barriers long before arcover or puncture takes place. This is true of either 60-cycle or impulse voltage. For obtaining the data shown in Figs. 14 and 15 the electrodes were symmetrically located on both sides of the barrier.

To see if I could check the constant impulse voltage curve, shown in Fig. 10 of Mr. Putman's paper, tests were made on a barrier in oil as near like this one as possible. Fig. 16 shows that with a fast-chopped wave the impulse voltage increased quite definitely with an increase in extension. For the ½/5 full wave the voltage increased more than with the chopped wave. For a still slower wave the increase in impulse voltage was more marked. Mr. Putman did not define the wave he used. I would not say that the constant voltage *versus* creepage can not be obtained though I feel that a condition that gives this result is not a practical one.

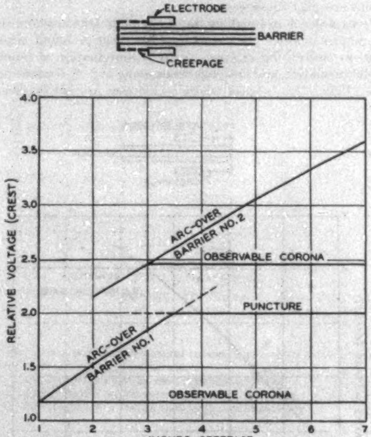

Fig. 15

On account of the possibilities of damage from corona it has been a long established practise with the General Electric Company to proportion the insulation barriers and to insulate the (electrodes or) ends of coil stacks by the use of taped buffer coils so that no corona occurs during any dielectric test. Dangerous corona in the oil usually results in the surface of the oil being disturbed.

In conclusion I want to emphasize two points—(1) that creepage distance if properly used can be depended upon for a certain insulation strength even with impulse voltage. (2) That no matter what tests on barriers show, impulse breakdown tests are finally made on complete transformer windings by testing them to destruction.

Transformers covering the complete voltage range have been tested to destruction in our laboratory by impulse voltage.

J. R. Gaston: Mr. Putman has obtained apparent success in building a surge-proof "pan-cake coil" shell type transformer. Unquestionably the design is successful between limits that will be shown. But to assume that the impulse tests signify, for this type of power transformer, the results in the degree that the paper seems to imply, fails to be impressive because of the omission of the following important data, namely:

1. *Polarity of Impulses*. An eleven-million-volt negative impulse is not necessarily as effective as a five-million-volt positive impulse. Both the paper and bibliography fail to give the polarity of the impulses.

2. *Temperature of Transformer*. The dielectric strength of both oil and fibrous material decreases with a rise in temperature, while their conductivity may increase several thousand fold or more in the operating temperature range of the average transformer. The paper does not give these data, and therefore from engineering point of view we must assume that the surge tests were made cold.

In commenting on Appendix II, it appears that explanations for Figs. 9 to 11 are in error, as follows, namely:

1. Breakdown creepage distance varies logarithmetically with the voltage, and not directly.

2. Fig. 11 would be correct if K for oil and the fibrous insulating materials were the same; the deduction being that unless the corners are heavily reinforced breakdown would result, and after all, that is the purpose of "angle rings" or "flanged collars" as used on core type transformers.

I agree with Mr. Palueff that under the present system of impulse testing the purchaser's property is more apt to be unnecessarily exposed to incipient breakdown, if tested hot, and I do not consider a negative cold surge test of much significance, with the present data available.

A. A. Emlen: In connection with the suggestion that the machinery committee investigate the possibility of incorporating in the Institute Rules a "surge test for transformers," I would offer the following comments:

It is not clear from the data given on surge tests as actually applied to transformers at what temperature these tests were made. Since the temperature of the dielectric, whether oil or solid, has a decided bearing on the voltage a transformer will stand (either 60 cycle or impulse) it would be interesting to learn whether the results obtained were at low temperatures or at 75 deg. cent.

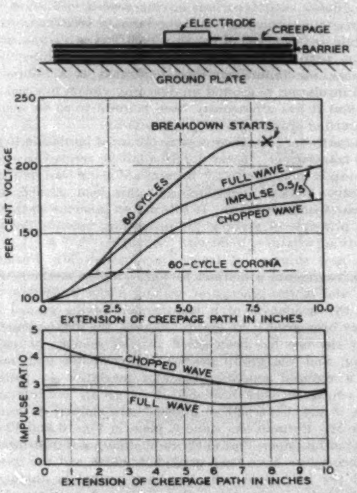

Fig. 16

If studies are to be made regarding surge tests for commercial transformers it seems that the temperature at which these tests are made is a very important point to consider.

Considerable progress seems to have been made in the development of apparatus to determine the existence of incipient corona or breakdown. However, till these detectors are practically foolproof it would not seem advisable to apply surge tests of any magnitude to commercial transformers.

J. Murray Weed: It is most gratifying to me to see the progress which has been made and which still is being made in the development of surge-proof transformers. When the pioneer paper on this subject was discussed by the Institute in 1922,[1] the consensus of opinion was that, although it possessed great theoretical interest, its application was either unnecessary or impracticable. In closing that discussion, I put the best face I could on the situation by saying, "I have gone as far as I have been able to go in pointing the way and I would not be surprised to see the application of the general method here illustrated carried much further than seems to be expected by most of those who have discussed the paper."

The designers of the transformer described in the present paper must be given credit for having made an approach to a uniform gradient for surge voltages in shell type transformers. It is interesting to observe that, however much the arrangement here shown appears to differ from any of those shown in my paper referred to or from the non-resonating transformers which have been built by others, the fundamental principle involved is the same in all cases. As stated in my paper, the method consists in so embedding the winding within the dielectric of a condenser, whose plates are connected to the winding terminals, that the potential imparted to each turn through the dielectric (by condenser action) corresponds to a uniform distribution of voltage from turn to turn. If, in this case, the line static plate and the tongue iron be recognized as the chief plate elements of the condenser and the plot of electrostatic field shown in Figs. 3 and 4 be completed to cover the region occupied by the winding, this will give a true picture of the initial surge voltage distribution. The potential of each turn would be indicated by its position with respect to the equipotential lines.

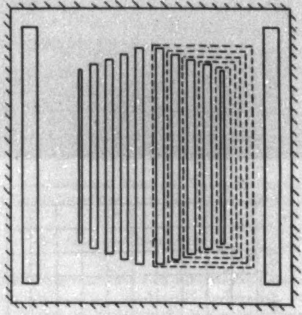

FIG. 17—SINGLE-GROUP SHELL TYPE TRANSFORMER DESIGNED FOR ELIMINATION OF CREEPAGE SURFACES

If the measurements of the surge voltages shown in Figs. 5 to 8 were made on the outside turns of the respective coils, the part of the field external to the coils would need some correction, of course, to correspond with the actual voltage distribution. If the whole field were correctly plotted, I fear that the location of the equipotential lines would reveal voltages on some of the interior turns of the coils which would fall a considerable distance from the curves in Figs. 5 to 8. It is readily seen, however, that the initial surge voltage distribution near line end of the winding has been improved by the introduction of the plate.

F. J. Vogel: The primary importance of Mr. Putman's paper rests in the principle that to obtain the highest impulse voltage strength, it is necessary to eliminate creepage surfaces. This is not a new principle, since in the insulation between the high- and low-voltage windings of all transformers, barriers of insulating material are common practise. What is new is the further application of this principle between coils of a single winding to increase the insulation strength of the winding to the greatest extent.

One of the most interesting features of this development is the broad extent of its application. It is not limited to designs using a single high-voltage group, nor even to designs with grounded neutral. It can be applied to transformers grounded through reactance at the neutral or fully insulated star- or delta-connected transformers. To demonstrate this, the possibility of elimination of creepage between coils of a shell type transformer winding capable of withstanding a high dielectric test to ground is indicated in Fig. 17.

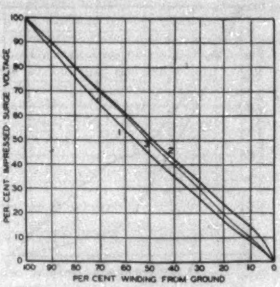

FIG. 18—SURGE VOLTAGE DISTRIBUTION

In single-group shell type transformer surge entering one end—1½—40 wave
Curve 1—Initial distribution
Curve 2—Maximum oscillation

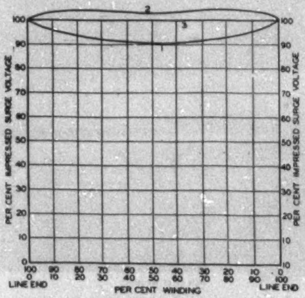

FIG. 19—SURGE VOLTAGE DISTRIBUTION IN SINGLE-GROUP SHELL TYPE TRANSFORMER

Equal surges entering both ends—1½—40 wave
Curve 1—Initial distribution
Curve 2—Maximum oscillation

The extent of the application of such a group as shown in Fig. 17 is very important. For example, the distribution of surge voltages is just the same, whether the surge comes in from either end of the winding due to the symmetry of its arrangement. Fig. 18 shows the initial distribution of voltage and maximum voltage to ground when the surge enters from one end, and Fig. 19 shows both the initial distribution and the maximum voltage to ground when equal surges enter both ends of the winding. Therefore, it is easily possible to apply this type of design to delta-connected transformers and obtain surge proof characteristics.

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[1] A.I.E.E. TRANS., Vol. XLI, p. 151.

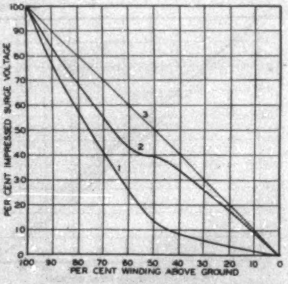

Fig. 20—Surge Voltage Distribution in Two-Group Shell Type Transformer

Surge entering one end—1½—40 wave
Curve 1—Initial distribution
Curve 2—Maximum oscillation

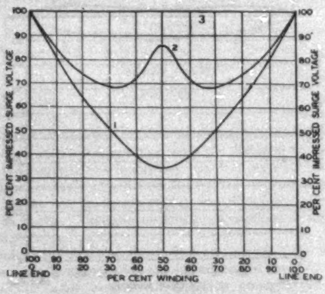

Fig. 21—Surge Voltage Distribution in Two-Group Shell Type Transformer

Equal surges entering both ends—1½—40 wave
Curve 1—Initial distribution
Curve 2—Maximum oscillation

It is but a step from the fully insulated single-group design, to the fully insulated two-group design. As previously brought out in a paper by Mr. J. K. Hodnette, (Trans. A.I.E.E., Jan. 1930, p. 68) if the surge enters from one line, nearly all the stress from the initial distribution will be concentrated in the one group, and will be nearly uniform in this one group. This is shown in Fig. 20. This one group can be made as resistant to insulation failure during the period of initial voltage distribution as the one group design previously described. The voltages to ground resulting from a long wave are also shown in Fig. 20. This shows that the stresses to ground at the middle of the winding will not be excessive, particularly since a wave as long as this will be much lower in magnitude than the very shortest ones. The initial distribution and maximum voltage to ground, for simultaneous equal surges on both ends of the winding, are shown in Fig. 21 taken from test data. Thus, the possible application of the design to delta connected transformers is clearly known.

One further step is the application of this principle to a grounded neutral design, but with insulation graded in proportion to the induced stress at the interleaved low-voltage coil and in the group connected to ground. Such an arrangement is shown in Fig. 22. The distribution within the winding for both short and long waves would be similar to that indicated in Fig. 20. Taking into consideration the relative magnitudes of such waves (approximately 14 to 7 times normal), it can be readily seen that the insulation at the interleaved part is not unduly stressed. (Fig. 23.)

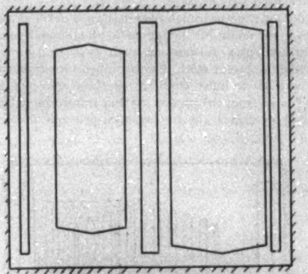

Fig. 22—Outline of Graded Insulation in a Two-Group Shell Type Transformer Eliminating Creepage

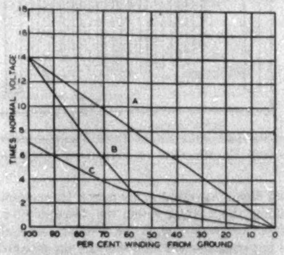

Fig. 23—Surge Voltage Distribution

Two-group shell type transformer with graded insulation
Curve B—Voltage in transformer due to highest line surges
Curve C—Voltage in transformer due to longest line surges (just low enough not to flash over line insulation)

The natural period of interleaved shell type transformer windings, although not in itself of great importance, is of interest due to its effect in reducing the magnitude of oscillation of such windings. A comparison of the oscillations within a single group, and also of an interleaved winding is shown in Fig. 20. The natural advantages of the shell form of construction are shown quite clearly by the fact that with a single group, good distribution is obtained, and hence the oscillation, although short in period and relatively completely developed is not important (Fig. 19). When interleaved, good distribution is obtained in the line group and the high ground capacity at the interleaving prevents high oscillation and also increases the natural period so that unless the surge is extremely long, the maximum oscillation can not be developed. For example the maximum potential by oscillation at the mid point of the winding as shown by Fig. 25 and by Fig. 20, curve 2, is 38 per cent of the impressed surge.

The curves of voltage distribution and oscillograms shown in this discussion were taken during the development of some 4,500 and 20,000-kva. transformers for the Pennsylvania Railroad. These transformers are single-phase and insulated for a 309,000-volt test from the high-voltage to low-voltage windings and ground. They are also designed to be co-ordinated with a 41-inch gap for impulse strength. To demonstrate this, the first transformers completed have been subjected to impulse voltage tests, with waves of both short and long duration. No indications of weakness either during the test or by complete disassembly and inspection of the units were found.

Fig. 24—Oscillogram Showing Voltages to Ground in a Single Shell Type Group at Points 1/3 and 2/3 from the Line End

No violent oscillations are indicated

Thus the application of the principle has been made to fully insulated transformers which are operated single-phase, and which could equally well be operated in delta.

H. V. Putman: Several of the discussors appear to be much concerned about the core-type transformer. They have rushed to its defense, but I have made no attack on it. My company builds core-type transformers—many more core types than shell. We recognize the merits of core type construction, but also its limitations. We recognize the merits and limitations of shell type construction also. Our engineering policy is to use the type best suited for any given application. Our development work for many years has been carried out on both types in parallel. We have done quite as much surge voltage investigational work on core types as on shell. We have millions of kva. of each type in service.

In discussing differences between core and shell, some of my critics are at a disadvantage, since their experience has been largely with core types. For instance, Mr. Clem condemns all shell type construction on the basis of experiments on ten round coils ten inches wide—a mere toy. Our knowledge of both core and shell types represents results on real transformers—some of them among the world's largest.

This background of parallel experience on both types places my associates, Bellaschi, Vogel, Fortescue and Hodnette, in a favorable position to speak authoritatively on the *differences in characteristics* of the two types.

Creepage Surfaces

Mr. Palueff's discussion is characterized by sketches representing his idea of Westinghouse transformers. These sketches are incorrect. No transformers have ever been built employing the winding arrangements shown in his Fig. 6, *B* and *C*, nor employing the insulation structure shown in his Fig. 7. He does not see why I prefer the new insulation arrangement, because, according to him, "creepage paths still exist at the top and bottom of the insulation structure (his Fig. 7), and the stress across these creepage paths is now three to eight times as great as in the old design." I prefer the new arrangement, because, based on actual tests, it has set a new standard of performance in the matter of resistance to surge voltages.

The insulation barriers employed in the new surge-proof transformers do not extend straight up above the coils (as in Mr. Palueff's Fig. 7), but they are turned over at the top substantially coincident with the equipotential surfaces. The creepage paths shown by Mr. Palueff and referred to by Mr. Peek therefore do not actually exist in the transformer. I fear neither of these gentlemen really understands what I mean by eliminating creepage surfaces. I mean simply the location of the insulation barriers so that their surfaces are substantially coincident with the equipotential surfaces of the electrostatic field. I do not claim to have enclosed each coil in a sealed box of insulation. It is, of course, possible to trace a devious path from one coil to any other coil. But with my arrangement, the direction of voltage stress is approximately normal to the insulation surfaces at all points, and this is the condition that gives the highest impulse strength.

Fig. 25—Oscillogram Showing Voltage to Ground at the Midpoint of a Two Group Shell Type Winding

Note long natural period of over 300 in. due to interleaved windings

Mr. Palueff states that in my new design I have crowded the full line-to-ground voltage into 1/3 to 1/8 the space used for it in the old. This statement is based on the winding arrangements imagined by Mr. Palueff in his Fig. 6, B and C, which, of course, never have been and never would be used. He appears to refer to the voltage lengthwise of the high-voltage stack, and presumably to the surge voltage, since there would be no question about the normal voltage. His statement would not be true even if the winding arrangements in his Fig. 6 were used, since in any case practically the whole surge voltage would appear across the group connected to the high-voltage line. This is due to the large ground capacity to the low-voltage coil group at the end of the first high-voltage group.

In any transformer there are only a few inches of space between the high-voltage winding and the low, and across these few inches the entire surge voltage exists. Here, indeed, is a high gradient. Mr. Palueff is greatly concerned about gradients of much lower value lengthwise of the high-voltage stack; why isn't he concerned about this high gradient in the few inches between the high and low? Simply because in these few inches lie puncture barriers separated by oil ducts—the best kind of insulation known. It is this kind of insulation—puncture barriers separated by oil ducts—that I have used throughout my entire insulation structure *lengthwise of the stack*, as well as between high- and low-voltage windings. The gradients through the barriers lengthwise of the stack are, of course, appreciably less than through the barriers in the space between high and low.

Both Mr. Montsinger and Mr. Palueff insist that creepage surfaces are entirely dependable if used intelligently within their limitations. I fully agree (we use them in our core type transformers), but is that any reason for not discarding them in favor of something superior wherever practical?

It is only when creepage surfaces are involved, that designers begin to compare gradients in different types of transformers and worry about volts per inch. These matters do not concern our new surge-proof transformers; they belong to the past.

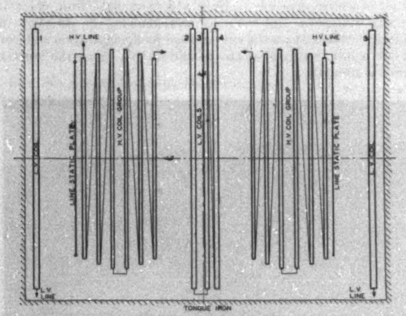

FIG. 26

Surges Induced in Low-Voltage Windings. I desire to correct a statement made by Mr. Palueff regarding the surge voltages induced in the low-voltage windings of shell type transformers. He states, "Another undesirable feature of the non-oscillating transformer of the interleaved type is the *extremely* high lightning voltage induced electrostatically in the low-voltage windings, from the high-voltage winding and static plate shown in Fig. 10. This happens because the low-voltage winding, instead of being placed next to the core, as in core type transformers, is placed in close proximity to the high-voltage line coil and static plate. From the approximate distribution of the electrostatic field produced by a lightning wave, Fig. 10, it follows that the low-voltage winding rises above ground to some 60 or more per cent of the amplitude of the lightning wave impressed on the high-voltage terminal."

It can be shown that this statement is incorrect. Of all the surge-proof transformers we have built, the 4,500-kva., 138-kv. fully insulated, single-phase transformers recently built for the Pennsylvania Railroad, had relatively the highest surge voltage induced in the low-voltage winding. These transformers have interleaved windings, as shown in Fig. 26.

Surely one could not conceive of a winding arrangement in which the low-voltage and high-voltage windings are more effectively coupled electrostatically. Having chosen the worst possible winding arrangement, let us also choose for investigation the worst possible case which occurs when a surge enters both high-voltage terminals simultaneously. Note that the high-voltage winding is not connected to ground at any point. Both ends are raised to the potential of the incoming surge and the remainder of the winding is free to float; closely coupled to it is the low. Even the terminals of the low are not solidly grounded but are connected to ground through 450 ohms to simulate the surge impedance of the low-voltage circuit. Under these conditions, let us examine the voltage induced in the low.

During the design of these transformers, Mr. Bellaschi made such an examination. His plot of the electrostatic field is shown in Fig. 27. The induced voltages shown by this plot are given in the figure. In Fig. 28 are also shown the test results for the potentials at all points in both the high and low voltage windings. It will be noted by reference to the table below that the maximum induced potential found in the low is 18 per cent of the lightning surge voltage as compared wtih 20 per cent calculated by Mr. Bellaschi.

CALCULATED AND MEASURED VALUES FOR VOLTAGES AT DIFFERENT POINTS IN THE L.V. WINDING

	Measured	Calculated
Coil No. 1—Line end	6%	
Coil No. 1—Finish Coil No. 2—Finish	15%	17%
Coil No. 2—Start Coil No. 3—Start	16%	
Coil No. 3—Finish Coil No. 4—Start	18%	20%
Coil No. 4—Finish Coil No. 5—Finish	16%	17%
Coil No. 5—Line end	6%	

It may be remarked that the measured values shown above were obtained by spark gap measurements between the various points in the winding and ground, and therefore represent maximum values, while the calculated values are for the initial voltages. However, it can be easily shown that under these conditions the maximum potentials in the low occur as a result of the initial distribution and not as a result of any subsequent oscillation.

Both the calculated and measured values reported above show that the surge voltages induced in the low of an interleaved shell type winding are not even of the same order of magnitude as Mr. Palueff believes.

In making his analysis, Mr. Palueff made the mistake of assuming that the potential of the points in the space occupied by the windings are unaffected by the presence of the windings themselves. He takes no account of the fact that the windings are made up of heavy copper conductors along which the charges may move—indeed, do move very appreciably in even a small fraction of a microsecond. When the presence of the windings themselves is taken into account, the shape of the field is materially changed, as can be seen from Bellaschi's sketch, Fig. 27, which is made correctly. In reaching conclusions in matters of this kind, practical experience with real transformers is desirable, since a superficial analysis can so easily be wrong.

In comparing the induced surge voltages in the low in core and shell type transformers, care should be taken to compare transformers of like specifications. For instance, Palueff says that in core types the low-voltage winding is placed next to the core, but in a core type transformer, to meet the low 4 per cent reactance required in the 4,500-kva. transformers, it is necessary to place a section of the low-voltage winding outside the high-voltage stack as well as next to the core. The low-voltage winding thus consists of two concentric cylinders with the high between the two. With this winding arrangement, which is common in core types, designed for low reactance, the electrostatic coupling between high- and low-voltage windings is quite as good as in an interleaved shell type. In our experience we have found that core and shell type transformers are on a par so far as surge voltages induced in low-voltage windings are concerned.

Distribution of Surges in Individual Coils. From the incorrect field plot in his Fig. 10, Mr. Palueff draws another surprising conclusion relative to the distribution of surge voltages within the individual high-voltage coils. He says, "However, the actual distribution shown by the solid wavy curve is extremely non-uniform with the middle of each coil at much higher potential than its edges, and with as much as 30 to 40 per cent of the transformer terminal voltage appearing across half of a coil near the ground end of the stack." This conclusion is, of course, absurd. Let Palueff refer to his own Fig. 13. According to the title, these curves represent actual measurements of the surge voltage distribution in a stack of four very wide coils. The voltage distribution within the individual coils is not uniform, and the distribution curves are characterized by what we call "saw teeth." The magnitude of these saw teeth is greater the fewer the number of coils in the group, so that in a group of only four, these saw teeth would be very pronounced.

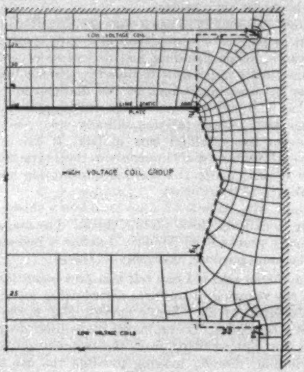

Fig. 27

An examination of these curves shows that the potential of the mid point of the coil at the ground end is about 5½ per cent for connection No. 1 and about 7 per cent for No. 2, instead of the 30 to 40 per cent mentioned earlier by Palueff. A further examination of these curves shows that in every case except coil No. 4, connection No. 1, the potentials at the mid points of the coils lie either in between those of the edges of the coils or else are lower; whereas Palueff concluded from his analysis, the potentials of the mid points would be *very much* higher than the potentials of the edges, as illustrated in his Fig. 10.

Effect of Coil Connections on Surge Voltage Distribution. Mr. Clem suggests that cross connections are hazardous and have been abandoned in shell type transformers. Mr. Peters' statement in this connection, quoted by Mr. Clem referred to transformers (core type, for instance), where the coils are spaced close together and sufficient room for properly insulated cross connections is difficult to obtain. In shell type transformers it is usually possible to obtain sufficient clearance for start-finish (or cross) connections and, in fact, this often results in the more economical winding. Our insulation tables cover both start-finish and start-start connections, and the designer employs the one best adapted to the particular design at hand.

In Mr. Palueff's discussion of Hodnette's paper referred to by Mr. Clem, Palueff claims that our results of surge voltage distribution were incorrect because we failed to measure the surge voltages on the *inside* edges of the coils and predicted that when we did so we would find the extremely non-uniform distribution claimed by him. Mr. Palueff's assumption in this case was unwarranted, because the measurements were made on a winding having start-finish connections, in which the inside edges of each coil are connected directly to the outside edges of each adjacent coil. Measurements were therefore made on both inside edges and outside edges simultaneously. Much of our early work was done with this type of winding because it did make it possible to measure the potentials on the inside edges. And it was the inside edge potentials in which we were vitally interested, because they established the potential distribtion along the creepage surface lengthwise of the group, along which failures took place when tests to destruction were made.

Uniform Distribution in Shell Type Stacks. Both Mr. Clem and Mr. Palueff discuss at great length the relative merits of core and shell type groups from the standpoint of initial surge voltage distribution. They present again the same old arguments and data which Mr. Palueff has repeated time and again in the A.I.E.E. discussions. These gentlemen apparently do not yet understand what we mean by saying that in a shell type group the surge voltage distribution is inherently substantially uniform. I am going to explain this statement very carefully and clearly. But before doing so, let us orient ourselves in this discussion to see what it is all about, and what practical conclusion it is desired to reach.

An ordinary transformer of either core or shell type construction, when tested to destruction with surge voltages, will usually fail lengthwise of the high-voltage stack, the failure starting at the inside edge of the line coil and extending along the creepage path on the insulation barrier adjacent to the coils. If the surge voltage stress is uniformly distributed along this creepage path, it is obvious that the maximum gradient will be lower and its resistance to breakdown will be greater than in a similar transformer, where the distribution is non-uniform. So the point under discussion is whether the distribution *along the creepage surface* lengthwise of the high-voltage stack is or is not substantially uniform in a shell type group. It is well known that the initial distribution along this creepage surface in a core type is extremely non-uniform, there being a very high gradient at the line end. Indeed, the use of shielding in a core type has as one of its principal objects, the elimination of this non-uniform stress distribution.

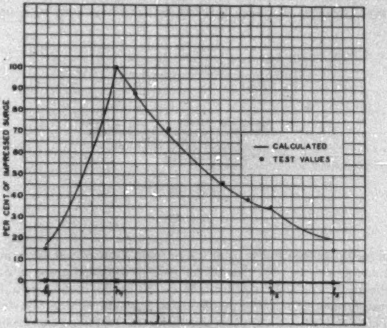

Fig. 28

But in an ordinary shell type, there appears still to be a difference of opinion about the uniformity of the gradient along this creepage surface. I shall therefore at this point present a careful explanation of this situation in a shell type group.

In Fig. 29 I illustrate a group of nine coils. In Fig. 4, Mr. Clem shows the initial distribution in such a group of nine coils for both start-finish and finish-finish connections. In an actual transformer of conventional construction, the line coil would have comparatively few turns heavily reinforced, so that the percentage of the total surge voltage appearing across it would not be as large as shown by Mr. Clem's curves, but the principle is the same and his curves will serve our purpose. In an actual transformer also, the duct space between the first two coils would be made larger than between the others, and additional insulation barriers employed in this duct.

The creepage surface in such a winding exists on the insulation barrier inside of the coils at the crown above the iron. It extends between points 2 and 4 as illustrated by the dotted line in Fig. 29. We have tested a number of conventional shell type transformers to destruction with surge voltages and they invariably fail along the path shown by the dotted line. We have never experienced a failure either across the face of the first coil nor between the first two coils at the outside, that is, between points 1 and 3, Fig. 29. The reason for this is that the great width of the shell type coil, which might be 20 or 30 inches in a large transformer, effectively protects the line coil against flashover. This width may often be greater than the length of the creepage path along the stack, so that there is not even a remote possibility of flashover across the face of the line coil. Furthermore, failures have not been experienced between points 1 and 3, Fig. 29, because there is no creepage surface here, solid insulation barriers being used, just as in the space between the high- and low-voltage windings. It is clear, therefore, that no difficulty has been experienced in insulating the first coil and duct for the surge voltages appearing across them.

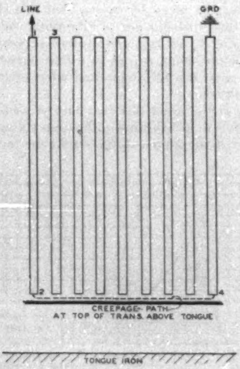

Fig. 29

The real problem in shell type transformers has been the gradient along the creepage surface just as in the core type.

In Fig. 30 I have plotted the voltage along this creepage surface, using the values from Mr. Clem's curves, Fig. 4, to illustrate the point that this gradient *is* substantially uniform. In plotting these curves, I have used the potentials on the inside edges of the coils which are adjacent to the creepage path and which establish the gradient along it, reading the data from Mr. Clem's curves.

These curves clearly show the uniformity of the gradient along the creepage path, which is characteristic of shell type groups. The difference between the two types is that in the core type this gradient is far from uniform, while in the shell type it is substantially uniform. The same result can be obtained from any of the other distribution curves for shell type groups presented by Palueff and Clem. With this point clearly established, let us examine its practical significance. It is this; shielding, however desirable in high-voltage core type transformers, would be totally ineffective in shell type so far as improving the dielectric strength of the creepage path lengthwise of the stack is concerned, since the gradient along this path is already uniform. And if any improvement is to be made in a shell type transformer, it must be made here, since this creepage surface is its weakest point. That improvement has been made a major improvement through elimination of the creepage surface itself. Shielding would obviously be worse than useless.

Mr. Palueff has sought to show that the distribution in a shell type is just as bad as or worse than in a core type; that a shell type, therefore, needs shielding just as bad as a core type; but that it is practically impossible to shield it. He has beclouded the entire issue by drawing curves of the distribution within the winding, that is, in and out among the turns, rather than along the creepage path, where failures actually take place. I believe this discussion has clarified this matter. It has shown that shielding could fulfill no useful purpose in shell type transformers. It has also shown clearly the logic of eliminating creepage surfaces in this type of transformer.

Static Plate. Mr. Palueff asks me to define a shield. Surely he is not going to call my static plate a shield. The static plate was patented many years ago by Moody. I define it just as he did and use it for the same purpose. I employ no shield.

Mr. Palueff also asks if I can tell him how essential the static plate is to the securing of uniform distribution along the stack. It is not essential at all. Clem's curves, Fig. 4 show this. Its slight effect is negligible. It eliminates the voltage drop across the face of the line coil resulting from the penetration of the surge into the winding, thereby making possible the use of ducts all of the same width and a uniform conductor insulation throughout. The use of a static plate with shell type transformers of conventional construction would result in general in lower impulse strength, because it would place the entire surge voltage across the creepage path lengthwise of the group, whereas without the plate this voltage across the creepage path *is* reduced by the voltage drop across the line coil. The line coil is much more able to stand this voltage than the creepage path.

Distribution in Core Type Stacks. Mr. Clem presents certain data in his Fig. 4 which show that the distribution along a stack of core type coils only two inches wide is substantially uniform. In fact, Fig. 4 shows a perfectly uniform distribution along the inside edges of the coils, and hence along the creepage surfaces. These data are, to say the least, most misleading. Surely he would not have any one ascribe any practical significance to such results. If he is correct, then transformer designers have been grossly misled these many years in reinforcing the insulation between coils at the line end of core type transformer windings and there is no purpose in Palueff's shielding. Such results can be obtained in a stack of coils without ground capacity, but not in a real transformer.

Surge Voltage Tests. I would like to say a few words about the relative severity of our lightning tests. My claim to first place was based on the G.E. published data relative to its New England power test. However, comparing data as Palueff has done in Fig. 11 is rather fruitless until it can be established why a Pittsfield volt is not as large as a Sharon volt. Our laboratories will no doubt eventually be able to measure the same voltage for the flashover of the same gap. At present, however, when dealing with short surges of the order of 5 microseconds, Pittsfield measures voltage from $\frac{1}{4}$ to $\frac{1}{3}$ higher than Sharon and

Trafford. Thus a given voltage measured by us as 1,500 kv. would be measured by Pittsfield as approximately 2,000 kv. If my test on the 42,000-kva. Baltimore Consolidated transformer were calibrated in Pittsfield volts, it would register close to 3 million volts, while our measurements indicated 2¼ million. This question might well have been deferred until the data published by our laboratories show better agreement.

Messrs. Paluett, Montsinger and Peek have all referred to the pinhole punctures between turns resulting from repeated surge voltage tests. This arouses my curiosity, because we have not found these in our experience. If such failures have actually been experienced, it must indicate some weakness in conductor insulation. We have built and tested experimentally many full size power transformers. Some of these were tested to destruction, others not; in all cases they were torn down and examined. Many were rebuilt, retested, and re-examined the second time. In some cases our coils were examined under a magnifying glass in an effort to find some of these pinholes, but without success.

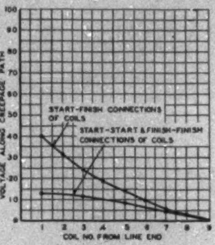

FIG. 30

I might say also that in all of our experience in testing shell type transformers to destruction with surge voltages, we have never seen a coil fail across its face. There need be no concern about the unequal distribution of surges within individual coils.

Mr. Palueff asks if we have torn down and examined commercial transformers after impulse tests. The Baltimore transformer, on which the measurements of distribution within the winding were made, was torn down sufficiently to attach the necessary connections between coils. We were able to examine those parts of the insulation structure which our experience had indicated would be the first to show any marking. Every piece of insulation was clean.

Some of the 4,500-kva. fully insulated transformers mentioned previously were disassembled and minutely examined. One of these transformers was given a total of 106 surges, all above the flashover of the protective gap. On one occasion the coils and insulation were laid out on the assembly floor, where they were examined by a party of 40 or 50 visiting engineers.

If was after making a number of successful surge voltage tests on commercial transformers that we recommended early last fall to the N.E.L.A. the use of surge tests to demonstrate guarantees of impulse strength. I am glad to see that others are now also coming to recognize that low-frequency dielectric tests do not constitute any demonstration of impulse strength and to agree that surge tests are practicable. I feel sure that we have already reached a satisfactory solution to the problem of failure detection mentioned by Mr. Peek.

Messrs. Gaston and Emlen ask about the temperature at which impulse tests have been made. All of our commercial transformers have been tested hot. Tests on our experimental transformers have been made both hot and cold.

Corona. Messrs. Montsinger and Peek mention the undesirability of corona during the low-frequency test. Since high-voltage transformers came into use, we have been building trnasformers which in all probability have some slight corona during the high-voltage test. Our experience is that in some types of insulation the corona point is very close to the breakdown point. Here the presence of corona during test would indicate overstressed insulation, and we are careful to avoid it. On other types of insulation structures, the corona point occurs at a much lower voltage than breakdown, and here slight corona during test need give no alarm. Copious corona discharges will usually produce visible marking on the insulation, and should, of course, be avoided. Experienced witnesses, who have seen insulation tests on many transformers of different makes, have often complimented us on the quietness of our transformers during the insulation test and on the lack of disturbance in the oil.

Barrier Test. I am much interested in Mr. Montsinger's comments on the barrier test. I went through the same process of reasoning myself at first and concluded that the impulse strength must increase with barrier extension. But repeated tests with very short surges (we used 1 microsecond or less) showed only very slight increase. Fig. 31 shows a complete set of time lag curves for barriers 1 inch thick, and with different extensions. These curves converge together for the very short waves, giving the result indicated in Fig. 10 of my paper. Mr. Montsinger's curve of impulse ratio against extension shows that the impulse ratio varies from 4.5 for zero extension to 2.7 for 8 inches extention for a ½-5 wave. With a much shorter surge, my Fig. 10 shows the impulse ratio to vary from 4.5 to 2. With surges of the same length, our data would not be far apart.

Internal Oscillations. Mr. Dann's discussion answers this question of internal oscillations in shell type transformers effectively, and I hope finally. One of the important differences between core type transformers and shell type transformers having interleaved windings is the long, naturel period of the latter, which effectively prevents internal oscillations of high magnitude. As an example, the natural period of the 4,500-kva. transformers previously mentioned was found from test to be 300 microseconds.

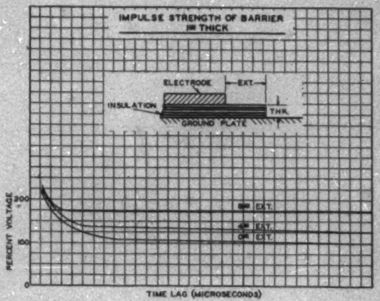

FIG. 31

Fully Insulated Transformers. One of the fine things about the improvements in shell type transformers which I have described is that they are in no way limited to grounded neutral transformers. Mr. Vogel in his discussion shows their application to fully insulated transformers and to transformers of the interleaved type.

Conclusions. The following paragraphs present a few important facts on which there is no difference of opinion. Core type transformers of conventional construction are characterized by an extremely non-uniform distribution of surge voltage lengthwise of the coil stack with high concentration at the line, and by relatively severe internal oscillations which produce correspondingly severe voltage stresses in the major insulation between high- and low-voltage windings, even with fairly short surges. These characteristics are recognized by all manufacturers of core type transformers, and the design of the insulation is made to take account of them.

There are two schools of thought regarding the best method of handling these core type insulation problems. One is represented by Palueff and his associates, who advocate graded insulation and the use of shielding to secure uniform distribution and prevent internal oscillations. The other is represented by European engineers, who prefer to utilize the space required by the shielding structure for full major insulation between high and low, and properly to reinforce the end turns rather than insulate a shielding structure at line potential from the entire high-voltage winding. This reinforcement or padding of end turns and coils insulates for the non-uniform initial distribution. The use of major insulation provides adequate dielectric strength against internal oscillations. The European engineer attempts to know the surge voltage stress at every point in his winding and to insulate for it directly. This is sound engineering and its simplicity commends it.

Both schools of thought are one in recognizing these insulation problems in core type transformers. They differ only in the method of solution advocated. For low-voltage transformers, the simpler European method of solution is used universally. It is in the higher voltage classes that these insulation problems become paramount and require careful consideration.

In these higher voltage classes we prefer to avoid entirely these difficult insulation problems by employing the simple construction disclosed in my paper. Reinforcement of insulation on end turns and coils is not required because the distribution of surge voltage stress throughout the entire insulation structure is substantially uniform and there is no creepage surface to protect. Where grounded neutral operation is required, graded insulation can be utilized with perfect assurance because severe internal oscillations cannot occur.

The simplicity of this new construction is apparent. Our claims of surge strength are backed by sound theoretical analysis and by tests on complete transformers. Operating engineers are not likely to be impressed by purely theoretical discussions. Only by actual demonstrations with surges representative of those produced by natural lightning will they be convinced of the surge-proof qualities of transformers.

Effect of Transient Voltages on Power Transformer Design—IV

Transition of Lightning Waves from One Circuit to Another Through Transformers

PART I—BY K. K. PALUEFF*
Member, A.I.E.E.

PART II—BY J. H. HAGENGUTH*
Associate, A.I.E.E.

Summary of this and preceding papers.*—Three preceding papers of the same general title have dealt with transient voltage stresses developed within high-voltage transformer windings. The most important conclusions arrived at by the author and presented in these papers and the discussions are:*

1. Transformers of all conventional constructions undergo oscillations when subjected to lightning or switching surges.

2. The amplitudes of these oscillations may be dangerously high, depending on the amplitude and the shape of the applied voltage. Their frequencies range from a few thousand to a few hundred thousand cycles per second.[1,2]

3. A lightning wave of a given shape produces very different stresses in different transformers. [1,2]

4. In practical design, neither amplitude nor frequency of these oscillations can be controlled by arrangement or proportions of windings.[1,2]

5. A lightning wave chopped by flashover of line insulation can produce stresses in transformer windings equal to or even in excess of those produced by a long wave of the same amplitude and front. [1,2]

6. Unless means are taken to obtain uniform distribution under all lightning conditions it is entirely possible to design transformers that will pass A.I.E.E. test which are inadequate for service conditions. This follows because test voltages in neither magnitude nor distribution of stresses correspond to that produced by transient voltages in transformers of ordinary construction. This difference is particularly great in transformers with solidly grounded neutrals, where the potential test allows the insulation from high-voltage winding to low-voltage winding and ground to be reduced as the neutral is approached.[1,2]

7. If a transformer winding is equipped with a properly designed electrostatic shield, voltages of all surges will distribute uniformly, and oscillation or resonance will not occur. Over 2,000,000 kva. of such non-resonating transformers are in successful operation.[1,2,4]

To demonstrate the lightning strength of such a transformer, one of fourteen 13,000-kva., 230-kv. transformers was subjected to artificial lightning in August 1930.[4,10] *Over 200 high-voltage surges were applied. Forty were in excess of the flashover voltage of the 64-inch coordinating gap. The majority of these were full waves just below the flashover of a string of 14 standard suspension insulators. A few were full waves just below flashover of the transformer bushings. Also a few waves high enough to flash over the bushing were applied. To be certain that no damage was done by these tests the transformer was completely disassembled for examination, since there is no other positive practical method available at present for determining a partial or local damage to the insulation inside of the windings that may be caused by lightning tests. These tests were preceded and followed by complete A.I.E.E. Acceptance Tests, including a 460-kv. induced potential test.*

8. A neutral impedor has been developed, extending the application of non-resonating design to transformers with partially or entirely isolated neutral.[3]

9. A non-resonating design has been adapted to auto-transformers.[5]

10. It has been shown theoretically and later experimentally, that a choke coil is worthless as a protective device, and may actually increase stresses.[1,2]

In this paper, the phenomena of the transmission of voltage surges through transformers to the connected secondary circuits are critically examined, a quantitative theory for their calculation is developed, and general conclusions on their magnitude are presented.

Part I

INTRODUCTION

THE impact of a surge on the high-voltage terminals of a transformer produces transient voltages not only within that winding, but also within the low-voltage winding, as well as at its terminals. The latter voltage is impressed on the network and apparatus connected to this winding and is the subject of this paper.

Some occasional failures, during lightning storms, of rotating apparatus connected to lines through transformers, indicate the possibility of transmission of high-transient voltages through transformers.

Dry insulation used in rotating machinery has much lower impulse strength than transformer insulation for the same operating voltage. On account of this it is more difficult to protect generators from transient voltages than transformers.

The first part of the paper deals with the subject from a practical standpoint. It establishes certain approximate relations between circuit constants, that are generally available, and transient voltages.

The second part is the mathematical treatment of some of the problems dealt with in the first. It shows first: that the approximations used in the first part are justified; second that in some cases these approximations are unavoidable because in practise most of the constants of the complete equations ((**1a**) for example) can not be determined.

FOUR COMPONENTS OF TRANSIENT ON THE SECONDARY CIRCUIT

It is convenient to consider that the surge voltage produced at the low-voltage terminals of a transformer by a rectangular traveling wave is a result of superposition of four components. Fig. 1.

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*Both of Power Transformer Dept., General Electric Co., Pittsfield, Mass.

1, 2. See Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

1. At the moment of impact of a lightning wave, an *electrostatic* voltage is produced at the terminals of the low-voltage winding which depends upon the relation of the distributed capacitances of the low-voltage winding to high-voltage winding and ground, and is independent of turn ratio.

2. A *free oscillation in the high-voltage winding*, consisting of a number of space harmonics,[1,2] induces a corresponding oscillation in the low-voltage winding. The induction is accomplished by means of the electrostatic and electromagnetic fields of these harmonics, and is dependent upon the distributed constants and turn ratio of the windings.

3. A *free oscillation in the low-voltage winding* follows immediately after the impact of the wave. The resulting voltages depend upon the distributed constants of the low-voltage winding.

4. Through a *purely electromagnetic induction* between the two windings, a unidirectional surge is produced on the secondary circuit. Its voltage rises from zero to a certain maximum, then decays to zero. The voltage is directly proportional to the turn ratio and is independent of the distributed capacitance. It is a simple function of the short-circuit inductance of the transformer and of the surge impedances of external circuits.

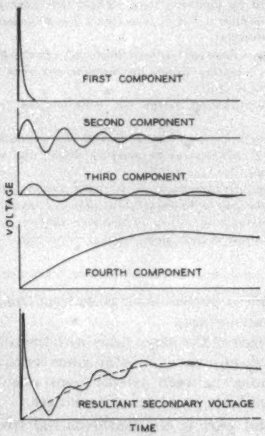

FIG. 1—SECONDARY TERMINAL VOLTAGE AND ITS FOUR COMPONENTS

All four components depend, to a different degree, upon surge impedances and lengths of external circuits connected to the transformer.

RELATIVE IMPORTANCE OF FOUR COMPONENTS

Let $2E$ be the voltage produced across the high-voltage ("primary") winding of a transformer by a rectangular wave of E volts and of infinite length, E_2' the crest voltage of the wave transmitted to the low-voltage ("secondary") circuit, Z_1 and Z_2' the surge impedances of external circuits connected respectively to the primary and secondary of the transformer, t_2' the *total* length in microseconds of the transmitted secondary wave, and t_1 the length of primary wave which would contain the same amount of energy as the secondary wave. It is assumed that the secondary circuit is at least $t_2'/2$ microseconds long.*

It can be shown that for an exponential secondary wave with a sheer front and a tail that falls from E_2' to $E_2'/2$ in t_2 microseconds, the maximum secondary voltage is

$$E_{2e}' = 0.59 \times 2E \sqrt{\frac{Z_2' t_1}{Z_1 t_2}}$$

Similarly, for a secondary wave, half of a cycle of a sine wave in shape with a length t_2 during which the amplitude is above $E_2'/2$, it can be shown that

$$E_{2s}' = 0.58 \times 2E \sqrt{\frac{Z_2' t_1}{Z_1 t_2}} \cong E_{2e}'$$

While for oscillatory secondary voltage which falls to zero in say five cycles, the crest voltage of the first half of a cycle is

$$E_{2o}' = 0.41 \times 2E \sqrt{\frac{Z_2' t_1}{Z_1 t_2}} = .71\, E_{2e}'$$

Voltages produced by the first and the fourth components are exponential and those produced by the other two are essentially sinusoidal and oscillatory.

Simple calculations show that for each of the first three components, t_1 is less than *two-* or *three-tenths* of a microsecond. It follows that t_2 for a given component must be exceedingly short to permit that component to produce a secondary voltage that is comparable to $2E$. (Fig. 2.)

Fig. 2 permits estimate of maximum possible secondary voltage where order of magnitude of t_2 is known. It also gives the maximum possible length t_2 of the secondary wave where its crest is known.

Under the assumption that the primary circuit is an overhead transmission line, curve 4.0 corresponds to a case where the secondary circuit is a winding of a small rotating machine, curve 1.0 where it is an overhead transmission line, and curve 0.1 where it is an underground cable. The range of large rotating machines is between the last two curves.

Time t_2 is materially shorter for the first component than for the second or third, due to the difference in the nature of these transients, but generally speaking the energy (proportional to t_1) involved in the first component is somewhat less. The value t_2 for the first com-

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*By the "length" of a given circuit is meant the length of *time* T_1 required for a traveling wave to traverse the circuit from end to end. In one microsecond, a wave travels 1,000 ft. on an overhead transmission line and about 500 ft. on an underground cable. Propagation along a generator winding depends on many design factors, and is of the order of 100 ft. per microsecond.

ponent ranges between a fraction of a microsecond and say 5 microseconds. Furthermore, since the voltage of this component is induced purely electrostatically it obviously can not exceed the voltage applied to the primary ($2E$) but is a fraction (K_e) of this voltage. K_e depends only on the electrostatic relation of windings. The voltage produced by the first component becomes more dangerous as the rated secondary voltage, and correspondingly the insulation of secondary apparatus are reduced, because the voltage of the transmitted wave is independent of the turn ratio.

The time t_2 for the second and third components is relatively long, being in the order of a few microseconds to several hundred microseconds, particularly in a case where energy of these components can not be transmitted to the secondary circuit in one-half cycle of their respective frequencies as generally is the case. Keeping in mind the fact that t_1 in each case is a small fraction of a microsecond (t_1 of the second component is much greater than t_1 of the third), it follows from the equations above that E_2' for these two components must be a very small fraction of $2E$, Fig. 2.

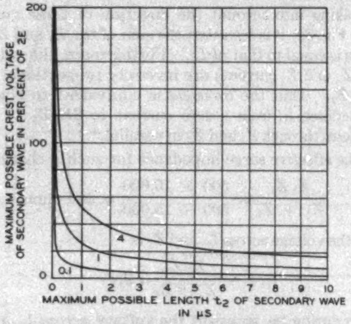

FIG. 2—RELATION BETWEEN THE MAXIMUM SECONDARY VOLTAGES AND THE MAXIMUM LENGTH OF THE SECONDARY WAVES CONTAINING THE SAME AMOUNT OF ENERGY

Curves based on: $t_1 = 0.2$ microseconds. Numbers on curves indicate the corresponding value of Z_2'/Z_1

Thus it appears that under average service conditions it is possible but not probable that secondary waves of high amplitude may be produced by the second and the third components of the secondary transient. However, high voltages may be produced if Z_2' is abnormally high or if the secondary winding is not connected to its network.

The amount of energy that can be transmitted to the secondary network by the fourth component may reach the order of magnitude of that contained in a *few hundred* microseconds of the primary wave. If t_2 were small compared to t_1, it would follow that E_2' could be many times E. However, the mechanism of creation of the secondary wave of this component, is such that t_2 is necessarily greater than t_1, to the extent that E_2' is never more than $2E/r$ in case the secondary circuit is sufficiently long and $4E/r$ in case the secondary circuit is very short (r is the ratio of turns of the primary winding to the secondary winding). It is seen, therefore, that very powerful secondary surges of dangerously high voltage can be produced by purely electromagnetic transformation of a lightning wave.

Where the secondary circuit is short in comparison with the length of secondary wave voltages in excess of those of Fig. 2 can be produced.

ELECTROMAGNETIC TRANSFORMATION OF TRAVELING WAVES

A. Single-Phase Transformer

1. Infinite Rectangular Traveling Wave

If the secondary winding were perfectly interlaced with the primary, then when a rectangular wave of infinite length is applied, the secondary voltage would rise instantly to a value equal to the primary voltage divided by the turn ratio.* The voltage of both windings would fall gradually to zero as the current in the primary circuit increases and approaches a certain constant value.

In an actual transformer, however, there is considerable leakage or "short-circuit" inductance between the two windings. This causes the shape, and to a small extent the amplitude of the secondary wave to be different from that stated above. The rise of voltage is not instantaneous, but requires from a fraction of one microsecond to a few hundred microseconds.

For the analysis of the electromagnetic transient it is convenient to think of it as consisting of two parts. The first is the comparatively rapid rise of secondary voltage. This is dependent almost entirely upon the *short-circuit inductance* L_s of the transformer, and will be referred to as the L_s transient. The second is the comparatively gradual decay of secondary voltage to zero. This is dependent almost entirely upon the *self inductance* L_1 of the primary winding, and will be referred to as the L_1 transient.

As is customary for mathematical analysis and laboratory tests, surge impedances of transmission lines, cables, generator windings, etc., may be replaced with resistances of equal ohmic values, and the voltage E of the traveling wave replaced by a d-c. voltage of $2E$. This permits the circuit consisting of the transformer (with both windings grounded) and its primary and secondary surge impedances, (Fig. 3a) to be replaced by that in Fig. 3b. The application of this method in the laboratory has known limitations which, however, do not apply in this case.

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*The effect of mutual inductance on transient phenomena is well known. The mathematical derivations and resulting equations however are quite complex and difficult to visualize. Here, by taking advantage of relative values of transformer constants affecting the phenomena, the solution is arrived at in a most elementary manner. The results obtained agree very closely with the exact solution given in Part II.

To take care of the effect of turn ratio between primary and secondary windings, all of the constants of the secondary circuit may be replaced by equivalent constants which would produce the same effects in the primary circuit if the turn ratio were unity. This requires that the actual secondary surge impedance Z_2' be replaced by an equivalent value Z_2 which is equal to $Z_2'r^2$. Also this permits the circuit to be redrawn as in Fig. 3c, in which L_1, L_2 and M are respectively the self inductance of the primary and secondary (equivalent) windings and the mutual inductance between them. This may be simplified to the approximate equivalent circuit of Fig. 3d.

After a solution is obtained for an equivalent circuit, all secondary current and voltage values (I_2 and E_2) *must be corrected by the turn ratio to obtain actual values* (I_2' *and* E_2') *for the secondary circuit.*

To make the analysis more concrete, it will be followed in parallel with a numerical example. E is assumed to be 500,000 volts, Z_1 is 500 ohms. Single-phase transformer, 60 cycles, 22,500 kva., 66,000 volts (115,000 volts Y) to 13,200 volts, 6 per cent leakage reactance, L_s = 0.031 henrys, L_1 = 0.31 henrys, r = 5, Z_2' = 600 ohms, Z_2 = 25 × 600 = 15,000 ohms.

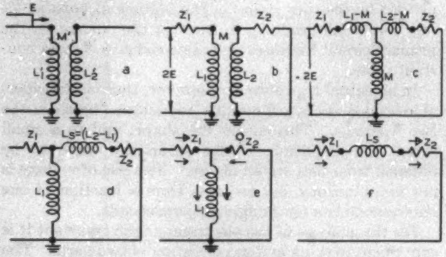

FIG. 3—DEVELOPMENT OF EQUIVALENT CIRCUIT FOR SINGLE-PHASE TRANSFORMER

⬅ Final current ← Transient current

L_1 Transient. Neglecting the effect of leakage inductance, it becomes possible to investigate the L_1 transient alone (See Fig. 3e).

At the moment of impact of a rectangular wave, L_1 acts as an open circuit because of the steepness of the front of the wave. A current equal to $2E/Z_1 + Z_2$) = (2 × 500,000)/(500 + 15,000) = 64.5 amperes will flow through Z_1 and Z_2, and no current will flow through L_1. The voltage across Z_1 will be $2E\ Z_1/(Z_1 + Z_2)$ = 64.5 × 500 = 32,250 volts. The voltage across Z_2 and L_1 will be $E_2 = 2E\ Z_2/(Z_1 + Z_2)$ = 64.5 × 15,000 = 967,750 volts. These are called initial values.

With $2E$ maintained sufficiently long across the circuit, a constant current will finally be established through it. To such a current the reactance of L_1 will be zero. Therefore, all the current will flow through L_1 and none through Z_2. The full voltage $2E$ or 1,000,000 volts will appear across Z_1 and no voltage will appear across L_1 or Z_2. The current through Z_1 and L_1 will be $2E/Z_1$ = 1,000,000/500 = 2,000 amperes, and that through Z_2 will be zero. These are called the *final* values.

Since the amplitude of the transient currents and voltages is the *difference* between the final and initial values, it follows that the amplitudes of transient currents are:

$$\text{in } Z_1,\ \frac{2E}{Z_1 + Z_2}\left(\frac{Z_2}{Z_1}\right) = \frac{2 \times 500{,}000}{500 + 15{,}000}\ \frac{(15{,}000)}{500}$$

$$= 1935.5 \text{ amperes}$$

$$\text{in } Z_2,\ \frac{2E}{Z_1 + Z_2} = \frac{2 \times 500{,}000}{500 + 15{,}000} = 64.5 \text{ amperes}$$

$$\text{in } L_1,\ \frac{2E}{Z_1} = \frac{2 \times 500{,}000}{500} = 2{,}000 \text{ amperes.}$$

Taking into account the direction of these currents (See Fig. 3e) it is seen that the sum of the Z_1 and Z_2 currents is equal to that of L_1. Furthermore, the values of the Z_1 and Z_2 currents are inversely proportional to Z_1 and Z_2. Thus the *transient* is equivalent to charging inductance L_1 from a zero current to $2E/Z_1$ = 2,000 amperes through Z_1 and Z_2 in parallel.*

The effective surge impedance for such a charge is

$$\frac{Z_1\,Z_2}{Z_1 + Z_2} = \frac{500 \times 15{,}000}{500 + 15{,}000} = 484 \text{ ohms}$$

and the voltage across L_1 and Z_2 is

$$E_2\, e - \frac{Z_1\,Z_2}{Z_1 + Z_2}\, t/L_1$$

Thus during the transient the voltage across L_1 and Z_2 falls exponentially from $E_2 = 2E\ Z_2/(Z_1 + Z_2)$ = 967,750 volts to zero. (Curve 1, Fig. 4.) From the laws of exponential functions, the voltage falls 95 per cent of this value in

$$T_1 = \frac{3\,L_1}{\dfrac{Z_1\,Z_2}{Z_1 + Z_2}} \text{ seconds} = \frac{3 \times 0.31}{484} \text{ seconds}$$

$$= 1{,}860 \text{ microseconds.}$$

L_s Transient. As was previously stated, the leakage inductance L_s (See Fig. 3d) prevents the voltage across Z_2 from rising instantly to its maximum value. During the time required for this voltage to rise only an exceedingly small current has time to build up in L_1, due to the fact that in transformers L_1 is generally much larger than L_s. For the analysis of the L_s transient this small current can be neglected and the equivalent circuit becomes that of Fig. 3f.

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*This is in accordance with one of the useful rules of transients which permits an imaginary short-circuiting of all points between which voltage of a given transient is zero during the entire period. In this case voltage between terminals 1 and 2 is maintained constant ($2E$) and therefore has no transient component. This rule alone permits writing the expressions for E_2 and T_1 directly from inspection of the circuit of Fig. 3a.

Since the front of the incident wave is infinitely steep, at the first instant the entire applied voltage appears across L_s, and none across Z_1 and Z_2. No current will flow. This is the *initial* condition.

With $2E$ maintained sufficiently long across the circuit, the current will increase and approach a constant value. To this constant current L_s will offer zero reactance. The full voltage $2E$ will appear across Z_1 and Z_2 in series, and the current through them will be $2E/(Z_1 + Z_2) = 2 \times 500{,}000/(500 + 15{,}000) = 64.5$ amperes. The voltage across Z_2 will be $E_2 = 2E\,Z_2/(Z_1 + Z_2) = 2 \times 500{,}000 \times 15{,}000/(500 + 15{,}000) = 967{,}750$ volts. This is the *final* condition.

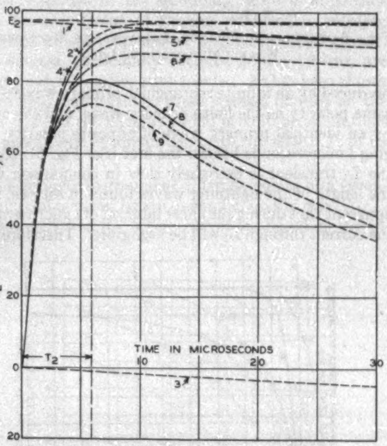

FIG. 4—SECONDARY TERMINAL VOLTAGE AS OBTAINED FROM EQUIVALENT CIRCUITS OF FIG. 3 FOR THE NUMERICAL EXAMPLE

Infinite rectangular primary wave
1. Voltage across Z_2 due to L_1 transient
2. Voltage across Z_2 due to L_s transient
3. Difference between curve 1 and E_2
4. Resultant secondary voltage obtained by superposition of curves 2 and 3

5 and 6. Resultant secondary voltage (using limits of $L_1 = M$ and $L_2 = M$ respectively) obtained by rigorous solution of circuit Fig. 3c (equation (2), Part II)

Twenty-five microsecond exponential primary wave
7. Resultant secondary voltage in circuit of Fig. 3f

8 and 9. Resultant secondary voltage for circuits Fig. 3c
(8) $L_1 = M$ (9) $L_2 = M$ (equation (9a), Part II)

Since the transient is caused by the *difference* between the final and initial conditions, the amplitude of transient current is $2E/(Z_1 + Z_2)$ and the amplitude of transient voltage across Z_2 is $2E\,Z_2/(Z_1 + Z_2)$. The transient consists therefore of charging the inductance L_s from a zero current to $2E/(Z_1 + Z_2)$ through Z_1 and Z_2 in series. It follows that the transient voltage across Z_2 rises exponentially from zero to $2E\,Z_2/(Z_1 + Z_2) = 967{,}750$ volts. It will reach 95 per cent of its ultimate value in $T_2 = 3L_s/(Z_1 + Z_2) = 3 \times 0.031/(500 + 15{,}000)$ seconds or in 6 microseconds. Thus,

$$e_2 = 2E\,\frac{Z_2}{Z_1 + Z_2} - 2E\,\frac{Z_2}{Z_1 + Z_2}\,e^{-\frac{Z_1 + Z_2}{L_s}t}$$

$$= E_2\left(1 - e^{-\frac{Z_1 + Z_2}{L_s}t}\right) \qquad \text{Curve 2, Fig. 4}$$

Resultant Z_2 Voltage. The voltage across Z_2 would be as shown by curve 2, Fig. 4 if the ultimate voltage across Z_2 were E_2. However, on account of the presence of L_1 the voltage across Z_2 gradually falls below E_2 and finally reaches zero (curve 1, Fig. 4). Therefore, voltage across Z_2 at any moment will be lower than that shown by curve 2 by the amount very nearly equal to the difference between E_2 and curve 1, *i. e.*, by:

$$-\left(E_2 - E_2\,e^{-\frac{Z_1\,Z_2}{Z_1 + Z_2}t/L_1}\right) = -\,E_2\left(1 - e^{-\frac{Z_1\,Z_2}{Z_1 + Z_2}t/L_1}\right)$$

(curve 3, Fig. 4).

The resultant voltage across Z_2 therefore (curve 4, Fig. 4) is obtained by superposition of curves 2 and 3 of Fig. 4.

$$i.\,e.,\ e_2 = E_2\left(e^{-\frac{Z_1\,Z_2}{Z_1 + Z_2}t/L_1} - e^{-(Z_1 + Z_2)\,t/L_s}\right)$$

Compare this with exact solution equation (**1b**) and (**2**) of Part II.

It can be stated that the secondary voltage is essentially a copy of the voltage across the primary terminals reduced by the turn ratio (curve 1 of Fig. 4) with the front slanted to T_2 microseconds due to the effect of leakage inductance.

To arrive at the above solution the approximate equivalent circuits of Figs. 3e and 3f were used in analyzing the L_1 and L_s transients, and the effect of the transients on each other was neglected. To illustrate the degree of accuracy obtained by this simple method, curves 5 and 6 of Fig. 4 are shown, which were obtained by rigorous solution of the exact equivalent circuit of Fig. 3c for two extreme cases (one $L_1 = M$ and the other $L_2 = M$). (See equation (**2**) in Part II.)

In T_2 microseconds the voltage of the L_s transient will rise to 95 per cent of its maximum value (E_2) while the voltage of the L_1 transient will hardly begin to fall from the same numerical value. Therefore, the first exponential term of the above equation is approximately equal to unity for at least T_2 microseconds and

$$e_2 = E_2\left(1 - e^{-\frac{Z_1 + Z_2}{L_s}t}\right)$$

This means that at $t = T_2$ $e_2 = 0.95\ E_2 = 919{,}350$

$$\left(\text{divide it by turn ratio to get actual secondary voltage}\quad \frac{919350}{5} = 183{,}830 \text{ volts}\right).$$

For the sake of simplicity this value will be considered as the practical crest of secondary voltage, and T_2 as the time of its occurrence. Fig. 5.

It is important to realize the relative duration of the L_1 and L_s transients as expressed by the ratio

$$\frac{T_1}{T_2} = \frac{L_1}{L_s} \frac{(Z_1 + Z_2)^2}{Z_1 Z_2} = \frac{1860}{6} = 310$$

In practise L_1/L_s for lightning voltages is possibly never less than five and generally is above ten. Irrespective of values of Z_1 and Z_2, $(Z_1 + Z_2)^2/Z_1 Z_2$ cannot be less than four and generally is more than ten. It follows that the L_1 transient is at least twenty times slower than the L_s transient. This justifies the above approximation of the maximum secondary voltage, and the neglecting of the effect of the above transients on each other which made possible the independent use of the simplified equivalent circuits of Figs. 3e and 3f.

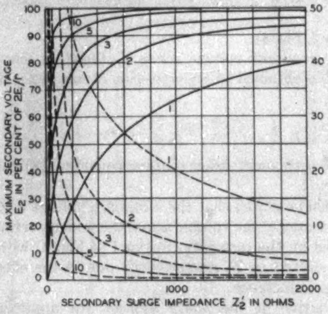

FIG. 5—EFFECT OF SECONDARY SURGE IMPEDANCE Z_2' AND TURN RATIO ON MAXIMUM VOLTAGE AND FRONT OF SECONDARY WAVE

Primary surge impedance: Z_1 = 500 ohms
Short-circuit inductance: L_s = 0.01 henrys*
Numbers on curves indicate the corresponding value of turn ratio
Solid lines = maximum voltage
Dotted lines = wave front
*(For any other value of L_s the length of the front has to be multiplied by $L_s/0.01$)

Secondary Winding Not Grounded. In the above, one terminal of the transformer secondary winding was connected to ground, and the other to a surge impedance Z_2. If both terminals were connected to identical circuits of surge impedance Z_2, then in effect the surge impedance of the secondary circuit would be doubled. Then

$$E_2 = 2E \frac{2Z_2}{Z_1 + 2Z_2} \text{ and } T_2 = \frac{3L_s}{Z_1 + 2Z_2}.$$

A voltage to ground of $\frac{E_2}{2}$ would appear on each of the two surge impedances.

Transmission of Surges Through Auto-Transformers. The relations derived above for single-phase transformers apply equally well for auto-transformers. The transmission of surges through auto-transformers was described in a previous publication.[5]

2. Finite Rectangular Primary Wave

If a rectangular primary wave has a finite length of t microseconds, the secondary wave produced is the resultant of superposition of two secondary waves, one produced by an infinite rectangular primary wave of the same polarity as the finite primary wave, and the other by an identical primary wave of opposite polarity lagging t microseconds behind the first (See Fig. 6). Since the L_1 transient is extremely slow in comparison with the length of the lightning waves found in service, it is apparent that during the short interval of t microseconds the current through L_1 will be negligible. Therefore, for the analysis of conditions found in practise, *the effect of L_1 can be completely neglected*, and the equivalent circuit of Fig. 3f is sufficiently accurate, as shown by comparison of curve 7 with 8 and 9 of Fig. 4.

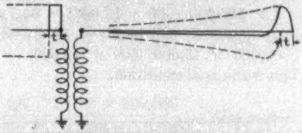

FIG. 6—SECONDARY VOLTAGE PRODUCED BY RECTANGULAR WAVE, t MICROSECONDS LONG

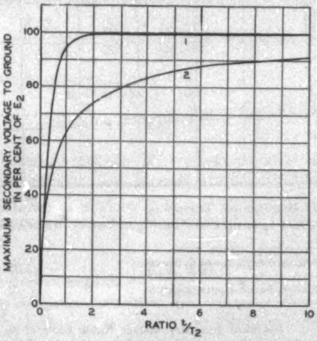

FIG. 7—EFFECT OF THE LENGTH t OF PRIMARY (APPLIED) WAVE ON THE AMPLITUDE OF THE SECONDARY WAVE

1. Finite rectangular wave applied
2. Wave with exponential tail applied

E_2 is secondary voltage produced by infinitely long rectangular wave
T_2 is secondary front produced by infinitely long rectangular wave

From curve 1, Fig. 7, it is apparent that the secondary voltage will not reach 0.95 E_2 unless the length t of the rectangular primary wave is at least equal to the length $T_2 = 3L_s/(Z_1 + Z_2)$ of the front of the secondary wave.

The fraction of this value that will be reached depends upon the ratio t/T_2 as shown in curve 1 of Fig. 7.

3. *Effect of Shape of Primary Wave*

The maximum secondary voltage produced by the primary wave having vertical front and exponential tail is shown on curve 2, Fig. 7. In this case the length t of the primary wave is measured at the level $E/2$ as is commonly done.

The steepest front T_2 of the secondary wave is produced by a rectangular primary wave. The length of secondary wave front increases as the length of the primary wave front is increased, and the two become approximately equal when the latter is more than $3T_2$.

The steepness of the secondary wave is important from the standpoint of stresses produced in rotating machines or other apparatus in the secondary circuit. The stresses, particularly between adjacent turns, are much greater for steep wave fronts. As has been shown in previous publications, these stresses depend on the ratio of T_2 to the natural period of voltage oscillations of a given piece of apparatus, or to the length T_3 of its circuit.

The voltages to ground at different points of a secondary circuit of given length depend upon the ratio T_2/T_3 as shown by Fig. 8. These curves are based on the assumption that the far end of the secondary circuit is kept at ground potential, so that secondary waves suffer negative reflection from that point. It follows from the curves that the secondary voltage cannot reach the maximum value at any point whose "distance" from the far end is less than $T_2/2$. As a result of this phenomenon, that part of Z_2 nearest the line end is subjected to high voltages more often than the rest.

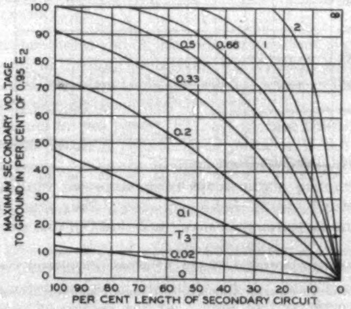

FIG. 8—EFFECT OF LENGTH OF FRONT T_2 OF SECONDARY WAVE ON MAXIMUM VOLTAGE ALONG EXTERNAL SECONDARY CIRCUIT

Total length of secondary circuit is T_3 microseconds. Far end grounded Numbers on curves indicate corresponding values of T_3/T_2

B. *Effect of Transformer Bank Connection and the Number of Phases Struck by Lightning*

Relations between the circuit constants and the crest E_2 and front T_2 of the secondary wave were established above for a single-phase transformer. Since installations of power transformers are seldom single-phase, it is important to determine the secondary voltage obtained in three-phase banks. It is apparent that the results obtained for the single-phase transformer apply directly in case both windings and generator are connected in Y with all neutrals grounded, because in such a case each phase is independent of the others. Table I shows the effect on the secondary voltage of other bank connections. For convenience of comparison the front (T_2) and crest (E_2) obtained in a single-phase transformer is taken as a reference. Figures without brackets give relative voltages and wave fronts for banks which have the same ratio of *line* voltages, while figures in brackets give the corresponding values for banks which have the same turn ratio or ratio of *phase* voltages.

TABLE I

BANK CONNECTIONS			*RELATIVE MAXIMUM SECONDARY VOLTAGE TO GROUND		AMPLITUDE FACTOR AND TIME CONSTANT (EQUATION 13-b)	
	PRIM.	SEC.	AMPLITUDE	FRONT LENGTH	A	B
1	Y⏚	Y⏚	100	100	$\frac{1}{r}\frac{q}{1+q}$	$\frac{Z_1}{L_s}(1+q)$
2	Y⏚	Y	67 (67)	100 (100)	$\frac{2}{3}\frac{1}{r}\frac{q}{1+q}$	$\frac{Z_1}{L_s}(1+q)$
3	Y	Y⏚	67 (67)	100 (100)	$\frac{2}{3}\frac{1}{r}\frac{q}{1+q}$	$\frac{Z_1}{L_s}(1+q)$
4	Y	Y	67 (67)	100 (100)	$\frac{2}{3}\frac{1}{r}\frac{q}{1+q}$	$\frac{Z_1}{L_s}(1+q)$
5	▷	▷	67 (67)	33 (33)	$\frac{2}{3}\frac{1}{r}\frac{q}{1+q}$	$3\frac{Z_1}{L_s}(1+q)$
6	▷	Y⏚	57 (100)	300 (100)	$\frac{1}{r}\frac{q/3}{1+q/3}$	$3\frac{Z_1}{L_s}(1+\frac{q}{3})$
7	▷	Y	57 (100)	300 (100)	$\frac{1}{r}\frac{q/3}{1+q/3}$	$3\frac{Z_1}{L_s}(1+\frac{q}{3})$
8	Y	▷	57 (33)	11 (33)	$\frac{1}{r}\frac{q}{1+3q}$	$\frac{Z_1}{L_s}(1+3q)$
9	Y⏚	▷	57 (33)	11 (33)	$\frac{1}{r}\frac{q}{1+3q}$	$\frac{Z_1}{L_s}(1+3q)$

*Amplitudes and lengths of fronts of the bank connections (2 to 9) are expressed in per cent of corresponding values of bank connection (1) as the base

Figures without brackets are based on equal ratio of operating line-to-line voltage

Figures in brackets are based on equal transformer turn ratio

To illustrate the simplicity of the method used in obtaining this table, the solution for secondary voltage will be analyzed for single-phase excitation of a Y-delta bank with neutral isolated and generator connected in Y.*

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*There is considerable similarity between the method used here and that used in analysis of single-phase excitation of a three-phase system at operating frequency.

Construction of the equivalent circuit of the bank is accomplished in three steps in Fig. 9. From the relations of currents of the secondary and primary lines it is evident that in spite of the fact that each individual transformer was reduced to an equivalent 1:1 transformer the combined effect of interconnection of the three units resulted in a 1.5:1 ratio of transformation from the primary terminals (*a* to *b* and *c*) to the secondary terminals *d* to *e*). Therefore, in the final equivalent circuit, the surge impedance Z_2' is multiplied not by r^2, but by $(1.5\,r)^2$ or $2.25\,r^2$. Furthermore, since none of the secondary lines is grounded, the voltage to ground at *d* is $+\dfrac{e_2}{2}$ and at *e* is $-\dfrac{e_2}{2}$. The current in each corresponding generator phase is

$$\frac{e_2}{2 \times 2.25\,Z_2};$$

which means that in the equivalent circuit $2 \times 2.25\,Z_2$ must be substituted for the secondary surge impedance.

Since the primary current flows first through one transformer and then through the other two in multiple, the total inductance of the circuit is equal to $L_s + \dfrac{L_s}{2}$. For the same reason the combined surge impedance of phases *b* and *c* is equal to $\dfrac{Z_1}{2}$. These considerations lead to the final equivalent circuit of Fig. 9. This circuit is very similar to that of Fig. 3f and therefore we can write the following expressions:

$$E_2 = \frac{2E}{1.5\,Z_1 + 4.5\,Z_2}\,4.5\,Z_2 \times \frac{1}{1.5}$$

$$= \frac{2E}{Z_1 + 3Z_2} \times 2Z_2 \text{ volts}$$

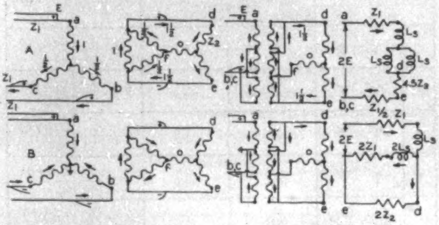

FIG. 9—DEVELOPMENT OF EQUIVALENT CIRCUITS FOR Y-DELTA TRANSFORMER BANKS CONNECTED TO A GENERATOR (BASED ON ONE-TO-ONE TRANSFORMER RATIO)

A—Primary neutral isolated
B—Primary neutral grounded

$$T_2 = \frac{3L_s}{Z_1 + 3Z_2} \text{ seconds.}$$

The voltage from one phase (*f*) of the secondary circuit to ground is zero, and from the other two (*d* and *e*) is $\pm E_2/2$.

Where $Z_1/3Z_2$ is very small, $E_2 = 2E/1.5$ and $T_2 = L_s/Z_2$. Thus the maximum possible line-to-line voltage in this case is 67 per cent and the line-to-ground voltage is 33 per cent of that for a single-phase transformer, and the front of the wave is three times as steep.

To obtain the actual secondary voltage, E_2 must be divided by the turn ratio *r*.

Since the two secondary waves are equal but opposite in polarity, the resultant voltage to ground will be zero at the point of the secondary circuit where the waves meet as at the neutral of a generator.

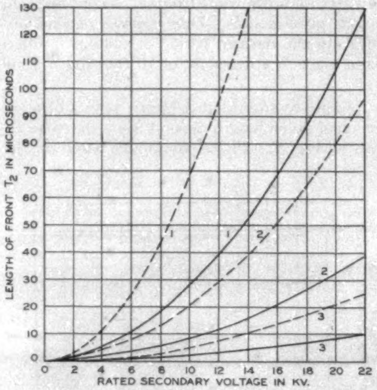

FIG. 10—LENGTH OF FRONT OF SECONDARY WAVE AS A FUNCTION OF RATED SECONDARY LINE-TO-LINE VOLTAGE

Solid curves $Z_2' = 500$ ohms. Dotted curves, $Z_2' = 200$ ohms

1. Transformer capacity 1,000 kva. per phase. Short-circuit reactance 5 per cent
2. Transformer capacity 5,000 kva. per phase. Short-circuit reactance 7.5 per cent
3. Transformer capacity 25,000 kva. per phase. Short-circuit reactance 10 per cent

Frequency 60 cycles per second. Z_2 is very large compared to Z_1
These curves apply to Y-Y connection with both neutrals grounded
For other bank connections see Table I

If identical lightning waves arrive simultaneously on two phases (*b* and *c*) with no wave on the third phase (*a*), the voltage from *b* and *c* to *a* will be the same as above (Fig. 9) and the secondary voltages therefore will be the same.

When identical lightning waves arrive simultaneously on all three phases in banks having one side connected in delta, no voltage is produced on the secondary side because the voltages in the three phases of the delta are equal and in phase.

C. *Finite Secondary Circuit*

All voltage relations established in the paper were arrived at under assumption that the time length T_3 of the secondary circuit is at least half of the total length t_2' of the secondary wave. In case the secondary circuit is shorter than that, the crest and the front of the secondary wave may be essentially different from those given above. This is caused by the phenomena of repeated reflections of the secondary wave from both ends of the circuit.

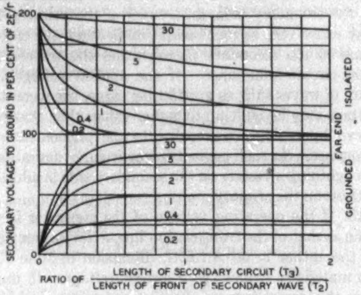

FIG. 11—SECONDARY TERMINAL VOLTAGE DUE TO REPEATED REFLECTIONS IN FINITE GROUNDED AND ISOLATED SECONDARY CIRCUITS. NUMBERS ON CURVES INDICATE CORRESPONDING VALUES OF Z_2/Z_1

In case the far end of the secondary circuit is grounded directly or through a relatively (to Z_2') low surge impedance the resultant voltage will be lower. This is the most common condition in practise because, as was explained in the paper (Fig. 9), in case the secondary external circuit is connected in delta or Y the point in that circuit, (like the neutral for example), where two secondary waves (caused by a single primary wave) meet, acts as if it were grounded (except bank connection in No. 1, Table I with primary wave on more than one phase). Fig. 11. Zone grounded.

In case the far end of the secondary circuit is open or grounded through a relatively (to Z_2') high surge impedance the voltage will be higher and its front steeper than was given previously in this paper. Such a condition is found, for example, where low voltage (secondary) buses are left connected to transformers but are disconnected from generators. Fig. 11. Zone isolated.

In case a generator is connected to transformer by means of a cable, the repeated reflections in the cable may produce voltages higher than those produced in case the generator is connected to the transformer directly. However, this voltage will be lower than that produced in the cable with generator disconnected. Fig. 12. The shape and the amplitude of these voltages can be calculated with sufficient accuracy on the assumption that the cable of surge impedance Z_2 and of a given length is equivalent to a condenser C_2 whose capacitance is equal to that of the cable. (Fig. 12.)

CONCLUSIONS

The voltage on the secondary terminals of a transformer produced by an impact of a lightning wave on the primary side consists of four components. The magnitude of the voltage due to the fourth component generally is much higher than that produced by the second and third components and may be exceeded only by that produced by the first, under certain special conditions.

When the surge impedance Z_2' of the external secondary circuit is very high or the circuit is short, amplitudes of each of the four components may be dangerously high.

The voltages due to the first three components depend on the relative position and distribution of windings and of the core, and may be essentially different in transformers of identical operating characteristics, like short-circuit reactance, kva. rating, etc., but of different types of design.

The fourth component, the electromagnetic, depends only on the short-circuit reactance of a transformer and is the same for transformers having the same reactance (in henrys) and the same turn ratio no matter how different may be their design features.

For the above reasons, in a general discussion of characteristics of lightning voltages transmitted to the secondary network, little can be said about the first three components that would be applicable to all transformers, particularly because the essential design features of a given transformer are generally known only to its manufacturer. On the other hand, the value of short-circuit reactance and of turn ratio of transformers is generally known and therefore very definite and specific conclusions can be drawn in regard to the voltage produced by the fourth component.

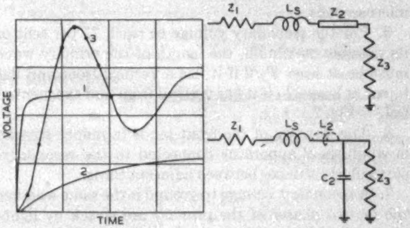

FIG. 12—EFFECT OF CABLE AND CAPACITOR ON SECONDARY VOLTAGE TO GROUND (CALCULATED)

1. Generator (Z_3) connected directly. $Z_2 = 0$
2. Generator (Z_3) connected through cable (Z_2)
   Repeated reflections are neglected
3. Generator (Z_3) connected through cable (Z_2)
   Repeated reflections are taken into account. Dots indicate points calculated by method of repeated reflections. Curve is calculated with total capacitance (C_2) and total inductance (L_2) of cable substituted for its surge-impedance (Z_2)

The following conclusions may be drawn concerning voltages produced on secondary circuits by the *electromagnetic components.*

1. The effect of a transformer on lightning voltages, transformed electromagnetically to the secondary network, is practically the same as that of an inductance equal to the short-circuit inductance of the transformer, connected between the primary and secondary lines (in the equivalent circuit reduced to 1:1 ratio). Figs. 3 and 4.

2. The ratio of crests of primary and secondary voltage is practically equal to the ratio of turns of the respective windings, where surge impedance of primary circuit is negligible in comparison with the effective surge impedance ($Z_2 = Z_2' r^2$) of the secondary circuit, as often is the case. (Fig. 5.)

3. On overhead transmission lines the amplitude of a relatively long (20 microseconds or more) traveling wave may be six times the system operating voltage (crest). It follows that such a wave may produce a secondary voltage of 3.4 to 6 times the secondary operating voltage, depending upon the transformer bank connection used. (Table I)

4. Although the front of an incident traveling wave may be vertical, the front (T_2) of the secondary wave is always slanted due to the effect of short-circuit inductance of the transformer. The front (T_2) is shortest where transformers of large kva. capacity and of relatively low voltage and reactance are connected to circuits of high-surge impedance, and is longest where the opposite is the case. In general the length of the front ranges from a few to a hundred microseconds (Fig. 10), but in exceptional cases may reach several thousand microseconds.

5. For the secondary voltage to reach 75 per cent of its possible maximum, the length of the primary wave must be at least $T_2/2$ if it has a vertical front and flat top or at least $2T_2$ if it has vertical front and exponential tail. (Fig. 7.)

6. The slanting of the front tends to reduce stresses in windings of apparatus connected to the secondary, particularly stresses between adjacent turns.

7. A secondary voltage to ground is the same whether one or two phases of the primary are struck by lightning. If either primary or secondary, or both, are connected in delta, then, when all three primary phases are struck by equal traveling waves, no secondary voltages will be produced.

8. With the exception of the Y-Y bank with both neutrals grounded, all three-phase bank connections produce zero lightning voltage at the isolated neutral of generators connected in Y.

9. A few hundred feet of metal-clad cable, connected between a transformer and generator, may materially slope the front of a secondary voltage wave, but substantially increase the amplitude if the primary wave is sufficiently long. The same is true of relatively small capacitances connected from the generator terminals to ground. (Fig. 12) (See Part II)

10. A neutral grounding resistance on the primary side tends to reduce the amplitude and shorten the front of the secondary wave. A reactance tends to increase the length of the front and thereby reduce the maximum amplitude produced by relatively short waves. In practise, the effect of the resistance will be small, while that of the reactor may be appreciable.

11. Protective gaps and lightning arresters are more effective in reducing secondary voltage when connected on the primary side. When connected on the secondary side, the sudden arcing of the gap or of the arrester may cause considerable stresses between turns of connected apparatus. This is not true in case the arrester has the proper negative volt-ampere characteristics.

12. Where capacitances or metal-clad cables are used on the secondary side, an arcover of the primary line may cause very serious oscillation between the capacitance of the secondary network and the transformer short-circuit inductance. If the applied voltage is a train of waves such as a switching surge, the secondary voltage may be built up to higher values than that produced by a single wave, due to the phenomenon of resonance. In such cases it is particularly desirable to use lightning arresters on the secondary side in addition to those on the primary.

13. If the operating voltage of the generator is the same as that of the transmission line, a device described by the author in an A.I.E.E. discussion in June 1930 can materially reduce the transient voltage. In many cases this device is more effective and economical than a 1:1 delta Y transformer bank that reduces voltages to 57 per cent (or less) of the value of the incoming wave.[6]

14. Where the "time length" (T_3) of a secondary circuit is short, in comparison with the length of the secondary wave, voltages in excess of that mentioned in conclusion 2 may result (Fig. 11). For this reason, particularly high secondary voltages may be expected on low-voltage buses disconnected from the apparatus and the network.

15. The above analysis was made for a transformer. It is obvious, however, that the same formulas apply also to the analysis of a rotating machine and of an induction regulator due to the presence of "transformer action" between windings.

Part II

When a lightning wave strikes the terminal of the primary winding of a transformer, four distinct phenomena contribute to the transient in the secondary winding. These are discussed in the first part of the paper. The electromagnetic transfer of the lightning wave has predominating influence on the secondary transient as far as secondary terminal voltages are concerned, provided lines, cables or generators of relatively low surge impedance are connected to the secondary winding.

A. SINGLE-PHASE TRANSFORMER WITH GROUNDED NEUTRAL

Calculations for single-phase connection are chosen to derive simple equations for the electromagnetic transformation of lightning waves in inductive windings. They can be applied directly for Y-Y connected transformer banks with both neutrals grounded, and are the basis for solution of other transformer connections.

Fig. 3a shows the diagrammatic sketch of a two-winding transformer, with primary and secondary transmission lines. This can be simplified to Fig. 3b, where the surge impedances are reduced to resistances (provided the lines are long) and the applied voltage has twice the amplitude of the traveling wave of Fig. 3a. The internal resistances of the windings are in general too small to affect the result and therefore can be neglected. If in any case it should appear necessary to take these resistances into account, they may be inserted in series with the corresponding surge impedances of the lines. The fundamental equations for voltages and currents in operational form are:

$$2E = (Z_1' + pL_1')\, i_1 \pm pMi_2'$$
$$0 = (Z_2' + pL_2')\, i_2 \pm pMi_1'$$

where $p = \dfrac{d}{dt}$ is Heaviside's operator. Solving these two simultaneous equations for i_2 gives

$$i_2 = \mp \frac{2E\,M'}{L_1'L_2' - M'_2}\,\frac{p}{p^2 + 2\alpha' p + \omega_o'^2};$$

$$e_2 = i_2 Z_2' = \mp \frac{2E\,M'Z_2'}{L_1'L_2' - M'^2}\,\frac{p}{p^2 + 2\alpha' p + \omega_o'^2}$$

The solution[8] of this equation is

$$e_2 = \mp \frac{2E\,M'Z'_2}{L_1'L_2' - M'^2}\,\frac{\epsilon^{-m't} - \epsilon^{-n't}}{n' - m'}$$

$$= \mp \frac{2E\,M'Z_2'}{\sqrt{(L_1'Z_2' - L_2'Z_1')^2 + 4Z_1'Z_2'M'^2}}\,(\epsilon^{-m't} - \epsilon^{-n't}) \qquad \text{(1a)}$$

where

$$n' = \frac{L_2'Z_1' + L_1'Z_2' + \sqrt{(L_2'Z_1' - L_1'Z_2')^2 + 4Z_1'Z_2'M'^2}}{2\,(L_1'L_2' - M'^2)};$$

$$m' = \frac{L_2'Z_1' + L_1'Z_2' - \sqrt{(L_2'Z_1' - L_1'Z_2')^2 + 4Z_1'Z_2'M'^2}}{2\,(L_1'L_2' - M'^2)} \qquad \text{(1b)}$$

The same solution has been published by various authors. In a practical case the sign should be determined by the polarity of the transformer. It will be assumed positive in the rest of the paper. The constants used in the equation are all physical constants, which should be calculated from the physical configuration of the coils and transmission lines. The equation can be rewritten using constants which are converted to the primary side (or secondary) as for the equivalent transformer circuit (Fig. 3c).

$Z_1 = Z_1'$; $L_1 = L_1'$; $M = rM'$; $L_2 = r^2L_2'$; $Z_2 = r^2Z_2'$

r = ratio of primary turns to secondary turns.

Then

$$e_2 = \frac{2E}{r}\,\frac{Z_2M}{\sqrt{(L_1Z_2 - L_2Z_1)^2 + 4Z_1Z_2M^2}}\,(\epsilon^{-mt} - \epsilon^{-nt}) \qquad \text{(2)}$$

$$e_2\, max = 2E\,A\,C \qquad \text{(3)}$$

where amplitude factor

$$A = \frac{1}{r}\,\frac{Z_2M}{\sqrt{(L_1Z_2 - L_2Z_1)^2 + 4Z_1Z_2M^2}} \qquad \text{(4)}$$

and crest factor C is the maximum value of $(\epsilon^{-mt} - \epsilon^{-nt})$ as t is varied. The expressions for n and m are identical to those for n' and m' using constants of the equivalent circuit in place of the physical constants. Values of Z_1 and Z_2, representing surge impedances of transmission lines, cables and generators can be obtained quite readily, while the evaluation of L_1, L_2 and M requires involved calculations or tests. In any given transformer the values of L_1, L_2 and M are not constant due to saturation and skin effect in the core. Fortunately results within engineering accuracy can be obtained although the absolute values of inductances are not known. The following transformation will show that a wide variation in absolute values of inductance produces only a small variation in the amplitude of the secondary wave and therefore a simplified equation can be used.

The amplitude factor (4) can be rewritten as

$$A = \frac{1}{r}\,\frac{1}{\sqrt{\left(\frac{L_1}{M} - \frac{Z_1}{Z_2}\,\frac{L_2}{M}\right)^2 + 4\,\frac{Z_1}{Z_2}}}$$

and substituting:

$c = \dfrac{M}{L_1}$; $d = \dfrac{M}{L_2}$; $q = \dfrac{Z_2}{Z_1}$ = effective ratio of secondary to primary surge impedance

$$q = r^2 s;\ s = \frac{Z_2'}{Z_1'}$$

and

$L_s = \dfrac{L_1L_2 - M^2}{L_2} \cong L_1 + L_2 - 2M$ = Short-circuit inductance in henrys.

$$A = \frac{1}{r}\,\frac{q}{\sqrt{\left(\frac{q}{c} - \frac{1}{d}\right)^2 + 4q}} \qquad \text{(5)}$$

The crest factor is a function of n/m

$$n = \frac{Z_1}{2L_s}\left(1 + \frac{dq}{c} + \sqrt{\left(1 + \frac{dq}{c}\right)^2 - 4\,\frac{L_s}{M}\,dq}\right)$$

$$m = \frac{Z_1}{2L_s}\left(1 + \frac{dq}{c} - \sqrt{\left(1 + \frac{dq}{c}\right)^2 - 4\,\frac{L_s}{M}\,dq}\right) \qquad \text{(6)}$$

Approximate Solution. In concentric construction either L_1 or L_2 is approximately equal to M, depending on the position of the coils with respect to each other and the core, and therefore either c or d will be approximately equal to unity. In interleaved construction the value of L_1 is approximately equal to L_2 and therefore c is approximately equal to d. In the following tables the variation of the maximum secondary voltage is given for two conditions, *i.e.*, $M/L_1 = 1$ and $M/L_2 = 1$. The short-circuit inductance L_s remains constant at 0.2 henrys, while it is arbitrarily assumed that the primary or secondary inductances change from 1 henry (air-core) to 100 henrys (iron-core). Amplitude factors, crest factors and maximum secondary voltages obtained with air-core inductance are expressed in per cent of the corresponding values obtained with iron-core inductance

Constants	L_1 henrys	L_2 henrys	M henrys	$C = \frac{M}{L_1}$	$d = \frac{M}{L_2}$	$\frac{L_s}{M}$
Iron-core (i)	100	M	99.8	0.998	1	0.002
Air-core (a)	1	M	0.8	0.8	1	0.25
Iron-core (i)	M	100	99.8	1	0.998	0.002
Air-core (a)	M	1	0.8	1	0.8	0.25

$q = \frac{Z_2}{Z_1}$	$\frac{A(a)}{A(i)} \times 100$	$\frac{C(a)}{C(i)}$	$\frac{e_2 \max (a)}{e_2 \max (i)} \times 100$	
1	99.4	78.8	78.4	$L_2 = M$
100	80.3	99.5	80.0	
1	99.2	75.5	74.8	$L_1 = M$
100	98.3	98.0	97.3	

This table shows that the maximum secondary voltages for air-core are 75 to 97 per cent of those for iron-core or in other words, the secondary voltage varies not over 25 per cent even for the extreme conditions assumed here. In nearly all practical cases L_s/M will be smaller than 0.1, which will decrease the variation of maximum voltage to 10 per cent or less. Considering the narrow range between air-core and iron-core values of maximum secondary voltages, it is justifiable to substitute $c = 1$ in equations (**5**) and (**6**). The equation for secondary voltage then is reduced to a very simple form (exponent m becomes so small compared to n that ϵ^{-mt} equals 1 for all practical purposes).

$$e_2 = 2E \, \frac{1}{r} \, \frac{q}{q+1} \left(1 - \epsilon^{-\frac{Z_1}{L_s}(1+q)t}\right) \tag{7}$$

This equation gives a good approximation for the front and the maximum amplitude of the secondary wave and e_2 is reduced to a function of only the effective ratio of secondary to primary surge impedances, the primary surge impedance and the short-circuit inductance of the transformer. In the case of infinitely long wave this equation would obviously give incorrect results for the tail of the secondary waves. However, for waves of the length found in practise, good approximate results are obtained when the approximate equations (**8b**) and (**9b**) are used (Fig. 4). The circuit representing equation (**7**) is shown in Fig. 3f and consists of two transmission lines joined by the short-circuit inductance of the transformer.

Finite Wave. Equations developed so far represent the secondary wave when the lightning wave has rectangular front and infinitely long tail. By means of Duhamel's theorem or Heaviside's shifting theorem[8] equations can be obtained for the secondary voltage, when the applied wave is not rectangular. Equations (**8a**) and (**9a**), give voltages for the circuits of Fig. 3c and (**8b**) and (**9b**) for circuit Fig. 3f.

If the applied wave at the terminals of the equivalent circuits

$$e = 2E\,(\epsilon^{-at} - \epsilon^{-bt})^*$$

then

$$e_2 = 2E\,A \left\{ \left(\frac{a}{m-a} - \frac{b}{m-b}\right)\epsilon^{-mt} - \left(\frac{a}{n-a} - \frac{b}{n-b}\right)\epsilon^{-nt} + \left(\frac{a}{n-a} - \frac{a}{m-a}\right)\epsilon^{-at} - \left(\frac{b}{n-b} - \frac{b}{m-b}\right)\epsilon^{-bt} \right\} \tag{8a}$$

$$e_2 = 2E\,A \left\{ \left(\frac{n}{n-a}\right)\epsilon^{-at} - \frac{n}{n-b}\,\epsilon^{-bt} - \left(\frac{a}{n-a} - \frac{b}{n-b}\right)\epsilon^{-nt} \right\} \tag{8b}$$

or if the applied wave is

$$e = 2E\,(\epsilon^{-at})^*$$

$$e_2 = 2E\,A \left\{ \left(\frac{a}{n-a} - \frac{a}{m-a}\right)\epsilon^{-at} + \frac{m}{m-a}\,\epsilon^{-mt} - \frac{n}{n-a}\,\epsilon^{-nt} \right) \right\} \tag{9a}$$

or $$e_2 = 2E\,A_1\,(\epsilon^{-at} - \epsilon^{-nt}) \tag{9b}$$

where $$A_1 = \frac{n}{n-a}\,A = \frac{n}{n-a}\,\frac{1}{r}\,\frac{q}{q+1}$$

B. THREE-PHASE BANKS

In the previous discussion it is shown that the transformer circuit of Fig. 3a can be simplified to that of Fig. 3f without appreciable error. The same principle can be applied to three-phase connections of transformers. Fig. 9 shows the equivalent circuits for Y-delta connected banks. Only the solution for the condition of grounded primary neutral is discussed here.

The general solution for the voltage e_b across surge impedance Z_b is

$$e_b = 2E\,\frac{1}{r} \left\{ \frac{Z_b Z_c}{Z_a Z_b + Z_a Z_c + Z_b Z_c} \left(1 - \frac{n\epsilon^{-mt} - m\epsilon^{-nt}}{2\omega}\right) + \frac{Z_b}{Z_a}\,\frac{\epsilon^{-mt} - \epsilon^{-nt}}{2\omega} \right\} \tag{10}$$

---

*These applied waves are equivalent to traveling waves of $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$ and $e = E\,\epsilon^{-at}$ respectively.

and the general solution for voltage e_c is

$$e_c = 2E\frac{1}{r}\,\frac{Z_bZ_c}{Z_b+Z_aZ_c+Z_bZ_c}\left\{1-\frac{1}{2\omega}(n\epsilon^{-mt}-m\epsilon^{-nt})\right\} \tag{11}$$

where $m = \alpha - \omega; \quad n = \alpha + \omega$

$$\alpha = \frac{1}{2}\,\frac{L_a(Z_b + Z_c) + L_b(Z_a + Z_b)}{L_aL_b};$$

$$\omega_o^2 = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{L_aL_b}; \omega = \sqrt{\alpha^2 - \omega_o^2}$$

By inspection the following relations will be found (all constants are transferred to the primary phase on which the lightning wave travels).

$Z_a = Z_1; Z_b = 2r^2Z_2' = 2r^2sZ_1 = 2qZ_1; Z_c = 2Z_1$

$L_a = L_s$ = short-circuit inductance of one transformer.

$L_c = 2\,L_s$

Substitution of these values in equation (**10**) gives

$$e_b = 2E\,\frac{1}{r}\,\frac{2q}{1+3q}\,(1 - \epsilon^{-nt}) \tag{12}$$

The value of n is given in Table I.[9] Keeping in mind that e_b is the voltage across the secondary winding and that Z_b represents the surge impedance of two secondary lines in series it will be found that the waves traveling on two secondary lines (the voltage at the third secondary line is zero) have an amplitude of one-half the value of e_b and are of opposite polarity.

$$e_2 = \pm\,2E\,\frac{1}{r}\,\frac{q}{1+3q}\,(1 - \epsilon^{-nt}) \tag{13a}$$

or $e_2 = \pm\,2E\,A\,(1 - \epsilon^{-nt})$ (**13b**)

Due to this fact the voltage at the generator neutral, where the two waves meet will be zero. The same reasoning can be applied to other bank connections, with the exception of the Y-Y bank with both neutrals grounded. Similarly the super-position of positive and negative waves in the phases of a delta-connected generator will cause a great reduction of voltage, where the secondary waves meet.

Equation (**13b**) is the solution for secondary voltages when identical lightning waves come in on one line only or on two lines simultaneously. If identical lightning waves come in on all three primary lines simultaneously no voltages will appear on the secondary lines. The same is true for all other bank connections of Table I with the exception of Y-Y bank with both neutrals grounded. Equation (**13b**) applies equally well for all bank connections, when tabulated values of amplitude factors (A) and time constants 1/11 are substituted.

Equation (**13b**) has the same form as (**7**) for single-phase connection (or Y-Y bank with both neutrals grounded) and equations (**8**) and (**9**) for waves with finite front and tail apply equally well requiring only the substitution of the correct amplitude factors and time constants.

The voltage on either of the two unexcited high-voltage lines becomes:

$$e_c = 2E\left(\frac{q}{1+3q} - \frac{1}{3}\,\epsilon^{-mt} + \frac{1}{3}\,\frac{1}{1+3q}\,\epsilon^{-nt}\right);$$

$$m = \frac{Z_1}{L_s} \qquad n = \frac{Z_1}{L_s}\,(1+3q) \tag{14}$$

For an infinite rectangular wave the final value of e_c is r times as great as that of the secondary wave. The fronts of the waves on the high-voltage lines are, however, in general much slower than the fronts of the secondary waves. The maximum value of the wave is at the most 1/3 that of the lightning wave.

C. Repeated Reflection

In drawing the simplified circuit of Fig. 3b it was assumed that primary and secondary lines are very long. This assumption made possible the change from surge impedance to pure resistance. Although this assumption may be justified for the primary lines, it would give wrong results when secondary lines consist of short cables or overhead lines between the secondary of the transformer and the generator terminal or the finite length of generator windings. In such cases the replacement of the surge impedance is valid only until the wave returns to the transformer terminal after being reflected from the far end of the secondary line. From that instant on the returned wave and its reflection on the transformer terminals should be added to the initial secondary wave (see Fig. 12). If the reflection operator on the far end of the line is R_g and the reflection operator on the transformer terminal is R_T, then the resulting secondary voltage for single-phase transformation due to reflection can be written as:

$$e_{2res} = e_2 + (R_ge_2 + R_TR_ge_2)_{(t-2T)} + (R_g^2R_Te_2 + R_g^2R_T^2e_2)_{(t-4T)} + (R_g^3R_T^2e_2 + R_g^3R_T^3e_2)_{(t-6T)} + \ldots \tag{15}$$

where $2T$ indicates the time interval during which the original wave travels twice the length of the secondary line, taking into account the velocity of propagation, which is different for transmission lines (1,000 ft. per microsecond) cables (500 ft. per microsecond) and generators (100 ft. per microsecond). The subscript $(t - 2T)$ indicates that the two waves in the brackets have to be added to the previous wave at time $t = 2T$. Suppose the secondary line or cable with surge impedance Z_2 is starting at the far end to a generator with a surge impedance of Z_3 ohms then

$$R_g = \frac{Z_3 - Z_2}{Z_3 + Z_2}$$

In Y-connected generators, Z_3 is the surge impedance of one phase, in delta-connected generators Z_3 = one-half the surge impedance of one phase.

For the simplified circuit of Fig. 3f the reflection factor R_T at the transformer terminals becomes

$$R_{\mathrm{T}} = \frac{Z(p) - Z_2}{Z(p) + Z_2} = \frac{pL_s + Z_1 - Z_2}{pL_s + Z_1 + Z_2}$$

$$= \frac{p + \dfrac{Z_1 - Z_2}{L_s}}{p + \dfrac{Z_1 + Z_2}{L_s}} = \frac{p + \beta}{p + \alpha}$$

The solution for R_{T} for an infinite long wave with rectangular front is

$$R_{\mathrm{T}} = \frac{\beta}{\alpha} + \epsilon^{-\alpha t}\left(1 - \frac{\beta}{\alpha}\right)$$

The applied wave in this case is, however, the original secondary wave (equation (7)) reflected at the generator end and equals

$$R_g e_2 = 2E\,\frac{Z_3 - Z_2}{Z_3 + Z_2}\,A\,(1 - \epsilon^{-\alpha t})$$

Using the Heaviside shifting theorem or Duhamel's theorem the reflected wave resulting when $R_g e_2$ is applied becomes

$$R_g R_{\mathrm{T}} e_2 = 2E\,\frac{Z_3 - Z_2}{Z_3 + Z_2}\,A\left\{\frac{\beta}{\alpha} - \epsilon^{-\alpha t}\left[\alpha t\left(\frac{\beta}{\alpha} - 1\right) + \frac{\beta}{\alpha}\right]\right\}$$

This wave travels out on the line, is reflected at the generator, and a wave $R_g^2 R_{\mathrm{T}} e_2$ returns. $R_g^2 R_{\mathrm{T}} e_2$ is the wave to be used as applied wave in this case.

$$R_g^2 R_{\mathrm{T}} e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_3 + Z_2}\right)^2 A\left\{\left(\frac{\beta}{\alpha}\right) - \epsilon^{-\alpha t}\left[(\alpha t)\left(\frac{\beta}{\alpha} - 1\right) + \frac{\beta}{\alpha}\right]\right\}$$

and the reflected voltage at the transformer becomes

$$R_g^2 R_{\mathrm{T}}^2 e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_3 + Z_2}\right)^2 A\left\{\left(\frac{\beta}{\alpha}\right)^2 - \frac{\epsilon^{-\alpha t}}{2}\left[(\alpha t)^2\left(\frac{\beta}{\alpha} - 1\right)^2 + 2\alpha t\left\{\left(\frac{\beta}{\alpha}\right)^2 - 1\right\} + 2\left(\frac{\beta}{\alpha}\right)^2\right]\right\}$$

or in general the nth reflection at the transformer terminal becomes

$$R_g^n R_{\mathrm{T}}^n e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_3 + Z_2}\right)^n A\left\{\frac{\beta^n}{\alpha^n} - \epsilon^{-\alpha t}\left[\frac{(\beta t)^n}{\underline{\lfloor n}} + \sum_0^n{}^k\left\{\left(\frac{\beta}{\alpha}\right)^k \frac{(\beta t)^{n-k}}{\underline{\lfloor n-k}} - \frac{\underline{\lfloor n}}{\underline{\lfloor n-k}}\,\frac{\beta^{n-k}}{\underline{\lfloor k}}\right.\right.\right.$$

$$\left.\left.\left.\sum_0^{k-1}{}^\gamma\,\frac{\alpha^{k-\gamma}}{\underline{\lfloor n-\gamma}}\,\frac{t^{n-\gamma}}{\underline{\lfloor k-1-\gamma}}\,\frac{\underline{\lfloor k-1}}{\underline{\lfloor \gamma}}(-1)^{k-1-\gamma}\right)\right\}\right]\right\} \quad (16)$$

or if the applied wave is of the form $e = 2E\,\epsilon^{-at}$

$$R_g^n R_{\mathrm{T}}^n e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_2 + Z_3}\right)^n A_1\left\{\left(\frac{\beta - a}{\alpha - a}\right)^n \epsilon^{-at}\right.$$

$$- \epsilon^{-\alpha t}\left[\frac{\beta - a}{\alpha - a}\,\frac{((\beta - a)t)^{n-1}}{\underline{\lfloor n-1}} - \frac{(-(\alpha - a)\,t)^{n-1}}{\underline{\lfloor n-1}}\right.$$

$$+ \frac{(\beta t)^n + (-\alpha t)^n}{\underline{\lfloor n}} + 1$$

$$+ \sum_1^{n-1}{}^k\left(\frac{\beta - a}{\alpha - a}\,\frac{((\beta - a)t)^{n-1}}{\underline{\lfloor n-1-k}}\,\frac{1}{((\alpha - a)t)^k}\right.$$

$$- \frac{(-(\alpha - a)t)^{n-1-k}}{\underline{\lfloor n-1-k}\;\underline{\lfloor k}}\,\frac{\underline{\lfloor n-1}}{\underline{\lfloor n-1-k}}$$

$$+ \frac{\underline{\lfloor n}}{\underline{\lfloor n-k}}\,\frac{(-\alpha t)^{n-k}}{\underline{\lfloor n-k}\;\underline{\lfloor k}} - \frac{\underline{\lfloor n}}{\underline{\lfloor n-k}}\,\frac{(\beta - a)^{n-k}}{\underline{\lfloor k}}$$

$$\sum_0^{k-1}{}^\gamma\left\{\frac{(-(\alpha - a)^{k-1-\gamma}}{\underline{\lfloor n-1-\gamma}}\,\frac{t^{n-1-\gamma}}{\underline{\lfloor k-1-\gamma}}\,\frac{\underline{\lfloor k-1}}{\underline{\lfloor \gamma}}\right\}$$

$$+ \frac{\underline{\lfloor n}}{\underline{\lfloor n-k}}\,\beta^{n-k}\sum$$

$$\left.\left.\left.\sum_0^k{}^\gamma\left\{\frac{(-\alpha)^{k-\gamma}}{\underline{\lfloor n-\gamma}}\,\frac{t^{n-\gamma}}{\underline{\lfloor k-\gamma}}\,\frac{\;}{\underline{\lfloor \gamma}}\right\}\right)\right]\right\} \quad (17)$$

$$A_1 = A\,\frac{\alpha}{\alpha - a};\ \beta = \frac{Z_1 - Z_2}{L_s} = \frac{Z_1}{L_s}(1 - q);$$

$$\alpha = \frac{Z_1 + Z_2}{L_s} = (1 + q)\,\frac{Z_1}{L_s}$$

a = exponent of the applied wave.

Equations (16) and (17) can be used for single-phase connections and also for those bank connections, which can be represented by equivalent circuits similar to Fig. 9A.

The equation for equivalent circuit of Fig. 9B for rectangular primary wave when a short cable or line (Z_2) is placed between the generator (Z_3) and the secondary winding of the transformer is

$$R_g^n R_{\mathrm{T}}^n e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_3 + Z_2}\right)^n A\left[D^n(1 - \epsilon^{-\alpha t})\right.$$

$$+ \alpha D^n\,\epsilon^{-\alpha t}\sum_1^n{}^k\left(\frac{\underline{\lfloor n}}{\underline{\lfloor n-k}}\left(\frac{B}{D}\right)^k \frac{1}{\underline{\lfloor k}}\sum_0^{k-1}{}^\gamma\right\{$$

$$\left.\left.\sum_0^{k-1}{}^\gamma\left\{\frac{(-\alpha)^{k-1-\gamma}}{\underline{\lfloor \gamma}}\,\frac{t^{k-\gamma}}{\underline{\lfloor k-\gamma}}\,\frac{\underline{\lfloor k-1}}{\underline{\lfloor k-1-\gamma}}\right\}\right)\right] \quad (18)$$

$$D = \frac{1-3q}{1+3q}; B = \frac{6q}{1+3q}; \alpha = \frac{Z_1}{L_s}(1+3q); q = \frac{Z_2}{Z_1}.$$

Simplified Solution. A simpler method of solution for secondary voltages due to repeated reflections consists in the replacement of the surge impedance of the secondary line or cable by their capacitance to ground (Fig. 12). In case the inductance of the line or cable is comparable to the short-circuit inductance of the transformer it should be added to L_s. The solution for single-phase transformation and all bank-connections, which can be represented by an equivalent circuit similar to Fig. 3a, is for

$e = E$ (traveling wave)

$$e_2 = 2E\,A\left\{1 - \frac{\omega_o}{\omega}\epsilon^{-\alpha t}\sin\left(\omega t + \tan^{-1}\frac{\omega}{\alpha}\right)\right\}$$

$$\text{if } \omega_o^2 > \alpha^2 \qquad \textbf{(19a)}$$

$$e_2\,max = 2E\,A\left(1 + \epsilon^{-\frac{\alpha}{\omega}\pi}\right)$$

since $t_1 = \dfrac{\pi}{\omega}$ is the time at which the maximum occurs.

$$e_2 = 2E\,A\left\{1 - \frac{1}{n-m}(n\epsilon^{-mt} - m\epsilon^{-nt})\right\} \text{ for } \alpha^2 > \omega_o^2$$

$$\textbf{(19b)}$$

for $e = E\,\epsilon^{-at}$ (traveling wave)

$$e_2 = 2E\,A\left\{\frac{\omega_o^2}{(\alpha-a)^2+\omega^2}\left[\epsilon^{-at}\right.\right.$$

$$\left.\left. - \frac{\sqrt{(\alpha-a)^2+\omega^2}}{\omega}\epsilon^{-\alpha t}\sin\left(\omega t + \tan^{-1}\frac{\omega}{\alpha-a}\right)\right]\right\}$$

$$\text{if } \omega_o^2 > \alpha^2 \qquad \textbf{(20a)}$$

$$e_2 = 2E\,A\frac{\omega_o^2}{(\alpha-a)^2+\omega_1^2}\left\{\epsilon^{-at}\right.$$

$$\left. - \frac{(a-m)\,\epsilon^{-nt} - (a-n)\,\epsilon^{-mt}}{n-m}\right\}$$

$$\text{if } \alpha^2 > \omega_o^2 \qquad \textbf{(20b)}$$

where

$$\alpha = \frac{1}{2}\left(\frac{Z_1}{L_s} + \frac{1}{C_2 Z_3}\right); \omega_o^2 = \frac{Z_1 + Z_3}{Z_3}\;\frac{1}{L_s C_2};$$

$$\omega = \sqrt{\omega_o^2 - \alpha^2}$$

$$n = \alpha + \omega_1; m = \alpha - \omega_1; \omega_1 = \sqrt{\alpha^2 - \omega_o^2}$$

It should be noted that in this case $q = r^2 \dfrac{Z_3'}{Z_1'}$.

Solution by means of equations (**19**) and (**20**) shows a good agreement, with the solution obtained by method of repeated reflections when the front of the secondary wave is longer than twice the length of the line or cable. If the front of the secondary wave is shorter than double the length of the line or cable only approximate results are obtained since in such a case the repeated application of steep fronts is neglected. Equations (**19**) and (**20**) can also be used, to calculate the voltage rise at the generator terminals, when a protective capacitor is used.

CONCLUSIONS

1. For the purpose of calculation of the electromagnetic transformation of lightning waves, the complicated transformer circuit can be replaced by an inductance equivalent to the short-circuit reactance of the transformer. The results obtained are in nearly all practical cases within 10 per cent of the real values.

2. Similarly bank connections can be reduced to simple equivalent circuits, which allow ready calculation of secondary voltages in networks.

3. Repeated reflections on short secondary lines or cables which are connected to generators or transformers on the far end may materially increase the amplitude of the secondary voltage over that obtained if the generator had been connected directly although the rate of rise of secondary voltage may be lower. The same is true, if small protective capacitors are used.

ACKNOWLEDGMENT

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Discussion

F. W. Peek, Jr.: The paper gives a practical method of determining the voltages that pass through the low side when the high side of a transformer is subjected to lightning. By means of this method the stresses on generators and other low side apparatus can be predetermined and adequate protective measures taken. This paper is one of a pioneer series by Mr. Palueff and his associates (Messrs. Weed, Blume, Boyajian, Brand and others) that has placed the subject of transients in transformer windings upon a scientific and engineering basis. The results of this study have been more than a theoretical treatment. From it came the now well-known non-resonating transformer in which the voltage stresses in any part of the winding and through the insulation structure are always uniform and relatively the same whether the applied voltage is at operating frequency, lightning impulses or high frequency. This ideal transformer thus has the same relative strength under all types of voltages. The designer knows just how much insulation to place at any particular part. He also knows that, whatever the type of test—60 cycles or lightning—the stresses are the same. Thus for this transformer the usual A.I.E.E. acceptance tests are equivalent to lightning tests.

I bring up the matter of testing because I feel that in view of the new knowledge resulting from this series of papers the commercial testing of transformers at operating frequencies should be reviewed by the Machinery Committee of the Institute and recommendations made for revision, if it seems advisable. A method of determining the corona point in oil in the 60-cycle test should be considered for reasons that I will bring out later. *I further recommend that this same committee set up methods, waves, etc., for making a lightning test on transformers* if such a test seems desirable. I suggest an A.I.E.E. committee for this work because of the highly technical problems involved. High-voltage testing is rather expensive and, since much depends upon it, should be made as effective as possible. I do not hesitate to recommend the consideration of commercial lightning testing after years of experience with such testing on complete transformers for the purpose of design. This testing was necessary because we have been building transformers lightning proof for a number of years. Someone has referred to impulse tests made in Europe. These tests are of such a mild value as to be useless.

One of the most important problems in establishing lightning tests is to devise means for detecting incipient turn-to-turn, coil-to-coil, or other failures. This is so because the operator must feel assured beyond any reasonable doubt that the transformer is as good after the test as it was before the test. Follow-up 60-cycle tests will not necessarily detect such injuries but they will be developed later by lightning strokes incidental to operation. Detection is difficult because of the short duration of the stroke. Long experience has shown us that in obtaining design data it is of the utmost importance to know when damage starts. This follows because the effect of lightning on partly damaged highly stressed insulation is cumulative.

A casual or superficial consideration of the subject would probably lead to the conclusion that the present A.I.E.E. tests should be abolished if lightning tests are established. Experience shows that such conclusions are erroneous. The tests at commerical frequencies are applied long enough to establish the strength of the major insulation, to detect manufacturing mistakes and, in case of over-stress, to indicate it by copious corona discharges. It is true that tests at operating frequency may also cause deterioration in poor designs, but such designs are easily detected by copious corona formation during such tests. Such discharges not only indicate high stresses but also a weak transformer for lightning. For example, corona between coils would indicate a design giving a high gradient between coils and short life under lightning. For this reason it has been our practise to design so that corona does not occur over the test range and to apply methods of detection of corona during tests.

F. W. Gay: The time is probably not far distant when all transformers to be used on high-voltage lines will receive a surge test and some manufacturers are even now waiting for the standardization of tests before purchasing one or more surge generators.

Transformers used on lines having arcover distances greater than 30 inches will probably be equipped with some means for assisting the equalization of stresses among winding turns and layers.

The electrical engineer is supplied by nature with three fundamental tools which he may use if he will in solving all his problems, *i. e.*, resistance, inductance, and capacitance. It is however, a characteristic of the electrical engineer to try and complete an undertaking while using only one of his tools.

It is entirely possible to provide a transformer with a resistance path in parallel with a winding to be protected and electrostatically couple the resistance path with the winding and with the transformer terminals, so that relatively very great currents will pass to charge the winding for high-frequency surges but relatively insignificant current will pass through the resistance at normal frequency.

It is also entirely possible to wind in juxtaposition with the many turn low-frequency high-voltage winding a surge winding of relatively very few turns and couple this high-frequency winding to the transformer terminals by condensers.

It is obvious that a shunt path of very low surge impedance is thus provided through the condenser and high-frequency few-turn surge winding. A surge of steep wave front will instantly pass through the condenser and pile up across the surge winding. The surge current will build up very fast in this surge winding and will induce a voltage in the low-frequency many-turn winding. This voltage in the low-frequency winding will serve to charge its turns and layers.

Mr. Palueff solves a problem in which the building up of the main magnetic flux in his primary winding is on the order of 2,000 microseconds. I propose to apply a surge winding about and close to a high-voltage winding having a corresponding time element on the order of a few tenths of one per cent as long; for example, having a time element on the order of say 10 microseconds instead of on the order say of 2,000 microseconds.

The time element required for voltage to build up on a secondary winding due to capacity effect between a primary and secondary winding is given for Mr. Palueff's transformer as on the order of 0.3 of one per cent of the long time element. With a surge winding such as I have outlined the distribution of voltage in the many-turn high-voltage winding, due both to inductance and capacitance, should be completed in a few hundredths of a microsecond.

A winding such as I have outlined should include in its circuit so great a series resistance that oscillation is completely damped. The success of such a surge winding requires:

1. The distribution of stress throughout the many-turn high-voltage winding so fast that substantially no oscillatory energy is stored in the high-voltage winding.
2. The natural period of oscillation of the surge winding designed so long that the many-turn high-voltage winding is able to maintain a substantially uniform voltage distribution as the voltage across the surge winding (relatively) slowly collapses to substantially zero.
3. Enough resistance in the surge winding to damp out oscillations. It would appear that considerable study could profitably be given to the use of resistance and reactance as a means of equalizing stresses in high-voltage transformers.

It is obvious that the above discussion applies primarily to those surge phenomena which Messrs. Palueff and Hagenguth classify under components 1 and 4.

In order to squelch components 2 and 3 more capacity must be used than can be justified for this purpose alone. It becomes necessary therefore to cause the capacity so employed to function also for power factor correction and voltage regulation; and to still further distribute costs a method of transformer winding switching has been developed whereby a substantial saving can be made in transformer material, and transformer efficiencies can be materially improved, while a single control is common to all the above functions. This switching of transformer windings becomes more important as transformer iron is improved and the per pound cost of iron with respect to copper tends to increase while the total copper loss to total iron loss ratio simultaneously increases.

In practise what amounts to a six-phase transformer is used to accomplish all the above results but for simplicity in the following explanation and to avoid design discussion six single-phase transformers are assumed as follows:

1. The six single-phase transformers are assumed to be designed for 13.2-kv. to 4-kv. operation at a magnetic density of 15,000 gausses. Each transformer is assumed to have permanently connected across its 4-kv. terminals a capacitor of 60 per cent of the transformer kva. capacity.

2. The above six transformers are permanently connected in pairs in series on the high side and in series on the low side and these three pairs of transformers are then connected in delta on a 13.2-kv. circuit high side and in delta on a 4-kv. circuit low side. With this connection all six transformers and their associated capacitors are operating at one-half voltage. The magnetic density in the transformer iron will then be 7,500 gausses.

3. A separate two-pole single throw switch is provided for each pair of series connected transformers. Those two high-voltage windings connected in series across two 13.2-kv. phase wires have their common junction point connected to one pole of the switch so that on closing, the common junction point is joined to the third phase wire and the two high-tension windings are connected in open delta by the closing of the switch. In a similar manner the two corresponding low-voltage windings connected in series across a corresponding two of the 4-kv. phase wires have their common junction point connected to one pole of the switch so that on closing, the common junction point is joined to the third 4-kv. phase wire and the two low-voltage windings together with their connected capacitors are thus connected in open delta.

By successively closing the three two-pole single-throw switches the three transformers and their associated capacitors are successively changed from half-voltage to double-voltage operation until when all three switches are closed both 13.2-kv. and 4-kv. windings and their associated capacitors are connected in parallel delta.

The following results are obtained.

1. Surges under the Palueff-Hagenguth 2 and 3 classification are squelched.
2. For a given 80 per cent power factor load the active material required in the transformers is less than half what would be required in a normal transformer of high light load efficiency.
3. The transformer losses are reduced for very light loads and for overloads and are substantially the same for intermediate loads.
4. Transformer reactances are reduced at heavy loads.
5. 13.2-kv. line carrying capacity is increased by improved power factor.
6. Voltage regulation may be obtained by controlling the closing of the three two-pole switches.
7. Economy of material is obtained by combining the above functions in one device.

L. V. Bewley: The authors have shown that the transient transmitted to the secondary circuit consists of three distinct components:

1. The initial very fast electrostatic impulse.
2. The long slow electromagnetic transient.
3. The oscillatory component, of small amplitude, oscillating about (2) as an axis.

They have further shown that each of these components may be considered separately, due to the great disparity in their time constants, and have gone into considerable detail concerning the electromagnetic transient.

Some time after the above development was made by the authors, I undertook a mathematical analysis of the internal oscillations in the primary and secondary windings of a transformer. It was found that complete solutions were possible only for grounded or isolated terminal conditions; but that the initial distributions could be found for general terminal impedances, and that the equations which determine the axes of internal oscillations reduce to those given in Part II of the paper by Palueff and Hagenguth. It is interesting to attempt a verification of their conclusions by deductions from the general equations given in my paper.* As stated in the closing discussion thereto, one of the principal uses to which these general equations may be put is to examine the validity of approximate equivalent circuits. In submitting this discussion I do not wish to convey the idea that I am adding anything essentially new, but merely to correlate the work of the authors with the general equations mentioned above.

THE ELECTROSTATIC COMPONENT

If both the primary and secondary neutrals are grounded, and the other secondary terminal is connected to a surge impedance z, then in the notation of my paper $Z_1 = Z_2 = O$, $Z_3 = z$, and by equations (**26**), (**27**), (**28**) and (**29**), there are:

$$\left.\begin{aligned}
E &= P\epsilon^{\alpha} + Q\epsilon^{-\alpha} + R\epsilon^{\beta} + S\epsilon^{-\beta}\\
O &= P + Q + R + S\\
O &= mP + mQ + nR + nS\\
O &= (1+a)\,mP\epsilon^{\alpha} + (1-a)\,mQ\epsilon^{-\alpha} + (1+b)\,nR\epsilon^{\beta} + (1-b)\,nS\epsilon^{-\beta}
\end{aligned}\right\}$$

in which $a = zK_1\alpha p$ and $b = zK_2\beta p$. Solving for the integration constants, and substituting in (**24**), there results for the secondary voltage:

$$e_2 = mn\left[\frac{\sinh\alpha x(\sinh\beta + b\cosh\beta) - \sinh\beta x(\sinh\alpha + a\cosh\alpha)}{n\sinh\alpha(\sinh\beta + b\cosh\beta) - m\sinh\beta(\sinh\alpha + a\cosh\alpha)}\right]E$$

At the terminal $x = 1$, the above equation reduces to

$$e_2 = mn\left[\frac{(\beta\sinh\alpha\,.\,\cosh\beta - \alpha\sinh\beta\,.\,\cosh\alpha)}{(n\beta\sinh\alpha\,.\,\cosh\beta - m\,\alpha\sinh\beta\,.\,\cosh\alpha)}\right]\frac{p}{p+\gamma}E$$

$$= \frac{E\,mn\,(\beta\sinh\alpha\,.\,\cosh\beta - \alpha\sinh\beta\,.\,\cosh\alpha)}{(n\beta\sinh\alpha\,.\,\cosh\beta - m\alpha\sinh\beta\,.\,\cosh\alpha)}\,\epsilon^{-\gamma t}$$

where

$$\gamma = \frac{(n-m)\sinh\alpha\,.\,\sinh\beta}{zK_2\,(n\beta\sinh\alpha\,.\,\cosh\beta - m\alpha\sinh\beta\,.\,\cosh\alpha)}$$

Now α and β are large enough in practical cases so that $\sinh\alpha \cong \cosh\alpha$ and $\sinh\beta \cong \cosh\beta$. Hereby the above equation simplifies to

$$e_2 \cong \frac{E\,mn\,(\beta - \alpha)}{(n\beta - m\alpha)}\,\epsilon^{-\frac{(n-m)t}{zK_2(n\beta - m\alpha)}}$$

Considering the case given in Mr. H. L. Rorden's discussion of my paper, and $z = 500$; there is $\alpha = 19.0$, $\beta = 11.7$, $m = -1.617$, $n = 0.618$, $K_2 = 1 \times 10^{-11}$. Then

$$e_2 \cong 0.193\,\epsilon^{-11.8t}$$

Thus the electrostatic component is an extremely short exponential impulse.

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*A.I.E.E. TRANS., June 1932, p. 299.

THE OSCILLATORY COMPONENT

For a grounded neutral and short-circuited secondary the general equations (**53**) and (**54**) become (dropping subscripts)

$$e_1 = E + \Sigma\,(A\cos\omega t + A'\cos\Omega t)\sin\lambda x$$
$$e_2 = O + \Sigma\,(rA\cos\omega t + r'A'\cos\Omega t)\sin\lambda x$$

and from equation (**49**)

$$(i_{L2} + i_{K2}) = I_2 + \Sigma\,[\omega A\,(rC_2 + rC_3 - C_3)\sin\omega t$$
$$-\,\Omega A'\,(r'C_2 + r'C_3 - C_3)\sin\Omega t]\,\frac{\cos\lambda x}{\lambda}$$

where I_2 is an integration constant with respect to x and therefore a possible function of t. In Appendix II it is identified as the electromagnetic component of current. The oscillatory components are a maximum for $x = 0$ or $x = 1$ (the terminals). Now if the secondary is closed through an impedance the current will be *less* than that given above for a dead short circuit. Therefore the voltage transferred to the secondary by the oscillatory components can not exceed

$$e_2 = zi \leq z\,\Sigma\left[\frac{\omega A}{\lambda}\,(rC_2 + rC_3 - C_3)\sin\omega t\right.$$
$$\left.-\,\frac{\Omega}{\lambda}\,A'\,(rC_2 + r'C_3 - C_3)\sin\Omega t\right]$$

In practical cases this is of the order of 1 per cent of E for $z = 500$ ohms. For example, in the case given in Mr. Rorden's discussion the fundamental component of the above equation becomes

$$e_2 \leq 0.0103\,E \text{ for the fundamental } (s = 1)$$

THE ELECTROMAGNETIC COMPONENT

The basic equations used by the authors in their investigation of the voltage and currents transmitted to the secondary are identical with the ordinary conventional equations of the transformer. However, they point out the difficulty of arriving at accurate numerical results using self and mutual inductances, and have circumvented this difficulty by showing how these equations for mutually coupled circuits may be replaced (on a 1:1 turn ratio basis) by a simple series inductance, which approximation tremendously simplifies the calculation of the electromagnetic transient. In Appendix II of my paper* it is shown that their basic equations are a natural consequence of the general differential equations of the complete circuit of the transformer (involving distributed inductances and capacitances). I would point out that while in my paper complete solutions are obtained only for zero and infinite terminal impedances, that the particular derivation given in Appendix II is general, and applies to any terminal conditions; so that the currents fixing the axes of oscillations are on conformity with the relationships

$$\left.\begin{aligned} E &= p\,(L_1'I_2 + M'I_2) + Z_1 I_1 \\ O &= p\,(M'I_1 + L_2'I_2) + (Z_2 + Z_3)\,I_2 \end{aligned}\right\}$$

The secondary terminal voltage is a maximum when it is open-circuited ($I_2 = 0$), in which case for a grounded neutral it is in the turn ratio:

$$e_2 = \frac{M'}{L_1'}\,E \cong \frac{n_2}{n_1}\,E$$

But under this condition the electrostatic component persists for a much longer time, so that the secondary terminal voltage will be in excess of that given by the turn ratio, and should be insulated accordingly. When the secondary is open-circuited, the general equations describe the complete transient.

THREE-PHASE BANKS

In order to determine the effect on the secondary voltage of the transformer bank connection, and the number of phases struck by lightning, the authors reduced each particular connection to an equivalent circuit and then solved these equivalent circuits. Two examples are given in their Fig. 9, one of which is discussed in Part I of their paper, and the equations for the other derived in Part II, equations (**10**), (**11**), (**12**) and (**13**). However, it is not at all necessary to reduce to equivalent circuits, and considerable time can be saved by writing the equations directly from the bank connections. The two cases which they discuss in detail will suffice as illustrations:

Fig. 9a. Writing the voltage drops from phase a to phase b directly from the bank connection, there is

$$(2E) = (Z_1 + pL_s + 0.5pL_s + 0.5Z_1 + 3Z_2 + 1.5Z_2)\,I$$

and the secondary voltage is

$$e_2 = \frac{1.5IZ_2}{r} = \frac{(2E)}{r}\,\frac{1.5Z_2}{1.5\,(Z_1 + pL_s + 3Z_2)}$$
$$= \frac{(2E)}{r}\,\frac{q}{1 + 3q}\left(1 - \epsilon^{-\frac{Z_1}{L_s}(1+3q)t}\right)$$

Fig. 9b. Let the current in phase a be I and that in phase b be aI. Then writing the voltage drop from phase a to phase b directly from the bank connection, there is

$$(2E) = [Z_1 + pL_s - aZ_1 - apL_s + (1 - a)\,2Z_2 + (1 - a)\,Z_2]\,I$$
$$= (1 - a)\,(Z_1 + pL_s + 3Z_2)\,I$$

and the secondary voltage is

$$e_2 = \frac{IZ_2\,(1 - a)}{r} = \frac{(2E)}{r}\,\frac{(1 - a)\,Z_2}{(1 - a)\,(Z_1 + pL_s + 3Z_2)}$$
$$= \frac{(2E)}{r}\,\frac{q}{1 + 3q}\left(1 - \epsilon^{-\frac{Z_1}{L_s}(1+3q)t}\right)$$

Incidentally, it may be mentioned that neither the procedure followed by the authors, nor that given above, takes cognizance of the fact that traveling waves on the transmission lines affect each other through the mutual surge impedances.

K. K. Palueff: Such tests as those suggested by Mr. Peek are very desirable indeed; they would weed out unsuitable transformer constructions. The principal difficulty in adopting an impulse test, as it appears to me, is to find reliable and positive methods of determining the effect of the test on all parts of transformer insulation. This is important because of the possibility of local damage resulting from such a test in transformers not properly designed or manufactured. Such damages, while of prime importance from the standpoint of the dielectric strength of the transformer, are often so extremely small physically that it is exceedingly difficult to locate them, or even to detect their presence.

For example, on some occasions in the past upon examining transformers damaged in service by lightning, I observed, in addition to the principal damage, which is generally augmented by subsequent power flow through the short-circuit, numerous "pinhole" punctures of the insulation. These holes could not be detected without complete disassembly of the transformer and the use of an especially bright light.

I fear that Mr. Gay's method of protection of the main transformer winding by means of an auxiliary or shunt winding connected to the terminals of the high-voltage winding through a capacitance, may prove to be not only impractical but also undesirable, as it would cause a substantial increase in the local voltage stresses in the main winding of the transformer. On account of this possibility I would suggest that Mr. Gay work out a numerical example of application of his device to an arbitrarily chosen transformer, showing at least the order of magnitude of the effect of the protective scheme.

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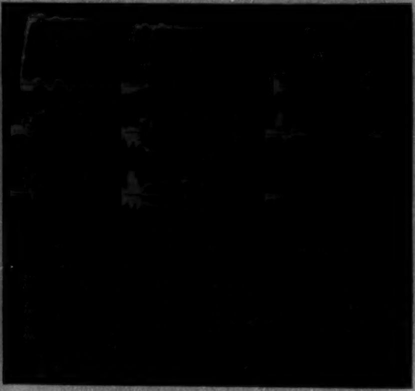
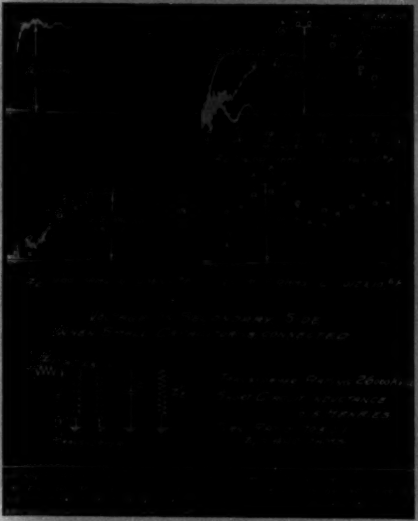
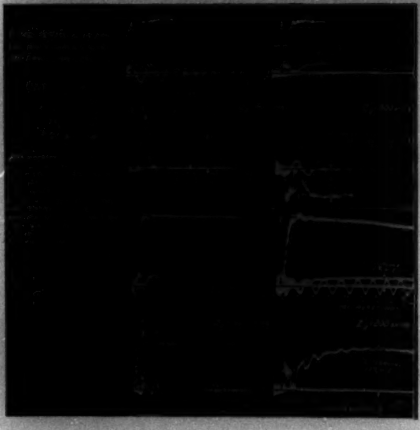
*Loc. cit.







Several years ago, we were confronted with the problem of the transmission of lightning through transformers and autotransformers. Considering the phenomenon which takes place within the transformer winding in its entire complexity, it became obvious that a general solution, permitting computations with engineering accuracy, was not possible. Consequently, we directed our efforts toward evaluation of the relative practical importance of the four components discussed in the first part of the present paper. The considerations presented led to the conclusion that the electromagnetic component dominates the phenomenon. Once this was understood, the solution of the problem became not only mathematically elementary but sufficiently accurate numerically as to be used in practise as shown in the paper. The results of this solution were presented by me in January 1931. (See reference 5 of the bibliography.) On account of space limitations it was impossible at that time to include the material of the present paper.

While Mr. Bewley's paper is devoted to the study of transient voltages *within* transformer windings, our present paper deals with transient voltages that get *through* a transformer to secondary circuit. For this reason, Mr. Bewley's discussion is particularly interesting as it ties these two papers together.

The chief limitations of Mr. Bewley's solutions for oscillatory and electromagnetic components lie in the practical impossibility of sufficiently accurate evaluations of some principal constants (like L_1, L_2, M). This essentially limits the practical applications of these solutions.

His solution for electrostatic components, I believe is quite satisfactory for windings concentrically located, but not for windings that are interleaved. In the latter case capacity constants like K_1, K_2, C, C_2, C_3 can not be analytically determined. For a number of years, therefore, we were obliged to use an entirely different method for the calculation of this component. The results of this calculation were published in my previous papers and discussions.

It is my conviction that while equivalent circuits are rarely necessary for the solution of a problem, they are extremely helpful in visualizing and describing the phenomenon. It is for this reason that the equivalent circuits are given in this paper.

The oscillograms of this discussion illustrate the relative importance of the four components. They also show the degree of agreement between the calculations made in accordance with simple equations given in our paper and the actual values measured.

The Proximity Effect

Its Application to the Concentration of Heating Currents in Predetermined Strips

BY EDWARD BENNETT*

Fellow, A.I.E.E.

Synopsis.—*This paper describes methods of concentrating heating currents used for any industrial operation, such as welding, in predetermined strips of conducting plates, pipes, or other shapes. The method is to so place the shapes in close proximity to each other, or to auxiliary conductors, and to so interconnect the shapes with sources of alternating, or oscillatory, current of moderately high frequency that the heating current concentrates largely in adjacent strips in close proximity, in which the current flows in opposite directions in the two adjacent strips. Briefly stated, the method is to use an enhanced "proximity effect" to control the distribution of heating currents in bodies.*

The paper contains curves which have been worked out to illustrate the control of the distribution of the heating current densities by means of the proximity effect.

I. The Proximity Effect

Any variation in the value of the electric current in a wire is attended by, or has associated with it, a definite distribution of induced *electric intensities* at all points of the surrounding space. The value of the induced electric intensity at the point P which is associated with variations in the current in the wire W of Fig. 1 may be computed as follows. Divide the wire into segments, each so short that it may be regarded as a short straight filament. Take the vector contribution, F, to the electric intensity at P which is made by each short segment of wire of length s to be as expressed by equation (1), and find the vector sum of the contributions of all the segments which make up the wire W.†

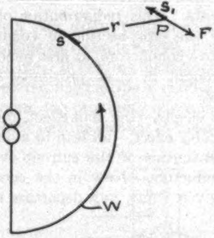


Fig. 1—Electric Intensity Due to a Segment of Current

$$\mathbf{F} \text{ (volts per cm.)} = -\, \mathbf{s}_1 \frac{\mu_o\, s}{4\, \pi\, r} \frac{di}{dt} \text{ (amperes) (sec.)} \qquad (1)$$

μ_o represents the permeability of free space in weber ampere-turn cm. units. $\mu_o = 4\,\pi\,10^{-9}$

$\mathbf{s}_1$ represents a unit vector parallel to the segment s and pointing in the arrow direction along s.

The significant features of this relation are that the electric intensity at P associated with the current in the segment is directly proportional to the *rate of increase* of the current, *inversely proportional to the distance* r from the segment to the point P and is parallel with but in a direction *opposite to the direction of acceleration* of positive charge in the segment.

By applying equation (1) to compute the electric intensity at a point P, (Fig. 2) lying close to a long straight filament of current and in its mid plane, the following expression is derived for the electric intensity:

$$\mathbf{F} \text{ (volts per cm.)} = -\, \mathbf{s}_1 \frac{\mu_o}{2\,\pi} \log \frac{s}{h} \frac{di}{dt} \text{ (approx.)} \qquad (2)$$

If the current flowing in these filaments is the sinusoidal current,

$$i = I \sin 2\,\pi\, f\, t \qquad (3)$$

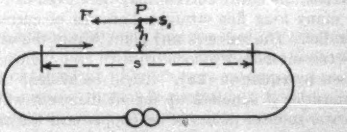


Fig. 2—Electric Intensity Due to a Long Straight Filament of Current

the expressions for the induced electric intensities accompanying this current become,

$$\mathbf{F} \text{ (volts per cm.)} = -\, \mathbf{s}_1 \frac{\mu_o\, s\, f\, I}{2\, r} \cos 2\,\pi\, f\, t \qquad (1a)$$

and

$$\mathbf{F} \text{ (volts per cm.)} = -\, \mathbf{s}_1\, \mu_o\, f\, I \log \frac{s}{h} \cos 2\,\pi\, f\, t \text{ (approx.)} \qquad (2a)$$

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†The fact that the electric intensities existing at P at the instant t are to be found from the value of di/dt in the segment at an instant r/c seconds earlier than the instant t, has been omitted from this statement. It is not material to the situation to be discussed.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wis., March 14-16, 1932.

In other words, a long straight filament of alternating current is attended by induced electric intensities, or electric forces, which tend to cause current to flow in all neighboring conductors in a direction *opposite* to the direction of increase of the current in the filament. These forces increase in direct proportion to the increase in the frequency of alternation of the current, and increase in logarithmic fashion as the point P is taken closer to the inducing current filament.

The induced electric intensities, whose relation to the current is expressed by equation (**1**), serve to account in a quantitative way for such electrical effects as transformer action between primary and secondary coils, the *skin effect*, the *edge effect*, and the *proximity effect* in conductors carrying alternating currents.

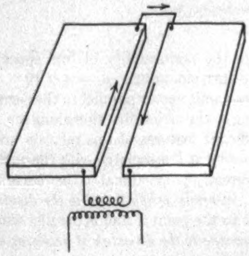

FIG. 3—CONNECTIONS FOR HEATING EDGES OF PLATES

Consider, for example, a long straight wire carrying an alternating current and let us suppose that the alternating current is uniformly distributed over the circular cross-section of the wire. For the purpose of computing the induced electric intensities within the body of the wire which attend the variations in the current, the entire current may be viewed as consisting of many long fine straight filaments of current all in parallel. The value at any point within the wire of the electric intensities associated with each filament is that given by equation (**2a**). It will be evident that these intensities if summed up for all filaments will yield a greater sum for points in the copper near the axis of the wire than for points more remote from the axis. That is to say, filaments near the axis are subject to greater induced electric intensities than filaments more remote, and these intensities are directly proportional to the frequency of alternation and at each instant are in a direction tending to lessen the rate of increase or decrease of the current. It follows that the current densities will be higher in filaments near the outside surface of the wire than in filaments near the axis. At high frequencies, this effect is so pronounced in conductors of large cross-section that an alternating current is confined to the filaments lying in a thin surface layer or *skin*, and the effect is called the *skin effect.*

If the alternating current flows in a conductor whose cross-section is a long narrow rectangle, as in a bus-bar of thin wide strap copper, it will be evident from the above considerations that the current densities will be much higher near the two edges of the rectangle than near the central portion of the strap. This increase in the current density toward the edges of rectangular sections is called the *edge effect.*

If now two long wires parallel each other, and if the wires constitute the outgoing and return conductors of an alternating current circuit, then at each instant of time the currents in the two conductors are opposite in direction. Accordingly, an increase in the current in conductor A results in induced electric forces in the filaments of conductor B which are in the direction in which the current in B tends to increase. But equation (**2a**) shows that these forces are greatest in the filaments of B which lie closest to A. In like manner the current B gives rise to assisting electric forces in the filaments of A. It follows that the current densities will be higher in the closely adjacent filaments of two circular conductors than in the more remotely separated filaments. The less the separation of the conductors and the higher the frequency, the greater will be the variation of the current density from the closely adjacent to the more remote filaments. This redistribution of the current densities over the cross-section of conductors which occurs when two conductors, at first remote from each other, are brought in close proximity is called the *proximity effect.*

These three effects,—the skin effect, the edge effect, and the proximity effect,—all lead to a departure from a uniform distribution of the current over the cross-section of conductors. Now in the conduction of a current of a given value, any departure from uniform current densities over the cross-section of homogeneous conductors means that the so-called I^2R loss, or the Joulean power expenditure, in the conductors is greater than for uniform densities. It is this wasteful aspect of these effects which seems to have led to many of the mathematical and experimental studies which have been made of the distribution of alternating current densities in conductors. It would seem that, in the main, the proximity effect has hitherto been viewed as an undesirable wasteful effect, and that thought and effort has been principally directed toward the minimizing of the effect.

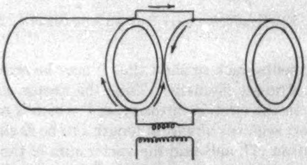

FIG. 4—CONNECTIONS FOR HEATING ENDS OF PIPES

II. THE USE OF THE PROXIMITY EFFECT TO CONCENTRATE HEATING CURRENTS IN PREDETERMINED STRIPS

While considering existing methods of welding pipe, it occurred to the author that the proximity effect could readily be magnified or enhanced to the point at which it might serve a very useful purpose in such industrial heating operations as those involved in welding, forming, hardening, etc. In heating operations requiring the heating of predetermined strips of conducting bodies or structural shapes, the proximity effect may be used to confine an electric heating current to these strips.

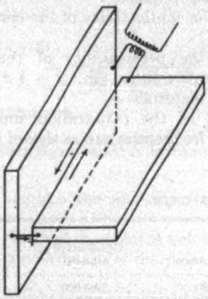

FIG. 5—CONNECTIONS FOR HEATING AN EDGE AND A STRIP

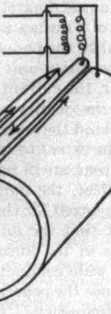

FIG. 6—CONNECTIONS FOR AN AUXILIARY CONDUCTOR WHICH DETERMINES THE PATH OF THE HEATING CURRENT

The confining of the heating currents to predetermined strips is accomplished by using heating currents whose frequency is in the range known as the audio range and by the electrical connections and the space arrangements illustrated in Figs. 3 to 8. An examination of these arrangements will show that they may be grouped under the following classification.

CLASSIFICATION OF CONNECTIONS FOR APPLYING THE PROXIMITY EFFECT

I. Current conductively conveyed to the body to be heated.
  A. The shapes and the connections of the heated bodies alone determine the strips in which the current concentrates. See Figs. 3, 4, and 5.
  B. An auxiliary water-cooled conductor is used to determine the strips of concentration. See Figs. 6 and 7.

II. The current is *induced* in the strip to be heated. (A water-cooled inducing coil lies in close proximity to the strip and there is no conductive connection with the body to be heated.) See Fig. 8.

The connections and the relations illustrated by the figures are so clear that they require little explanation. In each diagram, the transformer symbol represents the source of audio frequency current. The arrows represent the directions of the currents at a given instant of time in the different parts of the path. For the sake of clearness, it has been necessary to draw the parts with some considerable distance between the adjacent edges, but in the actual equipment this gap is of the order of a few millimeters or centimeters depending upon the thickness of the edges. Fig. 3 shows an arrangement in which the heating current is confined to layers a few millimeters to several centimeters deep along the adjacent edges of two steel plates. In Fig. 4, the heating current is confined to similar layers on the ends of two steel pipes. In Fig. 5, the heating current is confined to the edge of one plate and to a strip extending along the middle of the second plate.

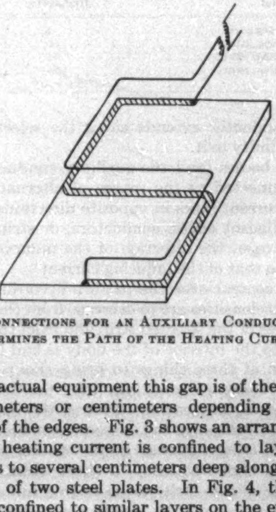

FIG. 7—CONNECTIONS FOR AN AUXILIARY CONDUCTOR WHICH DETERMINES THE PATH OF THE HEATING CURRENT

Figs. 6 and 7 show arrangements in which an auxiliary water-cooled tubular conductor is connected in series with the body to be heated. This auxiliary conductor parallels the strip to be heated in close proximity to it and thus serves to bring about the desired concentration of the heating current in the strip. The first sight of the red-hot strip of steel conforming to the sinuosities of the water-cooled copper tube mounted above the steel plate illustrated in Fig. 7 is found to be somewhat startling even to engineers who have witnessed the heating effects obtained with the Figs. 3, 4, and 5 arrangements.

The two views in Fig. 8 serve to illustrate the group of arrangements in which no conductive connection is made with the body to be heated. In this group an inducing current of suitable frequency flows in a tubular water-cooled inducing coil. By mounting this coil so that the entire coil or one leg or segment of the coil lies in *close proximity* to the strip, heating currents are induced along the selected strip.

It will be seen that the features common to all of these arrangements are as follows:

a. The strip of the conducting body in which heating current is to be caused to concentrate is selected and predetermined by so shaping, proportioning, and mounting another electric conductor that some portion of this second conductor extends along the selected strip in close proximity to it.

b. The bodies (and the auxiliary conductor, if any) are so connected to the source of alternating current that the current flows in opposite directions in the two closely adjacent edges, conductors, or strips. (In the inductive case, the direction of the induced current is opposite to that of the inducing current.)

c. The control over the pattern according to which the current densities are to decrease from the center line to the edges of the selected strip and from the surface of the strip to the interior of the body is had by the joint adaptation of three things to bring about the desired pattern. These three things are, the frequency of alternation of the heating currents, the distance from the surface of the strip to the second conductor, and the width and cross-sectional shape of this second conductor. A high frequency, a short distance between the surface of the strip to be heated and the second conductor, and a narrow second conductor lead to the concentration of the heating current in a narrow strip.

III. Illustrations of the Control Over the Distribution of the Current Densities

In dealing with the distribution of alternating currents over the cross-section of conducting bodies, it is very helpful to express all linear dimensions, not in centimeters, but in terms of a unit of length appropriate to this problem. We will call this unit of length the *penetration unit* for the material at the frequency in question. The advantage of using this unit is that it enables us to represent the distribution of the current for a wide range of conducting materials, spacings, and frequencies by a few curves.

The length of the *penetration unit* is,

$$1 \text{ P. unit (penetration unit) (in cm.)} = \frac{1}{\sqrt{\pi f \mu \gamma}} \tag{4}$$

f represents the frequency.

γ represents the conductivity of the conductor in mho-cm.

μ represents the permeability of the conductor in weber ampere-turns cm. $\mu = 4\pi\ 10^{-9}$ for non-magnetic materials.

The values of the penetration unit for different materials and frequencies are as shown in the following table.

TABLE I—VALUES OF THE PENETRATION UNIT

Material	Relative Resistivity	Relative Permeability	Frequency	Conductivity mho-cm.	P. unit cm.
Copper 20 deg. cent	1	1	960	580,000	0.214
Steel 20 deg. cent	11	100	960	52,700	0.070
Steel 800 deg. cent	94	1	960	6,240	2.06
Steel 1,300 deg. cent	100	1	960	5,800	2.14
Earth	5.8×10^9	1	60	10^{-4}	65,000

In order that a physical significance may attach to the penetration unit as defined in equation (4), the following statements may be noted. The effective resistance to an alternating current of frequency f of a conductor of circular cross-section whose radius is five or more times the penetration unit of the material at the frequency f, is approximately equal to the ordinary ohmic resistance of a surface layer of the conductor having a depth equal to the penetration unit. Again, at a depth of one penetration unit below the surface of such a large conductor, the current density is approximately 36.8 per cent of the density at the surface, and it lags by one radian behind the current density at the surface.

To illustrate the extent to which heating currents can be caused to concentrate in narrow strips by enhancing the proximity effect, the current densities have been worked out and plotted for the arrangement illustrated in Fig. 7, except that the auxiliary conductor has no sinuosities but is in the form of a long straight wire mounted over a wide conducting slab whose thickness is five or more times the penetration unit of the material at the operating frequency. The radius of the auxiliary conductor has been assumed to be very small in comparison with its mounting height h, say 5 per cent or less.

The current density C at any point A in the slab may be computed from the following Fourier integral.

$$C = \frac{j\,2\,I}{\pi} \int_0^\infty \frac{1}{\sqrt{b^2 + 2\,j} + b\,\mu_r} \cos xb \; exp\,(-\,hb - y\,\sqrt{b^2 + 2\,j})\, db \tag{5}$$

in which

- C represents the current density at A in amperes per square penetration unit.
- b represents the variable of the integrand. It disappears from the integral upon substituting the two limits 0 and ∞.
- I represents the value of the heating current in amperes.

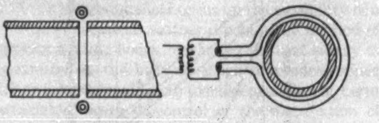

Fig. 8—Strip Heating by an Inducing Coil

- h represents the height of the auxiliary conductor above the surface.
- y represents the depth of the point A below the surface.
- x represents the x coordinate of the point A, as shown in Fig. 9.
- μ_r represents the relative permeability of the conducting slab.
- j represents $\sqrt{-1}$

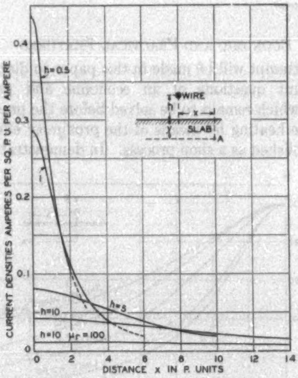

Fig. 9—Current Densities at the Surface of a Slab for Different Mounting Heights

Current densities are expressed in amperes per sq. penetration unit per ampere of heating current

The distances x and h are expressed in penetration units

$exp\ (-hb)$ represents the exponential series in $(-hb)$. (The values of x, y, and h are to be expressed in P units).

Equation (**5**) is valid for materials and frequencies in which the conduction current density is 100 or more times as great as the displacement current density. For copper at a frequency of 1,000 cycles per second, the conduction current density is 10^{15} times the displacement current density.

If the slab is of non-magnetic material or of ferromagnetic material at temperatures above the point at which the material becomes non-magnetic, μ_r has the value unity, and equation (**5**) reduces to a form equivalent to that given by Carson.[*]

$$C = -\frac{I}{\pi}\int_0^\infty [\sqrt{b^2+2j}-b] \cos xb \ exp\ (-hb-y\sqrt{b^2+2j})db \tag{6}$$

For mounting heights for the auxiliary conductor in which h is greater than 20 penetration units, equation (**6**) for non-magnetic materials evaluates to the simple form

$$C = \frac{1+j}{\pi}\ \frac{h}{h^2+x^2}\ (exp - y)\ (\cos y - j \sin y)\ I \tag{7}$$

An examination of Table I will show that the value of the penetration unit for steel at a frequency of 960 cycles per second lies between 0.07 cm. for steel with an assumed relative permeability of 100 and 2.14 cm. for steel at a temperature of 1,300 deg. cent. For an auxiliary conductor mounted at a height of one cm. above the conducting slab, the important range of mounting heights for this steel would lie between $h = 0.5$ and $h = 20$ penetration units.

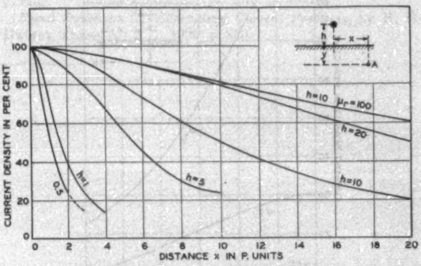

Fig. 10—Current Densities at the Surface of a Slab

For each height of wire, the current densities are expressed in per cent of the current density existing immediately under the wire at that height

The distances x and h are expressed in penetration units

Fig. 9 contains curves, each worked out for a different mounting height of the auxiliary conductor, which show the manner in which the current density at the surface of the slab falls off on each side of the center line of the heated strip. The values shown by the curves are the current densities in amperes per square penetration unit per ampere of heating current flowing in the slab. All the curves have been plotted for material in the non-magnetic state save the curve marked $\mu_r = 100$, in which the relative permeability has been taken to be 100. The curves in Fig. 10 likewise show the manner in which the current density at the surface of the slab falls off on each side of the center line, the curves in each case showing the current densities expressed in per cent of the current density immediately below the auxiliary conductor. In Fig. 10 the effect of the permeability of the steel in confining the current to a thinner surface layer of the steel and thereby causing a wider spread of the current is to be noted.

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[*] "Wave Propagation in Overhead Wires with Ground Return," John R. Carson, *Bell System Tech. Jour.*, Oct. 1926.

A mounting height of one penetration unit means that the height of the auxiliary conductor above the slab would be about two cm. for a steel slab with a strip at its welding temperature and a 960 cycle heating current. For this important case, Figs. 11 and 12 show the distribution of current densities at the surface and at points in planes located at depths of one and two penetration units below the surface. These figures also show the phase of the current density at these points, relative to the phase of the (total) heating current. The current density along the center line of the strip is seen to be 60 deg. in advance of the total current. At points in planes one and two penetration units beneath the surface, the current density lags substantially one and two radians behind the current density at corresponding points on the surface, and has decreased to roughly $(1/\epsilon)$th and $(1/\epsilon)^2$ of its value at the surface. In the surface at a distance of two penetration units from the center line of the strip, the current density is seen to be 40 per cent of the value on the center line, and thus the power expenditure per unit volume is only 16 per cent as great as along the center line.

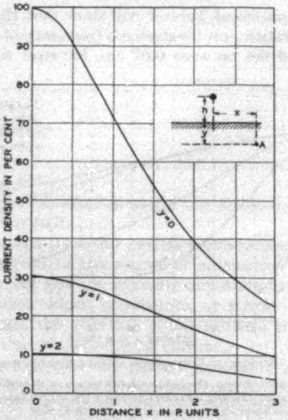

FIG. 11—CURRENT DENSITIES FOR A MOUNTING HEIGHT OF ONE PENETRATION UNIT

The current densities are given in per cent of the current density directly below the wire. x, y, and h are in penetration units

The relative effect of close proximity of the auxiliary conductor and of a high frequency of alternation in contributing toward the concentration of the heating currents in narrow strips is brought out more clearly by plotting the current densities at the surface against the ratio of the x distance of the filament to the mounting height h of the auxiliary conductor, as in Fig. 13.

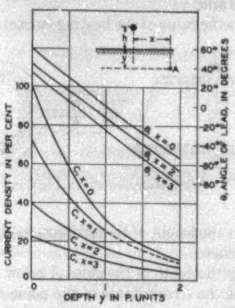

FIG. 12—CURRENT DENSITIES AND PHASE ANGLES FOR A MOUNTING HEIGHT OF ONE PENETRATION UNIT

The current densities are given in per cent of the current density directly below the wire

x, y, and h are in penetration units

The phase angles are the angles by which the current density leads the heating current

ECONOMIC AND PRACTICAL FEATURES

No attempt will be made in this paper to discuss the important questions of an economic and practical nature which remain to be solved before the practise of selective heating by means of the proximity effect can be established as a shop process. In demonstrating the method, the author has been limited to the experience to be had with a condenser-mercury-spark-gap source of high-frequency currents having a rating of 35 kw.

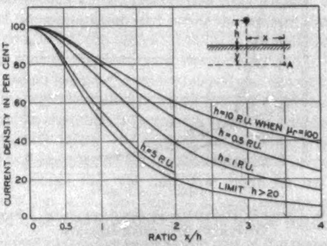

FIG. 13—CURRENT DENSITIES AT THE SURFACE OF A SLAB

For each height of wire, the current densities are expressed in per cent of the current density existing immediately under the wire at that height

The distance h is expressed in penetration units

The most economical range of frequencies for use in the process remains to be determined. Many factors which vary from operation to operation, such as the kind and size of material to be heated, the nature of the desired temperature pattern, and the relative cost of generating energy at the different frequencies will serve to determine the range of frequencies which will be the most economical. Present experience would indicate that the economic frequencies may be expected to center upon 960 cycles per second.

Unquestionably the control of the distribution of heating currents which is made possible by the application of the proximity effect opens up the possibility of a new range of industrial heating effects. The method of selectivity heating bodies by causing the heat energy to be generated in the body in strips of predetermined width and depth would seem to make possible a hitherto unattainable nicety of control of the distribution of temperatures in bodies. The control of the temperature patterns is obtained by adapting both the rate at which energy is supplied and the current density pattern to the desired end. For example, if in the welding of plates edge to edge, the induced heating current is for all practical purposes confined to a strip a few millimeters deep on the edge of the plate or plates, and if the power delivery is made large by using large inducing currents, portions of the plate a few centimeters from the edge may be substantially at room temperature when the edge reaches a welding temperature. On the other hand, if the energy is delivered at a more moderate rate, that is if smaller heating currents are used, the temperature gradient in the plate at the time the edge reaches a welding temperature may be made much more gradual. It will be seen that this control of the temperature patterns results from the fact that the application of the proximity effect makes it possible to confine the delivery of energy in rather definite fashion to predetermined strips of the body.

Bibliography

Bibliographies of the literature dealing with the distribution of current densities over the cross-section of conductors will be found at the end of the following papers. The second bibliography contains references to papers appearing since the preparation of the first bibliography.

Experimental Researches on Skin Effect in Conductors, A. E. Kennelly, F. A. Laws, P. H. Pierce, TRANS. A.I.E.E., 1915, p. 1953.

Bessel Functions for Alternating Current Problems, by H. B. Dwight, TRANS. A.I.E.E., 1929, p. 820.

Toll Switching Plan for Wisconsin

BY W. C. LALLIER[1]

Associate, A.I.E.E.

***Synopsis.**—This paper outlines the general plan which is being employed in the handling of intrastate toll traffic of the Wisconsin Telephone Company, including many of the transmission features involved and touching upon the relationship of the plan to that for handling countrywide toll connections. A brief discussion is included of the present and proposed toll cable network in Wisconsin.*

THE growth in long distance telephone traffic has been rapid from the inception of the telephone business. Developments of telephone plant and technique have constantly extended the distance over which it is technically possible and commercially practicable to furnish toll telephone service.

The result of this extension in the range of transmission has been to increase the number of possible commercial toll connections and to stimulate growth in toll service. A graphic picture of the past and expected near future growth in toll business by five year periods is given in Fig. 1.

The enlarging field of toll telephone operations, together with the particularly rapid rate at which the long haul toll traffic has grown, requires that circuits be provided of such numbers, arrangements, and design as to permit the prompt establishment of the desired connections with a satisfactory grade of transmission. The general basis under which this is being accomplished is known as a toll switching plan. The toll switching plan for Wisconsin is outlined briefly in this paper, including some features of the transmission design requirements and the present and proposed toll cable network and the relationship of the plan to that for handling countrywide toll connections. A description of a toll switching plan for the Bell System was given in a paper* presented at the summer convention of the A.I.E.E., 1930.

The intrastate toll traffic in Wisconsin is composed of over 3,500 individual items, that is, traffic between two specific toll centers. Many of these traffic items are not of sufficient volume to justify direct circuits between the terminal exchanges and the traffic must therefore be switched at one or more points. In accordance with the toll switching plan, the switching of multiswitched toll traffic is generally restricted to selected toll centers arranged for introducing transmission gains into the switched connections and having toll circuits of such transmission characteristics connecting them to other toll centers as to facilitate the provision of reasonably low transmission losses between toll switchboards on connections within the State and on connections to points reached via the Bell System network.

When we speak of the net transmission loss (or net equivalent) of a toll circuit or a complete toll connection, we mean the net difference between the sum of the attenuation losses in the circuit or connection and the total transmission gain introduced by telephone repeaters. It may not at first be obvious why transmission gains are introduced at the switching points in switched toll connections, that is, why each of the links involved in the switched connections is not normally adjusted to have sufficiently low losses as to result in satisfactory overall losses on the complete toll connections without introducing transmission gains at the points of switching. The answer to this question lies in the fact that it is generally impracticable for noise, crosstalk or repeater singing reasons, as will be explained later, to work individual repeatered toll circuits to sufficiently low transmission losses. As an example, it would not be advisable to work a 2-wire cable circuit loaded with 88 millihenry coils spaced 6,000 ft. apart from Milwaukee to Madison to less than about 5 or 6 db. on terminal business for singing and crosstalk reasons, whereas it is practicable and desirable to work such a circuit to a loss of 1 or 2 db. when used as an intermediate link in multiswitched toll connections.

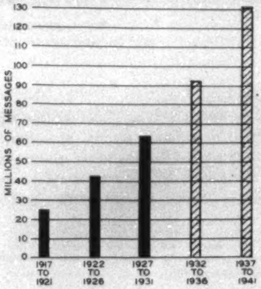

FIG. 1—TOTAL TOLL MESSAGES ORIGINATING IN WISCONSIN INVOLVING BELL SYSTEM CIRCUITS

There are two methods which are commonly used as a means of introducing transmission gain into switched toll connections, that is, the cord circuit repeater method and the pad control method.

The cord circuit repeater method, which employs special cord circuits containing telephone repeaters in connecting the various circuits together to form complete toll connections, has been used extensively in the past in providing improved transmission on switched connections. This is illustrated in Fig. 2. It has the disadvantage that the routine whereby the cord circuit repeaters are inserted into the switched connections by the toll operators is necessarily somewhat cumbersome, involving considerable expense for operators' labor and an increase in the time required to set up the switched toll connections. Furthermore, due to the human element involved, it is not possible to insure that the cord circuit repeaters will always be used when and as required by the routing instructions.

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[1] Wisconsin Telephone Co., Milwaukee, Wis.

**A General Switching Plan for Telephone Toll Service*, by H. S. Osborne, TRANS. A.I.E.E., Oct. 1930, p. 1549.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wis., March 14-16, 1932.

The method which is now being employed in connection with all new repeater switching offices, and which is also superseding the cord circuit repeater method of operation in many existing cord circuit repeater offices, is the pad control method of introducing transmission gain into switched toll connections. In this method, terminal repeaters and suitable loss pads are provided in the toll circuits at the pad control switching offices and are so associated with relay equipment that the loss pads are automatically cut out of the toll circuits at the pad control switching office when two circuits are connected together in a switched connection. This method of introducing gain into switched toll connections is also illustrated in Fig. 2.

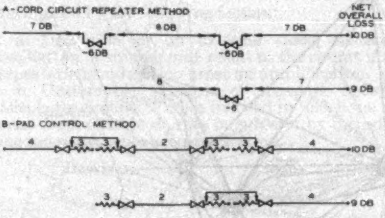

Fig. 2—Illustrations of Cord Circuit Repeater and Switching Pad Methods of Introducing Gain in Switched Connections

Two factors which have played an important part in justifying the extensive application of the pad control method of introducing transmission gains into switched toll connections are (a) important reductions which have occurred in the cost of terminal repeaters and (b) the substantial increase in the number of terminal repeaters required on toll circuits for other reasons such as, for example, in connection with the increased application of cable to the toll plant and the improved transmission efficiencies to which toll circuits are now generally being worked.

When the switching pad arrangement for gain control is employed at a switching office, pads are generally provided on all toll circuits which may be switched at this office in order to avoid repeater singing or undesirable noise or crosstalk effects when the circuits are used for terminating business. These pads are generally adjusted to have a loss of 3 db. except as higher loss values appear desirable to avoid noise, crosstalk or repeater singing or except as lower loss values are desirable to meet satisfactory transmission loss requirements and are satisfactory from the standpoint of noise, crosstalk, and repeater singing.

Switching Arrangements of Plan for Intrastate Traffic

The next item which will be discussed is the arrangement of switching offices and circuit groups employed in accordance with the toll switching plan for intrastate traffic.

Several primary switching points called "primary outlets" have been selected which are completely interconnected with direct circuit groups. Every toll center is to be provided with direct circuits to at least one of these primary outlets. With this arrangement, connections between any two toll centers can be handled with not more than two switches. As indicated in Fig. 3, the primary outlets which have been selected in Wisconsin are located at the following offices, each of which is a telephone repeater office and is arranged for introducing transmission gain into switched toll connections: Appleton, Eau Claire, Madison, Milwaukee, Stevens Point.

A number of "secondary switching points" has been selected for handling items of traffic where the length of haul can be shortened or other economies obtained by switching traffic at other than the above primary outlets, and for alternate and emergency routes. Each of these offices is also required to be a telephone repeater office arranged for introducing transmission gain into switched toll connections and to have such direct circuits to primary outlets, secondary switching points, and other toll centers as are warranted. Referring to Fig. 3, the following offices are now secondary switching offices: Ashland, Green Bay, La Crosse, Marinette, Rhinelander, Rice Lake.

In addition to the points just mentioned, it will be noted that Duluth, Minnesota; Ironwood, Michigan; and Dubuque, Iowa, all of which are repeater offices equipped with switching pads, are considered as secondary switching points in the toll switching plan for Wisconsin.

The following additional offices are planned to be secondary switching points as soon as the necessary telephone repeater and switching pad arrangements are provided, and are shown as such in Fig. 3: Fond du Lac, Janesville, Lake Geneva, Prentice.

The toll centers which are neither primary outlets nor secondary switching points are called terminating toll centers. Switching at these toll centers is limited to such single switched connections as can be made without exceeding certain transmission limitations as illustrated in Fig. 4. It will be noted from this figure that the permissible overall loss between terminal toll switchboards is 10 db. where the toll terminal losses of the toll centers are each 7 db., and that correspondingly higher overall toll circuit losses are permitted in cases where the toll terminal losses are less than 7 db.

Relation to Switching Arrangements for Countrywide Connections

Each of the five primary outlets in Wisconsin is provided with direct circuit groups to at least one "regional center," which is a backbone switching point in the nationwide toll traffic network. The regional centers are analogous in the countrywide toll switching plan to the primary outlets in the plan for Wisconsin in that they are arranged for introducing transmission gains into switched toll connections and are to be completely interconnected by direct circuit groups. Fig. 5 shows the present arrangement of regional centers and primary outlets in the Bell System. Every primary outlet in the country is to be provided with direct circuits to at least one of the regional centers and every toll center in the country is to be connected to at least one primary outlet. With the complete realization of this plan, therefore, it will be possible to talk between any two toll centers in the country with a total of not more than four switches involving five toll circuit links.

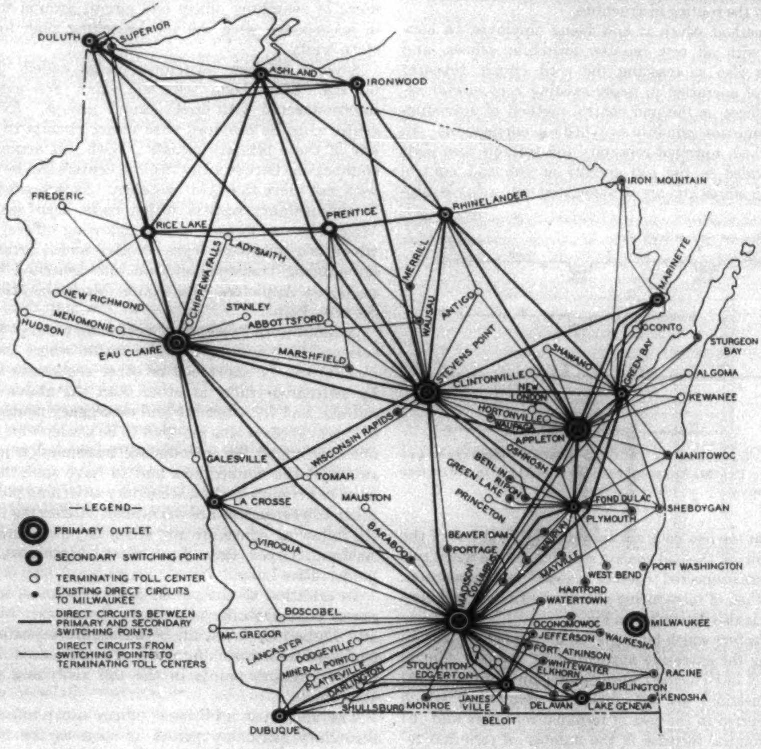
DULUTH
SUPERIOR
ASHLAND
IRONWOOD
IRON MOUNTAIN
FREDERIC
RICE LAKE
PRENTICE
RHINELANDER
LADYSMITH
CHIPPEWA FALLS
MERRILL
MARINETTE
NEW RICHMOND
STANLEY
WAUSAU
ANTIGO
HUDSON
MENOMONIE
ABBOTTSFORD
OCONTO
STURGEON BAY
EAU CLAIRE
MARSHFIELD
STEVENS POINT
SHAWANO
GREEN BAY
ALGOMA
CLINTONVILLE
NEW LONDON
KEWANEE
HORTONVILLE
WAUPACA
APPLETON
WISCONSIN RAPIDS
OSHKOSH
MANITOWOC
BERLIN
RIPON
GREEN LAKE
GALESVILLE
TOMAH
PRINCETON
FOND DU LAC
SHEBOYGAN
MAUSTON
LA CROSSE
PLYMOUTH
—LEGEND—
PRIMARY OUTLET
BARABOO
BEAVER DAM
SECONDARY SWITCHING POINT
PORTAGE
WAUPUN
VIROQUA
TERMINATING TOLL CENTER
MAYVILLE
PORT WASHINGTON
EXISTING DIRECT CIRCUITS TO MILWAUKEE
COLUMBUS
WEST BEND
MADISON
HARTFORD
WATERTOWN
DIRECT CIRCUITS BETWEEN PRIMARY AND SECONDARY SWITCHING POINTS
BOSCOBEL
MILWAUKEE
OCONOMOWOC
DIRECT CIRCUITS FROM SWITCHING POINTS TO TERMINATING TOLL CENTERS
MC. GREGOR
JEFFERSON
WAUKESHA
LANCASTER
DODGEVILLE
FORT ATKINSON
MINERAL POINT
WHITEWATER
STOUGHTON
ELKHORN
PLATTEVILLE
EDGERTON
RACINE
DARLINGTON
BURLINGTON
KENOSHA
SHULLSBURG
MONROE
JANESVILLE
DELAVAN
LAKE GENEVA
BELOIT
DUBUQUE

Fig. 3—Toll Switching Plan for Wisconsin

Note: As shown in the code, circuit groups to Milwaukee are indicated by dots at the distant terminals

The regional center to which all of the primary outlets in this portion of the country are connected is Chicago. Most of the secondary switching points in Wisconsin have direct circuits to Chicago, in which case they may be considered as "secondary outlets" in the countrywide toll switching plan.

Transmission Requirements of Toll Switching Plan

As mentioned previously, the transmission loss of a toll circuit or complete toll connection is the net difference between the total attenuation loss in the circuit or connection and the total transmission gain introduced by telephone repeaters. In practise, the amount by which it is possible to decrease the transmission losses of toll circuits or connections by introducing repeater gains is limited by the following factors:

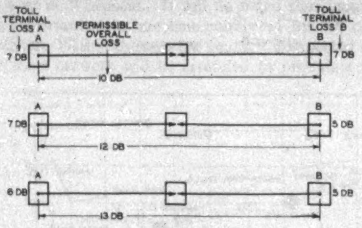
CIRCUITS TO BE SWITCHED AT TERMINATING TOLL CENTERS
TOLL TERMINAL LOSS A
PERMISSIBLE OVERALL LOSS
TOLL TERMINAL LOSS B
A
B
7 DB
7 DB
10 DB
A
B
7 DB
5 DB
12 DB
A
B
6 DB
5 DB
13 DB

Fig. 4—Transmission Requirements of Toll Switching Plan for Wisconsin

a. Distortion or, in extreme cases, sustained oscillation or singing may result in the circuit if the repeaters introduce too great an amplification.

b. Undesirable amounts of crosstalk between telephone circuits or noise induced in telephone circuits from outside sources may result by introducing too much repeater amplification.

c. Disturbing effects may be caused by the amplification of echo currents, *i. e.*, those portions of the speech currents which are reflected back from the distant end of the connection or from intermediate points, if the repeater gains are such as to result in too low an overall transmission loss.

On the longer toll connections, echo considerations are almost always the controlling factor in determining the minimum overall transmission loss which may be employed, whereas on the shorter toll connections, repeater singing, crosstalk or noise considerations generally are controlling. This is due to the fact that the echo effects on individual circuits increase more rapidly with length than do crosstalk and noise, the disturbing effect of echo currents being governed to a large extent by the amount of time elapsing between the transmittal of the speaker's voice and the return of the echo currents. This time interval is directly proportional to the length of the circuit. Singing tendencies also increase at a rapid rate with increase in length on two-wire circuits but tend to be independent of length on four-wire cable and carrier telephone circuits, which are used to a large extent in circuits between the primary outlets, between primary outlets and regional centers, and between the regional centers. Furthermore, when two or more toll circuits are connected together, the echo effects of the individual circuits add together almost directly, whereas the effects of crosstalk, singing and noise in-

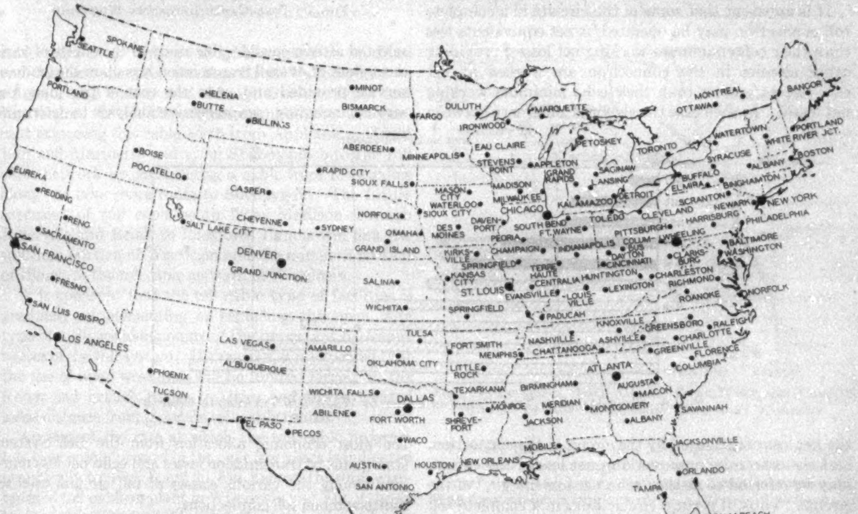
SEATTLE
SPOKANE
PORTLAND
HELENA
BUTTE
MINOT
BISMARCK
FARGO
BILLINGS
DULUTH
IRONWOOD
MARQUETTE
MONTREAL
OTTAWA
BANGOR
PORTLAND
WHITE RIVER JCT.
WATERTOWN
BOISE
POCATELLO
ABERDEEN
MINNEAPOLIS
EAU CLAIRE
PETOSKEY
TORONTO
SYRACUSE
BOSTON
ALBANY
EUREKA
REDDING
RENO
CASPER
RAPID CITY
SIOUX FALLS
STEVENS POINT
APPLETON
GRAND RAPIDS
SAGINAW
LANSING
BUFFALO
ELMIRA
BINGHAMTON
MASON CITY
WATERLOO
MADISON
MILWAUKEE
CHICAGO
KALAMAZOO
DETROIT
SCRANTON
NEWARK
NEW YORK
SACRAMENTO
SAN FRANCISCO
SALT LAKE CITY
CHEYENNE
SYDNEY
NORFOLK
SIOUX CITY
DAVENPORT
SOUTH BEND
TOLEDO
CLEVELAND
HARRISBURG
PHILADELPHIA
DES MOINES
OMAHA
PEORIA
FT. WAYNE
PITTSBURGH
WHEELING
BALTIMORE
WASHINGTON
FRESNO
DENVER
GRAND JUNCTION
GRAND ISLAND
KIRKSVILLE
CHAMPAIGN
INDIANAPOLIS
COLUMBUS
DAYTON
CINCINNATI
CLARKSBURG
SPRINGFIELD
TERRE HAUTE
CENTRALIA
HUNTINGTON
CHARLESTON
RICHMOND
SALINA
KANSAS CITY
ST. LOUIS
EVANSVILLE
LOUISVILLE
LEXINGTON
ROANOKE
NORFOLK
SAN LUIS OBISPO
WICHITA
SPRINGFIELD
PADUCAH
KNOXVILLE
GREENSBORO
RALEIGH
TULSA
NASHVILLE
ASHVILLE
CHARLOTTE
LOS ANGELES
LAS VEGAS
AMARILLO
FORT SMITH
MEMPHIS
CHATTANOOGA
GREENVILLE
FLORENCE
ALBUQUERQUE
OKLAHOMA CITY
LITTLE ROCK
ATLANTA
COLUMBIA
PHOENIX
TEXARKANA
BIRMINGHAM
AUGUSTA
MACON
TUCSON
WICHITA FALLS
DALLAS
MONROE
MERIDIAN
MONTGOMERY
SAVANNAH
ABILENE
FORT WORTH
SHREVEPORT
JACKSON
ALBANY
EL PASO
PECOS
WACO
MOBILE
JACKSONVILLE
AUSTIN
HOUSTON
NEW ORLEANS
ORLANDO
SAN ANTONIO
TAMPA
PALM BEACH
MIAMI
● REGIONAL CENTERS
• PRIMARY OUTLETS

Fig. 5—General Toll Switching Plan

Location of points at present employed as regional centers and primary outlets in the United States and Canada crease at a much less rapid rate. It follows, therefore, that when toll circuits are connected together in multi-switched connections, the overall combinations can, in general, be operated at transmission losses as limited by echo effects. Therefore, in establishing satisfactory transmission efficiencies for overall toll connections in accordance with the toll switching plan, each link must be designed not only with a consideration of its contribution to the overall transmission loss on the connection, but also with a consideration of its "echo minimum working net loss," *i. e.*, its contribution to the minimum working net loss of the overall connection from the echo current standpoint.

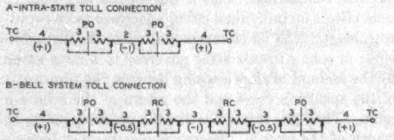
A–INTRA-STATE TOLL CONNECTION
TC
4
(+1)
PO
3
3
2
(−1)
PO
3
3
4
(+1)
TC
B–BELL SYSTEM TOLL CONNECTION
TC
4
(+1)
PO
3
3
3
(−0.5)
RC
3
3
3
(−1)
RC
3
3
3
(−0.5)
PO
3
3
4
(+1)
TC

Fig. 6—Transmission Requirements of General Toll Switching Plan Toll Circuits Used in Multiswitch Toll Connections

A. Figures above line indicate maximum transmission loss in db.

B. Figures below line in parenthesis indicate minimum echo net loss margins

It is apparent that some of the circuits in a complete toll connection may be operated at net equivalents less than their echo minimum working net losses, providing other circuits in the connection are worked at net equivalents greater than their echo minimum working net losses. In each case the algebraic difference between the net loss contributed by the circuit in the connection, and the echo minimum working net loss of the circuit, may be referred to as the "echo net loss margin" of the circuit. Thus, if certain circuit links in a complete toll connection have positive echo net loss margins, then other links may be permitted to have negative echo net loss margins in that connection, as long as the sum of the echo net loss margins of all of the links in the connection are in no case less than zero.

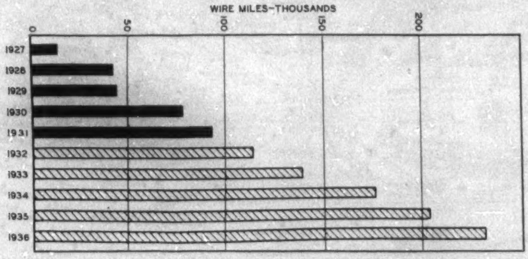

Fig. 8—Wire Miles of Toll Cable in Plant of Wisconsin Telephone Company

Fig. 6 shows the net equivalent and echo net loss margin requirements which have been set up for the various classes of toll circuits commonly used in multi-switched toll connections. These requirements were selected after a considerable study of the effects of various values of overall transmission loss upon the grade of service provided and upon the cost of providing the service, including comprehensive analyses to determine the most economic allocation from the Bell System standpoint of transmission losses and echo net loss margins among the various classes of toll circuits used in multiswitched toll connections.

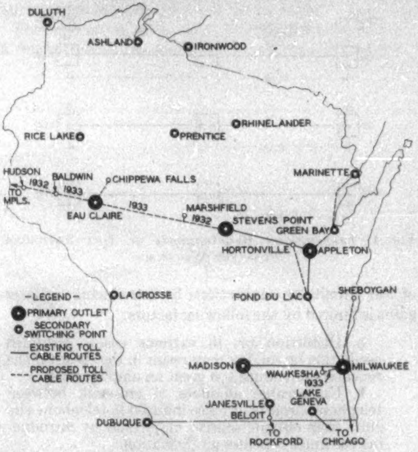
DULUTH
ASHLAND
IRONWOOD
RHINELANDER
RICE LAKE
PRENTICE
MARINETTE
HUDSON
1932
BALDWIN
1933
CHIPPEWA FALLS
TO MPLS.
EAU CLAIRE
1933
MARSHFIELD
1932
STEVENS POINT
GREEN BAY
HORTONVILLE
APPLETON
–LEGEND–
PRIMARY OUTLET
SECONDARY SWITCHING POINT
EXISTING TOLL CABLE ROUTES
PROPOSED TOLL CABLE ROUTES
LA CROSSE
FOND DU LAC
SHEBOYGAN
MADISON
MILWAUKEE
WAUKESHA
1933
LAKE GENEVA
JANESVILLE
BELOIT
DUBUQUE
TO ROCKFORD
TO CHICAGO

Fig. 7—Toll Cable Layout in Wisconsin

With the transmission limitations indicated on Fig. 6, the maximum transmission loss between terminal toll switchboards on overall toll connections should not exceed 10 db. on intrastate traffic in Wisconsin involving a maximum of three toll circuits, or 17 db. on traffic involving a maximum of five toll circuits in the Bell System network.

TOLL CABLE NETWORK IN WISCONSIN

Fig. 7 shows the present and contemplated toll cable network in Wisconsin. It will be noted that many of the important cities are now connected by toll cable facilities. Within the next few years, it is expected that the cable network will be extended to provide an all cable route from Chicago and Milwaukee to St. Paul and Minneapolis over the Fox River Valley route to Appleton and thence west through Stevens Point and Eau Claire. In addition, future requirements will warrant extending this cable north from Appleton to Green Bay and Marinette and tying Sheboygan into the toll cable network by constructing a cable from Milwaukee along the lake shore route to Sheboygan. The future extension of toll cable south from Madison through Janesville and Beloit to Rockford, Illinois will link the southern portion of Wisconsin and the northern portion of Illinois to the toll cable network in Wisconsin.

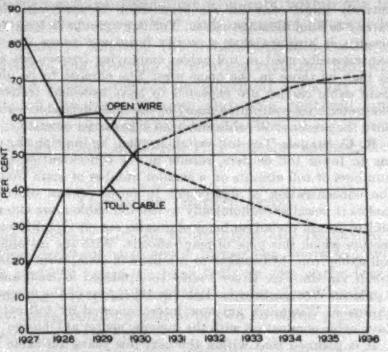

FIG. 9—PERCENTAGE OF TOLL WIRE MILES IN TOLL CABLE AND OPEN WIRE—WISCONSIN TELEPHONE COMPANY

It is apparent that the toll cable type of facilities is gradually supplementing or replacing the open wire type of facilities along many of the important backbone routes of the toll system. Ultimately, it is expected that the use of open wire lines will be limited largely to the feeder and branch routes to cities and towns located some distance from the main toll cable routes.

An idea of the rate at which the toll cable program has been going forward in the past few years is indicated graphically by Figs. 8 and 9. Fig. 8 shows the total wire-miles of toll cable in plant in Wisconsin and Fig. 9 shows the trend in the percentage of the total wire-miles in the toll plant which are in cable and in open wire.

The application of superimposed carrier systems to open wire lines has also been an important factor in providing for the increasing toll traffic during the last few years. Thus it may be seen from an inspection of Fig. 10 that a large majority of the principal open wire lines in the state now are arranged to have superimposed carrier systems operating on them.

Discussion

E. O. Neubauer: One of the important results of the application of the toll switching plan is the improved toll service to small communities located somewhat away from the larger cities. While the volume of toll traffic to and from these communities is comparatively small, it is necessary to include them in any plans for universal telephone service. As brought out in Mr. Lallier's paper a maximum of two switches and three toll lines is provided in the switching plans for complete intrastate service so that a satisfactory limit is automatically placed on the grade of toll service given to the smallest and most remote telephone exchange.

The large cities originate and receive such large volumes of toll traffic that large numbers of direct circuits are justified but the small community may have only a one circuit outlet. In Illinois under the toll switching plan, 87 per cent of the intrastate messages are completed over direct circuits, 12 per cent with one switch and only 1 per cent with two switches. It is this last 1 per cent of the traffic that is the largest beneficiary of the toll switching plan because previous to the switching plan there was no maximum limit to the number of switches and, therefore, no satisfactory service limit to the small community.

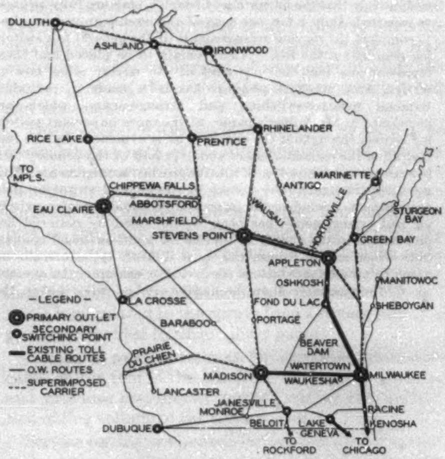
DULUTH
ASHLAND
IRONWOOD
RHINELANDER
RICE LAKE
PRENTICE
TO MPLS.
MARINETTE
ANTIGO
CHIPPEWA FALLS
ABBOTSFORD
EAU CLAIRE
MARSHFIELD
WAUSAU
HORTONVILLE
STURGEON BAY
STEVENS POINT
GREEN BAY
APPLETON
OSHKOSH
MANITOWOC
- LEGEND -
LA CROSSE
FOND DU LAC
SHEBOYGAN
PRIMARY OUTLET
SECONDARY SWITCHING POINT
BARABOO
PORTAGE
EXISTING TOLL CABLE ROUTES
BEAVER DAM
PRAIRIE DU CHIEN
O.W. ROUTES
WATERTOWN
SUPERIMPOSED CARRIER
MADISON
WAUKESHA
MILWAUKEE
LANCASTER
JANESVILLE
MONROE
RACINE
DUBUQUE
BELOIT
LAKE GENEVA
KENOSHA
TO ROCKFORD
TO CHICAGO

FIG. 10—PRINCIPAL TOLL CABLE, OPEN WIRE AND CARRIER ROUTES OF 1931—WISCONSIN TELEPHONE COMPANY

To make the switching plan fully operative, it is necessary only to have one connection from each terminal toll center to a primary outlet and to have the primary outlet interconnected. With these minimum requirements only 126 circuit groups would be needed to accommodate the 116 toll centers in Illinois. Actually there are over 500 such groups in service; the surplus groups affording more than one route between toll centers. The justification of these surplus groups is upon the basis of economy; *i. e.*, additional direct groups beyond the 126 must show economies in operation. Incidentally they afford a better grade of toll service than sought for under the switching plan. As the volume of toll traffic increases more of these surplus groups will be installed resulting in a gradual reduction in the number of messages requiring switches and a continuous improvement in toll service. However, to reach the other extreme would be to provide direct circuits between all toll centers but with 116 toll centers in Illinois, this would require nearly 6,700 circuit groups as against a little over 500 in service. The need for switching arrangements such as discussed in Mr. Lallier's paper is, therefore, a permanent one.

In regard to the replacement of cord circuit repeaters with the terminal repeaters and pads, this change in many situations has required a major investment and toll board rearrangement. Some offices showed ultimate savings by the conversion. These savings are brought about by the reduced number of toll positions required with a reduction in operating costs. Where ultimate economies could not be shown the conversion was made at Illinois switching points on the basis of the toll service improvements obtained with the pad system. Since the operation of the pads is automatic the operator is not required to determine the conditions under which pads are left in or removed as is the case with cord repeaters.

There are 4 primary outlets and 1 regional center in the switching plan for Illinois and all of these except one have already been converted. Centralia will be completed in the near future.

Glen Ireland: Although the general toll switching plan has only been in effect for about two years, sufficient progress has already been made and sufficient experience obtained to show conclusively that the advantages of such a plan are fully as great as expected. Only a few per cent of the circuit groups required to complete the routing arrangements of the general toll switching plan within the Bell System remain to be placed and these represent less than one per cent of the circuit miles now in service. Also, excellent progress has been made in providing terminal repeater-switching pad arrangements, which are mentioned in Mr. Lallier's paper, at the more important switching centers throughout the country, as is evidenced by the fact that all of the regional centers and over half of the primary outlets are now equipped with the improved arrangements. This provision throughout the system of the general switching plan arrangements, including the improved terminal repeater-switching pad arrangements, has been accompanied by material improvements in the quality of service on multi-switched connections which are, of course, the most difficult types of calls on which to provide satisfactory service. For example, the average speed of service interval on the multi-switched calls, that is, the time required from the placing of a toll call to the response of the called party or until a definite report is made by the operator, has been reduced by 45 per cent. Likewise there has been a marked improvement in transmission involving toll connections, particularly on the multi-switched toll connection.

In the latter part of Mr. Lallier's paper he points out a very interesting trend in the design of toll plant in Wisconsin which is also generally true throughout the Bell System, that is, that there has been a considerable increase in the use of toll cable and open wire carrier facilities. In this connection it is interesting to note that consideration has been given to and development is actively under way on the proposition of applying telephone carrier to long distance cables. For large groups of long distance circuits it appears that a carrier frequency range can be advantageously used in toll cables employing frequencies at least as high as those in the open wire. The circuits to be obtained from cable carrier are expected to have excellent transmission characteristics combining many of the most desirable features of both the present toll cable and open wire carrier circuits.

R. C. Siegel: The toll switching plan, by limiting the switching to fewer toll centers, results in the concentration of larger numbers of toll circuits on a smaller number of main routes and the concentration of equipment in fewer central offices. This makes it possible economically to use toll cable more extensively and to secure the advantages of greater circuit stability and protection which this type of plant affords. With the installation of toll cable from Milwaukee to Madison in 1928, from Milwaukee north via the Fox River Valley to Appleton in 1930 and from Appleton West to Stevens Point in 1931, four of the five primary outlets in Wisconsin are now interconnected by toll cable and have cable connection with the regional center at Chicago.

It is planned that within the next few years toll cable will be extended from Stevens Point west to Minneapolis, thereby completing the cable route from Chicago to Minneapolis and providing complete interconnection by means of toll cable of all primary outlets in Wisconsin. The construction of additional toll cables extending from these primary outlets to other important cities in Wisconsin is also contemplated in the future.

The procedure in reaching the objectives of the toll switching plan has been a gradual one and does not contemplate major replacements or rearrangement of plant, except those which may be required for other reasons. It is necessary, however, that the toll cable plant, central office buildings and telephone repeater equipment be so planned, constructed and arranged to permit of flexibly and economically caring for the requirements over a long period in the future.

Weather Resistant Covering for Line Wires

BY C. F. HARDING,* Fellow, A.I.E.E.
L. L. CARTER,* Associate, A.I.E.E.
and J. W. OLSON* Non-member

INTRODUCTION

THE problems involved in producing insulation for high-voltage cables have been of much interest to all electrical engineers for many years. Great advances have been seen in the art of producing insulation for higher voltage cables but it is only within the last two years that any unified and organized attempt has been made to improve the weathering and insulating qualities of the covering of weather resistant line wire. Previous to this time the development work lay in the hands of the manufacturer and was centered upon cutting costs to meet the demand for a cheaper product, which would meet the very meager specifications available.

ORIGIN OF INVESTIGATION

The concentration of effort upon the reduction of weather resistant wire costs during the years following 1918 or 1920 began to bear fruit in the early loss of saturating compound and the shedding of braids, in many cases, within a period of two to three years after the erection of the lines.

As more and more early failures of weather resistant wire coverings were seen to occur in extremely short lengths of time, the sentiments of many operating engineers swung toward discontinuing a weather resistant covering on overhead lines and adopting bare conductors for all service except where tree wire was necessary or where highly insulated conductors were required at points of inadequate spacing or clearance. Certain engineers opposed the policy of abandoning the use of weather resistant coverings on overhead lines of 4,000 volts line to line and less, for the reason that such a covering might reduce the number of service interruptions by preventing grounds and short circuits. This group believed that an attempt should be made to improve weather resistant coverings before making a decision to discontinue using such coverings.

At the insistence of the engineers among the companies represented by the Utilities Research Commission, that weather resistant wire coverings should be developed to provide longer life and higher insulating values, the Commission initiated a study of this problem at Purdue University. This study, Case 28, of the Utilities Research Commission was started October 1, 1929 under the direction of the Electrical Division of the Engineering Experiment Station with the Chemical Division cooperating.

PRELIMINARY INVESTIGATION OF PROBLEM

Shortly after the work was started a visit was made to the majority of the weather resistant wire manufacturing plants in the United States to observe the processes of manufacture, inspect the wire plant laboratories, and to discuss the problem with the men engaged in the production of weather resistant wire. This preliminary visit and discussion disclosed the fact that there was no adequate equipment available in the wire plants to test the comparative life of different weather resistant wire coverings and that there was only a meager knowledge of the reasons for early failure of weather resistant coverings and of means to increase the life of such coverings.

This preliminary survey emphasized the need for the purchase or construction of complete testing equipment at Purdue University.

ORGANIZATION AND EQUIPMENT OF LABORATORIES

The equipment purchased or constructed for this work includes apparatus for both accelerated and natural weathering of finished wires or wire covering materials and for complete physical and chemical tests of such materials. This equipment is being used to study the faults of present materials and to test new coverings and new materials.

The accelerated weathering equipment for testing the comparative length of life of completed wires, asphalts and other saturants, yarns, etc., exposes the samples to the light from a carbon arc and to a water spray at a constant wire temperature of 175 deg. fahr. during one part of the daily weathering cycle. The samples are subjected to freezing by moving the wire holding drum to a large refrigerator and to thawing and drying by moving back to the light exposure chamber. Distilled water is circulated to the sprays from a storage tank and pressure provided by an auxiliary pump.

A natural weathering rack for completed wires is located on the roof of the Electrical Engineering Building and provides continuous current loading and application of line voltage to the wire samples.

Service testing equipment for duplicating installation and service conditions on weather resistant wire includes apparatus for subjecting the wire to abrasion such as occurs when drawn over a cross arm, or that caused by rubbing on insulators, wrapping and bending at freezing temperatures, and for producing decentralization of the conductor in the covering such as occurs in service.

All the standard physical tests of asphalts, such as the determinations of softening or fusing point, viscosity at saturating temperatures, hardness at wire operating temperatures, and pliability at low temperatures are applied to weather resistant wire saturants.

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The fusing point is determined by the standard ball and ring method used by the asphalt industry. According to this test the fusing point is the temperature at which a steel ball of specified weight drops through a mold of the material under test, supported on a brass ring and immersed in water, the temperature of which is raised at a slow and definite rate.

The viscosity of the asphalt at saturating temperatures is determined by the Stormer method. Viscosity, by this method, is expressed as the number of seconds required for 100 revolutions of a cylinder propelled by a 100-gram weight and immersed in the material to be tested. It may be seen that the higher the time required for 100 revolutions the higher is the viscosity.

The consistency or hardness is determined by use of the Abraham consistometer. This equipment measures the pressure required to force various size plungers into the material under test, at a constant rate. The ends of these plungers are flat circular disks of larger size than the body of the plunger so that there is no friction between the asphalt and the shank of the plunger. The higher the consistometer hardness number the harder is the material under test. This hardness number is defined as the third root of the number of grams which must be applied to a circular flat plate, one square centimeter in area, to cause it to displace the substance under test at the rate of one centimeter per minute.

The pliability at low temperatures is determined by bending a mold, of dimensions ¼ in. by 1 in. by 4 in., broadside about a round mandrel ⅜ in. in diameter while the sample and apparatus are held at the low temperature of test. The angle of bend is increased at the rate of five degrees per minute and the angle of bending required to break the mold is taken as a measure of the pliability.

MAJOR FAULTS OF PRESENT WEATHER RESISTANT WIRE COVERINGS LIE IN THE SATURANTS

The work at Purdue University indicates positively that the poor physical properties of the saturants used in weather resistant wire coverings in the past have been the cause of the extremely early failure of weather resistant wire. The coverings of wires which last well are eventually caused to fail by a combination of physical changes, by the saturants shifting in the coverings, etc., and chemical changes in the saturant caused by exposure to light, oxygen of the air, etc., but in very early failures the effects of light and oxygen are of less importance than the effects of poor physical properties which cause the saturant to shift in the braid when hot and to crack and split when cold.

These poor physical properties of the saturants are, namely, too low fusing points, extreme softness when warm, great changes in hardness with changes in temperature, extreme brittleness at even moderately low temperatures, and a marked tendency to evaporate. The majority of saturants used have had softening points (as determined by the ball and ring method) ranging from 130 to 150 deg. fahr., lower than maximum wire operating temperatures. These materials shift in the braids leaving the upper braids unprotected at ordinary summer temperatures, which are well below the fusing point. The extreme brittleness of these saturants, which results in cracking and breaking of the braids in cold weather, may be illustrated by the fact that less than two degrees bending is required to break molds of these saturants in the pliability test at 0 deg. fahr.

Many of the poor physical properties, particularly the disappearance or evaporation of the saturant from the braids, have been caused by the addition of excessive amounts of waxes, wax tailings or low melting fluxes to obtain fluidity and ease of saturation. This addition was usually made by the workman operating the saturating tank and resulted in a saturant of variable properties. These low boiling materials soon leave the saturant, causing the body of the asphalt to disintegrate. The saturants vary in their resistance to the weathering effects of light and oxygen and the worst are subject to excessive surface deterioration and erosion.

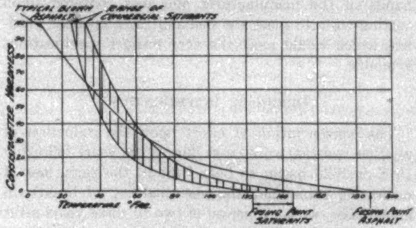

FIG. 1—COMPARISON OF INITIAL HARDNESS

The result of all these factors is the loss of compound, particularly from the upper side of the braids, with the resultant breaking of the braids on the upper side and the familiar festooning.

The use of materials subject to such faults has been permitted by specifications which did not specify the source nor physical properties of saturants. Proper specifications have not been written because of a lack of knowledge essential to the writing of such specifications. The work at Purdue University is to provide such a specification.

The commonly used drip and stain tests cannot be regarded as tests of the physical properties of the saturants. The high melting wax used as an outer coating prevents staining and the yarns hold the asphalt, preventing dripping, even though the saturant may be quite soft. The drip test may also favor an undersaturated wire.

Finishing waxes cannot be regarded as of great protection to the covering because they evaporate or crack and flake off within a short time or are damaged in installation. The wax has cracked, chipped off, or simply disappeared from the surface of the wire, on the various samples in the natural exposure rack, in from three to eight months.

COTTON YARNS

Have Minor Effect Upon Length of Life of Wire Coverings

It has been found that the cotton yarns are not so important in producing a long lived covering as are the saturants used. It is important, however, that sufficient cotton be present in the two inner braids to absorb and hold the saturant and that the outer braid should be closely woven to resist abrasion and erosion.

The one fault of the braided construction is that it does not, consistently, afford insulation beyond that of the air spacing provided, because of the interstices always present. It is desirable to have a more solid and homogeneous material next to the conductor.

IMPROVED SATURANTS

Intensive experimental work, in the past year and a half, with many asphalts of different types from different sources, tested in both accelerated and natural weathering exposures, has shown conclusively that the length of life of weather resistant wire coverings may be materially increased by using, for a saturant, blown petroleum asphalts having a ball and ring softening point of 180 to 200 deg. fahr., free of low boiling materials and without the addition of wax tailings in the wire plant.

Several of the weather resistant wire makers have offered to produce a quantity of experimental weather resistant wire on a commercial scale. It has been decided, therefore, to produce a carload of wire, using as a saturant, the asphalts which have been found to provide the longest lived wire coverings. This saturant will be a blown petroleum asphalt having the above range of ball and ring softening points; a hardness, as determined by the needle penetrometer, of 35-25 at 77 deg. fahr. with 100 grams loading for 5 seconds; chosen for the greatest pliability at low temperatures and for freedom from low boiling materials. The experience gained in the production of this wire will be an aid in the writing of a specification for an improved weather resistant wire covering.

Such an asphalt, of greater stability at higher wire temperatures, will greatly reduce the shift of the saturant in the braids and will be less subject to evaporation and erosion. In addition, these asphalts are less susceptible to temperature changes and are much less brittle at low temperatures. Such asphalts may be bent many degrees in the pliability test while commercial saturants cannot be bent more than two degrees.

The differences in hardness of these materials may be noted by referring to the hardness curves in Fig. 1. Table I gives comparative values for fusing point, hardness, pliability, and temperature susceptibility.

TABLE I—PROPERTIES OF COMMERCIAL SATURANTS COMPARED TO A TYPICAL BLOWN ASPHALT

	Range of saturants	Blown asphalt
Ball and ring fusing point	120–150 deg. fahr.	192 deg. fahr.
Pliability at 0 deg. fahr	½–1½ deg	23 deg.
Consistency-Abraham hardness:		
20 deg. fahr	Above 100	80
60 deg. fahr	17–40	35
100 deg. fahr	4–9	15
Loss by evaporation 100 hr. at 325 deg. fahr	0.25–4.0	none
Ball and ring fusing point after heating	190–230 deg. fahr	224 deg. fahr.
Consistency after heating:		
20 deg. fahr	above 100	95
60 deg. fahr	43–90	42
100 deg. fahr	18–55	18
Viscosity: Temperature at which a viscosity of 100 sec. per 100 rev. is obtained	260–278	357

The presence of undesirable low boiling fluxes and waxes is detected by testing the changes in the physical properties of the asphalts and measuring the evaporation after they have been heated at 325 deg. fahr. for 100 hours. It is expected that this test gives, in a short time, a measure of the changes which take place in the saturants in the wire covering, over a period of years. The changes taking place during heating have been of much help in eliminating the poorer saturants. The comparative changes with heating, for commercial saturants and a typical blown asphalt, may be noted by comparing Fig. 2 with Fig. 1.

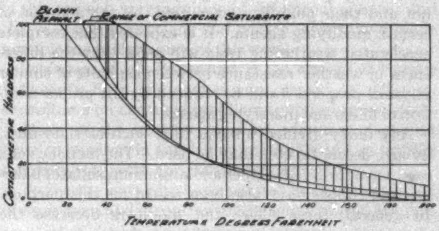

FIG. 2—COMPARISON OF HARDNESS AFTER HEATING

Heated 100 hours at 325 deg. fahr.

While the shifting of the saturants in the braids can be greatly reduced by choosing asphalts of the proper physical properties, such physical properties as may be checked in the laboratory in a few hours, and while saturants containing low boiling point materials can be eliminated by the heating test requiring several days, the accelerated weathering test is still required as a guide in choosing saturants least subject to weathering when exposed to light and air.

An exhaustive study of the manner in which asphalts weather, which have been selected from all sources and of different physical properties, is to be conducted to learn if it is possible to use only physical tests and the changes in their physical properties with heating as checks upon the life of such asphalts. If this is possible, then a specification for weather resistant wire which sets forth the properties of the saturant used will provide long life for such coverings. The asphalts can be tested within a reasonably short time to see that they meet the specification.

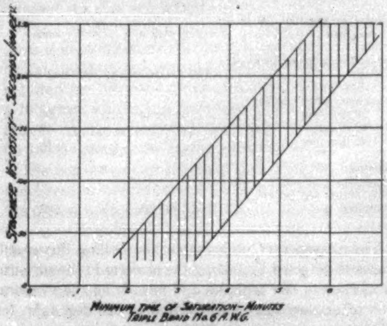

FIG. 3—RELATION OF VISCOSITY AND SATURATING TIME

MODIFICATION OF ASPHALTS

After asphalts of the proper physical properties have been chosen it will be found that these asphalts may be further improved, both in regard to their stability when hot and their pliability when cold, by the addition of certain modifying agents. It is expected that complete accelerated weathering tests will show that the differences in weather resistance between asphalts of similar physical properties will be reduced by the proper addition of fillers and modifying agents.

Any finely divided mineral filler such as slate dust, fly ash, dolomite, etc., may be used. The metallic salts used have included copper and aluminum sulfate; bakelite and rubber have also been tested for this purpose. In general, these fillers and modifiers decrease the changes in physical properties of the asphalts with changes in temperature and improve their resistance to the effects of weathering.

TEMPERATURES OF SATURATING BRAIDED COVERINGS *Must be Increased if Saturants of More Satisfactory Qualities are to be Used*

To obtain thorough saturation of triple braid coverings with saturants of a fusing point of 180 to 200 deg. fahr. at present saturating speeds it is necessary to use higher saturating temperatures than are used today. Laboratory tests have shown that to obtain saturation of three braids on No. 6 A. W. G. conductors in five minutes requires a saturant viscosity of 100 deg. Stormer or less. Fig. 3 shows graphically the time required to saturate the triple braid covering on No. 6 A. W. G. conductors with asphalts of a very wide range, at any given viscosity. It should be noted that the time of saturation is not the same for all saturants at a given viscosity but varies with the asphalt, some requiring more and some less time, as indicated by the shaded area.

Reference to Fig. 4 discloses that temperatures of 375 to 400 deg. fahr. are required to provide this viscosity in the case of a typical blown asphalt having a fusing point of 190 deg. fahr. A change would be required in the heating equipment for the saturating tanks in the wire plants, if such saturants are to be used.

SATURATING TEMPERATURES OF 400 DEG. FAHR. *Are Not Harmful to Cotton Yarns or Copper Conductors*

To determine definitely that saturating temperatures as high as 400 deg. fahr. can be used without damaging the cotton yarns nor too greatly annealing the copper conductors, a number of tests was made to determine the effects of this temperature.

The cotton yarns were tested for tensile strength, heated in air for different times, allowed to regain normal moisture content and again tested. The tensile strength of oil soaked yarns was also tested and compared with that of similar yarns heated in hot oil for varying times. No. 6 A. W. G. copper conductors, both medium hard drawn and hard drawn, were tested for tensile strength after heating in hot oil for varying times and compared with tests made on unheated conductors.

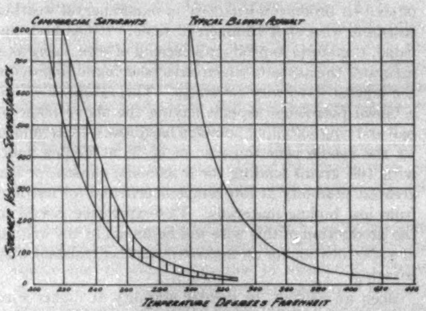

FIG. 4—COMPARISON OF VISCOSITY

The results of the cotton and copper tests are presented in Table II.

It may be seen that the loss of strength of the cotton yarn is of no importance in even the longest saturating time. There is no appreciable annealing of the copper conductors in the saturating time required in the modern continuous saturating tanks which are now generally used. In the case of the older type "reel in and reel out" saturating tank the time required for saturation

TABLE II—EFFECT OF SATURATING TEMPERATURE ON THE TENSILE STRENGTH OF COTTON YARN

Yarn heated in air (constant temperature oven)

Sample	Tensile strength	Loss	Per cent loss
Original yarn	2,001 gm.		
15 min. at 392 deg. fahr	1,760 gm.	241 gm.	12.0
30 min. at 392 deg. fahr	1,526 gm.	475 gm.	23.7
90 min. at 392 deg. fahr	1,360 gm.	641 gm.	32.0

Yarn heated in oil

Sample	Average tensile strength	Per cent loss		
		A	*B*	*C*
Dry yarn	1908 gm.			
Dipped in cold oil	1917 gm.			
Heated 15 min. 212 deg. fahr	1809 gm.	5.62		
Heated 30 min. 212 deg. fahr	1731 gm.	14.5	4.32	
Heated 15 min. 302 deg. fahr	1683 gm.	12.2	7.00	
Heated 30 min. 302 deg. fahr	1623 gm.	15.3	10.30	3.57
Heated 7½ min. 392 deg. fahr	1637 gm.	14.6	9.55	2.52
Heated 15 min. 392 deg. fahr	1593 gm.	16.85	11.98	5.35
Heated 30 min. 392 deg. fahr	1546 gm.	19.3	14.55	8.13

A Loss in strength considering the cold oiled sample as a standard.

B loss in strength considering the sample heated 15 min. at 212 deg. fahr. as a standard.

C loss in strength considering the sample heated 15 min. at 302 deg. fahr. as a standard.

All samples heated at 392 deg. fahr.

Kind of sample	Tensile strength lb. per sq. in.	Loss in strength lb. per sq. in.	Per cent loss
Hard drawn No. 6 copper wire			
Unheated wire	63,500		
Heated 10 minutes	62,590	970	1.53
Heated 30 minutes	61,140	2,420	3.96
Medium hard drawn No. 6 copper wire			
Unheated wire	51,430		
Heated 10 minutes	50,780	650	1.27
Heated 30 minutes	49,490	1,940	3.93

may be long enough to anneal the conductor slightly, requiring that the conductors be drawn somewhat harder initially.

These tests, it is believed, have definitely cleared the way for the use of higher saturating temperatures in so far as likelihood of any damage to yarns or conductors is concerned.

New and Different Types of Coverings

While the development work carried on thus far has been concentrated upon providing an improved asphaltic base saturant which could be applied to the present triple braid covering without great changes in the weather resistant wire plant, experimental work with other types of weather resistant saturants and with coverings other than triple braid, has been carried on.

Bakelite and other synthetic resins, Harvel compounds and varnishes, and other materials have been applied to weather resistant wire coverings. The use of these materials as saturants for a single outer braid, applied over an insulating cover on the conductor, provides a highly protective coating because of the excellent weathering quality of such materials.

Experiments with conductors having a paper tape covering next to the conductor and a single outer braid weatherproofed with these very high quality saturants have shown that coverings of consistently high insulating value can be obtained and that the paper next to the conductor does not deteriorate appreciably in the time of one accelerated weathering test.

Most of these high grade saturants can only be applied to one braid because of their high cost.

It is expected that future work will provide a weather resistant wire covering of higher insulating value and longer life than the present triple braid construction but of smaller size and weight. There are many interesting possibilities of improvement in this type of covering.

Conclusion

The present triple braid weather resistant wire covering may be materially improved by the use of higher fusing point blown asphalts, free of waxes and low boiling fluxes, as saturants. Such a saturant will provide a covering of uniformly long life.

It has been proved that the higher saturating temperatures required to apply such saturants may be used without damage to the cotton yarns or excessive annealing of the copper conductors.

Specifications for weather resistant coverings will soon be written as a result of this work, the terms of which will guarantee to the utility, weather resistant wire of long life.

Future work will provide a covering of lighter weight and smaller size but of higher insulating value and longer life than present construction.

Acknowledgment

Acknowledgment is gratefully extended to Mr. G. W. Hamilton, Chairman, and to the members of the committee of the Utilities Research Commission in charge of this project, in recognition of the encouragement and the valuable suggestions and advice which have made possible the performance of this work.

Professors H. C. Peffer and R. N. Shreve of Purdue University have rendered invaluable aid in acting as consultants on chemical problems involved in the work.

Many of the weather resistant wire manufacturers have cooperated in furnishing materials and in frankly discussing the work and its application to the plant.

To the students who have assisted in the construction of equipment and in routine testing many thanks are due.

Discussion

C. D. Brown: The development of an improved saturant for weather resistant covering of line wires is a decided step in the right direction and Mr. Harding is to be complimented for his research work on this subject.

In some cases, triple braid weatherproof insulation has little insulating qualities after it has been up for four or five years, and a great many companies do not place any confidence in this type of insulation on primary distribution feeders. The result is that many companies only use weatherproof insulation where it is required by the code, and their construction is such that reliable service can be maintained even though the insulation drops off entirely, except where tree conditions are encountered.

Tree wire is used under much less favorable conditions than the ordinary weatherproof wire and the weather resistant covering of tree wire must not only be depended upon to protect the rubber insulation against weather conditions but also against abrasion. If the insulation of tree wire fails, it often results in an interruption to service. Our experience has indicated that the outer covering of tree wire has in many cases the same deficiencies with respect to impregnating compound, as are brought out in the paper regarding weatherproof insulation.

Has any investigation been made of the properties of the weather resistant covering for tree wire, and if so, what have been the results of the investigation?

James H. Foote: It is indicated in the paper that seven years' life of weatherproofing on overhead wire would seem to be reasonable, and certain economic considerations given in the paper are based upon this life. In my experience this would seem to be an abnormally short period of time. I have in mind a distribution system which was entirely rebuilt in 1914 using triple braid weatherproof wire manufactured to no special specifications, which does not today after 18 years show any appreciable amount of raggedness or deterioration easily visible from the ground. In another city the entire street lighting system was rebuilt in 1917 using triple braid weatherproof wire obtained on the market, which today shows no deterioation visible from the ground after 15 years of life.

The question may arise as to whether the results obtained with the lighter braids ordinarily used on the smaller sizes of wire used by the authors of the paper in their experiments will also apply to the considerably thicker cotton covering usually applied to the larger sizes of wire ordinarily used in distribution, such as 0 or 0000 A.W.G.

It is mentioned in the paper that many engineers have swung towards discontinuing the weather resistant covering on overhead lines. While it would seem that the omission of insulating covering may well be justified on the higher voltage conductors, especially primary lines of over 5,000 volts, still the weatherproofing has a real value as insulation in the case of overhead secondary lines run on close spacing such as is common where secondary racks are used instead of crossarms, and especially where the secondary lines are run through trees where the tree branches may push the wires together during storms. It would seem entirely reasonable to omit the weatherproofing from the neutral wire of a three-wire secondary, leaving the weatherproofing on the outside wires. Service wires should also have weatherproofing or other insulation since they very frequently are so strung as to swing together under certain conditions. Thus, there appears to be an adequate market for weatherproofed wire in the future and it would seem that the investigation carried on by the authors is certainly very well worth while.

Frank Kingburne: While some of the findings may be revolutionary, largely because of the dearth of available information, this centralized, unbiased research should excite the manufacturers toward improving the general character of this phase of their business.

The contention that the saturant is of first importance is quite correct, but at the same time the final improved product will depend on both the saturant and the braid construction. To that end a better knowledge of braids and braiding construction should be important; a braiding schedule specifying the following: size of cotton; number of plys; number of ends; picks per inch to attain a so-called 100 per cent braid covering according to size or diameter of wire. The wording "closely woven braid" in present specifications is not very definite and lends itself readily to controversy. Again, in normal times, cotton is quite an item in the quoted price in comparison with the present saturant, although it may not be with a high quality saturant.

The statement of having "a more solid and homogeneous material next to the conductor" should also indicate that flexibility in the material is desirable. The product of the Peerless Wire and Cable Company at Pennington, N. J. may be somewhat along this line, also the experimental work done by the American Steel & Wire Co., Worcester, Mass., on a similar type of line wire, *viz.:* paper tape next to the conductor and a single cotton braid overall, weatherproofed with a high quality saturant, recorded in the paper.

The matter of using a high saturating temperature, 400 deg. fahr., with a high quality saturant should not act as an impediment, or be detrimental to the tensile strength of the cotton braid, provided proper precautions and refinements are used during the saturating process. Locomotive headlight wires for steam engines are saturated under conditions quite similar with good results. Standardization of this practise, high saturating temperatures, would undoubtedly mean revamping the present apparatus found in most wire plants.

It may be of interest to know that the railroads purchase large amounts of line wire, usually in accordance with A.R.A. specifications. These specifications require two cotton braids over the conductor, the conductor being either iron wire, copper, or copper alloy. This leads to the question as to why the utilities should require three braids, or their equivalent, for their purposes. True, the operating conditions, voltage for instance, are different, yet from contact with operating men it has been my impression that the utilities in replacing festooned line wire were not worried about the lowered insulating value, but to comply with municipal ordinances, or public opinion, or perhaps to establish an underground system. So from this why would not a similar wire be just as good, regardless of whether rural or urban use was contemplated.

C. F. Harding, L. L. Carter, J. W. Olson: Many reports are received of weatherproof wires that have been in service for periods of 15 to 35 years without shedding the braided covering. In marked contrast to this there are many reports of weatherproof wire coverings that have failed in periods as short as two years, particularly in hot climates. It should be remembered that the majority of the wires that have failed quickly have been produced in the last 10 or 12 years whereas the longer lived wires were produced in the years previous to 1920 or previous to the war period.

A graphic illustration of the differences in the weathering qualities of the recent product and the older weatherproof wire is the fact that certain commercial wires placed in the outdoor weathering exposure at Purdue in September of 1930 appear to be in poorer condition than wires installed in an outdoor weathering rack in Chicago in 1916.

BRIEF HISTORY OF WEATHERPROOF WIRE PRODUCTION METHODS

To explain the reasons for this variation in life and the reasons for the changes in weatherproof wire covering materials and production methods, which have caused this change in life, it seems desirable to give a brief history of weatherproof wire production methods and materials.

Before the start of the last decade, weatherproof wire coverings were usually built up of braids made from high strength, long fiber, cotton yarn, often saturated after each braid was applied and usually saturated with natural asphalts rather than petroleum base asphalts. Production methods were slow and the materials used more expensive than it is now possible to use.

With the start of the last decade the increased demand for electric service caused a very great increase in demand for weatherproof wire. Coupled with this demand for more wire was an insistence that the cost of the wire be lowered. The increased demand alone, if the earlier materials and production methods had been used, would probably have resulted in an increased cost of the product. We thus had two factors, namely, the increased demand and the demand for lower prices, that forced the substitution of cheaper materials and speedier production methods in the weatherproof wire industry.

The weatherproof wire producers began to use cheaper grades of cotton yarn and looser braids and they turned to petroleum by-product asphalts for saturants. They further changed from saturation of each braid after its application to the application of all three braids in one step and the saturation of these braids as a second step. To obtain the production speeds required and to saturate three braids in one step at the temperatures available in the wire plants (that of low pressure steam) the use of low melting point petroleum asphalts of low viscosity at saturating temperatures became general.

In this short description of the change on the industry we have the background for the short-lived weatherproof wire coverings produced in the past 10 or 12 years. This background of the changes in the weatherproof wire industry is of much importance to anyone interested in developing a longer lived covering.

FUNDAMENTAL PROBLEM INVOLVED IN RESEARCH AT PURDUE

The real problem in the research work at Purdue has been that of finding a material which could be used as a saturant, that would provide equal or better life than was had by the weatherproof wire coverings produced years ago but at a very slight increase over the present cost of production.

The answer to the question "if long lived wire coverings were produced 15 to 20 years ago, why cannot they readily be produced again" is very simple. Such coverings could not be produced with the old materials and the old methods at a price that could, in most cases be economically justified.

IMPROVEMENT IN WIRE COVERINGS CAN BE MADE AT LITTLE INCREASED COST

The use, as weatherproof wire saturants, of the air blown asphalts meeting the preliminary specification which has been developed, has been proven in the laboratory to yield a greatly increased length of life of the covering on any size wire, over that provided by the usual commercial saturants and to provide a greatly increased flexibility of the finished wire. The physical properties of these asphalts and the present commercial saturants are given in Table I. The preliminary specification follows:

> The saturating compound shall consist of a pure blown petroleum asphalt and shall not be mixed with any material such as paraffin, wax tailings or fluxes. It shall have the following physical properties:
>
> *Ball and Ring Fusing Point*: 180-200 deg. fahr.
>
> *Penetration*: 77 deg. fahr. 100 g., 5 sec. 35 or harder, tenths of mm.
>
> *Pliability*: A mold of the asphalt ¼ in. by 0.1 in. by 0.4 in. shall not break when subjected to an angle of bending of 15 deg. around a mandrel of 3/16 in. radius at a temperature of 0 deg. fahr. This test shall be made in the Reeve and Yeager pliability testing equipment, at a rate of bending of 10 deg. per minute.
>
> *Loss By Evaporation*: A 100 gram sample of the compound heated for 100 hr. at a temperature of 325 deg. fahr. shall not lose more than 1.0 g. in weight.
>
> *Rise in Fusing Point*: The difference in the ball and ring fusing point determined before and after the above heating test shall not be greater than 40 deg. fahr.
>
> *Change in Penetration After Heating*: The penetration of the sample subjected to the above heating test shall not become harder by more than 10 points. The crust formed on the sample during the heating test may be removed before determination of the penetration after heating.

Adequate proof of the improvement obtained by the use of these materials is given by comparing the wires in Fig. 1 with those in Fig. 2, all having had the same exposure in the accelerated weathering test. Fig. 1 shows the condition of wires saturated with the present commercial saturants after having been exposed in the accelerated weathering test at Purdue.

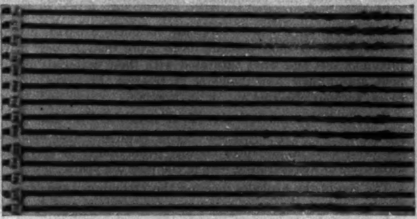

FIG. 1—CONDITION AFTER ACCELERATED WEATHERING TEST OF WIRES SATURATED WITH COMMERCIAL SOLVENTS

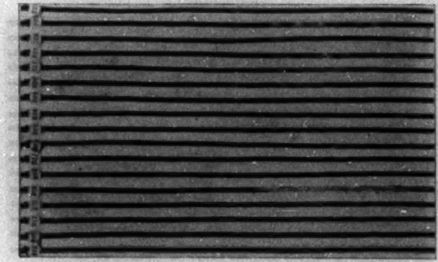

FIG. 2—CONDITION AFTER ACCELERATED WEATHERING TEST OF WIRES MEETING VARIOUS PARTS OF PRELIMINARY SPECIFICATIONS

Wires 815-8, 820-6, and 830 meet these specifications in every detail

The four wire coverings that have held up well on this test are saturated with very hard and brittle materials. The asphalts with which these four wires are saturated bend less than two degrees in the pliability test. This emphasizes the importance of applying both the laboratory pliability test and the accelerated weathering test in choosing the most satisfactory

TABLE I—COMPARISON OF PHYSICAL PROPERTIES, COMMERCIAL SATURANTS AND ASPHALTS PROPOSED AS SATURANTS

	Range of physical properties		Average of physical properties	
	Commercial saturants	Proposed saturants	Commercial saturants	Proposed saturants
B & R fusing point				
Before heating	119-158°F.	186-208°F.	143.2°F.	196.1°F.
After heating	176-245	209-232	198.4	220.7
Penetration: 77°F., 100 grams, 5 seconds				
Before heating	3-66	21-33	25.5	27.5
After heating				
With crust	0-23	0-4	8.7	1.0
Crust removed	0-26	17-29	11.8	22.3
Loss in weight during heating	0.12% gain, 7.17% loss	0.03-0.97% loss	1.63%	0.36%
Pliability	0.41° -13.3°	16.2°-48°	3.21°	26.5°
Specific gravity				
Before heating	1.001-1.200	0.987-1.044	1.071	1.011
After heating	1.021-1.208	0.988-1.042	1.080	1.013

weather resistant coverings. The wires in Fig. 2 have coverings saturated with air blown asphalts meeting the specification for melting point and penetration, but in the case of samples 819, 826, 827, 828, 829 and 831 not meeting other parts of the specification such as that for pliability or changes in physical properties with long continued heating. It may be readily seen that the wires meeting the preliminary specification, in every detail, provide a vast improvement in weather resistance over the commercial saturants. These wires are numbers 815-818, 820-826 and 830. All of these asphalts bend more than 15 deg. in the pliability test and wires saturated with these materials may be bent about their own diameter at freezing, or lower in some cases, without breaking any of the braids.

Sample 819 is saturated with a material that has shown peculiar breakdown when exposed to light and heat and that has been used in the lower melting point ranges in very great quantities of weatherproof wire in the past ten years.

The use of the asphalts meeting the preliminary specification requires only a change of saturating temperature and does provide a remarkable improvement in the weather resistance of the covering.

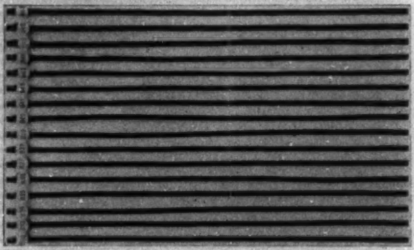

Fig. 3—Wires Illustrating the Improved Weather Resistance of Air

Blown asphalts to which proper inert mineral fillers have been added

Fig. 4 illustrates the smaller change in hardness of air blown asphalts as compared with the commercial saturants and also the greater stability at the higher operating temperatures. It may be seen that the air blown asphalts are harder, as an average, at temperatures of 60 deg. fahr. and higher, but are softer than the commercial saturants below this temperature.

Fig. 5 shows the smaller change in the hardness of the air blown asphalts as compared with the commercial saturants after being heated for 100 hours at 325 deg. fahr. It will be noted that the blown asphalts have not changed greatly while the commercial saturants have become much harder. This lack of change with long heating has been proven to correlate rather closely with the resistance of the wire covering to weathering.

Fig. 6 shows that higher temperatures are required to obtain saturation of the braided coverings with the air blown asphalts than are required in the case of the commercial saturants. Viscosities lower than 100 seconds per 100 revolutions are required to obtain saturation of triple braid coverings at present saturating speeds and temperatures of 360 deg. and higher are required for saturation with these asphalts.

These "air blown asphalts" are produced by blowing air through the heated petroleum residue, as compared to the production of the usual commercial saturant by blowing steam through the petroleum residue. Certain changes in the structure of the asphalt are produced by the air blowing which results in a much more stable asphalt of lower temperature susceptibility. The asphalts illustrated in Fig. 2 are stock products. They are not special materials in any sense. There is no reason to think that the increased use of air blown asphalts meeting these specifications should result in a marked increase in cost of the materials (which now cost very little more than the steam reduced asphalts) because the petroleum residue is just as readily blown to meet this specification as to meet one of a lower melting point.

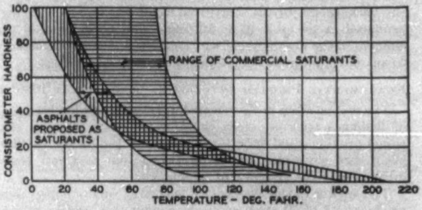

Fig. 4—Comparison of Initial Hardness

Commercial saturants and asphalts proposed as saturants

Modification of Asphalts

Fig. 3 shows the improvement that may be made in the weather resistance of the air blown asphalts by the use of the proper inert mineral filler. Wires numbers 832-835 are saturated with the same materials as wires 815-818 in Fig. 2 except that 30 per cent by weight of 325 mesh slate dust was added to the asphalt before saturation. Wires 840-842 in Fig. 4 are also asphalts to which certain modifying agents have been added. The use of such modifying materials provides an additional and very desirable improvement in the weather resistance of the wire covering and also may slightly reduce the cost of the saturant because of the cheapness of the slate dust. The use of the filler is particularly helpful in the case of the asphalt on the surface of the wire and in the case of the finishing materials.

Improved Finishing Materials

Finishing materials that are more flexible than the present waxes may be had and are much more desirable, particularly when some filler is used as a coating or an anti-stick in these finishes. Mica flake is particularly valuable as an outer finish both to prevent sticking and to improve weathering, reduce cracking, etc.

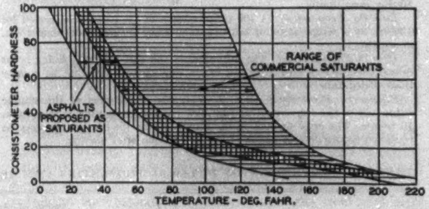

Fig. 5—Comparison of Hardness

Heated 100 hr. at 325 deg. fahr. Commercial saturants and asphalts proposed as saturants

Accelerated Weathering Test is Still Required in Choosing the Saturants and Finishers

While all of the asphalts which weather poorly in the wires in Fig. 2 would have been eliminated by requiring them to meet the preliminary specification, we cannot be certain as yet that this will always be true and we must still depend upon the accelerated weathering test to eliminate the poor materials. In the operation of the weather-ometer on the standard cycle used by the Bureau of Standards the commercial saturants all show extreme checking or cracking in only one day whereas the asphalts recommended show no signs of deterioration in this time, except in one instance, thus checking the tests of the asphalts in wires as illustrated in Figs. 1, 2, and 3.

Asphalt Analysis

The analysis of asphalt and the determination of chemical unsaturation are now being carried on to learn if it is possible to correlate some result of this analysis with the variation in weathering of the asphalts.

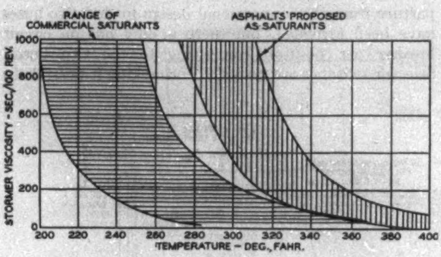

Fig. 6—Comparison of Viscosity

Commercial saturants and asphalts proposed as saturants

Additional Testing Equipment Designed or Constructed

The design for an improved Abraham consistometer and a Reeve & Yeager pliability test equipment especially adapted to this work has been completed. The rate of penetration in the consistometer and the rate of bending in the pliability tester will be quite constant as both are to be electrically driven and the degree of bending in the pliability test will be determined by automatic electric timing of the start and the finish of the bending period. The consistometer hardness will be read from a pressure gage which indicates the pressure on the table of the consistometer supporting the asphalt sample under test.

In the testing of finished wire a small electrically heated braid press and a pliability tester in which very small samples of asphalt may be tested have both been completed. The braid press is used to extract the saturant from the two inner braids of a finished wire. Sufficient asphalt is obtained from a two foot length of No. 6 wire for the standard ball and ring melt point test and the penetration determination. The pliability test can be made with this new equipment upon the small sample extracted from the braids.

The accelerated weathering equipment is still depended upon to indicate certain types of failures in the asphaltic saturant.

New and Improved Coverings

The much superior insulation of paper tape covered samples having a single weatherproofed outer braid has been shown repeatedly by the slower decrease in insulation resistance of this covering when immersed in water, as compared with the triple braid construction. The use of this paper tape and single braid covering provides a conductor of lighter weight and much smaller overall diameter, which means that sleet and wind loads will be lower, and also of consistently high dielectric strength even with considerable exposure of the braids to the weather.

Conclusion

This discussion is intended to answer the questions raised at the time of presenting this paper and also to summarize the work done since the writing of the original paper.

The work done since writing the paper has given ample proof that the materials recommended will provide longer life for weatherproof wire coverings and also that these materials may be obtained within a reasonable distance of any weatherproof wire plant in the United States.

It is felt that the experience and equipment built up at Purdue as a result of this work should prove of value in the development of improved weatherproof braids as applied to other types of wire than the ordinary weatherproof wire. This is especially apparent when it is pointed out that in looking for improved materials, the coverings of tree wires and other insulated overhead line wires were investigated to learn if the weatherproofing compounds used in the braids on these wires were superior to the ordinary weatherproofing compounds, and it was found that while these coverings were made of exceptionally strong and high grade braids they were not saturated with compounds in any way superior to the present commercial weatherproof saturants.

Electrical Design Features of Waukegan Station

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THE Waukegan Station of the Public Service Company of No. Illinois is located at the north limit of the city of Waukegan on the west shore of Lake Michigan. This steam generating station is the chief energy supply point between the Illinois-Wisconsin State line and Chicago. It also forms one of the supply points for the Company's 132-kv. transmission system which forms an outer belt around Chicago. The major substations and interconnections with other companies are shown in Fig. 1. The main one-line diagram of the station is presented in Fig. 2.

Some features of the design anticipated the general use of similar equipment in other stations. The general design of the station, however, paralleled the advance of the art, as the station has developed from a single 25,000-kw., 12,000-volt unit in 1923 to five units having a total capacity of 290,000 kw. in 1931. The fifth of these is a 115,000-kw. unit generating at 18,000 volts.

Waukegan Station is unique in a number of its design features, some of which result in a material capital saving, some in a greater safety to personnel, and others in greater operating reliability. Probably the best way of presenting these features is to review the development of the station in three fairly distinct periods.

INITIAL INSTALLATION

In the first period (units 1 and 2), a double 12-kv. bus was started using 15-kv. truck type oil circuit breakers and cell-type construction to enclose the buses, disconnecting switches, and instrument transformers, as well as the oil circuit breakers. A Cory lock system was provided to prevent improper operating sequence of the disconnecting switches. The auxiliary equipment operated at 480 volts, and an ordinary open bus was provided for this purpose. All cables were run in conduits from the single bus used for the turbine and boiler-room auxiliaries. Reserve for the auxiliaries was provided by a tie between the unit 1 and 2 sections of the auxiliary bus. Feeders were run from the 480-volt auxiliary bus to *VV* switch groups located at various load centers along the coal-preparation and conveying system.

METAL-CLAD SWITCHGEAR USED IN SECOND PERIOD

The second period started with the installation of a unit twice the size of the original one, and a radical departure from conventional design was introduced by the use of metal-clad switchgear for the 12,000-volt equipment, and for the 2,400-volt and 480-volt auxiliary equipment. Fig. 3 shows the 12,000-volt, type M switchgear.

The bus arrangement of the 12,000-volt metal-clad switchgear used for units 3 and 4 is somewhat of a departure from the conventional design in that two buses have been provided, but there is only one oil circuit breaker per position connecting to the two buses. Selector switches are provided for making it possible to place the oil switch on either the main or the reserve bus, as desired. There are fourteen of these units, which range in carrying capacity from 600 to 4,000 amperes. The interrupting capacity of all these switchgear units is 1,500,000 kva.

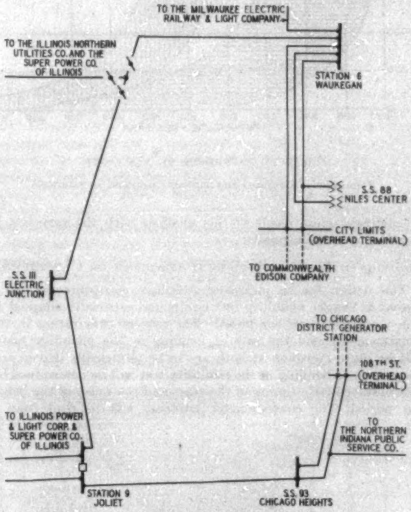
TO THE MILWAUKEE ELECTRIC RAILWAY & LIGHT COMPANY
TO THE ILLINOIS NORTHERN UTILITIES CO. AND THE SUPER POWER CO. OF ILLINOIS
STATION 6 WAUKEGAN
S.S. 88 NILES CENTER
CITY LIMITS (OVERHEAD TERMINAL)
TO COMMONWEALTH EDISON COMPANY
S.S. III ELECTRIC JUNCTION
TO CHICAGO DISTRICT GENERATOR STATION
108TH ST. (OVERHEAD TERMINAL)
TO THE NORTHERN INDIANA PUBLIC SERVICE CO.
TO ILLINOIS POWER & LIGHT CORP. & SUPER POWER CO. OF ILLINOIS
STATION 9 JOLIET
S.S. 93 CHICAGO HEIGHTS

FIG. 1—132-KV. SYSTEM AND INTERCONNECTIONS OF THE PUBLIC SERVICE COMPANY OF NORTHERN ILLINOIS

There is a number of interlocks on this switchgear which prevent incorrect operation due to errors, oversights, or carelessness when switching operations are performed. For instance, the isolating or selector

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*Elec. Engr., Public Service Co. of No. Illinois.

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switches cannot be opened or closed unless the main oil circuit breaker is open. Also, the oil circuit breakers cannot be lifted from their tanks, nor can the selector and isolating switch tanks be lowered, when current carrying parts are alive.

AUXILIARY EQUIPMENT SIZE CAUSES OPERATING VOLTAGE CHANGE

The use of larger turbo-generator units required boilers of greater steaming capacity and larger auxiliary equipment. As a result of the increased size of auxiliary equipment, larger motors were used; so the operating voltage was changed from 480 to 2,400 volts for these motors. Also, the larger capacity of the boilers made it necessary to provide more flexible operation of the stoker-feed mechanism on the boilers of unit 3. To provide this flexibility, a-c. brush-shifting motors were used. This arrangement provides almost as much flexibility as could be obtained by the use of the customary d-c. motors and control.

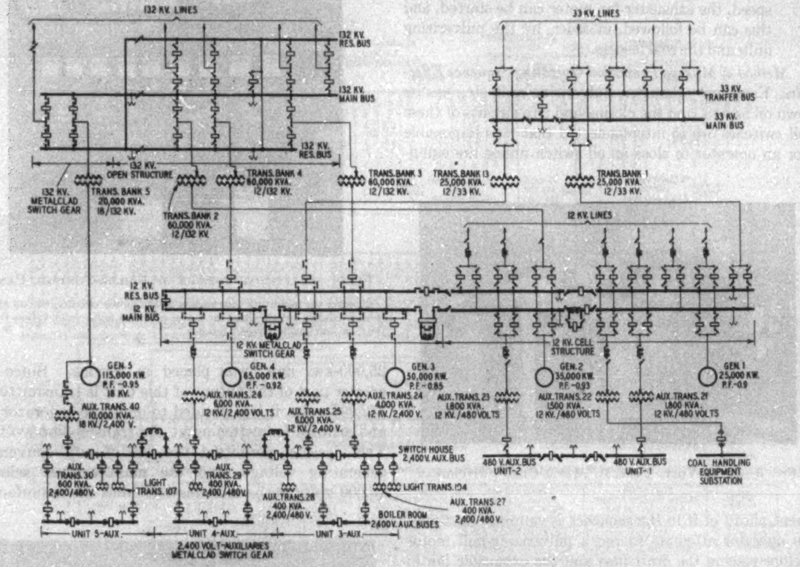

132 KV. LINES
132 KV. RES. BUS
132 KV. MAIN BUS
132 KV. RES. BUS
132 KV. OPEN STRUCTURE
132 KV. METALCLAD SWITCH GEAR
TRANS. BANK 5 20,000 KVA. 18/132 KV.
TRANS. BANK 2 60,000 KVA. 12/132 KV.
TRANS. BANK 4 60,000 KVA. 12/132 KV.
TRANS. BANK 3 60,000 KVA. 12/132 KV.
33 KV. LINES
33 KV. TRANFER BUS
33 KV. MAIN BUS
TRANS. BANK 13 25,000 KVA. 12/33 KV.
TRANS. BANK 1 25,000 KVA. 12/33 KV.
12 KV. LINES
12 KV. RES. BUS
12 KV. MAIN BUS
12 KV. METALCLAD SWITCH GEAR
12 KV. CELL STRUCTURE
GEN. 5 115,000 KW. P.F. -0.95 18 KV.
GEN. 4 65,000 KW. P.F. - 0.92
GEN. 3 50,000 KW. P.F. - 0.85
GEN. 2 35,000 KW. P.F. -0.93
GEN. 1 25,000 KW. P.F. -0.9
AUX. TRANS. 40 10,000 KVA. 18 KV./2,400 V.
AUX. TRANS. 26 6,000 KVA. 12 KV./2,400 VOLTS
AUX. TRANS. 25 6,000 KVA. 12 KV./2,400 V.
AUX. TRANS. 24 4,000 KVA. 12 KV./2,400 V.
AUX. TRANS. 23 1,800 KVA. 12 KV./480 VOLTS
AUX. TRANS. 22 1,500 KVA. 12 KV./480 VOLTS
AUX. TRANS. 21 1,800 KVA. 12 KV./480 VOLTS
SWITCH HOUSE 2,400 V. AUX. BUS
480 V. AUX. BUS UNIT-2
480 V. AUX. BUS UNIT-1
COAL HANDLING EQUIPMENT SUBSTATION
AUX. TRANS. 30 600 KVA. 2,400/480 V.
LIGHT TRANS. 107
AUX. TRANS. 29 400 KVA. 2,400/480 V.
AUX. TRANS. 28 400 KVA. 2,400/480 V.
LIGHT TRANS. 104
AUX. TRANS. 27 400 KVA. 2,400/480 V.
BOILER ROOM 2,400 V. AUX. BUSES
UNIT 5-AUX.
UNIT 4-AUX.
UNIT 3-AUX.
2,400 VOLT-AUXILIARIES METALCLAD SWITCH GEAR

FIG. 2—SINGLE-LINE DIAGRAM OF THE WAUKEGAN POWER STATION

Main one-line diagram of connections

UNIT 4 DESIGN FEATURES

Although the same general electrical design was used for unit 3 and 4 installations, there are a few features of the unit 4 installation that are of interest. Since unit 4 has a capacity of 65,000 kilowatts, the carrying capacity of its metal-clad switchgear increased; the size and number of auxiliary motors increased because of the use of larger auxiliaries and also because unit-type coal-pulverizing mills were installed. Centralized control of the boiler coal and air supply auxiliary motors was provided on panels similar to the one shown in Fig. 4; and the electrical control system was designed for either manual or automatic operation, in response to the demands of the automatic combustion-control equipment which regulates the coal and air supply for the boilers.

PROTECTION ON PULVERIZED FUEL SYSTEM

The use of automatic combustion-control equipment and of pulverized coal made it necessary to exercise unusual care in designing the control and protective system. By designing the system so that a negative pressure or suction must be present inside the furnace when it is in service, the possibility of the pulverized coal's igniting and flashing back into a pulverizing mill and causing an explosion, are eliminated. This is the principle followed in designing, for the boiler-feed auxiliary equipment, an electrical interlock system which prevents any deviation from the prescribed operating procedure.

STARTING SEQUENCE

The prescribed starting sequence of the equipment that must be followed whenever a boiler is fired up is as follows:

1. Start low-speed, induced-draft fan, wound-rotor motor. When the resistance is all out of the secondary circuit, the drum controller automatically closes the high-speed squirrel-cage motor oil switch.
2. When the induced draft fan is up to speed, the forced-draft fan oil switch can be closed.
3. After the forced-draft fan motor is up to speed, the exhauster-fan motor can be started, and this can be followed, in order, by the pulverizing mills and the coal feeders.

Method of Making Prescribed Operating Sequence Effective. Each fan, exhauster, pulverizing mill, etc., has its own oil switch, and the closing and trip circuits of these oil switches are so interconnected that it is impossible for an operator to close an oil switch unless the equipment ahead of it in the sequence is running; that is, if an operator attempts to run a pulverizing-mill motor before placing the draft fans and the exhauster fan in service, the oil circuit breaker controlling the pulverizing-mill motor cannot be closed.

FIG. 3—12,000-VOLT, TYPE M, METAL-CLAD SWITCHGEAR

TWO-MOTOR DRIVE FOR INDUCED-DRAFT FANS

To provide the maximum operating flexibility and reliability, two motors are used on each induced-draft fan. The fan starts on the low-speed wound rotor motor; after it is up to its maximum speed, the drum controller completes a circuit which closes the high-speed motor oil switch and trips out the low-speed motor oil switch. If the high-speed motor should fail to start or fail in service, the operation of the overload relays would function to place the low-speed motor in service. If the low-speed motor should fail, the protective relay operation would cause the high-speed motor oil circuit breaker to close. This arrangement, of course, complicates the interlock scheme, but this problem was met by using centrifugal switches on the motor shafts to complete or open auxiliary switch circuits on the oil switches as the motor speed increases or decreases.

UNIT 5 INSTALLATION CONSTITUTES THIRD PERIOD OF DEVELOPMENT

In the third period, the unit illustrated in Fig. 5, which is over four times the capacity of the original 25,000-kw. unit, was placed in service. Since the greater part of the output of this unit is transmitted at 132,000 volts, it was decided to omit the generator bus and arrange for switching at the transmission voltage. This decision permitted the use of any convenient generating voltage, and the manufacturer selected 18,000 volts as the most feasible from the manufacturing standpoint. Subsequent units, of course, may generate at other voltages. As the generator and its associated 120,000-kva. transformer bank constitute an operating unit with no intervening bus, the generator leads had to be tapped to supply the 10,000-kva. auxiliary transformer. The high-voltage and low-voltage switchgear units provided for this transformer are

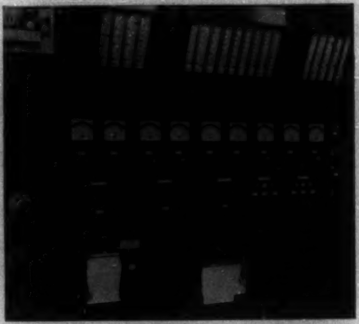


FIG. 4—ELECTRICAL PORTION OF BOILER-CONTROL PANEL

Showing the indicating instruments in the motor circuits, the oil circuit-breaker control switches, and the motor-operated boiler inlet valves feeding the main and auxiliary steam headers

FIG. 5—115,000-KW., 18,000-VOLT GENERATOR 5

tripped by differential relays in case internal trouble develops. In this way, interference with the operation of the generator is avoided.

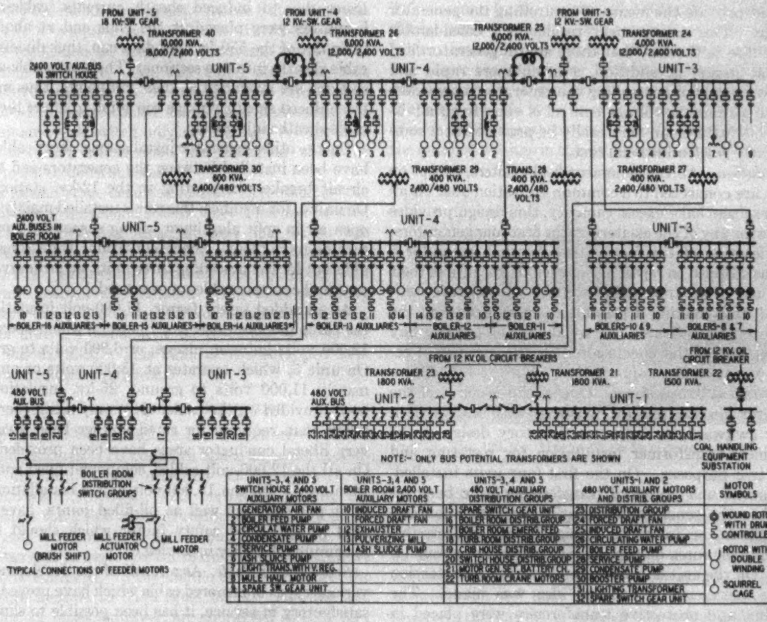
Auxiliary System

The low side of this 10,000-kva., 18,000/2,400-volt transformer is connected to the main 2,400-volt station auxiliary bus. This bus can be sectionalized by units if desired, or it can be operated as one bus. Reactors have been provided between sections of the auxiliary bus for fault current control. Power for starting the auxiliaries on units 3, 4, or 5, or for emergency operation in case one of the station auxiliary transformers is out of service, is obtained from the station auxiliary bus. Fig. 6 gives a one-line diagram of the station auxiliary system.

The present auxiliary power installation provides 9.4 per cent of the generator capacity in auxiliary transformers. The active and reserve transformer capacities are 7.4 per cent and 2 per cent respectively. These transformer capacities have been selected so that we have 20 to 30 per cent unused capacity in each transformer under normal demand conditions. This normally unused capacity provides:

1. For equipment not included in the original design but which appears desirable to install later.
2. A reduction of the investment in reserve transformers, thus saving foundations, switching and control equipment, and installation costs for an "unproductive" transformer.
3. The excess capacity required for station overloads.

The general arrangement, design, and type of auxiliary switchgear used for units 1 and 2 and for the succeeding three units offers quite a contrast, as well as showing the development of the art. The auxiliary system used for units 1 and 2 was of an open-bus type with its connections extending from the oil switches up to the disconnecting switches and then to the overhead bus, and considerable space was occupied for a comparatively small number of pieces of auxiliary equipment. All of the switching equipment for the turbine and the boiler-room was supplied from a single bus in the switch-house. On the other hand, for units 3, 4, and 5, 2,400-volt metal-clad switchgear has been used for the main auxiliaries, and separate buses have been provided for the equipment in the turbine-room and in the boiler-room. Two feeders are provided from the main bus


FROM UNIT-5
18 KV-SW. GEAR
TRANSFORMER 40
10,000 KVA.
18,000/2,400 VOLTS
2400 VOLT AUX. BUS
IN SWITCH HOUSE
UNIT-5
FROM UNIT-4
12 KV-SW. GEAR
TRANSFORMER 26
6,000 KVA.
12,000/2,400 VOLTS
UNIT-4
TRANSFORMER 25
6,000 KVA.
12,000/2,400 VOLTS
FROM UNIT-3
12 KV-SW. GEAR
TRANSFORMER 24
4,000 KVA.
12,000/2,400 VOLTS
UNIT-3
TRANSFORMER 30
600 KVA.
2,400/480 VOLTS
TRANSFORMER 29
400 KVA.
2,400/480 VOLTS
TRANS. 28
400 KVA.
2,400/480
VOLTS
TRANSFORMER 27
400 KVA.
2,400/480 VOLTS
2400 VOLT
AUX. BUSES IN
BOILER ROOM
UNIT-5
UNIT-4
UNIT-3
BOILER-16 AUXILIARIES
BOILER-15 AUXILIARIES
BOILER-14 AUXILIARIES
BOILER-13 AUXILIARIES
BOILER-12 AUXILIARIES
BOILER-11 AUXILIARIES
BOILERS-10 & 9 AUXILIARIES
BOILERS-8 & 7 AUXILIARIES
FROM 12 KV. OIL CIRCUIT BREAKERS
FROM 12 KV. OIL CIRCUIT BREAKER
TRANSFORMER 23
1800 KVA.
TRANSFORMER 21
1800 KVA.
TRANSFORMER 22
1500 KVA.
UNIT-5
UNIT-4
UNIT-3
480 VOLT
AUX. BUS
480 VOLT
AUX. BUS
UNIT-2
UNIT-1
COAL HANDLING
EQUIPMENT
SUBSTATION
NOTE:- ONLY BUS POTENTIAL TRANSFORMERS ARE SHOWN
BOILER ROOM
DISTRIBUTION
SWITCH GROUPS
MILL FEEDER
MOTOR
(BRUSH SHIFT)
MILL FEEDER
ACTUATOR
MOTOR
MILL FEEDER
MOTOR
TYPICAL CONNECTIONS OF FEEDER MOTORS

UNITS-3, 4 AND 5 SWITCH HOUSE 2,400 VOLT AUXILIARY MOTORS	UNITS-3, 4 AND 5 BOILER ROOM 2,400 VOLT AUXILIARY MOTORS	UNITS-3, 4 AND 5 480 VOLT AUXILIARY DISTRIBUTION GROUPS	UNITS-1 AND 2 480 VOLT AUXILIARY MOTORS AND DISTRIB. GROUPS	MOTOR SYMBOLS
1 GENERATOR AIR FAN	10 INDUCED DRAFT FAN	15 SPARE SWITCH GEAR UNIT	23 DISTRIBUTION GROUP	WOUND ROTOR WITH DRUM CONTROLLER ROTOR WITH DOUBLE WINDING SQUIRREL CAGE
2 BOILER FEED PUMP	11 FORCED DRAFT FAN	16 BOILER ROOM DISTRIB. GROUP	24 FORCED DRAFT FAN	
3 CIRCULAT. WATER PUMP	12 EXHAUSTER	17 BOILER ROOM EMERG. FEED	25 INDUCED DRAFT FAN	
4 CONDENSATE PUMP	13 PULVERIZING MILL	18 TURB. ROOM DISTRIB. GROUP	26 CIRCULATING WATER PUMP	
5 SERVICE PUMP	14 ASH SLUDGE PUMP	19 CRIB HOUSE DISTRIB. GROUP	27 BOILER FEED PUMP	
6 ASH SLUICE PUMP		20 SWITCH HOUSE DISTRIB. GROUP	28 SERVICE PUMP	
7 LIGHT TRANS. WITH V. REG.		21 MOTOR GEN. SET, BATTERY CH.	29 CONDENSATE PUMP	
8 MULE HAUL MOTOR		22 TURB. ROOM CRANE MOTORS	30 BOOSTER PUMP	
9 SPARE SW. GEAR UNIT			31 LIGHTING TRANSFORMER	
			32 SPARE SWITCH GEAR UNIT	

Fig. 6—Single-Line Diagram of the Station Auxiliary System of Waukegan Power Station

over to the boiler-room auxiliary bus. These feeders are protected differentially, and the bus in the boiler-room, as well as the main bus in the switch-house basement, can be sectionalized.

Generator Field Control

Another point in which the change in practise is apparent is in the method of controlling the generator field. Instead of using generator field rheostats, as had been done for the preceding four units, a main and a pilot exciter have been provided on the shaft of generator 5. The exciters and the generator field are so interconnected that variations of the main exciter field rheostat provide the means of controlling the generator voltage. This arrangement results in a considerable saving due to the fact that there are no generator field rheostat losses; in addition, a much more rapid and flexible method of controlling the generator is provided. In case automatic voltage control is considered advisable, this feature can very easily be provided at a comparatively small additional cost.

Excitation. All the generator field exciters, except on unit 5, are connected to a common excitation bus. As all these exciters have excess capacity, this design provides reserve for any of the exciters on the first four generators. Consideration of the facts, however, that generator 5 must be stopped to repair either exciter, because of their locations; that the main exciter will operate self-excited, thus making it possible to operate without the pilot exciter, and also the reliability of present day exciters, led to the conclusion that the expense of reserve excitation was unjustified.

Instrument Transformers. One of the features of the generator installation for which it has always been rather awkward to make a satisfactory design is the instrument-transformer installation for metering and protective purposes. On the first four units installed, the current and potential transformers have been installed in the generator foundations in rather inaccessible places, or in rather unsightly and inaccessible transite enclosures on the side of the generator foundations. On the unit 5 installation, however, an extension of the metal-clad switchgear idea was made. The metering and protective transformers were placed in metal-clad petrolatum-filled compartments in a recess in the side of the generator foundation in such a way as to make a safe, compact, accessible, and well-appearing installation. The metal-clad instrument-transformer compartments were designed with potheads on the lower side ready for the installation of the generator cables.

Generator Cable and Pothead Installations and Insulation

On the unit 5 installation four single-conductor, 3,000,000-circular mil, paper-core, 25-kv. paper-insulated, lead-covered cables are used for each phase. These cables extend from the generator to the 120,000-kva. transformer bank, in fiber ducts. This 16-duct bank was designed in the form of a square surrounding a large, square tile placed in the center of the duct bank to permit air circulation thus equalizing the internal and external temperatures of the ducts.

Special care in design was used to make the cables carry equal load currents and to avoid injurious sheath voltages. The cable phases in the duct have been so arranged that the electromagnetic fields surrounding the cables tend to neutralize each other, thus equalizing reactances and avoiding reductions in carrying capacity.

As it was considered more economical to have induced voltages on the cable sheaths than to suffer the losses due to induced sheath currents, cable-sheath insulators were placed at the ends and at about the midpoint of the 550-ft. fiber-duct run, thus dividing the cable sheaths into two sections. These two cable-sheath sections are grounded at their midpoints, thus making the induced sheath voltage too small to affect the lead-cable sheaths injuriously.

On the other generator installations, short cable runs have been installed between the generators and the oil circuit breakers connecting to the 12-kv. station bus. On unit 4, for instance, this run is installed mainly in the open air in split aluminum tubing suspended from the turbine-room floor. Since conditions were somewhat different for the preceding three units, an ordinary fiber-duct and concrete cable run was used.

As an added safety factor, 17,000-volt insulation on the cable has been used for the units which operate at 12,000 volts between phases, or 6,900 volts to ground. On unit 5, which operates at 18,000 volts or approximately 11,000 volts to ground, 25-kv. insulation has been provided on the cable. Not only has higher insulation than required for rated voltage been used, but very liberal conductor areas have been provided also. On all the 12,000-volt cables, compound-filled potheads were used. On the 18,000-volt unit 5 installation, oil-filled potheads, as well as oil-filled joints, have been used as an added safety factor which should insure maximum reliability in service.

Small Power Cable. As a result of the development of various types of armored cable which have proved quite satisfactory in service, it has been possible to eliminate the very expensive conduit runs which were needed to protect cable of the ordinary type from mechanical injury. Instead, troughs suspended from the ceiling are provided. The armored cables are laid in these troughs much more quickly than cable could be pulled through conduit. As a result of this change in practise, the cost of our cable installations has been materially reduced because of the saving in drafting-room and field work.

Protective Relays

Various types of relays have been used to protect tne equipment. The generators, cables, and transformers are protected with differential relays. The transformers are also equipped with overcurrent-relay protection. Some of the motors are equipped with phase-balancing relay protection and others with overcurrent protection, depending on the particular duty. The 132-kv. line protection consists of impedance relays which operate in conjunction with overcurrent and undervoltage auxiliary relays. This arrangement makes it unnecessary to change current-transformer ratios in light or heavy-load periods.

OUTDOOR INSTALLATIONS

The outdoor installations at Waukegan have gone through two successive stages of design similar to those involving the indoor equipment. At the time unit 1 was installed, it supplied a 25,000-kva. 12/33-kv. transformer bank, and an open-type main and transfer-bus switching structure. At this time, also, the first 132,000-volt installation was made. This consisted of a 30,000-kva. water-cooled transformer bank provided with an air-break switch only, on the high-voltage side. The associated transmission line extended about 28 miles to a step-down substation.

The second stage occurred in 1927 as a result of the installation of the second 132-kv. line. This necessitated the 132-kv. switching structure and bus, which was installed in conjunction with this unit. At this time, a 60,000-kva. water-cooled transformer equipped with load-ratio control was installed. The steel switching structure was added to as additional lines were needed, and for unit 4, a 60,000-kva. oil air-cooled transformer bank was installed. This bank is also equipped with load-ratio control equipment.

132-KV. SWITCHING STRUCTURE

The 132,000-volt structure has a number of rather unusual features. To conserve space in a north-and-south direction, it was necessary to use more in an east-and-west direction. This permitted placing the equipment for a line and a transformer in a bay 38 ft. 6 in. long from north to south, but much wider than this from east to west. As the main bus is located above the other equipment in the middle of the structure, the structure appears to have three buses. The line and transformer buses, however, are connected so that they form a rectangular "ring" bus with the main bus in the middle. The line and transformer or reserve bus may be sectionalized by means of an oil switch, or by means of load-break disconnecting switches.

132-KV. METAL-CLAD SWITCHGEAR

The first 132,000-volt switchgear ever used has been placed in service as part of the unit 5 installation. Two double bus units were installed as shown in Fig. 7, one for switching the high-voltage side of the 120,000-kva. transformer bank supplied by generator 5, and one for an additional 132,000-volt transmission line. The buses of these two units are connected to, and form an extension of, the main and reserve buses in the open-type steel structure. These two double bus switchgear units are arranged as indicated in Fig. 8, and consist of oil-filled cable buses and interconnections, special motor-driven disconnecting switches enclosed in metal housings, oil switches equipped with potheads instead of outdoor bushings, and special bus connections and disconnecting devices for the instrument transformers.

As this switchgear is the first of its kind, the details of its capacity and design features are of interest. The switchgear is rated to carry 1,200 amperes and the oil circuit breakers will interrupt 2,500,000-kva. at rated voltage. The capacity of the buses may be increased to 2,400 amperes by installing another set of buses. The present buses consist of 3,020,000-cir. mil., oil-filled, paper-insulated, lead-covered, copper-armored cable. Isolated racks support the bus cable about nine feet above the switchgear foundations as it loops from switchgear unit to switchgear unit. The ends of each of these bus cable loops are insulated, and the midpoint of each cable sheath in each loop is grounded to keep the induced voltage in the cable sheath at a minimum.

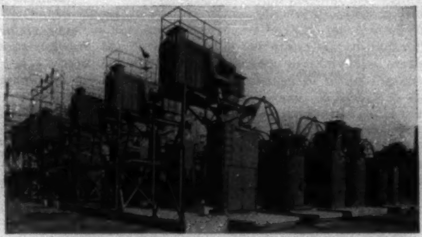


FIG. 7—THE 132-KV. METAL-CLAD SWITCHGEAR AT WAUKEGAN

The disconnecting switch consists of contacts in an oil-filled tank equipped with entrance bushings which are, in turn, enclosed in an outer housing. The tank, as well as the bushing on the bus side, is raised and lowered by a motor-driven mechanism, and overtravel is prevented by limit switches. When the tank is lowered, the circuit is opened under oil and the bus contact on the entrance bushing withdraws from its bus receptacle, thus providing a visual separation, which may be observed from the outside of the housing through a sight glass. The bus receptacle consists of a hollow porcelain shell that forms a part of the main structure into which the entrance bushing on the disconnecting switch tank enters. The opening in the lower end of this porcelain shell is automatically covered by a metal cover when the disconnecting switch is withdrawn. These switches are operated in groups from the oil circuit breaker control panel, or individually at the disconnecting unit.

As it is seldom necessary to disconnect a potential transformer, this disconnecting device is not motor operated. A portable motor has been provided, however, for raising or lowering the potential transformer "coupler" tank whenever necessary.

132-Kv. Switchgear Safety Features. There is a number of interlocks provided to prevent operating errors and to protect personnel. For instance, one of the interlocks prevents opening the upper door in the disconnecting switch house until the disconnecting switch is opened, which causes the shutter preventing access to live parts to close. Another interlock prevents operation of any or all disconnecting switches on both sides of the breaker unless the breaker is open. This interlock is effective on each disconnecting switch on individual operation, as well as on group control. The breaker cannot be closed unless all the disconnecting switches on both sides of the breaker are fully open or completely closed. An interlock also prevents operating any of the line disconnecting switches unless the line cable test switches are open. Conversely, no line cable test switch can be operated until line disconnecting switches have been opened. An interlock also prevents the lowering of any potential transformer coupler tank unless the ground switch and test switch on that phase are closed. An interlock prevents opening any ground switch or test switch until the potential transformer coupler tank is in the connected position. This is accomplished by means of a system of Cory locks.

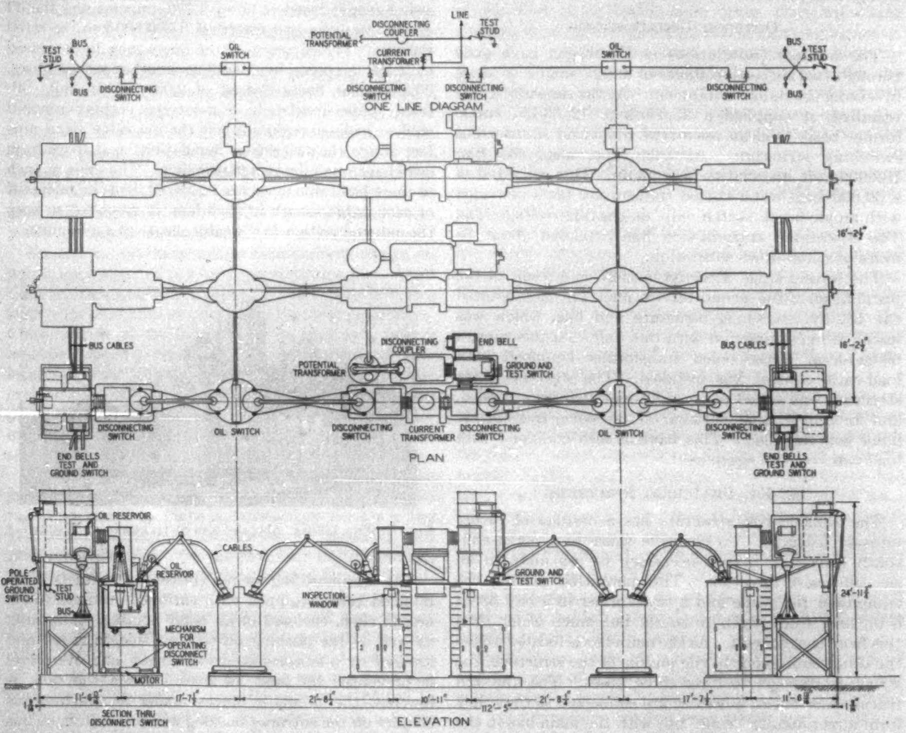

LINE
TEST STUD
DISCONNECTING COUPLER
POTENTIAL TRANSFORMER
CURRENT TRANSFORMER
TEST STUD
BUS
BUS
DISCONNECTING SWITCH
OIL SWITCH
DISCONNECTING SWITCH
DISCONNECTING SWITCH
OIL SWITCH
BUS
TEST STUD
BUS
DISCONNECTING SWITCH
ONE LINE DIAGRAM
16'-2½"
16'-2½"
BUS CABLES
DISCONNECTING COUPLER
END BELL
POTENTIAL TRANSFORMER
GROUND AND TEST SWITCH
BUS CABLES
DISCONNECTING SWITCH
OIL SWITCH
DISCONNECTING SWITCH
CURRENT TRANSFORMER
DISCONNECTING SWITCH
OIL SWITCH
DISCONNECTING SWITCH
END BELLS TEST AND GROUND SWITCH
PLAN
END BELLS TEST AND GROUND SWITCH
OIL RESERVOIR
CABLES
OIL RESERVOIR
POLE OPERATED GROUND SWITCH
TEST STUD
BUS
INSPECTION WINDOW
GROUND AND TEST SWITCH
24'-11½"
MECHANISM FOR OPERATING DISCONNECT SWITCH
MOTOR
1⅛"
11'-6 13/16"
17'-7½"
21'-8¼"
10'-11"
112'-5"
21'-8¼"
17'-7½"
11'-6 13/16"
1⅛"
SECTION THRU DISCONNECT SWITCH
ELEVATION

Fig. 8—Layout of the 132-Kv. Metal-Clad Switchgear

Showing a plan and elevation, the internal features of the disconnecting switches, and a one-line diagram of connections which conforms to the physical arrangement of the equipment

A-C. Car-Mule Drive

One of the recent additions to the coal-handling system at this station is the coal-car mule, which pushes the loaded cars up an incline to the rotary car dumper. The special service requirements of the car-mule drive would normally dictate the use of d-c. motors, but, as direct current has not been necessary for the other auxiliaries, except for control purposes, it was decided to avoid its use for this device, if possible. Accordingly an a-c. system was developed which is, perhaps, the first of its type used in this country. Solution of the problems of securing large variations in torque with large variations in speed, of obtaining a flexible control system, and of taking the slack out of the hoisting cable to prevent breakage when the load is suddenly applied, have made the use of a-c. motors possible.

The essential feature of this installation is the use of a slow, two-speed, 440-volt, high-torque motor. Coupled directly to the torque motor is the 500-hp., 2,300-volt, wound-rotor, main driving motor. The drive shaft to which the motors are coupled is geared to the car-mule cable-hoisting and lowering drum.

The control system of the equipment which pushes a loaded coal car up the incline is complex, but to hoist a car automatically, it is only necessary for the operator to move a master control switch to the final point. This closes contactors which put the torque motor on its high-speed winding and thus bring the car mule out of the pit at a speed of 150 ft. per minute. As the car mule approaches the coal-car which has rolled over the hump, a limit switch operates and energizes intermediate circuits that put the torque motor on its low-speed winding, so that the mule engages the coal-car at a speed of 50 ft. per minute. This operation also places the main motor on the line with all its starting resistance inserted. After the car has been picked up, another limit switch operates to energize a contactor which accelerates the main motor. This process continues in six steps until the main motor reaches its maximum speed of about 300 ft. per minute. As the car approaches the car-dumper cradle, the car is slowed down to 150 ft. per minute by inserting automatically all of the main motor resistance, and the torque motor is put on its high-speed winding. If the cradle is properly seated, the car and mule will proceed to the final mule limit; at this point the motors stop automatically and the brakes set. By reversal of the master switch the motors operating the track gates, through which the car mule leaves or enters the pit, change the position of the gates and the car mule is lowered automatically.

Discussion

A. H. Lovell: To one who is not a resident of this territory of Chicago and its adjacent western surroundings, it is very puzzling that Waukegan, built for heavy energy production, in relation to the Chicago load, should have chosen units of only 25,000 kw. in 1923 and 65,000 kw. in 1930. Will the author discuss this in relation to the size of the load in Chicago at those times?

W. S. Monroe: The history of the Waukegan power station from the time the first unit was ordered in 1921, represented, in a large measure, the development of the power station industry in the Chicago District so far as steam pressure and steam temperature are concerned, and the change of boiler furnaces from the chain grate stoker to pulverized coal. Referring to the size of the units, the first two represented the largest units that it seemed desirable to put into the station, considering the load available in the district, and the last three units, Nos. 3, 4 and 5, of 60,000, 75,000 and 105,000 kw. respectively, were each the largest single shaft unit available at the time they were ordered.

115,000-Kw. Turbo-Alternator

BY R. B. WILLIAMSON*

Fellow, A.I.E.E.

Synopsis.—*Due to the increasing use of large single-shaft steam turbine units, it has been necessary to design very large generators operating at 1,800 r.p.m. The present paper describes a 115,000-kw. unit recently placed in operation. The generator is wound for 18,000 volts and the whole output of the machine, with the exception of a small part used for the operation of auxiliaries, is stepped up to 132 kv. and all switching is done on the high-voltage side of the transformers. It was possible, therefore, to design the generator for a voltage that would permit a simple two-conductor per slot winding.*

On account of the great axial length of the machine, ventilation required careful study and a new scheme of rotor ventilation was developed to secure uniform cooling throughout the length of the long rotor body. The stator ventilation is of the inward-outward parallel flow type with fourteen parallel paths in each half of the stator.

Air circulation is provided by four single inlet motor-driven blowers mounted under the generator. A fin type radiator cooler is mounted under the generator yoke and condensate is circulated through the cooler. Excitation is furnished from a 350-kw., 250-volt coupled, shunt-wound exciter and a 7½-kw. overhung pilot exciter is used to excite the fields of the main exciter.

THE development of large turbo-alternators has in recent years been characterized by the great increase in size of machines for operation at 1,800 r.p.m. The later designs of large steam turbines tend toward single shaft units and consequently the generator has to be large enough to carry the full output of the steam turbine. With units of the compound or triple-expansion type, the total output was divided between two or three generators and the maximum size of generator required was considerably smaller than with single shaft units. There has, therefore, been a demand for very large generators operating at 1,800 r.p.m. and the designer has had to solve a number of problems, mechanical and electrical, because of the great axial length of these machines and the large weights involved, particularly in the rotor.

FIG. 1—115,000-KW. UNIT INSTALLED

Following is a description of a 115,000-kw., 0.95-power factor, 121,000-kva., 1,800-r.p.m. generator put into operation in September, 1931, in the Waukegan Station of the Public Service Company of Northern Illinois. This generator is wound for 18,000 volts, 3,884 amperes, three-phase, 60 cycles. A general view of the unit as installed in the station is shown in Fig. 1.

STATOR CONSTRUCTION

The stator frame is of welded construction and made in two sections for convenience in handling, the halves being bolted together in a plane, normal to the shaft, at the center of the machine. Fig. 2 shows an end view of the stator core and yoke with the stator laminations in place, and Fig. 3 shows the completed winding. The axial length of the laminated core is 281 in. (714 cm.) between end heads. The punchings are clamped between heavy finger plates of non-magnetic cast steel. In stacking the core, the laminations were pressed down, by means of hydraulic jacks, as the work progressed. The core punchings are of silicon steel having a loss not to exceed 0.75 watt per lb., at a density of 10,000 c.g.s. lines per sq. cm., 60 cycles. A sectional view of the generator is shown in Fig. 4.

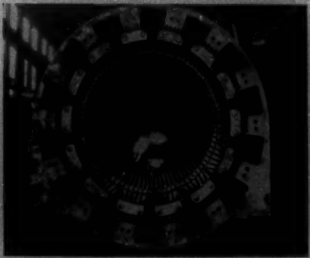


FIG. 2—END VIEW OF STATOR WITH LAMINATIONS IN PLACE

The stator winding is placed in 84 open slots, 21 per pole, and connected in two parallel circuits. In selecting the voltage for this generator, 18,000 volts was decided on after careful study, both from the operating point of view and from considerations of generator design. The whole output of the machine with the exception of a small part used for the operation of station auxiliaries, is stepped up and all switching done on the

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Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wisconsin, March 14-16, 1932.

high-voltage side of the transformers, hence the voltage of this unit did not have to correspond with that of the older units in the station, and it was possible to wind the generator for 18,000 volts in order to obtain the best type of stator winding. Two conductors per slot give the simplest form of winding and eliminate practically all danger of insulation breakdown between turns. The coils are made in halves, each conductor being built up of asbestos-covered, varnish-treated strands transposed to eliminate stray currents. The half coils are connected together at each end of the stator with solid clips, to form one-turn coils. Since the winding is in two parallel circuits, each conductor carries one-half of the total current output.

Fig. 3—Stator with Completed Winding

The coils are insulated with mica tape, with asphaltum base binder. The insulation is molded in place in such manner as to secure a dense insulation free from air inclusions. A layer of asbestos tape treated with varnish is placed on the outside of the coils to form a corona sheath. The whole winding was subjected to the standard A.I.E.E. test of 37,000 volts for one minute and there was very little evidence of corona at this voltage. Fig. 5 shows the stator slot and arrangement of conductors. The projecting ends of the coils, Fig. 6, are supported by lashing to insulated, non-magnetic brackets which form ring supports and at the same time avoid closed circuits for stray currents induced by leakage fluxes.

The terminals of the winding are brought out at each side of the end housing so as not to interfere with the air ducts, and also to render them accessible above the floor line of the unit. Line terminals are brought out on one side and neutrals on the other.

Fig. 7 shows the arrangement of the special compound filled terminal boxes on the line side of the generator. These are mounted on the foundation wall directly below the outlet on the generator. Each of the leads is provided with a terminal box in which current transformers for differential protection are mounted. A tap for auxiliary power supply is taken off directly from the main leads as indicated, and a current transformer in the

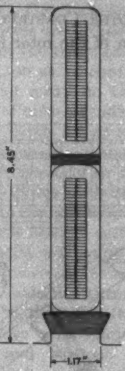

Fig. 5—Stator Slot Section Showing Conductor Arrangement

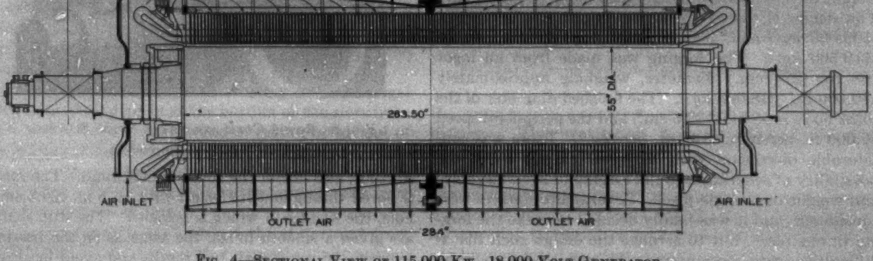

Fig. 4—Sectional View of 115,000-Kw., 18,000-Volt Generator

auxiliary tap is provided in the terminal box together with potential transformers for the machine. The whole arrangement is compact and affords a high degree of insulation. The boxes are filled with soft compound and the potential transformers are oil-insulated. The cable potheads on the terminal boxes are oil-filled, and lead-covered cables are carried down the side wall of the foundation and protected by a casing extending to the basement floor. This casing is made of aluminum plates bolted together so as to be readily removable. The stator end enclosures are of non-magnetic cast iron. The stator with core and windings complete weighs 350,800 lb. net, (159,000 kg.) and was shipped on a special car designed for 400,000 lb. (181,000 kg.) load.

ROTOR CONSTRUCTION

The construction of the rotor for such a large high-speed machine, required a great deal of study as the weight of the finished rotor is 228,650 lb. (103,600 kg.). The weight of the rough turned forging was 240,000 lb. (109,000 kg.) and, before rough turning, 264,000 lb. (119,500 kg.). The forging was made from an ingot 108 in. (274 cm.) diameter, weighing approximately 480,000 lb. (218,000 kg.). The finished diameter of the rotor body is 55 in. (139.6 cm.) and the peripheral speed 26,000 ft. per min. (132 m. per sec.). There was considerable discussion with the steel makers as to the possibility of securing a sound forging of such large size and weight in a single piece. Several alternatives were considered, and it was finally decided to make the forging in one piece, but to arrange the design such that if difficulties were encountered, the stub-shafts could be cut off and stubs bolted to the body, thus making a three-piece construction. Fortunately, this was not necessary as the steel maker produced a single-piece forging which, after careful physical tests and inspection by boring, proved satisfactory in every respect.

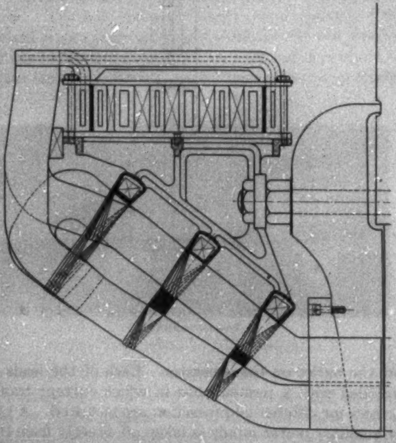

FIG. 6—METHOD OF SUPPORTING PROJECTING ENDS OF STATOR COILS

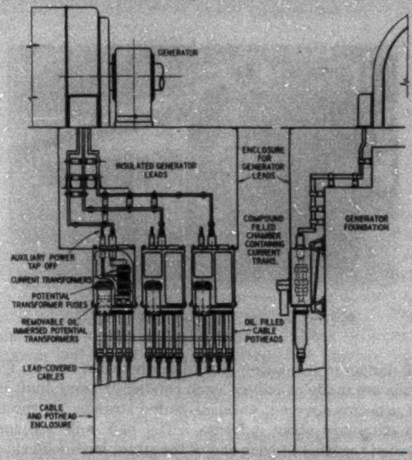
GENERATOR
INSULATED GENERATOR LEADS
ENCLOSURE FOR GENERATOR LEADS
COMPOUND FILLED CHAMBER CONTAINING CURRENT TRANS.
GENERATOR FOUNDATION
AUXILIARY POWER TAP OFF
CURRENT TRANSFORMERS
POTENTIAL TRANSFORMER FUSES
REMOVABLE OIL IMMERSED POTENTIAL TRANSFORMERS
OIL FILLED CABLE POTHEADS
LEAD-COVERED CABLES
CABLE AND POTHEAD ENCLOSURE

FIG. 7—TERMINAL BOXES FOR STATOR LEADS

The material in the rotor forging is open hearth, carbon steel, with a small percentage of vanadium, thoroughly annealed. The body is approximately 24 ft. (732 cm.) long and the distance between bearing centers 420 in. (1,070 cm.). Fig. 8 shows the rotor body after slotting; also one of the coil retaining rings. The rotor is bored throughout its length with an 8 in. (20.3 cm.) hole, the surface of which is polished. The stub shafts are given a smooth finish, the same as on the bearing journals, to remove all tool marks and avoid localized

FIG. 8—ROTOR CORE AND SHAFT AFTER SLOTTING

stresses that might start cracks during long-continued service. For the same reason, all fillets on the rotor and end rings are carefully polished. The rotor end rings are of high tensile strength, non-magnetic alloy steel in order to reduce stray loss.

In general, the rotor construction does not differ materially from that used for similar large rotors. All insulation is of mica and asbestos. On account of the large diameter of the stub shafts it was not advisable to mount the collector rings between the outboard bearing and the generator, as the rings would have to be of such large diameter that the rubbing speed at the brushes would be too high. The rings were, therefore, mounted outside the bearing and leads brought through the shaft.

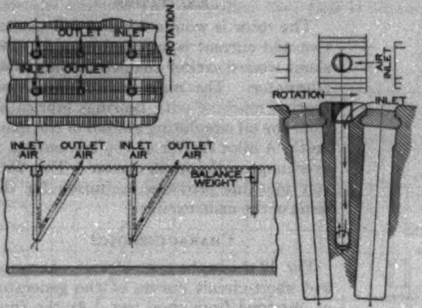

FIG. 9—METHOD OF ROTOR VENTILATION

A new method of ventilation was used on this rotor because, on account of its great axial length, the methods heretofore used would not be effective in ventilating the central part. The usual method of rotor ventilation has been to provide longitudinal channels under the coils, into which air can be drawn from each end of the rotor. Holes drilled down through the teeth connect with these channels and when the machine is in operation, air is drawn in from each end through the channels under the rotor coils and thrown out of the holes by centrifugal action. This method works well for rotors of medium length, but for very long machines it is difficult to secure enough inlet duct area under the coils to supply adequate ventilation, and what ventilation is obtained is not uniform along the length of the rotor body.

In the present case, instead of using the centrifugal effect and feeding air in at the bottom of the slots from each end, a method was devised of taking up air out of the air gap at the circumference of the rotor and utilizing the impact head to force air down through the ventilating holes and return it to the air gap. Air from separate ventilating fans is supplied under pressure to the air chambers at each end of the stator and from there passes into 14 inlets (Fig. 3) spaced equally around the back of the stator punchings. From these chambers it blows inward in a radial direction against the rotor, makes a U-turn and passes out radially through the yoke. Thus, air is blown in against the rotor at 14 points and this air is comparatively cool, as it has traveled only a short distance from the inlet. A small portion of the air is by-passed back of the coils in order to cool the part of the stator core behind the teeth, but the greater part blows in against the rotor. There is thus a strong circulation of air through the air gap at all points throughout the length of the rotor.

The scrubbing action of the surface and the ventilation at the ends of the rotor removes a large part of the heat, but it is also necessary to provide additional ventilation as stated above. Fig. 9 shows the method used in this case. Radial holes are drilled to within a short distance of the roots of the teeth and holes are drilled at an angle as shown, so as to connect with holes at the bottom. Plugs cut away on the under side are screwed into the tops of holes as indicated, and a short channel is cut in the tooth lip as shown, to connect with the slot in the under side of the plug. The plugs are of course locked in position so as to prevent any possibility of turning. When the rotor is rotating, at a peripheral speed in this case of 26,000 ft. per min. (132 m. per sec.) the impact head is sufficient to force air down through the radial holes and out through the holes at an angle. Numerous tests made on models, showed that the air velocity through the ventilating holes is at least 6,000-8,000 ft. per min. (30.5-40.7 m. per sec.). Also tests made with an experimental rotor, with and without these ventilating holes, showed a decided gain in cooling effect with the rotor run in a machine under exactly similar conditions. The ventilation does not depend on any centrifugal action, as the centrifugal effects on the air in the inlet and outlet holes are balanced. With this arrangement, as the length of the rotor is increased, the number of ventilating units and the air carrying capacity are also increased in proportion, and a long rotor can be ventilated as effectively as a short one. It might

FIG. 10—TWO OF THE FOUR MOTOR-DRIVEN BLOWERS

be thought that this method of rotor ventilation would be noisy, but such has not proved to be the case as the rotor makes no more noise than one provided with the usual centrifugal ventilation.

AIR SUPPLY

Air is supplied to the generator by four separate motor-driven blowers mounted within the foundation space, directly beneath the generator and operating on a closed system. Fig. 10 shows two of the blowers as installed, the other two being similarly located on the opposite side of the foundation. Separate fans were used for several reasons. With a rotor of such large weight and length of body, it was necessary to keep the stub shafts as short as possible and the use of self-contained fans with their air inlets adds considerably to the length of the shaft. Moreover, the large shaft restricts the air inlets to such an extent that self-contained fans could not deliver the amount of air required, without excessive inlet velocities and consequent loss in efficiency. Separate fans are more efficient and permit better mechanical design of the rotor, besides being more quiet in operation. Each fan can deliver 50,000 cu. ft. (1,420 cu. m.) of air per minute against 9 in. (22.8 cu. m.) static pressure. The fans operate at 1,160 r.p.m. and each requires a maximum of 104 hp. To allow ample margin, 150 hp. line-start, squirrel cage motors are provided. The fans are single inlet type with runners mounted directly on a shaft extension of the motor. All four fan inlets open directly into the chamber under the coolers into which the air is discharged. With the generator delivering 70,000-80,000 kw. it is found that two fans furnish sufficient air, and four are operated only under full load conditions. Outlet dampers are provided on each fan so that it can be cut off when not in use. Fig. 11 shows the general arrangement of the blowers in relation to the generator and air coolers.

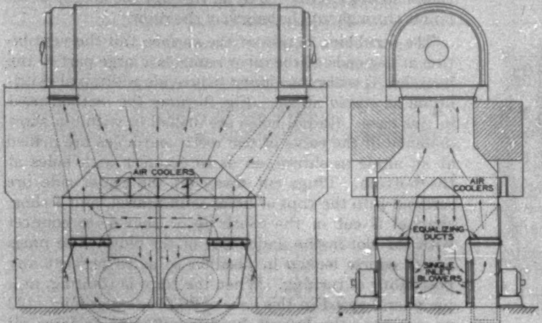

FIG. 11—GENERAL ARRANGEMENT OF MOTOR-DRIVEN BLOWERS

For cooling the air, condensate is circulated through a fin type radiator cooler mounted directly under the generator. The cooler is designed to handle 200,000 cu. ft. (5,670 cu. m.) of air per minute with a pressure drop not exceeding 1½ in. (3.81 cm.) water. With cooling water temperature not exceeding 80 deg. fahr. (26.6 deg. cent.), it will absorb 6,985,000 B.t.u. per hour and deliver air to the generator inlets at a temperature not exceeding 104 deg. fahr. (40 deg. cent.).

EXCITATION

The rotor is wound for 250-volt excitation and current is supplied by a 350-kw. shunt wound exciter coupled to the turbo-alternator. The main exciter is a two-bearing machine with bearings lubricated from the oil circulating system of the main unit. A pilot exciter of 7½ kw. output is overhung on the main exciter. This 250-volt machine provides excitation for the fields of the main exciter.

CHARACTERISTICS

Fig. 12 shows the open-circuit saturation and short-circuit curves of the generator as obtained from shop test. At the time of preparing this paper, complete tests for efficiency, heating, etc., had not been made, but regular operation on the station load indicates that the temperature rise of the generator at full load is well within the guarantees of 60 deg. cent. rise by detector on the stator, and 85 deg. cent. rise by resistance on the rotor. The estimated efficiency of the generator at rated load, including power required to drive the separate fans is 98.1 per cent.

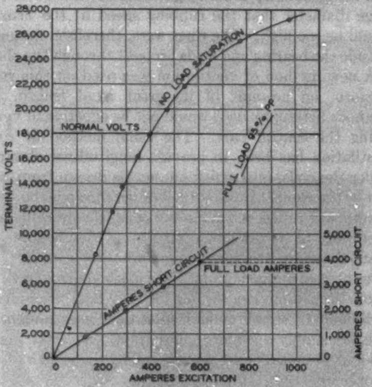

FIG. 12—OPEN-CIRCUIT SATURATION AND SHORT-CIRCUIT CHARACTERISTICS

Discussion

S. H. Mortensen: If in the design of a machine of the size described in the paper the effect of vibrations, stray fields, heating, etc., could be forgotten it should be possible to determine its most economical all around design from a careful analysis of the relative cost of labor and material involved, combined with the operating cost per kilowatt after installation. From such an analysis the corresponding generator efficiency could then be derived to serve as a guide for the design.

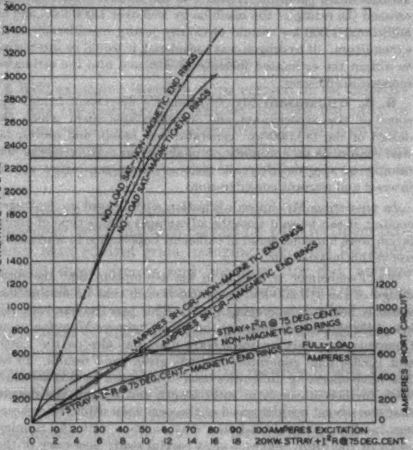

Fig. 1—No-Load Saturation and Synchronous Impedance Curves and Stray Loss

On a 2,500-kva. turbo-generator tested with magnetic and also non-magnetic coil support rings

However, vibrations, stray fields, heating, etc., if forgotten are apt to become exceedingly real and furthermore as present day tendencies combined with competition have made it necessary to give efficiency guarantees which approach those of transformers, it becomes incumbent on the designer to combine his best skill with that of the best material available for the generator construction. For the machine in question this has resulted in the use of silicon core steel and non-magnetic material wherever practicable. The use of silicon steel which is brittle when cold, makes it necessary to omit sharp corners from slots and dovetails and to provide core clamping devices which eliminate the danger of vibrations and loose core iron. Fig. 2 of the paper shows that this machine has sectionalized core clamping plates. As each section only covers 1/14 of the circumference they can be pulled up independently and compensate for variations in core length. To reduce stray losses the clamping plates and the spacers on the teeth of the center ventilating segments are made from non-magnetic material as are also the stator end covers. To further reduce stray fields and losses the material in the stator coil support rings is steel rendered non-magnetic by alloying it with manganese, nickel and chromium. On physical tests, this material had an ultimate strength of 135,000 lb. per sq. inch, a yield point of 92,000 lb. with an elastic limit of 56,000 lb. Its reduction in area was 42 per cent and the elongation 36 per cent. The effect of substituting non-magnetic for magnetic coil support rings on the leakage fields and stray losses of a 2,500-kva., 3,600-r.p.m. generator is depicted in Fig. 1. The respective curves show that fewer field ampere turns are required to generate a given voltage or short-circuit current with non-magnetic rings, which, interpreted in kva. means a larger output for a given rotor heating. In addition, the short-circuit losses with non-magnetic rings were only 58 per cent of those corresponding to magnetic rings. This, of course, means a decrease in the operating cost which, under circumstances may justify the increased first cost inherent to the use of the more expensive non-magnetic material in the support rings.

In connection with the operating voltage of the 115,000-kw. machine it is of interest to note that it was chosen so 1-turn coils could be used in its armature windings. It would seem that in installations where generator and step-up transformers can be considered as a unit the value of the generator voltage should be chosen to give the most favorable generator design, irrespective of voltage standards.

Prior to adopting the rotor ventilating scheme proposed by Mr. Williamson and shown in Fig. 9, a number of tests was made on models and also on a 4,500-kva. machine without and with the impact ventilating holes. The respective tests showed the new scheme to give better cooling than obtained with any of the schemes previously used, such as ventilating ducts in the rotor body, axial ducts under the rotor coils, axial slots in the pole center, etc., combined with the grooving of the rotor surface which Mr. Williamson introduced prior to 1911, to increase heat radiation from turbo-rotors. Curve *A*, Fig. 2 shows the rotor heating before, and curve *B* the results obtained after the surface on the 4,500-kva. rotor had been finished along the lines shown in Fig. 9. The respective curves in Fig. 2 show that for 85 deg. cent. rise the new method of ventilation increases the rotor ampere turns 8 per cent. It also increases the volume of generator ventilating air and the rotor windage loss. The effectiveness of the impact method of ventilation increases with the axial rotor length and a preliminary test on the 115,000-kw. machine indicates its radiating capacity to be 15 to 20 per cent higher than the best results previously obtained on shorter rotors.

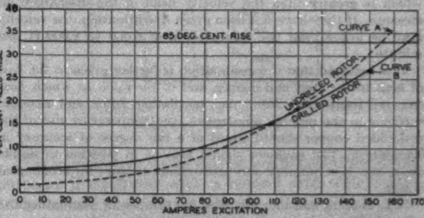

Fig. 2—Rotor Heating on a 4,500-Kva. Turbo-Generator Before and After Its Surface was Drilled

The short-circuit ratio of the 115,000-kw. machine is less than that of the smaller machines previously installed in the Waukegan station. The short-circuit ratio of a machine does not materially alter its efficiency, but if no other restrictions are involved, it does affect its size and cost and should for that reason be kept as low as experience and careful analysis of system characteristics will permit. For generators operated with full field excitation, a short-circuit ratio of 0.5, as has been used in Europe, or even less would probably give ample stability on metropolitan systems. Further for equal stability a double winding generator could have a lower short-circuit ratio than a single winding machine as it is less affected by disturbances.

The heavy weight and large diameter of the forging required for the rotor of the 115,000-kw. machine taxed the art of steel making to the limit. The loss of such a forging due to a blemish would be a serious matter and increase the cost of the machine. Until further progress is made in steel making it seems economically necessary for still larger rotors, to develop a construction reducing weights and diameters of the forgings for the component rotor parts.

Carl J. Fechheimer: When considering the construction of a machine of the large output and high speed described in this paper, it is well to regard the limitations imposed. It is well known that the limits are in the rotor, and they are imposed by mechanical stresses, critical speed, and temperature rise. The use of the better grades of steel, annealed or heat treated, with greater assurance of homogeneity, has warranted the adoption of higher stresses. Annealing or other heat treatment of the steel insures having material of ample ductility so as to avoid failure, if the elastic limit is exceeded at parts where there is still liable to be stress concentration, and plastic flow then secures more uniform stress distribution. Although the author does not so state, in all probability the running speed is above the first critical speed, and as this practise has been common for two-pole, 60-cycle machines of the larger outputs, there is no reason why the same practise should not be followed for four-pole machines. It is important to consider that the influence of the foundation may be to lower the critical speed, and there is danger of encountering a higher critical speed near the running speed.

The scheme adopted for rotor ventilation is ingenious. Determination of the air volumes that flow through the ventilating holes by means of models is in accordance with the desirable methods in use when such a complicated phenomenon as air flow is to be studied. Model tests are, however, not always easy to make in such a way that the conditions in the machine are completely simulated, and measurements are ofttimes difficult and may lead to false conclusions. As the author has not described the model and methods of measurement, it is difficult to comment at length. In this case it is well to consider, in addition to the flow in the rotor ducts as such, the effect of the variable back pressure in the air gap, and the loss of head at emerging, which is greater than the velocity head of the discharged air streams. If they have not been given due attention, the velocity may be lower than the values given of 6,000 to 8,000 ft. per minute.

It is believed that if consideration is further given to the flow of heat in the steel and copper, and to the rates at which heat can be transferred from the ventilating holes and external surfaces to the moving air streams, the temperature rise of the embedded copper should admit of a fair degree of accuracy in calculation. The engineer should also be in position to decide upon the best number and proportions of holes for economical design.

It is well to consider also the importance of ventilating the end windings of the rotor adequately. Cases have arisen in which the length of the outer coils is sufficiently great to cause large temperature differences when the ends were not ventilated. It is of interest to note that lower temperatures can be secured by reducing the sections, thereby raising the air velocities, even though the air volumes are reduced somewhat thereby.

As the rotor temperature rise depends upon the losses, it is interesting to note that the short-circuit ratio has been reduced about 0.67. This is in accordance with present tendencies in design. It is doubtful, however, whether a further reduction is warranted, as there would be but little reduction in excitation, and other effects, such as stability, must be given adequate consideration.

Philip Sporn: Was the use of hydrogen given any consideration in connection with the design of the machine in question? It seems to me that in a machine of this size there was a considerable margin to work on in order to make hydrogen justifiable. Besides the undoubted improved performance of an 18,000-volt winding due to the use of hydrogen, it would seem as if there was a possibility in this case of raising the efficiency to possibly as high as 99 per cent. If this was the case, it would have increased the rating of the machine by well over 1,000 kw., and an additional 1,000 kw. would obviously justify considerable extra expenditure. If hydrogen was considered, it would be interesting to obtain the estimated increase in cost and also the estimated increase in efficiency.

R. B. Williamson: Mr. Mortensen's discussion brings out some of the experiments and investigations upon which the design of the 115,000-kw. generator was based, and serves, for that reason, as an amplification of the paper. His remarks pertaining to the short-circuit ratio of turbo-generators are of interest and warrant consideration.

In answering Mr. Sporn's question with regard to hydrogen cooling, it can be said that the advantages of this type of ventilation were not considered of sufficient importance to warrant its use in this installation.

Mr. Fechheimer is correct in his assumption that the rotor design of the 115,000-kw. generator is such that its first critical speed is below its operating speed. This practise has been followed for the last 20 years by the Allis-Chalmers Manufacturing Company, not only on the 3,600-r.p.m., but also on the 1,800- and 1,500-r.p.m. generators and has given good results. In no case have operating difficulties been experienced on that account. With further increases in rotor length and weight, the danger that the second critical speed of the combined rotor, rotor bearings, and generator foundation may coincide with the running speed of the rotor, makes a most careful study necessary of the design and proportioning not only of the generator rotor but also of its bearings and foundation.

We agree with Mr. Fechheimer that it is difficult to make a model test for rotor ventilation which anywhere nearly represents actual conditions. Although the model mentioned in the paper was no exception to this, nevertheless the air velocities and general results obtained seemed sufficiently encouraging to warrant the application of the rotor ventilating system, described in the paper, to the rotor of the 4,500-kva. machine mentioned in Mr. Mortensen's discussion. The test data obtained on the 4,500-kva. machine were used as a basis for the calculations of the rotor heating on the 115,000-kw. machine. Subsequent tests bear out that the advantages of this method of rotor ventilation increase with the axial rotor length. The results thus far obtained, indicate that this type of ventilation is adequate to cool rotors of considerably greater axial length than the rotor for the 115,000-kw. machine.

The Mercury Arc Rectifier Applied to A-C. Railway Electrification

BY OTHMAR K. MARTI*

Member, A.I.E.E.

***Synopsis.**—A new commutatorless motor with series characteristics has been developed and is briefly reviewed. The motor is particularly suitable for traction purposes and can be supplied from an overhead trolley at the usual a-c. electrification voltages and any commercial frequency.*

It is shown how it is possible, by the use of a grid-controlled rectifier, to eliminate not merely the commutator of the motor, but practically all of the expensive control, switching, and reversing equipment commonly used on a-c. and d-c. locomotives. Regenerative braking is also made inexpensive and practicable.

The first part of the paper explains the general theory of the grid-controlled rectifier which is necessary for an understanding of the commutatorless motor.

The data and layout of a 1,000-hp., 50-cycle, 15,000-volt, single-phase locomotive, now under construction, are given.

* * * * *

INTRODUCTION

THE extensive research work carried on for improving the mercury arc rectifier has led not only to improvements in its operation but also to entirely new applications of the device, and to new applications of the valve action on which its rectifying property is based.

In recent years considerable experimental and development work has been carried on with rectifiers provided with control grids, utilizing the grids for interrupting short circuits and backfires, for d-c. voltage control, for the conversion of direct current to alternating current, etc. In this paper is described a new application of grid-controlled mercury arc rectifiers in connection with a new single-phase motor for a-c. railway service. By the use of the rectifier it was found possible to eliminate the commutator of the motor and practically all the expensive switching and control equipment commonly used on a-c. locomotives, which will result in considerable simplification of the control, and a reduction in cost.

The experiments carried out on a locomotive equipment with this type of motor have demonstrated the practicability of this scheme. Besides the advantages from the point of view of locomotive operation, this scheme makes it possible to use a power supply of any available frequency, whereas at the present time frequencies of 25 cycles or less are generally required. While it is too early to predict to what this development may lead, the marked advantages of a locomotive equipped with motors of this type, which are already apparent, may revolutionize the practise of railway electrification.

THE PRINCIPLE OF ACTION OF ENERGIZED GRIDS

The phenomena and characteristics of the mercury vapor arc in vacua have been treated in great detail in various scientific journals. Briefly stated, in a tank containing highly rarefied ionized gases only a small potential in a given direction is required to cause current to flow between two electrodes of certain types, while a much larger potential must be applied to the electrodes in the opposite direction before an appreciable current flows.

Fig. 1 shows an evacuated vessel with two electrodes: an electron-emitting electrode, *c*, called the cathode, and an electrode which does not emit electrons, *a*, called the anode. The cathode can be made to emit electrons by raising its temperature, by imposing a sufficiently high voltage between the anode and the cathode, or by striking an arc by contact. By using a highly evacuated tank, and by using mercury for the cathode material, a mercury arc power valve is obtained, of a high current-carrying capacity.

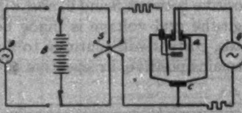

FIG. 1—ELEMENTARY DIAGRAM OF A GRID-CONTROLLED MERCURY ARC VALVE

This mercury arc valve can be made useful for many purposes if equipped with a control grid which is introduced into the path of the arc. This grid is insulated from the anode *a*, and may be energized from an outside source of potential, such as a battery, auxiliary transformer, generator, or a combination of these sources. Non-energized grids or screens are also used to a large extent in mercury arc rectifiers for a number of purposes.*

In all considerations stated below it will be assumed that the cathode is brought into a condition such that it emits electrons. If the battery *B* is so connected to the grid as to make it positive with respect to the cathode, then current from the generator *G* will flow from the

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*Engr.-in-Charge, Rectifiers and Rly. Equipment, Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wisconsin, March 14-16, 1932.

*See Marti and Winograd, "Mercury Arc Power Rectifiers," (McGraw-Hill), pp. 42, 227, 232, 397.

anode *a* to the cathode *c*. Reversing the polarity of this applied potential, so that the grid is negative with respect to the cathode, will prevent any current from flowing from the anode to the cathode, provided the current is not already flowing when the negative grid potential is applied.

If the sequence of the changing of the grid potential is according to curve 1-2-3-4-5-6-7-8 in the lower part of Fig. 2, and the voltage e_g of the generator *G* of Fig. 1 is as shown in the upper part of Fig. 2, then the operation of the valve is as follows:

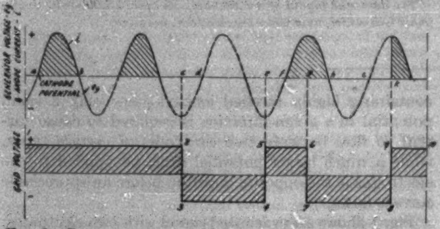

FIG. 2—CONTROL OF ARC BY MEANS OF POSITIVE AND NEGATIVE GRID POTENTIALS

Current will pass during the interval 1 to 2 every time the impressed generator voltage is positive. Since the grid is negatively charged from 3 to 5, no current will flow when the generator voltage becomes positive at *d*. However, current will start to flow at *f*, as the grid voltage is positive between 5 and 6, and will continue to flow although the grid voltage is made negative at *g*; in other words, current will continue to flow as long as the anode remains positive, irrespective of the potential of the grid. Due to the fact that the grid voltage is negative from 7 to 8, current can start to flow only at *k*, and will continue as long as the anode is positive; the anode therefore passes current only during one-half of the positive alternation.

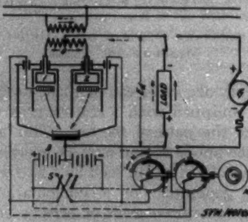

FIG. 3—CONTROLLED-GRID VALVE THAT CAN BE USED EITHER AS A RECTIFIER OR A D-C. TO A-C. CONVERTER

Applying the above considerations to a simplified two-anode rectifier, as shown in Fig. 3, the working of energized grids can readily be followed, and the many possibilities of such a controlled rectifier become evident. Let the primary side of the transformer be impressed with a sinusoidal a-c. voltage and let the grids be non-energized. Then the anodes will be positive during alternate half-cycles. During the half-cycle that anode 1 is positive, current will flow from anode to cathode through the load circuit, as indicated by the solid arrows. When anode 2 is positive, current will flow from anode to cathode through the load circuit, as shown by the dotted arrows. The current therefore flows through the load circuit in the same direction during both halves of the cycle. The same rectifying action will take place if a positive potential is applied to the grids during the intervals when the voltages of the corresponding anodes are positive. This could be accomplished by throwing the switch *S* (Fig. 3) from one position to the other after each half-cycle, making the grid of anode 1 positive during the half-cycle when anode 1 is positive, etc., or by rotating the commutator *C* at synchronous speed by means of the motor *M*.

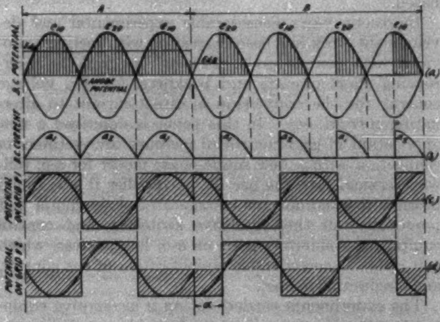

FIG. 4—VOLTAGE REGULATION AND CONTROL BY MEANS OF ENERGIZED GRIDS

This is illustrated in Fig. 4. Section *A* shows the anode voltages e_{10}, e_{20}. The shaded portions represent the d-c. voltage wave (neglecting the arc drop). Section *B* represents the d-c. wave for a resistance load. Sections marked *c* and *d* indicate the potentials applied to the grids; the shaded blocks represent the potentials if direct current is used, with a reversing or rotating switch, as shown in Fig. 3, and the sine waves represent the potentials if alternating current is used.

In *A* of Fig. 4 is shown the condition when the grids are made positive at the instant when the corresponding anodes become positive. Each anode then operates during half a cycle, and the average d-c. voltage is shown by line E_{d_A}. In *B* of Fig. 4 is shown the condition obtained by shifting the brushes of the rotating switch *C*, Fig. 3, through the angle α. The grid potential then becomes positive α electrical degrees after the anode potential, and the point at which the anodes start firing is delayed by the angle α, which reduces the d-c. voltage to the average value E_{dB}.

It can therefore readily be seen that by shifting the brushes of the commutator *C*, that is, by changing the timing with respect to the voltage of the anodes, the rectifier can be stopped from passing current, without interrupting the primary supply (see Fig. 2, interval 3 to 4), or the voltage can be changed from the maximum value E_{dA}, to a value E_{dB}, or as low as zero. The source of potential for energizing the grids, the battery *B*, can be replaced by an a-c. source, which could be an auxiliary transformer, a small generator, or the like.

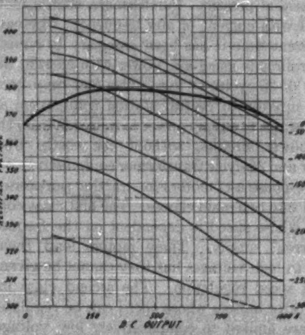

Fig. 5—Grid-Controlled Six-Phase Rectifier (without Inter-Phase Transformer)

Six-phase voltage superimposed on a d-c. grid potential, lagging anode voltage by 60 deg.

For controlling the grids of power rectifiers only a fraction of an ampere per anode is required. It therefore follows that the arrangement for controlling large units makes use of very simple and inexpensive equipment, doing away with much costly control, regulation, and protective apparatus. These advantages will become particularly evident in the description of the new motor and locomotive.

Voltage Regulation and Compounding of Commercial Rectifiers

Fig. 5 shows some experimental curves obtained by imposing a voltage on the grids of a six-phase rectifier. The diagram of connections is shown in Fig. 6. The grid voltage consisted of a negative d-c. voltage, on which was superimposed a six-phase a-c. voltage lagging the respective anode voltages by 60 degrees. The rectifier transformer was connected in six-phase star, without an interphase transformer. The light lines represent the regulation curves of the rectifier with the d-c. component of the grid potential varying from zero to 300 volts. The steep regulation curves are characteristic of this type of rectifier transformer connection.

The heavy line has been constructed from calculations based on the experimental regulation curves, and shows the regulation curve that would be obtained if the d-c. component of the grid potential were decreased uniformly as the load on the rectifier increased. As can be seen from this curve, such a control of the grid potential would give a flat-compounded rectifier, with a regulation curve very similar to the type obtained by flat-compounded generators. The high peak on this curve is of course due to the high regulation obtained with this particular transformer connection.

It is quite obvious that other combinations of d-c. and a-c. potentials can be applied to the grids, so as to obtain automatically almost any form of regulation desired. The broken horizontal line shows a regulation curve that could be obtained by the proper control of the grid potential. Such a curve could be obtained by varying the d-c. potential, the a-c. potential, or the phase angle of the a-c. potential on the grids with respect to the anode voltage. The d-c. voltage of the rectifier can be controlled as a function of the d-c. load, or could even be maintained constant, irrespective of any fluctuations of the a-c. voltage.

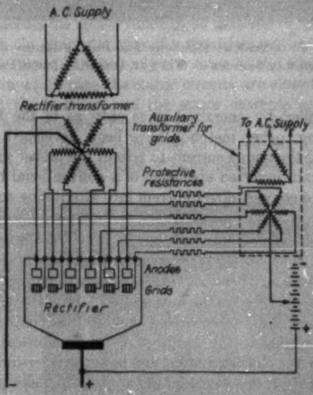

Fig. 6—Connections Used in Obtaining Curves Shown in Fig. 5

Fig. 7 shows a 3,000-kw. mercury arc rectifier of standard design with bushings and connections for grid control, at the West Allis Works of the Allis-Chalmers Manufacturing Co.

Inverted Rectifier

In the arrangement shown in Fig. 3, let us replace the load by a d-c. generator. If a positive voltage is applied to the grid of anode 1, current will flow through the left-hand portion of the secondary winding of the transformer, inducing a voltage in the primary winding.

After a certain time interval let the grid of anode 2 become positive, and current will start to flow through the right-hand portion of the secondary winding of the transformer. Let us assume that at the same time the current in anode 1 can be reduced to zero by some means or other, then the rectifier and associated equipment would be nothing else than a converter feeding power from the d-c. into the a-c. network. Although the flow of current from generator G through the anodes can be controlled by energized grids only as to the time of starting, other methods have been developed for reducing the anode current to zero in the proper sequence to obtain inverted operation of a conventional rectifier so as to convert direct current into alternating current.*

Fig. 7—3,000-Kw. Mercury Arc Power Rectifier for Railway Service on Test at Allis-Chalmers Plant

This feature opens up great possibilities for the now fully developed mercury arc rectifier, and many new applications for the mercury arc valve. One of the most important of these applications to become practicable will be described in the next section. It will be seen how the controlled valve action and voltage control have been applied to the design of a commutatorless motor with series characteristics, and how the inversion of the rectification process has been applied to make possible regenerative braking of a locomotive equipped with such motors.

Commutatorless Single-Phase Railway Motor for any Characteristic and any Frequency

For some time portable substations equipped with mercury arc rectifiers have been built abroad and in this country. As far back as 1914 a 250-hp. motor car, equipped with a mercury arc rectifier, was in service in this country, on the New York, New Haven and Hartford R. R. As some experience is therefore already at hand, it will not be a novelty to put a rectifier on a locomotive and use it as a mobile electric power converter.

During the last few years the mercury arc rectifier has influenced the electrification of railroads to a very great extent, because of its ability to convert a-c. to d-c. power at high voltages and with a high degree of economy and reliability. Again, it is the rectifier which may give another impetus to railroad electrification, for, as indicated in the subtitle to this section, it is now possible, by the use of a commercial mercury arc rectifier equipped with the grid control described above, to build a commutatorless motor which does not require the elaborate and costly switching, reversing, and control equipment used at present. Furthermore, such a motor can be used with current of any frequency, whereas up to the present time it has been necessary to supply electrified railroads using a-c. power with current at a frequency of 25 cycles or less. This fact may be of particular interest to central station companies, as they would be able to supply power to railroads from their 60-cycle power circuits, without the costly frequency changer and 25-cycle transformer equipment.

Single-phase a-c. motors for traction purposes can be divided into two main classes. The first class comprises motors whose speed is dependent on the frequency of the current supply. These motors lend themselves readily to high voltages but they cannot be given a continuous speed regulation. The second class includes commutator motors whose speed can be regulated, but which are limited to comparatively low voltages and frequencies, due to the presence of the commutator. The commutator also makes the construction of the motor and its control very expensive, and involves the troublesome difficulties inherent in commutation.

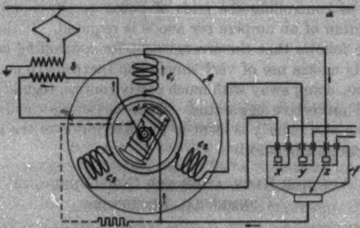

Fig. 8—Elementary Commutatorless Motor

In a motor of the second class, it is possible to replace the commutator by a series of electric valves, *viz.*, a mercury arc rectifier, by the use of which the above-mentioned disadvantages are obviated. As will be seen below, this scheme can be applied to single-phase motors so that relatively high voltages can be used, thus reducing the size and weight of the copper used in the connections. At the same time, a very desirable speed-tractive effort characteristic can be obtained. Further-

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*See Kern, *Bulletin Schweizerischer Elektrotechnischer Verein*, 1931, p. 538.

more, all expensive control, switching, and reversing equipment is dispensed with, while only a very simple and cheap grid control is added. As explained above, only a very small current is needed for the grid control, and the cost of the equipment is naturally low.

In Fig. 8, *a* designates a single-phase a-c. supply line, and *b* a single-phase transformer. The transformer is connected to the stator winding *c* and rotor *d* of the motor *e*, both of which are connected in series, using two slip rings. The rectifier *f* should be located as near as practicable to the motor. It is equipped with grids and control, as shown in Fig. 3. The motor is shown as a salient pole machine, but does not have to be such. In the following description it will be assumed that the field of the motor is rotating, and that the armature is stationary. It is evident, however, that this could be reversed. Each armature winding is connected to one anode of the rectifier, each anode being provided with control grids for regulating the flow of current.

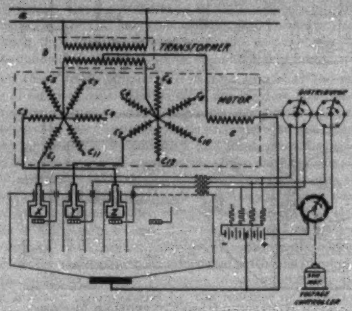

FIG. 9—COMMUTATORLESS MOTOR UTILIZING BOTH HALVES OF VOLTAGE WAVE

In operation, let it be assumed, first, that the rectifier equipment is in the position shown, and that one grid (of anode *z*) is positively energized through the distributor. This grid accordingly permits current to flow through the anode which it controls. All other grids will be negatively energized thus preventing current from flowing through their anodes. As the primary winding of transformer *b* is energized from the line, a half-wave current will flow through the winding of the rotor *d* and through c_1, as indicated by the arrows. As can readily be seen, the field produced in the windings c_1 and *d* will produce a torque which causes the field *d* to rotate and reach, with respect to winding c_2, the position which it previously had with respect to winding c_1. At the same time, the shaft of the motor *e* brings the brushes of the distributor to such a position that the grids of anodes *z* and *x* are now negatively energized, while the grid of anode *y* is positively energized, permitting current to flow from the anode *y* into the winding *d*, and through c_2. The above process is repeated in sequence for all windings of the stator, thereby producing a continuously rotating motion of the rotor.

It can be seen from the above that the motor simply constitutes a load for a conventional rectifier *f*, which may therefore be provided with all improvements known to the rectifier art. In particular, the voltage impressed on the stator windings may be regulated by any of the various means of voltage regulation used with rectifiers, and which have been described above. This voltage regulation can be used here, the voltage being reduced to a low value when the motor is started, and later on being adjusted for purposes of speed regulation. There are also other means for changing the speed or the characteristics of the motor, by a control of the anode grids.

As stated above, this arrangement utilizes one-half of the applied voltage wave. The arrangement shown in Fig. 9 will permit the utilization of both halves. The regulation of this motor is also effected by means of energized grids, and the speed can be varied and the characteristics altered by the same means.

The two motors mentioned above both have series characteristics which are desirable for traction purposes. In the first motor, a resistance could be connected across the stator winding, so that the motor can be given the characteristics of a repulsion motor. It is also possible to connect only the rotor to the source of power, in which case the stator circuits are closed only through the rectifier, and the stator currents are obtained by induction through the rotor. The stator, being then no longer connected to the circuit, can be wound for whatever voltage is found most desirable.*

FIG. 10—COMMUTATORLESS RECTIFIER MOTOR ON TEST

The operation of the transformer, rectifier, and motor, does not require a source of any definite frequency. This makes it possible to operate the unit with power from either 25-cycle or 60-cycle commercial lighting and power circuits. Moreover, if properly designed, a single motor can be operated on lines of different frequencies. It may even be possible to omit the transformer, and feed the motor directly from the line, through the recti-

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*See Kern, *Elektrische Bahnen*, Nov. 1931.

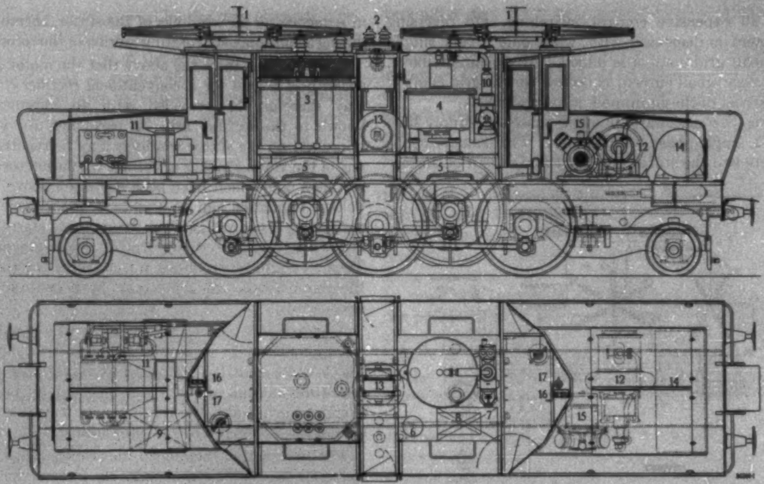

FIG. 11—GENERAL LAYOUT OF SINGLE-PHASE TEST LOCOMOTIVE WITH RECTIFIER GRID-CONTROLLED MOTORS

1. Current collector
2. Main switch
3. Transformer
4. Rectifier
5. Driving motors
6. Control button
7. Central controller
8. Contactor
9. Motor switch
10. Vacuum pump for control valve
11. Cooling set for control valve
12. Cooling set for transformer
13. Fan for driving-motors
14. Converter for auxiliary services
15. Air compressor
16. Driving controller
17. Reversing switch for running and recuperation

fier. This would result in an arrangement particularly desirable for suburban lines using multiple-unit cars. Only one rectifier is required to control all the motors of one locomotive or motor car. We therefore have a very simple arrangement with the greatest possible range of control, namely, from zero to maximum, using a very light and inexpensive grid-control equipment, and without the least degree of energy loss.

With a suitable arrangement of apparatus, this type of motor can also be used for regenerative braking while the locomotive or car is proceeding down grade.

Fig. 10 shows the first experimental model of such a commutatorless motor directly coupled to a generator as load, and with a commercial rectifier of very large capacity. The control hand-wheel is visible on the right.

In this connection it is of interest to note that Brown, Boveri & Co., Ltd., of Baden, Switzerland, is at present building a single-phase locomotive to be operated directly with single-phase power of 50 cycles at 15,000 volts. The locomotive is of standard gage, and is equipped with two motors of 500 hp. one-hour rating, for a maximum speed of 90 km. per hr. (56 mi. p. hr.). (See Fig. 11.) It is designed for regenerative braking. This locomotive has no commutators, tap switches, starting resistances, nor reversing switches, in the motor circuit. All such equipment is replaced by a high-voltage mercury arc rectifier with control grids. The tests with this locomotive will be highly significant in view of the possible utilization of alternating currents of commercial frequencies for the electrification of main-line railroads.

Discussion

L. R. Ludwig: For a great many years engineers have appreciated the advantages which may be gained by introducing grids into mercury arc rectifiers for the purpose of controlling them. A large number of patents has demonstrated the realization of the possibilities inherent in this type of device; in particular, the early fundamental patent on inverters (L. W. Chubb, Re. No. 17693) and later patents by Slepian have helped lead the way in these developments. Today the average design engineer accepts the possibility of a grid-controlled rectifier, perhaps without enough reservation as to the difficulties which the grid carries with it. Inherently, a grid-controlled rectifier is similar to a uni-directional switch, provided suitable extinguishing means are supplied if a d-c. circuit is to be interrupted. However, the grid-controlled rectifier suffers from the very serious disadvantage that the grids frequently lose control without apparent cause. In this respect the grid-controlled rectifier is much inferior to a switch because, as pointed out in a paper by Slepian and Ludwig,* the loss of grid control, which is most often in the nature of a backfire, does not occur above certain definite limits but may occur at random throughout any operating range of current or voltage. In other words, the loss of control of the grids constitutes a major problem, the solution of which today is more incomplete than that of the backfire problem in general. Another serious cause of loss of control of the grids when such devices are used as inverters is the fact that a relatively long deionizing time is necessary for a current zero in order for the grid to gain control. Careful measurements of this time made by Slepian and Crago in 1923 indicate hundreds of microseconds. In the inverter circuit the necessity for this deionizing time requires the use of special commutating means such as condensers, which must be comparatively large.

From these brief comments, it can be seen that the requirements of an ideal controlled rectifier are freedom from backfires and freedom from a long time lag necessary before the grid can gain control of the discharge to its respective anode. The large amount of research work which has been done by electrical manufacturers toward the perfection of the mercury arc rectifier and its grid-controlled brother will probably soon make the use of such devices technically possible. However, if one considers what is really required of the rectifier or inverter in an electrical circuit, it will readily be seen that the large and rather cumbersome devices which are on the market today really represent a rather crude development as compared with the device which may be imagined as being sufficient to accomplish the purpose which is really accomplished by the rectifier or inverter. The present size and complexity of this type of apparatus are no doubt due to the as yet imperfect understanding of the nature of backfire. For example, the ideal rectifier may be quite small, having very low loss and free from the complexity of the various grids and shields as employed at present. The ideal converter might well be imagined as a similar device in which satisfactory control is obtained perhaps with means other than grids. Consequently, there may be some question as to whether the present rectifier and converter must not be very materially improved before startling changes in the transmission of electrical power can be expected from their use.

It has also been recognized for some time that rectifiers may be used instead of commutators in conjunction with rotating electrical machinery. However, it would be no more correct to state that the rectifier can be used to replace the commutator than it would be to state that a gasoline engine can be used to replace the horse. Both gasoline engine and horse may be used as a source of power for locomotion, but certainly no one would expect to hitch a gasoline engine to the shaft of a wagon for the purpose of pulling it. Similarly, the rectifier is not the exact counterpart of the commutator. Two differences exist: (1) the rectifier is a uni-directional switch, whereas the commutator and brush will pass current in either direction, and, (2) the rectifier for practical reasons must have a comparatively small number of anodes or segments, whereas the commutator usually has a large number. The result is that when a rectifier is used in place of a commutator, as suggested by Mr. Marti, the utilization of the windings on the motor is not nearly so good as the utilization if a commutator is used. For this reason, one may question whether it would not be better practise to use a controlled rectifier merely as a frequency-changer without direct regard to the motor, and to use with it some type of motor in which the utilization of copper is considerably better. Certainly with this poor utilization, the requirements of the controlled rectifier as to size, cost, and reliability seem to be much greater than can be met at the present time.

R. E. Hellmund: The possibility of using rectifiers on locomotives running on an a-c. system has been rather intriguing to engineers ever since the conception of the mercury arc rectifier. This is evidenced by the early operation of a rectifier car by the Westinghouse Company on the New York, New Haven and Hartford system in 1914. The possibility of commutatorless motors operated by means of rectifiers was conceived very soon after the advent of the rectifier, as is evidenced by the early work of Chubb, Slepian, Hazeltine, Alexanderson, and others. The failure to introduce such arrangements for commercial operation was primarily due to the fact that they could not economically compete with arrangements using the ordinary single-phase commutator motor, and in the case of the schemes of operating commutatorless motors the development of suitable rectifiers in the early days was perhaps not far enough advanced to give sufficiently reliable operation. In the meantime, considerable progress has of course been made in the development of rectifiers and similar electronic devices, but probably even greater advances have been made in the design of single-phase motors by those engineers who realized the possibility of this type of motor for railway work. As a result of such progress in both lines of activity, their relative position today is about the same as in the early days, namely, the present rectifier arrangements cannot compete economically with single-phase commutator motors.

The fact that the single-phase commutator motor presents certain engineering problems in connection with commutation and that as a matter of course such problems have been discussed a great deal by the interested engineers, has led to the impression that if only a commutatorless motor could be devised for an a-c. railway the millennium would have been reached. In other words, the fact that the single-phase motor offers one major problem in its design has distracted attention from the fact that it offers some very marked advantages over any other type of motor for conditions prevailing in railway work. The very feature which has been introduced in the single-phase motor design to take care of the commutation problem, namely, the use of a large number of poles, brings with it the possibility of obtaining a higher rating within the definitely limited space available and at a lower weight than is possible with other motors.

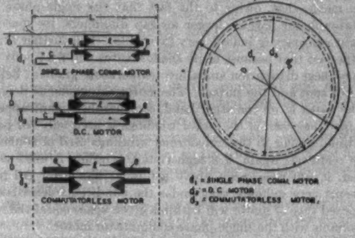

Fig. 1

This can best be pointed out by reference to Fig. 1. With an available outside diameter D and a large number of poles, the core naturally will be shallow, giving an air-gap diameter d_1 in the case of a single-phase motor. In a d-c. motor on a locomotive, at least an appreciable part of the total line voltage has to be applied to each motor, which necessitates large distances between brush-holders and, as a consequence, a small number of poles. This in turn requires appreciable space for the yoke section and the field coils, so that the air-gap diameter d_2 will have to be very much smaller. In most commutatorless motors the number of poles must also be made smaller because such motors, unlike the single-phase commutator motor, are unable to operate above synchronous speed. In the case of induction and synchronous motors this is of course evident, but it is also true with the motor described in the paper unless additional complicated

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Backfires in Mercury Arc Rectifiers, A.I.E.E. Trans., March 1932, p. 92.

means are introduced. While with the arrangement given, a number of cycles can be used in succession for each of the windings, thus making it possible to operate at low speeds; there seems at present to be no simple and inexpensive way of stopping the current flow in any one winding before the end of the half cycle. This means that with a given number of poles the speed cannot be increased indefinitely, which in turn means that in order to obtain high speeds a small number of poles has to be chosen. As a consequence, the depth of the stator core will be greater than with the commutator motor and therefore the air-gap diameter d_3 will be smaller. The same figure gives a comparison of the three types of motors with regard to filling the limited width available in railway work. The latter is usually taken up by some clearances, by the core width, the length of the coil-end connections, and the commutator where there is any. In the upper figure showing the single-phase motor, the commutator is long, but on account of the large number of poles the coil ends are short. In the second figure illustrating a d-c. motor, the commutator is shorter but the end connections are longer. In the third figure, representing the commutatorless motor, there is of course no commutator but the coil ends are long. If slip-rings are used inside of the bearings, as usually will be the case, further space may be taken by the slip-rings and the brush-holders, unless it is possible to locate them under the winding. In other words, the length of core available for the three types of motors probably does not vary to any great extent. This then means that, owing to the fact that the single-phase commutator motor permits larger air-gap diameters, its space and weight efficiency is indeed very favorable. It is further favored by the fact that because of its shallow cores and field coils there is good heat dissipation and a low temperature gradient between the hot spot and the surface. The motor arrangement described in the paper is, on the other hand, considerably handicapped by the fact that each of the stator windings is used only for part of the time, similar to the transformer windings used in connection with rectifiers. Again, if a high-voltage winding is used as proposed, it will necessarily prove a further handicap. All in all, it seems very unlikely that under the conditions described the proposed type of motor will within a given space equal the rating obtainable with the single-phase commutator motor.

With this picture of the motors before us, it is not difficult to see that, with no gain in the motor situation, the addition of a rectifier to the locomotive in the present state of the art is not likely to lead to an all-round economic solution from either the first-cost or the maintenance point of view. It is true that commutators as well as switches require some maintenance, but this is also true to a greater extent with the rectifier. Further consideration must be given to the fact that rectifier arrangements will be handicapped by the necessity for filtering equipments on the locomotive if telephone disturbances are to be avoided.

It is suggested in the paper that the proposed scheme might be used for 60-cycle electrification, thereby obviating the necessity for 25-cycle systems. Here again it so happens that although the considerations of the motor commutating problem have brought about the use of 25 cycles for railway work, such a system is advantageous from various points of view. The early claims that the 60-cycle system would eliminate rotating conversion apparatus have not materialized because it has been found that phase-converting apparatus would be necessary anyway. Furthermore, the use of 25 cycles under railway conditions, with the heavy loads of changing location, has shown quite some superiority from a voltage regulation point of view and also as far as telephone interference is concerned.

This discussion of the economic conditions is not intended to convey the idea that studies like that presented in Mr. Marti's paper are futile. However, the scheme presented, like many others previously suggested for the utilization of electronic devices both for traction and other purposes, will be dependent to a great degree upon further developments of electronic devices. Many of them will not find commercial utilization until the electronic devices have been developed to the point where they are considerably less expensive and where a greater reliability of various grid control schemes has been obtained.

E. F. W. Alexanderson: The vacuum tube technique has many ramifications and promises to become an important factor in most branches where electric power is used. One of those possibilities is to substitute grid control rectifiers or thyratrons, as we call them, for the copper and carbon brush commutator in the operation of adjustable speed motors. The largest field for the use of adjustable speed a-c. motors is undoubtedly railroad electrification and much thought and research have been given to this subject in the last ten years or more by those interested in the vacuum tube technique. Those who look at this development from outside may well ask: Why, if the problem has been clearly understood for so long, is the practical solution so slow to be realized? The answer to this is apparently that the vacuum tube technique is not a simple engineering problem but an evolution and like all evolutions it is necessarily slow. It involves the work of the physicists who are contributing new knowledge of electricity and matter. It involves the manufacture of the vacuum tubes themselves, and the evolution of a new technique for embodying those new principles in practical devices and finally, it involves the electrical engineers who must interpret the usefulness of these devices with relation to the more conventional types of electrical machinery.

The history of this evolution for the last ten years is that new principles have constantly been introduced in the manufacture of superior vacuum tube devices and these devices in their turn have made possible new and improved methods of engineering application. The commutatorless a-c. motor developed by the Brown-Boveri Company and described by Mr. Marti is one of the steps in this evolution. From personal experience I can say that motors of this type will operate with satisfactory characteristics and I do not doubt for a moment that a motor just as described can be designed to haul a locomotive.

Another question is: Will such a locomotive be economically competitive with other and existing types of electric locomotives? Serious efforts are being made to answer that question. Personally I am of the opinion that this development will lead to such a practical solution and my reason for this belief is that progress of the vacuum tube technique is rapidly leading to new and superior forms which in their turn make possible improved methods of correlating the power system, the vacuum tube, and the motor. From the point of view of engineering research, this subject is of intense interest. But no matter how great our enthusiasm, I think it is better and wiser to admit that from the point of view of practical railroad electrification it is not possible to discuss this matter intelligently until we have before us a locomotive with vacuum tube commutation which is not only operative but is economically competitive with the now existing types. If, and when, these hopes are realized it will mean a substantial boost to the spread of railroad electrification because locomotives of the new type may be operated side by side with locomotives of the old type on the same lines, each type filling its place according to its own merits. From this point of view, it is fortunate that in this country a power frequency of 25 cycles rather than the lower frequency of $16^{2}/_{3}$ cycles has been adopted for railroad electrification because 25 cycles is better adapted to various uses. The anticipation of new and improved types of locomotives is thus no reason for delaying practical progress in electrification, because the new methods when realized, will fit in with the old plans.

R. G. McCurdy: The developments described by Mr. Marti on rectifiers provided with control grids open up some very interesting possibilities in the way of new problems for the telephone companies as well as possible advantages to power and electric railway companies.

We have had, of course, no experience with the new applications described by Mr. Marti but must base our comments on our experiences with these devices as used for conversion from alternating to direct current.

Considerable information has been published in the technical press, including the TRANSACTIONS of the Institute, regarding the effect of mercury arc rectifiers on the wave shape of systems to which they are connected and the resulting effects on coordination with telephone lines. This has been more particularly in reference to the effects on the d-c. side. The joint studies which have been made of the problem by the electrical manufacturing and telephone companies have led to the development of filters for the d-c. side which reduce the harmonic voltages and currents to values which permit satisfactory coordination between the d-c. systems and nearby telephone systems. These have been employed in situations where it appeared that conditions of proximity with the telephone plant were such as to justify their use.

The a-c. side of the device presents a more difficult problem. As compared to the d-c. side the percentage harmonic voltages and currents are much greater in magnitude. Moreover, there is a larger number of harmonics present. Both of these factors considerably increase the cost and difficulty of filtering. Where the rectifier is close to a source of power of relatively large size and supplied over lines not involved in appreciable exposures to telephone plant, these relatively large harmonic currents are not of importance. The impedance of the system under such conditions is low enough so that no appreciable increase takes place in the harmonic voltages and the telephone interference factor to which they contribute. On the other hand when the rectifier load is a relatively large part of the total power rating of the system, or when lines and transformers of considerable impedance intervene, or if the line supplying the rectifier is involved in telephone exposures, the difficulties in coordination are greatly increased.

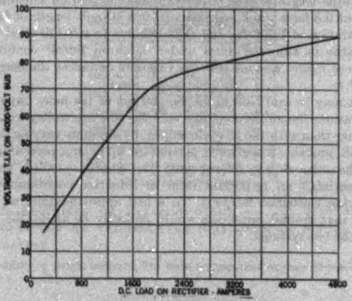

FIG. 2—VARIATION OF VOLTAGE TELEPHONE INTERFERENCE FACTORS ON A 4,000-VOLT BUS

With load on rectifier fed from same transmission system

On balanced polyphase power systems mercury arc rectifiers make no direct contribution to the residual voltages and currents of the system. Thus the usual methods of transposing the lines are applicable. In many cases, however, a distribution system involving numerous single-phase branches involved in exposures with telephone circuits may be supplied from the same system as a mercury arc rectifier. Under these conditions the harmonic voltages introduced by the rectifier will be residual so that means other than power transpositions will need to be resorted to in order to reduce the influence.

In Fig. 2 are shown some values of telephone interference factors on a 4,000-volt bus supplying distribution circuits, as a function of the load on a large mercury arc rectifier supplied from the same high-voltage transmission circuit as this 4,000-volt bus. It will be noted that under maximum load conditions on the rectifier, the bus voltage telephone interference factors are increased by a factor of more than 5:1. It will be apparent that coordination measures which are adequate for the minimum telephone interference factors values may be entirely inadequate when the load on the rectifier is large.

It is well known that the magnitudes of the harmonic voltages and currents on both the a-c. and d-c. sides are dependent in a large measure on the number of phases involved. A 12-phase rectifier for example has materially lower values of certain harmonics than the 6-phase rectifier. The wave shape of the two devices may be made substantially equivalent on the d-c. sides by filtering, a more effective filter however being required with the 6-phase device. On the a-c. side the 12-phase rectifier involves somewhat smaller fifth, seventh, seventeenth, and nineteenth harmonics. The eleventh and thirteenth harmonics are about the same for the 12-phase and 6-phase devices. The telephone interference factors so far observed have been in the ratio of about 1:3 in favor of the 12-phase device.

This matter is being actively studied by the Project Committee on Wave Shape of the Joint Subcommittee on Development and Research of the N.E.L.A. and Bell Telephone System. It is hoped that technical information which will permit the making of quantitative estimates of the effects of rectifiers on the wave shape of supply systems will be soon forthcoming from this committee.

In examining the effect of the controlled grid rectifier as described by Mr. Marti, it seems evident that the load regulation by these means must result in further reaction in wave shape. This may be seen by examining the graphs shown in Fig. 4 of Mr. Marti's paper. The effect on the wave shape of the d-c. side may, of course, be compensated for by suitable modification of the d-c. filter.

While I have not made a detailed quantitative examination of the wave shape of the current of the commutatorless single-phase railway motor described by Mr. Marti, it seems apparent that harmonics will exist in the a-c. side comparable at least with those experienced with the polyphase rectifiers previously mentioned.

Noise frequency induction has not in general been a problem with single-phase railway electrifications due to the small wave-shape distortion experienced, induction at the fundamental frequency being a controlling factor. No doubt the exacting commutation requirements of the single-phase motor have been a controlling factor in this situation. The controlled grid mercury arc rectifier would apparently seem to solve the commutation problems, but at the expense of a considerably increased wave-shape distortion. This may be expected to complicate the problems of coordination between single-phase a-c. railways and telephone circuits. It is hoped that the joint development effort previously referred to in securing inductive coordination with the conventional type of mercury arc rectifiers will lead to methods which may be applicable in the a-c. railway case.

H. M. Trueblood: The possibility of direct utilization of 60-cycle energy on the contact systems of electrified railways has engaged the attention of engineers occasionally in the past, and has led to certain preliminary exploration of its probable reaction on inductive coordination at fundamental frequencies. Mr. Marti's paper brings this question up again in an interesting way. In line with American practise in electric railway and commercial power engineering, the following general observations are based upon a comparison of the use of single-phase 25-cycle and 60-cycle energy upon the contact wire of an electrified railway using the running rails for return. Several factors are involved in a rather complicated way, and simplifying assumptions underlie this preliminary comparison. These are: first, the power supply is unidirectional *i. e.*, it is what is generally known as "stub-end feed;" second, the currents—load or short-circuit—are of the same magnitude in the two systems.

There are involved the following main factors:

Average earth current throughout the inductive exposure (it is chiefly this current, as distinguished from the current remaining in the rails, that causes fundamental frequency induction in communication circuits in the usual case). In addition to differences in track-earth circuit characteristics at the two frequencies, this point involves also the length of feed and the length of exposure.

Relative magnitudes of coefficients of induction at 25 and at 60 cycles. This depends upon the resistivity of the earth and upon the separation between the railroad and the communication line.

The efficacy of shielding conductors at the two frequencies. Such shielding conductors may exist, irrespective of any inductive coordination questions, in proximity to either the communication conductors (for example, cable sheaths) or the railway system (for example, structure ground-wires), or they may be installed primarily for purposes of coordination. Here again the earth resistivity is a factor, and, as exemplified particularly in the case of underground communication cables, the amount of metal in the shielding conductors and its distribution are important.

Estimates have been made of the effects of these factors, using reasonable figures for items of design, and, in the main, experimental data for physical characteristics such as rail impedance, track leakage, etc. It is not proposed here to attempt more than brief summaries. These estimates indicate:

1. That for a representative value of track leakance per track, the ratio of average earth current at 60 cycles to that at 25 cycles does not depart from unity by more than about 10 per cent for lengths of feed ranging from some 3 miles up to 20 miles or more.

2. That the ratio of the mutual impedance of two ground-return circuits at 60 cycles to the same quantity at 25 cycles ranges from 2.2 for 50 ft. separation and earth resistivity of 1,000 meter-ohms to about 1.2 for 1,500 ft. separation and earth resistivity of 10 meter-ohms.

3. That for optimum conditions (zero impedance ground connections at the ends of the exposure) the ratio of the reduction in induced voltage due to cable-sheath shielding at 60 cycles to the corresponding reduction at 25 cycles ranges from about 2.0 for one full-sized cable to about 1.5 for two full-sized cables in adjacent duct runs. (This is for an earth resistivity of 100 meter-ohms.)

4. That the efficacy of track feeders in reducing the amount of current in the earth, for long feeds, is some 5 to 10 per cent (depending on the number of feeders and the number of tracks) better for 60 cycles than it is for 25 cycles.

On the basis of the estimates that have just been summarized and under the above-stated simplifying assumptions, certain statements become possible with regard to the induced longitudinal voltages:

5. The ratio of the longitudinal induced voltage at 60 cycles to the longitudinal induced voltage at 25 cycles is greater when the track leakance, the length of feed, the separation, and the amount of shielding provided are small than when they are large.

6. For an earth resistivity of 100 meter-ohms this ratio ranges from about 2.4 for a track leakance of 0.5 mho per track mile, a 3-mile length of feed, a 50-ft. separation and no shielding, to about 0.8 for a track leakance of 1.0, a 10-mile length of feed, a separation of 1,500 ft., and shielding due to two full-sized cable sheaths in adjacent ducts under optimum (ideal) conditions. (These figures assume that the exposure embraces the length of feed.)

It may be said that in the general run of the more important situations the induced voltage with the 60-cycle trolley would be higher than with the 25-cycle trolley, by amounts varying from a small, probably a negligible, percentage, up to large percentages—of the order of 50 to 100 per cent. This ignores so-called "ground potentials," an important item in inductive coordination involving electrified railways. To a first approximation, we may say that ground potentials would probably be about the same for the two systems.

Insulator Sparkover

Factors Affecting the Sparkover Voltage of Insulators Used on High-Voltage Transmission Systems

BY W. L. LLOYD, Jr.*
Member, A.I.E.E.

Introduction

While the mechanical properties and strength of the various parts entering into the design of a modern transmission system are quite well known, the electrical characteristics of the various parts are often not so definitely understood. This is particularly true of the electrical properties of the insulating parts, including the line and station insulation and bushings used on the station equipment. It is the purpose of this paper to present data upon the sparkover strength of the various parts used for the insulation of modern high-voltage transmission systems.

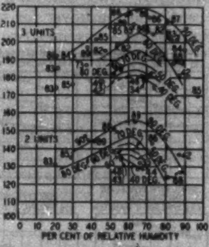

Fig. 1—Effect of Relative Humidity and Dry Bulb Temperature on the 60-Cycle Sparkover of Suspension Insulators

Number on points indicates dry bulb temperature at time of test. Air density = 1.00

Temperature and Barometric Pressure. The effect of temperature and barometric pressure upon the 60-cycle sparkover voltage was determined a number of years ago for sphere-gaps, point-gaps, bushings and various types of suspension and pin type insulators.[1] In this early work it was found that the sparkover voltage of gaps, varies with varying air density factor δ.

$$\delta = \frac{3.92\,b}{273 + t} \text{ in metric units, where}$$

b = barometric pressure in centimeters of mercury
t = temperature in degrees centigrade

At 25 deg. cent. and 76 cm., the air density factor is unity.

Or,

$$\delta = \frac{17.9\,b}{459 + t} \text{ in English units, where}$$

b = barometric pressure in inches of mercury
t = temperature in deg. fahr.

At 77 deg. fahr. and 29.9 inches, the air density factor is unity.

It is generally sufficiently accurate to assume that the change in sparkover voltage is directly proportional to the change in air density. Although the effect varies somewhat with the design of the insulator or bushing, the above rule will usually apply over the practical operating range. For example, the sparkover voltage of three and four unit insulator strings has been found to vary directly with the air density factor down to and including an air density 50 per cent of that occurring at standard temperature and barometric pressure (77 deg. fahr. and 29.9 inches).

Air density affects the lightning sparkover of gaps and insulators in the same general way as for 60 cycles. However, the effect for lightning is usually somewhat less.

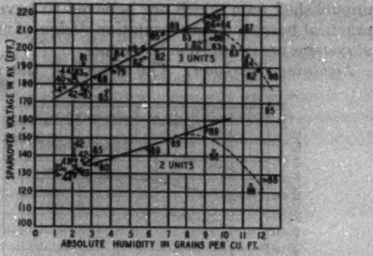

Fig. 2—Effect of Humidity on the 60-Cycle Sparkover of Suspension Insulators

Note that plotting sparkover voltage against absolute rather than relative humidity results in a single curve for all dry bulb temperatures (as indicated by number on points). Air density = 1.00

All sparkover voltages in this report have been corrected to an air density of unity by using the air density factor as given above.

Humidity. Only a small amount of data giving the effect of humidity has heretofore been published.[2,3] Tests made in different laboratories under identical conditions, in so far as A.I.E.E. specifications are concerned, often fail to produce identical results. Data included in this

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*High Voltage Engineering Laboratory, General Elec. Co., Pittsfield, Mass.

1. For references see Bibliography.

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paper indicate that the larger part of these differences can usually be attributed to the effect of humidity or water vapor in the atmosphere in which the tests are made. A humidity of 6.5 grains per cu. ft. (65 per cent at 77 deg. fahr.) has been used by this laboratory as the standard condition of humidity in giving out sparkover data. This is equivalent to a vapor pressure of 0.608 in. (15.4 mm.) of mercury.

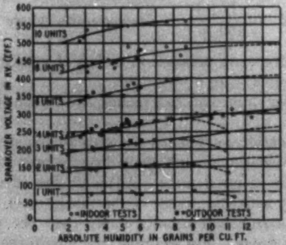

Fig. 3—Effect of Humidity on the 60-Cycle Sparkover of Suspension Insulators

10-in. diameter, 5¾-in. spacing, standard duty. Air density = 1.00

Smoke, Steam, Dew, Fog, Rain, and Surface Dirt and Moisture. While the effects of moisture and dirt have been recognized for a much longer time, specific quantitative data have often been lacking. Some heretofore unpublished data which are believed to have a direct practical bearing on the insulation design of many lines of systems are included in this paper.

Lightning Sparkover. The effects of the preceding factors on the lightning sparkover of various insulations are discussed and some data submitted.

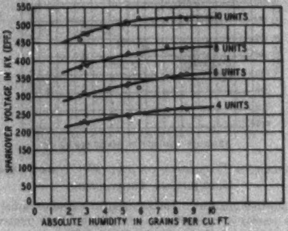

Fig. 4—Effect of Humidity on the 60-Cycle Sparkover of Suspension Insulators

10-in. diameter, 4¾-in. spacing, standard duty. Air density = 1.00

HUMIDITY

Where the humidity for a given sparkover test has been specified it has been common to express the humidity in terms of relative humidity. No confusion would result by such a procedure if all sparkover tests were made at approximately the same temperature. The effect of humidity can then be indicated by plotting sparkover voltage as a function of relative humidity. Sparkover tests are, however, sometimes made over a wide range of temperature and results so plotted are of little value in comparing the humidity effect and in finding an agreement in the sparkover voltage at some given humidity. A single curve of sparkover voltage as a function of relative humidity for different temperatures cannot be drawn. (See Fig. 1.) The effect can only be shown by drawing in a group or family of curves, each individual curve pertaining to a particular temperature. Since humidity has generally been expressed in terms of relative values, it has been difficult to interpret its true effect and different laboratories have found it difficult to agree upon the sparkover voltage at any given humidity. Note that for a relative humidity of 65 per cent, the sparkover for two units varies from 130 kv. to 155 kv., and for three units from 180 kv. to 220 kv. for a temperature variation of 40 deg. to 90 deg. fahr. or plus or minus approximately 10 per cent in voltage even after correction for temperature.

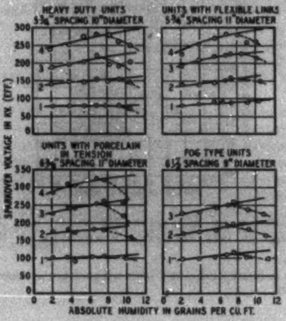

Fig. 5—Effect of Humidity on the Sparkover of Suspension Insulators, 60 Cycles

Air density = 1.00

Fig. 2 shows the same data when plotted in terms of the absolute humidity. The family of curves reduces to a single curve indicating the true effect of humidity. The dip in the curve beyond about 9 grains per cubic ft. will be discussed below.

Fig. 3 presents the sparkover voltage for 10-inch diameter standard-duty suspension insulators having a spacing of 5¾ inches. These tests have been made with four different sets of testing equipment in four laboratories. Three of the laboratories employed artificial humidity control; the fourth was an outdoor laboratory in Pittsfield using natural humidity occurring from day to day and season to season. All these tests were conducted in accordance with A.I.E.E. Standards No. 41. The importance of recording absolute rather than relative humidity is most apparent when the same relative humidity is measured in a spring or fall outdoor test as compared with a summer test. Although the relative humidity may be the same, the absolute humidity because of a considerable change in temperature may be considerably different. Under such conditions, unless the absolute humidity is used, considerable variations are indicated.

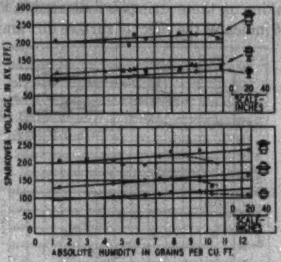

FIG. 6—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF PIN AND PEDESTAL TYPE INSULATORS

Air density = 1.00

The effect of humidity on the sparkover voltage of 10-inch diameter standard-duty suspension insulators having a spacing of 4¾ inches is shown in Fig. 4. The effect of humidity on the sparkover for various types of suspension insulators is shown in Fig. 5 while Fig. 6 presents the effect of humidity on certain sizes of pedestal insulators and a large pin type insulator. Fig. 7 presents similar data for certain sizes of small pin type insulators. The effect of humidity is not great on single suspension disk insulators or on these small pin type insulators.

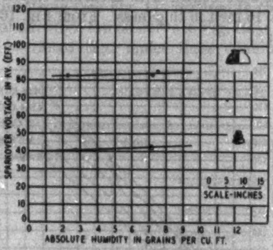

FIG. 7—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF PIN TYPE INSULATORS

Air density = 1.00

Fig. 8 shows the effect of humidity on the sparkover of an oil-filled bushing.

Fig. 9 is a typical humidity curve for the sphere gap. The sphere gap appears to be practically independent of the humidity. Fig. 10 presents the effect of humidity on the 60-cycle sparkover voltage of the needle gap. The data in Figs. 9 and 10 are in agreement with previous data on the effect of humidity on the sparkover of spheres and needles.[3]

Fig. 11 is a plot of the data from Fig. 10 in sparkover voltage as a function of the spacing. The standard needle gap curve is also indicated.

It will be noted that several of the humidity sparkover curves for insulators indicate a lowering of the sparkover for absolute humidities above approximately 9 or 10 grains per cu. ft. That this is an effect of surface moisture or condensation of the water vapor from the atmosphere on the porcelain surfaces at these high absolute (and generally relative) humidities and not an effect in the air gap, is indicated by the absence of this effect in the case of the needle gap and sphere gap, particularly the former. This conclusion is also indicated by the data in Fig. 12 where tests were made on chilled and slightly heated insulators. It is also indicated by the difficulty commonly experienced when tests on insulators at high humidities show a constantly rising sparkover in repeated tests. The surface moisture results in a low sparkover in the initial test. Repeated sparkovers remove this surface moisture and the insulator gradually acquires the higher sparkover resulting from the high humidity. Frequently this increase in sparkover continues for twenty-five or more tests before the sparkover voltage becomes constant at the upper value.

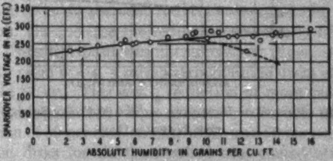

FIG. 8—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF AN OIL-FILLED BUSHING

Air density = 1.00

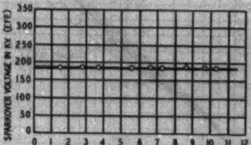

FIG. 9—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF A 25-CM. SPHERE GAP SPACED FOR 187 KV.

Air density = 1.00

Smoke, Steam, Dew, Fog, Rain, and Surface Dirt and Moisture

The 60-cycle sparkover of clean 10-inch diameter standard-duty suspension insulators of 5¾-inch spacing for dry, light rain, and heavy rain conditions is shown in Fig. 13. The tests apply to insulators in a vertical position. Rain tests with the insulators in a horizontal position indicate no lowering of the sparkover by rain with the rain directed either at right angles to the string or from, or towards, the line end at an angle of 45 deg. to the string and always at 45 deg. from the vertical. On a horizontal string rain sufficiently improves the poor voltage distribution along the string so as to raise the rain sparkover to that of the dry. The drip along the vertical string overbalances this effect and actually lowers the sparkover.

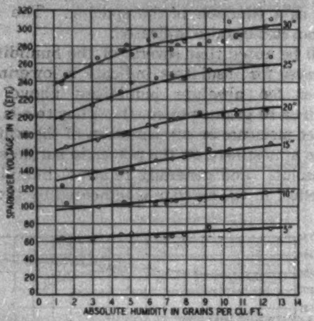

Fig. 10—Effect of Humidity on the 60-Cycle Sparkover of the Needle Gap

Air density = 1.00

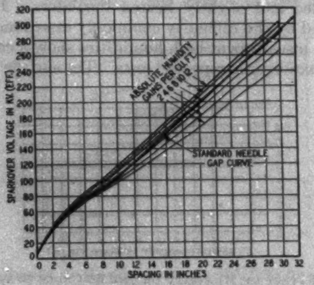

Fig. 11—Sparkover of the Needle Gap. Effect of Humidity on Sparkover. 60 Cycles

Air density = 1.00

Fig. 14 shows the effect upon the sparkover voltage of the resistivity of the water and the rate of precipitation used in making artificial rain tests on clean insulators and bushings.

That the dry 60-cycle sparkover of 10-inch diameter insulators strings having the same overall length is apparently independent of the spacing of the individual units making up those strings is indicated in Fig. 15.

Fig. 16 indicates the effect of surface dirt and moisture upon the 60-cycle sparkover of suspension insulators. The dirt used was that resulting from long exposure of the insulators to locomotive steam and smoke until the insulators were coated with a carbon deposit perhaps 1/32 to 1/16 inch in thickness. From Fig. 16 it will be observed:

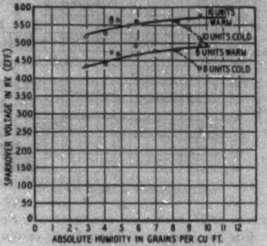

Fig. 12—Effect of Humidity on the 60-Cycle Sparkover of Suspension Insulators

10-in. diameter, 5¾-in. spacing, standard duty. Air density = 1.00

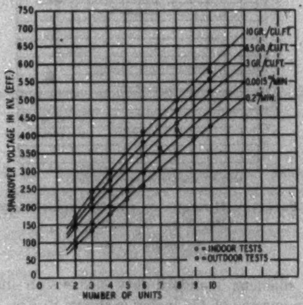

Fig. 13—Effect of Humidity on the 60-Cycle Sparkover of Suspension Insulators

10-in. diameter, 5¾-in. spacing, standard duty. Air density = 1.00

1. That this deposit of dirt when dry had practically no effect upon the insulator sparkover.

2. That the sparkover of the dirty insulators with top surface moistened was only slightly lower than the clean insulators with top and bottom surfaces moistened.

3. That the clean and dirty insulators had about the same sparkover under artificial rain conditions of 0.2

in. per minute at 45 deg. from the vertical, as specified by the A.I.E.E., presumably because some washing of the insulators took place.

4. That the dirty insulators with top and bottom surfaces moistened (by light spraying) had a very low sparkover voltage, and finally

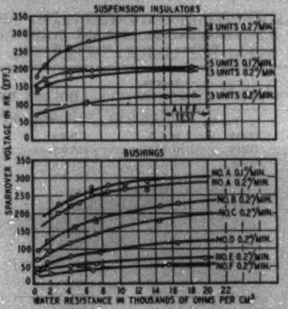

Fig. 14—Effect of Water Resistance on the Wet Sparkover of Suspension Insulators and Bushings. 60 Cycles

Air density = 1.00

5. That this latter condition combined with an atmosphere of heavy steam and locomotive smoke surrounding the string (see Fig. 17) resulted in the lowest sparkover of all. (See lowest curve in Fig. 16.)

It may be noted that under these conditions the sparkover of 14 disks was only 200 kv. effective, a reduction of 73 per cent.

Table I shows the effect of dew upon the sparkover voltage of clean and salt-coated suspension insulators.

The salted insulators in Table I were lightly coated with salt (sodium chloride) by hand-spraying with a saturated salt solution. The condensation of moisture on the porcelain surfaces was accomplished by exposure of the insulator string to the warm and somewhat humid atmosphere of the laboratory after cooling in an iced compartment. The duration of exposure in minutes is given in the table. Very little reduction in sparkover took place in five minutes. The maximum reduction was reached in about 10 or 15 minutes. The strings were observed to start to dry again after about 30 minutes of testing.

Surface moisture was found to have only a very slight effect upon the clean insulators. On the salt-coated strings a 30 to 40 per cent reduction in sparkover was common. In one case the reduction was nearly 50 per cent. The salt deposit was not excessive. Upon spraying with clean water, a salted string of nine disks would not hold 87 kv. This was a reduction of more than 85 per cent in sparkover. This reduction compares with a reduction of 50 per cent on nine disks and 60 per cent on fourteen disks for the locomotive soot-coated strings with top and bottom surfaces moistened.

Table II gives somewhat similar data on four small pin type insulators on which a deposit of mill dust had been accumulated as a result of a long time exposure in the vicinity of a Pittsburgh mill.

TABLE I—DEW TESTS ON CLEAN AND SALT-COATED INSULATORS
60 Cycles

String	No. of disks	Surface condition	Duration of exposure	Sparkover voltage	Sparkover reduction
No. 1	13	Clean and dry	0 min.	699 kv.	0 per cent
No. 1	13	Clean but moist	20 min.	692 kv.	1 per cent
No. 4	13	Salted and moist	6 min.	610 kv.	13 per cent
No. 4	13	Salted and moist	7 min.	531 kv.	24 per cent
No. 4	13	Salted and moist	8 min.	465 kv.	34 per cent
No. 4	13	Salted and moist	14 min.	425 kv.	39 per cent
No. 4	13	Salted and moist	24 min.	455 kv.	35 per cent
No. 4	13	Salted and moist	Held 127 kv. (eff.) for 7 minutes. No flashover.		
No. 4	13	String drying	31 min.	595 kv.	15 per cent
No. 4	13	String drying	44 min.	659 kv.	6 per cent
No. 5	13	Salted and moist	21 min.	466 kv.	33 per cent
No. 5	13	String drying	37 min.	594 kv.	15 per cent
No. 2	9	Clean and dry	0 min.	537 kv.	0 per cent (4)
No. 2	9	Clean but moist	6 min.	542 kv.	0 per cent
No. 3	9	Clean but moist	6 min.	300 kv. dried line unit in about 5 minutes	
No. 6	9	Salted and moist	7 min.	326 kv.	39 per cent
No. 6	9	Salted then sprayed with clean water		Less than 87 kv.	84 per cent
No. 7	5	Clean and dry	0 min.	345 kv.	0 per cent
No. 7	5	Salted and moist	6 min.	188 kv.	45 per cent
No. 7	5	Salted and moist	15 min.	175 kv.	49 per cent
No. 7	5	Salted and moist	20 min.	178 kv.	48 per cent
No. 7	5	Salted and moist	25 min.	201 kv.	42 per cent
No. 7	5	String drying	35 min.	301 kv.	13 per cent

From Table II, it may be seen that the reduction in sparkover voltage was about 66 per cent for the top and bottom moistened condition.

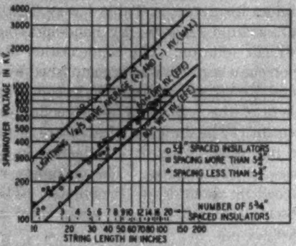

Fig. 15—Sparkover of Suspension Insulators

10-in. diameter. 5¾-in. spacing. Standard duty. Air density = 1.00

Lightning Sparkover

The effect of humidity on the lightning sparkover of different insulators is indicated in Fig. 18. Similar tests have been made on various gaps and a corresponding effect noted. Taking 6.5 grains per cu. ft. as standard, the variation in sparkover between 3 grains per cu. ft. and 10 grains per cu. ft. was found to be approximately plus and minus 10 per cent for the sizes tested. This is of the same order as the effect with 60 cycles.

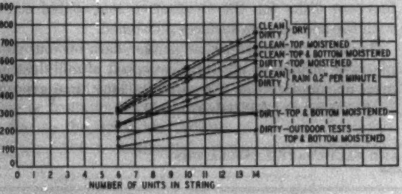

Fig. 16—Effect of Surface Moisture and Dirt Upon the Sparkover Voltage of Suspension Insulators

10-in. diameter, 5¾-in. spacing, 60 cycles. Air density = 1.00

TABLE II—60-CYCLE SPARKOVER OF DIRTY PIN TYPE INSULATORS DRY, WET AND UNDER ARTIFICIAL RAIN

Sample No.	Dry	Top surface moistened	Top and bottom moistened	Rain at 45 deg. (0.2 in. per min.)
1	58.5	29.0	19.0	27.8
2-A	59.0	33.4	23.9	25.1
2-B	58.7	26.8	25.9	27.6
3-A	59.7	32.3	21.2	25.0

Note: On the foregoing insulators with top and bottom surfaces moistened corona was visible in a darkened room at the following voltages:

sample No. 1	13.4 kv.
sample No. 2-A	10.7 kv.
sample No. 2-B	less than 9 kv.
sample No. 3	12.8 kv.

No corona was visible below 15 kv. on these insulators dry or with only the top surface moistened by spraying

That dew, fog, rain, and surface moisture have little effect upon the lightning sparkover of insulators is indicated by the data in Table III. In general, the effect of dew, fog, rain, and surface moisture decreases as the length of the wave decreases.

TABLE III—LIGHTNING SPARKOVER OF VARIOUS INSULATORS DRY AND WET

Positive Polarity Lightning Wave

Sample	Condition	Approx. wave shape	Dry	Rain 0.2 in. per min.
(a) Pin Type Insulators (Same as in Table II)				
No. 1	Dirty	0.5/5	110 kv.	106 kv.
No. 2-A	Dirty	0.5/5	113 kv.	113 kv.
No. 2-B	Dirty	0.5/5	112 kv.	109 kv.
No. 3-A	Dirty	0.5/5	122 kv.	110 kv.
(b) Pedestal Insulators				
6 units	Clean	1.5/40	1,600 kv.	1,490 kv.
(c) Suspension Insulators (Various Types)				
8 disks	Clean	1.5/40	875 kv.	845 kv.
8 disks	Clean	1.5/40	890 kv.	880 kv.
9 disks	Clean	1.5/40	945 kv.	885 kv.
12 disks	Clean	1.5/40	1,300 kv.	1,280 kv.
14 disks	Clean	1.5/40	1,370 kv.	1,310 kv.
16 disks	Clean	1.5/40	1,740 kv.	1,670 kv.
16 disks	Clean	1.5/40	1,730 kv.	1,560 kv.
18 disks	Clean	1.5/40	1,800 kv.	1,740 kv.

TABLE IV—COMPARISON OF THE DISRUPTIVE STRENGTH OF WATER AND OF AIR

Gap in cm.	Impulse kv. water	Impulse kv. air	Ratio water to air
1-Inch Diameter Spheres			
0.1	49.5	3.5	14.1
0.2	86.0	7.0	12.3
0.3	105.5	11.0	9.6
0.5	126.0	17.5	7.2
0.7	137.0	24.0	5.7
1.0	149.0	33.0	4.5
1.3	159.0	42.0	3.8
1.5	165.0	46.0	3.6
60 Deg. Points on 1/8 In. Rods			
1.0	56.5	25.0	2.05
2.0	72.0	35.3	2.15
3.0	92.0	41.0	2.25
4.0	113.0	46.5	2.42
5.0	134.0	51.5	2.61
6.0	156.0	56.5	2.76

Tests were made inside a 6-in. diameter glass sphere. Resistance of water was measured at 1.0 cm. setting of gap:

R between spheres = 20,000 ohms
R between points = 40,000 ohms

Tests on wood poles and crossarms with and without porcelain insulators mounted on the crossarm give the same indication. Wood poles, crossarms and guy insulator sticks of different cross-sectional areas have a lightning sparkover of approximately 175-kv. per ft. of length whether the wood is dry or wet with clean water or saturated salt water and brine.[8] That water has so little effect upon the lightning sparkover of porcelain insulators and insulations such as wood is probably because water is a fairly good insulator for impulse voltages as compared with air in which these materials are ordinarily tested. Table IV from "Dielectric Phenomena in High-Voltage Engineering," by F. W. Peek, Jr., Third Edition, p. 391, gives the relative disruptive strength of water as compared with air for impulse voltages.

ACKNOWLEDGMENT

Some data included in this paper have been obtained from each of the following laboratories:

High Voltage Engineering Laboratory at the Pittsfield Works of the General Electric Company.

Pittsfield Works Laboratory of the General Electric Company at Pittsfield (Outdoor and Indoor Laboratories).

Insulator Testing Laboratory at the Baltimore Works of the Locke Insulator Corporation.

FIG. 17—SMOKE TEST ON SUSPENSION INSULATORS

Switching locomotive used to supply heavily laden atmosphere of steam and smoke

The valuable assistance of the various members of the staffs of the above laboratories in making the many tests included in this paper is hereby acknowledged. The author also wishes to acknowledge the valuable guidance of Mr. F. W. Peek, Jr. under whose direction the various investigations reported above were conducted.

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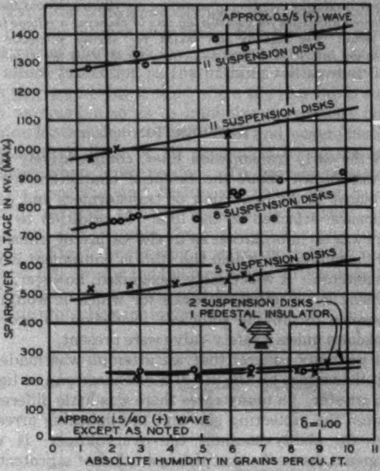

FIG. 18—EFFECT OF HUMIDITY UPON THE LIGHTNING SPARKOVER OF INSULATORS

Air density = 1.00. Approximately 1.5/40 (+) wave except as otherwise noted

Discussion

For discussion of this paper see page 697.

An Improved Type of Limiting Gap
for Protecting Station Apparatus

BY A. O. AUSTIN*
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Synopsis.—*The wide range of transients to which station apparatus is subjected is pointed out and the desirability shown for having adequate protection. The different breakdown characteristics of the various insulating members of a station are illustrated and the importance emphasized for considering this in choosing a voltage limiting device. The need is also shown for having protective gap discharges reduced to a minimum in order to limit the number of service interruptions.*

The characteristics and limitations of several forms of gaps are shown and discussed. A new form of limiting gap is described in which it is possible to change the sparkover characteristic over a considerable range to allow for the differences in time lag and polarity characteristics of station insulation. The advantage of using a limiting gap in multiple with a lightning arrester to provide maximum protection and reduce interruptions to a minimum is discussed.

* * * * *

INTRODUCTION

In the early transmission lines, comparatively little was known about the essential characteristics required in the insulation of transformers and other equipment attached to the line. In addition to this, there was no information as to the nature of transients and the number to which the station equipment would be subjected. It was clearly recognized, however, that lightning could impose stresses for which it would be impossible to provide sufficient insulation to prevent breakdown unless a safety valve were present.

It was only natural that an attempt was made to limit the voltage by the use of a discharge gap or lightning arrester. In many cases there was little difference between the protecting gap and the lightning arrester as to the ability to limit the overvoltage. It was recognized, however, that the operation of a protective gap generally caused an interruption due to the flow of normal frequency current following the high-voltage discharge. The desire to prevent the power outage following a discharge of the gap has led to many schemes for limiting the flow of current in the gap or for interrupting it.

It was recognized that the ideal lightning arrester should have a high discharge rate, small time lag, and should prevent the flow of normal frequency current, thereby giving it a great advantage over a limiting gap which would cause an interruption or material drop in voltage.

Records, as well as operating experience, show that stations may be subjected to a wide variety of transients. Although in practise it is necessary to limit the magnitude of the transients which may cause damage to the equipment, it is very desirable that any limiting device does not function unnecessarily and cause an interruption. This is particularly true where a protecting gap is used which does not have the ability to clear the line or prevent the flow of normal frequency current following discharge. This leads to the use of a limiting gap of relatively small time lag for transients of very high magnitude and steep front, in parallel with an arrester of greater time lag which will function for the many transients of lower magnitude without interrupting service.

FACTORS IN THE PROTECTION OF STATION INSULATION

It is exceedingly difficult to coordinate station insulation for the wide range of transients likely to be imposed, as the flashover characteristics may be quite different as to time lag and polarity, even though in close agreement at normal frequency. This is evident by reference to Fig. 1.

Owing to difference in time lag, a very high overpotential suddenly applied may cause one type of insulator to flash over, whereas a transient having a much lower crest value may cause another insulator to arc over. Furthermore, the difference in breakdown voltage for a negative and a positive transient may be quite large for bushings, bus insulators and the internal insulation of transformers and circuit breakers. A bus insulator or a disconnecting switch may have a high effective flashover under a negative impulse and a much lower flashover under a positive impulse. However, for some other piece of equipment, the reverse may be true. In station insulation it is therefore not only necessary to take into account the time lag in flashover, but the effect of polarity and oscillations as well.

In operation one station may be subject to very severe overpotentials, whereas the transients affecting another station may be made up largely of disturbances of lower magnitude. The transients imposed upon a given station will necessarily depend upon the storms affecting the station or the connecting lines, the number and magnitude varying greatly with the location and the line insulation.

A lightning arrester or a limiting gap which will protect station insulation against overvoltage transients has long been regarded of great economic importance, for it is evident that if a gap or lightning arrester may be depended upon to protect the insulation similar to the safety valve on a boiler, it is possible to lower the level of insulation and thereby effect a very material saving. To be fully effective, however, the limiting device must

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interrupt the discharge when the overvoltage is relieved without interfering with operation.

CHARACTERISTICS OF SEVERAL FORMS OF LIMITING GAPS

Fig. 2 shows the impulse flashover values for a simple type of gap formed by two 5/8 in. rods. Fig. 3 shows the flashover characteristics for a ring gap. Fig. 4 shows the time lag characteristics for a 22-in. horn gap.

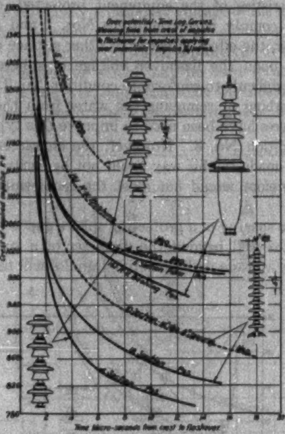

Fig. 1—Overpotential—Time Lag Curves

In considering the several types of gaps shown above, it will be noted that the flashover voltage tends to increase very rapidly for short waves or for short time intervals. Since equipment needs protection most for very severe direct strokes at or in the immediate vicinity of the station, it is evident that many gaps may not afford the desired protection, although having an arcing voltage comparatively low for less severe impulses or those of longer duration. It is also evident that the characteristics of some gaps may be widely different for a positive and a negative impulse. If the gap is to furnish efficient protection it is therefore essential that it have characteristics corresponding to those of the insulation in parallel with it, both with respect to polarity and time lag.

Fig. 5 shows the impulse time lag characteristics for a bushing and several types of limiting gaps. Typical time lag curves for only one polarity are shown in order to avoid confusion.

In order to provide protection for the bushing having the characteristics shown in curve *A*, it will be assumed that the voltage limiting gap should flashover at not more than 90 per cent of *A* for all time lags as indicated by curve *B*. The characteristic curve of the horn gap shown as *C* has approximately the same general shape as *A* for negative impulses but tends to depart somewhat for the shorter time lags. Reference to Fig. 4, however, shows that the difference between positive and negative sparkover voltages is likely to be very large for that type of gap.

By reference to Fig. 5, it will be noted that curve *D* for the ring gap intersects curve *A* at approximately 585 kv. For transients having a crest below 585 kv., the ring gap sparks over first. For transients having crest values above 585 kv., however, the ring gap does not afford protection as the bushing will arc over first. It is therefore evident that where the ring gap is set as high as possible so as to minimize unnecessary interruption for transients of lower crest and longer duration, it will afford little or no protection for severe overvoltages when most needed. To provide protection, the ring spacing and flashover voltage must be materially reduced thereby causing unnecessary interruptions. For example, the minimum flashover voltage shown in Fig. 5 for the ring is approximately 440 kv., as against 520 kv. for the bushing and a further reduction of the flashover voltage will be necessary in order to provide protection for high overpotentials.

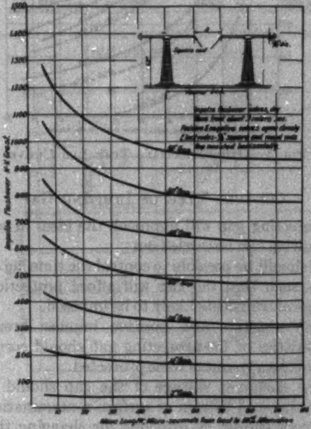

Fig. 2—Impulse Flashover Values, Dry

Curve *E* shows the characteristics of a sphere gap of negligible time lag having the same minimum flashover voltage as the gap of curve *B*. As the time of voltage application is decreased, however, insulators and most equipment will withstand much higher impulses. It is therefore desirable that the impulse flashover of the limiting gap be higher for short time lags. If a limiting gap having the rather uniform flashover characteristics of a sphere gap as shown in curve *E* is used, unnecessary flashovers will result owing to the sparkover of the gap for the shorter time lags where the insulation can well withstand higher voltages. Of course, it is possible to modify materially the characteristic of the sphere gap by changing the relative sizes of the spheres with respect to each other and with respect to the gap spacing. As will be discussed later, this principle is made use of with the control type of gap in order to obtain the characteristic of curve *A′*, Fig. 5, closely approximating the ideal characteristic curve *B*.

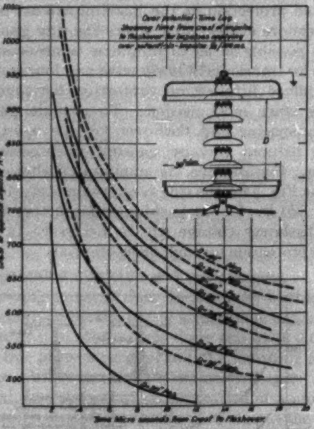

FIG. 3—OVERPOTENTIAL—TIME LAG CURVES

CONTROL TYPE OF LIMITING GAP

A protecting gap will approach the ideal in so far as the following can be carried out:

1. It should be possible to control the time lag characteristics such that the gap will afford protection over the range of transients likely to be imposed.

2. The negative and positive impulse breakdown characteristics of the protecting gap should correspond to that of the apparatus to be protected.

3. The flashover voltage of the gap should not be seriously lowered by rain or surface contaminations.

4. Means should be provided for changing the relative negative and positive flashover voltages where it is desired to work within close limits or to protect apparatus or insulation whose properties are not fully known.

5. It should be possible to change the time lag characteristics without materially affecting the minimum flashover voltage.

6. Where the discharge of the gap will cause a serious service interruption, a fuse or other device should be used for clearing the normal frequency current following discharge.

7. The gap should not discharge under high frequency wave transients of comparatively low magnitude due to switching or arcing grounds.

A gap conforming to the above requirements should have improved characteristics over types in general use in the past and a description is given below of the development of such a gap.

A protecting gap using spheres of proper size in relation to the gap spacing and of unequal diameters to provide for the difference in negative and positive characteristics of the equipment protected, could be used to comply with conditions Nos. 1 and 2 above. However, the effect of water or other surface contaminations upon the flashover voltage of spheres would make it difficult to conform to condition No. 3. Means for absorbing or removing the water from the face of the spheres have been used to improve this type of gap for condition No. 3. However, the characteristics of the gap made in this way could not be easily changed and therefore would not meet the requirements Nos. 4 and 5.

As the spacing of a protecting sphere gap is increased to lengths well beyond the diameter of a sphere, the flashover voltage under positive impulse will tend to be lower than under negative impulse. This can be offset by using a larger sphere for the live terminal and a smaller one for the ground terminal. Where the electrostatic fields of the two terminals of the gap are of equal intensity, the flashover will start from the positive terminal as it takes a lower density to start discharge with a positively charged electrode. It is therefore apparent that by controlling the flux density at the

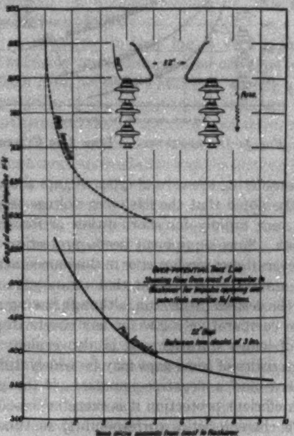

FIG. 4—OVERPOTENTIAL—TIME LAG CURVES

surface of a gap terminal, it is possible to vary the discharge voltage.

This last principle is made use of in the newly developed limiting gap, one form of which is shown on Fig. 5. The control shields shown in the vicinity of the gap terminals can be moved forward or backward to change the field density and regulate not only the time lag but the difference in negative and positive sparkover characteristics.

The results of tests in Fig. 6 show that the relative negative and positive flashover voltage of the protecting gap may be readily controlled by changing the flux density of the arcing tip which is positive. It is evident that any screen or shield may be used to accomplish this. It is however, important that the shield does not cause erratic performance by becoming one of the electrodes so that an arc will be struck between shields or between one of the tips and a shield.

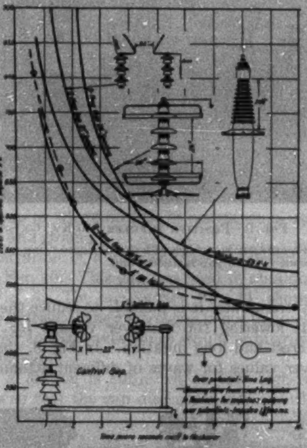

Fig. 5—Overpotential—Time Lag Curves

By using the insulated type of shield, this difficulty is prevented, even though the shields may be nearer together than the discharge points. This is particularly important under wet conditions or where the gap may be subjected to an oscillation, for under these conditions a gap having bare metal shields generally has a very low flashover voltage. Reference to Fig. 6 shows that the control of the flux density of the discharge points by means of the insulated shields is very effective in changing the relative arcing voltage for negative and positive impulse. In addition, it is seen that the arcing voltage under a damped wave or oscillation compares favorably with the arcing voltage under impulse thereby reducing unnecessary discharges.

In any discharge gap in which the electrodes are similar, the electrode or terminal on the live side generally has a much higher voltage gradient than that on the grounded side of the gap. It has been pointed out that increasing the size of this electrode would make it possible to reduce the gradient. Increasing the effective size of one electrode will also tend to increase the stress or lines of force on the electrode connected to the opposite side.

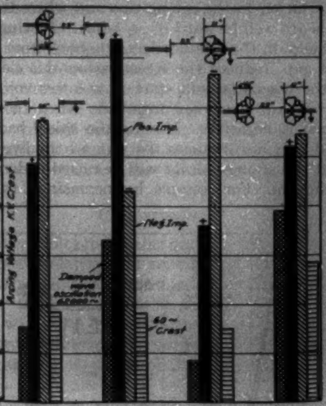

Fig. 6—Effect of Insulated Control Screens Upon Arcing Voltages of 22-In. Gap

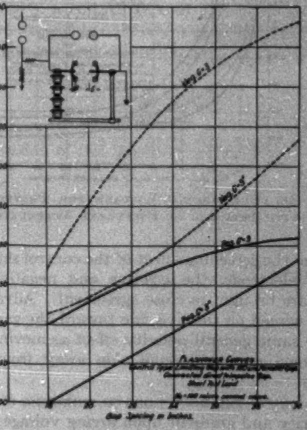

Fig. 7—Flashover Curves

Fig. 7 as well as Fig. 6 shows the change in relative negative and positive impulse sparkover values which may be effected by changing the flux density on the electrodes with insulated control shields. Reference to Fig. 7 shows that the arcing voltage for negative impulse is approximately 75 per cent greater than for positive impulse where the controlled screens have the same relation to the arcing tips and where the control screens are well forward (C = 3 in.). Moving the control shield on the ground electrode back from the point of the gap, reduces the screening on this electrode so that the discharge will start at a lower voltage for a given distance between terminals when the opposite terminal is negative. Moving the shield back on the ground side also reduces the flux on the live terminal so that the arcing voltage will be raised under the positive impulse. For example, by increasing C to 9 in. and reducing the screening effect of the control shield on the ground electrode, the positive and negative impulse values are brought in close agreement. Advancing the control screen on the live side toward the gap opening, has the same general polarity effect as moving the control screen on the ground side away from the gap opening.

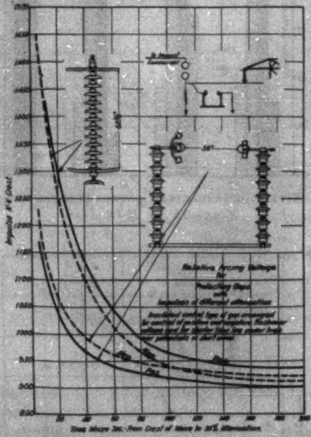

FIG. 8—RELATIVE ARCING VOLTAGE FOR PROTECTING GAPS WITH IMPULSES OF DIFFERENT ATTENUATION

The control gap allows one to make use of the effect of polarity and gradient upon arcing voltage similar to that with the point-to-plate discharge. It is evident that the arcing voltage will be increased for a given gap spacing where the stress upon the arcing tips is reduced by advancing the control shields. Therefore, by using the control shields, it is possible to reduce the gap spacing and the time lag for a given flashover voltage. It necessarily follows that the characteristics of the gap may be changed materially, both as to time lag and the effect of polarity upon discharge voltage by simply adjusting the control shields. As the projecting insulated control screen is affected but little by high frequency transients, it is possible to reduce the time lag without lowering the flashover voltage for waves of lower crest and long duration. In Fig. 8, the impulse flashover values for waves of different attenuation are given for a control type of gap and a ring equipped suspension string.

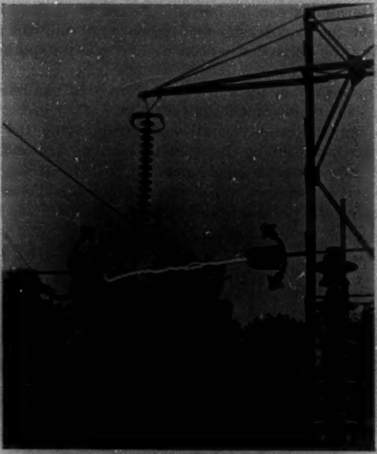

FIG. 9—POSITIVE IMPULSE 950-KV. CREST

Arcing-control gap in multiple with ring-equipped string. Wave ¼ x 100 microseconds. Control gap in artificial rain

FIG. 10—SMALLER TYPE OF CONTROL GAP

The control type of gap may be made in a number of different forms with plain shields or rings used in place of the insulated control type of screen. The latter has the advantage in that the gap does not have a low flashover value for oscillations and is affected but little by rain or conditions which may cause a gap having bare metal shield to be very erratic. Other forms of the gaps embodying the above principles may, in some cases, work out to much better advantage.

Fig. 9 shows the gap of Fig. 8 under test. The insulated support of the ground terminal may be replaced by a metal support where the change in field will not disturb the gap or where no fuse or resistance is used between gap and ground.

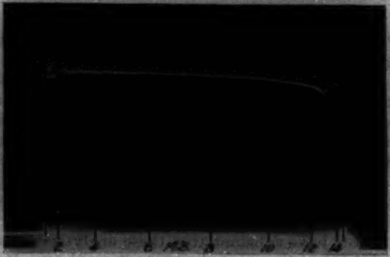

FIG. 11—NEGATIVE IMPULSE 460-KV. 1¼ x 100 WAVE ARCING 22-IN. CONTROL TYPE PROTECTING GAP

An illustration of a smaller type of control gap is shown in Fig. 10. This gap was used in obtaining the data in Fig. 5. The oscillogram, Fig. 11, shows that the arcing voltage of the control gap is not unnecessarily low even for longer waves which would ordinarily arc a horn or ring gap.

Where a station is subjected to few high-voltage transients so that the gap discharges infrequently, the clearances and setting of the gap may be of little importance, provided satisfactory protection is obtained. However, where the station must withstand many transients, it may be desirable to adjust the gap to take all discharges and, thereby, protect the equipment fully. This can only be accomplished by trial in many cases. It is therefore necessary to be able to change the sparkover characteristic of the gap.

Where the limiting gap must operate frequently some form of clearing device may be added, such as a magazine fuse or circuit breaker. In general attention should be given to the use of a lightning arrester for limiting transients without causing an interruption, using the limiting gap only where necessary to afford absolute protection or where the expense of an arrester is not warranted.

CONCLUSION

1. An ideal voltage limiting gap for station use is one wherein both polarity and time lag characteristics are capable of adjustment over a sufficient range to allow protection of all apparatus in parallel with it for any transient wave form imposed upon the station.

2. A form of limiting gap has been developed making use of adjustable electrostatic field controls for the gap electrodes. This allows the breakdown characteristics of the gap, both as to time lag and arcing voltage for positive and negative impressed waves, to be varied over a wide range so as to conform to the characteristics of the station insulation.

3. The use of a control type of gap for limiting the most severe transients only and a lightning arrester for the remainder will, undoubtedly, provide the best station protection and operation available at present.

Discussion

For discussion of this paper see page 697.

Suspension Insulator Assemblies

Their Design and Economic Selection

BY J. J. TOROK[*] and C. G. ARCHIBALD[†]
Associate, A.I.E.E. Non-member

***Synopsis.**—This paper presents a summary of data from extensive impulse and 60-cycle laboratory flashover tests upon suspension insulator strings relative to unit shell diameter and unit spacing. The insulating qualities and the economic advantages of the various sizes of units are discussed with due regard to the most recent lightning protection theories. Both steel tower and wood pole high-voltage transmission line insulation are given consideration from the electrotechnical and economic standpoint.*

INTRODUCTION

THE pioneer high-voltage transmission lines were insulated to the extent deemed sufficient for the normal operating voltage. The designers based their selections upon the experience gained from operating lower voltage lines. The performance of these lines is responsible to a large extent for the arbitrary quantities of insulation provided for various voltage classes. However, the average operating record of these higher voltage lines was not, in general, satisfactory. A cursory analysis of the faults and their causes showed that the majority were caused by lightning.

The formation of the majority of the lightning surges was explained at that time by the "induced voltage theory." The possibility of strokes terminating upon conductors was admitted but it was believed that these were infrequent and impossible to protect against. Several methods were tried in an attempt to reduce the lightning disturbances. At first increasing the number of suspension insulators, or "over-insulation" was tried. The resulting improvement in performance was inappreciable and not commensurate with the increase in insulation. At the same time ground wires were installed as a possible remedy. In many of these latter cases material improvement in reduction of line outages was noticed but in others no great improvement resulted. A satisfactory explanation of these apparent discrepancies did not appear until the "direct stroke" theory was proposed.[1] Following this, extensive field tests[2] were made, in which the effectiveness of ground wires and tower footing resistances were determined. Subsequent analytical studies[3-4] have disclosed the wave form and magnitude of lightning voltages which may appear across the line insulation.

With the extensive and detailed knowledge of the voltage for which a high-voltage transmission line must be insulated, the designer is now able to make a more logical choice of insulation rather than rely upon tradition. However, there are economic influences as well as electrical characteristics to be considered in the design of high-voltage lines.

STANDARDS FOR COMPARISONS

Past and present papers dealing with studies of lightning phenomena have brought us to a point where we can make a logical approach to the selection of sizes of units and their spacings in suspension insulator assemblies. In a paper presented by Messrs. Fortescue and Conwell[3] and a discussion[4] on this paper, it was shown that on a transmission line equipped with ground wires, the reflections from tower footings altered the wave form so that the resultant voltage across the insulators had the approximate shape of the wave shown in Fig. 1A. Field tests have substantiated these analytical results[4] as can be seen by comparing Figs. 1A and 1B. If the insulators are to be subjected chiefly to this type of wave it is essential to determine how the insulators will perform under these types of waves.

During the early search for the proper amount of insulation for high-voltage transmission lines, various sizes of suspension insulator units, as well as different numbers of units per string, were used over a period of a decade or more. At the present time, operating and construction companies continue to insulate their new lines with the same size units previously employed, partly because of the advantages of interchangeability and partly due to acquiescence to a standard established some years ago before the requirements for the insulation of lines against lightning were understood. In the past the actual advantage of one size over another had not been definitely established despite the multitude of sizes operating more or less satisfactorily under a wide range of conditions. Consequently, an extensive group of laboratory tests was arranged to study the effects shell diameter and unit spacing have upon the electrical and economic characteristics of suspension insulator string assemblies applicable to high-voltage service.

To make an investigation of the characteristics of insulators comprehensive, all tests should be made on a volt-time basis, that is, waves of long duration should be applied to the insulators and the magnitude varied so that flashover will take place in time lags ranging from 16 or more microseconds down to one or two. The performance of the insulation should then be judged according to its application. If it is to be used on ground wire installations the performance of the insulation should be judged by the flashover voltage at short periods of time. If it is to be used on lines not equipped

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1. For references see Bibliography.

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with ground wires its performance should be judged by long time lags as well as short. However, the majority of the transmission lines now being planned and constructed are to be equipped with ground wires. Therefore, the emphasis of this analysis will be placed on short time flashovers.

LABORATORY TESTS

Tests Establishing Comparison Basis. Laboratory tests were made upon suspension insulator assemblies and bushings with the voltage wave shown by the oscillogram of Fig. 2. This voltage wave very closely resembles the calculated curve shown in Fig. 1A. These tests were conducted to determine the relation between flashovers under this type of wave and those given by standard volt-time curves.

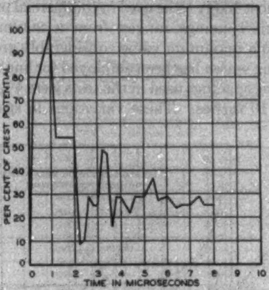

FIG. 1A—CALCULATED WAVE SHAPE

The wave shown in Fig. 2 was obtained by shunting the discharge circuit of a surge generator, arranged for the production of a smooth 1½-40 microsecond positive wave, with a needle gap having a resistance in series to ground. The needle gap flashing over near the crest of the wave results in an abrupt drop in potential across the insulators under test. However, the resistance in series with the gap limits the voltage to approximately 37 per cent of the crest voltage. Thereafter the voltage is reduced to a negligible value in five to six microseconds because of the energy dissipated by the resistor. In this manner, the wave shown in Fig. 1B was simulated.

The following crest voltages are the minimum values causing flashover of insulator assemblies made up of 8, 10, 12, and 14 units, 10-in. diameter and 5¾-in. spacing, when subjected to this type voltage wave.

No. units	Crest voltage (kv.)
8	1,050
10	1,240
12	1,550
14	1,800

It can be seen from volt-time curves shown in Fig. 5 that these voltages correspond to flashovers at some time lag lying between one and two microseconds on 1½-40 positive wave volt-time curves. The minimum flashover voltage decreases and corresponds to longer time lags when longer duration test waves are used. It has been shown in previous papers[4] that longer waves than shown in Fig. 1A result when tower footing resistances are high. These papers showed that effective ground wire protection can only be had with tower footing resistances below 10 ohms. The voltages appearing across the insulators with higher tower footing resistances become so high that it is uneconomical to insulate against even moderate intensity strokes. Because of these reasons, the comparison of insulators for use upon lines properly equipped with ground wires should be made at two microseconds as taken from volt-time curves.

It is realized that special field conditions may require a different wave upon which to base analyses; consequently, the insulator characteristic tests disclosed in this paper were made with a 1½-40 microsecond positive wave. The wave magnitude was varied to give time lag of flashover ranging from one to twenty microseconds. These results were then plotted in the form of volt-time curves. Insulator performance can be ascertained from these volt-time curves for any predetermined wave shape. In the event that some peculiar local condition necessitates the choice of an irregular wave as a basis of test, it is only necessary to test one combination of insulators with this wave. The flashover voltage values then may be referred to the volt-time curve to ascertain the corresponding time lag. The comparison of various type insulators can be made at the corresponding time lag from their respective volt-time curves.

FIG. 1B—ACTUAL WAVE SHAPE FROM FIELD TESTS

If the high-voltage transmission line is not protected by ground wires or if the ground wire protection is inadequate, then the voltage wave form across the insulators is not attended by reflections from tower footings; therefore, the wave shape approximates that of the lightning discharge. The wave form in this case is of longer duration, ranging from 10 to 50 microseconds. Since the high waves will invariably cause flashover at short time lags the impulse strength under these conditions is not important and therefore insulator characteristics should be compared under long time conditions rather than under a wave form such as shown in Fig. 1B.

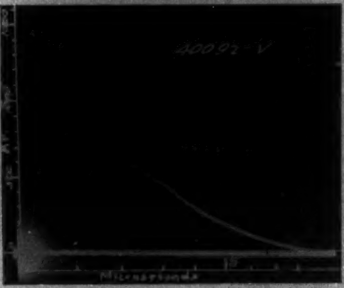

Fig. 2—Laboratory Test Wave

Diameter and Spacing Tests. Suspension insulators of the cemented cap and pin type were made up in a variety of shell diameters and unit spacings at the Westinghouse Porcelain Insulator Plant for pursuance of a series of tests to determine the effects these dimensions have upon the flashover characteristics of suspension insulator assemblies. The porcelain shells were standard in head, petticoat, and shed droop design except a special 10½-in. diameter unit designed for coastal duty. This unit had 10 per cent greater shed droop than the standard design. The heads of all these units were the same size such that all hardware was identical. This head size was that of a standard strength unit now most commonly employed upon high-voltage lines. The fixation of these details in dimension and shape limited as much as possible any variations in electrical characteristics other than those due to the unit diameters and spacings. The specific shell diameters made up for test purposes were 7, 8½, 10, 10½, 12, and 14 inches. The detail construction of the various size units is shown in Fig. 3. The design of the porcelain shells is based upon the requirements of practical application. These designs of porcelain shells are either those in common use or those most likely to be used in commercial production.

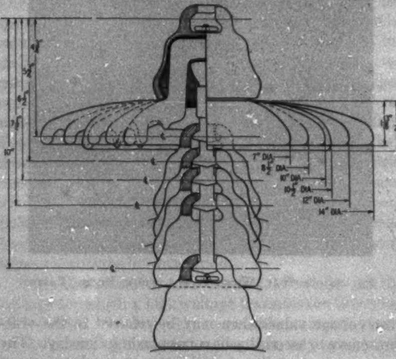

Fig. 3—Illustration of Insulators Tested

The unit spacings ranged from a minimum value possible to a maximum value considerably greater than any spacing which has been employed for line insulation. The specific spacings used in the tests were 4¾, 5¾, 6½, 7½, and 10 inches. These variations in spacing were obtained by pins of various lengths being screwed into a threaded socket provided on the stud. This arrangement materially reduced the number of units required for the tests and afforded more comparable data because identical shells of each diameter were tested at all spacings.

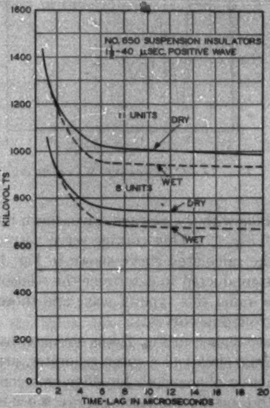

Fig. 4—Impulse Tests on Wet and Dry Insulators

Conventional 60-cycle dry and wet flashover tests were performed upon strings of 4, 8, 12, and 16 units of each dimension combination. That is, the units in each string assembly were all of the same diameter and the same spacing. No non-uniform string assemblies were tested because these conglomerations have been found to be very impractical line assemblies. These tests were made in accordance to A.I.E.E. Standard No. 41 pertaining to the conduct of normal frequency flashover tests.

Impulse flashover tests were made upon the same string assemblies with a 1½-40 microsecond lightning wave of positive polarity. A cathode ray oscillogram was made of each impulse flashover for accurate record. Over 700 oscillograph records were obtained in these tests. These data were corrected for atmospheric temperature and density, but no correction was made for the recorded relative humidity which ranged between 10 and 15 per cent. Correction factors for humidity have not been established. The question of humidity correction factors is at present before the Lightning and Insulator Subcommittee of the A.I.E.E. It is the opinion of the authors that the variation in impulse flashover voltage caused by changes in humidity during the tests is negligible. This opinion is based upon wet and dry impulse flashover tests. The average variation in time lag curves caused by standard rainfall is illustrated in Fig. 4. These curves were plotted from more than 25 points each. It can be seen that the wet and dry curves practically coincide for values less than two microseconds, and that the effects of moisture upon impulse flashover become appreciable only for time lags of three or more microseconds.

A portion of the data obtained from the impulse tests on various insulators is given in the replotted curves shown in Fig. 5. These curves were obtained from time lag curves, each of which was plotted from an average of 13 points. Some combinations of unit dimensions were not tested under the impulse wave as the test would become too long. The omitted combinations were so selected that they could be readily interpolated from the other tests. A basis for this consideration may be readily seen in Figs. 6, 7 and 8. A large number of tests upon the 14-in. diameter unit was omitted because these units were found to be impractical with present service requirements and manufacturing knowledge.

It is to be remembered that in making high-voltage measurements it is difficult to keep the limit of error

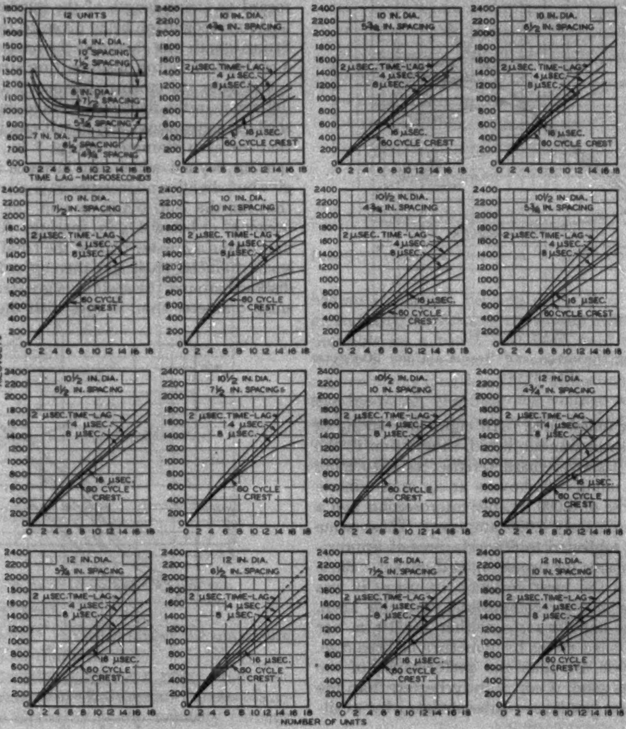

Fig. 5—Flashover Characteristics of Suspension Insulators

below about 5 per cent, even though great care is exercised. For example, on the curve showing an insulator string to have a flashover of 1,000 kv., the flashover may, under some conditions, be 1,025 or 975 kv.

The units found to be most effective under impulse tests were later subjected to arc tests to check the behavior of these diameter shells under high current impulse and power follow arcs. An eight-unit string of 12-in. diameter shells was subjected, in addition to the normal impulse flashover characteristic tests, to more than 100 severe impulse flashovers at an approximate rate of voltage rise of 6,000 kv. per microsecond and a current of approximately 5,000 amperes. These same units were then subjected to 60-cycle arcs of approximately 3,000 amperes for two seconds duration. Since the 10-in. unit is the most common in usage it was chosen as a reference for comparison with the 12-in. unit. Under the impulse tests neither the 10-in. nor the 12-in. units failed either by puncture or breakage. Under the 60-cycle arc tests the performance of the 12-in. unit was as good, if not slightly better, than the 10-in. unit. These results are substantiated by available service data.

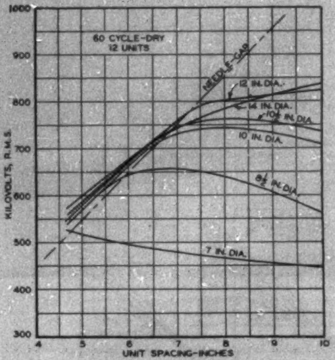

FIG. 6—EFFECT OF DIAMETER AND SPACING UPON 60-CYCLE FLASHOVER

DIAMETER AND SPACING

Impulse. The effect of unit shell diameter and unit spacing upon the flashover characteristics of suspension insulator strings is shown by representative data in Figs. 7 to 11. The economic characteristics of these factors are shown in Fig. 12. The data shown in these figures are representative in that they conform to average steel tower line conditions. A two-microsecond time lag basis of comparison is used because that value of time lag corresponds to a tower footing condition of 10 ohms or less. Higher tower footing resistances permit the lightning voltages to rise well above any economic steel tower insulation.

Figs. 6 to 10 show flashover characteristics of string assemblies consisting of 12 units. A string of 12 units was arbitrarily chosen to give a representative example of the characteristics of an insulator string with reference to shell diameter and unit spacing.

Figs. 11 and 12 show impulse flashover and economic characteristics of the various unit combinations with regard to string length of the insulator assembly itself without recognition being given to the number of units. String length was chosen as the basis of reference as it is a controlling factor in supporting structure dimensions.

60 Cycles. At normal frequency there is but slight difference in the flashover voltage due to an increase in shell diameter above 8½ in. It can be seen from Fig. 6 that for each diameter, except 7 in., there is a definite spacing beyond which any increase in the spacing will give but slight or no increase in flashover. It is probable that for the 7-in. diameter unit the knee of the curve occurs at or below 4¾-in. spacing. The head size of the unit forms a greater part of the total diameter with the small diameter shell and thereby reduces the flashover distance around the shell by a greater proportion at the small diameter than it does with the larger diameters.

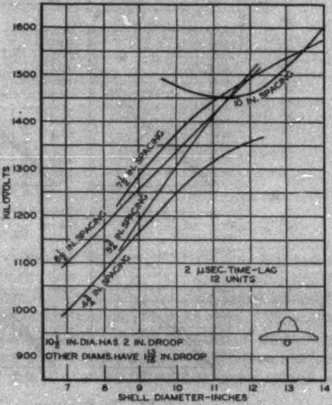

FIG. 7—EFFECT OF DIAMETER UPON FLASHOVER

It is to be noted from Fig. 6 that if the ratio of unit spacing to shell diameter is increased beyond 50 to 65 per cent, the flashover voltage falls below that of an equally spaced needle gap.

Wet flashover characteristics are of similar shape and relative position, being 60 to 75 per cent in voltage value.

It appears from Fig. 6 that at approximately 6½-in. unit spacing the 60-cycle flashover voltage will be about the same for all diameters from 10 to 14 in. Since the cost of larger diameter units is considerably higher, varying as some function of the diameter squared, it appears that large diameter units are not economical from a purely normal frequency standpoint. However, high-voltage insulation is no longer entirely judged upon its characteristics at normal frequency. The major consideration is now its behavior under impulse voltages.

Insulator Shields. The tests reported in this paper were made without auxiliary electrodes such as rings and horns. The effect of such devices has already been investigated.[11]

Cost Considerations. The designer is confronted by two major considerations in the selection of high-voltage transmission insulators. First, the permissible cost of line insulation. Second, the selection of the maximum insulation available for that cost

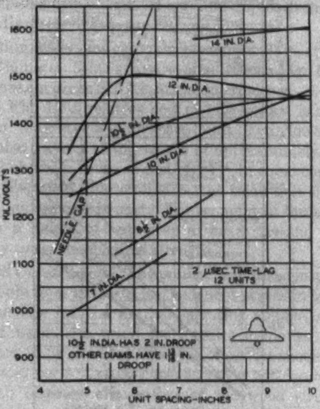

Fig. 8—Effect of Spacing Upon Flashover

The first consideration comprises many variables, such as the importance of the circuit, the necessary reliability, and the character of the territory through which the lines pass. All these factors are characteristic of each installation, and require individual study of the combination peculiar to each line. The second consideration entails the selection of the proper insulator assemblies.

Steel Construction. It must be premised that a steel tower line be properly equipped with ground wires to provide lightning protection because it is not economically feasible to protect a line by over insulating.

Once the permissible cost of insulation has been decided upon, the economic suspension insulator unit can be selected from examination of insulation and cost characteristics similar to those shown in Figs. 11 and 12.

Fig. 11 shows the impulse flashover characteristics with respect to string length of various assemblies of 10-in. and 12-in. diameter shells. These flashover voltages were taken at a two microsecond time lag for reasons described previously. It is to be noted that for a given unit spacing and flashover voltage strings comprised of 12-in. shells are appreciably shorter.

As has been pointed out previously, the string length has an important bearing upon tower design. For a given flashover voltage, the shorter string length is more desirable on a tower cost basis as it may reduce the length of the arm and the height of the tower, or, with a given string length and the utilization of a more effective insulator, the flashover voltage will be increased and the number of outages due to lightning will be decreased. For example, in Fig. 11, a 1,500-kv. flashover at two microseconds time lag is provided by a string length approximately 65 in. when composed of 12-in. diameter units spaced 4¾ in. and by a string length approximately 91 in. when composed of 10-in. diameter units spaced at 6½ in. A difference of 26 in. exists between the length of these strings. A steel tower design having the same tower clearances and height required by the long string would, of course, be appreciably heavier and more expensive than a tower designed for the short string. Using a string length of 91 in., the twelve 4¾-in. units have a flashover of 2,050 kv. as

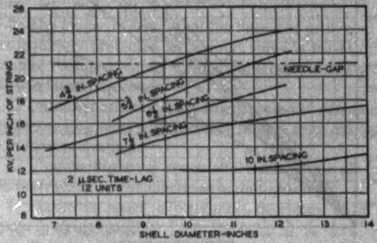

Fig. 9—Effect of Diameter Upon Flashover Efficiency

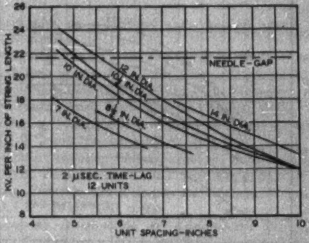

Fig. 10—Effect of Spacing Upon Flashover Efficiency

against 1,500 kv. for the ten 6½ units, or an increase of 550 kv. by using the more efficient combination of diameter and spacing. These factors should exert considerable influence upon the choice of properly dimensioned suspension units.

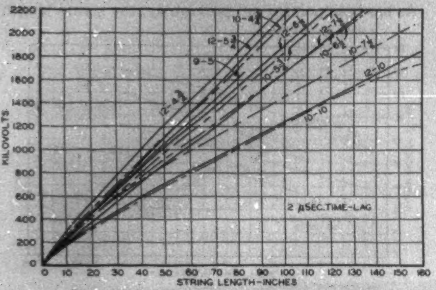

Fig. 11—Flashover Characteristics with Respect to String Length

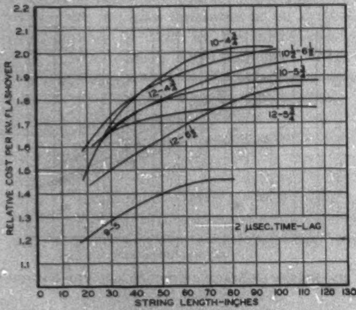

Fig. 12—Relative Insulation Cost of Various Sizes of Units

Economic Selection. The two-microsecond time lag flashover characteristics of Fig. 11 were incorporated with the cost of units to provide the economic comparison curves of Fig. 12. The cost values employed were those to a purchaser with packing and shipping expenses allowed to a shipping point 500 miles from the factory. This price is based upon the physical dimensions of the unit. The factors determining the price are, head size, shell size, and total volume, as given by the maximum dimensions of the porcelain shell. The head size determines the hardware cost factor. The shell size determines the amount of plastic material and consequently the size of moulds, etc., used in manufacturing operations. The total volume indicates the necessary kiln volume for firing, type of packing or crating, and space for storage and shipping. Fig. 12 discloses that up to approximately 70 in. string length, a 12-in. diameter unit spaced 6½ in. provides two microseconds insulation at the least insulator cost. This point corresponds roughly to the amount of insulation considered as economically balanced for 138-kv. transmission upon steel towers adequately protected by ground wires.

Beyond a string length of 70 in., a 12-in. diameter unit spaced 5¾ in. shows the lowest insulation cost. Ten-in. diameter units spaced at 4¾ or 5¾ in. are those dimension combinations most commonly used at the present time. However, these units show much higher relative costs than 12-in. diameter units at the same spacing for string lengths above 30 in. Incidentally, a string length of 30 in. corresponds roughly to economically balanced insulation commonly employed for 69 kv. Examination of these flashover and cost curves clearly indicates a wide range of selection open to a high-voltage transmission line designer for choice of the proper insulation.

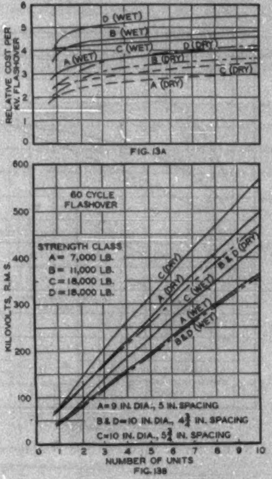

Fig. 13—Flashover and Cost Characteristics at 60 Cycles

It must be noted that Figs. 11 and 12 must be employed simultaneously to determine the maximum insulation for the least cost. For example; at approximately 65-in. string length, a 12-in. diameter 5¾-in. spaced unit provides 1,500 kv. flashover at a relative cost of 1.76, or an arbitrary total cost of 2,640, whereas a 10-in. diameter 6½-in. spaced unit to provide 1,500 kv. flashover must have a string length of approximately 91 in. This gives an arbitrary total cost of 1.93 times 1,500 which is 2,900, approximately 10 per cent higher insulation cost with 32 per cent undesirable increase in string length.

Very low cost units have been proposed for light duty purposes such as wood construction and lower voltage transmission lines. The economic characteristics of such a unit are shown by the 9-in. diameter, 5-in. spacing curve in Fig. 12. Obviously, for light duty service requiring insulation amounts corresponding to 60-in. string length (approximately 138 kv.) such units are economically desirable.

These unit dimensions were predicated upon the electrical and economic characteristics disclosed in these studies. The electrical characteristics compare favorably with those of much larger and more costly units as shown in Figs. 12 and 13. Impulse flashover string length characteristics of this unit are higher than a 12-in. diameter 6½-in. spaced unit and approach that of a 10-in. diameter 4¾-in. spaced unit.

Economically, for example, presume a 40-in. string of 9-in. diameter 5-in. spaced units. From Fig. 11 this combination gives 875 kv. at two microseconds time lag, and in Fig. 12 a relative cost of 1.35 per kv. is shown. A 12-in. diameter 5¾-in. spaced unit to provide 875 kv. must also have a string length of approximately 40 in. Fig. 12 shows a relative cost of 1.7. In equal comparison of electrical characteristics, the small unit shows a 20 per cent saving in cost. The economy of this smaller unit is a result of reduction in head size and consequently lower mechanical strength.

Wood Construction. The present practise on insulating wood pole lines is to use just sufficient porcelain to insulate the line for the normal operating voltage, and to depend upon the wood for impulse insulation. The insulator to be used for this type of service should have a low cost per kv. of 60-cycle flashover and mechanical strength slightly above that of the wood structure. Calculations of ultimate strengths of wooden structures now in use show that a unit having a mechanical strength of 7,000 lb. would be adequate. Fig. 13 shows the 60-cycle electrical and economic relationship of the various units most applicable to wood construction. The light duty unit cost is considerably below the other units, while the 60-cycle flashover per unit is only slightly below that of the popularly dimensioned unit of 10-in. diameter and 5¾-in. spacing.

CONCLUSIONS

1. The major factor of steel tower transmission line design is the shielding of the power conductors with properly located ground wires and maintaining the tower footing resistance as low as possible.

2. For towers of low footing resistance (on the order of 20 ohms) the wave is of short duration such that the minimum flashover voltage of the attached insulation corresponds to its breakdown voltage at two microseconds with a flat topped wave.

3. Suspension insulators should be judged upon their impulse flashover characteristics at two microseconds in applying insulation to well protected high-voltage lines.

4. The choice of insulation for lines not protected or inadequately protected by ground wires is based upon impulse flashover values at long time lags, of the order of 10 to 50 microseconds.

5. Impulse flashover space effectiveness of suspension insulators improves with increase in unit shell diameter at moderate spacings.

6. Impulse flashover space effectiveness for all unit diameters falls off with increase in unit spacing.

7. Flashover space effectiveness decreases with increase in string length.

8. The cost for a given impulse flashover voltage is generally lower for larger diameter units and is generally higher for larger unit spacings.

9. Available service data and laboratory tests show that the 12-in. unit is as serviceable as a 10-in. unit.

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Discussion

For discussion of this paper see page 697.

Normal Frequency Arcover Values of Insulators
as Affected by Size and Humidity

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INTRODUCTION

DATA covering the electrical characteristics of comparable insulators as received from different sources are frequently conflicting. A study has been made to establish rules whereby such characteristics can be estimated from the dimensions of the insulators with a reasonable degree of certainty. A long series of tests has been made covering several months with repeated checks upon the same insulator units.

The variation of dry arcovers due to the influence of humidity changes also is considered with approximate correction ratios.

The equipment used for these tests in the High Voltage Laboratory of the Locke Insulator Corporation at Baltimore consists of three transformers in cascade to give 60-cycle voltages up to 1,050,000 r.m.s. Voltages less than 350,000 were obtained by using only one of the three transformers shown in the accompanying illustration. One meter spheres were used as the primary standards for voltmeter calibration for subsequent observations. (A.I.E.E. Standard 4-53 to 57 inclusive.) The laboratory which is fully enclosed, can be kept at any desired humidity and temperature above that of the outside atmosphere. Special equipment is provided for this purpose. Routine tests upon insulators during cold weather were made at standard conditions, 25 deg. cent., (77 deg. fahr.), 65 per cent relative humidity; 0.608 in. mercury water vapor pressure. (A.I.E.E. Standard 410-100,—150). No air density corrections were made. (Standards 41-60 footnote.) Frequent calibrations from the spheres assured that insulators and sphere gaps, which are nearly equally affected by changes in air density, were under the same conditions of test. Thus insulator values were automatically referred to standard air density conditions.

TESTING EQUIPMENT—HIGH-VOLTAGE LABORATORY

Each recorded flashover measurement is the average of at least five successive arcover observations. Considerable variation was frequently noted.

TYPICAL OBSERVATION

Insulator	Pin type 35 kv. rating	Suspension 8–10 x 4¾
Total number observations	8.0	10.0
Minimum voltmeter reading	103.0	255.0
Maximum voltmeter reading	114.0	276.0
Average voltmeter reading	108.0	271.0
Per cent variation	11.0	8.2
Arcover value, kilovolts	111.0	428.0
Approximate transformer ratio	1,000 to 1	500 to 1
Barometer, inches mercury	30.14	30.01
Temperature, dry bulb	78.5 fahr.	80.0
Temperature, wet bulb	68 fahr.	68.0
Water vapor pressure	0.572	0.556

SUSPENSION INSULATORS

Tests have been made upon suspension insulators of various spacings and diameters in the vertical position. (A.I.E.E. Standard 41-150, 151.) Insulators tested are:

1. Standard strength cemented 10 in. diameter, 4¾ in., 5⅛ in., 5¾ in. spacing.
2. High strength cemented 10 in. diameter 5¾ in. spacing.
3. Extra high strength cemented 12 in. diameter 7 in. spacing.
4. Hewlett insulators 10½ in. diameter 5½ in. spacing.

The 60-cycle dry and wet flashover values of 10-in. cemented type suspension insulators of all standard spacings are shown on Fig. 1, plotted against actual arcing distance, the shortest air path between electrodes. All dry values regardless of the insulator spacing, fall upon one graph within expected errors of observation. The wet values are affected slightly by design.

The characteristics of the high strength and extra high strength insulators are shown in Fig. 2. The graph for dry measurements is identical with that for the standard insulators, Fig. 1. The wet values again show variations due to design.

The characteristics of Hewlett insulators are shown in Fig. 3. Again the graph for dry values is the same as that of Fig. 1.

Dry arcover values, therefore, for all standard insulators depend solely upon the minimum arc length or tight string distance about the insulators. The wet values

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*Locke Insulator Corp., Baltimore, Maryland.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wisconsin, March 14-16, 1932.

are affected by design. In general, insulators with the greatest clear arcing distance between shells show the highest wet values. The remarkable field performance of insulators with fish tail shaped hoods even in dirty locations suggests that insulators should not be judged as much by wet values as by dry. The dry arcover values are measurably affected by the proximity of grounded objects in the neighborhood of the insulator string. This is shown by the double graph at the upper end of the dry curves. The upper graph was obtained with clearances at least twice the length of the insulator string.

The wet values of Figs. 1, 2, and 3 are plotted as a function of the overall arc length. The most usual design basis is as a function of the wet striking distance or the summation of the air gap distances beneath the insulator assuming the wetted surfaces above are short-circuited out by the water film. A more satisfactory check is so obtained as shown on Fig. 4.

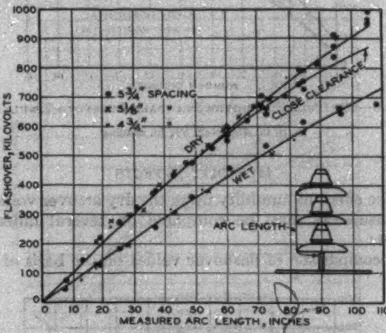

Fig. 1—60-Cycle Flashover Standard Strength Insulators 10 In. Diameter

Errors of individual observations and minor differences due to slight variations in manufacture are removed by obtaining insulator data from the above curves. Expected dry and wet arcover values of various standard and high strength insulators are given on Figs. 5, 6, and 7, plotted as a function of the number of units in the string, calculated from Figs. 1, 2, and 3.

Standard and high strength cemented insulators were tested also in the anchor position at an angle of about 10 degrees from the horizontal. The dry and wet values obtained also are shown upon Figs. 5, 6, and 7 in comparison with the arcover values for the vertical suspension insulators. The dry values are materially lower than for the insulators in the vertical. Wet arcover values are much higher in the horizontal than in the vertical. When the entire surface of the insulator is uniformily wetted a better voltage division between insulator units is obtained with the resultant higher voltage necessary to cause surface breakdown. The wet values in the horizontal are actually equal to, or in some cases with long strings, higher than the dry values in the horizontal.

Most of the tests in the horizontal were made without a jumper loop under the string or a conductor at right angles to it. Insulators in the horizontal position with the conductor at right angles showed dry values materially higher than without it, yet not equal to the values obtained for the vertical suspension insulators.

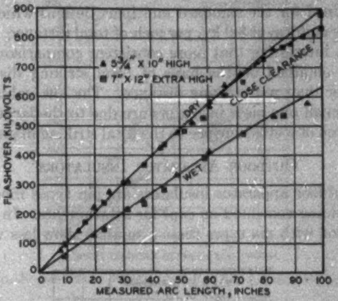

Fig. 2—60-Cycle Flashover High Strength Insulators

Pin Type (Transmission) Insulators

Standard pin type insulators show fairly uniform arcover values in proportion to the distance between electrodes as shown upon Fig. 8. The small contact area of tie wires upon the porcelain causes heavy dielectric flux concentration upon the tie wires with resultant active corona streamer formation at voltages near arcover. Observed data show least uniformity of any of the insulator types because of the erratic action of these streamers. The results of this are shown in the above tabula-

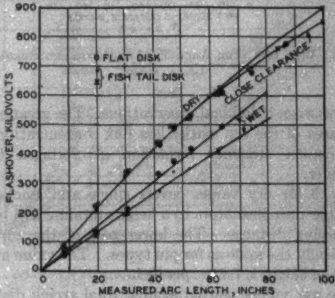

Fig. 3—60-Cycle Flashover Hewlett Suspension Insulators

tion of observed data and may be seen later on in the humidity studies. The dry flashover voltage per inch varies from 14 kv. for the small one piece and the smallest multipart insulators. It thence drops at a fairly uniform rate to 11.4 kv. for the largest multipart insulators nominally rated at 70 kv. For the single piece insulators the wet arcover value is uniformly 7 kv. per inch of total striking distance. A higher value is obtained for the standard multipart designs which are quite uniform at 8.9 kv. per inch of total distance. It is most interesting that more consistent comparisons are here obtained on the basis of total striking distance rather than wet striking distance. The low values for the small one piece insulators are due to the large proportion of wetted surface to the total striking distance.

OUTDOOR APPARATUS INSULATORS

Outdoor apparatus insulators are pin types modified by adding cemented-on caps for attachments. In comparison with pin types these insulators show less variation between individual observations. The greater contact area of the cap causes less flux concentration and less corona streamer formation at voltages below flashover.

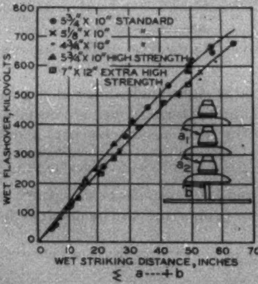

FIG. 4—WET 60-CYCLE ARCOVER ALL-CEMENTED SUSPENSION INSULATORS

Fig. 9 for dry values covers single and stacked or multiple unit insulators to a height totaling 116 inches dry arcing distance. The lower part of the curve is practically the same as for pin types. The value at 100 inches of height is 7.3 kv. per inch.

The wet values plotted against total arcing distance show considerable variation but less when plotted against the wet striking distance. Both graphs are shown on Fig. 9.

EQUIPMENT BUSHINGS, SOLID TYPE

The performance of porcelain equipment bushings (not oil filled) is shown in Fig. 10, for ratings from 7.5 to 69 kv. The field established about the bushing determines the dry values. It is important that the conductor be in place inside the insulator as it will be in service. Much higher values will be obtained without it. The wet values, however, are the breakdown voltages for the water film upon the surfaces. For the larger bushings the wet value is higher than the dry.

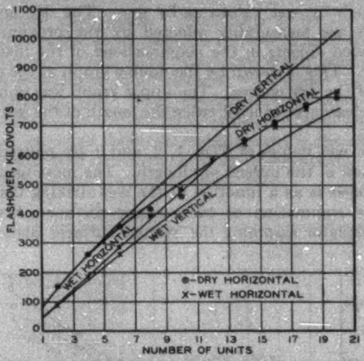

FIG. 5—60-CYCLE FLASHOVER STANDARD STRENGTH INSULATORS

10 in. diameter, 5¾ in. spacing

HUMIDITY EFFECTS

The effect of humidity upon the dry arcover voltages of insulators may be compared on several different bases:

A comparison of flashover values on the basis of per cent humidity is not satisfactory, for it alone does not account for the total quantity of water in the air.

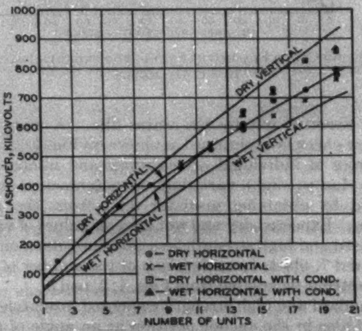

FIG. 6—60-CYCLE FLASHOVER STANDARD STRENGTH INSULATORS

10 in. diameter, 5⅛ in. spacing

A comparison on the basis of absolute humidity or mass of water per unit of air volume may be satisfactory, but does not account for the molecular activity of the water.

A comparison of flashover may be made on the basis of water vapor pressure in some convenient units such as the barometric method, inches of mercury. This not only accounts for the number of water molecules present, but their velocity as well.

The water vapor pressure as the standard is recommended by A.I.E.E. Standards (41-100). All observations in the Locke Laboratory have been compared upon this basis.

No difficulty has been experienced in maintaining the laboratory at standard temperature (25 deg. cent.—77 deg. fahr.), during the cold months. The laboratory is not equipped to reduce the temperature below that of the outside atmosphere. All tests made during the winter are at the standard temperature and with humidity adjusted as desired. In warm weather tests were made at the outdoor temperature and humidity sometimes as high as 95 deg. fahr., and 85 per cent relative humidity.

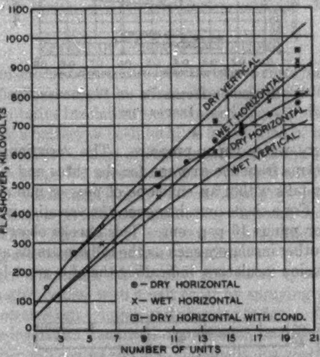

FIG. 7—60-CYCLE FLASHOVER HIGH STRENGTH INSULATORS

10 in. diameter, 5¾ in. spacing

The effect of humidity upon the dry flashover of standard suspension insulators is shown in Fig. 11. Average dry flashover curves have been drawn for strings of various numbers of units. By plotting upon semi-logarithmic paper equal ordinate distances give equal percentage variations. It is thus easy to compare percentage corrections for different strings and percentage variations from the average of individual observations.

The slope of all these average graphs is practically the same. When referred to 100 per cent at the standard water vapor pressure, 0.608 inches mercury the correction at 0.2 inches mercury is − 8 per cent. The correction, therefore, is approximately 2 per cent for each 0.1 inch of mercury variation of water vapor pressure.

The points of observation would be enclosed within an envelope of plus and minus 5 per cent from the curve of averages.

It, therefore, follows that even with the correction for humidity variations a variation of plus or minus 5 per cent in flashover values of suspension insulators must be expected. The insulators used were the same throughout the entire test.

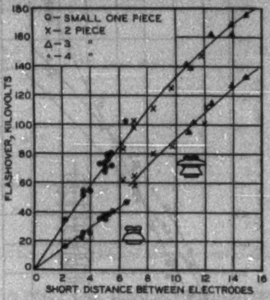

FIG. 8—60-CYCLE FLASHOVER PIN TYPE INSULATORS

The effect of humidity on flashovers of typical pin type transmission insulators is shown in Fig. 12. But one insulator of each type was tested. Variations shown are not due to manufacturing variations in different insulators. The individual observations fell within an envelope of plus or minus 6 per cent, the broadest of any insulator type tested. The variation in flashover of all insulators between vapor pressures of 0.2 and 0.608 is about 12 per cent. Frequently in testing at standard temperature with relative humidity above 65 per cent, erratic performance and marked fall in flashover was observed. A typical example is shown for the 27 kv. rated insulator. This is believed to be due to the formation of invisible water film upon the surface of the insulator at these higher humidities.

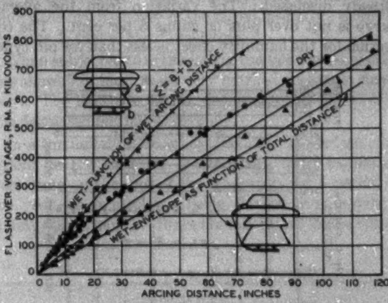

FIG. 9—60-CYCLE FLASHOVER OUTDOOR APPARATUS INSULATORS

Fig. 13 shows the effect of humidity upon typical outdoor apparatus insulators. The points of observations shown upon these graphs in comparison with those for the pin types, Fig. 12, show much more stable performance. This is due to absence of corona streamers as already discussed. The variation from the average curve is not over plus or minus 5 per cent. The correction for the humidity effect is less than for pin types.

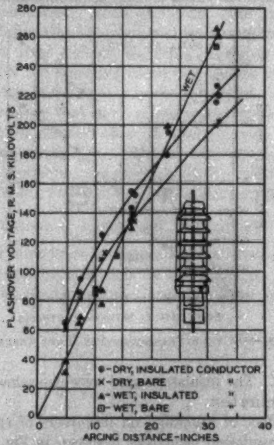

FIG. 10—60-CYCLE FLASHOVER SOLID TYPE EQUIPMENT BUSHINGS

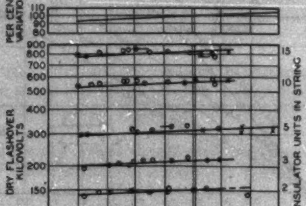

FIG. 11—EFFECT OF HUMIDITY STANDARD SUSPENSION INSULATORS

10 in. diameter, 5¾ in. spacing

It is 8 per cent between vapor pressures of 0.2 and 0.608, the same as for suspensions. Points at the upper right hand ends of these graphs are for temperatures well above standard, yet falling closely upon the same graph, indicating that comparison on the basis vapor pressures is quite satisfactory.

A most interesting observation upon an apparatus insulator of the multiple unit type for stacking is shown in Fig. 14. The correction for humidity is the same for this insulator as for the other apparatus and for the suspension insulators, 8 per cent from 0.2 to 0.608 inches mercury water vapor pressure. The variation of observations from the average for the three and six unit stacks falls within an envelope of plus or minus 5 per cent. For a single unit, however, the variation is nearly plus or minus 10 per cent. This checks observations upon other insulators such as the tall apparatus and the suspension insulators, indicating that for great lengths the percentage variations are somewhat less and the performance of the insulators more stable.

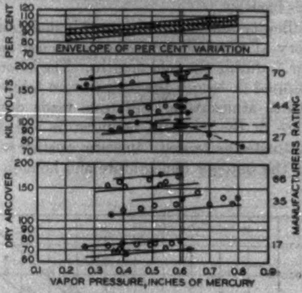

FIG. 12—EFFECT OF HUMIDITY ON FLASHOVER PIN TYPE INSULATORS

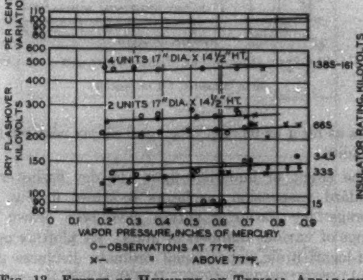

FIG. 13—EFFECT OF HUMIDITY ON TYPICAL APPARATUS INSULATORS

The performance of porcelain equipment bushings with varying humidity is shown in Fig. 15. The correction for humidity with these bushings is the same as for all others except the pin types.

A tabulation has been made, summarizing the above data as follows:

TABLE I

Insulator type	Dry value Arc length Kilovolts / Inch	Wet value Arc length Kilovolts / Inch	Wet value Wet striking distance Kilovolts / Inch	Humidity correction per 0.1 in. mercury water vapor pressure
Suspension:				
0 to 100 in.				
Standard cemented	10.5 to 9	7 to 6.8		2 per cent
High strength cemented	10.5 to 9	7 to 6.3	14 to 10.6	
Hewlett flat disk	10.5 to 9	7.5		
Hewlett fish tail	10.5 to 9	6.5		
Pin types				
Small	14	7.0		3 "
Multipart	14 to 11.4	8.9	(Erratic)	3 "
Apparatus insulators	11 to 7.3	8 to 5.7 (Erratic)	13 to 10.4	2 "
0 to 110 in. dry arcing distance				
Equipment bushings				
Solid type	13.6 to 6.75	8.3		2 "
(With bare conductor)	12.4 to 6.1	8.3		

Where two figures are given above first applies to small insulators, second to the largest with fairly uniform steps in between, except as noted as erratic.

EFFECT OF SURFACE CONDITIONS

In an effort to determine how the above rules for insulator flashover might be affected by surface conditions, representative insulators were exposed above the smoke boxes of a battery of beehive kilns for a period of several months. The surfaces of these insulators at the expiration of this period were thoroughly covered with a dirt deposit consisting largely of carbonaceous material mixed with a certain amount of clay dust and ordinary floor dirt. The coating was rather hard, due to the baking action of the heat from the kilns.

Other insulators of exactly the same type were thoroughly coated with a salt crust by spraying with a concentrated salt solution and allowing to dry.

The insulators were then flashed over, first clean and dry, then salt coated and dry, and finally dirt coated and dry. Tests were repeated with the insulators in a steam chamber filled with steam at approximately 100 deg. temperature to simulate fog conditions. Finally some of the insulators were tested under A.I.E.E. Standard Wet Flashover Test. The results of the tests are shown in the following tabulation.

It will be noted that there is very little reduction in the dry flashover of the insulators when dirty. Dry tests on the salt coated insulators were made at two humidity conditions. The first group of tests yielding the higher values in the tabulation were made at approximately 77 deg. dry bulb temperature, 65 per cent relative humidity. The second group of tests made at approximately 77 deg. dry bulb temperature and 79 per cent relative humidity gave flashover values materially lower, probably due to the absorption of water by the salt, at this relative high humidity. At the lower humidity the dry flashover of the salted insulators is nearly the same as that of clean insulators.

When subjected to artificial fog the salt coated insulators showed in most cases very marked decline in flashover. The flashover of the dirt coated insulators was also decreased but not so severely.

In a further effort to study the effect of surface conditions on the flashover of insulators, other insulators were hung on the roof of the laboratory for a period of one year, each insulator being flashed over approximately once a month. Since the laboratory is in a heavy

TABLE II

Nominal rating or number of units	Type of insulator	Dry				In A.I.E.E. standard wet flashover test			In steam fog		
			Salt coated								
		Clean	79% humid.	65% humid.	Dirty	Clean	Salt coated	Dirty	Clean	Salt coated	Dirty
20 kv	Pin type A	67	41	72					22.5	23	
20 kv	Pin type B	60	62	77					30	20–26	
35 kv	Pin type	120	97	112	116		34–63		56	24	48
44 kv	Pin type	135	97	114	128	95	52–80	67–74	74.5	34	68
55 kv	Pin type	137	90	140	136				78–89	48	85
66 kv	Pin type	161		164.5					132	51	
70 kv	Pin type	178	185	189	178				132	55	83
1 unit	5¾ x 10 in. disk	77	34	75	69	48	27–44	36–53	33	23	32.5
3 units	5¾ x 10 in. disk	204			205				139		114
9 units	5¾ x 10 in. disk	535		479							
9 units	5⅛ x 10 in. disk	485		454							

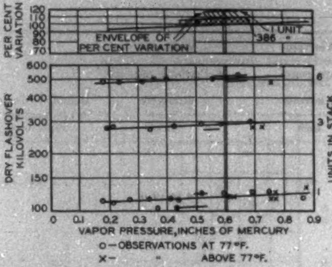

Fig. 14—Effect of Humidity on Apparatus Insulator Multiple Unit Type

10 in. diameter, 9¾ in. high

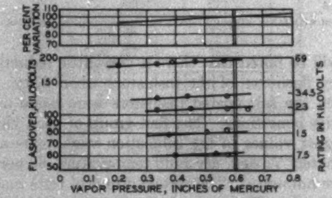

Fig. 15—Effect of Humidity on Porcelain Equipment Bushings

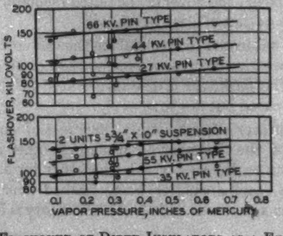

Fig. 16—Flashover of Dirty Insulators as a Function of Humidity

industrial district the insulators were subjected to soft coal smoke and industrial gas fumes. The results of the tests over a period of one year are tabulated. All flashover measurements were made at a time when the insulators were apparently thoroughly dry.

These tests were, of course, made at such humidity and temperature as happened to be existing at the time. The points have been plotted on Fig. 16. It will be noted that the curves are identical with those of Figs. 11 and 12. There are certain points on each of the curves which fall materially below the average. It is believed that in these cases the dirt film on the surface was very slightly moistened. Since most of the tests were made in the morning it is quite possible that in these cases there was still a certain amount of invisible dew remaining on the surface of the insulators.

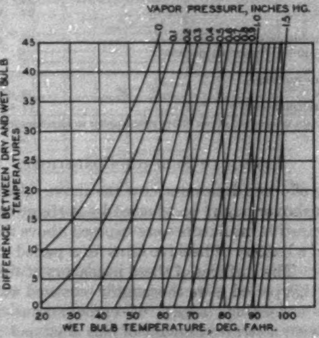

Fig. 17—Psychrometric Chart

From Smithsonian Meteorological Tables. Barometer = 30.00 in. Hg.

Conclusions

The relations found between arcing distance and insulator flashover and between humidity and insulator flashover are indicated in the foregoing curves and tabulations. At very high relative humidities there is in some cases a scattering of test results with points falling below the humidity curves. This is believed to be due

TABLE III

Time of exposure	0	1 Month	2 Months	3 Months	4 Months	5 Months	6 Months	7 Months	8 Months	12 Months
Dry bulb temp	82	65	50	43	27	30	44	52	61	74
Wet bulb temp	68	56	48	41	24	27	38	48	56	69
Vapor pressure	0.533	0.352	0.314	0.235	0.092	0.111	0.163	0.292	0.396	0.659
Barometer	30.04	30.15	30.25	29.91	30.09	29.94	29.57	30.06	30.00	29.96
Rating and type of insulator										
27 kv. pin type	89	86.5	78	68	83.5	88	82.5	86.5	85.5	95
35 kv. pin type	109	101.5	94.5	89.5	96.5	104.5	102.5	108	106	109
44 kv. pin type	123	111	100	88	103	105.5	110	101	108.5	127
55 kv. pin type	132	125	111	95	112	122	124.5	113	120	136
66 kv. pin type	164	152	135	116	136	141	152	126.5	151	164
2 units 5¾ in. x 10 in. suspension type	148	142	135	122	136	132	134	129.5	143	144

to the precipitation of a fine moisture film on the surface of the insulators.

Throughout the common range of temperatures (20 to 100 deg. fahr.) the relation between vapor pressure (expressed in manometric inches of mercury) and absolute humidity (expressed in grains per cu. ft.) is such as to make no practical difference which of them is used as the basis of measurement.

The same humidity corrections are applicable to dry dirty insulators as to clean insulators.

Dry dirt only slightly lowers the flashover of insulators. A small amount of moisture greatly reduces the flashover of dirty insulators, particularly if the dirt is of a saline nature.

At high relative humidities the flashover of salt coated insulators is materially reduced due to absorption of water by the salt.

Insulators can be checked for flashover from data given in Table I which also gives humidity correction factors. Deviation of individual flashover voltages from the average are not fully explained by humidity variations alone. After correction for humidity has been made a variation of about plus or minus 5 per cent remains in some cases.

NOTE: Vapor pressures may be obtained from barometric and dry and wet bulb thermometer readings with the assistance of the Smithsonian Meteorological Tables from which Fig. 17 has been drawn.

From this vapor pressures may be obtained with sufficient accuracy for the purpose.

Discussion

INSULATOR SPARKOVER

(LLOYD)

AN IMPROVED TYPE OF LIMITING GAP

(AUSTIN)

SUSPENSION INSULATOR ASSEMBLIES

(TOROK AND ARCHIBALD)

NORMAL FREQUENCY ARCOVER VALUES OF INSULATORS

(FREY AND HAWLEY)

A. O. Austin: The paper by Messrs. Torok and Archibald calls attention to a subject which is of great economic importance to the transmission system. It has been evident for many years that length efficiency was frequently of far more importance than any difference in the cost of the normal insulator. With the more efficient insulator it is possible to reduce the clearances and the height of tower. In the two-circuit tower this is exceedingly important as the height of the structure can be reduced so that the probable number of hits to the line will be less. With closer clearance the line reactance is reduced and capacitance increased so that more power may be transmitted and the stability increased.

Increasing the section length or the size of the metal parts to obtain a higher ultimate without increasing the diameter and flashover voltage of the disk lowers the length efficiency of the string. Impulse tests show that the wet flashover of the string as compared to the dry flashover voltage drops off materially as the insulation of the string is reduced. The ratio of flashover voltage in the disk to the voltage imposed upon same in the string under the most severe condition is the first consideration. As an increase in diameter raises the flashover voltage it will tend to improve the string performance providing this is not offset by increased drip water and section length. It would therefore seem that insulators having higher flashover voltages than the normal 10-in. diameter disks can be used to advantage.

The 10-in. disk insulator originally had a section length of 5¼ in., a flashover voltage of 92 kv. and a mechanical ultimate of 8,000 lb. The tendency has been to increase the mechanical ultimate and section length only, and in many cases the flashover voltage probably does not exceed 70 kv. This tends to reduce the time lag and length efficiency of the string under impulse.

It is feasible to use insulators which have a higher flashover voltage even though they are considerably more expensive. Where a whole string is made up of insulators of large diameter it will be found that the drip water has a material effect upon the flashover voltage. The large projected area of the insulators increases the amount of drip water, which has a material effect upon starting a flashover under impulse. By using insulator units of large diameter along with smaller intermediate units, not only is the drip water reduced but the short-circuiting effect of the stream of drip water is much less. Strings of this kind show a greatly improved performance, and when the advantage is given due credit it will be seen that the higher cost is more than offset.

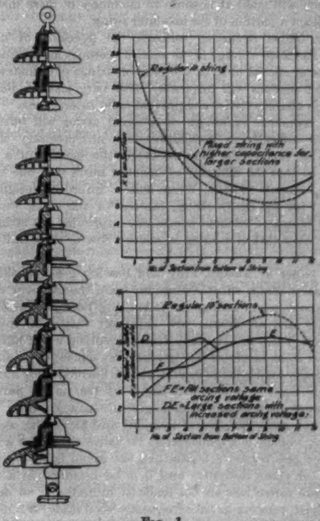

FIG. 1

The necessity of using mixed diameters in the shells of pin type insulators has long been recognized and the treatment is even more effective in the long string of suspension insulators, as accumulated drip water tends to short-circuit the lower portion of the string.

I believe that Mr. Torok should extend his investigation to include longer tailed waves, as many systems do not have a low effective ground resistance or capacitance, and an effective counterpoise may be considered too costly.

It must also be remembered that many strokes of lightning consist not of a single impulse but a number in rapid succession so that if tests are confined to a wave between one and two microseconds duration as shown by the oscillogram in the paper, the insulator may give a poorer performance in service than would be expected from the tests.

Some interesting insulators of larger diameter and higher flashover voltage were shown in a paper presented at the Pasadena Convention in 1924. Fig. 1 shows some of these insulators and the effect upon the ratio of flashover voltage to duty imposed, which may be termed the unit factor of safety. The priming or shunting of the surface insulation of the disk under a very high overvoltage makes it possible to use insulator sections of higher flashover and improved characteristics to advantage. The insertion of a few disks of large diameter in a string of short spaced 10-in. diameter units will frequently increase the flashover voltage from 15 to 25 per cent even though the insulators used do not represent the maximum possibilities as to flashover voltage.

While larger and more efficient sections have been available for many years, their higher cost has prevented their general use. Recognition of the economic advantages of the high length efficiency will make it possible to use many of these insulators for at least a portion of the insulator string.

With higher operating voltages, the necessity of using a higher flashover voltage in the section becomes more important. The use of special units at the ends of strings of ordinary 10-in. diameter sections will frequently effect very material improvements in characteristics and prevent radio interference.

The economic advantage of a mixed string will offset any assumed difficulty in installation. It is not necessary that all strings be exactly the same to obtain the benefits of a mixed string, so that the difficulty of handling more than one type of section amounts to little. While units of improved design could be used for the entire string, the cost of the string would be greatly increased without necessarily improving the string materially over that of a string in which a few of these units were used.

The paper by Messrs. Frey and Hawley gives considerable information on the effect of humidity upon the flashover voltage of various types of insulators. It has long been recognized that humidity and air density are important factors in insulator flashover. There are so many factors involved which affect the flashover of an insulator that general laws cannot be established which apply to all types of insulators. The correction for humidity therefore must be determined largely by test on the particular insulator or one approximating its general characteristics.

In many cases the effect of humidity and air density upon the flashover voltage is approximately the same as that for a needle gap. For this reason a needle gap has given fairly consistent results for many insulators and is to be preferred to a sphere gap where corrections are not made for humidity or air density. In the past, corrections have frequently been made for a measuring gap without giving the insulator the benefit of the same corrections. This practise has led to more confusion than any other item in insulator testing. In making careful dielectric tests, corrections of course should be made so as to obtain the absolute voltage applied.

As pointed out in the paper of Messrs. Frey and Hawley, the field set up by a bushing has a considerable effect on the flashover voltage. It would seem, however, that the higher wet flashover shown in Fig. 10 applies only to bushings which have a very poor surface gradient.

Many properly designed bushings have a higher flashover voltage with the conductor in place then when the conductor is removed. This is due to the fact that the design of the bushing makes use of the field set up by the metal parts in screening the surface from shunting discharge streamers.

In regard to Mr. Lloyd's paper, where the humidity is high it is very important that the insulators be at room temperature in making the tests, otherwise large discrepancies may arise.

C. L. Fortescue: It will probably take some time for the industry to appreciate the significance of the work that has been done by Messrs. Torok and Archibald. The research was started purely with the idea of finding out the influence of diameter and spacing in the impulse strength of suspension insulators. Since the object of the research was to determine the insulating value of suspension insulators of different diameters and spacings to withstand lightning surges on transmission lines it was necessary to determine only a suitable wave form to represent the actual impulsive stresses to which these insulators would be subjected under operating conditions and the wave form shown in their paper was the result.

To those not familiar with the more recent theories resulting from the lightning field investigations of the last few years I wish to point out that the direct stroke theory has been quite generally accepted by all engineers. According to this theory the outages on high-voltage transmission lines, that is to say, 66 kv. and above, are solely due to lightning strokes which hit the ground wire or line wire; the induced surge due to a nearby stroke is too low to be of any consequence. The surge that appears across an insulator when a stroke takes place at a tower is of very steep front and the high part is of very short duration. This is due to the fact that the tower itself acts as a short length of transmission line and the lightning stroke builds up the potential at the tower top during the time required to travel back to the top of the tower. The approximate wave form may easily be computed by the method of reflections and the computations result in the series of waves which was shown by the authors for different values of tower footing resistance. It will be noted that as the tower footing resistance is increased the magnitude and duration of the high part of the wave is also increased. With a very low tower footing the high part will be of low magnitude and its duration will be quite short, as short as one microsecond.

The criterion set up was the ability to withstand surges of the wave form shown and the resulting data have been given in the plotted curves in the paper. These curves show that larger diameter, namely 12 inches, and medium spacing, namely 5¾ inches, will result in the most economic insulator to apply to high-voltage transmission lines. The important gain comes about by reduction in string length which with equivalent air clearances results in a lower and cheaper tower. I wish to emphasize also that there is nothing untried in the type of insulators proposed. Insulators of this diameter and larger have been in use on the Pacific Coast and have given good service. They are just as sturdy as the standard 10-inch suspension insulator.

In discussing the Torok-Archibald paper regarding the latest point of view in the application of insulators in transmission line design, Mr. Austin has laid great stress on what might occur after flashover takes place. We prefer to lay more stress on the ability of an insulator assembly to withstand an impulse without flashing over. The protection of transmission lines is now reaching the point where the protective level is so high that more than two outages per 100 miles per year is not considered a very good record. Under such conditions it seems to me that the result of a flashover occurring once or twice on a 100-mile line per year is not of much consequence since as a general rule after the flashover occurs the line goes into service at once. A broken shed on two or three insulators is not of any great importance and the insulators can be changed at any convenient time after the occurrence of the flashover. Therefore, we are more concerned with obtaining high economic efficiency in the application of insulators than in any other feature such as what occurs after flashover takes place. We may even reach the ideal where a flashover is so rare that it may happen, due to lightning, only once in every few years.

J. J. Torok: There are two very important points in Mr. Austin's paper, first in that he points out the difference between the positive and negative flashover characteristics of various gaps and its importance in coordination, and second, the relations of flashover between waves of various durations and the volt-time curve obtained by a flat-topped wave.

Mr. Austin in his various curves points out the essential difference between the positive and negative characteristics in the volt-time curves of various forms of insulation and gaps. It is evident from these curves that careful consideration must be given to coordinate the gaps and insulators under both the positive and negative waves. At the same time he offers a possible solution to the question of gaps in coordination.

In obtaining the curves shown in Figs. 2 and 8, Mr. Austin must have spent considerable time in making the studies illustrated in these curves. The abscissa designate the time in microseconds from crest to 50 per cent attenuation. The curves vary from approximately 5 microseconds to 100 microseconds. To obtain waves of varying duration it is necessary to change the set-up for each desired wave thus making a study of this nature very tedious. Having these data it is then possible to make a comparison between curves obtained with flat-topped waves in which the magnitude is varied and the duration of the time of flashover determined and curves made with waves of varying duration. From such data it is possible to make a comparison between the flashover under waves of different duration with different times on the volt-time curve. Thus a basis of comparison of different flashovers of insulation can be established by choosing particular times on the volt-time curves.

N. B. Obbard: The paper by Messrs. Torok and Archibald is one of the first in which the economic relation of the insulator string to the cost of the tower has been taken into consideration. The usual procedure in the past has been to design the tower to suit the insulators, which have usually been designed and often purchased, before any thought is given to the towers which must support them. In its way, designing towers to suit insulators is just as incorrect, as designing insulators to fit a specific tower. The two factors, as shown by this paper, are absolutely inter-dependent, and should be considered simultaneously if the best and most economical transmission line is to be obtained.

The large clearances, high ground wire and long string lengths which are demanded by the best present day practise will increase the cost of transmission towers very considerably, unless care is taken to take advantage of the knowledge which has been recently made available to designers. The string length is a matter of particular importance in large single-circuit lines, where the center conductor must pass through the body of the tower and where it is desired to keep the outer conductors as close to the center as possible in order to reduce both torsion on the tower and right-of-way requirements.

Some recent studies showed a difference of around 10 per cent in the cost of the towers and anchors, designed for identical electrical and mechanical conditions, but with different types of insulators. It is, therefore, well worth while to consider the design of the insulators and towers at the same time and not as separate problems.

Philip Sporn: Messrs. Torok and Archibald's paper naturally falls into two parts: the first dealing with the question of the type of wave to be employed for impulse testing where the results are to be given consideration in the selection of insulators, and second, with the characteristics of insulators themselves under different arrangements of spacing and for different diameters of shells.

Regarding the question of wave shape, it is very interesting to note that the authors now believe that insulation can be compared on the basis of a single test wave (represented in Fig. 2) and this wave is substantially the same as the 1/2-5 microsecond wave of Mr. Peek. It is quite true that it is specified that this is the wave that should be used only in cases where ground resistance of the order of 10 ohms and where ground wires are employed. In this connection it is stated in the paper that if the ground wire protection is inadequate, then the wave form to be used for checking the performance of the insulators is one having a duration from 10 to 50 microseconds. I do not believe that adequate proof has been presented to justify the above statement.

The second part of the paper deals extensively with the question of diameter and spacing of insulator units. I believe that the value of this information could have been greatly enhanced if the tests had been confined to practical designs and to spacings and diameters within the commercial range. I cannot, for example, see the benefit obtained by testing 10-in. shells on 10-in. spacing that is, upon shells that have never been built for commercial use and have never really passed even the laboratory stage. There is so much more information that is badly needed now regarding the performance of standard insulators within the commercial range that it seems to me that the effort could well have been directed in those other channels with greater benefit to the industry. For example, referring to Fig. 6, it is hard to see where much more than that portion of the nest of curves between the abscissas of 4¾ and 5¾ in. can be any real value either at the present time or in the future. It is to be noted that if data within those limits are examined, and if allowance is made for the limits of test accuracy obtained, data presented in Fig. 6 will check the information presented in a number of the other papers as to the flashover value of commercial insulators when plotted against arcing instead of string length.

Referring to the impulse flashover data presented in Fig. 11, it seems to me that the weakness of the entire series of data is the fact that all the information was gathered on the basis of non-shielded strings. Within limits of spacing and shell diameter likely to be employed if grading shields were to be taken into consideration, I believe it would show that there is very little difference in flashover characteristics in a given length of distance between the rings, regardless of the insulator spacing or the insulator diameter. Beyond those limits (for example, in the case of a 10-in. shell with a 10-in. spacing, difficulties with wet flashover performance would most certainly be encountered. This, of course, is not in agreement with the statement in the paper but it is to be noted that the authors present no data to substantiate their claim.

The economics of insulator flashover per kv. is undoubtedly of importance, although quite secondary to insulator performance. I have compared the relative cost values given in the authors' Fig. 12, with insulator prices today and find there is a variation of nearly 70 per cent between insulators of different manufacture ranging in commercial size and spacing from 4¾ to 5¾-in. spacing. For the same range the authors show only a 7 per cent variation in cost. It is therefore apparent that the cost analysis made by the authors has little value except under the special conditions which they assumed in their analysis and is of very little practical importance in the selection of insulation for an actual transmission line.

In discussing wood construction, the authors state that calculations or ultimate strengths of wooden structures now in use show that a unit having a mechanical strength of 7,000 lb. would be adequate. I presume that the 7,000 lb. refers to the combined M & E test. I do not know where the authors obtained their information but there are many lines that have been built on the systems of companies with which the writer is associated where the units can be subjected to a maximum stress of 4,000 lb. M & E under class *B* loading, and we most certainly would not think of using a 7,000-lb. unit for those conditions. Nor do we believe that there will be found many other practical engineers who would be willing to sponsor construction of that type. We think that this statement discloses a total unfamiliarity with actual conditions involved in the field.

The conclusions given at the end of the paper suffer, it seems to me, from a lack of data to substantiate their validity. There is, for example, no information presented with regard to the choice of impulse waves of 10 to 50 microseconds under the conditions outlined in conclusion No. 4. Nor is any more information given to substantiate claim No. 9 with regard to the service performance of 12-in. units. While the 12-in. unit may be satisfactory, it seems to me, it is not fitting that a conclusion be established as to its adequate performance in service when not a single record is cited in the paper to substantiate it. With literally millions of 10-in. suspension insulators now in service from 5 to 17 years, ranging in spacing from 4¾ to 5¾ in. and with almost perfect performance given by these units, on a test of 90,000 units on lines of various subsidiaries of the American Gas and Electric Company where insulators have been in service from 3 to 12 years, a total of 12 bad units showed up out of the 90,000 units tested, or a little over 1/100th of 1 per cent. I think one should hesitate a great deal before changing practise on the basis of conclusion No. 9, unsupported by operating experience.

The data presented by Mr. Lloyd are very interesting in view of the fact that discrepancies in 60-cycle flashover of insulators as given by different manufacturers were pointed out forcefully in a paper* presented in New Haven in 1928. Subsequent work of the Lightning and Insulator Subcommittee has shown that this is largely due to the different humidity conditions under which the tests were made. Mr. Lloyd's paper definitely confirms this and shows how consistently the results show up when plotted against absolute humidity and not against relative humidity. The information given with regard to performance of insulators under salt spray conditions is extremely interesting as all this has been known for some time, but this is the first time that definite numerical information has been furnished. The data with regard to increased value of impulse flashover value of insulators under conditions of absolute humidity are new so far as is known, and show very definitely the need of incorporating a humidity clause in connection with any specifications for impulse value testing.

The paper by Messrs. Frey and Hawley very closely checks the information furnished by Mr. Lloyd. Mr. Hawley plots his flashover values of various insulators within the commercial range of diameter and spacing against measured arc length and it again is very interesting to note how beautifully all the data obtained fit together when plotted against this particular set of coordinates. In obtaining his information on the behavior of insulators under different humidity conditions Mr. Hawley has gone one step beyond Mr. Lloyd and has plotted his data against abscissas of vapor pressure, which is certainly the most scientific manner of presenting data, but as he points out in one of his conclusions, within the common range of temperatures it makes substantially no difference whether absolute humidity or vapor pressure is used. The information given in Fig. 10 with regard to the performance of a solid type bushing under wet and dry conditions is interesting, particularly the increased flashover obtained on higher lengths of bushing under wet conditions. This difference is probably due to a better dielectric flux distribution under the wet conditions.

W. L. Lloyd, Jr.: The front of the wave given in Fig. 1B of the paper by Messrs. Torok and Archibald and labeled "actual wave shape from field tests" is smooth and rises to its maximum in less than half a microsecond. As far as a test-piece would be concerned, the total duration of this wave from zero to maximum voltage value and then to 50 per cent of maximum on the tail of the wave is less than one microsecond. Sparkover of a test-piece takes place at or near the crest of such a wave, or not at all, and high sparkover values and impulse ratios are obtained.

The laboratory wave shown in Fig. 2 has about twice the length of front and duration of its main portion as the wave shown in Fig. 1B and this laboratory wave shape should result in lower sparkover values and impulse ratios than the actual wave shape from field tests shown in Fig. 1B. If Fig. 1B is a typical field wave, the wave in Fig. 2 should give lower values than those measured in the field. This is apparently actually the case. The voltage values given are somewhat lower than those which are generally measured with natural lightning on actual lines in the field.

The authors state that the wave shown in Fig. 2 was obtained by shunting the discharge circuit of a surge generator, arranged for the production of a smooth 1.5/40 microsecond positive wave, with a needle gap having a resistance in series to ground. The needle gap flashing over near the crest of the wave results in an abrupt drop in potential across the insulators under test. This satisfactorily accounts for the fall in voltage on the tail of the wave but does not account for the shortening of the front. Fig. 2 shows a wave having less than a one microsecond front rather than a 1.5 microsecond front. Would the authors please describe what was used to shorten the front or if this is the usual front of their 1.5 microsecond wave, data for which are given in the remaining figures of their paper. An overvoltage thrown on such a combination of discharge circuit and needle-gap with series resistance would result in a shorter front if the needle gap were to be sparked on the rising front of the wave. Fig. 2 does not indicate, however, that such was the case. Unless the capacitance of the generator used was extremely limited, sparkover took place at the point of abrupt fall in voltage indicated at approximately 1.7 microsecond. Such sparkover would not affect the front.

While the authors have made quite an extensive study of the sparkover of various spaced insulators, their efficiency and relative costs, I feel that the value of the data would have been considerably enhanced if the actual test points had been given. Without the test points on the curves, it is simply a "take it or leave it" proposition. There is no chance of gaging the accuracy of the work and therefore its value. This same comment also applies to Mr. Austin's paper, the value of which might possibly also have been enhanced if the measured test points had been given. Messrs. Hawley and Frey have given their test values in detail.

With further reference to Mr. Austin's paper, I cannot agree with all of his conclusions. Conclusions No. 1 and No. 3 give the requirements which an ideal gap should have and its use in the station. These conclusions may be proper, but in my opinion, a great many tests will be necessary before one can be certain that conclusion No. 2 is justified. So many variables enter between the different figures of this paper that it is difficult to check the data and draw definite conclusions.

I cannot agree with Mr. Austin's statement regarding the flux control gap being less erratic than the ordinary type of gap with bare electrodes. I have never heard before of the bare electrode gap being erratic when tested with *reliable equipment* under definite and known conditions.

The author states that it will be necessary to put up his type of gap and try it, and then change its characteristics, etc., until the desired characteristics are obtained. I do not think it desirable to put up a gap which must be played with. In the first place, one does not know which polarity surge will strike the station next, nor would there be time to adjust the gap if there were some way of predetermining the polarity, duration, voltage, etc. I feel that most operating engineers will want to have its characteristics more definitely known before it is placed in service as a protecting means. Many further tests will be required before we can feel that we have sufficient knowledge of the characteristics of this gap and I do not feel that the data in the paper are sufficient to prove the claims which have been made.

J. T. Lusignan: In his control type of limiting gap, Mr. Austin very ingeniously provides for a wide variation of the electrostatic field in order to secure practically any sparkover characteristic desired. In carrying this out, the following two principles are involved:

1. The effect of uniformity of voltage gradient across a gap upon the *time lag* of the gap.
2. The effect of an unsymmetrical electrostatic field upon the *polarity* at which a gap will have a lower breakdown voltage.

**Rationalization of Transmission System Insulation Strength,* Philip Sporn, A.I.E.E. TRANS., No. 4, 1928, p. 998.

Reference will be made to several simple gap arrangements to illustrate the manner in which the above time lag and polarity principles are applied.

Time Lag. The typical property of a needle gap is the non-uniform voltage distribution between electrodes as shown in *A* of Fig. 2 of this discussion. On the other hand, the sphere gap *B* is known to have a much more uniform voltage distribution across it. As a result of the higher voltage concentrations at the electrodes of the needle gap, local breakdown or corona starts there and proceeds outward long before there is any sign of breakdown in the sphere gap. Consequently, for 60-cycle voltages, or impulse voltages longer than a fraction of a microsecond, the needle gap must be set at an appreciably greater spacing for the same sparkover voltage. This increased gap length that must be broken down is an important factor in increasing the time lag of a needle gap.

For impulse breakdowns occurring in a microsecond or less, a needle gap must be set at a spacing approximately the same or smaller than that of a sphere gap in parallel with it. This is also due to the non-uniformity of field and therefore the higher time lag of the needle gap, as the following reasoning will show:

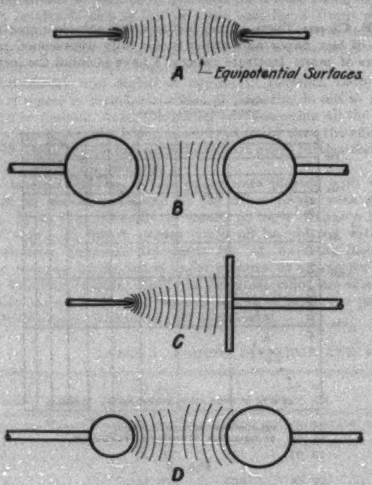

FIG. 2

When air breakdown starts at the electrodes of the sphere gap, it is only a negligible time later before the remainder of the gap is also broken down, due to the fact that practically the same voltage gradient prevails all the way across as shown in *B*. With the needle gap, on the other hand, the intense field near the electrodes causes an early failure of the air there, but until streamers from the electrodes have extended out appreciably and applied sufficiently higher voltage gradients to the center of the gap, the latter air remains undisturbed.

From the above, it is apparent that any tendency to approach a sphere gap condition by increasing the effective area of the electrodes of a gap tends to distribute the voltage gradient more evenly throughout the gap, allows the spacing to be reduced for the same sparkover voltage, and lowers the time lag. Mr. Austin accomplishes this in his control type of gap by arranging adjustable control shields about each electrode.

Polarity. It has long been recognized that sparkover between a point and a plane, as shown in *C*, always takes place at a lower value with uni-directional voltages when the point is positive; and with alternating voltages, always on a half cycle when the point is positive. In the same way the unsymmetrical field of the sphere gap *D* will cause it to display the same polarity effect and have a lower breakdown voltage when the smaller sphere is positive. The general rule for the above action is that sparkover always occurs at a lower voltage when the electrode in the more intense field is positive.

Mr. Austin develops this same polarity effect by arranging his control shields behind each electrode so as to increase or decrease its effective area and have the proper unsymmetrical electrostatic field condition across the gap.

It is apparent from Fig. 1 of Mr. Austin's paper that the insulating members of a station have line electrodes of small effective areas in comparison with the ground electrodes and, consequently, have appreciably lower positive than negative sparkover voltages. Also, Figs. 1 and 5 indicate that their time lag properties would lie between those of a needle gap and a sphere gap. With his adjustable field controls, Mr. Austin is able to secure any of the electrostatic field conditions of gaps *A*, *B*, *C* or *D* or variations of them and thereby arrange his gap to have the proper sparkover characteristics for protecting any or all station insulating members for all incoming transients, regardless of wave shape or polarity.

J. F. H. Douglas: Mr. A. O. Austin's paper shows how an analysis of the electrostatic fields can be used to achieve a sound solution of insulator problems. The different behaviour of points and spherical electrodes, with positive and negative electrification is clearly shown to be in accord with electrostatic field theory, and is verified amply by experimental test. Future experiment might be well directed toward some of the details which are not yet fully accounted for.

E. Hansson: Mr. Austin, in his paper, has outlined a method of controlling the electrostatic field around a relief gap so that the time lag of the gap will follow that of the insulation to be protected. That such coordination is necessary if full protection with a minimum of outages is to be obtained is indicated by the experience we had by reducing the line insulation ahead of some of our transformer banks.

In 1926 we reduced the insulation at both ends of two and at the station end of a third of our 66-kv. circuits from 8 to 5 units, O. B. Catalogue No. 25622. The transformer bushings which we wanted to protect were G. E. type F-1 rated at 88 kv. Our information as to the 60-cycle flashover values is 290 kv. for the bushings, and 267 kv. for the line insulators.

During 1926 and the early part of 1927, we had 22 cases of flashovers involving either bushings or reduced line insulation. Transformer bushings flashed in 7 cases, and the reduced insulation in 15 cases. Undoubtedly, the gap spacing was too large to protect the bushings under all conditions. We have no means of determining how many times the gaps operated unnecessarily. Insulation was restored to 8 units in June 1927.

Early in 1929, we installed fused arcing rings at both ends of one of the above mentioned circuits. The gap between the fuse and ring was set for 22 inches. During 1928 and 1929 we had four cases of bushing flashovers. In two of these cases, fuses were blown adjacent to the transformer and in two cases the fuses were intact.

During the same period another parallel circuit was equipped with a 12-in. gap between line and ground and in series with this gap we had an automatic fuse bank. We have had no bushing flashovers on this circuit since installation of this gap but we have, of course, blown a good many fuses. However, the fuse action is so rapid that the blowing of the fuse causes no disturbance on the system.

Joseph C. Rah: An explanation is given in the paper by H. A. Frey and K. A. Hawley that a better voltage division is obtained if the whole surface of the porcelain insulator is wetted, and therefore in the horizontal position, higher wet flashovers are obtained. This is probably true under certain conditions, but if the authors believe a better voltage distribution is obtained, then in explaining a lower wet flashover on a string of insulators by the fact that the wet surface on the top may be considered as short-circuited and only the dry air space effective, makes the two statements contradictory.

If a wet surface on the top of the insulator is short-circuited by a film of water, then the horizontally placed string will be completely short-circuited by the water film and a much lower value of flashover should be obtained, which is contrary to test results.

The results obtained on insulators from experiences of many observers, including myself, indicated that air spaces are responsible for reduced flashover rather than wet surfaces, and therefore, the wet flashovers are entirely a function of design.

Dry flashovers to a certain extent are also a function of design, as indicated by the differences in flashovers of pin type and equipment type insulators. The metallic cap on pin type insulators makes the porcelain insulator more reliable and steadier in flashover value due to a decrease in streamers, which are local flashovers. Therefore, if an insulator is designed so that streamers do not appear until up to the flashover value of the insulator assembly, then the insulator is a function of design, and not only the striking distance alone, as was proven by recent designs.

Fig. 1 gives flashovers with respect to spacings indicating that there is very little effect of spacing between disks. This may be so if the variation of spacings is one inch only where the inaccuracies of observation are great enough to offset the spacing effect. With smaller spacings however, there is a pronounced difference in flashover. The disk diameter and spacings can be adjusted to obtain well observable and definite variations in flashover values.

In the part describing equipment bushings, the authors claim that the field established around the bushing determines the dry flashover value, and by putting a conductor inside the bushing, lower flashover values are obtained.

This appears inconsistent, as introducing a conductor inside the bushing should and does improve the field, provided the interior and exterior designs are harmonious and do not interfere with one another. I have obtained flashovers considerably higher with conductors inside the bushings than without conductors with the proper exterior and interior construction.

When testing glass insulators of the same shape as porcelain, in regard to the effect of humidity, a lower flashover is obtained. This is due to the increased dielectric constant, as air nearer the insulator is overstressed. If we can lower flashover by introducing a material of a higher dielectric constant, we can also increase flashover by introducing the porcelain insulator into a humid atmosphere, which has a resultant dielectric constant higher than air, and in some cases may be higher than porcelain. This is, in my opinion, one reason of the increased flashover due to humidity. I also believe the absolute humidity is responsible for the increase of flashovers. In testing, I have found that on short insulators, the change in humidity has practically no effect on flashovers. The longer the insulator, the more effect the absolute humidity has upon flashovers. This suggests that the air is laden with moisture particles which line themselves up in the field and produce a multiplicity of condensers. This multiplicity of condensers is only effective in the part of the field which is nearly uniform. Therefore, on shorter insulators where the field is not uniform, there is little effect of the moisture particles to increase flashover potentials. On long insulators with a much longer uniform part of the field, moisture content in the air has an effect. As we increase air humidity we get more condensers in series, and therefore, the air is rendered electrically stronger until a time where there are so many moisture particles that they touch each other, and this is the dew point at which the flashover potential falls down sharply.

Corroboration of this theory may be observed in the behavior of the needle gap. The farther apart needle points are set, the larger the variation due to absolute humidity is apparent, as with increase of length the uniform field is made stronger by particles of moisture.

As far as the effect upon the sphere gap is concerned, the following may explain the apparent lack of effect of humidity. When the sphere gap is put into an atmosphere laden with moisture, the dielectric constant of the air plus the moisture is increased, and therefore the field becomes more uniform. The equipotential surfaces straighten out and instead of a sphere gap of smaller dimensions, a gap of larger dimensions is produced. If it were not for the fact that moisture is being drawn into the densest part of the field at the center line of the spheres, actual flashover of the spheres in humid atmosphere would be higher. In other words, we get a resultant strength of the air, and this resultant strength appears to be very close to the original strength of dry air.

J. E. Clem: There is one factor which enhances greatly the value of any paper and which is not fully appreciated by the authors of many Institute papers. I have in mind the inclusion of actual test points in curves. When test points are given the reader can readily form an idea of the variation to be expected in the use of the material and can also make his own analysis and form his own conclusions. I think it is almost self-evident that papers giving curves actually drawn through test points are much more valuable than those in which curves are shown but with no supporting data.

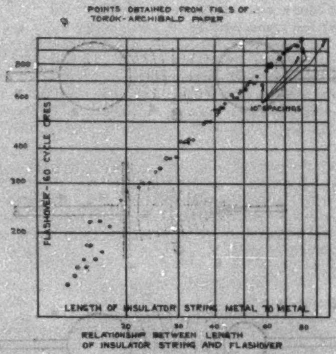

Fig. 3

The most important item of interest in the Frey-Hawley and the Lloyd papers is the effect of humidity on the flashover voltage of various types of insulators and gaps.

In Fig. 9 Mr. Lloyd shows a curve indicating that the flashover voltage of sphere gaps is not affected by changes in humidity for values of humidity of less than 11 grains per cu. ft. This is probably due to the fact that the field between the spheres is uniform and there is no point of flux concentration. It seems quite reasonable to believe that if the spacing between the spheres was large in proportion to the sphere diameter, this could no longer hold and the flashover voltage would be affected by humidity.

The type of gap which has characteristics departing most widely from those of a sphere gap is the needle gap. It would be natural then to expect that the needle gap would be greatly affected by humidity and further that needle gaps of small spacings might show less effect than long gaps. Reference to Mr. Lloyd's Fig. 10 indicates that such are the facts. Over a range of humidity from zero to 10 grains per cu. ft. the 5-in. gap increases about 18 per cent in flashover and the longer gaps increase in the order of 33 per cent over the same range of humidity.

The effect of humidity on the flashover of insulators is very important and it is also highly desirable that some standard method of making humidity corrections be made available as soon as possible. It seems reasonable to believe that insulator assemblies having a long flashover distance and having end terminals relatively small in proportion to the arcover distance, should behave approximately as a needle gap; and that small insulators should behave approximately as a sphere gap as far as the humidity effects are concerned. This is confirmed by reference to the curves in both the Frey-Hawley and Lloyd papers where humidity effects as high as 33 per cent for a long string of suspension insulators and as low as 4 per cent for a 1-unit string can be found. Fig. 12 in the Frey-Hawley paper gives an increase of about 35 per cent for pin type insulators over the same range of humidity as previously mentioned.

Unfortunately, the degree of agreement between these two papers in regard to the relative effect of humidity is not as great as could be desired. After comparing and analyzing all the data in both papers the conclusion must be reached that the effect of humidity is not completely settled and that much further thought must be given to this problem.

The humidity curves in one paper were plotted with vapor pressure as the abscissa and in the other paper with grains per cu. ft. as the abscissa. It would be very desirable if humidity curves in future papers could all be plotted with the same abscissa. Institute Standards No. 41, covering insulator testing, recommend the use of vapor pressure as a basis for comparing humidity values. Until the Institute rules are changed, and they should be if good reasons can be offered, the present Institute recommendations should be followed.

In reading through the Torok-Archibald paper one would come to the conclusion that the spacing of insulator units and the shell diameter of the insulator units have a marked effect on the flashover of an insulator string of a given length. In Figs. 1 and 2 of the Frey-Hawley paper and Fig. 15 of the Lloyd paper there are curves indicating that the unit spacing and shell diameter have very little effect on the arcover voltage of an insulator string of a given length. However, further examination of the data given in Fig. 5 of the Torok-Archibald paper leads to the conclusion that the flashover voltage of an insulator string of a specified length is approximately the same with the spacings from 4¾ to 7½ in. or with the shell diameter from 10 to 12 in.

In Table I of this discussion there is shown the 60-cycle flashover voltage of the various insulator units as read from the curves of Fig. 5 of the Torok-Archibald paper. These data are plotted in Fig. 3 of this discussion. From this figure it is seen that there is no consistent variation in flashover for varying spacings between 4¾ and 7½ in. and shell diameter between 10 and 12 in. Therefore, since the 60-cycle flashover shows no variation for these factors, it is rather difficult to understand how the impulse flashover can show any variation for the same factors, as claimed in the Torok-Archibald paper. Until sound reasons can be advanced for a difference in effect of unit spacing and shell diameter on 60-cycle and impulse flashover, extreme caution should be exercised in designing transmission line insulation based on this difference.

Fundamentally, no great difference in the flashover of an insulator string of given length for units of different spacing or different shell diameter should be expected within the ordinary limits of these dimensions. The arcing distance is the metal-to-metal length from the top of the top unit to the bottom of the bottom unit. Increasing the spacing between units increases this distance or adding a unit increases it. Whichever way the distance is increased the effect should be the same. The effect of increasing shell diameter should be greatest for a single unit. However, if the shell diameter is increased too much without increasing the thickness, no improvement would be obtained on account of the overstressing of the porcelain.

TABLE I—ARCING DISTANCE AND 60-CYCLE FLASHOVER FOR SPACING INDICATED

No. units	Dia.	4¾ in. Arc Dis.	Kv.	5¾ in. Arc dis.	Kv.	6½ in. Arc dis.	Kv.	7½ in. Arc dis.	Kv.	10 in. Arc dis.	Kv.
2	10	12.75	130	13.75	150	14.5	180	15.5	220	16	230
4		22.25	290	25.25	330	27.5	370	30.5	425	32	450
6		31.75	425	36.75	490	40.5	530	45.5	625	48	640
8		41.25	550	48.25	630	53.5	680	60.5	800	64	800
10		50.75	660	59.75	760	66.5	840	75.5	950	80	910
12		60.25	780	71.25	900	79.5	980			96	1,000
14		69.75	880								
16		79.25	1,000								
2	10½	13.5	140	14.5	160	15.25	180	16.25	220	19.0	260
4		23.0	300	26.0	340	28.25	370	31.25	420	38.5	500
6		32.5	430	37.5	500	41.25	540	46.25	610	58.0	680
8		42.0	560	49.0	640	54.25	690	61.25	800	77.5	850
10		51.5	680	60.5	780	67.25	850	76.25	950	97.0	980
12		61.0	800	72.0	910	80.25	1,000				
14		70.5	900								
16		80.0	1,010								
2	12	14.85	150	15.75	160	16.5	170	17.5	210	20	280
4		24.25	300	27.25	340	29.5	375	32.5	420	40	550
6		33.75	430	38.75	500	42.5	500	47.5	610	60	760
8		43.25	580	50.25	650	55.5	730	62.5	790	80	920
10		52.75	700	61.75	800	68.5	880	77.5	940		
12		62.25	810	73.25	930						
14		71.75	920								

Based on readings from Fig. 5 of Torok-Archibald paper.

However, when the unit spacing exceeds the arcover distance of a single unit, conditions may be somewhat different. In this case each unit of the string serves as a flashover unit and there should be more liability for the strings to cascade. This is no doubt the reason for the rapid falling off in the flashover voltage of the insulators with 10-in. spacing shown in the Torok-Archibald paper.

Further evidence that the flashover of a string of insulators is independent within ordinary limits of the unit spacing or the shell diameter is given in Fig. 4 of this discussion. The plotted points represent the 60-cycle flashover of pin and pedestal type insulators. The units tested cover a wide range in design features, but it is clearly evident from the points that the total flashover voltage is a function of the total flashover length. If there were any fundamental effect on flashover voltage, of unit spacing or shell diameter, one would expect it to be as pronounced or even more pronounced in an assembly of pedestal insulators than in an assembly of suspension insulators. It seems reasonable then to conclude that within the ordinary limits of these factors there is no effect on the flashover of a string of insulators of given length whether the spacing varies from 4¾ to 7½ in. or the diameter varies from 10 to 12 in.

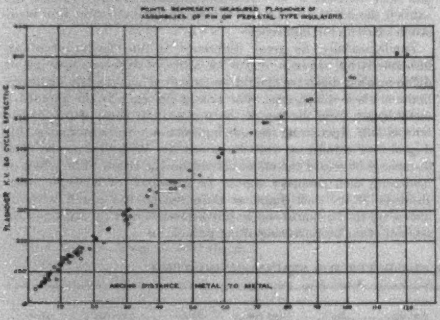

FIG. 4

In Mr. Austin's paper there is a lack of test points and oscillograms to show what impulse wave was used for obtaining the test data.

Mr. Austin would have us believe that the use of a gap for coordination leads to a very intricate and difficult problem. However, such is not the case. The problem in coordination is to select the gap setting so that the flashover of the gap will be less than the breakdown strength of the insulation for those waves which are considered most dangerous. The present tendency is to believe that the short waves of high magnitude are most likely to injure the insulation and further that there is little likelihood of obtaining long waves in sufficiently high voltage to be of much trouble. Therefore, in selecting the coordinating gap the thing to do is to base the setting on the gap characteristics for short time lags.

After comparing certain of the curves given in Mr. Austin's paper, I am doubtful as to whether his complex gap structure has any advantage over the common point gap. In Fig. 5 and Fig. 8 he shows the lag curves for the complex gap and in the text intimates that the adjustment is such as to give very desirable characteristics. For both figures about the same time lag curve could be obtained from the plain point gap, as can be verified by reference to Fig. 2 in Mr. Austin's paper. For instance, for comparison with Fig. 5, a 22-in. gap would have a flashover of about 470 at a 10 microsecond lag and about 600 at a 2 microsecond lag. Likewise for comparison with Fig. 8 a 60-in. gap would have a flashover of about 930 at a time lag of 100 microsecond and about 1,180 for 10 microsecond. It is quite clear that nothing has been gained by the use of a complicated gap structure for these two conditions.

W. L. Lloyd, Jr.: Absolute humidity may be expressed either in grains per cubic foot or in terms of the associated vapor pressure in inches of mercury. "Grains per cubic foot" appears to be the term more commonly used by engineers and it was for this reason that it was used in my paper. An absolute humidity of 6.5 grains per cubic foot corresponds to a vapor pressure of 0.608 inches of mercury at 77 deg. fahr. Therefore, where values in terms of vapor pressure in inches of mercury are desired approximate values can be obtained by dividing the grains per cubic foot by ten. The more exact figure is $\frac{6.5}{0.608} = 10.7$, but the difference in sparkover voltage associated with the resulting error in humidity value obtained is extremely small.

The A.I.E.E. Standards No. 41 dated March 1930, state that "flashover shall be determined at, or corrected to, a standard humidity corresponding to a vapor pressure of 0.6085 in (15.45 mm.) of mercury. This is equivalent to a relative humidity of 65 per cent at 77 deg. fahr. (25 deg. cent.) and a barometric pressure of 30.0 in. (76.2 cm.) of mercury." It might also have been added that this corresponds to 6.5 grains per cubic foot. In any future revision of A.I.E.E. Standards No. 41 it might be desirable to omit the reference to a relative humidity of 65 per cent at 77 deg. fahr. (25 deg. cent.) and a barometric pressure of 30.0 in. (76.2 cm.) of mercury since tests have since been made and data are given in this paper which indicate that the absolute humidity rather than the relative humidity is the determining factor.

Mr. Austin discussed the importance of maintaining the insulators at room temperature in making tests at high humidity. This is very important as indicated by Fig. 12 of my paper where a slight chilling of the insulators resulted in condensation on the porcelain surfaces and reduced sparkover voltages. Precautions against condensation were taken in making the tests up to 14 grains per cubic foot on 4 units in Fig. 3 and up to 16 grains per cubic foot on an oil-filled bushing in Fig. 8.

A. O. Austin: The several discussions make it appear that length efficiency in the insulator is a comparatively new thought, whereas the length efficiency or flashover voltage for a given length of insulator string is one of the oldest and most important considerations in the design and selection of insulators. The effect of length efficiency was treated in a paper before the World Engineering Congress in Tokio—November, 1929. Some examples showing the very material benefits possible with high length efficiency in the insulator string were treated in a paper before the International High Tension Congress in June, 1931.

Tower and conductor clearances are regarded as prime factors in transmission line costs. Unnecessary spacing not only increases the reactance of the line but increases the cost of the structure. High length efficiency in the insulator permitting of minimum tower clearance and height of structure therefore is of very great economic value. This matter was given very careful consideration by Mr. H. H. Cochrane on the lines of the Montana Power Company in 1909, the insulators and towers being selected on the basis of high length efficiency in the insulator.

Most of the work carried on at the outdoor laboratory of the Ohio Insulator Company has been devoted to increasing the flashover voltage or length efficiency of the insulator string, or in so constructing the tower as to provide maximum flashover between conductor and support. The economic advantages in many cases may be much greater than those pointed out by Mr. Obbard, particularly in the case of two-circuit towers. The use of an insulator of good length efficiency together with wood arms may double or even treble the effective flashover voltage between conductor and ground for a given vertical spacing or size of tower. Taking advantage of the high flashover voltage will permit operation at a much higher operating voltage and the increased kva. of the line may save additional circuits. Under these conditions the corona point of the conductor is likely to be the limiting factor.

The possibility of using small conductors coated with an insulating paint or varnish promises very important economic advantages where it is desired to raise the voltage of an old system or to take advantage of the close conductor spacing or the lower stress due to smaller conductors. Under these conditions the additional cost of an insulator string of increased insulation would be rather small compared to the economic advantage.

In the two-circuit tower the high length efficiency in the string permits a material reduction in the height of the tower. This results not only in a lower cost of the structure but in fewer number of probable hits to the line which should receive due credit.

More attention should be given to low effective height, not only for the conductors but for the ground wires. Shorter spans, cross connection between ground wires, and the use of increased ground capacitance or counterpoises should all be given consideration, as well as the effective flashover in the string or the flashover between the conductor and tower. Savings due to increased length efficiency in the insulator and to the increased flashover due to the use of wood arms are the most important economies to be considered at this time.

With the exception of the height of the structure, the above advantages apply to practically any transmission line even though lightning is not a consideration. The design of the transmission line is a compromise and it will be found that any method which will increase the length efficiency and flashover voltage for a given clearance will result in a material saving when applied.

In recent years there has been a tendency to develop mechanical strength in the insulator at the expense of reduced insulation in the string. It is therefore gratifying to note that length efficiency in the insulator is likely to receive some of the attention it deserves.

Mr. Sporn has raised a very pertinent question as to the maximum working load of the insulator. An insulator designed for a 4,000 lb. maximum working load will have a higher length efficiency, lower thermal stresses and a longer economic life for the same cost than an insulator designed for a 7,000 lb. maximum working load. The use of an insulator having an unnecessarily high ultimate is therefore questionable from the standpoint of economy and reliability.

Mr. Fortescue apparently has misunderstood some statement, as I have always been in favor of increasing the effective insulation as the most economical means of improving reliability rather than the use of arcing devices such as rings or horns.

The use of wood makes it possible to develop very high effective flashovers for lightning transients even for the lowest voltage and most inexpensive lines. To design station equipment so it will match the flashover voltage of the line, however, would be economically impossible particularly for the lower voltage installations. It may therefore be assumed that the station equipment will be subjected to transients which will cause flashovers or damage to some of the equipment unless the voltage is limited by an arrester or some form of limiting gap. It is much better to provide a path for the discharge which will produce little or no damage than to permit the discharge at a point where considerable damage may be caused, even though a few more interruptions may take place due to the lowered flashover value of the gap. Mr. Hansson has covered this point very effectively and shows that by equipping the gap with a quick acting fuse the discharge may have little or no effect upon the operation of the system.

A long discharge path or a resistance in series with the discharge which tends to limit the current is very effective in causing a recovery of the resistance in a discharge path. This recovery tends to prevent the follow up of a power arc and is effective in eliminating interruptions either on the line or for station equipment. Increasing the insulation level of the station decreases the probable number of interruptions, but unfortunately high insulation levels are very costly. If a high insulation level could be obtained as easily for station insulation as for the line, there would be little need of protective equipment. However, since this is impossible some device which will limit the voltage is of great economic importance particularly for the lower voltage systems.

I quite agree with Mr. Fortescue that the damage of a few insulator sections is of little moment compared to reducing the number of interruptions. However, in the case of station insulation it is frequently very difficult and costly to replace a part damaged by a power arc such as a bushing. Therefore protective apparatus such as rings or horns which lower the flashover, while being economically wrong for the transmission line, may be justified for the station.

I wish to call attention to the fact that the flashover voltage of the disk for a given section length is of far greater importance than diameter. Some of the 10-in. disks shown in the slides, which were made in 1909, have flashover values as high as 150 kv. for a 7-in. section length. An insulator string made up of these high flashover disks will have a higher insulation ratio or factor of safety than an insulator string made up of units of the same section length and a flashover value of 90 kv. even though the diameter of the sections is 14 or 15 in.

Under rain, the performance of units of larger diameter will be much poorer, particularly for an impulse having a steep wave front, comparable to a direct hit on the line.

At the present time a high length efficiency or flashover voltage for lightning can be most economically provided by a mixed string. Once the advantages of a mixed string are appreciated, it will be readily seen that these more than offset any possible disadvantages even from the maintenance standpoint, as the few replacements required could be made with either diameter of section without material effect upon the operation of the system.

In order to reduce unnecessary interruptions due to the discharge of a gap to a minimum, it is essential that the gap will not flashover under oscillations or when wet at very low values. Mr. Lloyd apparently has overlooked the fact that the flashover voltage of a horn or point gap will vary considerably under these conditions. The flashover of a sphere gap will also be influenced by water, the familiar weather protection used for sphere gaps on lightning arresters being proof of this.

Reference to Fig. 6 shows that the controls make it possible to reduce the difference in flashover voltage for various conditions, the variation being much less than for a plain point or horn gap. While the sphere gap would have less variation, the presence of water makes this gap very erratic under some transients, particularly where fairly large spheres are used.

If the station insulation has a materially higher flashover voltage with a transient of one polarity, it is possible to take advantage of this by setting the control gap accordingly, which would be impossible with other types of gaps without making a change in the parts.

In installing a control type of gap the available information of course would be used in the setting of the gap. In the case of a plain gap it would not be possible to take any advantage of the difference in flashover under negative and positive impulses, whereas the control gap could be set to take advantage of any material difference. With the control type of gap it is also possible to reduce the time lag without lowering the flashover voltage of the gap unnecessarily for low grade transients by simply adjusting the control screens and the tips.

If a plain gap is used the only change possible will be to reduce the clearance and flashover voltage. If a favorable setting is obtained when the control gap is first installed, no further change will be necessary. However if it is desired to change the characteristics of the gap to reduce the number of discharges, means are available for making this change. It is also possible to change the characteristics of the gap should the insulation of the station be changed at any time. A control type of gap certainly would require no more attention than a plain gap and even with the very meager information available it would seem that a better result would be possible without any adjusting of the gap after the initial setting.

Inability to take advantage of additional information has rendered much insulation and protective apparatus obsolete. It would seem that the possibility of taking advantage of this information certainly would not be a detriment and could not be held against the control type of gap.

The intensity and magnitude of transients may vary greatly on different systems, and there certainly is no disadvantage in being able to conform to these conditions. The polarity of a stroke of lightning of course cannot be controlled. However, if the minimum flashover voltage of the insulation is much lower for one polarity than for the other, the control gap can be adjusted for this. What is really needed is more information as to the flashover or maximum voltage which may be imposed upon the equipment. Lacking this information the control gap may be used to obtain approximate information which otherwise might not be available.

J. J. Torok: Mr. Sporn in discussing Mr. Archibald's and my paper assumes that most of the work has been confined to the short wave which he has simulated to the ½-5 microsecond wave. It was the purpose of this investigation to analyze insulator performance on a broad basis rather than any specific condition such as an investigation with a short wave. A long wave was used, commonly termed the 1½-40, and the volt-time characteristics of the insulators obtained with this wave. With such a volt-time curve the characteristics of the insulators are known for short waves as well as for long waves and an analysis of the relative characteristics of insulators can be made for any length of wave desired. Thus if it is desired the relative characteristics of the insulators can be obtained for the three preferred waves from these volt-time curves. In the paper a comparison is made using the short waves for illustrative purposes and because short waves will be encountered in ground wire installations. Also, the comparisons are made for long durations such as 60 cycles.

Mr. Sporn pointed out that the authors did not make an investigation of insulators and arcing rings. In the body of the paper the authors very explicitly stated that arcing rings were not included in these tests because the effects of rings have already been investigated and reported in other papers and that inclusion of rings would introduce such a variable as to make the data required so large as to be out of the question.

Mr. Sporn commented that the cost analysis given in the paper was at variance with present price quotations and that the discrepancy in some cases was as much as 70 per cent. The cost analysis given in the paper is based on an average cost extending over a period of five or more years and is therefore only an average cost. At present costs of raw materials and finished products are undergoing a rapid change. It is therefore to be expected that the cost of porcelain products should also change. It is very likely that at the resumption of stable prices the ratios of costs as given in the paper will be approximately correct.

Mr. Lloyd questioned the nature of the wave shape used for the short time investigation. He stated that a chopped wave should be of the saw-tooth type. The reason why this wave is of altered shape is that with such high voltages the streamer formation prior to flashover requires such heavy currents that the voltage wave is considerably distorted. In lower voltage circuits the streamer formation is considerably less and the distortion of the voltage wave is negligible.

Mr. Lloyd stated that the points should have been placed on the curves. The authors in making the curves realized that because of space limitation they would have to be greatly reduced. Consequently, the significance of such points would be lost in greatly reduced figures. Again, these points are taken from oscillograms and since the oscillograms could not be printed it was logical to draw in the curves without the points.

H. A. Frey and K. A. Hawley: We would like to point out that the humidity data contained in Mr. Lloyd's paper and that in our paper may be compared very readily since, for the practical temperatures involved, the vapor pressure expressed in inches of mercury is numerically very nearly equal to the absolute humidity expressed in grains per cubic foot. Such a comparison shows that while there are some variations which will require further study the general agreement is remarkably close.

Mr. Austin has made some statements in his discussion with which we cannot quite agree. Mr. Austin expresses the opinion that general laws for humidity correction in insulator flashover cannot be established and that such correction must be determined,—"largely by tests on the particular insulator or one approximating its general characteristics." We believe that while this opinion may be true when viewed from a strictly rigorous standpoint yet even with the data now at hand it is possible to apply a humidity correction to the flashover value of any commercial insulator which is correct to within the necessary practical limits of accuracy.

Mr. Austin also states, in commenting on certain of the bushing data, that "many properly designed bushings have a higher flashover voltage with the conductor in place than when the conductor is removed." We have experimented with *solid type* bushings of several different types and makes and have failed to find any in which the presence of the conductor did not lower the flashover value. It is quite true, however, that oil bushings with built-in flux distributors may have a higher flashover with the conductor than without. Such bushings are of an entirely different type than the solid bushings for which data are given in the paper.

The Parallel Type Inverter

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Member, A.I.E.E.

***Synopsis.**—The parallel type inverter is one of many forms which are undergoing development at the present time. It gives promise of becoming of considerable importance and therefore the principles of operation should be better and more widely known. The object of this paper is to make a qualitative analysis of the operation of this type under different conditions of loading, and to present the results largely in the form of complete sets of oscillograms for further study.*

Inverters of the types now being studied involve the use of hot-cathode mercury-vapor tubes called thyratrons. The characteristics of these tubes are given briefly and the data necessary for an understanding of their operation in these circuits are presented.

The principle of operation of the inverter circuit is developed by means of simple diagrams and then the actual conditions throughout the circuit are studied by means of oscillograms showing the voltages and currents in all of the essential parts. It is shown that the method of operation may be considered in a different manner from that usually assumed in the development of the circuit, inasmuch as the capacitor can be thought of as performing the function of giving the correct phase relation between the input current and the induced transformer primary voltage, rather than providing a sudden reversal of potential on the anode of the tube being stopped. This reversal is provided by the induced primary voltage. The current conditions for the various steps of operation are shown by simple diagrams.

The effects of low power factor loads with both leading and lagging current are shown in sets of oscillograms taken under these conditions. Another oscillogram gives the effects of improper circuit constants when operating at low power factor.

The output wave form under conditions of full load, unity power factor, is analyzed and it is shown that with the proper circuit constants the voltage wave is a fair approximation of sine wave. The output wave forms change, for any given set of circuit constants, with the character and degree of loading.

When reasonably good output wave forms are secured, the voltage regulation is poor but the efficiency is high. Changes in the circuit which give better regulation cause poorer wave form so that in the design of this type of inverter circuit a balance must be chosen between these two factors. Curves of efficiency and regulation, the latter showing the effects of different values of capacity, are given and briefly discussed.

INTRODUCTION

THE development of the thyratron or three-element, hot-cathode gas-filled tube has given electrical engineering a new tool. Many interesting and valuable applications have been devised, among the most interesting of these being the inverter. Its purpose is the inversion or changing of direct current to alternating current, the reverse process of the more usual rectification. The inversion process is possible with the ordinary types of three-element vacuum tubes, but due to their low efficiency and high voltage drops it has not been practical to secure alternating current in this manner. With the advent of the thyratron, with its high efficiency, low voltage drop, and its ability to handle comparatively large amounts of power, the inverter gives promise of becoming of commercial importance. Hitherto, inversion from direct to alternating current has been possible only by means of rotating machinery. If the inverter is successfully developed, as it seems to give promise of being, a method of inversion will be provided which does not require rotating machinery, but which accomplishes its purpose by means of starting and stopping electron flows. This operation can be performed at high voltages, thus making high-voltage direct-current power transmission possible, the energy being generated as alternating current, rectified to direct current for transmission, and then inverted to alternating current at the receiving end of the line. It is easy to conceive of many other interesting applications of this circuit.

Inverter circuits may be divided into two main types, series and parallel. While considerable data have been published and many different circuits devised for the former type, very little beyond the fundamental circuit has been given for the latter. The purpose of this paper is to present the results of a study of the parallel type inverter which was undertaken to secure a better understanding of the function of each part and the operation as a whole. There is presented the necessary information concerning the operation of thyratrons in order that their operation in the circuit may be understood. The principles of operation of the inverter are developed by means of simple diagrams, and the actual operation under various conditions of loading is shown and discussed from complete sets of oscillograms. It is shown that the function of the condenser in the complete circuit may be considered to provide a phase relation necessary for operation, thereby differing from the description of its function that has hitherto been made. The regulation and efficiency will be indicated and briefly discussed.

THYRATRON CHARACTERISTICS

It is beyond the province of this paper to cover completely the characteristics of the thyratron but since the operation of the inverter cannot be understood without a knowledge of these, the essentials are given. Various types of thyratrons have been developed differing in their characteristics. The technical information concerning the more common and smaller types is listed in Table I. Much larger thyratrons than those listed are made for greater plate currents and higher anode voltages.

The thyratron, being a grid-controlled arc in low-pressure gas, differs considerably in operation from the

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Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

TABLE I

Type	FG-27	FG-57	FG-67
Main use	Controlled rectifier	Controlled rectifier	Inverter
Number of electrodes	3	3	3
Cathode—Volts	5.0	5.0	5.0
Amperes	6.0	5.0	5.0
Type	Coated	Indirect heater	Indirect heater
Max. peak inverse voltage	1,000.0 v.	1,000.0 v.	1,000.0 v.
Max. peak forward voltage	1,000.0 v.	1,000.0 v.	1,000.0 v.
Max. peak plate current	5.0 a.	15.0 a.	15.0 a.
Max. average plate current	2.5 a.	2.5 a.	2.5 a.
Max. tube voltage drop	24.0 v.	24.0 v.	24.0 v.
Min. tube voltage drop	10.0 v.	10.0 v.	10.0 v.
Deionization time (Microsec.)	1,000.0 approx.	1,000.0 approx.	100.0 approx.

ordinary vacuum tube. The characteristics essential for an understanding of its operation in the inverter circuit are given below.

1. The thyratron is a gas-filled tube dependent upon ionization of the gas by the passage of current to remove the space charge, thus allowing large currents to flow with very small and nearly constant tube voltage drops and consequently having a high efficiency with high anode voltage.

2. The potential of the grid controls the starting of the tube, but when the current has started and the gas has become ionized the grid has no further control. The grid has no control over the stopping of the flow of current nor of the amount of current flowing. The potential of the grid which will allow the current to start or that will prevent it from starting is dependent upon a number of factors, among them being the anode voltage, the temperature of the tube, and the inherent characteristics of the tube. Inverter tubes usually require the grid to be positive before the current will start.

3. Flow of current through the tube may be stopped in either of two ways; the anode may be made negative, causing the current to cease, deionization to take place and the grid to resume control, or the plate current may be momentarily interrupted giving the same effect. When alternating current is applied to the anode it becomes negative each half cycle, thus giving the first condition, and returning control to the grid each cycle. With direct current applied to the plate its potential must be made momentarily negative or the current interrupted for a sufficient length of time for deionization to take place before the grid can resume control.

4. The amount of current passed through the tube is dependent upon the characteristics of the external circuit, the average tube voltage drop of the order of fifteen volts having little effect. The safe amount of current is determined by the voltage drop across the tube and is limited by destructive ionic bombardment of the cathode. It has been found that as long as the current passed through the tube does not exceed the emission of the cathode, destructive bombardment will not take place.

The thyratron is inherently a rectifier, passing current in one direction only, thus it is often convenient to think of the device as a one-way contactor with an average drop of 15 volts irrespective of the amount of current. This view will help in the understanding of the operation of the inverter.

FUNDAMENTAL CIRCUIT

The development of the inverter from simple circuits involving direct-current-operated thyratrons has been described by several writers on the subject. A similar method will be used here with the aid of simple diagrams. The connections shown in Fig. 1 are indicative of a means of stopping the current in a direct-current-operated thyratron by momentarily making the anode potential negative. A condenser *C* is connected in series with a resistance across the load. The grid of the tube is normally biased negatively, thus no current can pass. If a positive impulse is applied to the grid circuit sufficient to overcome the bias the current will start. The grid has now lost control as the gas is ionized by the passage of the current. Assuming 500 volts being supplied, the drop across the load will be 485 volts as the tube drop is approximately 15 volts. The condenser being across the load will be charged at this potential. Now assume the switch *S* is closed; the right hand side of condenser *C*, which was at a positive potential of 485 volts, is suddenly dropped to zero volts. Due to the transient impedance of the load the left hand terminal of the condenser momentarily undergoes a similar drop which brings the anode potential to the value of 470 volts negative. If this negative voltage is maintained for a sufficient length of time, deionization takes place, the current is stopped, and the grid again assumes control.

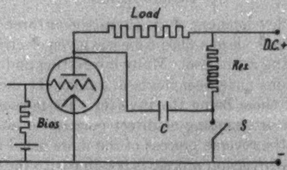

FIG. 1—METHOD OF STOPPING A THYRATRON OPERATED ON DIRECT CURRENT

First stage in inverter operation

The next step in the development of the inverter circuit is the substitution of another thyratron for the switch. The connections are shown in Fig. 2. Here the grids are shown connected to a transformer in such a manner as to cause them to be alternately positive and negative. If the grid of No. 1 tube is positive, current will be passing through it and through the load, No. 2 tube not operating because its grid is negative and the conditions will be the same as in the previous discussion with the switch open. Now as the grid potentials change, that of No. 1 will become negative, which will have no effect upon it, while that of No. 2 will become positive. As the potential applied to the anode of No. 2 is positive, current will flow through this tube and the conditions will be the same as in Fig. 1 with the switch closed, except that there is a tube drop of 15 volts. If the same supply potential is assumed as for the first case, the left hand terminal of condenser *C*, instead of falling to zero volts, now falls to positive 15 bringing the anode potential of No. 1 tube to 465 volts negative. Tube No. 2, it is seen, has performed the same function as the switch in Fig. 1. If the grids are made alternately negative and positive the current may be transferred back and forth between the two tubes, the major requirement for the operation being that the anodes are negative for a sufficient length of time for deionization to take place.

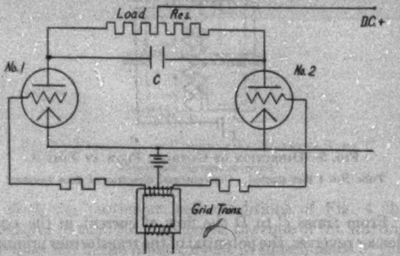

Fig. 2—Method of Stopping a Thyratron Operated on Direct Current by Means of Another Thyratron

Second stage of inverter operation

Substituting for the load and resistance of Fig 2, the primary of a transformer, gives the fundamental circuit of the parallel type inverter. This is shown in Fig. 3 with the addition of a choke coil in the supply lead to keep the alternating-current component out of the direct-current circuit. The actions taking place in this circuit are considerably more complicated than those in the circuit of Fig. 2, due to the addition of an inductance coil, the leakage reactance of the transformer, and the fact that considerable capacitance must be used. Another factor, which was not present in the circuit of Fig. 2, is the induced voltage of the transformer primary which greatly affects the operation of this circuit. A bias battery is shown so connected that the grids are biased negatively, but experience indicates that this is not needed. The frequency of the output is determined by the frequency applied to the grid transformer, which in this case is shown supplied from an external circuit. It is possible to make the circuit self-exciting, in which case the frequency is governed by the constants of the grid circuit.

OPERATION

The sequence of events in the operation of the circuit is illustrated by the oscillograms of Fig. 4 which give the voltages and currents in the different parts at approximately full-load unity power factor. In the tests herein described FG-67 inverter tubes were used, the condenser capacity being 4.5 μf., the inductance being 0.2 h., and the applied voltage of the order of 500 volts. While with this applied voltage the peak-inverse and peak-forward voltages were, under some conditions, rather more than the values given in the tube rating, no trouble was experienced from this cause. The circuit constants were chosen after much experimentation to give the best output wave form under widely varying conditions of load regardless of other factors such as regulation. In taking the oscillograms the input voltage was adjusted until the output voltage was 110, the grid voltage frequency being 60 cycles. The connection diagrams associated with the oscillograms of Fig. 4 show the positions of the oscillograph galvanometer elements in the circuit, and the various curves are numbered accordingly. The results are purely qualitative as no attempt was made to calibrate the elements.

It may now be of interest to analyze the conditions in each part of the circuit step by step throughout the cycle of operation. Referring to Fig. 4 at time *A*, curve 1 shows that the grid potential was zero, which at the applied plate potential was sufficient to permit the plate current to start as shown in curve 2. The condenser immediately charges through half of the primary in the transformer as shown in curves 3 and 4. The current in this half would have become zero except for this charging current. The current in the other half of the primary, which was slightly negative, becomes positive as shown in curve 5. A high potential difference between the anode and cathode of the tube drops to a low value as soon as the current starts, as shown by curve 7, the anode being positive. This tube voltage drop of between 10 to 24 volts does not show on the oscillograms as the scale required by the inverse voltage was too

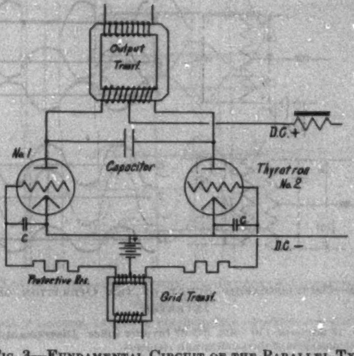

Fig. 3—Fundamental Circuit of the Parallel Type Inverter

large. The curves of 8, 9, 10 and 11 are the voltages of the halves of the transformer primary, the whole primary, and the secondary. The current in curve 12 is the output current which, when compared with the voltage of curve 11, shows that the load was non-inductive. The flow of current in the circuit an instant after time *A* is illustrated by Fig. 5, the arrows indicating the direction of flow from positive to negative according to the usual manner.

At time *B* the transformer primary and secondary voltages and the output current are zero. Comparing these with curve 2 indicates that this condition does not exist until some time after the current through the tube has started, thus showing that the latter leads the induced voltages. This is a characteristic of this type of inverter that will be found throughout its operation and it is the function of the condenser in the circuit to provide this phase relation. It will be shown later that when the power factor of the load is sufficiently leading the condenser is no longer needed, the necessary phase relations for inverter operation being caused by the load.

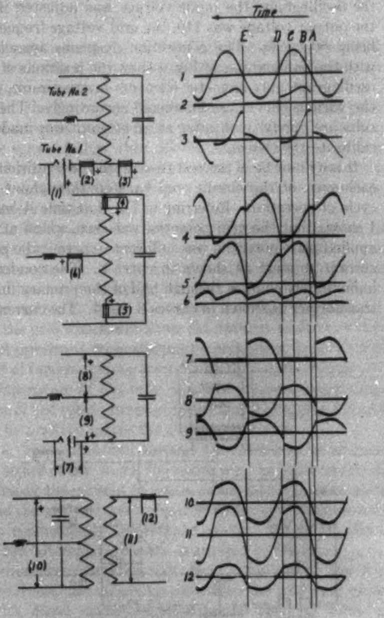

FIG. 4—OSCILLOGRAPHIC STUDY OF THE OPERATION OF THE INVERTER

Load, 18 amperes, 110 volts, FG-67 inverter tubes. Diagrams at the left show the positions of the oscillograph elements

Time *C* is interesting as at this time the flow of current into the condenser has stopped as indicated by curve 3, and consequently the current through one-half of the primary has also stopped as shown in curve 4. It is also noted from curves 8, 9, 10, 11 and 12 that at this time the transformer voltages and output current are at nearly their maximum value. The current flow conditions at this time are illustrated in Fig. 6.

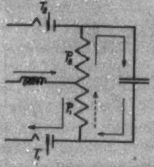

FIG. 5—DIRECTION OF CURRENT FLOW AT TIME *A*

Tube No. 1 just started. Conventional direction of flow assumed

From times *C* to *D* the flow of current in the condenser reverses, the potential of the transformer primary falling, thus leaving the condenser at a higher voltage; this flow of current naturally follows. The reversal of current in the condenser is shown in curve 3 and the consequent reversal of current through one-half of the transformer primary in curve 4. This reversed current continues until the other thyratron starts. The current conditions at this time are shown in Fig. 7.

Time *D* illustrates the stopping of thyratron No. 1 and the starting of No. 2. Curve 1 shows that when the grid of No. 2 becomes zero, current starts to pass through this tube, the flow through thyratron No. 1 stops as indicated by curve 2 and the condenser charges in the reverse direction as shown by curve 3. Curve 7 indicates the negative potential which appears between the anode and cathode of No. 1 thyratron which permits deionization to take place. The time that this anode remains negative is given by the distance between the sudden negative drop to the place where the curve again crosses the zero line in the positive direction. This time, while short, is long enough for the gas content of the tube to become deionized as this requires only approximately 100 microseconds for the tubes being used. From the time of starting of the other

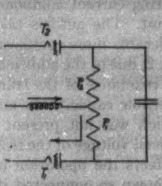

FIG. 6—DIRECTION OF CURRENT FLOW AT TIME *C*

Condenser fully charged

tube the conditions in the circuit are similar to those already described, tube No. 2 replacing No. 1 and the other half of the transformer primary carrying the tube current. It is seen that only half of the primary is active at a given time as far as output is concerned. This fact must be taken into account when choosing the transformer ratio desired for an inverter circuit. The current conditions at time *D* are illustrated by Fig. 8.

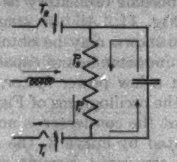

Fig. 7—Direction of Current Flow After Time *C*

Condenser current reversed

With the particular load conditions of Fig. 4 the stopping of tube No. 1 occurs at the time of the starting of tube No. 2, the input current in consequence of this action not reaching zero as shown by curve 6. With other conditions of load one tube stops conducting some time before the other starts, as shown by the other sets of oscillograms. The stopping of current flow through the thyratrons can be explained in a different manner than was assumed in the description of the development of the inverter circuit as the condenser may be thought of as performing a different function in the complete circuit. It is seen that the potential of half the transformer primary is shown in curve 9, the anode end being negative to the center when the tube is operating and positive when no current is passing. Voltage only appears between the cathode and anode on the oscillograms under the condition of no current flow, the tube drop being too small to show. If due to the starting of tube No. 2 there was a sudden change in voltage across the condenser terminals it would appear in curve 10, but there is no evidence of such change. The method of stopping the operation of one tube when the other starts in the case of the conditions shown in Fig. 4 may be analyzed as follows. Just an instant before time *D*, tube No. 1 is conducting as shown by curve 2 and the voltage of its half of the primary is shown by curve 9. At time *D* the grid of tube No. 2 becomes zero, which is sufficient to allow it to start carrying current, the anode potential being positive as is seen in curve 8. When this occurs the full potential of the primary less the tube drop of No. 2 is impressed on tube No. 1, the cathodes, being connected together, completing the circuit. This action causes the cathode of No. 1 tube to become for a time more positive than its anode, thus stopping the current flow. The voltage impressed on this tube is shown in curve 10. The anode remains negative to the cathode for a sufficient length of time for deionization to take place, and when it does become positive the grid potential is sufficiently negative to prevent the current from starting. From this analysis it follows that the action of the condenser is to give the correct phase relations for operation, which differs from the earlier conception of its purpose in the description of the development of the circuit.

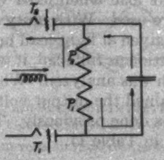

Fig. 8—Direction of Current Flow at Time *D*

Tube No. 2 starts conducting, tube No. 1 stops

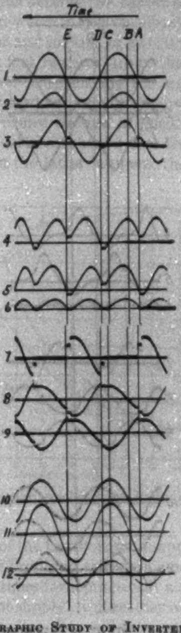

Fig. 9—Oscillographic Study of Inverter Operation at Light Load

Output, 5 amperes, 110 volts, non-inductive. Positions of oscillograph elements as in Fig. 4

Oscillograms for light-load operation are shown in Fig. 9. It is seen that the shapes of the curves are in many cases greatly altered, and that the output waveform is better. An analysis of the operation in this and in succeeding sets of oscillograms does not differ materially from that just made. In the case just discussed one thyratron stopped only when the other started. Curve 2 of Fig. 9 shows that No. 1 tube stops before No. 2 starts. This is apparently due to the constants of the whole circuit including the load. When this happens the anode potential becomes negative due to the potential of one-half of the primary coil, the latter being shown by curve 9 and the sudden drop to this value appearing in curve 7. When the second tube starts putting the full primary potential on the first tube, the anode potential becomes still more negative as shown by the second drop in curve 7. In other words, the anode voltage becomes practically first that of curve 9 and then that of curve 10.

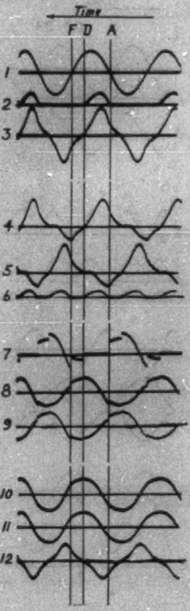

Fig. 10—Oscillographic Study of Inverter Operation with Inductive Load

Output, 19 amperes, 110 volts, 11.5 per cent power factor, condenser 6¾ μf., inductance 0.20 h.

Low Power Factor Loads

The parallel type inverter will carry low power factor loads when the circuit has the proper constants. An example of the operation with lagging current is given in the oscillograms of Fig. 10. The output voltage and current, curves 11 and 12, show that the current was lagging and that the power factor was very low. In this case, as in the case just described, the current through one tube stops some time before that through the other starts. This operation is characterized by the two distinct negative voltage drops appearing between the cathode and anode of the stopped tube as shown by curve 7.

With incorrect constants and low power factor loads the circuit may become oscillatory as is shown in the oscillograms in Fig. 11. All the stages from correct operation to that shown may be obtained by varying the values of the inductance and capacity.

Operation with a low power factor, leading-current load is shown in the oscillograms of Fig. 12. It is interesting to note that the condenser is not needed in this case, as is indicated by curve 3, the necessary phase relation between the input current and the induced primary voltage being caused by the load. The fact that this circuit will operate under this condition is proof that the condenser is needed only to give the necessary phase relation and not to fulfill the function assumed in the development of the circuit.

Fig. 11—Output with Inductive Load, Incorrect Circuit Constants

Wave Form

The output wave forms differ with the amount and power factor of the load when fixed values of capacity and inductance are used. With unity power factor, the best wave form is given at light load but as this is not as interesting a condition as full load, it was the wave form of the latter which was analyzed. When the curves for analysis were obtained the output voltage was not held at 110 volts as was done previously. The results of the analysis are given in Table II.

Regulation and Efficiency

The value of the capacitor used in the circuit has great bearing on the voltage regulation of this type of inverter. It also has a large effect on the output wave form so that a reasonable compromise must be made. A low capacity gives the best voltage regulation and the poorest wave form and may cause instability. Capacity of too large a value reduces the output voltage to a considerable extent.

The efficiency of the inverter using thyratrons is high due largely to the high efficiency of the tubes them-

TABLE II

Effective voltage	122 volts
Average voltage	115 volts
Form factor	1.06
Peak factor	1.30
Maximum value of the 3rd harmonic	16.75 volts
Maximum value of the 5th harmonic	6.35 volts
Maximum value of the 7th harmonic	2.80 volts
Maximum value of the 9th harmonic	1.18 volts

selves. With an average tube voltage drop of only 15 volts and with reasonably high anode voltage, the losses in the tube are small and the efficiency high. Fig. 13 shows efficiency and voltage regulation curves in which it is seen that the regulation with 4½ μf. capacity is much poorer than with 1⅛ μf. However, to secure a reasonably good output wave form it was necessary to use the higher value.

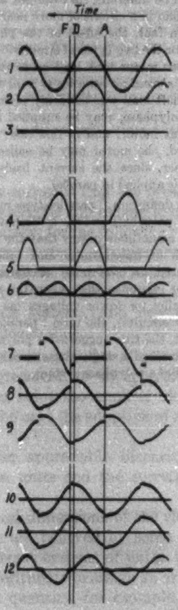

Fig. 12—Oscillographic Study of Inverter Operation with Capacitative Load

Output, 18 amperes, 110 volts, 10.1 per cent power factor, no condenser, inductance 0.20 h.

Conclusions

The oscillograms give in a graphical way the complete operation of the inverter under widely varying load conditions. An analysis indicates that the operation differs somewhat from the descriptions previously made, and it is felt that a more complete picture of the functions of each part and of the operation as a whole is here presented than has hitherto appeared. The circuit is a most interesting one and already has been applied in a commercial way for supplying alternating-current power to alternating-current radio sets where operation on direct-current distribution systems is required.

Nothing has been said about parallel operation of inverters with systems supplied by synchronous machines. This is a perfectly feasible method of operation and has the advantage that no condenser is required in the inverter circuit, thus making the operation more stable.

The two factors which may give difficulty are the poor voltage regulation and the lack of stability under certain conditions. The first requires, up to the present time, that some method of regulating the input voltage be used. The second, which largely disappears in parallel operation, will require the use of quick acting circuit breakers to protect the tubes in case the circuit stops operating. When the latter happens, the counter electromotive force of the transformer disappears, and both the tube voltage drop and the resistance of the circuit being small, the tubes form practically a short circuit on the direct-current supply. Unless they are very quickly removed from the line they will be injured.

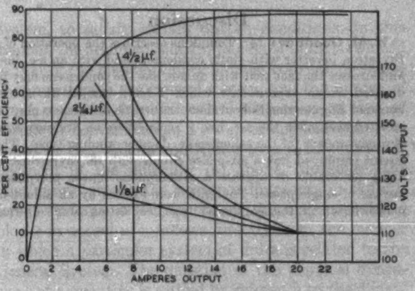

Fig. 13—Efficiency and Voltage Regulation Curves

There is nothing to indicate that the difficulties of operation are insurmountable, and since the growth of electrical engineering seems to be in the direction of an increasing use of thermionic devices it may be expected that circuits of this nature will rapidly become of commercial importance.

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Discussion

W. M. Goodhue: Mr. Tompkins describes the operation of a thyratron inverter with static condensers and with an a-c. line, and stresses the fact that with an a-c. line the condensers may be omitted. Some experiments which I have carried on at the Harvard Engineering School show that static condensers may be omitted also when the a-c. line is replaced by an ordinary synchronous motor (of the salient-pole, damper-winding type). The circuit employed was a six-phase circuit with six thyratrons, it being preferable to operate a synchronous motor polyphase, instead of single-phase. The grids were excited by an induction phase-shifter of the polyphase type, consisting of a slip-ring induction motor (1/2 hp.), with the rotor turned by a handwheel and worm drive, and insulating transformers for the rotor output. *This phase-shifter was supplied directly from the terminals of the synchronous motor*, so that the inverter was permanently *locked in step* with the motor, and would automatically follow any speed changes. Fundamentally, the phase-shifter determines the *power factor* of the inverter, while the d-c. field of the motor enables adjustment of the a-c. voltage. The a-c. current is nearly proportional to the d-c. current, so that voltage, current, power factor, *and hence power* delivered to the synchronous motor may be adjusted, and if this power exceeds the requirements of the motor, the motor will speed up and vice versa. Hence the speed *and frequency* may be varied by the phase-shifter. The d-c. current and voltage are related by the volt-ampere characteristic of the d-c. supply circuit, so that the frequency *vs.* phase-shifter characteristic will be different, for example, in a constant-potential d-c. circuit than in a circuit of more nearly constant-current characteristics. In fact, the effect of the phase-shifter is often almost opposite for the two types of circuits.

The synchronous motor took load up to the maximum rating of the tubes, and *showed no tendency to hunt or to fall out of step*, even when the shaft load was suddenly changed. An a-c. load, single-phase or polyphase, may be supplied from the terminals of the synchronous motor, and in this case, if the shaft of the motor is unloaded, the motor may be called, as usual, a synchronous condenser, since the current leads, unless a highly capacitative load is applied in parallel.

Concerning the definition, "parallel-type inverter," some confusion arises. In polyphase work, a "series-type rectifier (or inverter) circuit" sometimes means that the different phases of a polyphase bank of transformers and tubes have their d-c. terminals connected *in series*, to accommodate the higher voltages, with the inference that when the various phases are connected in parallel for lower voltages (as in multiple-anode polyphase railway circuits), the term "parallel-type" should be employed. In fact, the title suggested a polyphase circuit, while Mr. Tompkins described a single-phase circuit, such as might be a single-phase unit of a polyphase system.

A General Theory of Systems of Electric and Magnetic Units

BY VLADIMIR KARAPETOFF*

Fellow, A.I.E.E.

***Synopsis.**—A system of electric and magnetic units may be shown to be characterized by five parameters, namely: a numeric, n, which gives the ratio between the density of electric displacement used in that system and the so-called theoretical density; a numeric, p, which gives the corresponding ratio for magnetic flux densities; a physical conversion factor, k, which converts a given volume of current into the corresponding magnetomotive force; the absolute permittivity, κ; and the absolute permeability, μ. On the basis of these five parameters, the principal fundamental equations of electricity, magnetism, and electromagnetic waves are written in what the author calls the general system of units, without assigning definite values to these parameters. He also shows that the five parameters must satisfy the equation* $vk = \sqrt{np/\kappa\mu}$, *where v is the velocity of propagation of electromagnetic waves in that particular medium to which κ and μ refer; otherwise the five parameters may be chosen arbitrarily.*

By giving these parameters specific values, seven different systems of units are derived, namely, the electrostatic, the electromagnetic, the practical, the Gauss system, the Heaviside-Lorentz system, one which the author calls the compromise system, and the ampere-ohm system introduced by him some twenty years ago.

The characteristics, the advantages, and the disadvantages of each system are briefly discussed, and it is shown how to deduce a new system of units from the general system, to conform to certain desired specifications.

At the end of the article a discussion is given on the physical dimensions of various electric and magnetic units. The author states that four fundamental units are necessary for a system, so that if a certain length, mass and time interval are taken as three fundamental units, a fourth electric or magnetic unit must be added, for example, the permittivity or the permeability. In the ampere-ohm system two mechanical fundamental units are used, the centimeter and the second, and two electrical units, the ampere and the ohm.

* * * * *

IT IS generally conceded that the present situation in regard to the electric and magnetic units is far from being satisfactory, for the following reasons:

a. Two or three systems of units are in general use (c.g.s. and practical), at least two more are employed by some prominent writers (the Gauss system and the Heaviside-Lorentz system), and new ones are being proposed from time to time.

b. The absolute magnitudes of some units fixed by international agreement, or in general use otherwise, are considered as being too small or too large by some investigators, who urge the adoption of smaller or larger units.

c. There is an appreciable disparity between the "absolute" c.g.s. units and the corresponding "international" units.

d. The physical dimensions of the electric and magnetic quantities are believed by some scientists to be different in different systems of units. Some physicists even believe it legitimate to assign an arbitrary physical dimension to a quantity, for example, to assume the absolute permeability of any medium to be a numeric; see Section IX below.

It is not the purpose of the present investigation to extol one of the existing systems of units in preference to the others, or to propose a new system of units. The author merely intends to show that all the existing systems are special cases of a General System of Units and may be obtained from it by assigning definite numerical values to certain parameters. Moreover, new systems of units may be produced out of this General System, to satisfy certain desired conditions. In this manner, the argument about systems of units is reduced to that of the possible choice of most convenient values of parameters in the general system, and the burden of proof is shifted from a general discussion to that of particular convenience in a group of applications. It is hoped in this way to contribute to a satisfactory adjustment of the difficulty (a) above. Moreover, the relationship among the corresponding units in the different systems, which is one of the main stumbling blocks in actual use, follows directly from the manner in which a particular system of units is related to the General System. The question of the physical dimensions of a quantity is settled by reference to the dimensions of that quantity in the General System of Units. The same physical quantity can hardly be expected to have two different physical dimensions in two special systems derived from the same General System. This question is discussed more fully in Section IX below.

The difficulty (b) above has existed and always will exist in many branches of quantitative natural sciences. It is clear that atomic dimensions and interstellar distances cannot be conveniently measured with the same yardstick, any more than the current of a photoelectric cell and the current through a large metallurgical steel furnace. This difficulty is minimized by using a multiplying factor of ten to a convenient integral power. From the point of view of applications it is immaterial whether the millimeter or the meter is used as a primary unit, or whether a millisecond or a megasecond is chosen as the primary unit of time. The same applies to units of magnetic flux, power, etc. The use of the factor ten to an integral power is not only inevitable, but quite convenient in applications.

The difficulty (c) is in the process of being rectified.

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Due to a better experimental technic, it ought to be possible before long to reach a new agreement in regard to the slight corrections to the present international values of the ampere and the ohm, so as to bring them more closely in accord with the corresponding multiples of the c.g.s. values.

* * * * *

In order to derive a consistent set of equations of the principal phenomena in electricity and magnetism, we shall abandon the historical point of view and look upon these phenomena as being known to us without any chronological precedence of discovery. Besides, we shall introduce factors of proportionality in a general form, without ascribing to them definite values, such as 1, 4π, c, etc. In this manner, it will be possible to derive some general relationships first and obtain various existing systems of units as special cases later. The kinship of the systems and their salient features will then become apparent.

I. ELECTROSTATIC RELATIONSHIPS

We begin with the recognized fact that an electrostatic field may be established, say in vacuum, by suitably placing electric charges here and there. We agree to measure the intensity of such a field at a point by the mechanical force exerted by the field upon a small concentrated electric charge, say an electron, placed at that point. Thus, one of our starting points is the equation

$$f_e = Gq \tag{1}$$

where f_e is the mechanical force, G the field intensity (voltage gradient) and q the probe charge.* The subscript e stands for "electrostatic." The force f_e is to be measured in any chosen mechanical units which are to go with the electric and magnetic units. G thus becomes defined as the mechanical force per unit charge. Of course, equation (1), by itself, is not sufficient to define both unknowns, q and G, and it is to be looked upon only as one of a set of simultaneous equations which together are sufficient for defining the principal electric and magnetic quantities; see Appendix. Consequently, it is immaterial whether equation (1) or equation (8) is introduced first, the latter being the familiar Coulomb's law of inverse squares. The fundamental quantities are not defined "step-by-step," but all the fundamental phenomena and the equations which express them are considered on a par in the set of final formulas.

For purposes of definition of units equation (1) may be said to contain two unknown quantities in the same sense as the familiar relationship in elementary mechanics

$$\text{force} = \text{mass} \times \text{acceleration}.$$

Here the acceleration may be assumed to be known from a more elementary subject, namely kinematics; however, the unit mass must be also defined before this equation, alone, can be used for the definition of unit mechanical force.

All the equations which are to be used in the deduction of the quantities in the proposed General System of Units must be based either on immediate results of some measurements, or on conclusions derived from observed facts and from an assumed mechanism at work. Some physicists may be inclined to consider equation (8) to be more fundamental than equation (1), yet the latter is being constantly verified in various measurements on electronic velocities, accelerations, and impacts. With the advance in the physical and chemical theories, certain ultra-microscopic mechanisms have to be postulated that lead to relationships which can be measured in bulk only. Thus, the Boltzmann-Maxwell distribution of molecular velocities in a gas, the mean free path, etc., can be verified only indirectly. From this point of view, equation (1) is also an experimental relationship and not an *a priori* assumption.

As another fundamental phenomenon, let us consider the dielectric flux and its density due to an electric charge, say Q, concentrated upon a sphere of very small radius ζ. It is an experimental fact that the total dielectric flux due to such a charge is the same through any imaginary concentric sphere or other surface which encloses the charge. In the language of theoretical physics, the divergence of an electrostatic flux in a space devoid of electric charges is equal to zero. The simplest assumption to make is that this flux is numerically equal to the charge which causes it, since experiment shows the two to be proportional to each other. We shall call the value of the flux expressed in such units as to be numerically equal to the corresponding charge the "theoretical" value. In a given system of units it may be necessary to multiply this value by a numerical factor in order to obtain the actual flux in that system of units. At a distance $R > \zeta$ from the center of our charge, the area of the imaginary concentric sphere is $4\,\pi R^2$, so that the "theoretical" density of displacement is

$$D_t = Q/(4\,\pi R^2) \tag{2}$$

where the subscript t stands for "theoretical." In a chosen system of units another value of density of displacement, say D, proportional to D_t, may be used, so that, generally speaking,

$$D = nD_t \tag{3}$$

where n is a numeric. Thus, for example, in the c.g.s. electrostatic system, as used by Maxwell, $n = 4\,\pi$.

The physical properties of a dielectric in which an electric displacement takes place are expressed by its so-called absolute permittivity, which we shall denote by κ. Except under very special laboratory conditions, there is no saturation in dielectrics, and experiment shows that G is proportional to D. We thus have, as a definition of κ,

$$D = nD_t = \kappa G \tag{4}$$

As a special case, for vacuum,

$$D_v = nD_{vt} = \kappa_v G \tag{5}$$

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*For complete Notation see page 726.

It is convenient to express the absolute permittivity of a medium by indicating the number of times by which it is greater than that of vacuum, in the same system of units. We may thus write

$$\kappa = K\kappa_v \qquad (6)$$

where κ is the absolute permittivity of the dielectric in question, κ_v is that of vacuum, and K is known as the dielectric constant or specific inductive capacity. It is also proper to call K the relative permittivity of the dielectric. K is a numeric, whereas κ and κ_v are physical quantities which have, or may have, physical dimensions. Substituting for D_t its value from equation (2) in equation (4), gives

$$G = nQ/(4\pi\kappa R^2) \qquad (7)$$

Equation (1) therefore becomes

$$f_e = nQq/(4\pi\kappa R^2) \qquad (8)$$

This is the familiar Coulomb's law of inverse squares in an isotropic dielectric whose absolute permittivity is K. Instead of being accepted as a fundamental experimental fact, this law has been deduced here from two other general experimental facts, namely, (a) in a homogeneous isotropic dielectric which is free of charges an electrostatic flux is solenoidal, that is, its divergence is zero; and (b) an electrostatic stress at a point and the corresponding density of displacement are proportional to each other. For the purposes of this paper it is immaterial whether equation (8) is assumed as fundamental and the conclusion that the flux is solenoidal deduced from it, or *vice versa*. In either case, experimental facts are strictly adhered to, and the reader need not have an uneasy feeling that physical truths are being derived from mental ideas.

As a digression, it may be of interest to mention that the attraction or repulsion between two point charges in a three-dimensional space being inversely as the square of the distance, may be thought of as a consequence of the fact that the areas of two concentric spheres are as the squares of their radii. In other words, the density (and consequently the intensity) of the electric field due to one of the charges varies inversely as the square of the distance, because the flux has no divergence. Similarly, in a two-dimensional space with solenoidal fields, the mechanical force between two point charges may be expected to vary as the first power of the distance, because circles take the place of the spheres. This is confirmed by the fact that the mechanical force between two parallel infinitely long filamental charges, per unit length, varies inversely as the distance between them. If it may be assumed that the electric flux in a Euclidean space of Z dimensions is also solenoidal, the mechanical force between two concentrated charges should be taken as inversely proportional to $(Z-1)$st power of the distance, because of the geometric properties of the space.

Let Φ_e be an electrostatic flux expressed in some system of units and let Φ_{et} be the same flux in "theoretical" units, defined above by the condition that

$$\Phi_{et} = Q \qquad (8a)$$

From equation (3) we then obtain

$$\Phi_e = nQ \qquad (8b)$$

The latter expression is Gauss' theorem in the General System of Units.

II. Magnetic Relationships

On the assumption that free magnetic poles actually exist, a set of formulas may be written down by analogy with those developed for electric charges above. True, such an assumption is out of harmony with the modern ideas of molecular structure, and a theory of magnetism can be built up without bringing in the idea of free poles at all. However, the proposed General System of Units must include the c.g.s. electromagnetic system as a special case, and since the latter has the Coulomb law of mechanical force between free magnetic poles for its starting point, it is advisable to include the concept of magnetic pole in our General System of Units. The formulas containing such poles explicitly need not be used in applications.

By analogy with G, we define the magnetic (or magnetizing) intensity, H, at a point in a magnetic field as the mechanical force exerted at that point upon a unit magnetic pole. This gives the relationship.

$$f_m = Hm \qquad (9)$$

analogous to equation (1). Here m is the strength of a probe magnetic pole, and the subscript m signifies that a magnetic phenomenon is meant. By analogy with equation (2) we have

$$B_t = M/(4\pi R^2) \qquad (10)$$

where M is a concentrated magnetic pole and B_t is the so-called "theoretical" density of magnetic flux which M produces at a distance R from its center. Equation (5) becomes

$$B = pB_t = \mu H \qquad (11)$$

Here B is the magnetic flux density used in a chosen system of units, p is a numerical coefficient which gives the ratio between this flux density and the above-defined theoretical flux density; μ is the absolute permeability of the medium. Of course, μ may have definite physical dimensions, although at present they cannot be expressed through those of length, time, and mass alone. By analogy with the absolute and relative permittivity, we introduce the notation

$$\mu = \mu_r\mu_v \qquad (12)$$

Here μ is the absolute permeability of a medium, μ_r its relative permeability, and μ_v is the absolute permeability of vacuum. The quantity μ_r is a numeric. By analogy with equations (7) and (8), we may write

$$H = pM/(4\pi\mu R^2) \qquad (13)$$

$$f_m = pMm/(4\pi\mu R^2) \qquad (14)$$

The latter equation is Coulomb's law of inverse squares for magnetic poles. By analogy with equations (8a) and (8b), Gauss' law for magnetic poles becomes:

$$\Phi_{mt} = M \qquad (14a)$$

$$\Phi_m = pM \qquad (14b)$$

III. THE TWO LAWS OF CIRCUITATION

When electricity is in motion along certain paths, magnetic lines of force are formed in such a manner as to be linked with the paths of the current. This general experimental fact may be quantitatively expressed by the following relationship:

$$k[(1/n)\,(\partial\Phi_e/\partial t) + IN] = \oint H\,ds\cos(H, ds) \qquad \textbf{(15)}$$

In this equation the right-hand side represents the line integral of magnetic intensity H along a closed path; this line integral is known as the magnetomotive force. The relationship is similar to that between an electric intensity, G, and its line integral known as the electromotive force. On the left-hand side of the equation we have the total volume of current threading the closed curve chosen on the right-hand side. In the most general case, in an imperfect dielectric, the current is partly that of displacement of electric charges and partly a true conduction current. Φ_e is an electrostatic flux due to moving charges; dividing it by n converts it into the theoretical flux or actual charge, according to equations (**8a**) and (**8b**). The rate of change of this charge with the time gives the displacement current. The term NI represents the volume of the conduction current, where I is the current itself and N the number of turns through which it is flowing. The coefficient k characterizes a chosen system of units and converts a current into the corresponding magnetomotive force. In a perfect dielectric the IN term is equal to zero. In a metal conductor the term containing Φ_e may be omitted. The factor k may be assumed to have definite physical dimensions, because it converts an electric quantity into something different. What the dimensions of k are, we do not know.

In a special case of an infinitely long straight conductor carrying current I, the lines of magnetic force are concentric circles. At a distance R from the center line of the conductor, the length of a line of force is $2\,\pi R$, so that, with Φ_e = const. and $N = 1$, equation (**15**) becomes

$$kI = 2\,\pi RH \qquad \textbf{(16)}$$

which is the general form of the law of Biot and Savart.

An equation similar to equation (**15**) may be written for a change of a magnetic flux with the time (Faraday's law of induction). In the General System of Units this law becomes:

$$-\,(k/p)\,(\partial\Phi_m/\partial t) = \oint G\,ds\cos(G, ds) \qquad \textbf{(17)}$$

The right-hand side is the line integral of electric intensity G over a closed curve, in other words, the electromotive force induced along this curve. The magnetic flux threading through this curve is Φ_m, and its rate of change with the time is experimentally known to be proportional to this e.m.f. The factor p converts the flux expressed in a particular system of units into the "theoretical" number of lines of force, equation (**14b**). The factor k characterizes the system of units used and converts a time rate of change of a flux into an e.m.f. At first sight, no reason is evident why the factor k should be the same in equations (**15**) and (**17**), but this is proved below from energy considerations. In equation (**17**) the term corresponding to NI is omitted, because no magnetic current is known to exist which corresponds to an electric conduction current.

As a special case of equation (**17**), consider a uniformly wound torus ring connected to a source of direct current through a switch. The ring is also provided with a concentric secondary winding consisting of N turns and connected to a ballistic galvanometer. Let the primary circuit be initially open and then suddenly connected to a source of direct current. The absolute magnitude of the secondary induced voltage at an instant t will be $(k/p)\,N(\partial\Phi_m/\partial t)$. This voltage is used up in the resistance drop ir and the inductive drop $L\,di/dt$ of the secondary circuit. Thus, for an interval of time dt we have:

$$(k/p)N\,d\Phi_m = ir\,dt + L\,di \qquad \textbf{(18)}$$

Integrating this expression with respect to time, from the instant at which the primary switch is closed until the flux has practically reached its final value and the secondary current vanished, we obtain:

$$(k/p)\,N\Phi_m = rQ \qquad \textbf{(19)}$$

where Q is the total electric discharge through the ballistic galvanometer (integral of idt) in units of electric charge. The term with L vanishes because $i = 0$ at both limits. Equation (**19**) is more convenient in some applications than equation (**17**), and moreover it connects a magnetic flux with an electric charge, which is of importance in studying units.

IV. STORED ENERGY

The energy stored in a charged condenser may be looked upon either as concentrated on the metal electrodes, or as distributed in the dielectric in the form of electrostatic stresses and displacements. Of course, numerically the two expressions for the stored energy are identical, but the second form permits us to introduce the useful concept of density of stored energy, or energy stored per unit volume of the dielectric. Consider in particular a parallel-plate condenser which is being gradually charged from zero to a charge Q. At some intermediate instant, t, let the instantaneous accumulated charge be q and let dq be the charge added during the interval dt. If the stress in the dielectric is G and the distance between the plates a, the work done in bringing this additional charge will be:

$$dW_e = aG\,dq \qquad \textbf{(20)}$$

The true electrostatic flux density is q/A, where A is the area of either plate, so that, in accordance with equation (**4**),

$$G = (q/A)\,(n/\kappa) \qquad \textbf{(21)}$$

Consequently

$$dW_e = anq\,dq/A\kappa) \qquad \textbf{(22)}$$

or, after integration,

$$W_e = anQ^2/(2\,A\kappa) \tag{23}$$

Dividing both sides of this equation by the volume of the dielectric, aA, gives the density of electrostatic energy

$$W_e' = W_e/(aA) = n(Q/A)^2/(2\,\kappa) = nD_i^2/2\,\kappa \tag{24}$$

From this expression we may immediately write

$$W_e' = nD_i^2/(2\,\kappa) = D^2/(2\,n\kappa) = GD_i/2 = GD/(2\,n) = \kappa G^2/(2\,n) \tag{25}$$

Since the magnetic concepts introduced above have been assumed to be formally analogous to those in electrostatics, we may also write

$$W_m' = pB_i^2/(2\,\mu) = B^2/(2\,p\mu) = HB_i/2 = HB/(2\,p) = \mu H^2/(2\,p) \tag{26}$$

V. PROOF THAT THE FACTOR k IN EQUATIONS (15) AND (17) IS THE SAME

Let us calculate the magnetic energy stored in a torus ring excited with direct current. This energy is equal to the electric energy put into the exciting winding during the process of bringing the flux from zero to its final value. According to equation (17), that portion of the applied voltage which at an intermediate instant t balances the counter e.m.f. of the flux, is:

$$e = (k/p)N\,d\Phi_m/dt \tag{27}$$

Multiplying both sides of this equation by $i\,dt$, where i is the instantaneous value of the exciting current, we obtain

$$eidt = (k/p)Ni\,d\Phi_m \tag{28}$$

The left-hand side of this equation gives the energy contributed from the source of electric power. Consequently, the right-hand side represents the increase in the stored magnetic energy. In order to be able to integrate the right-hand side, between zero and the final value of the flux, we shall express both i and Φ_m through the magnetic intensity H. If the cross-section of the flux is A and the flux density B, we have

$$\Phi_m = BA = \mu HA \tag{29}$$

Let the average length of the lines of force within the torus ring be a; then the total magnetomotive force is aH, and according to equation (15) it is equal to $k'iN$. Here the value of k is temporarily denoted by k', and it is required to prove that it is equal to the value of k in equation (17). Thus,

$$Ni = aH/k' \tag{30}$$

Substituting the values from equations (29) and (30) in equation (28) and integrating between the limits $H = 0$ and $H = H$, we obtain for the stored magnetic energy the expression

$$W_m = (k/k')\mu H^2 \,.\, aA/(2\,p) \tag{31}$$

so that the density of energy is

$$W_m' = (k/k')\mu H^2/(2\,p) \tag{32}$$

Comparing this expression with formula (26), we see that $k' = k$.

VI. RELATIONSHIP BETWEEN THE PARAMETERS OF A SYSTEM AND THE VELOCITY OF LIGHT

So far the five parameters n, p, k, κ, and μ of a system of units have been assumed to be entirely arbitrary and independent of one another. It will now be shown that they must satisfy the experimental fact that the velocity of propagation of electromagnetic waves in a medium characterized by κ and μ is equal to the velocity of light in that medium. Consider a plane electromagnetic wave propagated in a perfect dielectric in the direction of the X-axis, the electric and magnetic intensities being parallel to the Y and Z-axes respectively. For any point in the medium, equation (15) becomes (references 12, 14, 16)

$$(k/n)\partial D/\partial t = \partial H/\partial x \tag{33}$$

while equation (17) gives

$$(k/p)\partial B/\partial t = \partial G/\partial x \tag{34}$$

In these equations, D may be replaced by κG and B by μH, so that only the functions G and H remain. H can be eliminated by taking a partial derivative of equation (33) with respect to t and a partial derivative of equation (34) with respect to x. The result of elimination is the following well-known partial differential equation of wave propagation for G as a function of t and x:

$$v^2\partial^2 G/\partial x^2 = \partial^2 G/\partial t^2 \tag{35}$$

An identical equation may be deduced for H. In these equations the new parameter, v, is determined by the relationship

$$vk = \sqrt{np/(\kappa\mu)} \tag{36}$$

The parameter v may be shown to be the velocity of propagation of electromagnetic waves in the particular medium characterized by the absolute permittivity κ and absolute permeability μ. For the vacuum the well-known magnitude of the velocity of propagation of light or of electromagnetic waves is usually denoted by c, so that we have

$$ck = \sqrt{np/(\kappa_v\mu_v)} \tag{37}$$

This is the relationship which must be satisfied by the parameters n, p, k, κ_v and μ_v of a system of units. In other respects the parameters may have arbitrary values. Thus, we are free to choose four out of five parameters, and the fifth must satisfy equation (37).

It must be understood that a choice of numerical values of four out of the five parameters merely singles out a group of systems of units, and not a specific system of units, because one is still at liberty to select desired "non-electrical" fundamental units, such as those of length, time and force (or mass). Therefore, it is not meant that in equation (37) the velocity of light should necessarily be measured in centimeters per second or in some such universally used units. Having chosen arbitrarily the values of four electromagnetic parameters, some units of length and time must also be chosen independently, and the velocity c

expressed in these units before equation (**37**) may be applied. Thus, the velocity of light is placed on no special or preferred basis.

The use of equation (**37**) will become more clear in the applications in the next section.

VII. TABULATION OF THE PARAMETERS FOR SOME EXISTING SYSTEMS OF UNITS

In Table I the values of the five parameters which characterize a system of units are shown for the principal systems in use or proposed. The "eruption of 4 π's," using Heaviside's expression (reference 5), is apparent in the first four systems, and has been one of the reasons for the introduction of the later systems indicated in columns 5 and 7. The table may be conveniently studied in connection with the list of the principal formulas given in the Appendix. These formulas are explained in the text above and the numbering is the same.

It is not the purpose of this paper to advocate or to defend any particular system of units, but only to show how all the existing systems and an infinite number of new ones can be derived by giving different values to the five parameters which satisfy equation (**37**) and also choosing desired units of length, time and, say, mass. Looking first at the values of n and p in Table I it will be seen that the existing systems may be divided into those in which the value of these parameters is 4 π, and those in which this value is equal to unity. The first group of systems may be called "classical," the second "rational," or "rationalized." The term rational has been often used by the defenders of the latter systems, and is employed here in that specific sense, without in any way implying that the 4 π systems are "irrational." The author is almost tempted to suggest the adjective "pi-less" in place of "rational."

The advantage claimed for a rational system ($n = p = 1$) is that the flux density in such a system (both electrostatic and magnetic) becomes equal to the theoretical flux density; equations (**3**) and (**11**). This means that the total electrostatic flux due to a charge is both numerically and dimensionally equal to that charge; equation (**8b**). The same applies to a magnetic pole and the flux issuing from it; equation (**14b**). Thus, Gauss' theorem simply becomes $\Phi_e = Q$ and $\Phi_m = M$.

As another advantage of a rationalized system it is pointed out that in dealing with geometric shapes of fields, the factor π appears with cylinders and spheres, as is to be expected, and does not appear with rectangular shapes. The opposite is the case with the classical systems. For example, in the c.g.s. electrostatic system of units the capacitance of a sphere in vacuum is equal to its radius R, and the capacitance of a plane-parallel condenser is $\kappa A/4\pi a$. On the other hand, in the Heaviside-Lorentz system these values are $4\pi R$ and $\kappa A/a$ respectively. Again, in the electromagnetic system, the density of stored magnetic energy in a non-magnetic body is $B^2/8\pi$, whereas in the ampere-ohm system it is $B^2/2\mu_v$. The fact that μ_v in the latter system contains 4π is not supposed to weaken the argument. The user of this system of units simply knows that μ for ordinary non-magnetic bodies is equal to 1.257×10^{-8}, this being an experimental physical constant, and the presence of the factor 4π need not be explained theoretically, as is the case with the classical systems.

Of course, an expert who has been using the classical systems for a number of years finally learns to place the factor 4π where it belongs, but undoubtedly this factor is a stumbling block to a student and a young investigator. In a rational system, the factor 4π is retained in the expression for Coulomb's law of inverse squares, equations (**8**) and (**14**), for the reason that 4π enters in the formula for the surface of a sphere in equations (**2**) and (**10**).

Substituting the values of the five parameters of the electrostatic system in equation (**37**), we obtain

$$4\pi c = \sqrt{(4\pi)^2 c^2}$$

which is an identity. In the same manner it may be shown that the parameters of the other six systems given in the table satisfy equation (**37**).

It will also be seen that no particular units of length and time need be specified so long as it is only desired to express the electromagnetic parameters in terms of c

TABLE I—VALUES OF THE PARAMETERS IN SOME TYPICAL SYSTEMS OF UNITS

System number	1	2	3	4	5	6	7
Parameter	Electrostatic	Electromagnetic	Practical	Gauss	Heaviside-Lorentz	Compromise	Ampere-ohm
k	4π	4π	4π	$4\pi/c$	$1/c$	$4\pi/c$	1
n	4π	4π	4π	4π	1	1	1
p	4π	4π	4π	4π	1	1	1
κ_v	1	$1/c^2$	$1/c^2$ (not used)	1	1	$1/(4\pi)$	$10^9/(4\pi c^2)$
μ_v	$1/c^2$	1	1	1	1	$1/(4\pi)$	$4\pi/10^9$
Fundamental units	cm gram sec κ	cm gram sec μ	10^9 cm 10^{-11} gr sec μ	cm gram sec κ, μ	cm gram sec κ, μ	cm gram sec κ, μ	cm. ampere ohm sec.

algebraically. Thus, one may say that there is an infinite number of electrostatic systems of units, all characterized by the values $k = n = p = 4\pi$ and $\kappa_v = 1$. For all such systems $\mu_v = 1/c^2$. With the centimeter and the second independently chosen as fundamental non-electrical units, $c = 3 \times 10^{10}$, and therefore $\mu_v = (1/9) \times 10^{-20}$. With the kilometer and the microsecond as such units, $c = 0.3$ and $\mu_v = 1/0.09$. In some investigations it is convenient to choose $c = 1$; in the corresponding electrostatic system $\mu_v = 1$.

A similar reasoning applies to all the columns in Table I, each column (with the exception of the last) standing for an infinite number of systems of units all characterized by the same values of four out of five parameters, the fifth being expressed in terms of the velocity of light, c. The last horizontal row in Table I indicates which particular units of length, time, and mass have been chosen in the system of each type actually in use. These non-electrical units may be changed at will, still retaining the general electric and magnetic characteristics of the system.

The symbol κ written at the bottom of column 1 means that historically the electrostatic system of units was built on the assumption of a definite magnitude for the absolute permittivity of the vacuum, more particularly $\kappa_v = 1$. The symbol μ at the bottom of the second and third columns signifies that these systems were built on the assumption $\mu_v = 1$. In the systems shown in the columns 4 and 5 it was simultaneously assumed that $\kappa_v = 1$ and $\mu_v = 1$. In column 6 it is postulated that $\kappa_v = \mu_v = 1/(4\pi)$, for reasons of symmetry explained later. In the ampere-ohm system no attempt is made to fix the values of κ_v and μ_v, this particular system having been devised to take advantage of the existing values of the ohm and the ampere (irrespective of their historical origin) and of the simplicity of having $k = n = p = 1$.

It is left for the reader to substitute the values of the parameters given in Table I in the various formulas given in the Appendix and to satisfy himself that familiar expressions are obtained as a result. It may be of interest here to mention briefly the reasons for which the systems shown in the table have been proposed and used.

In the beginning, say late in the eighteenth century, no connection was known between electric charges and magnetic poles; therefore, there was no necessity for expressing the magnetic and the electrostatic quantities in one and the same system of units. Since the concept of "action at a distance" was prevalent among the French savants of that time, a deduction of formula (**8**) in a manner similar to that given in the text above was not possible. Two charges or two magnetic poles were found by experiment to exert upon each other a force inversely as the square of the distance, so that the natural assumption was (especially for the mathematical school of physicists) to write $f_e = Qq/R^2$ and $f_m = Mm/R^2$, for air or vacuum. The added assumption was that in the electrostatic system κ_v was equal to unity and in the electromagnetic system μ_v was equal to unity. Later the values of relative permittivity and permeability were added as a factor in the denominators of these formulas, to account for newer experimental results. These two formulas thus served as a basis for the c.g.s. electrostatic and electromagnetic systems of units respectively. In our notation, the omission of the factor 4π in the denominators of equations (**8**) and (**14**) is equivalent to the assumption $n = p = 4\pi$; it is in this way that the factor 4π came into the mathematical theory of electricity and magnetism and has kept physicists struggling with various systems of units ever since.

Later, Biot and Savart found experimentally that the magnetic field due to an infinitely long straight current-carrying conductor could be expressed by the formula $H = 2I/R$, and Laplace gave his familiar differential expression for the interaction between a magnetic pole and an element of current, in the form $H = ids \cos \theta/r^2$. This was equivalent to assuming $k = 4\pi$ in equations (**15**) and (**16**), and also meant $k/p = 1$ in equations (**17**) and (**19**), as was later found by Faraday. Of course, those investigators did not think in terms of the General System of Units here described, but their assumptions based on other arguments were equivalent to assigning the values of the parameters indicated in Table I. Having four out of five parameters fixed, the remaining ones (μ_v in the electrostatic system and κ_v in the electromagnetic system) also became determined.

Later it was found that the ratios of the various quantities in the two systems, 1 and 2, were simple functions of the velocity of light, c, although the reason for this did not become clear until after Maxwell had published his electromagnetic theory of light. This relationship between κ_v, μ_v and c is the one given by equation (**37**) which connects the five parameters with the velocity of light. It is because of this relationship that the absolute magnetic permeability of vacuum in the electrostatic system is found to be numerically (although not necessarily dimensionally) equal to $1/c^2$, and the same value comes out for the permittivity of the vacuum in the electromagnetic system.

Consider again equation (**8**), first written in an electrostatic system of units and then in an electromagnetic system, the fundamental units of length, time, and mass being the same in the two systems. Assume equal charges, $q = Q$ and $q' = Q'$, in vacuum; let primed letters refer to the electromagnetic system. Let R be the same in both cases, and let q and q' be physically the same charge, so as to have the same force on the left-hand side. We then have

$$q^2 = c^2(q')^2$$

where the factor c^2 comes in on account of the value of the permittivity in the second system. Thus, the unit of electric charge in an electromagnetic system is c times that in an electrostatic system with the same fundamental non-electrical units. In the particular case of the two c.g.s. systems the ratio equals 3×10^{10}, this being the numerical value of c. By a similar reasoning, the ratio of the units in any two systems in Table I can be readily determined, it being only necessary to write the same formula in the two systems of units.

The so-called practical system of units, with the volt, the ampere, and the ohm as the fundamental units, was originally introduced as a modification of the c.g.s. electromagnetic system, because of inconvenient sizes of the units of potential, current, and resistance in the latter system. The actual assumed ratios are as follows:

one volt = 10^8 c.g.s. electromagnetic units.
one ampere = 10^{-1} c.g.s. electromagnetic units.
one ohm = 10^9 c.g.s. electromagnetic units.

With the magnitudes of any two of these units fixed, the third one is determined by Ohm's law. At the bottom of column 3 in the table, two of the basic non-electrical units of the practical system are given as 10^9 cm. and 10^{-11} gram. While theoretically this would be so if the practical system of units were used as a *pure* system, in reality it is used as a *hybrid* system. Most practising engineers would be surprised were they told that the current density in a generator or transmission line should be measured in amperes per quarter of the earth's meridian squared. In applications, current density and flux density are measured per sq. cm., or per sq. mm., and no one measures magnetizing force in gilberts or ampere-turns per quarter of earth's meridian. Moreover, the practical system of units has not been universally extended to the magnetic quantities; magnetic fluxes are often expressed in maxwells, with the factor 10^{-8} added at proper places. The same is true of many electrostatic computations which are conducted in the c.g.s. system and only the results are converted say into microfarads.

VIII. SOME MORE RECENT SYSTEMS OF UNITS

Since the systems characterized in columns 4 to 7 in Table I are less familiar than the first three systems, it may be of interest to analyze briefly their raison d'être from the point of view of the formulas given in the Appendix.

A glance at column 4 will show that those who prefer the system associated with Gauss' name (refs. 2 and 15) do so because they find it convenient to assume both the absolute permittivity and the absolute permeability of the vacuum to be equal to unity. This, together with the assumption $n = p = 4\pi$, means that in this system electric charges are measured in electrostatic units and magnetic poles or fluxes in electromagnetic units. The other quantities come out in one or the other system of units depending upon the method of their derivation, that is, whether through the parameter n or p. It will be noted that the parameter k has the factor c in it, since c has been omitted from both κ_v and μ_v. This automatically adjusts the equations in which both electrostatic and electromagnetic quantities enter. For example, on the left-hand side of equation (16), when written in the Gaussian system, I is expressed in electrostatic units, being derived from Q, whereas H is in electromagnetic units. Dividing I by c (which enters in k) converts it into electromagnetic units. In equations (17) and (19), dividing Φ_m by c converts it into electrostatic units, to agree with the quantities on the right-hand side of the equations. In other words, the Gaussian system enables one to express a quantity in that system from which it has been fundamentally derived; the factor c is put explicitly in the circuitation equations, thus doing away with converting the quantities before substituting them in these equations.

A system of units in which $n = p$ and $\kappa_v = \mu_v$ may be called a symmetrical system, and the Gaussian system, as well as those shown in columns 5 and 6, are symmetrical systems. One of their advantages is that the fundamental formulas for electrostatic and magnetic relationships become formally identical. Thus, formulas (7) and (13), as well as (8) and (14) have the same factor $n/\kappa = p/\mu$. The same is true of the circuitation equations (15) and (17), provided that $I = 0$, and of the energy equations (25) and (26). This symmetry is considered by some physicists to be quite valuable in teaching and in some investigations.

Heaviside also believed such characteristics of a system of units to be desirable, especially in theoretical investigations of general nature where a formal symmetry of magnetic and electric equations and of the two circuital relationships is important. He objected only to 4π which enters in three parameters in the Gaussian system. What he did was equivalent to changing the values of n and p from 4π to 1 (making the system "rational" in his sense). As a result, 4π also dropped out of k, in accordance with equation (37). An inspection of equation (8) will show that, in effect, Heaviside left f_e and the denominator on the right-hand side unchanged, but omitted the factor $n = 4\pi$ in the numerator. Consequently, the numerical values of q and Q had to become $\sqrt{4\pi}$ times greater, or, which is the same, the unit of charge made smaller in the same proportion. The same reasoning applies to equation (14), so that in the Heaviside system of units all the principal electric and magnetic quantities differ from those in the two c.g.s. systems by factors containing $\sqrt{4\pi}$. Moreover, the quantities themselves are either in electrostatic or in electromagnetic units (*not* c.g.s.), depending upon the method of their derivation, precisely as the Gauss' system, and the factor c is introduced in the value of k. The Heaviside system of units has been used by some prominent writers in Europe and in the United States (refs. 5, 9, 11, 13) because of the simplicity and symmetry of the formulas. In general theoretical deductions, such as the properties of electromagnetic waves, motions of electrons, and the like, the magnitude of the units is of no particular consequence until the final formulas have been reached. In substituting the numerical values, the factor $\sqrt{4\pi}$ has to be properly taken into account. A detailed tabulation of the ratios between the classical and Heavisidian units will be found in reference 11.

To the author's knowledge, the system tabulated in column 6 and called the compromise system has been neither proposed nor used heretofore. It is mentioned here, not because he is in favor of it, but merely to show how easily a new system of units could be devised on the basis of the General System of Units in which the equations in the Appendix are expressed. Assume a hypothetical investigator or teacher who is dissatisfied with the two c.g.s. systems, first because the user constantly has to convert quantities from one to the other, and secondly because of the presence of 4π in the values of n and p, making it necessary to consider carefully whether this factor enters in the numerator or denominator of a particular formula. He is not satisfied with the Gauss system either, because of the same 4π, and he objects to Heaviside's system of units because one has to convert the usual c.g.s. units into his units with the aid of the factor $\sqrt{4\pi}$. So, to accomodate this scientist, we devise a new "rational" system of units, that is, one for which $n = p = 1$. The value of κ_v in equation (8) has to be equal to $1/4\pi$ in order to have the same unit of charge as in the c.g.s. electrostatic system. The same is true of μ_v in equation (**14**). Having the values of the four parameters thus prescribed, equation (37) gives $k = 4\pi/c$.

It may be stated in passing that the occurrence of 4π in the expressions for κ_v and μ_v is not nearly so objectionable from the beginner's point of view as in the parameters n and p. After all, κ_v and μ_v are physical constants, and the user or the student merely needs to know that with the universally accepted values of units of current and resistance these constants come out numerically equal to 0.07958 ($= 1/4\pi$). He need not question the reason for this value any more than he does the numerical value of the conductivity of copper, the electronic charge, or any other physical constant. On the other hand, when a flux is taken to be 4π times the charge which causes it, and when 4π enters everywhere with parallel planes and disappears with cylinders and spheres, much more explanation is necessary. Perhaps a frank admission that 4π should have been used in the initial statement of both Coulomb's laws is the easiest and most honest way out. In Goethe's words, "woe unto you because you are a grandson!" In other words, had Heaviside been born fifty years earlier, before the two c.g.s. systems and their units became firmly established, his views and his system of units would probably have been universally accepted.

The dilemna then is, either to use new fundamental units for current, resistance, etc., as Heaviside did (column 5), or to preserve the present units and accept 4π in the values of κ_v, μ_v, and k, as is done in the compromise system (column 6). Had Heaviside urged his system before the older values of Q and M came into general use, his solution would have been perfect. We simply would have had different values of units of current, e.m.f., etc., than we do at present. A glance at column 5 does not show a single 4π.

The ampere-ohm system (refs. 7 and 8) shown in the last column has been laid out to satisfy the following requirements:

1. The new system of units must be "rational," that is, $n = p = 1$, but for practical purposes it need not be symmetrical, that is, μ_v need not be equal to κ_v, as in the columns 4, 5, and 6.
2. It must be based upon the generally recognized international units, to wit, the ampere, the ohm, and their derivatives, the volt, the farad, the henry, the ampere-turn, etc. However, the historical connection of these units with the corresponding c.g.s. units must nowhere appear explicitly.
3. There should be only one set of units, for both electromagnetic and electrostatic computations, because in modern physics and engineering both kinds of quantities often enter in the same equations.
4. The velocity of light c and the factor 4π must be completely eliminated from the working formulas so that their presence need not be explained theoretically.
5. The question of inconvenient sizes of the fundamental units need never arise, because the prefixes milli, micro, mega, etc., are to be freely used, as well as 10 to suitable integral powers (a hybrid system). For practical uses, a system need not be pure. A hybrid system with milliamperes, megohms, square decimeters, etc., is just as convenient, if not more, than a pure consistent system with inconvenient magnitudes of the units.
6. The values of κ_v and μ_v are to be taken as physical constants, with definite physical dimensions, and need not be simple integers, such as 1 or 2, any more than we expect other physical constants to be simple integers in a chosen system of units.
7. With the ampere, the ohm, and the second as the fundamental units, the joule becomes the natural unit of energy, and therefore one joule per centimeter the fundamental unit of mechanical force; the author has used the name *joulecen* for this unit of force since 1910 (ref. 8).
8. The mechanical force being included under the fundamental units, no unit of mass need be specified. The centimeter and the second are to be taken as the two other non-electrical fundamental units.

In laying out such a system, we assume from the beginning that $n = p = k = 1$, thus rationalizing it completely in so far as the general theory of electricity and magnetism is concerned. The factor π is to appear only in problems involving surfaces or volumes of cylinders and spheres. The ampere-turn becomes the unit of magnetomotive force, and H in equations (**15**) and (**16**) is to be expressed in ampere-turns per centimeter. The right-hand side of equation (**17**) being in volts, and

$k/p = 1$, the unit flux is such a flux which induces one volt per turn when this flux disappears or is built up at a uniform rate exactly within a second. In other words, fluxes are measured in volt-seconds; the name "weber" has also been proposed for such a unit.

To deduce the value of μ_v in the ampere-ohm system theoretically, consider equation (**11**), $B = \mu_v H$, written in this system, and compare it with the same relationship, $B' = H'$, in the c.g.s. electromagnetic system. Since the unit of flux in the ampere-ohm system is 10^8 times that in the other system, we have $B' = 10^8 B$. The magnetizing force, H, in the ampere-ohm system is equal to NI/a, where I is in amperes. In the c.g.s. electromagnetic system, $H' = 4\pi(NI/a)\ 10^{-1}$, because in this system $k = 4\pi$ and the unit of current is equal to ten amperes. Thus, we have

$$10^8 B = (4\pi/10) NI/a \qquad \textbf{(38)}$$

and also

$$B = \mu_v NI/a \qquad \textbf{(39)}$$

Comparing these two expressions, we find that in the ampere-ohm system

$$\mu_v = 4\pi/10^9 \qquad \textbf{(40)}$$

To deduce the value of κ_v in the ampere-ohm system theoretically, use equation (**8**) in application to two equal charges at a distance R, in vacuo. In the ampere-ohm system

$$f_e = Q^2/(4\pi\kappa_v R^2) \qquad \textbf{(41)}$$

whereas in the c.g.s. electrostatic system

$$f_e' = (Q')^2/R^2 \qquad \textbf{(42)}$$

Here f_e is expressed in joulecens and f_e' in dynes. Since a joule is equal to 10^7 ergs, a joulecen is equal to 10^7 dynes; that is, $f_e' = 10^7 f_e$. We first convert Q into c.g.s. electromagnetic units (abcoulombs) by dividing it by 10, and then convert abcoulombs into electrostatic units by using the conversion factor c. Thus, $Q' = cQ/10$. Substituting this value in equation (**42**) and comparing the result with equation (**41**) we get

$$\kappa_v = 10^9/(4\pi c^2) \qquad \textbf{(43)}$$

It will be seen that the values of the five parameters given in column 7 of Table I satisfy equation (**37**). In fact, having determined μ_v, the value of κ_v could have been deduced directly from equation (**37**).

The reader will thus see that with the general equations of electricity and magnetism written in the form given in the Appendix, new systems of units can be multiplied indefinitely, as shown on a few examples above. Each system so far developed satisfies certain "specifications" and may be the best system for a particular purpose or set of problems. It is hardly feasible to speak of the best all-around system of units; at least none has been universally accepted as such so far, despite numerous discussions and arguments. With the General System of Units expounded in this paper, all the systems now in existence, and possibly some of the new ones to be evolved in the future, may be looked upon as specific cases of this system. Therefore, the argument about systems of units becomes localized along a few definite lines, such as the following: (a) Is it desirable to have a "rationalized" system of units; (b) Should the best system be also "symmetrical"; (c) Should any of the existing international electric or magnetic units be made part of it; (d) What are to be the fundamental non-electrical units; (e) Should it be a pure or a hybrid system of units; etc. It will be readily seen that anything like a universal agreement among the physicists and engineers of the various countries on all these points is a remote possibility. A more hopeful initial program may consist in a gradual elimination of the older and less used system of units.

IX. PHYSICAL DIMENSIONS OF QUANTITIES

The broad question of physical dimensions of units is not necessarily connected with the introduction of the General System of Units, and persons holding quite opposite views on dimensional analysis (ref. 21) may find the proposed derivation of various systems of units from a single more general system useful. A brief discussion of physical dimensions is included in the paper, partly for the sake of completeness and partly to bring out a subsequent discussion of conflicting views on the subject of dimensional formulas. The controversy hinges on whether the physical dimensions of a quantity, say mechanical force or inductance, are inherent in that quantity or are a matter of convenience and somewhat arbitrary. The author takes the point of view that for those investigators whose field of activity lies primarily in applied problems, it is both simpler and safer to consider the physical dimensions of a quantity as inherent in its nature (although perhaps unknown to us). With this assumption, the ratio of the magnitudes of the same quantity in two different systems of units can be only a numeric. It would be out of the question to state that the ratio of the units of electric charge in the two c.g.s. systems is equal to the velocity of light; this ratio is an abstract quantity and can only numerically equal the velocity of light. Otherwise it could be claimed that the ratio of a square meter to a square foot is equal to 10.76 gram-seconds!

The opposite point of view is ably defended by some leading physicists skilled in dimensional analysis, and the author can best indicate their general attitude by quoting an esteemed colleague who saw this paper in manuscript form: "Many people seem slow to realize that physical dimensions are purely a matter of convenience, as Bridgman has so ably shown in his little book on Dimensional Analysis. I think Professor Karapetoff becomes almost mystical in this last section of his paper. However, he has plenty of company in the attitude he takes. . . . In several places throughout the paper in speaking of dimensions the author speaks as if dimensions were inherent objective properties amenable to physical investigation. This attitude toward dimensions is similar to the attitude taken in a paper by Professor Kennelly on a related subject published a year or two ago. It will appear to many readers, including the present one, that the assignment of dimensions is altogether arbitrary and subject to no physical investigation; it is suggested, therefore, that a discussion should be included of this matter, proving either that dimensions are inherent and amenable to physical investigation or else that they are purely arbitrary. . . . . As an example, the dimensions of volume may be considered. It is customary to assign the dimension L^3 to volume for the reason that the volume of a cube, for instance, is obtained by raising its side to the third power. It would be equally correct to say that the volume of the cube equals a constant of numerical value unity but having some arbitrary dimension assigned to it multiplied into the cube of the side. In this case we are impelled to make the constant dimensionless by the feeling that the result will be greater simplicity rather than by the knowledge that volume inherently possesses the dimension L^3."

The author grants some strength to this argument and is the first one to welcome a generalization of our customary doctrines in the hands of scientists of the first rank who work upon an extension of the fundamental physical concepts and the underlying philosophy of ideas in physics. He only feels that for the rank and file of the profession it is safer and wiser to adhere to a more limited attitude of inherent physical dimensions. The remarks which follow represent distinctly this point of view, and the author hopes that more advanced and flexible views upon the subject may be contributed to the discussion of this paper.

In our present state of knowledge, it is impossible to express the physical dimensions of electric and magnetic quantities in terms of mechanical units alone, for example, of mass, length, and time. The dimensions of at least one electric or magnetic quantity have to be assumed as fundamental, in addition to the mechanical units. For example, in the electrostatic system of units the numerical value of κ, is put equal to unity, but its physical dimensions should be taken as something fundamental and not reducible to the units of length, mass, and time. This is indicated in the last horizontal row of Table I by stating whether κ or μ is used as the fourth fundamental unit. In the systems 4, 5, and 6, the dimensions of some units are expressed through κ, of others through μ. In the ampere-ohm system the physical dimensions as well as the magnitudes of the ohm and the ampere are considered as fundamental. For this reason, only two mechanical fundamental units are necessary, those of length and time. Energy and force are expressed in terms of electric power, I^2r.

It is also possible to take the physical dimensions of voltage, inductance, electrostatic flux, or any desired electric or magnetic quantity as fundamental, and to express the physical dimensions of the rest of the quantities in its terms. For an engineer and applied physicist it is safer to adhere to the view that in nature the physical dimensions of an electric or magnetic quantity are intrinsic in that quantity and do not depend upon a human system of units or upon a choice of the fundamental units. While outwardly these dimensions may look different in different systems of units, this may be simply due to our ignorance of the intrinsic nature of the phenomena involved. It is therefore safer not to maintain (as is sometimes done) that in the electrostatic system of units absolute permittivity is a numeric, whereas in the electromagnetic system its physical dimensions are those of a velocity to the power of minus two.

From a practical point of view, a more conservative statement is as follows: In the electrostatic system of units the physical dimension of permittivity is a fundamental dimension. Therefore, calling the numerical value of an absolute permittivity (κ) and its physical dimension $[\kappa]$, we have

$$\kappa = (\kappa)\,[\kappa] \tag{44}$$

In the electromagnetic system of units μ is a fundamental unit. From equation (**36**), the dimension of permittivity (assuming for the present k to be a numeric) is

$$[\kappa] = [\mu]^{-1}\,[v]^{-2} \tag{45}$$

Therefore, in this system

$$\kappa = (\kappa)\,[\mu]^{-1}\,[v]^{-2} \tag{46}$$

Intrinsically, however, the physical dimensions indicated in equations (**44**) and (**46**) are identical; their product has the dimensional formula (time/length)^2, to agree with equation (**37**).

In the ampere-ohm system, the dimensions of κ may be deduced from equation (**5**). The flux density has the dimensions of a quantity of electricity divided by an area, or $[\mathrm{D}] = [i][t][A]^{-1}$. The dimensions of the voltage gradient are those of a voltage divided by a length, or $[G] = [I]\,[r]\,[a]^{-1}$. Consequently,

$$[\kappa] = [D][G]^{-1} = [t][r]^{-1}[a]^{-1} \tag{47}$$

The physical dimensions of various quantities in the principal systems of units in use will be found in refs. 4, 11, and 17. For the corresponding tabulation in the ampere-ohm system see refs. 7 and 8. It may also be added that from the point of view maintained by the author the ratio of unit charges in the two c.g.s. systems is only *numerically* equal to the velocity of light in vacuo; the ratio itself can only be a numeric.

The problem of physical dimensions is further complicated by the question of the physical dimensions of the parameter k. In equation (**15**) this parameter changes a current into a magnetomotive force, and in equation (**17**) it converts a rate of change of flux into a voltage. Since the purely physical side of these transformations is still enshrouded in mystery, it may be just as well for the present *provisionally* to consider k to be a numeric. From the physical point of view this may be equivalent to the assumption that there are no transformations in kind, so that a current *is* a magnetomotive force and a change in flux *is* an induced e.m.f.

Nevertheless, it is well to keep in mind the possibility of a later discovery of the intrinsic nature of these circuital relations. For this reason, it seems both wise and considerate towards future generations not only to preserve this parameter, but to grant beforehand the possibility of its having definite physical dimensions. This may forestall for our posterity a difficulty similar to the one we are having with 4π.

The author therefore heartily seconds Dr. W. Jaeger's proposal, (ref. 3, table on page 35) of including the dimensions of k with those of the magnetic quantities. When studying his table the reader should keep in mind that Jaeger's k is different from the one used in this article, being defined by his equations (28) and (29), even though the general purpose of both parameters is the same.

NOTATION

A	cross-section of an electric or magnetic path
a	length, usually of a line of force
B	magnetic flux density
c	velocity of light in vacuum
D	dielectric flux density, or density of displacement
e	as a subscript, means "electrostatic"
f	mechanical force
G	potential gradient, or electric field intensity
H	magnetizing force, or magnetic field intensity
I, i	electric current
k	a parameter which characterizes a system of units with respect to the two laws of circuitation, equations (15) and (17)
K	relative permittivity, or dielectric constant
L	inductance of a circuit
M, m	strength of a magnetic pole
m	as a subscript, means "magnetic"
N	number of turns in a winding
n	a parameter which characterizes a system of units and gives the ratio of the dielectric flux density used in that system to the so-called theoretical flux density; equation (3)
p	a parameter which characterizes a system of units and gives the ratio of the magnetic flux density used in that system to the so-called theoretical flux density; equation (11)
Q, q	magnitude of an electrostatic charge
R	distance measured from the center of a charge or conductor
r	electrical resistance of a circuit
r	as a subscript, means "relative"
s	variable length along a closed curve
t	time
t	as a subscript means "theoretical"
v	velocity of electromagnetic waves in a perfect dielectric
v	as a subscript, means "vacuum"
W	stored electrostatic or magnetic energy
W'	same, per unit volume
x	direction of propagation of a plane electromagnetic wave
κ	absolute permittivity of a dielectric
μ	absolute permeability of a magnetic medium
Φ	electrostatic or magnetic flux
ζ	radius of a small sphere

Appendix

TABULATION OF THE PRINCIPAL FORMULAS IN THE GENERAL SYSTEM OF UNITS

The equation numbers are the same as in the text.

$$f_e = Gq \tag{1}$$

$$D_t = Q/(4\pi R^2) \tag{2}$$

$$D = nD_t \tag{3}$$

$$D = \kappa G \tag{5}$$

$$\kappa = K\kappa_v \tag{6}$$

$$G = nQ/(4\pi\kappa R^2) \tag{7}$$

$$f_e = nQq/(4\pi\kappa R^2) \tag{8}$$

$$\Phi_e = nQ \tag{8b}$$

$$f_m = Hm \tag{9}$$

$$B_t = M/(4\pi R^2) \tag{10}$$

$$B = pB_t = \mu H \tag{11}$$

$$\mu = \mu_r \mu_v \tag{12}$$

$$H = pM/(4\pi\mu R^2) \tag{13}$$

$$f_m = pMm/(4\pi\mu R^2) \tag{14}$$

$$\Phi_m = pM \tag{14b}$$

$$k[(1/n)(\partial\Phi_e/\partial t) + IN] = \oint Hds \cos (H, ds) \tag{15}$$

$$kI = 2\pi RH \tag{16}$$

$$-(k/p)\,(\partial\Phi_m/\partial t) = \oint Gds \cos (G, ds) \tag{17}$$

$$(k/p)N\Phi_m = rQ \tag{19}$$

$$\begin{aligned} W_e' &= nD_t^2/(2\kappa) = D^2/(2n\kappa) = GD_t/2 \\ &= GD/(2n) = (\kappa G^2/(2n) \end{aligned} \tag{25}$$

$$\begin{aligned} W_m' &= pB_t^2/(2\mu) = B^2/(2p\mu) = HB_t/2 \\ &= HB/(2p) = \mu H^2/(2p) \end{aligned} \tag{26}$$

$$(k/n)\,\partial D/\partial t = \partial H/\partial x \tag{33}$$

$$(k/p)\,\partial B/\partial t = \partial G/\partial x \tag{34}$$

$$v^2\partial^2 G/\partial x^2 = \partial^2 G/\partial t^2 \tag{35}$$

$$v^2\partial^2 H/\partial x^2 = \partial^2 H/\partial t^2 \tag{35a}$$

$$vk = \sqrt{np/(\kappa\mu)} \tag{36}$$

$$ck = \sqrt{np/(\kappa_v\mu_v)} \tag{37}$$

Literature References

For citation of numerous articles on units in periodicals, see references 3 and 4 below.

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10. J. C. Maxwell, "A Treatise on Electricity and Magnetism," see under Units in the Index at the end of Vol. II.

11. Leigh Page, "Introduction to Theoretical Physics," 1928, p. 316; pp. 421 to 425.

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13. Leigh Page and N. I. Adams, "Principles of Electricity," 1931, pp. 435-443; p. XII.

14. *Ibid.*, pp. 551 to 569.

15. G. W. Pierce, "Electric Oscillations and Electric Waves," 1920, pp. 359 and 510.

16. *Ibid.*, pp. 370 to 381.

17. "Standard Handbook for Electrical Engineers," Section 1.

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27. A. C., "Un système d'unités pratiques doté de quatre dimensions fondamentales," *R.G.E.*, Vol. 23, p. 113 (1932).

28. Louis Joly, "L'état actuel de la question des unités électriques et magnétiques," International Congress of Electricity, Paris, 1932, Section II, Report No. 1.

Discussion

Lloyd Champlin Eddy: Such statements on the general relations of units as were presented by Mr. Karapetoff may prove to be of ultimate value for more thoroughly understanding universal relations, even of certain forms of life also. Who knows what manifestations of life are not fundamentally electric in nature? Perhaps two physical phenomena most resembling life are radio-activity and magnetism, both electrical phenomena. Both of these phenomena apparently depend on certain relations or numbers of electrons or atoms much as animal life itself appears mainly where carbon is present in more or less definite relations with other elements. The extension of our knowledge of the general relations of units in electrical engineering as well as other fields, therefore, may be of considerable interest also to those interested not only in the fundamental units of science and electrical engineering but in universal relations of physical life.

A Proposal to Abolish the Absolute Electrical Unit Systems

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***Synopsis.**—The specification of a physical quantity must include numerical value and unit. The expression "dimension" is defined as the general unit of a quantity and it is shown that applied mathematics has to deal with physical quantities rather than pure numbers, so that each equation yields a relation between numerics and dimensions. It is possible to reduce all dimensions to a set of a few fundamental dimensions, called a "dimension system." The application of this concept to the various fields of physics is shown, and it is proved that the so-called "absolute" dimension systems in electromagnetism are incorrect from this point of view and lead to inconsistent results. The proposal is made, therefore, to abolish the "absolute" dimension systems in electromagnetism and to introduce two new dimension systems as extensions of the two well-known mechanical systems by adding just one electric fundamental dimension. The new proposal retains all the familiar units but puts them on the sound basis of correct dimension systems, which, it is hoped, eliminates the unfortunate discrepancy in electromagnetism which has caused so much trouble ever since the "absolute" systems were introduced. The adoption of the new proposal, however, requires the cooperation of physical as well as electrical engineering societies, especially with regard to possible international action.*

* * * * *

INTRODUCTION

THE fundamental discovery that the ratio of the "absolute" electrostatic unit to the "absolute" electromagnetic unit of electric charge, predicted as a velocity, was apparently identical with the velocity of light,[1] brought about an unforseen confusion in the field of electrical units. The use of various "absolute" dimension systems, expressing the electric and magnetic quantities by means of the three mechanical fundamental dimensions, influenced the writing style of formulas and caused the introduction of factors in various places where they seemed quite arbitrary. At the same time that A. E. Kennelly delivered a paper on "magnetic reluctance"[2] and caused a long discussion as to the position of the factor 4π, Heaviside in London prepared a paper on "the eruption of the 4π's."[3] Heaviside was apparently the first to notice that the dielectric constant and the permeability of free space ought to be considered as physical quantities, but he made no proposals as to their definite dimensions.

Since then a veritable flood of articles and papers has been published in the important periodicals of the world, concerned either with a new system of electric and magnetic units or dimensions, or with a comprehensive basic system of mechanical units. None of them has succeeded as yet, because of the *attitude of respect toward the classical absolute systems*, which have exerted an unfortunate power of tradition, especially in the field of physics. This paper aims to prove conclusively the incorrectness of these classical dimension systems and to propose *natural systems* which, it is hoped, may finally settle the differences and put the electrical unit systems on a sound basis.

In order to provide the means for a logical proof and to furnish the necessary elements for building up the new systems, the paper deals first with the fundamental conception of the dimension of a physical quantity, in so far as it is necessary for the scope of this paper.

I. Physical Quantities, Their Units and Dimensions

The aim of all physical science is to obtain a description of the inorganic world in terms of mathematics. The scientific type of human mind tends to bring all relations into forms which are more or less mathematical and in doing this is concerned fundamentally with the process of measurement.

A. THE DIMENSION OF A PHYSICAL QUANTITY

The following statements are a brief summary of the fundamental conception of physical quantities in so far as they differ from pure numbers and must form the true basis of all considerations concerning the problem of units and unit systems. In order to emphasize the importance of each statement, the special arrangement in form of letter paragraphs has been chosen.

α. To determine definitely the value of a physical quantity, it is necessary to make *two statements*; one indicating the *unit* which is used in measuring the quantity, and the other giving the *number of units* involved in the result of that measurement. For example, to state "the length of the path is 4" is an incomplete statement because the unit may be *m*, cm., inch, foot, mile, or so on, each of the units being widely different. The theoretical number of possible units, obviously, is infinity.

β. However, all the various units of length form an infinite group of similar quantities, each one being a definite length itself, so that $[l]$ may stand as a symbol for any one unit of length. The statement, "the length of the path is $4 \cdot [l]$ "is still incomplete, but in this case there is at least the indication that a physical quantity is involved and it is therefore necessary to specify a length unit.

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1. For references see Bibliography.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

γ. At this point the *dimension* of a quantity is defined by the infinite reservoir of possible units for that quantity. A physical quantity is then generally designated by

$$N \cdot [D], \tag{1}$$

where N is the numeric and $[D]$ is the symbol for the dimension. A special value of that quantity is obtained by specifying the values of N and $[D]$. The conception of a physical quantity as the product of numeric times dimension corresponds to the process of measuring, which essentially consists in *first* fixing $[D]$ as a definite unit (usually with a definite name), and *then* comparing the given quantity with this unit.[4,5]

From this point of view, dimension is as necessary as the numerical value and can not be abstracted from a physical quantity without reducing it to a pure number.* It should be noted that the word dimension is often used with other meanings, as for example: three-dimensional space, four-dimensional world, and so on. This should not be confused with the definition given above.

δ. Comparison of the same quantity, measured with two different units leads to the mathematical relation

$$N_1 \cdot [D]_1 = N_2 \cdot [D]_2,$$

where $[D]_1$ and $[D]_2$ designate definite units. If the following unit relations exist

$$[D]_1 = n_{12} \cdot [D]_2, [D]_2 = n_{21} \cdot [D]_1 \tag{2}$$

then

$$\begin{aligned} N_1 \cdot [D]_1 = N_1 \cdot n_{12} \cdot [D]_2 &= N_2 \cdot [D]_2 \\ &= N_2 \cdot n_{21} \cdot [D]_1 \end{aligned} \tag{3}$$

illustrating the process of changing units. It must be emphasized, however, that the conversion factors n_{12}, n_{21} *must be pure numbers*, in order to make the units comparable at all, *i. e.*, all units of the same quantity must have the same dimension. This statement is identical with that in β or γ and is extremely important in passing on the correctness of unit systems.

B. Applied Mathematics and Physics

Pure mathematics operates with symbols designating numbers and numbers only. It is closely related to geometry in so far as all expressions may be interpreted in terms of coordinates, and their relations producing geometrical forms, such as surfaces, curves or points. The extension of these interpretations to hyper spaces is well known.

Applied mathematics, however, uses the laws and methods deduced by pure mathematics in order to solve the problems arising in physics, chemistry, and the related engineering sciences. All physical relations involve physical quantities, since they can be found or proved only experimentally. The only means for finding quantitative laws is by measuring the various quantities involved. This conclusion is thus identical with the statement at the beginning of the previous chapter and it is obvious now, that before experimental investigations can proceed, it is necessary that the means for measurement, *i. e.*, the units, must be available.

In this paper no attempt is made to give a historical review of the development of measuring quantities and determining units in the various fields of physics. However, a brief introductory review of the fields of physics may be given in order to prepare for the following presentations.

The dominating law of physics is undoubtedly the law of conservation of energy. It is known that energy appears in various forms, but that the total quantity of energy can not be changed. Energy is therefore the *only true fundamental quantity* which ties together all the various fields of physics.

A fundamental scheme for distinguishing between the various fields of physics would appear to be the subdivision according to the different forms of energy. This method would lead to the following classification:

Mechanics. The mechanical energy appears in the two forms—potential energy and kinetic energy.

Heat. In this case there is only the single form energy which may be called heat energy or molecular energy.

Electricity. Here the energy manifests itself as the electric field energy and magnetic field energy.

These three fields of physics are the only ones which have been discovered so far. It is unnecessary at this point to speculate as to the possibility of a common origin of these various forms of energy.

It is obvious from what has been said in (A) that, whatever unit we choose for energy, all such units in the various fields of physics must have the same dimension and can be converted into one another. Since the research work in the various fields was carried on independently it is but natural that different units of energy were chosen. Table I gives a short review of the units most frequently used and the relations existing between them.

TABLE I

Mechanical energy......	1 kg force m	= 9.806 joules	= 9.806×10^7 ergs
Heat energy............	1 cal.	= 10^3 cal	
Electromagnetic energy .	1 wattsec		

Inter-relations

1 kg force m = 2.34 cal = 9.806 wattsec.

II. The Mathematical Forms Used in Physics

Theory in any respect is descriptive. The mathematical type of human mind searches for relations between quantities, progressing in each field toward the fundamental laws upon which the final theory may be built. The laws are expressed in mathematical forms and the whole chain of relations, *the theory*, is then the *mathematical picture* of the respective physical field.

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*However, the dimension has nothing to do with the "ultimate nature" of physical quantities. See also reference 37, chapter 2.

Only when experiment and theory conform exactly can the theory be said to be satisfactory.

The fundamental mathematical form is the equation. There are but two different types of equations* which occur in theoretical work: the arbitrary definition and the statement of experimental proportionality. Because of the importance of these two kinds of equations they will be considered in some detail in this section.

A. The Arbitrary Definition

Many physical laws can be expressed in simpler form, if use is made of defined quantities rather than measured quantities. Thus, the quotient of two physical quantities, Q_1 and Q_2

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \cdot \left[\frac{D_1}{D_2} \right] = N \cdot [D],$$

results as the product of a numerical value N and a dimension $[D]$, *i. e.*, as a physical quantity itself which may or may not have a true physical meaning. At any rate it can be named and used in physical relations as an arbitrarily defined quantity.

The arbitrary definition can also take the form of differential and integral relations, as for example

$$v = \frac{ds}{dt},$$

the definition of velocity being the differential quotient of path and time. Even differentials are parts of the physical quantity, although infinitely small. In general, mathematical operations on physical quantities again give physical quantities.

It is this fact which justifies the comparison of the dimensions on the two sides of any physical relation, or more exactly, any relation in applied mathematics, and such a comparison is a direct consequence of dealing with physical quantities rather than with pure numbers. Thus, each equation can be split up into a relation of numerics, and a relation of dimensions or units depending on whether or not definite units are specified.

B. The Statement of Experimental Proportionality

Our knowledge of natural phenomena is due primarily to the research of experimental physics, and the mathematical expression of these experimental investigations is always the statement of a proportionality. For example, the relation

$$F = k \cdot \frac{Q_1 Q_2}{r^2}$$

represents the experimental relation describing the force existing between two masses, two electric charges, or two magnetic poles, if their centers are separated a distance $r \cdot [l]$. In all three cases the proportionality factor k serves to express the proper quantitative dependence. This means, however, that k has *two* functions, namely, to balance the numerical values as well as the units used for the known physical quantities on both sides of the equation. It is obvious, therefore, that generally k itself appears as a *physical quantity*.

In many cases this fact is recognized by giving a characteristic name to the proportionality constant. In some important cases attempts were made to specify quite arbitrarily the meaning of the proportionality factor, thereby emphasizing the least important quantity in the whole equation. It will be shown that this procedure leads to inconsistent as well as incorrect results.

III. On Dimension Systems and Unit Systems

A. Definitions of Dimension Systems and Unit Systems

In order properly to express its value, a physical quantity must be definitely specified as to its numerical value and unit. However, it would be too confusing to use as many unit names as there are physical quantities. As pointed out above, there are many relations between the various quantities, either as definitions or as experimental proportionalities, which permit a reduction in the number of unit names.

Since each relation between quantities can be split up (see chapter II A) into one relation of numerics and another one of dimensions (as general units), it is possible to combine all known relations of dimensions. In setting up these relations, all proportionality factors must be taken as physical quantities. If there are m *independent* relations known, and $(m + p)$ dimensions involved, it indicates that m dimensions can be expressed in general by any p "fundamental" dimensions chosen arbitrarily.

This set of p "fundamental" dimensions is then called a *dimension system*. From the theory of numbers therefore, it is known that one generally has a choice of $\binom{m+p}{p}$ possible dimension systems. Thus, if $p = 3$, $m = 3$, then one has $\binom{6}{3} = 20$ different possibilities. A necessary condition, however, is that *each* independent relation involves at least $(p + 1)$ dimensions. If this is not the case, then the number of possible dimension systems is less, so that $\binom{m+p}{p}$ indicates the *upper* limit.

Furthermore, in order to arrive at some method of properly expressing physical quantities it is necessary to choose a unit for each of the p fundamental dimensions. Each set of p units so obtained is a "*fundamen-*

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*E. Bennett[6] distinguishes 4 types of relations; however, for the purpose of this paper it is but necessary to distinguish according to the mathematical types.

tal unit system." Any unit of the $(m + p)$ quantities can then be expressed in terms of the p fundamental units.

Since each fundamental dimension yields an infinity of possible units, there result "infinity to the p, raised to the $\binom{m+p}{p}$ power," possible fundamental unit systems. This vast array of possibilities must emphasize the desirability of international agreement before too much use is made thereof.

Any dimension system chosen in the described manner, is consistent, as well as correct and never leads to ambiguity with respect to the expression of physical quantities.

B. The Dimension Systems in Mechanics

No attempt is made to give a complete history of the mechanical units. The investigations of this chapter only serve to develop a thorough basis for the derivations of the following ones, and present selected points of interest.

It follows from Appendix A that there are at least $\binom{6}{3} = 20$ possible dimension systems. Since space and time are admittedly the fundamental perceptions for investigating physical phenomena, let $[l]$ and $[t]$ be two of the three possible dimensions. The third dimension could be chosen from $[k]$, $[m]$, $[f]$, $[W]$, thereby restricting the number of possibilities to four. There is no dimension system known, which uses the proportionality constant $[k]$ as a fundamental dimension, but the use of $[W]$ as the third fundamental dimension has been proposed by W. Ostwald[7] without much success. If, then, $[f]$ is taken as the third fundamental dimension, the system becomes the so-called gravitational system, in which the fundamental dimensions are $[l]$, $[t]$, $[f]$. Because of the fact that this system is used to a large extent in engineering, it will be referred to in this paper as the *technical dimension system.* On the other hand, if $[m]$ is chosen for the new dimension there results the so-called dynamical system whose dimensions are $[l]$, $[t]$, $[m]$. Because of the frequent use of this system in physics it will be called the *physical dimension system.* In the latter system the dimension of force becomes

$$[f] = [m] \cdot [l]\,[t]^{-2}. \tag{5}$$

These two dimension systems of mechanics are consistent because they are composed of the required three fundamental dimensions which can be used to specify all the other quantities.[27]

Table II shows, for convenience, the dimensions of various mechanical quantities in both dimension systems, if use is made of the relations given in Appendix A.

The conversion from one system into the other is easily made if the unit systems are specified within the dimension systems. Consider, for example, the same amount of work measured in both dimension systems. In the technical dimension system the equation could be written in the form

$$W = a \cdot ([f]\,[l])_u,$$

where a is the numerical value, and the symbol $(\ \ )_u$ designates the unit. In the physical dimension system the work could be given by

$$W = b \cdot ([m]\,[l]^2\,[t]^{-2})_u,$$

where b is now another numerical value and $(\ \ )_u$ the other unit. Since the same amount of work is involved, equating these two quantities gives the unit relation

$$1\,([f]\,[l])_u = \frac{b}{a}\,([m]\,[l]^2\,[t]^{-2})_u.$$

As emphasized above, units have the same dimensions as the quantities themselves. Equating the dimensions on both sides there results

$$[f] = [m]\,[l]\,[t]^{-2},$$

a dimension relation which is also given in Table II. *No new dimensional relations are ever obtained by measuring quantities in various dimension-systems as long as such systems are consistent.*

It follows from chapter III A that there are "infinity cubed" unit systems possible in each dimension system. In spite of this fact, however, when the two systems were introduced, exactly the same unit names were chosen in both systems. This startling coincidence is perhaps comprehensible, but it is the cause of much confusion in attempting to distinguish the *kg*, *g*, etc., as a unit of force in the one system and as a unit of mass in the other. It has been proposed to call the technical system unit, *kg*-force, and for the sake of clarity this designation will be used here. (See Table I.)

C. The Dimension Systems in Heat

It is surprising to note that in the field of heat a single dimension system has been developed and no confusion whatsoever exists as to its fundamental dimensions. There are, however, many inconveniences as to the units; these difficulties may be settled internationally when scientific minds grow international.

TABLE II

Symbol	Quantity	Technical system	Physical system
	Fundamental dimensions	$[f],[l],[t]$	$[m],[l],[t]$
m	Mass	$[f][l]^{-1}[t]^2$	$[m]$
f	Force	$[f]$	$[m][l][t]^{-2}$
k	Gravitation constant	$[f]^{-1}[l]^4[t]^{-4}$	$[m]^{-1}[l]^3[t]^{-2}$
W	Work, energy	$[f][l]$	$[m][l]^2[t]^{-2}$
P	Power	$[f][l][t]^{-1}$	$[m][l]^2[t]^{-3}$

As may be seen from Appendix B the two fundamental laws in heat involve 6 dimensions, thus requiring 4 fundamental dimensions for a dimension system.[3] Again we assume the two basic perceptions of space and time to be natural fundamental dimensions, and since temperature had been measured long before a theory of heat existed, it is justifiable to take the temperature as a third fundamental dimension $[\theta]$. The fourth dimension can be chosen from

$$[k], [R], [m], [f], [W],$$

if use is made of the definitions in Appendix B whereby $[W]$ can be replaced by either $[m]$ or $[f]$.

Of all these possibilities, the only important system is the one involving $[m]$ and it may be referred to as the "thermophysical" dimension system, since it is based upon the mechanical-physical system. No use has been made of the system involving $[f]$ which might similarly be called the "thermotechnical" dimension system. As mentioned above, the system using $[W]$ had been proposed by Ostwald[7] without success.

The proportionately constant R of Boyle-Charles's law is known as the universal gas constant and its dimension can be read from the law in Appendix B as[9]

$$[R] = [l]^2 \cdot [t]^{-2} \cdot [\theta]. \qquad (6)$$

In concluding this chapter it may be emphasized that, thus far, none of the proportionality factors has been mistreated or taken as fundamental dimensions.

TABLE III—COMPLETE ELECTRICAL DIMENSION SYSTEMS

		Additional fundamental dimension			
		$[Q]$	$[I]$	$[k_e]$	$[k_m]$
	Dimension systems	$[m],[l],[t],[Q]$	$[m],[l],[t],[I]$	$[m],[l],[t],[k_e]$	$[m],[l],[t],[k_m]$
Quantity	Symbol				
Electric charge	Q	$[Q]$	$[I][t]$	$[m]^{1/2}[l]^{3/2}[t]^{-1}[k_e]^{-1/2}$	$[m]^{1/2}[l]^{1/2}[k_m]^{-1/2}$
Current	I	$[Q][t]^{-1}$	$[I]$	$[m]^{1/2}[l]^{3/2}[t]^{-2}[k_e]^{-1/2}$	$[m]^{1/2}[l]^{1/2}[t]^{-1}[k_m]^{-1/2}$
	k_e	$[m][l]^3[t]^{-2}[Q]^{-2}$	$[m][l]^3[t]^{-4}[I]^{-2}$	$[k_e]$	$[l]^2[t]^{-2}[k_m]$
	k_m	$[m][l][Q]^{-2}$	$[m][l]\,[t]^{-2}[I]^{-2}$	$[l]^{-2}[t]^2[k_e]$	$[k_m]$
Electric field strength	E	$[m][l]\,[t]^{-2}[Q]^{-1}$	$[m][l]\,[t]^{-3}[I]^{-1}$	$[m]^{1/2}[l]^{-1/2}[t]^{-1}[k_e]^{-1/2}$	$[m]^{1/2}[l]^{1/2}\,[t]^{-2}[k_m]^{1/2}$
Magnetic field strength	B	$[m][t]^{-1}[Q]^{-1}$	$[m][t]^{-2}[I]^{-1}$	$[m]^{1/2}[l]^{-3/2}[k_e]^{1/2}$	$[m]^{1/2}[l]^{-1/2}[t]^{-1}[k_m]^{1/2}$

IV. The So-Called Absolute Electric and Magnetic Dimension Systems

Confusion seems to have reigned unconfined in the dimension systems of electricity and magnetism, even to the present day. It is the purpose of this section to apply the fundamental principles deduced above to an analysis of these absolute dimension systems.

A. The Electromagnetic Dimension Systems

As indicated in Appendix C, the three independent laws of electromagnetics give relations between seven dimensions, so that four, and not less than four, fundamental dimensions compose an electromagnetic dimension system. Again, the basic perceptions of length and time would seem to justify the choice of $[l]$ and $[t]$ as fundamental dimensions. Since Coulomb's and Ampere's laws are closely related to mechanics, the choice of a mechanical dimension as a third one suggests itself. Inasmuch as both of these laws involve only the force $[f]$, the two definitions given in Appendix C can be used to express $[f]$ in terms of either $[m]$ or $[W]$, so that the third fundamental dimension can be chosen from $[m]$, $[f]$, $[W]$, as in the mechanics. The first two possibilities are the only ones of practical importance as seen from chapter III B.

The fourth fundamental dimension in electromagnetics can then be chosen from

$$[Q], [I], [k_e], [k_m].$$

The last two quantities are the proportionality constants of Coulomb's and Ampere's laws and as such, have no special physical significance. However, even though such a specification of this dimension system is entirely natural and quite convincing, it has been customary to specify as fundamental dimensions[12] those of the proportionality factors k_e and k_m, so that the electric charge and current appear as derived dimensions. This entirely arbitrary procedure is hardly justifiable on the grounds of a logical development of dimensions.

As previously mentioned, each of the four quantities, Q, I, k_e, k_m, can be taken as the new fundamental dimension. The four dimension systems thus obtained are shown in Table III, where the first two columns give the dimensions with integer exponents, and the last two columns have exponents of fractional orders. This again makes it preferable to choose either $[Q]$ or $[I]$ as fundamental dimensions instead of $[k_e]$ or $[k_m]$.*

It is possible to convert from the one electrical system into the other if the units are fixed. Since there are four different systems, four relations for the dimensions of the units are obtainable, as for example, in the case of current:

$$[Q][t]^{-1} = [I] = [m]^{1/2}[l]^{3/2}[t]^{-2}[k_e]^{-1/2} = [m]^{1/2}[l]^{1/2}[t]^{-1}[k_m]^{-1/2}$$

which fits perfectly into the respective columns of Table III. There is not the least ambiguity as to the fundamental dimensions; *i. e.*, the *unit relations are unique*. This is also the reason for calling these systems complete dimension systems, and as such, they cannot have less than four fundamental dimensions.

It may appear possible to obtain a complete set of four fundamental dimensions by choosing *two electrical fundamental dimensions* in addition to length and time. Such an "electrical" set of dimensions has been used by several authors in which current, resistance, length and time are the four fundamental dimensions.[13,14] In this case it becomes necessary (see Appendix C) to introduce the mass as a derived electrical dimension, and so one would "electrify" the mechanical system. From the point of view of international agreement between physicists and engineers, however, such an arrangement would appear to be quite unsatisfactory and undesirable.

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*In the field of electrical engineering, E. Bennett[8] used a complete practical system, as given in the third column.

B. The "Absolute" Systems and the System of Gauss

The mechanical system of dimensions composed of the three fundamental dimensions, $[m]$, $[l]$, $[t]$, was often called the "absolute system" and it was thought possible to reduce all the dimension systems in the various fields of physics to the same absolute system. This was shown to be impossible in the case of heat, and from the discussion above (see Table III) it ought to be impossible in the case of electromagnetism. However, the first experimental law (Appendix C) presents an almost irresistible temptation to be made the basis of an absolute system. It was thought possible to assign any dimension whatsoever to the proportionality factor and consequently the choice was so made that $[k_e] = 1$; *i. e.*, k_e *degenerated* into a pure number. The third column in Table III gives then the absolute dimensions of the electromagnetic quantities if $[k_e] = 1$. For convenience the result is shown again in the first column of Table IV. Since Coulomb's law occurs in electrostatics, the dimension system has been called the *absolute electrostatic dimension system.*

Another absolute dimension system was specified on the basis of the fundamental law of magnetism, or steady currents. By choosing $[k_m] = 1$ the proportionality factor became a pure number (similar to k_e in the electrostatic system) and Ampere's law (Appendix C) could then be used for determining the absolute dimension of the current. The resulting dimensions may be found in the last column of Table III, introducing $[k_m] = 1$ and is given for convenience in the second column of Table IV. The dimension system based upon this arbitrary choice of k_m is called *absolute electromagnetic dimension system.*

Although each of the above dimension systems is quite arbitrary, it is apparent that they are unsymmetrical, in spite of the fact that the two proportionality factors have the same respective position and are supposed to play the same role. A comparison of the last two columns in Table III will show that the ratio of the dimensions for any quantity in both systems is given by

$$\frac{[k_e]}{[k_m]} = [l]^2\,[t]^{-2}, \tag{7}$$

which is evidently the dimension of the square of velocity.

In order to obtain a symmetrical system, Gauss introduced a new proportionality factor defined by the equation

$$k_m' = k_m \cdot v^2, \tag{8}$$

where v denotes a quantity having the dimension $[l]/[t]$. Because of this substitution the factors k_e and k_m' have then the same dimensions. By specifying $[k_e] = [k_m]' = 1$, *i. e.*, defining both constants as pure numbers, the remarkable result is obtained that all the electric quantities, including the current, are in the electrostatic dimension system, and the magnetic quantities are in the electromagnetic system, as shown in the last column of Table IV.

C. Objections to the Absolute Electrical Dimension Systems

In introducing *arbitrary statements about the proportionality factors* k_e and k_m, *absolute systems have been obtained but the dimensional relations are incorrect.* The proof of this statement can be presented by summing up the more important objections to the various attempts at reducing the number of required fundamental dimensions to less than four.

α. The dimension of the proportionality factors k_e *or* k_m *can not be chosen arbitrarily.* It has been shown in Appendix C that there are three independent relations between mechanical and electrical quantities, involving, however, four new electrical dimensions which must be determined. This means, as explained above, that it is necessary to introduce a fundamental electrical dimension. Defining k_e or k_m as pure numbers, however, means exactly the same as defining the gravitation constant k, in law 2 of Appendix A, as a pure number. On the basis of this supposition, then, the three laws in mechanics would involve only 5 dimensions and there would be only two fundamental dimensions. This means, in other words, that mass, force and energy could be expressed in powers of $[l]$ and $[t]$ alone.

TABLE IV

Quantity	Symbol	Absolute dimension systems Electrostatic $[k_e] = 1$	 Electromagnetic $[k_m] = 1$	 Gauss $[k_e] = [k_m'] = 1$
Electric charge	Q	$[m]^{1/2}[l]^{3/2}[t]^{-1}$	$[m]^{1/2}[l]^{1/2}$	$[Q]_e$
Current	I	$[m]^{1/2}[l]^{3/2}[t]^{-2}$	$[m]^{1/2}[l]^{1/2}\,[t]^{-1}$	$[I]_e$
	k_e	1	$[l]^2[t]^{-2}$	1
	k_m	$[l]^{-2}[t]^2$	1	$[k_m]_e$
Electric field strength	E	$[m]^{1/2}[l]^{-1/2}[t]^{-1}$	$[m]^{1/2}[l]^{3/2}\,[t]^{-2}$	$[E]_e$
Magnetic field strength	B	$[m]^{1/2}[l]^{-3/2}$	$[m]^{1/2}[l]^{-1/2}[t]^{-1}$	$[B]_m$

However, it would not be necessary to stop there, since other arbitrary statements could be made so that the number of dimensions could be further decreased. There is no logical reason for such a procedure and it never can be justified. The theory of dimensions and dimension systems as developed in these previous chapters give, so it is hoped, a definite guide for obtaining proper dimension-relations.

β. *The arbitrary designation of k_e and k_m as pure numbers actually converts the absolute electrical systems into the mechanical dimension system.* Hence, absolute electric, as well as absolute magnetic dimension systems, are in fact parts of the mechanical system. Consequently, since all quantities are measured in the same system, unit relations of the type $1m = 100cm$ ought to hold. However, in both systems *the dimensions are different.** There is thus an *obvious contradiction* which can not be reconciled. It would be similar to a situation, for example, in which velocity had different dimensions if measured in the physical or in the technical mechanical dimension systems. This would be immediately recognized as being incorrect, so that the same conclusion should be drawn in the case of the absolute electrical systems.

γ. *The absolute electrical dimension systems as parts of the mechanical dimension system lead to strange relations.* Since all the absolute electromagnetic systems are based upon the same three fundamental dimensions of mechanics, all electric and magnetic quantities appear as mechanical quantities of the same type. As in the case of the two mechanical dimension systems (chapter IIIB), and as in the case of the complete electrical systems (chapter IVA), the dimensions in the absolute systems may be compared. If these dimension systems are perfect, no new dimensional relations should be obtained.

However, equating the dimensions of Q, I, E, and B, as given in Table IV, gives the strange result

$$[l] \cdot [t]^{-1} = 1,$$

and equating the dimensions of k_e and k_m there results $[l]^2\,[t]^{-2} = 1$. Or, in other words, it ought to be possible to express length by time or vice versa. Needless to say again, the arbitrary statements of k_e and k_m as pure numbers are quite incorrect.

As may be seen from Table IV, even Gauss' System does not succeed in obtaining a mechanical system in electromagnetics. Since the electric quantities are measured in the electrostatic system and the magnetic quantities in the electromagnetic system, exactly the same objections can be made.

δ. *The ratio of the electrostatic to the electromagnetic unit of charge is not equal to the velocity of light.* It has been explained in almost every textbook† that the ratios of electrostatic to electromagnetic units involve a fundamental quantity—the velocity of light, approximately equal to $c = 3.10^{10}\ \dfrac{\text{cm}}{\text{sec}}$. It is known that within the mechanical system, the multiplication of two physical quantities changes the dimension and in general the physical meaning. It is, therefore, a surprising statement that in the absolute electric systems the same electric quantities have units which differ in their *dimensions by a velocity* or some power thereof.

However, introducing the absolute electrostatic unit of charge as defined by Coulomb's law with $k_e = 1$ and $f_e = 1$ dyne, $r = 1$ cm, it follows that

$$1\,([m]^{1/2}\,[l]^{3/2}\,[t]^{-1})_e = 1g^{1/2}\ \text{cm}^{3/2}\ \text{sec}^{-1},$$

which is a very small unit. For practical purposes, the *coulomb* has been introduced so as to give

$$\begin{aligned} 1 \text{ electrostatic unit of charge} &= 1g^{1/2}\ \text{cm}^{3/2}\ \text{sec}^{-1} \\ &= \frac{1}{3.10^{9}} \text{ coulomb.} \end{aligned} \tag{9}$$

On the other hand, the absolute electromagnetic unit of current follows from Ampere's law with $k_m = 1$ and $f_m = 1$ dyne, $r = 1$ cm, $l = 2$ cm, as

$$1\,([m]^{1/2}\,[l]^{1/2}\,[t]^{-1})_m = 1g^{1/2}\ \text{cm}^{1/2}\ \text{sec}^{-1}.$$

The practical unit is defined as the *ampere*, so that

$$1g^{1/2}\ \text{cm}^{1/2}\ \text{sec}^{-1} = 10 \text{ ampere,} \tag{10}$$

and converting to the electromagnetic unit of charge by means of the relation $1cb = 1a \cdot \text{sec}$, there results

$$\begin{aligned} 1 \text{ electromagnetic unit of charge} &= 1g^{1/2}\ \text{cm}^{1/2} \\ &= 10 \text{ coulomb.} \end{aligned} \tag{11}$$

Hence, the ratio of (21) to (23) becomes

$$\frac{1 \text{ electrostatic unit of charge}}{1 \text{ electromagnetic unit of charge}} = \frac{\text{cm}}{\text{sec}} = \frac{1}{3.10^{10}}. \tag{12}$$

and this surprising result shows that the ratio of the units of electric charge gives *the unit of velocity;* but *the ratio of the numerical values of electric charge* gives the *reciprocal of the numerical value of this velocity.*

The last two terms of equation (12) may be written as[18]

$$3.10^{10}\ \frac{\text{cm}}{\text{sec}} = 1. \tag{13}$$

This rather unusual result is a direct consequence of using absolute unit systems. These contradictions have been sanctioned by physics since their introduction and almost no protest has been raised against such relations.

(ϵ) *The form of Faraday's induction law.* In Appendix C, Faraday's induction law is given as a dependent relation and the proportionality factor β which was arbitrarily introduced, turns out to be a pure numeric, if use is made of the general dimension relations given by the 3 independent laws. This indicates that the factor $1/c$ which is necessary in Gauss' dimension system in order to balance both sides of the induction law, has that position only because of the incorrect dimension of either B or E. Again, the inconsistency shows up quite clearly and brings to light the various ingenious ways of correcting equations in order to maintain artificial dimension systems.

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*The different dimensions always caused difficulties. See references 17, 18, 37.

†As illustrations, some of the most recent valuable textbooks on physics and electrical engineering may be named. See bibliography references 19, 30, 21, 22, 23, 24.

D. CONCLUSIONS AS TO THE ABSOLUTE ELECTRICAL DIMENSION SYSTEMS

All attempts to reduce the required four fundamental dimensions of electromagnetism to the three fundamental dimensions of mechanics by arbitrary statements as to the proportionality factors have failed. Moreover, the consequences of such statements are not in agreement with the normal laws of mathematics as shown in chapter IIIA. The results, therefore, lead to the following conclusion:

The absolute electrostatic, electromagnetic and Gauss' dimension systems in the field of electromagnetics are incorrect and have to be abandoned finally and completely. Reference can *no longer* be made to *c.g.s. systems or to "absolute units" of any quantity in electromagnetics,* since the proper electrical dimension system must have four fundamental dimensions. The notation, *c.g.s. system, occurs only in mechanics* and *has to be restricted to this field of physics,* where it represents just one possible system of units within the "physical" dimension system.

If this denial of the possibility of c.g.s. systems in electromagnetics is granted—and the above proof justifies this conclusion—it must follow at once that the proportionality factors k_e and k_m can not be considered as pure numbers. Their dimensions and numerical values follow from the special type of the dimension system which shall be adopted, if possible, internationally. If k_e and k_m are physical quantities then the *dielectric constant* (permittivity) and the *magnetic constant* (permeability) of free space (being linearly depending on k_e and k_m) are physical quantities. As such they must have numerical value and dimension, similar to the *gravitation constant* k, and the gas constant R, and must be considered to be *of the same universal importance.*

Due to the fundamental importance of the subject and because of the fact that even the most recent textbooks of physics and engineering are using these absolute electrical systems,* international authority in physics and engineering will be required to banish the absolute systems and to introduce a dimension system appropriate to the field of electromagnetism. The various committees could consider the specification of units and unit names only after the absolute systems have been abandoned and a proper dimension system introduced.

There has been much discussion as to the "rationalization" of units; *i. e.*, shifting the factor 4π, involved in Maxwell's field equations, into more natural positions. However, it is meaningless to talk of "rationalized absolute" or "unrationalized absolute" units when the very existence of such absolute units is unjustifiable.

V. On Proper Dimension Systems in Electromagnetism

The fact that the absolute system has been proven to be incorrect makes it necessary to propose a proper dimension system in electromagnetics—one which presents at least some promise of being considered internationally. A selection has to be made between the four complete dimension systems given in chapter IVA, since these systems are directly deduced from the fundamental laws of electromagnetics. However, before making a definite proposal, it will be feasible to consider the so-called practical unit system.

A. THE SO-CALLED PRACTICAL UNIT SYSTEM

The "absolute electrostatic" unit of charge and the "absolute electromagnetic" unit of current (see chapter IVB) were the basis of two unit systems, the c.g.s. electrostatic and the c.g.s. electromagnetic unit systems. Later, both unit systems were found to be quite inconvenient and for practical purposes other units were introduced and defined quite arbitrarily, as shown by the following examples.

1 coulomb $= 3.10^9$ c.g.s. electrostatic units of charge
$= 10^{-1}$ c.g.s. electromagnetic units of charge
1 farad $= 9.10^{11}$ c.g.s. electrostatic units of capacitance
$= 10^{-9}$ c.g.s. electromagnetic units of capacitance
1 henry $= \frac{1}{9} \cdot 10^{-11}$ c.g.s. electrostatic units of inductance
$= 10^9$ c.g.s. electromagnetic units of inductance

Only a few relations are given here for a special purpose, and even these few show a surprising variety of numerical factors.

The practical units are but definitions of units without regard to their dimensions. And, indeed, it would be difficult to state dimensions for these practical units. Take for example, the charge. As shown in equations (**9**) and (**11**) the *coulomb* is related to both dimension systems with the numerical factors given above. Equating (**9**) and (**11**), since the *coulomb* has to be the same unit in both cases, gives

$$3.10^9\, g^{1/2}\, \mathrm{cm}^{3/2}\, \mathrm{sec}^{-1} = 1 \text{ coulomb} = \frac{1}{10}\, g^{1/2}\, \mathrm{cm}^{1/2}. \quad (\mathbf{14})$$

The question naturally arises as to which dimension may now be chosen for *coulomb*. Furthermore, equation (**13**) may be read again if the extreme left-hand and the extreme right-hand terms are considered. Similar contradictions can be obtained for the dimensions of each practical unit.*

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*See as examples, references 19, 20, 22, 23, 24.

Another strange relation can be obtained for the dimensions of *farad* and *henry*. The following dimension relations can be taken from any conversion table of the absolute electric and absolute magnetic dimension systems:

[c.g.s. electrostatic unit of capacitance] = cm,
[c.g.s. electromagnetic unit of inductance] = cm.

Using the above numerical factors it is permissible to write

$$1\text{ cm} = \frac{1}{9}\cdot 10^{-11}\text{ farads} = 10^{-9}\text{ henrys} \qquad (15)$$

so that it should be possible to express *farads* in terms of *henrys* and vice versa—another proof of the inconsistency of the absolute dimension systems.

In many cases the absolute c.g.s. units were designated as abcoulomb, abohm,[26,27,36] or the abbreviation e.s. volt, and e.m. volt were used for the c.g.s. unit of voltage in the electrostatic and electromagnetic dimension systems.[6] Again, the "practical" unit of magnetic flux was called the pra-maxwell,[28] but an extension of this method would have resulted in complications. All these attempts were quite unsatisfactory but serve as an indication of the many difficulties which were caused by the simple statements that k_e and k_m were to be considered as pure numbers.

The *conclusion stated in chapter IVD* has to be supplemented, therefore, by the condition *that the practical units be completely separated from the absolute systems* and that the conceptions of the dimensions of the practical unit systems be clarified. It is a unique fact that *here is a unit system in daily use and well known* even to nonexperts, but *without the background of a proper dimension system.* In no other field of physics is there such a well-built unit system with such distinguished unit names as in electromagnetics, and yet there is no fundamental basis to justify this system.

Although the practical unit system is complete as to the unit names, it has not been customary until recently to use it as a complete system in the case of Maxwell's field equations. A mixture of the absolute electrostatic, absolute electromagnetic and practical units has been used most frequently, especially in the literature of electrical engineering. Accordingly, Maxwell's field equations are amply supplied with numerical factors in the technical literature.[29] This was the cause of an attempt to choose the practical units ampere, ohm, centimeter and second as a basis for a separate and independent set of electrical units.[13,14] However, the above four units are not a unit *system* and consequently, they are not at all a dimension system as frequently assumed. (See chapter IVA).

This proposal resulted as a reaction against the mixture of absolute systems and as such, took the extreme position of separating the field of electromagnetics completely from mechanics. These attempts have the invaluable merit of having caused a spirited discussion for over twenty years, without, however, achieving a definite result.

B. REQUIREMENTS OF NEW PROPER DIMENSION SYSTEMS

Thus far it has been explained and proved that all existing dimension systems are quite unsatisfactory, the classical systems themselves being incorrect. Greatest care has to be taken in proposing new dimension systems, since such systems should be satisfactory for international consideration.

The *main points in preparing new dimension systems*, according to the results indicated in this paper, can be enumerated as follows: *First,* a single fundamental electrical dimension has to be combined with the three mechanical fundamental dimensions in order to obtain a proper dimension system. There is no possibility of transposition of a purely mechanical dimension system into electromagnetics. *Second,* the expressions of *absolute* units, *absolute* dimensions and *absolute* systems have to be entirely suppressed. *Third,* it is necessary to adhere to the trend in electrical engineering of using the so-called practical *unit* system, but with due regard to a proper and adequate *dimension* system. *Fourth,* the independently defined absolute units have to be introduced into a physical dimension system. These physical units must be related to the so-called practical units by numerical factors. The velocity of light as an important multiplier must disappear finally and completely. *Fifth,* it should be possible to write Maxwell's field equations in such a general way that, by introducing the different unit systems, all the various schemes for writing the equations are obtained. In all cases only numerical conversion factors are involved—the factor 3.10^{10} is in no case to be considered as a velocity. *Sixth,* the dielectric constant and the permeability of free space are not to be taken as numerics, but rather as physical quantities with dimensions derived from the dimension system specified.*

At first glance it would seem almost impossible to find a solution containing all these requirements, especially since no comprehensive solution has been suggested throughout the last forty years of active discussion. However, a satisfactory solution of the problem is possible and the following section contains proposals for such "natural dimension systems."

Before submitting a plan of solution of the problem, it will be convenient for future use to indicate the fundamental equations of the electromagnetic field in homogeneous media.

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*Tables of such comparative types may even be found in the literature. See reference 25.

*O. Heaviside[3] emphasized this point of view, but did not assign definite dimensions.

$$f_e = k_e \frac{Q_1 Q_2}{r^2} \qquad f_m = k_m \frac{I_1 I_2}{r} l \tag{16}$$

$$k_e = \frac{1}{4\pi} \cdot \frac{1}{\Delta} \qquad k_m = \frac{1}{2\pi} \cdot \Pi \tag{17}$$

$$\Delta = \Delta_0 \cdot \epsilon \qquad \Pi = \Pi_0 \cdot \mu \tag{18}$$

$$\bar{\mathbf{D}} = \Delta \cdot \bar{\mathbf{E}} \qquad \bar{\mathbf{E}} = \rho \bar{\mathbf{G}} \qquad \bar{\mathbf{B}} = \rho \cdot \bar{\mathbf{H}} \tag{19}$$

$$\psi = \iint D_n d\sigma; \quad I = \iint G_n d\sigma; \quad \phi = \iint B_n d\sigma \tag{20}$$

$$W_e = \frac{1}{2} \iiint \bar{\mathbf{E}} \bar{\mathbf{D}} d\tau; P_R = \iiint \bar{\mathbf{G}} \bar{\mathbf{E}} d\tau; W_m = \frac{1}{2} \iiint \bar{\mathbf{H}} \bar{\mathbf{B}} d\tau \tag{21}$$

$$= \frac{1}{2} CV^2 \qquad = RI^2 \qquad = \frac{1}{2} LI^2 \tag{22}$$

$$\text{curl } \bar{\mathbf{H}} = \bar{\mathbf{G}} + \frac{\partial \bar{\mathbf{D}}}{\partial t} \tag{23}$$

$$\text{curl } \bar{\mathbf{E}} = - \frac{\partial \bar{\mathbf{B}}}{\partial t} \tag{24}$$

These forms hold in *any case* where the *same unit system* is used throughout. If units of different systems are employed on both sides of any equation, numerical factors enter the relations, and it will be shown later that these numerical factors are the same as those found in the various forms of equations used for decades. The above *given forms* may be called *rationalized*; but it will be seen later, that the factor 4π can be shifted around simply by choosing the respective combination of units of the various systems in order to obtain all the familiar forms. The factor 4π and the problem of rationalization is not an essential problem for correct expression of the equations; but it is essential that the velocity of light no longer appears in the fundamental laws.

It should be emphasized at this point that the proposals made in the following section are in no sense to be considered as the ultimate or final solution of the problem; but that any other complete analysis must follow this same line of thought. This first discussion then, will confine itself to the fundamental requirements—no final proposal being made as to the notation and symbols to be adopted.

C. Proposal as to Natural Dimension Systems

In all the attempts to settle the confusion of the various unit systems, one fact seems to have been consistently disregarded; namely, that there *already are two mechanical systems* in general use, a technical and a physical one (see chapter IIIB) with their respective fundamental dimensions [*f*], [*l*], [*t*] and [*m*], [*l*], [*t*]. It is but natural to ask—why not supplement both systems by a fundamental electrical dimension and call them respectively, the *electrotechnical and electrophysical dimension systems?* As has been shown in chapter IIIB, both mechanical dimension systems are perfect. If, therefore, an electrical dimension is added, it is to be expected that perfect electrical dimension systems will be obtained (see chapter IVA). This is the substance of the following proposal, for which the dimensional relations and units are given in Tables V and VI.

The *new fundamental electrical dimension* is taken to be the *electric charge*. As previously pointed out in chaper IVA, there are many advantages in specifying the electric charge as a fundamental dimension. In addition to the *natural unit* existing in the electron, the electrotechnical unit *coulomb* is fixed internationally and can be realized electrochemically quite readily. The basis of the electrophysical definition of unit charge is Priestley's law (**16**). By taking the numerical value of k_e for free space equal to unity (but still having a dimension assigned to it), a unit of charge is defined as such a charge, acting on an equal charge at a distance of 1 *centimeter* with a force of one *dyne*. This unit is here called the *priestley* according to the true discoverer of the law[10] and is the fundamental unit.

The dimensions of the true electrical quantities in the third group of the tables are the same in both systems and are expressed in [*l*], [*t*], [*Q*], only. In addition, the dimensions of all other mechanical-electrical quantities (related to the mechanical forces in the electromagnetic field) as well as the parameters are integer powers of all four fundamental dimensions. Finally, from the physical point of view, electricity is nothing more than electric charges at rest or in motion and as such are the cause of the electric and magnetic fields.

The separation into two dimension systems extending the two mechanical systems into electromagnetics gives *the solution* in so far as avoiding the interference due to the requirements of physics and engineering. In the former field the fundamental units *cm, gram, sec, priestley,* represent the extension of the extremely valuable c.g.s. system into the c.g.s.p. system for electrophysics. On the other hand, the fundamental units *cm, newton, sec, coulomb,* represent the extension of the similarly valuable technical-mechanical system into the c.n.s.c. system. The electrical units employed in the electrotechnical system are all the familiar, so-called "practical" units; while the units employed in the electrophysical system are all of the so-called absolute units, using the latest decisions of the International Electrotechnical Commission.[23] It is surprising to note, that during the last few meetings the International Electrotechnical Commission considered the completion of unit names in the electrophysical system. It is

TABLE V—ELECTROTECHNICAL DIMENSION SYSTEM

Fundamental Dimensions: $[f]$, $[l]$, $[t]$, $[Q]$

Group	Quantity	Symbol	Dimension	Electrotechnical unit	Expressed in electrophysical units
1. (Fundamental)	Length	l	$[l]$	centimeter	1 centimeter
	Time	t	$[t]$	second	1 second
	Force	f	$[f]$	newton = 10.2 kg force	10^7 dynes
	Electric charge	Q	$[Q]$	coulomb	$3 . 10^9$ priestleys
2. (Mechanical)	Energy	W	$[f]\,[l]$	joule = 1 newton cm	10^7 ergs
	Power	P	$[f]\,[l]\,[t]^{-1}$	watt	10^7 ergs/sec
	Mass	m	$[f]\,[l]^{-1}\,[t]^{-2}$	gram-seventh = 10^4 kg	10^7 grams
3. (Electrical)	Electrostatic flux	ψ	$[Q]$	coulomb	$3 . 10^9$ priestleys
	Displacement	D	$[Q]\,[l]^{-2}$	coulomb/cm²	$3 . 10^9$ priestleys/cm²
	Current	I	$[Q]\,[t]^{-1}$	ampere	$3 . 10^9$ priestleys/sec $= \frac{1}{10}$ gilbert
	Current density	G	$[Q][l]^{-2}[t]^{-1}$	ampere/cm²	$\frac{1}{10}$ gilbert/cm²
	Magnetizing force	H	$[Q][l]^{-1}[t]^{-1}$	ampere/cm	$\frac{4\pi}{10}$ oersted $= \frac{1}{10}$ gilbert/cm
	Magnetomotive force	F	$[Q][t]^{-1}$	ampere	$\frac{4\pi}{10}$ oersted cm $= \frac{1}{10}$ gilbert
4. (Electrodynamical)	Magnetic flux	ϕ	$[f][Q]^{-1}[l][t]$	voltsec	$\frac{1}{300}\ \frac{\text{erg sec}}{\text{priestley}} = 10^8$ maxwell
	Induction	B	$[f][Q]^{-1}[l]^{-1}[t]$	voltsec/cm²	10^8 gauss
	Electric field intensity	E	$[f][Q]^{-1}$	volt/cm	10^8 maxwell/cm sec
	Voltage	V	$[f][Q]^{-1}[l]$	volt	10^8 maxwell/sec
5. (Electromagnetic parameters)	Capacitance	C	$[f]^{-1}[Q]^2[l]^{-1}$	farad	10^{-9} sec/kirchhoff
	Absolute dielectric constant	Δ	$[f]^{-1}[Q]^2[l]^{-2}$	farad/cm	10^{-9} sec/kirchhoffcm
	Resistance	R	$[f][Q]^{-1}[l][t]$	ohm	10^9 kirchhoff
	Inductance	L	$[f][Q]^{-2}[l][t]^{-2}$	henry	10^9 kirchhoffsec
	Absolute permeability	Π	$[f][Q]^{-2}[t]^2$	henry/cm	10^9 kirchohffsec/cm

UNIVERSAL CONSTANTS

Absolute dielectric constant of free space: $\Delta_0 = \frac{1}{4\pi} . \frac{1}{9.10^{11}}$ farad/cm $= 0.884 . 10^{-12}$ farad/cm

Absolute permeability of free space: $\Pi_0 = 4\pi . 10^{-9}$ henry/cm $= 1.256.10^{-8}$ henry/cm

necessary, however, that a final agreement be reached between physicists and engineers.

The unit relations contain numerical factors and not factors with dimensions of velocity or any power thereof. This is due to the clear specifications of four fundamental dimensions.

In order to simplify the notations, three new unit names have been suggested: priestley, newton and kirchhoff. Whether or not these names are adopted is irrelevant, but it is necessary that these or other names be chosen to complete the tables. For some time the need has been felt for a unit of force equal to 10.2 *kg-force*,* which provides a simple relation between the technical and physical force units, as well as the respective energy and power units. With respect to the *priestley*, it is obvious that a name should be given to such a fundamental unit of the entire electrophysical system. The *kirchhoff* has been introduced due to its importance for the last group of parameters in Table VI.

It remains to show how the various forms of writing Maxwell's field equations merely depend upon the units employed. Consider, for example, the first field equation in the simplified form:

$$\text{curl}\ \overline{\text{H}} = \overline{\text{G}}. \tag{25a}$$

In the electrotechnical system the units are amp/cm for $\overline{\text{H}}$, amp/cm² for $\overline{\text{G}}$, and no numerical factor is involved. Now, if $\overline{\text{G}}$ is left in amp/cm² and $\overline{\text{H}}$ is given in *oersteds*, Table VI gives the relation

$$1\ \text{oersted} = \frac{10}{4\pi}\ \text{amp/cm},$$

so that

$$\left.\begin{array}{lll} & \text{curl}\ \dfrac{10}{4\pi}\ \overline{\text{H}} = \overline{\text{G}} & \\ \text{or} & \text{curl}\ \overline{\text{H}} = \dfrac{4\pi}{10}\ \overline{\text{G}} & \end{array}\right\} \begin{array}{l} \overline{\text{H}}\ \text{in oersteds,} \\ \\ \overline{\text{G}}\ \text{in amp/cm}^2. \end{array} \tag{25b}$$

In this case, where different units are to be used on the two sides of the equation, it has to be written explicitly as shown in **(25b)**, and a numerical factor enters the equation.

Consider now the complete relation in **(23)** and introduce $\overline{\text{D}}$ in *priestley*/cm². If $\overline{\text{H}}$ is expressed in *oersted*, it is necessary to use the unit relations

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*This has already been emphasized by V. Karapetoff.[14]

TABLE VI—ELECTROPHYSICAL DIMENSION SYSTEM

Fundamental dimensions: $[m]$, $[l]$, $[t]$, $[Q]$

Group	Quantity	Symbol	Dimension	Electrophysical unit	Expressed in electrotechnical unit
1. (Fundamental)	Length	l	$[l]$	centimeter	1 centimeter
	Time	t	$[t]$	second	1 second
	Mass	m	$[m]$	gram	10^{-7} gram-seventh
	Electric charge	Q	$[Q]$	priestley	$\frac{1}{3} \cdot 10^{-9}$ coulombs
2. (Mechanical)	Energy	W	$[m][l]^2[t]^{-2}$	erg	10^{-7} joule
	Power	P	$[m][l]^2[t]^{-3}$	erg/sec	10^{-7} watt
	Force	f	$[m][l][t]^{-2}$	dyne	10^{-7} newton
3. (Electrical)	Electrostatic flux	ψ	$[Q]$	priestley	$\frac{1}{3} \cdot 10^{-9}$ coulombs
	Displacement	D	$[Q][l]^{-2}$	priestley/cm² $= \frac{1}{3.10^{10}}$ gilbert sec/cm²	$\frac{1}{3} \cdot 10^{-9}$ coulombs/cm²
	Current	I	$[Q][t]^{-1}$	gilbert $= 3.10^{10}$ priestley/sec	10 amperes
	Current density	G	$[Q][l]^{-2}[t]^{-1}$	gilbert/cm²	10 ampere/cm²
	Magnetizing force	H	$[Q][l]^{-1}[t]^{-1}$	oersted $= \frac{1}{4\pi}$ gilbert/cm	$\frac{10}{4\pi}$ ampere/cm
	Magnetomotive force	F	$[Q][t]^{-1}$	gilbert $= 4\pi$ oersted cm	10 ampere
4. (Electrodynamical)	Magnetic flux	ϕ	$[m][Q]^{-1}[l]^2[t]^{-1}$	maxwell $= \frac{1}{3.10^{10}} \frac{\text{erg sec}}{\text{priestley}}$	10^{-8} voltsec
	Induction	B	$[m][Q]^{-1}[t]^{-1}$	gauss	10^{-8} voltsec/cm²
	Electric field intensity	E	$[m][Q]^{-1}[l][t]^{-2}$	maxwell/cm sec	10^{-8} volt/cm
	Voltage	V	$[m][Q]^{-1}[l]^2[t]^{-2}$	maxwell/sec	10^{-8} volt
5. (Electromagnetic parameters)	Capacitance	C	$[m]^{-1}[Q]^2[l]^{-2}[t]^2$	sec/kirchhoff	10^9 farad
	Absolute dielectric constant	Δ	$[m]^{-1}[Q]^2[l]^{-3}[t]^2$	sec/kirchhoff/cm	10^9 farad/cm
	Resistance	R	$[m][Q]^{-2}[l]^2[t]^{-1}$	kirchhoff $= \frac{1}{9.10^{20}} \frac{\text{erg sec}}{\text{priestley}^2}$	10^{-9} ohm
	Inductance	L	$[m][Q]^{-2}[l]^2$	kirchhoff sec	10^{-9} henry
	Absolute permeability	Π	$[m][Q]^{-2}[l]$	kirchhoffsec/cm	10^{-9} henry/cm

UNIVERSAL CONSTANTS

Absolute dielectric constant of free space: $\Delta_0 = \frac{1}{4\pi} \frac{\text{priestley}^2}{\text{erg cm}} = \frac{1}{4\pi} \cdot \frac{1}{9.10^{20}}$ sec/kirchhoffcm

Absolute permeability of free space: $\Pi_0 = 4\pi \cdot \frac{1}{9.10^{20}} \frac{\text{erg sec}^2}{\text{priestley}^2\text{ cm}} = 4\pi$ kirchhoff sec/cm

$$1 \text{ priestley/cm}^2 = \frac{1}{3.10^{10}} \text{ gilbert sec/cm}^2$$

$$= \frac{4\pi}{3.10^{10}} \text{ oersted sec/cm,}$$

which lead to

$$\text{curl } \overline{\text{H}} = \frac{4\pi}{10} \overline{\text{G}} + \frac{4\pi}{3.10^{10}} \cdot \frac{\partial \overline{\text{D}}}{\partial t} \qquad \text{(25c)}$$

Where $\overline{\text{H}}$ is in *oersteds*, $\overline{\text{G}}$ in *amp./cm²* and $\overline{\text{D}}$ in *priestleys/cm²*. It appears that the degree of familiarity of the result depends upon the variety of units involved.

D. THE PROBLEM OF A UNIQUE COMPREHENSIVE UNIT SYSTEM

Quite a long series of papers has been devoted during the last three decades to a problem which may be formulated as: *Find, among the "infinity to the fourth" various unit systems* possible in electromagnetics, *a single one* which satisfies all requirements of practicability in all fields of physics, by means of merely taking powers of 10 of the "fundamental units."*

No such system has been found as yet and it would appear that *there is no real need for such a comprehensive system.* The two systems as proposed here may, however, approximate quite closely such a comprehensive system, if the *newton* is introduced as a new force unit. Then the electrical and mechanical quantities are connected by simple relations involving powers of 10, but the mass unit is somewhat large. This objection is not so serious since the mass is only a *derived* unit in the electrotechnical system.

The electrophysical system, with the unfortunate tendency of introducing factors 4π and 3.10^{10} naturally does not involve such simple relations. If there were, however, the possibility of defining a fundamental unit of electric charge as

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*For information pertaining to the various attempts to solve this problem see references 26, 30, 31, 32, 25, 33, 6, 34, 35.

$$1 \text{ cavendish} = 3 \text{ priestleys},$$

then the factors 3 and 9 or $\frac{1}{3}$ and $\frac{1}{9}$ would disappear in all unit relations but would combine with the universal constants as

$$\left.\begin{aligned} \Delta_0 &= \frac{1}{4\pi} \cdot \frac{1}{9} \frac{\text{cav}^2}{\text{erg cm}} = \frac{1}{4\pi} \cdot \frac{1}{9} \cdot 10^{-11} \text{ farad/cm} \\ \Pi_0 &= 4\pi \cdot 10^{-20} \frac{\text{erg sec}^2}{\text{cav}^2 \text{ cm}} = 4\pi \cdot 10^{-9} \text{ henry/cm.} \end{aligned}\right\} \quad (26)$$

Then, if the unit of magnetizing force be taken as 1 gilbert/cm instead of the new oersted, the relations between the two electrical systems of units would only involve integer powers of 10. This would require that, for free space $k_e = 9 \frac{\text{erg cm}}{\text{cav}^2} = 1 \frac{\text{erg cm}}{\text{priestley}^2}$; or the unit charge 1 *cavendish* would be defined as a charge exerting a force of 9 *dynes* on an equal charge at a distance of 1 *cm;* or a force of 1 *dyne* at a distance of 3 *cm*. Such definitions might not meet with the approval of physicists but they do indicate the possibility of a comprehensive unit system.

CONCLUSION

The absolute dimension systems expressing the electric and magnetic quantities by the 3 dimensions of mechanics have been shown to be incorrect, since the consequent use of proper mathematical statements leads to impossible results. However, the definitions of the absolute units, as well as the practical units in electromagnetics have sense quite independent of the dimension systems. It is shown that the simplest and as it seems apparently, the only true solution, is to extend the two mechanical dimension systems, known as technical and physical systems, into the field of electromagnetics, by adding a new fundamental electric dimension. This gives two electrical systems—the electrotechnical dimension system with all the practical units and the electrophysical dimension system with all the absolute units.

The continuous use of the *absolute* dimension systems in physics as well as electrical engineering textbooks, requires final international action and the proposals summarized in Tables V and VI may be considered as a basis for such action. However, for the present, it would seem desirable to restrict the discussion to the fundamental idea outlined in this paper. The introduction of new unit names and other such details may well be deferred to some later time. The achievement of final international agreement is highly desirable, but such a goal will require the cooperation of physicists and the International Electrotechnical Commission.

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Appendix A

THE DIMENSION SYSTEMS IN MECHANICS

Fundamental Measurements:

Distance $[l]$
Time $[t]$

Important Definitions

Velocity $v = \frac{ds}{dt} \therefore [v] = \frac{[l]}{[t]}$

Acceleration $a = \frac{dv}{dt} \therefore [a] = \frac{[l]}{[t^2]}$

Momentum $M = mv \therefore [M] = [m] \frac{[l]}{[t]}$.

Independent Laws in Mechanics:

1. Force on a moving body $f = m \cdot a \therefore [f] = [m] \frac{[l]}{[t^2]}$
2. Gravitation law $f = k \frac{m_1 m_2}{r^2} \therefore [f] = [k] \cdot \frac{[m^2]}{[l^2]}$
3. Kinetic energy $W = \frac{1}{2} mv^2 \therefore [W] = [m] \cdot \frac{[l^2]}{[t^2]}$

Some Dependent Relations:

Force from energy $f = -\frac{\partial W}{\partial s} \therefore [f] = \frac{[W]}{[l]}$

same as (**1**) and (**3**).

Power $P = \frac{dW}{dt} \therefore [P] = \frac{[W]}{[t]}$

From the three independent laws it is seen that 6 dimensions are involved, so that $\binom{6}{3} = 20$ possible *dimension systems* exist.* It would, however, be possible to use any one of the definitions (only few are listed) in order to replace a certain dimension in the laws 1, 2, 3 by a defined dimension. For example, $[l]$ could be replaced by $[v]\ [t]$, thereby obtaining a wider choice of fundamental dimensions. However interesting the result may be, no use will be made of these possibilities at this point.

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*Since each independent law involves 4 dimensions, this result is correct.

Appendix B

The Dimension Systems in Heat

Fundamental Measurements:

Distance	$[l]$
Time	$[t]$
Temperature	$[\theta]$

Important Definitions:

Force $\quad f = m \cdot a \quad \therefore [f] = [m] \cdot \dfrac{[l]}{[t^2]}$

Pressure $\quad p = \dfrac{df}{dS} \quad \therefore [p] = \dfrac{[f]}{[l^2]}$

$$= \frac{[m]}{[l]\,[t^2]}$$

Energy $\quad H = W = m\dfrac{v^2}{2} \therefore [W] = [m] \cdot \dfrac{[l^2]}{[t^2]}$

Independent Laws in Heat Theory:

1. Boyle-Charles's law for gases

$$p \cdot V = m \cdot R \cdot \theta, \therefore \frac{[l^2]}{[t^2]} = [R] \cdot [\theta]$$

2. Conduction of heat power

$$\frac{dH}{dt} = k \cdot S \cdot \frac{d\theta}{dn} \therefore \frac{[W]}{[t]} = [k]\,[l]\,[\theta]$$

Some Dependent Relations:

Specific heat $\quad H = s \cdot m \cdot \delta\,\theta$

$$\therefore [s] = \frac{[W]}{[m]\,[\theta]} = \frac{[l^2]}{[t^2]\,[\theta]}$$

In the two independent laws given above, 6 dimensions are involved and, therefore, 4 fundamental dimensions are required to form a dimension system in the theory of heat. The upper limit $\dbinom{6}{2} = 15$ dimension systems is not reached here, since the first law only involves 4 dimensions. However, it would be possible to use the definitions in order to replace $[W]$ by either $[f]$ or $[m]$, thereby again extending the choice.

Appendix C

The Dimension Systems in Electromagnetism

Fundamental Measurements:

Distance	$[l]$
Time	$[t]$

Important Definitions:

Force $\quad f = m \cdot a \quad \therefore [f] = [m]\dfrac{[l]}{[t^2]}$

Work $\quad W = \int \bar{\mathbf{f}} \cdot \overline{\mathbf{ds}} \therefore [W] = [f] \cdot [l]$

Independent Laws in Electromagnetism:

1. Coulomb's law* (for two point charges)

$$\bar{\mathbf{f}}_e = \left(k_e \cdot \frac{Q_1}{r^2} \cdot \frac{\bar{\mathbf{r}}}{r}\right) \cdot Q_2 = \bar{\mathbf{E}}_1 \cdot Q_2 \therefore [f]$$

$$= [k_e] \cdot \frac{[Q^2]}{[l^2]}$$

2. Ampere's law* (for two parallel infinitely long wires)

$$\bar{\mathbf{f}}_m = \left[\bar{\mathbf{I}}_2 \times \left(k_m \cdot \frac{\bar{\mathbf{I}}_1}{r} \times \frac{\bar{\mathbf{r}}}{r}\right)\right] \cdot l$$

$$= (\bar{\mathbf{I}}_2 \times \bar{\mathbf{B}}_1) \cdot l \therefore [f] = [k_m] \cdot [I^2]$$

3. Law of conservation of electricity†

$$I = \frac{dQ}{dt}, \qquad \therefore [I] = \frac{[Q]}{[t]}$$

Some Independent Relations:

Faraday's induction law:

$$-\frac{d}{dt}\iint \bar{\mathbf{B}} \cdot \overline{\mathbf{ds}} = \beta \cdot \int \bar{\mathbf{E}} \cdot \overline{\mathbf{ds}} \therefore [k_m] \cdot [I]\frac{[l]}{[t]}$$

$$= [\beta] \cdot [k_e]\frac{[Q]}{[l]}$$

so that: $[\beta] = 1$.

Again there are 3 independent laws as in mechanics, but in this case 7 dimensions are involved so that the highest number of possible dimension systems would be $\dbinom{7}{3} = 35$, each dimension system being composed of 4 fundamental dimensions. However, two of the laws involve only 3 dimensions, so that the number of possibilities is reduced to

$$\binom{3}{2} \cdot \binom{4}{2} = 3 \times 6 = 18.$$

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*These two laws can also be written as $f_e = k_e \dfrac{Q_1Q_2}{r^2}$ and $f_m = k_m \dfrac{I_1I_2}{r} \cdot l$. In this scalar form the main difference between the two fields described by the laws is not sufficiently emphasized. The electrostatic field is a scalar-potential field, produced by scalar quantities, the electric charges; the magnetostatic field is a vector potential field, produced by vector quantities, the currents. The electric field strength $\bar{\mathbf{E}}$ is a gradient, whereas the magnetic field strength $\bar{\mathbf{B}}$ is a curl. It would seem inadvisable to choose similar mathematical expressions for two physical concepts which are so extremely different.

†This law can be interpreted either as the link between electrostatic charge and the discharge current for a condenser, or as the drift of electrons in metallic conduction.

As in Appendix B, it would be possible to extend the choice of dimension systems by using the definitions given above and expressing $[f]$ in terms of either $[m]$ or $[W]$.

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Performance Calculations on Induction Motors

Practical Straightforward Means for Calculating Performance of Single-Phase or Polyphase Induction Motors

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***Synopsis.**—Practical means for calculating the performance of both single-phase and polyphase induction motors from previously determined constants are developed in a straightforward manner. The methods are suitable for accurate calculations but can be simplified for approximate calculations. The methods given are complete and no auxiliary charts or sets of curves are required, nor need any diagram be constructed.*

The single-phase calculation method is based on the cross field theory presented by H. R. West. The polyphase calculation method is based on the equivalent circuit.

I. INTRODUCTION

RECENTLY there have been presented two methods of calculating the performance of polyphase induction motors. One of these was developed by Mr. W. J. Branson[1] and is based upon an analytical solution of a circle diagram, although it might have been based upon an equivalent circuit, and requires about 100 pages of families of curves. The other method was proposed by Mr. P. L. Alger[2] and is based upon the expansion of the impedance of the equivalent circuit into the form of an infinite power series, the number of terms of which are used being dependent upon the accuracy desired. Four or five years ago, Mr. H. R. West[3] developed and proposed a symbolic method for calculating the performance of a single-phase induction motor.

In this paper are presented two calculating methods, one for the performance of single-phase induction motors and one for polyphase induction motors. Both of these methods are developed by means of complex algebra and the finished calculating sheets are quite similar in character. Neither of the methods requires an auxiliary chart, circle diagram, or set of curves, and both methods are believed to have certain distinct and unique advantages over other methods proposed to date.

The basis for the calculating method for polyphase motors proposed in this paper is the equivalent circuit of Fig. 2 and no new theories or concepts are introduced. This method can be followed to give exactly the same identical results in every respect as this equivalent circuit, for any and all values of any of the given constants. Such exactness, however, is ordinarily not necessary, and virtually all other methods which have been proposed previously as practicable contain one or more simplifying approximations, some methods making one approximation and some another.

It is unfortunate, however, that what may be safe approximations for one motor may be quite inadequate for another motor; *e. g.*, the influence of primary resistance on the primary current can usually be neglected in large motors but not in small motors. Moreover, in some cases it is desirable to make a number of approximations in order to obtain rough results quickly, and in other cases, more exact results are needed and a greater expenditure of time is justifiable.

The calculation method presented in this paper is so arranged and described that the Steinmetz equivalent circuit of Fig. 2 can be followed with absolute exactness or with varying degrees of approximation as desired, the approximations resulting in a saving in labor in calculating the constants for applying the method but not changing the general scheme of the method. This feature is felt to possess very distinct advantages also, the fact that it is neither necessary to construct a circle diagram nor to use any auxiliary charts is a not-to-be-over-looked advantage.

The single-phase calculation sheet offers, though to a more limited extent, the same general advantages as does the polyphase calculation sheet.

Since the purpose of this paper is only to present a simple, straightforward method of calculating the performance of either a single-phase or a polyphase induction motor and not to introduce any new or unfamiliar concepts, the paper itself deals only with the methods themselves, the assumptions upon which they are based, and how to make practical use of them. These methods can only be applied after the constants of the motor have been determined either from design constants, or by test, or by any other suitable means. The determination of these constants is a sufficient problem in itself and one with which this paper will not be concerned. The developments of the calculating methods are recorded in Appendixes II and III.

II. NOTATION

Before proceeding with an explanation of the calculation sheet, the constants appearing on these sheets will be briefly summarized and discussed.

E = impressed volts per phase

r_1 = primary resistance per phase

r_2 = secondary resistance per phase referred to the primary

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1. For references see Bibliography.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

r_f = resistance representing core loss when connected in parallel with X_m

r_m = resistance representing core loss when connected in series with X_m

x_1 = primary leakage reactance per phase

x_2 = secondary leakage reactance per phase referred to the primary

Xm = magnetizing reactance per phase

X = ideal short-circuit reactance of motor ($r_2 = 0$)

$$= x_1 + x_2 \frac{Xm}{Xm + x_2}.$$

This is the reactance the motor would have with short-circuited secondary if the iron loss and secondary resistance were zero. This expression, however, does take into account the shunting of the secondary leakage reactance by the magnetizing reactance

Xo = $Xm + x_1$ = reactance of the primary winding with the secondary open-circuited

$$Kp = \frac{Xm}{Xo} = \sqrt{\frac{Xo - X}{Xo}} \text{ (used only when } x_1 = x_2\text{)}$$

I_1* = primary current per phase

I_2 = secondary current per phase for polyphase motors—secondary current in transformer axis for single-phase motors

I_3 = secondary current in cross field axis (single-phase motors only)

Im = total current in branch of equivalent circuit representing core loss and magnetizing current, (polyphase motors only)

Io = no-load current per phase

$$i_m = \frac{E}{Xo}$$ = primary current per phase which would flow if secondary were open-circuited, either single-phase or polyphase motors, (in polyphase motors $i_m = Io$, approximately)

$$S = \frac{\text{actual speed}}{\text{synchronous speed}}$$ (used with single-phase motors)

$$s = \frac{\text{slip r.p.m.}}{\text{synchronous r.p.m.}}$$ (used with polyphase motors)

ϕ = number of phases

Torque. The formulas for torque given in this paper are for torque in oz-ft. These formulas may be changed to give torque in other units by substituting for the quantity 112.6, the following quantities:

For lb-ft.—7.04

For kg-meters—0.974

III. Single-Phase Induction Motor Performance Calculations

Theoretical Basis

In the development of the calculating method for single-phase induction motors, extensive use has been made of West's[2] theory. The assumptions upon which this calculating method is based are thus similar to his in many respects. The angle of hysteretic lag between flux and m.m.f. is neglected, and the magnetomotive force or current required to produce a certain flux is assumed to be proportional to the flux just as in his paper. Sinusoidal voltages and currents, of course, are also assumed.

The general basis of these methods is that the voltage equations for each of the three circuits (shown in Fig. 6) *viz.*, the primary circuit, the transformer field circuit in the secondary, and the cross field secondary circuit, are set up in accordance with Kirchhoff's law. The solution of these three simultaneous equations yields expressions for the currents in each one of these circuits. From these currents and the known reactances of the motor, the fluxes, hence torque and mechanical power developed by the rotor, are obtained. This developed mechanical power is item 30 on the calculation sheet, Fig. 1.

From this point on, the treatment differs somewhat from West's method. From the power developed by the rotor is subtracted the friction and windage and the iron loss due to the cross field. (The cross field loss is assumed to be half or slightly less than half the total iron loss.) The main field iron loss is assumed to be supplied directly from the stator circuit. This is a logical procedure when the motor is considered from the standpoint of the cross field theory,[3] though to be strictly correct, it should be remembered that the core loss is due, not to the separate effects of components of flux, but to the actually existing resultant flux.

As stated before, the main field iron loss is assumed to be supplied directly from the primary power supply, and the stator current is corrected for this additional current to supply the iron loss. This correction is item 14 on the calculating sheet. The input watts are obtained by adding the primary and secondary copper losses and main field iron loss to the developed mechanical output. Power factor and efficiency can then be figured in a conventional manner. It is highly desirable to determine the efficiency by summation of the losses as this at once tells the designer how the losses are distributed in the machine.

Since, in this method of calculation, speed is the independent variable, some means for determining the speed at maximum torque are necessary. In order to accomplish this, use was made of a Branson[4] diagram for single-phase motors as shown in Fig. 7, to determine a general analytical expression for the speed at pull-out as a function of r_2/X, upon which factor the pull-out speed principally depends. The pull-out speed has been

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*Note: I_1, I_2, I_3, and I_m represent currents which vary with load; I_o and i_m, for any given motor, remain constant.

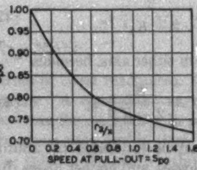
					From diagram	No load	Max. torq.
Constants of motor	1	$S = \frac{\text{r.p.m.}}{\text{syn.}}$	0.962	0.956			0.81*
Line volts $= E$ $= 110$	2	S^2	0.925	0.913			0.655*
Total leakage $= X$ $= 8.3$	3	$(1-S^2)$	0.075	0.087			0.345*
$X_0 = 110$	4	$(1-S^2)r_1$	0.29	0.33			1.31*
$r_1 = 3.8$	5	F_1	4.98	4.98			4.98*
$r_2 = 4.65$	6	$U = (4)+(5)$	5.27	5.31			6.29*
$K_p = \sqrt{(X_0-X)/X_0} = 0.964$	7	$(1-S^2)X$	0.62	0.72			2.86*
$r_2/X = 0.56\, r_2/X_0 = 0.0423$	8	F_2	0.52	0.52			0.52*
$i_m = \frac{E}{X_0} = 1.00\; i_m r_2 = 4.65$	9	$W = (7)-(8)$	0.10	0.20			2.34*
$F_1 = (2-K_p^2)r_1 = 4.98$	10	$\sqrt{U^2+W^2}$	5.27	5.31			6.72*
$F_2 = (2\,r_1+r_2)(r_2/X_0) = 0.518$	11	$(1-S^2)E$		9.58			
$F_3 = (i_m r_2)(r_2/X_0) = 0.197$	12	F_3		0.20			
$F_4 = (i_m r_2)2 = 9.30$	13	$M = (11)-(12)$		9.38			
$F_5 = (i_m r_2)K_p = 4.48$	14	$F_5 U$		0.48			
$F_6 = [(i_m r_2)K_p]^2 r_2 = 93.3$	15	$N = (13)+(14)$		9.86			
$F_7 = EK_p = 106.0$	16	$\sqrt{N^2+F_4^2}$		13.54			
$F_8 = (EK_p)^2 r_2 = 52{,}200$	17	$I_1 = (16)/(10)$		2.55	2.49	1.865	
$F_9 = \frac{Fe \text{ loss } (m)}{E} = 0.091$	18	$(1-S^2)F_7$		9.22			
	19	$\sqrt{(18)^2+F_5^2}$		10.26			
	20	$I_2 = (19)/(10)$		1.90	1.86	0.898	
	21	SF_5		4.29			
	22	$I_3 = (21)/(10)$		0.807		0.898	
	23	$(1-S^2)F_8$	3920	4540			18,000*
	24	F_6	90	90			90*
	25	$(23)-(24)$	3830	4450			17,910*
	26	Prim. cu. loss $= I_1^2 r_1$		24.7	24	13.2	
	27	Sec. cu. loss $(m) = I_2^2 r_2$		16.8	16	3.8	
	28	Sec. cu. loss $(c) = I_3^2 r_2$		3.0	3	3.8	
	29	*Fe* loss (m)		10.0	10	10.0	
	30	$(25)\times(2)/(10)^2$	127.5	144.0		19.0	260*
	31	Input $= (26)+(27)+(28)+(29)+(30)$		198.5	195	49.8	
	32	*Fe* loss (c) + (F and W)	19.0	19.0	19.0		19*
	33	Output $= (30)-(32)$	108.5	125.0	123		241*
	34	R.p.m. $= S \times$ syn.		1721	1721		1460*
	35	Torque $= \frac{112.6\times(33)}{(34)}$		8.16	8.05		18.6*
	36	Eff. $= (33)/(31)$		63.0	63.2		
	37	P.F. $= (31)/EI_1$		70.8	71.0		
	38	App. eff. $= (36)\times(37)$		44.6	44.7		
	39						
*Terms to get output and torque	40						

FIG. 1—PERFORMANCE CALCULATIONS ON A SINGLE-PHASE INDUCTION MOTOR

plotted against r_2/X on Fig. 1 and this method has been found to give generally good results as the error in pull-out speed can be quite large without materially effecting the pull-out torque.

This curve of pull-out speed against r_2/X was calculated for values of the latter up to 2.4 and approached an asymptote of 0.7, indicating that 70 per cent of synchronous speed is the lowest speed at which the pull-out torque of a single-phase induction motor can be made. It is, however, possible to lower the speed at pull-out somewhat below this value by unusual design, though it is not possible to obtain anything like the same control over it as is obtained over the pull-out speed in polyphase motors.

Simplified Calculation Sheet

On the calculating sheet, Fig. 1 it has been further assumed that the primary and secondary reactances were equal, *i. e.*, $x_1 = x_2$. This assumption is not absolutely necessary, as will be shown subsequently, but it is usually justifiable and greatly simplifies matters.

The use of this calculation sheet will be illustrated by means of an example. A 110-volt, 60-cycle, 4-pole squirrel-cage motor was picked at random and the values of E, X, Xo, r_1 and r_2 entered on the sheet. The total core loss for this motor was 19 watts which was arbitrarily distributed, 10 watts to the main field (entered in item 29) and 9 watts to the cross field, which, added to the 10 watts friction and windage, gave 19 watts for the cross field loss plus friction and windage (item 32). From the given constants, Kp etc., and the F constants were figured.

The next step is to select the speed at which the performance characteristics are desired. If the full load performance be desired, the full load slip should be estimated. Thus, it will usually be more satisfactory to attempt to estimate the quantity $1 - S^2$, which is approximately twice the slip for small slips. Just what value of $1 - S^2$ to assume is best governed by experience, as this depends generally upon the size of the motor, ratio of maximum to full load torque, etc. Ordinarily, a value somewhere between 0.05 and 0.08 may be assumed for $1 - S^2$.

In this particular case, 0.075 was assumed for $1 - S^2$, and the output for this value of $1 - S^2$ was figured by computing items 1-10, 23-25, 30 and 33. It will be observed that it is not necessary to figure all the items if only the output is desired. Those items which are necessary are marked by an asterisk in the extreme right hand column. (For a short cut to computing item 10, see Appendix I.)

Now that the output has been found for this motor for a particular value of $1 - S^2$, the full load speed or slip can be estimated with sufficient accuracy for the output can be assumed to be proportional to the quantity $S^2(1 - S^2)$. Using this principle, the full load speed can now be approximated very closely. In the example chosen, an output of 108.5 watts corresponds to a value of $0.925 \times 0.075 = 0.0694$ for the quantity $S^2(1 - S^2)$. The value of the latter quantity for an output of 124.3 watts (full load) is, therefore, by direct proportion, 0.0796. The corresponding value of $1 - S^2$ can be figured from any known value of $S^2(1 - S^2)$ by means of the equation

$$1 - S^2 = \frac{1 - \sqrt{1 - 4\,[S^2(1 - S^2)]}}{2} \qquad \textbf{(41)}$$

In the chosen example, this becomes

$$1 - S^2 = \frac{1 - \sqrt{1 - 4 \times 0.0796}}{2} = 0.087$$

which is the value of $1 - S^2$ corresponding to full load. (Actually $1 - S^2$ may be more readily determined from the quantity $S^2(1 - S^2)$ on a slide rule without the use of the formula given.)

Using 0.087 as the value of $1 - S^2$, the complete performance was calculated by computing items 1 to 38 inclusive. The performance of this same motor was also calculated by means of Branson's[4] circle diagram and the results given in the third column.

The maximum torque was obtained by noting from the curve that the pull-out speed for this motor ($r_2/X = 0.56$) was approximately 0.81 and figuring the torque at this speed.

No-Load Conditions. The no-load currents can be checked approximately by taking $S = 1.0$. With this assumption, the no-load primary current becomes approximately

$$I_0 = \frac{F_4}{F_1} = \frac{2}{2 - K_p^2}\, i_m \qquad \textbf{(42)}$$

and the secondary current, in either axis, becomes

$$I_2 = I_3 = \frac{F_5}{F_1} = \frac{K_p}{2 - K_p^2}\, i_m \qquad \textbf{(43)}$$

The no-load input can then be estimated by adding friction and windage, iron losses, and copper losses due to the currents given by the above expressions. Calculation of no-load current and watts is illustrated in Fig. 1.

For working up constants from test, the following relations obtained from equation (**42**) are useful

$$\left.\begin{aligned} I_0 &= \frac{2E}{Xo + X} \\ Xo &= \frac{2E}{I_0} - X \\ Xm &= XoKp = \sqrt{Xo^2 - XXo} \end{aligned}\right\} \qquad \textbf{(44)}$$

Performance Calculations When x_1 Differs From x_2

The procedure to be followed in this case differs only in the computation of the "F constants." The reactances which must be known are x_1, x_2, and Xm. In Table I are listed the values of the "F constants" in

TABLE I—"A CONSTANTS" AND "F CONSTANTS" FOR SINGLE-PHASE MOTORS

F constant	A constant	Expression for constant $x_1 \neq x_2$	Expression for constant $x_1 = x_2$
F_1	A_1	$r_2\left[\frac{x_2 X_m - r_1 r_2}{(X_m + x_2)^2} + \frac{X_m + 2x_1}{X_m + x_2}\right]$	$r_2\left[\frac{X_0 + X}{X_0} - \frac{r_1 r_2}{X_0^2}\right]$ or $r_2\left[2 - K_p^2 - \frac{r_1 r_2}{X_0^2}\right]$
F_2	$-A_3$	$r_2\left[\frac{r_2(X_m + x_1)}{(X_m + x_2)^2} + \frac{2r_1}{X_m + x_2}\right]$	$(2r_1 + r_2)\ (r_2/X_0)$
F_3	$-A_5$	$E\left(\frac{r_2}{X_m + x_2}\right)^2$	$(i_m r_2)\ (r_2/X_0)$
F_4	$-A_7$	$\frac{2Er_2}{X_m + x_2}$	$(i_m r_2)\ 2$
F_5	$-A_9$	$\frac{EX_m r_2}{(X_m + x_2)^2}$	$(i_m r_2)\ K_p$
F_6	$-A_{11}$	$\frac{E^2 r_2^3 X_m^2}{(X_m + x_2)^4}$	$[(i_m r_2)\ K_p]^2\ r_2$
F_7	A_8	$\frac{EX_m}{X_m + x_2}$	EK_p
F_8	A_{10}	$\left(\frac{EX_m}{X_m + x_2}\right)^2 r_2$	$(EK_p)^2\ r_2$
F_9	A_{12}	$\frac{Fe \text{ loss } (m)}{E}$	$\frac{Fe \text{ loss } (m)}{E}$
	A_4	$x_1 + x_2 \frac{X_m}{X_m + x_2}$	X
	A_2	r_1	r_1
	A_6	E	E

terms of these reactances. (No attention need be paid to the "A constants" as these were merely used in deriving the methods.) In place of X appearing among the given data and in item 7, Fig. 1, should be entered the value of A_4, which is the short-circuit reactance.

For the sake of comparison, the reactance of the motor in the chosen example was assumed to be unequally distributed, 60 per cent of the reactance being in the primary. The "F constants" were computed from the formulas given in Table I and used in the same manner as previously. The final results were found to agree within one per cent.

General Remarks

In general, the above method gives quite satisfactory results. This method is especially convenient when only a part of the performance is desired. For example, if a speed torque curve be desired, only the same items used to figure the maximum torque need be figured. When figuring a speed torque curve, the cross field iron loss and friction should be made to decrease to zero with the speed; otherwise a following of the calculation sheet would result in a negative torque at zero speed. For want of better information, it is suggested when figuring torques below pull-out, that item 32, as given on the sheet, be multiplied by S^2.

Sometimes power factor or efficiency in themselves are unimportant, but the apparent efficiency or full-load current is wanted. In this case, all that need be figured in addition to the output are items 11-17 inclusive, giving the primary output at that point. The apparent efficiency, of course, is

$$\text{apparent efficiency} = \frac{\text{output watts}}{EI_1} \qquad (45)$$

For rough calculations on large motors where the primary resistance is small, the effect of the latter on the primary current can easily be neglected by taking r_1 equal to zero.

IV. POLYPHASE INDUCTION MOTOR PERFORMANCE CALCULATIONS

Theoretical Basis

The generally accepted form of equivalent circuit representing the performance of a polyphase induction motor is as shown in Fig. 2, and this is the circuit proposed by Steinmetz and used by Mr. Alger in his recent paper. In this circuit, the iron loss is represented by a resistance in parallel with the magnetizing reactance. Mr. Branson, in his recent paper,[1] represents the iron loss by a resistance, r_f, connected just inside the primary resistance.

The equivalent circuit of Fig. 2 can be reduced to the form shown in Fig. 3 which is exactly equivalent to that of Fig. 2 if the following values are chosen for r_m and X_m, where r_f and X_m' are taken from the circuit of Fig. 2

$$r_m = \frac{r_f X_m'^2}{r_f^2 + X_m'^2} = \frac{X_m'^2}{r_f} \times \frac{1}{\left[1 + \left(\frac{X_m'}{r_f}\right)^2\right]} \tag{46}$$

$$X_m = \frac{r_f^2 X_m'}{r_f^2 + X_m'^2} = X_m' \times \frac{1}{\left[1 + \left(\frac{X_m'}{r_f}\right)^2\right]} \tag{47}$$

The first approximation made in this paper is that the term in brackets in equations (**46**) and (**47**) is assumed equal to unity. With ordinary motors, this assumption merely means a difference of one or two per cent in reactance only of the core loss and magnetizing branch of the equivalent circuit, but means an appreciable saving in labor when calculating performance.

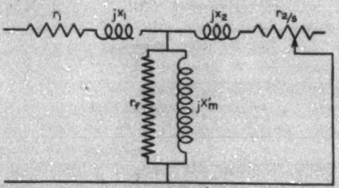


FIG. 2—STEINMETZ EQUIVALENT CIRCUIT OF A POLYPHASE INDUCTION MOTOR

Usually the constants of a motor corresponding to the circuit of Fig. 2, are not known with sufficient accuracy to make a distinction between X_m' and X_m. If desired, however, and r_f and X_m' are known, r_m and X_m can be calculated by means of equations (**46**) and (**47**) and applied directly. In this paper, however, r_m is calculated by the simpler formula

$$r_m = \frac{\text{Core loss}}{\phi \times i_m^2} \tag{48}$$

The method of deriving the calculation method was briefly as follows. Expressions for the primary and secondary currents in terms of the constants of the motor and slip were obtained by ordinary circuit analysis. The mechanical power developed by the secondary, or the secondary output is

$$I_2^2 r_2 \phi \times \frac{1-s}{s} \tag{49}$$

This appears as item 20 on the calculation sheet, Fig. 5. Subtraction of the friction and windage from the secondary output gives the output of the motor. The input is obtained by adding the core loss, the secondary and primary copper losses to the secondary output. The core loss is assumed constant so that this summation gives a slightly higher input than would be obtained by strictly following the equivalent circuit. To follow this equivalent circuit exactly, the copper loss in the resistance r_m can be calculated for each load point and used in place of the core loss. To do this, it is necessary to calculate I_m for each load point. For any load point, I_m is given by

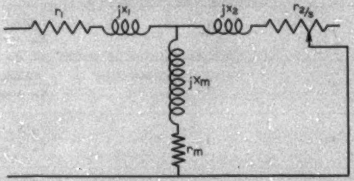


FIG. 3—ANOTHER FORM OF CIRCUIT OF FIG. 2

$$I_m = -\frac{\sqrt{(12)^2 + A_{10}^2}}{(10)^*} \tag{50}$$

where A_{10} has the value given in Table II. I_m could thus be figured and for each individual load point, and the quantity $\phi I_m^2 r_m$ entered in item 19 instead of the core loss, and thus the input obtained would be exactly the same as the power input to the equivalent circuit (real component of $\phi \times \check{E} \times$ conjugate of $\check{I}_1$).

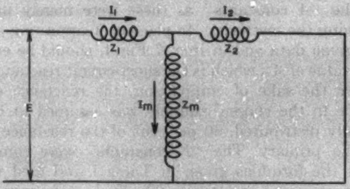


FIG. 4—GENERALIZED FORM OF EQUIVALENT CIRCUIT OF THE POLYPHASE INDUCTION MOTOR

Simplified Calculation Sheet of Fig. 5

In deriving the simplified calculation method of Fig. 5, a further assumption has been made—namely, that the primary reactance equals the secondary reactance. The calculation sheet is virtually self-ex-

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*(12) and (10) refer to items 12 and 10 in Fig. 5.

TABLE II—"A CONSTANTS" AND "F CONSTANTS" FOR POLYPHASE MOTORS

F constant	A constant	Expression for constant $x_1 \neq x_2$	Expression for constant $x_1 = x_2$
F_1	$-A_1$	$x_1 + x_2 \dfrac{X_m}{X_m + x_2} - \dfrac{r_1 r_m}{X_m + x_2}$	$X - \dfrac{r_m}{X_0} r_1$
F_2	A_4	$\dfrac{(X_m + x_1) r_2}{(X_m + x_2)}$	r_2
F_3	A_2	$\dfrac{(r_1 + r_m) r_2}{X_m + x_2}$	$r_2 \left(\dfrac{r_1 + r_m}{X_0} \right)$
F_4	A_3	$\dfrac{r_m (x_1 + x_2)}{X_m + x_2} + r_1$	$r_1 + \dfrac{2}{1 + K_p} \dfrac{r_m X}{X_0}$ or $r_1 + \dfrac{r_m}{X_0} X$
F_5	A_5	$\dfrac{E r_m}{X_m + x_2}$	$i_m r_m$
F_6	A_6	$\dfrac{E r_2}{X_m + x_2}$	$i_m r_2$
F_7	A_9	$\dfrac{E \sqrt{r_m^2 + X_m^2}}{X_m + x_2}$	$E \sqrt{K_p^2 + \left(\dfrac{r_m}{X_0} \right)^2}$ or $E K_p$
	A_7	E	E
	A_8	$\dfrac{E X_m}{X_m + x_2}$	$E K_p$
	A_{10}	$\dfrac{E x_2}{X_m + x_2}$	$\dfrac{i_m X}{1 + K_p}$

planatory. The known constants of the motor, namely, E, X, X_o, K_p, r_1, r_2, friction and windage and core loss are entered at the left side of the sheet in the spaces provided. The F constants are then computed from these given constants. The slip at which the performance is desired is assumed in item 1. In order to get the output, it is unnecessary to figure each item. Only those items marked by an asterisk are necessary for determination of the output corresponding to the assumed slip. If the first value of slip assumed gives nearly full load output, the next value of slip can be found by assuming the output proportional to the slip. (A useful short cut for figuring items 10 and 14 is described in Appendix I.)

The slip at maximum torque can be found from the expression

$$s_m = \frac{r_2}{\sqrt{r_1^2 + X^2}} \tag{40}$$

Maximum torque is, of course, the torque figured at this slip and starting torque is the torque figured for a slip of unity. The formula given in item 26 can not be used for starting torque since both the output and speed are zero, but starting torque is readily found by putting $s = 1$ in equation (35), Appendix III, obtaining

$$\text{starting torque} = \frac{112.6}{\text{syn. r.p.m.}} I_2^2 r_2 \phi$$

$$= \frac{112.6}{\text{syn. r.p.m.}} \times (18)\ s = 1 \tag{51}$$

The use of this calculation is illustrated by means of a numerical example.

Performance Calculation When x_1 Differs From x_2

If x_1, x_2 and X_m are known, the "F constants" should be calculated in terms of the latter by using the suitable expressions from Table II. It can then be forgotten how the "F constants" were obtained and the performance calculated exactly as shown in Fig. 5.

If it is desired, an equivalent circuit of the form of Fig. 2 can be used as a basis for the calculation of the performance by first reducing it to a circuit of the form of Fig. 3 by means of equations (**46**) and (**47**). Moreover, the core loss can be assumed to vary with load exactly as does the copper loss in the resistance representing core loss in the equivalent circuit by calculating A_{10} and following the process just described.

Approximate Calculations

In addition to the approximations mentioned, other assumptions can be made to simplify calculations for rough work. The calculation sheet is so arranged, for example, that either the primary resistance or iron loss can be neglected by putting r_1 or r_m or both equal to zero.

V. CONCLUSIONS

In this paper, practical methods for calculating the performance of both single-phase and polyphase motors have been developed by straightforward means. Either calculating method consists of the filling in of the blank spaces of a calculation sheet especially prepared for this purpose. The performance can easily be calculated roughly or with practically any desired degree of accuracy by the expenditure of a trifle more labor, depending upon the accuracy with which the constants of the motor are known. The methods are thus suited for either rough design calculations or for more refined calculations, the same form being used for either, the use of the same form for either type of calculation being a distinct advantage.

Moreover, neither of the methods presented in this paper requires the constructing of a circle diagram nor any auxiliary charts or sets of families of curves.

The author wishes gratefully to acknowledge helpful suggestions from Messrs. C. R. Boothby, E. P. Codling, and H. E. Ellis.

Appendix I

There is a method, the origin of which is somewhat uncertain, of evaluating the square root of the sum or difference of two squares which so simplifies this operation for slide rule calculations as to make it well worth a description here since this operation is performed several times in the calculating methods described in this paper.

E = volts per phase = 127

ϕ = no. of phases = 3

X = s. c. reactance (per ph.) = 5.0

X_0 = o. c. reactance (per ph.) = 98.0

$K_p = \sqrt{\dfrac{X_0 - X}{X_0}}$ = 0.970

Friction and windage (watts) = 20

i_m = no load amps = E/X_0 = 1.297

Core loss (total) = 37

$r_m = \dfrac{\text{core loss}}{\phi \times i_m^2}$ = 7.32

r_1 = pri. res. per phase = 2.40

$F_2 = r_2$ = 1.55

$F_3 = r_2 \dfrac{r_1 + r_m}{X_0}$ = 0.1537

$F_1 = X - \dfrac{r_m}{X_0} r_1$ = 5.82

$F_4 = r_1 + \dfrac{r_m}{X_0} X$ = 2.85

$F_5 = i_m r_m$ = 9.49

$F_6 = i_m r_2$ = 2.01

$F_7 = EK_p$ = 123.2

Stg. torque = $\dfrac{112.6}{\text{syn. r.p.m.}} \times (18)_{s=1}$ (in oz-ft.)

Slip at max. torque = $s_m = \dfrac{r_2}{\sqrt{r_1^2 + x^2}}$ = 0.240

*Terms to get output and torque

Item		Full load	Stg. T.	Max. T.
1	Slip = s (assume)	0.0297	1.00	0.24*
2	$1/s$	33.7	1.00	4.17*
3	$1 - s$	0.970	0.00	0.76*
4	F_1	5.82	5.82	5.82*
5	$F_3 \times (2)$	5.17	0.15	0.64*
6	$U = (5) - (4)$	−0.65	−5.67	−5.18*
7	F_4 =	2.85	2.85	2.85*
8	$F_2 \times (2)$	52.3	1.55	6.46*
9	$W = (7) + (8)$	55.2	4.40	9.31*
10	$\sqrt{U^2 + W^2}$	55.2	7.17	10.66*
11	F_5	9.49		
12	$F_6 \times (2)$	67.75		
13	(11) + (12)	77.24		
14	$\sqrt{(13)^2 + E^2}$	148.6		
15	$I_1 = (14) \div (10)$	2.69		
16	$I_2 = F_7 \div (10)$	2.235	17.20	11.56*
17	Pri. loss = $I_1^2 \times r_1 \times \phi$	52		
18	Sec. loss = $I_2^2 \times r_2 \times \phi$	23.4	1375	620*
19	Core loss	37		
20	Sec. output = (18) × (2) × (3)	766		1961*
21	Input = (17)+(18)+(19)+(20)	878		
22	Friction and windage	20		15*
23	Output = (20) − (22)	746		1946*
24	R.p.m. = (3) × syn. r.p.m.	1746		1369*
25	Torque = 112.6 × (23) ÷ (24)	48.1	86	160*
26	Eff. = (23) ÷ (21) × 100	84.9		
27	P.f. = (21) × $100/EI_1 \cdot \phi$	85.7		
28	% full load	100		

FIG 5—PERFORMANCE CALCULATIONS ON A POLYPHASE INDUCTION MOTOR

Assume that we wish to evaluate $\sqrt{a^2 + b^2}$. By simple algebra

$$\sqrt{a^2 + b^2} = a\sqrt{1 + \left(\frac{b}{a}\right)^2}$$

Either b or a may be the larger, but it will generally be easier to divide the larger number by the smaller one.

To illustrate, let us evaluate $\sqrt{2.34^2 + 6.29^2}$. Divide 6.29 by 2.34 using the C and D scales. The square of the quotient, $\left(\frac{b}{a}\right)$, is located under the left index of the B scale which reads 7.22. Add one mentally, obtaining 8.22. Move the slide until the left index of the B scale is opposite 8.22. Now, move the runner to the smaller number or 2.34 on the C scale and read 6.72, which is the desired result, on the D scale. The method is simpler to apply than explain. Similarly, the square root of the difference between two squares can be determined by subtracting one mentally from the square of the quotient of the larger number by the smaller one, extracting the square root and multiplying this root by the smaller number.

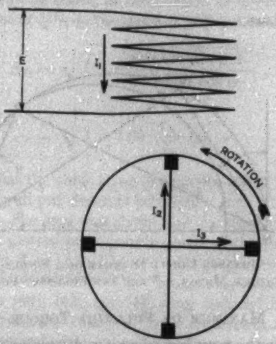

FIG. 6—THE SINGLE-PHASE INDUCTION MOTOR VIEWED FROM THE CROSS FIELD BASIS

Appendix II

THEORY OF THE SINGLE-PHASE MOTOR

Determination of the Currents. A single-phase motor can be represented diagrammatically as shown in Fig. 6.

Using Kirchhoff's law that the applied voltage must equal the impedance drops, Mr. West writes, for the primary,

$$E = r_1I_1 + jx_1I_1 + jX_m(I_1 - I_2) \quad (1)$$

Similarly, in the rotor, the sum of all the induced voltages in each circuit must independently be equal to zero, giving the following two equations:

$$-jX_m(I_1 - I_2) - S(X_m + x_2)I_3 + r_2I_2 + jx_2I_2 = 0 \quad (2)$$

$$j(X_m + x_2)I_3 - SX_m(I_1 - I_2) + Sx_2I_2 + r_2I_3 = 0 \quad (3)$$

These three equations can be re-arranged and solutions readily obtained for I_1, I_2, and I_3 by means of determinants. The determinant for the common denominator reduces to

$$U_1 + jW_1$$

Where

$$\begin{aligned} U_1 = & -r_1r_2^2 + 2r_2x_1(X_m + x_2) + r_2X_m(X_m + 2x_2) \\ & + (1 - S^2)\,r_1(X_m + x_2)^2 \end{aligned} \quad (4)$$

$$\begin{aligned} W_1 = & -r_2^2x_1 - 2r_1r_2(X_m + x_2) - r_2^2X_m \\ & + (1 - S^2)\left[x_1(X_m + x_2)^2 + x_2X_m(X_m + x_2)\right] \end{aligned} \quad (5)$$

The numerator of I_1 is

$$E\left[-r_2^2 + (1 - S^2)(X_m + x_2)^2 - j2r_2(X_m + x_2)\right] \quad (6)$$

The numerator of I_2 is

$$EX_m\left[(1 - S^2)(X_m + x_2) - jr_2\right] \quad (7)$$

The numerator of I_3 is

$$-SEX_mr_2 \quad (8)$$

To make calculations on different motors more nearly alike, it is best as Mr. West points out, to divide equations (4) to (8) by $(X_m + x_2)^2$ obtaining the following expressions for the three currents

$$I_1 = \frac{A_5 + (1 - S^2)A_6 + jA_7}{U + jW} \quad (9)$$

$$I_2 = \frac{A_8(1 - S^2) + jA_9}{U + jW} \quad (10)$$

$$I_3 = \frac{A_9S}{U + jW} \quad (11)$$

Where

$$\begin{aligned} U &= A_1 + A_2(1 - S^2) \\ W &= A_3 + A_4(1 - S^2) \end{aligned} \quad (12)$$

and A_1, A_2, etc., have the values given in Table I. Equations (10) and (11) are the basis for items 20 and 22 on the calculation sheet. The primary current, however, is obtained from equation (18).

Output. Mr. West shows that the net torque in synchronous watts developed by the rotor is, in the symbols of this paper

Torque (in synchronous watts)

$$= \frac{E^2X_m^2r_2S\left[(1 - S^2)(X_m + x_2)^2 - r_2^2\right]}{U_1^2 + W_1^2} \quad (13)$$

$$= \frac{S(1 - S^2)A_{10} + SA_{11}}{U^2 + W^2} \quad (14)$$

Where A_{10} and A_{11} have the values given in Table I. The mechanical power developed by the rotor, exclusive of core loss and friction and windage is, therefore:

$$\text{power developed in watts} = \frac{(1 - S^2)\, A_{10} + A_{11}}{U^2 + W^2} \times S^2 \qquad (16)$$

This equation is the basis for item 30 on the single-phase calculation sheet, Fig. 1A. From this quantity is subtracted the cross field iron loss and friction and windage to obtain the output. The input is obtained by adding to the former quantity the main field iron loss and the primary and secondary copper losses.

Correction of Primary Current for Main Field Core Loss. In order to correct the primary current for the main field core loss, it is assumed that slightly more than half the core loss is supplied directly from the stator power supply. The method of accounting for the core loss cannot be illustrated directly either by a diagram or by an equivalent circuit. The iron loss is considered as being supplied by an in-phase component of current which flows through the primary circuit. This in-phase component of current which supplies the main field core loss is

$$A_{12} = \frac{\text{main field iron loss}}{E} \qquad (17)$$

Adding this current to I_1 as given in equation (**9**), gives for the primary current corrected for iron loss

$$I_1 = \frac{A_5 + (1 - S^2)\, A_6 + A_{12}U + j\,(A_7 + A_{12}W)}{U + jW} \qquad (18)$$

It will be found, however, that in normal motors, W is usually small compared with U up to well above full load, so that for all practical purposes, the effect of the term A_{12} on I_1 cannot be found in ordinary slide rule computations, and it is therefore justifiable to neglect it. On the calculation sheet, the primary current is figured from equation (**18**), neglecting the term $A_{12}W$.

Simplification of the Constants. If the leakage reactance be assumed to be equally divided between the primary and secondary, the constants A_1 to A_{12} inclusive can be reduced to expressions very much easier to calculate numerically, and the final results will be little different than if account were taken of the actual distribution of leakage. In the majority of cases, the amount of leakage chargeable to the primary and that to the secondary is not accurately known since no method has yet been devised to separate these leakages by an actual test. It is therefore justifiable, for ordinary purposes, to assume the $x_1 = x_2$. The reactance of the primary with the secondary open-circuited will be, therefore

$$X_m + x_1 = X_m + x_2 = X_0 \qquad (19)$$

The short-circuit reactance of the motor, taking into account the shunting of the secondary leakage by the magnetizing reactance is

$$x_1 + x_2 \frac{X_m}{X_m + x_2} = x_2 + x_1 \frac{X_m}{X_m + x_1} = X \qquad (20)$$

From the above relations, a further interesting relation can be shown to be exactly true when $x_1 = x_2$

$$\frac{X_m}{X_0} = \sqrt{\frac{X_0 - X}{X_0}} = K_p \qquad (21)$$

The primary current with the secondary open is

$$i_m = \frac{E}{X_0} \qquad (22)$$

Using equations (**19**) to (**22**) the values obtained for the A constants can be much simplified as shown in Table I. In expressions in this table, no approximations have been made although on the calculation sheet the term $-\dfrac{r_1 r_2}{X_0}$ has been dropped from F_1 as it is insignificant here.

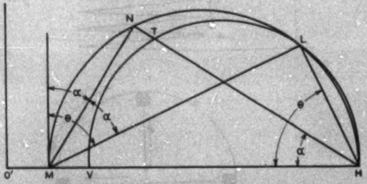

Fig. 7—Branson Circle Diagram of a Single-Phase Induction Motor, Locked and Pull-Out Points

MAXIMUM OR PULL-OUT TORQUE

Perhaps the most logical way to determine the maximum torque would be to write for the developed or internal torque, the following expression from (**16**)

$$\text{Internal torque} = \frac{112.6 \times S}{\text{syn. r.p.m.}} \times \frac{(1 - S^2)\, A_{10} + A_{11}}{U^2 + W^2} \qquad (23)$$

differentiate it with respect to S and equate to zero to obtain the speed at maximum torque. Such a procedure, however, leads to a sixth degree equation (really a third degree equation as only the even powers of S exist) which is too cumbersome for practical use. Since it is not necessary to know the speed at pull-out with a high degree of accuracy in order to obtain the maximum torque sufficiently close, the following alternative method was used to obtain the speed at pull-out.

Fig. 7 shows a typical Branson circle diagram of a single-phase motor. $O'M = i_m$, $O'V$ no-load current and $O'H$ the locked current, also $< LMH = \tan^{-1} \dfrac{r_2}{X}$. For any load point T, the speed, as a fraction of synchronous speed is, from Branson

$$S = \sqrt{\frac{TH - MNr_2/X}{TH}} = \sqrt{1 - \frac{MNr_2/X}{TH}} \qquad \textbf{(24)}$$

The point T for maximum torque is taken so that the line NH bisects the arc VL. This amounts to the same thing as bisecting the arc ML. We can now calculate MN and NH as follows for

$$\theta = \cot^{-1} \frac{r_2}{X} \text{ (complement of } < LMH)$$

$$\alpha = \frac{\theta}{2} = \frac{1}{2} \cot^{-1} \frac{r_2}{X} \qquad \textbf{(25)}$$

and

$$MN = MH \sin \alpha$$
$$NH = MH \cos \alpha$$

but TH can be taken, in most cases, as approximately 95 per cent of NH, so the speed at pull-out becomes

$$S_{po} = \sqrt{1 - \frac{(MH \sin \alpha)\, r_2/X}{0.95\, MH \cos \alpha}}$$

$$= \sqrt{1 - 1.05 \frac{r_2}{X} \tan \alpha} \qquad \textbf{(26)}$$

where α has the value given in equation (**25**). Thus, the speed at pull-out depends principally upon r_2/X. The value of *Spo* was calculated for different values of r_2/X and plotted on the curve given on the calculation sheet, Fig. 1.

No-Load Conditions. The various relations given in equations (**42**), (**43**), and (**44**) were deduced by simple algebra from the values of the F constants and need not be repeated here.

Appendix III

THEORETICAL BASIS OF POLYPHASE CALCULATION SHEET

Fig. 4 shows the more general form of equivalent circuit for either Fig. 2 or Fig. 3. The currents, in terms of the impedances, are:

$$I_1 = \frac{E\,(Z_m + Z_2)}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \qquad \textbf{(27)}$$

$$I_2 = \frac{E Z_m}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \qquad \textbf{(28)}$$

$$I_m = \frac{E Z_2}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \qquad \textbf{(29)}$$

It is now a simple matter to substitute the individual resistances and reactances in the above equations. For the sake of convenience in numerical computations, it will be found highly desirable to divide numerators and denominators by the quantity $(X_m + x_2)$. This done, the following equations are obtained

$$I_1 = \frac{A_5 + A_6/s + jA_7}{U + jW} = \frac{F_5 + F_6/s + jE}{U + jW} \qquad \textbf{(30)}$$

$$I_2 = \frac{A_5 + jA_8}{U + jW} = \frac{F_7}{\sqrt{U^2 + W^2}} \text{ (numerically)} \qquad \textbf{(31)}$$

$$I_m = \frac{A_6/s + jA_{10}}{U + jW} \qquad \textbf{(32)}$$

where

$$U = A_1 + A_2/s = F_3/s - F_1 \qquad \textbf{(33)}$$
$$W = A_3 + A_4/s = F_4 + F_2/s \qquad \textbf{(34)}$$

The values of A_1, A_2 etc., and F_1, F_2, etc., are given in Table II. All of the A constants were not used on the final calculation sheet (especially when $x_1 = x_2$) and so those that were used were rearranged and the symbols F_1, F_2, etc., were assigned to those that were used.

Simplification of the Constants

If the primary and secondary reactances be assumed equal, the relations of equations (**19**) to (**22**) apply directly. The values of the various expressions in Table II were determined for $x_1 = x_2$ and are listed in this Table. No approximations have been made and each one of these expressions is exact except the second value given for F_4 and F_7; the first value is exact and it can be seen that each approximation is insignificant.

The procedure to be followed from this point in obtaining the complete performance is amply described in part IV of the paper with the exception of how the slip at maximum torque was obtained, a matter which is developed in the following paragraphs.

Slip at Maximum Torque

The torque developed by a polyphase motor, neglecting friction and windage, is

$$T = \frac{112.6}{\text{syn r.p.m.}} \times \frac{I_2^2 r_2 \times \phi}{s} \qquad \textbf{(35)}$$

$$= \frac{112.6}{\text{syn r.p.m.}} \times \frac{F_7^2 \times r_2 \times \phi}{\left[\left(\frac{F_3}{s} - F_1\right)^2 + \left(\frac{F_2}{s} + F_4\right)^2\right] s}$$

The torque will be a maximum when the expression

$$\left[\left(\frac{F_3}{s} - F_1\right)^2 + \left(\frac{F_2}{s} + F_4\right)^2\right] s$$

is a minimum, a condition which can be found by differentiating the latter and setting equal to zero, whence the slip at maximum torque is found to be

$$s_m = \sqrt{\frac{F_2^2 + F_3^2}{F_1^2 + F_4^2}} \qquad \textbf{(37)}$$

In terms of the fundamental constants of the motor, the above expression becomes

$$s_m = r_2 \sqrt{\frac{1 + \left(\dfrac{r_1 + r_m}{X_0}\right)^2}{\left(X - \dfrac{r_m}{X_0} r_1\right)^2 + \left(r_1 + \dfrac{2}{1+K_p} \times \dfrac{r_m}{X_0} X\right)^2}} \qquad (38)$$

If iron loss be neglected, $r_m = 0$ and (**38**) becomes

$$s_m = \frac{r_2}{\sqrt{r_1^2 + X^2}} \times \sqrt{1 + \left(\frac{r_1}{X_0}\right)^2} \qquad (39)$$

For most practical cases, the quantity under the second radical is so nearly unity that it may be neglected and thus

$$s_m = \frac{r_2}{\sqrt{r_1^2 + X^2}} \text{ (approx.)} \qquad (40)$$

Maximum Torque. The maximum torque in terms of the F constants can be found by substituting equation (**37**) in equation (**36**) obtaining

$$\text{maximum torque} = \frac{112.6}{\text{syn. r.p.m.}}$$

$$\times \frac{F_7^2 \times r_2 \times \phi}{2\left[\sqrt{(F_3^2 + F_2^2)\,(F_1^2 + F_4^2)} + F_2F_4 - F_1F_3\right]} \qquad (41)$$

If the values of the F's in Table II be put into the above expression and r_m be assumed equal to zero, equation (**41**) reduces to

$$\text{maximum torque} = \frac{112.6}{\text{syn. r.p.m.}}$$

$$\times \frac{(EK_p^2)\,\phi}{2\left[\sqrt{r_1^2 + X^2} + K_p^2 r_1\right]} \qquad (42)$$

This latter equation is quite useful in itself for figuring the maximum internal torque of a polyphase induction motor. It is also valuable as a check on the method of Fig. 5 to determine if the F constants are numerically correct.

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Discussion

H. R. West: It seems that many designers object to the use of the general method for calculating the performance of a machine as described by Mr. Veinott because it does not enable them to follow the processes with physical interpretation. The answer is, of course, that the purpose of a method of calculation is to enable the designer to predetermine machine performance with the desired accuracy in the least time. Such methods as the circle diagram or the equivalent circuit are, of course, invaluable to anyone who has much to do with motors, but not necessarily for purposes of calculation. That method of calculation should be used which gives the desired accuracy with the least labor, and I believe that the results of algebraic analyses, arranged in systematic form such as Mr. Veinott is using, can profitably be used more than they are at present.

In connection with the correction for the core loss component of primary current in the single-phase motor; Mr. Veinott considers that this component, in calculating the primary copper loss, causes a voltage drop which is not taken into account in figuring the input to the rotor. This means that the calculated motor output is too large by a very slight amount. The error is certainly very small. Has this error been found to be negligibly small in comparison to the error that would be made if the total core loss were subtracted from the output and no correction were made for the primary copper loss?

R. E. Hellmund: Those who designed induction motors many years ago and obtained at that time a rather close agreement between calculations and load tests undoubtedly wonder why new and improved calculating methods such as that presented by Mr. Veinott and others (see bibliography given in paper) should still be an active subject of discussion at Institute meetings. At the same time, it may be a surprise to many to learn that the discrepancy between the results obtained from load tests and those obtained by calculating methods from no-load values and other motor constants has been greater during recent years than it was, say, 20 to 25 years ago. There are no doubt various reasons for this and a brief discussion of a few of them is given in the following.

A comparison of present-day designs and those made about 20 years ago indicates a marked decrease in motor size. This change has taken place gradually, so that many of the effects of it have not been evident at any one time, and, furthermore, they probably have been obscured by various individual changes in the numerous variables entering into the design. Considering, however, that now, as then, the torque of the motor is proportional to the field and the ampere conductors, one cannot escape the conclusion that generally speaking there must be with the smaller dimensions higher flux densities and also more ampere conductors within a unit of space in the up-to-date motors than was found in the older ones; most likely this also means in general more ampere conductors per slot than in the older designs. This means that the zigzag and slot leakage fluxes increase, which will tend toward greater saturation at load, especially in the tooth tips of partially-closed-slot machines; to this must be added the fact that the main flux densities in the same parts have also increased. If we consider now that all theories deriving motor performance from the motor constants neglect the effect of saturation, it is not surprising that in the modern designs there should be a greater discrepancy between such calculated results and tests made under actual load conditions.

Let us, for the sake of argument, assume the extreme case of complete saturation of the tooth tips under load condition by the combined effect of main and leakage fluxes, while at no-load the main flux by itself does not bring about an appreciable saturation of these same parts. Under these assumptions, many of the conditions under load will approach those obtained with an open-slot machine, while at no-load there would be the full benefit of the partially-closed-slot construction. It follows then that the magnetizing current of the main flux under load would be larger, so that the power factor is undoubtedly different from that derived by calculations based on no-load conditions. It follows further that the flux distribution under load in the air-gap will approach that of an open-slot machine and thus cause extra core losses in the surface and in the teeth of both members similar to those that would be obtained if both members were constructed with open slots, and which, as every designer knows, will appreciably differ from those obtained with partially-closed slots. In other words, it is evident that the full-load core losses will be different from the no-load losses which are made the basis of the usual performance calculations. Naturally these extreme assumptions do not exist in actual practise, but it is quite evident that a closer approach to them is generally obtained with the smaller up-to-date motors than with the very much larger motors of the past.

Other changes in design might be cited which would further explain certain differences between the old and the present-day designs. In pointing out these differences, it should not be inferred that this is meant as a criticism of the newer designs. On the contrary, the designers and manufacturers have rendered very valuable service not only by making appreciable reductions in the cost of the motor, but also by greatly reducing the motor dimensions with resultant greater ease of application of motors for all purposes, all of which, of course, is of benefit to the user. This discussion is merely intended to indicate that by the continuous improvement and refinement of motors, the designer has created new problems for himself and perhaps brought about the necessity for a different point of view. Rather than priding himself upon the close agreement between his calculated results and those obtained from actual load tests, he will have to continue the refinement and applicability of his calculating methods such as has been done in Mr. Veinott's paper, as well as of his test methods under load, for the purpose of permitting a more correct determination and analysis of the difference between the two. It is only through such efforts that the effect of changed conditions upon the various constants can be correctly analyzed and so pave the way for still further improvement in motor design.

P. H. Trickey: I have used Mr. Veinott's method for over a year for both single-phase and polyphase motors and for ratings between 2 hp. and 3 watts output, and I have had very satisfactory results.

P. C. Smith: Anyone spending the entire day in the construction of circle diagrams would, no doubt, welcome the easier, faster method presented by Mr. Veinott. The analytical method lacks the visual picture given by the diagram, but it is less tedious, which goes a long way towards off-setting the advantage of the visual picture. The analytical method published by Mr. G. T. Smith in the *Electrical Journal*, December 1922, has been in use since that time on larger polyphase motors, but it was devised particularly for large motors and approximations were made, for simplicity's sake, which make it unsuitable for fractional horsepower sizes. Mr. Veinott's method is more universal, permitting quick approximations or accurate calculations as desired.

The paper goes into great detail in dealing with the current through the shunt circuit, showing the influence of core loss but apparently neglects friction and windage. Do not friction and windage influence this current? The paper also indicates that core loss decreases with the load. This is not always the case.

Item 18 of the polyphase calculation makes use of the secondary resistance in determining the secondary loss. In cases where the actual tested slip is available, it might be advisable to substitute this for the secondary resistance.

C. G. Veinott: Mr. West has raised a question about the relative correctness of the method of correcting the primary current for core loss as presented in this paper with merely subtracting the total core loss from the output. The latter mentioned method assumes all the core loss power to be transferred across the air gap and hence, the power developed by the rotor at full load is slightly too large. The method presented in this paper has been found to give satisfactory results. Indeed, the results are so nearly correct, that it would be difficult to determine if Mr. West's suggestion would be more accurate or not.

Mr. Smith asks whether or not friction and windage influence the current through the shunt circuit. Friction and windage are a mechanical load on the motor and do not influence the current in the shunt circuit.

I am not certain that I understand Mr. Smith's point about secondary resistance. The secondary resistance can be corrected in cases where the actual tested slip is available by multiplying the calculated secondary resistance by the ratio of tested slip to calculated slip. If this is Mr. Smith's idea, it is a very good suggestion.

Measurement of Stray Load Loss
in Polyphase Induction Motors

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Synopsis.—*The purpose of this paper is to present the results of experiments carried out to establish a practical method of measuring stray load loss in induction motors. The nature of stray load losses is discussed, methods of measuring these losses are described, and proposals for the revision of the Institute Standards to include stray load losses are made.*

DEFINITION OF STRAY LOAD LOSSES

As the name indicates, the stray load losses are those losses in addition to the I^2R losses in the motor windings, which occur as a result of the load currents. These losses occur in all types of machines, and their inclusion in efficiency tests on synchronous machines has long been required by the A.I.E.E. Standards.

For d-c. machines, however, only an arbitrary allowance of 1 per cent, and for induction motors no allowance, for these losses is made in the Standards for lack of a satisfactory method of measurement in either case. Were this difficulty removed the desirability of including the stray load losses in efficiency determinations could not be questioned.

The magnitude of stray load losses in polyphase induction motors is not well known at present, but in general it is of the order of from 2 to 5 per cent, in squirrel-cage motors of from 1 to 5 hp. and it decreases gradually with increase in motor rating until it is less than 1 per cent for motors of several hundred horsepower. The "conventional efficiency" of polyphase induction motors as determined by the A.I.E.E. Standards is thus always higher than the actual motor efficiency by the amount of the stray load losses.

This has resulted in awkward situations at times, particularly in the case of motors used to drive apparatus such as pumps or generators. The overall efficiency of the pumping or generating unit being known by measurement and the motor efficiency being determined by the manufacturer on the basis of conventional efficiency, the existence of stray load loss in the motor operates to lower the apparent efficiency of the pump or generator.

Besides removing these inconveniences, the development of any easy method of measuring stray load losses should permit the losses themselves to be studied more intensively and perhaps ultimately to be greatly reduced. The experiments on polyphase induction motor stray load loss determination here described were undertaken with these objects in view.

POSSIBLE SOURCES OF STRAY LOAD LOSS

The losses included in the calculation of conventional efficiency are, primary copper losses as determined by the load current and the measured d-c. resistance of the primary, core loss as determined under running light conditions, friction and windage usually also determined from the running light or excitation tests, and the secondary copper loss determined from the measured slip under load.

These leave as a possible source of stray load loss (a) additional primary copper loss due to eddy currents in the primary conductors, (b) eddy losses induced in the end shields, clamping fingers, etc., by the leakage fields of the primary coil ends, etc., (c) increase in the iron or core loss attendant upon the increase of the stator current from its running-light to its full-load value, and the similar change in the rotor current.

As this paper is principally concerned with stray load losses in moderate size polyphase motors, (c) is the principal item with which we are concerned. Additional losses under item (a) are now so thoroughly understood by designers that they are not permitted to reach large values, and even though corrective measures to reduce them are not undertaken, methods of calculating them are well known and may be relied upon to determine their magnitude. Losses under heading (b) are very difficult to calculate. These losses may be appreciable, especially in high-speed motors where the end leakage reactance is high.

CHANGE OF CORE LOSS WITH LOAD

The running-light core loss of any induction motor is usually several times as large as the "fundamental frequency" loss calculated from the volumes of the stator core and teeth and the magnetic densities existing in these parts. Part of this difference may be due to increased fundamental frequency loss as the result of burrs or punching strains, but the major portion is the tooth-frequency core loss of the motor. It is the tooth frequency core loss which changes with load and forms the principal seat of load loss in moderate size induction motors. The fundamental and tooth-frequency elements of the core loss may be experimentally separated by measurement of the running-light slip of an induction motor.

Although it is difficult to calculate the tooth-frequency core loss in an induction motor under load or for that matter in a motor running light, it is not difficult to show that the tooth-frequency core loss is a function of the load and that its increase with load constitutes a real reason for the observed stray load losses.

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Consider for example the losses in the rotor surface and teeth of a slip-ring induction motor. These are induced by the stator tooth harmonic flux. This flux may be considered as consisting of two component fluxes, the two component fields having the same number of poles. The first of these is independent of load and is the harmonic flux produced by the action of the fundamental m.m.f. on the stator slot openings. The second varies with load and is due to the tooth harmonic magnetomotive force produced by the concentration of the stator current in slots. It is the increase of this tooth harmonic magnetomotive force with load which accounts for the change of the tooth-frequency core loss with load.

In a squirrel-cage motor, all of the tooth-frequency core loss need not necessarily occur in the laminations. Inasmuch as each rotor tooth of a squirrel-cage motor is embraced by a low-resistance electric circuit, little pulsation of flux can occur in a rotor tooth. Instead, a high-frequency current circulates in the rotor bars and the flux is forced to pulsate in the stator teeth even when the motor is running light. The loss due to this high frequency current in the rotor bars is as much a part of the tooth-frequency core loss of the motor as the eddy and hysteresis losses in the laminations themselves.

Indirect Methods of Measuring Stray Load Loss

All methods of measuring stray load losses may be classified as either direct or indirect. Indirect methods aim to measure the total loss in the motor, subtraction of the conventional losses yielding the stray load loss. Some of these methods are:

a. *The input-output method.* In this method the motor output is measured directly on a dynamometer and the input by wattmeters. The difference is the total motor loss. If it could be carried out accurately the input-output method would of course be the one infallible method of determining stray load loss. Actually in even moderate size motors, both the input and the output have become such large numbers compared to their difference that even a small error in either the one or the other results in very poor accuracy, by the time the necessary subtractions have been made to obtain the stray load loss.

Some of the results presented later in this paper were obtained by the input-output method. The results were obtained with good accuracy only by virtue of the highly developed technique and very accurate equipment, resulting from study and experience in making input-output tests over a period of several years. The equipment was refined to a high degree, the dynamometers being provided with oscillating bearings and the dynamometer errors being accurately determined by running each of three dynamometers against the other two.

b. In order to avoid the necessity of making mechanical measurements of the output an identical motor may be used as a generator, the output then being measured electrically. The load losses are then divided equally between the two machines, after allowing for their different frequencies.

c. *Calorimeter methods.* These consist in measuring the amount of heat carried away from the motor by the cooling air. They are only applicable to large motors and those in which the cooling air is forced to flow in definite passages and even here they yield only fair accuracy.

Other ingenious methods of this nature may be devised such as running the motor in a box with a known amount of air passing through, etc., but these methods are hardly applicable to ordinary shop conditions.

Direct Methods of Measuring Stray Load Loss

Direct methods of measuring stray load loss are based on a procedure by which either the rotor or stator winding is excited with direct current, the unexcited winding being short-circuited. The rotor is revolved at synchronous speed and the amount of power required to do this is measured. This is usually accomplished by coupling the motor to a dynamometer or by driving it from a calibrated direct-current motor. For accurate results, the rating of the driving motor should not be more than 10 per cent of the induction motor rating.

Under these conditions a fundamental frequency current is induced in the conductors of the unexcited member. This current will be of such magnitude that its m.m.f. will very closely equal and oppose the m.m.f. of the direct-current excitation. The only fluxes existing in the motor then are the tooth harmonic fluxes which produce the tooth-pulsation and surface losses the motor experiences under load. In order to find the amount of these losses, it is necessary to subtract from the amount of power required to drive the rotor, the friction and windage loss and also an amount of power equal to the I^2R loss produced by the fundamental frequency current in the unexcited member. The remainder is the surface and tooth-pulsation loss caused by the opposing currents and is approximately equal to the actual stray load loss.

These methods have the advantage that they are usually easy to carry out and the inaccuracies in test are not greatly increased by the subsequent calculations as is the case with the input-output method. Their adaptability to shop conditions makes these methods suitable for approval by the A.I.E.E. if their validity can be established.

Accepting the results of these tests as the true stray load loss of the motor is equivalent to assuming that the tooth-frequency core loss due to the load current m.m.f. may be separately determined as in the direct method, and added to the running-light core loss to obtain the total core loss under load. Whether or not this is a justifiable assumption must be proved experimentally, although it has been found approximately true for synchronous machines. The actual tooth harmonic fluxes which exist in a motor under load are difficult to express analytically and there is little possibility of proving the validity of the direct method by analytical means. Recourse must then be made to comparative tests by direct methods and dynamometer tests. These comparative tests have been made on a number of motors. The results are given in Table I.

TABLE I—LOAD LOSS TESTS

			Direct methods	
	Input-output method	Method *A*	Methods *B* and *C*	Method *D*
Motor No. 1—Full load	306	200	315	
25% overload	498	270	459	
Motor No. 2—Full load	165	87	174	
25% overload	234	145	281	
Motor No. 3—Full load	109	120	122	
25% overload	190	165	197	
Motor No. 4—Full load	321	360	914	
25% overload				
Motor No. 5—Full load	153	100	230	
25% overload	228	160	369	
Motor No. 6—Full load	315		246	
25% overload	505		586	
Motor No. 7—Full load	487		680	500
25% overload	835		1,015	860
Motor No. 8—Full load	324	240	420	
25% overload	619	380	720	
Motor No. 9—Full load	606	210	530	
25% overload	884	410	940	
Motor No. 10—Full load	526	165	462	
25% overload	957		732	
Motor No. 11—Full load			210	146
25% overload			396	287

Direct Methods Applied to Slip-Ring Motors

In making measurements on slip-ring motors, it is customary to excite the rotor with direct current, the stator terminals being short-circuited. An ammeter is included in one of the stator phases. With the rotor revolving at synchronous speed, the rotor excitation is regulated to give a fundamental frequency current in the stator corresponding to the particular load for which the load loss measurement is being made. The amount of loss in the stator winding is then known from the direct current resistance of the stator winding.

It should be noted that when the rotor is excited with direct current, fundamental frequency current flows in the stator windings and the stray load loss which is measured includes, in addition to the tooth pulsation and surface losses, those additional losses mentioned under sources (a) and (b). As these are legitimately a portion of the motor stray load loss, it is preferable, wherever possible, to excite the rotor or secondary winding with direct current rather than the stator. When the stator is excited with direct current the (a) and (b) losses are not present and the indicated stray load loss will on this account be less than actual.

The above method of measuring stray load loss in slip-ring motors will be recognized as nothing more than the accepted method of measuring short-circuit core loss in synchronous motors and generators. A slight difference exists in the case of induction motors, however, and this must be corrected for in testing squirrel-cage motors as well as slip-ring motors.

Direct methods of measuring stray load loss will, of course, give zero load loss for zero stator current. Actually the motor stray load loss is, by definition, zero when the stator current has its running-light or excitation value. This discrepancy may be corrected for by assuming that the exciting current causes a small amount of tooth frequency core loss which is already included in the running-light core loss. The amount of this loss may be found by making measurements with the stator current adjusted to its running-light value. The value of the stray load loss for this value of current being already included in the running-light core loss, should be subtracted from the stray load loss for all other values of current.

Although this method of correcting for the effect of the magnetizing current may not be strictly accurate, the actual amount of the correction is usually small, except for very low-speed motors.

Direct Methods Applied to Squirrel-Cage Motors

In testing squirrel-cage motors, the stator winding is, of course, the only member which can be excited with direct current.

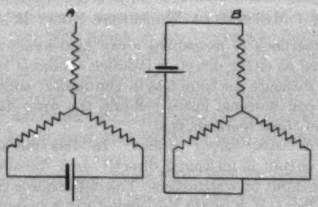

Fig. 1

It is necessary, in order to carry out this method, to determine what value of direct-current excitation corresponds to any particular value of stator load current and also the value of the loss in the rotor due to the fundamental frequency current in the rotor bars. This latter loss must be subtracted from the power required to rotate the rotor or the "rotational watts" in the same way that the stator I^2R loss is subtracted in the case of the slip-ring motor.

The necessary relation between d-c. excitation and r.m.s. load current depends on the way in which the stator terminals are connected. In either case direct-current and alternating-current values are equivalent when they produce the same magnetomotive force in the air-gap. If the connections are as in Fig. 1A

$$I_{\text{D-C.}} = I_{\text{A-C.}} \sqrt{2} \times \frac{\sqrt{3}}{2} = 1.22\, I_{\text{A-C.}} \qquad (1)$$

or if the connections are as in Fig. 1B

$$I_{\text{D-C.}} = I_{\text{A-C.}} \sqrt{2} = 1.414 \text{ A-C.} \qquad (2)$$

It is immaterial which connection is used as long as the proper equation above is used.

In order to determine the loss in the rotor due to the fundamental-frequency current circulating in the rotor bars, four methods are available. Three of these were described in an article by H. E. Linck which was published in the *Archiv fur Elektrotechnik*, Nov. 5, 1929. The methods described in this article seemed promising and furnished one of the incentives for these experiments.

Method A. This makes use of data obtained from a blocked rotor, or impedance test. Simultaneous values of stator current and input watts are taken, with the rotor blocked at standstill and balanced alternating voltages of rated frequency applied to the terminals. If from the input watts under these conditions the stator I^2R loss is subtracted, the remainder is principally the rotor loss. This should be identical with the rotor loss during the stray load loss test if the impedance watts and rotational watts are based on equivalent values of stator current.

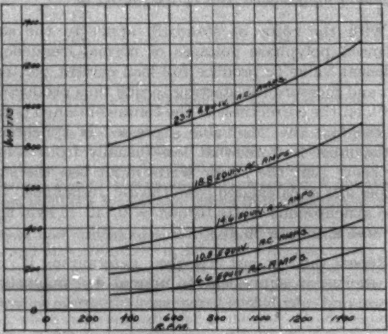

Fig. 2

The rotor loss determined in this way is too large, however, on three accounts. In the first place, if the airgap is small the squirrel-cage currents will not be uniform but will have a tooth harmonic pulsation in their space distribution. The rotor copper loss will thus be larger than that due to the fundamental rotor current which alone determines the full-load slip. Also, since there are actually stator losses of types (a) and (b), and likewise a certain amount of core loss due to the stator leakage flux which exists both under impedance test and under actual load conditions, the rotor loss determined by this method is too large by their amount also. These errors, making the rotor loss too large, make the difference between the total rotational loss and the rotor loss too small. Thus the stray load loss determined in this way is always less than the true stray load loss occurring under short-circuit conditions. As the effect of superposition of load and no-load fluxes is generally to reduce to total loss below the sum of the open- and short-circuit core losses, due to saturation, the error in the loss determination is partially corrected for.

The stator losses of types (a) and (b) and the core losses due to the stator leakage flux above mentioned can be separately determined by repeating the impedance test on the stator with the rotor removed. These losses are usually so small in comparison with the total stray load loss however, that this additional test is not warranted except possibly for very large motors.

This method *A* has an advantage over method *B*, to be described, in that it gives more correct results for double squirrel-cage types of motors.

Method B. This method of determining the rotor loss consists in taking data for a series of curves as shown in Fig. 2. These are curves of rotational watts plotted against speed for various values of stator excitation. For each value of stator excitation the intersection of the rotational watts curve with the axis of zero speed gives the value of rotor I^2R loss.

If the rotor bar resistance varies with frequency an error is introduced and the method becomes useless unless a calculated correction for the change in rotor resistance is made. If this correction is not made this method gives too low a rotor loss and the load loss so determined is correspondingly high. This correction becomes very large when the rotor bars are of the deep bar and double squirrel-cage type.

Since in using this method the stator is excited with direct current, stray load losses of types (a) and (b) will not appear in the result. These are small however, in motors of moderate size.

Method C. This method requires that only one rotational watts curve at synchronous speed be determined by test. The rotor loss is assumed to be equal to the slip loss under load, which is available from the measurements of watts input and slip made when the motor is tested under load. The slip loss should be measured at about 25 per cent overload in order to minimize the effect of the magnetizing current, and the rotor loss is assumed to vary as the square of the exciting current.

It is obvious that the method is practically the equivalent of method *B*, so that the remarks pertaining to double squirrel-cage rotors and losses of types (a) and (b) apply equally here.

The slip loss of the motor under load for any value of stator current is found with sufficient accuracy from the usual formula,

Slip loss = (input — core loss — primary copper loss) slip.

Method D. This, like method *A*, makes use of data obtained from a blocked rotor test. The actual torque developed by the motor at standstill, with balanced alternating voltages of fundamental frequency applied to the stator, is measured by means of a brake arm, or wheel and pulley.

The rotor loss is then equal to the corresponding value of synchronous kilowatts, or to the product of the measured standstill torque in foot pounds by 0.142 times synchronous speed in r.p.m. The value so obtained is less, and more nearly correct, than that given by method *A*, as it does not include the stator losses of types (a) and (b). There is still a slight error due to the net torque produced by the harmonics in the rotor current distribution, but this is normally small.

The standstill torque should preferably be measured at a current of 100 to 150 per cent of the full-load value, and for other loads it may be assumed to vary as the square of the current. The torque at full-load current is normally only 2 to 10 per cent of full-load torque, or it is of the same order as the static friction torque. For this reason, very careful torque readings with the rotor turned both with and against the torque must be taken to average out the friction and the irregularities of the torque position curve.

RESULTS

A summary of the results of measurements of the stray load loss of a number of induction motors using both input-output and direct methods is contained in Table I.

A review of these results indicates that for squirrel-cage motors the best results are obtained using method *C*, or preferably method *D*. Method *C* is easily carried out, and gives accurate results for all small motors which have no appreciable eddy current effect in the rotor bars at starting. When double squirrel-cage rotors are used the method is inaccurate. Motor No. 4 was built with a double squirrel-cage rotor.

Using method *D*, the only error in the determination of the true stray load loss is the omission of the stator losses of types (a) and (b). This error is normally small and may be balanced by the consideration that the stray load loss under actual load conditions is probably somewhat less than that at short circuit because of saturation.

For slip-ring motors the most satisfactory method appears to be to measure the rotational loss with the rotor excited with direct current and to subtract the stator I^2R loss as determined from the measured stator current and the d-c. resistance.

Detailed outlines of these preferred tests are given in Appendix A. It is proposed that after confirmation or modification by other investigators, these methods be taken as the basis for a revision of the Institute Standards to require the inclusion of induction motor stray load losses in efficiency determinations.

Appendix A

PROPOSED METHOD OF DETERMINING STRAY LOAD LOSS IN INDUCTION MOTORS

Slip-Ring Motors

1. Measure the power required to drive the motor at synchronous speed with the primary winding short-circuited and sufficient d-c. excitation applied to the secondary winding to cause full-load current to circulate in the primary; the power required is designated W. From the d-c. resistance (making any corrections necessary for differences in temperature) determine the primary I^2R loss and designate this as T.

2. Repeat the test just outlined with the primary current equal to the magnetizing current; and determine the rotational power required and the I^2R loss; these latter are designated W_o and T_o.

3. The stray load loss is then equal to $W-W_o-T+T_o$.

Squirrel Cage Motors

1. Apply d-c. excitation to the stator using two of the stator terminals and leaving the third open; the amount of direct-current required is 1.22 times the full-load alternating-current per terminal. Under these conditions determine the power required to rotate the rotor at synchronous speed; this is designated W.

2. With the rotor at standstill apply to the stator balanced polyphase voltages of normal frequency of sufficient value to produce full-load current per terminal, and measure the starting torque thus produced. A check on the correctness of the torque obtained is given by the fact that its value expressed in synchronous watts, must be somewhat less (usually from 5 to 20 per cent) that the stator watts input less the stator copper loss as calculated from the d-c. resistance. Then T = synchronous watts torque = 0.142 × syn. speed (r.p.m.) × torque (lb. ft.).

3. Repeat the test just described, determining W_o with d-c. excitation equivalent to normal a-c. magnetizing current and calculate

$$T_o = T_s\left(\frac{\text{Magnetizing current}}{\text{Full-load current}}\right)^2$$

4. The stray load loss is then $W - W_o - T + T_o$.

In practise, the starting torque varies closely as the square of the primary current; it is desirable therefore to measure the value of T at only one value of current, selected to obtain the greatest accuracy of test, and to obtain the values of T at other points by proportionality. The best value of current to select for this test will depend somewhat upon the type and size of motor. If the current is too large, overheating takes place and errors arise due to incorrect conductor temperature determinations. On the other hand, if too low values of current are used, the small torque produced in comparison with the static friction makes it difficult to obtain accurate values. It is recommended that full-load current normally be used for squirrel-cage and ball-bearing motors, while values of 150 per cent full load current are recommended for normal use on sleeve-bearing motors of less than 25 hp.

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Discussion

Orrin C. Rutledge: Since Mr. Koch's tests were made on machines of different sizes and types a more comparative idea of the results can be obtained by placing them on a percentage basis. Taking the values in Table I obtained by input-output method as 100 per cent in each case we obtain the following figures for the direct methods.

Method	% Load	Number of machines	Average	Minimum	Maximum
A	100	8	68.2%	31.4%	112.2%
	125	6	63.5	46.4	86.8
B and *C*	100	9	110.4	78.1	150.5
	125	9	112.6	76.5	161.0
D	100	1	103.0		
	125	1	103.0		

Method *A* gives low values and Mr. Koch has explained the reason for this.

Method *D* is the one proposed by Mr. Koch for adoptions in the standards but from Table I it is only possible to compare this with the input-output method on one machine.

Methods *B* and *C* give permissable average values but as may be seen there is a considerable spread from minimum to maximum.

I have been making some tests on a 10-hp. wound rotor machine and have endeavored to determine stray load loss by both input-output and direct methods, computing copper losses by d-c. resistance and measured currents in all cases. Plotting loss against line current the input-output method gives the curve (1) Fig. 1, which varies approximately as the cube of load component of line current.

The direct method gives approximately the same results whether the excitation is placed on rotor or stator, using either connection *A* or *B* in Mr. Koch's Fig. 1. This is shown in this discussion as curve 2, Fig. 1. It will be noted that the curve passes through the origin and has a value of about 60 watts when the equivalent current has a value equal to the no-load magnetizing current. It is considered that this value corresponds to the no-load pulsation loss but another test, proposed by Mr. Alger in 1920, showed a no-load pulsation loss of about twice this value.

If this is a true value of no-load pulsation loss it should be subtracted from all subsequent points moving the curve vertically downward into the position shown as curve 3, since by definition there is no "load loss" at no-load.

I would suggest consideration of the possibility that the a-c. equivalent of the d-c. current is comparable to the load component of line current under normal operation rather than the total line current. In that case we would add exciting component vectorially for each point to get it in terms of line current. This would have the effect of moving the curve nearly horizontally to the right instead of vertically downward, and of increasing the slope somewhat, (see curve 4).

The same process would be applied to Mr. Koch's method *D* for determining copper loss of the unexcited member. This loss would then also be zero when the line current equals the exciting current rather than when the line current is zero.

The curves indicate that in the vicinity of full load the loss by direct measurement is of the same order of magnitude as that by input-output determination. This checks with Mr. Koch's results. However, the shape of the curve would still leave open the question as to whether or not the pulsation loss by the d-c. method is a correct representation of the true pulsation loss occurring under load.

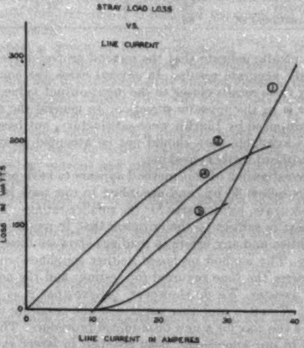

FIG. 1

O. C. Schoenfeld and S. F. Henderson: We have been studying the subject of stray load losses with the object of determining their value in our designs and are glad that it has been brought up for discussion. We, also, have been testing motors by the input-output method for a considerable period, but found, as did Mr. Koch, that many refinements were necessary to obtain results of the degree of accuracy desired for a study of the stray load losses.

The method outlined by Mr. Koch has some merits, but it must be admitted that it is not theoretically correct, nor could we check it experimentally. We believe that proper consideration has not been given to the effect of saturation. This factor alone can have considerable influence and as a result the d-c. short-circuit method gives values of losses in excess of those obtained under normal load conditions. A theoretical discussion as well as an experimental check of this method and the effect of saturation is given in an article by L. Dreyfuss in the July 7, 1928 issue of *Archiv für Elektrotechnik.* Making use of a paper entitled "Iron Losses for Simultaneous Magnetization at Two Different Frequencies" written by F. Schroeter and published in the 1924 issue of the same magazine, Linckh, in the article mentioned by Mr. Koch, explains this influence and proves it experimentally by tests on a series of 10 motors. The tests which we made check the observations of these authors.

Linckh found that correction factors were necessary and states, "It is probable that for practical purposes the accuracy of the constants is sufficient. However, the values have been found by readings on a comparatively small number of motors and it should be desirable that others should make similar investigations, especially on machines of larger ratings." The correction factors proposed are

$$\text{slip-ring motors } 0.5\ \frac{I^2 - I_0^2}{I^2}$$

$$\text{squirrel-cage motors } \frac{I^2 - I_0^2}{I^2 DC}$$

where the currents are stator currents. The method is, otherwise, essentially the same as that proposed by Mr. Koch and it will be noted that the corrections are of such magnitude that they cannot be neglected.

The following is a brief tabulation of our tests.

Full load efficiencies and stray load losses

Motor No.	Brake values	Linckh		Koch		
		B	C	B	C	D
1 Stray loss watts	115.0	140.0	165.0	225.0	265.0	431.0
% Efficiency	86.5	86.1	85.7	85.0	84.4	82.4
% Conventional eff.	88.4					
2 Stray loss watts	140.0	216.0		347.0		
% Efficiency	86.5	85.3		83.8		
% Conventional eff	88.7					
3 Stray loss watts	402.0	410.0		643.0		
% Efficiency	82.3	82.2		79.5		
% Conventional eff	88.6					
4 Stray loss in watts	157.0	170.0		270.0		372.0
% Efficiency	85.9	85.5		84.3		83.0
% Conventional eff	88.3					
5 Stray loss in watts	87.0	77.0		130.0		220.0
% Efficiency	86.2	86.4		85.3		83.6
% Conventional eff	88.3					
6 Stray loss watts	112.0	145.0		225.0		362.0
% Efficiency	86.2	85.8		84.8		83.0
% Conventional eff	88.1					

These results indicate that the method proposed does not give sufficiently accurate results. In several cases the conventional efficiency gave results closer to the input-output value, although the error is in the opposite direction. In general the correction factors proposed by Linckh were satisfactory, but based on the few tests available they should not be accepted as sufficiently correct without further checking.

The investigation of this method appears to have been limited to ratings below 30 hp. and according to our tests is not sufficiently accurate, particularly for the smaller ratings. The input-output test is subject to the criticism that it requires accurate test facilities and any errors in readings show up directly in the efficiency. The method proposed neither simplifies the facilities nor shortens the time required for testing, and the accuracy is not any greater. We do, however, believe that it is a step in the right direction and this paper and the resulting discussion may pave the way for a new or modified procedure. The method has not, to our knowledge, been checked in larger ratings and it may prove to be sufficiently accurate in this range where a dynamometer is not available. This can be determined only by experimental tests, and it appears that further investigation is warranted.

L. E. Hildebrand: The present conventional efficiency ignoring load loss does not give correct efficiency.

The theoretically correct input-output test requires a highly trained crew, highly developed equipment and highly developed technique to obtain satisfactory results and even then, it is expensive and subject to question.

In all practical work a conventional efficiency test *including* a conventional load loss is certainly superior to ignoring load loss. Also in the hands of an average crew with average equipment it should be superior to the impractical input-output test providing a method is used for determining a conventional load loss which gives consistently repeatable results approximately equal to the true load loss. We may as well face the fact that there has been developed no method for accurately determining the real load loss. A conventionally determined load loss should average close to the true load loss if proper account is taken of the factors which might vitiate the results.

Mr. Koch has pointed out those connected with the determination of the subtractable true secondary I^2R loss. His method for slip-ring motors is certainly essentially sound. His methods *B*, *C* and *D* all seem equally sound for single-cage motors and *C* is more convenient than *B*, so we should use either *C* or *D*. *C* is unsound for double cage and other rotors with deep bar effect. His method *D* would seem the best in general if it were not for two factors, first, the determination of the true starting torque at full load current and operating temperature is not quite as simple as it sounds due to the friction and heating factors. Second, method *C* sometimes gives closer to the true facts with single-cage motors.

It would seem that a method for determining a conventional efficiency including a conventional load loss should be legalized. Mr. Koch's proposed method *D* (with *C* as a permissible alternative for single-cage motors) seems practical for a tentative permitted method.

H. L. Barnholdt: The subject treated in Mr. Koch's paper is a very timely one and a satisfactory solution to this difficult problem is much to be desired.

That the problem of directly measuring stray load losses in induction motors is a difficult one is evidenced by the wide discrepancies shown by the test results. For squirrel-cage induction motors it is stated that method *D* is preferred, but only one test is indicated comparing it with the input-output test and while this checks quite closely, it is entirely possible that if a number of tests was made according to method *D* they would show as wide discrepancies as methods *A*, *B*, and *C*. Since the several methods admittedly are based on experimental data only, it is believed that further tests and greater consistency of test results are necessary before attempting standardization.

The method proposed for squirrel-cage motors does not take into account additional primary copper losses due to eddy currents in the primary conductors or the eddy losses induced in the end shields, etc., by the leakage fields of the primary coil ends, although the latter is admitted to be appreciable, especially in high-speed motors. Hence, even after making all these tests, the validity of the test results would still be open to question by the pump builders, particularly since high-speed squirrel-cage motors are used mainly by them.

For these reasons, it would probably be better, on smaller motors, to determine the efficiency by the input-output dynamometer method which, if properly carried out, could not be open to question. This would take care of the bulk of the motor production and would cover the class of machines where the stray load losses are the greatest.

In the case of larger induction motors, it is known that the stray load losses are relatively of less importance than on smaller machines. For example, some years ago the writer had occasion to make input-output efficiency tests of two 7,000-hp. induction motors and the efficiency by this method checked to within one-quarter of one per cent with the efficiency computed from the separate losses determined in the usual manner. Since it is impracticable to apply the dynamometer test regularly to larger motors and since here the stray load losses are relatively small, an arbitrary allowance say ¾ per cent could be made to cover stray load losses on large induction motors. Input-output tests could be made as opportunities presented themselves, with the idea of revising the allowance as soon as more test data would become available. This would have the advantage of simplicity, would inflict no hardship on anyone and would probably give results fully as accurate as those obtained by the proposed method.

C. G. Veinott: Mr. Koch is to be highly commended for his endeavor to measure the stray load loss in polyphase induction motors, however inaccurate his method may be. That such losses do actually exist there can be no doubt and it is essential for the designer to have some idea of their magnitude before they can be successfully reduced.

Suppose that the stray load losses were known, how would Mr. Koch suggest account be taken of these in calculating the performance of the motor by the equivalent circuit?

Mr. Koch recommends that either his method or some other method for determining stray load losses be added to A.I.E.E. "conventional" efficiency calculations. At present the Institute recognizes directly measured efficiency and conventional efficiency but rule 9-302 states that "Unless otherwise specified the conventional efficiency shall be employed." The Institute thus makes no distinction between large and small motors. I think Mr. Koch's paper brings out reasons why the Institute should make a distinction between large and small motors and should, in my opinion, recommend the directly measured efficiency (input-output) for the latter. This opinion is stated on the grounds that, for small motors, stray load losses are most important, the input-output method is easier, cheaper and more reliable to make. Just where the dividing line should be put is perhaps doubtful, but it certainly should be above 3 hp.

P. L. Alger: It is very desirable to establish a standard method for determining induction motor load losses, and while Mr. Koch's proposals are fundamentally sound, there is much to be done in the way of study and tests before any method can be adopted as a standard.

Measurement of the rotational losses by driving the motor at normal speed with direct current in the stator winding is an essential element in any direct measurement of stray load loss. A difficulty arises, however, in the determination of the correct secondary I^2R loss to be subtracted from the total rotational losses. If the rotor I^2R loss is determined on the basis of the resistance corresponding to normal load slip, the value obtained is too small, since it does not include the full frequency rotor iron and copper losses. These errors are very large on deep bar or double squirrel-cage motors, and for this reason methods *B* and *C* are regarded as undesirable for general use.

While determination of the rotor losses by measurement of the standstill torque at full load current gives theoretically almost correct results, it is very difficult to carry out in practise on small motors, because of the irregularity of the torque position curve and the errors due to bearing friction.

Determination of the rotor losses by measuring the impedance watts input and subtracting from it the primary I^2R loss is convenient and approximately correct in the case of overhung slot motors, but it gives a value of rotor loss which is slightly too large, and therefore gives a value of stray load loss somewhat too small. This error is somewhat offset by the existence of saturation in the flux leakage paths under normal load conditions. It seems practical, therefore, to get accurate results by this method, if a simple correction factor of perhaps 10 per cent is applied to the primary copper loss, to be subtracted from the impedance watts input.

In conclusion method *A* should form the basis of standards for stray load loss measurement in the case of overhung primary slot motors of 1 hp. and larger, with the addition of a multiplying factor of perhaps 1.1 for the primary copper loss on the standstill impedance test.

Method *D* should form the basis of standards for stray load loss measurement in the case of open primary slot motors (which are generally built in ratings of 20 hp. and larger).

Direct input-output tests should remain standard for fractional hp. motors and should remain optional for larger motors.

The Induction Motor Subcommittee of the Committee on Electrical Machinery is now reviewing this problem, and plans to submit some definite recommendations on the subject to the Standards Committee in the near future.

C. J. Koch: We are fortunate in having discussion contributed by gentlemen who have had the opportunity to test in their factories the procedures described in this paper. The result of their experiences leaves in doubt the desirability of adopting the proposals of this paper at least for motors of moderate size. It is perhaps true that for motors below 50 hp. a direct input-output test on a dynamometer will give the true efficiency with sufficient accuracy and with less expense than the no-load test required to find the conventional efficiency and the separate determination of the stray load loss. For large motors the dynamometer is quite impracticable however and it is believed the proposed methods will be of real value.

A few specific questions have been asked. It is true that using any of the proposed methods for squirrel-cage motors, stray load loss due to additional eddy current losses in the stator conductors, clamping fingers, etc., will not be included as the stator is excited with direct current. These losses in modern designs are usually small but if a question arises regarding their magnitude they may be easily checked by exciting the stator with alternating current and measuring the watts loss with the rotor removed.

There is no simple way of including the stray load loss in the equivalent circuit. For any value of load it should be subtracted from the gross output of the motor along with the friction and windage losses. This is based on the assumption that the stray load loss is all increase in tooth frequency core loss. (See Alger and Eksergian.) This procedure although correct would not be really an inclusion of the stray load loss in the equivalent circuit as for other loads the equivalent circuit would not tell the magnitude of the stray load loss. This would be found by other means.

Torque-Angle Characteristics of Synchronous Machines Following System Disturbances

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***Synopsis.**—A method of predetermining the torque-angle characteristic of a synchronous machine following a system disturbance is of importance in calculating the transient stability limit and in evaluating the effect on this limit of additional rotor circuits or of a change in excitation. In the first part of this paper equations are derived which are of sufficient generality to evaluate such factors. Two types of system disturbances are considered:*

1. Switching in or out of a connecting line,

2. Occurrence of a balanced three-phase system fault.

In the second part, an actual case of switching out a connecting line is calculated and the results compared with field tests taken on the New York Power and Light Corporation's system.

No attempt has been made to draw any general conclusions as to the effect of amortisseurs or changes in excitation. A subsequent paper will present results which have been obtained from the application of the method presented in this paper.

INTRODUCTION

THE determination of the behavior of synchronous machines during transient conditions subsequent to a system disturbance is becoming increasingly interesting and important. In determining this behavior it is essential to know the torque-angle characteristic of the given machine under conditions resulting from disturbances of various sorts. The most important of such disturbances are variations in (1) mechanical torque, (2) excitation voltage, (3) amount of external reactance, (4) system voltage. System voltage may vary either in magnitude or in angular position.

The cases of cyclic variation of impressed torque, sudden angular displacement and synchronizing out of phase have already been treated by Doherty and Nickle.[4] Equations for the damping and synchronizing components of pulsating torque caused by a given small angular pulsation of the rotor for any number of rotor circuits have been derived by Park.[2]

A problem of particular interest to operators is that of the variation in torque caused by a change in external reactance due either to the switching in or out of a line, or due to the occurrence of a system fault. This problem has been discussed in some detail by various authors[5,6,7,8] under the subject of system stability. The purpose of this paper is to establish general equations from which may be derived specific equations applying to any of the particular conditions mentioned above. Starting with the fundamental relations developed by Park,[2] general equations are derived for the positive phase sequence torque of a synchronous machine subsequent to the following system disturbances:

1. Switching in or out of a connecting line.
2. Occurrence of a balanced three-phase system fault.

The evaluation of the torque characteristics following such disturbances is of particular importance in the determination of transient stability limits. A step-by-step method of calculation is given so that the electrical torque of a machine having any number of rotor circuits can be determined at any time of its swing. The formulas and step-by-step method presented make it possible to predetermine such phenomena as that produced by amortisseur windings, high-speed excitation, etc.

ASSUMPTIONS

In order to simplify the mathematical work and to permit the use of operational solutions certain assumptions which appear to be reasonable and practical have been made. These are:

1. The machine is connected to a relatively large system; *i. e.*, it is assumed that the machine terminals are connected to an infinite bus through external impedance.
2. The machine is ideal as defined in reference 1.
3. Armature and line resistance are negligible.
4. The effect of the direct-current components of transient armature current, and all induced currents resulting therefrom, may be neglected in computing the torque.
5. The component of transient armature voltage due directly to the rate of change of magnitude of the rotor flux may be neglected in comparison with the voltage due to the rotation of this flux with respect to the armature. This assumption is justifiable for all ordinary machines.
6. The variations in rotor speed due to its oscillation with respect to the connected system may be neglected in the calculation of induced voltages.

I—Mathematical Analysis

FUNDAMENTAL RELATIONS

Using the two-reaction method of analysis it is possible to determine the characteristics of a synchronous machine from a consideration of two circuits. One of these, the direct axis circuit, represents those circuits which are symmetrical about the axis of the poles and the other, the quadrature axis circuit, represents those circuits which are symmetrical about the axis between the poles.

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*Both of the Central Station Engineering Dept., General Electric Company, Schenectady, New York.

4. For references see Bibliography.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

The fundamental relations upon which are based the equations developed in this paper were derived by Park[3] and are as follows:

$$\left.\begin{aligned}\Psi_d &= G(p)E - x_d(p)i_d \\ \Psi_q &= -\,x_q(p)i_q\end{aligned}\right\} \qquad (1)$$

$$T = i_q\Psi_d - i_d\Psi_q \qquad (2)$$

$$\left.\begin{aligned}i_{do} &= \frac{E_{do} - \Psi_{do}}{x_d} \\ i_{qo} &= -\,\frac{\Psi_{qo}}{x_q}\end{aligned}\right\} \qquad (3)$$

$$\left.\begin{aligned}\Psi_d &= e\cos\delta \\ \Psi_q &= -\,e\sin\delta\end{aligned}\right\} \qquad (4)$$

$$\left.\begin{aligned}e_d &= p\Psi_d - ri_d - \Psi_q p\theta \\ e_q &= p\Psi_q - ri_q + \Psi_d p\theta\end{aligned}\right\} \qquad (5)$$

The operational impedance of the direct-axis circuit met by the current resulting from a unit function change of terminal linkages in the direct axis is called $x_d(p)$. Similarly $x_q(p)$ is the operational impedance of the quadrature axis circuit. $G(p)$ is an operator which relates the field voltage with the current and linkages in the direct axis. A method of deriving these operators from the equivalent circuits of the direct and quadrature axes has been presented.[13]

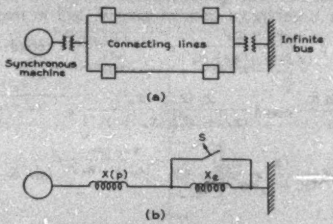

FIG. 1—SYSTEM DIAGRAM AND EQUIVALENT CIRCUIT FOR SWITCHING OPERATION

Case 1. Switching Out a Connecting Line

Consider the case of a synchronous machine connected to a very large system through two or more parallel transmission lines as shown in Fig. 1a. The machine is operating with a steady excitation voltage E_{do} and displacement angle δ_o before $t = 0$. If, at the instant $t = 0$, one of the lines is suddenly opened, there will be an instantaneous change in the amount of reactance between the machine and the infinite bus. An equivalent circuit representing this condition is shown in Fig. 1b where a reactance x_e is suddenly inserted by opening the switch S; $x(p)$ represents $x_d(p)$ for the direct and $x_q(p)$ for the quadrature axes; $x(p)$ includes the machine reactance plus the external reactance between the machine terminals and the infinite bus before the switch S is opened.

The effect of opening a switch which is carrying a current i' may be simulated by leaving the switch closed and impressing a current equal and opposite to i' through it so that by the superposition of the two, the resultant current through the switch is zero. Since no current was flowing through x_e before opening the switch the component of the impressed current which flows through x_e is the resultant current flowing between the machine and the infinite bus. We have, then:

$$i = \frac{x(p)}{x_e + x(p)}\,i' \qquad (6)$$

where

i = the value of current after opening the switch

and

i' = the value of current that would flow through the switch S if it were not opened.

The components of i' are:

$$\begin{aligned}i_d' &= i_{do} + \Delta i_d \\ i_q' &= i_{qo} + \Delta i_q\end{aligned} \qquad (7)$$

where from equation (1):

$$\begin{aligned}\Delta i_d &= -\,\frac{1}{x_d(p)}\,\Delta\Psi_d + \frac{G(p)}{x_d(p)}\,\Delta E \\ \Delta i_q &= -\,\frac{1}{x_q(p)}\,\Delta\Psi_q\end{aligned} \qquad (8)$$

Substituting equations (3) and (8) in equations (7):

$$\begin{aligned}i_d' &= \frac{E_{do} - \Psi_{do}}{x_d} - \frac{1}{x_d(p)}\,\Delta\Psi_d + \frac{G(p)}{x_d(p)}\,\Delta E \\ i_q' &= -\,\frac{\Psi_{qo}}{x_q} - \frac{1}{x_q(p)}\,\Delta\Psi_q\end{aligned} \qquad (9)$$

Equations (9) may now be substituted in equation (6) giving the direct and quadrature components of transient current of the machine subsequent to the switching operation. After re-arrangement and substitution of equations (4) in (9):

$$\begin{aligned}i_d &= \frac{E_{do} - e\cos\delta_o}{x_d}\;\frac{x_d(p)}{x_e + x_d(p)} \\ &\quad - \frac{e}{x_e + x_d(p)}\,(\cos\delta - \cos\delta_o) + \frac{G(p)}{x_e + x_d(p)}\,\Delta E \qquad (10) \\ i_q &= \frac{e\sin\delta_o}{x_q}\;\frac{x_q(p)}{x_e + x_q(p)} + \frac{e}{x_e + x_q(p)}\,(\sin\delta - \sin\delta_o)\end{aligned}$$

The operators contained in equations (10) may be expanded by Heaviside's expansion theorem when operating on a unit function. However, the second terms of both equations contain an operator which is operating on a function of the machine angle which may vary with time. If the relationship between the angle and time is known it would be a simple matter to expand the expression by means of Duhamel's integral. However, since this relation is not generally known it is usually necessary to obtain the solution by means of a step-by-step calculation.

One form of Heaviside's expansion theorem is:

$$\frac{Y(p)}{Z(p)} = \frac{Y(o)}{Z(o)} + \left[\frac{Y(\infty)}{Z(\infty)} - \frac{Y(o)}{Z(o)} \right] \sum a_n \epsilon^{p_n t} \tag{11}$$

where

$$a_n = \frac{Z(\infty)Z(o)}{Y(\infty)Z(o) - Z(\infty)Y(o)} \cdot \frac{Y(p_n)}{p_n Z'(p_n)}$$

p_n = root of equation $[Z(p) = 0]$

and $\Sigma a_n = 1.0$.

Applying this theorem to the operators in equations (10):

$$\begin{aligned} i_d = {} & \frac{E_{do}}{x_e + x_d} - \frac{e \cos \delta}{x_e + x_d} - \frac{E_{do} - e \cos \delta_o}{x_d} \\ & \cdot \frac{x_e (x_d - x_d'')}{(x_e + x_d)(x_e + x_d'')} \sum a_{dn} \epsilon^{-\alpha_{dn} t} \\ & - \frac{e (x_d - x_d'')}{(x_e + x_d)(x_e + x_d'')} \sum b_{dn} \epsilon^{-\alpha_{dn} t} (\cos \delta - \cos \delta_o) \\ & + \frac{G(p)}{x_e + x_d(p)} \Delta E_d \\ i_q = {} & \frac{e \sin \delta}{x_e + x_q} - \frac{e \sin \delta_o}{x_q} \cdot \frac{x_e (x_q - x_q'')}{(x_e + x_q)(x_e + x_q'')} \sum a_{qn} \epsilon^{-\alpha_{qn} t} \\ & + \frac{e (x_q - x_q'')}{(x_e + x_q)(x_e + x_q'')} \sum b_{qn} \epsilon^{-\alpha_{qn} t} (\sin \delta - \sin \delta_o) \end{aligned} \tag{12}$$

Substitution of equations (4) and (12) in equation (2) gives:

$$\begin{aligned} T = {} & \frac{E_d e \sin \delta}{x_e + x_d} + e^2 \frac{x_d - x_q}{2 (x_e + x_d)(x_e + x_q)} \sin 2 \delta \\ & - \frac{E_{do} e - e^2 \cos \delta_o}{x_d} \sin \delta \frac{x_e (x_d - x_d'')}{(x_e + x_d)(x_e + x_d'')} \sum a_{dn} \epsilon^{-\alpha_{dn} t} \\ & - e^2 \sin \delta \frac{x_d - x_d''}{(x_e + x_d)(x_e + x_d'')} \sum b_{dn} \epsilon^{-\alpha_{dn} t} (\cos \delta - \cos \delta_o) \\ & - \frac{e^2 \sin \delta_o}{x_q} \cos \delta \frac{x_e (x_q - x_q'')}{(x_e + x_q)(x_e + x_q'')} \sum a_{qn} \epsilon^{-\alpha_{qn} t} \\ & + e^2 \cos \delta \frac{(x_q - x_q'')}{(x_e + x_q)(x_e + x_q'')} \sum b_{qn} \epsilon^{-\alpha_{qn} t} (\sin \delta - \sin \delta_o) \\ & - \frac{e \sin \delta}{x_e + x_d} \sum c_{dn} \epsilon^{-\alpha_{dn} t} \Delta E \end{aligned} \tag{13}$$

The above expression is the equation for the torque developed by a synchronous machine when an external reactance is suddenly switched in, accompanied by a change in angle from δ_o to δ and a change of excitation voltage ΔE. This expression for torque is general, within the limitations of the original assumptions, in that it includes the effect of any number of rotor circuits as well as any change in excitation voltage.

If the variations of δ and E as functions of time are known, Duhamel's integral may be applied and the electrical torque determined directly at any time after the disturbance. However, if they are not known, as is the more usual case, the step-by-step method of calculation presented in Appendix A may be used to determine the angular swing of the machine.

For one rotor circuit in each axis equation (13) becomes:

$$\begin{aligned} T = {} & \frac{E_d e}{x_e + x_d} \sin \delta + e^2 \frac{x_d - x_q}{2 (x_e + x_d)(x_e + x_q)} \sin 2 \delta \\ & - \frac{E_{do} e - e^2 \cos \delta_o}{x_d} \sin \delta \frac{x_e (x_d - x_d')}{(x_e + x_d)(x_e + x_d')} \epsilon^{-\frac{x_d + x_e}{(x_d' + x_e) T_{do}'} t} \\ & - e^2 \sin \delta \frac{x_d - x_d'}{(x_e + x_d)(x_e + x_d')} \epsilon^{-\frac{x_d + x_e}{(x_d' + x_e) T_{do}'} t} (\cos \delta - \cos \delta_o) \\ & - \frac{e^2 \sin \delta_o}{x_q} \cos \delta \frac{x_e (x_q - x_q')}{(x_e + x_q)(x_e + x_q')} \epsilon^{-\frac{x_q + x_e}{(x_q' + x_e) T_{qo}'} t} \\ & + e^2 \cos \delta \frac{(x_q - x_q')}{(x_e + x_q)(x_e + x_q')} \epsilon^{-\frac{x_q + x_e}{(x_q' + x_e) T_{qo}'} t} (\sin \delta - \sin \delta_o) \end{aligned} \tag{14}$$

Case 1a. Constant Flux Linkages Maintained in Both Axes

In order to simplify calculations the approximations are frequently made that constant flux linkages are maintained in the predominant rotor circuit of each axis at any instant after the occurence of the disturbance and that the effect of additional rotor circuits is negligible. Such an assumption simply specifies that the predominant rotor circuit of each axis has an infinite time constant whereas all additional rotor circuits in both axes have zero time constants.

If such an assumption is made equation (13) may be greatly simplified as all of the exponentials will become either unity or zero depending upon whether the time constant of the particular circuit it represents is infinite or zero. Also $x_d'' = x_d'$; $x_q'' = x_q'$. Equation (13) becomes:

$$\begin{aligned} T = {} & \frac{x_d' E_{do} e}{x_d (x_d' + x_e)} \sin \delta + \frac{e^2 (x_d' - x_q')}{2 (x_d' + x_c)(x_q' + x_e)} \sin 2 \delta \\ & + \frac{e^2 (x_d - x_d')}{x_d (x_d' + x_e)} \cos \delta_o \sin \delta - \frac{e^2 (x_q - x_q')}{x_q (x_q' + x_e)} \sin \delta_o \cos \delta \end{aligned} \tag{15}$$

If x_e is zero equation (**15**) corresponds to equation (**46**) of reference 4 for sudden angular displacement. Also, making $x_e = 0$, $E_{do} = e$ and $\delta_o = 0$, (the last two relations being the conditions for a machine initially open-circuited) equation (**15**) reduces to equation (**61**) of reference 4 for synchronizing out of phase.

In making stability studies involving several machines the assumptions are commonly made that $x_d' = x_q'$ and $x_d = x_q$. Equation (**15**) becomes:

$$T = \frac{x_d' E_{do} e}{x_d (x_d' + x_e)} \sin \delta + \frac{e^2 (x_d - x_d')}{x_d (x_d' + x_e)} \sin (\delta - \delta_o) \qquad (\mathbf{16})$$

By trigonometric transformations equation (**16**) may be readily shown to be equal to the more common form:

$$T = \frac{E' e}{x_d' + x_e} \sin \delta' \qquad (\mathbf{17})$$

Where E' = the voltage back of transient reactance.
δ' = the angle between E' and e.

Case 1b. Switching in a Connecting Line

If a connecting line is switched in, the equivalent one line circuit of Fig. 1b may be used by giving x_e a negative value. The above equations may therefore be applied for switching in a connecting line, as well as for switching out a connecting line, providing x_e is given the proper sign.

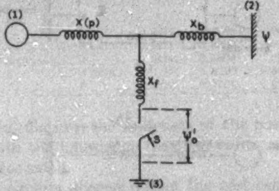

Fig. 2—Equivalent Circuit Representing Occurrence of Balanced Fault

Case 2. Occurrence of a Balanced Three-Phase System Fault

The calculation of the transient stability limit frequently involves the determination of the relation between torque, machine angle and excitation following the occurrence of a balanced three-phase system fault. Faults on a system connecting one machine and an infinite bus can be reduced to an equivalent circuit of the form shown in Fig. 2. The occurrence of the fault is represented by closing the switch, S. $x(p)$ is the machine reactance plus the reactance of that part of the equivalent Y-circuit which extends from the machine terminals to the junction point of the Y.

In keeping with assumptions 3, 4, 5, and 6, equations (**5**) reduce to the following simple relations:

$$e_d = -\Psi_q$$
$$e_q = \Psi_d$$

Switching operations may then be made by suddenly applying flux linkages equal in magnitude to the corresponding voltages. In the network of Fig. 2, the effect of closing the switch may, therefore, be simulated by leaving it open and, at the instant $t = 0$, impressing across it a value of flux linkages, Ψ_o', equal and opposite to the value that existed before $t = 0$. At $t = 0$, δ and E may also change, depending upon the conditions of the problem.

Before $t = 0$, the current flowing through x_b to the infinite bus is

$$i_{do} = \frac{E_{do} - \Psi_{do}}{x_b + x_d}$$

$$i_{qo} = -\frac{\Psi_{qo}}{x_b + x_q} \qquad (\mathbf{18})$$

After $t = 0$, the component of current in x_b due to closing the switch is

$$i_{d1} = -\frac{\Psi_{do}'}{x_{d23}(p)} = -\frac{i_{do} x_b + \Psi_{do}}{x_{d23}(p)}^{*}$$

$$i_{q1} = -\frac{\Psi_{qo}'}{x_{q23}(p)} = -\frac{i_{qo} x_b + \Psi_{qo}}{x_{q23}(p)} \qquad (\mathbf{19})$$

The component of current in x_b due to the change in δ is:

$$i_{d2} = -\frac{\Delta\Psi_d}{x_{d22}(p)}$$

$$i_{q2} = -\frac{\Delta\Psi_q}{x_{q22}(p)} \qquad (\mathbf{20})$$

The component of current in x_b due to a change in excitation voltage is:

$$i_{d3} = \frac{G(p)\Delta E_d}{x_{d12}(p)}$$

$$i_{q3} = 0 \qquad (\mathbf{21})$$

By superposition, the resultant current in x_b, after $t = 0$, is the algebraic sum of the components given by equations (**18**) to (**21**). Then

$$i_d = \frac{E_{do} - \Psi_{do}}{x_b + x_d} - \frac{i_{do} x_b + \Psi_{do}}{x_{d23}(p)} - \frac{\Delta\Psi_d}{x_{d22}(p)} + \frac{G(p)\Delta E_d}{x_{d12}(p)}$$

$$i_q = -\frac{\Psi_{qo}}{x_b + x_q} - \frac{i_{qo} x_b + \Psi_{qo}}{x_{q23}(p)} - \frac{\Delta\Psi_q}{x_{q22}(p)} \qquad (\mathbf{22})$$

Substituting equations (**22**) and (**4**) in (**2**) and simplifying

$$T = \frac{E_{do} e \sin \delta}{x_b + x_d} \left[\frac{x_b + x_d(p)}{x_{d12}(p)} \right]$$

---

[*] $x_{12}(p)$, $x_{13}(p)$, and $x_{23}(p)$ used hereafter are transfer impedances in operational form. $x_{22}(p)$ is a driving point impedance. See Appendix III of reference 6.

$$- \frac{e^2 \sin\delta \cos\delta_o}{x_b + x_d}\left[1 + \frac{x_d}{x_{d23}(p)}\right]$$

$$- \frac{e^2 \sin\delta}{x_{d22}(p)}(\cos\delta - \cos\delta_o)$$

$$+ \frac{e^2 \cos\delta \sin\delta_o}{x_b + x_q}\left[1 + \frac{x_q}{x_{q23}(p)}\right]$$

$$+ \frac{e^2 \cos\delta}{x_{q22}(p)}(\sin\delta - \sin\delta_o) + e \sin\delta \frac{G(p)}{x_{d12}(p)}\Delta E_d \qquad (23)$$

If the operators in the above expression are expanded by theorem (**11**), the equation for transient torque becomes

$$T = \frac{E_d e \sin\delta}{x_{d12}} + e^2 \frac{x_d - x_q}{2\, x_{d12} x_{q12}} \sin 2\,\delta$$

$$+ \frac{E_{do} e \sin\delta}{x_b + x_d} \frac{x_b\,(x_d - x_d'')}{x_{d12} x_{d12}''} \sum a_{dn} \epsilon^{-\alpha_{dn} t}$$

$$+ \frac{e^2 \sin\delta \cos\delta_o}{x_b + x_d} \frac{x_d\,(x_d - x_d'')}{x_{d12} x_{d12}''} \sum b_{dn} \epsilon^{-\alpha_{dn} t}$$

$$- e^2 \sin\delta \frac{x_d - x_d''}{x_{d12} x_{d12}''} \sum c_{dn} \epsilon^{-\alpha_{dn} t} (\cos\delta - \cos\delta_o)$$

$$- \frac{e^2 \cos\delta \sin\delta_o}{x_b + x_q} \frac{x_q\,(x_q - x_q'')}{x_{q12} x_{q12}''} \sum a_{qn} \epsilon^{-\alpha_{qn} t}$$

$$+ e^2 \cos\delta \frac{x_q - x_q''}{x_{q12} x_{q12}''} \sum b_{qn} \epsilon^{-\alpha_{qn} t} (\sin\delta - \sin\delta_o)$$

$$- \frac{e \sin\delta}{x_{d12}} \sum d_{dn} \epsilon^{-\alpha_{dn} t} \Delta E_d \qquad (24)$$

The above expression is the equation for the electrical torque developed by a synchronous machine subsequent to the occurrence of a balanced fault on the system accompanied by a change in angle from δ_o to δ and in excitation of ΔE. As in *Case 1* the electrical torque can be determined at any time of the rotor swing either by use of the step-by-step process or by the application of Duhamel's integral.

When x_f includes the equivalent reactance for unbalanced faults determined by the method of symmetrical components,[6,8] equation (**25**) will give the electrical torque if the additional assumption is made that the negative and zero phase sequence losses are negligible. If these losses are appreciable their effect may be included by the method given in Appendix B.

Case 2a. Constant Flux Linkages Maintained in Both Axes

As in *Case 1a* calculations may be greatly simplified, when justifiable, by making the assumption that the predominant rotor circuit of each axis has an infinite time constant and that all additional rotor circuits have zero time constants. If this assumption is made equation (**24**) may be simplified to the following expression:

$$T = \frac{E_{do} e}{x_{d12}'} \frac{x_b + x_d'}{x_b + x_d} \sin\delta + \frac{e^2}{2} \frac{x_d' - x_q'}{x_{d12}' x_{q12}'} \sin 2\,\delta$$

$$+ \frac{e^2}{x_{d12}'} \frac{x_d - x_d'}{x_b + x_d} \sin\delta \cos\delta_o$$

$$- \frac{e^2}{x_{q12}'} \frac{x_q - x_q'}{x_b + x_q} \cos\delta \sin\delta_o \qquad (25)$$

If $x_b = 0$ and $x_f = \infty$ equation (**25**) corresponds to equation (**46**) of reference 4 for sudden angular displacement.

II—Application of Mathematical Analysis

In the latter part of 1929, field tests* were made to determine the effect of amortisseur windings on the transient behavior of waterwheel generators following switching disturbances. Two machines were available for testing purposes; one had no additional rotor circuits other than a top field collar, and the other was provided with an incomplete copper amortisseur. The machines were otherwise identical. The results of these tests provide a means of checking the mathematical analysis given in Part I.

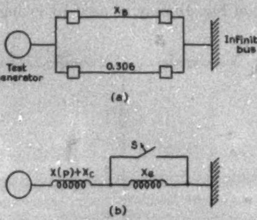

FIG. 3—TEST ARRANGEMENT AND EQUIVALENT CIRCUIT FOR SWITCHING OPERATION

The machine being tested was delivering power to a relatively large 110-kv. system through several circuits. A disturbance was created by opening all the circuits but one, which is a condition similar to that of *Case 1* in the first part of this paper.

In order to make use of equation (**13**) two assumptions as to the test set-up were necessary. These are:

a. The point of interconnection with the 110-kv. system can be considered as an infinite bus. Actually the infinite bus is farther back in the system. To

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*A companion paper, *Field Tests to Determine Damping Characteristics of Synchronous Generators*, by F. A. Hamilton, Jr., see page 775 of this issue, describes the test machines, set-up, and results.

TABLE I—SWING CURVE CALCULATION OF MACHINE WITH AMORTISSEUR

	Before 0	After 0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$15.13\ T_a$	0	0	2.098	2.602	0.4418	−1.636	−2.840	−2.763	−1.610	0.0290	1.439	2.135
$\Delta\delta$	0	0	2.098	4.700	5.142	3.506	0.666	−2.097	−3.707	−3.678	−2.239	−0.104
δ	19.62	19.62	21.718	26.418	31.560	35.066	35.732	33.635	29.928	26.250	24.011	23.907
2δ	39.24	39.24	43.44	52.84	63.120	70.13	71.46	67.27	59.86	52.50	48.02	47.81
$\sin \delta$	0.3358	0.3358	0.3701	0.4450	0.5234	0.5746	0.5840	0.5540	0.4989	0.4423	0.4069	0.4053
$\sin 2\delta$	0.6326	0.6326	0.6876	0.7970	0.8920	0.9405	0.9481	0.9223	0.8648	0.7939	0.7434	0.7409
$\cos \delta$	0.9410	0.9419	0.9290	0.8956	0.8521	0.8185	0.8118	0.8325	0.8666	0.8969	0.9135	0.9142
$\cos \delta_n - \cos \delta_{n-1}$	0	0	−0.0129	−0.3334	−0.0435	−0.0336	−0.0067	0.0208	0.0341	0.0303	0.0166	0.0007
$B = -A(\cos \delta_n - \cos \delta_{n-1})$	0	0	0.0207	0.0154	0.0138	0.0129	0.0123	0.0118	0.0113	0.0108	0.0104	0.0100
$A_n = 1.599$				0.0535	0.0397	0.0356	0.0333	0.0317	0.0304	0.0292	0.0280	0.0269
$A_{n+1} = 1.187$					0.0695	0.0516	0.0463	0.0433	0.0412	0.0395	0.0379	0.0364
$A_{n+2} = 1.064$						0.0538	0.0399	0.0358	0.0335	0.0319	0.0306	0.0293
$A_{n+3} = 0.9959$							0.0107	0.0079	0.0071	0.0066	0.0063	0.0061
$A_{n+4} = 0.9475$								−0.0332	−0.0246	−0.0221	−0.0207	−0.0197
$A_{n+5} = 0.9082$									−0.0545	−0.0405	−0.0363	−0.0340
$A_{n+6} = 0.8722$										−0.0483	−0.0359	−0.0322
$A_{n+7} = 0.8374$											−0.0265	−0.0197
$A_{n+8} = 0.8048$												−0.0011
C	1.317	0.9597	0.9802	0.9863	0.9896	0.9920	0.9940	0.9957	0.9973	0.9989	1.000	1.002
$B + C$	1.317	0.9597	1.0009	1.0552	1.1126	1.1459	1.1365	1.0930	1.0417	1.0060	0.9938	1.004
$a = (B + C) \sin \delta$	0.4422	0.3222	0.3704	0.4695	0.5823	0.6584	0.6637	0.6055	0.5197	0.4449	0.4044	0.4069
$b = 0.2149 \sin 2\delta$	0.2496	0.1359	0.1478	0.1713	0.1917	0.2021	0.2037	0.1982	0.1858	0.1706	0.1598	0.1592
$\sin \delta_n - \sin \delta_{n-1}$	0	0	0.0343	0.0749	0.0784	0.0512	0.0094	−0.0300	−0.0550	−0.0567	−0.0354	−0.0016
$E = D(\sin \delta_n - \sin \delta_{n-1})$	0	0	0.0107	0.0022	0.0005	0.0001	0.0000	0	0	0	0	0
$D_n = 0.3135$				0.0235	0.0049	0.0011	0.0003	0.0001	0	0	0	0
$D_{n+1} = 0.0654$					0.0246	0.0051	0.0012	0.0003	0.0001	0	0	0
$D_{n+2} = 0.0153$						0.0160	0.0033	0.0008	0.0002	0	0	0
$D_{n+3} = 0.0036$							0.0029	0.0005	0.0001	0	0	0
$D_{n+4} = 0.0009$								−0.0094	−0.0020	−0.0005	−0.0001	−0.0
$D_{n+5} = 0.0002$									−0.0173	−0.0036	−0.0008	−0.0002
$D_{n+6} = 0.0000$										−0.0178	−0.0037	−0.0009
											−0.0111	−0.0023
												−0.0050
F	0	−0.0471	−0.0098	−0.0022	−0.0005	−0.0001	−0.0001	0.0000	0	0	0	0
$E + F$	0	−0.0471	0.0009	0.0235	0.0295	0.0222	0.0076	−0.0076	−0.0180	−0.0219	−0.0157	−0.0084
$c = (E + F) \cos \delta$	0	−0.0444	0.0008	0.0210	0.0251	0.0182	0.0062	−0.0063	−0.0164	−0.0196	−0.0143	−0.0077
$T_e = a + b + c$	0.6918	0.4137	0.5190	0.6618	0.7991	0.8787	0.8736	0.7974	0.6891	0.5959	0.5499	0.5584
$T_a = 0.691 - T_e$	0	0.2773	0.1720	0.0292	−0.1081	−0.1877	−0.1826	−0.1064	0.0019	0.0951	0.1411	0.1326

simplify calculations the finiteness of the bus was neglected with the knowledge that the value used for x_e would be too small.

b. The circuits opened during the test can be considered and treated as one equivalent circuit. These circuits actually consisted of a system network with other connected machines. The reactance of this equivalent circuit was then determined by the division of power. Before switching, the power factors of the two circuits were practically the same, so that this assumption is reasonable.

These assumptions made it possible to reduce the system to the one-line diagram in Fig. 3a which may be represented by the equivalent circuit in Fig. 3b. The rating of the test machines (9,000 kva.) was selected as the base per unit kva.

CHECK OF POWER OUTPUT OF MACHINE EQUIPPED WITH INCOMPLETE COPPER AMORTISSEUR

The following operational impedances of the machine equipped with an amortisseur were determined from the design data:

$$x_d(p) = 0.91 - \frac{1339p^5 + 778.2p^4 + 148.2p^3 + 10.98p^2 + 0.2545p}{1730p^5 + 1030p^4 + 202p^3 + 15.6p^2 + 0.396p + 0.000361}$$

$$x_q(p) = 0.524 - \frac{122.3p^3 + 32.4p^2 + 2.134p}{783.7p^3 + 263.2p^2 + 28.09p + 0.9202}$$

It was not necessary to determine $G(p)$ as the tests were made with constant field voltage ($\Delta E_d = 0$).

The inertia constant H was calculated from the design data and is equal to 1.79. Also $E_d = 1.19$, $e = 1.06$, $x_b = 0.058$, $x_c = 0.049$, $x_e = 0.257$ and initially $\delta_o = 19.62°$ and $T = 0.691$.

Substituting the machine and system impedances and the initial quantities in equation (13) the expression for torque output was obtained. The swing curve work sheet was set up (Table I) and the step-by-step calculations carried out according to Appendix A.

Fig. 4 presents a comparison between the calculated and test curves of electrical power output. The calculated initial drop in power was not quite as great as the test drop in power. The calculated oscillations are damped out somewhat faster than the test oscillations. Both of these errors can be largely accounted for by the fact that the assumed infinite bus is actually finite, and that the armature circuit resistance has been neglected.

Fig. 5 shows the calculated torque angle ellipse which describes the oscillation of the rotor for the first two swings. The marked points on the ellipse are points of equal time intervals.

CHECK OF POWER OUTPUT OF MACHINE WITH NO ADDITIONAL ROTOR CIRCUITS OTHER THAN A TOP FIELD COLLAR

From the design data of the second generator the following operational impedances were determined:

$$x_d(p) = 0.91 - \frac{0.7329p^2 + 0.1298p}{p^2 + 0.1994p + 0.0001940}$$

$$x_q(p) = 0.524$$

Also $H = 1.79$, $E_d = 1.19$, $e = 1.06$, $x_b = 0.046$, $x_e = 0.04$, $x_s = 0.266$ and initially $\delta_o = 19.36°$ and $T = 0.693$.

In a manner similar to that described in the preceding case, the swing of the rotor and the output of the machine were calculated. See Fig. 6 for a comparison between the test and calculated values of power output. Fig. 7 is the torque angle characteristic of this machine for the first two oscillations.

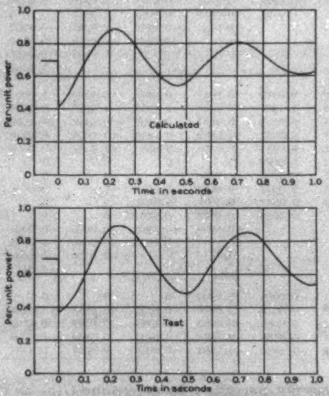

Fig. 4—Comparison of Calculated and Test Values of Transient Power (with Amortisseur)

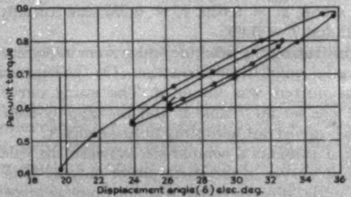

Fig. 5—Calculated Torque-Angle Characteristic (with Amortisseur)

Conclusion

Equations have been derived in this paper by which the transient behavior of a synchronous machine following the common types of system disturbances may be calculated. They are of sufficient generality that the effect of amortisseur windings or of excitation may be evaluated. Only such assumptions have been made in their derivation as are necessary to reduce them to a practical basis. If additional assumptions are made the equations may be reduced to the less general but more familiar equations of electrical torque derived by other authors. A comparison of the calculated and test results shows good agreement. The general equations may also be used as a basis for the derivation of more specific equations applying to particular conditions.

Appendix A

Step-by-Step Method

Step-by-step methods have been developed and explained in several other papers.[5,6,7] However a method well adapted for use with the general torque equations given in Part I is obtained from Duhamel's theorem* expressed as a summation rather than as an integral.

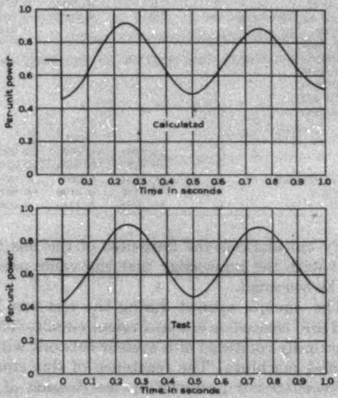

Fig. 6—Comparison of Calculated and Test Values of Transient Power (without Amortisseur)

In Fig. 8 the curve $F(t)$ represents any function of time which is to be operated on by an operator. In equation (**13**), for example, there are three such functions $(\cos \delta - \cos \delta_o)$, $(\sin \delta - \sin \delta_o)$ and ΔE. Let $T(t)$ represent the component of electrical torque produced by any one of these terms.

Then

$$T(t) = f(p)F(t) \qquad \textbf{(1a)}$$

where $f(p)$ = an operator

Let

$$F(\Delta t)_n = F(n\Delta t) - F[(n-1)\Delta t] \qquad \textbf{(2a)}$$

The curve $\delta(t)$ may be approached by assuming that at $t = 0$ a unit function $\delta(o)$ is suddenly impressed. After a time interval (Δt) a second unit function

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*See Chapter XIV of "Heaviside's Operational Calculus," by E. J. Berg.

$\delta(\Delta t)_1$ is added. At the end of a second time interval a third unit function $\delta(\Delta t)_2$ is added, etc. The value of the function $T(t)$ at any time t is then equal to the sum of the torques due to $\delta(o)$ beginning at $t = 0$, $\delta(\Delta t)_1$ beginning at $t = \Delta t$, $\delta(\Delta t)_2$ beginning at $t = 2\Delta t$, etc. The torque due to $\delta(o)$ is $\delta(o) \cdot \phi(t)$ where $\phi(t)$ is the solution with unit function impressed at $t = 0$. The torque due to $\delta(\Delta t)_1$ is $\delta(\Delta t)_1 \cdot \phi(t - \Delta t)$. The torque due to $\delta(\Delta t)_2$ is $\delta(\Delta t)_2 \cdot \phi(t - 2\Delta t)$, etc.

$$\therefore T(t) = \delta(o) \cdot \phi(t) + \sum_{n=1}^{n=\infty} \delta(\Delta t)_n \cdot \phi(t - n\Delta t) \qquad \textbf{(3a)}$$

Equation (**3a**) provides a step-by-step method for obtaining the expansion of any terms containing operators operating on functions of the machine angle. All other terms containing operators operating on unit function alone may be expanded directly by equation (**11**). The procedure from this point on is similar to the ordinary swing curve calculation using the following equations given in Appendix IV of reference 6.

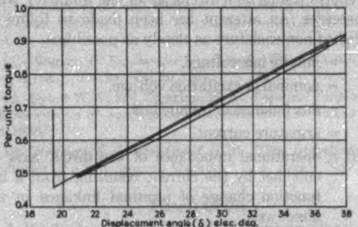


Fig. 7—Calculated Torque-Angle Characteristic (without Amortisseur)

$$T_a = T_m - T_E$$

$$\Delta\delta_1 = \frac{kT_a}{2}$$

$$\delta_1 = \delta_o + \Delta\delta_1$$
$$\Delta\delta_2 = \Delta\delta_1 + kT_1$$
$$\delta_2 = \delta_1 + \Delta\delta_2, \text{ etc.}$$

The details of the calculation may best be understood by reference to the sample swing curve calculation given in Part II.

Appendix B

Method for Determining the Torque Characteristics when Resistance is Included

It may be desirable to determine the influence of line and armature resistances upon the torque characteristic. Cases also arise in which the negative phase and zero phase sequence losses may be appreciable; as, for example, in the case of a machine having a high resistance amortisseur, or of a transformer having its neutral grounded through a resistor. The following method of analysis may be used to evaluate such factors.

If assumptions 4, 5, and 6 are made, equation (**5**) becomes

$$e_d = -ri_d - \Psi_q$$
$$e_q = -ri_q + \Psi_d \qquad \textbf{(1b)}$$

The effect of armature and line resistances may then be included in equivalent circuits for the direct and quadrature axes as shown in Fig. 9. Since an induced voltage and its corresponding flux linkages are in time quadrature with each other, current flowing through a resistance in one axis causes an equivalent flux linkage drop in the other axis. The directions in which these equivalent flux linkage drops are acting are indicated by arrows.

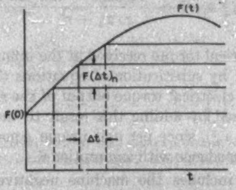
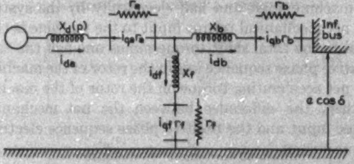
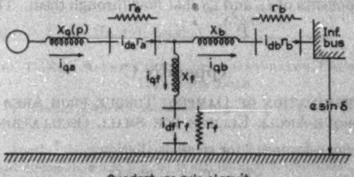


Fig. 8—Illustration of Step-by-Step Process



Direct axis circuit



Quadrature axis circuit

Fig. 9—Equivalent Flux Linkage Circuits Including Effect of Armature Circuit Resistance

As a simple example assume that $x_f = \infty$, $x_b = 0$, $r_b = 0$, constant flux linkages are maintained in the main field winding and there are no additional rotor circuits. Then

$$x_d(p) = x_d'$$
$$x_q(p) = x_q' = x_q$$

From Fig. 9 the following relations can be written

$$\Psi_d' - e\cos\delta = i_q r + i_d x_d'$$
$$e\sin\delta = i_q x_q - i_d r \qquad \textbf{(2b)}$$

where

Ψ_d' = constant value of flux linkages in the main field.

Solving equations (**2b**) simultaneously

$$i_d = \frac{\Psi_d' x_q - e x_q \cos\delta - er\sin\delta}{x_d' x_q + r^2}$$

$$i_q = \frac{\Psi_d' r - er\cos\delta + e x_d' \sin\delta}{x_d' x_q + r^2} \qquad \textbf{(3b)}$$

The electrical torque received at the infinite bus may be obtained by substitution of equations (**3b**) and (**4**) in (**2**). The electrical torque output of the machine can be determined by adding this result to the resistance loss $r\,(i_d^2 + i_q^2)$, since per unit torque equals per unit power in accordance with assumption 6.

When r_f includes the machine negative phase sequence resistance (r_2), half of the negative phase resistance loss in the rotor is supplied mechanically by the machine shaft and half electrically by the system. The net mechanical torque input to the machine is then equal to the total shaft torque minus one-half the total negative phase sequence loss in the rotor of the machine. The net accelerating torque on the rotor of the machine is, then, the difference between the net mechanical torque input and the positive phase sequence electrical torque output.

The zero phase sequence loss can be determined by including in r_f the proper components of zero phase sequence resistance and by evaluating the zero phase components of i_{df} and i_{qf} that flow through them. Then

$$P_o = r_o\,(i_{dfo}^2 + i_{qfo}^2) \qquad \textbf{(4b)}$$

Appendix C

DETERMINATION OF DAMPING TORQUE FROM AREA OF TORQUE-ANGLE ELLIPSE FOR SMALL OSCILLATIONS

From reference 2 for small oscillations:

$$\Delta T = T_s\Delta\delta + T_d p\Delta\delta \qquad \textbf{(1c)}$$

Since the oscillation of one machine with respect to a system is nearly harmonic, the angle of oscillation about the mean may be approximately expressed by

$$\Delta\delta = -\,[\Delta\delta]\cos st \qquad \textbf{(2c)}$$

where $[\Delta\delta]$ is the maximum deviation and $s/2\pi$ is the frequency of oscillation.

Substituting equation (**2c**) in (**1c**)

$$\Delta T = -\,T_s\,[\Delta\delta]\cos st + sT_d\,[\Delta\delta]\sin st \qquad \textbf{(3c)}$$

The area of the ellipse is:

$$A = \int_{\theta=0}^{\theta=2\pi} \Delta T d(\Delta\delta) \qquad \textbf{(4c)}$$

where $\theta = st.$

Substituting (**3c**) in (**4c**):

$$A = \int_0^{2\pi} \{-\,T_s\,[\Delta\delta]\cos\theta + sT_d\,[\Delta\delta]\sin\theta\}\,[\Delta\delta]\sin\theta\, d\,\theta \qquad \textbf{(5c)}$$

which reduces to $A = \pi s T_d\,[\Delta\delta]^2$

giving

$$T_d = \frac{A}{\pi s\,[\Delta\delta]^2} \qquad \textbf{(6c)}^*$$

NOMENCLATURE

All quantities are expressed as a fraction of their normal values which are per unit values as defined in reference 3. The subscripts d and q refer to the direct and quadrature axes respectively. The subscript o specifies initial steady state conditions. The coefficient Δ denotes the increment change of the quantity which it precedes. An attempt has been made to follow established nomenclature as closely as possible.

e = infinite bus voltage.

E = nominal or excitation voltage.

Ψ = flux linkages of infinite bus.

i = armature current.

$x_d(p)$ = operational impedance of the direct axis circuit met by the current resulting from a unit function change of terminal linkages in the direct axis.

$x_q(p)$ = operational impedance of quadrature axis circuit.

$G(p)$ = an operator which relates field voltage with current and linkages in the direct axis.

p = an operator d/dt.

δ = displacement angle of the nominal voltage from the infinite bus voltage expressed in electrical radians. It is the actual space phase lag or lead of the rotor.

s = per unit frequency of rotor oscillations.

θ = angle in electrical degrees between the axis of the rotor and a stationary axis in space.

t = time expressed as a fraction of the time to pass one electrical radian at fundamental frequency. Unit time is thus $1/2\pi f$.

n = a subscript denoting any field circuit, also denoting any time interval.

r = armature circuit resistance.

T_o' = open circuit time constant in radians.

x† = synchronous reactance.

x'† = transient reactance.

x''† = sub-transient reactance.

---

*This expression was derived by I. H. Summers.

†NOTE: These values of reactance include certain reactances external to the machine terminals which differ depending upon the conditions of the case considered.

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Discussion

J. W. Butler: Messrs. Crary and Waring have done a creditable piece of work in making available equations for determining the transient characteristics of synchronous machines, giving in particular, due consideration to every rotor circuit. The power oscillations were checked accurately by a step-by-step solution using the general torque equation (13) of the text. The solution was perfectly general within the assumptions, without dividing the torque into its components, generally spoken of as synchronizing and damping torques.

However, if it is desired to calculate the damping torque for comparatively large sustained oscillations, it can be obtained from the general equation (13) as follows:

Let the change in the angle δ, be of the form

$$\Delta\delta = [\Delta\delta] \sin st$$

Then, by use of the superposition theorem, the response of the system can be found to this variation in angle. In order to integrate the resultant expression, represent the sine and cosine terms, by the first two terms of the series, such as

$$\cos [(\Delta\delta) \sin st] = 1 - \frac{[(\Delta\delta) \sin st]^2}{2\,!}$$

$$\sin [(\Delta\delta) \sin st] = (\Delta\delta) \sin st - \frac{[(\Delta\delta) \sin st]^3}{3\,!}$$

Using these forms in the transient terms of equation (13), due to the unit function change in δ, and applying the superposition theorem there is obtained an expression for torque, for a sinusoidal change in angle. Due to the fact that a sustained oscillatory condition is being considered only the sustained terms are to be retained in this expression. Integrating the sustained terms, with respect to st, from o to 2π, the area of the torque angle ellipse is obtained. Dividing the resultant expression for area by $\pi s\,(\Delta\delta)^2$ as shown in Appendix C, the expression for the damping torque coefficient is obtained. The above procedure was carried out and is too bulky to include here but the final result is

$$T_d = e^2 \frac{x_d - x_d''}{x_d x_d''} \sum b_{dn}\alpha_{dn} \left\{ \frac{\sin^2\delta_o'}{\alpha_{dn}^2 + s^2} \left[1 - \frac{(\Delta\delta)^2}{8} \right] \right.$$

$$\left[1 - \frac{(\Delta\delta)^2}{6\,(\alpha_{dn}^2 + 9s^2)} \left(\frac{3\alpha_{dn}^2 + 3s^2}{4} + 6s^2 \right) \right]$$

$$\left. + \frac{(\Delta\delta)^2}{4} \frac{\cos^2\delta_o'}{\alpha_{dn}^2 + 4s^2} \left[1 - \frac{(\Delta\delta)^2}{12} \right] \right\}$$

$$+ e^2 \frac{(x_q - x_q'')}{x_q x_q''} \sum b_{qn}\alpha_{qn} \left\{ \frac{\cos^2\delta_o'}{\alpha_{qn}^2 + s^2} \left[1 - \frac{(\Delta\delta)^2}{8} \right] \right.$$

$$\left[1 - \frac{(\Delta\delta)^2}{6(\alpha_{qn}^2 + 9s^2)} \left(\frac{3\alpha_{qn}^2 + 3s^2}{4} + 6s^2 \right) \right]$$

$$\left. + \frac{(\Delta\delta)^2}{4} \frac{\sin^2\delta_o'}{\alpha_{qn}^2 + s^2} \left[1 - \frac{(\Delta\delta)^2}{12} \right] \right\}$$

It is of interest to note that this expression, letting $[\Delta\delta] = o$, checks Park's results in reference 2, derived in a different manner.

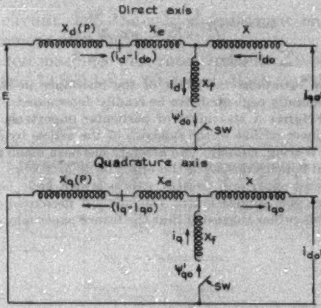

FIG. 1—EQUIVALENT CIRCUIT OF A GENERATOR FEEDING AN IMPEDANCE LOAD

Closing switch *SW* simulates a three-phase fault on feeder

Another thing which I feel should be emphasized is the equivalent circuit shown in Fig. 9. It can be used to calculate any sort of a transient in a machine, including resistance, within the assumptions made. One particular use I have made of the circuit is in calculating decrement curves, for a fault on a generator feeding an impedance load, considering the effect of the load resistance in the time constants and magnitudes of the current. For this application the circuit reduces to that shown in Fig. 1.

Relative to Fig. 1, the following linkage equations may be written.

$$\Psi_{do}' + i_{qo}r = xi_{do} + i_d x_f$$
$$\Psi_{do}' = (i_d - i_{do})\,[x_e + x_d\,(p)] + i_d x_f$$
$$\Psi_{qo}' - i_{do}r = i_q x_f + i_{qo}x$$
$$\Psi_{qo}' = i_q x_f + (i_q - i_{qo})\,[x_e + x_q\,(p)]$$

where

i_d = fault current direct axis
i_{do} = load current direct axis
i_q = fault current quadrature axis
i_{qo} = load current quadrature axis

x = load reactance
r = load resistance
x_f = fault reactance
x_e = external reactance
$x(p)$ = operational impedance of generator
Ψ' = linkage at point of fault, before fault occurred.

Solving these, there is obtained for the fault currents

$$i_d = \frac{\Psi_{do}' \left[x\,(x + a)\,x_{q31}\,(p) + r^2\,(x_f + b)\right] + \Psi_{qo}' rab}{D(p)}$$

$$i_q = \frac{\Psi_{qo}' \left[x\,(x + b)\,x_{d31}\,(p) + r^2\,(x_f + a)\right] - \Psi_{do}' rab}{D(p)}$$

where

$$a = x_e + x_d\,(p)$$
$$b = x_e + x_q\,(p)$$
$$D(p) = x^2 x_{d31}\,(p)\,x_{q31}\,(p) + r^2\,(x_f + a)\,(x_f + b)$$

Solving these operational equations by the expansion theorem there is obtained

$$i_d = I_d + (I_d'' - I_d) \sum b_{dn}\,\epsilon^{-\alpha_n t}$$
$$i_q = I_q + (I_q'' - I_q) \sum b_{qn}\,\epsilon^{-\alpha_n t}$$
$$\alpha_n = \text{roots of } D(p) = o$$
$$b_n = \frac{1}{(I'' - I)}\;\frac{Y\,(\alpha_n)}{\alpha_n\,Z'\,(\alpha_n)}$$
$$\sum b_n = 1.0$$

From these equations the effect of the resistance in the load, which is generally neglected, can be readily determined.

S. H. Wright: A statement of particular importance to the utility engineer is that in the synopsis of the article by Messrs. Crary and Waring, namely, "No attempt has been made to draw any general conclusions as to the effect of amortisseurs . . . . A subsequent paper will present results which have been obtained from the application of the method presented in this paper." This rather suggested that the future paper may discuss the more general problem of determining the most suitable type of damper winding for specific applications. In this regard, the following discussion is thought to be pertinent.

Certain of the prime advantages of low and high resistance dampers are as follows:

1. Low-resistance dampers tend rapidly to damp out swings which occur during symmetrical circuit conditions (that is where current magnitudes are equal in the three phases).

2. For two-machine stability problems (or multi-machine problems, where there are no serious compound oscillations) high-resistance dampers give higher transient stability limits, especially for the longer clearing times, such as greater than 16 cycles where f = 60 cycles per sec. This was brought out by C. F. Wagner, (see item 11 of bibliography of Crary and Waring paper, particularly Fig. 3 of this reference). However, this gain is apparently negligible for fault durations of the order of 8 cycles or less, and 8 cycles clearing time is being successfully obtained with modern high-speed circuit breakers and relays. The higher the speed of clearing any type of line fault, the more the entire occurrence approaches merely the condition of switching out a circuit.

In years to come it is likely that the average high-voltage system will reduce its clearing time to 8 cycles or less and it is possible that at the same time, interconnections and other factors will make compound system oscillations more important; in such a case the greater damping ability, under symmetrical circuit conditions, of low-resistance dampers would become of paramount importance.

S. B. Crary: Mr. Butler has made an interesting application of the general torque equations. His solution is limited to a forced oscillation such as is obtained in a compressor drive and is interesting in that it can be applied to large angular deviations.

Mr. Wright's conclusions concerning the influence of decreased switching times on the gains to be realized from high and low resistance amortisseur windings correspond with our opinions. When the switching times are very short, a high resistance amortisseur is of slight benefit in improving stability limits. On the other hand, the low resistance amortisseur is of aid in damping out all angular oscillations and is generally beneficial.

Field Tests to Determine the Damping
Characteristics of Synchronous Generators

BY F. A. HAMILTON, Jr.*

Associate, A.I.E.E.

***Synopsis.**—Practical system tests of the damping action of generators with and without low resistance damper windings, but otherwise identical, have been made. The effects of normal variations of generator field circuit resistance, field current, and loading, were investigated. Data for checking calculated machine constants also were obtained. The tests results are valuable from a quantitative, as well as from a qualitative standpoint.*

* * * * *

INTRODUCTION

IN the School Street Station of the New York Power and Light Corporation there are two waterwheel-driven generators which, originally, were identical 40-cycle units. Early in 1927 one of these machines was rebuilt for 60-cycle operation. Later in the same year the rebuilding of the companion unit was undertaken.

Meanwhile arguments for and against the use of damper windings were advanced; it was becoming apparent that comparative tests on an actual system might help to determine their effect upon transient power limits.

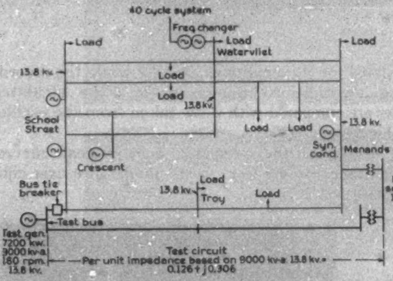
40 cycle system
Freq changer
Load
Load
Watervliet
Load
13.8 kv.
Load
Load
13.8 kv.
School Street
Load
Load
13.8 kv.
Syn. cond.
Menands
Crescent
Load
Bus tie breaker
13.8 kv.
Troy
Load
Main system 110 kv. bus
Test bus
Test gen. 7200 kw. 9000 kv-a. 180 rpm. 13.8 kv.
Test circuit
Per unit impedance based on 9000 kv-a 13.8 kv.= 0.126+j0.306

FIG. 1—ARRANGEMENT OF CIRCUITS AND EQUIPMENT

The idea of adding damper windings to the second machine to be rebuilt for 60-cycle operation, and of comparative tests on this and on the machine already rebuilt without them, was broached to the New York Power and Light Corporation. As a result, low-resistance damper windings were installed in the second machine to be rebuilt, and plans for testing were developed.

Force of circumstances ultimately prevented extensive tests to establish relative power limits for various types of short circuits of different durations. But less severe tests to check the validity of existing methods of analysis, and to further the development of damping theory, were finally completed late in 1929.

This paper describes these tests, and includes a resumé of the data obtained. Evidence of the accuracy of available methods of calculating machine constants is presented. The effectiveness of damper windings in rapidly smoothing out balanced disturbances is shown. In a companion paper, a method of predicting generator behavior during disturbances is developed, and calculations based on this method verified by comparison with the test data.[1]

DESCRIPTION OF TESTS

The circuits, and the major equipment employed, were arranged as in Fig. 1.

Both test machines were rated 7,200 kw., 9,000 kva., 180 r.p.m., 13,800 volts, 60-cycles. Both machines had top field collars. One had no damper windings. The other was equipped with low-resistance damper windings of the type shown in Fig. 2.

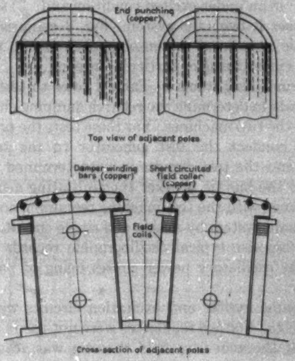
End punching (copper)
Top view of adjacent poles
Damper winding bars (copper)
Short circuited field collar (copper)
Field coils
Cross-section of adjacent poles

FIG. 2—TYPE OF DAMPER WINDINGS EMPLOYED

Each test machine was subjected to a series of tests, those made on one machine being duplicated as nearly as possible on the other.

The first item in Table I refers to tests which were made to obtain data for checking calculated machine

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*Central Station Engg. Dept., General Electric Co., Schenectady, N. Y.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

[1] For references see Bibliography.

TABLE I—SUMMARY OF TESTS

Item	Type of test	Damper windings	Initial and final load kw.	*Initial and final field current amps.	Final power factor	Field circuit resistance ohms	T_d (see Appendix)
1	Machine constant	Yes	0	15	0	0.8	
		No	0	15	0	0.8	
2	Damping	Yes	6,300	285	0.985 lag	0.8	5.78
		No	6,300	290	0.991 lag	0.8	1.69
3	Damping	Yes	6,300	260	0.995 lag	0.8	5.34
		No	6,300	265	0.996 lag	0.8	2.02
4	Damping	Yes	6,300	240	1.0	0.8	5.34
		No	6,300	255	1.0	0.8	1.95
5	Damping	Yes	6,300	225	0.995 lead	0.8	6.1
		No	6,300	230	0.995 lead	0.8	2.09
6	Damping	Yes	6,300	200	0.980 lead	0.8	5.22
		No	6,300	210	0.980 lead	0.8	2.14
7	Damping	Yes	6,300	185	0.965 lead	0.8	5.22
		No	6,300	195	0.965 lead	0.8	1.71
8	Damping	Yes	6,300	225	0.994 lead	1.0	6.1
		No	6,300	230	0.994 lead	1.0	1.73
9	Damping	Yes	6,300	225	0.994 lead	1.2	6.3
		No	6,300	230	0.994 lead	1.2	1.59
10	Damping	Yes	0+	180		0.94	5.63
		No	0+	200		0.92	1.12

*Field current required to produce rated open-circuit voltage is approximately 180 amperes.

constants. For these tests the procedure was to (1) connect the test generator to the system, (2) operate it at no-load in an under-excited condition and, 3) suddenly disconnect it from the system. Fig. 3 comprises oscillographic records of the ensuing phenomena, the field current curves being of particular interest.

The remaining items in Table I refer to tests which were made to determine the relative damping characteristics of the two machines. For these tests the procedure was to connect the test generator to the test bus, load it and the test circuits in predetermined fashion, and trip the bus tie breaker thus initiating a balanced disturbance with the test machine remaining connected to the main system by means of a single circuit. Figs. 4 and 5 comprise typical oscillographic records of such tests, the oscillatory power curves being of particular interest.

The same exciter and excitation circuits were used for all tests. Voltage regulators were not used. Prior to each test the generator field rheostat was adjusted to give the desired value of generator field circuit resistance. The desired value of generator field current was then obtained by manual manipulation of the exciter field rheostat.

The voltage of the main system bus varied but little during the testing period, and during each individual test remained practically unchanged.

MACHINE CONSTANTS

The constants of each test machine were calculated from design data in the manner described by Linville.[2] Using these values and an operational method outlined by Park,[3] the action of each machine's field current was calculated for the conditions existing while the records of Fig. 3 were being obtained. In Fig. 6 the calculated curves are compared with those taken from the oscillographic records of Fig. 3.

In developing the calculated field current curves, direct axis constants were used; quadrature axis constants were not used because the tests to be checked were made at no-load. The excellent agreement between the calculated and test curves indicates the accuracy of the method used to derive the direct axis constants. The same method was used to calculate the quadrature axis constants. Thus, the reasonable-

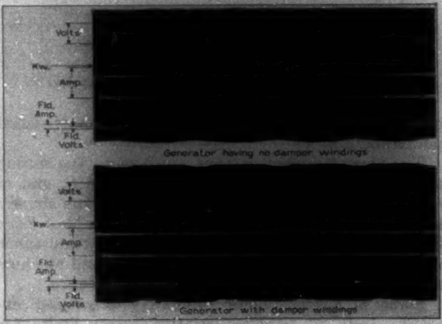


FIG. 3—OSCILLOGRAPHIC RECORDS OF MACHINE CONSTANT TESTS

ness of the calculated machine constants, used by the authors of the companion paper[1] in theoretically checking the power swing records of Fig. 4, is indicated.

The more important constants of the two test machines are given in Table II.

DAMPING TORQUE COEFFICIENTS

The time required for the damping of a balanced disturbance to $\frac{1}{\epsilon}$ of its initial value is inversely proportional to the damping torque coefficient, T_d, which can be determined from test records by the method given in the Appendix. In this way the damping torque coefficient was determined for each of the tests referred to in items 2 to 10 of Table I.

These results are plotted in Figs. 7 and 8, both of which show the relative damping action of machines with and without low resistance damper windings of the type shown in Fig. 2. The damping torque coefficients for the damping winding machine are, approximately, three times those of the other machine. The major portions of power oscillations involving the damper winding machine were damped out in about one-third of the time required for the damping out of like oscillations involving the machine not equipped with these windings.

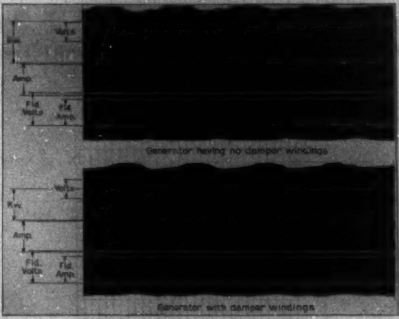


FIG. 4—OSCILLOGRAPHIC RECORDS OF DAMPING TESTS—LOADED CONDITIONS

TABLE II—CALCULATED MACHINE CONSTANTS

Constant	With damper windings	No damper windings
x_d	0.91	0.91
x_d'	0.26	0.26
x_d''	0.14	0.177
x_q	0.524	0.524
x_q'	0.524	0.524
x_q''	0.368	0.524
T_{do}'	2.86 (sec.)	2.86 (sec.)
T_{do}''	0.055 (sec.)	0.0136 (sec.)
T_{qo}''	0.044 (sec.)	
H	1.78 (sec.)	1.78 (sec.)

A comparison of the power swings shown in Fig. 4, and a comparison of those shown in Fig. 5, provide a more graphic illustration of the increased rapidity with which balanced disturbances are damped out when machines are equipped with low-resistance damper windings.

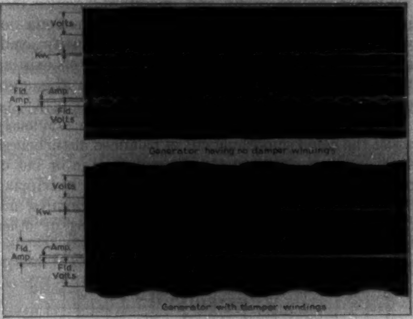
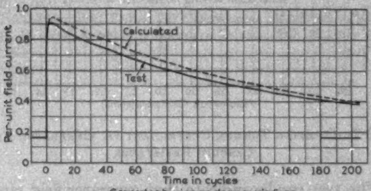
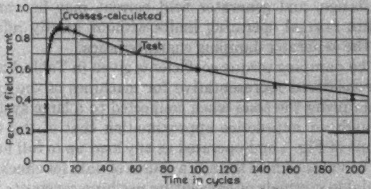


FIG. 5—OSCILLOGRAPHIC RECORDS OF DAMPING TESTS—LIGHTLY LOADED CONDITIONS



Generator having no damper windings



Generator with damper windings

FIG. 6—MACHINE CONSTANT TESTS—COMPARISON OF CALCULATED AND TEST RESULTS

Fig. 7 also shows that damping varies but little with normal values of generator field circuit resistance. Fig. 8 also shows that damping varies inappreciably with values of generator field current within normal operating limits.

The effect of generator loading can be judged by comparing the values of T_d given in item 10 of Table I with values of T_d tabulated for other tests. For the machine with damper windings T_d was approximately the same, regardless of the average load. For the machine having no damper windings, T_d during no-load operation was only about one-half as great as with the machine loaded.

CONCLUSIONS

Field tests are useful and important because they provide graphic evidence concerning phenomena not well understood, and data with which to verify available methods of analysis, or extensions of existing theory. Both purposes were fulfilled by the tests described.

Low-resistance damper windings quite obviously assist the rapid smoothing out of balanced disturbances such as occur owing to (1) the switching in or out of lines, (2) synchronizing, (3) load changes or the tripping of loaded generators, (4) three-phase short circuits, and (5) the occurrence and subsequent clearing of other types of faults.

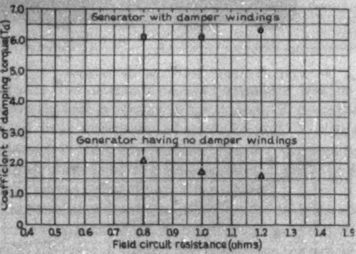


FIG. 7—DAMPING TORQUE COEFFICIENT VS. GENERATOR FIELD CIRCUIT RESISTANCE

The test results indicate that generator field circuit resistance, and generator field current may be varied within normal operating limits without greatly affecting the generator damping characteristics.

The damping torque coefficients given in Table I may be considered representative for machines of the type used during the tests when connected to a large system through an impedance approximating that of the test circuit of Fig. 1 and operated at any load up to full load.

The value of field tests as a source of data for checking available methods of calculating machine constants has been shown. The importance of field tests as a source of data for verifying the theory employed to predetermine the action of machines during system disturbances is shown by the companion paper.[1]

The New York Power and Light Corporation is deserving of much credit for the results of these tests, for which it made available a portion of its system, and in the carrying out of which Mr. Powell, Mr. Bromeson, and other members of its staff provided material assistance.

Appendix

DETERMINATION FROM TEST RECORDS OF DAMPING TORQUE COEFFICIENTS

The electrical torque of a synchronous machine during an oscillation may, in an approximate manner, be separated into two components. These are the damping and the synchronizing torques. If the oscillations are small the synchronizing torque at any instant is very nearly proportional to the angular displacement of the generator rotor. Under such conditions this, torque may be represented by a constant, T_s, times the angular displacement. The damping torque depends upon the rate of change of angle and may be represented by a constant, T_d, times the rate of change of angular displacement.

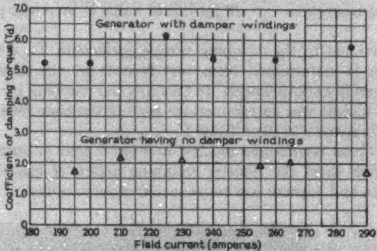


FIG. 8—DAMPING TORQUE COEFFICIENT VS. GENERATOR FIELD CURRENT

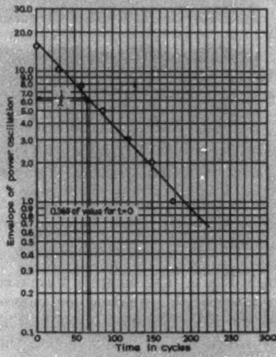


FIG. 9—DETERMINATION OF TIME CONSTANT OF POWER OSCILLATION DECREMENT

Multiply cycles by 2π to obtain $\frac{1}{l}$ in radians before substituting in equation (3)

The differential equation for the motion of a generator rotor with constant mechanical torque input, and small angular displacement, is approximately

$$4\pi f H \frac{d^2\delta'}{dt^2} = -T_s\delta' - T_d\frac{d\delta'}{dt} \tag{1}$$

where

f = system frequency in cycles per second.

H = inertia constant of the machine, or

$$\frac{0.231\ WR^2\left[\frac{\text{r.p.m.}}{1{,}000}\right]^2}{\text{Base kw.}}\ (\text{Sec.}).$$

δ' = angle in electrical radians of the rotor from the final angle of equilibrium.

T_s = coefficient of synchronizing torque, *i. e.*, per unit torque per radian of angular displacement.

T_d = coefficient of damping torque, *i. e.*, per unit torque per radian of angular displacement per radian time.

t = time measured in radians, *i. e.*, unit time is the time required for the rotor to pass one electrical radian at fundamental frequency (f).

The solution of equation (**1**), when $\frac{d\delta'}{dt} = 0$, and $\delta' = -\delta_0'$, at $t = 0$, is

$$\delta' = -\delta_0'\,\epsilon^{-lt}\cos st \tag{2*}$$

where

δ_0' = the initial angular displacement of the rotor from the final angle of equilibrium,

$$l = \frac{T_d}{8\pi f H} \tag{3}$$

$$s = \sqrt{\frac{T_s}{4\pi f H} - \left[\frac{T_d}{8\pi f H}\right]^2} \tag{4}$$

Since the maximum and minimum values of displacement angle will occur approximately simultaneously with the maximum and minimum values of electrical power, the time constant of the decrement of the power envelope equals $\frac{1}{l}$ which is determined from a plot on semi-log paper of the power oscillation as shown by Fig. 9. Therefore, T_d can be obtained directly from equation (**3**) and the test oscillographic record of electrical power.

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Design of Capacitor Motors for Balanced Operation

BY P. H. TRICKEY*

Associate, A.I.E.E.

INTRODUCTION

THE present methods of calculating the full load performance of capacitor motors are quite long and tedious. This paper presents a simple, convenient method of calculating full-load conditions.

It is generally agreed that a capacitor motor should be designed to operate under nearly balanced conditions at full load, meaning by this that there shall be no transfer of power from one phase to the other and balanced currents in the rotor. With such a design, each phase produces equal flux and the rotor operates as if in a two-phase stator.

METHOD OF OBTAINING BALANCED CONDITIONS

An attempt is sometimes made to approximate balanced conditions by making the voltages applied to each phase 90 electrical degrees apart and proportional to the turns in the windings. However, it is obvious that in order to obtain true balanced conditions, the field produced by each phase must be of the same magnitude and 90 degrees apart. If this is true, the rotor currents will also be of the same value and displaced 90 degrees. Under such conditions, the induced voltage, and not the applied voltage must be balanced. The magnetizing current being proportional to the induced voltage, it will, therefore, also be balanced and the primary currents as well as the secondary currents will be 90 degrees displaced and inversely proportional to the effective turns. Fig. 1 shows the vector diagram for a capacitor motor.

As a general rule the main winding has been determined by the necessary maximum torque of the motor. This in turn determines the primary resistance, and the current in the main phase. It remains, therefore, to determine the number of turns in the capacitor phase, the size of wire and the impedance of the capacitor itself. Since the resistance of the capacitor is controlled by the available space for it, and the allowable loss, the designer can usually estimate fairly closely as to its value. It will be treated hereafter as a known constant and if necessary the motor may be redesigned when the resistance of the capacitor differs from the assumed value. Thus, there are three variables to be determined, number of turns, size of wire, and capacitive reactance.

SOLUTION OF THE VECTOR DIAGRAM

Under the first necessary condition, the two fluxes must be alike. If this is the case the induced voltages in each phase must be 90 deg. apart, and their magnitudes must be in the ratio of the effective conductors in each phase.

$$e_{2S} = jKe_{2m} \tag{1}$$

where

e_{2S} = induced voltage in the capacitor phase

e_{2m} = induced voltage in the main phase

$K = \dfrac{\text{effective conductors in capacitor phase}}{\text{effective conductors in main phase}}$

j = operator to revolve vector 90 deg.

Under the second necessary condition, the two primary currents must be 90 deg. apart and their magnitudes must be in the inverse ratio of the effective conductors.

$$I_{1S} = \frac{j}{K}\, i_{1m} \tag{2}$$

Where

i_{1S} = primary current in capacitor phase

i_{1m} = primary current in main phase

$$E_S = e_{2S} + i_{1S}Z_{1S} \tag{3}$$

Where

E_S = capacitor phase voltage

Z_{1S} = primary impedance of capacitor phase

But

$$E_S = E_M - i_{1S}Z_c \tag{4}$$

Where

E_m = main phase and line voltage

Z_c = impedance of capacitor

(*a*) *Solution of equation* (**3**)

$$E_S = jKe_{2m} + \left(\frac{j}{K}\, i_{1m}\right)(Z_{1S})$$

But

$Z_{1S} = r_{1S} + jX_{1S}$

Where

r_{1S} = primary resistance of capacitor phase

X_{1S} = primary reactance of capacitor phase

If the winding distribution is the same in each phase

$$X_{1s} = K^2X_{1m}$$
$$r_{1s} = KK_a r_{1m}$$

Where

$K_a = \dfrac{\text{area of conductor in main phase}}{\text{area of conductor in capacitor phase}}$

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32-106

r_{1m} = primary resistance of main phase
X_{1m} = primary reactance of main phase

$$E_s = jKe_{2m} + \left(\frac{j}{K}\, i_{1m}\right)(KK_a r_{1m} + jK^2 X_{1m})$$

$$= jKe_{2m} - KX_{1m} i_{1m} + jK_a r_{1m} i_{1m}$$

But

$$e_{2m} = E_m - i_{1m} Z_{1m}$$

$$E_s = jKE_m - jKi_{1m}(r_{1m} + jX_{1m}) - KX_{1m} i_{1m} + jK_a r_{1m} i_{1m}$$

$$= jKE_m - jKi_{1m} r_{1m} + jK_a r_{1m} i_{1m}$$

It will be noted that X_{1m} drops out at this point eliminating the question of dividing the leakage reactance into primary and secondary components.

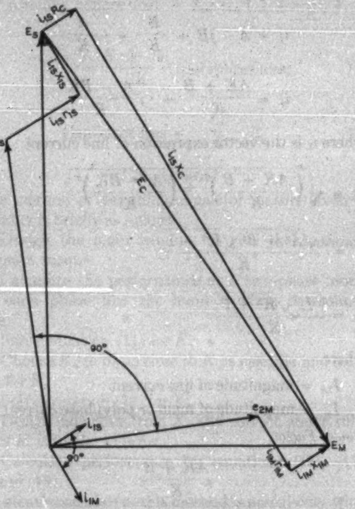

Fig. 1—Vector Diagram of Capacitor Motor Under Balanced Operating Conditions

For simplification, let

$$i_{1m} = A - jB$$

Where
A = power component of main phase primary current
= I_m times power factor
B = reactive component of main phase primary current
= $\sqrt{I_m^2 - A^2}$

I_m is the phase current as determined by calculating the motor as a two-phase motor with each phase identical to the main winding of the capacitor.

$$E_s = K_a B r_{1m} - KBr_{1m} + j(KE_m + K_a A r_{1m} - KAr_{1m}) \qquad (5)$$

(*b*) *Solution of equation* (**4**)

$$E_s = E_m - i_{1s} Z_c$$

$$Z_c = R_c - jX_c$$

Where
R_c = effective resistance of capacitor
X_c = effective capacitive reactance of capacitor (positive)

$$E_s = E_m - \frac{j}{K}(A - jB)(R_c - jX_c)$$

$$= E_m - BR_c/K - AX_c/K + j(BX_c/K - AR_c/K) \qquad (6)$$

Since these two equations for E_s must be identical, the components must be equal.

Horizontal Components

$$E_m - BR_c/K - AX_c/K = K_a B r_{1m} - KBr_{1m}$$

$$KE_m - BR_c - AX_c = KK_a B r_{1m} - K^2 B r_{1m} \qquad (7)$$

Vertical Components

$$BX_c/K - AR_c/K = KE_m + K_a A r_{1m} - KAr_{1m}$$

$$BX_c - AR_c = K^2 E_m + KK_a A r_{1m} - K^2 A r_{1m} \qquad (8)$$

Multiplying by B and A respectively in equations (**7**) and (**8**) and eliminating X_c.

$$K^2[AE_m - r_{1m}(A^2 + B^2)] - K[BE_m - K_a r_{1m}(A^2 + B^2)] + (A^2 + B^2) R_c = 0 \qquad (9)$$

Since this quadratic equation gives a difficult solution to use, a cut and try method is followed.

$$K[AE_m - r_{1m}(A^2 + B^2)] = [BE_m - K_a r_{1m}(A^2 + B^2)] - (A^2 + B^2) R_c/K$$

$$K = \frac{BE_m - K_a r_{1m}(A^2 + B^2) - (A^2 + B^2) R_c/K}{AE_m - r_{1m}(A^2 + B^2)} \qquad (10)$$

But $A^2 + B^2 = I_m^2$ where I_m is the magnitude of i_{1m}

$$K = \frac{BE_m - K_a r_{1m} I_m^2 - (R_c/K) I_m^2}{AE_m - r_{1m} I_m^2} \qquad (11)$$

In the above equation everything has been determined by the design of the main winding except K and K_a. The method of using this equation is as follows: K_a must, of course, be a function of $\sqrt[3]{2}$ since wire sizes vary in this definite ratio. The usual motor will have one, two, or three wire sizes smaller than the main winding, and thus K_a will be 1.26, 1.59, or 2.00. One of these is assumed, and since, as will be shown later, K should be somewhere near equal to K_a, the value of K on the right-hand side of equation (**11**) is taken equal to the assumed value of K_a. The equation is then solved for K. The value of K obtained is substituted for the assumed value and a second solution is made. Since the third term is usually small, it is seldom necessary to make more than two solutions. For every value of K_a, there will be a definite value of K, or in other words, there are as many balanced motors as there are wire sizes. However, the change in K for different values of K_a is very small. Fig. 2 shows the effect of changing K_a upon the constants and performance of a typical motor. It will be noted that the only values which change in any great degree are the weight of wire and the current density, and that whatever gain is made in reducing capacitor phase copper is made at the expense of a higher current density in the capacitor winding. It would seem desirable, therefore, to choose K as nearly equal to K_a as wire sizes will permit.

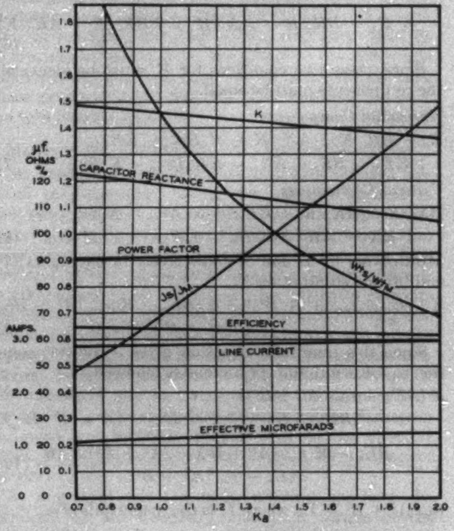

Fig. 2—Effect of K_a on Capacitor Motor Performance

$Ka = \frac{\text{area of conductor in main phase}}{\text{area of conductor in capacitor phase}}$

$K = \frac{\text{effective conductors in capacitor phase}}{\text{effective conductors in main phase}}$

$\frac{J_s}{Jm} = \frac{\text{current density in capacitor phase}}{\text{current density in main phase}}$

$\frac{Wt_s}{Wtm} = \frac{\text{total weight of wire in capacitor phase}}{\text{total weight of wire in main phase}}$

Having determined K from equation (**11**), equation (**7**) gives,

$$X_c = \frac{KE_m - BR_c - KK_a Br_{1m} + K^2 Br_{1m}}{A} \qquad (12)$$

$$C = \frac{10^6}{2\pi f X_c} \text{ or } \frac{2650}{X_c} \text{ for 60 cycles} \qquad (13)$$

where C = effective microfarads

$$E_c = \frac{I_m Z_c}{K} = \text{voltage across capacitor} \qquad (14)$$

Performance Calculation of Balanced Motor

Since the motor has been designed to have rotor conditions identical to a two-phase motor with each phase like the main phase of the capacitor motor, the motor may be calculated like a polyphase motor. There will be, however, differences in some of the performance values.

Line Current

$$i_L = i_{1m} + i_{1c} = i_{1m} + \frac{j}{K} i_{1m}$$

$$i_L = A - jB + \frac{B}{K} + j\frac{A}{K}$$

$$i_L = \frac{AK + B}{K} + j\frac{A - BK}{K}$$

where i_L is the vector expression of line current

$$I_L = \sqrt{\left(\frac{AK + B}{K}\right)^2 + \left(\frac{A - BK}{K}\right)^2}$$

$$= \frac{\sqrt{A^2 + B^2}\sqrt{K^2 + 1}}{K}$$

$$I_L = \frac{I_m\sqrt{K^2 + 1}}{K} \qquad (15)$$

where

I_L = magnitude of line current

I_m = magnitude of main or polyphase current

Power Factor

$$\text{P. F.} = \frac{\frac{AK + B}{K}}{\sqrt{\left(\frac{AK+B}{K}\right)^2 + \left(\frac{A-BK}{K}\right)^2}}$$

$$= \frac{AK + B}{\sqrt{A^2 + B^2}\sqrt{K^2 + 1}}$$

$$\text{P. F.} = \frac{AK + B}{I_m\sqrt{K^2 + 1}} \qquad (16)$$

or

$$\frac{A + \frac{B}{K}}{I_L} \qquad (17)$$

The losses of the capacitor motor will differ from the losses in the polyphase motor only in primary copper loss, and the additional loss in the capacitor. The main phase loss will be the loss per phase in the polyphase motor.

$$\text{Main phase loss} = \frac{\text{polyphase loss}}{2}$$

$$\text{Capacitor phase loss} = i_{1s}^2 r_{1s}$$

Only the magnitude of current is of interest

$$\text{Capacitor phase loss} = \left(\frac{I_m}{K}\right)^2 r_{1s}$$

$$= \left(\frac{I_m^2}{K^2}\right) K K_a r_{1m} = (I_m^2 r_{1m}) \frac{K_a}{K}$$

$$\text{Total primary copper loss} = I_m^2 r_{1m}\left(1 + \frac{K_a}{K}\right)$$

$$= \frac{K + K_a}{2K} (\text{polyphase loss}) \qquad (18)$$

$$\text{Capacitor loss} = \frac{I_m^2}{K^2} R_c \qquad (19)$$

CONCLUSION

The method of designing capacitor motors given in this paper is briefly as follows:

1. Design the main winding to give the necessary maximum torque.
2. Calculate the performance as a two-phase motor with each phase like the main winding determined above.
3. Solve equation (**11**) for K.
4. Choose K_a to be as close to K as possible and solve again for K.
5. Solve equation (**12**) for X_c.
6. Design capacitor from X_c and correct above solutions if R_c is too far in error.
7. Calculate performance by equations (**15**), (**17**), (**18**) and (**19**).

As mentioned above, this method is useful only when the starting torque required is not too great.

In many cases, starting torque requirements determine the windings, and the motor must operate slightly unbalanced at full-load with the resultant circulating current and lowered efficiency. A sample calculation on a typical motor is given in the appendix.

LIST OF SYMBOLS

E_m = main winding and line voltage
E_s = capacitor winding voltage
E_c = capacitor voltage
e_{2m} = induced voltage in main winding
e_{2s} = induced voltage in capacitor winding
i_{1m} = primary current in main winding
i_{1s} = primary current in capacitor winding
I_M = current per phase in identical two-phase motor
I_L = line current of capacitor motor
A = power component of main winding primary current
B = reactive component of main winding primary current
J_m = current density in main winding
J_s = current density in capacitor winding
Wt_m = weight of wire in main winding
Wt_s = weight of wire in capacitor winding
$K = \dfrac{\text{effective conductors in capacitor winding}}{\text{effective conductors in main winding}}$
$K_a = \dfrac{\text{area of conductor in main winding}}{\text{area of conductor in capacitor winding}}$
r_{1m} = primary resistance of main winding
X_{1m} = primary reactance of main winding
Z_{1m} = primary impedance of main winding
r_{1s} = primary resistance of capacitor winding
X_{1s} = primary reactance of capacitor winding
Z_{1s} = primary impedance of capacitor winding
R_c = effective resistance of capacitor
X_c = effective reactance of capacitor (positive)
Z_c = effective impedance of capacitor
C = effective capacity of capacitor (in microfarads)

Appendix

Following is given an example of a capacitor motor design illustrating the method described in this paper. It is assumed that the main winding has been determined from the maximum torque requirements, and that the performance as a two-phase motor with each phase like the main winding has been calculated.

E_m = 110 volts (applied voltage)
r_{1m} = 1.89 ohms primary resistance of main phase
I_m = 2.39 amperes current per phase
Primary copper loss = 21.6 watts
Input = 273.6 watts
Output = 186.7 watts (1/4-hp., 110-volt, 60-cycle, two-phase, 4-pole)
Losses 86.9 watts
Efficiency 68.2 per cent
Power factor 52.0 per cent
R_c = 8.00 ohms
$I_m^2 = 5.7$
$A = I_m PF = 2.39 \cdot 52 = 1.24$
$B = \sqrt{I_m^2 - A^2} = 2.045$
K_a and K are assumed = 1.26

$$K = \frac{2.045 \times 110 - 1.26 \times 1.89 \times 5.7 - \dfrac{8.0}{1.26} \times 5.7}{1.24 \times 110 - 1.89 \times 5.7}$$

$$= \frac{225 - 13.57 - 36.2}{136.1 - 10.79} = \frac{175.2}{125.31} = 1.40$$

Second trial

$$K = \frac{225 - 13.57 - \dfrac{8.0\ 5.7}{1.42}}{125.31} = 1.430$$

$$X_c = \frac{1.43 \times 110 - 2.045 \times 8 - 1.43 \times 1.26 \times 2.045 \times 1.89 + 1.43 \times 2.045 \times 1.89}{1.240}$$

$$= 114.3 \text{ ohms}$$

$$C = \frac{2650}{114.3} = 23.2 \text{ effective microfarads}$$

$$I_L = \frac{2.39}{1.43}\sqrt{1.43^2 + 1} = 2.915 \text{ amperes}$$

$$\text{P. F.} = \frac{1.24 + \left(\dfrac{2.045}{1.43}\right)}{2.915} = 91.5 \text{ per cent}$$

$$\text{Primary loss} = \left(\frac{1.43 + 1.26}{2\ \ 1.43}\right) 21.6 = 20.3$$

(decrease of 1.30 watts)

$$\text{Capacitor loss} = \left[\frac{2.39}{1.43}\right]^2 8 = 22.4$$

Total losses 86.9 − 1.3 + 22.4 = 108.0

Output = 86.7

Input = 294.7

$$\text{Efficiency} = \frac{186.7}{294.7} = 63.3 \text{ per cent}$$

Fig. 2 is plotted from the values obtained in this example.

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Discussion

L. A. Doggett: In this article as well as all previously published articles, the theory of the capacitor motor has been developed on the assumption of sine waves. The correctness of this assumption is challenged by the oscillogram of Fig. 1 which shows the current waves when a *RKS*-126, 2-hp., 1,800-r.p.m., 220-volt, 8.6-ampere, single-phase, 60-cycle, G. E. capacitor motor is operated from a practically sinusoidal source. (Analysis: $Y = \sin x + 0.01 \sin 2x + 0.00 \sin 3x + 0.01 \sin 4x + 0.01 \sin 5x$). These current waves are for the motor running light, curve $I_l = 4.4$ shows the line current; curve $I_c = 6.1$ shows the capacitor phase current; and curve $I_m = 2.4$ shows the main phase current

That these currents depart widely from sinusoidal shape throughout the whole range of operation is brought out by an I^2R, I^2X analysis of the load run test data. Since wattmeters measure the average power, not only of the fundamental but also of the harmonics, a reasonably close check was to be expected of the I^2R analysis. The I^2X values were calculated from the expression,

$$I^2X = (\overline{EI}^2 - [I^2R]^2)^{1/2}$$

which is true for sinusoidal cases only. The degree by which the I^2X analysis fails to check is a measure of the departure of the actual waves from sine waves. The wide discrepancies in the I^2X values, shown by Table I, indicate the presence of large harmonics.

So long as there is a transfer of power from one phase to the other there are bound to be large harmonics present. If the designs of Mr. Trickey avoid this transfer of power, oscillograms from motors of his design should show much better sine waves than those of Fig. 1. It is to be hoped that Mr. Trickey will submit oscillograms for some of his motors.

If there is a transfer of power, then one phase becomes a generator and the other phase becomes a receiver. It will hardly be claimed that such a generator is a sine-wave generator. To make matters worse, a sizeable condenser is included in the series circuit between the generator and the source. Thus the stage is set for the generation of harmonics, wide in variety and large in amplitude.

TABLE I*

Line			Cond. phase			Main phase		
I^2R	I^2X	I^2Z	I^2R	I^2X	I^2Z	I^2R	I^2X	I^2Z
1,020	−387	1,090	1,460	−716	1,598	−440	203	485
1,410	−387	1,462	1,420	−591	1,535	− 40	176	180
1,620	−412	1,673	1,400	−490	1,482	240	000	240
1,880	−490	1,944	1,360	−544	1,463	520	245	576
2,110	−374	2,140	1,320	−505	1,415	800	122	809
2,280	−400	2,313	1,280	−485	1,370	1,000	173	1,015
2,500	−374	2,525	1,240	−424	1,310	1,260	255	1,285
2,640	−316	2,660	1,200	−453	1,282	1,440	141	1,445
1,020	−387	1,090	1,480	−678	1,598	−480	232	533

SUMMARY OF ABOVE

Line I^2R	Branches I^2R	Line I^2X	Branches I^2H
1,020	1,020	−387	−513
1,410	1,380	−387	−415
1,620	1,640	−412	−490
1,880	1,880	−490	−299
2,110	2,120	−374	−383
2.280	2,280	−400	−312
2,500	2,500	−374	−169
2,640	2,640	−316	−312
1,020	1,000	−387	−446

I^2R = wattmeter reading.

I^2Z = product of voltmeter and ammeter readings.

$-I^2X$ – condensive reactance volt amperes.

*The data of this table were prepared by A. R. Mengel and D. W. McGill, Enrolled Students A.I.E.E. at Pennsylvania State College.

C. G. Veinott: Anyone who has designed, or has attempted to design capacitor motors recognizes that the problem is tremendously involved and that there is a considerable number of factors which must be taken into account. Mr. Trickey's paper throws much useful light on this complicated problem in a simple manner. He shows the conditions which must be met in order to obtain operation equivalent to that of a two-phase motor and incidentally brings to light some interesting and important conclusions. For example, in Fig. 1, the winding ratio (ratio of capacitor phase turns to main phase turns) is of the order of the tangent of the power-factor angle of the motor when considered as a two-phase motor with both phases like the main winding. It is, therefore, necessary to have control of the winding ratio as well as the external microfarads in order to obtain balanced operation. This means that any polyphase motor cannot necessarily be operated as a polyphase motor from a single-phase source merely by the addition of a capacitor of suitable value in one phase. This fact seems to be often misunderstood by those not familiar with capacitor motors.

Fig. 1

P. H. Trickey: Regarding the advisability of operating a capacitor motor at its balanced operating point, Fig. 2 shows the effect of changing the voltamperes of the capacitor on efficiency, power factor, and apparent efficiency of a typical ¼-hp., 110-volt, 60-cycle motor. The balanced point is at *A*.

Having designed a motor for balanced operation, and calculated its performance by the method of this paper, the designer can choose actually to operate the motor at any value of capacitor voltamperes he wishes. Point *B* is the maximum efficiency point which will coincide with point *A* if $R_c = O$ and if $K = K_a$, point *C* gives the maximum power factor and point *D* gives the maximum apparent efficiency. The operator may operate at any one of these points, or if cost is of greatest importance the point *E* might be chosen, which gives a motor within 1½ per cent of the maximum efficiency and has only 59 per cent as many voltamperes. However, the power factor has dropped from 80 to 69 per cent.

If the designer chooses to operate at any point between *E* and *D*, this method is still of use to him in predetermining performance especially on new and radically different designs. Knowing the efficiency and power factor quite accurately at the balanced point, he can estimate very closely the performance at the actual operating point, from the general shape of the curves of Fig. 2.

In reply to Mr. Doggett's discussion the most prominent cause of harmonics in capacitor motor currents is saturation of iron, either in the motor itself or in the capacitor transformer. The transformer is the most likely cause because it has no air gap and on any voltage the magnetizing current may have a very bad third harmonic component. Unless a capacitor motor is returning power to the line through its main phase, the main phase current will have nearly the wave shape of the applied voltage except a slight distortion due to saturation in motor iron. The capacitor phase current will be distorted depending on the transformer saturation and since at no load the capacitor phase current is such a large proportion of the line current, the line current will also be distorted. The capacitor phase current will be symmetrical about a vertical axis, and since it adds approximately in quadrature to the main phase current, the line current will be distorted toward one side. Under no-load, overvoltage conditions this distortion may be very bad. In fact if the voltage is raised high enough, the capacitor phase fundamental magnetizing current will just balance the leading condenser current, and leave the third harmonic practically alone.

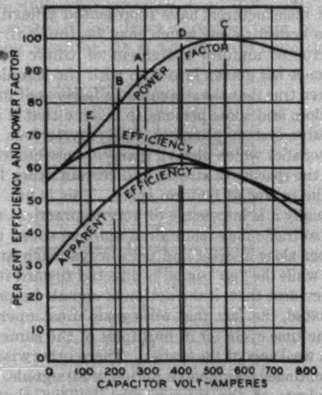

Fig. 2—Effect of Changing Capacitor Voltamperes on Capacitor Motor Performance

A. Balanced point
B. Maximum efficiency
C. Maximum power factor
D. Maximum apparent efficiency

However, I believe the wave shapes in a capacitor motor designed for balanced operation at full load, and with inductions sufficiently low in the transformer for operation at the usual overvoltages, will be close enough to a sine-wave for all practical purposes except at very light loads. At very light loads the motor is inherently unbalanced, and inefficient and a few harmonics more or less will not make it much worse.

The Flexible Progressive Traffic Signal System

BY H. I. TURNER*
Associate, A.I.E.E.

A FLEXIBLE progressive traffic signal system has been defined as one which provides a means of timing signals at different street intersections for operation in any desired relation with respect to each other, subject to the one limitation that the time of each cycle of operation must be the same at all intersections of the system. Such a range of adjustments usually makes it possible for an engineer to time the signals so that traffic may move east and west as well as north and south at certain predetermined uniform speeds without being obstructed by a "stop" signal after once having passed a "go" signal.

Most manufacturers have represented a flexible progressive system as one which may be timed to permit uninterrupted uniform movement of traffic after the traffic once has gotten into step with the signals. This is so often true that the statement is fairly well justified, but it does lead some persons to believe that a flexible progressive system possesses some mysterious timing characteristics which always make it possible,irrespective of the spacing of streets and certain other factors, for traffic to flow at uniform speed. This is not true as occasionally it is impossible or very impractical to time signals so traffic may maintain a uniform speed in both directions along a street and yet always reach an intersection while the "go" signal is directed toward it.

It is evident that, if the action of signals be properly coordinated, the fact that all signals must operate on the same time cycle (or in multiples of the same cycle) is not a weakness of the system. There otherwise could be no continued coordinated action of all signals. There are students of the subject who believe that it is theoretically impossible to provide any timing of signals within a given area which is an improvement over the timing that is possible in a genuine flexible progressive system, providing, of course, that facilities are furnished for readily varying timing to meet changes in conditions which develop during different times of the day. Some engineers believe that a single intersection, considered by itself, may often be dealt with more satisfactorily with signaling devices which are actuated by traffic than by any pretimed signals. A careful study of what can be accomplished by a properly-proportioned set timing of signals will, nevertheless, frequently indicate that even for many types of isolated intersections the controller with set timing deals with conditions as well or very nearly as well as the most elaborate system of traffic-actuated devices. Coordinated action of signals located at intersections of streets leading to a square having several entering streets is nearly always the most satisfactory way of dealing with these situations and the advantage of such a system becomes more evident as the volume of traffic entering the square increases.

In any event, traffic-actuated devices can rarely be introduced into the average flexible progressive system without upsetting the uniform flow of traffic to a more or less serious degree. In a great majority of congested districts the coordination of the movement of traffic through the entire district is of incomparably greater importance than the most efficient movement of traffic at any few intersections taken by themselves.

Signals may be timed for flexible progressive service by two general types of systems: *i. e.*, that which employs a central office type controller with no timing devices located at individual intersections, and that which employs a master controller at a central point and an individual controller at each intersection. A most outstanding example of the former is the controller which times the signals in the Chicago Loop District. Examples of the latter are found on Massachusetts Avenue, Cambridge; Olive Street, Saint Louis; Wilshire Boulevard, Los Angeles; Broad Street, Philadelphia, and in many other cities. All of these controllers have facilities for the following adjustments which entitle them to be classed as truly flexible progressive systems:

1. The time of each cycle of operation of the signals at any intersection may be adjusted from a central point. A cycle time anywhere between 30 and 120 seconds may be found useful in the average system.

2. The percentage of the cycle taken by the "go" signal and the "stop" signal may be adjusted over a wide range and adjustments made at one intersection are made independently of adjustments at any other intersections. In practise it has been found that ability to adjust both the "go" and "stop" periods from 15 to 85 per cent of the total cycle is all that is necessary.

3. The start of the cycle of operation of the signals at any intersection is 100 per cent adjustable with respect to the start of the cycle of operation of the signals at any other intersections of the system. The start of a cycle may be considered the instant when the "go" signal appears on any signal face and the total time of one cycle is the time which elapses between the appearance of one "go" signal and the appearance of the next "go" signal on the same face.

The chart A, Fig. 1, shows how it became necessary to time signals on one street to provide a progressive flow of traffic. The wide black marks indicate that traffic on the main street is facing a "stop" signal. At First Avenue a 25-second "stop" period for traffic on the main street was necessary. A 35-second "go" period was likewise necessary. In other words, during 58 per cent

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*Eagle Signal Corporation, Moline, Ill.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

of the cycle the "go" signal shows to the main street. In order for traffic to flow in both directions without a stop, it was necessary for the signals at Second Avenue to show "go" to the main street during 75 per cent of the cycle. The adjustments necessary at the other avenues can be seen from the chart. These signals were so timed that traffic could start at either end of the street and proceed at 15 miles per hour past the other three intersections without being obstructed by a "stop" signal. A little study of the chart will make clear to the reader that traffic must not move much faster nor much more slowly than this speed or it will encounter a "stop" signal. This condition is one of the important features of a flexible progressive system—it discourages the movement of traffic at speeds either above or below that for which the system is timed.

Chart B shows what happens when the time of each cycle of the signals is changed from 60 seconds to 30 seconds. Traffic must now move 30 miles per hour to avoid "stop" signals instead of 15 miles per hour. The operator at the central station can, therefore, time the signals so as to discourage the movement of traffic at any speed other than the one he desires. No police officers on the street can so effectively control the speed of traffic as a well-engineered flexible progressive system. Traffic officers may restrict speed, but a flexible progressive system will actually make a driver try to hold his speed at that for which the signals are set. It is obvious that daily changes in traffic volume, slippery streets, and many other conditions which are constantly developing bring about excellent applications of the operator's ability to influence speed from a central point. Usually it has been found that a much larger volume of traffic can be handled by a comparatively low speed and systematic timing than by higher speeds without well-coordinated operation of signals.

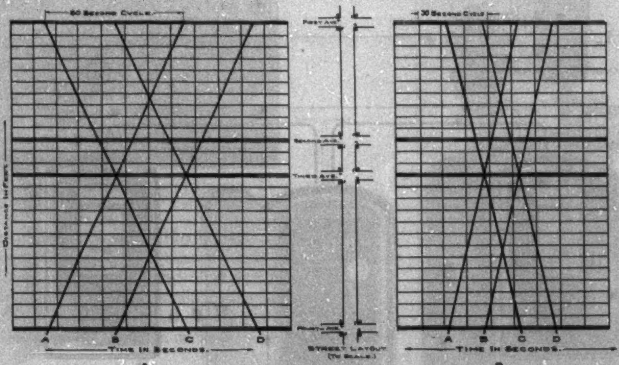

Fig. 1—Traffic Movement Chart

The following features or characteristics are important in any flexible progressive control devices:

1. Those which keep maintenance expenses and service interruptions at a minimum, such as: simplicity, sturdiness, smooth quiet operation, freedom from impact between parts.

2. Facilities for manual control. Usually the manual control of signals at one intersection of a system is objectionable because no officer can time the movements anywhere near as efficiently as can the controller itself. There are times when manual control is essential, however, such as immediately after an accident has occurred, when school children are leaving a nearby school and during times when very abnormal traffic conditions exist.

3. Facilities for shutting down the signals at each local controller and from a central point by merely throwing a switch. It is advisable to cause signals to flash amber lights when not operating as "stop" and "go" signals. Therefore, facilities should be provided at the central control point for causing signals to flash amber when not in normal operation.

4. Adjustments should be easily understood and of wide range. It is preferable that the controller be designed so adjustments may be made on a sliding scale and not in steps. A vehicle moving 30 miles per hour travels 44 feet per second and it is obvious that unless a controller can be adjusted to change its signals at the correct second it is not possible to handle traffic as smoothly as is possible when adjustments are set accurately as dictated by a traffic flow chart.

5. The duration of the amber light or warning that the signal is about to change should not increase as the total time cycle increases and should not decrease when the total time cycle is decreased. Fig. 1 shows that traffic has to travel faster when the time cycle is decreased in order to keep in step with the signals. If

decreasing the time cycle also decreases the amber or other warning signal, traffic, when it is traveling fast, receives less warning that it will be required to stop than when it is traveling more slowly. Obviously this is wrong. The amber period should either remain constant or else increase as traffic is speeded up and should either remain constant or decrease when traffic is slowed down. Any controller which causes its amber signal to increase or decrease as the total cycle increases or decreases will either provide too short an amber period at short cycles or too long an amber period at long cycles.

6. Means for exhibiting emergency signals by control from a central point. This usually consists of causing all signal faces to show red lights only and is for the benefit fo the fire department.

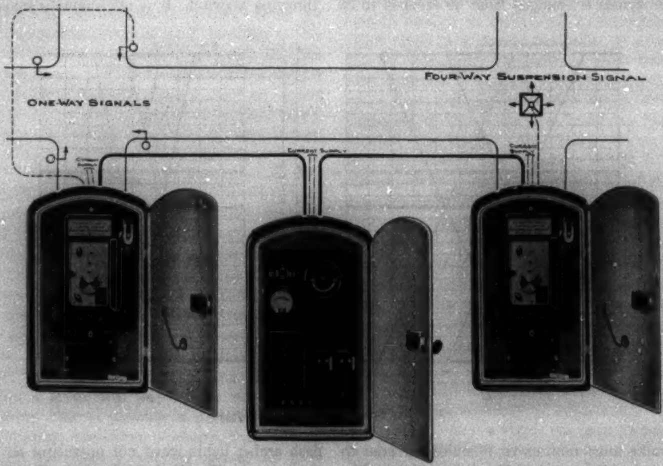

Fig. 2—Typical Master Controller Connected to Two Local Controllers

7. Means for aiding traffic movement during times of the day when traffic is very heavy on a particular street. Seldom does it happen that the ideal timing for conditions existing during most of the day is also ideal for peak hours. At such times facilities for increasing the "go" signals showing on the main thoroughfare or facilities for changing with respect to each other the times at which "go" signals at different intersections appear may meet the situation. If the "go" period is increased for the benefit of the main street, it gives cross street traffic less chance to move. If "go" signals at different intersections are shifted in time relation with respect to each other in order to favor traffic in one direction, whatever benefit is derived by traffic in that direction is at the expense of traffic in the other direction.

A description and brief history of two typical flexible progressive systems follow:

Chicago Loop System

This system was one of the earliest genuine flexible progressive systems to be placed into successful operation. It was so very substantially constructed that for six years it has functioned every day without an interruption worthy of mention. The flexible progressive system was installed in preference to other systems partly to minimize the peak load on the power station supplying power for the street railway system, as limited progressive and synchronized systems tend to make motormen start and stop many cars at the same instant at many street intersections.

From Fig. 3 the general construction of the controller is evident. It consists of individual units coupled together by a train of gears and driven by a motor. All of these units rotate at the same speed so that a complete cycle of signal changes at each intersection is accomplished in the same length of time irrespective of the relation of "go" periods to one another. Each dial shown in the illustration has associated with it four contacts, No. 3 closes circuit to north and south red lights and east and west green lights, No. 2 closes circuit to north and south green lights and east and west red lights. No. 1 and No. 4 close the circuit to amber lights in all four directions. The No. 3 contact and No. 2 contact are adjustable with respect to each other by arms in back of the dial, and by moving them to the proper positions, the red and green signals may be caused to show for any desired percentage of the cycle and the signals controlled by one unit may be caused to operate at any time with respect to the signals controlled by any other unit.

The total cycle may be increased or decreased by means of either one of two power units, each power unit consisting of a motor, an adjustable speed transmission and clutch which drive the entire train of gears. One advantage of the central office type controller is that as many and as wide variations in timing of signals as may be required to meet variations in traffic conditions during the day may quickly be effected by the operator in charge. The controller is located near the central point of the Chicago Loop District and all power for the lights is conducted over wires issuing from this central office. This system is probably the most sturdy and simplest flexible progressive system in existence.

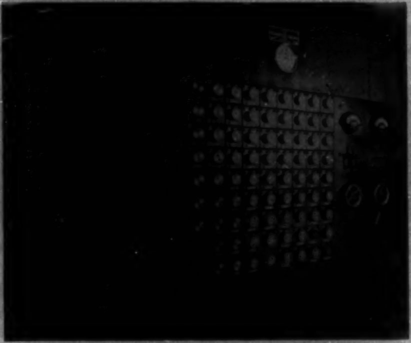

FIG. 3—CENTRAL OFFICE TYPE CONTROLLER OF CHICAGO LOOP SYSTEM

WILSHIRE BOULEVARD, LOS ANGELES, CALIFORNIA

The flexible progressive system was specified for Wilshire Boulevard only after a long and intensive study. It has been stated that both the density and speed of traffic on this thoroughfare are higher than on any other avenue in the country. In any event, Los Angeles claims more automobiles in proportion to its population than any other large American city and this is reflected in an unusual traffic problem on Wilshire Boulevard.

Wilshire Boulevard uses the Coordiplex system. This system employs at each intersection controllers having two induction disk motors. One of these motors drives a cam shaft intermittently and its rotation causes the lights to change from "stop" signal to "go" signal or vice versa. The other motor called the "cycle motor" closes the contacts which start the first motor when it is time for signal lights to change. The cycle time is controlled by a master timer located at a central point. Once during every cycle of operation of the lights, all cycle motors are brought to a stop for about one second and by means of an impulse sent out from the master timer are caused to start into motion at the same instant. Errors in the speed of these motors are thereby cancelled by the beginning of each cycle of operation. The speed of these motors as well as the speed of the master timer which sends out the re-synchronizing impulse is varied by the voltage impressed on the system by an adjustable voltage auto-transformer located at the master switchboard. This system provides flexible progressive operation in its strictest sense. The time of the "go" and "stop" periods, also the timing of one controller with respect to others is set by self-locking arms on the cycle motor dial of each intersection controller. The system enables an operator instantly to cause all signals to show red lights for the benefit of the fire department. It may be operated at late hours of the night with flashing amber showing on Wilshire Boulevard and flashing red showing on the cross street. Amber lights preceding the color change may be set at any desired interval, but when once set the interval does not vary when the cycle time is changed at the master controller. All controllers may be operated manually and one position on the manual control provides for red lights in all directions at the intersection under manual control.

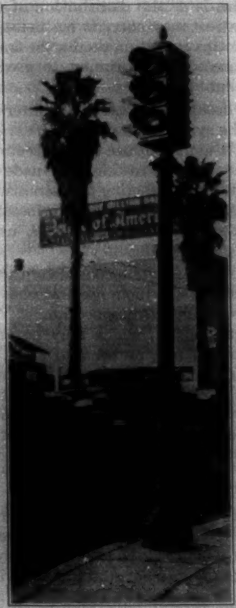

FIG. 4—WILSHIRE BOULEVARD SIGNAL

The Electro-Matic Traffic Dispatching System

BY H. A. HAUGH, Jr.*
Associate, A.I.E.E.

THE electro-matic vehicle-actuated traffic-dispatching system, as its name implies, is a system of traffic control by which the traffic in a given area controls itself. In its operation it approaches the ideal system in which the right-of-way or "green light" period is assigned to the approaches to an intersection in proportion to the volume of traffic on each approach. For example, if at the junction of A and B streets there is twice as much traffic on A street as on B street, the timing of the traffic-light cycle will be so adjusted that, within certain limits, the A street green light period will be twice as long as the B street green light period. A few minutes later the distribution of traffic may be reversed. B street may carry twice as much traffic as A street. When this condition occurs the traffic light cycle will be readjusted in its timing to give B street the longer green light period.

Originally designed for use at the intersection of only two streets the system has since been expanded and improved until it is now in use at intersections where five or six streets meet. Provision is made for handling trolley cars and for protecting pedestrians. Under traffic conditions varying from instant to instant it operates to produce a smooth flow of vehicles, using the intersection at its greatest efficiency.

The electro-matic system consists of three parts:

1. The detectors, which are switches operated by vehicles approaching the intersection.
2. The signal lights, which may be any one of the standard types of red, amber and green traffic lights. The amber light may be eliminated if so desired.
3. The dispatcher, which receives electrical impulses from the vehicle detectors and controls the signal lights in accordance with the traffic requirements as indicated by the detector impulses from all streets.

THE DETECTOR

The detector may be any one of several types such as a mechanical switch, an electromagnetic unit, or a beam of light which will be interrupted by approaching vehicles. Experiments have been made with all of these types of vehicle detectors and all of them are now in use in electro-matic systems.

The mechanical type of detector is the simplest, cheapest and the most practical. The mechanical detector used in this system consists of a pair of thin flexible steel plates separated by a strip of rubber around the edge which holds them normally one-quarter of an inch apart. The passage of a vehicle over the detector flexes the upper plate so that it makes contact with the lower plate thereby causing a current to flow to the dispatcher. These steel plates are completely enclosed in a rubber package which is vulcanized to them, thus making the whole a water-tight unit. A lead wire is attached to each plate and brought out from the package through a rubber pig tail. Fig. 1 is an illustration of a detector package. The detector package is laid in an iron casting, a strip of rubber one-half inch thick is bolted over the top of this casting to protect the package from mechanical damage by the cars which cross it, and the casting is ready to be placed in the street for service. At one end of the iron casting is a junction box in which the wires running to the dispatcher are connected by a water-tight joint to the leads from the detector package. This construction gives an extremely simple detector unit absolutely water-tight and thoroughly reliable (there are no levers, pinions, or bearings to wear or get out of order). Operating with an electro-matic dispatcher this type of detector will "pick up" a vehicle traveling at sixty miles an hour. Fig. 2 shows a mechanical detector imbedded in the pavement.

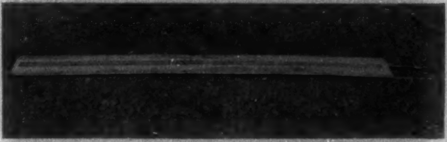

FIG. 1—A DETECTOR PACKAGE

FIG. 2—A DETECTOR IMBEDDED IN THE PAVEMENT

The casting which holds the detector package is twelve inches wide, two inches deep and is made in three lengths, four, six, and eight feet. The shortest is used between the rails of a trolley track and the other two sizes are used to "cover" streets of different widths. In "covering" the side of a street on which vehicles approach an intersection it is not necessary to place the detectors immediately adjacent to each other. If the street is wide enough to require two or more detectors they are located along a line at right angles to the curb and are spaced four feet apart end to end. Since the

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*Vice President, Automatic Signal Corp., New Haven, Conn.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

tread of a standard vehicle is more than four feet, one wheel is bound to cross one of the detectors thus sending an impulse to the dispatcher. If a vehicle crosses the detector in such a way as to send in several impulses some caused by the front wheels and some by the rear wheels, the operation of the system is not impaired.

Occasionally a street will be so narrow that vehicles going in either direction must perforce travel the middle of the highway. To prevent the sending of impulses by vehicles leaving such an intersection a special "directional" detector is used. The directional detector sends an impulse to the dispatcher from those vehicles approaching the intersection but does not send any impulse when crossed by a vehicle leaving the intersection. This effect is accomplished by splitting the top steel plate lengthwise and bringing out three leads from the detector package instead of the usual two. One of these leads is attached to each half of the top plate and the third is attached to the single bottom plate. By longitudinally splitting and electrically separating the top plate into the two parts, one on each side of the center line of the detector package, two impulses are sent to the dispatcher by each vehicle crossing the detector. Calling these impulses *A* and *B* it is evident that a vehicle crossing the detector will send in the two impulses, one shortly after the other. If a car moving toward the intersection sends the impulses in the order *A* followed by *B*, a car moving away from the intersection will send them in the reverse order, *B* followed by *A*. A relay system is arranged to pass on to the dispatcher the impulse *A* followed by *B* but it will stop that of *B* followed by *A*. Thus the directional effect is achieved.

THE DISPATCHER

The outstanding characteristic of the electro-matic system is its extreme flexibility. The timing of the signal light cycle is constantly changing to meet the irregular demands of traffic. The secret of this flexibility is to be found in the dispatcher, which is essentially a group of time-delayed relays combined with a switching mechanism. The switching mechanism operates the traffic signal lights. It receives and stores until used all impulses sent from the vehicle detectors. It also switches the group of time-delayed relays and vehicle detectors into different parts of the circuit as the green light is shifted from street to street. The time-delayed relays act as watchmen who keep an eye on each street while it has the green light, to see that it does not hold the right-of-way longer than is necessary. These "watchmen" will go so far as to stop a line of moving cars, when occasion requires, in order to give traffic on a cross street a chance to move. The functioning of these relays is worthy of some consideration.

There are three time-delayed relays in this group. One of them measures the amber light period and the "initial period" which will be discussed later in this paper. The operation of the second of these relays (called the vehicle interval relay) signifies that a line of cars has completely passed the intersection and puts the system into such a condition that a "call" from the cross street will cause immediate transfer of the right-of-way thereto. This transfer, however, takes place only when cross traffic "requests" it.

The operation of the third time-delayed relay (called the maximum-interval relay) signifies that a car has been kept waiting on the cross street for a period of time sufficiently long to warrant arbitrarily stopping the moving traffic to let it move into the intersection. When the maximum-interval relay operates it produces an effect beyond that of the vehicle-interval relay. The vehicle-interval relay merely makes the right-of-way subject to call by cross-street traffic. The maximum-interval relay, on the other hand, causes immediate transfer of the right-of-way to the cross street.

The vehicle-interval relay serves to protect a moving line of cars against unwarranted interruption by cross traffic. The maximum-interval relay serves to protect waiting cross traffic against being unreasonably delayed by a "solid" line of moving cars.

As might be expected from the difference in their functions these two relays are energized through separate circuits. The vehicle interval relay is energized through a rather peculiar connection in order to permit a "solid" line of moving cars to protect its right-of-way by preventing this relay from operating. As each street is given the green light the vehicle detectors located in that street are connected by the dispatcher switching mechanism to the vehicle-interval relay in such a way that impulses from these detectors will reset this relay. Hence the vehicle-interval relay will not operate to yield the right-of-way to cross traffic so long as cars coming into the green light continue to reset it by operating the detectors at short intervals. A break in the line of moving cars will, however, give the vehicle-interval relay time to operate and put the system into a condition to respond immediately to "calls" from cross-street traffic. Whenever the intersection is free of moving traffic this relay causes the right-of-way to be subject to call by cars on the cross street. But, on the other hand, it will not operate and therefore will not release the right-of-way to cross traffic so long as cars continue to move through the intersection.

The maximum-interval relay is energized as soon as a car has indicated its presence on the cross street by operating the cross-street detector. This relay is not reset by moving cars. Therefore, if at the end of a predetermined period subsequent to the arrival of cross traffic, there has been no break in the moving traffic, the maximum period relay will operate to transfer the right-of-way to the waiting cross traffic.

It can be seen that the vehicle-interval relay and the maximum-interval relay working together cause the right-of-way to be yielded to cross-street traffic when the intersection is clear, or even when it is not clear if the cross traffic has waited for a predetermined period, but they never interrupt moving traffic unless the right-of-way has been called to the cross street by waiting traffic or a "pedestrian period."

TIME INTERVALS

The time intervals measured by this group of time-delayed relays are basic units in the electro-matic system and may be described as follows:

1. The *amber interval,* the period during which the amber light is shown.
2. The *vehicle interval,* the time required for a single vehicle in motion to pass through the intersection.
3. The *initial interval,* an additional interval inserted at the beginning of each green light period in order to give those cars which are not in motion a little extra time to get under way.
4. The *maximum interval,* the time during which the traffic in one street can hold the right-of-way against waiting cross traffic.

The vehicle interval is such an important unit in the electro-matic system that it merits more detailed consideration.

It is measured by the vehicle-interval relay and is the means by which moving cars can extend the right-of-way period between the minimum limit (set by the initial interval plus one vehicle interval) and the maximum limit fixed by the maximum interval. Each car which crosses the detector in the street which has the right-of-way cancels, by resetting the vehicle interval relay, the remainder of the vehicle interval allotted to the preceding car and starts a new vehicle interval to hold the green light while it passes through the intersection. The cancellation of the remainder of each vehicle interval by the succeeding vehicle is a very important factor in providing for the highest efficiency in the use of the intersection. In operation, this cancellation means that the cross traffic does not have to wait for a series of accumulative vehicle intervals to expire. Right-of-way will be transferred to the waiting cross traffic immediately after the last car of a moving line has passed through the intersection.

To demonstrate how these time intervals are used to produce a smooth flow of traffic assume four different traffic conditions at the intersection of A street and B street. These traffic conditions may be designated as cases 1, 2, 3, and 4, described as follows:

1. Cars moving on A street with no traffic on B street.
2. Cars moving on A street with one car waiting on B street.
3. Cars moving on A street with several cars waiting on B street.
4. Heavy traffic on both A street and B street.

In case 1, the cars on A street have the right-of-way and there is no waiting traffic on B street. The maximum interval is not being measured since no B street traffic has called for the right-of-way. The initial interval has expired. The vehicle interval is being intermittently renewed through the resetting of the vehicle interval relay by the cars crossing the detectors on A street. The vehicle interval relay is operating from time to time. However, since the right-of-way has not been called to B street, the operation of the vehicle interval relay does not act to transfer the right-of-way thereto, the green light continues to shine on A street and A street traffic flows without interruption.

In case 2 traffic is moving out of A street and a single car is waiting on B street. The A street initial interval has expired. Let us assume that the cars on A street are crossing the detectors at sufficiently short intervals to reset invariably the vehicle interval relay before it can operate. The vehicle interval relay therefore tends to hold the right of way on A street. However, the A street maximum interval is being measured because the car in B street, when it crossed the B street detector, energized the maximum interval relay. The transfer of the right-of-way to B street will be made in this case when the A street maximum interval has expired. If, on the other hand, we assume that the cars on A street are "straggling" or that the A street line of traffic completely passes the intersection before the maximum interval expires, the right-of-way will be yielded by the operation of the vehicle interval relay as soon as the A street traffic breaks. It can be seen that, with waiting cross traffic, the vehicle-interval and the maximum-interval relay are both energized, the right-of-way being yielded by the first to operate. Inasmuch as in this case there is only one car on B street, the green light will revert to A street by action of the vehicle-interval relay, after the initial interval and one vehicle interval have elapsed.

In case 3 the right-of-way will be transferred to B street exactly as in case 2. The instant that the B street green light is illuminated the timing of the initial interval begins. This will be succeeded by the first vehicle interval. However, in this case there are several cars waiting on B street. Before the end of the first B street vehicle interval this line of cars will be in motion and will be crossing the B street detector thereby always resetting the vehicle interval relay each time before it has time to operate. These cars will therefore hold the right-of-way on B street until they have had sufficient time to pass through the intersection, the time being assumed to be less than the maximum interval allowed for B street traffic against waiting A street traffic. As the last B street car passes the intersection the right-of-way will be transferred to A street by the operation of the vehicle interval relay.

In case 4 where traffic was assumed to be heavy on each street, the vehicle interval relay will be repeatedly reset by successive cars thereby holding the right-of-way on each street until the operation of the maximum relay yields it. It is surprising how seldom this condition occurs even when traffic seems to be very heavy. Interruptions constantly occur in the flow of traffic. A trolley or a bus must, in providing the public with adequate service, stop at any street corner when passengers require it. The electro-matic system permits cross traffic to take advantage of all such breaks in the moving traffic.

The electro-matic dispatcher has been described in some detail from a functional standpoint. A short description of its actual makeup may prove interesting. Fig. 3 is a front view of a dispatcher. The dispatcher was described earlier in this paper as a combination of a switching mechanism with a group of time-delayed relays. The switching mechanism consists of a shaft on which are fixed several cams and a separate group of contacts which are either opened or closed by the action of these cams. A cam shaft with some associated contacts is shown in Fig. 4. In completing one signal cycle this cam shaft makes one revolution. It is operated by a solenoid-actuated ratchet and pawl, Fig. 5, which turns it through an angle of 60 deg. at a time. The solenoid which drives the cam shaft is in turn controlled by the time-delayed relays. Hence, when the time-delayed relays say in effect that traffic conditions warrant it, the solenoid advances the cam shaft into its next position thereby also advancing the signal lights one step in their cycle. In addition, as stated before, the vehicle detectors and the time-delayed relays are by this act switched into the particular place in the circuit which they should occupy during this step of the traffic signal cycle. Presently the time-delayed relays will again act to advance the cam shaft into its next position thereby advancing the system one more step in its cycle. Six such steps constitute a complete cycle which is repeated as often as traffic conditions may require it. In following the operation of the cam shaft through one cycle assume a standard red, amber and green color cycle with amber displayed on both streets between the red and the green periods.

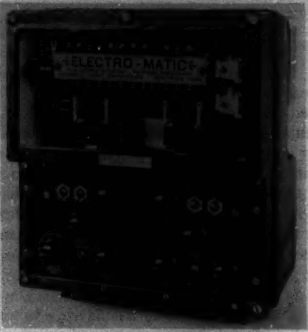

Fig. 3—The Front View of a Dispatcher

In each cam-shaft position one of the time intervals upon which the electro-matic system is based is being measured to determine when to move the shaft into its next position. Again referring to two intersecting streets as A and B streets, let us designate as position 1 the position which the cam shaft holds when A street has just been given the right-of-way. The shaft remains in position 1 during the A street initial interval. The time-delayed relay measuring this interval is energized continuously while the cam shaft is in position 1. To end this period the initial-interval relay operates the solenoid to move the cam shaft into position 2. No change in the signal lights accompanies the change from position 1 to 2. Position 2 might aptly be called a waiting position. The cam shaft does not move again until traffic requires the right-of-way to be shifted to B street. While the shaft is in position 2 the vehicle interval relay is energized and if there is waiting cross traffic the maximum interval relay is also energized. When the right-of-way is to be transferred to B street either the vehicle-interval relay or the maximum-interval relay will cause the solenoid to move the cam shaft into position 3. In this position the cam contacts close the circuit to the amber lights, energize the amber interval relay and open the circuits feeding all red or green lights. At the end of this period the amber interval relay operates to cause the solenoid to advance the cam shaft to position 4. Positions 4, 5, and 6, correspond respectively to positions 1, 2, and 3. Inasmuch as the latter three positions differ from the former only in that the right-of-way is being transferred to A street instead of to B street, there is no need to follow this operation through positions 4, 5 and 6.

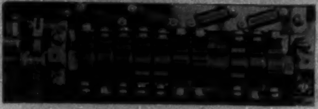

Fig. 4—A Cam Shaft with Some Associated Contacts

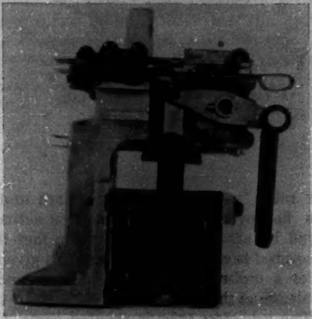

Fig. 5—The Cam Shaft Solenoid Ratchet and Pawl

THE TIMING UNITS

It is desirable that each of the four basic time intervals of the electro-matic system should be variable in magnitude as they are applied to traffic in different streets. A slight grade on one approach may cause cars entering the intersection from that street to be somewhat slower in starting than those on the cross street. To compensate properly the initial interval and perhaps the vehicle interval on the "slow" street should be somewhat longer than the corresponding intervals on the other. Or it may be desirable to allow a longer maximum period for one street than for the other. "Static timing" as used in the electro-matic system greatly facilitates such variations.

The static timing unit (the type of time-delayed relay now used in the electro-matic system) consists of a Strowger relay, a condenser, a resistance unit and a grid-glow flashover tube. The timing is actually accomplished by applying the well-known fact that the period required to charge a condenser to a given potential, under a uniform impressed e.m.f., depends upon the magnitude of the resistance in series with the condenser. The grid-glow tube is shunted across the condenser through the coil of the Strowger relay. When the condenser voltage reaches the critical voltage of the tube the tube "breaks down" and becomes a conductor. This permits the energy stored in the condenser to discharge through the relay coil which causes the relay to operate, thus signifying that the interval being measured has expired. As this timing unit is switched from place to place in the circuit, the charging resistance in series with the condenser is changed to meet the different timing requirements. Thus, for instance, the vehicle-interval timing unit, which serves each street in turn, may be set for a five second period on one street, whereas on another street it may operate after eight seconds. In a like manner the initial- and the maximum-interval relays can be used to time their particular intervals for one street after another, giving a different period for corresponding intervals on each street. The parts of these timing units are so located in the dispatcher that no photograph can include them all. Three condensers, one for each timing unit, with their associated Strowger relays may be seen in Fig. 6.

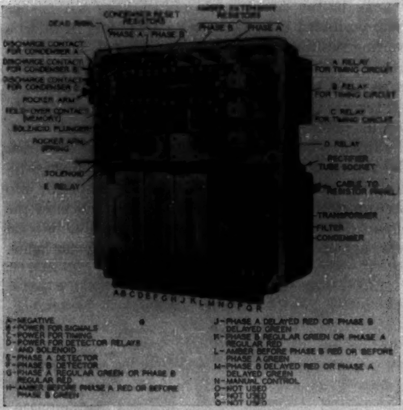
CONDENSER RESET RESISTORS
AMBER EXTENSION RESISTORS
DEAD RING
PHASE A
PHASE B
PHASE B
PHASE A
DISCHARGE CONTACT FOR CONDENSER A
DISCHARGE CONTACT FOR CONDENSER B
DISCHARGE CONTACT FOR CONDENSER C
ROCKER ARM
HOLD-OVER CONTACT (MEMORY)
SOLENOID PLUNGER
ROCKER ARM SPRING
SOLENOID
E RELAY
A RELAY FOR TIMING CIRCUIT
B RELAY FOR TIMING CIRCUIT
C RELAY FOR TIMING CIRCUIT
D RELAY
RECTIFIER TUBE SOCKET
CABLE TO RESISTOR PANEL
TRANSFORMER
FILTER
CONDENSER
A B C D E F G H J K L M N O P Q R
A-NEGATIVE
B-POWER FOR SIGNALS
C-POWER FOR TIMING
D-POWER FOR DETECTOR RELAYS AND SOLENOID
E-PHASE A DETECTOR
F-PHASE B DETECTOR
G-PHASE A REGULAR GREEN OR PHASE B REGULAR RED
H-AMBER BEFORE PHASE A RED OR BEFORE PHASE B GREEN
J-PHASE A DELAYED RED OR PHASE B DELAYED GREEN
K-PHASE B REGULAR GREEN OR PHASE A REGULAR RED
L-AMBER BEFORE PHASE B RED OR BEFORE PHASE A GREEN
M-PHASE B DELAYED RED OR PHASE A DELAYED GREEN
N-MANUAL CONTROL
O-NOT USED
P-NOT USED
Q-NOT USED

FIG. 6

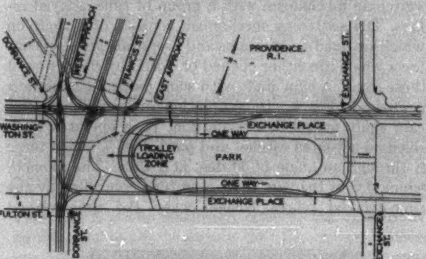
DORRANCE ST.
WEST APPROACH
FRANCIS ST.
EAST APPROACH
PROVIDENCE, R.I.
EXCHANGE ST.
WASHINGTON ST.
EXCHANGE PLACE
ONE WAY
TROLLEY LOADING ZONE
PARK
ONE WAY
EXCHANGE PLACE
FULTON ST.
DORRANCE ST.
EXCHANGE ST.

FIG. 7

The important function of resetting the vehicle-interval relay is accomplished by discharging the condenser in this timing unit by action of a vehicle-detector relay.

Each timing interval, the initial, the vehicle, the maximum and the amber interval is independently adjustable for each street. This adjustment is made by turning a knob on a panel until the arrow thereon points to the number of seconds desired for the particular interval controlled by that knob. As a knob is turned it introduces resistances of different magnitudes into the condenser charging circuit which it controls thereby changing the time required to charge the condenser to the critical voltage of the grid-glow tube.

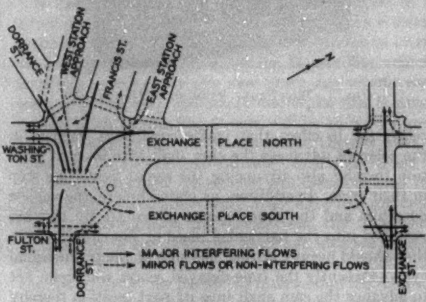
DORRANCE ST.
WEST STATION APPROACH
FRANCIS ST.
EAST STATION APPROACH
N
WASHINGTON ST.
EXCHANGE PLACE NORTH
EXCHANGE PLACE SOUTH
FULTON ST.
DORRANCE ST.
MAJOR INTERFERING FLOWS
MINOR FLOWS OR NON-INTERFERING FLOWS
EXCHANGE ST.

FIG. 8—PRESENT TRAFFIC FLOWS EXCHANGE PLACE

This paper describes a standard two-phase system which provides complete vehicle actuation for signals governing two interfering lines of traffic. By using a somewhat more complicated switching mechanism in conjunction with the three basic timing units the system has been adapted to control traffic at intersections where as many as six or seven interfering flows cross. Furthermore, adjacent intersections may be interconnected so as to operate with a certain degree of coordination without unduly interfering with the free vehicle actuation of each. Exchange Place in the center of Providence, Rhode Island, provides a good example of this type of installation. The location of vehicle detectors and signal lights is shown in Fig. 7. The various traffic flows are shown in Fig. 8, a study of which shows that there are several one way streets.

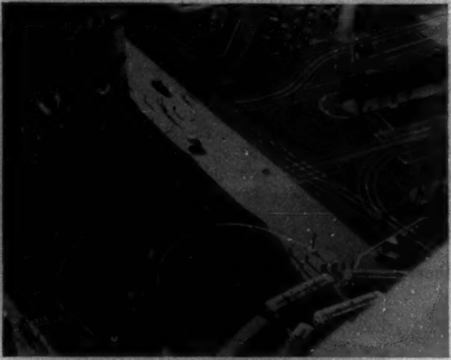

Fig. 9—Exchange Place Showing Trolley Loading Station

The traffic in each corner of Exchange Place is controlled by a separate independent vehicle-actuated dispatcher which gives the right-of-way in general to north-south traffic and to east-west traffic independently in order to eliminate crossing flows. The right turn traffic out of Washington Street, that from the southern end of Exchange Place into Fulton Street, and that out of Dorrance Street South, as well as the left turn traffic around the central section, moves at all times except during pedestrian protection periods. Pedestrians are a major factor in Exchange Place traffic. A trolley loading station located in the southern end of the Place, shown in Fig. 9, makes it necessary for thousands of people to cross the streets clustered about this point. To give these people a maximum of protection with a minimum of inconvenience to the motorists a "progressive" pedestrian period is provided. This period is inaugurated by showing the red light on all streets at the southwest corner, thereby stopping all traffic inbound to the southern end of Exchange Place. At the same instant the dispatcher controlling the traffic at the southeast corner begins to limit signal changes there in order to be ready for its pedestrian period which will commence 15 seconds later. This 15 seconds between the inception of the pedestrian periods at the two corners permits cars which were already in the southern end of Exchange Place when the pedestrian period was started at the southwest corner to pass out of this district without being stopped at the southeast corner. Hence the whole southern end of Exchange Place is cleared of traffic to give the pedestrians unhampered use of this district. At the termination of the pedestrian period traffic is first allowed to move at the southwest corner. A few seconds later, just as the first cars moving through the southern end reach the southeast corner, the pedestrian period is ended there and traffic again flows smoothly.

A Recent Development in Traffic Control

BY H. W. VICKERY* and V. W. LEONARD*
Non-member Non-member

***Synopsis.**—The large number of automobiles in use today has introduced new problems in the control of vehicular traffic. Timing devices with increased flexibility are required so that signals can be operated more nearly in accordance with varying conditions of traffic flow. A recently developed timing device, containing new features and having increased flexibility, offers a solution of the problems involved. The timer is described and its advantageous features are discussed.*

INTRODUCTION

CONGESTED traffic conditions of the present day indicate the necessity for greater flexibility in timing apparatus so that signals can be set in step with traffic and adjusted to accommodate variations in traffic flow. Rapidly changing traffic conditions emphasize the need for timing devices capable of adjustment to meet future requirements. As a solution of these problems, a timing device, with new characteristics and extremely wide ranges of adjustment, has recently been developed.

TYPES OF TRAFFIC CONTROL SYSTEMS IN USE

Traffic control systems now in use are of the following general types: first, the isolated system for controlling traffic at a single intersection; second, the non-interconnected progressive system in which timing devices at various intersections, though not interconnected in any way, can be adjusted to operate the signals in a progressive relation; and third, the interconnected progressive system in which not only a progressive relation of the signals is maintained but, in addition, certain features are obtained by remote control, such as change of total time cycle, selection of various progressive relationships, shut down, flashing amber and fire control.

A fourth type of interconnected system is that in which timing apparatus at the central point alone controls signals at a number of intersections. One form, in which a single timer controls the system, is extremely limited as far as flexibility is concerned. In its other form, where a timer for each intersection is located at the central point, the installation cost is so high as almost always to be prohibitive.

The discussion, therefore, is confined to the first three systems mentioned, in which individual timing devices are located at each intersection.

ADVANTAGES OF SYNCHRONOUS MOTOR DRIVE

Timing devices in non-interconnected progressive systems must, of necessity, be driven by synchronous motors in order that the progressive relation between intersections be maintained.

The advantages of synchronous motor drive in isolated and in interconnected progressive systems are brought out by consideration of a practical form of timing device. Such a device is shown in Fig. 1.

The synchronous motor (Fig. 2) operates on the hysteresis principle. Thus, unusual torque is developed with the expenditure of a small amount of power. The rotor is unconventional in the fact that it simply consists of hardened steel laminations spun into a drawn brass shell. All windings are stationary, surrounded and protected by the rotor. This construction produces a sturdy motor unit, insusceptible to damage in operation or handling as opposed to the inherently frail disk type of motor commonly used.

Any setting which may be made on a synchronous timer, whether it be total time cycle, proportion of periods in the color sequence, or progression setting is exact. All dials are marked in exact values due to the fact that the speed of a synchronous motor is independent of voltage, temperature and even fairly wide friction changes.

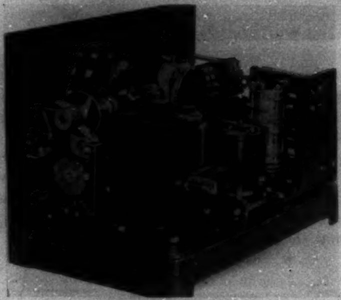

FIG. 1—REAR VIEW OF SYNCHRONOUS TIMER WITH COVER REMOVED

Any setting which may be made on an induction timer, on the other hand, is inexact. The total cycle dial must be calibrated, that is, various total cycles must be measured by means of a stop watch or some other standard and the dial marked to correspond. Changes in temperature, voltage or friction render such dial markings inaccurate.

Induction timers in interconnected systems require a compensating period, which is introduced once each

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cycle, to correct for the inaccuracies in speed of the various timer motors. The master controller is set to operate at a time cycle slightly longer than that of the intersection timers. The intersection timers complete one total cycle at which time they stop and wait for the master controller to overtake and release them.

Synchronous timers in interconnected systems, on the other hand, due to their time keeping accuracy, require no waiting or compensating period. A short resynchronizing interval is provided once each cycle, simply as a supervisory check, so that any timer which may be out of step, due to servicing, tampering or other reasons, will immediately be placed back in step. The master controller is merely a supervisory device, whose function is not to keep the timers in step, but simply to place them in step.

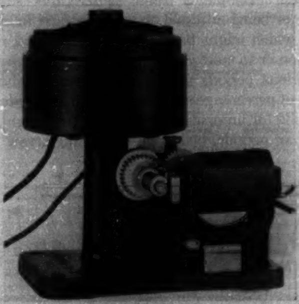

FIG. 2—SYNCHRONOUS MOTOR UNIT

While it is true that interconnected induction timer systems, in some cases, continue to operate even though central power fails, they usually speed up in case of such failure and, due to their non-synchronous principle, are no longer able to maintain progressive relationship. The interconnected synchronous timer system, on the other hand, not only continues to operate when central power fails but, due to its synchronous principle, maintains the same total cycle, the same progressive relationship and the same percentage values as before.

Another advantage of synchronous motor drive is the fact that flashing signal lights can be made to operate at a given, constant and exact speed, regardless of any change in total cycle which may be made. Such operation is not possible with an induction timer because in such case, the rate of flashing changes in proportion to the total cycle.

CHANGE OF TOTAL TIME CYCLE

The gear shift unit used for changing the total time cycle is shown in Fig. 3. It consists of a cone of gears of varying diameters and an idler gear which can be shifted to engage any one of the cone gears. The gears can be shifted manually at each timer or, in the case of interconnected systems, from a remote point. The total cycle range is flexible, usually consisting of ten steps with five seconds between steps.

While it is true that an induction timer, in which the total time cycle is usually changed by varying the motor speed, can apparently be set in steps much smaller than five seconds, it is probable that, due to voltage, temperature and friction changes, the actual total time cycle varies as much if not more than five seconds from the time cycle indicated on the dial.

SELECTION OF PROGRESSIVE RELATIONSHIPS

In order that traffic may move along a street at a given uniform speed and with a minimum number of stops, it is necessary that the signals be set in step so that traffic arrives at each intersection during the green or "go" indication. Setting the signals in step gives a progressive movement to traffic, the traffic moving in groups or platoons along the street.

To give ideal progressive movement to traffic in a given direction, it is necessary that the beginnings of the green indications of the signals occur successively one after the other in the given direction, the time elapsing between the beginning of the green indication of any one signal and the beginning of the green indication of the next adjacent signal in the given direction being equal to the time required to travel the distance between the two signals at the given speed.

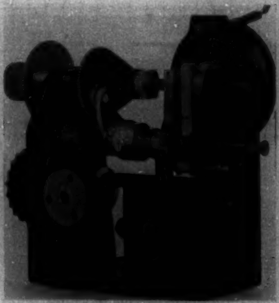

FIG. 3—GEAR SHIFT UNIT

Unfortunately, due to unequal distances between signals and due to the particular speed at which it is desired to have traffic move, setting the signals in step to give ideal progression in one direction usually results in very poor progression in the opposite direction (Figs. 4 and 5). The beginnings of the green indications of the signal, almost invariably, do not occur in a sequence in the opposite direction to allow as large a number of automobiles to travel without stopping, as is possible in the direction of ideal progression.

The usual procedure, where only one progressive relation of the signals is available, is to set the signals in step so as to give average progression in both directions (Fig. 6). With average progression, however, the number of automobiles which can travel in either direction without stopping is smaller than would be possible in the direction of ideal progression as shown in Figs. 4 and 5. Furthermore, continual stopping and starting of some of the traffic, due to the beginnings of the green indications of the signals being out of step, creates confusion and delay, thereby diminishing the number of automobiles which otherwise would be able to proceed without stopping.

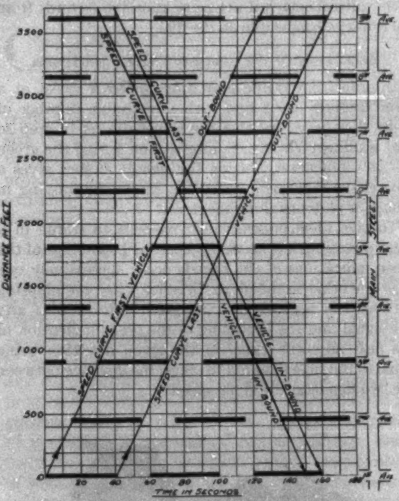

Fig. 4—Schematic Timing Diagram Showing Ideal Progressive Relation Favoring Out-Bound Traffic

Traffic speed—20 miles per hour. Total time cycle—60 seconds
Main street "green"—40 seconds indicated by heavy lines

Since, in large urban districts, traffic on a given street may be mostly inbound in the morning, mostly outbound in the later afternoon, and about equally divided in both directions at other hours of the day and since the volume of traffic in such districts is usually much greater during the morning and late afternoon than at other hours of the day, it is obvious that a single progressive relation of the signals does not meet these three fundamental traffic conditions in the best manner possible. A means, therefore, has been developed in the timer to give three separate and distinct progressive relations of the signals. Thereby, the progressive relation can be changed remotely to give ideal progression to inbound traffic in the morning, ideal progression to outbound traffic in the late afternoon and average progression in both directions at other hours of the day.

Adjustment Flexibilities

Since traffic conditions change from month to month and from year to year, sufficient flexibility should be built into traffic signal timing apparatus to insure that such apparatus shall not shortly become obsolete. In other words, a traffic signal timer should have sufficient flexibility to allow of adjustment to meet changing and, as far as possible, new requirements which may develop. For example, a traffic signal timer should be capable of being adjusted to meet changes in color sequence which might be authorized by federal or state legislation or to meet certain special local requirements. The various periods of the color sequence should be capable of percentage adjustment over very wide ranges to provide for unforseen conditions which might occur. Pedestrian periods, bell or green turning arrow indications which might later be required should be easily obtained by minor changes in the timer as opposed to intricate relay combinations used at present to secure such features.

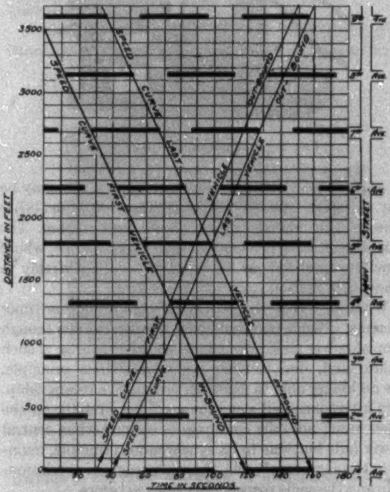

Fig. 5—Schematic Timing Diagram Showing Ideal Progressive Relation Favoring In-Bound Traffic

Traffic speed—20 miles per hour. Total time cycle—60 seconds
Main street "green"—40 seconds indicated by heavy lines

A solution of these problems is represented by a drum unit (Fig. 7), a rotating switch attached to the gear shift unit in Fig. 3 and a cycle percentage dial having 100 slots and containing a predetermined arrangement of easily removable keys as shown on the timer panel in Fig. 8. The drum contains a predetermined arrangement of cams for opening and closing the signal light contacts and is rotated in steps by means of a solenoid-operated ratchet. As the finger of the rotating switch strikes a key in the cycle percentage dial, contact is momentarily made and an impulse applied to the operating solenoid of the drum, thus causing the drum to advance to a new position or period in the color sequence.

Extreme flexibility is obtained by such construction inasmuch as any of the modern color sequences can be obtained merely by setting up the proper arrangement of cams on the drum. Also, any desired percentage adjustments of the periods in the color sequence can be obtained simply by proper arrangement of the keys in the cycle percentage dial.

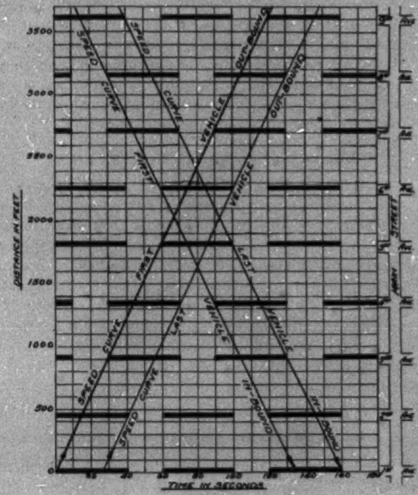

Fig. 6—Schematic Timing Diagram Showing Progressive Relation for Average Traffic Flow

Traffic speed—20 miles per hour. Total time cycle—60 seconds
Main street "green"—40 seconds indicated by heavy lines

The drum unit is mechanically independent of the cycle percentage dial as opposed to conventional construction in which fanned cams are used, not only to determine the length of each color period, but, by means of drops in these cams, to change the lights from one condition to another. Such fanned cam construction is usually limited as far as changes in color sequence or percentage adjustment of color periods is concerned.

A unique feature of the cycle percentage dial is the fact that there are no moving parts on the front of the timer and yet direct visual indication is given at all times, not only of the relative length of each color period, but also of the position of each period in the color sequence. This is not true of fanned cam construction since, to obtain the length of the cross-street green period, it is necessary to subtract the sum of all of the other periods from 100 per cent. Furthermore, with such construction, no indication whatever is given of the positions of the periods in the color sequence. Also, in some cases, the existence of certain auxiliary periods, such as warning periods, pedestrian periods, or bell periods, is not indicated at all on the front of the timer, inasmuch as such periods are usually obtained by the internal adjustment of slotted cam sectors inside the timer.

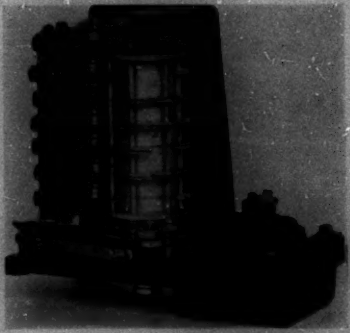

Fig. 7—Drum Unit

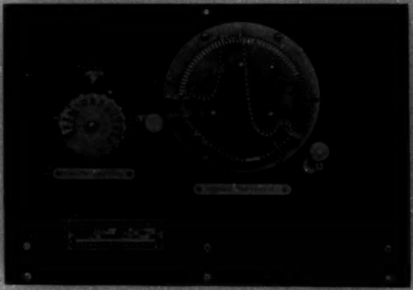

Fig. 8—Front View of Timer Panel

The operation of the drum and rotating switch and the visual picture of the color sequence as presented by the cycle percentage dial are exceedingly simple and easy to understand as opposed to the somewhat complex and inflexible fanned cam construction.

MANUAL CONTROL

Manual control of the signal lights is made possible by means of a push-button switch attached to a flexible cord. Successive, momentary depressions of the push-button switch impart electrical impulses to the drum solenoid and cause the drum to advance through the various periods in the color sequence. An important point is the fact that the traffic officer is thus forced to operate the lights in exactly the same sequence as is obtained with automatic operation. He cannot skip certain change periods as officers are sometimes prone to do with manual handles of the conventional type.

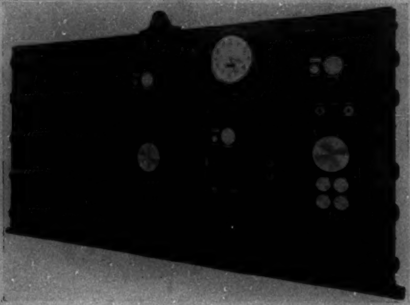

FIG. 9—TRAFFIC CONTROL SUPERVISORY BOARD IN LARGE EASTERN CITY

INHERENT DESIGN FEATURES

Auxiliary features, such as remote shut down, flashing amber and fire control, are usually obtained in interconnected systems by means of relays in each control box, one additional interconnecting cable or wire being required for each of these three features. Elimination of such relays and additional interconnecting cables is made possible by an "off, flash, fire" device in each synchronous timer. This device, while being essentially a multi-contact relay, is operated over the same wires used for remote shifting of the gears, thus resulting in a very important saving in the number of interconnecting cables required.

The timer is of universal application, that is, timers of the same identical construction can be used at isolated intersections, in non-interconnected progressive systems and interconnected progressive systems. This is of extreme importance where it is desired to operate timers in a non-interconnected progressive system initially, interconnection to be made at a later date as warranted by traffic conditions, since the necessity of replacing present timing apparatus with new and radically different construction is avoided.

The master controller, due to the fact that its supervisory feature is obtained by means of a simple pair of contacts, can, with the exception of these contacts, be of the same identical construction as an intersection timer, if so desired. Thus it can be made to serve as an intersection timer to control a set of signal lights in addition to performing its supervisory function.

INSTALLATION IN LARGE EASTERN CITY

A complete traffic signal installation, having all the flexibility described, has recently been installed in a large eastern city on one of the most congested traffic arteries in the world. The system is interconnected and automatically controlled from a supervisory board (Fig. 9). Ideal progression is given to the inbound traffic peak in the morning. Outbound traffic is similarly accommodated in the late afternoon, while average progression is maintained at other hours of the day. Program clocks automatically change the timing in accordance with the desired daily schedule for each day of the week.

CONCLUSION

It is felt that the synchronous timing device described is a real contribution in the art of efficient, flexible traffic control. Being built upon principles which are fundamentally sound and correct, it should take care of traffic requirements for years to come.

The Traffic Flow Regulator

BY C. H. BISSELL* and J. G. HUMMEL*
Member, A.I.E.E. Non-member

THE annual cost of automobile accidents in the United States is more than the annual cost of public school education, five times the country's average yearly fire loss, and more than half the amount required to maintain all the agencies of the federal government. In an attempt to promote safety, many cities have installed a large number of traffic signals at isolated corners. While the installation of these signals has perhaps promoted safety it has at the same time severely penalized traffic by needless delays and congestion during the major portion of the day, thus contributing to the two billion dollar loss previously mentioned. This discussion is confined primarily to the reduction of this loss through the installation of an efficient and properly timed control.

During certain hours of light traffic a "go" period of 15 seconds on each street may be found very satisfactory, while during rush hours, especially on Saturdays and Sundays, a "go" period of 60 seconds on each street may be demanded. With the equipment available to date, the only solution is a compromise, with the result that generally a timing of 30 seconds on each street is used. This compromise is maintained all day long, and every day of the week. During the hours when a 15-second "go" is best, the 30-second compromise "go" is 100 per cent too long, and during the times when a 60-second "go" is best, the 30-second compromise "go" is 100 per cent too short. This practise quite naturally leads to the severe criticism of causing needless delays and congestion, and the continued operation of signals in this manner may break down the obedience of motorists to traffic signals in general.

There has always been a great deal of discussion about the extreme variation in traffic. One group of adherents has contended that there was absolutely no regularity in traffic flow and has cited several freak instances of momentary variation to bear out its contention in an attempt to justify the expenditure of large amounts of money. Another group took traffic surveys and counts and found that these freak conditions occurred at not more than one intersection in twenty, or not over 5 per cent. These surveys also showed the existence of very definite peaks and valleys in the volume of traffic.

For the past six years an independent and impartial traffic survey has been carried on by the University of Wisconsin in the research of the Barney Link Fellowship and the result has just been published in its 1931 report. This report contains the result of six years of traffic research and study including the counting of millions of cars in a large number of different cities of various sizes, studied during all kinds of weather. The report shows that the occurrence of traffic peaks and valleys at 95 per cent of all intersections is so regular that definite formulas for determining the vehicular traffic have been derived for various size cities. In fact, characteristic curves for the traffic flow can be obtained by taking only five traffic counts of 12 or 18 hours each. The characteristic curve is normally typical of the traffic on all boulevards and primary streets for the city in which the data are taken.

It is a well-known fact that traffic counts and surveys have demonstrated that there are very definite traffic peaks and valleys for every week-day as well as Saturdays and Sundays. Experience has shown that these variations in traffic occur with definite regularity each day of the week. By this we mean that the light and heavy movements of traffic at each intersection occur with definite regularity at that intersection. While the traffic may differ very materially on a week-day from that on Saturday and Sunday, the average flow on every normal Monday, for instance, will be the same. Likewise the peaks and valleys of the traffic flow will occur with definite regularity on each and every day of the week, perhaps different for week-days and week-ends but with a very definite repetition. It has not been found practical or economical to send out officers to change the timing on every isolated traffic timer in the city or county to take care of each changing condition.

The only attempt to take care of this variation to date has been an attempt to control the timing simultaneously in accordance with the number of vehicles on each street. This requires some sort of electrical eye or ear or a mechanical finger in the street or highway, generally requiring the breaking up of the highway and the installation of considerable underground ducts, cables, etc., with the attendant expense of installation and equipment varying from approximately one to five thousand dollars per intersection depending upon the complexity of the installation. The result has been that few cities are able, or have felt justified, in going to this expense to take care of momentary or simultaneous variations. The most recent reaction has been that certain cities which arranged to install vehicle-actuated equipment on a rental basis feel that the expense is not justified and are looking for a more economical manner of controlling traffic efficiently.

During the past few years a group of traffic engineers has been studying this problem in an attempt to solve it in an economical manner. This study lead to the traffic flow regulator shown in Fig. 1. This device makes it possible to vary automatically the timing to

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take care of these regular traffic peaks and valleys that observation or a traffic survey and count show so distinctly. Essentially this device consists of the following principal units each of which is labeled in Fig. 1:

1. An automatic timing switch.
2. One or more time allowance units.
3. An electrically-wound time switch.
4. A seven-day program unit.

The function of the automatic timing switch is the same as with any automatic traffic signal, that is, to change the signal indications at predetermined intervals and in a desired sequence.

The function of the time allowance unit is to prolong the duration of the desired or preset signal indications or to introduce indications different than those normally used.

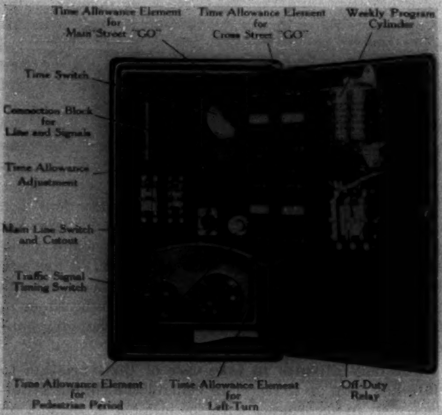

FIG. 1

The function of the time switch is to cause the desired alterations in signal indications to occur at a desired time and to continue as long as predetermined.

The function of the seven-day program unit is to permit the setting up of a program for an entire week. This program, therefore, may be different for different days of the week.

When traffic is light on both streets, the Traffic Flow Regulator operates on a short total period thus passing vehicles and pedestrians safely and rapidly. As the traffic increases, the timing is automatically extended to take care of this increase. When the rush is over, the timing is automatically shortened.

Briefly, there are fifteen different cases, or symptoms, a traffic engineer (or doctor) may be called upon to prescribe for. The nine average conditions which are encountered at practically every intersection are:

1. *The Period of Operation* by which we mean the time at which the signal is turned "on" and "off" or on a steady or flashing indication for off-duty.
2. *The Early Morning Light Traffic* which may extend up to 7:30 a. m.
3. *The Morning Rush Period* generally occurring from 7:30 to 9:00 a. m.
4. *The Morning Off-Peak Period* extending from 9:00 to 12:00 noon.
5. *The Noon-Hour Rush* generally occurring from 12:00 to 1:30 p. m.
6. *The Afternoon Off-Peak Period* occurring from 1:30 to 4:30 p. m.
7. *The Afternoon Rush* lasting from 4:30 to 6:00 p. m.
8. *The Dinner Off-Peak Period* usually evident from 6:00 to 8:00 p. m.
9. *The Evening Pleasure Driving Period* extending from 8:00 to midnight.

In the above we have simply assumed certain hours, however, it should be pointed out that there is no limitation whatsoever. Any time can be selected for the various periods; likewise, the number of periods can be varied.

In addition to the above regular conditions that are met at practically every intersection there are certain special conditions that depend more upon the location of the intersection. For instance, a timer located adjacent to a school should be capable of allotting protection to the school children while passing to and from school and to regulate the traffic expeditiously and safely at all other times. A timer located adjacent to a church should be capable of giving protection to the church-goers on Sunday and at all other times control traffic in a manner dictated by the traffic conditions.

At certain intersections a great deal of out-of-town traffic must be cared for on Saturdays, Sundays and holidays.

At an intersection where a particularly bothersome left-turn is encountered it should be possible to insert a period to take care of this left-turn whenever, and as often as desired. Likewise, the control should be capable of inserting a period for pedestrians at a corner where the pedestrian travel is severe.

At intersections where there are more than four streets or entrances, it should be possible to control the traffic in two movements during hours of light traffic and during rush hours the timer should automatically change to three or four movements or phases of control.

All of the above conditions are taken care of by the Traffic Flow Regulator. Its flexibility is almost limitless and it is possible to furnish a Traffic Flow Regulator to take care of any traffic condition.

Double Conductors for Transmission Lines

BY H. B. DWIGHT* and E. B. FARMER*
Fellow, A.I.E.E. Non-member

Synopsis.—*When two overhead conductors a few inches apart are used for each phase of a transmission line circuit, instead of one larger conductor, the reactance is reduced 20 per cent or more. Where voltage drop determines the maximum power load, as it does in many cases, reduced reactance is advantageous and increases the allowable power rating. Current-carrying capacity and corona voltage are increased. The advantages are to be balanced against the extra cost due to mechanical features, including hangers, increased cost of stringing and additional wind and ice load. Since increase in ice load is possibly the greatest disadvantage, the use of double conductors is of most interest for southern districts, where ice load is not encountered.*

BY the term double conductors employed in this paper, is meant the use of two overhead conductors instead of one, for each phase. The construction is the same as in usual transmission lines except that a second conductor is hung a few inches below the first by metallic hangers, similar to those used in electric railway catenaries. The two conductors are therefore electrically in parallel and form one effective conductor of large cross-section, without increasing the number of insulators or cross-arms.

The advantages and disadvantages of this type of construction and the classes of lines on which it might be advantageous, will be described. Formulas will be given for the calculation of the electrical characteristics, which will enable comparisons to be more easily made.

REACTANCE

An overhead double-conductor line has approximately 20 per cent less reactance than a single-conductor line of the same weight of conducting metal. In many usual cases, especially where there is not complete control of the voltage by synchronous condensers, the reactance is the most important item in determining the power rating of the line, for both the voltage drop and the stability limit of the load depend principally on the reactance. Therefore, in many instances, without increasing the weight of conductor metal, a line can be built for about one-fifth greater power rating at very little increase in cost where ice load is absent, by using double-conductor construction. This is particularly advantageous where the cost of the right-of-way is large, and it is desirable to transmit as much power per circuit as possible.

The reduction in reactance, amounting to approximately 20 per cent on practical lines, obtained by using double-conductor construction, is graphically shown in Fig. 1. The formula for the reactance of a double-conductor line, suitable for usual overhead spacings, is quite simple and is given in Appendix I, equation (1). Fig. 1 is drawn for copper conductors. It may be noted that hollow conductors and aluminum conductors of equivalent resistance have from 5 to 7 per cent less reactance than the values shown for single conductors in Fig. 1. The two small conductors of the double-conductor line may be of aluminum if desired. A formula and curve for calculating reactance of hollow conductors are given in Appendix II.

The effect of changing the distance between the two wires of the same phase, in a double-conductor line, is shown in Fig. 2. It is seen that for larger spacings than 6 inches there is little further improvement. Accordingly the discussions in this paper for the most part are based on a value of 6 inches. Curves for other conductor sizes and for other spacings between phases are very much the same as the curves of Fig. 2.

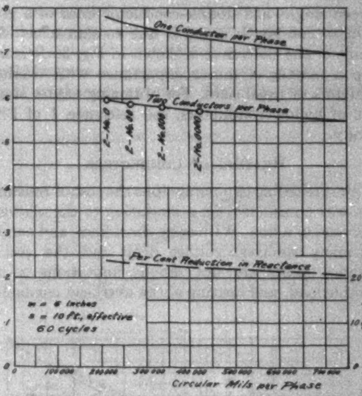

FIG. 1—COMPARISON OF REACTANCE OF SINGLE- AND DOUBLE-CONDUCTOR LINES

RESISTANCE

Another limit to the power rating of a transmission line is the cost of resistance losses. The greatest advantage of double-conductor construction in this connection is in the case of a line already built, which has higher resistance than desired and which has some mechanical margin of safety. It is possible to add a second conductor and cut the resistance in half without the necessity of taking down or scrapping the old conductor. In other words it is possible to make the reduction in resistance with only one-half as much new conductor metal as would be needed if single large-size conductors were installed. It is of course necessary that the line be strong enough to stand the extra weight including wind and ice on the second conductor. The proposition to add to the conductor metal of existing lines may be attractive at a time when the cost of conductors per pound is low.

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*Massachusetts Institute of Technology.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

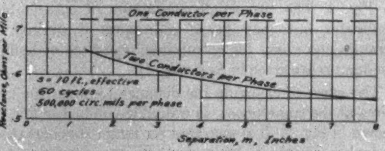

FIG. 2—CHANGE OF REACTANCE WITH SEPARATION OF PARALLELED CONDUCTORS

For new transmission lines the difference in resistance between single- and double-conductor lines of a given weight of conductor metal is due to skin effect. The amount of the difference is shown in Fig. 3. It is seen to be a matter of minor importance with overhead conductors of usual sizes. For obtaining values of skin effect of hollow conductors, see reference 1 of Bibliography.

HEATING OF CONDUCTORS

In the preceding two paragraphs the cost of resistance losses was considered, as a limitation to the allowable power load of a line. With some short lines, and notably with many tie-lines, voltage drop and the cost of resistance losses are not serious limitations to the allowable load, and heating of the overhead conductors is the practical limit encountered. In many cases, twice the normal load must be carried by one circuit as an emergency condition. It is stated in reference 4 that hard-drawn copper starts to anneal at 100 deg. cent. and that there is a reduction of 12 per cent in tensile strength due to a temperature of 130 deg. cent. for 72 hours, while a temperature of 150 deg. cent. for the same time results in a reduction of tensile strength of 30 per cent. The possibility of oxidation of joints in connectors is another reason for keeping the conductor temperature down. The current-carrying capacity of overhead conductors listed in reference 5 is based on 40 deg. cent. rise above 40 deg. cent. ambient temperature.

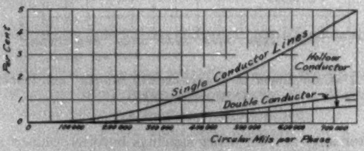

FIG. 3—INCREASE IN RESISTANCE DUE TO SKIN EFFECT AT 60 CYCLES

Tie lines have been built with very heavy overhead conductors for the purpose of avoiding overheating of the conductors. A 132-kv., 30-mile tie line from Chicago to Waukegan has been built with 750,000-cir. mil hollow copper conductors. It is stated in reference 6 that the main condition in designing the line was the heavy current to be carried.

The curves of Fig. 4 have been drawn from the table of current ratings for bare conductors, outdoor service, published in reference 3, page 466, using the ratings for 40 deg. cent. rise. It is seen that double-conductor construction increases the current-carrying capacity nearly 30 per cent, for a given weight of conductor metal. This is because the two small conductors have more surface than the single large conductor. If current ratings from the table for indoor service were used, the percentage difference between single- and double-conductor construction would be nearly the same. Hollow copper conductors, because of their large diameter, show an increased current-carrying capacity, over corresponding standard cables, though the increase is usually a small percentage. Aluminum conductors also have larger diameter and therefore greater current-carrying capacity than standard copper cables of the same resistance.

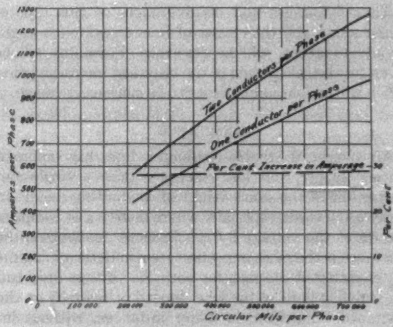

FIG. 4—CURRENT-CARRYING CAPACITY

CAPACITANCE

As is shown by Fig. 5, the capacitance of an overhead double-conductor line is 20 per cent or more greater than that of a single-conductor line of the same weight of conductor metal. This is an advantage in the case of power networks in well-settled parts of the country, where the loads and generating stations are scattered to some extent throughout the network. In such networks it is a matter of experience that synchronous condensers are used almost entirely with strong field currents, and very little with lagging power factor load and weak field currents. Consequently, an increase in line capacitance of 20 per cent means a saving of a definite amount of synchronous condensers which would otherwise need to be installed.

In other cases, where transmission lines are very long and the loads are widely separated from the generators, special attention is required to handle the line capacitance current. A sufficient number of generators must be kept always connected to the system, so that the no-load current is not an overload for them and so that the field current at rated voltage and leading power factor load does not become so small as to endanger the stability of the system. While these matters require attention, yet they seldom involve expense, particularly in the case of water-power systems where they are most likely to be noticeable, for there are practically always plenty of generators available. Accordingly even in such cases, an increase in line capacitance by using double-conductor construction, is not an economic disadvantage.

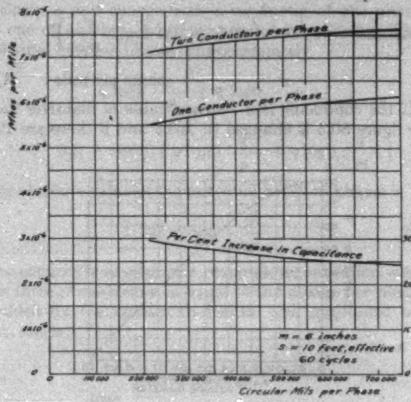

FIG. 5—COMPARISON OF CAPACITIVE SUSCEPTANCE OF SINGLE- AND DOUBLE-CONDUCTOR LINES

CORONA

Values of corona voltage, that is, the voltage at which corona starts, are shown in Fig. 6 for one and two conductors per phase, where *m*, the separation of the two conductors of the same phase, is 6 inches. Approximate formulas for calculating these results are given in Appendix IV. Very little increase in corona voltage is obtained by using two small conductors of the same type as the single large one. This is in general true at other separations than 6 inches. More increase in corona voltage is obtained by using 3 or more conductors per phase. The use of a hollow or cored conductor gives a considerable increase in corona voltage.

Measurements of the corona voltage with two and three transmission line conductors per phase are given by F. W. Peek, Jr., in "Dielectric Phenomena in High Voltage Engineering" (reference 8), page 82, and also in the first edition of the same book, pages 71 and 72, published in 1915.

If a double-conductor line and a single-conductor line have the same calculated corona voltage, the operating voltage of the double-conductor line should be kept lower than that of the other, because when corona does start, as in stormy weather, the double-conductor line has twice as many conductors on which loss takes place.

Reference should be made to the paper by P. H. Thomas, TRANS. A.I.E.E., 1909, (reference 7), in which a number of features relating to the use of two and three conductors per phase was discussed. A rather large spacing between the paralleled conductors, namely 18 inches, was assumed.

A transmission line with two conductors per phase could be constructed using a type of hanger already developed and tried out for electric railway catenaries. It would be more expensive than a single-conductor line of equal mechanical strength. The duty on the towers, and their height, would be greater due to approximately twice as much wind and ice load. More clearance would be required and shorter spans might be needed. More margin of safety would be required in the tensile strength of the conductors, for although each might be pulled up to the proper tension, it would not be certain that one conductor would not take more than its share of the tensile load at some later time. The cost of stringing the conductors would be greater. Since the increase in ice load is perhaps the greatest disadvantage, the use of double conductors is of most interest for southern districts, where ice load is not encountered.

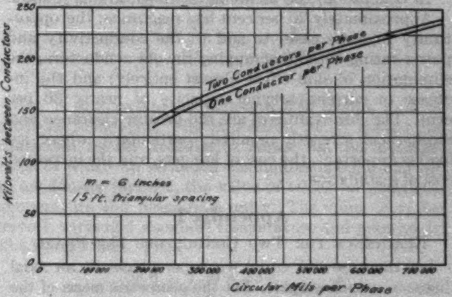

FIG. 6—CORONA VOLTAGE AT SEA LEVEL

The additional cost due to the mechanical features of the double-conductor construction is to be balanced against the value of the advantages to be obtained. If circumstances are such that the voltage drop in the line must not be more than a certain percentage, then 20 per cent reduction in reactance permits a corresponding increase in the allowable power load. The additional cost, is, in such a case, to be compared with 20 or 25 per cent of the entire cost of the transmission line. Such a case assumes that methods of controlling the voltage as by synchronous condensers are not considered applicable, and that the only way in which the power can be increased is by decreasing the reactance or providing additional circuits.

An increase in current-carrying capacity or in corona voltage might be an additional advantage, but such an increase would probably not by itself justify the special construction, for ratings now in use. The current-carrying capacity or corona voltage of a single-conductor line could usually be increased 30 per cent without increasing the cost of the entire transmission line more than a few per cent, by using a heavier conductor or one of about 30 per cent larger diameter. The cost of the conductors is only a fraction of the cost of the entire line. Such a conductor, however, will not give 20 per cent less reactance. The large reduction in reactance is therefore the most important feature of the double-conductor construction.

In conclusion, the advantages shown are as follows: Approximately 20 per cent less reactance; the opportunity in some cases to add to the conductivity and power rating without scrapping the old conductors; the elimination of skin effect almost entirely; and the increase in current-carrying capacity of nearly 30 per cent. The disadvantages are the greater clearance and mechanical strength required, particularly where ice load is important, the cost of hangers, and the increased cost of installation.

Appendix I

REACTANCE FOR TWO CONDUCTORS PER PHASE

Let there be two equal wires per phase, at an axial distance m inches apart. Let the geometric mean of the spacings between the phases be s inches. Then the reactance will be approximately the same as for equilateral spacing of s inches. If s is considerably larger than m, the current will divide practically equally between the two wires of each phase.

Reactive drop in one wire is approximately, for a three-phase system,

$$= j2\pi f\,10^{-9}\left[\frac{I}{2}\left(\frac{1}{2}+2\,\text{logn}\,\frac{p}{\rho}\right)+\frac{I}{2}\,2\,\text{logn}\,\frac{p}{m}\right.$$

$$\left.+I(-0.5+j0.866)\,2\,\text{logn}\,\frac{p}{s}+I(-0.5-j0.866)\,2\,\text{logn}\,\frac{p}{s}\right]$$

volts per cm.

where ρ is the radius of the wires, and p is a certain large distance up to which flux is counted. As p cancels out, the result is the same no matter how large p may be. The term logn denotes natural logarithm. Reactive drop

$$= j\,\frac{2\pi f\,10^{-9}}{2}\,I\left(\frac{1}{2}+2\,\text{logn}\,\frac{s}{\rho}+2\,\text{logn}\,\frac{s}{m}\right)$$

$$= j\pi f I\,10^{-9}\left(\frac{1}{2}+2\,\text{logn}\,\frac{s^2}{m\,\rho}\right)\text{ volts per cm.}$$

Changing units,

$$\text{Reactance} = \pi f\,10^{-6}\left(80.5+741\,\log_{10}\frac{s^2}{m\rho}\right)$$

$$\text{ohms per mile} \qquad (1)$$

If the conductor is a 7-strand cable of diameter 2ρ, the figure 80.5 is changed to 103.3 and if there are 19 strands, the figure becomes 89.3. (See reference 9, page 127, for other numbers of strands.)

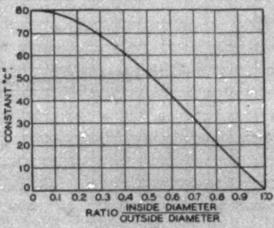

FIG. 7—CONSTANT c FOR USE IN EQUATION (2), REACTANCE OF TUBULAR CONDUCTOR

Appendix II

REACTANCE OF TUBULAR OR HOLLOW CONDUCTOR

The reactance of a tubular conductor is

$$2\pi f\,10^{-6}\left(c+741\,\log_{10}\frac{s}{\rho}\right)\text{ ohms per mile} \qquad (2)$$

where s is the geometric mean of the axial spacings between phases, 2ρ is the diameter of the conductor and c is read from Fig. 7. The formula used in drawing the curve for c is given in reference 9, page 137, equation (5), or in other books.

Formula (2) is in most practical cases a close approximation for the reactance of a hollow stranded conductor. It may be noted that the reactance of a standard conductor having 37 strands, and 18 strands in the outer layer, is given very closely by the formula for a solid wire.

Appendix III

CAPACITANCE FOR TWO CONDUCTORS PER PHASE

Let there be a three-phase system with two equal conductors per phase of radius ρ and at an axial distance m. Let the geometric mean of the spacings between the phases be s. Then the capacitance will be approximately the same as for equilateral spacing s. If s is considerably larger than m, the charge will divide almost equally between the two conductors of each phase. Let the charge on each of the conductors of the first phase be $q/2$ per cm. For the fundamentals of this type of calculation see reference 9, pages 142, 143 and 152, or other books.

Carry a unit charge from a conductor of one phase to one of another, assuming equilateral spacing. The potential between phases is approximately

$$q \operatorname{logn} \frac{s}{\rho} + q \operatorname{logn} \frac{s}{m}$$

$$-(-0.5 + j.866)\left(q \operatorname{logn} \frac{s}{\rho} + q \operatorname{logn} \frac{s}{m}\right)$$

The absolute value of the potential between phases is

$$1.732\, q \operatorname{logn} \frac{s^2}{m\rho} \qquad (3)$$

The capacitive susceptance required in connection with the three-phase circuit is equal to the charge per phase divided by the voltage to neutral and multiplied by $2\pi f$, and is

$$\frac{2\pi f}{\operatorname{logn} \dfrac{s^2}{m\rho}} \text{ in absolute units.}$$

$$\text{Changing units,} \quad b = \frac{2\pi f \times 38.8 \times 10^{-9}}{\log_{10} \dfrac{s}{\sqrt{(m\rho)}}} \qquad (4)$$

$$= \frac{4\pi f\, 38.8 \times 10^{-9}}{\log_{10} \dfrac{s^2}{m\rho}} \text{ mhos per mile} \qquad (5)$$

The capacitance of a double-conductor line, to the first approximation, is therefore the same as that of a single-conductor line the radius of whose conductors is $\sqrt{(m\rho)}$.

Appendix IV

CORONA VOLTAGE

By corona voltage is meant the voltage at which corona starts to form on transmission line conductors in fair weather. This is an advisable limit of operating voltage. It depends on the potential gradient in volts per cm. at the surface of the conductors.

If in a double-conductor transmission line s/m is comparatively large, the charge may be taken to be equally divided between the two conductors per phase. If $q/2$ is the charge per cm. of conductor, the gradient in absolute units at the surface of one of the wires is $\dfrac{q}{\rho}$ due to the charge on that wire. This component of the gradient is, for equilateral triangular spacing between phases,

$$\frac{q}{\rho} = \frac{e_o}{2\rho \operatorname{logn} \dfrac{s}{\sqrt{(m\rho)}}} \text{ in absolute units,}$$

$$\text{since} \qquad e_o = 2q \operatorname{logn} \frac{s}{\sqrt{(m\rho)}} \text{ to neutral,}$$

from equation (3), Appendix III.

In order to include the component of gradient due to the other conductor of the same phase, the above value of gradient should be multiplied by $1 + \dfrac{2\rho}{m}$ according to equation (**15**) and Table II of the paper *Three-Phase Multiple-Conductor Circuits*, by Edith Clarke, see page 809 of this issue. This factor was given for use when the value of $\dfrac{m}{2\rho}$ is as low as 5.

Then if g_o is the gradient at which corona forms,

$$e_o = \frac{2\, g_o\, \rho}{1 + \dfrac{2\rho}{m}} \operatorname{logn} \frac{s}{\sqrt{(m\rho)}}$$

If g_o is in kilovolts per inch and ρ in inches, e_o is in kilovolts to neutral. The expression for e_o should be multiplied by an irregularity factor m_o, an average value of which for stranded conductors is 0.85, and by an air density factor δ, the value of which for standard temperature and pressure is unity. The value of g_o for smooth wires and standard temperature and pressure is 53.6 kv. per inch. Then

$$e_o = \frac{53.6 \times 2.30\, m_o \delta \rho}{1 + \dfrac{2\rho}{m}} \log_{10} \frac{s^2}{m\rho} \qquad (6)$$

For stranded conductors at normal temperature and pressure,

$$e_o = \frac{105\, \rho}{1 + \dfrac{2\rho}{m}} \log_{10} \frac{s^2}{m\rho} \text{ kilovolts to neutral,} \qquad (7)$$

for double-conductor lines, dimensions being in inches.

By equation (35′), page 302, reference 8,

$$e_o = 105\, \rho \log_{10} \frac{s}{\rho} \qquad (8)$$

for single-conductor lines, with $m_o = 0.85$. For a description of calculations of corona, see reference 8, Chapter X.

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Discussion

For discussion of this paper see page 821.

Three-Phase Multiple-Conductor Circuits

BY EDITH CLARKE*

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Synopsis.*—During the past year the subject of power transmission by multiple-conductor circuits has received considerable attention. The formulas and estimating curves given in the paper may therefore be of interest at this time.*

Formulas are developed for the inductance and capacitance to neutral per phase and the approximate corona starting voltage of perfectly transposed, multiple-conductor, three-phase transmission lines having any number of conductors per phase.

For certain special arrangements of the conductors, curves are given for the 60-cycle reactance, capacity susceptance, and corona starting voltage. These curves show the effect of variations, throughout their practical range, in conductor diameter, spacing between phases and between conductors of the same phase. Two, three, four and five conductors per phase are considered.

A comparison is made of multiple- and single-conductor circuits with respect to charging current, voltage at no load, power which can be carried with the same voltage drop, and stability.

* * * * *

INTRODUCTION

MULTIPLE or split conductors were proposed in 1909 by P. H. Thomas in a paper[1] before the Institute. In this paper Mr. Thomas pointed out the advantage of split conductors in increasing the power which may be transmitted at a given voltage drop. Since then, at irregular intervals, the use of two or more conductors per phase has been suggested by transmission engineers for various contemplated projects. At such times it was found advantageous to have available for estimating purposes, curves giving the 60-cycle reactance, the capacity susceptance and the disruptive critical voltage for various proposed arrangements of the multiple-conductor circuits. These curves were made up from time to time as new arrangements of the conductors were considered.

The recent revival of interest in the subject has suggested that these curves be assembled and brought to the attention of those who may find use for them in estimating the performance of multiple-conductor circuits. The curves include various arrangements of two, three, four, and five conductors per phase.

I—Electrical Characteristics of Multiple-Conductor Circuits

TRANSPOSED LINES

When the conductors of a three-phase transmission line are unsymmetrically arranged, unless the conductors are transposed, the inductance and capacitance to neutral of the three phases will be unequal and the currents under normal operating conditions unbalanced. When a multiple-conductor line is so transposed that each conductor occupies each of the tower positions or its equivalent for equal distances, the phases being rotated in cyclic order, the total inductance and capacitance to neutral of the three phases will be equal. Also, the total inductance and capacitance of each conductor will be the same. Therefore, with balanced voltages applied, the phase currents will be balanced and each will divide equally among the conductors of that phase. When the line is not transposed, or only partly transposed, an exact solution may be obtained by the use of symmetrical components. In the general case, with balanced voltages applied, there will be negative and zero sequence currents as well as positive sequence currents. Approximate results may be obtained by considering only the positive sequence currents and by using for inductance and capacitance to neutral the average of the three positive sequence inductances and capacitances, respectively. This is equivalent to using the values of inductance and capacitance which the line would have if it were perfectly transposed.

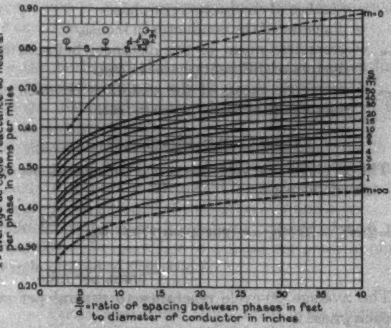

FIG. 1—TWO CONDUCTORS PER PHASE ARRANGED VERTICALLY

60-cycle reactance as affected by S/d and S/m

S = horizontal distance between adjacent phases in feet
m = distance between conductors of the same phase in feet
d = diameter of the conductor in inches

In the work which follows a completely transposed line will be assumed.

INDUCTANCE

A formula for the average inductance to neutral per mile of each conductor of a perfectly transposed three-phase transmission line having n conductors per phase

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1. For references see Bibliography.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

is derived in Appendix A. The inductance will vary along the line with the transpositions, but the average inductance per mile of all conductors to neutral will be the same. Since the n conductors of the phase are in parallel, the average inductance per phase will be the *nth* part of the inductance per conductor.

It has been found convenient in using curves for estimating purposes to express the diameter of the conductors in inches while the spacings between phases and between conductors of the same phase are in feet.

From (**9a**), Appendix A, the average inductance to neutral per mile of each conductor is

$$L_c = 0.08047 + 0.74113 \log_{10} \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}$$

millihenrys per mile (**1**)

The average inductance per phase, L, will be the *nth* part of the inductance per conductor, therefore

$$L = \frac{1}{n}\left[0.08047 + 0.74113 \log_{10} \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}\right]$$

millihenrys per mile (**2**)

where

n = number of conductors per phase.

d = diameter of the conductor in inches.

S_{gm} = geometric mean of the distances between conductors of different phases in feet.

m_{gm} = geometric mean of the distances between conductors of the same phase in feet.

60-CYCLE REACTANCE

The average 60-cycle reactance to neutral per mile of each conductor is

$$x_c = 2\pi f L_c = 377\,L_c = 0.03034 + 0.27941 \log_{10} \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}$$

ohms per mile (**3**)

The average 60-cycle reactance to neutral per mile of each phase is

$$x = 377\,L = \frac{x_c}{n}$$

$$= \frac{1}{n}\left[0.03034 + 0.27941 \log_{10} \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}\right]$$

ohms per mile (**4**)

SOLID, STRANDED AND HOLLOW CONDUCTORS

The internal inductance[2] of a stranded conductor is greater than that of a solid conductor of the same outside diameter, while that of a hollow conductor is less. The internal 60-cycle reactance of a solid conductor in ohms per mile is given by the first term on the right hand side of (**3**), while the external reactance is given by the second term.

Table I gives a plus or minus correction, ϵ, to be added to the 60-cycle phase reactance calculated by (**4**) to obtain the correct reactance per phase for stranded and hollow multiple conductors. These corrections were calculated by using for stranded conductors the formulas given in "Transmission Line Formulas," by H. B. Dwight, page 127, and for hollow conductor, equation (**14**) p. 827, Pender's "Handbook for Electrical Engineers."

For hollow or stranded multiple conductors

$$x = \epsilon + \frac{1}{n}\left[0.03034 + 0.27941 \log \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}\right]$$

ohms per mile (**5**)

CAPACITANCE

Dr. Kennelly in his discussion[3] of Mr. Thomas' paper,[1] by applying the theorem of the propagation of electric disturbances in free space at the speed of light to multiple conductor circuits, points out that the capacitance to neutral in statfarads per cm. is the reciprocal of the external inductance to neutral in abhenrys per cm. The first term on the right hand side of (**8a**), Appendix A, is the external and the second the internal inductance. Therefore, for a perfectly transposed multiple-conductor circuit the average capacitance to neutral per cm. for each conductor will be

$$C_c = \frac{1}{2 \quad \log_e \dfrac{2\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \text{ statfarads per cm.} \quad (\mathbf{6})$$

where d, S_{gm} and m_{gm} are expressed in the same units.

Expressing (**6**) in practical units with S_{gm} and m_{gm} in feet and d in inches, the average capacitance to neutral per mile for each conductor is

$$C_c = \frac{0.03883}{\log_{10} \dfrac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \text{ microfarads per mile} \quad (\mathbf{7})$$

TABLE I—ϵ—CORRECTION FOR STRANDED AND HOLLOW CONDUCTORS

Conductors per phase	Stranded conductors				Hollow conductors					
	No. of strands				Ratio of internal to external diameter					
	7	19	37	61	0	0.20	0.4	0.6	0.8	1.0
1	0.0086	0.0033	0.0017	0.0010	0	−0.0022	−0.0075	−0.0145	−0.0222	−0.0303
2	0.0043	0.0017	0.0009	0.0005	0	−0.0011	−0.0038	−0.0073	−0.0111	−0.0152
3	0.0029	0.0011	0.0006	0.0004	0	−0.0007	−0.0025	−0.0048	−0.0074	−0.0101
4	0.0021	0.0008	0.0004	0.0003	0	−0.0005	−0.0019	−0.0036	−0.0056	−0.0076
5	0.0017	0.0007	0.0003	0.0002	0	−0.0004	−0.0015	−0.0029	−0.0044	−0.0061

The average capacitance to neutral per mile, C, for each phase will be n times the capacitance per conductor, therefore

$$C = \frac{0.03883\, n}{\log_{10} \dfrac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \text{ microfarads per mile} \qquad (8)$$

60-Cycle Capacity Susceptance

The average capacity susceptance to neutral per mile of each conductor for a frequency of 60 cycles will be

$$b_c = 2\pi f C_c = 377 C_c = \frac{14.64}{\log_{10} \dfrac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \text{ micromhos per mile} \qquad (9)$$

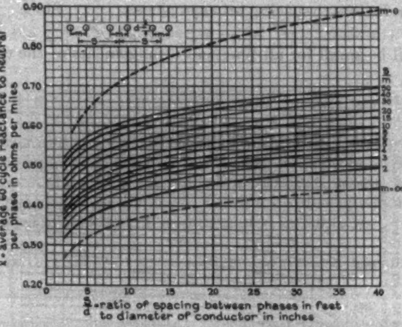

FIG. 2—TWO CONDUCTORS PER PHASE ARRANGED HORIZONTALLY

60-cycle reactance as affected by S/d and S/m

S = distance between centers of adjacent phases in feet

m = distance between conductors of the same phase in feet

d = diameter of the conductor in inches

The average 60-cycle capacity susceptance to neutral per mile of each phase, will be

$$b = 377C = nb_c = \frac{14.64\, n}{\log_{10} \dfrac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \text{ micromhos per mile} \qquad (10)$$

Relation Between 60-Cycle Reactance and Capacity Susceptance

If the product of capacitance and external inductance is unity in absolute units, it follows that there is a definite relation between 60-cycle reactance and capacity susceptance.

Transposing (5) and multiplying by (10) there results

$$b\left[x - \frac{0.0303}{n} - \epsilon\right] = 4.09 \qquad (11)$$

When x, the 60-cycle reactance to neutral per phase in ohms is known, b, the capacity susceptance per phase in micromhos may be calculated from the following formula:

$$b = \frac{4.09}{x - \left(\dfrac{0.0303}{n} + \epsilon\right)} \text{ micromhos per mile} \qquad (12)$$

Should b be known, x may be obtained, thus,

$$x = \frac{4.09}{b} + \frac{0.0303}{n} + \epsilon \text{ ohms per mile} \qquad (13)$$

60-Cycle Reactance and Capacity Susceptance Curves for Multiple-Conductor Circuits

Equations (4) and (10) give the average 60-cycle reactance and capacity susceptance to neutral per mile for each phase of a perfectly transposed multiple conductor circuit, having any number of conductors per phase and any arrangement of the conductors.

For certain symmetrical arrangements of phases and conductors in the phases, the reactance and capacity susceptance may be expressed in terms of the ratios S/d and S/m, where

S = the horizontal distance between the center lines of adjacent phases in feet.

m = the distance between adjacent conductors of the same phase in feet.

d = the diameter of the conductor in inches.

In Appendix B, formulas for 60-cycle reactance and capacity susceptance for special multiple-conductor circuits are given in terms of the ratios S/d and S/m.

The following cases have been considered and the average 60-cycle reactance to neutral per phase in ohms per mile plotted in Figs. 1 to 7:

1. Two conductors per phase
    a. Vertical arrangement of conductors in the same phase.
    b. Horizontal arrangement of conductors in the same phase.
2. Three conductors per phase
    a. Vertical arrangement of conductors in the same phase.
    b. Conductors of the same phase at the corners of an equilateral triangle.
3. Four conductors per phase
    a. Vertical arrangement of conductors in the same phase.
    b. Conductors of the same phase at the corners of a square.
4. Five conductors per phase
    a. Vertical arrangement of conductors in the same phase.

These curves are applicable for all practical conductor diameters, spacings between phases and between conductors of the same phase. Since they are for solid conductors, the correction, ϵ, Table I, should be applied when necessary.

Fig. 8 gives the average 60-cycle capacity susceptance to neutral per phase in micromhos per mile for two conductors per phase, arranged vertically, case *Ia*. Fig. 1 gives the corresponding 60-cycle reactance.

Since 60-cycle capacity susceptance is readily calculated from equation (**12**) when the reactance is known, curves for the capacity susceptance of the other conductor arrangements are not given.

CORONA

In a perfectly transposed multiple-conductor circuit, since each conductor occupies each of the tower positions for equal distances, the corona-starting voltage for all conductors will be approximately the same. As a conductor is changed from one tower position to another, in general, it will have a different corona-starting voltage in each position.

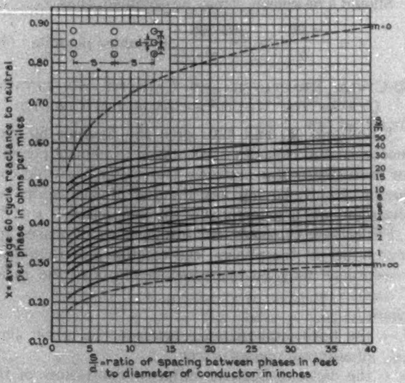

FIG. 3—THREE CONDUCTORS PER PHASE ARRANGED VERTICALLY

60-cycle reactance as affected by S/d and S/m
S = horizontal distance between adjacent phases in feet
m = distance between adjacent conductors of the same phase in feet
d = diameter of the conductor in inches

In Appendix C, by Mr. S. B. Crary, a formula is developed for the disruptive critical voltage, or the corona-starting voltage, for any conductor of a multiple-conductor circuit. If the conductor which has the lowest disruptive critical voltage is designated *II-1*, as in Fig. 10, the lowest disruptive critical voltage of the system will be given by (**8c**), if 1 and *1* are interchanged.

Introducing the irregularity and air density factors,[4] M_0 and δ, respectively, and multiplying numerator and denominator of the fraction in (**8c**) by d, e_0, the lowest corona starting voltage of the system will be given by (**14**).

$$e_0 = \frac{61.5\,M_0\delta d\log_{10}\dfrac{2\sqrt{(S_{11}.S_{12}\ldots S_{1n})(S_{11}.S_{12}\ldots S_{1n})}}{dm_{12}\ldots m_{1n}}}{1+\sqrt{\left[\sum\dfrac{d}{m_{1n}}\cos\theta_{1n}\right]^2+\left[\sum\dfrac{d}{m_{1n}}\sin\theta_{1n}\right]^2}}$$

kv. to neutral (**14**)

where

e_0 = line-to-neutral corona starting voltage in kv.
M_0 = irregularity factor = 0.98 − 0.93 for roughened or weathered wires. = 0.87 − 0.83 for seven-strand cables.
δ = air density factor = $3.92b/(273+t)$ = 1.00 at 76 cm. barometer and 25 deg. cent.
where b = barometric pressure in cm. and t = temperature in deg. cent.

d, m, and S are in inches.

The formulas for disruptive critical voltage, given by (**8c**) and (**14**), are based on the assumptions that the charges on the three phases are balanced, and the charge per unit length is the same for all conductors of the same phase. The field distortion in the vicinity of a conductor due to charges on conductors of the other two phases will depend upon the ratio of diameter of conductors to spacing between phases, $\frac{d}{S}$. Since $\frac{d}{S}$ will be small for all practical arrangements of the conductors in a multiple conductor circuit, the field distortion due to the charges on the other two phases will have a negligible effect on the corona starting voltage. The field distortion due to the charges on conductors of the same phase will depend upon $\frac{d}{m}$, the ratio of diameter of conductors to spacing between conductors of the same phase, and may have an appreciable effect upon the corona starting voltage. This is evident from

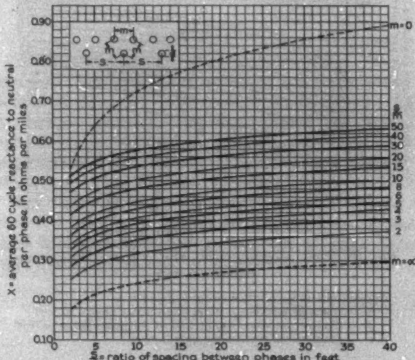

FIG. 4—THREE CONDUCTORS PER PHASE ARRANGED AT THE CORNERS OF AN EQUILATERAL TRIANGLE

60-cycle reactance as affected by S/d and S/m
S = distance between centers of adjacent phases in feet
m = side of triangle in feet
d = diameter of conductors in inches

inspection of the denominator of the fraction in (**14**). As the ratio of diameter to spacing between conductors decreases, the field distortion will decrease and finally become negligible for very small values of $\frac{d}{m}$. When $\frac{d}{m}$ is small relative to unity, the denominator of the fraction in (**14**) approaches unity; the numerator of the fraction will then give the corona starting voltage.

As a first approximation, the corona starting voltage may be calculated from the numerator of the fraction in (**14**). When greater accuracy than this is required, the approximate corona-starting voltage may be corrected by dividing it by D, the denominator of the fraction in (**14**).

Let

$$D = 1 + \sqrt{\left[\sum \frac{d}{m_{1n}} \cos \theta_{1n}\right]^2 + \left[\sum \frac{d}{m_{1n}} \sin \theta_{1n}\right]^2}$$

$$= 1 + k\frac{d}{m} \qquad \mathbf{(15)}$$

where d and m are in the same units and k depends upon the arrangement and the number of conductors in the phase. Values of k corresponding to seven special arrangements of conductors in multiple conductor circuits were calculated by (**15**) and tabulated in Table II.

TABLE II—VALUES OF k AND x_k TO BE USED IN CONNECTION WITH FIG. 9 TO OBTAIN CORONA STARTING VOLTAGE

Conductors per phase	Arrangement of conductors in the phases	k To be substituted in equation (15) to obtain D	x_k To be substituted in equation (18) to obtain x_m
2	vertical	1.0	0.056
2	horizontal	1.0	0.056
3	vertical	1.5	0.112
3	at corners of an equilateral triangle	1.732	0.084
4	vertical	1.833	0.179
4	at corners of a square	2.121	0.112
5	vertical	2.083	0.251

If the numerator of the fraction in (14) is compared with the formula for minimum 60-cycle reactance given in (**11a**), Appendix A, it will be noticed that the argument of the logarithm is the same in both equations. For convenience, corona-starting voltage will be expressed in terms of x_m, the minimum 60-cycle reactance of any conductor.

Replacing the logarithm in (14) by its value from (**11a**), and replacing the denominator by D,

$$e_0 = \frac{61.5\, M_0 \delta d \, \dfrac{x_m - 0.03034}{0.27941}}{D} \qquad \mathbf{(16)}$$

Substituting 0.87 for M_0 and 1.0 for δ in (**16**) and multiplying both sides of the equation by $\sqrt{3}$, the line-to-line corona-starting voltage, E_0, is obtained.

$$E_0 = \frac{332\, d\, (x_m - 0.03034)}{D} \text{ kv. line-to-line} \qquad \mathbf{(17)}$$

In Appendix B, formulas are given for x, the average 60-cycle reactance per phase for special arrangements of the multiple conductor circuits. These are plotted in Figs. 1 to 7. In Appendix A it is shown that x_m the minimum 60-cycle reactance for a conductor in ohms per mile may be obtained from x_c its average 60-cycle reactance by subtracting a value, x_k, which is approximately constant for any given number of conductors per phase and arrangement of conductors in the phase. Values of x_k for special conductor arrangements are derived from (**13a**), Appendix A and tabulated in Table II. x_m may be calculated from Figs. 1 to 7 and Table II, by means of (**18**).

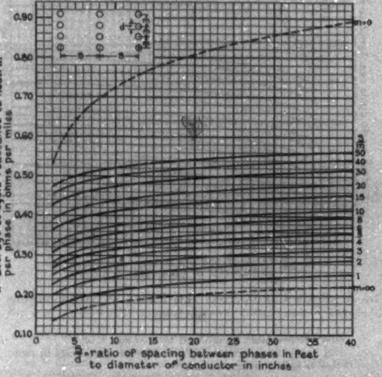

FIG. 5—FOUR CONDUCTORS PER PHASE ARRANGED VERTICALLY

60-cycle reactance as affected by S/d and S/m

S = horizontal distance between adjacent phases in feet

m = distance between adjacent conductors of the same phase in feet

d = diameter of the conductor in inches

$$x_m = x_c - x_k = nx - x_k \qquad \mathbf{(18)}$$

Fig. 9, plotted from (**17**) when $D = 1$, gives a first approximation of the line-to-line corona-starting voltage in kv., expressed in terms of minimum 60-cycle reactance to neutral of any conductor in ohms per mile, and the diameter of the conductor in inches. It is applicable to both multiple- and single-conductor circuits. The reactance to be used is that for solid conductors of the given diameter and spacings.

From Fig. 9 and Figs. 1 to 7 the approximate diameter which is required to avoid corona at specified spacings and voltage for any of the special conductor arrangements considered, may be determined by the following procedure:

a. Assume *d*, the diameter of the conductor.

b. From Figs. 1 to 7 read x, the average 60-cycle reactance per phase corresponding to the specified number of conductors per phase, arrangement of conductors, and calculated values of S/d and S/m.

c. Multiply x, the reactance per phase, by n, the number of conductors per phase, to obtain x_c, the average reactance per conductor.

d. From x_c, the average reactance per conductor, calculate x_m, the minimum reactance, by subtracting the appropriate value of x_k given in Table II. (See equation (18).)

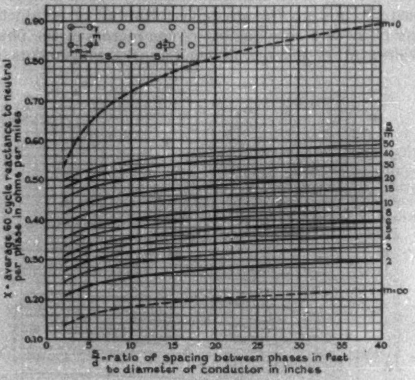

FIG. 6—FOUR CONDUCTORS PER PHASE AT THE CORNERS OF A SQUARE

60-cycle reactance as affected by S/d and S/m
S = distance between centers of adjacent phases in feet
m = side of square in feet
d = diameter of the conductor in inches

e. From Fig. 9 read E_0', a first approximation of the line-to-line corona-starting voltage corresponding to the assumed diameter, *d*, and the minimum reactance, x_m. This will be very nearly the correct value of the corona-starting voltage when the ratio of d/m is small relative to unity.

f. Calculate d/m and read k from Table II corresponding to the given number and arrangement of conductors. Substitute d/m and k in (15) to obtain D, d and m being in the same units.

g. Divide the value of E_0' read from Fig. 9 by D to obtain the corrected corona-starting voltage, E_0.

h. If E_0 obtained from (*g*) is greater than the operating voltage, corona will be avoided. If E_0 is less than the operating voltage, a larger diameter should be selected and the process repeated.

II—Comparison of Multiple- and Single-Conductor Circuits

REACTANCE AND CAPACITY SUSCEPTANCE

The upper dotted curve in Figs. 1 to 7 for $m = o$ gives the average 60-cycle reactance to neutral of the three phases of a single-conductor circuit with flat arrangement of conductors. It also gives the average reactance per mile of each phase of the perfectly transposed line. The reactance is plotted in terms of S/d, the ratio of the distance between adjacent conductors in feet to the diameter of the conductor in inches. The curve, which is the same on all seven figures, has been repeated for convenience in comparing the reactance of the single-conductor circuit with that of multiple-conductor circuits. Since the curve is for solid conductors, the correction from Table I corresponding to $n = 1$ should be applied for stranded or hollow conductors.

As m increases from 0 to ∞, the reactance per phase of the multiple conductor decreases. The lower dotted curves for $m = \infty$ apply when the conductors of the phase are so far apart that all mutual effects disappear and there are n separate parallel circuits.

From Figs. 1 to 7 it may be seen that the reactance per phase of a multiple-conductor circuit is always less than that of the single-conductor circuit, the diameter and spacing between adjacent phases being the same. The reactance of one conductor of the multiple-conductor circuit, however, is greater than that of a conductor of the single-conductor circuit.

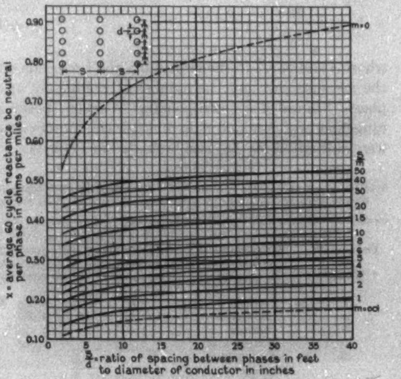

FIG. 7—FIVE CONDUCTORS PER PHASE ARRANGED VERTICALLY

60-cycle reactance as affected by S/d and S/m
S = horizontal distance between adjacent phases in feet
m = distance between adjacent conductors of the same phase in feet
d = diameter of the conductor in inches

In Fig. 8 the lower dotted curve gives the 60-cycle capacity susceptance to neutral of the single-conductor circuit. The upper dotted curve gives the upper limit of the capacity susceptance per phase as the spacing between the conductor of the phase is indefinitely increased. It is evident that the capacity susceptance per phase for the multiple-conductor circuit is greater than that of the single-conductor circuit for the same S/d, while the capacity susceptance per conductor is less.

DIAMETER TO AVOID CORONA

Fig. 9 gives the approximate diameter required to avoid corona at a given voltage in terms of x_m, the minimum 60-cycle reactance of any conductor. Since the reactance per conductor is greater in the multiple- than in the single-conductor circuit, from Fig. 9 it follows that a smaller diameter is required to avoid corona in the multiple- than in the single-conductor circuit. With the same conductor diameter, the multiple-conductor circuits can be operated at a higher voltage without corona than the single-conductor circuit.

CHARGING CURRENT AND RISE IN VOLTAGE AT NO-LOAD

At a given voltage and frequency the charging currents on the multiple- and single-conductor circuits are directly proportional to their respective capacitances to neutral per phase. Since the larger current flows through the smaller inductance and vice versa, the rise in voltage at no-load is practically the same for both the multiple- and single-conductor circuits. These statements follow from the transmission line equations[5] (**19**) and (**20**) under no-load conditions, *i.e.* $I_r = 0$.

$$e_s = e_r \cosh\sqrt{ZY} + I_r Z\left(\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}}\right) \qquad \mathbf{(19)}$$

$$I_s = I_r \cosh\sqrt{ZY} + e_r Y\left(\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}}\right) \qquad \mathbf{(20)}$$

where

e_s and e_r = voltage to neutral at sending and receiving end of line respectively in volts.

I_s and I_r = the corresponding phase current in amperes.

$Z = l\,(r + jx) = l\,(r + j\,2\pi fL)$ = total impedance of one phase to neutral in ohms.

$Y = l\,(g + jb) = l\,(o + j\,2\pi fC)$ = total admittance of one phase to neutral in mhos.

r, L, g and C = resistance, inductance, leakance and capacitance respectively to neutral per mile.

l = length of line in miles.

Replacing the hyperbolic terms in (**19**) and (**20**) by their equivalent series, and setting $I_r = 0$, at no load

$$e_g = e_r\left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \ldots\ldots\right) \qquad \mathbf{(21)}$$

$$I_g = e_r Y\left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \ldots\ldots\right) \qquad \mathbf{(22)}$$

For single-conductor circuits the product of inductance and capacitance varies but slightly throughout the practical range of conductor diameters and spacings. Therefore, for a given frequency and length of line, the terms in parenthesis in (**21**) and (**22**) are substantially constant. The effect of resistance, and variations in the product of inductance and capacitance, on the hyperbolic terms used in transmission line calculation are shown graphically in "Simplified Transmission Line Calculations" by the author, *General Electric Review*, May 1926. In all practical cases these effects are small.

In multiple-conductor circuits the inductance per phase is decreased and the capacitance increased over that of the single-conductor circuit in approximately the same ratio, therefore the product of inductance and capacitance will not be greatly changed. This product, in general, will be slightly less for multiple- than for single-conductor circuits, but it will not be more than 5 per cent less. The effect of this variation on the terms in parenthesis will be insignificant except for very long lines. It is therefore evident from (**22**) that the charging current varies directly with the capacitance, and from (**21**) that the rise in voltage at no-load for the single- and multiple-conductor circuits is substantially the same.

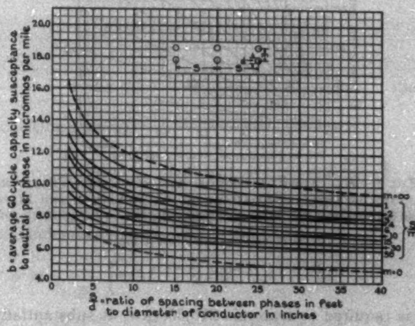

FIG. 8—TWO CONDUCTORS PER PHASE ARRANGED VERTICALLY

60-cycle capacity susceptance as affected by S/d and S/m

S = horizontal distance between adjacent phases in feet

m = distance between conductors of the same phase in feet

d = diameter of the conductor in inches

POWER TRANSMITTED UNDER GIVEN TERMINAL CONDITIONS

For the same receiver voltage, equation (**19**) shows that I_r may be increased in the same ratio that Z is decreased without affecting e_g, since the hyperbolic terms are substantially constant for a given frequency and length of line for both single- and multiple-conductor circuits. It follows, therefore, that when resistance is small, or when the ratio of resistance to reactance is the same for both circuits, the power which can be transmitted at a specified voltage drop, receiver voltage, and power factor, varies inversely as the respective reactances of the single- and multiple-conductor circuits.

For unequal ratios of resistance to reactance, this simple relation will hold only approximately, but in any case it may be used to advantage in estimating the power which can be transmitted over a multiple-conductor circuit. It may also be used to determine approximately the spacings required with a proposed arrangement of conductors to deliver a given amount of power under specified terminal conditions.

For example: Suppose it is known that 150,000 kw. can be delivered over a certain 230-kv. single-conductor line with given terminal voltages and receiver-end power factor. The reactance of the line is 0.8 ohms per mile.

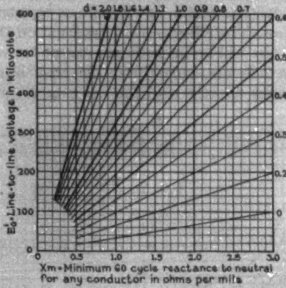

FIG. 9—E_0', FIRST APPROXIMATION OF CORONA STARTING VOLTAGE AS AFFECTED BY DIAMETER OF CONDUCTOR AND MINIMUM REACTANCE PER MILE AT 60 CYCLES

d = diameter of conductor in inches

$$E_0 = E_0'/D, \text{ where } D = 1 + k\frac{d}{m}$$

It is required to transmit 210,000 kw. at substantially the same terminal conditions over a circuit of the same length having two conductors per phase, vertically arranged.

To increase the power in the ratio 210/150 the reactance must be reduced to approximately $0.8 \times 150/210 = 0.571$ ohms per mile. With two conductors per phase the average reactance per conductor, x_c, will be $2 \times 0.571 = 1.143$ ohms per mile. From Table II $x_k = 0.056$, and from (**18**) $x_m = x_c - x_k = 1.087$ ohms per mile. From Fig. 9 the diameter to avoid corona at 230 kv. must be greater than 0.66 inches, how much greater depending upon the ratio d/m.

Fig. 1 shows all possible combinations of the ratios S/d and S/m which will give the required reactance. The choice of spacings between phases and between conductors of the same phase is limited by mechanical considerations; consequently, it may not be possible to use spacings which would give the best results electrically. The following combinations of S, m and d are among those which will give a reactance of approximately 0.571 ohms per mile, and have a corona starting voltage above 230 kv., the operating voltage.

S-ft.	m-ft.	d-inches	S/d	S/m
15	1.25	0.728	20.6	12.00
20	2.00	0.806	24.8	10.00
25	3.00	0.855	29.3	8.3

STABILITY

The methods now in use for determining the stability limits of systems composed of single-conductor circuits may be applied with equal rigor to systems containing one or more multiple-conductor circuits.

A multiple-conductor circuit can be designed to carry more power than a single-conductor circuit of the same length and operating voltage. Accordingly, fewer multiple- than single-conductor circuits are required to transmit a given amount of power from a generating plant to its load center under specified terminal conditions.

The relative stability of a proposed system with multiple- or single-conductor circuits would depend upon whether the operating conditions are to be the same for the two schemes with all lines in service or in the emergency conditions of one line out of service. When the two systems are designed to operate under the same terminal conditions with all lines in service, the total reactance and capacity susceptance between line terminals will be substantially the same whether multiple- or single-conductor circuits are employed. Consequently, under normal operating conditions the steady-state power limit and margin of stability would be the same for either arrangement. With a line out of service, the steady-state power limit with multiple-conductor circuits would be less than with single-conductor circuits. Should the systems be designed to operate under the same terminal conditions during emergency conditions, the relative steady-state stability with multiple-conductor circuits would be increased.

A fault on a conductor of a multiple-conductor circuit will be a less severe shock to the system than one on a conductor of a single-conductor circuit. But after the fault has been cleared, the loss of a multiple-conductor circuit would reduce the synchronizing torque between generator and load to a greater extent than the loss of a single-conductor circuit. The stability of the two systems under transients conditions would depend upon the location and duration of the fault, the number of conductors involved and the relative amounts of power carried by a multiple- and a single-conductor circuit.

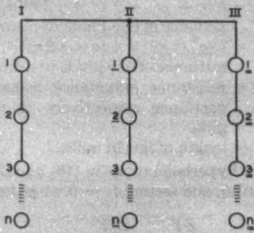

FIG. 10—THREE-PHASE MULTIPLE-CONDUCTOR CIRCUIT HAVING *n* CONDUCTORS PER PHASE

ACKNOWLEDGMENT

The author wishes to express her appreciation of the assistance given by Mr. R. N. Slinger and Mr. M. A. DeFerranti in the preparation of the curves.

Appendix A

INDUCTANCE AND 60-CYCLE REACTANCE OF MULTIPLE-CONDUCTOR CIRCUITS

When a three-phase multiple-conductor transmission line is completely transposed so that each conductor occupies each of the tower positions, or its equivalent, for equal distances, the phases being rotated in cyclic order, the total inductance to neutral of any conductor will be the sum of the inductances in each position.

Let

n = number of conductors per phase.
l = total length of each conductor.
$l/3n$ = length of conductor in each position.
I = current in phase A.
I/n = current in one conductor of phase A.

$$\frac{I}{n}\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)=\frac{I}{n}a^2$$

= current in one conductor of phase B

$$\frac{I}{n}\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)=\frac{I}{n}a$$

= current in one conductor of phase C

The phase positions on the tower will be designated by I, II, and III, and the conductor positions in the phase positions by 1, 2 . . . n, *1, 2* . . . *n*, and **1, 2** . . . **n** respectively. Fig. 10.*

Starting with phase A in position I, the conductor of phase A which first occupies the position I-1 will in turn occupy each of the $3n$ positions for equal distances, $l/3n$.

For sine waves of current of frequency f, the total inductive voltage drop in any conductor will be the sum of the voltage drops in the conductor for the $3n$ positions and may be expressed in terms of self-inductance, L_1, and mutual inductance, M.

With the conductor in position I-1

$$V_{\mathrm{I}-1}=2\pi f\left[\frac{\mathrm{I}}{n}(L_1+M_{12}+\ .\ .\ .+M_{1n})\right.$$
$$+\frac{\mathrm{I}}{n}a^2(M_{\mathit{11}}+M_{\mathit{12}}+\ .\ .\ .+M_{\mathit{1n}})$$
$$\left.+\frac{\mathrm{I}}{n}a(M_{\mathbf{11}}+M_{\mathbf{12}}+\ .\ .\ .+M_{\mathbf{1n}})\right]\quad\textbf{(1a)}$$

There will be a total of n equations in (**1a**), one for each of the conductor positions in phase position I.

There will also be n equations when phase A occupies phase position II, and likewise n equations for position III. One equation for position II is given by (**2a**) and one for position III by (**3a**)

$$V_{\mathrm{II}-\mathit{1}}=2\pi f\left[\frac{\mathrm{I}}{n}(L_1+M_{\mathit{12}}+\ .\ .\ .\ M_{\mathit{1n}})\right.$$
$$+\frac{\mathrm{I}}{n}a^2(M_{\mathit{11}}+M_{\mathit{12}}+\ .\ .\ .\ M_{\mathit{1n}})$$
$$\left.+\frac{\mathrm{I}}{n}a(M_{\mathit{11}}+M_{\mathit{12}}+\ .\ .\ .\ M_{\mathit{1n}})\right]\quad\textbf{(2a)}$$

$$V_{\mathrm{III}-\mathbf{1}}=2\pi f\left[\frac{\mathrm{I}}{n}(L_1+M_{\mathbf{12}}+\ .\ .\ .\ M_{\mathbf{1n}})\right.$$
$$+\frac{\mathrm{I}}{n}a^2(M_{\mathbf{11}}+M_{\mathbf{12}}+\ .\ .\ .\ M_{\mathbf{1n}})$$
$$\left.+\frac{\mathrm{I}}{n}a(M_{\mathit{11}}+M_{\mathit{12}}+\ .\ .\ .\ M_{\mathbf{1n}})\right]\quad\textbf{(3a)}$$

Replacing a by

$$\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right),\ a^2\text{ by}\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)$$

and adding the voltage drops in the $3n$ positions, the total inductive voltage drop, V, is obtained.

$$V=j2\pi f\frac{\mathrm{I}}{n}\left[3nL_1+2(M_{12}+\ .\ .\ .\ M_{1n}+\ .\ .\ .\right.$$
$$+M_{\mathit{12}}+\ .\ .\ M_{\mathit{1n}}+\ .\ .\ M_{\mathbf{12}}+M_{\mathbf{1n}}+\ .\ .\ )$$
$$-(M_{\mathit{11}}+\ .\ .\ M_{\mathbf{1n}}+\ .\ .\ +M_{\mathit{11}}+\ .\ .\ M_{\mathit{1n}}$$
$$\left.+\ .\ .\ M_{\mathbf{11}}+\ .\ .\ M_{\mathbf{1n}}+\ .\ .\ )+j\frac{\sqrt{3}}{2}(0)\right]\quad\textbf{(4a)}$$

From equations (**1a**), (**2a**), (**3a**), (**4a**) and Fig. 10 it may be seen that there will be $\frac{n(n-1)}{2}$ mutuals between conductors in the same phase position, (some of which may be equal) and a total of $\frac{3n(n-1)}{2}$ for the three phase positions. The number of mutuals between conductors of different phases will be $3n^2$.

From equation (**4a**) the inductance per conductor, L_t, may be obtained by dividing through by $j2\pi f\frac{\mathrm{I}}{n}$.

$$L_t=\frac{V}{j2\pi f\dfrac{\mathrm{I}}{n}}=\left[3nL_1\right.$$
$$+2\sum\left(M_{aa},\ \frac{3n(n-1)}{2}\text{ terms}\right)$$

---

*The conductors may have any desired arrangement in the phase.

$$- \sum (M_{ab},\, 3n^2 \text{ terms}) \Big] \qquad \textbf{(5a)}$$

where M_{aa}, M_{ab} represent the mutual inductances between conductors of the same and different phases respectively.

The following formulas in absolute units for self and mutual inductance of non-magnetic wires in air or other non-magnetic medium follow from those given in the "Bulletin of the Bureau of Standards," Vol. 8, No. 1, p. 150-151, when the length, l', is great in comparison with the diameter, d, and the distance between wires, S:

$$L_1 = 2l' \left[\log_e \frac{4l'}{d} - \frac{3}{4} \right] = \text{self-inductance} \qquad \textbf{(6a)}$$

$$M = 2l' \left[\log_e \frac{2l'}{S} - 1 \right] = \text{mutual-inductance} \qquad \textbf{(7a)}$$

Replacing l' in (**6a**) and (**7a**) by $l/3n$, the length of conductor in each position, substituting (**6a**) and (**7a**) in (**5a**) and simplifying, the total inductance to neutral per conductor may be obtained in absolute units, thus:

$$L_t = l \left[2 \log_e \frac{2\,(S_{gm})^n}{d\,(m_{gm})^{n-1}} + \frac{1}{2} \right] \text{abhenrys} \qquad \textbf{(8a)}$$

where

S_{gm} = the geometric mean of the distances between conductors of different phases; or, the $3n^2$ root of the product of the $3n^2$ independent distances between conductors of different phases.

m_{gm} = the geometric mean of the distances between conductors of the same phase; or, the $3n(n-1)/2$ root of the product of the $3n(n-1)/2$ independent distances between conductor of the same phase.

d = diameter of conductors.

l = length of conductors.

Expressing (**8a**) in practical units, with S_{gm}, m_{gm} and d in the same units, the average inductance to neutral per conductor is

$$L_c = 0.08047 + 0.74113 \log_{10} \frac{2\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}$$

$$\text{millihenrys per mile} \qquad \textbf{(9a)}$$

The average 60-cycle reactance to neutral per conductor will be

$$x_c = 0.03034 + 0.27941 \log_{10} \frac{2\,(S_{gm})^n}{d\,(m_{gm})^{n-1}} \qquad \textbf{(10a)}$$

MINIMUM 60-CYCLE REACTANCE

In a completely transposed line the inductance of a conductor will vary with its position on the tower. Since each conductor occupies each tower positions, the minimum, as well as the average inductance, of all conductors will be the same. The inductance of the conductor will be smaller when it is in the center phase position than when in either of the two outside positions. It will be a minimum when the conductor is in the center phase position and in that conductor position which has the greatest geometric mean distance to the other conductors of the phase. If such a position is designated II-*1*, the inductance for the conductor in this position may be obtained from (**2a**). When the conductor in II-*1* is symmetrically located with reference to the conductors of the other two phase, the j term of the inductance will disappear; in any case it will be small and will therefore be neglected.

The minimum 60-cycle reactance to neutral of any conductor of a perfectly transposed multiple conductor may be written

$$x_m = 0.03034 + 0.27941 \log_{10}$$

$$\frac{2\sqrt{(S_{I1}\,.\,S_{I2}\,.\,.\,.\,S_{In})(S_{I1}\,.\,S_{I2}\,.\,.\,.\,S_{In})}}{d\,(m_{12}\,.\,.\,.\,m_{1n})} \text{ ohms per mile}$$

$$\textbf{(11a)}$$

An equation giving, x_k, the difference between the average and minimum 60-cycle reactance in ohms per mile for any conductor, may be obtained by subtracting (**11a**) from (**10a**),

$$x_k = x_c - x_m = 0.27941 \log_{10}$$

$$\frac{(S_{gm})^n\,(m_{12}\,.\,.\,.\,m_{1n})}{(m_{gm})^{n-1}\sqrt{(S_{I1}\,.\,S_{I2}\,.\,.\,.\,S_{In})(S_{I1}\,.\,S_{I2}\,.\,.\,.\,S_{In})}}$$

$$\textbf{(12a)}$$

When the ratio S/m is 5 or more, the ratio of $(S_{gm})^n$ to the radical in the denominator of (**12a**) approaches $(\sqrt[3]{2S})^n/S^n = (\sqrt[3]{2})^n$, and (**12a**) becomes approximately

$$x_k = x_c - m_m = 0.27941 \log_{10} \frac{(\sqrt[3]{2})^n\,[m_{12}\,.\,.\,.\,m_{1n}]}{(m_{gm})^{n-1}}$$

$$\textbf{(13a)}$$

Appendix B

AVERAGE 60-CYCLE REACTANCE AND CAPACITY SUSCEPTANCE

Equations (4) and (10) give respectively the average 60-cycle reactance, x, and capacity susceptance, b, to neutral per mile for each phase of a completely transposed multiple-conductor circuit. Seven special symmetrical arrangements of conductors in multiple-conductor circuits are shown in the inserts in Figs 1 to 7. For these arrangements, when S_{gm} is expressed in terms S and m, and m_{gm} in terms of m, x and b will be expressed in terms of the two ratios S/d and S/m.

I. TWO CONDUCTORS PER PHASE

a. Vertical Arrangement of Conductors in the Same Phase. From Fig. 1.

$$(S_{gm})^n = (S_{gm})^2 = \left(\sqrt[12]{S^4\,(2S)^2\,(S^2+m^2)^2\,(\overline{2S}^2+m^2)} \right)^2$$

$$= \sqrt[3]{4}\,S^2 \sqrt[6]{\left[1+\left(\frac{m}{S}\right)^2\right]^2 \left[1+\left(\frac{m}{2S}\right)^2\right]} \qquad \textbf{(1b)}$$

$$(m_{gm})^{n-1} = m \qquad \textbf{(2b)}$$

Replacing $(S_{gm})^n$ and $(m_{gm})^{n-1}$ in (**4**) and (**10**) by their values in terms of S and m from (**1b**) and (**2b**), the average 60-cycle reactance and capacity susceptance to neutral per mile for each phase are, respectively,

$$x = \frac{2\pi f L}{n}$$

$$= \frac{1}{2}\left\{ 0.03034 + 0.27941 \log_{10} 24\sqrt[3]{2}\,\frac{S}{d} \right\}$$

$$+ 0.1397 \log_{10} \sqrt[3]{2}\,\frac{S}{m}$$

$$\sqrt[6]{\left[1+\left(\frac{m}{S}\right)^2\right]^2\left[1+\left(\frac{m}{2S}\right)^2\right]}$$

ohms per mile (**3b**)

$$b = 2\pi f C n$$

$$= \frac{29.28}{\log_{10} 24\sqrt[3]{2}\,\frac{S}{d} + \log_{10}\sqrt[3]{2}\,\frac{S}{m} \times \sqrt[6]{\left[1+\left(\frac{m}{S}\right)^2\right]^2\left[1+\left(\frac{m}{2S}\right)^2\right]}}$$

micromhos per mile (**4b**)

For the other six cases the average 60-cycle reactance equations only will be given.

b. Horizontal Arrangement of Conductors in the Same Phase, Fig. 2.

$$x = \frac{1}{2}\left\{ 0.03034 + 0.27941 \log_{10} 24\sqrt[3]{2}\,\frac{S}{d} \right\}$$

$$+ 0.1397 \log_{10} \sqrt[3]{2}\,\frac{S}{m}$$

$$\sqrt[6]{\left[1-\left(\frac{m}{S}\right)^2\right]^2\left[1-\left(\frac{m}{2S}\right)^2\right]}$$

ohms per mile (**5b**)

II. THREE CONDUCTORS PER PHASE

a. Vertical Arrangement of Conductors in the Same Phase, Fig. 3.

$$x = \frac{1}{3}\left\{ 0.03034 + 0.27941 \log_{10} 24\sqrt[3]{2}\,\frac{S}{d} \right\}$$

$$+ 0.09314 \log_{10} \left(\frac{S}{m}\right)^2$$

$$\sqrt[9]{\left[1+\left(\frac{m}{S}\right)^2\right]^5\left[1+\left(\frac{m}{2S}\right)^2\right]^2\left[1+\left(\frac{2m}{S}\right)^2\right]^2}$$

ohms per mile (**7b**)

b. Conductors of the Same Phase at the Corners of an Equivalent Triangle. Fig. 4.

$$x = \frac{1}{3}\left\{ 0.03034 + 0.27941 \log_{10} 24\sqrt[3]{2}\,\frac{S}{d} \right\} + 0.09314 \log_{10} \sqrt[3]{4}\left(\frac{S}{m}\right)^2$$

$$\sqrt[9]{\left[1-\left(\frac{m}{S}\right)^2\right]^2\left[1-\left(\frac{m}{2S}\right)^2\right]\left[1+\left(\frac{m}{S}\right)^2+\left(\frac{m}{S}\right)^4\right]^2\left[1+\left(\frac{m}{2S}\right)^2+\left(\frac{m}{2S}\right)^4\right]}$$ ohms per mile (**8b**)

III. FOUR CONDUCTORS PER PHASE

a. Vertical Arrangement of Conductors in the Same Phase, Fig. 5.

$$x = \frac{1}{4}\left\{ 0.03034 + 0.27941 \log_{10} 24\sqrt[3]{2}\,\frac{S}{d} \right\} + 0.06985 \log_{10} \frac{(S/m)^3}{\sqrt{3}}$$

$$\sqrt[12]{\left[1+\left(\frac{m}{S}\right)^2\right]^8\left[1+\left(\frac{2m}{S}\right)^2\right]^4\left[1+\left(\frac{3m}{S}\right)^2\right]^2\left[1+\left(\frac{m}{2S}\right)^2\right]^3\left[1+\left(\frac{3m}{2S}\right)^2\right]}$$

ohms per mile (**9b**)

b. Conductors of the Same Phase at the Corners of a Square. Fig. 6.

$$x = \frac{1}{4}\left\{ 0.03034 + 0.27941 \log_{10} 24\sqrt[3]{2}\,\frac{S}{d} \right\} + 0.06985 \log_{10} \sqrt{2}\,(S/m)^3$$

$$\sqrt[12]{\left[1-\left(\frac{m}{S}\right)^4\right]^4\left[1-\left(\frac{m}{2S}\right)^4\right]^2\left[1+4\left(\frac{m}{S}\right)^4\right]^2\left[1+\frac{1}{4}\left(\frac{m}{S}\right)^4\right]}$$

ohms per mile (**10b**)

IV. FIVE CONDUCTORS PER PHASE

a. Vertical Arrangement of Conductors in the Same Phase, Fig. 7.

$$x = \frac{1}{5}\left\{0.03034 + 0.27941 \log_{10} 24 \sqrt[3]{2}\,\frac{S}{d}\right\} + 0.05588 \log_{10} \frac{(S/m)^4}{\sqrt[3]{4}\,\sqrt[5]{3^4}}$$

$$\sqrt[15]{\left[1+\left(\frac{m}{S}\right)^2\right]^{11}\left[1+\left(\frac{2m}{S}\right)^2\right]^{7}\left[1+\left(\frac{3m}{S}\right)^2\right]^{4}\left[1+\left(\frac{4m}{S}\right)^2\right]^{2}\left[1+\left(\frac{m}{2S}\right)^2\right]^{4}\left[1+\left(\frac{3m}{2S}\right)^2\right]^{2}}$$

ohms per mile **(11b)**

Appendix C

DETERMINATION OF THE DISRUPTIVE CRITICAL VOLTAGE OF A MULTIPLE-CONDUCTOR TRANSMISSION LINE

By S. B. Crary

For a transmission circuit which has one conductor per phase, the assumption that the potential gradient is uniform around the surface of the conductor is justifiable, when the ratios of diameters of conductors to spacing between phases, $\frac{d}{S}$, is small. However, a conductor of a multiple-conductor circuit may be located relatively close to the other conductors of the same phase. An appreciable field distortion may result which will lower the disruptive critical voltage. The following analysis determines the disruptive critical voltage, assuming that the field due to the conductors of the other phases is negligible and that the field produced by the other conductors of the same phase may be considered uniform in the region of the conductor under consideration.* By comparison with an exact solution for two equally charged conductors, developed by Dr. H. Poritsky, the latter assumption was found to be essentially correct for a ratio of spacing to diameter of conductor as low as 5 to 1.

Let

E_{o1} = the field produced by the other conductors of the same phase in the region of conductor 1. Conductor 1 may be any conductor in any phase.

$$\therefore E_{o1} = \sqrt{\overline{\Sigma E_x}^2 + \overline{\Sigma E_y}^2} \qquad \textbf{(1c)}$$

The intensity at a distance m from a line charge of Q per unit length is

$$E = \frac{2Q}{m} \qquad \textbf{(2c)}$$

Equation (**1c**), therefore, becomes

$$E_{o1} = 2Q\sqrt{\left[\sum \frac{1}{m_{1n}} \cos\theta_{1n}\right]^2 + \left[\sum \frac{1}{m_{1n}} \sin\theta_{1n}\right]^2} \qquad \textbf{(3c)}$$

where

m_{1n} = the distance between conductor 1 and the nth conductor in the same phase.

θ_{1n} = the angle between a line joining the centers of conductors 1 and n and the reference axis.

At the surface of a conducting cylinder in a uniform field the maximum field intensity is $2E_{o1}$, and the total maximum intensity at the surface when the conductor is charged is obtained by adding to $2E_{o1}$, $\frac{2Q}{r}$†. Therefore the maximum intensity is

$$E_{m1} = 2E_{o1} + \frac{2Q}{r} \qquad \textbf{(4c)}$$

where

r = the radius of the conductor.

It will be assumed that the charge Q per unit of length is the same on all the conductors of the same phase.

Substituting (**3c**) in (**4c**) and replacing r by $\frac{d}{2}$ equation (**5c**) is obtained.

$$E_{m1} = 4Q\left\{\sqrt{\left[\sum \frac{1}{m_{1n}} \cos\theta_{1n}\right]^2 + \left[\sum \frac{1}{m_{1n}} \sin\theta_{1n}\right]^2 + \frac{1}{d}}\right\} \qquad \textbf{(5c)}$$

If the phase positions on the tower are designated by I, II and III and the conductor positions in the phases by 1, 2, — — n, *1*, *2*, — — *n*, **1**, **2**, — — **n** respectively, Fig. 10, the potential of conductor 1 in phase I will be:‡

$$\begin{aligned} V_1 = {} & P_{11} Q_1 + P_{12} Q_2 \text{— — — — —} + P_{1n} Q_n \\ & + P_{\mathit{11}} Q_{\mathit{1}} + P_{\mathit{12}} Q_{\mathit{2}} \text{— — — — —} + P_{\mathit{1n}} Q_{\mathit{n}} \qquad \textbf{(6c)} \\ & + P_{\mathbf{11}} Q_{\mathbf{1}} + P_{\mathbf{12}} Q_{\mathbf{2}} \text{— — — — —} + P_{\mathbf{1n}} Q_{\mathbf{n}} \end{aligned}$$

When the ratio of $\frac{2H}{S}$ is large (H = height above ground) the coefficients become

$$P_{11} = 2 \log_e \frac{4H}{d}$$

---

*Suggested by M. L. Henderson.

†Page and Adams, "Principles of Electricity," pp. 97-98.

‡It is apparent that the equations developed are general and apply to any arrangement of conductors and phases. However, a specific arrangement as that shown in Fig. 10 and a particular conductor (1) in a particular phase (I) has been chosen for clearness. The equations can be applied when conductor 1 is any conductor and phase I is any phase.

$$P_{1n} = 2 \log_e \frac{2H}{m_{1n}}$$

$$P_{1n} = 2 \log_e \frac{2H}{S_{1n}}$$

$$P_{1n} = 2 \log_e \frac{2H}{S_{1n}}$$

For a three-phase system the charge on the other two-phase conductors is $-nQ$ per unit of length when the charge is nQ per unit of length on phase 1. Assuming that the charge on each of the other phase conductors is $-\dfrac{Q}{2}$ when the charge on conductor 1 is Q, the potential of conductor 1 of phase 1 becomes

$$V_1 = 2Q \log_e \frac{2\sqrt{(S_{11}\, S_{12} \;.\;.\;.\; S_{1n})\,(S_{11}\;.\;S_{12}\;.\;.\;.\;S_{1n})}}{d\,(m_{12}\;.\;.\;.\;.\;.\;.\;.\;.\;m_{1n})} \tag{7c}$$

Substituting the value of Q determined from equation (5c) in equation (7c) and 21.1 kv/cm.* for E_{m1} the following expression is obtained for the disruptive critical voltage in effective kv. to neutral of conductor 1:

$$V_1 = \frac{21.1 \log_e \dfrac{2\sqrt{(S_{11}\, S_{12}\;.\;.\;.\;S_{1n})\,(S_{11}\;.\;S_{12}\;.\;.\;.\;S_{1n})}}{d\,(m_{12}\;.\;.\;.\;.\;.\;.\;.\;.\;m_{1n})}}{2\left\{\sqrt{\left[\sum \dfrac{1}{m_{1n}} \cos\theta_{1n}\right]^2 + \left[\sum \dfrac{1}{m_{1n}} \sin\theta_{1n}\right]^2} + \dfrac{1}{d}\right\}} \tag{8c}$$

In any arrangement (on the basis of the preceding assumptions) the conductors of the center phase have a lower disruptive critical voltage than those of the two outside phases, as the numerator of the argument of the logarithm of equation (8c) has a smaller value for the center phase conductors than it has for the outside phase conductors. Further from an analysis of this equation, it is apparent that a certain conductor of the center phase may have the lowest critical voltage depending upon the arrangement of the conductors of that phase. If the arrangement of the conductors in a given phase is symmetrical, however, the disruptive critical voltage will be the same for all the conductors of that particular phase.

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Discussion

DOUBLE CONDUCTORS FOR TRANSMISSION LINES

(DWIGHT AND FARMER)

THREE-PHASE MULTIPLE-CONDUCTOR CIRCUITS

(CLARKE)

Hillel Poritsky: The case of equal charges of opposite sign is a familiar one from two-phase lines or from considerations of the cylindrical condenser. As is well known, the field may be expressed in terms of logarithmic functions, and yields circles both for the lines of force and the equipotentials. On the other hand, it will be seen that the case of equal charges of the *same* sign requires doubly periodic or elliptic functions for its treatment.

This discussion is confined to the case of cylinders of the *same* radius a. Let Q be the charge on each cylinder per unit length of axis, and $2b$ the (shortest) distance between the cylindrical surfaces. In a plane perpendicular to the cylinder axes and cutting the cylindrical surfaces in circles C_1, C_2 place a coordinate system with the x-axis intersecting the cylinder axes and the origin half way between the center of circles C_1, C_2. The problem resolves itself into finding a harmonic function u (the potential) which takes on the same constant value along the circle C_1, C_2, and whose conjugate harmonic function v (or flux function) increases by $4\pi Q$ for a path which encircles either C_1 or C_2 alone in the positive sense; moreover, at infinity u is to become infinite like $2Q \log (x^2 + y^2)$. This problem is treated best by means of complex quantities and the theory of analytic functions.

In the xy-plane we introduce the complex variable

$$z = x + iy,\; i^2 = -1.$$

As is known the potential u and its conjugate harmonic function v define an analytic function,

$$w = w(z) = u + iv. \tag{1}$$

First we shall carry the problem from the z-plane to a z_1-plane, where

$$z_1 = x_1 + iy_1 = \log \frac{c + z}{c - z},\; c = \sqrt{b\,(b + 2a)}\,. \tag{2}$$

This carries the circles C_1, C_2 into the straight lines

$$x_1 = \pm d, \tag{3}$$

where

$$d = \log \frac{\sqrt{b + 2a} + \sqrt{b}}{\sqrt{b + 2a} - \sqrt{b}} \qquad d > 0. \tag{4}$$

The region outside the circles is transformerd into the region between the lines (3) and the original field goes into a field which is periodic in the y_1-direction of period 2π. Since the potential u is constant along $x_1 = \pm d$, the field may be extended across each of these boundaries to a width $2d$ on each side, then extended across the new boundaries, etc., resulting finally in a doubly periodic field, the periods being $4d$ and $2\pi i$. This field consists of sinks of lines of force at $z_1 = \pi i$, the image of $z = \infty$, and of its congruent points, and of sources at $z_1 = 2d + \pi i$ and its congruent points. At each source $8\pi Q$ lines start, and an equal number ends at each sink.

The field in question is described by means of

$$w = u + iv = 4Q \log \left[\Theta_1(z_2)/\Theta_1\left(z_2 - \frac{1}{2} \right) \right]. \tag{5}$$

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*F. W. Peek, Jr., "Dielectric Phenomena in High-Voltage Engineering."

where

$$z_2 = (z_1 - \pi i)/4d, \tag{6}$$

and Θ_1 is defined by means of

$$\Theta_1 (v,\omega) = 2 \sum_{n=0}^{\infty} (-1)^n q^{\left(n+\frac{1}{2}\right)^2} \sin [(2n+1)\pi v], \; q = e^{\pi i \omega} \tag{7}$$

and

$$\omega = 2\pi i/4d. \tag{8}$$

The variable z_2 is introduced merely as a change of scale, reducing the periods $4d$, $2\pi i$ of the z_1-plane to unity and ω (given by (8)) in the z_2-plane; moreover, the origin has been shifted to one of the sinks correspondingt o $z = \infty$. The function $\Theta_1 (z_2)$ (the second argument ω has not been indicated) has roots at $z_2 = m + n\omega$ (see "Functions of a Complex Variable," by Pierpont, pp. 428, 429), hence

$$\Theta_1 (z_2)/\Theta_1 \left(z_2 - \frac{1}{2} \right)$$

has roots and poles at the sinks and sources respectively of the above field; it may also be shown to be doubly periodic.

To obtain the field intensity in the z-plane we need to know dw/dz. This may be evaluated from the formula

$$\frac{dw}{dz} = \frac{dw}{dz_2} \, \frac{dz_2}{dz_1} \, \frac{dz_1}{dz}.$$

This maximum field intensity occurs at $z = b + 2a$ and comes out to be

$$2\pi Q \frac{\sqrt{b}}{ad \sqrt{b + 2a}} \; \frac{1 + 3q^{1\cdot 2} - 5q^{2\cdot 3} - 7q^{3\cdot 4} + 9q^{4\cdot 5}}{1 - q^{1\cdot 2} - q^{2\cdot 3} + q^{3\cdot 4} + \ldots}$$

The case of two cylinders which touch one another may be handled either as a limiting case of the above by letting b approach zero, or may be treated directly by means of the transformation

$$z_1 = 1/z$$

in place of (2). The field, after reflection across the straight lines corresponding to C_1, C_2 and further reflection is now simply periodic and may be described by means of circular functions. Thus

$$w = 4Q \log \left[\sin z_2 / \sin \left(z_2 - \frac{\pi}{2} \right) \right],$$

where $z_2 = \dfrac{\pi a}{2} z_1$,

and the maximum intensity is $\dfrac{Q\pi}{a}$

S. B. Crary: In determining the voltages of transmission lines which have only one conductor per phase, the assumption is usually made that the potential gradient around the surface of the conductor is uniform. This is justifiable when the ratios of diameter of conductor to spacing between phases is small, which is usually the case. However, a conductor of a multiple-conductor circuit may be located relatively close to the conductors of the same phase. An appreciable field distortion may result. This field distortion is produced by the proximity of conductors having charges of equal magnitude and of the same sign.

The preceding discussion, by Dr. H. Poritsky, describes the method in which he obtained the exact solution for the maximum potential gradient of two cylinders having charges equal in magnitude and of the same sign. This formula may be applied in order to determine the critical disruptive voltage of a double-conductor transmission circuit. The exact solution for more than two conductors would undoubtedly lead to a result more involved and difficult to apply.

In order to obtain a result of sufficient engineering accuracy for the practical cases formula (5c) given in Miss Clarke's paper was developed. This formula corrects for the field distortion due to the conductors of the same phase by making two assumptions that are applicable to practical multiple-conductor transmission circuits. These are:

1. The field distortion due to the conductors of the other phases and the ground plane is negligible.
2. The field produced by the other conductors of the same phase may be considered uniform in the region of the conductor under consideration.

The formula of Dr. Poritsky's provided an excellent means of determining the correctness of the second assumption. Fig. 1 shows graphically the relative accuracy obtained by using the approximate formula (5c) given in Miss Clarke's paper as compared with the result obtained by using Dr. Poritsky's formula. The approximate formula is seen to be essentially correct for a ratio of m/d as low as 5 to 1, which covers the practical range of spacings for multiple-conductor circuits.

The lower curve shows the relative gradient obtained when the field distortion is neglected. For a ratio of m to d of 5, an error of 15 per cent would be obtained if the field distortion were not considered.

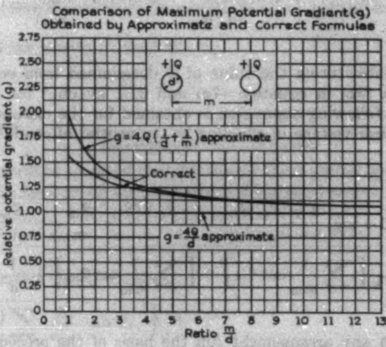

FIG. 1

Good agreement was found between the test results given in the text by F. W. Peek, Jr., "Dielectric Phenomena in High Voltage Engineering," Figs. 57 and 59, for the visual corona for two conductors at the same potential and for a triangular arrangement of conductors. Equation (8c) for the disruptive critical voltage was used, modified for single-phase and multiplied by the factor $\left(1 + \dfrac{0.301}{\sqrt{r}} \right)$ in order to determine the visual corona voltage.

W. W. Lewis: The General Electric Company has been studying the use of multiple conductors from time to time for a great many years. When the economical side is worked out, multiple conductors do not look so attractive. This is mainly because the large weight of conductors in this scheme requires heavy towers and short spans. In order to obtain continuously the theoretical advantages in the way of decreased reactance, increased capacitance and increased corona starting voltage, it is desirable to maintain as closely as possible the spacing between conductors in the same phase and in the different phases. This requires special insulating arrangements and supporting and separating yokes, which increase the expense of the installation.

About two years ago, we made a study of a transmission system, intended to deliver 500,000 kilowatts over a distance of 300 miles. Split conductors, 345-kv. and 230-kv. lines were considered and compared with conventional lines of the same voltage. The power limit of each arrangement was on the basis of steady state stability.

With the split conductor, 345-kv. line, two conductors were used per phase, each 1,192,500 cir. mils A.C.S.R. Various spacings between conductors and between phases and various lengths of span were considered and the most economical arrangement adopted. This line would carry 500,000 kw. The investment, including line, step-up and step-down transformers, synchronous condensers, switchgear and substations, was $48.40 per kw. and the yearly charge $7.90 per kw. Two conventional lines (one conductor per phase) for 345 kv., would carry 660,000 kw. and entail an investment of $49 per kw. and a yearly charge of $7.60 per kw.

Two split conductor 230-kv. lines would carry 470,000 kw. at an investment cost of $63.30 per kw. and a yearly charge of $10.70 per kw. Four conventional 230-kv. lines would carry 600,000 kw. at an investment cost of $64.20 per kw. and a yearly charge of $10.50 per kw.

From these figures it appears that at 345 kv., two conventional lines have about the same cost per kw. and same yearly charge as one split conductor line. The two conventional circuits have a greater total capacity and have the obvious advantage that the total flow of power is not dependent upon one tower line. At 230 kv., the cost per kw., and the yearly charge on two split conductor lines and four conventional lines is about a toss-up. However, the capacity of the two split conductor lines is somewhat less than required in this particular case; while the four conventional lines have a considerable margin in capacity.

Since the split conductor line is untried in practise and has certain constructional and operating disadvantages with no advantage in cost, it seems desirable to adhere to the conventional construction for the present.

W. S. Moody: The possibilities of using higher voltages than 220,000 are very welcome to all who have contributed to the gradual and steady rise in transmission voltages from 10,000 to 220,000 volts. For some 20 years this increase, as we all know, was roughly but quite regularly some 10,000 volts per year. Since this progress stopped some ten years ago, transformer engineers have been "chafing at the bit" like race horses before the barrier. The only comfort was in the fact that this restraint was the result of there being no more worlds to conquer, at present, rather than any inability to keep up the progress on the part of the engineers.

Higher voltages than 220,000 are *commercially* practical only when very large power is to be transmitted a long distance before distribution of any of this block of power begins. Such opportunities will doubtless come along soon after commercial conditions are again normal notwithstanding constant improvement in steam plants. Advantage should be taken of present conditions therefore to discuss and solve the problems involved so as to have them ready before the first opportunity to use them arrives.

H. B. Dwight: The question whether double conductors may sometimes be advantageous for transmission lines is most naturally investigated by comparing designs which use single- and double-conductor construction. Unless some very cogent reason is encountered, all other features of the alternative designs should be alike—they should have the same voltage, the same resistance per phase, that is, approximately the same conductor weight, etc. The influence of extraneous features should be avoided. The comparisons in our paper, which does not deal with record-breaking voltages, are on the above basis and it seems easy and natural to make them so.

With the comparisons so direct, the following conclusions for the range considered seem to stand forth:

a. There is approximately 20 per cent advantage in kw. rating in favor of double-conductor construction, when reactance determines the allowable load.

b. This advantage may or may not be neutralized by extra cost due to special mechanical features, depending particularly on whether provision is to be made for a large ice load.

c. Very little advantage as regards corona is obtained from double-conductor construction, if the disruptive critical voltage e_0 is used as the criterion.

If now hollow conductors are used, part of the advantage as regards reactance is taken away, and probably all of the advantage as regards corona, at present voltages. None of the papers at this meeting seems to have discussed definitely the question as to how thin and of how large diameter a hollow conductor may be, for successful use. Without an exact answer to this question, it is very difficult to make comparisons at extreme, record-breaking voltages.

However, it seems that a transmission line using single, hollow conductors could be built for ratings much beyond those now used, and in cases where this is so, comparisons of double-conductor lines should be made on the basis used in our paper, of equal voltage and conductor weight per phase. Comparisons become much easier when it is admitted for a certain case that a single-conductor line can be built to duplicate a proposed double-conductor line exactly except for the number of ohms of reactance.

The reactance seems to be the most important feature at present and it may, in favorable cases, make double-conductor construction advisable.

Solution of Circuits Subjected to Traveling Waves

BY HAROLD L. RORDEN*
Associate, A.I.E.E.

Synopsis.*—The major purpose of this paper is to illustrate how accurately the physical assumptions made in applying conventional traveling wave theory are substantiated by test results.*

Analytical solutions for traveling waves at a transition point, as given in "Traveling Waves on Transmission Systems," A.I.E.E. TRANS., *1931 (L. V. Bewley) are used in this analysis. These derivations involve the solution of circuits that may be used to represent equivalent terminal conditions of transmission systems, when simplifying assumptions are made, and when the free traveling wave can be expressed in exponential form. Discussion of types and specifications of such waves, attenuation and distortion, and the reflection and refraction operators, is limited to that essential for an understanding of the procedure. A method of calculating successive reflections on a short line, and a step-by-step solution involving a variable parameter at a transition point are included.*

* * * * *

GENERAL

THE conventional theory of traveling waves is based on a single-conductor circuit and supposes the transmission line to be ideal. That is, the line may be represented as a pure surge impedance $\sqrt{L/C}$, where L and C are to be determined on the assumption that ground is a true zero potential surface. The losses of the line are negligible for waves below the critical corona point. Coupling with adjacent conductors is ignored, and the waves are calculated by the simple single-conductor theory, *i. e.*, the associated electrostatic and electromagnetic fields exist between the conductor and its image an equal distance below the ground surface.

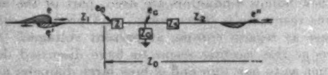

FIG. 1—EQUIVALENT CIRCUIT OF GENERAL NETWORK

Comparative tests conducted to determine the relative error involved in the preceding assumptions illustrate several conclusive points:

1. Traveling waves on single-conductor circuits, if below the corona limit, can be calculated with excellent engineering accuracy when the parameters of the transition point are known.
2. Successive reflections can be calculated accurately by means of superposition, and the calculation facilitated by lattices.
3. Step-by-step calculations are applicable to traveling wave problems involving transition points with variable parameters.

The effect of traveling waves on terminal apparatus is usually most easily solved by means of Heaviside's operational calculus. Solutions have been given for various terminal apparatus that may be encountered in service, by considering them to be simple lumped impedances. These are intended to approximate the general conditions of a transmission system, which may be reduced to the equivalent circuit of Fig. 1. In Fig. 1, Z_1 represents the surge impedance of the line on which the transient wave originates, Z, Z_s, and Z_g, are terminal impedances, and Z_2 the surge impedance of all outgoing lines in parallel.

With the cathode ray oscillograph valuable information has been obtained relative to the wave shape and other peculiarities of traveling waves. Many irregularities exist in these waves, but generally they may be reproduced to a fair enough degree of accuracy so that their effect may be determined both analytically and experimentally.

WAVE SHAPES

Transients of the order of lightning waves may be approximately represented by the difference of two exponentials, merely by a proper adjustment of parameters. This is of considerable importance in effecting the general solution of traveling waves. Fig. 2 illustrates different types of waves that may be expressed as the difference of two exponential terms, of the form $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$. The parameters a and b may be adjusted to give any of the wave shapes shown in Fig. 2. Probably the most common of these waves are of the

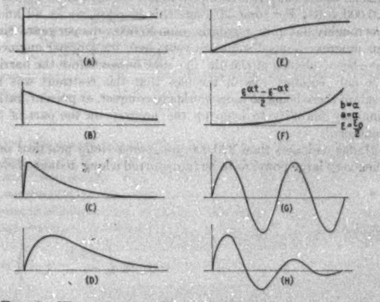

FIG. 2—WAVE SHAPES GIVEN BY $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$

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*Power Transformer Dept., General Electric Co., Pittsfield, Mass.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

form illustrated by C and D of Fig. 2, where both a and b are real and positive, and they are the simplest to duplicate in the laboratory.

A set of curves has been devised by Mr. T. Brownlee whereby a wave of the exponential form in which a and b are real and positive can be defined completely, knowing only the time (t_1) for the wave to reach its crest, and the total time (t_2) to fall to half its crest voltage. These curves were plotted empirically on the assumption that for a given ratio of t_2/t_1, there would be a fixed value of b/a. However, the truth of this relation may be derived rationally. By differentiating the equation for e, and equating to zero for the maximum, it may be shown that the time to reach the crest is

$$t_1 = \frac{\log (b/a)}{b - a} \tag{1}$$

Therefore,

$$at_1 = \frac{\log (b/a)}{\dfrac{b}{a} - 1} = B \tag{2}$$

The crest voltage E_1 is given by

$$E_1 = E\ (\epsilon^{-at_1} - \epsilon^{-bt_1}) \tag{3}$$

$$\frac{E_1}{2} = E\ (\epsilon^{-at_2} - \epsilon^{-bt_2}) \tag{4}$$

Substituting (2) in (3), there is

$$\frac{E_1}{E} = \epsilon^{-B} - \epsilon^{-(b/a)B} = R \tag{5}$$

The main function of the parameter b is to determine the front of the wave, and for all excepting the cases where t_2 approaches t_1, it is so large compared to a that $\epsilon^{-bt_2} \cong 0$.

Solving (4) for at_2 and combining with (1), there results

$$t_2/t_1 = \frac{\log \dfrac{2}{R}}{B} \tag{6}$$

from which it follows that t_2/t_1 is a function of (b/a), since both R and B have been shown to be functions of (b/a). Thus the three expressions at_1, E_1/E, and t_2/t_1 may be plotted against the common term b/a. Fig. 3 shows such a set of curves, from which a wave can be completely specified. As an example, let us suppose the record of a lightning wave shows it to reach its crest in 3 microseconds and to fall to half-crest value in 21 microseconds. Then $t_2/t_1 = 7$, and from Fig. 3, $b/a = 28.5$. For this value of b/a, $at_1 = 0.122$ and $E_1/E = 0.852$.

Solving,

$$a = 0.041,\ b = 1.15$$

The wave is then completely defined as

$$e = E\ (\epsilon^{-.041t} - \epsilon^{-1.15t})$$

where the crest voltage will reach 85.2 per cent of E, and t is measured in microseconds. The ratio E_1/E is particularly valuable in laboratory studies where capacitors are used as a lightning generator, charged to a voltage E. Otherwise it is necessary to determine the effect of waves on terminal impedances, as will be shown later. From the incident wave the resultant stresses on terminal apparatus may be completely determined, as will also be shown later.

The curves of t_2/t_1 in Fig. 3 are plotted on the assumption that $\epsilon^{-bt_2} = 0$. The error thus involved becomes appreciable when b/a is less than 2.5. By expressing equation (4) in the form

$$\frac{E_1}{2E} = \epsilon^{-B(t_2/t_1)} - \epsilon^{-B(b/a)(t_2/t_1)}$$

and plotting a family of curves for the lower values of t_2/t_1 together with $R/2$ from equation (5), it may be shown rationally that t_2/t_1 remains a function of b/a throughout its lower range. To extend the curves of Fig. 3 to the lower limit where $b = a$ involves a trial and error method, and does not extend the lower limit of t_2/t_1 appreciably.

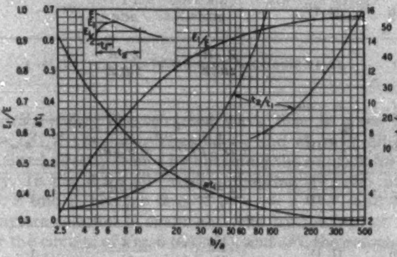

FIG. 3—GRAPHICAL DETERMINATION OF WAVES EXPRESSED BY $e = E\ (\epsilon^{-at} - \epsilon^{-bt})$

GENERAL SOLUTION

By considering terminal impedances to consist of lumped constants, the determination of voltages and waves resulting from an incoming free traveling wave can be effected in terms of the reflection and refraction operators. With the symbols as defined in Fig. 1, these operators are

$$\left(\frac{Z_o - Z_1}{Z_o + Z_1}\right) = \text{reflection operator.}$$

$$\left(\frac{Z_G}{Z_G + Z_S + Z_2}\right)\left(\frac{2Z_2}{Z_o + Z_1}\right) = \text{refraction operator.}$$

All impedances are expressed in operational form, the subscript (p) being dropped for simplicity, and they derive their respective voltages when applied to the incident wave. The total voltage e_o at the transition point (Fig. 1) is the sum of the incident and the reflected waves ($e_o = e + e'$) for which the operator is $2Z_o/(Z_o + Z_1)$.

TABLE I—CIRCUITS OF FIG. 4

$e_0 = e + e'$

	e'	α	β	A
A........	e			
B........	$\frac{R-Z}{R+Z}e$			
C........	$-e$			
D........	I, III	$\frac{Z}{L}$	$\frac{Z}{L}$	1
E........	I, III	$\frac{1}{CZ}$	$\frac{1}{CZ}$	-1
F........	I, III	$\frac{ZR}{L(R-Z)}$	$\frac{ZR}{L(R+Z)}$	β/α
G........	I, III	$\frac{R-Z}{ZRC}$	$\frac{R+Z}{ZRC}$	-1
H........	II, IV	$-\frac{1}{2CZ}$	$\frac{1}{2CZ}$	-1
I........	II, IV	$\frac{Z-R}{2RCZ}$	$\frac{Z+R}{2RCZ}$	-1
J........	$e - Zi_0$			
K........	I, III	$\frac{1}{C(Z-R)}$	$\frac{1}{C(Z+R)}$	$-\beta/\alpha$
L........	I, III	$\frac{Z-R}{L}$	$\frac{Z+R}{L}$	1
M........	II, IV	$\frac{-Z}{2L}$	$\frac{Z}{2L}$	1
N........	II, IV	$\frac{R-Z}{2L}$	$\frac{R+Z}{2L}$	1
O........	Same as *N* before gap sparkover Same as *C* after gap sparkover			

By the refracted wave is meant the voltage, as a function of time, that passes onto the line Z_2, and is equal to e''.

All of the circuits in Figs. 4, 5 and 6 may be solved by one of two general equations,[1] the one to apply depending on whether or not the impedances are such as to make oscillations possible. For an incident wave of the form $e = E\epsilon^{-at}$ these expressions are

$$AE\left[\frac{a+\alpha}{a-\beta}\epsilon^{-at} - \frac{\alpha+\beta}{a-\beta}\epsilon^{-\beta t}\right] \text{ for the non-oscillatory circuits} \qquad \text{(I)}$$

and

$$AE\left\{\frac{\omega_o^2 - 2a\alpha + a^2}{\omega_o^2 - 2a\beta + a^2}\epsilon^{-at}\right.$$

$$\left.+\frac{2(\alpha-\beta)\epsilon^{-\beta t}}{\omega(\omega_o^2 - 2a\beta + a^2)}\left[(\omega_o^2 - a\beta)\sin\omega t + a\omega\cos\omega t\right]\right\}$$

for the oscillatory circuits (II)

where $\omega_o^2 = 1/LC$ and $\omega^2 = \omega_o^2 - \beta^2$, for the oscillatory circuits. α, β, and the amplitude factor A, are functions of the impedances. In an earlier publication[2] it was shown that for an incident wave containing two or more exponential terms, the above expressions could readily be expanded to correspond. Thus for an incident wave of the form $e = E(\epsilon^{-at} - \epsilon^{-bt})$ the expressions (I) and (II) become

$$AE\left[\frac{a+\alpha}{a-\beta}\epsilon^{-at} - \frac{b+\alpha}{b-\beta}\epsilon^{-bt} + \frac{(\alpha+\beta)(a-b)}{(a-\beta)(b-\beta)}\epsilon^{-\beta t}\right] \qquad \text{(III)}$$

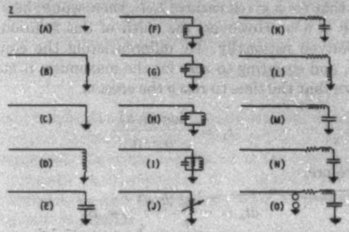

FIG. 4—TERMINAL IMPEDANCES

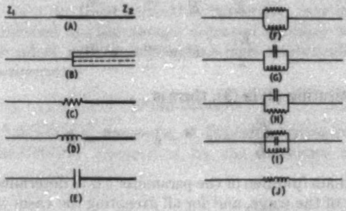

FIG. 5—JUNCTIONS BETWEEN CIRCUITS

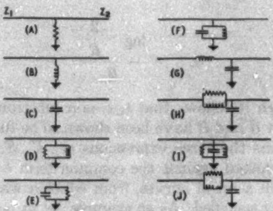

FIG. 6—JUNCTIONS BETWEEN CIRCUITS

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1. For references see Bibliography.

and

$$AE\left\{\frac{\omega_o^2 - 2a\alpha + a^2}{\omega_o^2 - 2a\beta + a^2}\epsilon^{-at} - \frac{\omega_o^2 - 2b\alpha + b^2}{\omega_o^2 - 2b\beta + b^2}\epsilon^{-bt}\right.$$

$$+ \frac{2(\alpha - \beta)\epsilon^{-\beta t}}{\omega(\omega_o^2 - 2a\beta + a^2)}[(\omega_o^2 - a\beta)\sin\omega t + a\omega\cos\omega t]$$

$$\left. - \frac{2(\alpha - \beta)\epsilon^{-\beta t}}{\omega(\omega_o^2 - 2b\beta + b^2)}[(\omega_o^2 - b\beta)\sin\omega t + b\omega\cos\omega t]\right\} \tag{IV}$$

The values of ω_o and ω are as defined for (I) and (II). In (II) and (IV) where the combination of L and C is such that the circuit is more than critically damped, $\omega = j\sqrt{\beta^2 - \omega_o^2}$ and the resultant expression reduces by the familiar substitution of hyperbolic functions to a real term.

TABLE II—CIRCUITS OF FIG. 5

$e_o = e + e'$ $e'' = \dfrac{Z_2}{Z_1}(e - e')$

e'	α	β	A
A.. $\dfrac{Z_2 - Z_1}{Z_2 + Z_1}e$			
B.. $\dfrac{Z_t - Z_1}{Z_t + Z_1}e$	Z_t = Surge impedance of all outgoing lines in parallel		
C.. $\dfrac{Z_2 - Z_1 + R}{Z_2 + Z_1 + R}e$			
D.. I, III	$\dfrac{Z_1 - Z_2}{L}$	$\dfrac{Z_1 + Z_2}{L}$	1
E.. I, III	$\dfrac{1}{C(Z_1 - Z_2)}$	$\dfrac{1}{C(Z_1 + Z_2)}$	β/α
F.. I, III	$\dfrac{R(Z_1 - Z_2)}{L(R - Z_1 + Z_2)}$	$\dfrac{R(Z_1 + Z_2)}{L(R + Z_1 + Z_2)}$	$\dfrac{R - Z_1 + Z_2}{R + Z_1 + Z_2}$
G.. II, IV	$\dfrac{1}{2C(Z_2 - Z_1)}$	$\dfrac{1}{2C(Z_2 + Z_1)}$	β/α
H.. I, III	$\dfrac{R - Z_1 + Z_2}{RC(Z_1 - Z_2)}$	$\dfrac{R + Z_1 + Z_2}{RC(Z_1 + Z_2)}$	$\dfrac{Z_2 - Z_1}{Z_2 + Z_1}$
I.. II, IV	$\dfrac{Z_2 - Z_1 + R}{2RC(Z_2 - Z_1)}$	$\dfrac{Z_2 + Z_1 + R}{2RC(Z_2 + Z_1)}$	$\dfrac{Z_2 - Z_1}{Z_2 + Z_1}$
J. I, III	$\dfrac{Z_1 - Z_2 + R}{L}$	$\dfrac{Z_1 + Z_2 + R}{L}$	1

Most of the circuits shown in Figs. 4, 5, and 6 have been calculated in this analysis. In the circuits of Fig. 4 where Z_2 becomes infinite, there is no transmitted wave, and since the relation $e_o = e + e'$ is general, the complete solution is obtained from the incident wave by calculating for the reflected wave only. In Fig. 5, Z_Q becomes infinite and the refraction operator is given by $2Z_2/(Z_o + Z_1)$. The refracted wave then is given by $e'' = (Z_2/Z_1)(e - e')$. Thus e_o, e', and e'', may all be obtained directly from e by obtaining the solution for e' only. In Fig. 6 for all circuits excepting (G), (H), and (J), Z and Z_o become zero, $e'' = e_o = e + e'$, and the complete solution is again obtained by solving for e'. For the circuits of Fig. 6 (G), (H), and (J), the complete solution becomes somewhat more lengthy.

TABLE III—CIRCUITS OF FIG. 6

$e_o = e'' = e + e'$ (Exceptions, G, H, J)

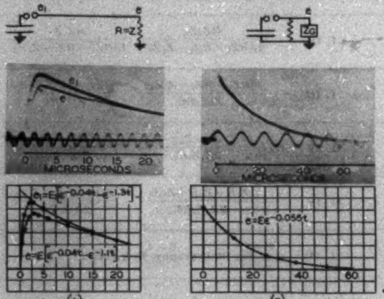
e'	α	β	A
A.. $\dfrac{Z_2R - Z_1R - Z_1Z_2}{Z_2R + Z_1R + Z_1Z_2}e$			
B........ I, III	$\dfrac{Z_1Z_2}{L(Z_2 - Z_1)}$	$\dfrac{Z_1Z_2}{L(Z_2 + Z_1)}$	β/α
C........ I, III	$\dfrac{Z_2 - Z_1}{Z_1Z_2C}$	$\dfrac{Z_2 + Z_1}{Z_1Z_2C}$	−1
D........ I, III	$\dfrac{RZ_1Z_2}{L(RZ_2 - RZ_1 - Z_1Z_2)}$	$\dfrac{RZ_1Z_2}{L(RZ_2 + RZ_1 + Z_1Z_2)}$	β/α
E........ I, III	$\dfrac{RZ_2 - RZ_1 - Z_1Z_2}{Z_1Z_2RC}$	$\dfrac{RZ_2 + RZ_1 + Z_1Z_2}{Z_1Z_2RC}$	−1
F........ II, IV	$\dfrac{Z_1 - Z_2}{2Z_1Z_2C}$	$\dfrac{Z_1 + Z_2}{2Z_1Z_2C}$	−1
I........ II, IV	$\dfrac{Z_1R - Z_2R + Z_1Z_2}{2Z_1Z_2RC}$	$\dfrac{Z_1R + Z_2R + Z_1Z_2}{2Z_1Z_2RC}$	−1
G H........ II, IV J	See Bibliography, Reference 2		

Since, as has been shown, the complete solutions may be obtained by solving for the reflected wave only, it is necessary to apply the parameters to the general equation only for this one solution. α, β, and A for the circuits of Figs. 4, 5, and 6 are given in Tables I, II, and III, by means of which the circuits may be completely solved for any incident wave expressed in the exponential form.

EXPERIMENTAL LINE—SURGE IMPEDANCE

For test purposes, a transmission line about two miles long (11½ microseconds) was used, this line consisting of two parallel conductors spaced eight feet apart and forming a loop so that both ends terminated at the laboratory where both the lightning generator and the oscillograph were located. For the circuits of Fig. 5 both lines were at first used, with the result that waves were badly distorted due to mutual effects between the two lines. Since it is not the purpose here to investigate the effects of mutually coupled circuits, the outgoing line from the terminal impedance was replaced by a resistance equivalent to the line surge impedance. That this is an exact substitution may be shown mathematically, and is verified by the results obtained.

In Fig. 7A the oscillograms e_1 and e are the waves recorded by the cathode ray oscillograph at both ends of the line, when a resistance $R = Z$ was connected to ground as is illustrated. The magnitude of the line surge impedance Z may be determined in several ways. Since inserting a resistance $R = Z$ at the far end of the line makes it equivalent to approximating a line of infinite length, R may be adjusted until there are no reflections, as observed with the oscillograph. A more accurate method is to send a wave of fast front on the line, regardless of the impedance at the far end, and obtain a record of the wave at the impulse generator. Then the surge impedance may be very accurately calculated from the equation representing the wave. As an illustration let us observe the wave in Fig. 7A. In the sketch directly below the oscillogram the solid line e_1 is a reproduction of the wave at the generator. It is readily seen that for points well down on the wave tail, the equation is given very exactly by $e = E\,\epsilon^{-at}$, where $a = 1/ZC$, since the circuit is that of a capacitance discharging into an equivalent resistance. Thus by arbitrarily choosing two points t_1 and t_2 on the tail, the surge impedance may be obtained from the relation



(A) (B)

Fig. 7—Incident Waves

$$Z = \frac{t_2 - t_1}{C \log \dfrac{e_1}{e_2}}$$

where C is the capacitance of the impulse generator. The surge impedance of the line used in these tests is 510 ohms, obtained by this method. From this result E may be found by substituting $a = 1/ZC$ in the single exponential form for the incident wave, or it may be found directly from the curves of Fig. 3 by scaling the time to crest and time to half voltage. The value of E thus obtained is indicated by the broken line of Fig. 7A, and the complete equation of the wave, as obtained by the curves of Fig. 3 plot the points in the circles.

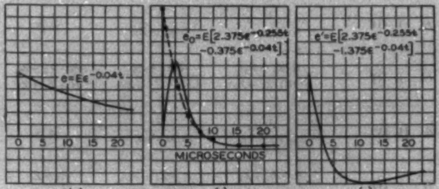
Where an oscillograph is not available, the surge impedance of a line may be approximated experimentally, but the method is not as accurate as the preceding one. This is done by sending a wave on the line from an impulse generator, measuring the voltage, and then substituting a resistance for the line, adjusting it until the measured voltage is the same as that obtained on the line. If the average height h of the line is known, and also the radius r of the conductor, the surge impedance may be found from the relation $Z = 60 \log (2h/r)$. Values thus obtained are found to check very well with those determined experimentally.

Incident Wave

The complete equation of a wave may be obtained readily by the method of superposition as well as by the curves of Fig. 3. Consider the complete wave to be made up of its component parts ϵ^{-at} and $-\epsilon^{-bt}$, plotting approximately into the wave tail and wave front respectively. Then a may be determined from the first term by the method described above, by arbitrarily choosing two points on the tail of the wave, and b may be found by treating the second term similarly.

The difference between the two waves of Fig. 7A is a measure of the voltage losses sustained by the wave in traveling about two miles. The lightning generator was charged to about 75 kilovolts. It will be observed that very little change has resulted in the time to reach crest voltage, but that the amplitude has dropped 17 per cent, and that the wave tail has lengthened. This phenomenon is not unusual, but it serves to illustrate what differences might be encountered if these losses were not considered in the mathematical analysis.

In this investigation the incident wave is that obtained directly at the terminal impedance when that impedance consists only of a resistance $R = Z$ to ground. But all oscillograms were not taken at the same time, and therefore the incident wave, although sent from the same generator and maintaining as nearly as possible the same voltage, has some slight differences. This, also, is not unusual.



(A) (B) (C)

Fig. 8—Error Due to Neglecting Wave Front

(See Fig. 10A)

Experimental Correlation

How precisely the mathematical solution of circuits involving necessary simplifying assumptions may agree with oscillograms is demonstrated in Figs. 9 to 17 inclusive. The labor of calculating the waves obtained at the terminal impedances would become greatly simplified if the incident wave could be considered of the single exponential form, *i. e.*, if the wave front were neglected. This is obvious from an inspection of the expressions (I), (II), (III), (IV). The labor is particularly tedious for the oscillatory cases. Whether or not the front can be neglected depends on the relative time constants of the circuit. In Fig. 8 is shown a case in which neglect of the front leads to serious error. Fig. 8A is the assumed form of incident wave, neglecting the front, where the actual wave is that of Fig. 7A. The solid line of Fig. 8B is a plot of the oscillogram of Fig. 10A and is the voltage wave e_o obtained at the terminal impedance when it consisted of an inductance of 0.002 henrys. The broken line joining the circles of Fig. 8B represents the calculated voltage, for which the actual voltage crest is in error 57 per cent. Fig. 8C is the reflected wave e', calculated only, such that $e' = e_o - e$.

In these tests no extraneous constants were in the circuit to produce a slow front wave, *i. e.*, the front deviated from reaching crest in zero time due only to physical limitations of the circuit. It would seem that in a study of traveling waves generally the front can not be neglected without involving a high degree of error.

The Figs. 9 to 14 inclusive illustrate the experimental and mathematical solutions of most of the circuits of Figs. 4, 5, and 6, for the specific impedances as given. The voltage scale of the plotted waves is comparable to the oscillographic deflection of the incident wave, rather than being given in kilovolts. For Figs. 9 to 13, the waves were sent over the line previously discussed, with an initial potential of about 75 kilovolts and the wave form e of Fig. 7A is taken as the incident free traveling wave. For Fig. 14 the waves were applied directly from the impulse generator, and the voltage e_o is of the form given in Fig. 7B. The waves of Fig. 14 have been published,[2] but since they serve as illustrations of several of the circuits of Fig. 6 they are repeated here. They serve the further purpose of illustrating the correlation that may be obtained when neglecting the front, with a voltage wave approximating as nearly as possible the single exponential form.

Points of interest of Figs. 9 to 14 will be discussed in order. For all the oscillograms and calculations the constants used in the impedances were as listed herewith, unless otherwise noted in the figures:

$$R = 1{,}000 \text{ ohms}$$
$$L = 0.002 \text{ henrys}$$
$$C = 0.0015 \times 10^{-6} \text{ farads}$$

For each case the circuit is shown at the top of the column, then the oscillogram of e_o, followed by the replot of the oscillogram and the calculated points (circles). The reflected wave e' is plotted from calculated values only, since the oscillographic record of this could only be obtained in these tests as the difference between e_o and e. However, they deviate from this only by the difference found in e_o, since the calculated incident wave checks the oscillogram very precisely. Next in the

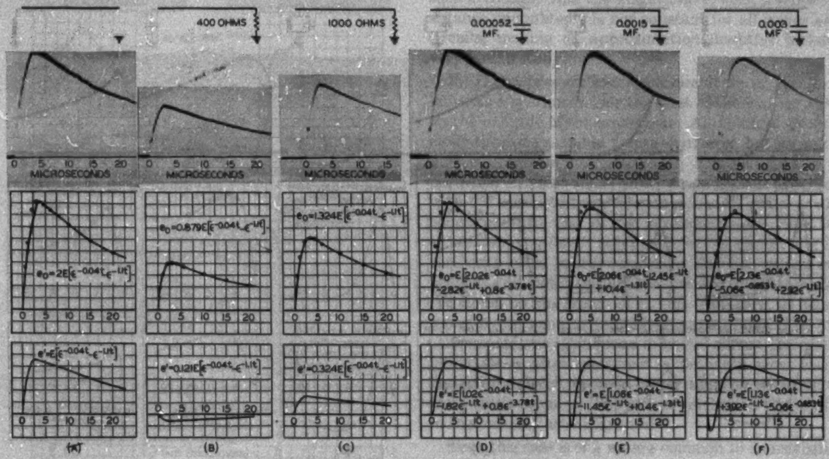


FIG. 9—CIRCUITS OF FIG. 4

$$e = E\,(\epsilon^{-0.04t} - \epsilon^{-1.1t}) \qquad e_0 = e + e'$$

column, where they exist, are given the oscillogram and calculations for the refracted wave e'', excepting where $e'' = e_o$.

FIG. 9—CIRCUITS OF FIG. 4

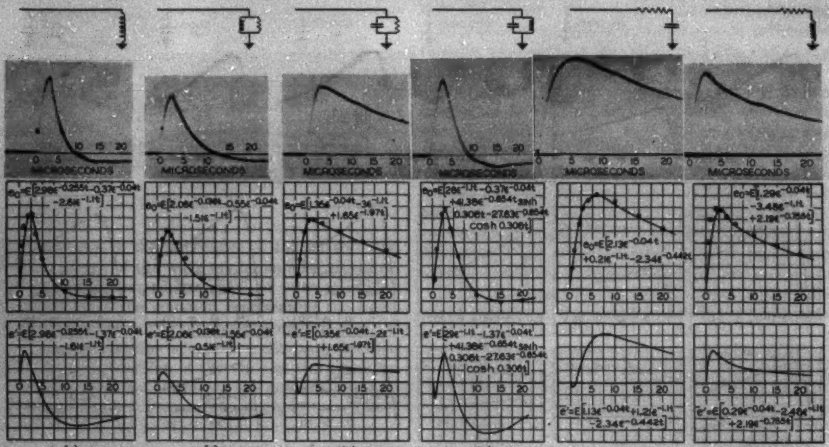
A. By superposition $e_o = 2e$ where the line is open at the far end, and the reflected wave $e' = e$, the incident wave.

B and C. With a resistance R less than Z the reflected wave is negative, and with R greater than Z the reflection is positive. These two waves were taken primarily to check the magnitude of the surge impedance obtained by the method previously described. From these two oscillograms, and the tentative value of 510 ohms for Z, the incident wave was calculated. Then with $R = 510$ ohms the wave e of Fig. 7A was obtained and found to check the calculation precisely.

mating the incident wave the mathematical and experimental results can be made to check to a surprising degree of accuracy. On this wave the only discrepancy to be found is a slight variation in the front. e' is the inverse of the form of Fig. 9F, showing the inductance to act as an open circuit for the initial, and a closed circuit for the final period of time.

B and C. These figures illustrate the effect of an inductance and capacitance respectively shunted by a resistance. e' illustrates the two opposite forms of reflected waves again.

D. This figure is the case of a combination impedance the equation for which comes under the oscillatory form, but is more than critically damped. e' illustrates the predominating effect of the capacitance in the initial stage and the inductance in the final stage.



(A) (B) (C) (D) (E) (F)

FIG. 10—CIRCUITS OF FIG. 4

$$e = E(\epsilon^{-0.04t} - \epsilon^{-1.1t}) \qquad e_0 = e + e'$$

D, E and F. These cases illustrate the effect of capacitances of different magnitudes on e_o and e'. The capacitance of 0.00052 μf. differs but little from an open circuit. As the capacitance is increased the negative loop of the reflected wave becomes larger, illustrating the well known principle that a capacitance acts as a short circuit for the first instant of time, and finally as an open circuit. The case of Fig. 4C is not illustrated since obviously $e_o = o$ and $e' = -e$. This represents an infinite capacitance, and the entire reflected wave is negative.

FIG. 10—CIRCUITS OF FIG. 4

A. This is a more exact solution of the circuit of Fig. 4D than that given in Fig. 8. By more exactly approxi-

E and F—are illustrations of Fig. 4K and L respectively.

FIG. 11—CIRCUITS OF FIGS. 4 AND 4O

This figure probably represents the most practical application to circuits found in service. It represents a terminal impedance consisting of a transformer at the end of a line and protected by a current limiting reactor. The series resistance illustrated is inherent in the reactor. The voltage scale of the oscillograms is reduced by a ratio of 1.73 to those of the preceding figures in order to obtain the complete record on the film. The sketches are plotted to the same voltage scale as the oscillograms for comparative purposes, being therefore reduced by the above ratio from the incident wave of Fig. 7A.

A. Illustrates the full wave impressed on the impedance. e_o is obtained as before, for the oscillatory case, and rises to 204 per cent of the crest value of the incident wave. The reflected wave has a crest of 124 per cent of the incident wave. The voltage across the capacitance e_c rises to 130 per cent of the incident wave and is not directly obtainable by the general equations, excepting that it may be considered a special case of e'' of Fig. 6J. And if this is done the simplification becomes so involved that it is more practical to derive the equation from the impedance operators. The calculated values of e_c become noticeably in error after about 12 microseconds, while the difference in e_o is not appreciable. In the test circuit, the series resistance was not inserted as is illustrated in the sketch, and e' and e_o were calculated as though the terminal impedance consisted of L and C only. The results of this calculation are the circles on the sketch of e_o, and the solid line of e'. The crosses on the sketch of e_c illustrate that voltage calculated by the same method. It is evident that the inherent resistance of the impedance combination has little effect on the wave on the line side, but can not be neglected for the voltage transmitted to the capacitance. By considering the reactor to have an inherent resistance of 250 ohms the circles on e_c are obtained, which seems near enough for practical purposes. The value of 250 ohms was obtained by considering the damping factor of the circuit to consist of the line surge impedance plus such a series resistance as to approximate the actual decrement of the damped oscillatory component of e_c.

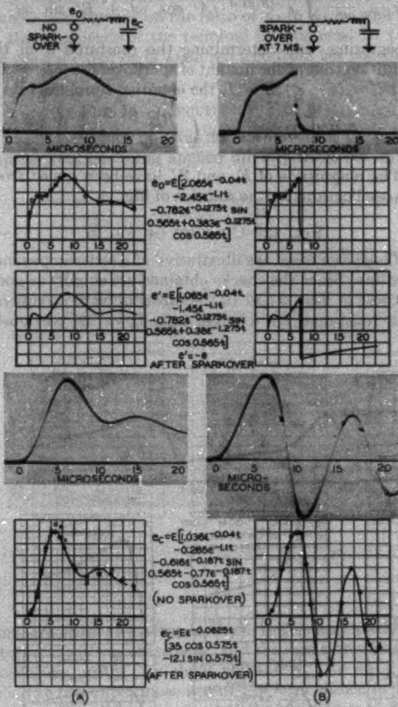


FIG. 11—CIRCUIT OF FIG. 4(O)

$e = E(\epsilon^{-0.04t} - \epsilon^{-1.1t}) \quad e_0 = e + e'$

Probably such a close approximation is not justified, since use of the voltage wave obtained by taking the series resistance equal to zero would be an approximation tending toward greater safety in protecting for such a voltage. And the magnitude of such a series resistance undoubtedly is not constant for all waves and circuits, making an approximation doubtful without very careful study.

B. This represents the same circuit as in Fig. 11A, but with a sparkover near the crest of the voltage wave on the line side of the impedance. All voltages are precisely the same as in 11A before sparkover, but present a different problem after sparkover. The voltage e_c is of particular interest since to obtain a reasonable check with the oscillogram the decrement factor could not be compared with that of the preceding case. Here the entire damping factor is inherent in the circuit and its constants after sparkover, since the line surge impedance does not enter into the circuit. In order to plot the circles on the sketch of e_c, this resistance, as determined by the same method of superposition of the wave components was again found to be 250 ohms. The calculation for e_c after sparkover is probably most easily derived by direct methods rather than by resorting to Heaviside's calculus. The axis of oscillation of e_c shifts to the zero axis after sparkover.

Since this case is of a nature common to transmission systems, it is of interest to include its derivation. The reflected wave at the line side of the impedance is obtained from the general equations, or it may be derived easily by the operational calculus. The impedance operator for Fig. 4N is

$$Z_o = R + pL + \frac{1}{pC}$$

The reflection operator is

$$\frac{LCp^2 + (RC - ZC)\,p + 1}{LCp^2 + (RC + ZC)\,p + 1}$$

which, when operating upon the voltage of the incident wave, derives the reflected wave. It is expedient to consider the incident wave of the form $e = E\epsilon^{-at}$, thereby obtaining equation (II). As previously mentioned, the solution for any other incident wave then follows directly. Substituting α and β as in Table I to reduce the expression to a familiar form, there results

$$e' = \frac{p^2 + 2\alpha p + \omega_o^2}{p^2 + 2\beta p + \omega_o^2}\,E\epsilon^{-at}$$

Applying the shifting theorem, this becomes

$$e' = E\epsilon^{-at}\,\frac{p^2 + 2\,(\alpha - a)\,p + (a^2 - 2\alpha a + \omega_o^2)}{p^2 + 2\,(\beta - a)\,p + (a^2 - 2\beta a + \omega_o^2)} \qquad (1)$$

the solution for which reduces to the expression (II).

The voltage e_c of Fig. 11A is derived similarly but for which the impedance operator is

$$\frac{Z_c}{Z_o} = \frac{1}{LCp^2 + RCp + 1}$$

e_c after sparkover, in Fig. 11B is given by

$$e_c = \frac{1}{C}\int i\,dt$$

The current is determined by solving the differential equation for a closed circuit consisting of a capacitance, an inductance and a resistance in series, given by

$$\frac{1}{C}\int i\,dt + L\frac{di}{dt} + Ri = 0$$

After differentiating this may be written in the form

$$\left(D^2 + \frac{R}{L}\,D + \frac{1}{LC}\right)i = 0$$

the solution for which is

$$i = A\epsilon^{(-R/2L + j\omega)t} + B\epsilon^{(-R/2L - j\omega)t}$$

This is reduced and substituted in the equation for e_c to give

$$e_c = \frac{1}{C}\int \epsilon^{-(R/2L)t}\,(A'\ \cos\ \omega t\ +\ B'\ \sin\ \omega t)\ dt$$

Integrating, and determining the constants from the condition that at the instant of sparkover $t = o$, $e_c = E_c$, and $i_c = (e - e')/Z = I$, the equation simplifies into

$$e_c = \epsilon^{-(R/2L)t}\left[E_c\cos\omega t + \left(\frac{RE_c}{2L\omega} + \frac{I}{\omega C}\right)\sin\ \omega t\right]$$

The magnitude of E_c and ZI have been scaled from the oscillograms for the solution of Fig. 11.

Fig. 12—Circuits of Fig. 5

These five cases are illustrative of a series impedance in a line. Since the checks obtained are equally as good as those of Figs. 9 and 10 the substitution of an equivalent resistance for a surge impedance as previously

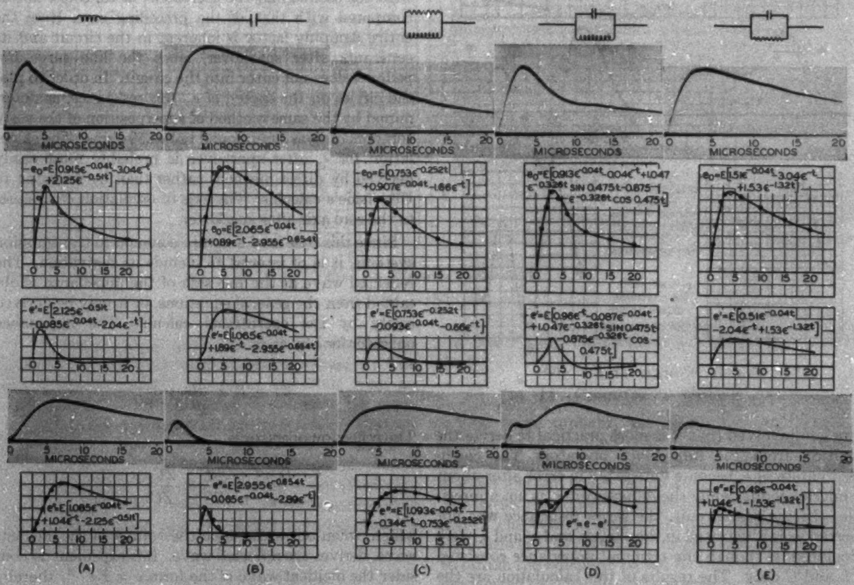


Fig. 12—Circuits of Fig. 5

$e = E\,(\epsilon^{-0.04t} - \epsilon^{-t}) \qquad e_0 = e + e'. \quad e'' = e - e'$

mentioned, is justified. For the figures shown, $Z_1 = Z_2 = 510$ ohms. The voltage deflection of the oscillograms is reduced by a ratio of 1.21 to that of the incident wave of Fig. 7A, but the sketches are plotted to the same scale as in Figs. 9 and 10 for comparative purposes.

FIG. 13—CIRCUITS OF FIG. 6

As in the preceding group the line Z_2 was replaced by a resistance of 510 ohms, so that $Z_1 = Z_2 = 510$ ohms. The reflected waves are predominately negative, but in general this depends on the ratio of Z_2 to Z_1, as well as on the magnitude of the terminal impedances. In (*E*) the general equation is again reduced from the complex form.

FIG. 14—CIRCUITS OF FIG. 6

The transmission line was not used in these tests, the impulse voltages being applied directly from the lightning generator to the impedances. In this way a wave of steeper front was obtainable, and the duration was controlled by means of a shunt resistance. The wave applied to the impedances was of the form shown in Fig. 7B but with different values of the parameter *a*. It is seen to have a front of sufficiently close to zero time to make such an assumption substantially correct. The fair check between the oscillograms and the calculated points further justifies the assumption, and is of particular interest to illustrate how closely a wave of zero time to reach crest may be approximated without serious error.

By using the single exponential form the labor involved in calculating the voltage is greatly reduced. In *A*, it is reasonable to conclude that the error could be accounted for in the inherent constants, the omission of which gives results on the side of safety. It is interesting to note the precise check in *B* where the oscillatory term becomes negligible and gives an abrupt change in the slope of the tail.

These circuits are special cases of Fig. 6, G, H and J, and the waves shown are discussed in more detail in their original publication.[3] The sketches of $e_o - e''$ represent the voltage across the inductance of a current limiting reactor. Since this is not a case of traveling waves, the incident wave becomes the total voltage at the reactor and there is no reflected wave.

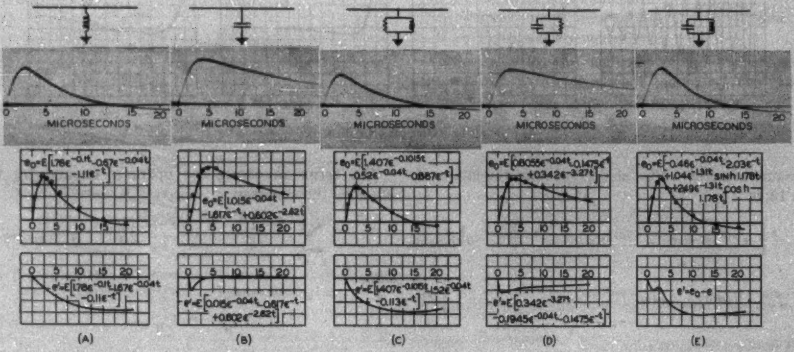


FIG. 13—CIRCUITS OF FIG. 6

$e = E(\epsilon^{-0.04t} - \epsilon^{-t}) \qquad e_0 = e'' = e + e'$

SUCCESSIVE REFLECTIONS BETWEEN A CAPACITANCE AND A RESISTANCE

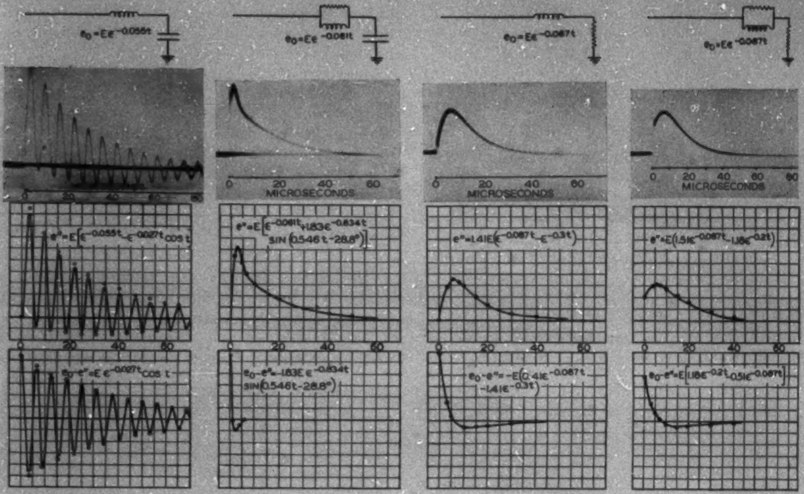
The solution for waves traveling between a capacitance and a resistance shows that the effect of the line surge impedance is eliminated by successive reflections. Fig. 15 illustrates a line of length τ (time units) and of surge impedance *Z*, having an impulse generator at one end and a resistance at the other. The reflection operators at *R* and *C* are respectively,

$$a = \frac{R - Z}{R + Z}$$

and

$$b = \frac{1/pC - Z}{1/pC + Z} = \frac{\alpha - p}{\alpha + p} \text{ where } \alpha = 1/ZC$$

The solution of the system is readily understood from the lattice method of solving successive reflections.[3] Attenuation is included in the general derivation and is assumed to consist of an exponential decrement, given by $\epsilon^{-\beta t}$. The number of reflections is taken as $(n + 1)$ for simplicity in the form of the final equation. Then the total voltage at any point, x distant from C,



(A) (B) (C) (D)

Fig. 14—Circuits of Fig. 6

(measured in time units) as derived from the lattice of Fig. 15, is

$$E_{(x)} = \Big[\epsilon^{-\beta x}{}_{(x)} + \epsilon^{-\beta(2\tau - x)}\, a_{(2\tau - x)} + \epsilon^{-\beta(2\tau + x)}\, (ab)_{(2\tau + x)}$$

$$+ \epsilon^{-\beta(4\tau - x)} (a^2 b)_{(4\tau - x)} + \epsilon^{-\beta(4\tau + x)} (a^2 b^2)_{(4\tau + x)} + \ldots \Big] f(t)$$

where $f(t)$ is the incident wave and the subscripts of the operators indicate the time at which they become effective. When $x = \tau$, this term reduces to

$$E_{(\tau)} = \Big[\epsilon^{-\beta\tau}\, (1 + a)_{(\tau)} + \epsilon^{-3\beta\tau}\, ab\, (1 + a)_{(3\tau)}$$

$$+ \epsilon^{-5\beta\tau}\, a^2 b^2 (1 + a)_{(5\tau)} + \ldots \Big] f(t)$$

of which the general term is

$$e_{(n+1)} = \Big[\epsilon^{-(2n+1)\beta\tau}\, (ab)^n\, (1 + a) \Big] f(t)$$

When operating on the incident wave this derives the voltage at R due to any reflection $(n + 1)$. Taking the incident wave of the form $e = E\epsilon^{-t/ZC} = E\epsilon^{-\alpha t}$, ignoring line distortion, and applying the shifting theorem, there results

$$e_{(n+1)} = E\,(1 + a)\, a^n\, \epsilon^{-(2n+1)\beta\tau}\, \epsilon^{-\alpha t}\, (2\alpha - p)^n \frac{1}{p^n} \quad (1)$$

The terms involving the operator expand by the binomial theorem into

$$\sum_{k=0}^{n} \frac{(-1)^k}{\underline{/k}}\, \frac{\underline{/n}}{\underline{/n-k}}\, (2\alpha)^{n-k}\, p^k\, \frac{t^n}{\underline{/n}},$$

since

$$\frac{1}{p^n} = \frac{t^n}{\underline{/n}}.$$

But

$$p^k t^n = [n(n-1)(n-2) \;.\;.\;.\;.\; (n-k+1)]\, t^{n-k} = \frac{\underline{/n}}{\underline{/n-k}}\, t^{n-k}$$

The voltage at R due to any one reflection $(n + 1)$ then is

$$e_{(n+1)} = E\,(1 + a)\, \frac{a^n}{\underline{/n}}\, \epsilon^{-\beta t}\, \epsilon^{-\alpha t'}$$

$$\sum_{k=0}^{n} \frac{(-1)^k}{\underline{/k}} \left(\frac{\underline{/n}}{\underline{/n-k}} \right)^2 (2\alpha t')^{n-k}$$

where t' indicates that the time function starts at $[t - (2n + 1)\,\tau]$ and t is counted from the instant the capacitance begins its initial discharge.

The total voltage at R at any reflection $(n + 1)$ is

$$E_{(n+1)} = \sum_{r=1}^{n+1} e_r\,(t')$$

where t' denotes the time functions to be displaced as defined above.

When the wave is shorter than 2τ, $e_{(n+1)}$ is the total voltage at R until the wave-length has been increased by successive reflections until it is greater than 2τ. If the wavelength is greater than twice the line length, the voltage at R increases in steps until the decrement is greater than $e_{(n+1)}$. For a wave very much longer than the line, the voltage increases in steps until the effect of successive reflections becomes negligible. Fig. 16 is an oscillogram of the voltage wave obtained at a resistor of 60 ohms, 600 ft. from the impulse generator, the capacitance of which was 0.75 μf. The line was the same as that used in the preceding tests, having a surge impedance of 510 ohms. Superimposed on the oscillogram is the calculated voltage wave, neglecting attenuation, for which $\beta = 0$. Reflections beyond the eleventh are neglected. The initial wave was 62 kilovolts, and the time between successive steps is 1.2 microseconds. It is evident from the oscillogram and from the equation, that the final voltage wave after the reflections become negligible, is independent of the line surge impedance.

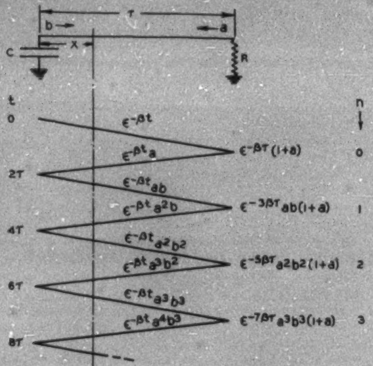

τ
b
a
X
C
R
t
0
2τ
4τ
6τ
8τ
n
$\epsilon^{-\beta t}$
$\epsilon^{-\beta t}a$
$\epsilon^{-\beta t}ab$
$\epsilon^{-\beta t}a^2b$
$\epsilon^{-\beta t}a^2b^2$
$\epsilon^{-\beta t}a^3b^2$
$\epsilon^{-\beta t}a^3b^3$
$\epsilon^{-\beta t}a^4b^3$
$\epsilon^{-\beta\tau}(1+a)$
$\epsilon^{-3\beta\tau}ab(1+a)$
$\epsilon^{-5\beta\tau}a^2b^2(1+a)$
$\epsilon^{-7\beta\tau}a^3b^3(1+a)$
0
1
2
3

Fig. 15—Successive Reflections Between a Capacitance and a Resistance Over a Transmission Line

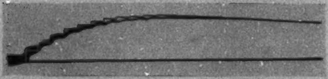


Fig. 16—Resultant Voltage at Resistor Due to Successive Reflections

STEP-BY-STEP SOLUTION FOR NEUTRAL GROUNDING IMPEDOR

The oscillogram of Fig. 17 illustrates voltage waves obtained at the line and neutral terminals of a transformer with thyrite as a neutral grounding impedor. Due to the variable characteristic of thyrite, the neutral voltage cannot be solved by any direct method as yet devised, but an approximate solution can be obtained by a step-by-step process, as derived from the differential equation. For the equivalent circuit of a transformer with an impedance consisting of R, L and C in parallel in the neutral, there is

$$2e - Zi = L_T \frac{di}{dt} + E$$

$$L \frac{di_L}{dt} = E$$

$$C \frac{de}{dt} = i_c$$

where

e = voltage of the incident wave.
Z = surge impedance of the line.
L_T = short-circuit inductance of the transformer.
E = voltage at the neutral.

For calculating the neutral voltage the inherent capacitances of the equivalent circuit of the transformer may be neglected without serious error.

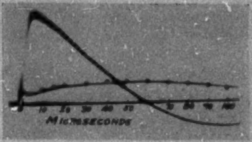


Fig. 17—Thyrite as a Neutral Grounding Impedor

Since R is variable these differential equations cannot be solved directly, but by substituting increments for differentials there results:

$$\Delta i = \frac{2e - Zi - E}{L_T} \Delta t$$

$$\Delta i_L = \frac{E}{L} \Delta t$$

$$\Delta E = \frac{i_c}{C} \Delta t$$

in which voltages and currents are averages over the interval Δt. There is also:

$i_L = \Sigma \Delta i_L$ = current in inductance.
$i = \Sigma \Delta i$ = total current.
$i_R = f(E)$ from the characteristic of thyrite.
$i_c = i - i_R - i_L$ = current in capacitor.
$E = \Sigma \Delta E$ = voltage at neutral.

From these relations a step-by-step tabulation may be compiled to any desired degree of accuracy. The

TABLE IV—STEP-BY-STEP SOLUTION OF FIG. 17

Δt	t	$2e$	E	Δi	i
	0	0	0		0
2				0.80	
	2	350	55		0.80
2				2.08	
	4	550	78		2.88
2				2.72	
	6	620	94		5.60
2				2.80	
	8	610	105		8.40
2				2.68	
	10	595	113		11.08
5				5.97	
	15	525	127		17.05
5				4.89	
	20	460	137		21.94
10				6.80	
	30	325	147		28.74
10				3.08	
	40	200	152		31.82
10				−0.31	
	50	80	151		32.13
10				−3.11	
	60	− 10	148		29.02
10				−5.27	
	70	− 90	140		23.75
10				−6.74	
	80	−140	127		17.01
10				−7.34	
	90	−165	109		9.67
10				−7.00	
	100	−165	76		2.67

trials are adjusted until all the given conditions are satisfied, which ordinarily may be obtained sufficiently accurate in three or four attempts. It is advisable to plot successive points as they are obtained and also to plot a curve of Zi, since extrapolation of the curves provides an excellent means of obtaining the next increment. This also insures against additive errors, since each point is directly dependent upon the preceding one. This method assumes all functions to be linear over the interval Δt, and therefore, if increments are assumed too large, successive points will alternate about their correct value in the region of rapid curvature, resulting in superimposed oscillations that do not exist.

The solution corresponding to the oscillogram of Fig. 17 is given in Table IV. Waves were applied directly from the impulse generator at 620 kilovolts to a transformer having an inductance of 0.368 henrys and with 100 disks of thyrite in the neutral to ground. The plotted circles in Fig. 17 are the solution for the neutral voltage obtained from Table IV, considering only the inductance of the transformer and the thyrite. Since a transmission line, and inductance and capacitance at the neutral were not present, the solution is greatly simplified, and successive points may be obtained fairly accurately on the second trial.

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Engineering Subjects, Electrical and Cognate, in the Four-Year College Program of Electrical Engineering

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IN the field of education for electrical engineering both the teacher and the practising engineer are vitally concerned; the teacher, because it is his life work, the practitioner because the young graduates are presumably trained to meet the specifications of his profession, to join its ranks, and to work beside him in the years to come. Obviously the working relations of man to man and the characteristics of the profession will be more or less determined by the nature of the training given to the young men who come into it at the rate of approximately 3,000 a year. In this sphere two great national organizations, the Society for the Promotion of Engineering Education and the American Institute of Electrical Engineers, have mutual interests and interlocking membership. It would seem well to set aside for the special consideration of the S.P.E.E., the methods of selection, the study of teaching methods, the operating details and organization of the curricula, etc., but certainly then, it is the special function of the A.I.E.E. to advise as to the economic and professional content and interrelations of the programs.

As a matter of background the study of engineering education by the S.P.E.E., 1923-1929, brought out a synthesis of opinion from teachers and engineers as to what engineering curricula should be, as follows:†

1. "Moderate diversity but tending away from specialization.
2. Dominance of the scientific and broadly technical content and emphasis.
3. Inclusion of a well-identified core of required subject matter.
4. Inclusion at all stages of subjects of purely cultural value.
5. Due emphasis (though not predominant) on the economic aspects of engineering and on its concern with administration and management.
6. Coherence of arrangement and coordination of related subjects.
7. Thoroughness rather than completeness of detail."

It will be remembered with regard to "specialization" that, instead of the general sense of the word as applied to the differentiation of instruction in accordance with the major divisions of engineering—civil, electrical, mechanical, etc., the S.P.E.E. used the term in the above discussion in a narrower sense as applied to a further degree of differentiation of curricula within the major fields themselves, such, for example, as the provision of distinct curricula in sanitary engineering, structural engineering and the like, within the major field of civil engineering. There was common agreement between the teachers and engineers that the primary purpose of instruction in specialized engineering subjects should be to teach fundamental principles and methods rather than to train for particular kinds of work. The engineers emphasized the opinion that specialization is out of place in undergraduate curricula and that thorough grounding in fundamentals is of paramount importance. Yet many of the teachers believed that in teaching specialized engineering subjects there is no essential conflict between preparation for particular fields of work and training in fundamental principles and methods, although the latter should always be the primary purpose. It has been a standard educational policy that principles are best taught in connection with their applications, and that the technical subjects proper should serve as media for teaching principles and the characteristic engineering methods of thought and analysis, instead of attempting preparation for specific types of practise.

The engineers were strongly in favor of extended training in economics and for a greater emphasis upon the economic phases in the engineering subjects proper. A large majority of the teachers, engineers, and graduates who were questioned indicated the belief that foreign languages, as taught at that time, were not of sufficient value to warrant their inclusion as requirements in the engineering curriculum. Both teachers and engineers were in agreement that shop work should not be required of all students, but only of those enrolled in curricula most closely related to the manufacturing industries. As a result of these definite criticisms and

SEMESTER HOURS AVERAGE REQUIREMENTS FOR GRADUATION IN ELECTRICAL ENGINEERING

S.P.E.E. Report, 1923-1929		S.P.E.E. Report, 1930	
Units	Subject Group	Subjects	Units
		Technological Subjects	
34.4	Electrical Engineering	Major Branch	39.0
42.1	Other Engineering	Other Technical Subjects	23.5
6.8	Engineering Electives	Electives	10.4
1.8	Non-Engineering Electives	Shop	4.6
0.1	Miscellaneous	Drawing	7.2
6.6	Phys. and Mil. Training	Phys. and Mil. Education	4.6
2.1	Foreign Language	Modern Foreign Language	1.2
4.4	Social Science	Economics	4.7
20.6	Physical Science	Physical Science	20.7
8.9	English	English	9.0
18.9	Mathematics	Mathematics	18.5
63.6	Total Non-Engineering		
83.3	Total Engineering		
137.6	Total Prescribed	Total Prescribed	133.0
9.3	Total Elective	Total Elective	10.4
146.9	Grand Total	Grand Total	143.4
55	Number of Institutions	Number of Institutions	51

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†Page 413, Report of the Investigation of Engineering Education, Vol. 1.

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recommendations, chiefly with regard to the preparatory and secondary courses in the curricula, many of the colleges have changed their requirements to meet the recommendations. The progress in this respect may be estimated by comparing the 1923-1929 table of graduation requirements for the first degree in electrical engineering with the corresponding table for 1930. Unfortunately, the two tabulations were not made in exactly the same subdivisions, but *Shop and Drawing*, listed as items in the 1930 report, are evidently included in the *Other Engineering* item of the 1923-1929 report, while *Engineering Electives, Non-Engineering Electives and Miscellaneous* of the 1923-1929 report are doubtless all combined into the item of *Electives* in the 1930 report. The other subjects, however, listed side by side appear to agree exactly.

The slight increase in English and economics with the corresponding decrease in modern foreign languages is notable. Still this does not show the full effect of the change since some colleges are still revising their curricula in this respect. For example the College of Engineering at the University of Michigan will change its requirements as of September, 1932, dropping 16 hours of foreign language or nontechnical electives in favor of 4 additional hours required in English, 6 hours required in economics, and 6 hours of free nontechnical electives.

It is believed then, that a great improvement has been made in the preparatory and secondary parts of the curriculum in electrical engineering as a result of the foregoing study and discussion. Some glaring omissions have been remedied and some old ideas have given way to new and present conditions. The fact that the recommendations after careful analysis by the colleges have been so widely adopted shows how valuable the recommendations of the practising engineers have been and how much they have achieved. We should expect, therefore, similar advantages to accrue from a study and discussion of the technical portion of our curriculum in something like the detail which was thus given to the preparatory and secondary subjects.

With regard to the technological portion of the curriculum in electrical engineering the tabulations of the total semester hours allotted to these subjects in the major branch of electrical engineering itself, those in other engineering fields, and those in engineering electives, in the 1923-1929 and the 1930 S.P.E.E. reports, are as follows:

SEMESTER HOURS REQUIRED FOR GRADUATION

1923-1929		1930				
Subjects	Units	Subjects	E.E.	M.E.	C.E.	Ch.E.
Elec. Engg.	34.4	Technological Major Branch	39.0	41.0	50.0	44.9
Other Engg.	42.1	Other Technical Subjects	23.5	22.9	18.4	21.6
Engg. Electives	6.8	Electives	10.4	9.3	8.4	6.7
Grand Total	146.9	Grand Total	143.4	145.5	144.0	146.4
No. of Institutions	55	No. of Institutions	51	44	51	41

It is noted that while there has been an increase in the central core of subjects in electrical engineering itself from 34.4 to 39.0, still the electrical curriculum contains less technical courses in its own field than any of the other major curricula, particularly those of chemical and civil engineering with their respective requirements of 44.9 and 50 semester hours in their own major branch. It would appear then, that the electrical engineering students take proportionately more of their technical work in associated departments and in electives, a situation which will heartily commend itself from the points of view of broad foundational training, and of freedom for the student to pursue his ambition in at least a slight degree into some particular field of his own choice.

The tendency to increase the number of semester hours of the technical requirements in electrical engineering was noted in the 1923-1929 S.P.E.E. report. That it prevailed and resulted in an 11 per cent increase is shown by the report for 1930. In view of the close approach to an upper limit should we not examine how scientific the training of a four-year electrical student should be? Doubtless we are all agreed that he must have, as an absolute minimum, say, some 16 semester hours of fundamentals in d-c. and a-c. machinery and steady-state circuits. But in the light of modern developments and in preparation for some known problems of the future how much further should we go?

The electrical engineers who were consulted in 1923-1929 as to which divisions of electrical engineering were of such importance as to warrant their inclusion in the training of all electrical students replied as follows:

Per cent favoring inclusion in all electrical engineering curricula	Subjects
67.5	Transmission and Distribution
59.0	Power Plant Engineering
57.0	Electric Machine Design
44.0	Industrial Power Engineering
38.8	Hydroelectric Engineering
33.5	Illumination
33.2	Electrochemical Engineering
28.4	Electric Railways
22.5	Telephony-Telegraphy
19.5	Radio Engineering

(It was taken for granted that a general foundation in the principles of electrical and magnetic units, laws, circuits and machines would of course be required in every case.)

In view of the developments of the past few years and of immediate future demands surely the subject of electronics and vacuum tubes should be added to the above list. Such a course should include an engineering approach to the theories of ionization, of the mechanisms of current flow and energy interchanges in ionized regions, and of thermionic, photoelectric, and other types of electron emission, as related to conducting gases at atmospheric and lower pressures. The study should cover thermionic vacuum tube characteristics and types of circuits and tubes suitable for rectifiers, amplifiers, detectors and oscillators. Not all of the

TABLE I—REQUIRED SEMESTER HOURS IN ELECTRICAL DIVISIONS
1931

Name of Institution	Fundamental Courses	Trans and Dist.	Power Plants	Elec. Mach. Design	Industrial Power	Illumination	Electrochem.	Elec. Ry.	Tel. and Tel.	Radio	Electronics
Alabama Pol	24	5	8	4E	4E	2E			6E	6E	
Armour Inst	34	4.5	4.5		2						
Brooklyn Pol. Inst	18	3		2		2		2	5	2	
Univ. of Cal	18	5	4	3E					5	6E	
Carnegie Inst	33	2	5	9					2	3	
Case School	27	5	1		3E	6		1	3		3E
Cornell Univ	33	3E	3	4E	2E	2E		2E	4E	3	
Georgia Tech	24	3	5E	2		3E		5E	2	3E	3
Harvard Univ	16	7		3E					3E	3E	3
Univ. of Ill	22	3E	4	2		1E				6E	
Univ. of Iowa	22	6E	5E	3		3E			6E	3E	
Iowa State	24	2	2	2E				2E	2	2E	
Johns Hopkins	30	2.5	2.5	4		2.5		2.5		5	
Univ. of Kansas	15.5	4	3		3E	2.5		2E	5E		
Kansas State Ag	26	3E		3		3E			3	3E	1E
Lehigh Univ	25	6E	3	6E	2E			3E	3E	3E	
Univ. of Maine	27	3	3	3		3		3	4.5	4.5	
Mass. Inst. Tech	19	5	6E	6E	3E	3E		6E	3E	3E	
Univ. of Mich	16	4	5	4	2E	2		2E	4E	6E	4
Univ. of Minn	27	4E	6E	4	2E	4E		2E	6E	6E	2E
Univ. of Nebraska	22	4E	4E			2E		3E	4E	3E	
Ohio State Univ	20	2E	2E	4E		4E		3E	6E	3E	2
Penn. State College	20	5E	2E	2E	2E	2		3E	3E	2E	
Univ. of Pittsburgh	17.5	3	2	2	4	3		2	1.5	1.5	
Princeton Univ	16	2	3E	6	2E		4E	2E			
Purdue Univ	26	6E	6E	5	3E	3E		3E	6E	6E	6E
Renss. Pol. Inst	21	4	3	4	2	2	3	2	1.5	1.5	3
Texas A. & M	32	3	3	6E	3E	3E			3E	3E	
Union College	28			3							
Virginia Pol. Inst	18	6E	6	3						6E	
Washington State	23	5	2E	2		3E		3	3		3E
Univ. of Wisconsin	24	6E	3E	3E		3E			3E	6E	3E
Yale Univ	32	3E	3E		3E				3E	3E	3E

E = elective.

subjects in the above tabulation are standard requirements in the college curricula but nearly all are represented in the programs as requirements or available electives, the exception being perhaps electrochemical engineering which in practically every case is offered in the departments of chemistry. The following study shows what a number of representative schools provide in required and elective courses in the above subjects:

It is notable that there is a great consistency for the first four columns of Table I in that nearly all colleges require a group of fundamental courses of not less than 16 hours and also provide one or two courses in transmission and distribution, power plants, and electrical machine design. The few schools whose catalogs do not show a course in electrical power plants provide a strong preparation in the courses they schedule in the heat power division of mechanical engineering. There is also strong agreement in using the field of power plant engineering as the vehicle for applying economics to engineering problems. In the remaining columns of the table the courses offered are mainly elective, the restriction of time allowing the undergraduate student probably not more than one, or at most two, courses in any division. Many colleges offer several sequential electives in the various divisions but the courses are principally for graduate students. There are difficulties in analyzing the curricula in these divisions. Illumination may in some cases be offered in the department of physics; telephony, telegraphy, radio, and electronics may be combined in various ways in courses given as communication engineering.

In order that the preparation shall have a proper breadth and that the graduate shall understand something of the problems facing his colleagues, what should be the training of electrical students in the fields of the associated technical departments of mechanical, chemical, and civil engineering?

In mechanical engineering the electrical student has need of a fundamental course dealing in elementary thermodynamics, steam, fuels, boilers, steam engines and turbines, condensers, internal combustion engines, and the general problem of a power plant. Some time in steam laboratory would be valuable to vitalize the work. It will be necessary to establish the standard ratings, efficiencies, methods of governing, and speed regulation of the prime movers here so that they may be transferred to the electrical courses concerned with the generator.

To chemical engineering we look for a knowledge of the engineering materials with which to work. For general structural and power work every student needs an elementary knowledge of the manufacture and properties of the ferrous and non-ferrous alloys, cements, clay products, protective coatings, fuels, boiler scale, and water softening together with associated laboratory experience in mechanical working and heat treatment of the various metals, welding practise, protection against corrosion, etc. In addition to these widely used materials which are generally covered in the

TABLE II—REQUIRED SEMESTER HOURS IN COGNATE ENGINEERING DIVISIONS
For Students of Electrical Engineering
1931

Name of Institution	Mechanical			Civil				Chemical
	Heat Power	Machine Design	Shop	Structural Design	Hydraulics	Surveying	Ry.	Engineering Materials
Alabama Pol	10		4		3	3		2
Armour Inst	9	4	6		4	1.5		1
Brooklyn Pol. Inst	9	1	2	2	3			1
Univ. of Cal	9	2	4		6	6		
Carnegie Inst			7		3	2		4
Case School	7	3	2		3	3		1
Cornell Univ	11	4	4		2	3		6
Georgia Tech	8		3		4	2		2
Harvard Univ	8	4	1		3			4
Univ. of Ill	11		6		3			
Univ. of Iowa	5		8		2	3		
Iowa State	8		4		3	2		2
Johns Hopkins	13				3	3		3
Univ. of Kansas	6.5	3	4		3	3		1
Kansas State Ag	9	3	5		4	2		2
Lehigh Univ	8				3	4		3
Univ. of Maine	7.5		3					
Mass. Inst. Tech	8		3		3	2		
Univ. of Mich	4	3	2	3	3	2		3
Univ. of Minn	6		5		3			
Univ. of Nebraska	7	6E	6E		3E	6E		
Ohio State Univ	6	6	6		2			
Penn. State College	10	4	4		2	1		2
Univ. of Pittsburgh	5	4	1		2	2		
Princeton Univ	10	3			3	2		2
Purdue Univ	7		4		4E	2E		
Renss. Pol. Inst	9	2	8	3	4	2	3	1
Texas A. & M	8		2		3E	2		
Union College	6				3	6		
Virginia Pol. Inst	6		6		3	2		
Washington State	9	2	4		3	3		4
Univ. of Wisconsin	9	3	4		3	3E		2
Yale Univ	6					3		2

E = elective.

common introductory course in chemical engineering, the electrical student has a special interest in the silicon steels, in oils and papers, in cold and hot flow materials, in phenol plastics, varnishes, tapes, and mica. Seemingly no course given at the present time covers these materials which are so essential to the electrical designer. They should be included in the more comprehensive list of fundamental materials for an electrical student.

In civil engineering a common basic structural design course may follow some eight semester hours of engineering mechanics or the same field may be covered as applied mechanics in the last named department. At Michigan the preparatory design courses in mechanics cover statics—3 hours, strength and elasticity of materials, 4 hours, laboratory in strength of materials, 1 hour, and dynamics, 3 hours. In a fundamental structural design course we feel that the student should have some elementary training in analysis of stresses, moments and shear, for wood, steel and concrete beams, roof trusses and columns; beams and slabs in reinforced concrete masonry; an introduction to soil bearing theory, bearing power of piling and settlement; together with some contact with electric welding of frames and machine structures.

The above tabulation of the curricula in electrical engineering for the schools listed, shows the semester hours required in cognate engineering subjects:

While there is almost a general agreement as to the necessity for courses in heat power, shop work, and hydraulics there is evidently a considerable difference of opinion as to the need for training in surveying and in the mechanical features of machine design. Again almost half of the programs offer no training in engineering materials and only three curricula call for a structural design course after a preparation in engineering mechanics.

In view of these differences of opinion among teachers not only as to the subjects to be included in the fundamental training of the student but as to the time spent upon them as well, I feel that consideration and study and the gathering of suggestions from the practising members of our profession would be very much worth while.

Discussion

S. M. Dean: Because it is necessary that an engineer, like anyone else, shall thoroughly enjoy his job if he is to be successful and because an increasingly large proportion of engineers are before many years finding themselves in administrative positions, I wish to discuss Professor Lovell's paper from a somewhat nontechnical point of view.

I agree that any electrical engineer must have a basic training in the fundamental concepts upon which electrical development and nomenclature are reared. Referring to Professor Lovell's Table I, I should include in this first and prescribed group the fundamental courses as well as transmission and distribution and also, as he has suggested, electronics. Having progressed that far I should offer as electives a rather wide range of the more specialized subjects by way of determining the "natural bent" of the student as well as showing him into what the various branches of the electrical engineering profession lead. A student's selection of these courses would, of course, be carefully guided.

Two very important characteristics of the successful administrative engineer are his ability to take a broad view of any situation and his ability to set forth his ideas in a logical and convincing manner.

It seems to me as I look back at my college training that I could have been greatly aided by having a better idea of the course of events by which electrical engineering came up and where it stood when I entered the field. It would, of course, have been the story of the surmounted obstacles and of the men who contributed to the successful development of the various steps, both from the strictly engineering side as well as the commercial. I feel very strongly that a carefully planned course of this kind should be included early in the student's technical course. Such a course would call for the abilities of a master teacher and its importance would justify it.

Contributing also to this breadth of view are the broad range of cognate subjects suggested by Professor Lovell. I believe I would go so far as to include the more important of them to the possible exclusion of some of the more highly specialized courses in the electrical list of electives, because it is less difficult later to acquire an understanding of matters within one's professional field than those outside.

Laurance E. Frost: Professor Lovell concluded his paper with a call for suggestions from practising members of the engineering profession. Of course, no one could take greater pleasure in responding to such an invitation than one who has had the privilege of being a student in Professor Lovell's own classroom as I have.

Professor Lovell refers to a list of ten major divisions of electrical engineering which have widely been considered as worthy of inclusion in the training of all electrical students. He also points out another one, electronics, which has come to the fore so much in the last few years that it can hardly be denied admission to the list. So the list has grown to eleven items. And who can doubt that future years will bring other new fields into this list faster than old ones are dropped from it? Reflecting for a moment that it takes the best part of a lifetime for a man to attain a mastery of the technical phases of one or two of these fields, we may well pity any student who hopes to know them all in four or five years.

It is gratifying to a practising engineer to find it apparent from Professor Lovell's paper and others that, in the technical realm, many engineering educators are still concentrating on the basic and indispensable concepts of physics, mathematics, and chemistry, which form the background of all engineering, rather than diluting the training over-much with studies of current engineering practises and applications. A college course hasn't room for much more technical material than these fundamentals if they be treated with the thoroughness they deserve. It seems to be agreed generally that industry is in a better position than the colleges to teach about current engineering applications.

In the eleven short years since my own graduation it has been my good fortune to come in contact with many young men soon after their entry from college into the practising profession. From careful observation of their work it has been clear that those who had acquired a thorough knowledge of basic engineering principles have been the ones best adaptable and best able to meet their engineering opportunities, in spite of initial unfamiliarity with the details of particular applications.

May I then add my voice to the plea that in the teaching of electronics, or any other new field, most of the teaching effort be directed toward the basic principles which will be pertinent to future development and usage rather than toward description of present day practises which are susceptible to rapid change.

R. E. Hellmund: The study and statistics presented in Professor Lovell's paper are very instructive, especially since they indicate a number of points in which improvements in the curricula seem to be very necessary. It is most surprising and regrettable that almost one-half of the schools listed in Table II do not give any training in the mechanical features of machine design, and also that a very large percentage do not offer any courses in engineering materials. It cannot be stressed too strongly that all component parts of electrical machines and apparatus present mechanical problems and that the greater part of the work of electrical design engineers has to do with the mechanical phase. Hence there can be no doubt that curricula for designers should include training in the mechanical features of design and engineering materials. Although work on heat power and hydraulics, courses which are so generally represented in the tabulation, is of course of importance to future central station engineers, it would seem well worth while to give some of the time now devoted to these subjects to mechanical design and engineering materials; at least it would seem that good courses on these subjects should be optionally available to those students wishing to go into design work. It is also believed that these courses should be taken by designers in preference to any work on surveying.

Professor Lovell's suggestion that this entire subject be given consideration and study is very timely.

D. C. Jackson, Jr.: Since some data for Professor Lovell's paper were gathered from the college catalogues for 1930-31, changes of curricula which were in process at that time are not included in his tables. Both the University of Kansas and Kansas State College made during the past academic year a number of changes in their electrical engineering curricula. In order that Professor Lovell's paper may be up to date as far as these two institutions are concerned, Tables I and II should read as follows:

TABLE I

	Fund. Courses	Trans. & Dist.	Power Plants	Elec. Mach. Des.	Indus. Power	Illumination	Tel. & Tel.	Radio	Electronics
Univ. of Kansas	18	4	3*		3E	2.5	3E†		
Kansas State	26	3E		1		3E	3	3E	3

*Required in power and machinery option, elective in communication option.
†Elective in power and machinery option, required in communication option.

TABLE II

		Heat Power	Mach. Design	Shop	Hydraulics	Surveying	Engineering Materials
Univ. of Kansas	Power and Machinery	6.5	3	3	3	3	1
	Communication	5	3E	3	3E	3	1E
Kansas State		9	3	4	3	2	2

The data with regard to Kansas State College were obtained from Professor R. G. Kloeffler, head of the department of electrical engineering at that institution, since I knew that certain changes had been made rather recently in his department.

The changes in the electrical engineering curricula of the Kansas State College of Agriculture and Applied Science, and of the University of Kansas, tend to substantiate the trend indicated by Professor Lovell. Kansas State is requiring electronics instead of making it elective and the hours allocated to this subject are now greater in number. Kansas University has a larger number of hours given over to fundamental courses in electrical engineering and now has rather more than sixteen, which Professor Lovell points out is the minimum for fundamental electrical engineering subjects in those institutions which he used in his study.

Lyman F. Morehouse: In Table I of the paper which shows for various schools the division of required semester hours in electrical engineering, and taking the University of Michigan as an example, it is noted that the required "fundamental" electrical courses total 16 hours; also, that the required courses in transmission and distribution, power plants, electrical machine design and illumination, total 17 hours. The 16 hours of fundamental courses are, obviously, fundamental courses in dynamo electric machinery, and would appear to me to be specialized courses in the power field rather than broad fundamental courses. Special courses outside of the power field are all elective. The same seems to be generally true of other institutions.

It may be questioned whether all of the required work should not be given in truly fundamental courses, in which the broad fundamentals would be illustrated from every phase of electrical engineering, or, if this is impractical, that the required courses other than those designated as fundamental cannot have a more general content and be more widely illustrative of modern electrical engineering in its broadest application. For example, transmission and distribution might well cover the entire range of frequencies utilized in modern electrical engineering, and include not only power distribution but telegraphy, signaling, telephony, radio, and perhaps television.

It is stated that the course in "electrical power plants" is usually employed as a vehicle for applied economics. It seems obvious that economic principles could be applied with just as great benefit to other types of installations as to steam power plants. The suggestion might be added, therefore, that in studying the contents of the electrical engineering part of the curriculum, attention might well be given to broadening it and balancing it in line with what appears to be the present trend of the electrical field—that is, the trend toward new things involving all parts of the electrical frequency range, rather than concentration on the production and distribution of 60-cycle current. It seems to me that this is what an education in electrical engineering of the future requires.

L. W. W. Morrow: A study of the technical content of engineering courses is very desirable and Professor Lovell presents data to sustain this suggestion. In my opinion this study offers the greatest opportunities for improvement in technical education and is most needed at this time. All aspects of engineering change rapidly both in theory and practise and it is essential that the technical contents of engineering courses keep abreast of the art. The scope and depth of fundamental science should be increased and changed each year and the applications of principles should be made to the current engineering problems encountered in practise. Yet I have heard it asserted that the colleges are at least five years behind the times in their technical course content as regards both modern theory and modern practise.

It is interesting to study Table I of the paper which shows the required semester hours in the electrical divisions of 41 leading colleges.

The data in this table raise several questions at first glance:

1. What reason is there for the hours devoted to fundamental courses to vary from a minimum of 15.5 to 34?

2. Why is there no correlation between fundamental course hours and the additional hours of either required work or electives?

3. Why are so many electives given in specialized branches of electrical engineering?

4. Why, for example, do courses in electric railways persist in a large number of colleges while comparatively few give courses in electrochemistry and electronics—live and active field engineering divisions?

5. Why does electric machine design persist so greatly especially since this is applied to machines that have been standardized for many years?

6. How can any student be broadly trained in electrical engineering if he is forced to specialize in limited branches just as soon as he finishes his fundamental courses?

7. Why is it necessary or even feasible to teach specialized courses such as power plants, illumination, transmission and communication in undergraduate courses?

A general reaction to this table as a statistical summary of technical courses is that it shows very vividly an antiquated, superficial and cumbersome content for the technical engineering courses. In my opinion, based upon my experience as a teacher and as an engineer who studies all field problems as an editorial observer there is a very great opportunity to improve the technical content of engineering courses. This should be based entirely upon the principle of teaching the latest and best theoretical principles by applying them to current engineering problems and practises. There should be no attempt to specialize and the technical content should consist of a basic fundamental course to be given all students in their junior and senior years with a few electives based upon choice of specialized theory rather than commercial divisions of engineering as found in the field.

Let us glance briefly at the field and see what problems confront practising engineers and then perhaps we can grasp the point of view I have expressed.

Power Plants. Hydro stations and all equipment are very greatly standardized with little opportunity for improved efficiency except in the design of governors, water passages and waterwheel controls. Steam stations at normal pressures are standardized except for control of temperature over wide load ranges. In the high pressure—high temperature range there is a problem of materials to be solved. The two-fluid heat cycles are in process of development. Vibration of turbine parts and shafts, clearances, effects of corrosion and erosion of blades—these are current technical difficulties. A detailed study of a Corliss valve or of a simple steam engine as taught in college helps not at all on these problems. Fundamental thermo- and hydro-dynamics and a knowledge of mechanics and materials are the tools an engineer needs.

Transmission and Distribution. We are now dealing with network circuits. Our problems are in control, relaying, metering and fault elimination. The circuit equations are complex because of the network and only new mathematical tools such as symmetrical components and hyperbolic functions can be used to get results. We have distorted electric and magnetic fields, multiple path electric circuits and a mass of equipment that makes impossible an approach by use of a simple circuit equation or even the Fourier series so well drilled into undergraduates. Beyond these current field problems transmission is in a development stage through the application of electronics and ionization principles to new types of apparatus. Even our transmission structural design problem is one of vibration and fatigue wherein the book formulas give little aid.

Industrial. Very little change in design or performance has been had in years with standard motors and generators. Their application is largely that of control and their design is largely that of working materials at maximum thermal limits so as to reduce costs. Possibly the two live problems today are control and electric heat. Electromagnetic controls of all types are confronted by the new vacuum tubes and photoelectric cells that may replace them. Electric heat problems are mechanical and metallurgical rather than electrical.

Electrochemical processes in industry are on the increase, chromium-plating, electro-deposition of white lead, the making of synthetic resins, electro-winning of ores—no field is so active in development or more promising. How many undergraduates know the first principles of electrochemistry?

In the realm of electronics a vast field is opening up. An executive of a large manufacturing company said to me the other day "I would like $10,000,000 and a group of our best scientists to specialize in electronics for a year or two. I believe the biggest future in industry lies in this field." Vacuum tubes, photo-cells, ionization phenomena, thermionics—all the new tools and new theories are ripe for use and students should know the principles and have sufficient experimental work to use them. Yet the course still is filled with motor design and standardized rotating machine tests about which no major problems are associated and in connection with which the market has an over supply of engineers.

One could touch illumination with gaseous conduction and ionization as basic elements instead of resistance filaments. Radio with its untouched possibilities for power transmission and for directive and selective energy control is another live topic. Even the new idea of air conditioning introduces new fundamentals for students.

I point out a few of these field conditions in order to repeat my points about the technical content of college courses.

1. Stick to theoretical principles and apply them to all current engineering applications.
2. Eliminate the old, the standardized and the obsolete principles and applications and replace them with the new and live theories and applications.
3. Do not specialize in terms of arbitrary and artificial field divisions of engineering, but require all students to take the same fundamental course with a few electives in specialized theory instead of practise.

A. E. Silver: The college education of engineers is a matter of mutual concern to the educator and practising engineer. Constructive suggestions of greater value for the keeping of principles and curricula apace with advances in the art, should be forthcoming from the latter group if they would give closer attention to these important problems and if their cooperation is enlisted. Few of us adequately reflect and weigh how our education has been extended and modified since leaving college or how the prescribed courses of study, or our selection of them, might have been improved for the purposes of meeting the constantly advancing requirements of experience.

The work of the Society for the Promotion of Engineering Education is commendable and the conclusions seem well founded. Possibly, however, because of the time lag in the evolving and applying of such principles, greater weight should be given to looking reasonably into the future. This should be beneficial if it is found that programs can be brought more nearly abreast of conditions as encountered by the graduates.

For the education of an electrical engineer, it is a real question whether the student should be required to take all of those subjects that are considered fundamental and basic for his broad technical, economic and cultural education, or whether a considerable degree of latitude should be allowed for specialization or the pursuing of courses to the particular liking of the individual student. If the student could be guided early to discover for himself his caliber as engineering material, it should be of great assistance to the individual as well as the educator in determining this question.

For the broadly promising engineering timber, there might be required basic courses in physics, mathematics and electrical engineering and, with proper guidance, a large portion of the balance of the college work selected from a well formulated group of fundamental courses. These would include practical use of the English language, economics, business, introductions to all the important branches of electrical engineering and familiarity with the allied engineering fields. Non-essential subjects could be made of secondary importance, *e.g.*, modern languages, assuming these reasonably covered in preparatory school, would be subjects for later specialization.

In my opinion, natural ability justifying a four year course should be devoted primarily to the laying of a foundation, learning the principles of accomplishment by the effective directing of the forces at hand, acquiring the trait of making one's knowledge available and of value to others and finding one's natural self, leaving specialization largely to graduate work and to one's later initiative and experience.

A. E. Knowlton: There should be generous support of Professor Lovell's plea that the electrotechnical content of the curricula should be surveyed. But much of the value of such a survey would be lost by placing too much emphasis on the statistical data about semester hours and division between prescribed and elective subjects or inferring too much from the catalogue descriptions of the courses. Every one who has ever been in the teaching profession knows that catalogue descriptions of courses are seldom a true index of their character or objectives. The students in them however devise labels that are often far more indicative although not always expressed in language that the faculty could employ. Catalogue statements and credit hours mean little.

We hear much about reversion to fundamentals and elimination of the curse of specialization. But there seems to be no agreement on what constitutes the fundamentals. Are the fundamentals to be merely the basic technologic facts upon which the early branches of the electrical arts were founded? If that is true then it accounts for the adoption and retention of a curriculum built on the concept of an industry that had two or three well-defined branches. Such a practise might have been appropriate while the branches of the electrical art were few and well-defined. But as sub-divisioning and specialization proceeded within the electrical engineering field, education followed suit after the normal time lag. The resulting specialization has caused some concern and much criticism.

One way out would appear to be to review the content of fundamentals. Apply to every item the test of its broad applicability over the whole gamut of energy magnitudes and diversified applications. Those phenomena and principles which have greatest adaptability to the widest range of electrical activities would appear most entitled to the rank of fundamentals. On this basis an extended discussion of commutation or of the circle diagram of the induction motor might be found unjustifiable. In fact much of machine design would suffer in the face of that kind of analysis.

The other phase of correction suggested by one who has had an opportunity both to teach fundamental and specialization courses and to observe at close range what practising engineers deem to be the pressing matters of the moment, is that more stress be placed on the functional significance of the electrical phenomena and the physical equipment devised to exploit them. No effort on the part of the teaching profession can keep the curriculum abreast, let alone ahead, of the kaleidoscopic changes in the field of practise as long as the courses follow a vertical cleavage in imitation of the industry divisions. But if the functional tasks are allowed to determine the course content the horizontal character of the set-up will give the subject matter an incidence upon every branch of the art which involves the exercise of those functions.

To make the point specific, there is the function of generation. This does not mean dynamos and turbo-generators alone; it means batteries, photo-electricity, transmitters, piezo-electricity. There is the function of magnetization. The function of conveyance of energy embraces transmission, distribution, radiation and waves. There is the function of insulation as contrasted with conduction. There is the function of control and the function of protection against extraneous influences. Most of these appear in some form in every significant branch of the industry. A student drilled in these fundamental functional concepts would acquire a confidence or aptitude and adaptability now lacking in many graduates. He would see the unity of electrical technology where he now too often sees only a narrow cellular structure.

Incidentally, if a further opinion is permissible, some one of these functional subjects could well be carried to just as thorough an exhaustiveness as time permits. It makes little difference whether it be metering, amplification or protection or something else as long as it goes as far as current literature and current thinking have gone in that one specialty field. The method by which engineering industry improves its practises and its economics is the crucial thing to incorporate and this can be done by the through-to-the-end analysis of some appropriate field. Take the function of protection for example. It involves lightning, grounding, fusing, relaying, insulation coordination, circuit breakers, shielding, flux control, reactances, stability, unsymmetrical vectors, arresters, traveling waves, inductive coordination, electrolysis. Anyone would have difficulty in suggesting a better way to add zest and comprehensiveness to electrical education than to place it on such a functional basis, especially if the project idea is employed to carry analysis to finality.

Reverting then to the topic of the paper, the plea is made that the curricula be studied as suggested by Professor Lovell. That little stress be placed on statistical data. That the bulk of the effort be devoted to disclosing the objective and the motivating purpose of each subject and the course as a whole. That the prescribed courses be so surveyed and recorded as to indicate clearly whether the so-called fundamental content would fall under a horizontal or vertical category. That a measure be established in advance to determine what are the common denominator fundamentals of electrical power, transportation and communication. That the content of each course be tested to determine the extent to which it includes these fundamentals and excludes other material. Such a procedure might succeed in silencing the accusation from alumni and industrial employers that colleges "specialize too much and do not stress the fundamentals."

W. W. Lewis: Engineering graduates are absorbed through three main channels: the large manufacturing companies, the large public utilities and miscellaneous small manufacturing companies, public utilities and industrial plants. The first two groups maintain more or less extensive training courses to equip the graduate for the requirements of the particular company, and this training may be broad or special, depending on the extent and activities of the company. The main requirement of the graduate for these groups is a broad training in the fundamentals, such as physics, mathematics, chemistry and a-c. and d-c. machinery. The third group requires the same broad training plus a specialized training, which will fit the graduate for immediate usefulness in the company or industry by which he is employed.

Since very few know until very shortly before graduation in which group they will be placed, obviously it is necessary to give all students the broad fundamental courses plus a number of specialized courses. The specialized courses have the advantage that principles are more readily understood and become more interesting when connected with a specific application. Interest in a special course may guide the student to seek employment in a particular industry. Specific knowledge applicable to an industry will help the student to swing into his work, even though the course may have been brief and the knowledge gained meager.

It follows that on account of the indefiniteness of the ultimate industrial destination of the student, the college is obliged to teach a diversity of subjects. It would be impossible to arrive at more than a general agreement as to the relative importance of the various subjects, as those in different industries are bound to have different points of view and the colleges themselves are subjected to local considerations.

We agree that the subject of electronics is becoming increasingly important and should be given a place in all electrical courses. Personally, we would favor the elimination from the electrical course of surveying as unnecessary. Foreign languages could also be eliminated, as the courses are usually too inadequate to be useful. Otherwise, practically all of the courses listed in Tables I and II are desirable.

Philip S. Biegler: At the conclusion of Professor Lovell's paper he makes the statement that suggestions from practising members of the profession would be very helpful in the formulation of electrical engineering curricula. With this same idea in mind the College of Engineering of the University of Southern California organized an Advisory Committee made up of prominent engineer-executives of this vicinity somewhat over a year ago. This committee has made a study of engineering curricula of the College of Engineering during the past year, meeting for two hours about every two weeks. We feel that the Advisory Committee has and is contributing a very valuable service in making a critical study of curricula and methods of instruction at this university. In general, these men agree that specialization in engineering instruction should be minimized and that our six curricula in engineering should be made as much alike as practicable. This is, of course, in line with present trends in engineering education. They believe that the training in English for engineering students is still inadequate. As one of the members put it: "Not all engineers use calculus in engineering practise, but they all do use English."

The Committee recommends the increase in training in economics and economic aspects of engineering. It doubts the justification of such a course (which might be called a specialized course) as power plant design. It recommends that rather than courses in structural design on reinforced concrete, other students than civil engineers take a single course in design dealing with wood, steel, and concrete. Our electrical engineering department has developed a four unit course in electronics or "high frequency" with the idea that this is a fundamental course and not specialized.

Alfred H. Lovell: The 16 hours of "fundamentals" quoted by Mr. Morehouse include:

Principles of electricity and magnetism	4 hours
Direct current apparatus and circuits	4 hours
Alternating current apparatus and circuits	8 hours

so that perhaps only 6 hours is confined to dynamo electric machinery while 10 hours is general. Even our transmission course (electromechanics-4 hours) is based on the general equation of the circuit with the examples taken from both the power and communication fields. Thus Mr. Knowlton's pithy comments on the names of courses are amply confirmed.

Obviously economic principles could be applied to other types of installations than to power plants but where will the teacher find in the current technical literature a body of data covering installation and performance costs, maintenance, depreciation, rates, etc., comparable to that given for the power field?

I believe that Mr. Morrow's criticism is very just,—our technical layout is antiquated. However the situation is not as bad as it appears. Here again names of courses are misleading. I think the majority of our so-called machine design courses really cover studies in heat storage, transfer and dissipation, field mapping and calculation, etc. We still offer electric railways because our Transportation Department requests it. Still I do not see how we can teach "the latest and best theoretical principles" without some proper ground work. How will a student appreciate that any theory is latest and best without a contact with the orderly development in that field and an appreciation of the economic factors that forced improvement and new design? Since the latest theory is very likely to be closely related to laboratory research there is also a serious limitation of laboratory equipment. Perhaps not half of our laboratories possess a cathode ray oscillograph. We now can bring the student interested in transmission through a well rounded foundation plan to a knowledge of the main elements of control, relaying and fault elimination, with ability to use hyperbolic functions and symmetrical components in a simple network. I feel that is all that can be expected of a four-year program.

Mr. Knowlton's suggestion of a horizontal division of electrical phenomena and related equipment is most interesting and startling. After a fundamental preparation such a plan would offer wonderful opportunities for development and coordination through the entire field of electrical engineering. Our course in electronics seems to be developing exactly along this line. It would appear to call for a complete reorganization of present teaching plans into a research plan such as where a new development is carried into all available fields. It would also call for fundamental changes in teaching staff assignments since present teachers have been selected largely on account of experience in some vertical cleavage of an industrial division.

I am sure that teaching staffs will give careful consideration to the recommendation that fundamental courses should be reexamined and retested for broad application. Also much will be gained through study of the other carefully considered criticisms and suggestions which have been given in such a fine spirit of cooperation.

Educational Aspects of Engineering and Management

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INTRODUCTION

SHALL a common educational program be employed as a preparation for engineering and for management, and if so shall it be planned primarily for engineers, as in the past, or for managers; or shall there be independent programs for each group? In view of clear trends of experience—of the movement of engineers into management—shall we conclude that technical engineering has proved to be an effective and sufficient training at college for those who may later enter the field of management? Or, shall we consider it necessary radically to modify all engineering curricula in order to prepare men *primarily* for management? Or, again, and from the same experience, shall we plan separately for each? And, having answered this, what should the plan be?

The general subject of engineering education has had searching and extensive study during the past fifteen years. The first broad study was by Dr. C. R. Mann for the Carnegie Foundation[1] in 1918; the second, by the National Industrial Conference Board[2] in 1923; then followed the comprehensive investigation and study by Dr. W. E. Wickenden for the Society for the Promotion of Engineering Education,[3] which was reported, together with the recommendations of the Board of the Society, in the period 1926-29. The S.P.E.E. study covered practically all phases of the subject including not only a study of the purely educational aspects, but also an analysis of function, salary, and opinion of practising engineers. Other studies relating to individual aspects—such as functional distribution, salary, etc.,—were also reported in this period by various organizations; Eta Kappa Nu,[4] American Telephone and Telegraph,[5] American Institute of Chemical Engineers,[6] Purdue University,[7] Yale University,[8] and the American Society of Mechanical Engineers.[9] The latter is a recent, very comprehensive statistical study dealing with functional distribution and salary of mechanical engineers. And finally, there has been extensive discussion of it all in the literature. Thus, there now exists a prodigious mass of material in all stages from raw data to carefully thought out conclusions and recommendations. As one attempts, by casual study, to form a rational opinion regarding educational policy, as this relates to engineering and management, one is likely to become lost. At least this was the writer's experience. Even the carefully framed conclusions and recommendations of the S.P.E.E. seem inadequate in certain respects, in view of later facts—for instance, those brought out by the A.S.M.E. survey; also one wonders whether all of the data collected by these various agencies are consistent, and, finally, whether the basic thoughts underlying proposed educational policy are all well taken. A careful study was therefore undertaken of the new facts, their historical setting, and of the basic views underlying prevalent thought. From the perspective thus arrived at, a future educational policy, which seems appropriate to the writer, was framed. Since there are doubtless many others interested in this problem who have experienced a similar confusion in the face of conflicting opinion and the great tonnage of undigested data, it seemed in order to present this study.

The proposed policy turns out to be in most respects consistent with the recommendations in the Mann Report and the S.P.E.E. Report, and also with those particular proposals of the National Industrial Conference Board Report which deal with the educational program for engineers. There are some respects, however, in which important differences do arise. These are largely questions of emphasis, some of which are raised as a result of the new facts. Yet there is one phase of the general problem which does not seem to have been considered seriously in previous studies, and that is the practicability of an effective training for general management in non-technical industry, by other subject matter than technical engineering. This possibility is recognized in the present study.

FACTS

The first step in the study was to ferret out and compare the major, relevant facts from the various surveys. The summary facts thus indicated, which apply certainly to mechanical, very probably to electrical, and possibly to most other engineering graduates, are as follows:

1. About one-half of all such engineering graduates by the time they have reached 40 years of age, have gone into industrial work primarily executive in character. These include two approximately equal groups.
    a. Those whose duties have *required* a substantial *technical engineering knowledge*.
    b. Those whose duties have not required such knowledge.
2. Only about one-quarter of electrical and mechanical engineering graduates have remained past 40 years of age in purely technical engineering work, involving practically no executive responsibility.
3. The rest, or roughly one-quarter, have gone ultimately into work such as teaching, selling and consulting, all of which involve dealing with people.

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1. For references see Bibliography.

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4. Those who have gone into work involving executive responsibility (group 1) have, on the whole, received greater financial reward than those in group (2) whose work has not involved such responsibility.

5. Those in *general* management, specified in (1b), whose duties have been entirely administrative and practically non-technical, have, on the whole, received much greater financial reward than any other group.

Three extremely important facts of general knowledge might be added.

6. Practically all college graduates, whether engineers or not, who have become successful in management, have learned all they have known about it, in practise. Formal college training in management, in the general sense, has existed only during recent years, and in only a few institutions. But it is steadily increasing.

7. Practically all engineering graduates, whether they have gone into some form of management or not, have encountered those human relations which are inevitable in the cooperative enterprises in which engineers engage.

8. The great technological progress of the last half century has been based, first, upon the extension of scientific knowledge by scientists; second, upon the careful and rational application of this knowledge to the solution of practical problems by engineers; and, third, upon the organization of capital, physical facilities, and men for production and distribution, by those in management.

RETROSPECT

Industrial management and engineering are twin brothers; they have developed together. With a constantly expanding science and a growing army of engineers and inventors who applied that science, American industry has developed from the simple English patterns of the 18th century to its modern, gigantic, highly organized branches, all of them interdependent, capable of turning out with incredible rapidity and perfection steam turbines, electric generators and motors, automobiles, radio sets, cigarettes and plastic products. During this development, both industrial management and engineering naturally experienced a correspondingly extensive metamorphosis. The range in responsibility and in requisite talent, ability, and knowledge was multiplied a thousand fold. This extension of industry was upwards, fan-like. The small factories and machine shops continued, but there was also tremendous growth both in size of organizations and in range of products in the expanding industries. This two-way growth necessarily demanded not only a commensurate increase in level of ability to handle merely the larger and consequently more complex management problems, but also a corresponding increase in technical knowledge and skill, on account of the ever widening spread in range of manufacturing processes and of products, from the simplest, well-understood forms to the most technical and complicated. In this expanding industry, including even those branches of non-technical character, the problems of both management and engineering called for less and less guess work, and more and more rational treatment.

Under these conditions of growth, the thing which might have been expected, happened. Practically all of the conditions were favorable for the engineer to be called to management positions. He was geographically on the job; his distinguishing characteristics—a scientific attitude of mind and method of approach—were among the foremost requisites, and, moreover, he was vitally interested, because he not only understood the processes and the products, they were his creations! Hence by normal growth and evolution many engineers have come into administrative positions.

What has been the past relation of education to management? To what extent has engineering training been appropriate and adequate for management; to what extent, essential? Formal college training in industrial management is of recent origin, and hence has had very little, if any, influence in determining the present personnel in management. On the other hand, the character of engineering education undoubtedly has influenced it. How much, relative to the influence of other existing forms of education, is not altogether clear. For instance, whether the number of engineering graduates engaged in industrial management, considered in its most exclusive sense, is a larger percentage of all engineering graduates than the corresponding figure for graduates of other courses, is not definitely known to the author. But that there are large numbers of the latter in industrial management is a well-known fact, and this should not be lightly dismissed in any comprehensive review of the relation of education to management. However, it is certain that a large percentage of engineering graduates are in industrial management positions of one form or another, and particularly in both those which require technical knowledge, and those which, although not requiring such knowledge, are more or less terminal positions from a line or promotion through the former. There are, of course, thousands of positions of these kinds in industry, and for these, certainly, engineering training has been, at least in part, a logical preparation. Its lacking elements have been the subject of many an utterance; and unquestionably essential phases of management—such as personnel and accounting, for instance—have been omitted. Yet it is also an indubitable fact that, quite consistent with his characteristic quality of finding out how to handle new problems, the engineer has acquainted himself, at least to a practical extent, with these other phases, and has done the job. How appropriate engineering training has been for those management positions for which technical knowledge *per se* has not been a requisite, either directly, or as a stepping stone, is largely a matter of opinion. Only this is certain: to the extent to which a quantitative method and a scientific attitude and approach are requisite or desirable in management, engineering training is highly appropriate. These qualities do seem desirable, and in the larger industries, requisite. The engineer's sequence of training and progress has been perhaps not as well ordered as it might have been, nor as it should be in the future. However, the point is that in that particular stage of industrial and educational development, it was a logical outgrowth, and it worked, as is attested by the growth of American industry.

In relatively recent years there has been a growing recognition in the educational policy of engineering colleges, of the all-pervading fact that the majority of engineering graduates have gravitated ultimately to some form of management. This recognition has largely taken the form of adding appropriate electives to the regular engineering curriculum, and in setting up new courses under such headings as industrial engineering, engineering administration, etc., in which the essential core has been engineering. This has been in accordance with the recommendations both of the Mann Report and of the S.P.E.E.

But there has also been a rapidly growing philosophy which would not recognize the essentiality of *technical engineering* subject matter in college preparation for general industrial management. It holds that a scientific attitude and rational method of analysis, which are recognized both as essential and also as characteristics of engineers who have succeeded in management, can be developed on some other subject matter than engineering. This general idea has not yet, to the writer's knowledge, taken full-fledged practical form, unless some of the business administration courses are so regarded; but it is being reflected in the gradual deletion of technical engineering matter from such courses as mentioned above, and the substitution of general management studies. So the presumably more direct and certainly less exacting college approach to management is gradually being set up.

The financial reward for technical leadership has been less than that for general managerial leadership. According to the statistics, the man who has been able to "run" things—to coordinate human activities effectively and to judge soundly, toward definite objectives—has been valued more highly by leading executives than the man whose natural talents have been in a technical vein. Moreover, according to the facts, even those who could manage men and things in technical activity have received less financial reward than those in "general" management, in which technical engineering knowledge *per se* is either not essential or is altogether secondary. Going still further, those who have remained in purely technical work with no executive responsibility, have been the most poorly paid. On the face of it, then, it would appear that in some fundamental way, mere association with technical matters relegates one to a lower stratum in financial reward.

However, that it has been so in the past should not be accepted as the establishment of an eternal fact. Random play of tremendous forces out of equilibrium have been the order of the day, and we must be wary in drawing conclusions from experience of such a character. But the clear implication is that something is wrong; technical engineering leadership has played too important a part in the past and obviously will be too much needed in the future, to be now overlooked and smothered into insignificance under our intense enthusiasm for industrial management, which it largely created. It is not a question of merely giving such leadership its due; simply we should be stupid trustees, indeed, to the next generation if we created a problem for it by letting the present trend slip ahead unabated. Our first problem in this connection is to try to understand the past better. It is complex enough, but a thoughtful review does indicate, here and there, a rational thread running through it all.

Supply and demand, as it relates to the present subject, seems to be one such thread. In the mad rush ahead, there has been a greater relative scarcity of managers, for whom no formal college training facilities had existed, than for technical leaders, for whom such facilities had been abundant. But scarcity for that reason, it should be observed, was only an accident; it was not a scarcity of natural management talent. The apparent result has been that much, but fortunately not all, of the best engineering brains necessarily has been conscripted for difficult service as pilots through these new and uncharted management areas. Thus the pressing and urgent demand during this industrial development era for men to fill administrative positions, dug deep into the upper levels of engineering ranks.

If much of the best brains turned to management, and, moreover, if there yet continued to be a scarcity of management ability, it is not at all surprising that this should be reflected in financial reward. By and large, if those who entered management could do the technical work the others were doing and were also capable of managing, were they not more valuable? Undoubtedly; to the extent that this was so. If they were better all-round men, naturally they could earn more.

On the other hand, the group of engineers who have remained in purely technical work—comprising about one-quarter of all engineering graduates—have been poorly paid relative to all others. This can probably be justified. What is the function of those who remain in purely technical activity, with no administrative responsibility whatever, and who are not technical leaders in the above sense? Routine technical activity. They have learned how to make certain routine calculations, drawings or tests, or to operate some machinery, and they do these things day after day in a routine and satisfactory manner. Their work is important—indeed the execution of such tasks is obviously essential. The point is that a four-year college education is hardly a necessary prerequisite to such tasks. Those who have remained in such positions are, for the most part, those who have not possessed the requisite qualities of a successful engineer; and doing the work merely of a trained technician, they have been paid accordingly.

However, there is one element in this financial reward matter which does not fit in so well. *Creative*, technical engineering *leadership* has not been appropriately rewarded. There are two sides of technical leadership.

One is technical management, which administers the day-to-day technical processes of operation, design or calculation. The other is *creative*, professional leadership in technical matters. In this leadership is that small group of 10 per cent or so of scientists and engineers who are identified by *creative achievement*. They are the ones who conceive the new technical possibilities—who can see through the technical mists which blind most of us; who thus provide the radically new concepts and methods, whether they be new forms of machines or structures, new ways of building these, or new principles and methods of predicting performance. These things are absolutely indispensable to technological progress, in the same sense that the products of creative, professional leadership in management are absolutely indispensable. One whose head is a fountain of *creative* ideas is thus a technical leader, even if he cannot himself do much toward putting them into practical or material form. If he can, he is simply that much more valuable as a leader. The one quality rounds out and supplements the other. Either may predominate in equally valuable men—valuable as measured in terms of achieved results. Both functions are professional in character and are complementary in relation to each other, and should therefore be regarded more nearly on the same basis so far as reward is concerned than they have been. It is a matter of fairly general knowledge, which the surveys have confirmed, that this phase of technical engineering leadership has not been, on the whole, appropriately rewarded. This state of things in which commensurable achievements are not weighed in the same scales does not fit well into any rational picture. Emerson said, "Money, which represents the prose of life, and which is hardly spoken of in parlors without an apology, is, in its effects and laws, as beautiful as roses. Property keeps the accounts of the world, and is always moral. The property will be found where the labor, the wisdom and the virtue have been." One wonders where he got his data.

However, there is a very definite, favorable trend in policy about these matters in some of the large industries, even if this is not reflected in the surveys; so there are indications that creative, technical engineering leadership is beginning to approach its proper estate. But one wishes to see this favorable trend spread about more rapidly.

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Thus looking back over the development of industry with the purpose of understanding the relation which has grown up between engineering and management, one discerns among other things, that the reason why engineers have trended in such numbers toward positions of administrative responsibilities in industry is not alone that they acquired technical knowledge at college. This, of course, has had its important influence, but the things which seem to have had a predominating influence are: (1) a pressing demand for managers in the rapidly growing industry; (2) the absence of any source of graduates formally trained in the fundamentals of management; (3) engineering graduates have possessed one of the requisite qualities of capable management—a scientific and utilitarian attitude of mind and an analytical method—even if their formal college training has lacked almost completely any consideration of the fundamental aspects of management; (4) they have been directly interested, geographically on the job and thus readily available; and, of outstanding importance, (5) they have been resourceful and able to acquire in one way or another, the necessary additional knowledge and ability for adapting themselves successfully to management.

One sees, in addition, an extremely wide range in industry, from the highly technical, on the one hand, to the practically non-technical, on the other; and consequently a correspondingly wide spread in the technical educational requirements for management throughout the range.

And finally, one finds that financial reward has been distributed in a reasonably rational way, with one glaring exception. It seems reasonable that engineers at the top in general management should receive higher reward than those at the top in technical management, and so on down the line; and that those in technical management should receive more than those in routine technical activity. But it does not seem reasonable that creative, technical leadership has not ranked more nearly on a par, throughout the range, with creative, managerial leadership. This, coupled with the creation of attractive courses leading presumably more directly to management, is rapidly removing incentive for the best minds to take technical engineering courses.

Looking to the Future

When we orient ourselves in the direction of present trends and follow out those few clearly discernible lines from the past, they seem to lead to certain fairly definite indications. The foremost of these is that the future will have no less need than the past for leadership both in the scientific and technical fields of engineering and in management. On the contrary, the need will probably be greater; everything is getting more complex; each year it is becoming more difficult to see from one end to the other of this growing, technical-economic-social crazy-quilt with such thousands of complicated, overlapping and changing patterns. Trained minds *for leadership* will be all the more essential. Inventors must more and more have advanced scientific knowledge; the cream of invention has already been skimmed from the more obvious aspects of science, and therefore inventions will unquestionably become, in increasing measure, the product of scientists and engineers. The real leaders in technical engineering must also be more thoroughly trained in the basic sciences and in the rational applications of scientific principles to practical problems. Moreover, they must have a better comprehension of the human aspects of their job; they must learn to deal more effectively with other people, for it is a part of their job. And leadership in industrial management must likewise become broader in its outlook, more scientific in its knowledge and more rational in its approach to general management problems. In a word, all professional leadership—not excluding law, banking, the clergy and the rest—must broaden its outlook, perennially develop its scientific base and improve its art; else, in the inexorable advance of complexity, the leaders will become even less able than they are now to see across the boundaries. We have all taken hold of the technological bull's tail, and it is too late to let go; instead we've got to plan pretty quickly how to guide this brainless, powerful brute toward a rational objective, or he will distribute us prostrate over the terrain.

Another indication is that the same forces which have moved engineers into management will continue to do so, in a somewhat modified form, in the future. Much of the industry will continue to be based upon engineering; and with this and its collateral conditions remaining, that movement will undoubtedly persist, but with modifications in two respects. In the first place it will probably be modified by the influx of a new group which has not existed during the past growth of industry, namely, college graduates who have had formal training in the fundamentals of general management, but who have had little, if any, technical engineering training. With the very wide spread of industry itself and of function within the larger industries, it is practically certain that such graduates, who will not be engineers, will be absorbed in those areas of industrial management where technical *knowledge* is not essential. Then also there will be the graduates of those existing courses which, although still designated as "engineering"—in most cases properly so—are yet intended primarily as fundamental preparation for industrial management. Altogether then, there will probably be a very large body of graduates, most of them falling under the heading of engineers, who will be available, alongside technical engineering graduates, for industrial management positions. The flow of technical engineering graduates to management will therefore probably be modified both by the withdrawal from technical courses of managerial-minded students, and by a competition after graduation which has not formerly existed; to what extent, will depend upon whether such management courses have successfully retained those elements which engender the method and attitude that have been characteristic of engineers, and which presumably have been the keynote of their success in management.

If such elements are retained, then there seems to be no reason why in the future those other graduates should not comprise a large percentage of those in management. The tremendously wide range in type of industry surely affords openings in management for men of all degrees of technical training. Even in the most technical industries there are many administrative positions which require only a very limited technical knowledge; and the further up one looks in the organization, the more this seems to be so. Take the General Electric Company for instance. A former president and a chairman was a non-technical man from the shoe industry; the present chairman is a lawyer by profession; there has always been a large fraction of its administrative officers who were not technical men; and many of them, including some who were technical engineering graduates, joined the organization in middle life. All of this has evidently reflected a wise policy, if a successful organization is any criterion; but the point to be illustrated is that technical engineering graduates are not altogether essential in the *general* management of a highly technical industry. Hence, if the colleges do a good job, it seems practically certain that, within another generation, graduates of engineering-management and other less technical courses will figure heavily in industrial management.

However, there is one point to be kept in mind. There should be at least some individuals in the high levels of general management who have an understanding and appreciation of technical matters and a sympathetic interest in research. The half-informed business executive who disparagingly avers that he can get all the engineers he needs at $40 per week, or who naively believes that creative processes can be put on a mass production basis, is a misfit in modern industry. Such views, fortunately, are gradually passing. There should be general management leadership which thoroughly understands and has a sympathetic interest in technical progress. This can come most effectively from former technical leaders.

What is the future for technical engineering graduates? Are they doomed? Not at all. On the contrary, their estate should become as attractive as any, if industry and the colleges adopt wise policies. There are two important phases of management which will indubitably be reserved almost exclusively for them; and, in addition, all other fields will be open to them just as they have been in the past, except that the competition will be more keen. One reserved field is technical administration which has always served as a stepping stone to higher levels. The other, and altogether foremost, is the creative, professional guidance and administration of technical and scientific progress. In this, which obviously is one of the most important management functions in society, one must be capable not only of creative technical leadership, as already discussed, but, in addition, be able to guide and manage others; in other words, be capable of management in a dignified and highly select field. And in this field also will be those technical and scientific leaders who may not be capable of management in its strict sense. These fields afford broad and extensive professional opportunities for technical engineering graduates; and adding to these fields the opportunity, as in the past, to enter any other phase of management, the future possibilities for such graduates can be highly promising. They may be in the minority in general management, but they will be adequately represented there, and, if all goes well, will also be in other enviable positions.

Thus looking to the future, the indications are that there will be even greater need than in the past for trained professional leadership in all fields. The same forces which have moved technical engineers into management will continue, but will be modified both by the withdrawal of managerial-minded students from technical courses and by the influx of graduates of engineering-management and other courses designed primarily as preparation for management. The latter graduates may preponderate in numbers, if proper elements are maintained in their college programs and the general estate of technical engineering graduates can be, and, in the interest of balanced progress should be, made as attractive as any.

FUTURE POLICY

Educational policy cannot be logically separated from industrial policy; there are too many respects in which they are tied together. Most engineering graduates enter industry of one form or another, hence engineering colleges have always taken into account the requirements of the industrial job. When industry wanted graduates who were "practical," the colleges built work shops and tried to make their courses practical; when it changed its mind and insisted rather upon more "fundamentals," they gradually swung in this direction; and when it put large numbers of engineers into management, they worked out courses which would be preparatory to management. Thus any rational educational policy must give appropriate recognition to this interlocking of interests between the industries and colleges.

Moreover, an educational policy which did not recognize the often expressed hope that the engineer might sooner or later take an active interest in those unsolved, complex, social and economic problems which his technology has largely created, would surely be inadequate. What should that recognition be? Sociology, industrial history, and more economics? Undoubtedly this would be valuable to him if he later took such an active interest; but those engineering graduates who have gone into business, banking, management, etc., have not done so because they have studied these subjects in college; forces of circumstance and, above all, a special type of qualification have combined to move them there. And in all probability if the engineer ever takes an effective hand in the solution of such social problems, it will not be because he studied subjects relating to them as a major part of his curriculum; for if he did, he wouldn't be an engineer. Something more must be expected from education—more than merely providing in addition to engineering, a proper balance of courses in sociology, industrial history, and the like. It must make such courses stimulating and distinctly purposeful. If they are successful in *starting* the engineer to read and to think about such matters with purpose, so that after graduation he will continue and thus acquire a full appreciation of human and economic values, then, in the writer's opinion, education will have done all it can do, but nothing more than it should do.

Another thought which should be stated at this point relates to the scope of activity which may be properly classified as engineering. The practise of engineering has *demanded* straight thinking and an unqualified respect for facts. The engineer can find no shelter under an alibi. His responsibility is ruthlessly definite. He has thus been obliged to exhaust every resource of science, all technical knowledge and every power of rational analysis at his command, in order to feel confident that his designs and predictions were sound. He has not done this by virtue of an extraordinary wisdom of a biologically unique group of individuals; he has simply had to do it in order to endure, and in a sense and to a degree that most members of other professions have not. Consequently not as many of them have become so skilled in scientific method. He deserves great credit both for his achievements and his methods, but in the interest of keeping our educational feet on the ground and without detracting at all from his achievements, let us recognize the special circumstances which have given rise to his distinguishing characteristics. Otherwise, our inferences may not be sound.

We may conclude that such characteristics cannot be developed by other professions; that engineering is the proper training for management, business, finance *et al*; and also that engineering graduates who are successful in these other fields are still practising engineering. Thus engineering would come to mean everything which is done thoughtfully, and would therefore mean nothing. Let us keep engineering as engineering; have pride, of course, in those engineering graduates who become successful in other professional fields, and perhaps trust that by their examples these other professions may be graced and benefited; however, when it comes to education, let us not be too presumptuous in our advice to other professions; let their schools introduce engineering content or method in their curricula if they wish to do so, but let us not try to stretch the engineering curriculum over the whole domain.

It has already been stretched to the breaking point in its affiliation with management. Here we have a perfectly legitimate reason for extension; many phases of the management of engineering enterprises logically can be, and certainly are, regarded as engineering. But as we approach general management, the engineering aspects gradually diminish, and in the limit, engineering *per se* has vanished. An engineer may be on the *general* management job using methods and habits of mind he acquired in engineering practise; but he is no more practising engineering, as the writer conceives it, than is his brother engineer who went into banking; or than the lawyer who went into management is practising law. The one is still an engineer, and the other still a lawyer; but their new function is neither engineering nor law. It is general management.

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Some general observations should now be stated regarding college educational process, as the writer views it. This is venturesome, because he is not qualified as either a professional educationist or psychologist, and may therefore violate technical terminology and perhaps more; but it seems necessary, for the purpose in hand, to state in a layman's terms the more or less distinct phases of educational process as they appear to him. There are thus five phases, as follows:

1. The first relates to the process of acquiring factual and definitive knowledge. One learns for instance, the factors in the "flexure" formula in mechanics; the *definition* of "marginal utility"; that Disraeli was an English statesman; that the base of napierian logarithms is 2.718; that inertial reaction = mass × acceleration. It is the process of building up one's mental encyclopedia. It is essentially a memory matter.

2. The second has to do with that type of reasoning in which the student is *led*, step by step, through logical processes either by a teacher, or by a text book. His part in this particular phase of his journey through the educational forest is not to move precariously along a trail which is only blazed; instead he rides on an educational sight-seeing bus over a modern, concrete highway of which even the curves and grades are reduced to a minimum. His only responsibility is to take due note of the scenery, and of the announcer's comments as he rides along. For instance, he may be thus escorted through the *logical development* of the "flexure" formula; of the *theory* of "marginal utility," or of the vector representation of alternating currents. This process is one of civilization's necessary short-cuts. In order to reach the frontier of new things, to acquire on the way an acquaintance with the methods used there, and yet have a reasonable portion of one's life still available for professional activity, some such rapid transportation seems absolutely necessary. In the process one learns to follow logically and to retrace the steps; one's mental muscles are developed somewhat, but they do not become sturdy and of good form by purposeful exercise under their own power; instead, they are massaged.

3. The third phase is the process of acquiring skill in manipulation. For example, one develops skill in algebraic transformations; in numerical calculation from formulas by the slide rule; in operating a machine; or in carrying out an experiment. It is a discipline in procedure.

4. The fourth phase relates to that type of reasoning in which the student himself takes the initiative. *He* establishes the trail. The objective may, at first, be defined for him, and perhaps its general direction and a few landmarks may be indicated; but he presses through alone. In clearing away the undergrowth, in fording or swimming the technical streams which cross his path, in retracing his steps for a new start when he encounters an impasse, he not only exercises his mental muscles in worthy activity and stretches them to the elastic limit, but also develops his sense of direction and his power of discerning helpful landmarks. Each successive trail thus established *by him* increases his pioneering powers; and presently he will be able, if it is in him, to blaze his own trail through new areas to new objectives. The S.P.E.E. Report says:

> A besetting danger in engineering education is that our effort to maintain a high standard of work, if misdirected, may lead us to prefer efficient routine to less certain but more educative self-directed effort.

5. The fifth has to do with the establishment of a natural continuity between the development of the individual at college and after college. The branches of a truly educational program will neither terminate in dead ends here and there during the college course, nor be chopped off at graduation. Instead, they will feed *primary educational stems* which will both continue and expand beyond graduation as long as the individual is on the ascending slope of his professional career. However, the extension of such stems after graduation is extremely difficult unless the student has already begun to extend them under his own steam before graduation. He must develop under guidance both the *desire* and *power* to extend them. The college can plan them and give him guidance; but *he* has to develop them. He cannot be expected to do this unless his course work is definitely planned with this general idea in mind; unless the idea receive continued emphasis, especially in the last two years, by repeated reference to the relation between his present studies and those which he will pursue after graduation. This means, then, first of all, a competent leader as a teacher; and secondly, not only a well balanced curriculum for the purpose, but, in addition, a plan obvious to the student, according to which every branch of study feeds one of the general educational stems which are to project and expand beyond graduation.

The underlying thought here is clearly recognized in the S.P.E.E. report:

> Money costs, the mating instinct and the time demanded for making adjustment to practical life, tend to set a time limit to education as a formal, scholastic process and to push a growing and important part of it over into the early years of productive life. This principle should be frankly accepted in engineering education by recognition of the fact that the process is divided into scholastic and post-scholastic stages.
>
> In this connection we would reaffirm our view that the proper time for specialization is after the undergraduate course, and that in a majority of cases specialization must accompany active experience rather than take the form of a further discipline in college. The provision of effective continuation education for this period remains one of our most urgent problems.

The great importance of establishing such educational stems, however, seems to the writer to demand much greater emphasis and definite planning.

Of the foregoing five phases, probably the first three are essential in the preparation for engineering work of any character; they are sufficient, it would seem, for *routine* technical activity, but the last two, in addition, are absolutely essential to progress toward any highly *professional* goal. If they are omitted at college, as they largely are, the graduate is obliged to provide them somehow after graduation, or else not reach the goal. It is probably a fair statement that not one out of twenty-five students is consciously forming an organic educational structure in his mind, as outlined in phase 5. They think that purposeful study and reading normally end at graduation. To correct this situation—to provide phases 4 and 5—is, in the writer's opinion, the foremost problem confronting pre-professional education. This is particularly so in engineering and management courses, because, unlike law and medicine, students have, as a rule, only four years of formal training.

The specific numbering of these five phases is not intended to imply that they should normally follow each other in practise in that order. On the contrary, they should largely run parallel.

Turning more specifically to the engineering-management aspects of educational policy, those students who would come within the scope of our subject can be classified, for the purpose here, under four divisions according both to natural qualifications and to appropriate education. It may still be very difficult practically to identify these groups completely, but they exist nevertheless, and every effort should be made to learn better to identify them. They are as follows:

Sub-Professional Group. Those who are primarily qualified by nature to carry out assigned tasks, either in technical work or in routine supervision. These comprise certainly one-half, probably more, of all college students which come under the scope of our subject. They comprise not only those whose intellectual qualifications limit them to such activities, but also those who don't want to do anything else. There are people of high intellectual ability who shrink from responsibility of any kind, or who are misfits, or who are plain lazy. They all seem to share a common level of activity with those who can't do more.

In accordance with the S.P.E.E. recommendations,[10] the educational program for this group, excepting the misfits and the lazy ones, should be that of the technical institute, which recognizes primarily the first three phases.

Professional General Management Group. Those who have outstanding natural powers of leadership and understanding, but relatively little technical leaning. This group may be expected to fill a portion of those positions of industrial and business administration—*i.e.*, general management—which do not require technical knowledge.

Just what their educational program should be is a question which is largely outside the writer's knowledge. However, they must be well educated men of reasonable culture; hence it is safe to say that there should be at least two—possibly three—stems: a cultural, and a social and economic, and possibly a slim technical one. The experiment of training men directly for general management is an interesting one, and should be encouraged. One great danger to be guarded against, of course, is the idea in student's minds that such a course provides the "royal road" to general manager or president. Moreover, if the experiment is not successful, it will certainly not be because there were no industrial positions which did not require technical engineering knowledge; it will be because such courses either were not successful with their materials, as engineering courses have been with theirs, in developing a scientific attitude of mind and method of approach; or because the fundamental point was overlooked that most of the professional aspects of management have to be learned on the job.

Professional Engineering Group. Those who have outstanding natural talents along technical lines, some of whom have, in addition, the natural qualities of human leadership. From this group we should expect the future technical leaders; to some degree, leaders in general management, and, possibly also a few who would become actively interested in the larger social and economic problems.

Their program at college should have three stems—one main and two auxiliary: technical, cultural, and social and economic. The first and main stem should be a rigorous and thorough technical discipline. It should recognize all five phases, and should be framed around the basic engineering "core" referred to in the Mann Report, the S.P.E.E. Report and more recently by Dean Kimball;[11] it should lay more emphasis upon the interpretation of fundamental physical laws and their application to the solution of illustrative problems than upon the memorization and use of standard formulas in connection with type form cases; it should continually consolidate the gains achieved in all branches, by feeding them into the main educational stem. For instance, in the junior and senior years it should make use of the results achieved in mathematics, physics and mechanics, not in separate problems in as many classifications, but in engineering problems involving all. Past achievement would thus be unified with respect to the main purpose.

The cultural stem necessarily must be slim. But it can be vital, and should be so. Perhaps the most that can be expected in the brief time available is to emphasize not only the technique of speech and writing (phase 3), but also and principally the fundamental purpose of most speech and writing—namely, to clearly express an idea—and the importance of this in professional life; and to *engender the habit and desire* of purposeful reading of historical, philosophical and other cultural literature (phase 5). In the writer's opinion, this has not been sufficiently emphasized in the reports referred to.

While the social and economic stem is like the cultural in two respects, it is nevertheless of relatively greater importance. It is like the latter both in that it is auxiliary in character and that the general educational purpose is the same, *i.e.*, gradually to establish, in accordance with phase 5, the habit and desire of reading and thinking in a given field. In this case, however, it is about such practically important matters as industrial history, human relations in industry, sociology and economics of engineering industry and business. Taking courses in such subjects and letting it go at that is not enough; it must all be purposeful.

The program for the *professional engineering group* also should anticipate that a moderate fraction of them will take formal post-graduate work either in college or in industry.

Professional Engineering-Management Group. Those all-round, capable men who have a definite technical leaning and also the natural qualities of leadership. This group, after graduation, may be expected to pass along the industrial, three-stage separator in which the first stage comprises the lower and moderate levels of technical activity; the second, technical management; and the third general management. Each stage will take its toll of men. And again, one would hope that some of these would ultimately become interested in general social and economic problems.

Their college program, like that of the *professional engineering group*, should have three stems—the same three stems; but there should be a fundamental shift in emphasis in two of them. This group should develop and retain the engineer's attitude and his respect for quantitative facts and measurements, both because that attitude and respect will always be highly useful, and also because technical engineering practise will presumably constitute his entree to management responsibilities. However, the latter, being the main objective, should be given a corresponding weight in the program. Thus the social and economic stem should, it seems, receive the same relative weight as the technical stem for the professional engineering group. The technical, for the present group, should lay relatively greater emphasis on the first two phases; the social and economic, on the last two phases. Analytical and pioneering powers should be exercised heavily on the subject matter of the social and economic stem, so that this will have a vital growth after graduation. For illustration, take human relations in industry. Can you teach in undergraduate courses the professional technique of personnel management, or the art of getting along with associates? It would seem to be quite hopeless. But you can introduce the student to fundamentals, and exercise his analytical powers on actual personnel problems—all with the idea of developing a purposeful and enduring interest, and, as Professor E. D. Smith says,[12] of making him "sensitive" to the lessons of experience.

The cultural stem should presumably be the same as that for the *engineering group*.

SUMMARY

During the development of industry, engineering and industrial management have evolved together. Certain phases of management, as the writer views it, have actually been a part of engineering; others have not. The direct administration of technical engineering activities *is* engineering, in the same sense that the administration of personnel and procedure in a legal matter is law. However, we should not become confused; the further movement of engineers from such technical management to general management has not made the latter become engineering, any more than the movement of numerous lawyers also into general management has made it become law. It is still general management. Engineering and management are thus closely related, but all management is not engineering.

Looking to the past, the facts are that something over a quarter of engineering graduates have gone into general management; a quarter have remained in technical management, which is engineering; a quarter, in purely technical activity; and the rest in sales, consulting, teaching, etc. Those who have gone into general management have received the greatest financial reward; those who have remained in purely technical activity, the least.

Looking to the future, it appears that, on account of the rapidly increasing complexity on all sides, even more capable leadership, both technical and managerial, must be provided for the future. It seems clear also that the forces which have moved engineers in such large numbers into positions of executive responsibility—both technical and general—will continue in the future. In other words, technical engineering graduates, it appears, will have the same opportunity as in the past for movement into such positions, excepting that they will be in competition with graduates of courses designed as fundamental preparation for executive positions. There is a very real danger that both the prospect of a greater financial reward, and the less exacting nature of these presumably more direct courses in preparation for management, will cause many to turn from technical engineering courses who are naturally qualified for such work, and who will be needed for technical leadership in the next generation. The problem of financial reward is one which the college cannot solve; industry must do it, not alone to give creative, technical leadership its due, but, in industry's own interest, to prevent the source of technological progress from drying up.

For the purpose of defining educational policy, the students who fall within the scope of our subject can be broken down into four divisions: a *sub-professional* group, and three professional groups, namely, *engineering, engineering-management*, and *general management*. The educational program for the sub-professional group should be that of the technical institute. It should recognize primarily the first three phases of educational process—the acquisition of factual and definitive knowledge; the development of technique in manipulation and procedure; and the understanding of basic theory. The programs for the professional groups, on the other hand, should recognize all five phases, and should emphasize particularly the last two—*viz.*, the exercise of analytical and pioneering powers; and the development of *primary educational stems*, and of the power and desire to extend and expand them after graduation. It is around these stems that the results of purposeful study and experience will be structurally articulated. The professional groups, it seems, should all have the same three stems: (1) cultural, (2) social and economic, and (3) technical. Merely the emphasis would be appropriately different.

The present inability completely to classify all students according to natural aptitudes should not discourage us. Such a classification is now possible to a limited degree. In any case *a* division is made; boys actually do choose to be engineers, lawyers, managers, etc., and college courses are set up to train them. A rational solution of the educational problem, it seems, can be approached only as we learn progressively to sift out types during the lower educational stages, and somehow help the individuals to find their way into educational programs and activities along the lines of their natural aptitudes.

In a summary word, let us recognize the necessity of providing appropriate educational preparation for industrial management; but in our enthusiasm for this, let us not misinterpret past experience and thus fail to recognize another equal necessity; we must also provide both an appropriate educational preparation and a promising outlook for technical engineering leadership. Its contributions to technological progress surely bespeak a greater recognition than it has had. But mere justice is not the point here. It is a question whether, by a stupid perversion of emphasis now, we shall encumber the next industrial generation with the problem of a dried up source of technical engineering leadership. Wide publicity of the new facts regarding attractive salaries in management, and the continued establishment—proper as this may be—of less exacting courses leading to management, will, it seems, inevitably discourage the more capable men from choosing technical engineering courses. The solution is not to suppress the facts or to discourage management courses, but for industry to recognize the situation, as some are beginning to do, and appropriately reward technical engineering leadership; in other words, to make such leadership a worthy and promising goal on its own score, and not merely as an entree to general management.

ACKNOWLEDGMENT

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Discussion

R. E. Hellmund: The study by Professor Doherty of the educational aspects of engineering and management brings out a great many very interesting facts and conclusions, practically all of which coincide closely with my own experience and observations in my work with the Westinghouse Electric and Manufacturing Company. I wish to emphasize in particular the point that our enthusiasm over the fact that in the past our engineering education has turned out personnel suitable for industrial management should in no way lead to a diminution of the engineering phase of the undergraduate program in an attempt to provide at the same time the education necessary for managerial work. Undoubtedly the undergraduate program should have the three stems mentioned in the paper; namely, technical, cultural, and social and economic, and I am confident that no mistake will be made in always placing the main emphasis on the technical phase and giving merely enough of the cultural and social and economic to arouse the interest of the students in these subject matters. The fact that only 25 per cent of the engineering graduates remain in purely technical work after the age of forty should not result in our overlooking the fact that by far the majority of all students during their earlier life, and especially, soon after leaving college, are called upon to carry on technical engineering; as a matter of fact, the greater part of all engineering work, not only of the routine type but also in the more progressive and advanced lines, is done by men during their early professional life. It would therefore be a great mistake and likely to reflect very unfavorably upon the quality of engineering and the technical progress made if the technical subjects of the educational program for engineers were reduced below a certain necessary standard.

The previous remarks should by no means be taken as indicating that industry is not badly in need of leaders and managerial talent and that educational programs should not take this into account. However, in so far as all those men who eventually work toward advanced and managerial positions usually carry on postgraduate work, either in college or otherwise, the emphasis on studies necessary for these positions should be left for such postgraduate work and as preparation for further advancement in industrial life. The student after entering industrial life will have a much keener appreciation of psychological and business problems than can be had in college. Furthermore, he will be in a better position by that time to sense whether his natural aptitude is toward purely technical or managerial work and will be more able to choose between the various studies suitable for either phase of the profession. I would therefore advocate that, rather than materially changing the undergraduate courses, considerable attention be given to the training of personnel for managerial work during the various postgraduate programs and to inducing a certain number of engineers to emphasize this part of the work during their postgraduate education.

D. C. Jackson, Jr.: Professor Doherty brings out that the really able men in engineering work in the industries are brought into management because of their ability and he suggests that the greater financial return accruing to these men, especially those in *general* management, is due to the fact that they are in management as such. In following the progress of able engineers in the industries during the past ten years, my opinion has come to be that it is not management *per se* which brings the larger remuneration, but the fact that men of outstanding ability in general and broad fields of endeavor are more necessary in management than in purely technical work. As a result, the able men who are given managerial duties receive higher salaries because of their ability which gives them increased value to the industry, rather than merely because they are in management instead of purely technical work.

Professor Doherty is correct in saying that the engineer in management work is still an engineer and continues to have the engineering trend of thought and to solve the problems he encounters with the logical and keen method of analysis which he has acquired in his engineering training and practise, but that such an engineer is not doing engineering work. A man doing work of this sort is still an engineer, although he is no longer actually in the field of engineering. It is this fact that has made the engineer particularly valuable in the industries (and in this term I include the public utilities, railroads, etc., as well as the manufacturing establishments which are frequently classed as the "industries"). It is my belief that engineering training will continue in the future to be valuable to men having managerial duties in the industries based on engineering. There may for a time be a trend of students toward courses of business or engineering administration or management which will carry some of our abler students away from the undergraduate engineering courses. However it is doubtful whether such courses of study can provide as satisfactory a training as do engineering courses in the "straight thinking and unqualified respect for facts" combined with logical analysis, all of which are so desirable and necessary in our present-day industries. It is my confident belief, if the engineering educators continue as progressive as they have in the past in eagerly making the engineering training particularly effective in the solution of modern problems, without sacrificing the sound principles fundamental to engineering, that the engineer will prove in the future to be even more valuable in both engineering and general management.

Ernest E. Johnson: I should like to present evidence of one point which Professor Doherty has made in his paper, namely, that in industry, men for responsible engineering management positions are and can be drawn quite largely from among those who have made substantial success at technical work.

For the past ten years I have been with the General Electric Company at Schenectady, seven years of which have been spent in connection with personnel and educational activities. I have seen it happen time and again that men who have demonstrated a high order of technical leadership have gradually come into positions of technical management by that route.

One man who was very successful in the technical design and development of mechanical devices is now engineer-in-charge of the Welding Engineering Department and is chairman of all welding activities for the entire company, an activity involving several millions of dollars annually.

Another man who demonstrated exceptional ability in the highest type of analytical engineering work was placed in charge of directing a group of ten or twelve men engaged in power system studies.

Another, a first-class designing engineer, is now in responsible charge of an engineering organization at Lynn, Mass., for the design of turbo-alternators.

Still another, of high talent in general engineering, has been placed in the manufacturing department at Schenectady in charge of waste and spoilage—an activity which calls for high leadership and which involves all of the manufacturing divisions at the Schenectady plant.

And there are many other cases. From this experience and from my frequently having acted as a buffer between over-zealous department heads on the one hand and the outstanding young men on the other, to prevent the latter from leaving technical leadership for technical management too early, it is quite evident to me at least, that there need be no fear that we will be taking away from the field of management by including in our engineering curricula a fundamental technical background.

I—Combined Reliability and Economy in
Operation of Large Electric Systems
THE DETROIT EDISON COMPANY

BY A. P. FUGILL*
Member, A.I.E.E.

WITH but few exceptions, the requirements for extreme reliability and the economical use of resources in any electric system are diametrically opposed. It is not particularly difficult to design and operate a system to give any practicable degree of reliability of customer service, if no consideration need be given to the price the customer must pay. On the other hand, a system could be designed and operated with an overall economy exceeding present values, if the service standards were lowered sufficiently. The real problem is to obtain the required reliability with the maximum operating economy and the minimum of investment.

These ideas are not particularly novel, and it may seem at first thought as if there is little to present on the general subject of reliability and economy which has not already been discussed. However, in the light of the experience of the past few years, perhaps the need for an excessively high standard of reliability has been overstressed; certainly in some cases a rather high price has been paid for marginal increments above an already rather high standard. Furthermore, the industry as a whole has so actively striven for low production cost, that almost every power plant which has been built has surpassed its predecessor in this respect. Unfortunately, the improvement has usually been obtained by additional capital investment with its consequent increased annual fixed charges. It has been recognized that the total system economy is a combination of investment economy and operating cost and a balance has been struck between the two to give the lowest overall cost of power. But too frequently, the basis for striking this balance has been the expected station loading for business conditions such as obtained in the decade previous to 1930. With the advent of less prosperous times, the balance was upset and the long-time economy for the station suffered. Since it seems inevitable that the curve of general business will continue to be a series of peaks and valleys, more weight should be given that factor in balancing investment against operating cost. Perhaps, also, the time has come for a more intense study of the important factors of transmission and distribution efficiency.

Accordingly, it seems well worth while to discuss the subject in the light of the modified ideas on the relative importance of the factors involved. This paper, therefore, will outline the practises of The Detroit Edison Company affecting reliability and economy in the hope that by comparison with the facts brought out by other operating companies in companion papers, some benefit will accrue to the companies represented and to the industry as a whole. To prevent the paper from becoming unnecessarily long, only the fundamental principles and major factors in their application will be discussed, details of practise being included only where they seem particularly pertinent.

GENERAL PRINCIPLES

Stated briefly, reliability of the service furnished by The Detroit Edison Company is assured; first, by the design and installation of facilities to reduce the probability of outage; and second, by a system layout and operating procedure based on segregation and sectionalization to minimize the effect of an outage. Operating economy is secured; first, by the fundamental design of each new installation for reasonably low production cost; and second, by so allocating the load between generating stations that the more economical machines carry a greater percentage of the total load in so far as the physical location of the load makes this possible.

DESCRIPTION OF THE SYSTEM

The following description of the system will be clarified by an inspection of Fig. 1 which shows the geographical location and electrical connection of the principal production and transmission facilities. The Detroit Edison Company supplies electric service to a territory of 4,580 square miles in Southeastern Michigan, the extreme limits of which are mostly included within a semi-circle with the City of Detroit as the center and the eastern coastline of the state as the diameter. This area divides logically into two sections; metropolitan Detroit which includes less than 5 per cent of the total territory but 76 per cent of the people and 75 per cent of the power requirements, and the suburban area surrounding the city. The load is supplied principally by steam turbine-generators located at four power houses with an aggregate installed capacity of 825 megawatts. Two plants, Delray and Connors Creek, are located on the Detroit River within the corporate limits of the City of Detroit. The other two plants are located in suburban territory; Marysville is on the St. Clair River about 50 miles northeast of Detroit and Trenton Channel is on the Detroit River about 20 miles south of the city.

The city load is supplied at 24 kv. from five major switching stations, Waterman, Connors Creek, North-

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east, Warren and Navarre, and three distributing substations, Chandler, Cortland, and Frisbie. Waterman (the switching station for Delray) and Connors Creek are located at the river's edge but convenient to large industrial areas. Warren and Navarre are located at the western edge of the city and Northeast just outside the corporate limits at the north. Chandler is adjacent to the industrial section on the east side of Detroit, and Cortland and Frisbie are close to the geometric center of the city. A 24-kv. underground cable transmission system ties these city stations together. The Waterman Station handles the entire output of the Delray power plant; and Connors Creek switching station the entire output of the Connors Creek plant. Northeast is supplied normally from the Marysville power house over a 120-kv. transmission system, and Warren and Navarre from Trenton Channel in the same manner. Chandler, Cortland, and Frisbie are supplied with bulk power at 24 kv. from the above major switching stations. The city load requires the entire output of Delray and Connors Creek, the bulk of the output of Trenton Channel and a goodly portion of the output of Marysville.

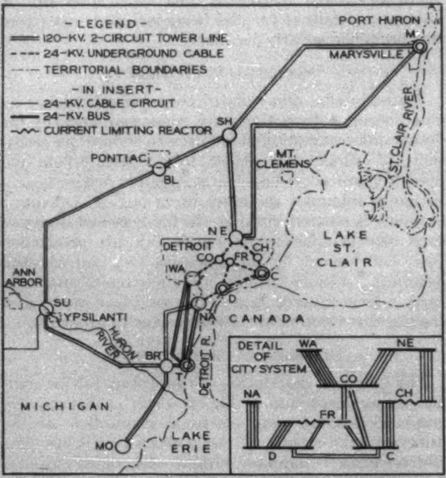

FIG. 1—THE PRODUCTION AND TRANSMISSION SYSTEM OF THE DETROIT EDISON COMPANY SHOWING GEOGRAPHICAL LOCATION OF FACILITIES

M—Marysville Power House
C—Connors Creek Power House
D—Delray Power House and Waterman Station
T—Trenton Channel Power House
BR—Brownstown Station
SU—Superior Station
BL—Bloomfield Station
SH—Shelby Station
NE—Northeast Station
WA—Warren Station
NA—Navarre Station
MO—Monroe Station
CH—Chandler Substation
CO—Cortland Substation
FR—Frisbie Substation

The suburban load is supplied by the Marysville and Trenton Channel power plants from a 24-kv. switching station at Marysville and from four 120-kv. to 24-kv. step-down transmission stations, Bloomfield, Superior, Brownstown, and Monroe, the first three of which are on a 120-kv. double-circuit transmission tower line which connects the two suburban power plants. The other station, Shelby, shown in Fig. 1 is merely a 120-kv. switching station and supplies no load.

TABLE I—DATA ON TURBINE GENERATORS

Power House	Gen. No.	Rating		Date in commission
		Mw.	Mv-a.	
Delray	6*	14	14	1909
	7*	14	14	1910
	8*	14	14	1911
	9*	15	18.75	1913
	10*	30	33.33	1919
	11	50	62.5	1929
	12	50	62.5	1929
	21	10	12.5	1931
Connors Creek	1	20	25	1917
	2	20	25	1915
	3	20	25	1915
	4	45	50	1918
	6	45	50	1920
	8*	30	37.5	1920
Marysville	1	10	12.5	1922
	2	30	33.33	1922
	3	10	12.5	1923
	4	30	37.5	1926
	5	30	37.5	1926
	6	50	62.5	1930
Trenton Channel	1	50	62.5	1925
	2	50	62.5	1924
	3	50	62.5	1924
	4	50	62.5	1926
	5	50	62.3	1927
	6	50	62.5	1929

*Emergency reserve only.

Table I lists pertinent data on the turbine generators and the one-line diagrams of Fig. 2 show the electrical connections at the power stations and at typical transmission stations.

A small amount of the load in the vicinity of Ann Arbor is furnished by hydroelectric generators located in stations along the Huron River. The aggregate installed generator capacity is only 9 megawatts.

APPLICATION OF GENERAL PRINCIPLES

Reduction in the Probability of Outage. It is the general policy of The Detroit Edison Company to be reasonably conservative in the application of facilities on the basis that an increased reliability of equipment results. In determining the capacity to be installed, the design is such that the equipment does not need to be crowded to the very limit of its output for a reasonable emergency. This is particularly true of items such as boilers, which are especially affected by overloading. For instance, at all of the plants, except Connors Creek, the boilers are conservatively rated at maximum plant output. This provides an overload capacity which, incidentally, was occasionally very useful in that period when it was next to impossible to obtain coal with the expected heat content and, as a consequence, the boilers could not reach their maximum rating. However, advantage is taken in emergencies of the inherent short-time overload capacity of certain equipment when it is felt that such operation does not reduce its reliability. For instance, transformers and induction regulators are permitted to operate at higher than their nameplate rating for a short-time peak load period in an emergency.

In general, a voltage insulation standard higher than the minimum acceptable, has been adopted because of its important bearing on equipment failure. For instance, cable insulation is thicker than usual, the bus insulators and bushings on transformers and breakers are the next higher voltage class, and clearances to ground and between phases are greater than the minimum standards.

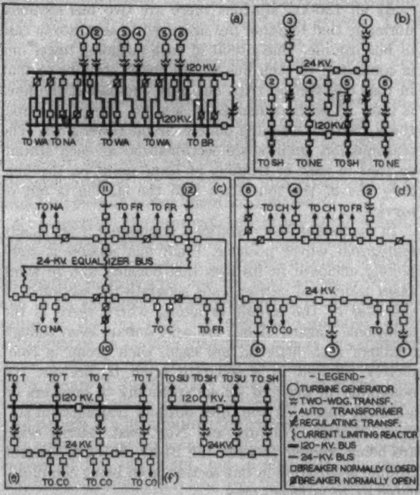

FIG. 2—ONE-LINE DIAGRAMS SHOWING ELECTRICAL CONNECTIONS AT THE POWER HOUSES AND AT TYPICAL TRANSMISSION STATIONS

(*a*) Trenton Channel Power House
(*b*) Marysville Power House
(*c*) Delray Power House and Waterman Station
(*d*) Connors Creek Power House
(*e*) Warren Station
(*f*) Bloomfield Station

It has been found that a large percentage of the trouble on electrical equipment is due to the mechanical rather than the electrical features. A study of maintenance records clearly shows any weak point in the design or manufacture of purchased items and the probable remedy can usually be suggested. By a close coordination among the maintenance, operating, and engineering departments, these defects are remedied in new equipment by designating acceptable construction for these particular features in the purchase specifications. In this way, as additional experience is accumulated, a specification is evolved which pretty well defines the acceptable mechanical construction. This often results in forcing the manufacturer to furnish somewhat special equipment, but this slight additional expense is entirely justified by the improvement in operating reliability and reduction in maintenance costs which result.

Realizing that circuit breakers are by far the most hazardous piece of electrical equipment involved in the station design, they have been eliminated wherever possible, as, for instance, in distribution stations fed at 24 kv. In these cases, cable and transformer are considered as a unit, being switched on the high-voltage side of the transformer at the sending station only, with no 24-kv. switching equipment or bus at the distribution stations. At the indoor 24-kv. switching stations, where it is impossible to eliminate breakers, a type with a minimum of oil is used on the basis that the real hazard is the large quantity of oil in the tanks. At the outdoor stations, both 24-kv. and 120-kv., oil filled breakers are used of necessity.

In power house design, two rather important features have, it is believed, materially reduced the probability of outage of production equipment. One is the use of low-voltage d-c. house service for all essential auxiliaries and its complete electrical isolation from the rest of the station. The small likelihood of trouble on the low-voltage system, and the ease with which maintenance work can be done with the bus energized, make a service interruption highly improbable; the inherently superior speed control characteristics of the d-c. motor recommend it from the ease of operation standpoint; and the economic studies indicate that for conditions as they exist on the system, based on past records, there is little cost advantage for the a-c. The second factor is the use of a unit exciting system for each machine, and the elimination of the main field rheostats. This allows the connections between exciter and machine to be as direct as possible and eliminates in this important circuit, long connections which introduce additional hazard. Furthermore, the excitation system is isolated from other circuits which, from their very extent, are more likely to give trouble.

Since no perfection of design can overcome defective installation, every effort is made to secure first class construction. A rigid inspection and test program insures that the work is properly done before the installation goes into service.

Reduction in the Effect of Outage. Since it is impossible completely to eliminate outage of major equipment no matter how much emphasis is placed on reducing its probability, the system is so designed and operated that failure will have the minimum effect on service to the customer. Throughout the system, the layout is such that the loss of any one production or transmission facility, be it boiler, turbine generator, transformer, transmission line or bus, need not occasion loss of service to a customer. This does not mean that there are no service interruptions but most of those that occur should rightly be charged to the distribution facilities. Obviously, the different requirements as to character of service which are imposed in supplying different classes of customers affect the distribution facilities provided. Thus for residential loads, voltage regulation is stressed but a momentary interruption is not particularly objectionable, while for industrial loads, the elimination of interruption is considered vital with close voltage regulation of secondary importance. Accordingly, residential load is supplied from a distribution station with two or more separate 4,800-volt buses fed radially from 24-kv. transmission stations. In case of failure of one source, service is interrupted to one bus for a second and a half, and is restored by automatic throwover to another 4,800-volt bus which is usually fed from a different power area. On the other hand, for industrial loads, the 4,800-volt bus sections are fed radially from different 24-kv. bus sections as before, but the distribution buses are connected together through reactors and a linkage bus so that the loss of one source does not cause an interruption but may cause some voltage drop. In this case, however, the whole station is fed from one power area in order to avoid reducing the impedance between power houses by ties through distribution stations.

At each power house the running reserve in boilers and turbine-generators is maintained at all times such that if one unit is lost, the remaining units, with the relief available from the system over the tie lines, can carry the station load. The steam piping is sufficiently interconnected so that each turbine can receive steam from several boilers. The mechanical equipment, auxiliaries, coal handling equipment and the like, are designed with reserve capacity, or duplicate equipment, so that a probable failure does not impose any limitation on the major equipment, the turbines and the boilers. At each transmission station, the transformers are operated normally at such loads that in case one bank trips out, the emergency rating of the remaining banks is sufficient to carry the station peak loads.

In the electrical system, the principle of electrical sectionalization and physical segregation is employed rather freely. The degree of sectionalization can best be determined by a study of the one line diagrams in Fig. 2, which show the electrical connections at the four power plant switching stations, at a typical 120-kv. city transmission station, Warren, and at a typical 120-kv. suburban transmission station, Bloomfield. It is evident that in every case a generator or transformer can be tripped out without any interruption of service to the bus. In the case of Waterman, for instance, the load would have to be carried over a reactor until a rearrangement of buses could be made. This might cause a slight drop in voltage at the bus but that is not serious, especially since it is usually compensated for by induction regulators at the distribution stations.

In all cases it will be noted that the 24-kv. bus can be sectionalized in an emergency so that a faulted portion can be cut out by protective equipment without shutting down the complete bus. Since, as stated above, any important load is fed from two or more bus sections, the loss of one is not serious. Normally, of course, all sections are electrically connected; solidly as at Warren, through reactors and an "equalizer bus" as at Waterman, or through the substations as at the present Connors Creek station. Shortly, the new Essex station will replace the present Connors Creek switching station, and, as soon as conditions warrant, an "equalizer bus" scheme similar to that at Waterman will be installed.

At all 120-kv. stations there are two bus sections, normally tied together but arranged to separate in case of bus trouble. One circuit of each outgoing tower line is connected to each bus at the power stations so that in case one section trips out only one-half of the lines is lost. At each transmission station, except Monroe, there are two incoming double circuit tower lines. The transmission circuits are sectionalized at each station so that in case of trouble, at some point on the line, only one of the four lines into the station is out of service.

In the indoor 24-kv. stations, equipment is physically segregated where possible. Each bus section is completely enclosed in its own bus housing and in some cases additional barrier walls separate the sections. In the case of the circuit breakers, where there is the additional hazard of oil, fire and smoke, even though breakers with little oil are used, each group is completely isolated in a separate room to prevent the possibility of fire and smoke spreading to breakers on adjacent buses. At the three outdoor transmission stations, Warren, Navarre and Northeast, barrier walls are being erected between the 24-kv. buses to prevent trouble originating in one section from being communicated to the adjacent ones.

Due to the normal operation of all transmission ties in pairs, differential protection is used on both the 120-kv. lines and the 24-kv. cables, to give very fast clearing of a faulted circuit. This minimizes the disturbance to the system and eliminates any danger of instability between power houses. Recently, bus differential relaying has been installed on most of the 120-kv. and 24-kv. outdoor buses to give the advantage of rapid clearing in case of bus trouble. In the newer 24-kv. indoor switching stations, as Chandler and Frisbie, the new section of Waterman, and the new Essex Station, all electrical equipment is completely enclosed in metal housings isolated from ground except through a current transformer at one point, and equipped with fault bus relaying which again is practically instantaneous.

As a further means of localizing serious trouble, the system is operated on the power area plan. Normally each area can carry its own load without assistance from the system, the transmission ties acting mainly as emergency reserve. In case a fault is not properly cleared by the local protective equipment and the trouble reaches serious proportions, the breakers on the transmission ties will open, as a last resort, isolating the affected section and allowing it to shift for itself, but at the same time removing the disturbance from the rest of the system.

System Planning. In planning the installation of additional facilities, the criterion for station capacity is the peak load and it must be possible to carry this load with a reasonable emergency existing. The probable total system load peaks are predicted for several years in advance from a study of past performance, the existing trend of load increase, national and local business conditions and other factors which affect load growth. The probable loads on individual substations are predicted from the same data supplemented by more detail information on the plans for industrial expansion or real estate development as affecting a particular substation. From a knowledge of the probable total system load, the probable individual substation loads, the proposed changes in load areas and the diversity factor between the peak loads of the substations in a given area, the peak load on each power house is estimated.

To determine the firm capacity of the total system, the emergency criterion which has been established is that peak load occurs when the largest machine is down for routine maintenance and the next largest trips out due to an emergency. This criterion is reasonable because of the necessity for periodic inspection and maintenance on turbine generators and is particularly so on a large system where the number of generating units makes it increasingly difficult to schedule this necessary inspection and maintenance work at light load periods. In determining the capacity of each plant, it is assumed that both the machines out of service are located at this one plant, but in this case, the relief which is available to that load area from the system over the transmission ties is added to the emergency rating of the remaining machines to determine the permissible load on the station. Obviously the firm capacity of the system is not the sum of the permissible loads at each plant as this would assume several emergencies existing simultaneously. Practically, a certain margin is maintained between the predicted peak load on the system and the firm capacity of the system in order to be able to supply an unexpected load of reasonable proportions or to handle a load growth somewhat in excess of expectations.

The emergency criterion for determining the capacity of the transmission station transformers is somewhat different because they are seldom down for routine maintenance for any length of time. In case of failure, the defective unit is replaced by the spare transformer available at each station and the bank is back in service rather quickly. Hence the permissible load on a transmission station is determined by the emergency rating of the remaining transformers with one bank down.

Provision for Reactive Kva. Provision has been made in the design of the generators to take care of reactive as well as real load. The load power factor on the high voltage side of the generator transformers may be as low as 82 per cent at times of system peak. By the time this is transferred to the generator terminals through the rather high transformer reactance and the effect of the transformer excitation is taken into account, the power factor is several per cent lower. It is quite possible that an 80 per cent power factor generator might be somewhat shy of reactive capacity in an emergency. Hence, in the later units excess reactive capacity in the generators has been obtained.

There are several small synchronous condensers and one of 30,000 kva. capacity on the system but these were installed mainly for voltage regulation. The large unit located at Northeast is available, however, to relieve Connors Creek generators of some of the wattless kva. of their load. The machines in this power plant were purchased at the time when load power factors were considerably higher than they are today and do not have the reactive kva. reserve present at the other plants.

Fundamental Design for Economy. In power plant design, a reasonable operating economy is sought, and the newer plants, of course, have considerably higher economy than the older ones. Because of the power area plan, each plant carries load which varies approximately in proportion to the system load with no station acting as a base load plant. Consequently exceptionally high economies at the expense of increased plant investment and possibly greater operating complications have not been sought, since they can be justified only by assuming base loading.

Load Allocation. To obtain the most economical operation, each load should be supplied from the power house which gives the lowest delivered cost of power including production and transmission costs. However, the ties between the power areas are limited in capacity so that the transfer of power from one area to another is correspondingly limited. With the scheme of operation adopted by The Detroit Edison Company the loads on the machines at a plant and on the tie lines into the plant must be maintained at such a point that the area load can be carried in case one machine trips out. To obtain economical loading on the machines, this requirement usually results in the transmission ties being loaded below their actual capacity. Furthermore, with the plants interconnected by the system network, it is not always possible to carry a particular load on any one power house because regulation at one plant is not entirely independent of the others and in trying to obtain the desired results at one plant, conditions at the others are adversely affected. Hence the method of allocation used is a compromise between maximum economy and practical operating conditions and is based on the station capacity weighted by the production economy and modified to take account of the physical location of the loads and the method of operation.

In applying the method, the active capacity at each plant is divided by the plant coal rate in pounds per kw-hr. Then as a first approximation, the percentage load on each plant is determined on the basis of these weighted figures. A modification is introduced to take into account the transmission loss. If this allocation does not meet the requirements of practical operation, it is further modified to take this factor into account. A specific example will explain the method more clearly. In Table II recent data for the four plants are listed.

TABLE II—METHOD OF LOAD ALLOCATION

Plant	Active generator capacity In mw.	Active generator capacity In % of total	Coal rate in lb./kw-hr.	Col. 2 / Col. 4	Load allocation in % of system load Approx.	Load allocation in % of system load Actual
Delray	110	15.3	1.12	98.2	15.3	20
Connors Creek	150	20.8	1.45	103.2	16.0	20
Marysville	160	22.2	1.07	149.7	23.3	19
Trenton Channel	300	41.7	1.03	292.1	45.4	41
Total	720			643.2	100.0	100

The active installed capacity at each plant is given in megowatts in column 2 and in per cent of the total in column 3. Column 4 shows the coal rate in pounds per kw-hr. Dividing the plant capacity in column 2 by the plant coal rate in column 4 gives column 5. Totaling these weighted figures and dividing each item by the total gives the approximate allocation of column 6. This percentage takes into account the effect of differences in production costs but does not include the effect of transmission loss. It happens that the load allotted to Connors Creek and Delray in this case is only about 75 per cent or less of the area loads of these plants. If the cost of transmitting the balance of these loads from the suburban power stations into the city load areas is considered, the city plants should carry a larger percentage of the total load. In addition, the availability of the transmission ties for emergency relief would also dictate a greater load on these stations. Without going through the detail procedure in this particular case, these factors indicated that 20 per cent each should be carried on Connors Creek and Delray. The actual load allocations are given in column 7.

The plant economy used is obtained from the records of the immediately preceding period. These figures apply only for the actual conditions which existed for that period but usually the conditions do not vary sufficiently to make correction necessary. A computation similar to the above is made periodically and the allocation changed when necessary. The same percentages are used regardless of system load because it is felt that variation with system load does not offer sufficient advantage to compensate for the operating difficulties involved.

At certain power plants there is a variation in economy among the machines. In these cases, the combination of machines which will give the highest plant economy is preferred, if this does not require some undesirable operating condition such as running a unit for an hour or two at a time.

INTERCONNECTIONS

The only central station interconnection of The Detroit Edison Company is that with the Consumers Power Company through a 30,000-kva. transformer bank and a single 140-kv. transmission line. This interconnection, of course, acts as emergency reserve and eliminates the necessity for a certain amount of running reserve in generators. Since the Consumers Power Company has a large amount of hydroelectric equipment, at times of excess water and light load on its system it can furnish dump power at an attractive cost so that a certain economy is effected at these times.

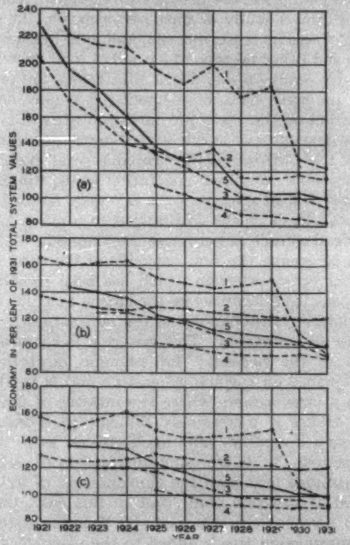

Fig. 3—Yearly Production Data of The Detroit Edison Company

(*a*) Cost in cents per net kw-hr.; 100 per cent = 0.334
(*b*) Coal rate in lb. per net kw-hr.; 100 per cent = 1.184
(*c*) Thermal efficiency in B.t.u.'s per net kw-hr.; 100 per cent = 15,780

Curve 1—Delray Power House
Curve 2—Connors Creek Power House
Curve 3—Marysville Power House
Curve 4—Trenton Channel Power House
Curve 5—Total System

In addition, there is an interconnection with the Ford Motor Company through two 30,000-kva. transformer banks. This again acts as running reserve to a certain point, but in this case the generating capacity consists of steam turbine units so there is no relatively cheap power involved as is the case with the hydro plants.

INSTALLATION OF NEW FACILITIES AHEAD OF LOAD REQUIREMENTS

In general, it has not been the practise of The Detroit Edison Company to install new facilities before the load has demanded it merely to secure some added operating economy. At times obsolete generating equipment has been replaced by some of higher economy. But this was done primarily because increased generating capacity was required at that time.

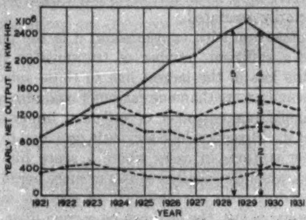

FIG. 4—YEARLY NET OUTPUT OF THE DETROIT EDISON COMPANY IN KW-HR.

1. For Delray Power House
2. For Connors Creek Power House
3. For Marysville Power House
4. For Trenton Channel Power House
5. For the total system

On the other hand, where the main reason was an improvement in reliability, there have been several instances where new substations have been built to replace old ones, or old stations rebuilt. The equipment in the old stations, principally circuit breakers, was becoming obsolete with the growth of the system, and there was some question about the ability of the stations to operate reliably under the changed system conditions. In these cases, the new or rebuilt station had more capacity than the old but the additional capacity was not always essential at the time the change was made.

RESULTS OBTAINED

Due to its rather indefinite nature, there is no absolute measure of the improvement in the reliability of service which can be directly credited to the production and transmission facilities. The average number of interruptions per customer, over a period of years, or some such criterion might be used, and would indicate a marked improvement. But the measure would be a truer indication of the improvement in distribution facilities than in transmission and production facilities. However, a study of the failures of transmission and production facilities which have occurred, indicates a definite improvement and shows that at the present time such failures are of minor consequence in affecting the reliability of service to customers.

A more definite proof of the results in operating economy of production can be presented. The curves in Fig. 3 show graphically the improvement in yearly production economy for the past decade. Data are given for each of the four plants and for the system as a whole in per cent of the corresponding value for the whole system for 1931. Presented in this form, the improvement at each plant and for the system is clearly shown as well as the relative efficiencies of the different plants. Since the cost of a kw-hr. is the absolute criterion of success, Fig. 3a shows the production cost in this form. This figure includes cost of coal, miscellaneous supplies, maintenance, labor, office and executive personnel and such items. Since this value includes various factors over which the operator has little control, such as the varying price of supplies and labor, a better comparison of the actual improvement in thermal efficiency which can be credited to the method of plant operation is given by the pounds of coal per kw-hr. and the B.t.u. per kw-hr., which are shown in Figs. 3b and 3c, respectively.

Since the system output and the shape of the daily load curve have an important bearing on the cost of operation, Figs. 4 and 5 are included. Fig. 4 shows the yearly system output in kw-hr. for the last decade together with its distribution among the four power stations. Fig. 5 shows the load curves for three days in January and February of this year. One is for a cloudy week day, one for a clear week day, and the third for a Sunday.

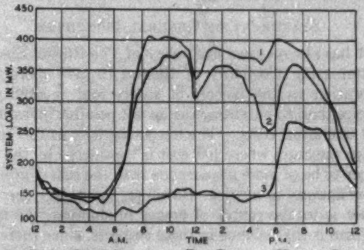

FIG. 5—DAILY LOAD CURVES OF THE DETROIT EDISON COMPANY

Curve 1—For cloudy weather week day
Curve 2—For clear weather week day
Curve 3—For Sunday

CONCLUSIONS

In view of the results accomplished, it is felt that the fundamental principles followed have been sound. Experience has shown that in certain details of design or operation, changes can be made to advantage, and, of course, are being made as they are discovered. However, these modifications are all in the detailed method of carrying out the fundamental principles and not in the principles themselves.

Discussion

For discussion of this paper see page 888.

II—Combined Reliability and Economy in Operation of Large Electric Systems

THE EDISON ELECTRIC ILLUMINATING COMPANY OF BOSTON

BY R. E. DILLON*

Non-member

INTRODUCTION

IN the development and operation of a successfully managed public utility reliability and economy must be based on the particular conditions inherent to the locality which it serves. As a result every company's problems must have individual study, and no set and fast rules can be established for their solution. However, the policies, practises, and experiences of each, are helpful in moulding the development of others.

The business of the utility is founded upon its ability to compete successfully with the private plant in reliability and economy from the standpoints of both operation and investment.

Customers do not always require the same balance between reliability and economy and, therefore, a company may find it necessary to make some variation in its distribution practise in order to compete with a contemplated plant at the customer's location.

STATEMENT OF GENERAL PRINCIPLES

It has always been the policy of The Edison Electric Illuminating Company of Boston to give more consideration to the factors of safety and reliability of service than to obtaining the lowest possible investment and operation costs. The few exceptions to the policy are in the cases where low cost is considered by the customer to be of more importance than the risk of possible interruptions to service. However, the nearer the customer is to the center of power production, the more uniform is the balance in these respects, since there the balance is set by customers who demand the greatest reliability.

The final plan for the addition of a large block of capacity in generating or distributing facilities is the result of a compromise between those features that seek a minimum of investment outlay, to be obtained by simplicity, and those that urge complications designed to give a high degree of operating flexibility in behalf of reliability.

Here again it should be pointed out that reliability is not necessarily sacrificed in the plan for less total investment. On some occasions reliability may be assured with smaller total investment by the installation of an increased amount of spare capacity using simplified equipment and at less cost per kilowatt.

In line with this company's policy to give the most reliable service possible at the point of production, sufficient capacity is operated to allow the loss of the largest turbine in the generating stations, or the largest group of tie lines on a single tower line, or the largest tie line of a group of tie lines on separate towers, including cases where these latter are on the same right-of-way, if satisfactorily separated.

In connection with transmission, it is the policy to operate a sufficient number of transmission lines to allow the loss of the largest line to those substations which supply more than one customer, and also to those that supply one customer of sufficient importance to warrant it. However, loop service is provided to single-customer stations in cases where a spare line is not justified, when the location of two or more permits it.

Substations also are designed so that in case one transformer is lost, the load can be carried upon the remaining transformers. Here again an exception must be made for the single-customer substation where cheap power is preferable to continuity.

Continuity of service must be provided by keeping in service adequate capacity in boilers, turbines, and transmission lines; and the next concern is to allocate properly the load to such generating stations and facilities as will give the best efficiency.

Although spare capacity is installed in substations according to the policy which has been outlined, nevertheless the capacity is operated with regard for the best dollar economy.

DESCRIPTION OF THE SYSTEM

There are two main generating stations located on tide water and several small steam and hydroelectric generating stations.

The Charles Leavitt Edgar Station of 158,000-kw. capacity is located at Weymouth which is 12 miles south of Boston. The L Street Station of 210,000-kw. capacity is located one mile south from the center of Boston.

The transmission system voltages range from 110 kv. down. A large part of the system consists of underground cable at 25 kv. and 14 kv. There are two underground direct ties between the two main stations operated at 14 kv. There are underground radial feeders at 14 kv. and 25 kv. from both stations to several of the substations and some of these can be used for loose coupling.

Two of the four points of interconnection with the New England Power Association System complete a 110-kv. ring around the territory served by the com-

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*Edison Elec. Illuminating Co. of Boston.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

pany. This extends from Edgar Station on the south, through Walpole, Medway, Millbury, Pratts Junction, and Tewksbury to Woburn at the north. At this point the voltage is stepped down to 14 kv. and the lines from there run underground, through several substations, to L Street Station.

Local areas are served by transformer substations along the ring.

The smaller generating stations are connected at various points to the d-c. system and to the a-c. distribution and transmission systems.

There are 116 substations, covering a territory which is spread over a radius of 30 miles from Boston. Some of these substations are customer-owned. The maps show this territory with the location of the generating stations and tie lines.

Application and Discussion of General Principles

It is possible to predict the requirements of each district of the territory with good accuracy by making a study of the system area loads over an extended period of years. In such estimates due consideration must be taken of the effect of the schedules of power rates established from time to time which are aimed to bring kilovoltampere closer to kilowatt and also to fill the valleys of the load curves.

The load density of the district served dictates the nature of the capacity which may be added, that is, by 1,500-kva. network vaults, 7,500-kva. semi-automatic substations, or larger attended substations.

The company-owned capacity requirements are affected by the interconnections. This comes as a result of planning and operating the Edison system with the New England Power Association system for the most economical mutual results without regard to separate ownership.

Cooperation extends with regard not only to the capacity requirements such as total capacity, but also to the pooling of spare capacities, to balancing of hydro capacity against steam for the purpose of conserving storage, to the spare capacity operated for protection, to the operation of the most economical units, and to the scheduling of apparatus for overhaul. The latter, due to the flexibility of rates designed to improve load factor, is today becoming a matter worthy of consideration.

The policy of always having a spare unit has been a factor in determining the size of units to install. Another factor is the relation between carrying charges on spare capacity and the decreased operating and investment charge on the larger units. Against better fuel economy and lower labor cost per kilowatt-hour in connection with larger units, there is the increased cost due to the operation of larger relaying capacity as a result of using larger units.

All of these factors are affected considerably only by the loads which are to be carried a major part of the time. These loads may be determined by a study of the load-duration curve.

On this system the last installation made has 87,500 kw. in compound capacity.

The company policy in the past has been to duplicate auxiliaries to turbines and boilers, making many connections between units, and in general, safe-guarding against all emergencies at a high cost per kilowatt of capacity installed. The policy of some companies, however, which depends upon simple installations at low cost per kilowatt of capacity and provides a greater number of spare units, is being watched very closely in order to determine whether in the final analysis this policy results in lower investment charges.

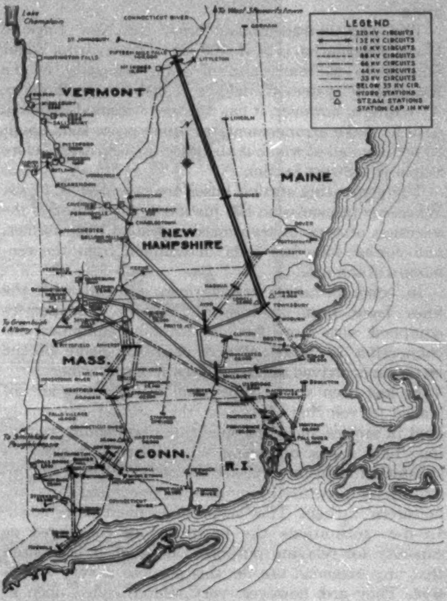

Fig. 1—Map Showing Generating Stations and Tie Lines

Because of our policy there has never been a loss of load due to either lack or loss of generating equipment.

Proper protection of the system by spare operating capacity has served to take care of the greater part of the reactive kilovoltamperes with units rated at unity power factor. Some of the later machines have been bought, however, with excess kilovoltampere over kilowatt rating.

When the northern tie with the New England Power Association system was put into service two rotary condensers of 12,500-kva. capacity each were installed at

Woburn. This was in view of the necessity of maintaining a good power factor on the 110/220-kv. ties to Fifteen Mile Falls Station.

The proper rate restrictions have kept the power factor on the Edison system approximately 94 per cent at the time of the peak.

Ties with other systems, together with transmission ties between generating stations, have become a part of the study of additional spare capacity and have been determined by this study. This has resulted in building the present tie lines between the generating stations for 33,000-kw. capacity, which approaches the size of the largest turbine in the station having the smallest size units.

The ring-tie-transmission from the New England Power Association is of 75,000-kw. capacity from Edgar Station to Millbury and of ample capacity in the Association system to deliver 75,000 kw. at Woburn. At Woburn, however, it has not been found advisable to date to make arrangements to utilize more than about 30,000 kw.; all of which is absorbed by local areas before reaching L Street Station.

The generating station buses are operated at 14 kv. Wherever transmission has justified higher voltages the selection of type of transmission system, transformer, and connections has been determined from the economics of the case.

In some instances transformers integral with the lines have been installed. In others, transformers equipped with more flexible switching equipment for connection to buses proved desirable. Some of the latter cases arrived as the outgrowth of the former.

As has been stated substation transformers are installed for relaying. However, they are not continually kept in service and their operation may require switching. This keeps operating cost down. Capacities of substation transformers are figured according to operating-temperature limits and not according to manufacturers' ratings. This keeps investment costs down. In d-c. substations the batteries are considered as capacity for relaying purposes, although it is realized that any extended use for this purpose is very expensive. They are, however, very seldom called upon in actual practise and, therefore, this use appears justified.

The responsibility for minute-to-minute control of actual operation of relaying capacity is lodged with the system operator. For this purpose he can supplement the capacity of the company equipment by the purchase of firm or secondary power from adjoining systems.

As has been pointed out the operation of the Edison system and the Association system is carried on practically as a unit from the standpoint of economy of generation. The allocation of load among the hydro and steam stations is handled by the system operators, they having continuous knowledge of system load conditions.

After the minimum load requirements of machines needed for relayed capacity have been fulfilled, and the primary-power-takings have been utilized to their utmost value, the balance of load of the combined system is handled by generation at the most economical stations, wherever located. This may result in the transfer of load from generating stations of high operating cost on one system to stations of low operating cost on another system; such transfer being termed "interchange" power.

This plan is not essentially different from that governing the operation of an isolated company, where every effort is made for maximum output from the most efficient and minimum output from the least efficient machines. The savings realized from the power interchange, after making proper deductions for transmission line losses, are divided equitably among the companies participating in the transaction.

The interchange is based upon the immediate differences in increment costs at every period of time between production of excess power on the sending system and replacement power on the receiving system.

Under these conditions power goes from the system which is most economical to the one least economical at the time.

Accurate determination of the increment costs which are utilized in such transactions are necessary. Capital charges, except for the lines, do not enter into consideration since it is an exchange of excess power that is concerned.

However, operating costs change from season to season, from day to day, from watch to watch, and from hour to hour, and to these extents the analysis of costs must be carried in order to arrive at representative figures.

Many of the charges which actually enter into the operation during a given hour are charges which have been deferred from previous hours of production; for example, those due to banked boilers and maintenance. These deferred charges are substantial and must be included.

The no-load loss is an example of a loss which is chargeable or not, according to conditions.

Increment costs differ depending upon the turbines to which the load is added and also upon whether this is added on primary or on secondary valves.

Addition of load to a generating station may produce added repair costs due to forcing of boilers. It may also produce abnormal increases in transmission system losses due to lines being already heavily loaded. These increased costs will be in addition to the coal, water, supplies, and normal repair costs. Reduction of load on a generating station may produce unbalanced loading of high and low pressure boilers with resultant lowered economy. It may also produce non-uniform temperature gradients through turbine rotors at light loads with possible abnormal wear. In addition it may produce fuel losses through non-uniform loading of boilers. Recognition of these various factors will prevent losses due to interchanging power on an increment cost margin which is too narrow.

In practise, the trading, and cost figuring is accomplished in the manner to be described as follows:

The load dispatchers of each system are informed of the amount of the excess power available. The tentative purchaser places a bid for a stated amount of power during stated periods. The seller quotes costs estimated from forecast conditions. The purchaser notes the spread between these costs and the estimated replacement costs on his system, and proceeds with the transaction accordingly.

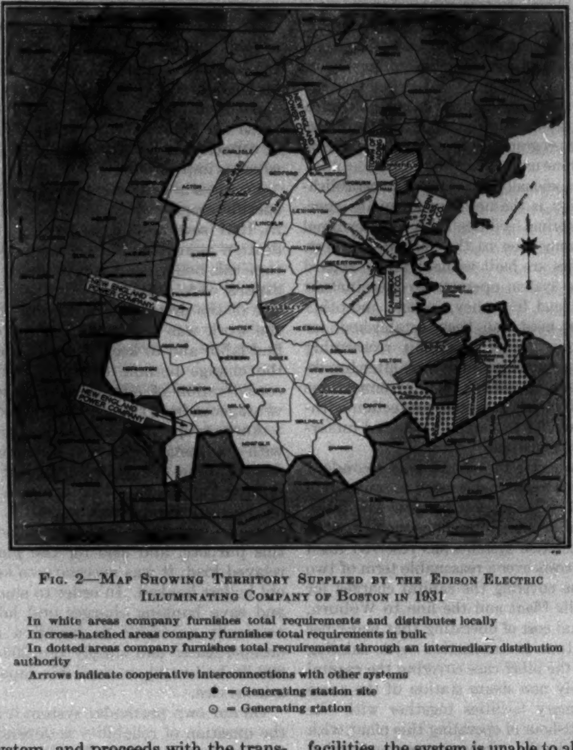
NEW ENGLAND POWER COMPANY
NEW ENGLAND POWER COMPANY
NEW ENGLAND POWER COMPANY
TOWN OF READING
EASTERN MASS. ST. RY. CO.
CAMBRIDGE EL. LT. CO.
CARLISLE
ACTON
BEDFORD
BURLINGTON
LEXINGTON
LINCOLN
WALTHAM
WATERTOWN
NEWTON
NATICK
NEEDHAM
MILTON
HOPKINTON
HOLLISTON
MEDFIELD
WALPOLE
CANTON
NORFOLK
SHARON

Fig. 2—Map Showing Territory Supplied by the Edison Electric Illuminating Company of Boston in 1931

In white areas company furnishes total requirements and distributes locally
In cross-hatched areas company furnishes total requirements in bulk
In dotted areas company furnishes total requirements through an intermediary distribution authority
Arrows indicate cooperative interconnections with other systems
● = Generating station site
◎ = Generating station

After the transfer of power has been made, the transaction is figured in accordance with definite knowledge of amounts and loading conditions, and with power charges or credits through changing conditions from hour to hour. The figuring is done according to a predetermined method, so that results are always on a comparable basis. The bills are made out in accordance with these figures. Finally, checks are made of the final results with the available figures of actual production costs.

Allocation of the load between Edgar Station and L Street Station from the standpoint of economy is not a complex problem. Edgar Station is more efficient than L Street Station, largely because of the high steam pressure equipment. It is, therefore, desirable to carry the base of the load at all times at Edgar Station. However, due to a retarded growth of load in the section surrounding Edgar Station, and to limited tie-line facilities, the system is unable to absorb full capacity of the station. The remainder of the normal load is taken on the turbine primary valves at L Street Station with fuel economy equal to that of the load taken on the secondary valves at Edgar Station.

Effect of Major Interconnections on Reliability, Investment, and Operating Economy

Justification of extensive interconnections between companies is sometimes difficult to establish on an earning-power basis unless the construction of such an interconnection is part of a primary power contract, as in the case with the Association and Edison primary ties, where the cost of this primary power plus the carrying charge of the interconnection may be credited as offsetting the carrying charge for the investment in new generating facilities which would have been made necessary in lieu of the primary power purchase. On the other hand the Millbury line is a case in which the carrying charge of an inexpensively constructed transmission line has been amply covered by the savings derived purely from secondary power interchange.

The support given a system through the medium of an interconnection during abnormal loss of capacity is of incalculable value in avoiding loss of service to customers.

The reliability of interconnecting transmission lines, considering modern design and construction and also routine sleet melting programs, approaches comparison with the modern turbine unit.

A factor that is occasionally lost sight of in establishing an interconnection is the need for adequate communicating and metering facilities from the terminal points of the interconnection to the system operator. Too often the tie lines are built without ample regard for the needs of the system operator to give instant knowledge of tie-line load, frequency, voltage, and other conditions. Also the need has been established for adequate synchronizing points to permit reassembly of the system in the shortest possible time after break-apart. The investment of millions in interconnections cannot pay proper dividends in system protection without the insurance provided by carefully selected metering and communication facilities.

The subject of economy in the operation of secondary power interconnections has been discussed in connection with the necessity of obtaining cost factors. The question of economy of the primary interconnection between the Edison and Association systems has resolved itself into a comparison of costs over a reasonable term of two alternative cases; one covering the capital charges for the Fifteen Mile Falls Plant and the line to Woburn, together with the total cost of operating that plant per kilowatt-hour of output in connection with existing generating facilities; the other case covering the capital charges for an entirely new steam station of the same capacity as the primary facilities together with the total cost per kilowatt-hour of operating this plant with an output equivalent to the primary power, and giving due consideration to its value for base load service.

As would be expected, operating costs are far in favor of the Fifteen Mile Falls Plant. This permitted a wide divergence in the investment cost of the hydro and steam plants before the project became uneconomical. The limit of difference in investment was not approached and the Association was able to set a price for the power from its system, augmented by Fifteen Mile Falls, which assured success to the project and a profit to the Edison system over a period of years.

INSTALLATION OF NEW FACILITIES AHEAD OF LOAD REQUIREMENTS TO SECURE ADDITIONAL SYSTEM ECONOMY AND RELIABILITY

On this system it has never been found profitable to install new facilities ahead of load requirements. None of the facilities displaced in service by this move would be displaced in book value. A scanning of the load duration curve discloses the fact that the percentage of the annual output which has to be produced by equipment installed prior to 1920 is relatively small. All equipment installed after 1920 is efficient according to present standards, and none of this can be profitably displaced.

To carry this reasoning still further, there is at best only a difference of about two mils between the operating cost of the poorest and the most efficient equipment on this system. The fixed charges per kilowatt of new capacity at the present time would be $17.00. To justify these fixed charges, therefore, would require operation at capacity for 8,000 hours per year which it is needless to say is impossible.

Changes in equipment are sometimes desirable and in specific cases they can be justified. In 1916 and again in 1921 several of the overfeed stokers at L Street Station were changed to underfeed stokers. At that time this boiler plant was in heavy service. Labor conditions were uncertain. The type of stoker and small size of the units required a large force. The nature of the work was heavy. It became difficult at times to find labor and to keep it. In the interests of reliability the change became desirable, but it also proved in consideration of advances made in stokers to be advantageous from the economic standpoint.

Another specific example is furnished in connection with the installation of a reducing valve at L Street Station. The modern extension at L Street Station is operated at 300 lb. steam pressure while the older part of the station generates at 200 lb. pressure. The older station is used now for idling capacity or standby. For this purpose, and also for the purpose of picking up relayed load, it was necessary to keep a number of the old boilers banked. In order to shut these boilers down and save banking charges and labor, it became economically advisable to install a reducing valve and desuperheater and supply the 200-lb. mains from the 300-lb. boilers where available capacity for the purpose is in service.

On our own particular system it has been found that the question of reliability is dependent upon two main factors; first, flexibility of design to permit loss of any one machine, transformer, line, or bus section without loss of customers' service; and secondly, the inherent reliability of the manufactured items such as cables, transformers, switches, machines, etc. The increasing reliability of the equipment itself has within the past few years reached a point where internal breakdown is extremely infrequent, and the adoption of routine methods of testing has permitted the location of weak spots in the insulations before these defects have had time to develop into service failures. In spite of these conditions, proper protection requires the installation of sufficient equipment to permit the loss of any one generating or transmitting unit during the maximum load period without materially affecting the service.

The load growth in one section of a system may be so rapid that the normal extension of facilities along existing lines may be attended by an indication that voltage irregularity or loss of service may result, in which case a new investment in transmission facilities must be made to improve reliability. However, such new investment is not justified far in advance of the actual need for capacity.

Wherever the service becomes unsatisfactory additional investment which is adequate for the type of load served seems imperative.

Brief Statement of Results

Effect of Failures in Production Facilities on Customers' Services. Since the majority of large manufacturing customers are furnished with duplicate-line service, generally underground, the occasions of total interruption of service to any large single customer or group of smaller customers due to loss of transmission facilities have been negligible. Occasionally voltage fluctuations produced by system disturbances from cable faults or lightning strokes may affect the uniformity of product of some textile or paper industry, but these occurrences are fortunately few. The rare occasions on which generating units have been taken out of service on emergency have found ample reserve protection on the system to carry the load without interruption.

Percentage Reduction in Production Costs Over a Period of Years. The change from vertical to horizontal generating units with a simultaneous addition of boiler capacity at increased steam pressures and temperatures was accompanied by a decided improvement in operating economy. Later the advent of very high pressure boilers and turbines at the Edgar Station permitted a further economy increase. Furthermore, automatic substations together with modern equipment have improved efficiency and reduced labor.

Instruments have been perfected and put into service greatly helping efficient operation.

These changes came about in the course of providing for the growth of load. They were attended also with increasing education on the part of the attendants towards efficiency and an increasing vigilance of all forces against losses.

All these contributing elements together with the policies which have been outlined here have resulted in a reduction in production cost. A comparison of total operating cost per kilowatt-hour covering generating stations, substations, and purchased power for the years 1920 and 1930 shows that a reduction of 62.3 per cent took place.

Whenever changes of particular equipment were advised a large gain had to be assured in order to win approval for the change and in all cases approved the figures left no question to be raised.

Most careful consideration has always been given to the problem of replacing capacity solely for better operating economy. Many replacements have been made in past years, but at the present time none can be justified because fixed charges which would be incurred on new equipment prove equal to a very large percentage of the total cost of operating the old equipment.

Discussion

For discussion of this paper see page 888.

III—Combined Reliability and Economy in Operation of Large Electric Systems

PHILADELPHIA ELECTRIC COMPANY SYSTEM

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Synopsis.—*This paper discusses the practises in use on the Philadelphia Electric Company system in attaining the maximum system economy consistent with high degree of service reliability. The system comprises not only steam electric stations of different efficiencies, but also a large hydroelectric plant and a major high-voltage interconnection with two large neighboring utilities, so that problems of operating economy are both highly important and complex in character.*

The paper also directs attention to problems of investment economy with a view toward securing the best balance of fixed charges, operating costs, and service reliability.

* * * * *

INTRODUCTION

MAXIMUM system economy is attained when the sum of operating costs plus fixed charges on investment is a minimum. Considerations of reliability usually lead to increased capital investment for sturdier facilities and for spare equipment, and consequently add to the fixed charge burden. Fortunately, however, operating costs are to some extent inversely proportional to fixed charges, and analyses of system performance will often reveal operating savings which can result from judicious use of investment in spare facilities. Indeed, it is frequently the case that new fixed charges can be deliberately incurred because they will be more than offset by the operating savings accruing from the new investment. However, it is also frequently the case that new investment must be made solely to improve service dependability.

It is the purpose of this paper to discuss practises followed on the Philadelphia Electric Company system from the standpoint of production facilities in attaining highest system economy consistent with high degree of service reliability. Production facilities include generating plants, 220-kv., 66-kv., and 13-kv. transmission tie lines, and transmission substations. For convenience, this combination will be referred to as the "equivalent generation system."

Statement of General Principles

In the development of the Philadelphia Electric Company system, certain general principles have been followed in the installation and operation of reserve facilities on the equivalent generation system. These general principles are briefly enumerated below and will be discussed more fully in a later section of this paper.

FROM THE STANDPOINT OF RELIABILITY

Generating Capacity. Reserve generating capacity shall be equal at least to the capacity of the largest generating unit. A major portion of the reserve should be located within the system with the remainder readily available from adjacent interconnected companies without imposing an undue burden on transmission facilities.

220-Kv. and 66-Kv. Transmission. Sufficient 220-kv. and/or 66-kv. transmission capacity shall be provided between major generation and transmission centers so as to insure that reserve generating capacity available at any point on the system can be made available as necessary at any other generating location and also to insure that upon failure of any one transmission line, the remaining capacity is sufficient to carry the system load.

Transmission Substation Transformer Capacity. Sufficient transformer capacity shall be provided so that failure of any transformer bank may occur without impairing the ability of the system to carry the load existing at the time of failure.

FROM THE STANDPOINT OF OPERATING ECONOMY

Allocation of Hydroelectric Energy. Run-of-river hydroelectric energy shall be allocated so as to reduce steam station costs to a minimum.

Allocation of Boiler Capacity for Reserve. Boiler capacity required for reserve shall be allocated in the most economical manner after having provided for local reserve requirements in accordance with the principle that sufficient boiler capacity shall be provided and so operated that failure of any piece of equipment at any time will not impair the ability of the system to carry the load existing at the time of failure.

Allocation of Steam Turbine Capacity for Reserve. Steam turbine capacity required for hot reserve shall be allocated in the most economical manner in accordance with the principle that the failure of any piece of equipment, at any time, will not impair service.

Allocation of Steam Station Load. Steam station load shall be allocated to individual steam generating stations in the most economical manner within minor transmission limitations so as to obtain minimum steam system cost per kilowatt-hour delivered from the transmission system.

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Transmission Tie-Line Losses. Transmission tie-line losses shall be considered in arriving at correct load allocations.

Description of the Philadelphia System

The Philadelphia Electric System has developed as a closely-knit system of the so-called solidly linked type, although for a number of years, 66-kv. lines have been paralleled at some substations only through low-voltage buses, and lately under certain conditions, the system has been even further sectionalized. The system as a whole now consists of seven major generating stations aggregating more than 900,000 kw. in generating capacity, ten 220-kv. and/or 66-kv. transmission substations, nearly 60 frequency changer, railway and distribution substations, and 315 circuit miles of 220-kv. and 66-kv. transmission, 1,500 circuit miles of 33-kv. and 13-kv. transmission, and 3,900 circuit miles of distribution circuits. In addition a major interconnection at 220 kv. has been established with the Pennsylvania Power and Light Company and the Public Service Electric and Gas Company of New Jersey. Two other connections have been made with the Public Service Company, one at 66 kv. and one at 26 kv. Fig. 1 shows the general plan of the system; major switching details are shown in Fig. 4 of another paper.[1]

All of the generating stations are connected directly or indirectly to the 66-kv. transmission system through transmission substations, and the 66-kv. transmission lines are essentially the backbone of the system.

Application and Discussion of General Principles

From the Standpoint of Reliability

Generating Capacity. Reserve generating capacity is a serious burden on investment economy, although in many cases the additional fixed charges so incurred can be largely offset through normal operation of spare capacity to secure maximum operating economy. Nevertheless it is frequently necessary to maintain a considerable amount of capacity in older and less efficient stations mainly for reserve capacity in event of failure of the more economical normally operated equipment during the annual system peak load period. From this standpoint, it would be desirable to install new generating units in small capacities.

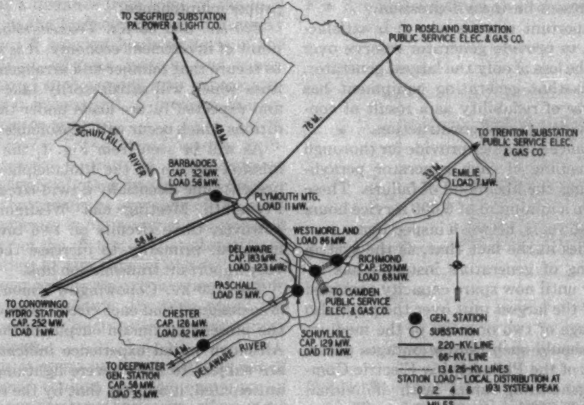
TO SIEGFRIED SUBSTATION
PA. POWER & LIGHT CO.
TO ROSELAND SUBSTATION
PUBLIC SERVICE ELEC. & GAS CO.
48 M.
78 M.
SCHUYLKILL RIVER
TO TRENTON SUBSTATION
PUBLIC SERVICE ELEC.
& GAS CO.
33 M.
EMILIE
LOAD 7 MW.
BARBADOES
CAP. 32 MW.
LOAD 56 MW.
PLYMOUTH MTG.
LOAD 11 MW.
58 M.
WESTMORELAND
LOAD 86 MW.
DELAWARE
CAP. 183 MW.
LOAD 123 MW.
RICHMOND
CAP. 120 MW.
LOAD 68 MW.
PASCHALL
LOAD 15 MW.
TO CAMDEN
PUBLIC SERVICE
ELEC. & GAS CO.
TO CONOWINGO
HYDRO STATION
CAP. 252 MW.
LOAD 1 MW.
CHESTER
CAP. 126 MW.
LOAD 62 MW.
SCHUYLKILL
CAP. 129 MW.
LOAD 171 MW.
14 M.
DELAWARE RIVER
TO DEEPWATER
GEN. STATION
CAP. 58 MW.
LOAD 35 MW.
● GEN. STATION
○ SUBSTATION
220-KV. LINE
66-KV. LINE
13 & 26-KV. LINES
STATION LOAD - LOCAL DISTRIBUTION AT 1931 SYSTEM PEAK
0 2 4 6
MILES

Fig. 1—Philadelphia Electric Company Main Transmission System—1931

However, investment economy is enhanced by installing the largest possible size of unit consistent with future load growth so as to secure minimum investment cost per kilowatt of generating capacity. In this connection, accurate forecasts of future loads are essential, for on present day interconnected systems an error of only 2 or 3 per cent may mean an excess or deficiency of 30,000 kw. to 50,000 kw. in generating capacity. Economies to be secured over a period of years by maximum utilization of existing and available sites through the adoption of the largest units justified by future growth may more than offset the disadvantage of maintaining the larger amount of generator reserve. The size of machine which is to be installed can be determined on an economic basis only by a balance of these factors.

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[1] For references see Bibliography.

With machines of given ratings installed on a system, it becomes a matter of careful judgment based on experience to determine the number of simultaneous machine outages for which reserve must be provided. On the Philadelphia Electric Company system, it has been found satisfactory to provide for the single loss of the largest unit. Only on three occasions since 1919 have difficulties arisen with generator units during the peak season, and in each case only one unit was involved. In one case, the unit was returned to service in about twelve hours, and in the other two cases, the units were able to resume service at reduced capacity pending complete repairs.

It should be noted that reserve capacity on the Philadelphia system at the present time is considerably greater than the largest unit. This is accounted for by the fact that system load growth has not yet overtaken the large installation necessary at Conowingo for the economical development of the Susquehanna River, and by the effect of the present business depression.

There are four important reasons why it is satisfactory in Philadelphia to provide generator reserve over the system peak for the loss of only the largest generator.

The first reason is that generating equipment has attained a high degree of reliability as a result of constant improvement in design and construction.

Secondly, maintenance schedules provide for thorough inspection and overhauling of each generator periodically, thereby reducing the likelihood of failure. These schedules are based on a minimum of 6,000 service hours and a maximum of two years, between inspections.

The third reason lies in the fact that, as the system has grown, the rating of generators installed has increased progressively until now spare capacity provided against the outage of the largest unit over the peak also provides for the outage of two or more of the medium or small size units, should such double outages occur. The generating plant of the Philadelphia Electric Company includes 25 generating units with individual ratings ranging from 20,000 kw. to 60,000 kw. The few units with ratings under 20,000 kw. are seldom operated and because of their relative unimportance, need not be considered here. There are only three units in the 60,000-kw. class—two at Richmond and one 58,000-kw. unit at Deepwater. These three units, although comprising 20 per cent of the total capacity, represent only 12 per cent of the total number of generators rated at or above 20,000 kw. However, 19 generators or 76 per cent of the total number, comprising 72 per cent of the total capacity, are rated from 30,000 to 36,000 kw. It is apparent, therefore, that should a double outage occur, the probability is that it would occur in the medium or small-unit class and, practically, reserve therefor is provided if reserve is maintained for single outage of the largest unit.

The fourth and probably the most important reason is found in the characteristics of the system load. As in other metropolitan areas, the annual peak load occurs about 5 p.m., generally on a December day, and is extremely sharp. The season of these extreme peaks is only from two to three months' duration. During the remainder of the year, a portion of the generating capacity must be idle. Moreover, even during the peak season, the variation in daily peaks is such that except on relatively few days, reserve capacity is available for greater outages than loss of the largest unit. Analysis of several peak seasons has shown that on more than 90 per cent of the days in the peak season (November, December, and January), the outage of an additional 30,000-kw. unit is provided for, and on more than 60 per cent of the days, the outage of an additional 60,000-kw. unit or two additional 30,000-kw. units is provided for.

In general, the shape of the daily load curve and seasonal variations in the daily peaks have an influence on justified generator reserve capacity particularly since, where high loads persist, there is less opportunity for proper maintenance.

220-Kv. and 66-Kv. Transmission. From the standpoint of investment economy, it is necessary, of course, to secure that number and arrangement of transmission lines which will satisfactorily take care of the present and expected future loads under those emergency conditions which occur with reasonable frequency.

As will be seen from Fig. 1, the high-voltage transmission system in the Philadelphia territory is at the present time essentially a twin-circuit system. Between Plymouth Meeting and Westmoreland Substations, however, three circuits on two tower lines have been installed, primarily to increase the reliability of this very important transmission link.

The 220-kv. Conowingo-Plymouth Meeting System is so designed that each circuit is capable of transmitting the present maximum output of the Conowingo plant. Although initial experience indicated that these lines are subject to more severe lightning hazards than was anticipated, it appears that by the recent application of high-speed oil circuit breakers and relays, the requirements of the design have been met with minimum hazard from instability. The remainder of the 220-kv. system, comprising the 220-kv. interconnection, is essentially a duplicate line system although only one line is provided between each two of the three companies. With this triangular arrangement, any one leg of the triangle may fail without severing the connection of the three parties.

On the 66-kv. transmission system, experience has indicated that outage of only one circuit need be anticipated. Of the total number of circuit miles comprising this system, about 74 per cent is double-circuit aerial construction with two ground wires, 6 per cent is single-circuit aerial construction with one ground wire (on double-circuit towers), and the remaining 20 per cent is double-circuit underground construction, that is, six single-conductor cables in one bank of ducts. On lines within the equivalent generation system, double-circuit failures resulting from lightning storms or from other causes such as accidental contact with foreign bodies, occur but rarely. Single-circuit failures are expected and do occur occasionally, particularly on the underground portion of the system, but it is the general experience that transmission failures on the high-voltage system do not result in interruptions to customers' service—and this would seem to be the criterion in judging the success or failure of the transmission system.

Since single-circuit outage only need be anticipated, the locations of generating plants and the arrangement and capacity of the high-voltage transmission system are such that any line may fail without impairing the ability of the system to carry the load existing at the time of failure.

It is to be noted that the capacity of the high-voltage transmission system is not necessarily determined by system peak loads. With the most economical generating plants located at a distance from some of the load centers, as they are in the Philadelphia system, it may be desirable to carry the heaviest loads on parts of the transmission system at off-peak periods, and consequently, examination of such periods may expose conditions which make it economical to increase the capacity of the transmission system.

Transmission Substation Transformer Capacity. Sufficient transformer capacity is provided at transmission substations for the supply of other transmission lines or large local loads so that failure at any time of the most heavily loaded transformer bank in any substation will not impair the ability of the system or the substation to carry the load existing at the time of the failure. In some cases transformer outage may require reallocation of load among generating plants. In other cases, the load on the faulty bank may be transferred to remaining banks or to other sources.

In this connection, it seems apparent that the use of forced air transformer cooling and the operation of transformers on a temperature basis can effect large savings in reducing the amount of spare transformer capacity which it is necessary to provide.

From the Standpoint of Operating Economy

Allocation of Hydroelectric Energy. The Conowingo Hydro Station with a generating capacity of 252,000 kw. generates in a normal hydro year approximately 40 per cent of the total energy supplied by Philadelphia Electric Company. Conowingo is a run-of-river plant, and, therefore, the energy output is dependent on the flow of the Susquehanna River. The approximate flow is forecast the latter part of each week for the following week in order to determine the steam station boiler schedule for that week. Each day an accurate estimate is made of the daily kilowatt-hour output from Conowingo for the following day by the hydrologer at Conowingo, who bases his estimate on the flow of the main branch and tributaries of the Susquehanna at various locations on the watershed, and a general knowledge of the actual and expected rainfall on the drainage area.

The most economical allocation of available hydro energy is obtained, first, by utilizing all the energy available from the river within the limits of plant capacity, and second, by utilizing this energy to obtain the maximum hydro capacity and thereby reduce steam station peak and energy costs to a minimum. This is accomplished by allocating a block of the system daily load curve, Fig. 2, of such proportions that the area of the block corresponds to the available energy to be generated from the total flow of water for the particular day and the ordinate of the block corresponds to the maximum hydro capacity.

Referring to Fig. 2, the higher the position of the Conowingo block on the load curve, the less will be the area of this block, which is proportional to the Conowingo energy output. The steam base load, indicated in Fig. 2, may be defined as that load above which Conowingo operates and takes all load variations up to the limit of its capacity. When the system load exceeds the steam base load plus the full capacity of Conowingo, the excess must be carried as steam peak, the steam then taking all variations in system load.

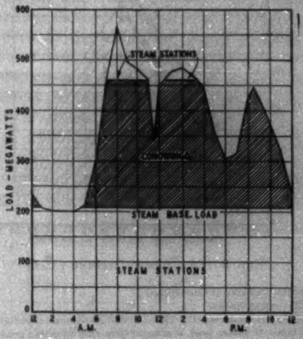


Fig. 2—Daily System Load Curve Showing Load Allocated to Conowingo

There is a definite relation between the steam base load and the Conowingo daily energy output for a given daily system load. This relation is utilized in determining the steam base load for each day by using the curves in Fig. 3, based on an available hydro capacity of 252,000 kw. The load dispatcher determines from these curves the steam base load corresponding to a given Conowingo energy output and the daily system energy generation which he estimates for the following day. The variations of the actual daily Conowingo and system outputs from the estimate are absorbed by allowing the amount of water stored in the Conowingo pond to vary. The curves in Fig. 3 are within the accuracy of predictions of river flow and system output, regardless of the seasonal variation in the shape of the daily load curve.

Allocation of Boiler Capacity. The scheduling of boiler capacity for each steam station is done on a weekly basis in order to reduce boiler starting losses to a minimum. The number of boilers required for the week is determined from the estimate of weekly system peak load and the average steam base load for the week, which is based on the Conowingo estimated output for the week. If the difference between the system peak and the steam base load is less than the available Conowingo capacity, the steam peak is then the steam base load, inasmuch as the excess of the system load above the steam base load can be carried entirely on Conowingo. This is the case during periods of low river flow. When the difference between the system peak load and the steam base load is greater than the available Conowingo capacity, the full hydro capacity can be utilized and the steam peak load is then the difference between the system peak load and the Conowingo capacity.

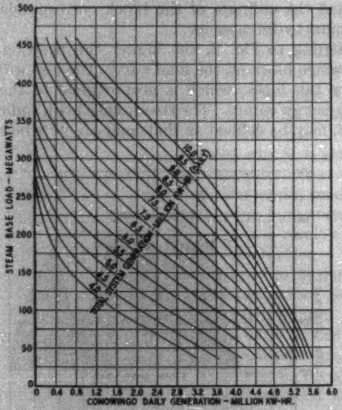


FIG. 3—CONOWINGO LOADING CHART

7 machines available at Conowingo
Capacity = 252,000 kw.
Available total peak capacity without additional steam capacity = 252,000 + steam base. If steam peak is greater than this available peak the steam peak capacity must be increased to make up the difference

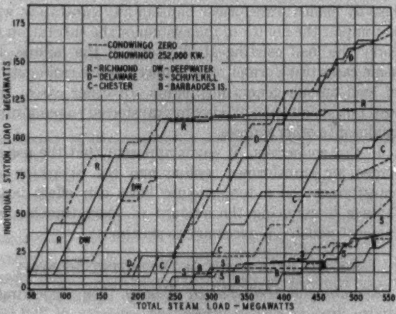
The total number of boilers required for system reserve is based on the steam peak load and the reserve required for the failure of any piece of equipment on the system.

The boilers are allocated to the individual generating stations as far as possible in the most economical manner, although it is generally necessary to operate some boiler capacity at less efficient stations in order to protect that portion of the system against the failure of any piece of equipment supplying it.

At times with a high steam station load factor, it becomes economical to operate additional boilers in excess of reserve requirements. The additional boilers reduce the average rating on each boiler with consequent improved efficiency during the heavy load periods which more than offsets the additional banking losses.

Allocation of Steam Turbine Capacity. Sufficient turbine capacity must be scheduled in the different parts of the system at all times so that the failure of any piece of equipment will not impair service. In economically scheduling turbine capacity in accordance with reliability requirements, the no-load costs as well as incremental costs on the turbines in each station are considered. A reduction in incremental costs can sometimes be effected by placing an additional turbine in service in excess of reserve requirements. This condition occurs when the turbines are being operated with the overload valves partly open, where their incremental costs are higher. The saving in incremental costs must be greater than the no-load costs on the turbine to justify placing it in service.

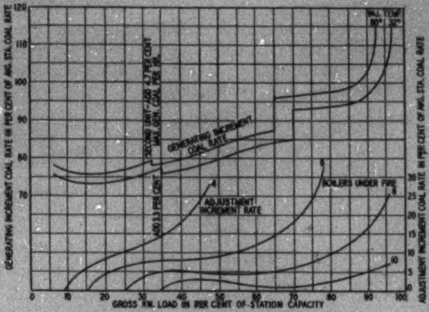
Allocation of Steam Station Load. The load dispatcher determines the most economical allocation of steam load and turbine capacity among the individual steam generating stations by means of a steam load allocation chart, Fig. 4. The objective in working out a steam load allocation chart is to obtain minimum system cost per kilowatt-hour delivered from the transmission system. This can be obtained in practise by considering fuel costs only, as the cost of fuel is by far the largest item of incremental cost and any other items of cost other than fuel tend to cancel each other on an incremental basis.



From *Electrical World*, Oct. 11, 1930.

FIG. 4—STEAM LOAD ALLOCATION CHART

Any correct load allocation must be made on an incremental basis, and is arrived at by allocating each increase in load to the station having the lowest incremental fuel rate corrected for transmission losses. The incremental coal rate may be defined as the pounds of coal required to produce an increment of one kilowatt-hour in the net station output at a given station load. The accuracy of a steam load allocation chart depends, in large measure, on the accuracy of the steam station performance data used in developing the steam load allocations.



From *Electrical World*, Oct. 11, 1930.

Fig. 5—Increment Coal Rate Curves

The incremental coal rate curves, Fig. 5, for each station provide the basic information not only for correct load allocation but also for the accurate cost data necessary in interchanging power economically between interconnected systems.. In Fig. 5 the incremental coal rates expressed in per cent of average station coal rate are shown in relation to station loading. Adjustments below the main curves are for the particular number of boilers which are under fire in the station and are added to the values given by the main curves to obtain the total incremental coal rate.

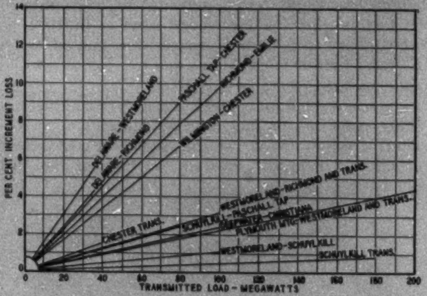
Transmission Tie-Line Losses. Economic loading of generating stations usually results in the transmission of considerable power over tie lines. Obviously, the transmission tie-line loss involved must be considered in determining correct load allocations. To facilitate the determination of the additional losses incurred in transmitting an additional block of power over tie lines from a generating source to a distributing point, transmission line increment loss curves, Fig. 6, have been developed. The per cent increment loss at a given transmission line load may be defined as the ratio of the incremental increase in loss to the incremental increase in load. If a transmission line is carrying a given load, *A*, and an additional block of load, *B*, is superimposed on the existing load *A*, the additional loss in per cent of load *B* resulting from load *B* is obtained directly from the incremental loss curve by entering the curve at a loading midway between the initial and final loading. If a given block of load is transmitted from a generating source to a distributing point over a number of transmission lines, all operating at different loadings, the net increase or decrease in loss resulting from transmitting the block can be readily determined by algebraically summating the percentage incremental losses on all the transmission lines. A net decrease in loss may result instead of a net increase, as it is entirely possible that by transmitting the block of load some of the transmission lines will be reduced in loading. The per cent incremental loss curves are useful not only in determining load allocations, but also in correcting costs of interchange power between interconnected companies for transmission losses.

The 220-Kv. Pennsylvania-New Jersey Interconnection

The Philadelphia Electric Company interconnection with the Pennsylvania Power and Light and Public Service Electric and Gas Companies at Plymouth Meeting has resulted in greater reliability to all the systems. The reliability results from the diversity of occurrence of failures on the individual systems. In other words, experience has shown that when a failure occurs on one system even to the extent of complete failure of a generating station, the other interconnected companies can render emergency assistance by supplying from their reserves the necessary load to maintain complete continuity of service.

Savings from the interconnection result from diversity in various forms as follows:

1. Diversity of the loads of the individual companies resulting in reduced combined peak load and consequent reduction in necessary plant capacity investment and in operating peak costs.



From *Electrical World*, Oct. 24, 1931.

Fig. 6—Transmission Line Increment Loss

2. Diversity of occurrence of failures resulting in lower combined reserve requirements and consequent reduction in necessary plant capacity and operating reserve costs.
3. Diversity of installation of new plant capacity resulting in staggered construction and economical utilization of available sites.

4. Diversity in utilization of economical plant capacity resulting in the economical interchange of energy.

Installation of New Facilities for Economy and Reliability

New facilities have been installed from time to time prior to load growth requirements in order to secure additional system economy or reliability or both. Each installation has been carefully studied, and either the estimated operating savings have shown ample margin over the additional fixed charges incurred through advancing the time of installation, or a commensurate gain in reliability has been realized. Specific examples of facilities which have been installed to secure additional system economy and/or reliability are (1) a 66/33-kv. transformer bank at Plymouth Meeting and (2) the third 66-kv. transmission circuit between Plymouth Meeting and Westmoreland Substations. The transformer installation permits shut-down of the Barbadoes Island Generating Plant during the greater portion of each year, thereby effecting operating economies in excess of the additional fixed charges. The transmission line, although effecting some operating savings (which, however, are not sufficient to offset the new fixed charges), was built primarily to increase system reliability, and operating experience during the summer following its installation fully justified this decision.

Results

The reliability of production facilities of the Philadelphia Electric System is indicated by an analysis of the factors affecting customers' service. In 1931, no interruptions to customers' service were caused by the failure of production facilities and experience in other years has been generally satisfactory.

The operating economies obtained have resulted in a steady reduction in fuel rate per kilowatt-hour. The following tabulation shows the pounds of coal burned per kilowatt-hour expressed in per cent of the 1926 values:

Year	Steam system fuel rate lb. coal per kw-hr. per cent of 1926 value
1926	100 per cent
1927	97 per cent
1928	99 per cent
1929	93 per cent
1930	89 per cent
1931	84 per cent

The only new and more efficient generating capacity added during the entire period was Deepwater Station, the Philadelphia Electric Company's share of which amounts only to 8.4 per cent of the total system steam capacity. The increase in fuel rate per net kilowatt-hour in 1928 was caused by the initial operation of the Conowingo Hydro-Electric Station, which necessitated running steam stations at greatly reduced load factors with abnormally large hot reserve to maintain reliability during the period of preliminary operation.

The effect of economical load and reserve allocation is indicated by the following tabulation showing the B.t.u. per net kilowatt-hour for various steam stations expressed in per cent of the steam station B.t.u. rate for the year 1931. The station capacity in per cent of the total system steam capacity and the energy generated by the various stations in per cent of total steam generation for the year 1931 are also shown.

Stations	B.t.u. per net kw-hr. year 1931 per cent of steam system B.t.u. rate	Station capacity per cent of total steam capacity	Energy generated year 1931 per cent of total steam generation
Deepwater	78 per cent	8.4 per cent	24.8 per cent
Richmond	88 per cent	17.4 per cent	36.4 per cent
Delaware	115 per cent	26.6 per cent	22.0 per cent
Chester	120 per cent	18.8 per cent	12.1 per cent
All other steam stations	183 per cent	29.3 per cent	4.7 per cent
Steam system	100 per cent	100 per cent	100 per cent

The B.t.u. rates for only two stations, Deepwater and Richmond, were better than that for the system in spite of the fact that the combined capacity of these two stations is only 25.8 per cent of the total steam capacity.

These results have not been obtained at the expense of abnormally high carrying charges. All facilities installed to increase operating economy have been required to show operating savings in excess of fixed charges.

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Discussion

For discussion of this paper see page 888.

IV—Combined Reliability and Economy in Operation of Large Electric Systems

CHICAGO DISTRICT

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Synopsis.—*The paper outlines the load characteristics, generating stations, and transmission lines of the Chicago territory and particularly the power supply of Chicago proper.*

The economics of the scheme of supply are discussed from the points of view of interchange energy agreements, comparative increment fuel costs of generating stations, increment losses of transmission lines, load schedules, remote metering and load dispatching, economics of 550- to 650-lb. units, automatic frequency regulation, control of wattless mva. voltage and phase angle.

The reliability of the system is outlined with respect to the design of generators, switchgear, transmission lines and cables, relay protection, methods of operating, and reserve capacity.

From the experience gained in the Chicago territory conclusions are drawn as to the policies of economy and reliability which have been adopted in operating the system.

* * * * *

INTRODUCTION

IN 1928 in a paper before the A.I.E.E., Mr. H. B. Gear[1] outlined the main features of power generation and interconnection for the Chicago District and the surrounding territory of the Middle West, and in January, 1930, Messrs. G. M. Armbrust and T. G. LeClair[2] outlined the main design features of the 25-cycle and 60-cycle systems of the Chicago District.

This paper deals essentially with the power generation features added since that date, and the improvement in economies that have been obtained by the methods followed in the Chicago District in the generation and transmission of power.

DESCRIPTION OF CHICAGO TERRITORY

Fig. 1 is a map of the territory embracing an area of approximately 20,000 square miles. The operating companies that supply this load which amounts to about 5,758 million kw-hr. per year are the following:

Commonwealth Edison Company with about 1 per cent of the area and about 73 per cent of the load serves Chicago proper, as shown in area *A* and in more detail in Fig. 2.

The Public Service Company of Northern Illinois, the Northern Indiana Public Service Company, and the Western United Gas & Electric Company serve the remainder of the territory embraced by *B*. The area of about 8,000 square miles supplied by these four companies is ordinarily called the Chicago District, and these companies work together by means of an Interchange Energy Agreement to facilitate minimum fixed charges and maximum economy in the operation of their interconnected system.

The remainder of the territory is supplied chiefly by the Illinois Northern Utilities Company, the Central Illinois Public Service Company, the Illinois Power & Light Corporation, the Central Illinois Light Company, and the Central Illinois Electric & Gas Company.

The power is supplied by the main generating stations of the various companies located as shown in Figs. 1 and 2. These naturally fall into two groups—the six closely linked stations around the Chicago area with a total of 1,480 mw. and the more loosely linked distant stations with a total of 538 mw. Practically all of these stations are interconnected by 66-kv. and 132-kv. underground and overhead ties of 60 mva. each and larger.

There are 132-kv. major connections to outside companies shown in Fig. 1 from Michigan City to the American Gas & Electric Company's system extending eastward, from Waukegan north to the system of The Milwaukee Electric Railway & Light Company centering at Milwaukee, and from Belvidere north to Janesville and Sheboygan, Wisconsin.

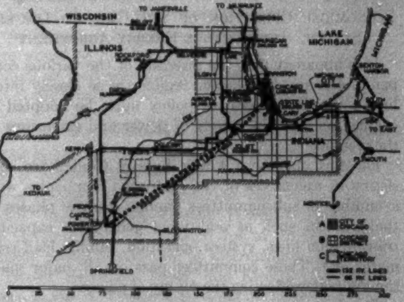

Fig. 1—Map of Chicago Territory

GROWTH OF LOAD IN CHICAGO

Fig. 3 shows the load growth since 1910 and indicates particularly the change in 25-cycle generation. The distribution system at 25 and 60 cycles has been described by others.[2]

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†Asst. Electrical Engr., Sargent & Lundy, Inc., Chicago, Ill.

1. For references see Bibliography.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

OUTSIDE SUPPLY TO CHICAGO

The importance of the interconnections in supplying power to Chicago is apparent from Table I, which indicates that in 1931, 41.3 per cent of the power consumed in Chicago was brought into the city from outside. The economics of this arrangement and the reliability of the supply from outside the city are therefore of the greatest importance.

TABLE I—DISTRIBUTION OF POWER GENERATED, PURCHASED, AND SOLD IN CHICAGO
(In Millions of Kw-Hr.)

Year	1928	1929	1930	1931
Item 1. Kw-hr. used in Chicago	3,851	4,276	4,191	4,024
Item 2. Kw-hr. generated by Commonwealth Edison stations for use in Chicago	3,542	3,466	2,855	2,364
Item 3. Kw-hr. purchased from Waukegan	276	232	236	289
Item 4. Kw-hr. purchased from State Line		418	812	952
Item 5. Kw-hr. purchased from Powerton	33	145	285	419
Item 6. Total purchased	309	810	1,336	1,660
Item 7. Purchased in per cent of kw-hr. used	8.0	18.9	31.8	41.3

Item 2 gives the amounts generated in the Chicago stations for use of customers in the city of Chicago. These stations generate some additional power that is used outside Chicago.

Item 6 gives the amounts brought into Chicago from the stations outside the city for use of customers in Chicago.

Item 7 gives the percentages of the kw-hr. brought into Chicago to the total amount used in Chicago.

ECONOMICAL OPERATION OF THE CHICAGO DISTRICT SYSTEM—INTERCHANGE ENERGY AGREEMENT

It is apparent that the interests of the various companies serving the Chicago region are so closely interlinked that a cooperative policy must be adopted in connection with the supply of power and extensions to the system.

For this purpose a Power Supply Committee consisting of executives of the interested companies with a number of subcommittees handles various phases of the problem such as additional generating capacity, transmission lines, system operation, billing, load estimates, etc. These committees pass on all major questions of policy affecting the supply of power.

The four companies supplying the area referred to herein as the Chicago District work together through an Interchange Energy Agreement which has now been in operation for some years. The basis on which it formerly operated was outlined by Mr. E. J. Fowler[3] in 1926. The aim of this agreement, which has since undergone some revisions, is equitably to allocate the following:

1. Demand charges in proportion to maximum demand of each party.
2. Fixed operating charges in proportion to the maximum demand of each party.
3. Fuel charge in proportion to the kw-hr. taken by each party.

LOAD SCHEDULES

In order properly to distribute the loads, ordinarily two load schedules such as those given in Fig. 4 for the closely linked stations of the Chicago area, and Fig. 5 for the outlying stations are used. These are revised from time to time to take care of varying conditions, such as seasonal load changes and overhauling of one or more units. The essential difference between these two load schedules is that the load schedule in Fig. 4 is fundamentally based on the operation of the distant metering system, as described by Mr. P. B. Juhnke.[4] This is supplemented by some telephone reports and instructions, since Waukegan supply of power to Chicago load is not included in the distant metering system.

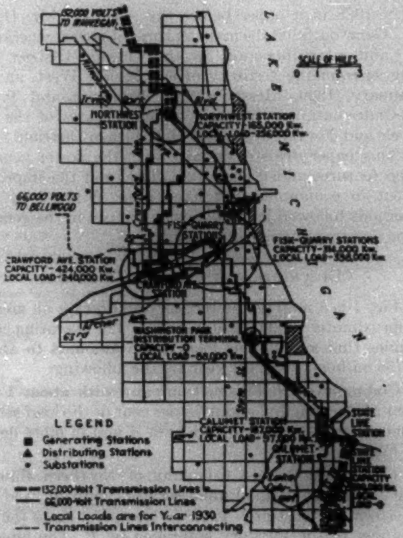
132,000 VOLTS TO WAUKEGAN
LAKE MICHIGAN
SCALE OF MILES
0 1 2 3
NORTHWEST STATION
NORTHWEST STATION CAPACITY-165,000 Kw. LOCAL LOAD-256,000 Kw.
66,000 VOLTS TO BELLWOOD
FISK-QUARRY STATIONS
FISK-QUARRY STATIONS CAPACITY-314,000 Kw. LOCAL LOAD-336,000 Kw.
CRAWFORD AVE. STATION CAPACITY-424,000 Kw. LOCAL LOAD-240,000 Kw.
CRAWFORD AVE. STATION
Archer Ave.
63rd
WASHINGTON PARK DISTRIBUTION TERMINAL CAPACITY-0 LOCAL LOAD-88,000 Kw.
CALUMET STATION CAPACITY-187,000 Kw. LOCAL LOAD-57,000 Kw.
CALUMET STATION
STATE LINE STATION
STATE LINE STATION CAPACITY LOCAL LOAD-0
LEGEND
■ Generating Stations
▲ Distributing Stations
• Substations
132,000-Volt Transmission Lines
66,000-Volt Transmission Lines
Local Loads are for Year 1930
Transmission Lines Interconnecting

FIG. 2—MAP OF CHICAGO

The advantage of the distant metering scheme is that the principal stations supplying the Chicago load automatically vary the amount that each generates without having to get telephone instructions from the load dispatcher, and furthermore can to a certain extent anticipate any departure in the day's load from the usual load during the season, due to such factors as sudden changes in weather conditions. This is of distinct value when a sudden oncoming storm materially increases the load beyond the usual requirements at the time of the daily noon dip.

Fig. 5 shows schedules for the outlying stations of Powerton and Michigan City. The Powerton schedule gives just the amount to be transmitted into the systems of the Public Service Company of Northern Illinois and the Commonwealth Edison Company and does not give the total amount to be generated at Powerton. In addition the Powerton Station supplies considerable load to the Central Illinois Public Service Company and the Illinois Power & Light Corporation.

Fig. 6 shows for a typical daily load what was the division of load supplied by the principal generating stations, in accordance with the schedules of Figs. 4 and 5.

It has been found practical, as discussed later, to transmit over the 132-kv. lines large amounts of power relatively long distances for this voltage. This can be done only by transmitting this power at practically unity power factor at delivery points, such as Waukegan, Crawford Avenue, and Calumet. It is also to be noted that the send-out voltage at Powerton to accomplish this has to be often 145.2 kv., the maximum operating voltage for the 132-kv. transformers with a voltage at the low-voltage delivery points, such as Northwest and State Line of about 125 kv.

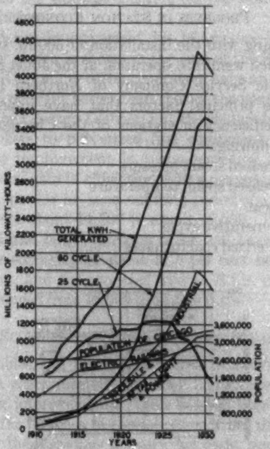

Fig. 3—Growth of Load in Chicago

At the present time more Powerton power gets into the Chicago District than would occur under usual business conditions when the industrial loads at many of the outlying industrial sections, such as Chicago Heights, Joliet, Electric Junction, Oglesby, and Kewanee would be higher.

Basis of Load Schedules—Increment Fuel Costs

The load schedules that have been discussed are based substantially on the comparative increment fuel costs of the various stations on the system. There are, however, two difficulties in adopting this practise too rigidly.

First: Reserve capacity must be carried at strategic points of the system to take care of emergencies. This is discussed later.

Second: During the early hours of the day from midnight to 4 a.m. and on Sundays and holidays, it is desirable to keep all major stations on the system with some load, and there are certain minimum practical loads that must be carried independent of the economics of the situation.

Fig. 7 gives the comparative increment cost per kw-hr. in per cent, as it varies with the station net output in megawatts for each of the five most recent stations, and also the proportionate cost of coal per B.t.u. The stations near Chicago have substantially the same cost of coal per B.t.u. but between the lowest value of the adjacent stations and the Powerton Station, the most distant station, there is a difference in favor of the Powerton Station of 15 per cent. Consequently in comparing the increment fuel consumption of the Powerton Station with the adjacent stations, not only must consideration be given to the very low fuel consumption for additional loads, but in addition, the advantage of 15 to 20 per cent lower fuel cost must be taken into account.

Neglecting the question of transmission for the moment, it will be seen that to carry an additional 10 mw. on the Powerton Station as compared with Waukegan there would be a differential in favor of Powerton depending on load conditions, but usually at least 33 per cent of the Powerton cost.

Increment Line Loss

The saving in costs due to more economical generation is partly offset by transmission losses. To take proper account of this, increment loss curves must be drawn. The conditions between Powerton and Waukegan are somewhat complicated due to the fact that ordinarily power is taken off at intermediate points such as Dixon, Belvidere, Joliet, and Electric Junction as shown in Fig. 1. To show the effect of the line loss in the power delivered at Waukegan for various amounts of power taken off at these intermediate points, Fig. 8 has been prepared. As stated above, the fuel cost at Powerton is usually 33 per cent less than at Waukegan. Hence, with all fixed charges of transmission to be paid in any event under rental agreements, the increment line loss on Powerton power can be as high as 33 per cent and the power delivered from Powerton to Waukegan, a distance of approximately 240 miles, will have a production cost the same as power generated at Waukegan. If the intermediate load is 60 mw. with the increment loss of 33 per cent, more than 60 mw. can be delivered to Waukegan with the same fuel cost as that generated at Waukegan. This is slightly below the reasonable stability limit which is shown in this figure by line $L_1 - L_1$, and, under ordinary conditions of operation, will be beyond the generating capacity available at Powerton. Similar studies show it is economical to send power through the State Line Station into Chicago.

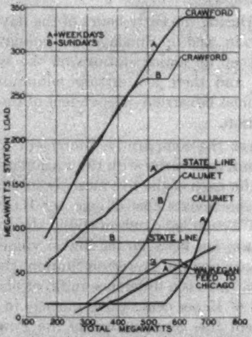

Fig. 4—Schedule for Main 60-Cycle Load for Adjacent Stations—Chicago District

Usually then the limit in the amount of power coming into Chicago is not the economics of the situation, nor the stability of the lines delivering this power, but instead in the present condition is the capacity available at Powerton for Chicago load.

Within the limits of the stations adjacent to the city of Chicago, that is from Waukegan to Michigan City, the increment line loss is usually not an important factor.

Relative Output of Generating Stations

The result of following the general policies determined from studies such as those outlined above is shown in Fig. 9, which has been made up to show the relative performance of the stations, the output and the per cent of the total load carried by each for the year 1931.

It will be noted that the Chicago territory, as a whole, produced 100 per cent of its power at an average B.t.u. per kw-hr. send-out of 15,006. Of this power 85.8 per cent was produced in stations with an average B.t.u. of 13,910. The capacity of these stations in per cent of the total capacity was 58.3, although, as stated above, they produce 85.8 per cent of the kilowatt hours. Older stations produced only 14.2 per cent of the power with an average B.t.u. per send-out of 21,603, although their capacity in the per cent of the total capacity was 41.7 per cent.

Of the 25-cycle power used in Chicago about two-thirds is generated at 25 cycles and about one-third is generated at 60 cycles and transformed through two 40,000-kw. frequency changers with a loss of about 6½ per cent. While the latest 25-cycle generating apparatus, of which the last unit was installed in 1920, has a B.t.u. per kw-hr. send-out of about 19,000 as compared with about 14,000 average for the 60-cycle generating apparatus, studies show that the 25-cycle generating apparatus cannot yet be replaced with new 60-cycle generating apparatus, because the fixed charges on this new generating apparatus, when added to the operating charges even at an economy of 12,000 B.t.u. per kw-hr. will give total charges on the new greater than the operating charges on the old apparatus. Also since the kw-hr. generated at 25 cycles is rapidly decreasing, the fuel consumption for this class of service is every year becoming a less important factor in the average production cost for the Chicago load.

A consideration of Fig. 9 will show the great advantage of interchange energy contracts and purchase contracts so drafted as to give all parties concerned the maximum practicable and equitable benefit of the most efficient stations. The same situation exists in the possibility of savings between central stations and large industrial plants.[5]

Progress in Station Economy

Beginning with the installation of 300-lb. steam turbines which went into operation at the Joliet Station of the Public Service Company of Northern Illinois in 1917, the principal factors that have improved the thermal efficiency in stations of the Chicago District are the following:

1. Increased steam pressure
2. Increased steam temperature
3. Reheat
4. Regenerative cycle
5. Pulverized coal furnaces

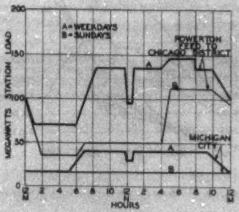

Fig. 5—Schedule of Load for Distant Stations—Chicago District

It must be borne in mind in comparing these economies with those obtained in stations of the east, particularly in those burning a better grade of coal than is used in the Chicago Region stations, that the Illinois coal runs about 12 to 15 per cent ash and 12 to 15 per cent moisture. The use of pulverized coal was under consideration for many years in the Chicago District, and as soon as actual experimental boilers installed at Calumet Station had shown that furnaces capable of burning this coal successfully were available, they were adopted for use in the Chicago Region stations.

Table II gives the dates of operation of units with these improvements in the Chicago District.

TABLE II—CHARACTERISTICS OF 550—650-LB. UNITS IN CHICAGO DISTRICT

Year in service	Station	Unit	Mw.	Gen. voltage	Throttle steam pressure lb/sq. in.	Throttle steam temp. fahr.	Reheat	Boilers
1924	Crawford	2	60	12,000	550	725	Boiler	Ch. grate
1925	Crawford	1	51	12,000	550	750	Boiler	Ch. grate
	Crawford	3	50	12,000	550	725	Boiler	Ch. grate
1926	Crawford	4	75	12,000	550	750	None	Ch. grate
1927	Waukegan	3	50	12,000	600	725	Boiler	Ch. grate
	Crawford	5	88	12,000	550	750	Steam	Ch. grate
1928	Crawford	6	100	12,000	550	725	Steam	Ch. grate
	Powerton	1	52	22,000	600	725	Boiler	Pulv. fuel
1929	State Line	1	200	22,000	650	730	Steam	Pulv. fuel*
	Powerton	2	52	22,000	600	725	Boiler	Pulv. fuel
1930	Waukegan	4	65	12,000	600	725	Boiler	Pulv. fuel
	Powerton	3	100	22,000	600	725	Boiler	Pulv. fuel
1931	Michigan City	1	68	22,000	650	750	Boiler	Pulv. fuel
	Waukegan	5	115	18,000	625	750	Boiler	Pulv. fuel

*Now uses 70 per cent dump gas.

Fig. 10 shows the improvement in the economy of the stations to the Chicago District for the years 1925 to 1931, inclusive.

The relative improvement due to advance in turbine and boiler design is shown in the case of the State Line Station. The unit 1 turbo-generators have an efficiency about 4.5 per cent better than that of the next previous units in the Chicago District, but the change from chain grate stokers to pulverized fuel equipment, together with that improvement in turbo-generator efficiency, made a total gain in efficiency of 10.4 per cent, reducing the B.t.u. per kw-hr. send-out for a period of a month or longer from about 14,500 to about 13,000. The 1,200-lb. No. 2 and No. 3 units on order will have a turbo-generator efficiency of about 11.5 per cent better than No. 1 unit, and the expected overall B.t.u. per kw-hr. will be reduced from 13,000 to 11,500 B.t.u.

ECONOMICS OF HIGH-PRESSURE STEAM

The tables and diagrams referred to above indicate the great economies effected with high-pressure steam and other improvements. Table III has been made up to show the increase in capital expenditure that will be offset by these savings in operating costs, and produce power for the same total cost including all charges.

The 600-lb. stations, as shown in this table, ran at an average capacity factor in 1931 of 53.2 per cent and produced the average kw-hr. sent out for 13,900 B.t.u. The saving in fuel costs shown in Table III has been made up with a variety of coal costs, so that the effect of changes in this figure, which varies considerably with conditions, will be apparent. The table gives the saving in the 600-lb. stations, as compared with a 350-lb. station on a capacity factor of 53.2 per cent mentioned above. Also it is apparent that as the 600-lb. equipment gets older it will naturally run at a lower annual capacity factor, but there seems to be no reason to believe that during its useful life its capacity factor in a large interconnected system need be less than the average in the Chicago system for the last ten years, which was 39.5 per cent. Therefore, the saving per year over its life should be at least from $1.18 to $2.36 per kw., justifying the expenditure to obtain this of from $10.75 to $21.50 per kw., based on 11 per cent fixed charges, and still give the same total overall cost of power.

TABLE III—1931 OPERATION OF 550—650-LB. UNITS

Station	Millions kw-hr. sent out	Thousands kw. capacity	Capacity factor	B.t.u. per kw-hr. sent out
Crawford Ave	1,880.4	424	50.6	14,914
Powerton	1,173.9	204	65.5	12,679
State Line	1,009.5	200	57.5	13,446
Waukegan	535.0	153.3(A)	31.9	13,890
Michigan City	269.8	58.5(A)	52.4	14,238
	4,868.6	1,039.8	53.2	13,900

(A) Average. One unit began service during year.

B.t.u. per kw-hr., average	13,900
B.t.u. per kw-hr. on 350-lb. units	17,500
Gain in 550—650-lb. units, B.t.u. per kw-hr	3,600
Coal, B.t.u. per lb	10,500
Gain in lb. coal per kw-hr	.343

Coal cost $ per ton	2.00	2.50	3.00	3.50	4.00
Capacity factor 53.2% (average of 550—650-lb. units)					
Hours operation	4,700	4,700	4,700	4,700	4,700
Saving per kw. per year	$1.61	$2.02	$2.42	$2.82	$3.22
Permissible increased investment per kw. at 11% fixed charges	14.70	18.40	22.00	25.70	29.40
Capacity factor 39.5% (average of Chicago District)					
Hours operation	3,460	3,460	3,460	3,460	3,460
Saving per kw. per year	$1.18	$1.48	$1.78	$2.08	$2.36
Permissible increased investment per kw. at 11% fixed charges	10.75	13.45	16.13	18.90	21.50

Still greater economies than those outlined above are expected from the 1,200-lb. equipment now on order for the extension to the State Line Station, which should produce a kw-hr. for 11,500 B.t.u.

FREQUENCY CONTROL

An important factor in the economical operation of the system is the method of frequency control in use. At the present time it is the normal practise to have Windsor, Philo, and Springdale Stations in the east under automatic frequency control, and when the tie line from Michigan City to South Bend is closed no automatic control is in service in the Chicago District. When this tie is open usually an automatic frequency controller at the Crawford Avenue Station is in service on one or two units, allowing the remaining stations to follow scheduled load. The former method of frequency control has, under normal conditions, held the frequency correct within 1/20 cycle and the time drift to about 2 seconds, with the eastern system connected with the western.

WATTLESS KVA., VOLTAGE, AND PHASE ANGLE CONTROL

Apart from generating station economics it is necessary to get the most efficient operation of the transmission system. The use of voltage and phase angle control here is of great assistance. Most of the 66-kv. and 132-kv. transformer banks in Chicago and at the distant generating stations are provided with load ratio control designed for a variation of plus or minus 10 per cent in voltage.

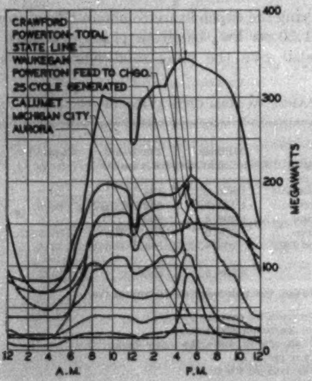

FIG. 6—STATION LOAD CURVES

These tap changers under load also serve the purpose of controlling wattless kva. on the lines, and in one case at least this has permitted carrying regularly an appreciably higher kw. load on a set of cable ties by operating the heavily loaded cable at unity power factor, and allowing the wattless kva. to be carried on the other cables with more margin in capacity.

It is of interest to note that the charging kva. of the overhead lines and also of the cables has a considerable effect on the power factor of the system. The charging kva. of the 132-kv. lines amounts to about 55 mva. and of the cable system about 83 mva. In addition there is a synchronous condenser capacity of more than 60 mva. Also in Chicago proper the 60-cycle rotary converters on the interconnected system of approximately 96 mw. are of assistance.

All these elements that tend to produce a good power factor, together with the normally high power factor of the metropolitan load of Chicago at the peak, enable large quantities of power to be brought into the city at high power factors without the use of large rotary condensers in Chicago.

With the installation of units 2 and 3 at State Line, phase shifters under load to give a displacement of 5.8 degrees will be used on the new 100-mw. 66-kv. feeders to the Commonwealth Edison Company to assure their carrying full load when in parallel with 60-mva. feeders.

RELIABILITY

There are two phases of the problem of reliability: first, the design of the individual stations, equipment, and transmission lines so as to reduce the possibility of outage; second, the operation of the system in such a way that any outage will not interfere with the supply of power to the consumer.

DESIGN FOR RELIABILITY

There is constant pressure on the designing engineer to reduce the first cost. Great savings could be effected in both generating station and substation design by sacrificing reliability, but this is not desirable. In power station design there is a definite limit to the cost in dollars per kw. if reasonable reliability is to be maintained.

GENERATOR DESIGN

All recent 60-cycle generators in the Chicago District were for 12,000 volts until in 1928 the Powerton Station began operation with a 52-mw. 22,000-volt generator, the first generator of that voltage to operate in the Western Hemisphere. There are now in the Chicago District 7 generators with 22,000-volt windings and 1 of 18,000 volts. At the time of writing this paper, none of these generators has given any insulation troubles. It might be mentioned that in none of these installations are there any outgoing feeders at generator voltage. The 8 units are of three different makes, are in four different stations, have the equivalent of about 17 generator years of service, and have demonstrated the entire success of high-voltage windings for generators. The aim has been to reduce the current necessary to handle in generator main cables and generator oil circuit breakers.

At State Line Station the 200-mw. unit is of the triple compound type giving the reliability of a two-unit station. This unit can continue to turn out power with any one or two members out of commission, except in the case of both low-pressure members. Since December 19, 1929, the unit has been supplying power continuously to the system, and the high-pressure member has run continuously for as long as 81 days.

Switchgear Design

The most outstanding developments from the electrical point of view in Chicago territory have been the use of isolated-phase and metal-clad switchgear.

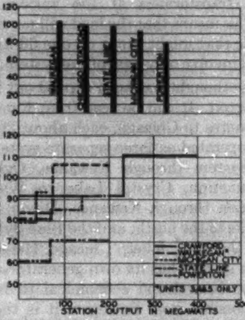

Fig. 7—Increment Fuel Costs

The importance of the Crawford Avenue Station in the Chicago District made advisable the further use of the isolated-phase principle, which had been first used at Calumet. This is now applied throughout the Commonwealth Edison system not only to 12-kv. equipment but also to 66-kv. outdoor equipment. The record of the Crawford Avenue Station has in the seven years of service demonstrated that a high degree of reliability has been attained by its use and faults confined to phase to ground in both 12 kv. at the station and 66 kv. on the system. The following points in design are to be noted as important factors in this record:

1. Fault bus protective scheme.
2. Reactors in all connections from generator buses.
3. Phases isolated at generator voltage and 3-ohm resistor in the neutral connection of one generator at a time.
4. Phases isolated at 66 kv. with circuits of single conductor underground cable and 30-ohm resistor in transformer neutral.

In addition to isolated-phase indoor and outdoor switchgear for 12 kv. and outdoor for 66 kv., the last four years have seen the rapid application of metal clad switching for 12 kv., 22 kv., 33 kv., and 132 kv.

In 1927 compound-filled indoor metal clad gear was used on the 12-kv. bus of unit 3 at Waukegan. In 1928 the first experimental outdoor metal clad equipment with oil-filled buses was installed at the Wheaton substation of the Public Service Company of Northern Illinois. This was followed by 22-kv. outdoor metal clad gear on the main generator bus at State Line and Powerton, additional 33-kv. substations, and in 1931 132-kv. equipment on unit 5 at Waukegan.

The chief advantages of metal clad gear are the following:

1. Personal hazards are practically eliminated.
2. The metal enclosed buses reduce the possibility of phase-to-phase short circuits.
3. Outdoor equipment eliminates building costs.
4. Factory assembled equipment reduces installation costs.
5. Fault bus protection is applicable for quick clearing of faults.

In 3 years of operation of the State Line Station, where the station auxiliary equipment is indoor metal clad and the main bus is outdoor metal clad, no member of the entire operating staff has met with any electrical personal injury accident of any kind. Also during this period with about the maximum range of temperature attainable at Chicago, the State Line metal-clad gear has developed no serious problems and the only minor difficulties encountered are those inherent in any electrically operated mechanisms whether installed indoors or outdoors.

The success of such equipment in the system of the Public Service Company of Northern Illinois has been outlined by Mr. J. L. Hecht of that company.[6]

Transmission Line Practise

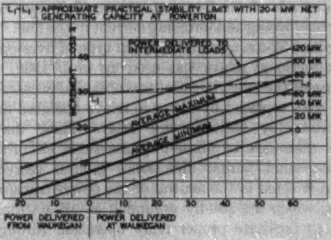

Fig. 8—Increment Loss Curves—Powerton—Waukegan

The experience with high-tension cable installations in Chicago has shown the very satisfactory performance that can be obtained with hollow core oil-filled cables as used in 132 kv. in service since 1927 or solid-core single-conductor cables provided with oil pressure reservoirs on 66 kv. in service since 1926. The use of oil reservoirs has also been adopted on the 22-kv. single-conductor cables at State Line and Powerton since they form part of the generator bus as it was felt that the additional cost was small and maximum reliability was essential. No cable failures have been experienced at either of these stations. The record of the 66-kv. and 132-kv. underground cables is annually given in the reports of the N.E.L.A. On overhead lines the greatest advance in reliability has been effected by the use of ground wires. The first few years service of 132-kv. lines was summarized by Mr. E. C. Williams[7] and further experience has confirmed the improvement outlined in his summary.

Relay Protection

The relay scheme of the territory is of the greatest importance in matters of reliability since it is very necessary to clear a short circuit in the minimum time.

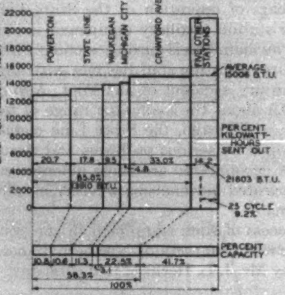

Fig. 9—Economies of Generating Stations

On all the 66-kv. cables in Chicago a reliable pilot wire protection scheme is used which opens the breakers at both ends of the line simultaneously in case of unbalanced ingoing and outgoing current in the line.

On the 132-kv. system the usual type of over-current and directional relays was used up until 1930, when because of the long time that was obtained in cascading, these were replaced by distance relays operated in certain cases by fault-detector relays. These distance relays take care of phase-to-phase short circuits. For line-to-ground faults, directional ground relays are used connected in the neutral of the power transformer banks, which are solidly grounded at all stations. In order to facilitate proper relay operation throughout the territory an Intercompany Relay Committee is in charge of relay settings, and recommends changes in types of relays so as to get the most satisfactory operation from the system.

Operating to Insure Reliability

The fundamental object in planning for system reliability is to operate in such a way that trouble in one section will not be allowed to spread to a large area, and also to provide sufficient reserve capacity to take care of any emergency.

The basic scheme which has been used in feeder arrangement has been a natural consequence of the isolated-phase idea. While in the station the various phases are kept separate, so that trouble will not spread from one to the other, in the system the transmission lines are divided into several sections so that one can be lost without affecting the load. In Chicago the 66-kv. cable system extending through the city from north to south is divided into two systems called the Red and Blue.[8] These are kept quite separate on the 66-kv. side and tied together only on the low-tension side of the step-down transformers at the various distribution centers. This means that the load will never lose supply, and trouble that might occur on one 66-kv. bus at any station will involve only part of the system.

The same thought has been carried out in the supply from Powerton. It can be seen from Fig. 1 that there are three routes to Chicago, each about 200 miles long. These are operated as three separate systems known as the Red System through Kewanee, Oglesby, Joliet, Electric Junction, Crystal Lake to Waukegan; the Green System through Kewanee, Dixon, Belvidere to Crystal Lake to the north; and the Blue System through Kewanee, Oglesby, Joliet, Chicago Heights to State Line. Each system has its own generator at Powerton. Since it has been found economical to feed as much power as possible from Powerton it is the practise to load these lines well up to a point not far from the stability limit, which represents about the full available output of the Powerton Station. If all three systems were bussed at Powerton the total supply from Powerton would have to be reduced in order to ride through short-circuit conditions.

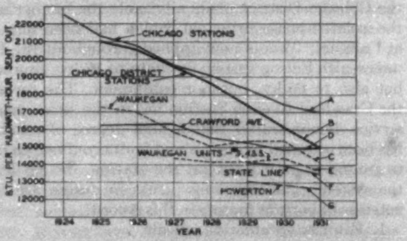

Fig. 10—Yearly Averages of Heat Rate—Chicago District

In the case of Waukegan, also, a loss of both lines to Chicago would be serious. It is therefore the practise usually to operate those lines separated, with one or more generators on each 132-kv. line to Chicago, and Powerton supply connected to only one section.

Reliability is further insured by the 132-kv. connection to the east tying in with the system of the American Gas & Electric Company. While there is normally little transfer of power between the two systems there is some mutual assistance in case of trouble on either system.

RESERVE CAPACITY

From the above it can be seen that consideration must be given to the possibility of certain line failures that would cause a loss of generating supply to Chicago. It is estimated that the maximum to be expected would occur in case of interruption of one of the lines from Waukegan to Chicago when there would be a loss of not only the machine on that line at Waukegan but also the Red and Green feeds coming through Waukegan from Powerton. This would be a maximum of about 90 mw. It is to be noted that this is little different from the loss of the largest unit within Chicago itself. Arrangements are made to have this amount of reserve capacity running on the stations of the Chicago District so that the load can be picked up without difficulty. It is apparent therefore that the total reserve is practically independent of the load on the system, but is dictated more by the size of generating units and tie lines. In all recent units the most economical point of operation is about three-quarters load, so that a reserve is available without loss of efficiency. This is shown in Table IV.

TABLE IV—RUNNING RESERVE IN CHICAGO DISTRICT IN MEGAWATTS

Adequate Boiler Capacity to Supply Same Must Be Kept Available

Week Days		Sundays
30 mw	Crawford	30 mw.
10 mw	Calumet	10 mw.
15 mw	Fisk	10 mw.
15 mw	Northwest	10 mw.
20 mw	Battery	10 mw.
15 mw	State Line	15 mw.
15 mw	Powerton	15 mw.
10 mw	Waukegan	10 mw.
6 mw	Michigan City	6 mw.
Total . . . 136 mw		116 mw.

On one occasion in which the entire supply from Waukegan and the Powerton Red and Green systems into Chicago was lost, the load was picked up with a momentary drop in frequency of only 4/10 cycle and no interruption in service to customers at any point.

CONCLUSIONS

From the practises outlined above the following general conclusions may be drawn:

1. When several companies are as closely linked as those in the Chicago territory, some working agreement must be established to produce the best overall economy of the system, as well as benefits of that economy to each party.

2. The most efficient types of equipment should be installed as rapidly as they become commercially available.

3. For any power station for a large interconnected system, there is a standard below which sacrificing efficiency produces no improvement in reliability, and reducing fixed charges at the expense of operating efficiency will increase the overall cost of energy. Furthermore, as manufacturing technique improves, this standard is continually advancing.

4. There has been a marked gain in economy by increasing steam pressures and temperatures without reduction in reliability.

5. The first unit or two built for higher pressure than has been previously employed does cost appreciably more per kw., but the third or fourth at the higher pressure costs appreciably no more per kw. than those of the lower pressure. The result then is that the average increase in cost for the four units is usually more than offset by the economy in fuel costs produced thereby. In periods of depression, such as the present, the efficient units are able to produce a far greater percentage of the total output than if the system is loaded to capacity, with a result that the saving in fuel costs more than offsets the increased fixed charges necessary to produce same.

6. Broad generalizations about the cost of power stations, or even of their operating costs, are of little value, because each one is a problem in itself. Both construction and operating costs are very considerably affected by location, which to a large degree determines such important factors as kind, character, and cost of fuel, type of foundations, extent of ground improvements, as well as by characteristics of the electric load, capacity factor, and amount of plant reserve required. The economic balance between the operating costs and fixed charges on the investment must be analyzed separately for each case.

7. It is recognized that interconnections of such different loads as electric railways, large industrial factories, and retail light and power in the same area from the same power supply has produced notable economies from diversity in load characteristics. Interconnection of large areas has produced still further economies from diversity in load.

8. The experience with the 132-kv. cable, which so far has been absolutely reliable from the electric failure standpoint, indicates that 220-kv. cable of ample reliability is feasible, so that in the choice of underground cable, economics rather than reliability largely dictates the choice of voltage.

9. With modern overhead steel tower transmission lines, skilfully constructed with low-resistance grounds, reliable ground wires, and coordinated relay protection, experience has shown that increasing amounts of power can be transmitted into Chicago without decreasing the reliability of supply to the distributing centers.

10. Load schedules for station output should be set up based on increment fuel costs, reliability, and reserve capacity.

11. Totalizing schemes for load dispatching are of considerable assistance in efficient operation of the system.

12. Automatic frequency control is a distinct advantage in conjunction with distant metering in enabling the individual stations to readily adhere closely to predetermined schedules.

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Discussion

I—COMBINED RELIABILITY AND ECONOMY IN OPERATION OF LARGE ELECTRIC SYSTEMS—THE DETROIT EDISON COMPANY

(FUGILL)

II—THE EDISON ELECTRIC ILLUMINATING COMPANY OF BOSTON

(DILLON)

III—PHILADELPHIA ELECTRIC COMPANY SYSTEM

(ANDERSON AND ESTRADA)

IV—CHICAGO DISTRICT

(PERRY AND SMITH)

R. L. Thomas: The paper of Messrs. Anderson and Estrada is of particular interest to anyone connected with a combined hydroelectric and steam-electric generating system and is a valuable contribution to technical literature, particularly with respect to the division of load between a run-of-river hydro plant and steam generating stations.

An interesting and complex phase of load division between steam plants and a run-of-river hydro plant with pondage is the economics of weekly drawdown—*i. e.*, a gradual drawing down of the pond say from Monday morning until Friday evening or Saturday, with a refilling over the week end. It is possible that the authors' omission of discussion of this is due in part to the fact that in the future, Conowingo operation in this respect will depend in some measure upon the method of operating the Holtwood and Safe Harbor developments on the Susquehanna River immediately above Conowingo. The Holtwood plant was first placed in operation in 1910 and now has a capacity of 111,000 kw. The Safe Harbor plant, which is thoroughly described in a series of papers presented at the Baltimore District meeting has an initial capacity of 168,000 kw., and was brought into service during the past few months.

The operation of the Holtwood plant of the Pennsylvania Water & Power Company in the combined Baltimore-Holtwood system has been described in several contributions to the technical societies. In general, the operation has been similar to that described for Conowingo, with the hydro plant carrying the peak loads during low flow, thus giving the Baltimore and Holtwood steam plants a "flat" load, and carrying the base load during periods of ample flow, with the steam plants taking the peaks. The Holtwood pond is too small to make weekly drawdown economical and it has been the practise to have the pond full every morning. However in hypothetical analysis for determining system generating investment programs weekly drawdown has been assumed. Set-ups were made to determine the maximum peak service or demand value obtainable from the hydro plant in case of the very unlikely occurrence of what has been dubbed the "pinch power condition"—*i. e.*, the coincidence of minimum river flow, maximum load and break-down of the largest generator units.

With the Safe Harbor plant in operation above Holtwood the problem has become much more complicated. The combined effect of greater area of pond, higher efficiency of generation and slightly better average head is that one foot of drawdown at Safe Harbor will yield about three times as much energy as one foot at Holtwood. Due to the "lapping" of heads a drawdown of the Holtwood pond is partially recovered in the form of increased head at Safe Harbor. On the other hand the Holtwood tailrace elevation is influenced by the amount of power house discharge to a much greater extent than the Safe Harbor tailwater. There are other practical considerations which affect the division of load between Holtwood and Safe Harbor, such as the fact that Holtwood supplies Baltimore at 25 cycles and Safe Harbor at 60 cycles, the amount of frequency changer capacity available, etc. Exhaustive studies which depend in part upon practical experience and the result of efficiency tests are in progress to determine the proper division of load between the two hydro plants. Investigation is also being made to determine to what extent weekly drawdown of the Safe Harbor pond is economical. Such drawdown, of course, results in a decrease in head and a loss of output at the Safe Harbor plant, but this may be more than offset by savings at the Baltimore and Holtwood steam plants—particularly a decrease in boiler banking losses. A possible method of operation is to give the steam plants a uniform flat load throughout the week, including Saturday and Sunday. It is more likely, however, that the "steam line" should be a horizontal line from Monday to Friday, but lower on Saturday and Sunday. It also appears probable that even on any one day, due to the relative characteristics of the Baltimore steam plants, the steam line should be a stepped line rather than a single horizontal line.

It may be of interest to state that for the seven days June 7th to 13th inclusive the weighted average daily load factor of the Baltimore steam plants was 95.9 per cent. The weekly load factor was 87.0 per cent. The Safe Harbor pond during this period was drawn down about 3½ ft. It is possible, however, that the studies will show that lower steam plant load factors would have been more economical.

It appears probable that the Holtwood pond should not participate in weekly drawdowns but should simply pass along to Conowingo the daily in-flow received from Safe Harbor. It follows that the operation of Safe Harbor plant as to weekly drawdown will influence to a great extent the operation of the plants below Safe Harbor.

The statement in connection with determination of investment programs that "accurate forecasts of the future loads are essential" raises a number of questions which may, under present conditions, border on the realm of speculation but which might well be the subject of engineering discussion and investigation.

Going back to the halcyon days of 1928-1929 no doubt every power company made forecasts of load for several, or even many, years ahead. When the depression came the residential and commercial load at first continued to grow, but the industrial load fell off. It was then the opinion of many operators that with a return of normal business conditions the recovery of industrial load, superimposed on the continuous growth of residential load, might bring about an abnormal rate of increase which might catch some companies unprepared. More recently, however, the residential and commercial load has ceased to grow or, in many cases, has actually decreased. What then may we expect in the future? Is the decrease in residential load due primarily to a practical cessation of new connections and an abnormally large number of cut-offs? To what extent have householders attempted to economize in the use of electricity? When growth in total load is resumed will it be at the previously normal rate, resulting in a shift of the load curve or a postponement of loads, or will the growth be at an accelerated rate tending to cause the load to "catch up" with the former estimates? The writer is inclined to think that, under present conditions, any forecaster who predicts loads for even a year or two in advance with "an error of only 2 or 3 per cent" may consider himself fortunate.

J. B. Baker: These papers well describe present methods of power system planning and operation and deserve a fifth or summary paper analyzing their similarities and differences. Within the last ten to twelve years, the subject of power generation has greatly broadened, so that one can no longer survey it without a comprehension of the relations within a group of stations. The development of interconnection has modified operating practise by bringing in aspects beyond that of the single power station, or a compact group of power stations. It is apparently not however, producing a widely connected geographical power scheme as was forecast by some of its early proponents. Instead the papers tend to show that the main reliance for power supply in any load area will continue to reside in power stations adjacent to the load. The true function of interconnection therefore seems to be of a supplementary nature.

Relatively long distance transmission of large blocks of power would seem from the papers to be limited to instances where hydro power is available at an attractive overall cost, and in the case of the Powerton station, where a low cost of fuel occurs in conjunction with a high fuel economy. The papers however substantiate each other in indicating that, at least in this part of the country, transmission from remote centers of production is wholly supplementary to local sources of supply in volume and operating reliability.

An interesting possibility that is appearing in the design of power stations is the unit plan of generators and boilers. The present papers lead one to anticipate that the adoption of the unit plan will in no way compromise operating reliability. Future papers on this subject will be welcome.

It is significant that the papers show a continued improvement in operating economy during the past two years despite a falling off in system load. It is understood that operating practises have played a large part in this continued improvement.

All of the current ideas regarding design specifications and operating practises to the end of reliability and operating economy, including purchased power commitments, are mentioned in these papers, and the student of the subject will be fully repaid for whatever critical analysis he may make of them.

Carl M. Gilt: The papers illustrate the progress that has been made toward securing continuity of service for each customer and the system as a whole. Equipment has been improved, lessening the frequency at which it fails, and systems and stations are sectionalized so that ordinary failures are limited in their effects. Much of this has been secured at little or no increased cost, and a certain amount of sectionalization has been adopted, partially at least, as a means of reducing costs.

However, there are failures, which fortunately are rare, but nevertheless are within the realm of possibility, which might very well result in the complete shut-down of a generating station. Such failures might be the rupture of a large steam line almost instantly filling the boiler house with steam, or a high pressure boiler feedwater line, or in the electrical end, a serious fire or explosion, possibly involving the control system. The fact that the community is fed from two or three stations may not greatly relieve the situation even though there are adequate ties between the stations. The sudden increase in load on the remaining stations of 100 per cent or even 50 per cent of what they were carrying, would, in many cases, be beyond the capacity of boilers, turbines, and generators to pick up quickly. Such a failure in a densely populated Metropolitan area would be a catastrophe that none of us would care to pass through.

There are two methods of more or less successfully meeting such a situation, after all reasonable precautions have been taken to avoid a complete station outage. The first divides the territory into sections, each fed by its own station, with ties to other stations for economy and ordinary contingencies, but designed with the intent that they shall open in serious emergency so that the failure of one station will not take others with it. The most important parts of the territory may be fed from the several sources and so limited in extent that the failure of one supply will not endanger the others.

The other method makes use of a high-tension network fed from so many sources of such relatively small capacity, that the complete loss of any station will not impose more load on the remaining stations than they can promptly pick up. It is essentially a system of decentralization of power supply with the high-tension network, a pool into which power is poured and from which it is drawn. Obviously, the network must be made up of such relatively small parts and so designed that an extended failure or break-up is inconceivable. While wide physical separation of stations is desirable it may not be practicable, but it is essential that the various "stations" be so completely isolated that any failure in one cannot communicate to another.

At first thought, costs of such a system may appear high; however, for the larger systems, such stations are of the order of magnitude of 500,000 kva. and make use of the most economical sized units available. Such stations frequently tax outlet facilities to the maximum so that larger concentrations of power may actually increase the cost per kw. of station and outlets. With feeders laid out primarily to cover the area served, not radially from each station, but essentially as a loop system, running from station to station, and load tapped off at intermediate points, the addition of new units or stations is accomplished essentially by cutting into an existing moderate voltage distribution system with but little additional line investment. Some difficulties may be experienced in securing proper division of load between lines, but these can be met by carefully distributing load among the feeders. It would appear that such a system would carry through any catastrophe short of one that would also wreck the community along with the electric system.

Philip Sporn: Previous to a year or so ago many power systems were taxing their production and transmission facilities to the limit to keep up with the heavy and increasing load conditions. The problem of design frequently centered about the question as to how soon additional facilities could be brought on the system and too frequently not enough attention was given toward bringing about a proper balance between cost and reliability. During the last year, however, many systems have been confronted with the problem of bringing about operating economies and have had to face the question as to how well the various parts of their system fit together with a view of permitting the maximum operating economies. They have had an opportunity to analyze some of the capital expenditures involved in making these extensions and re-evaluating the wisdom or the lack of wisdom of some elaborate features that were incorporated as part of the design.

From the system planning standpoint such an analysis ought to be particularly fruitful. With the decrease in load experienced on many systems it ought to be possible to check whether the system in the light of present conditions is still flexible and reliable enough to permit maximum utilization of the most efficient equipment—shutting down complete generating stations if necessary, and serving by transmission lines the entire load of areas formerly served direct from generating stations. If the system has been properly laid out, this economy ought to be possible without any appreciable sacrifice in reliability of service. Minor improvements or alterations, such as the speeding up of breaker operation to reduce the time required to clear faults, might be required before embarking on such a generation and transmission program but the main backbone of the system ought to be there to permit such a change in operating conditions. If the system lends itself to such operation, it has been well planned.

In our own case, under present conditions, many sections of territory that under more favorable load conditions were supplied by their own generating plants, are being handled strictly on a transmission basis. In this connection, it ought to be pointed out that whereas an economical limit to transmission of power is a very definite thing when cost of transmission and regulating facilities are taken into account, this limit automatically disappears once these facilities are already in and the question of their fixed charges plays no part in the picture. In other words, with the transmission facilities an established fact, the electrical transmission *versus* say the railroad transportation of coal has a much greater advantage and it is possible to transmit frequently hundreds of miles even at moderate voltages and obtain great economies over the equivalent rail transportation.

In connection with the question of increased economy and reliability, the proper method of attack in each case is to weigh all the conditions and not go after any additional economy unless there are definite indications that the operating savings will justify all the increased capital expenditures. However, it must not be forgotten that excessive cost per kw. of installed capacity and resulting economy need not necessarily be synonymous. It is entirely possible by eliminating, in many cases, the fancy fittings, architectural, mechanical and electrical, which are frequently allowed to go into a plant, and spending some of these savings in the form of high grade economical equipment, to not only get a highly economical plant, but to obtain it also at a very reasonable cost per kw. This is not to be taken as advocating a class of architecture in which ugliness is predominant; but it is worth while to remember that the object of a generating station is to supply kw. and generate kw-hr., and that the utilitarian motive ought to be predominant in all of its planning and designing.

Again, an analysis such as given in some of the papers should be particularly fruitful in determining the actual economies of some of the very elaborate features that were introduced in their construction. For example, in Mr. Dillon's paper it is stated that it has been his company's policy to duplicate auxiliaries to turbines and boilers, safeguarding against all emergencies, and while this has yielded a high cost per kw. of capacity, it has been justified because there has never been a loss of load due to either lack or loss of generating equipment. Of course, this does not mean that the same conditions might not have prevailed even without going to this duplication of facilities. Mr. Dillon states, it is true, that the policy of some other companies, with simpler installations and with a greater number of spare units, is being watched, but I do not see any comment upon the possibility of eliminating *all of these duplicate or spare features entirely* and relying upon the inherent and natural diversity between the various parts of the interconnected system and letting the interconnected system take care of such major (if any) catastrophe to prevent loss of load. Admittedly interconnecting a large group of power companies involves an expenditure of many millions of dollars, yet how can all these expenditures be justified if this splendid machine consisting of the interconnected system is not utilized at the very time when it can pay the most handsome dividends, namely in times of major distress?

On our own system we have had occasions where 100,000 kw. or more have been dumped on the interconnected system and while the system may have groaned slightly, it handled the load in most cases. Further, the giving or the receiving of aid has not been confined to any one particular section of the system, it generally happening that the company that needed aid at one time is the one that made possible the giving of aid to another of the group the very next time.

In connection with the same subject of reliability and cost, Mr. Fugill's comment with regard to the d-c. drive of auxiliaries is of interest. Mr. Fugill states that in his company d-c. drive of all auxiliaries has been justified by records which showed that there is little cost advantage for the a-c. system. This does not check with the great many studies that we have made on the same subject. Of course, if adjustable speed is called for in almost all drives, this may be the case, but here again our studies have shown that in very few applications in a power plant is adjustable speed drive really justified and on that basis the economies are most certainly in favor of the a-c. system. It would be very interesting if Mr. Fugill could furnish some information showing that the d-c. drive is almost as economical as the a-c. drive.

These papers mention some of the various ways in which interconnections are made use of to the fullest extent. In the case of one of our properties, which has a high seasonal summer peak, an arrangement is made annually with one of the interconnecting foreign companies under which our company receives 10,000 kw. standby without any charge during the period of June 15 to September 15, and furnishes a corresponding amount of standby to the other interconnecting company during the period of November 1 to February 1, the period when this other company invariably has its peak. This is cited as a very interesting case in which advantage is taken of seasonal diversity.

Messrs. Anderson and Estrada indicate the economy at the Deepwater Station as compared to the rest of their system. This, station is owned jointly by the Philadelphia Electric Company and the Atlantic City Electric Company, one of the subsidiaries of the American Gas and Electric Company. It may be of interest to give some actual data regarding the results obtained to date at the Deepwater Plant.

The average B.t.u. per kw-hr. output on the feeders for 1930 (the first full year's operation) was 12,874. For 1931 this was 12,050. The average for the first six months of this year is 11,795. The economy at Deepwater is exceptionally high for a plant employing a straight thermal cycle, but the most interesting part is that it has been obtained at a very low cost in investment per kw. Deepwater is a definite proof of the point made before that economy in operating cost and low investment cost are not necessarily contradictory and that their combination is perfectly feasible. It can not be done however by adhering to obsolete practise and by putting a double guard on each safety and protective feature.

A. H. Sweetnam: Loop service to high-tension customer substations is quite generally supplied except where the customer has generating capacity sufficient to carry his load. In such cases a tap from a transmission line is carried into the station.

Transformer ratings are figured on the basis of approximately 0 deg. C. ambient during the winter peak and 25 deg. C. during summer peak.

I. E. Moultrop: In some of these papers the statement is made that construction projects have been carried out in advance of load requirements for reasons of reliability or economy in operation.

We have undertaken construction work in advance of load requirements for reasons of reliability, but it has been confined mainly to the replacement of obsolete and inadequate equipment. We have not in general committed the Company to investments in advance of load requirements for reasons of economy in operation, feeling that the expected returns from such investments are not always fully reflected in the Company's balance sheet. The exception to this, however, is in the case of such items as converting attended substations to the automatic type where definite labor savings can be predicted and are inevitable.

Reference is made in one or two of the papers to the question of tie-line losses. We believe it to be fundamentally necessary, in order to arrive at accurate results in allocating loads to various sources of generation so as to obtain the largest overall economy, to take into consideration the incremental tie-line losses. This applies particularly in the case of interchange contracts. Ampere square meters for adjusting these losses at metering point are available and provide a simple means of correction for losses.

L. L. Perry: Conclusion 2 in the paper on the Chicago District applies to increase in capacity and not to replacement in existing capacity. Up to the present depression most metropolitan districts have had almost yearly to make increase in generating capacity in order to maintain adequate reserve. The less efficient equipment held in reserve, since it seldom runs more than a few hours per year, is usually much cheaper for reserve when fixed charges are included.

Electrical Equipment for Precipitation Service

BY H. SPEIGHT[1] Member, A.I.E.E. and V. G. RYDBERG[2] Non-member

THE application of a strong electrostatic field accompanied by corona discharge to the cleaning of gases was first shown by Hohlfeld in 1824. His observations were extended by Sir Oliver Lodge in 1884 who continued the work and made a practical installation at a lead smelter in Scotland.

Later, this work was extended by Dr. Cottrell in 1905-6, leading to the development of the electrical precipitation process for the separation of solids and liquids from gases.

FIG. 1—INSTALLATION IN NON-FERROUS FIELD

It was found that if the central or smaller electrode in a precipitator is made negative, corona can be obtained at a much lower voltage. Electrons in this field acquire such velocities they break up the gas molecules into positive ions and additional electrons. This augmented supply of electrons moves toward the outer or positive electrode and they are trapped by the solid particles which thereby obtain a negative charge. The charged particles are attracted to the positive pole and impinge upon it with considerable force. Here they release the trapped electrons and cling to the electrode surface by adhesion.

The early applications were cleaning of sulfuric acid mist and the cleaning of sulfur gases in the non-ferrous field to recover the non-ferrous metal particles which were being carried away in the discharge gases from the copper smelters. Later the field of application extended to cement, acid fumes, high temperature gas cleaning, detarring of combustible gases, removal of organic materials and miscellaneous applications.

For the best results in precipitation, uni-directional current is used, and a further fundamental condition is that the discharge electrode must be negative and the collecting electrode positive.

The extension of the field of application requires varying characteristics of power supply to the corona field. The electrical equipment usually consists of transformer (specially designed for this service), mechanical rectifier, motor-generator, rectifier motor, switchboard, and accessories. This apparatus is all specially designed to withstand the unusual and difficult conditions of the precipitation circuit, having been generally successful over a period of years. Fig. 1 illustrates an installation of electrical equipment for precipitation service in the non-ferrous field.

The electrodes for corona discharge consist of fine wires or rods hanging in the precipitator, the collecting electrodes are plates or pipes. The potentials used are between 20,000 and 75,000 volts, the actual voltage depending upon the design of the precipitator and the material to be treated. A rectifier connected to the high-voltage side of a step-up transformer is used to obtain the high-potential uni-directional voltage. In one form of the mechanical rectifier the stator consists of four segments or shoes spaced 90 deg. apart on a circle and mounted on fixed insulated arms. The rotor which is driven by a small induction motor designed to operate at synchronous speed consists of two insulated arms displaced 90 deg., on the ends of which the contact tips are mounted and connected in series.

There are many disadvantages to the mechanical rectifiers which may be summarized as follows:

1. The continuous spark discharges cause oscillating discharge and surges which create additional stresses on the transformer insulation.
2. The wave form is very irregular. Fig. 2 illustrates this.

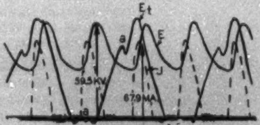

FIG. 2—WAVE FORM OF VOLTAGE MECHANICAL RECTIFIER

3. The contact tips and shoes commence wearing when the rectifier goes into service, continually decreasing the actual voltage at the precipitator. In addition a considerable amount of fixed resistance is necessary to assist in absorbing the energy of the surges caused by the spark discharge.
4. Equipment for suppression of radio interference is necessary.
5. Manual or automatic polarity control must be furnished, adding to the complexity of the control equipment.
6. It is noisy in operation.

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The expansion of the precipitation field has created a growing demand for a rectifier unit that will furnish a more even wave shape and maintain the same precipitator voltage over an extended period of time. With the improvement in the radio art, and the increasing use of rectifiers in power applications, the obvious alternative is tube rectification. By this method relatively smooth wave form of uni-directional current is obtained as illustrated in Fig. 3.

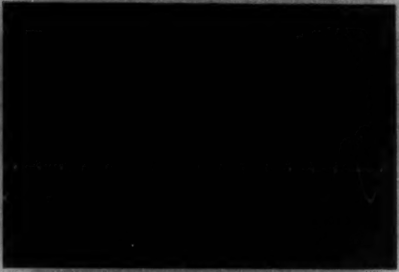

FIG. 3—WAVE FORM OF VOLTAGE TUBE RECTIFIER

The scientific study of the electrostatic field and corona discharge will be greatly assisted by tube rectification, which will permit assembling of data of the variables existing in precipitation applications, due to varying temperatures, conducting and non-conducting properties of the materials being treated.

In many applications the precipitator receives gases containing varying percentages of conducting and non-conducting particles. To meet these varying conditions, we should maintain constant current, which will necessitate raising or lowering the voltage, utilizing to the maximum efficiency the fixed spacing of the electrodes in the precipitator.

The load of the precipitator is very unusual and unique to this application. A negative lead from the rectifier connected to the wire electrodes in the precipitator is shown diagrammatically in Fig. 4. The corona discharge from the negative wire electrode ionizes the gases, thus reducing the normal dielectric insulation, thereby permitting the conveyance of the electron, by the particle, and increasing the normal capacity of the circuit.

The tube type rectifier furnishing uni-directional supply, to this unusual load will have the following advantages:

1. Less space.
2. No surges in the circuit due to rectification.
3. The wave form supplied will be regular.
4. The voltage will remain constant.
5. No radio interference from the rectifying equipment.
6. Polarity is automatically controlled.
7. Operation is very quiet.

The complete equipment for this application consists of transformer (including filament heating), tube rectifier, control, induction voltage regulator or tap changer, and sectionalizing switch for each compartment of the precipitator.

Due to the inherent characteristics of the precipitator the transformers must withstand heavy surges or transient stresses. If special arrangement were not made to avoid this condition these surges started by the flashover of the precipitator would occasion concentrated voltage stresses in the coils adjacent to the line terminal.

The severity of the transient is decreased by a series resistor between the tube and the precipitator, this reduces the voltage crest.

The concentration of voltage stress across the coils at the line end of the transformer windings is also prevented by increasing the effective capacitance between adjacent coils and largely eliminating the capacitance between these coils and the core and low-voltage windings. This is accomplished by designing the transformer so that the coils are located in a relatively uniform dielectric field or by increasing the electrostatic coupling between the coils, and a metallic plate interposed between these coils and the core.

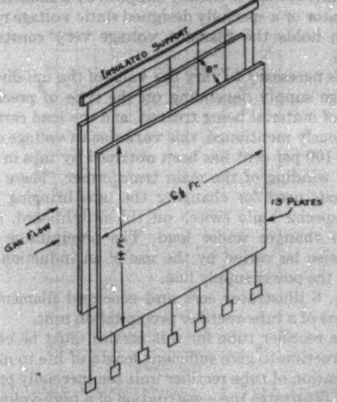

FIG. 4—DIAGRAMMATIC SKETCH OF PLATE TYPE PRECIPITATOR

For several years it was considered the best engineering practise to place a choke coil in series with the transformers, to choke or damp out any high frequency set up by surges in the circuit. Recently it was proved by laboratory tests and measurements that conditions may exist where the choke is the cause of a very high surge voltage being impressed across the transformer coils. It has also been proved that a choke coil shunted by a resistor gives a much greater damping effect than is obtained when using a plain choke coil. However, if both of these are replaced by a series resistance the best damping effect is obtained, and the smallest surges are impressed across the high-voltage terminals of the transformer. Therefore we see that the best protection is obtained by a shielded transformer and a series resistance between the rectifier and the precipitator.

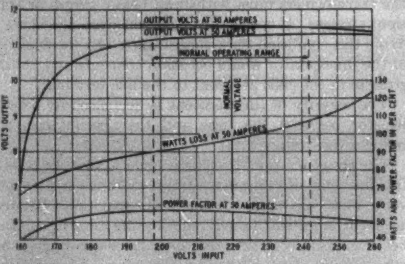

Fig. 5—Characteristic Curve of Static Voltage Regulator

The power supply for tube type rectification must furnish constant voltage for heating the filament. This constant voltage is usually supplied by a small induction regulator or a specially designed static voltage regulator which holds the filament voltage very constant. See Fig. 5.

It is necessary to vary the value of the uni-directional voltage supply depending on the type of precipitator, type of material being treated, and the load carried. As previously mentioned, this variation in voltage of about 50 to 100 per cent has been obtained by taps in the primary winding of the main transformer. There are two methods used for changing the taps bringing all taps to a special knife switch on the switchboard, or using a tap changer under load. The precipitator voltage may also be varied by the use of an induction regulator in the power supply line.

Fig. 6 illustrates core and coils and filament transformers of a tube-rectifier precipitation unit.

The rectifier tube for this service must be of sturdy construction to give sufficient length of life to make the application of tube rectifier unit commercially practical. Fig. 7 illustrates the construction of a high-voltage tube and Figs. 8 and 9 show the plate and filament characteristics. This tube is rated 750 milliamperes maximum peak current, 150 kv. maximum inverse potential.

The thermionic rectifier depends for operation upon the fact that when two electrodes (one which is heated) are placed in a high vacuum, current flows through the space between them, (when the heated electrode is negative). This is because the current is the electron stream emitted from the hot filament.

The peak plate current of a tube is limited by the electron emission from the filament or cathode. The emission in turn is dependent upon the cathode temperature and area. It is possible to operate the filament at less than rated voltage if the peak plate current drawn from the tube is sufficiently reduced. The lower filament temperature will result in slower evaporation of the filament material and consequently longer life.

If the filament voltage is reduced below that which is necessary to supply the peak plate current, the voltage drop between the filament and plate will increase. This will cause excessive heating of the plate and may result in damage to the tube.

The rated maximum crest inverse plate potential is the highest voltage that may safely be applied to the tube in the direction opposite the one in which the tube is designed to pass current. It is a measure of the insulation of the tube when the plate is at a negative potential. In addition to being limited by the distance between anode and cathode this rating is determined by the size and shape of the anode and its position with respect to the cathode. The anode or plate must be large enough to radiate the heat properly due to power losses in the tube. If it is heated too high in temperature there is a possibility of electrons emitting from it, causing an arc back from the cathode to the anode with consequent damage to the cathode. Hence the

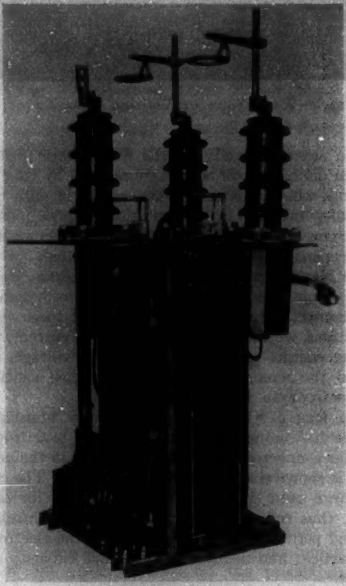

Fig. 6—Transformer for Tube Rectification

important constants of the tube are (a) filament voltage and current, (b) the peak voltage permissible between electrodes, (3) the peak plate current output.

The constancy of filament voltage and current will affect tube life. The emission increases with the temperature, a 2½ per cent increase in the terminal voltage being sufficient to double the evaporation of the filament material, thus the necessity of maintaining a constant filament voltage is emphasized.

The second constant of the rating to be considered is the maximum peak voltage meaning the highest peak voltage the rectifier tube can safely stand, in the direction opposite to the current flow. The value of the inverse potential will depend upon the voltage of the supply circuit and the type of circuit used for rectification.

Fig. 7—High-Voltage Tube—150,000 Volts Maximum, 750 Milliamperes Maximum

The circuits most frequently employed are:

1. Single phase, full wave, employing two tubes. See Fig. 10.
2. Single-phase, full-wave circuit in which four tubes are used.
3. Three-phase, half-wave circuit requiring three tubes.

The theoretical maximum voltage which can be supplied in a single-phase circuit is 1.4×r.m.s. value. However, the inverse potential may exceed this value. In the type of circuit under consideration, where capacity is coupled with a high insulation resistance, the circuit is charged to maximum potential of $1.4 \times E$ during the conducting period. When the tube is passing current this charge may not leak with sufficient rapidity so that a value nearly equal to $2 \times 1.4\ E$ may be obtained across the tube on the inverse half-cycle. Such things must be kept in mind when selecting a tube with the proper peak inverse potential rating for the application.

When using single-phase full-wave circuit with two tubes, (Fig. 11), when the right hand tube is conducting current, the drop across it is comparatively low, hence the effective voltage across the other tube will be $2E$, where E is the r.m.s. value obtained from one-half the transformer winding. The maximum peak voltage will therefore be 2×1.4 or 2.8 r.m.s. voltage from each half of the transformer.

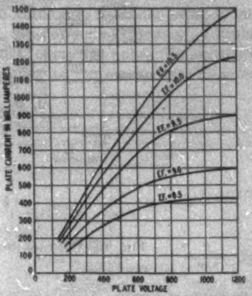

Fig. 8—Plate Characteristics of Tube Illustrated by Fig. 7

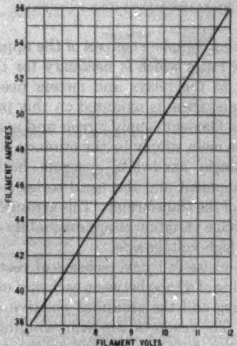

Fig. 9—Filament Characteristics of Tube Illustrated by Fig. 7

The input capacity of the transformer for single-phase full-wave rectification is 1.1 × the kilowatt output on the d-c. side, thus where the d-c. side is 10 kw. the transformer would be 11 kva. on the primary side since this is the measurable input.

A number of installations of tube rectification has been made utilizing various combinations of transformer connection and number of tubes. The description of one installation will be cited as an example.

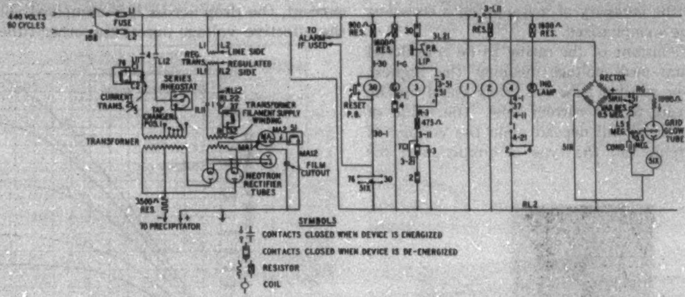
440 VOLTS
60 CYCLES
FUSE
L1
L2
108
4
L1
L12
76
C1
C2
CURRENT TRANS.
SERIES RHEOSTAT
TAP CHANGER POS. 1
TRANSFORMER
REG. TRANS.
LINE SIDE
REGULATED SIDE
IL1
IL2
IL11
RL12
RL22
37
TRANSFORMER FILAMENT SUPPLY WINDING
MA
MA2
MA1
MA12
51
V
FILM CUTOUT
NEOTRON RECTIFIER TUBES
3500Ω RES.
TO PRECIPITATOR
TO ALARM IF USED
900Ω RES.
1800Ω RES.
1-30
1-G
30
RESET P.B.
30-1
76
51X
30
G
4
3L21
P.B.
L1P
3
3-51
51
R-3
475Ω
3-11
TC1
3-21
3
2
3-L11
RES.
1
2
4
4-1
37
4-11
1
4-21
2
4
1800Ω RES.
IND. LAMP
RL2
RECTOX
51R
51R11
51
RG
1000Ω
VAR. RES. 0.5 MEG.
1.5 MEG.
0.5 MEG.
COND.
GRID GLOW TUBE
51X
SYMBOLS
CONTACTS CLOSED WHEN DEVICE IS ENERGIZED
CONTACTS CLOSED WHEN DEVICE IS DE-ENERGIZED
RESISTOR
COIL

Fig. 10—Control Scheme for Equipment Illustrated in Fig. 11

Item	Description	Item	Description
1	Filament supply contactor (2 pole)	51	H.T. D.C. (0.5 to 1.5A) short-circuit protection relay
2	Plate supply timing relay	51X	Aux. relay for 51
3	Master relay	76	Primary overcurrent relay (4-12 amperes)
4	Plate supply contactor (2 pole)	G	Green lamp—indicates "off"
108	Control power switch 2 P.S.T.	R	Red lamp—indicates "on"
30	Lockout relay (1*M*, 1*B*) 440-volt coil	TC1	Interlock on tap changer, closed when on No. 1 position. (Not used)
37	Filament undercurrent relay 2-6 amperes		

Fig. 11 illustrates an installation of commercial type vacuum rectifier. Note the compactness of the unit assembly giving complete control of the apparatus with wide voltage range which is necessary for the application. Added to the relays and meters needed in the control scheme, there is mounted on this panel a contactor for the filament supply circuit and a similar contactor for the main supply to the primary of the transformer. A single handwheel controls both the tap changing equipment and the usual series resistance in the primary of the transformer. This arrangement gives a smooth voltage variation of approximately 50 per cent range on the high-voltage side.

Fig. 11—Illustrates a Typical Installation of Tube Rectifier

The single-phase full-wave scheme of rectification is used, consisting of a tube between each end of the transformer winding and the grounded positive side of the precipitator. The center of the main winding is tapped for the negative high-tension lead to the precipitator circuit.

The control scheme is relatively simple and is self-protective. The main feature is interlocking to insure the filaments are energized a few seconds before the plate of the tube. This prevents flashover which might result, because the full voltage applied is effective in speeding up the electrons before the filament has reached normal temperature. This bombardment results in overheating of the plate. In addition to this interlocking feature the only protection necessary is against short circuits, overloads, and loss of filament.

Fig. 10 illustrates the control scheme. Closing of the knife switch energizes a time-delay relay which will complete the circuit for plate supply contactor designated 4. A relay with coil in the primary of the filament transformer must first be energized otherwise contactor 4 could not be energized. When contactor 4 closes, primary of the transformer is energized and voltage applied to the tube depending upon the setting of the tap changer. Rectification continues until the circuit is disconnected by the pushbutton or operation of protective relays. Relays to protect the tube from damage due to overload or short circuit will be connected in the control and will function in the usual manner. A frequent cause of short circuit and overload is charged to the character of the gas. The acid content of the gas, gas temperature, changes in dust or fume content, have a marked effect on the dielectric characteristics of the materials resulting in a change in load. Since the equipment should require no attendant it is not desirable to take it out of service whenever any momentary overload occurs. Therefore if the circuit is made to reclose automatically a certain number of times, protection is obtained for the tube, if the short circuit is only a momentary condition the equipment is permitted to continue in operation.

This protection is obtained on the installation by means of grid-glow auxiliary relays which consist of a cold cathode grid-glow tube, a small condenser, associated resistors and an auxiliary relay designated as 51-*X*. The grid-glow tube breaks down or discharges only at a predetermined voltage across the condenser. The over-current relay being instantaneous, will immediately close its contacts in case of short circuits, one set of contacts in the grid circuit of the grid-glow relay energizes the condenser circuit putting a certain potential on the condenser. This charge is not sufficient to cause discharge of the grid-glow relay but does not leak off for some time. After a short period the plate supply contactor is closed and if the short circuit is repeated the charge on the condenser is increased. The periodic opening and closing continues until the charge on the condenser reaches a sufficient value to energize the auxiliary relay 51-*X* when the equipment is automatically locked out.

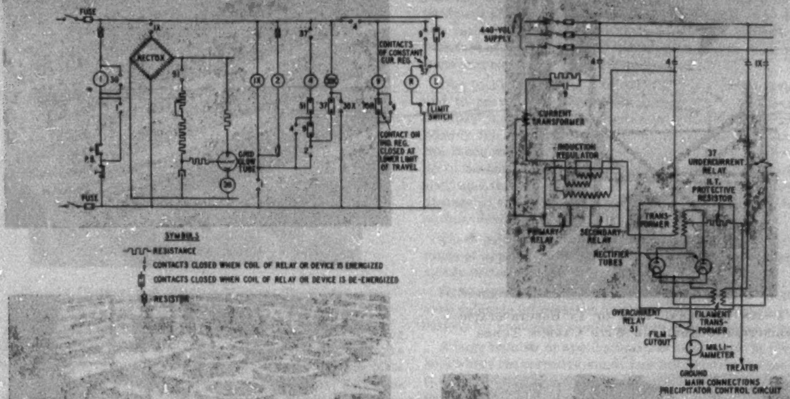
FUSE
1X
RECTOX
1
30
P.B.
51
GRID GLOW TUBE
30
FUSE
1X
2
37
4
30X
51
37
30X
9
4
2
70R
CONTACTS OF CONSTANT CUR. REG.
9
9
57
R
L
LIMIT SWITCH
CONTACT ON IND. REG. CLOSED AT LOWER LIMIT OF TRAVEL
SYMBOLS
RESISTANCE
CONTACTS CLOSED WHEN COIL OF RELAY OR DEVICE IS ENERGIZED
CONTACTS CLOSED WHEN COIL OF RELAY OR DEVICE IS DE-ENERGIZED
RESISTOR
440-VOLT SUPPLY
4
4
1X
9
CURRENT TRANSFORMER
INDUCTION REGULATOR
37 UNDERCURRENT RELAY
H.T. PROTECTIVE RESISTOR
PRIMARY RELAY 57
SECONDARY RELAY
TRANSFORMER
RECTIFIER TUBES
OVERCURRENT RELAY 51
FILAMENT TRANSFORMER
FILM CUTOUT
MILLIAMMETER
GROUND
TREATER
MAIN CONNECTIONS PRECIPITATOR CONTROL CIRCUIT

Fig. 12—Diagram of Control for Full Automatic Equipment

LEGEND

Item	Description	Item	Description
1	Master relay	37	Undercurrent relay
1X	Filament supply contactor	51	Overload relay
2	Timing relay	57	Constant current regulator (primary relay)
4	Main contactor	R	"Raise" coil of secondary relay
9	Control contactor	L	"Lower" coil of secondary relay
30	Lockout relay	70R	Interlock on induction regulator closed at lower limit of travel
30X	Auxiliary lockout relay (hand reset)		

The tube rectifier installation permits consideration of the possibility of an automatic control of the precipitator unit. A proposed scheme is shown in schematic diagram, Fig. 12. This regulating equipment would consist of a constant-current regulating device, responding to the changes in the primary current of the transformer. These changes are caused by changes in the condition of the gas supplied to the precipitator, the object of the automatic regulator being to maintain current of a pre-determined amount, calculated as the most efficient operating value. To accomplish this, the voltage applied to the precipitator must be controlled by a suitable regulating device. This device raises or lowers the voltage to keep the current constant.

The primary relay as controlled by the current of the circuit will control the induction regulator, thereby raising or lowering the voltage as required. The voltage regulation by this arrangement is smooth and gives quick response, thereby maintaining a constant current to the precipitator regardless of the changes in the characteristics of the gases.

Tube installations have given an operating life of over 4,000 hours for the tube. This is a satisfactory commercial life, and in addition to the advantages described above, the vital fact, judging by experience of installations to date, is a higher recovery per kilowatt input obtained than with mechanical rectification.

Discussion

W. C. Kalb: As a supplementary consideration to this valuable paper it may be of interest to consider developments of recent years relating to the adaptation of carbon tubes to electrostatic precipitation units operating on highly corrosive fumes, such as sulfuric, hydrofluoric, and phosphoric acids.

From a chemical standpoint, carbon is very inert in the presence of all reagents except those of high oxidizing power. It is attacked by free oxygen only at temperatures exceeding 600 deg. F. It possesses high electrical conductivity and mechanical strength and does not soften or melt under the influence of heat. These characteristics overcome many of the difficulties that have been experienced with other tube materials used for the precipitation of corrosive fumes or acid mist.

Fig. 1—An External View of an Electrostatic Precipitator Equipped With Carbon Tubes

Fig. 2—Carbon Tubes

Carbon tubes are now extensively used in the precipitation of phosphoric acid in the presence of hydrofluoric acid. They are also used for the collection of strong, hot acid mist in the manufacture of sulfuric acid by the contact process and in the concentration of sulfuric acid in drum type concentrators where the diluted acid is violently agitated by a blast of air at temperatures as high as 1,200 deg. F.

Carbon tubes lend themselves readily to the requirements of precipitator construction for chambers of either circular or square cross section. Two sizes are commonly used; the one 9¾ in. outside diam by 7¾ in. inside diam and the other 13 in. outside diam by 10 in. inside diam. These tubes are usually supplied to the customer in 6-ft. lengths which can be joined together by lap or threaded joints.

The precipitator equipped with carbon tubes shown in Fig. 1 is of rectangular cross section. The carbon tubes themselves are illustrated in Fig. 2. It will be observed that the ends of the tubes are threaded with male and female threads, and that one end of the tube supported by the hoist is recessed for the carbon blocks, which support it in the precipitation chamber. In precipitators of square cross section, the brick arch construction shown in Fig. 3 is commonly used for support of the precipitator tubes. Fig. 4 illustrates the method of supporting the tubes by means of carbon blocks carried on these brick arches. Fig. 5 shows the completed assembly viewed from the upper portion of the chamber.

Fig. 3—Brick Arch Construction

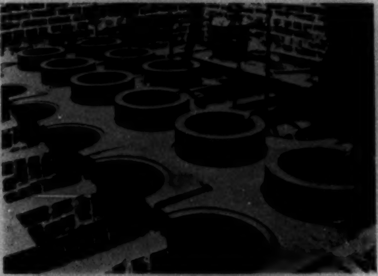

Fig. 4—Brick Arches Carrying Carbon Blocks for Supporting the Tubes

Inasmuch as the brick arch construction is not well adapted to precipitator chambers of circular section, lead covered "I" beams are used as supporting members for the tubes in this type of chamber. Except for this detail, the method of construction is practically the same as in chambers of rectangular cross section. In addition to carbon being used for tubes, carbon blocks are also used as a construction material for anchoring the bottom of the tubes and the lower end of the negative electrode wires.

Carbon tubes have now been used in precipitation service for several years and have conclusively demonstrated that they are cheaper, lighter, and more permanent than acid resistant metal tubes. They are more easily cleaned than lead tubes, do not get out of shape, and will not pit or locally fuse should arcing occur. The high electrical conductivity of carbon gives it a marked advantage over ceramic tubing which is dependent for its electrical conductivity upon a film of acid on its inside surface.

V. G. Rydberg: The design of high-voltage rectifier tubes for precipitation service has been aided by the experience obtained in rectifier tube applications for other fields of activity, such as radio and xray. In radio use the peak inverse voltage requirements of rectifier tubes is comparatively low, being about 50,000 volts maximum. The plate current, however, in the larger vacuum types is quite heavy, being of the order of several amperes, and in the mercury vapor type several times this much.

Fig. 5—Complete Assembly Viewed From Above

In such tubes the problem of insulating against the working voltages does not offer particularly serious difficulties. In the vacuum type rectifiers it is a problem, however, to cool the plate because of the heavy plate current and comparatively high tube voltage drop. In some tubes it is necessary to resort to water cooling to take care of the heat dissipation. In the mercury vapor type, however, the plate dissipation is low because of the low tube voltage drop.

Rectifier tubes used in xray work are designed for a very different type of service. In this application it is necessary to supply direct current at very high voltages while the plate current requirements are comparatively low. Tubes which withstand a peak inverse voltage of 250 kv. may not have to pass more than 100 milliamperes peak current, and are usually in service for very short periods of time at just a very few milliamperes average current.

Tubes for precipitation service, particularly the heavy duty types, must be designed for high peak inverse voltages and also relatively high peak plate currents which are applied continuously twenty-four hours of the day. This means that the plate must be sufficiently large to dissipate the heat losses in it and must also be spaced relative to the cathode and of such conformation as to prevent flash backs due to improper distribution of the electric field within the tube.

The arcing arm or distance between the cathode and anode terminals must also be sufficient to prevent flashover outside the tube. In this connection it is desirable to clean the bulbs periodically in service to remove accumulated deposits of conducting material.

In the WL-612 tube shown, it is noted the plate has curved edges to distribute properly the electric field and prevent concentration at the edges. A shield around the cathode serves partially to concentrate the electron stream and prevent bombardment of the bulb. There is also a shield extending down from this cathode shield over the cathode stem to prevent electrical puncture of the press. A shield over the anode press serves the same purpose for the seal at the other end.

Due to the heavy filament used in this tube to give long life and to provide the peak plate current, it is necessary to use auxiliary filament leads as the standard socket will not handle reliably the 50 amperes of filament current at 10 volts. If high resistance were introduced into the filament circuit so as to cut down the current appreciably, there would not be sufficient emission, and the plate voltage drop would go up with consequent damage to the tube. This contingency is eliminated by the use of these flexible leads.

These points of design provide a line of tubes to cover a wide range of applications in the precipitation field with assurance of steady performance and long life.

H. Speight: Mr. W. C. Kalb has supplemented the paper with very valuable information on the benefits on carbon tube applications for precipitator units operating in corrosive atmospheres.

A large number of applications has been made in the chemical field where the corrosive conditions fully justify the use of carbon tubes or plates and in recent years installations in combined metallurgical and chemical plants have resulted in the electrodes of precipitator units being corroded away within six months.

We believe the carbon tube applications described by Mr. Kalb for the chemical field might be advantageously used in other fields.

Mr. V. G. Rydberg has given some valuable data regarding the design of the tube for the severe precipitation service which design features have a bearing on the successful application of tubes in this field.

We believe that the improved wave form from tube rectification and closer control will result in higher operating efficiencies in precipitation service.

Oxide Coatings on Aluminum

BY J. D. EDWARDS, Non-member MARTIN TOSTERUD, Non-member and H. K. WORK Non-member

THE interest in oxide coatings on aluminum began with the discovery of the metal about 100 years ago, for the chemical attraction of aluminum for oxygen is one of the most important characteristics of the metal. At various times in the history of aluminum, attention has been focused on the oxide film, from the scientific standpoint or from the commercial standpoint. To the scientist, for example, the electrical characteristics of the oxide film have been of great theoretical interest and for many years have been the subject of experimental study and speculation. Important commercial use of the film in various electrical devices has followed upon the scientific studies.

Aluminum owes its stability and resistance to corrosion to the invisible oxide film always present. Aluminum, whenever exposed to air containing moisture, immediately becomes coated with a film of oxide; this film is ordinarily invisible and not more than a relatively few wavelengths of light in thickness. Ordinarily the film is quite impermeable, so that pure aluminum will stand exposure to water with little corrosive action. The presence of this oxide film is quite apparent when one attempts to amalgamate aluminum. Aluminum and mercury combine readily, but when covered by an oxide film the mercury will not wet and hence will not combine with the aluminum. Even if the aluminum is freshly abraded, as by filing, scratching and the like, and then plunged into mercury, no action takes place because the oxide film reforms instantly. If, however, the oxide film is removed by abrasion while immersed in the mercury and out of contact with air, amalgamation takes place.

A very important characteristic of this oxide film is its impermeability. The oxide formed from aluminum occupies a greater volume than the aluminum from which it is formed. This apparently results in a compact and non-porous film with high protective properties. Even on continued heating at high temperatures, the oxide film shows relatively little increase in thickness. Although impermeable to gases, the oxide film as naturally formed has a negligible electrical insulating effect and offers little resistance to the passage of current, even at very low voltages. This is fortunate, and permits the manufacture, for example, of variable air condensers with aluminum plates held in their mountings by mechanical pressure, with no increase in resistance over similar condensers of other metals in which the plates are soldered to the supporting pillars.

The asymmetric character of the film "artificially" produced, when aluminum is made anode in various electrolytes, has been of great scientific and considerable commercial interest. Since the rise of the radio industry, extensive use of film-coated aluminum has been made in the electrolytic rectifier and, more recently, in the electrolytic condenser. High capacity in small volume and at low cost has proved a great stimulus to the development of the electrolytic condenser. Originally marketed in the form of a cell containing electrodes immersed in a substantial volume of electrolyte, the trend for small condensers, at least, is now largely towards the so-called "dry" form, in which a limited volume of electrolyte is held absorbed in a paste kept in contact with the electrodes. In all of these devices, however, the oxide film—at least the active part—is quite thin.

The extensive use of oxide coatings having thicknesses comparable with those of paint films is a relatively recent development. If aluminum is made the anode in boric acid, the potential drop at the anode may be increased to a value of about 400 to 600 volts with very little current passing; the film, once formed, is thin and shows little tendency to decrease or increase in thickness. If, however, an electrolyte of different chemical characteristics is chosen, as for example, a dilute solution of sulfuric or oxalic acid, a characteristically different behavior is noted. Within limits, the oxide film may be built up to a substantial thickness, since the potential may not rise above 10 to 50 volts, and current continues to pass until thick films have been formed, say 0.001 in. or over. Films formed in this latter way, when removed from the electrolyte and dried, have desirable mechanical, chemical, and electrical properties. The film in respect to hardness partakes of the characteristics of corundum, which is a hard, crystalline oxide, Al_2O_3. Because of this hardness, it possesses wear and abrasion-resistant qualities which can be made use of commercially. The dielectric strength of these dry films can be made sufficiently high so that they can be used for purposes of electrical insulation on wire, sheet and foil, for conductors and condensers. In a chemical way, the films can be made with or without high adsorptive properties, so that they can be colored by absorption of dyes or mineral pigments, thus opening up a new and extensive field for the obtaining of decorative effects. These thick oxide films on aluminum can be obtained not only by electrolytic treatment, but also by direct chemical action of various solutions on aluminum. The properties of these thick oxide films will be discussed in succeeding paragraphs.

In the formation of oxide coatings on aluminum by the electrolytic action of current, the oxide formed is produced at the interface between the aluminum and any oxide coat already existing on it. It might be thought, therefore, that the oxide coat could be in-

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creased to any thickness as long as the coating remains sufficiently permeable to permit the passage of current. There is another limiting factor, however, and that is the solvent action of the electrolyte upon the oxide film. Most of the electrolytes which are useful for producing thick oxide coatings have an appreciable solvent capacity for aluminum oxide, usually increasing with rise in temperature, and the anodic coating process becomes a race between the oxidizing action of the current and the solvent action of the electrolyte. The oxide coating may also have its characteristics changed by specific chemical reactions with the electrolyte.

An interesting experiment with an electrolyte of low solvent action gave a series of eight coatings which were determined by direct measurement of microscopic sections to have an average thickness of 0.00038 in. Direct measurement also showed that the sheet had increased in thickness by 0.0002 in. as a result of oxidation on both sides, so that the increase in thickness attributable to a single oxide film is 0.0001 in. The aluminum, therefore, in forming aluminum oxide, increased in volume by about 35 per cent. This is a difficult measurement to make with precision, and values were obtained for the percentage increase in volume by oxidation, which ran from about 15 to 60 per cent. When, however, electrolysis was carried out with a solution of high solvent power for aluminum oxide, the sheet frequently showed a total decrease in thickness many times the thickness of the oxide coating, even though the coating itself was of substantial thickness.

The coatings described in the previous paragraph were subjected to x-ray examination by the powder method and were found to be amorphous—at least no evidence of crystallinity was observed. The water in the coating is therefore probably held by adsorption, and not as one of the crystalline hydrates of aluminum. However, by a special after-treatment the oxide can be converted, at least in part, into crystalline aluminum mono-hydrate. The change is accompanied by a marked decrease in the permeability of the film and an increase in its dielectric and protective properties.

From the standpoint of the electrical engineer, the possibility of using an oxide coat as insulation on aluminum conductors is a matter of considerable interest. The idea, of course, is quite old and some commercial use has been practised, particularly abroad. The present advanced technique and knowledge of oxide coatings now available has created a new interest in these coatings. With certain types of electrical machinery, the use of aluminum conductors would be advantageous from the standpoint of the weight reduction possible. Another advantage of the oxide-coat-insulation is the fact that it is non-combustible and will stand fairly high temperatures without deterioration. There is even a possibility of its use as a dielectric in dry condensers. The break-down voltage of oxide films is roughly proportional to the thickness of the film. The break-down characteristics, however, are also dependent upon the specific physical and chemical characteristics of the film. In Fig. 1 is shown a series of observations on the break-down voltage of oxide films on commercially pure aluminum, plotted as a function of the thickness of the oxide film. This particular film was made by anodic coating of aluminum in a 15 per cent solution of sulfuric acid held at a temperature of 75 deg. fahr. To produce an oxide film one mil in thickness required, under the operating conditions, a period of about one hour. By special after-treatment, without introducing any organic material into the film, such as that referred to in a previous paragraph, the break-down voltage of the film may be increased as much as 50 per cent or more. The oxide coatings can also be produced in very absorbent form, and in this condition can be impregnated with oils, waxes, enamels, etc., with a substantial increase in insulating value.

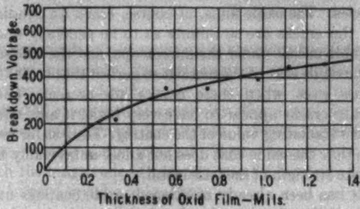

FIG. 1—BREAKDOWN VOLTAGE OF OXIDE FILMS ON COMMERCIALLY PURE ALUMINUM

That the oxide coat is very much harder than the base metal is at once obvious if an attempt is made to scratch the surface. Some quantitative figures have been obtained by subjecting oxide films of various thicknesses to an abrasion test which consists of abrading the surface with an aloxite wheel under light pressure. The abrasion resistance is measured in terms of the number of revolutions which the specimen makes against the abrasive wheel. These figures are given in the following table.

ABRASION RESISTANCE OF OXIDE COATINGS AS A FUNCTION OF THICKNESS

Thickness of coating Inches	Abrasion resistance—revolutions against Aloxite Wheel
0.00011	15 revolutions
0.00019	95 revolutions
0.00031	268 revolutions
0.00037	365 revolutions
0.00074	2.563 revolutions
0.00148	7,061 revolutions

According to these figures, the abrasion resistance of the oxide coating increases very rapidly as the thickness of the film increases beyond about 0.3 to 0.4 mil. For comparison with these figures, reference may be made to the abrasion resistance of enamel and varnish coats such as are applied in finishing metal furniture and office equipment. A typical finish of this type, consisting of primer, two ground coats, graining coat and three coats of finishing varnish, having a total thickness of 3 mils, showed with one sample an abrasion resistance of 306 revolutions, and a somewhat different finish of the same thickness showed an abrasion resistance of 1,095 revolutions. For equivalent thicknesses, therefore, the oxide coat is substantially harder than the baked enamel just mentioned.

Oxide films not over 0.1 to 0.3 mil in thickness, on thin sheet can be readily formed into bottle caps and similar articles. The oxide films, of course, crack on bending, but the cracks are so minute as to be generally undetectable except under the microscope, and the adhesion of the oxide coat to the metal is so perfect that no flaking occurs even when these cracks are present. These minute cracks appear to have very little, if any, effect on the insulating value of the coating. Tests so far indicate that the oxide coat does not affect unfavorably the fatigue strength of the base metal.

As has been already mentioned, oxide coatings may be applied to aluminum by chemical dip methods without the use of current. Such coatings, while usually relatively thin, nevertheless have been used commercially, both in the United States and abroad, for their insulating value. Because of their characteristics, they are only of use when subjected to low voltages; therefore they find their greatest application in motor fields coils and similar applications where the voltage drop between the conductors is low.

Certain types of oxide coatings are very absorbent and can be readily colored by dyes and mineral pigments. These methods of coloring the oxide coatings open up interesting possibilities in the electrical industry. Wires may be colored, for example, to indicate polarity of direct current. Where several wires are in a single conduit, the colors may be used to mark the various circuits. For rapid heat dissipation, busbars and other conductors may be colored black. The bottoms of aluminum utensils for use on electric stoves may be colored black for better heat absorption. The commercialization of these plain and colored "alumilite" finishes in the electrical industry is already under way. Oxide-coated and colored nameplates are finding increasing use. A tremendous number of aluminum flashlight cases in attractive colors have been manufactured. Practically the entire electric refrigerator industry has turned to the use of aluminum refrigerator trays and grids with the plain anodic coating. It is quite evident that these finishes are going to be of increasing value to the electrical industry in the future.

Calculation of Inductance and Current Distribution in Low-Voltage Connections to Electric Furnaces

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Non-member

***Synopsis.**—The special nature of the low-voltage circuit to electric furnaces is emphasized. The conductors employed vary in arrangement and cross section, and are relatively short. The very large currents carried give rise to reactive drops of greater magnitude than is usually supposed. For these reasons a detailed study of the inductance of such circuits is desirable.*

Fundamental theorems for deriving geometrical mean distances are first stated, leading to the well-known formulas for self- and mutual-inductance of circular and rectangular sections.

Practical applications of these formulas to the problem of calculating the approximate reactive drop in three-phase arc furnace circuits are worked up in great detail. The same calculations can be used to determine the approximate operating power factor of the furnace.

Unequal current distribution in multiple conductors, due to proximity and skin effects is discussed.

THE problem of calculating the inductance of electric furnace circuits so as to determine the reactive drop is one of considerable practical importance. Although the connections used are made as short as possible, the very large currents involved cause reactive drops which are very considerable and cannot be neglected by the designing engineer.

From the point of view of the designing engineer the value of such calculations consists in the ability which they give to predetermine the approximate power factor of the circuit. This is of importance because the thermal energy developed by the furnace depends only on the kilowatts delivered to it. Therefore, if the circuit has excessive reactance the power factor will be low, and equipment of greater kilovoltampere capacity is needed than would be necessary if the reactance were held to a reasonable figure. Low power factors also concern the power company when energy is purchased and will often materially increase the cost of power.

From another standpoint a certain amount of reactance is necessary and often purposely added for stabilizing the circuit. In such cases it is important to so design the circuit as to provide the reactance in the leads where possible and thus save the additional expense of reactors.

Problems of inductance of circuits and calculations of line reactance are familiar to engineers concerned with transmission lines. However, the physical conditions surrounding the electric furnace circuit are so diametrically opposite to those of the transmission line, that the same methods and formulas do not apply. In the first place transmission line reactances are figured as so many ohms per mile. In the electric furnace circuit on the other hand reactance is figured in ohms per centimeter or per inch. Again the transmission line is figured as a circuit of uniform cross section, in which the conductors are usually circular in section. Such circuits do not present the problems of variation in cross section and arrangement that are found in the electric furnace circuit, where connections are made up of straps varying in number, size and spacings, and where conductors such as flexible cables are bunched together in a restricted space so that proximity effects must be considered, and current distribution between conductors may become quite unequal. These reasons coupled with the magnitude of currents usually handled, make the furnace circuit a unique one.

FUNDAMENTAL FORMULAS

The formulas for figuring self- and mutual-inductance of parallel rectangular conductors are derived from the formulas for self- and mutual-inductance of straight cylindrical wires. A cylindrical wire may be assumed composed of an infinite number of filamentary wires or "elements." The self-induction of such a wire is defined as the integration of the infinite number of mutual inductances of every element to itself and to every other element of the conductor. Since the logarithm of the distance between the centers of each pair of elements is involved in the calculation of the mutual inductance of each individual pair, we find that all inductance calculations involve the summation of the logarithms of these distances. The expression "geometrical mean distance" is used to designate this summation.

It is thus seen that this quantity termed geometrical

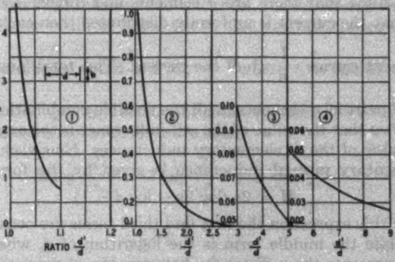

FIG. 1—CURVE SHOWING GEOMETRIC MEAN DISTANCE BETWEEN THIN STRAPS

Based on Formula 132. Bulletin 169, Bureau of Standards

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mean distance or g.m.d., as it is usually written, is the basis of our calculations.

GEOMETRICAL MEAN DISTANCE

This device for figuring self- and mutual-inductance was first introduced by Maxwell and has been used extensively since his time. The definition usually given of the g.m.d. either of a conductor from itself or from another conductor is not clear without some amplification. This definition states that the g.m.d. of a point P from a line is the nth root of the product of the n distances from the point P to the various points on the line, n being increased to an infinite value in determining the value of the g.m.d. To explain this we shall first state, and later prove, the formula for self-inductance of a filamentary cylindrical wire. The formula is

$$M = 2s\left(\log^{*}\frac{2s}{r} - 1\right)$$

where s is the length of the filament and r is the radius of the wire.

In applying this to the problem of determining the self-inductance of thin straight strips we make use of the theorem that the self-inductance of the circuit is equal to the sum of the mutual inductances of every element to itself and to every other element of the circuit. If we assume that there are n elements and further assume that the current is uniformly distributed then each element carries $\frac{1}{n}$th of the current. The total number of pairs of n elements in all the possible combinations is n^2 so that the value of the self-inductance is the average value of the n^2 elementary inductances. Now each elementary mutual inductance is given by the formula

$$M = 2s\,(\log 2s - \log d - 1)$$

in this expression the first and third terms are constant while the middle term is the logarithm of d, where d represents the n distances between the n^2 pairs of elements. If the average value of this logarithm be found, it will obviously enable the self-inductance of the strip to be calculated. As a result the formula for a thin strip will be represented by

$$L = 2s\left(\log\frac{2s}{R} - 1\right)$$

where $\log R$ = average value of the quantity $\log d_1 + \log d_2 + \log d_3 + \ldots \log d_n$ in which $d_1, d_2 \ldots$ represent the n distances between the elements. The average value of this quantity will be

$$\log R = \frac{1}{n}\log\,(d_1 d_2 d_3 \ldots d_n)$$

or

$$R = \sqrt[n]{d_1 d_2 d_3 \ldots dn}$$

So that the reason for the definition of R as the nth root of the product of the n distances between all the pairs of points in the line of length s representing the cross section of a very thin strip is now apparent. This formula

$$L = 2s\log\left(\frac{2s}{R} - 1\right)$$

can readily be established from first principles.

The above illustration has been used to fix the physical meaning of the term g.m.d. so that it will not become an arbitrary number to be determined and applied in the succeeding formulas for self- and mutual-inductance.

METHODS FOR FINDING G.M.D.

Limitations of space prevent the logical theoretical development at this time of the methods for finding g.m.d. This subject is treated in a general way by Gray,[†] but it is necessary to apply his general equations to the particular cases in which we are interested. There are also a few typographical errors in Gray's equations, and a lack of that detail of mathematical development which would assist the reader.

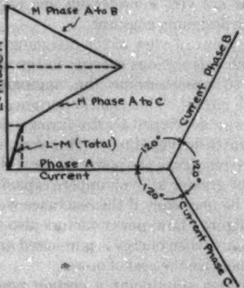

FIG. 2—VECTOR DIAGRAM OF SELF- AND MUTUAL-INDUCTANCES

G.M.D. OF RECTANGULAR SECTIONS

The theorem applying to g.m.d. of two parallel lines from each other is the basis for determining the mutual-induction of thin parallel straps. The cross section of these straps can be represented by the two parallel lines, and their g.m.d. R is given by the formula

$$\log R = \frac{d^2}{b^2}\log d + \frac{1}{2}\left(1 - \frac{d^2}{b^2}\right)\log\,(d^2 + b^2) + 2\,\frac{d}{b}\tan^{-1}\frac{b}{d} - \frac{3}{2} \qquad (1)$$

Where d is the spacing and b the width of the strap. This is equation 132 given in Bulletin 169 of the Bureau of Standards. It is used to determine the g.m.d. be-

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[*] All logarithms throughout the paper are to base ϵ.

[†] See Gray's Absolute Measurement in Electricity Vol. II, Part I.

tween parallel straps which in turn is used to calculate their mutual inductance.

The determination of the g.m.d. of a rectangular area from itself which is used in the formula for self-induction of a rectangular bar, is derived from the general equation for the g.m.d. of two parallel coplanar rectangles from each other. The general equation for this case is a very lengthy one, which reduces, for the special case of a rectangular area to itself, to the formula

$$\log R = \frac{1}{2}\log\left(a^2+b^2\right) - \frac{1}{12}\frac{b^2}{a^2}\log\left(1+\frac{a^2}{b^2}\right) - \frac{1}{12}\frac{a^2}{b^2}\log\left(1+\frac{b^2}{a^2}\right) + \frac{2}{3}\frac{b}{a}\tan^{-1}\frac{a}{b} + \frac{2}{3}\frac{a}{b}\tan^{-1}\frac{b}{a} - \frac{25}{12} \qquad (2)$$

Where a and b are the length and breadth of the rectangle.

However, this equation need not be used in numerical calculations since it has been found that there exists a very simple relation between the g.m.d. and the perimeter of a rectangle.

The table given below is taken from the Bureau of Standards Bulletin 169 and shows that for various ratios as a the length, to b the breadth of the rectangle the g.m.d.

$$R \text{ is very nearly equal to } 0.2235\,(a+b) \qquad (3)$$

This simple formula is used for determining the g.m.d. of a section.

TABLE I—GEOMETRIC MEAN DISTANCES OF RECTANGLES

Ratio	R	$\frac{R}{a+b}$
1.0 :1	0.44705a	0.22353
1.25:1	0.40235a	0.22353
1.5 :1	0.37258a	0.22355
2 :1	0.33540a	0.22360
4 :1	0.27961a	0.22360
10 :1	0.24596a	0.22360
20 :1	0.23463a	0.22346
1 :0	0.22315a	0.22315

Working formulas for rectangular sections are

(1) G.m.d. of rectangular section on itself

$$R = 0.2235\,(a+b) \qquad (3)$$

(2) G.m.d. of rectangular sections from each other

$$\log R = \frac{d^2}{b^2}\log d + \frac{1}{2}\left(1-\frac{d^2}{b^2}\right)\log\left(d^2+b^2\right) + 2\frac{d}{b}\tan^{-1}\frac{b}{d} - \frac{3}{2} \qquad (1)$$

G.M.D. of Circular Conductors

By reference to the theoretical treatment of g.m.d. for circular areas we find the g.m.d. between two circular areas is the physical distance between the centers of the two areas, or $R = d$.

In the case of a circular area from itself

$$\log R = \log r - \frac{1}{4}$$

or

$$R = r\epsilon^{-\frac{1}{4}}$$
$$= 0.779r \qquad (4)$$

Where r is the radius of the conductor.

These two equations are used in figuring the self- and mutual-inductance of cylindrical conductors.

Self-Inductance of Two Parallel Cylindrical Conductors

This is given by the well-known formula

$$L = 2\log\frac{D}{r} + \frac{\mu}{2} \qquad (5)$$

Where L is the self-inductance per cm. length of each wire. The first term of this equation is the inductance due to the external flux while the second represents the inductance due to internal flux of conductor with permeability $= \mu$.

If the conductor instead of being solid were a tube with a wall of infinitesimal thickness, the flux inside the conductor could be neglected so that the inductance of the conductor would be $2\log\frac{D}{r_1}$ where r_1 is radius of the tube with equivalent inductance. Since these two expressions for inductance are equal.

$$2\log\frac{D}{r_1} = 2\log\frac{D}{r} + 2\log\epsilon^{0.25}$$

$$= 2\log\frac{D\left(\epsilon^{0.25}\right)}{r}$$

So that

$$r_1 = 0.779r \qquad (6)$$

This is the expression for the g.m.d. of a circular area from itself. We can thus eliminate consideration of the internal flux by using the g.m.d. of the circular area from itself instead of r the physical radius of the conductor.

The inductance per cm. length is therefore

$$L = \frac{D}{r_1}$$

where r_1 is the g.m.d. of the conductor.

If the return circuit be considered infinitely distant the following equation in terms of length s and radius r will be found more convenient.

$$L = 2s\left(\log\frac{2s}{r} - \frac{3}{4}\right) \qquad (7)$$

MUTUAL-INDUCTANCE OF TWO PARALLEL CYLINDRICAL WIRES

The mutual inductance of two parallel wires of length s, radius r and distance apart d will be the number of lines of force due to unit current in one which cut the other when the current disappears.

In considering the self-inductance of a single wire, we found that the number of lines N outside the wire which will collapse upon the wire when the current stops, is found by integrating between the limits $x = r$ and $x = \infty$. In this case however the flux between the wires is ineffective because it produces effects which neutralize each other, so the integration is made between the limits $x = d$ and $x = \infty$.

Thus

$$M = 2\left[s \frac{\log s + \sqrt{s^2 + d^2}}{d} - \sqrt{s^2 + d^2} + d\right]$$

$$= 2s\left[\log \frac{2s}{d} - 1 + \frac{d}{s}\right] \quad (8)$$

SELF-INDUCTANCE OF A STRAIGHT RECTANGULAR BAR

The self-inductance of a bar with rectangular cross section is the same as the mutual inductance of two parallel straight filaments of the same length separated by a distance equal to the g.m.d. of the cross section of the bar.

$$L = 2s\left(\log \frac{2s}{R} - 1 + \frac{R}{s}\right) \quad (9)$$

Note that in this formula the g.m.d. R replaces the distance d in formula (8). The reason of course is that the physical distance between centers in the case of cylindrical conductors is equal to the g.m.d., whereas with the rectangular sections the distance between centers must be replaced by the g.m.d. of the section to itself. R in this formula is readily obtained from equation (3), which gives

$$R = 0.2235\,(a + b)$$

Where a is the length and b the width of the rectangular section.

MUTUAL-INDUCTANCE OF TWO PARALLEL RECTANGULAR STRAPS

The equation for this case is merely a modification of equation (8) in which the distance d between conductors is replaced by d' the g.m.d. between the rectangular areas.

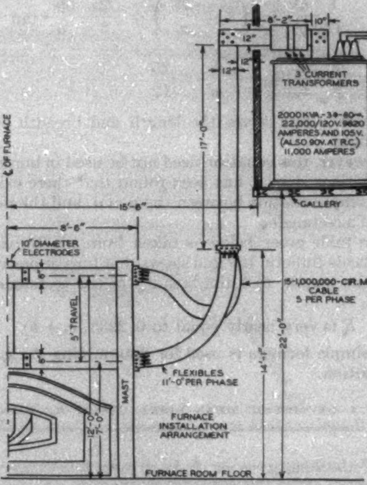
Hence

$$M = 2s\left[\log \frac{2s}{d'} - 1 + \frac{d'}{s}\right] \quad (10)$$

This equation is very frequently used.

MULTIPLE CONDUCTORS

All formulas so far have been concerned with a single conductor either circular or rectangular in section. However, the conductors met with in practise consist of a number of individual conductors connected in parallel. The calculations must therefore be modified to take this into consideration. One method is to find the mutual inductance of each conductor with respect to every other conductor in the multiple conductor and also with respect to itself, and then take the average of these mutual inductances. This is, however, a tedious job, and for most numerical calculations, except when buses are interlaced, it is simpler to replace the multiple conductor by an equivalent single conductor with a thickness equal to the sum of the thickness of the individual straps plus the distance between the straps, provided these distances are small compared to the width of the bar. This method is not used for interlaced connections because it is necessary to find the algebraical sum of the various individual inductances between straps, since adjacent straps carry current in opposite direction.



8'-2"
10"
12"
12"
12"
3 CURRENT TRANSFORMERS
2000 KVA.-3Φ-60~. 22,000/120V. 9620 AMPERES AND 105V. (ALSO 90V. AT R.C.) 11,000 AMPERES
17'-0"
℄ OF FURNACE
GALLERY
15'-6"
8'-6"
10" DIAMETER ELECTRODES
15-1,000,000-CIR. MIL. CABLE 5 PER PHASE
5' TRAVEL
22'-0"
14'-1"
FLEXIBLES 11'-0" PER PHASE
MAST
12'-0"
7'-0"
FURNACE INSTALLATION ARRANGEMENT
FURNACE ROOM FLOOR

FIG. 3—ARRANGEMENT OF 2,000-KVA. FURNACE CIRCUIT

In the following examples inductance has been expressed in terms of centimeters. To be strictly correct this should be centimeter flux lines, since inductance may be defined as the flux linking the total circuit per unit of total current. This value of centimeters flux lines must be multiplied by 10^{-9} to reduce to henrys.

PRACTICAL APPLICATIONS

In order to show how the foregoing principles and formulas may be applied to actual problems two circuits will be worked out in detail. Both of these are for existing installations and actual measurement of the impedance drop will be given as a check on the methods of calculation.

EXAMPLE I

The first furnace circuit has star-connected secondary leads of rather special arrangement. Because of constructional difficulties the furnace transformer was located in a transformer vault about 22 ft. above the furnace floor. The vertical leads from the transformer are interlaced for a distance of 17 ft. The interlacing, however, is not of the usual type, because the star is closed at the transformer thus eliminating the return conductors, and the phases are alternated thus a_1, b_1, c_1, a_2, b_2, c_2, etc., so that the mutual reaction between adjacent straps is between two straps in different phases rather than between two straps in the same phase, carrying current in opposite directions, as would be done in the usual type of interlaced circuit.

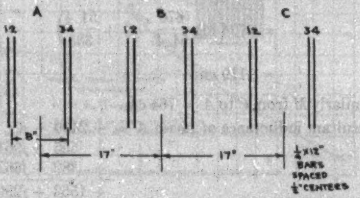


FIG. 4—HORIZONTAL LEADS FROM TRANSFORMER

Fig. 3 shows the general arrangement of the low-voltage bus layout. Since the measured drop was taken from a point close to the transformer terminals to the top of the electrodes, only the following sections will be calculated.

Section 1—Horizontal leads from transformer
Section 2—Interlaced bus
Section 3—Flexibles
Section 4—Copper bus on furnace

Section 1 consists of twelve 12 in. x ¼ in. straps arranged as shown in Fig. 4. To simplify calculation consider the two ¼ in. x 12 in. bars separated on ½ in. centers in each group as equivalent to one conductor ¾ in. x 12 in. This will make each phase consist of two parallel conductors ¾ in. x 12 in.

Self-inductance of either bar is given by formula

$$L = 2s\left(\log\frac{2s}{R} - 1 + \frac{R}{S}\right)$$

Where

S = 6 ft. - 2 in. = 188 cm.

R = g.m.d. of bar is given by the formula $R = 0.2235$ $(a + b) = 0.2235 \times 12.75 = 7.25$ cm.

L therefore $= 376 \times 2.984 = 1{,}120$ cm.

The mutual-inductance of straps at various separations is given by equation (**10**)

$$M = 2s\left(\log\frac{2s}{d'} - 1 + \frac{d'}{s}\right)$$

Where d' is the g.m.d. between the straps and s is the length in cm.

In order to find d' the g.m.d. between the straps readily curve Fig. 1 has been prepared from the formula giving the g.m.d. between rectangular sections. The ratio of $\frac{d'}{d}$ where d is the distance between straps and d' the g.m.d. is given for various values of the ratio $\frac{d}{b}$. By referring to this curve the g.m.d. for various values of $\frac{d}{b}$ can readily be found. Curve Fig. 5 has then been drawn by calculating a few values of M for various separations between bars of this particular section. The mutual inductance for any two straps can then be determined from the curve without further calculation. From Fig. 2 showing the vector relation between the inductances of the phases, we see that we must combine the inductance of phase A vectorially with that of phases B and C in order to get the total inductance.

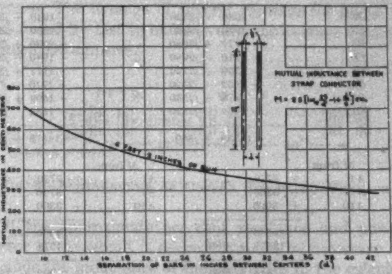


FIG. 5—CURVE SHOWING MUTUAL-INDUCTANCE BETWEEN STRAPS FOR SECTION NO. 1

$$\text{Total inductance} = L_A - \frac{1}{2}\,M_{AB} + j\;.866\;M_{AB}$$

$$- \frac{1}{2}\,M_{AC} - j\;.866\;M_{AC}$$

Lead (12) *A*

Self-inductance	1120 + *j*0	
Mutual from *A* (34) separation 8 in.	707 + *j*0	
Mutual from *B* (12) separation 17 in.		− 253 + *j* 430
Mutual from *B* (34) separation 25 in.		− 205 + *j* 355
Mutual from *C* (12) separation 34 in.		− 171 − *j* 296
Mutual from *C* (34) separation 42 in.		− 150 − *j* 260
	1827	− 779 + *j* 238
	= 1048 + *j* 238	

Leads (3.4) *A*		
Self-inductance	1120 + *j*0	
Mutual from *A* (12) separation 8 in.	707 + *j*0	
Mutual from *B* (12) separation 9 in.		− 338 + *j* 585
Mutual from *B* (34) separation 17 in.		− 253 + *j* 439
Mutual from *C* (12) separation 26 in.		− 200 − *j* 346
Mutual from *C* (34) separation 34 in.		− 171 − *j* 296
	1827	− 962 + *j* 382
		= 865 + *j* 382

The average of these two values = 957 + *j* 310

Since the two groups are connected in parallel

the total for phase *A* $= 478 + j\,155$

Phase *B* by a similar calculation $= 389 + jo$

Phase *C* same as phase *A* $= 478 - j\,155$

Section 2 consists of a 17-ft. run of interlaced connections shown in Fig. 6.

Self-inductance of one bar

$$L = 2S\left(\log\frac{2S}{R} - 1 + \frac{R}{S}\right)$$

$$= 4150 \text{ cm.}$$

Mutual-inductances for various distances between bars are given in curve Fig. 7.

Mutual-inductance of phase *A* (4 parallel-connected bars) to other two phases and to itself, using curve Fig. 7 and tabulating will be

M from	To Bar A_1	To Bar A_4	To Bar A_7	To Bar A_{10}
*B*2	− 2030 + *j*3520	− 1940 + *j*3360	− 1710 + *j*2980	− 1552 + *j*2690
*C*3	− 1940 − *j*3360	− 2030 − *j*3520	− 1777 − *j*3080	− 1599 − *j*2770
*B*5	− 1777 + *j*3080	− 2030 + *j*3520	− 1940 + *j*3360	− 1710 + *j*2980
*C*6	− 1710 − *j*2980	− 1940 − *j*3360	− 2030 − *j*3520	− 1777 − *j*3080
*B*8	− 1599 + *j*2770	− 1777 + *j*3080	− 2030 + *j*3520	− 1940 + *j*3360
*C*9	− 1552 − *j*2690	− 1710 − *j*2980	− 1940 − *j*3360	− 2030 − *j*3520
*B*11	− 1478 + *j*2560	− 1599 + *j*2770	− 1777 + *j*3080	− 2030 + *j*3520
*C*12	− 1452 − *j*2520	− 1552 − *j*2690	− 1710 − *j*2980	− 1940 − *j*3360
	− 13538 + *j*380	− 14578 + *j*180	− 14914 + *j*0	− 14578 − *j*180

M from		*M* from		*M* from		*M* from	
*A*4	+ 2710	*A*1	+ 3710	*A*1	+ 3300	*A*1	+ 3026
*A*7	+ 3300	*A*7	+ 3710	*A*4	+ 3710	*A*4	+ 3300
*A*10	+ 3026	*A*10	+ 3300	*A*10	+ 3710	*A*7	+ 3710
	14186		14870		14870		14186
	− 13538 + *j*380		− 14578 *j*180		− 14914 + *j*0		− 14578 − *j*180
	+ 648 + *j*380		+ 292 + *j*180		− 44 + *j*0		− 392 − *j*180

is given by formula

$$L = 2S\left(\log\frac{2S}{R} - \frac{3}{4}\right)$$

$$R = 3 \times 2.54 = 7.62 \text{ cm.}$$

$$S = 335 \text{ cm.}$$

$$L = 2490 \text{ cm.}$$

Mutual-inductance for circular area is given by equation (8).

$$M = 2s\left(\log\frac{2s}{d} - 1 + \frac{d}{s}\right)$$

Therefore mutual from phase *A* to phase *B*

$$= 670\left(\log\frac{670}{54.4} - 1 + \frac{54.4}{335}\right)$$

$$= 1110 \text{ cm.}$$

Similarly *M* from *C* to *A* = 764 cm.

$$\begin{aligned} \text{Resultant inductance of phase } A = {} & + 2490 + j0 \\ & - 555 + j961 \\ & - 382 + j662 \\ \hline & + 1553 + j299 \end{aligned}$$

Phase *A* average inductance $= \dfrac{\Sigma\ (L + M)}{4}$

$= + 126 + j94$

Phase *C* being symmetrical with *A* $= + 126 - j94$

Phase *B* calculated in same way as *A* $= + 68.5 + j0$

Section 3—Flexible Cables to Furnace.

Total length = 11 ft. = 335 cm.

Each phase consists of fifteen 1,000,000-cm. cables

Separation of flexibles 17 in. centers at start

27 in. centers on furnace

Equivalent separation $= \sqrt{17 \text{x} 27} = 21.4$ in. = 54.4 cm.

L for cables approximately diameter = 6 in.

Phase *C* is symmetrical with phase *A* $= + 1553 - j299$

$$\begin{aligned} \text{Phase } B = {} & + 2490 + j0 \\ & - 555 + j961 \\ & - 555 - j961 \\ \hline & + 1380 + j0 \end{aligned}$$

Section 4—Bus on Furnace. The actual arrangement of these buses is shown in Fig. 8.

In order to make the calculation shorter and less complicated consider the group of 4 conductors of each group with ¼ in. between each strap as equivalent to a single conductor of cross section 6 in. x 1 ¾ in. located on the center line of the group. Each phase will then consist of two groups of conductors each 6 in. x 1 ¾ in. and 5 in. apart.

Length of run = 6 ft. 7 in. = 200 cm.

Self-inductance of A_1

$R = 0.2235\,(6 + 1.75) = 1.73$ in. $= 4.4$ cm.

$$L = 400\left(\log\frac{400}{4.4} - 1 + \frac{4.4}{200}\right)$$

$$= 400 \times 3.532 = 1413 \text{ cm.}$$

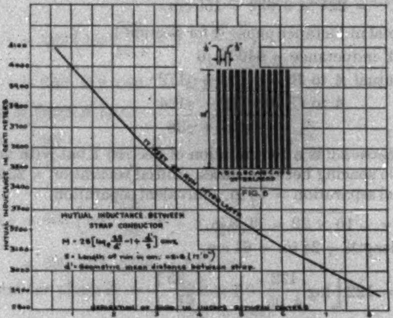


Fig. 6—Arrangement of Vertical Interlaced Bars

Fig. 7—Curve Showing Mutual-Inductance Between Straps for Section No. 2

The self-inductance and mutual-inductances for various separations between bars are tabulated below.

Lead $A1$

Self-inductance	1413 + j0
Mutual from $A2$ separation 5 in.	936 + j0
Mutual from $B1$ separation 22 in.	−247 + j425
Mutual from $B2$ separation 27 in.	−220 + j390
Mutual from $C1$ separation 56 in.	−148 − j256
Mutual from $C2$ separation 61 in.	−145 − j250
Total	1589 + j299

Lead $A2$

Self-inductance	1413 + j0
Mutual from $A1$ separation 5 in.	936 + j0
Mutual from $B1$ separation 17 in.	−265 + j495
Mutual from $B2$ separation 22 in.	−247 + j425
Mutual from $C1$ separation 51 in.	−155 − j268
Mutual from $C2$ separation 56 in.	−148 − j256
Total	1514 − j376

Average inductance of phase A = ½ the average of the two values = 751 + j174.

Phase B could be calculated in the same way. It should be noted that phase C will not in this case be the the same as phase A since the two spacings are not symmetrical.

The total inductance of phase A can now be tabulated.

Section	Inductance in cm.
1	478 + j155
2	126 + j 94
3	1553 + j299
4	751 + j174
Total	2908 + j722

The j component represents an actual energy component.

The inductive reactance $= 2\pi f L \times 10^{-9}$

$= 377 \times 2908 \times 10^{-9}$

$= 0.0011$ ohms

Test results—measured impedance drops on phase A.

Reading	Amperes	Volts
1	7,500	8.5
2	7,700	9
3	7,400	8.5
4	8,000	9.5
Average	7,650	8.9 volts

Calculated reactance—0.0011 ohms.

Drop when carrying 7650 amperes = 8.4 volts.

Resistance drop = 0.2 volts.

Impedance drop = 8.4 volts (approximately).

This compares with the measured drop within reasonable limits of accuracy, taking into consideration the fact that measured impedances when large currents are involved are not precise measurements.

Theoretically phase C should have the same inductance as phase A but since the j components for phase C are opposite in direction to phase A the actual drops will not be the same. A further unbalance exists in this case, since the neutral is not connected to the shell of the furnace. The average of several readings for phase C was found by test to be 7 ½ volts, the average current being 8,000 amperes.

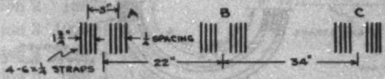


Fig. 8—Arrangement of Copper Straps on Furnace

If the approximate per cent reactance and power factor of the circuit is desired it would be necessary to include the inductance of the electrodes in the calculations. This would bring the total inductance to about 3700 + j789. Low tension voltage 120 volts Y = 69.3 volts to neutral.

Transformer reactance drop = 8.1 per cent = 5.62 volts

Current = 9620 amperes at 2000 kva.

Lead reactance $= 377 \times (3700 + j789)$

$= 0.00139 + j0.000297$ ohms

Total voltage drop at 120 volts = transformer + leads

$= 5.62 + 13.4 + j2.9$

$= 19 + j2.9$ volts

$$\text{Per cent reactance} = \frac{19}{69.3} \times 100 = 27.4 \text{ per cent}$$

Power factor = 96 per cent

j unbalance = 4.2 per cent

Example II

For this example a furnace with delta-connected secondary was chosen, the low-voltage leads being arranged as shown in Fig. 9.

In this example the portion of the low-voltage bus across which impedance drop was measured does not include any interlaced connections.

Section 1—Horizontal run on top of furnace. (Straight portion)

Section 2—Horizontal run on top of furnace. (Curved portion)

Section 3—Vertical run on furnace to point at which flexibles are connected.

Section 4—Flexible connections.

Section 5—Horizontal run from flexible leads to point at which transformer cables are connected.

Section 1—For arrangement of straps see Fig. 10.

$$\text{Length} = 44 \times 2.54 \text{ cm.}$$
$$= 112 \text{ cm.}$$

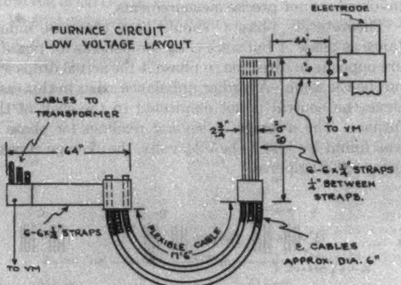


Fig. 9—Arrangement of Furnace Circuit

Self-inductance phase *A*

$$L = 2S\left(\log\frac{2S}{R} - 1 + \frac{R}{S}\right) = 224 \times 2.8553$$
$$= 640 \text{ cm.}$$

$$R = 223\,(6 + 2.75) \times 2.54 \text{ cm.}$$
$$= 4.95 \text{ cm.}$$

For mutual of *A* from *B* we have 22 in. separation

$$\frac{d}{b} = \frac{22}{6} = 3.66 \text{ from curve Fig. 1 } \frac{d'}{d} = 1.02$$

$$d' = d \times 1.02 \times 2.54$$
$$= 57 \text{ cm.}$$

$$M = 224\left(\log\frac{224}{57} - 1 + \frac{57}{112}\right)$$
$$= 224 \times 0.8861 = 198 \text{ cm.}$$

For mutual from *A* to *C* 44 in. separation.

$$\frac{d}{b} = \frac{44}{6} = 7.33 \text{ therefore } d' = 44 \times 2.54$$
$$= 112 \text{ cm.}$$

$$M = 224\left[\log\frac{224}{112} - 1 + \frac{112}{112}\right]$$
$$= 224 \times 0.693 = 155 \text{ cm.}$$

Total inductance phase *A* for Section 1

$$\begin{aligned}
\text{Self-inductance} &= 640 + j0\\
\text{Mutual } A \text{ to } B &= -\ 99 + j172\\
A \text{ to } C &= \underline{-\ 78 - j134}\\
&\quad\ 463 + j38
\end{aligned}$$

Section 2 is a continuation of Section 1 but with variable spacing between buses. Initial separation = 22 in., final separation = 10 in. equivalent separation,

$$= \sqrt{10 \times 22} = 14.8 \text{ in.}$$

length = 24 in. = 61 cm.

$$L = 122\left[\log\frac{122}{4.95} - 1 + \frac{4.95}{61}\right]$$
$$= 122 \times 2.2843$$
$$= 278 \text{ cm.}$$

For *M* from *A* to *B* separation = 14.8 inches

$$\frac{d}{b} = \frac{14.8}{6} = 2.46 \qquad \frac{d'}{d} = 1.04$$

$$d' = 1.04 \times 14.8 \times 2.54 = 39 \text{ cm.}$$

$$M = 122\left[\log\frac{122}{39} - 1 + \frac{39}{61}\right]$$
$$= 122 \times 0.7842 = 96 \text{ cm.}$$

For *M* from *A* to *C* separation = 29.6 in.

$$d' = 29.6 \times 2.54 = 75 \text{ cm.}$$

$$M = 122\left[\log\frac{122}{75} - 1 + \frac{75}{61}\right]$$
$$= 122 \times 0.7186$$
$$= 88 \text{ cm.}$$

Total inductance phase *A* for this section

$$\begin{aligned}
\text{Self-inductance} &= 278 + j0\\
\text{Mutual } A \text{ from } B &\ \ -\ 48 + j83\\
\text{Mutual } A \text{ from } C &\ \ \underline{-\ 44 - j76}\\
&\quad\ 186 + j7
\end{aligned}$$

Section 3—Vertical run on furnace.

Six 6 in. by ¼ in. straps on 10 in. centers

Length S = 105 in. = 256 cm.

$$L = 2S\left[\log\frac{2S}{R} - 1 + \frac{R}{S}\right]$$

$$R = 223 \times (6 + 2.75) \times 2.54$$

$$L = 532 \times 3.6915$$

$$= 1963 \text{ cm.}$$

For mutual of phase A from phase B

$$\frac{d}{b} = \frac{10}{6} = 1.6 \quad \frac{d'}{d} = 1.05 \quad d' = 1.05 \times 10 \times 2.54 = 26.8$$

$$M = 532\left(\log \frac{532}{26.8} - 1 + \frac{26.8}{266}\right)$$

$$= 532 \times 2.0867 = 1110 \text{ cm.}$$

For M from A to C

$$\frac{d}{b} = \frac{20}{6} = 3.33 \quad \frac{d'}{d} = 1.02 \times 20 \times 2.54 = 52$$

$$M_{AC} = 532\left(\log \frac{532}{52} - 1 + \frac{52}{266}\right)$$

$$= 532 \times 1.5254 = 810 \text{ cm.}$$

$$\begin{array}{lr} \text{Total for phase } A = & 1963 + j0 \\ & -\ 555 + j960 \\ & -\ 405 - j700 \\ \hline & 1003 + j700 \end{array}$$

Section 4—Flexibles.

Length—17 in. 6 in. = 210 in. = 534 cm.
Diameter = 6 in. Spacing 14 in.
$R = 7.62$ cm.

$$L = 1068\left[\log \frac{1068}{7.62} - \frac{3}{4}\right]$$

$$= 4475 \text{ cm.}$$

For mutual A from B, spacing is 14 in. or 35.6 cm.

$$M = 1068\left[\log \frac{1068}{35.6} - 1 + \frac{35.6}{534}\right]$$

$$1068 \times 2.4677 = 2640 \text{ cm.}$$

For mutual of A from C spacing = 71 cm.

$$= 1068\left[\log \frac{1068}{71} - 1 + \frac{71}{534}\right]$$

$$= 1068 \times 1.841 = 1960 \text{ cm.}$$

$$\begin{array}{lr} \text{For phase } A \text{ total} = & 4475 + j0 \\ & -\ 1320 + j2280 \\ & -\ 980 - j1700 \\ \hline & 2175 + j580 \end{array}$$

Section 5.

Length = 64 in. = 163 cm.
Spacing between phases = 14 in.
Each phase 6 straps 5 × ¼ with ¼ in. spacing

Equivalent conductor cross section 6 × 2 ¾ in.

$$R = 223 \times (6 + 2.75) \times 2.54 = 4.95 \text{ cm.}$$

$$L = 326\left[\log \frac{326}{4.95} - 1 + \frac{4.95}{163}\right]$$

$$= 326 \times 3.22 = 1050 \text{ cm.}$$

For M from A to B we have $\frac{d}{b} = \frac{14}{6} = 2.33$

$$\frac{d'}{d} = 1.04 \qquad d' = 1.04 \times 14 \times 2.54 \text{ cm.}$$

$$= 37 \text{ cm.}$$

$$M = 326 \times 1.414 = 460 \text{ cm.}$$

For M from A to C we have $\frac{d}{b} = \frac{28}{6} = 4.66$

$$\frac{d'}{d} = 1.015 \qquad d' = 28 \times 1.015 \times 2.54 = 72 \text{ cm.}$$

$$M = 326 \times 0.9681$$

$$= 315 \text{ cm.}$$

Total inductance of phase A for Section 5

$$\begin{array}{lr} \text{Self-induction} = & 1050 + j0 \\ \text{Mutual } A \text{ to } B = & 230 + j400 \\ \text{Mutual } A \text{ to } C = & -158 - j272 \\ \hline & 662 + j128 \end{array}$$

Total-induction for phase A

$$\begin{array}{lr} \text{Section 1} & 463 + j38 \\ \text{Section 2} & 186 + j7 \\ \text{Section 3} & 1003 + j260 \\ \text{Section 4} & 2175 + j580 \\ \text{Section 5} & 662 + j128 \\ \hline & 4489 + j1013 \end{array}$$

$$X_L = 2\pi f L \times 10^{-9}$$

$$= 377 \times 4490 \times 10^{-9} \text{ ohm}$$

$$= 0.00169 \text{ ohm}$$

Drop when carrying 8,400 amperes
= 14.19 volts

Test reading 13.9 volts impedance drop which agrees within reasonable limits of accuracy.

TEST RESULTS

Average of 8 readings on phase A = 4.2 amperes. CT ratio 2000/1

Average secondary current = 8,400 amperes

V^m readings of impedance drop phase A

Average of 8 readings = 13.9 volts

Calculated inductive reactance = 0.00169

Calculated reactive drop—14.19 volts

This agrees with the measured impedance drop within reasonable limits of accuracy.

DISTRIBUTION OF CURRENT IN MULTIPLE CONDUCTOR BUSES

The method of geometric mean distances is strictly accurate only when each section of the area of each individual conductor carries the same current. This assumption is not fulfilled in practise, because of the two phenomena usually called "skin effect" and "proximity effect." Skin effect is the result of the variation in the induced e.m.f. in the conductor due to its own field, and results in greater current density in the outer filaments of the conductor than in the inner filaments. As a result a conductor with a thickness much over ¾ is not economical, and for the same reason hollow conductors are superior to solid conductors. The effect on the current distribution in a conductor due to the currents in adjacent conductors, is called "proximity effect." For example the currents in the return conductor in a single-phase circuit and in the other phases of a three-phase circuit materially affect the current distribution of the conductor. Similarly in a multiple conductor the influence of each strap on the others must be considered. Taking the simple case of a single-phase circuit, the field of conductor B cutting conductor A causes a circulating current to flow down one side of the conductor and back along the other side, and this is superimposed on the main current so that the resultant density on the inner side of the conductor is greater than on the outside. However, when the current in the two conductors is in the same direction we find that the current crowds to the outer sides.

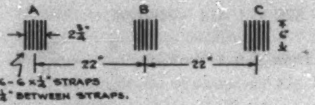


FIG. 10—ARRANGEMENT OF HORIZONTAL STRAPS ON FURNACE

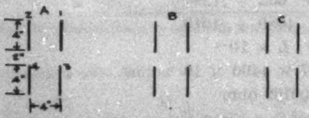


FIG. 11—THREE-PHASE MODIFIED HOLLOW SQUARE BUS

In a three-phase circuit the influence of the other two phases can be calculated and the current distribution of the phases determined. By reference to the vector diagram of the three-phase circuit it is seen that the proximity effect of the other two phases is approximately that due to a return current of half value combined with a quadrature component 0.866 in value. This quadrature component being opposite in one phase to what it is in the other will affect the current distribution accordingly.

When the conductors are built up of several spaced bars in parallel the current distribution is influenced by the proximity effect of the other conductors. For example if four straps are used the inner two straps will carry less current than the outer two when the proximity effects of the straps in that phase alone are considered. However, when the effects of the conductors in the other phases are considered the proximity effect will tend to cause the current to crowd to the side of the bar towards the other phases so that the combined effect of these two tendencies will determine the actual current distribution.

This unequal division of current is very materially affected by the spacing between phases. For instance if the distance between buses is doubled, the inequality between currents I_1 and I_2 is reduced almost in the same proportion so that to get the two conductors to divide current equally they should be well separated. As the spacing is decreased the current unbalance becomes greater, and therefore, the copper is not used economically or else part of the bus tends to overheat. In an effort to overcome this tendency a type of bus known as the modified hollow square bus has been advocated. The arrangement of this bus for four straps per phase is shown in Fig. 11. By putting conductors 1 and 3 vertically above each other so that they are the same distance from the conductors of the other two phases they will have practically no proximity effect on each other, and thus divide current equally. The unbalance will then be between the two pairs of conductors 1 and 3 and 2 and 4. Such a bus will carry the same current with less temperature rise than when the straps are arranged in the conventional manner all in line. Some curves illustrating the division of current with such an arrange-

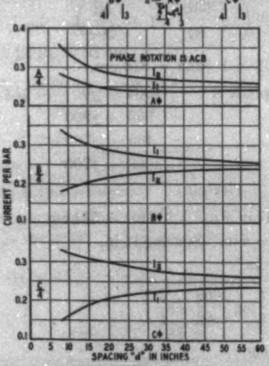


FIG. 12—DISTRIBUTION OF CURRENT IN CONDUCTORS OF HOLLOW SQUARE BUS

ment with varying spacing between phases emphasizes the necessity for separating the phases when very heavy a-c. currents are handled. This is a point that is seldom considered when low-voltage bus layouts are made, but it is of great importance when uninterlaced heavy capacity buses are to be designed. A bus capacity in excess of 6,000 amperes should not have less than 18 in. spacing.

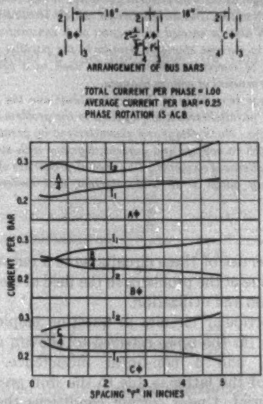


Fig. 13—Variation of Current Distribution with Change in Spacing

Fig. 12 illustrates the way in which the unequal division of current in the two groups of bars of a modified hollow square bus will vary with the spacing d between phases. It brings out very clearly the fact that increasing this distance will reduce the difference between the currents I_1 and I_2 in the two groups of bars.

Fig. 13 illustrates the variation of current distribution in the three phases with change in spacing f between the two groups of bars in each phase.

Conclusion

In the foregoing analysis of furnace circuits an attempt has been made to give the fundamental equations on which the calculations are based and typical calculations have been made. These calculations do not claim to be precise, but are sufficiently accurate for practical purposes.

They will enable the operating engineer to check approximate per cent reactance so that unnecessary reactance can be avoided. They will also assist the designer in laying out logical and reasonable arrangements of copper for carrying heavy currents and thus avoid subsequent criticism for poor arrangement.

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3. "Self- and Mutual-Inductance of Straight Cylindrical Wires," paper by E. B. Rosa, Bureau of Standards, Vol. IV, p. 301, etc.

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5. "The Efficient Utilization of Conductor Material in Bus Bar Sections," C. Dannatt and S. W. Redfearn, *World Power*, Vol. XIV, No. LXXXIII.

6. "Current Carrying Capacity of Bus Bars," by H. W. Papst, *Electrical World*, Sept. 21, 1929.

New Applications of Non-Linear Circuits
To Relay and Control Problems

BY C. G. SUITS*

Non-member

Synopsis.—*The fundamental relations between voltage and current for an iron core reactor are reviewed. The theory of the performance of such reactors in series and parallel resonance circuits may be qualitatively obtained by graphical means. For reactors which have nearly linear properties the graphical method is capable of yielding quantitatively accurate results. A fundamental analogy exists for the properties of series and parallel resonance circuits employing the same inductance and capacitance elements. It is shown that; (a) for the series circuit the current is functionally related to the voltage in the same manner as, for the parallel circuit, the voltage is related to the current, if (b) for the inductance element the current is functionally related to the voltage in the same manner as, for the capacitance element, the voltage is related to the current.*

It is stated that in the series or parallel circuit, means are available for causing the current, as the dependent quantity, to vary as a function of voltage, as the independent quantity, or vice versa. Because of this generalization, the series circuit which is sensitive to voltage changes can also be caused to function as a constant-voltage source, insensitive to current changes. Similarly the parallel circuit, which is sensitive to current changes, can be used as a constant-current source, independent of voltage changes.

The voltage sensitivity of the series circuit and the current sensitivity of the parallel circuit are applied to the problem of voltage and current relays. Such relays are characterized by great sensitivity to small changes in voltage and current, together with sturdy and economical mechanical elements. The special properties of the resonant-current relay adapt it to under-current and under-voltage relay applications.

ELECTRIC circuits comprising inductance, capacitance, and resistance elements in series and parallel resonance networks are fundamentally important to, and widely utilized in, the communication field, but their application to the problems in power transmission and distribution have been relatively limited. These circuits are important for their frequency characteristics, and if the magnitude of the inductance, capacitance, or resistance is independent of the current flowing in the circuit, these frequency-sensitive[1] properties are also independent of voltage or current. Circuits of this type may be said to be *frequency-sensitive, voltage-insensitive,* to distinguish them from another type of circuit about to be described.

This latter type of circuit similarly comprises reactive and resistive elements, but these elements are characterized by a dependence of the magnitude of the inductance, capacitance, or resistance upon current. Circuits employing elements of this general nature, of which an inductance with a closed iron core is a well-known and important example, will be referred to as *non-linear circuits.* This terminology is suggested by their alternating current volt-ampere characteristics which are convenient criteria of the kind and degree of non-linearity. There is a surprisingly large variety of non-linear inductances, resistances and capacitances, and of course the number of possible series and parallel combinations is very great.

It is intended to describe two fundamentally important non-linear circuits, and to show how the unusual properties of these circuits have a natural application to some of the problems in the field of electric power distribution and utilization.

The particular circuits in question employ a linear resistance (in the sense referred to above), a linear capacitance, and a non-linear inductance. The non-linearity of the latter is due to the iron present in the core material. Under certain conditions the flux density B in this core material is related to the magnetizing force H in the manner shown by the curves of Fig. 1, which were taken for a sample of silicon transformer steel. The flux density is thus an involved function of the magnetizing force, and because of this fact accurate analytical expression of the properties of circuits employing saturating reactors is attended with great difficulty. When this reactor is used as a circuit element, it is important to know how the inductance is related to the current. The instantaneous value e of the voltage between the terminals of the reactor is proportional to the rate of change of the flux ϕ through the windings,

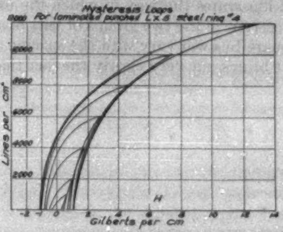

FIG. 1

$$e = K\frac{d\phi}{dt} = K\frac{d\phi}{di}\frac{di}{dt} = K_1\frac{dB}{dH}\frac{di}{dt}$$

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[1] In referring to the properties of circuits, *sensitive* is used here and subsequently to denote the ratio between the percentage change in the *dependent* quantity that follows from a certain percentage change in the *independent* quantity. Thus, a frequency-sensitive circuit is one for which the percentage change in current is large for a small percentage change in frequency.

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where i is the current, t the time, and K, K_1 are factors which depend upon the geometry.

It is assumed that all the flux is concentrated in the core material, that the distribution of flux is uniform, and that the resistance of the winding is negligibly small. These things are seldom true in practical cases. For a linear or constant inductance the coefficient $\dfrac{dB}{dH}$ would be proportional to the inductance; for this iron core reactor,

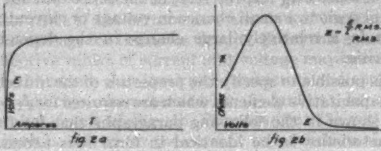

FIG. 2

$$K \frac{d\phi}{di} \left(\text{proportional to} \frac{dB}{dH} \right)$$

may be called the *incremental inductance*,[2] defined as

$$K \frac{d\phi}{di} = K \lim_{\Delta i \to 0} \frac{\Delta \phi}{\Delta i}$$

This quantity is proportional to the slope of the B-H curve of Fig. 1. Thus the incremental inductance of the saturating reactor is also a complicated function, and because the slope changes suddenly at each reversal of current (except at very high flux densities), this function has numerous discontinuities. The maximum and minimum values of the inductive function differ greatly in practical cases, a ratio of many thousand to one being common, while for some high-permeability materials a ratio of the order 40,000:1 is had.

It is interesting to note how this variation of incremental inductance with current influences the shape of the volt-ampere curve of a saturating iron core reactor,[3] taken for example, with a sinusoidal applied voltage. A curve of this kind is shown in Fig. 2a, where the effective current is plotted as a function of the effective applied voltage. The impedance, Z defined as

$$Z = \frac{E_{\text{RMS}}}{I_{\text{RMS}}} \text{ (sinusoidal)},$$

where E_{RMS} and I_{RMS} are respectively the effective value of voltage and current, is shown in Fig. 2b as a function of the sinusoidal applied voltage. For each value of voltage there is an effective inductance (proportional to Z) which is a sort of an averaged value of the incremental inductances throughout a cycle. By referring to the hysteresis loops of Fig. 1, it may be seen that at small magnetizing forces the incremental values of inductance corresponding to the horizontal segment of the loop are important in determining the average, whereas for the loop a the reverse is true; that is, the steep portions of the loop are of predominant importance. For very high magnetizing forces the horizontal, or saturation, values again become marked, resulting in a relatively low effective impedance and inductance. Thus the $Z = f(E)$ curve has a maximum, there being a predominating saturation effect at both very high and very low magnetizing forces.

It should be further stated that that component of the equivalent series resistance of the reactor, which is due to the hysteresis loss of the core, also depends upon current in a similar manner but to a greater degree.

There is much more to be said about the iron core reactor, but from the foregoing it should be clear that for analytical purposes this reactor, considered as a circuit element, is an extremely unobliging device, being nonlinear in all respects.

It is not surprising, therefore, that attempts have been made to approximate the magnetic characteristics (or the derivative thereof) by relatively simple analytic forms,[5,6] varying all the way from two straight lines[2] to hyperbolic functions and series expansions.[4] Most of these methods are accurate over a limited region; the complexity of the physical function precludes the possibility of any one of them being both simple and accurate for any great change in the variables.

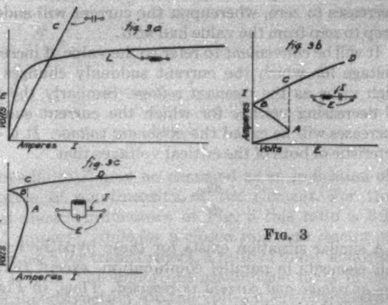

FIG. 3

PROPERTIES OF SATURATING REACTORS IN RESONANCE CIRCUITS

Let us consider a reactor for which the effective current is related to the voltage in the manner shown by the curve L of Fig. 3a, which was taken from test data.

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[2] Boyajian, A., *G. E. Rev.*, vol. 34, 1931, pp. 531 and 745.

[3] See discussion, C. G. Suits, A.I.E.E. TRANSACTIONS, Vol. 50, part 2, 1931.

[4] Peterson, *Bell Sys. Tech. Journal*, Oct. 1928, p. 762.

[5] Dreyfuss, *Arch. f. Electrot.*, 1913, p. 343.

[6] Biermann, *Arch. f. Electrot.*, 1915, p. 345.

For purposes of discussion, however, let us consider this reactor to be greatly idealized so that (a) for sinusoidal applied voltage the current is also sinusoidal, and (b) the power losses are zero. These assumptions are impractical but capable of an approximate experimental demonstration.

Fig. 3a includes the curve *C* which relates the current to the voltage for a capacitance, similarly assumed to be free from losses. Given these two curves of Fig. 3a, and bearing in mind the assumptions involved, one may obtain the volt-ampere characteristic for these circuit elements in series and parallel combination.[7]

In series combination, as shown by Fig. 3b, the current is *identical* in both elements. The difference in the ordinates of the two curves for any value of the abscissa is thus the voltage difference which must be supplied externally. If the current is plotted as a function of this externally applied voltage, the curve of Fig. 3b is obtained. Given these hypothetical circuit elements, the curve which would actually be obtained when they are used in series combination would depend a great deal upon the type of voltage source employed. In particular, it would depend upon the so-called "regulation." For example, if the applied voltage is substantially independent of the current taken by the load, the curve shown by the full line of Fig. 3b could not be obtained, for as soon as the point *A* is reached (for increasing voltage) the current will suddenly increase to the value at *C*. The portion *B-C-D* of the volt-ampere curve will thereafter be traced for any subsequent increase or decrease of voltage, except when the applied voltage decreases to zero, whereupon the current will suddenly drop to zero from the value had at *B*.

It will be convenient to refer to the value of increasing voltage for which the current suddenly changes to a high value as the *resonant voltage*. Similarly the value of decreasing voltage for which the current suddenly decreases will be called the *dissonant voltage*. It is characteristic of both of the critical voltages that

$$\frac{di}{de} = \infty.$$

A similar situation exists for these hypothetical circuit elements in parallel combination, except that the *role of voltage and current is reversed*. Thus, in Fig. 3c, is shown the curve of voltage *E*, as a function of current *I*, similarly obtained from Fig. 3a. For this case the voltage changes suddenly at critical values of current. It will be convenient to refer to these critical values of current, for which the voltage suddenly increases or decreases as the *resonant current* and *dissonant current* respectively, by analogy to the series circuit. It may be seen that in these non-linear resonance circuits there is a similarity between the characteristic of current as a function of the voltage in the series circuit and the voltage as a function of current for the parallel network. That is, if

$$I = f_1(E)$$

is the characteristic of the series circuit, and

$$E = f_2(I)$$

is the characteristic of the parallel circuit, f_1 and f_2 are similar functions.

The series and parallel non-linear circuits discussed above are thus characterized by *voltage sensitivity* and *current sensitivity* respectively, in the sense that for the critical regions a small change in voltage or current will result in a relatively large change in the dependent quantities.

It is possible to specify the properties of the inductive and capacitative elements which are required for $f_1 = f_2$. It is shown in the following paragraphs that for these characteristics to be identical in form it is necessary that the current be related to the voltage in the inductance in the same manner in which the voltage is related to the current in the capacitance. Thus, let i_L, e_L, i_c, and e_c be respectively the currents and voltages in the inductance and capacitance. Then, if

$$i_L = f(e_L)$$

it is required that

$$e_c = f(i_c)$$

where both of the *f*-functions are identical.

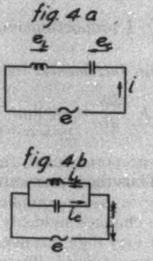

FIG. 4

Thus, consider the inductive and capacitance elements, assumed to be free from power losses, connected in series and parallel combination, as shown in Figs. 4a and 4b. The summation of voltages in the series circuit yields

$$e - e_L + e_c = 0, \qquad (1)$$

where *e*, e_L, and e_c are respectively the instantaneous value of the applied voltage, the inductive voltage, and the capacitance voltage. For the summation of current in the parallel circuit we have:

$$i + i_L - i_c = 0, \qquad (2)$$

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[7] This graphical method was probably first adapted to problems of this nature by Bethenod, *L'Eclairage Electrique*, 1907, p. 289. See also Starke, *Phys. Zeit.*, 18, 1917, p. 6. A comparison of experimental and graphical results is made by H. Gorges, *E.T.Z.*, 39, 1918, p. 101.

where i, i_L, i_c are the instantaneous values of the currents in the branches indicated in the diagram. Let the voltage e_L be related to the current i_L in the inductive element in the following manner:

$$e_L = f_{11}(i_L) \tag{3}$$

where f_{11} may be a linear or a non-linear function. Let the capacitance current i_c be related to the capacitance voltage in the following functional manner:

$$e_c = f_{12}(i_c). \tag{4}$$

Thus (1) becomes

$$e - f_{11}(i) + f_{12}(i) = 0 \tag{5}$$

since $i_c = i_L = i$. A solution of (5) will give the instantaneous value of the current as a function of the instantaneous voltage for the series circuit. From this solution the effective values of current and voltage may presumably be obtained. Let these same inductive and capacitative elements, characterized by (3) and (4), be used in parallel circuit of Fig. 4b. Suppose that (3) and (4) may both be solved for the current as a function of the voltage, and that these solutions are:

$$i_L = f_{21}(e_L) \tag{6}$$

and

$$i_c = f_{22}(e_c) \tag{7}$$

Substitute (6) and (7) in (2) and obtain

$$i + f_{21}(e) - f_{22}(e) = 0 \tag{8}$$

since $e_L = e_c = e$. A solution of (8) will give the instantaneous voltage across the parallel branch as a function of the total current in the series branch, and for this case, as in the above, the effective values of the variables may be obtained. The object is to relate the volt-ampere characteristic of the series circuit to the ampere-volt curve of the parallel circuit, in particular to specify the properties of the functions $f_{11}, f_{12}, f_{21}, f_{22}$ which will cause these characteristics to be identical. Substitute

$$i = e$$

and

$$e = i \tag{9}$$

in equation (5) for the series circuit, and obtain

$$i - f_{11}(e) + f_{12}(e) = 0. \tag{10}$$

Equation (10) will be identical with (8) if

$$f_{11}(\ ) = f_{22}(\ ) \tag{11}$$

and

$$f_{12}(\ ) = f_{21}(\ ). \tag{12}$$

For the *solution* of (10) to be identical with the *solution* of (8) it is sufficient that

$$f_{11}(\ ) = K f_{22}(\ ) \tag{13}$$

and

$$f_{12}(\ ) = K f_{21}(\ ), \tag{14}$$

where K is a constant. If the solutions are identical, the volt-ampere characteristic for the series circuit will be identical to the ampere-volt-characteristic for the parallel circuit. If (13) is true, (14) must be true from the manner of definition. The relation (13) is thus the condition to be satisfied for identical characteristics. From this we may conclude that the voltage of the inductance must depend upon current in the same manner in which the current of the capacitance depends upon voltage.

CORRELATION OF EXPERIMENT AND THEORY

For the purposes of the graphical construction a highly idealized iron core reactor has been assumed. No practical inductance is known which has precisely the properties that have been postulated for the curves of Fig. 3a. It is possible, however, by certain artifices to provide an inductance which for most purposes does have substantially a sinusoidal current for a sinusoidal applied voltage, as well as a non-linear volt-ampere characteristic. This may be accomplished, for example, by the use of an air core (or air gap, iron core) inductance in series with a closed iron core saturating reactor. The maximum effective impedance Z of the saturating element adds to the constant impedance of the linear element to produce a total impedance element which becomes linear for large currents. The departure of the wave form of current from a sinusoid is not marked for any value of current. A volt-ampere curve for a combined inductance of this type is shown as L of Fig. 5. The ratio of the maximum to minimum values of effective impedance Z may be regarded as an indication of the degree of non-linearity of the element. For the "almost-linear" inductance of Fig. 5 this ratio is 3:1 ($i < 0.5$ ampere), while for a closed iron core reactor of the type used for transformers, a ratio of many hundred to one is had. The important assumption made in the graphical construction for the volt-ampere characteristics of the series and parallel circuits (Figs. 3b and 3c) is that the current is sinusoidal for sinusoidal applied voltage. It may as well be admitted that this is not a very good assumption for a closed iron core reactor for which the incremental inductance experiences a 10,000 times change in a single cycle of alternating current. For this "almost-linear" reactor, however, the change in inductance is relatively small and the current distortion is quite negligible. Fig. 6 shows the volt-ampere curve for the series circuit employing this almost-linear reactor. The full line curve was obtained experimentally while the triangular points were made from the graphi-

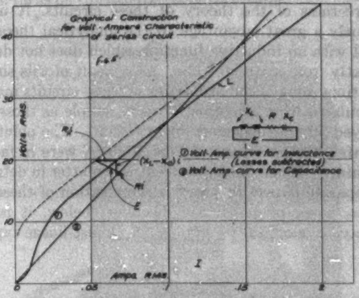

FIG. 5

cal construction of Fig. 5. It will be noted that the circuit resistance (including that component due to hysteresis and eddy current losses) has been added to the difference in reactive component. This detail of the construction follows from reasons that will be readily apparent. An entirely analogous construction may be made for the parallel circuit, with the result shown in Fig. 7. The close coincidence of the experimentally determined characteristics with the points obtained by the graphical construction is cited as proof of the essential correctness of the theory of these circuits. It is obviously a great advantage to the analytical theory to deal with an inductive function which does not depart greatly from a straight line. As a result of this simplification, some of the properties of these circuits may be calculated from algebraic forms as simple as those employed for the analogous linear circuits. The points indicated by the crosses of Figs. 6 and 7 were in fact so obtained. It is intended to treat this matter in detail in a separate paper. In practical application of these circuits, some of which are discussed below, it is not always economical to employ an almost-linear type of reactor, whereas in other cases, particularly for current relays, some important advantages result from their use.

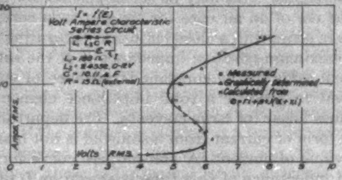

FIG. 6

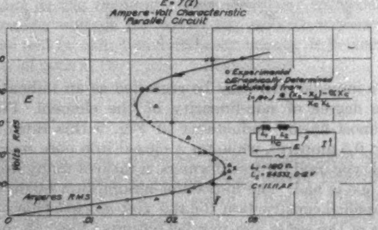

FIG. 7

VOLTAGE AND CURRENT SENSITIVITY

The characteristics of Figs. 6 and 7 are double valued in voltage and current respectively. Thus, when the series circuit is energized from a source of such characteristics that the voltage is substantially independent of the load, the current suddenly increases at the resonance voltage, for increasing voltage, and subsequently decreases suddenly as the voltage is reduced to the dissonance value. By a number of means, the simplest of which requires an increase in the resistance, the resonance and dissonance voltages may be brought together to produce a single valued volt-ampere characteristic. Curves of this type are shown in Figs. 8a and 8b. An almost linear reactor of the type described above was used in these experiments. The similarity of the volt-ampere curve for the series circuit to the ampere-volt curve for the parallel circuit is quite striking and follows fundamentally from the theorem proved above. The capacitor and reactor characteristics are both similar functions of the opposite variables.

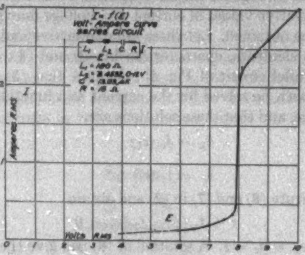

FIG. 8a

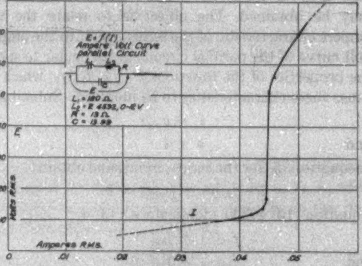

FIG. 8b

One important property of these circuits may be seen from these curves (Figs. 8a and 8b), *i.e.*, the *voltage sensitivity* of the series circuit and the *current sensitivity* of the parallel circuit. Thus, in the critical regions of voltage or current a small change in the independent quantity results in a large change in the dependent quantity.

It is not necessary that the voltage be the independent variable in the series circuit, or that current be the independent variable in the parallel circuit. For example, the series inductance, capacitance, resistance branch may be used in a circuit including a large ballast resistance so that a large fraction of the voltage supplied by the generator appears across this ballast. The voltage across the resonance branch will vary with current in precisely the manner shown by the curve of Fig. 8a. This application of the circuit is thus adapted to producing a *constant voltage*. In a similar manner the voltage may be made the dependent quantity for the parallel circuit, for which case a *constant current* circuit results.

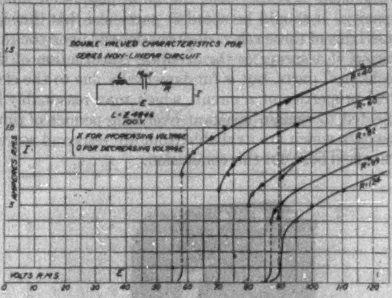

Fig. 9

In summarizing the important properties of these two interesting non-linear circuits it may be stated that the series circuit is adapted to the production of a constant voltage from a variable current, while the parallel circuit is capable of producing a constant current from a changing voltage. The series circuit is further characterized by voltage sensitivity in the sense indicated above, while the parallel circuit is sensitive to current in a like degree. It is the application of these latter two properties, namely, voltage sensitivity and current sensitivity, that will be treated in the subsequent paragraphs.

APPLICATION TO RELAYS

Voltage-sensitive and current-sensitive alternating-current relays are important control elements in a modern electric power system. Their function is usually to determine when voltage control and current control apparatus should operate. They are in many cases required to be as accurate and reproducible in their characteristics as a laboratory voltmeter or ammeter and preferably to cost a fraction as much. The series and parallel non-linear circuits described above have been applied to this problem of sensitive relays.

The fundamental advantages of relays employing resonant circuits follow directly from the large *percentage change in power* at the critical current or voltage. This power may be supplied to a load at an efficiency of the order 30 per cent to 60 per cent in typical cases, at 85 per cent for the best cases. As a result of the large percentage of difference in power which is available, a sensitive relay may be made by controlling a relatively sturdy and economical mechanical element by an electrically accurate circuit. Voltage-sensitive and current-sensitive relays of this general type are treated below.

VOLTAGE RELAYS

The critical change in current that follows from a small change in voltage has been shown in Fig. 8a above. The variation of this characteristic with series resistance R is shown in Fig. 9, which is reproduced from a previous publication.[8] It may be seen that the *resonant voltage* experiences a negligible change when resistance varies, while the *dissonant voltage* changes by a relatively large amount. This property might be inferred from the graphical construction of Fig. 5. It is susceptible to a simple calculation for almost-linear cases. For the present purpose it is sufficient to know that the dissonant voltage varies with R, while the resonant voltage remains substantially constant.

It is found by experiment and may be inferred from the graphical construction that both the resonant voltage and dissonant voltage increase together as the number of turns on the reactor becomes greater.

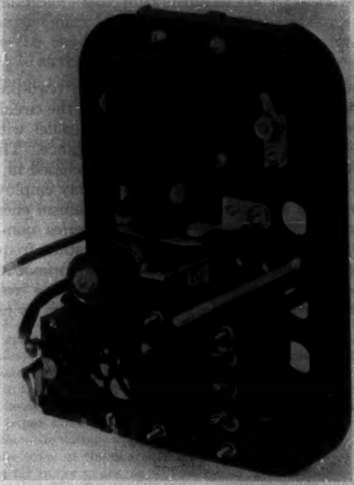

Fig. 10—Resonant Relay Employing Non-Linear Circuit

There are a number of optional means of energizing a contactor mechanism from this non-linear series circuit. In addition to the series current varying with voltage in the manner shown by Fig. 8a, the voltage across the resistance varies in a like manner, as well as the effective voltage of the condenser. The inductive voltage drop, however, varies in a much less critical manner, since in general a large change in current means a smaller change in flux because of the saturation properties of the iron. For purposes of controlling a contactor mechanism, therefore, power may be taken from the circuit by means of a load (a) in series with the circuit; (b) in parallel with the resistance, (c) in parallel with the capacitance, or (d) in parallel with any two elements. Which of these options is chosen in a practical case depends upon the characteristics of the contactor mechanism and its reaction upon the circuit. When this mechanism is a solenoid and armature (two-position, *not* "floating"), the preferred position in the circuit is in parallel with the capacitance or in parallel with the capacitance and a portion of the inductance. The reasons for this preference will not be discussed in detail here. In Fig. 10 is shown a resonant relay employing a solenoid and armature contactor mechanism energized in parallel with the capacitor of a series non-linear circuit. When used on a source of constant frequency this type of relay may be adjusted to a *minimum* difference between "pick-up" (resonant voltage and "release" (dissonant voltage) of ¼ per cent, and may, by change in resistance alone, be adjusted to a *maximum* difference of 30 per cent referred to the pick-up value. The maximum power requirement is 6 watts at 12 volt-amperes. The dependence of the "pick-up" and "release" characteristics of this relay upon the value of the series resistance and upon the number of turns of the reactor are shown by the curves of Fig. 11. In general, for resonant relays of this type, the adjustments of the percentage difference between the resonant and dissonant voltages is made by a change in resistance, while the voltage region in which they both lie is varied by taps on the windings of the reactor. These adjustments are not strictly independent, but they are found to be sufficiently so for small changes. The reactors used in these relays differ in no important respects from small bell-ringing transformers. Standard transformer steels are employed and conventional tolerances are found to be satisfactory. A relay of this kind is much more economical to build than any previously available device of the same accuracy and the same power-controlling capacity. It should be noted in particular that the high sensitivity of these relays does not depend upon the calibration of a spring or the accurate construction of the mechanical parts, but is fundamentally due to the electrical properties of a circuit which may reasonably be expected to remain constant over long periods of time. The data which are available from life tests and service in the field bear out this conclusion.

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[8] *Physics*, Vol. 1, No. 3, p. 171, Sept. 1931, Fig. 11, p. 178.

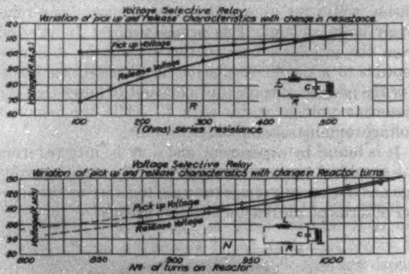

FIG. 11

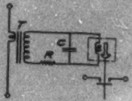

FIG. 12

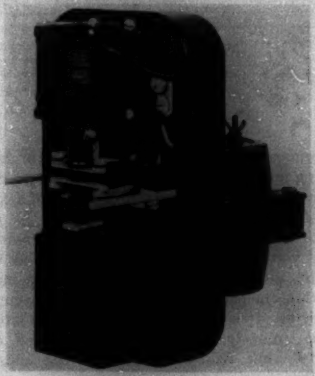

FIG. 13—G-E. RESONANT CURRENT RELAY

The pick-up and release characteristics of this voltage relay are subject to some variation with frequency. The frequency error is of the order 1 per cent in voltage for a 1 per cent change in frequency. Thus, for use on systems where 0.2 per cent is the frequency variation the change in relay characteristics from this cause alone will be a like order of magnitude.

CURRENT RELAYS

A resonant voltage relay may be used as a current relay by energizing the former from the voltage drop produced by passing the current through an impedance element. However, the volt-ampere burden of this impedance must be large compared to the volt-ampere requirements of the relay, so that this method is wasteful of apparatus, power, or both. In precisely the same manner in which the series non-linear circuit may be applied to the problem of voltage relays, the parallel circuit may be used for current relays. The useful property of the circuit for this purpose is shown by the curve of Fig. 8b above, and may be stated to be the great *percentage change* in voltage across the parallel network in response to a small percentage change of current to the parallel branch. This critical change in voltage may be used to energize a contactor mechanism as a load, and this load may be placed (a) in series with the inductance branch, (b) in parallel with the inductance, (c) in parallel with the resistance, or (d) in parallel with the condenser. It may be seen that in general it is required that the voltage across the current relay be a small fraction of the total voltage of the circuit in which it is used. This implies that the microfarad rating of the condenser must be very great for typical cases in which many amperes are to be passed by the relay on 110-volt or 220-volt circuits.

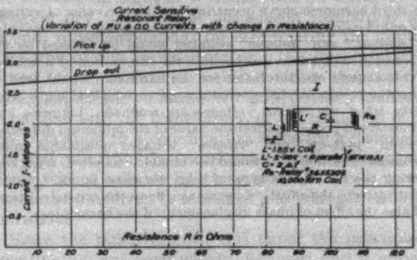

FIG. 14a

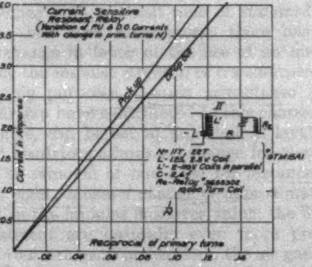

FIG. 14b

In the present state of the art it is uneconomical to build low-voltage, high-microfarad condensers, so that a transformer must be used to perform this transformation. It is feasible, however, to use a winding on the saturating reactor for this purpose, so that no additional equipment is required. The circuit in its practical form is shown in Fig. 12. A relay employing this circuit is shown in Fig. 13. For this particular relay the difference between the resonant current (pick-up) and the dissonant current (release) may be adjusted between limits of 3 per cent and 15 per cent. The resonant current and the dissonant current vary inversely as the number of turns on the transformer *T*. As in the case of the voltage relay previously discussed, the *percentage difference* between the pick-up and release characteristics, and the *magnitude* of these constants, are thus separately adjustable. These properties are shown by the curves of Figs. 14a and 14b. The total power input at the resonant current is of the order 6 watts at 6.5 volt-amperes.

It may be seen from the curve of Fig. 3c that for this parallel non-linear circuit employing a closed core reactor the voltage across the parallel network increases but little for a relatively large increase in current beyond the resonant value. This means that in practise a current relay which is rated at 5 amperes pick-up will stand a continuous current of a great many times this value. The voltage applied to the condenser and to the contactor mechanism never greatly exceeds the value had at the rated resonant current. The single requirement is that the primary winding of the transformer *T* (Fig. 12) must be built for whatever maximum current is anticipated. In the case of the relay of Fig. 13, the current may increase to 8 times the pick-up value before the condenser voltage becomes 1½ times the value had at the resonant current. It is relatively simple to provide this margin of safety in the design of these units, so that the problem of under-current relaying is well met with this type of equipment. By the use of a ballast resistance or reactance in series with the current relay an under-voltage relay is produced. This type of relay is capable of tolerating continuous voltages greatly in excess of the actuating voltage.

Discussion

E. S. Lee: In working with these non-linear circuits one is impressed with the consistency of performance which is entirely unlike that which might be expected to anyone familiar with the residual magnetism phenomena encountered in working with direct current, and the erratic behavior of similar circuits which have already been described in recent papers before the Institute by Messrs. Weller, Boyajian, LaPierre and others. From tests made in the laboratory it has been found that the phenomena of rapid increase and decrease in current shown in Fig. 2 of Dr. Suits' paper will repeat itself consistently within one-tenth of a volt when the voltage applied to the circuit is 100 volts.

While some of the characteristics of these circuits have been known for many years, as will be found from the references in the paper, it is only recently that much progress has been made in applying these properties to practical problems. The introduction of non-linear properties opens up a field in which another variable is put at our disposal in producing new and sometimes unexpected results. In the application described in the paper it results in a large increase in current for a small increase in voltage with the series circuit and large increase in current for a small increase in voltage under the proper conditions in the case of the parallel circuit.

Dr. Suits has not pointed out explicitly an essential difference between the series and parallel circuit which is, however, apparent from the curves he has drawn. This is as follows:

When a sine wave of voltage is applied to the series circuit and the voltage gradually raised the current will follow the volt-ampere curve closely up to the point A, Fig. 3; as the voltage is increased at this point it will be noted that there is no point on this portion of the curve corresponding to higher voltage and we might regard the condition as one of "instability" which results in the current jumping suddenly to the point C as the voltage is increased. With a sine wave of voltage applied to the parallel circuit, however, no such "instability" is encountered as appears evident from the volt-ampere curve. In the case of the parallel circuit, however, by inserting impedance in series with it "unstable" characteristics may be obtained as in the case of the series circuit when a sine wave voltage is applied over all.

The curves shown in the paper refer to the case of an almost linear circuit. In dealing with series circuits which show a large departure from linearity, such as a reactor with closed iron circuit, it has been our experience that the part of the curve up to A, Fig. 3, may be plotted reasonably well by the method given in the paper, *i. e.*, from the volt-ampere curve of the separate elements. The part of the curve from C to D is not so simply obtained. However, by making approximation to allow for the presence of harmonics a fair correspondence with tested values can be obtained. These curves deal only with steady-state conditions and no methods are available for giving the performance under transient conditions. We can also use these curves to show the effect of change of frequency which is to increase the ordinates of the reactor curve and decrease the ordinates of the condenser curve, resulting in a change in the location of the "unstable" portion of the combined volt-ampere characteristic.

From the practical point of view one of the important advantages of these non-linear circuits is that by using them in conjunction with a standard device we can greatly improve its characteristics and reliability within the limits of performance of the non-linear circuit.

A. C. Seletzky: An interesting problem in connection with the non-linear circuit type of relay arises from the presence of harmonic currents or voltages. If we consider, for simplicity the presence of a third harmonic only, the curves shown in Fig. 3 of Dr. Suits' paper undergo the following modifications. The capacity reactance line in Fig. 3a will have one-third the slope of that for the fundamental frequency; the inductive reactance curve will maintain substantially the same shape but its ordinates will be approximately three times greater. The net result is that the intersection of the two curves, which determines the point of resonant or "pick-up" voltage, will occur at a considerably higher value of voltage for the third harmonic than for the fundamental. The resultant volt-ampere characteristic will then be the addition of two curves, such as are shown in Fig. 3b, with the point A for the third harmonic curve occurring at a higher value of voltage. It would be expected, as a result of the foregoing, that the value of "pick-up" and "release" voltages would be affected by the presence of such harmonics because the summation of a number of volt-ampere characteristics for the harmonics would tend to make the resultant characteristic a curve which changes more slowly when the critical voltages are reached. Likewise harmonics would probably affect the value of the minimum difference between "pick-up" and "release" voltages obtainable under sinusoidal conditions. It would be valuable, therefore, when considering the use of this type of relay for close limits, to know quantitatively the effects of harmonics upon the critical voltages and also the effect of phase displacement of the harmonics.

The Coordination of Transformer Insulation with Line Insulation[1]

BY V. M. MONTSINGER[2]
Fellow, A.I.E.E.

and

W. M. DANN[3]
Fellow, A.I.E.E.

At the Toronto Convention of the Institute in June 1930, a paper[4] sponsored by the Transformer Subcommittee of the Electrical Machinery Committee was presented to bring before the membership the status of certain work done by the subcommittee in connection with the coordination of transformer and line insulation.

Two things of outstanding importance were included in this paper: first, a recommended practise of expressing the impulse strength of a transformer in terms of the 60-cycle dry flashover of suspension insulators, and second, recommendations for coordinating the impulse strengths of the transformer and the line insulation. These recommendations included either the use of a safety gap of known flashover characteristics connected in parallel with the transformer, or the reduction of the line insulation to a coordinating value for a distance of one-half mile from the transformer, or a combination of either with a suitable lightning arrester. These recommendations referred to standard line voltages of 69 kv. and above.

Since the publication of this paper the principle of coordination has been widely discussed and it appears to have become generally accepted throughout the industry. While practise has shown that the coordination obtained with one-half mile of standardized line insulation has been satisfactory, there has been a gradual change in favor of the use of an air gap for expressing the impulse strength of transformers and for coordinating purposes. Certain advantages possessed by the air gap have contributed to this change. A gap, for example, can be set at any desired level, while a string of insulators affords no chance for close adjustment. The removal of one disk, for instance, from a string of eight, causes a change of approximately 12½ per cent in the impulse flashover value; the difficulties of adjustment are naturally even more pronounced with shorter strings. Furthermore, the air gap localizes the flashover at a point in the system which can be made convenient for maintenance purposes or for the installation of lightning arresters. Where reduced line insulation corresponding to the original recommendations of the Transformer Subcommittee is being used for one-half mile out from the transformer bank, it is, of course, satisfactory for coordination purposes.

The Transformer Subcommittee has recognized the advantages of the air gap and it is the purpose of this brief progress report to bring to the attention of the Institute membership the following revisions which have been made in the previous recommendations:

1. The yard stick used to express the impulse strength of a transformer has been changed from the 60-cycle flashover value of a given line insulator to a coordinating gap with a given length in inches, as outlined in the following paragraphs and the coordinating principle has been extended to include rated circuit voltages as low as 15 kv.

IMPULSE STRENGTH OF TRANSFORMERS

a. Transformers Receiving Standard Tests as Specified in Paragraph 13-400, A.I.E.E. Standards No. 13

Apparatus conforming with the standards should be so designed that their impulse strength against lightning is greater than the impulse flashover voltage to earth of a coordinating air gap whose spacing[5] is in accordance with Table I.

TABLE I

Rated circuit voltage	Transformer 60-cycle test voltage	Spacing of air-gap
Kv.	Kv. r.m.s.	Inches
15	31	4.25
23	47	6.25
34.5	69	9.25
46	93	12.25
69	139	18.75
92	185	25.00
115	231	31.50
138	277	38.25
161	323	44.50
196	393	54.50
230	461	64.00

b. Transformers Receiving Other Than Standard Test

For transformers receiving a test different from that given for the rated circuit voltage in *a*, the gap spacing

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[1] Sponsored by the Transformer Subcommittee of the Electrical Machinery Committee, V. M. Montsinger, Chairman, W. H. Cooney, W. M. Dann, H. C. Louis, Basil Lanphier, L. C. Nichols, George Vaughan, F. J. Vogel.

[2] Research Engineer, General Elec. Co., Pittsfield, Mass.

[3] Asst. Engg. Manager, Westinghouse Elec. & Mfg. Co., Sharon, Pa.

[4] *Recommendations on Balancing Transformer and Line Insulations*, V. M. Montsinger and W. M. Dann, A.I.E.E. TRANS., Vol. 49, No. 4, 1930, p. 1478.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

[5] Between the ends of two square-cornered, square-cut coaxial rods not more than one-half inch thick up to 69 kv. circuit and not more than three-quarters inch thick for circuits above 69 kv. and so mounted that the rod overhangs its support at least one-half the gap spacing. Also, for circuits above 69 kv. other kinds of air gaps with the recommended spacings such as grading rings on suspension insulators, are satisfactory.

The insulators used as supports should meet the N.E.M.A.-N.E.L.A. 60-cycle flashover requirements for the line voltage.

should correspond to the test which the transformer will receive, in accordance with the relation between test and gap spacing given in *a*.

2. The *Recommendations for Coordinating Transformer Insulation with Line Insulation in the Field*, included in Part II of the Montsinger-Dann paper of June 1930, will be eliminated from the recommendations of the subcommittee. These recommendations involve the three methods of coordinating transformers with line insulation referred to in the opening paragraphs of this report. The coordinating gap referred to in (1) replaces the eliminated recommendations and serves the following purposes:

It establishes the standard of impulse strength required in the design of a transformer.

In practise it limits the magnitude of incoming surge voltages to values less than the impulse strength of the transformer, thereby establishing coordination.

A lightning arrester may be used in conjunction with the coordinating gap to prevent system outages.

Discussion

For discussion of this paper see page 929.

Insulation Coordination of Distribution Transformers

BY E. D. TREANOR* and W. H. COONEY*
Associate, A.I.E.E. Associate, A.I.E.E.

Synopsis.—*In this study of insulation coordination of distribution transformers, the operating conditions are reviewed and the requirements of transformers to be operated without protection are outlined. Test results on transformers designed to meet these requirements are given, showing that by the use of shields for the high-voltage windings and thyrite resistors for the low-voltage windings, transformers can be made in which the internal insulation will be protected by flashover of the bushings. The level of insulation chosen and the amount of system protection desired by the use of lightning arresters will determine the economic value of such transformers.*

GENERAL

Study of Coordination. As an alternative to the use of transformers of present levels of strength with arresters, connected either in the customary manner or interconnected, as suggested at the 1932 A.I.E.E. Winter Convention[6,7] and illustrated in Fig. 6, a study of the idea of coordination of insulation as necessary for the operating conditions of distribution transformers has been carried out and some of the results obtained are reported herein.

During 1924 and 1925, an extensive investigation of transient stress distribution was made on ordinary distribution transformer windings, measurements being made with sphere gaps, and a study of shielded windings was projected. These tests were later checked with the cathode ray oscillograph. In 1929, a number of shielded transformers for transmission line lighting was built and tested, first in the laboratory, and later on a transmission line in Michigan in the summer of 1930. Study of the complete set-up of coordinated distribution transformers involving both mechanical and electrical design has since been carried on, the results on one line forming the basis of this paper.

Operating Conditions. Primary distribution lines are commonly carried on wooden poles with relatively small insulators, the total insulation to ground being uncertain but probably not exceeding 400 kv. impulse. (A rather highly insulated line under actual test would not hold more than 150 kv. impulse.) From the transformer and, frequently, on the same poles run the low-voltage lines terminating in the customer's devices and with the neutral grounded to pipes or, at the least, to other grounds at the transformer or adjacent poles. If of appreciable length, these lines may have an exposure to lightning approaching that of the high-voltage lines. The tank may be grounded, but usually, on the basis of safety to linemen, is isolated. In such cases, the path of an impulse of sufficient magnitude to flash over the insulation of an unprotected transformer is from high-voltage winding to tank to low-voltage winding and thence to ground through the grounded neutral. This means, of course, that lead or major insulation is punctured. That this can and does happen occasionally is attested by records of transformer failures and examination of leads.

Requirements. The requirements in a coordinated distribution transformer, therefore, are that (a) if the tank is grounded, impulses will arc over the bushings of either winding to tank without damage to either bushings or winding, and (b) if the tank is ungrounded, that impulses will flash from one circuit to the tank and thence to the other circuit and ground without damage to windings or bushings. Furthermore, these flashover values should be, at least, of the same order as in present good distribution transformers so that no greater number of flashovers terminating in the secondary neutral will occur than in the case of standard transformers without protection.

The primary winding must be capable of withstanding impulses of any and all magnitudes, polarities, and shapes below the strength of the bushings under the conditions of installation. The secondary winding must be capable, similarly, of withstanding any impulses coming in on the secondary circuit or spilling over abruptly from the high voltage by way of the tank.

If capable of production, such transformers would largely eliminate transformer failures under lightning as a source of trouble. Fuse blowing as a result of flashover would still occur occasionally and arresters would be necessary primarily to prevent outages from this source and from damage to lines and poles. Without them, there would be no limit to the value of transients on lines except the impulse strength of the line structure.

Design. Shields, Windings, and Bushings. Transformers in the 2,400/4,160Y/2,500/4,330Y-volt line are in greatest use and the results presented here are confined to this line. The first step necessarily is to design the high-voltage coil so that its internal insulation, turn to turn, layer to layer, etc., is equal to or higher than the insulation strength to core and tank. A very extensive series of cathode ray oscillograph studies of internal distribution under different wave forms was made. The initial distribution and oscillations determined by the electrical constants resulting from the physical arrangement of the coils as outlined in the literature on transformers were measured. The application of shields, and design of simple windings which improve the initial distribution and minimize oscillations were then studied. It was found possible to obtain the advantages of the principle of shielding in the particular range referred to, that is, 2,400-volt transformers, 50 kva. and less, with the possibility of extending the practise to other lines.

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*General Electric Company, Pittsfield, Mass.

6, 7. For references see Bibliography.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

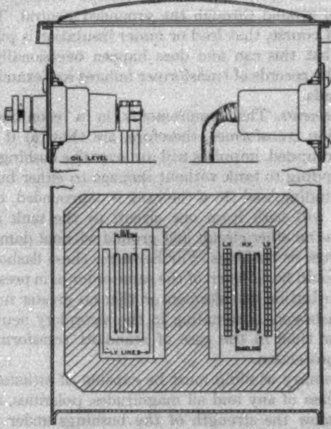

Fig. 1—Coordinated 2,400-Volt Distribution Transformer

Shielded high-voltage winding
High-voltage and low-voltage stud bushings

As this class of transformer is operated either between lines, or from line to grounded or ungrounded neutral, the shielding must be effective for either condition without change. Simple cylindrical shields as illustrated in Fig. 1 and a coil consisting of continuous cylindrical layers were found to provide an inexpensive, simple, and effective arrangement which permits the retention of all normal characteristics and, in addition, gives a practically uniform field under transients so that insulation can be applied for known stresses. This is an adaptation of one of the schemes originated by Weed.[2] The design reduces to a straightforward matter capable of proof and with factors of "experience" or guess work eliminated. Starting with a chosen level of strength, we may design the interior of the coil equal to or higher than the major strength to core and tank and protect both by a bushing which will flash over at a predetermined lower strength.

A high-voltage bushing having the best combination of electrical and mechanical features is necessary. For this service, such a bushing should be sturdy and as little likely to suffer mechanical damage in handling as possible. It should be replaceable, and splash-proof. On the other hand, it must have definite electrical characteristics so that all waves regardless of form or polarity which are dangerous to the winding will arc over externally. A tentative design which seems to meet requirements is illustrated in Fig. 1. The live parts can be taped or insulated to prevent contact by linemen, if this is deemed necessary, without affecting the flashover characteristics.

If flashover occurs with an isolated tank, the low-voltage winding is abruptly subjected to a high stress from the core and tank which must be relieved by a flashover from the tank to the low-voltage bushings, preferably the neutral bushing, which may be made slightly smaller to insure this. The low-voltage bushing, also, must be designed with definite arcover strength adapted to protect the low-voltage windings. Bushings having the desired electrical characteristics are shown in Fig. 1.

The low-voltage windings must be protected from a transient internal distribution standpoint primarily against those waves entering from the low-voltage lines in such a manner as to have an appreciable potential difference between lines or between lines and neutral. Due to the usual arrangement of the low-voltage circuit conductors, potential differences between the conductors of very large magnitude are not to be expected, although all conductors may be raised to quite high potentials. This fairly uniform potential to the core and tank is somewhat similar to that due to an impulse entering from the high-voltage side and the insulation must be coordinated with the bushing strength. For potential differences between secondary conductors, effective protection is offered by small thyrite resistors, without gaps, solidly connected between lines and neutral. This adds a small watt loss in the transformer but may be justified in cases of extensive secondary exposure. Small gaps may be used to eliminate this loss at some expense and reduction of reliability.

It seems desirable to point out in this connection also the benefits of another means of accomplishing internal secondary protection, that is, by the use of capacitors connected between lines and neutral. Such use of capacitors in suitable sizes should provide:

a. Protection, by absorbing and reducing the magnitude of secondary circuit impulses.

b. Reduction of lagging magnetizing current.

c. Reduction of radio interference passing through the transformer from external sources.

d. Reduction of the electromagnetically induced impulse in the secondary resulting from a primary circuit impulse.

Testing. Transformers made up along these lines have shown ability to stand up under apparently unlimited flashovers. This ample strength is believed to be necessary in view of the fact that the ratio of line strength to transformer strength is so high that during the life of a distribution transformer, it may be subjected to a number of waves, approaching or exceeding its strength, which is large compared to the number to which transmission line transformers are subjected. The internal insulation of a small transformer because of the small voltage per turn or layer may be punctured and repetition of the break may recur repeatedly without external evidence. All proof tests were, therefore, made with the transformer excited at full voltage and with full-load current at unity power factor flowing, so that current would not have to be established throughout the test system in order to produce a normal-frequency follow current in case the transient caused a puncture. On this account also, it was desirable to synchronize the impulse with the peak of the normal frequency wave. With all these precautions, it was necessary to tear down and subject to turn by turn examination many coils before there was assurance that no damage was caused in a long series of tests.

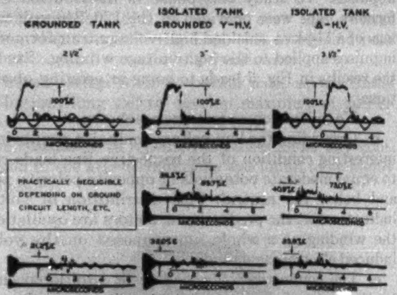

Fig. 2—Effect of Operating Conditions on Magnitude and Division of Voltages. Tests Made on a Coordinated Shielded Transformer Rated H-60-10-2,400-120/240

A —Voltage to cause high-voltage bushing flashover in equivalent rod gap* setting
B —Wave applied to high-voltage winding. Line to ground
C —Tank voltage to ground in per cent of applied voltage to ground
D —Low-voltage line to ground in per cent of applied voltage to ground
Low-voltage neutral grounded in all cases
* ½ diameter squared end rods

TEST RESULTS

Effect of Operating Conditions. The usual practise of isolating the distribution transformer tank leaves no ground potential point but the secondary neutral and by introducing tank capacitance to ground complicates considerably the capacitance relations between component parts of the transformer. The result is that the voltage required to flash over the high-voltage bushings to tank is affected not only by tank-ground capacitance but also by the connection of the high-voltage winding and the extension of the secondary circuit.

In Fig. 2 are summarized tests on a 10-kva., 2,400-volt transformer. Whereas in this particular case an impulse voltage equivalent to a 2½-in. gap flashes over to the grounded tank, isolation of the latter with the high-voltage winding connected from line to line requires a voltage equivalent to a 3½-in. gap. Grounding one high-voltage line reduces the flashover to 3 in. by holding the tank closer to ground potential through high-voltage winding-tank capacitance.

The oscillograms in Fig. 2 picture the sequence of the arcovers from the high-voltage line to tank and from tank to low-voltage circuit. Preceding the discharge over the primary bushing, the tank potential is raised through electrostatic coupling with the high-voltage winding, which the applied wave has entered. As the first flashover occurs abruptly, the tank potential is raised at a rate often exceeding the front of the initial impulse. The transformer core, metallically connected to the tank, rises likewise in potential, stressing the insulation to low-voltage winding, until flashover from the tank to the low-voltage circuit relieves this condition.

The flashover of the high-voltage bushing does not bring the isolated tank up to the full potential of the applied wave due in some degree to the capacity relations to the other windings, to energy losses in the arc, and to the beginning of low-voltage bushing flashover.

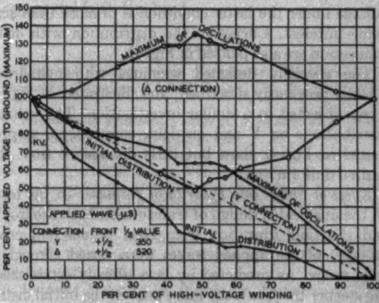

Fig. 3—Study of Transient Voltage Distribution in Unshielded High-Voltage Winding of Transformer Rated H-60-1½-2,400-120/240

In the oscillograms of secondary line potential to ground may be observed the first sudden rise caused by the impact of the incoming wave on the high-voltage winding transferred through the coupling between windings. This voltage quickly decays through the winding to the grounded neutral. The next sudden rise is due to the flashover of the high-voltage bushing and simultaneous rise of tank potential. This voltage is dissipated by the secondary bushing flashover, as well as through the winding to the neutral.

Internal Impulse Voltage Distribution High-Voltage Windings. Tests are presented in Fig. 3 on a type of distribution transformer which has been in extensive use. With the high-voltage winding in Y connection, the initial distribution and maximum of oscillating voltages are not unlike those already published for larger transformers.[1,3,4] Since distribution transformers for circuits below 8,700 volts are designed to operate in either Y or delta connection, the distribution of impulse voltage under the latter condition is of more than academic interest. Curves of the distribution with delta connection are also plotted in Fig. 3.

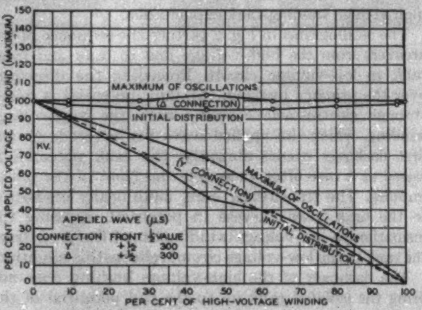

FIG. 4—STUDY OF TRANSIENT VOLTAGE DISTRIBUTION IN SHIELDED HIGH-VOLTAGE WINDING OF TRANSFORMER RATED H-60-1½-2,400-120/240

The specially designed windings with cylindrical shields, previously described, and illustrated in Fig. 1, were subjected to a similar investigation. The voltage distributions on Y and on delta connections, as presented in Fig. 4, show but little divergence from the desired uniform line.

Effect of Shape and Polarity of Impulse. In the tests on the unshielded design of transformer, both the fronts and the tails (duration from crest to one-half crest voltage) of the applied waves were varied through a wide range. As predicted by laws already established, the steeper the front, the more uneven the initial distribution; and the longer the tail, up to one-half the natural period of the transformer, the greater the maximum oscillations. Lengthening of tail beyond one-half the natural period increases the magnitude of oscillations in diminishing degree. The waves applied were gradually shortened in front and lengthened in tail, with corresponding increase in severity of distribution, until the maximum of oscillations under the conditions of Fig. 3 were attained.

In the case of the shielded windings, change of wave shape causes no measurable variation in results because the initial distribution under all conditions is very close to the final one and oscillations are practically eliminated.

Investigation of polarity by impressing negative, as well as positive waves on both types of designs indicated no measurable difference for this class of apparatus, so far as distribution is concerned.

Internal Impulse Voltage Distribution Low-Voltage Windings. Recently, attention has been drawn to the voltage concentrations which may be induced electrostatically and electromagnetically in low-voltage windings during the subjection of the high-voltage winding to impulses.[5] To study this phase in distribution transformers, taps were provided in the 120/240-volt windings of a 1½-kva. shielded high-voltage transformer and impulses applied to the high-voltage winding. Study of the results in Fig. 5 leads to some interesting observations:

a. No serious concentrations are shown.

b. Grounding the low-voltage midpoint results in the interesting condition of the respective line leads rising to equal moderate potentials of opposite polarity above and below the neutral. This distribution of voltages indicates that the predominant factors are oscillation of the winding as a whole superimposed on the voltage induced electromagnetically.

c. The connection of a load or of additional circuit to the low-voltage terminals reduces the voltage to ground at all points under all conditions to a more uniform value.

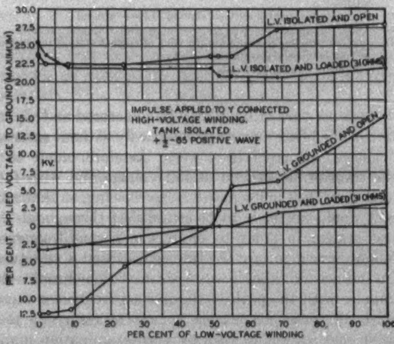

FIG. 5—STUDY OF TRANSIENT VOLTAGE DISTRIBUTION IN LOW-VOLTAGE WINDING (HIGH-VOLTAGE WINDING SHIELDED) OF TRANSFORMER RATED H-60-1½-2,400-120/240 TANK ISOLATED

Impulse Arcover Tests. Coordinated distribution transformers with shielded high-voltage windings were subjected to a series of impulse voltages under various combinations of the following conditions:

Connections:

a. High-voltage lines isolated.

b. One high-voltage line grounded.

c. With 60-cycle operating voltage, impulse synchronized at crest.

d. With rated load at unity power factor on secondary to provide follow current regardless of circuit inertia, should a break in insulation occur.

Tank:

a. Grounded.
b. Isolated.

Impulse:

a. Polarity: positive and negative.

b. Magnitude: adjusted to arc over high-voltage bushing about one-half the time, thus impressing both full and chopped waves on the transformer.

c. Shape: steep front to obtain most severe initial distribution; long tail to develop maximum of oscillations.

The resistance to repeated impulses under any combination of the above conditions was found to be highly satisfactory.

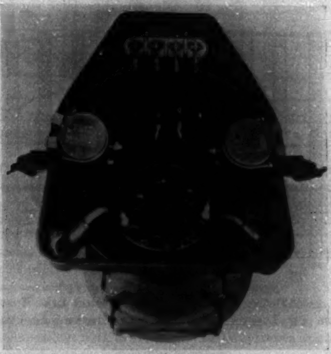

Fig. 6

Discussion

The transformers described have been built for what seems at present the desirable level of strength, that is, on the assumption that a two-in. rod gap for 4,330 Y-volts provides a logical extension of the already agreed upon coordination curve when extended to circuits below 5 kv., and the further assumption that approximately one inch is a proper level for the low-voltage bushings. These bushings must first be corrected for the effects of wave polarity, and the increase in internal stress upon the windings due to the series arcover in the case of the isolated tank must be factored. With these matters properly considered, it appears possible to build transformers at any desired level which will stand up indefinitely under waves which will flash over to the low-voltage circuit. Whether there is economic justification for the development remains to be seen as the level of coordination chosen will determine the cost of the transformer and, consequently, the balance between coordinated transformer and present type plus arrester protection. As the latter seems essential from a general service standpoint, the coordinated transformer may be of value only for marginal cases where the benefits of the arrester must be sacrificed.

Whatever considerations apply to the discharge of impulses into the secondary circuit must be taken into account, with the knowledge that such transformers have a maintained level of strength which will minimize these discharges. As distribution transformers coordinated at the level of line insulation would be prohibitively expensive, a compromise based on sound past practise as outlined above seems to present a logical improvement, largely eliminating transformer failures under lightning and leaving a further degree of perfection to be obtained by the addition of such protective devices as prove necessary.

Acknowledgment

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Discussion

THE COORDINATION OF TRANSFORMER INSULATION WITH LINE INSULATION

(Montsinger and Dann)

INSULATION COORDINATION OF DISTRIBUTION TRANSFORMERS

(Treanor and Cooney)

C. Francis Harding: With regard to the spark gaps proposed in both of these papers it should be noted that while such gaps "between the ends of two square-cornered, square-cut, coaxial rods not more than one-half inch thick . . . and so mounted that the rod overhangs its support at least one-half the gap spacing" may be suitable for a parallel discharge gap for the protection of transformers when installed, it is of questionable value as a laboratory testing standard as proposed. The standard sphere gap, already established by the Institute, has the advantage of no time lag and is relatively reproducible for either steep-wave-front surges or 60-cycle potential measurements, (particularly if provided with ultra-violet light). The latter gap has been found to provide an accuracy of from two to three per cent whereas the small sphere gaps operating with no ultra-violet light or those between plane surface electrodes with sharp edges vary as much as 15 to 20 per cent. Unless specified much more in detail than at present therefore such plane surface gaps should not be permitted to be used as a primary standard of high potential measurement, particularly upon steep front transient waves.

Furthermore, it should be clearly recognized by the purchasers of such surge-proof transformers that with the spark gaps or lightning arresters in parallel with the transformers it is the spark gap or lightning arrester which is being tested and not the transformer insulation. Unless additional tests are prescribed without the use of gaps, the real insulation strength and the ratio of insulation strength to gap potential will not be known and may have any value whatsoever above that of the gap. The latter potential, as previously pointed out, is none too definite nor is it easily reproduced as a reference standard.

Referring, in the paper by Messrs. Treanor and Cooney, to the proposed use of resistors between the secondary phase leads and the grounded neutral it should be noted that the possibility of thus relieving, by an amount of 60 to 70 per cent, the transient surge stresses induced upon the secondary was demonstrated in the paper* presented by Mr. C. S. Sprague and the writer at the Winter Convention in January, 1932.

The use of capacitance was also tried in these tests but in both cases the relief was only of local value, *i. e.*, it was only effective very near the point of connection of the resistance load or capacitance. In the latter case, the capacitance, to be sufficiently effective at the low secondary potentials, was considered to be excessively large and too expensive for general application, while the loss in a suitable resistance thus connected may prove a prohibitive expense.

However, with the more and more unsatisfactory lagging power factors recently being found upon transformers in residential distribution systems due to consumers' appliances and with the additional possibility of neutralization of the objectionable effects of lagging magnetizing currents it may be desirable, in the near future, to consider such capacitance connections, possibly for the sake of greater economy upon the primary side of distribution transformers, for the purpose of providing this three-fold advantage, (1) reduction of the magnitude of secondary induced surges, (2) improvement of low lagging power factor due to consumers' appliances, (3) neutralization of the low power factor of the magnetizing current of the transformer especially in cases where the all day efficiency is low.

J. K. Hodnette: Messrs. Treanor and Cooney present a very interesting discussion of the pertinent question of the impulse characteristics of distribution transformers in their analysis of insulation coordination of distribution transformers. In reading the paper, however, I was somewhat confused by the introduction of the question of shielding the high-voltage winding. Although the authors do not definitely state so, one gets the impression that the use of shields and the special arrangement of the high-voltage winding in single coil is necessary for obtaining coordinated insulation. In this connection it is interesting to note that a reasonably good distribution of surge voltage was obtained with an ordinary commercial distribution transformer as shown in Fig. 3 of the paper even with the unusually long test waves of 350 and 520 microseconds length. It is the usual practise to construct distribution transformers with the high-voltage winding divided into two or more parts or sections for the purpose of decreasing the voltage stress across the coils. If the transformers in question were designed with two sections, the initial voltage stress across the coil would be reduced from 100 to 77 per cent as compared to the coordinated design, and with four sections the stress would be reduced from 100 to 46 per cent. It appears from this material reduction in stress that the shields are not essential for building a transformer with coordinated insulation and that it could be accomplished without departing from the practises followed in the past simply by increasing or using more economically the present quantities of insulation if higher than present strength is desired.

The authors have found that transformers with bushings and windings coordinated in the manner described are able to stand up under apparently unlimited flashovers. What were the relative values of bushing flashover and winding insulation breakdown strength in these transformers? Certainly such a transformer could be built which would be self-protecting against surge voltages irrespective of the level of insulation chosen, so that the real problem is one of economics. Such a transformer would have certain definite merits but it has the distinct disadvantage that unless adequately protected by lightning arresters or other protective devices, the bushings will flash over, with the consequent blowing of the primary fuse and service interruption. Protecting the transformer against failure is but one of the major problems involved. Preventing service interruptions as a result of fuse outages is possibly of equal importance as evidenced by the fact that on an average one per cent or less of the transformers in service are broken down each year as a result of lightning, whereas in the neighborhood of 5 to 8 per cent suffer fuse outages. Since the coordinated transformer offers no solution to this serious problem, there is not much to recommend it over the usual type of distribution transformer.

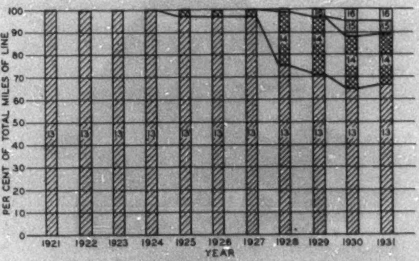

FIG. 1—220-KV. TREND OF TRANSMISSION LINE INSULATION NUMBER OF INSULATOR UNITS PER STRING—ACCUMULATIVE DATA

The whole problem of impulse characteristics of distribution transformers has been carefully investigated by the company with which I am associated during the past several years. Not only were the surge voltage stresses in different parts of the transformer measured by cathode ray oscillographs and sphere gaps, but the ultimate impulse strength of the various parts of the insulating structure determined under conditions simulating those encountered in service with the object of increasing the ability of the transformer to withstand impulse voltages and conditions encountered in service. The result of this analysis was the development of a surge and outage-proof distribution transformer. This transformer was briefly described at the Winter Convention of the A.I.E.E. It is self-contained and self-protecting. The low-voltage insulation is made stronger than the

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***Interconnection of Primary Lightning Arrester*, by C. Francis Harding and C. S. Sprague, A.I.E.E. TRANS., March 1932, p. 234.

flashover of the bushings and the high-voltage insulation stronger than the flashover of internally mounted deion protective gaps connected between the primary leads and the tank. The three insulated metallic circuits are so related through the protective devices in this transformer that dangerous voltage stresses cannot exist upon any part of the insulation. The protective gaps, in addition to coordinating the primary insulation, deionize the arc path of the surge. In conjunction with a series resistor they control the magnitude of the follow current and prevent blowing of primary fuses and service interruptions. In a transformer with coordinated insulation, a fuse outage will occur in a majority of cases of bushing flashover.

It is not necessary to flash over both the deion gap and the secondary bushing in order to discharge a surge to ground where the tank is insulated from ground. A discharge gap having a flashover voltage much less than that of the low-voltage bushing is connected directly to ground so that surges originating in either circuit are discharged to ground independently of other circuits.

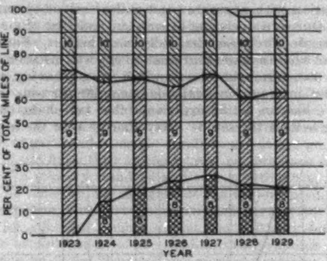

FIG. 2—132-KV. TREND OF TRANSMISSION LINE INSULATION NUMBER OF UNITS PER STRING—ACCUMULATIVE DATA

These transformers fulfil the major operating requirements for distribution transformers in a self-contained unit.

1. They are self-protecting against surge voltages.
2. They prevent fuse outages as a result of surge voltages.
3. They introduce no hazard to secondary apparatus, but provide an independent path to ground for surge currents originating in either circuit without involving the other.
4. They are universal in application. They may be operated single-phase or three-phase with tanks grounded or ungrounded.

R. N. Conwell: Messrs. Montsinger and Dann have indicated one of several methods for effecting coordination between transformer and line insulation, namely—the fixing of transformer insulation at a definite, arbitrary value and then relying upon the designer of the transmission line to coordinate with these values. Such a method surely cannot produce a well-balanced, satisfactory, economical system design. The transmission designer is handicapped at the very start by the necessity of meeting limits established without regard to service conditions or local factors such as the character of the terrain, lightning exposure, frequency of storms or the position of the line with regard to storm paths.

The table of protective gap settings given in the paper appears to be predicated on existing transformer design or manufacturing expediency rather than on a consideration of the trends of levels for insulation on lines and in equipment. Table I of this discussion shows the approximate equivalent number of standard insulator units to the air gap spacings given. These may have proved satisfactory for line insulation five or ten years ago but experience, lightning studies which have been recently completed, and demands for better service have all forced a gradual increase in line insulation as shown in Figs. 1, 2 and 3. A similar increase in insulation has been necessary in other voltage classes.

TABLE 1

Rated circuit voltage	Transformer 60-cycle test voltage	Spacing of air gap	No. of 5¾-in. units equivalent to air gap spacing
Kv.	Kv. r.m.s.	Inches	(Impulse sparkover)
13.8	28.6	4.25	0.6
23.0	47.0	6.25	1.0
34.5	69.0	9.25	1.5
46.0	93.0	12.25	2.1
69.0	139.0	18.75	3.3
92.0	185.0	25.00	4.6
115.0	231.0	31.50	5.9
138.0	277.0	38.25	7.3
161.0	323.0	44.50	8.6
196.0	393.0	54.50	10.6
230.0	461.0	64.00	12.5

The ultimate purpose of "coordination of insulation" is the improvement of service and the pre-determination of the location of failures. If flashovers can be prevented or made to occur at points where repairs can be made cheaply without interference to service, the criteria has been reached.

The major question involved in the solution of the problem of coordination is whether the insulation of the line or of the equipment, or a compromise between these two, is to be assumed as a base. We are of the opinion that generally line insulation should be taken as the base and insulation at the terminals graded from this value. Other schemes of coordination might be devised, however, depending on the particular bus layout and the relative importance of each piece of equipment. It is conceivable that service conditions may dictate the preferential loss of a bus section rather than a transformer bank. The low values of protective gap settings and transformer insulation eliminate this and other methods of coordination which might profitably be employed.

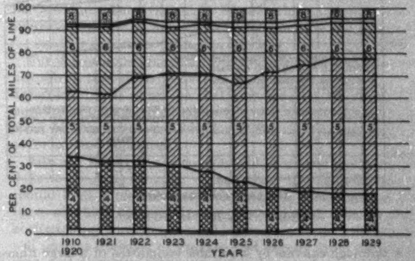

FIG. 3—66-KV. TREND OF TRANSMISSION LINE INSULATION NUMBER OF INSULATOR UNITS PER STRING—ACCUMULATIVE DATA

Merely as illustrative of other methods, Figs. 4 and 5 show the results of an effort to coordinate the insulation of equipment installed at the Roseland Switching Station of the Public Service Electric and Gas Company, with the connecting 132-kv. and 220-kv. lines. Roseland was designed in the winter of 1927-28 at which time very meager information on the surge characteristics of equipment and bushings was available. Complete information is still not available on apparatus bushings or the windings of transformers but the specifications for equipment carried a provision that the bushings should flashover prior to the failure of the windings. In lieu of more authentic data, the curve for the windings of the transformers has been obtained from the curve shown in Fig. 4 of Dr. Fortescue's paper* for needle gaps under

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**Rationalization of Station Insulating Structures*, by C. L. Fortescue, A.I.E.E. TRANS., October 1930, p. 1450.

oil using the induced test specifications of the transformer as a basis. Air gaps located in the station were used merely to correct discrepancies in the impulse characteristics of the various types or pieces of equipment.

The plan of coordination proposed by the authors will in most cases necessitate the installation of air gaps of relatively low breakdown value in vulnerable locations in the switch yard which unless protected by lightning arresters, will cause serious interruptions to service. The plan proposed will not meet service requirements generally and therefore the natural result will probably be the construction of a large number of so-called special transformers.

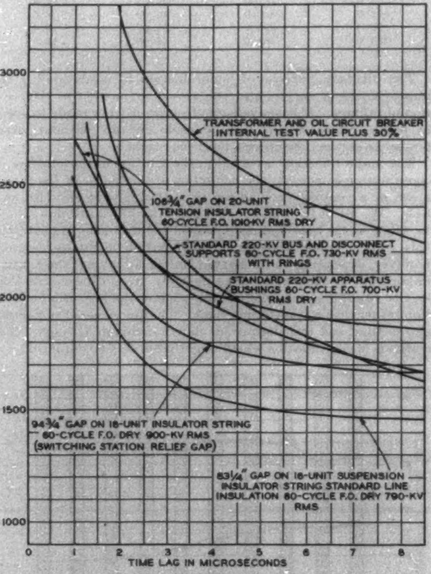

Fig. 4—220-Kv. Equipment

A thorough canvass by a suitable committee of a large number of operating companies to determine preferences on future insulation levels similar to those indicated in Figs. 1, 2, and 3 and methods to be employed in coordination should be made as a basis for the determination of insulation requirements of transformers. Thus a standard may be established which will eliminate the ordering of a large number of specially insulated transformers, allow greater freedom in the design of transmission lines and stations and a better balance in the service and economic features of the system effected.

Philip Sporn: The paper by Messrs. Montsinger and Dann is really nothing but a revision of their paper* presented at the Summer Convention of 1930. The only real differences are: (1) Table I of the original paper has been extended to include recommendations for circuit ratings from 46 kv. down to 13.8 kv.; (2) the other change is in the substitution of a square-cornered, square-cut rod gap for a series of 5¾ in. insulator disks recommended originally.

However, the objections made at that time to the principles of Messrs. Montsinger and Dann, which I stated in a companion paper†, are equally valid today. These principal objections are as follows:

1. *Codification of Insulation Levels.* While the proposed standards do not actually prescribe standard practise, the net effect would be the same as if they did. Not only is the general idea of codifying present practise objectionable, but it is almost impossible to obtain any uniform opinion as to what is a proper standard insulation level; there is on the contrary, a definite and well informed opinion today that no progress will be made in the subject of insulation coordination until the idea of specifying standard practise for any operating voltage is definitely abandoned.

2. There is a very definite lack of knowledge as to the performance of coordinating gaps, regardless of type, when installed in connection with apparatus and under various operating conditions. Our own operating experience, and that of many other companies, definitely and decidedly bear this out. For example, in one case where a 30-in. coordination gap was installed close to a potential transformer and an oil switch, the gap having, according to the best information available, some 20 per cent margin of safety, we had an actual experience that belied the theoretical reasoning behind the setup. Very shortly after the installation of the gap a lightning storm occurred, during the course of which two bushings failed, one on the transformer and one on the oil switch. The gap not only did not protect the equipment that it was designed to protect, but did not even flash over. The gap has now been set down to 24 in. but there is no definite guarantee that even this size gap will actually coordinate with the terminal equipment.

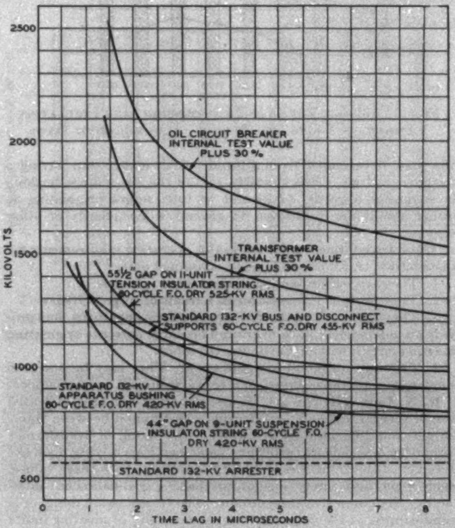

Fig. 5—132-Kv. Equipment

3. A very definite objection is the fact that this coordination is being carried out in terms of secondary standards. It is high

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*[*Recommendations on Balancing Transformer and Line Insulations*, A.I.E.E. Trans., Vol. 49, p. 1478.

†*Rationalization of Transmission Insulation Strength—II*, by Philip Sporn, A.I.E.E. Trans., 1930, Vol. 49, p. 1470.

time that we started to talk and measure impulse voltage in terms of volts and not in terms of gaps, insulators or some other secondary standard. Our experience in other fields of the lightning problem has shown that no real progress was made until elements of the lightning circuit were expressed in terms of such units as the volt, the ampere, the ohm and the second.

Besides, the whole problem of insulation coordination is much broader than the one envisioned by the authors. Apparently their standpoint aims primarily at insuring a set of operating conditions under all of which, when flashover occurred at a transformer, the transformer would still be safe. However, the problem of coordination is much broader than that, and in its broad aspects involves a much more important problem than one merely of the failure of a particular piece of apparatus. It involves the question of service and the rendering of service of the necessary high standard without going to a setup that involves ruinous expenditures for apparatus either in first cost or in its maintenance. The problem being one of this large scope, it would be far better to get back to fundamentals and to terms that could serve as a basis of reference for all makers of equipment. This means expressing impulse strength in terms of volts and time (although this could, for all practical purposes, be covered by a series of test waves). This would not necessarily preclude the use of gaps under service conditions, but the gaps in themselves would have their strengths and performance expressed in terms of a primary standard and such coordination as required would be done by the designer of the system upon whom the broad problem of system cost and system service has really been thrust for solution. Such a system, too, will very definitely permit the establishment of the fundamental principle of differing insulation levels for the rated or accepted operating voltages. A general discussion of this problem was presented before the Institute some time ago.* The principle has been under discussion now for four years and has been accepted as a workable principle by the joint NELA-NEMA Committee on Insulation Coordination. It does not seem as if the whole problem of coordination would be enhanced by having one group of the industry at the present time advocating coordination for a particular group of equipment at variance with this general principle.

J. B. Hodtum: Messrs. Treanor and Cooney mention that whether there is economic justification for the development of the coordinated distribution transformer described remains to be seen. We would like to point out that the Allis-Chalmers Co. developed a coordinated distribution transformer, known as type SB, which has been on the market for four years and has demonstrated itself satisfactorily from a service point of view and from the economic point of view. We found from experimental impulse tests, which have now been confirmed by four years of service operation, that we could build a coordinated transformer which would flash over the bushings before the transformer itself would fail. We have had over 5,000 of such transformers in service and have yet to learn of a single failure. Numerous cases are known where the bushings flashed over but the transformer did not fail.

We found that satisfactory coordination could be obtained without the necessity or the added complication of installing shields in the windings. If the desired service effect can be obtained without the addition of metal plates in the windings, we consider the transformer a better unit. On the other hand perhaps the use of shields might permit using less insulation than is used with the unshielded transformer, but it seems hardly feasible for 2,400-volt service at least.

We found that in order to obtain the service coordination referred to, that the bushing would not have to take on the peculiar form illustrated in Fig. 1 of the paper, but that on the other hand, the general form of bushing contour which is used with other insulators was entirely satisfactory. Perhaps the shape indicated in Fig. 1 was worked out to simplify the taping of the line conductor where it connects to the bushing. Practically, however, we have found a somewhat larger connection to be necessary, and we have found it desirable to furnish a solderless connector of the clamped type. These connections are often used bare, but where it is desired to cover them we have developed a heavy glass piece which slips over the line lead and makes a very satisfactory protecting cover.

As pointed out in Messrs. Treanor's and Cooney's paper, any coordination of distribution transformer bushing and winding will naturally result in blowing of the fuses when the bushings flash over. To eliminate this effect, lightning arresters or surge diverters of some nature must be furnished whether the transformer is coordinated or not. Surge diverters have been developed which can be applied to Allis-Chalmers stud bushing type transformers mounted either outside or inside of the case. The coordinated design obtained by the use of the stud bushing of proper flashover value with respect to the windings protects the transformer against abnormal voltages. The addition of the surge diverter eliminates fuse outages. This makes a completely satisfactory solution to the distribution transformer insulation problem without complications in the transformer structure itself and one which is right now economically practicable.

F. J. Vogel: The coordination of transformer insulation with the gap is of great value, not only to operating engineers, but to the manufacturer in providing a yardstick to measure the surge strength of station apparatus.

The operating engineer can proportion the station insulation to be above this benchmark, and rest assured that it will be protected. Likewise it provides a benchmark for the designer of station apparatus, and furnishes him an assurance that the design is correct if it is stronger than the specified gap.

It is not essential that the gap be used as the actual protective means in all cases; indeed it is very likely that it will be used merely as the level in the case of very low voltages. For this purpose the table given might be somewhat extended. Values suggested for this are 3 in. for 8,700 volts and 2 in. for 4,300 volts, for outdoor apparatus exposed to lightning.

For indoor apparatus, unexposed to lightning, long experience has shown that the requirements for outdoor apparatus are not necessary. It is suggested that standards for the class of apparatus for unexposed circuits might well be reduced one voltage class, for example to use 3 in. for 13,800 volts instead of 4¼ in. as for exposed apparatus.

Edward Beck: In the coordination of station insulation with definite air gaps for each voltage class, the lightning arrester should limit surge voltages to magnitudes somewhat less than those at which the coordinating gaps will flash over, so that if both gaps and arresters are used in the same station, the arresters will prevent flashover of the gaps and thereby avoid service outages, and if arresters alone are used, they can be depended upon to provide a voltage limiting device as good or better than the coordinating gap.

Lightning protective devices for electrical systems may be said to fall into two general classes. First, those intended for the protection of line insulation against flashover; these may have relatively high operating voltages because the insulation level of the transmission lines is generally high. Second, those intended for the protection of apparatus in stations; these must have relatively low operating voltages in order to coordinate with the gaps proposed for the various voltage classes.

In a discussion of the Lightning Symposium held at the Winter Convention in 1930, published in the TRANSACTIONS OF THE A.I.E.E., July 1930, page 936, the writer proposed that standard lightning arresters be used as a basis for coordination instead of the insulator strings then recommended because the characteristics of the lightning arresters are more uniform and lower than the flashovers of insulated strings such as were proposed for coordination purposes. At the present time, the use of insulator

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**Rationalization of Transmission System Insulation Strength*, by Philip Sporn, A.I.E.E. TRANS., 1928, Vol. 47, p. 998.

strings as a basis for coordination has been discarded and the coordinating medium is a gap in air. The relationship between these coordinating gaps and the lightning arresters holds equally well. A normal autovalve arrester of the same voltage class as the coordinating gap will limit the surge voltages to lower values than the flashover of the gap, as has been determined from numerous laboratory tests that have been made on the lightning arresters and coordinating gaps in parallel, as well as on comparison of data on the impulse flashover of the gaps and the performance of the arresters. Tests have been made for instance, on a lightning arrester of the 230-kv. class in parallel with the 230-kv. coordinating gap. Surges were impressed from the 3,000,000-volt surge generator at the Transformer Works. Similar tests have been made on the same devices of the 138,115, and lower voltage classes. In fact it is the practise in our laboratories at the present time to make all checks of performance of arresters with the recommended coordinating gaps in parallel with the arrester during test.

Comparing the impulse characteristics of the gaps with the performance of autovalve arresters, a typical case, for instance that of the 138-kv. circuit, shows a recommended gap of 38¼ in. with a minimum impulse flashover on a 1½-40 wave of approximately 575-kv. crest.

A 138-kv. class autovalve arrester for use on circuits with the neutral ungrounded or grounded through resistance or reactance would have a maximum permissible line-to-ground rating of 146 kv. This arrester will limit surges to approximately 525-kv. crest, providing a margin between the minimum flashover of the gap on a long wave and the arrester voltage. For surges of greater magnitude rising at the same or steeper rate, the flashover of the coordinating gap would be higher leaving a larger margin between its flashover voltage and the operating voltage of the arrester.

Considering another instance, a 23-kv. circuit, the recommended coordinating gap is 6¼ in. with a minimum impulse flashover on a 1½-40 wave of 120-kv. crest. A normal 23-kv. arrester with a maximum permissible line-to-ground voltage of 25 kv. will limit the surge voltages to between 95- and 100-kv. crest, leaving an ample margin between the flashover of the gap and the operating voltage of the arrester. A similar arrester for use on system with solidly grounded neutral would normally have a maximum permissible line-to-ground voltage rating of 80 per cent or 25 kv. or 20 kv. This arrester would limit the surge voltages to 80-kv. crest, indicating a still greater margin between the coordinating gap and the arrester.

In the coordination proposed in the Montsinger and Dann paper, the arrester falls into its designated place, that of providing a device with an operating voltage below that required to flash over the recommended gaps.

John O. Fenwick: The title of the report by Messrs. Montsinger and Dann indicates that transformer insulation is to be coordinated with line insulation. Therefore, the first logical step for the committee to take would seem to be to determine the insulation levels which operating companies desire to maintain on their lines. With this information at hand, the spacing of coordinating gaps to be used in conjunction with transformers could easily be calculated. It is entirely possible that such a procedure was followed by the committee but the report does not indicate this procedure.

The coordinating gaps used in conjunction with the transformers should be used for operation only and not as a basis of transformer design. Insulation deterioration and the necessity of a factor of safety demand impulse strengths of transformers to be much greater than the flashover value of the coordinating gaps. Operating companies should know and will desire to know the factor of safety that exists in the transformer insulation. Therefore, another set of air-gap spacings should be made standard for impulse testing of transformers, the gap settings taking care of the desired factor of safety.

Such a setup would seem to be most desirable from the standpoint of the operating companies.

C. S. Sprague: The statement is made in the paper by Messrs. Treanor and Cooney that taping the line parts of the bushing does not affect the flashover value. It has been the experience of the writer that such taping does affect the surge flashover; in fact, when determining the surge flashover of transformer bushings, the end having the lower flashover value has often been taped, this forcing the flashover to occur on the other end of the bushing. It would seem that sufficient tape to provide positive protection to the lineman would affect the impulse flashover.

In Fig. 2 the bushing flashover values are given in terms of the equivalent coordinating gap spacing. Also under *B* the time lag of the bushing (or bushings) increases from left to right. Since the coordinating gap itself has some time lag, concerning the extent and variability of which there is not universal knowledge, it seems, at present, that this is an unsatisfactory method of expressing quantitative values. It would be helpful to the reader to know the actual peak values of waves under *B* as well as the spacing of the coordinating gap.

As the authors state, the increased cost of the transformer with coordinated bushings and insulation may not justify its general use. For special locations, however, it seems that proper coordination, in the ratio of 2 or 3 to 1, together with good arrester protection, (which probably implies interconnection) would offer the ultimate in reliability and continuous service.

W. M. Dann: Professor Harding criticizes the use of rod gaps as a laboratory testing standard and recommends the use of sphere gaps on account of their greater accuracy. Granting that the sphere gap is somewhat more accurate it is quite desirable to use a gap with time-lag which a sphere gap does not have. The gap spacings as proposed in this paper are for the purpose of coordinating transformers in the field and not necessarily for testing transformers. Some other kind of gap or different gap settings may be used when the time comes for making impulse tests.

Regarding Mr. Conwell's criticism of the general setup of coordination, like so many others he has assumed that the manufacturers of transformers are attempting to establish the level for the line insulation. This is just the reverse of the intent of these recommendations.

The levels selected merely represent what the data in 1930 showed were the average line insulations used throughout the country. If the average line insulations have increased then it would be logical to increase the transformer insulation levels. Up to the present time we have not seen any concrete evidence that it is necessary to increase these levels which might add an unnecessary burden to the industry as a whole.

It is plainly stated in the paper that where different levels than those selected (for average conditions) are required, the transformer insulation level should be tested and made to fit the requirements. It appears that Mr. Conwell may be influenced by his own company's requirements in advocating that the average level be raised for the whole country. If the majority of operating engineers want the level raised, the manufacturers of transformers will be only too glad to do it, but at an added cost, of course.

We agree entirely with Mr. Sporn that it is almost impossible to obtain any uniform opinion as to what the standard level of insulation should be. It was on this account that we set up certain levels for what appeared to be average line insulation and then stated that for those requiring different levels the gap spacings and transformer test levels should be made to fit these special requirements.

Regarding Mr. Sporn's experience with the two bushings flashing over without the gap going over, a check up on the bushings shows that with the short distance involved (something like 30 feet) and the large difference mentioned in the flashover levels of the gap and bushings with any of the preferred impulse waves, it would be practically impossible to have this happen unless (a) the gap was improperly installed, or (b) there was a direct stroke on the terminals or bushings.

In connection with his objection to secondary standards, we have used the best available standards. As soon as the various laboratories get their differences ironed out, impulse volts can be used if desired.

The Subcommittee had the choice of either holding up its work until an agreement could be reached on impulse kilovolt flashover of gaps, disks, etc., or using this so-called secondary standard, which after all is what the man in the field wants to know when installing gaps.

Characteristics of Surge Generators for Transformer Testing

BY P. L. BELLASCHI*
Associate, A.I.E.E.

Synopsis.—*The requirements and limitations of surge generators for transformer testing can be summed up as follows:*

The voltage that can be obtained is, in the first place, largely determined by the capacity of the surge generator when the capacity of the transformer is great.

If a wave is desired without superimposed oscillations, it is necessary to insert resistance within the generator. The front of the surge then depends upon the inductance of the generator and the capacity of the transformer. This is a matter inherent to all surge generator test circuits. Thus, in general, waves of very steep front cannot be obtained with large transformers.

Further, the inserted resistance also limits the voltage by an amount dependent on the proportion of this resistance to the load resistance.

For transformers of low inductance, the length of the wave obtained is largely determined by the capacity of the surge generator. This results in a requirement of large generator capacity if very long waves are to be produced.

If the lead between the transformer and generator is appreciably long, other oscillations will be set up which are highly damped but are appreciable at the generator end. These oscillations occur on the front of the wave only, and will be prominent when measurements are taken at the surge generator.

INTRODUCTION

It is the purpose of this paper to consider the characteristics of surge generators in relation to their application to the surge testing of electrical apparatus, particularly transformers. Previously, surge testing has been mostly confined to gaps, insulators, bushings and similar insulation structures. Since the surge testing of transformers has been shown to be desirable,[1] the characteristics of surge generators in testing transformers become of great interest, particularly in the formulation of standards or rules for such testing.

CIRCUITS OF SURGE GENERATOR AND TESTED APPARATUS

The surge generator consists of a bank of condensers or a bank of groups of condensers, each condenser or group charged in parallel to a given potential and all condensers or groups discharged in series through gaps into an external resistance or series of resistors.[2] In the circuit diagram of Fig. 1a, these primary circuit elements are simply represented as a lumped surge generator capacity C_s, charged at potential E, and suddenly discharged through the gap or gaps G into the load or discharge resistance R. An inductance L_s is inevitably entailed in the condenser bank circuit and in leads connecting to the load resistance, also in leads connecting to tested apparatus. Likewise, a resistance R_s is present in the condenser bank circuit, connecting leads, etc., and, for reasons explained below, this resistance is purposely increased and adjusted to have a given proper value. Furthermore, the condenser bank structure and the accessory apparatus of the generator (spheres, leads, insulator strings, etc.) all have capacity to ground. This leakage capacity to ground may, for engineering purposes, be considered as a lumped capacity C_2. Thus, the circuit diagram of Fig. 1b represents in simplified form the equivalent circuit of the surge generator proper.

Apparatus tested, such as bushings, insulators, gaps, etc., are in effect lumped capacity loads. In case the apparatus connects to the generator through a short lead, the diagram of Fig. 1b also represents the equivalent circuit of the generator with load. The capacity C_T is the load capacity; C_L represents the leakage capacity to ground of the generator combined with the load capacity.

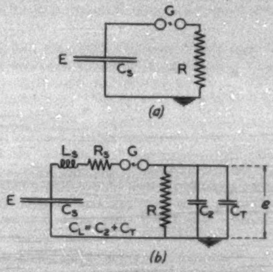

FIG. 1—CIRCUIT DIAGRAM OF SURGE GENERATOR WITHOUT LOAD AND WITH CAPACITY LOAD

Transformers and similar inductive apparatus introduce other elements in the test circuit of the surge generator, with resultant modifications in the surge generated. In the case of transformers with substantially uniform voltage distribution, the equivalent circuit of the transformer consists in effect of a capacity C_T shunted by an inductance L_T. The total load capacity due both to the surge generator and the transformer may be considered lumped in a capacity C_L, as shown in the circuit diagram of Fig. 2. Surge testing is also applied to transformers in which the voltage distribution departs from uniform.[1] The equivalent circuit of these transformers may be represented in simplified form as shown in Figs. 3 and 4, where the number of meshes depends on the type and construction of the transformer.

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*Westinghouse Electric & Mfg. Co., Sharon, Pa.

[1] For references see Bibliography.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

The diagram of Fig. 5 gives in a more generalized form the circuit of the generator, the apparatus tested and the intervening lead or other apparatus. Most always a simple lead connects the generator proper to the transformer tested. This connection is made as short as it is physically possible. Whenever there need be taken account of this additional element, the test circuit may be represented as shown in the diagram of Fig. 6a. For practical purposes this may generally be simplified to Fig. 6b and in turn to Fig. 2.

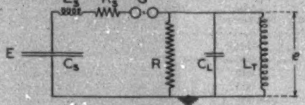


FIG. 2—CIRCUIT DIAGRAM OF SURGE GENERATOR WITH TRANSFORMER LOAD

L_T represents the effective inductance of the winding, taking into account mutual coupling between primary and secondary, and secondary load

AMPLITUDE OF SURGE GENERATED—CAPACITY REGULATION

It is desired that surges generated rise smoothly to crest value and decay uniformly to zero value. The surge generator capacity C_s and the load resistance R are both generally large compared to the other elements of the test circuit, so that the self-inductance L_s of the circuit in Figs. 1b and 2 tends to oscillate with the load capacity C_L, unless a proper damping resistance R_s is inserted in the circuit and adjusted to, or is greater than, critical value. The proper value for R_s is approximately

$2\sqrt{\dfrac{L_s}{C_L}}$, as discussed in Appendixes I and III.

In general, since the load resistance R is much greater than the damping resistance R_s, the maximum surge voltage generated and applied to the apparatus tested depends in a large measure on the total load capacity C_L relative to the surge generator capacity C_s. The capacity regulation of the maximum surge voltage generated may thus be simply expressed as:

$$\frac{e\,(\max)}{E} = \frac{1}{1 + C_L/C_s} \qquad (1)$$

Denoting the maximum voltage generated with no load as $e_o(\max)$, then the capacity regulation with load C_T is:

$$\frac{e(\max)}{e_o(\max)} = \frac{1}{1 + C_T/(C_s + C_2)} \cong \frac{1}{1 + C_T/C_s} \qquad (2)$$

Thus, in Table I are compared calculated and experimental results on regulation due to the capacity of the transformer tested. The curves in Fig. 7 give the characteristics of the C_s-R-C_L circuit, enabling one to calculate the regulation and to determine the load resistance R required to produce a given length of tail. These curves are calculated from equation (6) in Appendix I.

TABLE I—REGULATION OF SURGE GENERATORS DUE TO CAPACITY LOAD

Transformer load	Surge generated	C_T $\mu\mu f.$	C_s $\mu\mu f.$	Regulation due to capacity of transformer load compared to no-load	
				Calculated	Experimental
No. 1†	Long tail*	1,200‡	4,200	0.78	0.77
No. 2	Long tail	1,200	12,600	0.91	0.92
No. 3	Long tail	1,100	8,000	0.88	0.89
No. 4	Long tail	3,600	4,200	0.54	0.52

*In the order of and greater than 40 microseconds.
†The transformers tested were all of high inductance.
‡Transformer capacity.

CONTROL OF FRONT AND TAIL OF SURGE—CAPACITY LOAD ON SURGE GENERATOR

As stated before, the series resistance R_s of the surge generator is adjusted to approximately twice the oscillation impedance $\sqrt{\dfrac{L_s}{C_L}}$, to free the surge generated from superimposed oscillations. For the smooth wave, the series resistance R_s, the load capacity C_L and the circuit inductance L_s determine in a large measure the rise of the front.

The generator capacity C_s, initially at potential E, charges the total load capacity C_L from zero to maximum voltage, C_s corresponding to a large capacity relative to C_L. Neglecting for the moment the circuit inductance L_s, (for large values of load resistance R), the front of the voltage would rise to 50 per cent crest value in approximately a time 0.693 (R_sC_L) as shown in Fig. 8, curve for $L = O$. (Note: $C_e = C_sC_L/(C_s + C_L) \cong C_L$.) The effect of the circuit inductance on the rise of the front is also shown in Fig. 8, when R_s is adjusted for critical damping. The curves are calculated from equations (7) and (8) in Appendix I. Without damping resistance, the time required for the voltage to rise to the crest of the first oscillation is approximately

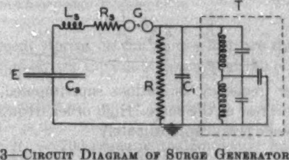


FIG. 3—CIRCUIT DIAGRAM OF SURGE GENERATOR WITH TRANSFORMER LOAD

C_1, load capacity
T, simplified circuit diagram corresponding to a two-group shell type winding (taking into account mutual coupling between primary and secondary)

$$\pi\sqrt{L_s C_L} = \frac{\pi}{2}(R_s C_L),$$

where R_s is the critical damping resistance.

It should be noted that the inductance L_s and the load capacity C_L are circuit constants inherent in all surge testing circuits and since these constants are not susceptible to appreciable control or modification, it follows that the shortest fronts generated, free from oscillations, are fixed within certain limits. The curves of Fig. 9 give the length or duration of the front as a function of the constants of the surge generator circuit. These curves are calculated from general equation (**3**) in Appendix I.

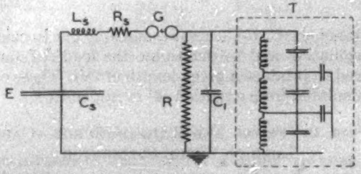


Fig. 4—Circuit Diagram of Surge Generator with Transformer Load

C_1, load capacity
T, simplified circuit diagram corresponding to a three-group shell type winding (taking into account mutual coupling between primary and secondary)

A lead connecting the surge generator proper to the apparatus tested introduces the circuit condition as shown in Fig. 6. The stray capacity C_2 of the surge generator proper is generally considerably smaller than the capacity C_T of the apparatus tested. The damping resistance

$$R_s = 2\sqrt{\frac{L_s}{C_L}},$$

where $L_s = L_{s1} + L_{s2}$ and $C_L = C_2 + C_T$, eliminates the low order frequency oscillations superimposed on the front and crest of the surge. High order frequency oscillations of a period approximately

$$2\pi\sqrt{\frac{L_{s1}L_{s2}}{L_{s1}+L_{s2}}C_2}$$

remain superimposed along the rise of the front, and these may be noted on a number of oscillograms. To eliminate these higher-frequency oscillations would require additional damping resistance particularly in the lead, but too large a resistance in the circuit between generator and apparatus tested would be objectionable, as for a given capacity load C_T this tends to lengthen the front. It should be noted, however, that these high frequency oscillations occur only on the front of the surge and they are of short duration; they occur in particular at the surge generator as discussed in Appendix III.

The time duration or length of the tail of the surge voltage to approximately half-crest value depends largely on the value ($0.693 \times RC_s$); to be more exact, the load capacity also contributes to the tail length, as is shown in Fig. 7. In the case of short tail surges, where R_s becomes appreciably large as compared to R, both resistances affect additively the tail length.

TABLE II—OSCILLOGRAMS AND DATA ON 3,000-KV. SURGE GENERATOR AT SHARON

Oscillogram	Load resistance	Inserted damping resistance*	Nature of load†‡
	(ohms)	(ohms)	(capacity load)
A	8,900	0	No load
B	8,900	100	No load
C	8,900	200	No load
D	8,900	600	No load
E	8,900	0	Capacity load C_T = 700 $\mu\mu$f. approx.
F	8,900	600	Capacity load C_T = 450 $\mu\mu$f. approx.
G	1,850	600	Capacity load C_T = 450 $\mu\mu$f. approx.
H	950	0	No load
I	950	300	No load
J	600	200	No load

*R_s = inserted resistance + inherent resistance of test circuit.
Inherent resistance of test circuit . . . 100 ohms approx.

†Total load capacity = stray capacity to ground of generator + capacity of apparatus tested.
C_L = $C_2 + C_T$, as shown in Fig. 6b.
Stray capacity of generator = ground capacity of condenser bank structure, capacity of sphere gaps, leads, etc. . . 600 $\mu\mu$f. approx.

‡Total inductance = inductance of generator proper + inductance of connecting lead . . . 100 μh approx.
L_s = $L_{s1} + L_{s2}$, as shown in Fig. 6b.
L_{s1} = 60 μh approx.
L_{s2} = 40 μh approx.

In Table II are listed a number of oscillograms corresponding to widely different conditions of capacity loads. Measurements were made at the generator. These experimental results refer to the 3,000,000-volt surge generator at the Sharon Laboratory. Calculations by the methods of this paper of the surge generator performance corresponding to several of the more important oscillograms have been made. The calculated curves corresponding to the oscillograms are included for comparison.

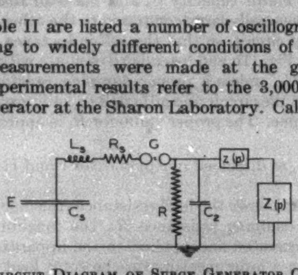
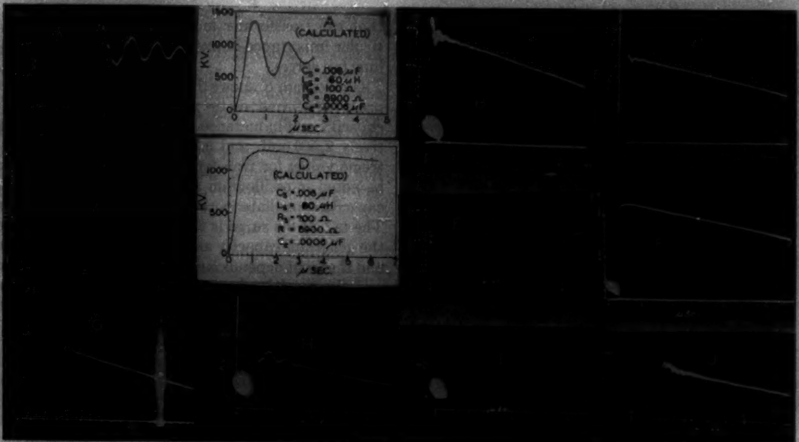
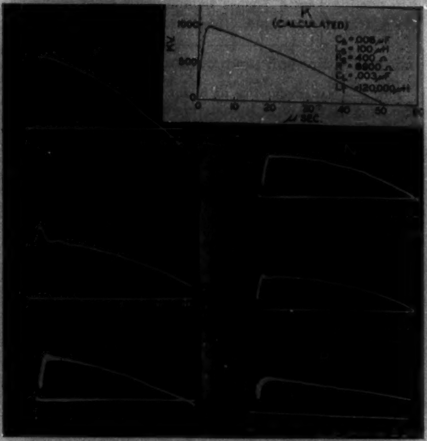


Fig. 5—Circuit Diagram of Surge Generator Connected to Load of Impedance $Z(p)$ Through Impedance $z(p)$

$Z(p)$ is the generalized impedance of the load and represents any type of load, for example a shell or core type transformer load

With the surge generator unloaded and with no series resistance added in the condenser bank, the surge voltage generated is given in oscillogram *A*. The test circuit corresponds to Fig. 6a with no load, however, connected to the surge generator lead. The inherent series resistance in the condenser bank for oscillogram *A* is not sufficiently large to dampen the oscillation superimposed on the unidirectional surge. Increasing the series resistance R_s to approximately twice the oscillation impedance $\sqrt{L_{s1}/C_2}$ practically eliminates the oscillation. The inductance L_{s1} and the capacity C_2 are approximately 60 μh and 600 $\mu\mu$f. Oscillograms *B*, *C* and *D* illustrate the point in question.





In the circuit set-up of Fig. 6a, connecting the generator with no damping resistance inserted in the condenser bank to apparatus equivalent to a capacity load C_T = 700 $\mu\mu$f, the surge generated is given by the oscillogram *E*. The lead connecting from the generator to the capacity load has an inductance L_{s2} in the order of 40 μh. As outlined in Appendix III, the periods of the superimposed oscillations are given from the relation in equation (**13**). Thus the calculated fundamental period of oscillation T_1 is 1.93 microseconds which compares closely to the experimental value. The period of the second harmonic T_2 calculates $\frac{1}{3}\, T_1$. There are also present ripples on the wave which are due to the distributed circuit constants throughout the entire test circuit.

In the circuit set-up for oscillogram *F*, the load capacity C_T is approximately 450 $\mu\mu$f. A total series resistance R_s = 700 ohms in the condenser bank completely dampens all superimposed oscillations. The surge indicated is practically a 2/55 microsecond wave, or approximately a $3\, R_sC_L/0.693\, R(C_s + C_L)$ wave as determined from curves of Figs. 7 and 9, where $C_L = C_2 + C_T$.

In the surge generator set-up from which oscillogram *F* was obtained, decreasing the load resistance from 8,900 to 1,850 ohms, R_s = 700 ohms, shortens the tail of the surge to approximately $(1{,}850 + 700)/(8{,}900) \times 100$ per cent $\cong$ 30 per cent. This point is clearly demonstrated in oscillogram *G*.

Oscillogram *H* shows the effect on the surge wave resulting from a further reduction of the load resistance from 1,850 to 950 ohms, where R_s = 100 ohms. Increasing the series resistance to R_s = 400 ohms dampens the superimposed oscillation as shown in oscillogram *I*. Oscillogram *J* corresponds closely to a ½/5 wave.

It should be noted that damping of superimposed oscillations in the equivalent circuits Figs. 1b and 2 may also be secured by a load resistance

$$R \leqq \frac{1}{2}\sqrt{\frac{L_s}{C_T}},$$

where R_s may be zero. However, the required damping load resistance would become unduly small in the practical test circuit set-up, so generally damping of oscillations is largely effected by series resistance. The damping resistance at the Sharon Laboratory is distributed along the entire condenser bank.

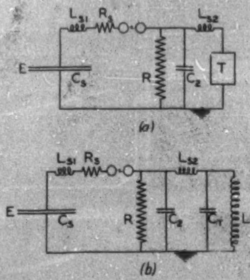


FIG. 6—CIRCUIT DIAGRAM OF SURGE GENERATOR CONNECTED TO TRANSFORMER LOAD THROUGH LEAD OF APPRECIABLE LENGTH

EFFECT OF TRANSFORMER LOAD ON SURGE GENERATED

Transformer loads introduce in various degrees modifying effects on the test circuit. In addition to those modifications on the surge front due to the total load capacity, discussed above, there results from the transformer inductance a modification in the surge tail.

With a transformer load, as shown in Fig. 2, the surge tail depends primarily upon the capacity $C = (C_s + C_L)$, the load resistance R and the transformer inductance L_T in case this circuit constant is relatively small. In Fig. 10 the length of the surge tail is expressed as a function of the test circuit constants that primarily determine the tail. Both the inductance and series resistance of the surge generator have a negligible effect upon the length of long tail waves. It is seen that the transformer inductance plays a major role for values $L_T < 4\,R^2C$, in which case the generator discharges largely through the inductance. In fact for values of L_T in the order of $0.4\,R^2C$, the generator and load capacities oscillate markedly with the transformer inductance, the surge voltage decreasing from crest to half value in a time approximately

$$\frac{1}{2}\sqrt{CL_T}\tan^{-1}\sqrt{\frac{4\,R^2C}{L_T}}.$$

The maximum possible tail length obtainable is with $R = \infty$, for which the tail reaches zero in $\frac{\pi}{2}\sqrt{L_TC}$, and half crest value in $\frac{\pi}{3}\sqrt{L_TC}$. It should be noted that the transformer inductance may become of particular importance in setting a generator test circuit for long tail surges.

Oscillogram *K* shows the effect of a 42,000 kva. surge-proof transformer on the wave of the surge voltage. The capacity and inductance of this transformer are respectively 2,500 $\mu\mu$f. and 0.12 henry; these represent extreme values for transformer units of this size. It is to be noted in oscillogram *K* that the generator and load capacities oscillate with the transformer inductance. The tail of the surge is therefore practically fixed from the generator capacity and the transformer inductance, and the front depends largely on the damping resistance and the total load capacity. The damping resistance R_s is 400 ohms as compared to 200 ohms for oscillogram *L*. This points out again the necessity of adjusting the damping resistance in the order of twice the oscillation impedance $\sqrt{\frac{L_s}{C_L}}$ to eliminate practically all oscillations superimposed on the surge wave. In this case the damping resistance R_s should be in the order of

$$2\sqrt{\frac{100 \times 10^6}{3,000}} \cong 360 \text{ ohms.}$$

More accurately, using C_s in place of C_L, the formula

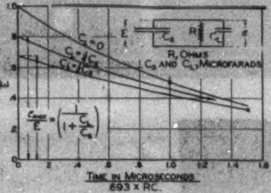


FIG. 7—CHARACTERISTICS OF SURGE GENERATOR CIRCUIT IN RELATION TO CAPACITY REGULATION AND LENGTH OF TAIL OF SURGE GENERATED

Example—Suppose $C_L = \frac{1}{4}C_s$, where $C_s = 0.008$ μf. Find: (a) regulation; (b) value of R to produce a tail of 40 microsec. to half crest value

$$\text{Regulation} = \frac{e\ \text{max}}{E} = \frac{1}{1 + \frac{C_L}{C_s}} = 0.8$$

Length of tail to half crest value (middle curve) = $1.25 \times 0.693 \times RC_s$ microseconds

$$R = \frac{40}{1.25 \times 0.693 \times 0.008} \cong 6{,}000 \text{ ohms}$$

gives 430, the difference being due to the large load capacity.

Oscillogram *M* gives the surge voltage with another large transformer load. With the load resistance removed, the length of the tail is increased to 125 per cent, as shown in oscillogram *N*. An increase in the generator capacity to 150 per cent further lengthens the tail to $\sqrt{1.5} \times 100$ per cent $= 125$ per cent, as given in oscillogram *O*.

Of further interest in regard to the surge testing of transformers is oscillogram *P*. In this case connecting the transformer produced practically no change in the shape of the surge, except for a lengthening of the front with the transformer connected. The transformer here tested has in the first place a large inductance L_T; second, the voltage distribution is non-uniform and there are marked oscillations in the winding. The oscillations in the winding do not set up any appreciable impedance drop in the generator circuit; that is, the generalized impedance $Z(p)$ of the winding, Fig. 5, is for practical purposes of a high order magnitude relative to the impedance of the surge generator. Surge testing on other transformers with non-uniform voltage distribution have given similar results, thus resulting in a smooth surge wave practically independent of the oscillations in the winding.

The high frequency oscillations due to the lead and the stray capacity of the generator are to be noted along the rising part of the front of the wave on a number of the oscillograms shown.

TABLE III—OSCILLOGRAM AND DATA ON 3,000-KV. SURGE GENERATOR AT SHARON*

Oscillogram	C_s	Load resistance	Inserted damping resistance	Nature of load
	(μμF)	(ohms)	(ohms)	(Transf. load)
K	8,000 All condensers in series	8,900	300	42,000-kva. power trans. $C_T = 2{,}500$ μμf.; $L_T = .12$ h
L	8,000 All condensers in series	8,900	100	42,000-kva. power trans. $C_T = 2{,}500$ μμf.; $L_T = .12$ h.
M	8,000 All condensers in series	9,200	300	22,250-kva. power trans.
N	8,000 All condensers in series	Load resistance removed†	300	22,250-kva. power trans.
O	12,000 Two-thirds of condensers in series	Load resistance removed†	300	22,250-kva. power trans.
P	8,000	9,200	700	1,000-kva. power trans.

*Reference to Table II for other test circuit constants.

†Load resistance present is that due to charging buses resistance only. This is very large.

In conclusion, the author desires to acknowledge the helpful assistance of Mr. F. J. Vogel for the experimental data obtained at the Sharon High-Voltage Laboratory, of Mr. H. V. Putman for his constructive criticism, and of other members in the Engineering Department of the Westinghouse Electric and Manufacturing Co.

Appendix I

The solution for the voltage generated in the circuit of Fig. 1b expressed operationally[3] is:

$$e = \frac{\dfrac{R}{RC_Lp + 1}}{\dfrac{1}{C_sp} + L_sp + R_s + \dfrac{R}{RC_Lp + 1}} E$$

$$e = \frac{\dfrac{p}{L_sC_L}}{p^3 + \left(\dfrac{R_s}{L_s} + \dfrac{1}{RC_L}\right)p^2 + \left(\dfrac{1}{L_sC_s} + \dfrac{1}{L_sC_L} + \dfrac{R_s}{L_sC_LR}\right)p + \dfrac{1}{L_sC_sC_LR}} E$$

Expressing the denominator in the form

$$(p + \alpha)\,[\,(p + \lambda)^2 + \omega^2]$$

and denoting

$$A = \frac{1}{L_sC_L},$$

$$e = \frac{Ap}{(p + \alpha)\,[\,(p + \lambda)^2 + \omega^2]} E,$$

which expands into:

$$e = \left[\frac{p}{p + \alpha} - \frac{\lambda - \alpha}{\omega}\,\frac{\omega p}{(p + \lambda)^2 + \omega^2} - \frac{p^2 + \lambda p}{(p + \lambda)^2 + \omega^2}\right] \frac{AE}{(\lambda - \alpha)^2 + \omega^2}$$

Thus the solution of the voltage (e) as a function of time (t) becomes:

$$e = \left[\epsilon^{-\alpha t} - \epsilon^{-\lambda t}\left(\frac{\lambda - \alpha}{\omega} \sin \omega t + \cos \omega t\right)\right] \frac{AE}{(\lambda - \alpha)^2 + \omega^2} \qquad (3)$$

Equation (3) states that an oscillation is superimposed on the unidirectional component $\dfrac{AE}{(\lambda - \alpha)^2 + \omega^2}\,\epsilon^{-\alpha t}$.

The oscillation is completely eliminated for the condition $\omega^2 = 0$, in which case equation (3) transforms into:

$$e = [\,\epsilon^{-\alpha t} - \epsilon^{-\lambda t}\,\{\,(\lambda - \alpha)\,t + 1\,\}\,]\,\frac{AE}{(\lambda - \alpha)^2} \qquad (4)$$

Whereas, in case ω^2 is negative ($\omega^2 = -\,\psi^2$), equation (3) transforms into:

$$e = \left[\epsilon^{-\alpha t} - \epsilon^{-\lambda t} \left(\frac{\lambda - \alpha}{\psi} \sinh \psi t + \cosh \psi t \right) \right] \frac{AE}{(\lambda - \alpha)^2 - \psi^2} \qquad (5)$$

Referring to the circuit of Fig. 1b, within the practical range of the circuit constants, the circuit elements: C_s, L_s, R_s and C_L constitute for practical purposes the oscillating circuit. As it is desirable to secure a smooth wave of the surge generated, to this end the series resistance is increased for practical purposes to a value in the order of

$$R_s \cong 2\sqrt{\frac{L_s}{C_o}} \cong 2\sqrt{\frac{L_s}{C_L}}; \quad C_o = \frac{C_s C_L}{C_s + C_L} \cong C_L$$

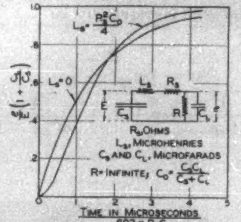


FIG. 8—EFFECT OF INDUCTANCE OF SURGE GENERATOR ON FRONT OF SURGE

A damping series resistance in this order practically eliminates the superimposed oscillation in equation (3).

Introducing the condition

$$R_s = 2\sqrt{\frac{L_s}{C_o}}$$

in the original operational expression above and expressing the time constants in term of $R_s C_L$, the operational expression becomes:

$$e = \frac{Ap}{p^3 + ap^2 + bp + c}$$

where

$$a = \left(\frac{1}{R_s C_L} \right) \left(4 \frac{C_L}{C_o} + \frac{R_s}{R} \right)$$

$$b = \left(\frac{1}{R_s C_L} \right)^2 \left(4 \frac{C_L}{C_o} \frac{C_L}{C_s} + 4 \frac{C_L}{C_o} + 4 \frac{C_L}{C_o} \frac{R_s}{R} \right)$$

$$c = \left(\frac{1}{R_s C_L} \right)^3 \left(4 \frac{C_L}{C_o} \frac{C_L}{C_s} \frac{R_s}{R} \right)$$

$$A = \left(\frac{1}{R_s C_L} \right)^2 \left(4 \frac{C_L}{C_o} \right)$$

Thus the solution for the voltage (e) is expressed in terms of the time constant $R_s C_L$. The curves in Fig. 9 are calculated in terms of $R_s C_L$ for various ratios of $\frac{C_L}{C_s}$ and $\frac{R_s}{R}$. These curves summarize the characteristics of circuit Fig. 1b, particularly the relation of the time constant $R_s C_L$ as a function of the duration of the surge front.

In case L_s and R_s are small relative to the other circuit constants, the crest and tail of the surge are given for practical purposes by the simple relation:

$$e = \frac{E}{1 + C_L/C_s} \epsilon^{-\frac{t}{R(C_s + C_L)}} \qquad (6)$$

The curves in Fig. 7 give the characteristics of this circuit.

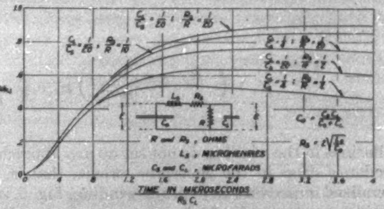


FIG. 9—CHARACTERISTICS OF SURGE GENERATOR CIRCUIT IN RELATION TO FRONT AND CREST OF SURGE GENERATED

Example—Capacity load, $C_L = 0.002$ μf.

Surge generator constants, $C_s = 0.008$ μf and $L_s = 100$ μh.

Problem is to find shortest front obtainable with a smooth surge having a 40-microsec. tail

Series damping resistance required,

$$R_s \cong 2\sqrt{\frac{L_s}{C_o}} \cong 2\sqrt{\frac{L_s}{C_L}} \cong 500 \text{ ohms}$$

From curves of Fig. 7 the load resistance required to give 40 microsec. tail is $R \cong 6{,}000$ ohms

$$\text{Then } \frac{C_L}{C_s} = \frac{0.002}{0.008} = \frac{1}{4}; \frac{R_s}{R} = \frac{500}{6{,}000} = \frac{1}{12}$$

From interpolation of the proper curves of Fig. 9 the time for the surge to reach full crest value is approx. $2.5\, R_s C_L = 1{,}250 \times 0.002 = 2.5$ microsec.

In case the load resistance R is very large and for

$$R_s = 2\sqrt{\frac{L_s}{C_o}},$$

equation (3) becomes:

$$e = \left[1 - \epsilon^{-\frac{R_s t}{2 L_s}} \left(\frac{R_s}{2 L_s} t + 1 \right) \right] \frac{E}{1 + \frac{C_L}{C_s}} \qquad (7)$$

and for zero inductance ($L_s = o$)

$$e = \left[1 - \epsilon^{-\frac{t}{R_s}\left(\frac{1}{C_s} + \frac{1}{C_L} \right)} \right] \frac{E}{1 + \frac{C_L}{C_s}} \qquad (8)$$

The last two equations are plotted in Fig. 8 to compare the rate of rise of the front with and without inductance.

Appendix II

A complete analysis of the circuit in Fig. 2 results in complex expressions. A clearer insight into the practical problem, and with a good degree of accuracy, is best gained by segregating the elements of the circuit that determine respectively the front and the tail of the surge. The front of the surge depends largely, for circuit conditions of practical interest, on the circuit elements discussed in Appendix I. The tail of the surge for large values of L_T relative to $4\,R^2C$ is also given for practical purposes in the equations of Appendix I. In case L_T is considerably less than $4\,R^2C$, the tail of the surge then depends in a large measure on the elements of the circuit in Fig. 10; the operational solution of this circuit is:

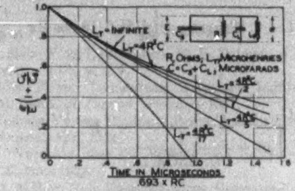


FIG. 10—EFFECT OF TRANSFORMER INDUCTANCE ON TAIL OF SURGE

Example—$C_s = 0.008\ \mu f$; $L_T = 0.12 \times 10^6\ \mu h$; load capacity $C_L = 0.002\ \mu f$; L_s and R_s disregarded

Problem: With $R = 6{,}000$ ohms, which was required to produce a 40 microsec. tail either without transformer connected or with transformer of high inductance connected, find length of tail with transformer of low inductance connected

$$\frac{L_T}{4R^2C} = \frac{0.12 \times 10^6}{4 \times 36 \times 10^6 \times 0.01} = \frac{1}{12}$$

Reading from the curves we get time for tail to reach half value $= 0.57 \times 0.693 \times RC = 24$ microsec.

The maximum possible tail obtainable is with $R = \infty$, for which the tail reaches zero in

$$\frac{2\pi}{4}\sqrt{L_TC} = 1.57\sqrt{0.12 \times 10{,}000} = 55 \text{ microsec.}$$

The slope of the tail under this condition is sinusoidal (see oscillograms *N* and *O*), so it reaches half crest value in 2/3 its total length or 37 microsec.

$$e = \frac{\dfrac{RL_Tp}{R + L_Tp}}{\dfrac{1}{Cp} + \dfrac{RL_Tp}{R + L_Tp}} \; \frac{E}{1 + \dfrac{C_L}{C_s}}$$

$$= \frac{p^2}{p^2 + \dfrac{1}{RC}\,p + \dfrac{1}{L_TC}} \; \frac{E}{1 + \dfrac{C_L}{C_s}}$$

Expressing the denominator in the form $(p + \alpha)^2 + \omega^2$,

$$e = \frac{p^2}{(p + \alpha)^2 + \omega^2} \; \frac{E}{1 + \dfrac{C_L}{C_s}},$$

which expands into:

$$e = \left[\frac{p^2 + \alpha p}{(p + \alpha)^2 + \omega^2} - \frac{\alpha}{\omega}\,\frac{\omega p}{(p + \alpha)^2 + \omega^2} \right] \frac{E}{1 + \dfrac{C_L}{C_s}}$$

Thus the solution of the voltage *e* as a function of the time *t* becomes:

$$e = \epsilon^{-\alpha t}\left[\cos \omega t - \frac{\alpha}{\omega} \sin \omega t\right] \frac{E}{1 + \dfrac{C_L}{C_s}} \tag{9}$$

where

$$\omega = \sqrt{\frac{1}{L_TC} - \frac{1}{4\,R^2C^2}}; \quad \alpha = \frac{1}{2\,RC}; \quad C = C_s + C_L$$

For the condition $L_T = 4\,R^2C$, equation (9) transforms to:

$$e = \epsilon^{-\alpha t}\,(1 - \alpha t)\,\frac{E}{1 + \dfrac{C_L}{C_s}} \tag{10}$$

These equations are evaluated in the curves of Fig. 10 for various values of L_T that are of practical interest.

Appendix III

The effect of the lead connecting the generator to the apparatus tested is of interest from various aspects.

In case the lead is short relative to the physical outlay of the generator, the equivalent test circuit is for practical purposes as in Fig. 2. In such a case, voltage measurements at the generator are practically identical to measurements at the apparatus. A smooth surge wave is secured by means of an appropriate damping series resistance.

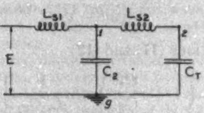


FIG. 11

A lead of appreciable length relative to the inductance of the generator proper introduces a condition in the test circuit as shown in Fig. 6. The solution of these circuits results in complex expressions, hence a physical analysis is considered below resulting in simpler terms and a clearer insight into the problem. With no damping resistance, the equivalent circuit of interest is as shown in Fig. 11; the operational solution of this circuit is:

$$e_{1g} = \frac{\dfrac{\dfrac{1}{C_2p}\left(L_{s2}p + \dfrac{1}{C_Tp}\right)}{\dfrac{1}{C_2p} + L_{s2}p + \dfrac{1}{C_Tp}}}{L_{s1}p + \dfrac{\dfrac{1}{C_2p}\left(L_{s2}p + \dfrac{1}{C_Tp}\right)}{\dfrac{1}{C_2p} + L_{s2}p + \dfrac{1}{C_Tp}}} E$$

$$e_{1g} = \frac{\dfrac{p^2}{L_{s1}C_2} + \dfrac{1}{L_{s1}L_{s2}C_2C_T}}{p^4 + \left[\dfrac{1}{L_{s2}C_2}\left(1 + \dfrac{C_2}{C_T}\right) + \dfrac{1}{L_{s1}C_2}\right]p^2 + \dfrac{1}{L_{s1}L_{s2}C_2C_T}} E$$

$$e_{1g} = \frac{\dfrac{p^2}{L_{s1}C_2} + \omega_1^2\,\omega_2^2}{(p^2 + \omega_1^2)\,(p^2 + \omega_2^2)} E.$$

The solution of the voltage (e_1g) as a function of time becomes:

$$e_{1g} = \left[1 + \frac{\dfrac{1}{L_{s1}C_2} - \omega_2^2}{\omega_2^2 - \omega_1^2} \cos \omega_1t \right.$$

$$\left. + \frac{\omega_1^2 - \dfrac{1}{L_{s1}C_2}}{\omega_2^2 - \omega_1^2} \cos \omega_2t \right] E \qquad (11)$$

In much the same way

$$e_{2g} = \frac{1}{L_{s2}C_Tp^2 + 1}\, e_{1g}$$

$$e_{2g} = \left[1 - \frac{\omega_2^3}{\omega_2^2 - \omega_1^2} \cos \omega_1t \right.$$

$$\left. + \frac{\omega_1^2}{\omega_2^2 - \omega_1^2} \cos \omega_2t \right] E \qquad (12)$$

where in equations (**11**) and (**12**)

$$p^4 + \left[\frac{1}{L_{s2}C_2}\left(1 + \frac{C_2}{C_T}\right) + \frac{1}{L_{s1}C_2}\right]p^2$$

$$+ \frac{1}{L_{s1}L_{s2}C_2C_T} = (p^2 + \omega_1^2)\,(p^2 + \omega_2^2)$$

$$= \left[p^2 + \left(\frac{2\pi}{T_1}\right)^2\right]\left[p^2 + \left(\frac{2\pi}{T_2}\right)^2\right] \qquad (13)$$

Thus for the circuit conditions:

Example 1: $L_{s1} = 60\ \mu h$ $\quad C_2 = 600\ \mu\mu f$
$L_{s2} = 40\ \mu h$ $\quad C_T = 2{,}000\ \mu\mu f$

$$e_{1g} = \left[1 - 0.68 \cos 2\pi \frac{t}{2.94} - 0.32 \cos 2\pi \frac{t}{0.72}\right] E$$

$$e_{2g} = \left[1 - 1.06 \cos 2\pi \frac{t}{2.94} + 0.06 \cos 2\pi \frac{t}{0.72}\right] E$$

Example 2: $L_{s1} = 60\ \mu h$ $\quad C_2 = 600\ \mu\mu f$
$L_{s2} = 40\ \mu h$ $\quad C_T = 600\ \mu\mu f$

$$e_{1g} = \left[1 - 0.82 \cos 2\pi \frac{t}{1.84} - 0.18 \cos 2\pi \frac{t}{0.63}\right] E$$

$$e_{2g} = \left[1 - 1.13 \cos 2\pi \frac{t}{1.84} + 0.13 \cos 2\pi \frac{t}{0.63}\right] E$$

For the loads considered, which correspond to the range of transformer capacities encountered, the periods of the fundamental and the higher frequency oscillations may be simply expressed with good approximation as:

$$T_1 = 2\pi \sqrt{(L_{s1} + L_{s2})\,C_T}$$

and

$$T_2 = 2\pi \sqrt{\frac{L_{s1}\,L_{s2}}{L_{s1} + L_{s2}}\,C_2}$$

The oscillation at the load is largely of fundamental period, whereas both the fundamental and the higher frequency oscillations appear in comparable proportion at the generator end.

A proper damping resistance in the order of

$$2\sqrt{\frac{L_{s1} + L_{s2}}{C_T}}$$

inserted in series with the inductance $L_s = (L_{s1} + L_{s2})$ practically eliminates the fundamental oscillation; this damping resistance may be distributed along the generator condenser bank. Inasmuch as the higher-frequency oscillation is largely confined between the connecting lead and the generator stray capacity, a proper resistance approximately in the order of

$$2\sqrt{\frac{L_{s2}}{C_2}}$$

inserted in the lead dampens this oscillation. From the viewpoint of the surge at the transformer load, inasmuch as the higher-frequency oscillation is confined between the connecting lead and the generator stray capacity, this oscillation is of practically no importance.

With a proper damping resistance in the generator condenser bank and a plain lead, voltage measurements at the generator show the higher-frequency oscillation superimposed along the rise of the front. Generally this oscillation disappears before the crest of the wave is reached. It should be noted that even with all superimposed oscillations eliminated, there always will be a difference in the voltage measured at the generator as compared to the voltage at the load; in case of short leads there can be only a theoretical difference between e_{1g} and e_{2g}, and this theoretical difference is confined only to the rise in the front of the surge.

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Discussion

C. L. Fortescue: The importance of this paper lies in the fact that Mr. Bellaschi has demonstrated that what may appear as extremely complicated circuits can be represented with a close approximation by simple circuits such as those shown in Figs. 1a, 1b, 2 and 6.

The circuits shown in Fig. 1 are straightforward and can be solved by the ordinary methods of differential equations or of the Heaviside Operational Calculus. The circuits shown in Figs. 2 and 6 look simple but are considerably more complicated than they appear. However Mr. Bellaschi has shown how a good approximation may be obtained by segregating portions of the circuits, analyzing them separately, and combining the results obtained in the proper manner.

In all experimental and testing work it is of great advantage to be able to check up by calculation results that may be expected under test. One is more apt to have confidence in test results which can be predicted with close approximation by calculations. Mr. Bellaschi has shown in detail in Appendixes 1, 2, and 3, the methods of carrying out the calculations which will apply to most practical cases and he has shown by computed curves *A*, *D*, and *K* how close these computed results check the actual cathode ray oscillograms obtained as marked *A*, *D*, and *K*.

When it was first suggested that standardization of wave form of surges for testing be undertaken I demurred, stating my reason that it would be necessary not only to standardize the waves but also to standardize the surge generator itself for the reason that two surge generators giving the same open circuit wave would not necessarily give the same wave under test conditions. This is no longer to be feared now that Mr. Bellaschi has shown how easy it is to predict the actual wave under test conditions and therefore we may go ahead with specifications for surge testing of apparatus without fear provided that measurements of the surge to which the apparatus is subjected are properly carried out and calculations are made beforehand to insure that the apparatus is receiving the proper test.

F. J. Vogel: There is a particular timeliness to the paper by Mr. Bellaschi. Surge testing of transformers to prove their surge strength is now a common procedure, and the Transformer Subcommittee of the Institute is even now engaged in the formulation of a code for such testing.

There has heretofore been no general understanding of the limitations of the means of testing. Probably most engineers have thought that the characteristics of surges could be easily altered by changing various constants in the generator. For example, data have been published using waves of various fronts and tails. Mr. Bellaschi has shown that specified characteristics can be obtained only within certain limits or with generators of given design. Two of these limitations are very important, particularly for use in the present work of the Transformer Subcommittee, and for the designers of such testing installations.

One limitation is in the steepness of the front of the wave. This is determined by the series resistance, the internal inductance of the generator and the capacity of the transformer. Of these constants, only the series resistance is subject to alteration, as the inductance is inherent in the generator, and the capacity of the transformer is of course fixed. The adjustment of the resistance is determined by the desirability of testing with a wave free from oscillation at the transformer. That this is desirable can be seen from the following facts:

1. The waves generally described in the past in surge testing have given the front and the length of the tail, which are perfectly definite things. If oscillations were permitted, the waves could not be so defined. The frequency and amplitude of the oscillations would vary, and require limitation. Therefore in the interests of simplicity and clearness, the elimination of oscillations is desirable.

2. The elimination of the oscillations will avoid variations in the reported performance of gaps, bushing and insulation. In the earlier surge investigations, the surges obtained for test were often calculated considering only the generator condenser capacity and the load resistance. Oscillations due to inductance and stray capacity were not taken into consideration, and when the voltage was measured by the sphere gap not only were fictitiously high results obtained, but agreement could not be obtained between different observers with different test apparatus due to the difference in oscillations obtained. Similarly if these oscillations are not damped out, the tests by different manufacturers will not be comparative, and they will furnish fictitious values for the tests made.

3. Oscillations will result in surge characteristics not easily predetermined for comparison with the oscillograms obtained by test.

Mr. Bellaschi has shown that oscillations may be eliminated by proper adjustment of the series resistance and that this limits the steepness of the wave fronts which may be obtained. It is further to be noted that this limitation is unavoidable, since there is a limit to which the inductance inherent in any generator can be reduced. However with the series resistance adjusted to damp oscillations at the transformer, wave fronts of from ½ to 2½ μsec can be obtained. These should be satisfactory for transformer testing.

Due to the fact that with certain combinations the crest of the wave is very flat, it is very difficult to make a definite statement as to the length of the entire front of the wave. A similar trouble often exists at the very beginning of the wave. Since these difficulties are often present, it is better to adopt some conventional method of defining the front which will lead to greater uniformity. One method which has been suggested has been to use the time from the beginning of the wave up to 90 per cent of its crest value. This time is in turn multiplied by the factor ten-ninths to obtain the entire front of the wave. Another method which has been suggested is based on the time elapsed between values of 10 per cent and 90 per cent of the crest value of the wave. This time would be multiplied by ten-eighths to obtain the full value of the front of the wave. Either of these two methods appears to be a great advantage in the standardization of wave fronts for transformer testing.

The other limitation is in the maximum length of the surge which can be obtained. In testing insulator strings or gaps the length of the tail of the surge is controlled by the adjustment of the discharge resistance of the surge generator. When testing transformers however, particularly large ones, the inductance of the transformer is so low compared to the discharge resistance that it becomes the controlling factor. The maximum length of the tail that can be obtained therefore depends on the internal capacity of the surge generator and the inductance of the transformer. It has been agreed that a 40 μsec. tail should be suitable for surge testing and the capacity of the Sharon generator was made large enough to secure this length even with the largest transformer.

C. Francis Harding: The profession is greatly indebted to the author of this paper for his detailed analysis of the characteristics of the discharge circuit of the surge generator, particularly with the surge-proof transformer used as a testing load. Obviously, the basic and unchangeable constants of such loads subjected to surge tests greatly affect the characteristics of the impressed wave. Hence the necessity of having cathode ray oscillograms of the exact wave impressed upon the test equipment accompanied by sufficient circuit data or check calculations to convince the purchaser that the oscillogram actually represents the surge potential to which the test specimen has been exposed.

In this connection it is well to point out to the prospective purchaser of surge tested equipment that the inductance of his equipment to be tested is not that which might be measured at 60 cycles nor necessarily that which might have been calculated from the Bureau of Standards formula based upon the geometry of the coils, but rather an equivalent inductance modified, as the result of the high frequency of discharge, by the distributed capacitance of the winding to be tested. Such an equivalent inductance, which the author considers correctly but which is frequently either neglected or incorrectly determined in such tests, may be determined either experimentally or by calculation if the proper rate of change of current is considered. Experimentally, it may be determined by measuring the potential and the discharge current by means of sphere gaps connected in parallel with the test load and a known series resistance respectively. Several such measurements at varying potentials, checked by means of a high frequency timing wave upon the oscillogram, permit the determination of the maximum discharge current and the time in microseconds at which the maximum current of the surge exists. These empirical values, when substituted into the fundamental discharge equations of the surge generator, will provide an approximate value of the equivalent inductance in the discharge circuit. Secondly, the equivalent inductance may be calculated from an oscillogram taken with a minimum of resistance in the discharge circuit. The resultant oscillatory frequency taken from the cathode ray oscillogram may then be used to determine the relation between the known induced potential across the inductive portion of the circuit and the inductance itself (expressed for the frequency of the actual surge discharge). A very close approximation has been secured at Purdue University with the use of these two methods in testing so-called surge-proof distribution transformers and other high-voltage equipment.

C. M. Foust: This paper on impulse testing of transformers is timely and valuable. The application of rigid mathematical analysis to discharge circuit conditions is to be commended. Added significance is obtained for sphere gap and cathode ray oscillograph measurements when they are interpreted in terms of capacitance, inductance and resistance constants of the impulse generator and connected circuits.

Several points in this paper invite questions but the interest of those engaged in impulse testing work is attracted particularly to the electrical mechanism of the discharge circuit and the circuit constants.

Referring to Table II and the oscillograms of the paper capacitance C_g is the ground capacitance of the condenser bank structure, capacitance of the sphere gap and leads and is taken as approximately 600 μμf. I assume that this value was chosen in accordance with the frequency and amplitude of the superimposed oscillations on the oscillograms. To me this 600 μμf. is considerably more than expected. A 150-cm. diameter sphere gap capacitance would not be in excess of 100 μμf. and a 100 ft. lead not in excess of 50 μμf. by calculation. This would leave 450 μμf. for the grand capacitance of the condenser bank structure. It is somewhat difficult to accept so high a figure in view of general experience with such circuit constants. On the basis of an 0.008 μf. capacitance for the impulse generator to ground this stray capacitance of approximately 450 μμf. is about 6 per cent of main capacitor C_s.

Some data and oscillograms taken in Schenectady are of interest on this point. Circuit constants of the impulse generator were arranged for a (1/2-5) wave. The capacitance C_s was 0.0083 μf., the inductance L_s 80 microhenrys and the load resistance 650 ohms. No added series resistance R_s was inserted. Oscillograms of wave shapes were taken as follows:

1. With the discharge circuit alone.
2. Discharge circuit with 8-unit string connected.
3. Discharge circuit with a 50-cm. diameter sphere gap connected.
4. Discharge circuit with the insulator string and sphere gap both connected.

The wave shapes are shown in Fig. 1. The oscillations are very evident on the crests of the waves. That they are less pronounced than several of those shown by Mr. Bellaschi is, of course, partly due to the use of a low load resistance R_L. Analysis of these oscillograms gave the following results:

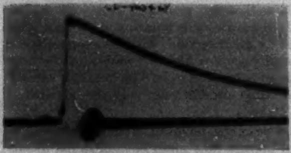
	Capacitance C_L from oscillograms μμf.
1. Circuit alone	65
2. Insulators	95
3. Sphere gap	90
4. Insulators and sphere gap	125

A more thorough presentation of these data is given in the *General Electric Review* for July, 1932.

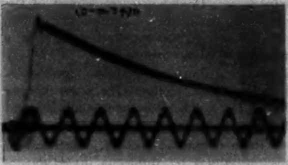
These data give a value of 30 μμf. for the insulator string, 25 μμf. for the 50-cm. diameter sphere gap and 60 μμf. for the two in parallel. Such values appear very reasonable when compared with calculated values. The 65 μf. stray capacitance to ground of the impulse generator is less than 0.8 of 1 per cent of the main capacitor C_s. In view of such experience it becomes difficult to accept the 600 μμf. value for C_L given by the author. This point appears to deserve further attention and any comments Mr. Bellaschi wishes to make will be appreciated.

W. L. Lloyd, Jr.: Mr. Bellaschi's paper is timely, well written and interesting but the subject is not particularly new to our laboratory staff. Since Mr. Peek's first lightning generator of 1913, the impulse voltages delivered by such a circuit have been subject to calculations. On page 1916 of the A.I.E.E. TRANSACTIONS for 1915 can be found a discussion by Mr. Peek of the effect of load capacitance upon the impulse generator test voltage. This effect and other variables were considered by Mr. Peek in 1915 when he first described his impulse generator at the San Francisco Convention.

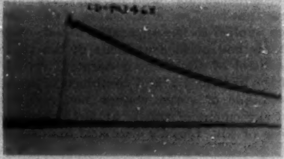
The calculations were first carried on by the laborious solutions of conventional mathematics and subsequently by the shorter methods of Heaviside's Operational Calculus. From the beginning these calculations could be, and were, checked by the sphere-gap. After 1925 the cathode ray oscillograph was used to check the calculations. When the cathode ray oscillograph was developed it was gratifying to see how closely it checked the previous calculations and test results. Of course, it *had* to check for the sphere gap had furnished a check on the calculated voltage. If the calculated voltage was correct then the calculated wave shape had to be correct also. But the independent check by the oscillograph was quite convincing and also accelerated the laboratory work. Oscillograms were often quicker to obtain than calculated results even by Heaviside's Operational Calculus. The electrons decide what they are supposed to do and do it in a few microseconds, whereas man's brain sometimes takes a few hours to predict their operation. The oscillograph, therefore, became a valuable laboratory tool and with the sphere gap has furnished important checks on the mathematical work upon the impulse generator circuit.



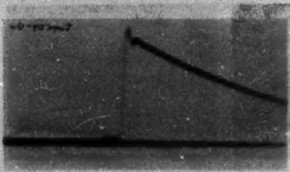
Discharge circuit only



Discharge circuit with insulator string load



Discharge circuit with sphere gap load



Discharge circuit with sphere gap and insulator string load

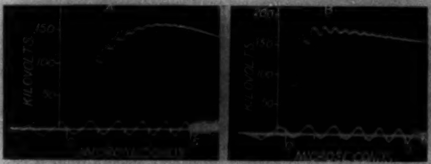
FIG. 1

These calculations have, however, been going on continuously since 1913 and much of the material which Mr. Bellaschi has given is fundamental and now pretty generally known. At least, in the laboratory, every-day use is made of the conclusions which he reaches in his paper. It is because of knowledge of these fundamental facts, that we have made so much progress on lightning research in the last ten years.

In our work we have found it necessary to go much more completely into the subject than Mr. Bellaschi has, particularly upon the effects of changes in the various circuit constants and different load conditions. This is especially necessary when impulse tests are made on large power transformers fully excited. A more thorough study on this subject has recently been made and the results are embodied in a paper entitled "The Production of Artificial Lightning in the Laboratory," by W. L. Lloyd, Jr. and J. C. Dowell which it is hoped will be published at an early date.

C. S. Roys: The ability to arrange a surge generator circuit in a manner such as to produce a wave of known form is becoming increasingly important in the testing of various electrical devices. Up to the present time it has been customary to consider a simple series circuit as the equivalent network of a surge generator and its test circuit under discharge conditions, determining the constants by methods that do not include the effect of capacity to ground, etc. Such a procedure does not give results that conform with observed ones, and this is especially true along a wave front, the most important portion of the wave in the majority of the tests.

By assuming an equivalent network in which consideration is given to the capacity to ground, although certain effects have been neglected, it is felt that the author has made a notable advance toward the complete understanding of the situation. So far as the tails of the waves are concerned, the validity of the assumptions is well established by the close agreement between the calculated waves and the ones determined by means of the cathode ray oscillograph. Unfortunately, however, the actual wave fronts are hardly discernible on the oscillograms and no close check can be made on the steepness of the wave fronts. Also, the nature of the high frequency oscillations and reflections is not apparent, these latter having been found to be of considerable importance in many instances.



OSCILLOGRAMS A AND B

In conclusion I wish to express the hope that the author will continue this line of investigation, being particularly careful to obtain clear records of the wave fronts. Only then will it be possible to say as to whether or not one can neglect the effect of reflections and high frequency oscillations under a similar set of conditions, and in consequence, assume the simple equivalent network of lumped constants.

C. S. Sprague: The author presents an excellent analysis, based upon lumped constants, of surge generator characteristics when the generator is connected to a capacity load, and shows that by the insertion of the proper value of series resistance, the oscillations due to the total load capacity and the inherent inductance of the generator circuit may be effectively damped. The actual oscillograms check well on the tail of the wave with the calculated curves from equations derived on the basis of lumped constants.

For surge testing of insulators, bushings and lightning arresters, the front of the wave is of prime importance, and the writer believes that equations derived on the assumption of lumped constants do not hold on the rising front of the wave. Oscillograms have shown that the finite length of the connections of the generator circuit gives rise to reflections across the load resistance, which reflections are not usually negligible when a smooth front is desired. In other words, during the first few instants following breakdown of the main generator gap, the circuit acts in a manner analogous to a transmission line, grounded through an impedance at one end. Reflections take place in the same manner except that due to the much shorter length of circuit involved, the frequency of reflection is much higher.

Oscillograms *A* and *B* illustrate these reflections, in this case the reflections being negative on both oscillograms. The following circuit, Fig. 2, illustrates the conditions under which these oscillograms were taken. The surge generator was not loaded, except for the capacity of the potentiometer and measuring sphere gap.

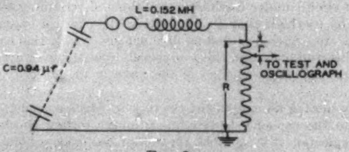


Fig. 2

Length of wire between generator and resistance = 171 ft.
For oscillogram *A*, *R* = 175 *r* = 19
For oscillogram *B*, *R* = 825 *r* = 69

Inductance *L* composed of 5 turns No. 22 wire, spaced 2 in., wound on an octagonal form, 10 ft. across corners. The value given above was calculated from Nagoska's formula assuming a circle of equivalent area

Each oscillogram shows a frequency of reflections of 2.5 megacycles. On the basis of the speed of propagation being 1,000 ft. per microsecond, these reflections would take place on a line 200 ft. long, whereas the measured length was 171 ft. between the generator and the resistance.

Oscillogram *A* shows that with 175 ohms series resistance, the wave requires about 10 reflections to reach a peak in approximately 3½ microseconds, while with 625 ohms, oscillogram *B* indicates that the peak is reached in three reflections in about 1 microsecond. Very obviously neither of these waves would be satisfactory test waves where a reasonably smooth front is desired. On the other hand sufficient capacity in the load would help to smooth out the ripples, but such loads as arresters, insulators or bushings would affect these waves very slightly.

In view of the above reasoning there seem to be two methods by which one may obtain a wave relatively free from oscillations on the front.

The first is analogous to grounding the end of a transmission line through a resistance equal to its surge impedance; in other words, so to adjust the total resistance in the circuit that reflections do not occur at the junction of the inductance and resistance. It is obvious that once this condition is obtained, only loads of low capacity and with short leads can be connected without changing the wave front. This method can be used only for the steepest waves and then the range of steepness may be quite limited for the usual possible variations in resistance.

The second method is to use a resistance low enough to make the individual reflections quite small compared with the main wave. This method is applicable to the slower waves. Oscillogram *C* illustrates a wave of 50 kv./μ sec front, obtained using this method.

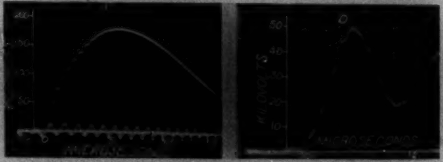
Using this latter method usually requires that additional series inductance be inserted in the discharge circuit. It has been the practise to construct such an inductance on a long, small diameter form, the purpose being to keep the volts per turn down to a small value. An inductance of this type is inefficient as regards the length of wire necessary to secure a given value of inductance. It has been determined experimentally and is in accord theoretically that to keep the reflections across the added series inductance of small magnitude, the inductance value should be obtained with a minimum length of wire. This means a large diameter, short length inductance with turn spacing as small as possible without flashover occurring between turns. This latter type gives a larger number of small reflections as compared with the long narrow inductance which gives a smaller number of large reflections.

It is unfortunate that so many of the cathode ray oscillograms appearing in this and other articles, lack detail on the front, and even if the front were visible the sweep is usually so slow that no accurate determination of the oscillations on the front of the wave is possible. The writer is fully aware that due to the high speed of beam travel across the film, this is the most difficult part of the wave to record, but believes that the advantage of recording the front of the wave is well worth the extra care and trouble required. Oscillogram *D* illustrates this point by showing a wave reaching a peak in about ½ microsecond, with a sweeping speed such that one microsecond occupies about 2 inches of the time axis. With an amplitude of approximately 3 inches, this spreads out the wave front so that oscillations may be accurately determined.

P. L. Bellaschi: The primary purpose of the paper was to analyze the fundamental factors that must be considered in any attempt to formulate a code for the surge testing of electrical apparatus, such as transformers. It was also the object of the paper to stimulate thought and criticism on the subject. The foregoing discussions are in themselves testimony that the results accomplished have well compensated our labors.

We cannot overemphasize the advantages of simple, but, none-the-less rigorous methods of analysis, particularly when dealing with engineering problems. Dr. C. L. Fortescue's advice and comments on this subject fully deserve our attention.

The questions on wave form discussed by Mr. Vogel are of vital importance. Results obtained by different investigators cannot be compared with the expectation of agreement, unless the same "yardsticks" are established and used by all.[1] The first yardstick in surge testing that requires establishing is the wave form of the surge voltage applied to the apparatus tested. The second yardstick is the method of measurement of the surge voltage. The correct control of the surge generated, supplemented with analysis of the test circuit, and combined with the proper method and technique of surge voltage measurement should facilitate general agreement of results between different investigators. Regardless of Mr. Lloyd's claims in his discussion, the experimental work on surge voltages done previous to the advent of the cathode ray oscillograph and previous to the extensive investigations of the past four or five years will remain, to be sure, of historical interest, but hardly to be used with full confidence at the present. We do not subscribe to Mr. Lloyd's forced reasoning, on the question of his calculations and measurements, that this and that *had* to check. Further discussion on the measurement of surge voltages will be duly con-



Oscillograms C and D

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[1] See, for example, *SEV Bulletin*, September 16, 1931, bottom of p. 473, second column, for disagreement in results between two different investigators, one using a smooth wave and the other unknowingly using a wave with superimposed oscillations.

sidered at the A.I.E.E. Winter Convention (1933) where the writer and others are presenting papers on the subject.

Professor C. F. Harding raises a pertinent question: the determination of the equivalent surge constants of the apparatus tested. These constants may be determined either experimentally by the proper methods, which he describes, or by calculation when the circuit constants under surge conditions of the transformer windings and also of the apparatus connected to the windings are taken into account and replaced by an equivalent simple circuit, when possible, as shown in the paper. The surge characteristics of transformer windings comprise a subject in itself of great importance.[2]

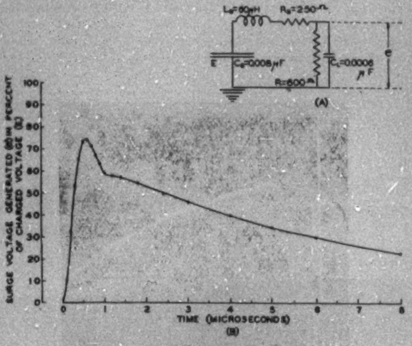

FIG. 3

The question raised by Mr. C. M. Foust on the circuit constants of the generator, particularly the point in regard to the stray capacity of generators is answered in full details in the following:

The surge generator capacity C_g, the load resistance R and the inserted resistance are determined by measurement on the individual elements. The effective stray capacity C_2 of the generator can be determined in a number of ways. Our estimated value of this capacity from the physical dimensions of the generator, taking into account the capacity of the auxiliary equipment connected to the generator and the capacity of the generator structure to ground, gives us a figure close to the value given in Table II of the paper. Experimentally, the capacity C_2 and inductance L_s may be determined closely from the relations

$$\frac{T_1}{T_2} = \sqrt{\frac{C_2}{C_2 + C_A}} \text{ and } T_1 = 2\pi\sqrt{L_s C_2}$$

where T_1 and T_2 are the fundamental periods of the superimposed oscillation on the wave without and with a known capacity load C_A. In such measurement all inserted resistance in the generator is removed and the load resistance is adjusted to a large value. The values of C_2 and L_s may also be determined from the fundamental period

$$T_1 = 2\pi\sqrt{L_s C_2}$$

in conjunction with the critical damping resistance

$$R_s = 2\sqrt{\frac{L_s}{C_2}}$$

The inherent resistance of the generator may then be estimated from the damping factor of the superimposed oscillation (the damping factor of a *C-L-R* series circuit is given by $\epsilon^{-\frac{R}{2L}t}$), provided all inserted resistance is removed and the load resistance is made sufficiently large relative to its critical damping value: $\frac{1}{2}\sqrt{\frac{L_s}{C_2}}$. A further check of L_s and of the inherent resistance may be had by short-circuiting the load resistance and removing all internal resistance. From the oscillogram thus obtained—*i. e.*, from the period and damping of the oscillation on the film, in conjunction with the known capacity C_g, the equivalent constants L_s and R_s (inherent) may be estimated. These are simple and practical methods based on lumped circuit constants which have proved adequate for engineering purposes; though, strictly speaking, in considering the general subject rigorously from a scientific viewpoint, under certain conditions and for certain cases discussed further on, consideration need be given to the distributed circuit constants of the condenser bank of the generator. In accordance with the engineering methods described above, the values of the circuit constants reported in the paper have been found. The close agreement between calculated surge voltages and the oscillograms justify the above methods for engineering purposes and for surge generators such as used for the purpose described.

We cannot compare surge generators unless they are on a par generally. The 3,000-kv. surge generator at Sharon is of liberal design. Accordingly the stray capacity of the generator alone is large. The value of C_2 is not only that of the generator alone, but includes the capacity of parts permanently connected to it, such as the 200-cm. sphere gap, insulators, lead, etc. The value of $C_2 = 600$ μμf. is the correct one to use for this generator. Inasmuch as loads tested may assume values for the load capacity C_T as large as five times the stray capacity of the generator, it would be immaterial on this score whether C_2 were greater or less than what it is. One of our smaller surge generators, which for comparison purposes falls within the classification of the one Mr. Foust describes, has a stray capacity $C_2 = 125$ μμf., including in this the structure of the generator, a 25-cm. sphere gap, an insulator string, a capacity potentiometer and a resistance potentiometer. Our figure for our small generator is identical to that given by Mr. Foust for his generator.

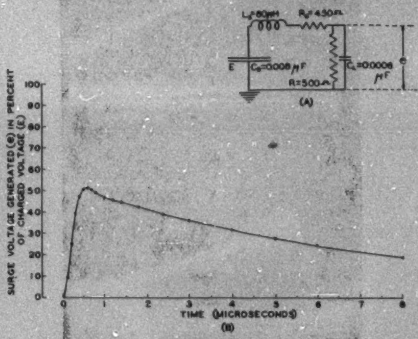

FIG. 4

Mr. Foust's oscillograms have not been reviewed by the writer, but very likely these are identical to those in Fig. 15 of his well-written article in the *General Electric Review* for July 1932. Mr. Foust states in his discussion that no added series resistance R_s was inserted. These oscillograms have an appreciable oscillation superimposed on the crest of the wave, particularly when an insulator string and a 50-cm. sphere gap are connected to the generator—which is the circuit condition that is of practical interest. We appreciate that the primary purpose of these oscillograms was to estimate or determine the effective capacity of the various insulation structure loads; such is our interpretation. We hope, however, that the wave with the marked superimposed oscillation is not being used as a standard wave. A similar example to that of Mr. Foust's is shown in oscillogram *Q*. The constants of the generator circuit for this oscillogram are given in Fig. 3A. Substitution of these circuit constant values in the fundamental equations in Appendix I of the paper, gives the following equation for the voltage generated:

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[2] See, for example, Appendix to P. L. Bellaschi's: "Transformatori a mantello antirisonanti" *L'Energia Elettrica*, April 1932-X.

$$e = \left[\epsilon^{-0.143t} - \epsilon^{-3.40t} \left\{ 0.606 \sin \frac{2\pi}{1.17} t + \cos \frac{2\pi}{1.17} t \right\} \right] 0.704\, E$$

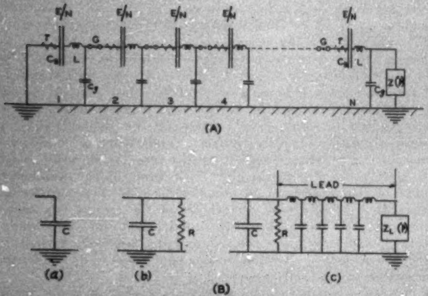
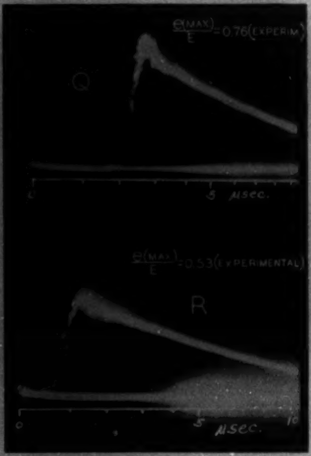
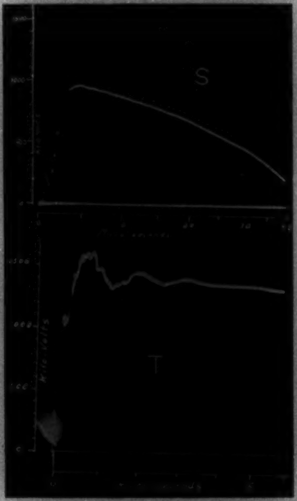


Fig. 5



Oscillograms Q and R

where t is in microseconds. This equation is plotted in the curve of Fig. 3B. This wave as well as the one of Mr. Foust, we have referred to, are unsuitable as a standard for surge testing of gaps, insulators and similar insulation structures for which the short wave would be used in addition to the (1½-40) wave. This type of wave may have been, and very likely it was, used in the *pre-cathode ray oscillograph epoch* to which Mr. Lloyd refers to and before the thorough analyses of the problem which have been made lately, but such a wave form is unsatisfactory at the present time.



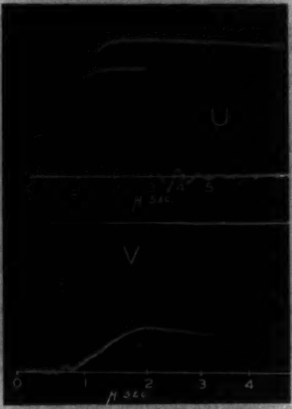
Oscillograms S and T

To give this wave a closer semblance to what the standard (½-5) wave should be, the generator circuit constants are adjusted as shown in Fig. 4A. Again, from the fundamental equations in Appendix I of the paper, the equation of the surge voltage generated for these circuit constants is:

$$e = \left[\epsilon^{-0.130t} - \epsilon^{-5.35t} \left\{ 1.05 \sin \frac{2\pi}{1.26} t + \cos \frac{2\pi}{1.26} t \right\} \right] 0.533\, E$$

where t is in microseconds. This equation plotted gives the curve in Fig. 4B. Oscillogram *R* gives the measured wave form and the regulation. This is a more desirable form of wave to use than any of the previous ones mentioned. The ripples or lead oscillations along the rising part of the front of the surges shown could be removed if that need be, likewise the regulation could be improved. These oscillograms are shown here for illustration purposes, to make clear our objectives in this question of standard wave form.

We are in sympathy with Messrs. C. S. Roys and C. S. Sprague on the question that published oscillograms do not always show the rising part of the front. The oscillations due to the lead and to the distributed constants of the condenser bank of the generator, mentioned in the paper, are present in our case only on a small part of the front along the rise; but unfortunately due to the relative rate of travel of the cathode beam, when the whole wave is recorded, the part on the front, as compared to that on the tail, is relatively faint on the original oscillograms so that in the course of reproduction and printing of the oscillograms the rapid oscillations in question are completely lost. Oscillograms S and T, here shown, are practically prototypes respectively of K and B in the paper, except that in S and T the time sweep is faster. Oscillogram S is of practical interest. It shows that the rapid oscillations disappear along the rising part of the front. In our case, these rapid oscillations are largely due to the lead mentioned in the paper and can be calculated, if need be, as shown in Appendix III.



Oscillograms U and V

We agree with Messrs. C. S. Roys and C. S. Sprague that the theoretical circuit of the condenser bank of the surge generator is a distributed circuit, as shown in Fig. 5A. We have always appreciated this fact and also the considerations connected therewith. We shall point out briefly that under certain conditions the condenser bank of a surge generator corresponds closely to a very short transmission line. In the case of a condenser bank disposed with respect to ground such that all condensers are equally spaced from ground, the circuit constants approach the uniformly distributed arrangement of the circuit in Fig. 5A, and provided there is no appreciable load effect due to $Z(p)$, this circuit would act closely to a line, more so if C_s is very large compared to C_g. In such a case, as stated correctly by Mr. Sprague, a terminal resistance equal to the impedance of the line would prevent reflections at the terminal end.[3] The natural oscillations of circuit Fig. 5A with load $Z(p)$ removed are determined from[4]

$$\cosh \sqrt{LC_g w^2 + \frac{C_g}{C_s}}\, N = \cos j \sqrt{LC_g w^2 + \frac{C_g}{C_s}}\, N = 0$$

where

$$j \sqrt{LC_g w^2 + \frac{C_g}{C_s}}\, N = K\frac{\pi}{2} \text{ and } K = 1, 3, 5, \text{ etc.}$$

The solution of the voltages in such a circuit is arrived at in much the same way as the solution for finite segments of a line, introducing boundary conditions as indicated from Fig. 5A. A detail analysis of this subject is beyond the scope of this discussion. The method of analysis is as outlined above.

The type of surge generator used is built vertically, comprising of three stairs arranged in zig-zag fashion.[5] In such a case the stray capacity of the generator alone, from the terminal end to ground, acts in effect largely as a lumped capacity in parallel with the generator. In addition, the stray capacity of parts permanently connected to the generator at the terminal end is appreciable, and this, too, acts as a lumped capacity. Any effect due to distributed capacity, which theoretically is always present, becomes in this case, practically speaking, of secondary importance in so far as the voltage at the terminal end is affected. The effect due to any distributed capacity present would appear as oscillations of higher frequencies compared to the natural fundamental oscillation superimposed on the wave, these being largely confined at the rising part of the front and reduced in magnitude at the terminal end due to the large effective terminal capacity. The distributed series resistance inserted in the generator would also tend to reduce the effect of these higher frequency oscillations by damping them. Furthermore in practise loads such as (a), (b), (c), indicated in Fig. 5B, modify considerably the effect at the terminal end due to any distributed capacity present. Fundamental considerations and experimental results thus show that for the type of surge generator and for the conditions of test discussed in the paper lumping the circuit constants of the generator is justified.

Using the ground end stair of our 3,000-kv. generator (equivalent to 1,000-kv.), with the large stray capacity C_s at the terminal end of the circuit and with the proper series resistance R_s, we obtain oscillograms as shown in U. The fronts here rise smoothly and are of the satisfactory type, as Messrs. Roys and Sprague mention, to test gaps and similar apparatus along the rise of the front. Oscillogram V was taken with a smaller generator, using a series inductance and a series resistance, in conjunction with the load capacity available, to secure a smooth front. The wave rises almost at a constant rate from zero to crest voltage. Increasing the crest voltage of this oscillogram automatically increases the rate of rise of the front. A more rapid sweep of the cathode beam along the time axis could have been used or a greater sensitivity of the voltage divider and cathode ray oscillograph could have been employed, if it were necessary.

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[3] See also, for example, article by J. Kopeliovitch on impulse testing of insulators, *SEV Bulletin*, Sept. 16, 1931, bibliographical reference (33), for investigation with cathode ray oscillograph on damping of 20 meter line.

[4] O. Heaviside, "Electromagnetic Theory," Vol. 2, p. 127, and sequel.

[5] O. Ackerman, "New Surge Generator for Testing Transformers," *Electric Journal*, February 1932, p. 61.

Characteristics of Load Ratio Control Circuits
For Changing Transformer Ratio Under Load

BY L. F. BLUME*

Associate, A.I.E.E.

Synopsis.*—The influence of saturation phenomena in series reactors on the circuit performance of load ratio control equipment is discussed. It is shown that unless care is exercised in the design of these reactors, gross distortion may be introduced into the circuit, resulting in obtaining across the reactor voltage peaks which on the one hand may endanger the insulation, and on the other hand greatly increase the rupturing duty on the circuit breaking devices. Formulas are developed which show what proportion of line voltage is absorbed by a series reactor subject to magnetic saturation. It is shown that this voltage can be expressed in terms of the reactive constants of which the circuit is composed, and that it is independent of the current or voltage wave form. Numerical solutions are given in the shape of curves, assuming that the magnetic circuit of the reactor is made up of typical transformer steel. The great influence of a small air gap in the magnetic circuit in reducing these voltages is shown.*

* * * * *

IN previous articles† the circuit characteristics of typical transformer connections used for changing transformer ratio under load have been discussed. It was shown that the performance of the equipment is dependent upon the design characteristics of the reactor employed, and also upon which positions are selected by the transformer designer, from the three possible, for the permanent operating position. It was shown that in the design of the reactor used, two conflicting considerations must be kept in mind, namely, that in the position where the reactor is connected across adjacent taps (bridging position) the resultant magnetizing current in the reactor should not be excessive, and on the other hand in the position where the reactor is in series with the load, (unsymmetrical or series position), the reactance introduced into the circuit should be maintained low. The reactor design is a compromise between these considerations. In other words, in order to avoid excessive series reactance being introduced into the series positions, it is necessary to design the reactor so that when connected across adjacent taps a comparatively large circulating current flows.

It was also shown that the best circuit performance is obtained when in the operating position the connection is made on one tap only, and with the reactor short-circuited. With permanent operation only on this connection, the voltage steps obtainable become uniform, and the minimum impedance is introduced into the circuit. This entails the provision in the transformer design of a transformer tap for every operating position. It also has the advantage that in the process of switching from one transformer tap to the next, the operation consists in the quick succession of a number of relatively small steps, which means that a smoother transition takes place during switching.

In the previous articles it was assumed that the reactor used possessed linear characteristics. The purpose of the present paper is to discuss the influence of magnetic saturation of the reactors on performance and to indicate the important part which a relatively small air gap provided in the reactor plays in preventing abnormal and dangerous voltage distortions.

SERIES REACTOR DESIGNED WITH A CLOSED MAGNETIC CIRCUIT‡

The importance of designing a reactor with a large magnetizing current—and with an air gap, so as to obtain essentially uniform inductance for all values of current—can best be appreciated by discussing the characteristics of a reactor having a closed magnetic circuit.

A reactor with a closed magnetic circuit can be readily designed so that the magnetizing current on the bridging position is kept small, say less than 10 per cent, and at the same time excessive reactance drop in the unsymmetrical or series position may be avoided by designing with high magnetic densities so that the load current will saturate the iron. Thereby the reactor becomes appreciably smaller and inherently less expensive.

By this means the effective value of the reactance drop can be maintained low, even with a load current equal to as much as ten times normal magnetizing current. However, when this is done excessive voltage peaks are introduced into the circuit, due to the distortion effects of magnetic saturation, which may result in dangerously high voltages being induced in the reactor.

This phenomenon is due to the fact that the ohmic value of the inductance of a closed magnetic circuit varies between very wide limits depending upon the instantaneous values of the flux density in the iron. When the current and therefore the flux is passing through zero, the iron is unsaturated and the inductance of the reactor may be several thousand times as much as when the densities approach or exceed 100 kilolines per square inch.

The simplest case to consider is a pure inductive load. In the unsymmetrical position of operation, Fig. 1, the inductance L of the load is in series with the variable inductance of series reactor L^1; and therefore the current being common to both inductances the division of

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*General Electric Co., Pittsfield, Mass.

†See items 1 and 4 of bibliography.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

‡For a general description of the effect of saturation on voltage distortion see "Theory and Calculation of Electric Circuits," C. P. Steinmetz, p. 146.

circuit voltage between the reactor and the load at any instant is directly proportional to the ratio of inductances. The voltage drop across the reactor is

$$\frac{e_r}{E} = \frac{L^1}{L + L^1} \qquad (1)$$

where

e_r = voltage across one-half of reactor instantaneous value
E = circuit voltage instantaneous value
L = combined inductance of load and transformer
L^1 = inductance of reactor, instantaneous values

The above formula for the instantaneous values of voltage absorbed by the reactor, is applicable to reactors having variable instantaneous value of inductance and also for reactors in which the inductance is constant. In other words the formula applies to both classes of reactors—those in which ample air gaps are provided and also those in which the magnetic circuit is entirely made up of iron. It can therefore be used to compare the two types of reactors from the point of view of the voltage distortion introduced by magnetic saturation of the iron. The formula, however, is strictly limited to pure inductive loads.

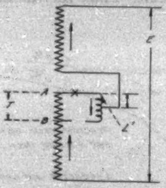

Fig. 1—Load Ratio Control Connection Showing Reactor L' Connected in the Unsymmetrical or Series Position

From the formula it is evident that the value of peak voltage is independent of the shape of the current wave. Saturation of the reactor iron results in a distortion of the current wave as it approaches and passes through zero, but this distortion has no influence on the magnitude of the voltage peaks. The amount of voltage distortion therefore is entirely determined by the variability in the magnetic circuit, that is, by the magnetization curve of the reactor. Consequently it is very much affected, first, by the value of the maximum flux density within the iron and, second, by the amount of air gap, if any, provided in the reactor.

Voltage distortion curves for various conditions are plotted in Fig. 2. Curves 1, 3 and 5 are for reactors having a closed magnetic circuit, whereas in the case of curves 2 and 4 the reactor is provided with a 1 per cent air gap. The great difference between the values of voltage peaks for reactors with and without a gap is illustrated by these curves. Comparing the three curves corresponding to zero air gap with each other, curve 1 corresponds to a maximum density within the iron of 75 kilolines per square inch, whereas curves 3 and 5 correspond to a maximum density within the iron of 100 kilolines per square inch, showing that increasing the maximum density to saturation values and beyond very greatly increases the voltage peak across the reactor. Curves 3 and 5 are identical except for the fact that the latter corresponds to a load current equal to the normal magnetizing current in the reactor, whereas curve 3 is a condition in which the load current is approximately three times normal magnetizing current.

The numerical values of these voltage peaks for various conditions are given in Table I. These values are arranged to show the influence on peak voltage of:

a. The amount of air gap provided in the reactor.

b. The maximum flux density for which the reactor is designed.

c. The value of normal voltage between adjacent transformer taps.

d. The amount of the zero power factor load, expressed as a function of the normal magnetizing ampere turns of the reactor.

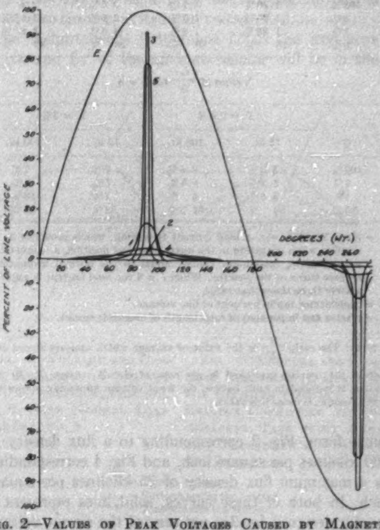

Fig. 2—Values of Peak Voltages Caused by Magnetic Saturation of Reactor

It is important to note that the values of $\frac{e_r}{E}$, given in this table are the voltages appearing across the reactor half, and as this voltage is induced in both halves of the reactor, the total voltage appearing across reactor terminals is twice this value. The table indicates that in the reactor unprovided with air gaps and designed normally for a voltage of less than 10 per cent of circuit voltage, it is possible to obtain across its terminals peak voltage considerably in excess of circuit voltage. This peak voltage not only is impressed on the internal insulation of the reactor, but also appears across the open contactor. As it appears across the contactor when it is opened during the process of switching, it is evident that the rupturing duty of the contactor is greatly increased.

The quantitative values given in the table have been obtained directly from the formula (4) derived in the appendix. This formula has also been expressed in curve form, Fig. 3 corresponding to a flux density of 100 kilolines per square inch, and Fig. 4 corresponding to a maximum flux density of 75 kilolines per square inch. In both of these curves, solid lines represent a reactor designed for 2½ per cent of the circuit voltage and the dotted lines for 5 per cent of the circuit voltage. Curves are given for zero gap and for 1 per cent, 2 per cent and 100 per cent gaps. These curves are plots of formula (4) derived in the appendix.

TABLE I

Values of $\frac{e_r}{E}$ for $l = 1$

	$T = 2½\%$		$T = 5\%$	
G	75 kl.	100 kl.	75 kl.	100 kl.
100%	1.26%	1.2%	2.5%	2.4 %
2%	1.26%	1.5%	2.5%	3.08%
1%	1.26%	1.8%	2.5%	3.6 %
0%	15 %	80 %	26 %	89 %

Values of $\frac{e_r}{E}$ for $l = 3$

	$T = 2½\%$		$T = 5\%$	
G	75 kl.	100 kl.	75 kl.	100 kl.
100%	3.5%	3.5%	7%	7%
2%	3.5%	4.5%	7%	9%
1%	3.5%	5 %	7%	10%
0%	34 %	92.5%	52%	96%

l = load. When $l = 1$, a load current is flowing, which produces in the reactor, when operating in the unsymmetrical position, a magnetizing ampere turns (maximum values) equal to the normal magnetizing ampere turns of the reactor. When $l = 3$ the load current is approximately three times that value.

T = transformer tap in per cent of line voltage.

G = reactor gap in per cent of total length of magnetic circuit.

Note: The ratio $\frac{e_r}{E}$ is the value of voltage which appears across one-half of the reactor expressed in per cent of circuit voltage E. As this voltage is induced in both halves, the total voltage appearing across the entire reactor, is twice this value.

The curves show graphically the effectiveness of a small air gap in reducing the peak voltage. It is apparent from the curves that the peak voltage across the reactor can be limited to relatively small values by:

1. Designing reactor for relatively low flux densities.
2. Using air gap which is at least 1 per cent of the total length of the magnetic circuit.
3. Designing the reactor with large magnetizing current, so as to avoid excessive voltages when overload currents are flowing.
4. It is also apparent by comparing solid with dotted curves that voltage peaks are reduced by providing the transformer with a larger number of smaller percentage steps so that the reactor may be designed for a smaller fraction of circuit voltage.

Influence of Power Factor. These curves give peak values for zero power factor load, the condition under which the peaks are maximum. With increasing power factor, the peaks are reduced appreciably in value, on account of the fact that current zero is shifted away from the instant of maximum circuit voltage. Voltage peaks are approximately proportional to the reactive factor of the circuit. Thus for an 80 per cent power factor circuit, the peak voltage will be 60 per cent of the value given in the curves.

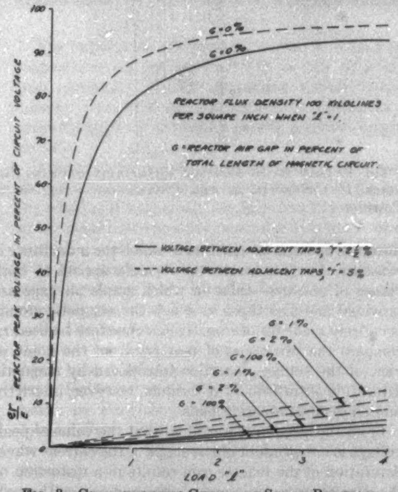

FIG. 3—CHARACTERISTIC CURVES OF SERIES REACTORS

Showing how very small air gaps greatly decrease the peak voltages induced in the reactor by magnetic saturation

Appendix

EFFECT OF MAGNETIC SATURATION IN REACTORS

Theoretical Determination

For the unsymmetrical position (Fig. 1) the division of circuit voltage E between the load and the reactor is given by equation (2). This equation holds for all instantaneous values of current voltage and inductance, and for any type of reactor. It is limited, however, to pure inductive loads.

$$\frac{e_r}{E} = \frac{L' \dfrac{di}{dt}}{L \dfrac{di}{dt} + L' \dfrac{di}{dt}} \qquad (2)$$

where

e_r = voltage across one-half of reactor (instantaneous value)

E = circuit voltage (instantaneous value)

Since the equation holds for all instantaneous values it can be used to determine the maximum values, *i. e.*, the voltage at the instant when the current is passing through zero. Since in the unsymmetrical position, the reactor and load current are identical, di/dt is the same for both reactor and load, and may be canceled from the numerator and denominator of equation (2), which therefore simplifies to

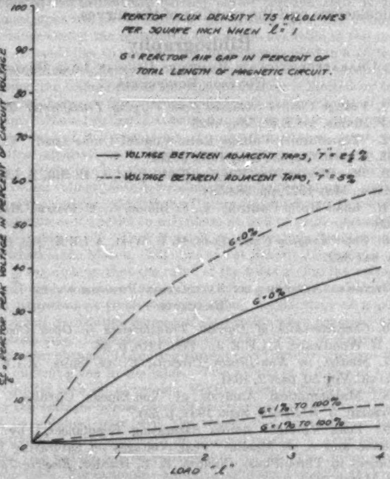

Fig. 4—Characteristic Curves of Series Reactors

Showing how very small air gaps greatly decrease the peak voltages induced in the reactor by magnetic saturation

$$\frac{e_r}{E} = \frac{L'}{L + L'} \qquad (3)$$

Load inductance L is constant for a given load and varies approximately inversely as the value of the load. Let the inductance of the normal load be defined as the value of inductance corresponding to the load which has the same peak current in the unsymmetrical position as the magnetizing current of the reactor at its rated sine-wave voltage, $\frac{T}{2}$. (See Figs. 5 and 6.) This is a convenient assumption because it makes the peak magnetizing current in the bridging position equal to one-half of the normal load current. If the actual load current is l times what is assumed here as "normal" load current, then

$$L = L_0/l$$

Equation (3) may then be written

$$\frac{e_r}{E} = \frac{L}{\dfrac{L_0}{l} + L'}$$

Let the constant L_0' be the inductance of an air-core reactor so chosen that the division of voltage for normal current, between reactor and load is

$$\frac{e_r}{E} = \frac{L_0'}{L_0 + L_0'} = \frac{T/2}{(E - T/2) + T/2}$$

The magnetization curve of such a reactor is given by the straight line in Fig. 7. Let curve *II* in Fig. 7 be the actual magnetization current curve of the iron core reactor, having the same peak magnetizing current as the air core reactor, at its rated voltage $T/2$.

The instantaneous inductance L' of the iron core reactor and L_0' of the air core reactor will be in the ratio of the slopes of the two curves at the particular current value

$$\frac{L'}{L_0'} = \frac{\text{slope of curve } II}{\text{slope of curve } I} = \tau$$

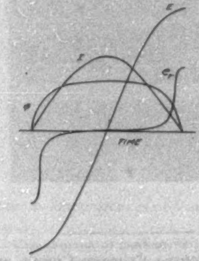

Fig. 5—Voltage and Current Relations in Reactor for Unsymmetrical Position, Fig. 1, with Normal Load Current. $l = 1$

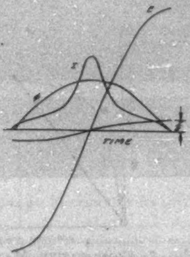

Fig. 6—Voltage and Current Relations in Reactor for Bridging Position, i. e., Reactor Connected Across Adjacent Taps, with Normal Magnetizing Current and Zero Load

E = Circuit voltage
e_r = Voltage across the reactor
T = Transformer tap voltage
I = Current through reactor
ϕ = Flux in reactor

Since our interest centers on the maximum voltage and the maximum voltages occur when the fluxes are passing through zero, therefore, to obtain the maximum reactor voltage, we should take the ratio of slopes at the

TABLE II

Maximum reactor flux density kilolines per sq. in.	Gap zero (all iron)	Values of r Gap 1% air	Gap 2% air	Gap 100% (all air)
0	1	1.00	1.00	1.00
20	6	1.00	1.00	
40	6	1.00	1.00	
60	7	1.00	1.00	
70	10	1.01	1.01	↓
75	14	1.01	1.01	
80	23	1.03	1.01	
85	50	1.07	1.03	
90	112	1.16	1.08	
95	207	1.31	1.15	↓
100	314	1.47	1.24	
105	488	1.74	1.37	
110	606	1.92	1.46	
115	829	2.25	1.64	
120	1,195	2.81	1.91	↓
125	2,380	4.61	2.80	
130	4,540	7.90	4.44	↓

origin. Table II gives this ratio for various values of normal flux density in the reactor, as determined from a typical d-c. magnetization curve of transformer steel for any maximum value of flux density.

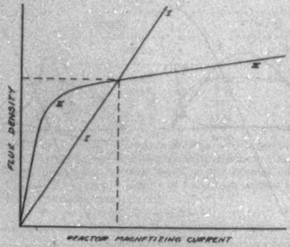

FIG. 7—COMPARATIVE MAGNETIZING CURRENT CURVES OF AIR CORE REACTOR (*I*) AND IRON CORE REACTOR (*II*) HAVING THE SAME NORMAL FLUX DENSITY

The general form of the equation giving the relation between peak reactor voltage, as a function of load and the permeability ratio r is

$$\frac{e_r}{E} = \frac{\frac{T}{2}\,r}{\dfrac{E - \frac{T}{2}}{l} + \frac{T}{2}\,r} \qquad (4)$$

where

T = voltage across adjacent taps
E = circuit voltage
l = load number
r = ratio of slopes on d-c. magnetization curve

Equation (4) is plotted in Fig. 3 and Fig. 4, in which e_r is the ordinate and l the abscissa.

Influence of Power Factor. Equation (4) is true for a zero power factor circuit. For other power factors, approximate values of voltage peaks may be obtained by multiplying equation (4) by the reactive factor of the circuit.

ACKNOWLEDGMENT

The author wishes to acknowledge the valuable suggestions given by Mr. A. Boyajian in connection with the theoretical development of the effect of magnetic saturation on voltage distortion, and also the assistance rendered by Mr. G. Heye in the numerical calculations and the plotting of the curves.

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Discussion

F. L. Snyder: Mr. Blume has demonstrated advantages to be derived from introducing a small air gap into the core of the reactor used in switching the load from one transformer tap to another. The paper deals with the method of tap changing described in papers which he has presented before the Institute heretofore.

It should be noted, however, that the introduction of an air gap into the core of the reactor increases its magnetizing current and consequently the duty on the transfer or load switches of the tap changer. This results because the switches must interrupt the vector sum of one-half the transformer load current and the reactor exciting current. For low-voltage circuits, this increase in current is not compensated for by the reduction in voltage. Furthermore, the point must not be lost sight of that some operating engineers prefer the method of tap changing using the position with the reactor spanning two taps, as an operating position. This is because fewer transformer taps are necessary, and a smaller number of points or contacts are required on the tap changer. When this simpler method of tap changing is used, the iron loss and exciting current of the reactor, which are added to that of the transformer on alternate positions, will be considerably greater with an air gap in the core of the reactor than when the core is not provided with such a gap.

I would like to supplement Mr. Blume's statement that the distortion of the current wave as it approaches and passes through zero is due to the saturation of the core. This distortion occurs only in case the voltage available in the circuit is not sufficiently high, with a sine wave current, to satisfy the fundamental law that the instantaneous voltage across the reactor is proportional to the rate of change of flux with respect to time. If the voltage available in the system at this instant is sufficient to satisfy the law with a sine wave current, no distortion will take place. This point is discussed by Messrs. Fahnoe and Maslin in their paper "Peak Voltages Across Saturating Reactances," published in the March 1932 issue of *The Electric Journal*.

Mr. Blume points out that the voltage across one-half of the reactor at the instant the current passes through zero, is dependent upon the power factor of the system, since this voltage can never exceed the instantaneous voltage available in the system. I would like to add that for tap changers used in a system for supplying a synchronous load, or used at a tie between two systems, the voltage across one-half the reactor is limited by the difference between the internal voltages at the two ends of the system. These two limitations of the voltage which can appear across one-half the reactor are of major importance, since, for practical applications, it is always the voltage available in the system at the instant the current passes through zero, and not the system voltage, that determines the voltage that may appear across the reactor. Mr. E. L. Harder has analyzed this quite thoroughly as it applies to polyphase as well as single-phase circuits, in his article "Peak Volages on Saturating Reactances," published in the March, 1932 issue of *The Electric Journal*.

It is my opinion that the value of the table giving the voltage across one-half the reactor would be enhanced if the load current were expressed as some constant value, rather than as a percentage of the exciting current of the reactor when spanning two adjacent taps. Changing the length of the air gap in the core of the reactor changes its exciting current and therefore changes the load current given in the table. This results in a different load current for each set of values. Furthermore, it should be noted that the voltages across one-half the reactor, given in the table, will be obtained only with the transformer supplying a pure reactive load with practically no external reactance. For normal loads, this voltage will be considerably less than the values given in the table. For example—applying equation 1 to a 25,000-kva., 72,000-volt, single-phase transformer equipped with load tap changer and reactor without an air gap in the core, which has been in service for several years, gives a value of $\frac{Er}{E}$ of 28.8 per cent when $L = 12$ and $T = 4$ and the power factor of the load $= 80$ per cent. It should be noted, therefore, that for actual applications, the value of $\frac{Er}{E}$ is in reality only about one-third of the theoretical value shown in the tables when the external resistance and reactance and power factor of the system are taken into consideration. Also, the measured voltage will, in every case, be less than the theoretical, calculated voltages, due to the fact that the space between the laminations of the core make up a small air gap, so that in reality, even the so-called solid core reactor has a small air gap.

In conclusion, I would like to point out that whether or not an air gap should be used in the core of the reactor depends upon the characteristics of the circuit which the designer wishes to secure, and that in any event, the tap changer, the transformer, and the reactor must be insulated for the voltages which may be impressed upon them under both normal and abnormal conditions. As pointed out in a paper "Tap Changing Transformers," published in the June, 1932 issue of *The Electric Journal*, the load tap changer should not only be capable of successful operation during normal conditions, but also under short-circuit conditions, since a tap changing operation may conceivably take place at the instant of line short circuit.

A. Boyajian: Attempts to limit a voltage rise by saturation result usually in just the opposite effect. Uncritical expectations based on rms. characteristics are brought to grief, and the very feature (saturation) depended upon for protection stings and pierces the insulation of the circuit by needle shaped voltage peaks.

Can saturation phenomena in an a-c. circuit be analyzed and understood physically in terms of elementary sine-wave concepts associated with linear circuits without recourse to advanced mathematics? It appears that this can be done, and it is very desirable for all technically inclined engineers to grasp it. The d-c. saturation curve of a closed iron core reactor may be represented with sufficient approximation by two straight lines meeting at a point designated by a current value I_t and flux density B_t, —transition values. For all values of current less than the transition value i_t, the reactor has a constant reactance X_1; and for all values of current above the transition value, the reactor has another constant reactance X_2. The wave shape of the behavior of the circuit then can be determined by ordinary sine-wave concepts, using X_1 during part of the cycle, and X_2 during the next, the beginning and ending of these parts or zones being determined by the transition current i_t. Adjoining zones are made to fit each other by elementary transient components, namely, by a d-c. transient, when capacitance is absent, and by a natural oscillation when capacitance is present.

Fig. 1—Oscillograms of Voltages on Preventive Reactors

This scheme of analysis is presented in considerable detail in an article in the *General Electric Review* for September and December 1931, with a number of illustrative examples. Strange wave shapes, which would have otherwise appeared most baffling for analysis, are found to consist of nothing but sections of two normal frequency sine-waves plus sections of two exponentially decaying d-c. transients. If capacitance is present, the damped d-c. transient is replaced by a damped oscillation at the natural frequency of the circuit. The physics of the phenomena is clarified by such a conception and analysis; and, in most instances, the final answers, that is, the equations of current and voltage, can be written down by inspection.

H. O. Stephens: In some of the early schemes for changing transformer taps under load split blade dial switches or straight line switches were used. Preventive reactors were connected between the split blades or contacts so that when the blades bridged adjacent taps during the transition periods the circulating current between the taps would be limited to safe values. It was observed that if these reactors were wound on completely closed magnetic circuits the arcing on the contacts was severe. Accordingly, tests were made at Pittsfield, Mass. in 1916 to determine the relative merits of preventive reactors, first with completely closed magnetic circuits, second with cores having air gaps and third with air core reactors.

These tests showed that when preventive reactors with closed magnetic circuits were used, decided peaks in the voltage waves across the reactors were obtained, the arcing on the contacts was severe and the contact life unsatisfactory. With air core reactors or with iron core reactors with suitable air gaps in the core so that substantially a straight line volt-ampere characteristic throughout the working range was obtained, the bad peaks in the voltage across the reactor were eliminated and the life of the contacts greatly increased. Comparative oscillograms taken during these tests show the marked superiority of the air core reactor over the closed iron core reactor (Fig. 1).

Since those tests in 1916 it has always been our policy at Pittsfield to recommend only reactors having substantially straight line volt-ampere characteristics throughout their working range when used as preventive reactors for transformer tap changing under load.

Frequently the preventive reactance is obtained by a suitable arrangement of the windings of the transformer itself in which case the volt-ampere characteristic of the preventive reactance is a straight line.

In the *General Electric Review* for December 1931, Mr. Boyajian gave a method of analyzing such non-linear function as the volt-ampere characteristic curve of iron core reactors with closed magnetic circuits. Mr. Blume's paper gives a simple conception whereby it is possible readily to determine the proper characteristics of preventive reactors and to determine quantitatively the effectiveness of gaps in cores and thereby avoid the hazards of peaked voltages across reactors and contactors when used for changing taps on transformers under load.

L. F. Blume: The primary purpose of the paper was to derive and express in mathematical language, the fundamental effect of magnetic saturation in tap changing devices and to give a simple means by which the maximum value of the peak voltage induced in a circuit can be easily determined for specified conditions. It is appropriate for Mr. Snyder to call attention to the fact that these voltages are very much affected by a variety of circuit conditions not considered in the paper. In addition to those which Mr. Snyder has mentioned, the circuit consisting of a parallel transformer bank, or the presence of line capacitance, will very materially modify the values of peak voltages given in the paper. Although it is admitted that the subject is not complete without considering these effects, nevertheless I cannot agree with Mr. Snyder in his apparent conclusion that such circuit conditions can be relied upon for the purpose of preventing or greatly reducing the intensity of the peaks. A transformer designer cannot be sure that his transformer will always be operated with special circuit conditions, and at least a transformer provided with a tap changing device, should be capable of operating under just as great a variety of conditions as an ordinary transformer. In other words, the fact that the tap changer is added to the circuit should not constitute a restriction of the conditions under which the transformer bank can be safely operated. The paper assumes that it is sufficient to derive a practical equation whereby the maximum values under the worst condition of service operation can readily be derived.

It is unfortunate that Mr. Snyder in the example in which he obtains a value of 28.8 per cent for voltage peak, fails to give all the conditions necessary under which this value was obtained. In addition to the constants which he gives, it is necessary to know the ratio of load current to normal reactor magnetizing current and the value of r, which measures the inductance of the reactor at the instant of current zero in terms of its normal inductance. To obtain the value for maximum peak voltage of only 28.8 per cent without using an air gap and assuming power factor of load equal to 80 per cent involves, according to equation (4) of my paper, restricting the normal flux density in the reactor and also the load current in terms of reactor magnetizing current, to rather small values. For example, let us assume that the normal maximum density in the iron of the reactor, that is the maximum density of the reactor when connected across adjacent taps, is 85 kilolines per square inch. From Table II the value of r corresponding is 50. Substituting this value together with the constants which Mr. Snyder assumes in equation (4) and equating to 28.8 per cent, with proper correction for the power factor of the load, results in a value of l approximately equal to unity. This means that the example is restricted to a load current not exceeding the normal magnetizing current of the reactor. As a primary purpose of avoiding air gaps in the reactor has been to reduce the exciting current to inappreciable values, as compared with the normal load current on the transformer, the conclusion follows that the example which Mr. Snyder cites applies only when the load on the transformer is a very small fraction of the rated load. Accordingly, I do not see any evidence whatever for the conclusion which Mr. Snyder draws that in practise the actual values of these peaks will be reduced to one-third of the theoretical values. Furthermore, the paper which Mr. Snyder refers to and which was published in the *Electric Journal* in March 1932, contains equations for calculating the peak voltages under conditions similar to those described in the present paper, and apparently there is no substantial difference between them. In this paper the specific statement is made that the equations given, although theoretically not precise, nevertheless will give substantially the correct values. I cannot understand the statement made by Mr. Snyder as follows: "Furthermore, it should be noted that the voltages across one-half the reactor given in the table will be obtained only with a transformer supplying a pure reactive load with practically no external reactance," unless Mr. Snyder meant to use the word "resistance" for the last word in this sentence instead of "reactance."

I am glad that Mr. Snyder has called attention to the possibility that a tap changing transformer may conceivably be called upon to operate under very heavy overloads, or under overloads approaching dead short circuit, and it is desirable that under this condition no harm come to either the transformer or the circuit upon which it is operating. Under such conditions, the impedance of the connected load is relatively low and almost entirely reactive. From the standpoint of the values of the peak voltages across the reactor, this condition is about the most severe that can be imagined, because, first, the ratio of unsaturated reactor constant to the reactance of the remainder of the circuit increases with the decrease of load inductance so that under short circuit, this ratio is maximum; and, second, short-circuit currents are generally of very low power factor, so that the peak voltage appears practically at the instant of the maximum circuit voltage. The effectiveness in reducing these peak voltages of an appreciable air gap, together with designing the reactor for a magnetizing current comparable to the rated current, is made evident by examining the curves given in Fig. 3 and Fig. 4. Assume, for example, that heavy short circuit will correspond to a value of l plotted on the horizontal scale equal to about 10. It is evident from these curves that any attempt to reduce greatly the reactor magnetizing current, or of eliminating the air gap, means that when the transformer is called upon to change a tap with a short circuit on the system, peak voltages will be induced across the reactor terminals practically equal to double circuit voltage.

Influence on Commutation of Brush Contact Drop

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Associate, A.I.E.E. Associate, A.I.E.E.

I. INTRODUCTION

IN designing electrical commutating machinery, an attempt is usually made to produce a linear change of current with time during commutation. In this case the current density under the brush is uniform, and therefore the voltage drop at the contact between brush and commutator will also be uniform over the brush surface. It is well known, however, that straight-line commutation[1] is rarely achieved. Furthermore, there is some doubt as to whether straight-line commutation is the most desirable form. In the case of non-linear commutation the current density is not constant across the brush surface, and consequently the contact drop will not be constant. This means that calculations made to determine the change in current in a commutating coil will be greatly in error if constant contact drop is assumed. Another difficulty is introduced by the fact that the contact drop will change with time in any small area on the brush surface, and therefore the static curve between voltage and current measured at the brush contact cannot be used and the transient characteristics of the brush contact drop must be known.

The usual lack of consideration of the nature of contact drop in dealing with commutation problems leads to the result that many observed phenomena cannot be explained by means of the calculation made. For example, the classical theory of commutation can in no way explain sparking underneath the brush, peculiarities found in brush and commutator wear, the choice of certain brushes in order to obtain good commutation, etc.

II. THEORY OF A SLIDING CONTACT

The electrical nature of a sliding contact is externally expressed if a volt-ampere characteristic of the contact is given. In the case of a graphite brush and a copper ring or commutator, the general form of the volt-ampere characteristic is well known, and the static characteristic is given as curve *a*, Fig. 1. There is evidence that this characteristic decrease in resistance of a sliding contact with increasing current may be explained on a thermal basis. Slepian[2] gives a formula

$$\left(T = \frac{E^2}{33.5\, K\, \rho} \right) \qquad (1)$$

for the temperature of a small current carrying contact area between two materials. He also cites tests in which contact drop is determined as a function of the specific resistance of the materials involved. Since a curve plotted between the logarithm of these two quantities is a straight line having a slope of 0.43, an exponential law is indicated.

To check the relation between contact drop and the resistivity of the brush material, a test was arranged in which a number of brushes, each of a different resistivity, and each carrying the same current, could be run simultaneously on the same copper slip-ring. By sanding the ring with fine sand paper each time shortly before taking a set of readings, very nice results could be obtained as shown by the curve of Fig. 2. The contact drop obtained on the freshly sanded ring might be called the primary contact drop for it has to do with the fundamental process by which current is conducted across a sliding contact. The difference between this drop and that obtained after a polished film has formed on the commutator and brush might be called the secondary contact drop for it depends upon how the brush polishes and how it rides on the polished surface of the commutator.

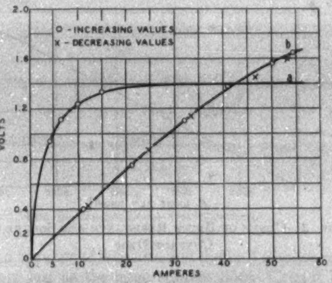

FIG. 1—STATIC AND TRANSIENT BRUSH DROP CURVES

Fig. 2 shows that the relation between the square of the contact drop and the resistivity of the brush material is nicely approximated by a straight line. Neglecting the difference in thermal conductivities of the different brush grades, this curve suggests that the contact points in all copper-carbon sliding contacts operate at approximately the same temperature. It further shows a very definite relation between the resistivity of a brush and its contact drop. This proves definitely that the contact drop is not produced, as is sometimes thought, by the current flowing uniformly through a thin layer of some high resistance material. The results of this test thus check further the hypothesis suggested by Slepian.

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[1] *Commutation Considered as a Switching Phenomena*, by R. E. Hellmund and L. R. Ludwig, A.I.E.E. Winter Convention, 1932.

[2] *Temperature of a Contact*, by J. Slepian, A.I.E.E. JL., October 1926.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

R. Holm[3] has shown that with stationary contacts the current is not carried across the entire contact area, but that it is carried through a relatively small number of points at which the contact is intimate. He describes the condition by speaking of "sieve resistance" at the contact surface. This condition is also true with sliding contacts, as later calculations and experimental results will show. Hence it may be said that the current is carried through small portions of the total brush surface at a comparatively small number of discrete points. Since the contact drop is a function of the resistivity of the brush material, the contact drop is not actually a difference in potential between contact points on the brush and on the ring or commutator, but is really a voltage drop in the brush itself due to the fact that the current density at the contact points is extremely high and that the brush material has considerable resistivity. In other words, the contact drop is merely a resistance drop near the surface of the brush. The volt-ampere characteristic previously referred to may then be explained as follows:

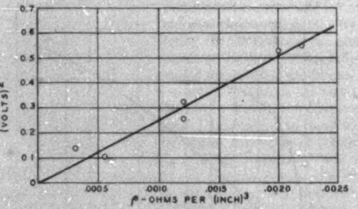

FIG. 2—CURVE OF BRUSH RESISTANCE AND SQUARE OF CONTACT DROP

If the current through the brush is increased, it will in turn tend to increase the temperature of the contact points, and whatever conditions exist at the surface which prevent complete intimate contact will be altered in such a way as to allow a greater total area of contact between the brush and the slip-ring or commutator, and consequently a lower resistance of the total contact.

The formula for the temperature of a contact point given by Slepian is derived for a contact between two electrodes of like material. In this case half the electrical drop occurs in each of the electrodes and the maximum temperature as expressed by the formula occurs at the point of contact. In the contact between a brush and a commutator substantially all the electrical drop occurs in the relatively high resistance brush, as just shown. The actual area of contact of the brush with the commutator must remain at the temperature of the commutator and will therefore have no temperature rise. The maximum temperature in this case is found to be the same as that given by Slepian's formula, but this maximum temperature occurs, not at the point of contact, but in a region of the brush very close to the point of contact.

Intimate contact at all points between the brush and ring is prevented by some form of film. The film must have properties such that if the temperature at a given contact point is increased, it will be driven back, thus allowing a somewhat greater area of actual contact. A film of adsorbed gas on the surface of the brush will have the required properties, for example, in that such a film would prevent close contact between brush and ring and at a certain temperature would be driven off, thereby allowing a greater contact area. The film may also be one of copper-oxide, or a uniform film of graphite material. It should be noted that in either case there is no stable equilibrium during operation; in other words, if a contact is operating with uniform current density and uniform temperature over the entire area, any slight increase of temperature at some point of this area will lower the resistivity at this point, so that very soon the current would not be uniformly conducted, but would again be carried at discrete points.

Tests by Baker[4] in hydrogen and nitrogen did not give contact drops appreciably different from those obtained in the air, and it seems therefore that the film is not in the nature of an oxide.

III. THE MECHANICAL NATURE OF THE SLIDING CONTACT

When a graphite brush runs on a copper ring or commutator which is smooth, in such a way that the mechanical conditions are good, the surfaces are found to have a high state of polish. This polishing, as described, for example, by Adam,[5] results in an amorphous layer of material on the brush and commutator surfaces. In fact, the polishing is so complete that the brush surface often resembles a mirror, and if a straight edge is placed across it a straight reflection is obtained on the surface. Thus, it would be expected that the number of contact points would be comparatively large in case these points are due to coincidence between the surfaces. As pointed out by Lamme,[6] material seems to be transferred across the contact and deposited on the opposite surface, the direction of transfer being the same as the direction of the current. The nature of this action is probably due to the formation and the destruction of small bridges between the materials, and may be somewhat as described by R. Holm[7] in the case of stationary contacts. The voltage required to build such bridges as given by Holm, is of the same magnitude as that of the contact drop. It is probable that the polishing action is considerably assisted by the flow of current and the deposition of material.

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[3] *Zeit. für Tech. Physik*, 1928, p. 454.

[4] *Electric Journal*, Feb. 1932.

[5] Adam "Physics and Chemistry of Surfaces," 1930, Clarendon Press, Oxford.

[6] Lamme "Electrical Engineering Papers," W. E. & Mfg. Co., E. Pgh., Pa., 1919.

It is interesting to note that the frictional loss is less when current is passing through the contact than otherwise. It may be supposed that this is due to the decreased coefficient of friction resulting because of the better polishing. The current flow is important to the polishing action as it provides a means for transfer of material and thus for a greater smoothness of the surfaces. Thus it is often found that when current is first applied to a contact even after it is well worn in, there will be a slow decrease in contact drop and at the same time a greater regularity in voltage regulations will be found. The time required for the contact to reach the steady state is comparatively long, in fact, too long to be accounted for by variations in temperature of the contact points themselves. It is more probable that a large number of contact points will be present after the polishing has been well established.

IV. Dimensions of a Contact

On the assumption that a point contact area between a brush and commutator is circular in shape it is possible to calculate the product of the number of contact points N and the radius of a contact area α. The derivation of Appendix I shows this product to be

$$N\alpha = \frac{\rho I}{2\pi E} \qquad (2)$$

where ρ is the electrical resistivity of the brush material, I is the total current through the contact and E is the voltage drop across the contact. Thus if either the number of contact areas or the radius of a contact is known, the other can be determined.

Little[8] has made some tests to determine the number of points of contact under a brush by discharging a condenser across the contact. The current during the discharge was sufficient to burn the ring and brush at the contact points. Following the discharge the ring was stopped and the number of contact points was counted. These tests probably give correct results because the current increase is so rapid that the contact resistance does not have time to change and hence the drop becomes so high that a small arc is formed.

Evidence was found that the number of contact points varied from ten to more than fifty. With this data it is possible to determine the size of a contact point necessary to account for the voltage drop observed.

Assume a brush with the following specification:

$N = 30$ points
$I = 50$ amperes
$\rho = 0.005$ ohms per cu. cm.
$E = 1.00$ volts

Substituting in equation (2)

$$\alpha = 0.00133 \text{ cm.}$$

If the number of contact points is reduced to three,

$$\alpha = 0.0133 \text{ cm.}$$

Even for the small number of larger points the calculated current density has the enormous value of 29,900 amperes per sq. cm. although as will be shown shortly, this value is quite feasible.

To test the feasibility of carrying very high current densities in a small contact, the face of a commercial sized brush was cut away very slightly to leave a small raised point 0.0013 sq. cm. in area in the center of the brush face to make contact with the slip-ring. It was found possible to raise the current density in this small contact to about 14,000 amperes per sq. cm. before any evidence of distress was observed. The volt-ampere characteristic of this small contact was similar in shape to that observed on a large contact and it must therefore be concluded that even this small contact still contains a number of separate and distinct smaller contact points just as the large contact does. Thus it becomes feasible to believe that an actual point of contact might very well carry a current density of 50,000 or even 100,000 amperes per sq. cm.

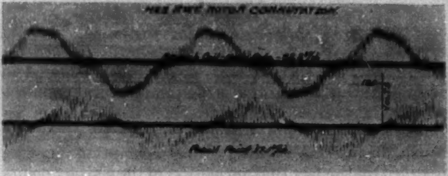

Fig. 3—Brush Drop Oscillograms

If a positive and a negative brush are both running on a slip-ring, the initial contact drop at both will be found to be the same shortly after voltage is first impressed. This is because the number of contact points is probably the same and the size must therefore be the same. This result has been observed when using direct currents, and furthermore the drop taken on a-c. machines is found to be the same with either direction of current through the brush, Fig. 3. After a short period of operation with direct current, however, the drop at one brush is found to become larger than at the other. This difference can only come about if the number of points on one brush becomes greater for some reason. The probable reason is that the difference in material transferred at the two polarities results in different degrees of smoothness. An increase in the number of contact points reduces the current to be carried per contact and thereby reduces the contact drop.

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[7] *Wiss. Ver. aus dem Siemens-Konzern*, X Vol. 4 Heft.

[8] A.I.E.E. Trans., Vol. 50, No. 2, 1931, p. 718.

V. TRANSIENT BRUSH DROP

It has been found experimentally that in case the current is rapidly increased through a contact, a volt-ampere curve will be obtained shown as curve *b* in Fig. 1. This curve corresponds to an almost constant contact resistance and indicates that the contact does not have time in which to adjust itself to the higher value of current, as the current is continually increased. Since the change in resistance is due to a thermal phenomenon, the time lag which exists in the contact must also be thermal, and is, in fact, due to the time required to raise the temperature of the contact point. There is also the possibility, of course, that there is some time delay in altering the nature of the contact surface even after sufficient temperature has been reached. For example, after an adsorbed gas film is considered to be present some time is required to drive off this film and thus allow a greater area of contact. However, calculations (not given) show that any time lag due to this or a similar phenomenon must necessarily be quite short. Calculations are made in the Appendix to determine the rate of heating of the contact points. The following equation may be used for this purpose:

$$\frac{\partial T}{\partial t} = \frac{E^2}{67.3\,\gamma\alpha^2} \qquad (3)$$

where γ is the specific heat of the brush material in calories per cu. cm., and T is the temperature rise of the contact in degrees centigrade. Such a calculation assumes that the contact point does not move on the brush during the heating. This assumption may be justified by the fact that no motion of the contact points would be expected to take place until the particular point had worn down, and this would require more time than the time necessary to heat the point.

The result is obtained that a point having the dimensions given in Section IV ($\alpha = 0.0133$ cm.) would require 0.032 sec. for heating to 62 per cent of its final temperature.

It is interesting to note from equation (3) that the rate of heating of a contact point depends on the properties of the brush and, of course, on the size of the contact point. This fact may open the door to the reasons which underlie the successful use of certain brushes under certain commutating conditions. A further discussion of this factor will be given in the next section.

VI. EFFECT OF TRANSIENT BRUSH DROP ON COMMUTATION

The current density under a brush varies from point to point, depending upon the voltage induced in a coil and the position of the associated bars under the brush. Except with straight-line commutation, there will be rapid changes in density at any point of the brush face; these changes occurring at commutator bar frequency. Conditions may be very severe in case of more than one coil per slot, because the curve along which the current changes for two consecutive coils may vary widely.

If such changes in density occur in practise, the voltage between a portion of the brush contact and the commutator becomes very high, and consequently, sparking under the brush will result. In Fig. 3 an oscillogram is shown of the voltage from the brush to a point on the commutator and it may be seen that a potential difference as great as 15 volts is reached. Such a potential difference is well above that required for the maintenance of a spark in air, which may require about 11 volts. Hence, in considering sparking during commutation, two kinds must be differentiated; first, sparking underneath the brush due to high transient voltages, and, second, the usual trailing edge sparking. The first form must not be confused with sparking due to mechanical roughness of the commutator.

In order to obtain good commutator and brush life, it is very important to avoid sparking under the brushes, which may occur even in the absence of the trailing edge sparking. An obvious way to avoid this difficulty is to design the machine so that no radical changes in density will take place. However, this cannot always be accomplished, in which event the designer often experiments with various grades of brushes and frequently finds one which operates satisfactorily. The reasons for the success of some brushes and the failure of others have not always been apparent. In these cases the explanation may be found in the fact that the successful brush has a very large number of contact points, and thus points of small area, which will heat and cool with sufficient rapidity so that high transient voltages will be avoided.

Another puzzling question in connection with commutation is that of why the current density in brushes must be kept so low when it is known that a much greater density may be employed in the laboratory without sparking. For example, a figure of 50 to 70 amperes per square inch is a fair design value, whereas densities of several thousand amperes per square inch have been used without sparking during laboratory tests. This matter is very important when it is realized that even doubling present densities would have the length of present commutators. In Section IV it was shown that the rate of heating of the contact points depends upon their size, which is a function of the current density. It is essential to keep the contact points small if sparking under the brushes is to be avoided, and therefore the real limitation of operating density becomes apparent as due to the necessity for keeping the current per contact small. In order to increase the present operating density, it would be necessary to make a brush which would operate at a higher density with small contact areas. This might be accomplished by making a brush having a very large number of contact points, or by greatly increasing the thermal conductivity without altering the electrical resistivity appreciably.

Appendix I

CALCULATION OF SIZE AND NUMBER OF CONTACT POINTS

The loss in a hemispherical shell of brush material around a point of contact is*

$$8\,\pi \times 0.0595\,\frac{E^2}{\rho}\,\frac{\alpha^2}{r^2}\,dr \text{ calories per sec.} \qquad (1)$$

E = volts drop in contact
α = radius of contact area in cm.
ρ = resistivity of brush material, ohms per cu. cm.
r = distance from point of contact.

The total loss for N points of contact is therefore

$$N\int_{\alpha}^{\infty} 8\,\pi \times 0.0595\,\frac{E^2}{\rho}\,\frac{\alpha^2}{r^2}\,dr \qquad (2)$$

$$= 8\,\pi \times 0.0595\,\frac{E^2\alpha N}{\rho} \text{ cal. per sec.} \qquad (3)$$

This must also equal

$$\frac{1}{4.19}\,EI \qquad (4)$$

where I is the total current carried by the contact. Therefore

$$\frac{1}{4.19}\,EI = 8\,\pi \times 0.0595\,\frac{E^2\alpha N}{\rho} \qquad (5)$$

or

$$N\alpha = \frac{\rho I}{2\,\pi E} \qquad (6)$$

Appendix II

RATE OF HEATING OF CONTACT POINTS

The heat generated per unit volume of the brush is*

$$\frac{\partial H}{\partial t} = 4 \times 0.0595\,\frac{E^2}{\rho}\,\frac{\alpha^2}{r^4} \qquad (7)$$

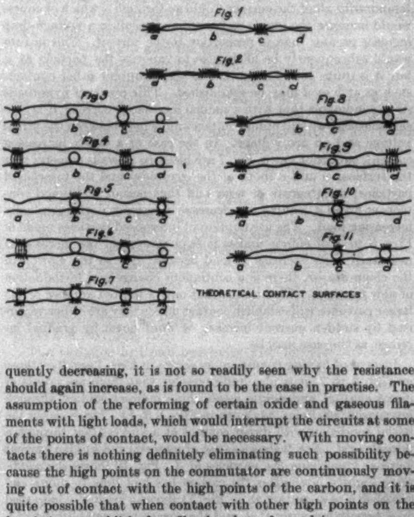
If γ is the specific heat of the brush material in cal. per deg. cent. per cu. cm., the initial rate of temperature rise at the point of maximum temperature ($r = 2a$) is

$$\frac{\partial T}{\partial t} = \frac{1}{\gamma}\,\frac{\partial H}{\partial t} = \frac{E^2}{67.3\,\gamma\alpha^2} \text{ deg. cent. per sec.}$$

Discussion

R. E. Hellmund: Contact phenomena and the characteristics of contacts under various conditions have been puzzling to electrical engineers for a great many years. While even yet a great many questions remain to be answered, considerable progress has been made in explaining these phenomena, and the paper presented by Messrs. Ludwig and Baker is a valuable contribution in this direction. It is especially interesting to note that while it frequently had been assumed that contact phenomena might require for their explanation entirely new and unknown basic principles, it seems now as though they might eventually be explained in a rather simple manner by assuming certain mechanical conditions of the contact areas and nothing but ohmic resistance in the materials proper and other well-known phenomena.

The theory discussed in the paper can possibly be illustrated by Figs. 1 and 2. If we have two irregular contact surfaces establishing small points of contact at *a* and *c*, as shown in Fig. 1, the drop is essentially caused by the ohmic resistance of the two materials in the neighborhood of the contact points. If the current suddenly increases before any change in this mechanical condition is possible, the resistance remains the same but the ohmic drop increases with the current, as indicated by curve 1*b* in the paper. If the increased current persists, it may be assumed that the heating of the small contact points increases to an extent which permits a certain disintegration of the materials at these points. This may result not only in increased contact surfaces at *a* and *c* as indicated in Fig. 2, but also in the establishing of additional points as shown at *b* and *d*, which would of course be possible if the carbon surfaces at *a* and *c* wear down a certain amount. This readily explains why with sustained load the resistance may decrease, keeping a fairly constant voltage drop. However, with this condition established and the current subsequently decreasing, it is not so readily seen why the resistance should again increase, as is found to be the case in practise. The assumption of the reforming of certain oxide and gaseous filaments with light loads, which would interrupt the circuits at some of the points of contact, would be necessary. With moving contacts there is nothing definitely eliminating such possibility because the high points on the commutator are continuously moving out of contact with the high points of the carbon, and it is quite possible that when contact with other high points on the brush is re-established, a film has been formed in many cases which will not be broken down except under certain conditions. In other words, even though many points of contact may have been established during operation with large current density, they do not necessarily continue to exist because of the motion of the commutator.



THEORETICAL CONTACT SURFACES

Instead of assuming direct contact between the two surfaces, another theory which assumes the existence of small loose particles between the surfaces, as shown in Fig. 2, might be advanced. The fact that such loose particles play an important part in the contact phenomena finds some support in the fact that both contact resistance and chattering noises can be materially changed merely by wiping the commutator surface with a dry piece of cloth. The assumption of the existence of such particles leads to some rather interesting possibilities. Let us start out again with current conduction at *a* and *c* on account of the larger size of the particles located there, while at *b* and *d* there is no contact, as shown in Fig. 3. If we assume now that the current suddenly increases up to a certain value, the resistance may not change, but voltage drop increases in proportion to the current. On the other hand, if the current is suddenly increased to a much greater extent, it can readily be imagined that the particles at *a* and *c* explode and evaporate and that, as shown in Fig. 4, there is temporarily no contact at all but an arc at the points *a* and *c*. This in turn would mean that there would be not only the ohmic drop in the materials near the points of the arc but also an arc voltage. This might at times explain the arcing which is noted under the brush. Soon after the arcs have formed, but not until the brush pressure has been able to accelerate the mass of the brush, we will obtain the condition in Fig. 5, where the current will now be conducted through the particles at *b* and *d* and the arc extinguished at *a* and *c* due to the fact that the ohmic drop through *b* and *d* is now below a value which would sustain an arc. The condition in Fig. 4 would of course mean a rather appreciable voltage drop.

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*See reference 2.

Instead of both particles at *a* and *c* vaporizing at the same instant, a condition as shown in Fig. 6 can be imagined. In this case it is assumed that the particle at *a*, on account of its smaller size, vaporizes before the particle at *c*. This would mean that temporarily all of the current would go through *c*, which of course would increase the ohmic resistance and result in a voltage-drop increase greater than in proportion to the current. This in turn would subsequently be likely also to vaporize the particle at *c*, but it is quite conceivable that in the meantime other contacts such as at *b* or *d* may be established. This previous hypothesis simply indicates that with a sudden increase in current, it is conceivable that we will obtain voltages anywhere between the initial voltage and an arc voltage. In the case of a slowly increasing current, conditions as in Fig. 7 are likely to prevail; namely, that the surfaces *a* and *c* (both of the electrodes and the interposed particles) disintegrate or wear and thus permit additional contacts at *b* and *d*, leading to a decrease of resistance with gradually increasing load. The assumption of interposed particles made in Figs. 3 to 7 of course makes it easy to explain the increase of resistance with decrease in load because, due to the motion of the commutator, there is a continuous change and introduction of new particles. This means that only a limited number of the larger particles will establish contact unless thay are either vaporized by sudden current increase or worn down by gradual increase, as the case may be.

Instead of having conditions as shown in Figs. 1 and 2 or those in Figs. 3 to 7, it is also quite possible that there are points of direct contact and points establishing indirect contact through interposing particles, as shown in Figs. 8 to 11. Fig. 8 shows the initial condition with direct contact at *a* and contact through a particle at *d*. Fig. 9 shows a case of material and sudden current increase vaporizing the particles at *d*. Fig. 10 shows the conditions slightly later after the carbon has had time to move and establish a new contact at *c*. Fig. 11 shows a gradually increased load permitting wear at *d*, and the consequent establishing of a new contact at *c*.

No claim is made that the above hypothesis has as yet been exhaustively checked with the results of experimental data, but it seems that there is nothing generally known which contradicts these possibilities. It may further be pointed out that the hypothesis advanced in connection with Figs. 3 to 6 assumes a certain motion of the carbons. This, if present, may lead to vibration and chattering, which in turn may bring about other conditions not discussed here in detail. Similarly, any roughness of the surfaces proper may lead to conditions which are not covered in this brief discussion. Further work along this line is very worth while, not only from a hypothetical and theoretical point of view but also because it is probable that great advances will not be made in the manufacture of contact materials until the contact phenomena are fully explained. In carrying out such work, it may be well to realize that possibly the ordinary oscillograms may not be fast enough for recording the maximum voltages for some of the transient conditions.

L. A. Heath: An instance of the ever changing conditions of contact between brush and commutator is that of brushes sparking when a cold rotary or generator is started and full load is carried immediately. Sparking at the brushes lasts for a period of say twenty minutes and then perfectly black commutation occurs and remains so for the balance of the run. This situation repeats every time the unit is started cold. During the initial period when sparking occurs the slope of the brush contact curve indicates too great an interpole field strength but after the sparking has ceased the contact curve is quite normal. The contact faces of the brushes when inspected during the first twenty minutes of operation, indicate that the brushes have changed their seating from that taken at the last shut down.

The question of an air film between brush and commutator or ring still presents a problem to be investigated thoroughly. In actual practise attempts are made to correct this fault by cutting slots in the faces of the brushes or drilling holes through the length of the brushes. Films or coatings on the collector surfaces may be formed in many ways, either by chemical fumes, oil or moisture in the air. Chemical fumes in sufficient concentration will form a film so heavy as to upset entirely the normal current collection and vicious sparking results. Also certain types of brushes will in due time absorb chemical fumes which destroy forever their customary current collecting properties.

During recent years the amount of moisture in the air also has been a factor in a number of brush operation problems. Just how great a part moisture plays in brush operation I am not prepared to say at this time. It has changed the friction losses considerably in a number of instances and has also altered the amount of the contact drop. At the present time there is under observation a brush test on a vacuum cleaner of a design such that the air is drawn across the motor. A record extending over several weeks shows that the color of the commutator and the degree of sparking vary with the relative humidity.

In part III of the paper reference is made to the desirability of a high polish on the contact faces of brushes. From observation in the field we have found that a high polish generally produces a higher friction and poorer commutation than a face slightly dull and pin pricked. Very often this high polish is produced when the machine is running at light loads and the current density is not great enough to cause a transference of brush material.

W. E. Stine: The "theory of a sliding contact" and data submitted by Messrs. Ludwig and Baker are in very close conformity with a theory which I advanced several years ago* to explain brush contact drop characteristics. I did not have any data as to the number of points in contact at any one instant of time but assumed that in all probability there were never less than three. It now seems that there are never less than ten points in contact.

The authors have not given us a detailed description of the kind of brush material or apparatus used; therefore there are a few questions raised in the following and a few points in which I express a different view.

Did the authors distinguish between brushes made of amorphous or electrographic carbon which have symmetrical specific resistance, and brushes made of graphitic carbon whose specific resistance measured in one direction may be many times greater than the specific resistance measured at right angles to that direction? Losses in the latter class of material cannot be calculated on the basis of hemispherical shells.

It seems that the authors have neglected the heat produced by friction when considering the temperature of the point of contact.

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*"Brushes for Electric Motors and Generators," *Jl. of the American Society of Naval Engineers*, May 1925.

If we consider that one-half of the friction loss appears as heat in these minute points of contact, will not the temperature of these points be considerably higher than the temperatures indicated by the formula?

Since carbon has an appreciable negative temperature coefficient of resistance, will not the resistivity of the brush material at the point of contact be a variable?

I do not believe that the authors are correct in assuming that the actual area of contact of the brush with the commutator must remain at the temperature of the commutator. If this were true a brake shoe would never have a higher temperature than the wheel upon which it is bearing.

It has been my observation that the friction loss is less when carrying current only when the brush is permitted to deposit a film of carbon on the ring of commutator. If the brush has sufficient abrasive action to keep the ring or commutator clean and bright, the flow of current does not cause a decrease in friction.

From my experience I cannot agree with the authors' observations as to the behavior of brushes that develop a higher contact resistance at one polarity than at the other. Some grades of brush material have a greater tendency to do this than others. If we take a set of brushes that have this as a marked characteristic, place them on a copper ring, pass alternating current through them and connect an average value meter across a brush contact, we will find that a reading is obtained, indicating that the contact drop during one-half cycle is greater than during the other half. This is obtained only while the ring is moving. As soon as the ring comes to rest the meter returns to zero or nearly so.

Brush material having the greatest tendency to deposit carbon on the ring shows the greatest difference in contact resistance at the negative and positive contacts respectively.

My observation of this phenomenon has led me to believe that the higher contact resistance at the negative (motor) brush is due to the fact that at this contact the brush material disintegrates at a greater rate than at the other contact and as it is carried from under the brush it lifts the brush from the ring causing the current to pass through these loose particles of brush material and thus introduce a longer high resistance path.

It has been found that a flow of one or two amperes is sufficient to bring about this difference in contact resistance at the negative and positive contacts.

Furthermore it makes no difference whether the brushes are trailing each other, which indicates that the deposited film is not the controlling factor.

L. R. Ludwig: In the paper the authors have attempted to describe a "film" on the brush surface which prevents intimate electrical contact between the brush and commutator except at discreet points. The description of this "film" was kept quite general, because at present its exact nature remains undetermined. Mr. Hellmund in his discussion has given further examples of what such a "film" may be like; for example he has supposed that it may consist of small carbon particles rolling between the brush and commutator. The properties which the "film" must have may be comparatively well defined, in that a diminution in the area of the film is to be expected with an increase in the temperature of the contact points, and that the phenomena must be reversible. Mr. Hellmund has explained this reversibility as due to motion of the commutator. This is a possible explanation, but not a necessary one. It is well known that an adsorbed gas film which has been driven off due to high temperature, will immediately re-form if the temperature is reduced.

Mr. Hellmund has mentioned the continuous motion of the points of contact between brush and commutator which probably takes place. It may be well to point out, however, that this continuous "shifting about" of the points of intimate contact is slower than the heating or cooling of the contact points. If this were not the case, variation in contact drop would be a function of the motion of the points only, and would not depend on temperature in the way shown by Fig. 2 of the paper.

The initial irregularity in brush contact when a commutator machine is first started has been pointed out by Mr. Heath. The authors have oscillograms of this phenomenon, and have explained it in section III of their paper as being due to the necessity of re-establishing of the "polishing" action.

Mr. Heath speaks of air films on the brush surface as faults which one may attempt to correct by slotting the brushes. With slip rings it is desirable to have the lowest possible contact drop at the brush surface and any insulating film may well be considered as a fault, but with commutators this film results in the contact drop which makes practical operation of commutating machinery possible. Without some form of film, it is doubtful if commutation could be satisfactorily accomplished at all. The films described in the paper of which an adsorbed (not absorbed) gas film is typical, are not to be gotten rid of by simply slotting the brushes. These films cling very tenaciously, and even in high vacuum, they are still to be found. The nature of the film and its behavior to variations in temperature depends on the gases present, and hence chemical fumes and moisture in the air may be expected to affect commutation as Mr. Heath has pointed out. Of course in practical operation extraneous films of oil for example, may be found on commutators which have an important effect on operation; but which play no part in the fundamental phenomena of the contact. Research done to establish the fundamental theory involved can only be begun with simple conditions in which cleanliness is essential.

The authors have used the term "polished surface" to devote one having a large number of regions at which electrical contact is possible. Although this would generally be expected to be the same as a mechanically polished surface, it seems possible that with some brush materials the "electrical polish" is better when the surface appears pin pinched as described by Mr. Heath. In the case he mentioned one would expect the number of contact areas to be less when running at light load than at full load, regardless of the surface appearance.

In answer to Mr. Stine, the authors used brushes having a specific resistivity independent of the direction of measurement.

The heat produced by friction was not neglected in the calculations which were made, but it plays no part in heating the contact points because most of the friction is not at these points, but is due to the presence of the "film" away from the points. This film has been found to withstand the pressure of brush on the commutator. (See paper by Schröter, *Archiv. für Elek.* 1927, p. 111 and 1931, July 15.)

The variation in brush resistivity with temperature was not taken into account in calculating the potential drop at the contact points, except that calculation showed this effect to be insufficient to explain the observed shape of the brush drop curves, in itself. The calculations given in the paper are therefore only approximate.

There seems to be no evidence for either electrical or thermal drop between a commutator and the contact point on a brush. A thermocouple placed at the brush surface (in so far as this is possible) would not measure the temperature of the contact points, because of the thermal as well as electrical drop in the material very near the contact points. This fact together with the fact that the friction is mainly away from the contact points proper readily explains why a brake shoe may have a temperature higher than the wheel upon which it is bearing. Actually, it is practically impossible to measure directly the temperature of the contact points themselves. It must be remembered too that mechanical contact persists at regions other than the region of electrical contact.

Mr. Stine has noted a non-symmetrical drop on alternating current by using a meter. This cannot be explained without more data, but the effect is not general as indicated by the symmetrical drop on alternating current shown in the oscillogram in Fig. 3 of the paper.

Single-Phase Short-Circuit Torque of a Synchronous Machine

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***Synopsis.**—This paper extends the analysis of synchronous machines previously published by Doherty and Nickle to include the calculation of torque due to single-phase short circuit. The torque is expressed as the sum of odd and even harmonic series which are simply related to those previously derived for the armature current. The effect of amortisseur windings is also taken into account.*

As an illustration of the application of the equations, the short-circuit torque is calculated for a 100,000-kva. generator.

The mathematical analysis is given in Appendixes as follows: A. Short-circuit armature current. B. Short-circuit torque. C. Decrement factors. D. Single-phase and three-phase reactances.

* * * * *

INTRODUCTION

THE analysis in this paper is based on those previously published by Messrs. R. E. Doherty and C. A. Nickle,[1,2,3,6] particularly on their paper *Synchronous Machines—IV,* which gives expressions for the currents resulting from single-phase short-circuit.

As the present paper is based on the theory and equations developed in *Synchronous Machines—IV,* the simplifying assumptions made therein are necessarily used here.

The most important of these assumptions are that:

1. Saturation is negligible.
2. The open-circuit voltage is sinusoidal.
3. Armature and field resistances are negligible except in determining decrements.
4. The short circuit occurs at no load.
5. Synchronous speed is maintained during short circuit.

Throughout the paper all quantities are expressed in the per-unit system unless otherwise noted, and instantaneous values of voltage, current, m.m.f., flux linkages, and torque are used.

GENERAL PROCEDURE

The synchronous machine is considered as having a main field winding in the direct axis and amortisseur windings in both the direct and quadrature axes, thus introducing both subtransient and transient reactances.

The general procedure employed in *Synchronous Machines—IV*[3] is used to determine armature current and rotor excitation, except that three-phase line-to-neutral reactances are used in place of single-phase static reactances.

The values of the armature current and m.m.f. before the effect of the decrements is appreciable are determined by the open-circuit armature voltage e_o, the position of the rotor when short-circuit occurs, and the subtransient reactances. The armature m.m.f. is resolved into components over the direct and quadrature axes and is expressed in terms of odd and even harmonic series. The fundamental of the odd-series current induces a transient direct current in the direct-axis rotor circuits; all other harmonic components induce alternating currents in the rotor windings. Consequently, on short circuit the d-c. rotor excitation is increased from the value existing just before short circuit to a new value which includes a component induced in the two direct-axis rotor windings by the armature current. The decay of the induced d-c. components of excitation and that of the odd-series component of the armature current are determined by the decrement factors of the rotor windings in the direct axis.

It is assumed that the decay of the unsupported rotor excitation may be represented with sufficient accuracy by two decrement factors, one applying to the decay of induced direct currents in the amortisseur winding, and the other applying to the decay of that part of the d-c. component of field current which the exciter does not maintain. The "rotor linkage factor," F, represents the total rotor linkages, at any time after short circuit, as a fraction of the rotor linkage just before short circuit. Equation (**15a**) of Appendix A gives F as a function of time, in terms of machine constants.

The linkages in the short-circuited armature winding are maintained, at the instant just after short circuit, by the induced d-c. component of armature current. These linkages must decay with the d-c. component of armature current. For convenience, an "armature linkage factor," A, is introduced. It represents the linkages of the short-circuited armature winding at any time after short circuit, as a fraction of the maximum linkages of that winding before short circuit. Equation (**16a**) of Appendix A gives the expression for A. As shown by Doherty and Nickle,[3] the even-series components of armature current and the odd-series components of rotor excitation are proportional to the d-c. component in the armature, so that their decay is also represented by the armature linkage factor, A.

The detailed derivation of the expressions for short-circuit armature current is presented in Appendix A.

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In equation (**17a**) the current is expressed in finite trigonometric form, while equation (**18a**) gives the equivalent Fourier series expansion.

The short-circuit torque is expressed in terms of the components of armature current and rotor excitation in the direct and quadrature axes. In Appendix B expressions for torque are derived, first in finite trigonometric form (equation **6b**), and second, as the sum of odd and even harmonic series (equation (**9b**).

Summary of Results. The per-unit single-phase short-circuit torque is

$$T = \frac{2FAe_o^2}{x_2 + x_d''}\{\sin(t+\alpha) + 3\,b\sin 3(t+\alpha) + 5\,b^2\sin 5(t+\alpha) + \ldots\}$$
$$-\frac{e_o^2}{x_2 + x_d''}\left[F^2\frac{x_2}{x_2 + x_d''} + A^2\frac{x_2 - x_d''}{x_2}\right]\{2\sin 2(t+\alpha) + 4\,b\sin 4(t+\alpha) + 6\,b^2\sin 6(t+\alpha) + \ldots\}$$

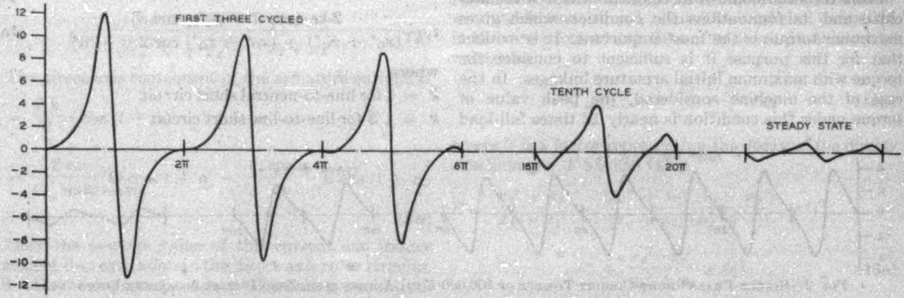

FIG. 1—SINGLE-PHASE SHORT-CIRCUIT TORQUE OF 100,000-KVA. ALTERNATOR, MAXIMUM INITIAL ARMATURE LINKAGES

$$\text{Torque} = -\frac{2F\,(F\cos t - A)\sin t}{H} - \frac{0.438\,(F\cos t - A)^2\sin 2t}{H^2}$$
$$= 5.25\,FA\,(\sin t + 0.669\sin 3t + 0.249\sin 5t + \ldots.)$$
$$-(3.21\,F^2 + 1.92\,A^2)(\sin 2t + 0.446\sin 4t + 0.149\sin 6t + \ldots)$$
$$F = 0.449 + 0.417\,\epsilon^{-0.0021t} + 0.134\,\epsilon^{-0.069t};\quad A = \epsilon^{-0.035t};\quad H = 0.515 - 0.219\cos 2t$$

where

e_o = per-unit armature voltage before short circuit

x_d'' = direct-axis subtransient reactance

x_q'' = quadrature-axis subtransient reactance

x_2 = $\sqrt{x_d''x_q''}$ = negative-sequence reactance

α = angular position of the direct axis at the instant of short circuit, measured in the direction of rotation from the axis of the armature winding

and

$$b = \frac{x_2 - x_d''}{x_2 + x_d''} = \frac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}}$$

F = rotor linkages as a fraction of their initial value (see equation (**15a**))

A = armature linkages as a fraction of their maximum value before short circuit, that is, as a fraction of their value on open circuit when the axis of the armature winding coincides with the direct axis of the rotor (see equation (**16a**))

From an inspection of the above equation it appears that the odd harmonic components of torque are due to the interaction of rotor and armature magnetic fields and that these components disappear as the trapped armature linkages die out. For the case of a short circuit with zero initial armature linkages no odd harmonic torques are present.

A part of the even harmonic torque is proportional to the square of the armature linkages and this part disappears when the subtransient reactances in the direct and quadrature axes are equal. This may be considered as a "reluctance torque" due to the variation in the permeance which the rotor offers to the trapped armature flux. Likewise the part of the torque which is proportional to the square of the rotor linkages is due to the variation in the permeance which the armature offers to the rotor flux.

It is interesting to note that, immediately after a short circuit with maximum armature linkages, the maximum value of second-harmonic torque due to the field linkages alone is $x_2/(x_2 + x_d'')$ times the maximum fundamental torque, while the maximum value of second-harmonic torque due to armature linkages alone is $(x_2 - x_d'')/x_2$ times the maximum fundamental torque. The maximum value of total second-harmonic torque just after a short circuit with maximum armature linkages is $(2\,x_q'' - x_d'')/(x_q'' + x_2)$ times the maximum value of fundamental torque. This ratio varies from 0.5 for $x_d'' = x_q''$ to 1.25 for $x_d'' = 0.2\,x_q''$. Thus the maximum values of fundamental and second-harmonic torque are of comparable magnitude just after short circuit.

Since the armature linkages eventually disappear, the steady-state torque is made up entirely of even harmonic components.

To illustrate the use of the formulas developed, numerical results have been calculated for a 60-cycle testing generator rated at 100,000 kva. The per-unit constants of the machine, as determined by test, are as follows:

$x_d = 0.615$; $x_d' = 0.207$; $x_d'' = 0.148$
$x_q = 0.412$; $x_q' = 0.412$; $x_q'' = 0.367$
$\sigma_a = 0.025$; $\sigma_f' = 0.0021$; $\sigma_f'' = 0.069$
Hence $x_2 = 0.233$; $b = 0.223$

Figs. 1 and 2 show the line-to-line short-circuit torque with maximum initial armature linkages and zero initial armature linkages respectively, as computed from the formulas derived.

From the standpoint of mechanical design of the machine and its foundations the condition which gives maximum torque is the most important. It is evident that for this purpose it is sufficient to consider the torque with maximum initial armature linkages. In the case of the machine considered, the peak value of torque under this condition is nearly 12 times full-load torque. It is to be noted that this high peak value is largely due to the presence of trapped armature linkages, since with no armature linkages the peak value of torque is only 4 times normal.

The trapped armature linkages are also responsible for all odd harmonic components of torque. Fig. 2 shows that with no trapped armature linkages the torque is made up entirely of even harmonics and in Fig. 1 it is seen that the odd harmonics decay gradually so that the torque curve approaches that of Fig. 2.

It appears from the equations in Appendix B that the short-circuit torque is the same for line-to-line and line-to-neutral short circuits. This is exactly true for an ideal machine and is true for actual machines to the extent that $x_o/2$ is negligible in comparison with x_d'' and x_q'' (see Appendix D). Although the armature *currents* resulting from the two types of short circuit of an ideal machine are different, the armature *m.m.fs.* are the same.

The consideration of machine losses would somewhat modify the torque as calculated in this paper. The correction for losses can be calculated approximately in a given case, but this correction is generally small enough to be neglected.

ACKNOWLEDGMENT

The authors gratefully acknowledge the many valuable suggestions of Mr. Alan Howard, regarding both the theory and its presentation, and the helpful criticisms of Mr. P. L. Alger and Mr. R. H. Park. They also wish to acknowledge the assistance of Mr. R. W. Smith in checking the equations, and of Mr. N. V. Cargill in calculating and plotting the curves.

Appendix A

SHORT-CIRCUIT ARMATURE CURRENT

Doherty and Nickle have shown[3] that the armature current resulting from single-phase short circuit is

$$i = \frac{2\,k e_o\,[\cos(t+\alpha) - \cos\alpha]}{(x_D'' + x_Q'') + (x_D'' - x_Q'')\cos 2\,(t+\alpha)} \qquad \textbf{(1a)}$$

where

$k = 1$ for line-to-neutral short circuit
$k = \sqrt{3}$ for line-to-line short circuit
e_o = armature voltage before short circuit
α = angular position of the direct axis at the instant of short circuit, measured in the direction of rotation from the axis of the armature winding

and x_D'' and x_Q'' are respectively the direct- and quadrature-axis single-phase static reactances of the short-circuited armature winding.

Changing to three-phase reactances, as in equation (**1d**) of Appendix D, the current is

$$i = \frac{3\,e_o\,[\cos(t+\alpha) - \cos\alpha]}{kH} \qquad \textbf{(2a)}$$

where

$$H = (x_d'' + x_q'') + (x_d'' - x_q'')\cos 2\,(t+\alpha) \qquad \textbf{(3a)}$$

As shown in reference 3, equation (**2a**) may be resolved into harmonic components as follows:

$$i = \frac{3\,e_o}{k\,(x_d'' + x_2)}\,\mathbf{O} - \frac{3\,e_o\cos\alpha}{2\,k x_2}\,\mathbf{E} \qquad \textbf{(4a)}$$

where $x_2 = \sqrt{x_d''\,x_q''}$ is the negative sequence reactance

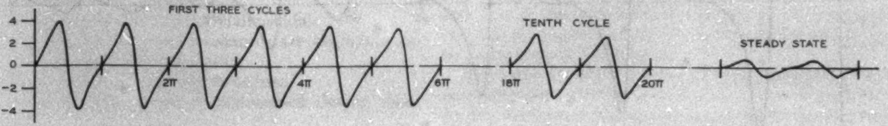

FIG. 2—SINGLE-PHASE SHORT-CIRCUIT TORQUE OF 100,000-KVA. ALTERNATOR, ZERO INITIAL ARMATURE LINKAGES

$$\text{Torque} = \frac{0.734\,F^2 \sin 2\,t}{H^2}$$
$$= 3.21\,F^2\,(\sin 2\,t - 0.446 \sin 4\,t + 0.149 \sin 6\,t - \ldots)$$
$$F = 0.449 + 0.417\,\epsilon^{-0.0021\,t} + 0.134\,\epsilon^{-0.069\,t}; \qquad H = 0.515 + 0.219 \cos 2\,t$$

of the machine applicable to the condition of dead short circuit, and where

$$\mathbf{O} = \text{odd series} = \frac{(x_d'' + x_2)\cos(t+\alpha)}{H}$$

$$= \cos(t+\alpha) + b\cos 3(t+\alpha) + b^2 \cos 5(t+\alpha) + \ldots$$

$$= \sum_{n=1,3,\ldots} b^{\frac{n-1}{2}} \cos n(t+\alpha) \tag{5a}$$

$$\mathbf{E} = \text{even series} = \frac{2x_2}{H}$$

$$= 1 + 2b\cos 2(t+\alpha) + 2b^2\cos 4(t+\alpha) + \ldots$$

$$= 1 + \sum_{n=2,4,\ldots} 2b^{\frac{n}{2}} \cos n(t+\alpha) \tag{6a}$$

and

$$b = \frac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}} = \frac{x_2 - x_d''}{x_2 + x_d''} \tag{7a}$$

The direct-axis component of the armature current is

$$i_d = \frac{2k}{3} i\cos(t+\alpha)$$

$$= \frac{2e_o}{x_d'' + x_2}\mathbf{O}\cos(t+\alpha) - \frac{e_o\cos\alpha}{x_2}\mathbf{E}\cos(t+\alpha) \tag{8a}$$

Only the average value of this current can induce transient d-c. excitation in the direct-axis rotor circuits. All other components induce alternating excitation. The average value of the even-series components of current is zero, and the average value of the odd-series components is

$$\frac{e_o}{x_d'' + x_2} \tag{9a}$$

The sudden appearance of this direct-axis armature current induces the initial d-c. rotor excitation

$$\Delta'' I_{de}\ (\text{d.c.}) = \frac{(x_d - x_d'')\,e_o}{x_d'' + x_2} \tag{10a}$$

as shown in *Synchronous Machines—III.*[2] Thus the total d-c. excitation due to currents in the direct-axis rotor windings at the instant immediately after short circuit is

$$e_o + \Delta'' I_{de}\ (\text{d.c.}) = e_o \frac{x_d + x_2}{x_d'' + x_2} \tag{11a}$$

The exciter voltage maintains only a part of this excitation, and the decay of the unsupported part is determined by the decrement factors of the rotor circuits.

Now if the decrement of the main field is very much smaller than that of the amortisseur winding, as is generally the case, the decay of the linkages supported by each of the circuits can be considered separately. Assuming the decay of induced direct current in the amortisseur to be practically completed before the decay of main field linkages is appreciable, the average value of the direct-axis component of armature current due to the linkages supported by the field winding alone is

$$\frac{e_o}{x_d' + x_2} \tag{12a}$$

The d-c. excitation due to the main field current alone is, therefore,

$$e_o + \Delta' I_{de}\ (\text{d.c.}) = e_o \frac{x_d + x_2}{x_d' + x_2} \tag{13a}$$

The difference between this excitation and the excitation e_o, which is supported by the exciter, decays according to the field decrement, σ_f'. Likewise the difference between the excitations expressed by (**11a**) and (**13a**) decays according to the direct-axis amortisseur winding decrement, σ_f''. Thus the total d-c. excitation in the direct axis at any time after short circuit may be written as

$$I_{de}\ (\text{d.c.}) = \frac{x_d + x_2}{x_d'' + x_2} F e_o \tag{14a}$$

where F is a factor representing the decay of the direct-axis linkages. F has the value

$$F = \frac{x_d'' + x_2}{x_d + x_2} + \frac{(x_d'' + x_2)(x_d - x_d')}{(x_d + x_2)(x_d' + x_2)}\epsilon^{-\sigma_f' t}$$

$$+ \frac{x_d' - x_d''}{x_d' + x_2}\epsilon^{-\sigma_f'' t} \tag{15a}$$

F is unity at the instant just after short circuit.

Since the odd-series components of armature current are due to rotor linkages the magnitudes of these components at any time after short circuit are F times their initial magnitudes.

The even-series components of armature current are due to the trapped armature linkages. Since these linkages are unsupported they decay in accordance with the armature decrement, σ_a, and the even-series components of armature current decay at the same rate. For convenience, the quantity

$$A = \epsilon^{-\sigma_a t}\cos\alpha \tag{16a}$$

is chosen to represent the trapped armature linkages at any time after short circuit occurs, as a fraction of the maximum armature linkages before short circuit.

Inserting the armature and field linkage factors, A and F respectively, in equations (**2a**) and (**4a**), the armature current at any time after short circuit is

$$i = \left(\frac{3}{2k}\right)\frac{2e_o[F\cos(t+\alpha) - A]}{H} \tag{17a}$$

or

$$i = \frac{3}{2k}\left[\frac{2e_o F}{x_d'' + x_2}\mathbf{O} - \frac{e_o A}{x_2}\mathbf{E}\right] \tag{18a}$$

where **O** and **E** are the series previously defined.

Appendix B

SHORT-CIRCUIT TORQUE

The general expression for the torque of a synchronous machine is given by R. H. Park.[5] Equations (**11**), (**12**) and (**19**) of reference 5 may be combined to give the torque in terms of the direct and quadrature-axis components of armature current and equivalent rotor excitation. The result is

$$T = i_q I_{de} - i_d I_{qe} - i_d i_q (x_d - x_q) \qquad \textbf{(1b)}$$

where i_d and i_q are the direct and quadrature-axis components of armature current, and I_{de} and I_{qe} are the equivalent rotor excitations in direct and quadrature axes respectively, including the excitation due to induced currents in field and amortisseur windings.

The armature current, i, is given by equation (**17a**). Its components in the direct and quadrature axes are

$$i_d = \frac{2k}{3} i \cos(t + \alpha)$$

$$= \frac{2e_o [F \cos(t + \alpha) - A] \cos(t + \alpha)}{H} \qquad \textbf{(2b)}$$

and

$$i_q = -\frac{2e_o [F \cos(t + \alpha) - A] \sin(t + \alpha)}{H} \qquad \textbf{(3b)}$$

where F, A and H are defined by equations (**15a**), (**16a**) and (**3a**) respectively.

The total equivalent rotor excitation in the direct axis is the sum of the initial excitation, multiplied by the field decay factor F, and the excitation induced due to the armature current. The total equivalent direct-axis rotor excitation is, therefore,

$$I_{de} = e_o F + i_d (x_d - x_d'')$$

$$= e_o F + \frac{2e_o (x_d - x_d'')[F \cos(t+\alpha) - A] \cos(t+\alpha)}{H} \qquad \textbf{(4b)}$$

Since there is no initial quadrature-axis excitation the equivalent excitation in the quadrature axis is

$$I_{qe} = i_q (x_q - x_q'')$$

$$= -\frac{2e_o (x_q - x_q'')[F \cos(t+\alpha) - A] \sin(t+\alpha)}{H} \qquad \textbf{(5b)}$$

Substituting these expressions in (**1b**) the torque is

$$T = -\frac{2e_o^2 F [F \cos(t + \alpha) - A] \sin(t + \alpha)}{H}$$

$$+ \frac{2e_o^2 (x_d'' - x_q'')[F \cos(t+\alpha) - A]^2 \sin 2(t+\alpha)}{H^2} \qquad \textbf{(6b)}$$

For the case of zero initial armature linkages, $\alpha = \pi/2$ and the above expression reduces to

$$T = \frac{2e_o^2 x_q'' F^2 \sin 2t}{H^2} \qquad \textbf{(7b)}$$

Equation (**6b**) is suitable for the calculation of the instantaneous torque at any time after short circuit. However it is also desirable to express the torque in terms of harmonic components. This may be done by substituting in (**6b**) the odd and even series, as defined by (**5a**) and (**6a**), for their equivalent trigonometric expressions. The result is

$$T = -\frac{e_o^2 F}{x_2} [(1 + b) F\mathbf{O} - A\mathbf{E}] \sin(t + \alpha)$$

$$+ \frac{e_o^2 (x_d'' - x_q'')}{2x_2^2} [(1+b) F\mathbf{O} - A\mathbf{E}]^2 \sin 2(t+\alpha) \qquad \textbf{(8b)}$$

since $\quad 1 + b = \dfrac{2x_2}{x_2 + x_d''}$ (see equation (**7a**))

This expression involves the squares and products of even and odd series. Expanding equation (**8b**) and collecting coefficients of each harmonic, the torque is

$$T = \frac{e_o^2}{x_2 + x_d''} \left\{ 2FA \sum_{n=1,3,\ldots} nb^{\frac{n-1}{2}} \sin n(t + \alpha) \right.$$

$$\left. - \left[F^2 \frac{x_2}{x_2 + x_d''} + A^2 \frac{x_2 - x_d''}{x_2}\right] \sum_{n=2,4,\ldots} nb^{\frac{n-2}{2}} \sin n(t + \alpha) \right\} \qquad \textbf{(9b)}$$

Appendix C

DECREMENT FACTORS

Doherty and Nickle have shown[3] that the decrement for the unsupported linkages in the main field, with the armature short-circuited single-phase, is

$$\sigma_f' = \frac{x_d + x_2}{x_d' + x_2} \sigma_o' \qquad \textbf{(1c)}$$

where σ_o' is the decrement for unsupported field linkages with the armature open-circuited.

The decrement for the induced direct current in the amortisseur winding is determined by the resistance and apparent inductance of the amortisseur. The apparent inductance is determined by the amortisseur linkages which depend on the presence of the amortisseur current. It may be shown that under single-phase short-circuit conditions this apparent inductance is proportional to

$$(x_d'' + x_2)/(x_d' + x_2),$$

while for three-phase short circuit it is proportional to x_d''/x_d'. Since the decrements for the two cases are inversely proportional to the respective inductances the decrement for induced direct currents in the amortisseur on single-phase short circuit is

$$\sigma_f'' = \frac{x_d'' (x_d' + x_2)}{x_d' (x_d'' + x_2)} \sigma_s'' \qquad \textbf{(2c)}$$

where σ_s'' is the amortisseur winding decrement for three-phase short-circuit at the machine terminals.

As shown in reference 3 (Appendix H) the apparent per-unit inductance of the armature winding just after short circuit is

$$\sqrt{x_D'' x_Q''} = \frac{2\,k^2}{3}\,x_2 \tag{3c}$$

The per-unit resistance of the short-circuited winding is

$$\frac{k^2 + 1}{2}\,r \tag{4c}$$

Therefore the decrement for the d-c. component of armature current is

$$\sigma_a = \frac{3\,(k^2 + 1)}{4\,k^2}\,\frac{r}{x_2} \tag{5c}$$

where r is the per-unit armature resistance, line to neutral.

Since, in the per-unit system, time is measured, not in seconds, but in electrical radians traversed at the normal frequency f, the decrements above are $1/(2\pi f)$ times the reciprocals of the respective time constants in seconds.

It will be noted that the influence of the resistance of one winding on the decrement of other windings has been neglected. Doherty and Nickle[3] have discussed and justified this approximation so far as the effect of armature resistance on field decrement is concerned, and *vice versa*. The neglect of mutual effects in determining decrements of the direct-axis rotor circuits is justifiable to the extent that the decrements of the two circuits, considered separately, differ considerably in order of magnitude.

Strictly speaking, the amortisseur winding of a commercial machine generally consists of a number of circuits. However their decrements are usually of the same order of magnitude, so that sufficient accuracy is obtained by considering the amortisseur winding as having a single decrement.

Appendix D

SINGLE-PHASE AND THREE-PHASE REACTANCES

Park and Robertson[4] have derived the relations between single-phase static reactances and the normal three-phase reactances of a synchronous machine. For example the direct axis single-phase subtransient reactance may be written

$$x_D'' = \frac{2\,k^2}{3}\,x_d'' \tag{1d}$$

for a line-to-line winding with $k = \sqrt{3}$

and
$$x_D'' = \frac{2\,k^2}{3}\left(x_d'' + \frac{x_o}{2}\right) \tag{2d}$$

for a line-to-neutral winding with $k = 1$. x_o is the zero-sequence impedance of the machine. Similar relations hold for the other reactances.

If $x_o/2$ is negligible compared with x_d'' equation (**2d**) evidently reduces to (**1d**). As this is nearly true for many machines, and as it simplifies the expressions, equation (**1d**) was used in changing from single-phase to three-phase reactances in Appendix A. If $x_o/2$ is not negligible, however, correct expressions for current and torque during line-to-neutral short circuit can be obtained by replacing, x_d'' by $x_d''+x_o/2$, x_d by $x_d+x_o/2$, etc. In this case x_2 must be replaced by

$$\sqrt{\left(x_d'' + \frac{x_o}{2}\right)\left(x_q'' + \frac{x_o}{2}\right)}$$

which is approximately $x_2 + x_o/2$. For line-to-line short circuit the expressions for current and torque are correct as given.

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6. *Synchronous Machines, Part V*, Doherty and Nickle, A.I.E.E. TRANS., Vol. 49, No. 2, 1930, p. 700.

Discussion

L. A. Kilgore: In this paper the authors have presented a relatively simple solution to the difficult problem of calculating the pulsating torque on single-phase short circuits. They have applied the extended two-reaction theory and the constant interlinkage theorem in order to include the affect of direct and quadrature axis damping circuits.

The form of the solution obtained shows up quite well the components of the torque. The main part of the double frequency component is due to the field flux and the single-phase alternating current in the stator. The fundamental frequency torque is due to the flux trapped in the armature circuit reacting on the current in the field. The other harmonics given by the series used in the paper are due only to the difference in the reactance on the two axes. For a machine with a damper winding, the differences are generally small and the harmonics are negligible, except where there is a possibility of mechanical resonance.

The assumptions made in obtaining this solution are sufficiently accurate in most cases, but it is worth while considering the limitations. The authors neglect armature and field resistance, except in determining decrement. This gives accurate results for the pulsating torque, except for a synchronous motor with a high resistance damper winding where the calculated value will be too high, since current is limited appreciably by resistance as well as reactance.

The average torque produced by the resistance losses may be small in comparison with the maximum values of pulsating torque, but it is not always negligible. In cases where there is appreciable external resistance or a high negative sequence resistance, the average torque may be several times the rated value.

Such a torque when suddenly applied can not be neglected in the practical problem of designing spring mountings of single-phase machines.

The authors assume that the difference between the direct and quadrature axis single-phase reactance varies as a second harmonic function of the rotor position. The actual variation with position may contain other harmonics, especially in the case of machines without dampers. These variations may introduce high harmonics of the same order of magnitude as those calculated by the formulas. However, this will not greatly affect the maximum value of the torque.

An earlier paper by G. W. Penney* dealt with the special case of machines without damper windings. Mr. Penney calculated the short-circuit torque from the change in stored magnetic energy. The authors of this paper use the equation (**1b**):

$$T = i_q I_{de} - i_d I_{qe} = i_d i_q \,(X_d - X_q),$$

and refer to a paper by R. H. Park (reference number 5) for the proof. In deriving this expression Mr. Park started with the fundamental relation (total power output) = (mechanical power input) + (rate of decrease in stored magnetic energy) − (total ohmic losses). In order to eliminate the consideration of stored magnetic energy, he calculated the power output assuming that all the stator and rotor currents remain fixed for the moment. Perhaps the authors can give a more direct physical explanation of this equation.

B. L. Robertson: In 1923 R. E. Doherty pointed out† that the short-circuit torque of a synchronous machine could be obtained from the time rate of change of the stored magnetic energy in the machine. Making use of the assumptions which Doherty had stated, and also employing relations previously given in discussions on mechanical forces between electric circuits, Penney presented in 1926 "Short-Circuit Torque in Synchronous Machines." The paper quite well analyzed in a qualitative manner the production of short-circuit torque for what he called an "ideal machine." The results showed that the torque was alternating in character, never pulsating; that it rose to very high values at regular intervals; and that single-phase short-circuits gave rise to torques greater than three-phase short-circuits.

As specifically pointed out by Penney, and as shown in his and the present paper, the torque of a short-circuited machine is not zero which is often thought to be the case. The average may be small (it is exactly zero if all losses are zero), but the actual torque periodically increases to very high values, and as has just been shown the peaks may be 12 times normal for the worst case of short circuit. Design, which must be based on these worst cases, must account for such peaks, part of this torque being transmitted to the shaft and to the stator supports.

It is perhaps of value to note the assumptions made in the earlier work on this subject. There were no resistance, round rotor, and no secondary windings on the field structure. In contrast, the present paper treats with the general salient pole machine, having an amortisseur winding in both direct and quadrature axes. Resistance in the circuit impedances of the machine is neglected as before, but it is taken into account in the decrement factors. Furthermore, the solution gives the complete transient short-circuit torque curve and hence does far more than merely get the end points, initial and steady-state. To realize the exactness of solutions when resistance is so included one is referred to a paper† by Doherty and Nickle which contains what is perhaps the most outstanding agreement between test and calculated results for complicated circuit problems.

It is interesting to note that whereas the armature current waves are decidedly different in form for the two transient cases of short-circuit at maximum and minimum flux linkages, the wave shapes for all torques bear a similarity to each other for each oscillation about the axis. It is realized, of course, that for short circuit at minimum linkages the torque curve for the first several cycles is of twice the frequency of that for short circuit at maximum linkages, and that over a certain period of the transient the similarity is somewhat lost. Although previous wave shapes are in general appearance like the present ones developed, the latter are more exact because of generalities and broader assumptions underlying their derivation.

In concluding this discussion I would like to point out the continued use of the theorem of constant linkages. This theorem was first given by Doherty in 1918, quite fully discussed by him in 1923, and soon thereafter applied by Laffoon, Franklin, and Karapetoff to synchronous machines, but only of the round rotor type. Doherty,‡ however, applied this theorem to the salient pole machine and in consequence it underlies this latest work on short-circuit torques. It is of considerable interest to see the remarkable results which this simple theorem has given through its use, and it is another example of the practical application of fundamental laws formulated in physics so long ago.

Alan Howard: It is well known that the short-circuit time-constant of the main field of a synchronous machine has a definite relation to the type of short circuit, this relation having been determined by several authors. The time-constant of the amortisseur winding of a synchronous machine is affected by the type of short circuit in a somewhat similar manner but the relations, such as given in equation (**2c**) of the paper, are less well known, and it may, therefore be of interest to derive them.

The amortisseur time-constant is the ratio of the apparent inductance of the amortisseur to its resistance. The short circuit does not affect the resistance, so the problem is to determine its effect upon the inductance. This will be done first for the case of a symmetrical three-phase short-circuit through a reactance x, under the same assumptions as are used in the paper. The time-constant under single-phase conditions is easily derived from this result.

The amortisseur linkages at any time may be represented as

$$\psi_k = a i_k + b i_f + c i_d$$

where a, b and c are constants, i_k and i_f are the amortisseur and field currents respectively, and i_d is the direct axis component of armature current.

The amortisseur linkages immediately after short circuit, before the amortisseur currents have decreased appreciably are equal to the linkages before short circuit which are,

$$\psi_{ko} = b i_{fo} = b e_o$$

After the amortisseur current has disappeared, but before the field linkages have changed appreciably, the direct axis armature and field currents will be

$$i_f = i_{fo} \frac{x_d + x}{x_d' + x} = e_o \frac{x_d + x}{x_d' + x}$$

$$i_d = \frac{e_o}{x_d' + x}$$

Thus the amortisseur linkages become

$$\psi_{k1} = \frac{e_o}{x_d' + x}\,[b\,(x_d + x) + c]$$

The change in linkages due to the disappearance of the amortisseur current is, then

$$\Delta\psi_k = \psi_{ko} - \psi_{k1} = \frac{-\,e_o}{x_d' + x}\,[c + b\,(x_d - x_d')]$$

The initial value of i_d is

$$i_{do} = \frac{e_o}{x_d'' + x}$$

**Short-Circuit Torque in Synchronous Machines without Damper Windings* A.I.E.E. Trans. 1929, Vol. 48, p. 1230.

†*A Simplified Method of Analyzing Short-Circuit Problems,* A.I.E.E. Trans. 1923, p. 841.

‡*Synchronous Machines—IV,* A.I.E.E. Trans. 1928, p. 457.

and since the circuits are linear, the initial amortisseur current is proportional to i_{do} or

$$i_{ko} = d\,\frac{e_o}{x_d'' + x}$$

where d is a constant.

The apparent amortisseur inductance is thus,

$$L_k = \frac{\Delta\psi_k}{i_{ko}} = \frac{x_d'' + x}{x_d' + x}\left[\frac{c + b(x_d - x_d')}{d}\right]$$

where the constants b, c, and d are undetermined.

The time-constant varies directly as the inductance with changes in external reactance since the amortisseur resistance is constant. Thus the ratio of the time-constant for a three-phase short circuit through an external reactance x to that for a terminal short circuit is

$$\frac{T_k}{T_{ks}} = \frac{L_k}{L_{ks}}$$

where T_k is the amortisseur time-constant and subscript s denotes quantities applying to a terminal short circuit.

The time-constant T_{ks} is usually referred to as *the* amortisseur time-constant and is usually the one for which the numerical value is most readily available. It is therefore convenient to express the time-constant applying to other short-circuit conditions in terms of T_{ks}. T_k thus becomes

$$T_k = \frac{x_d'\,(x_d'' + x)}{x_d''\,(x_d' + x)}\,T_{ks}$$

In terms of decrements, using the notation of the paper

$$\sigma'' = \frac{x_d''\,(x_d' + x)}{x_d'\,(x_d'' + x)}\,\sigma_s' = \frac{1}{T_k}$$

The time-constants and the average values of the direct axis currents in an unsymmetrical short circuit are the same as those occurring in a symmetrical three-phase short circuit through a definite value of series reactance. This series reactance is x_2 for a single-phase line-to-line short circuit, so that the amortisseur decrement in this case becomes

$$\sigma_f'' = \frac{x_d''\,(x_d' + x_2)}{x_d'\,(x_d'' + x_2)}\,\sigma_s''$$

which is equation (2c) of the paper. For a line-to-neutral short circuit the decrement would be,

$$\sigma'' = \frac{x_d''\,(x_d' + x_2 + x_0)}{x_d'\,(x_d'' + x_2 + x_0)}\,\sigma_s''$$

Wire Communication Aids to Air Transportation

BY H. H. NANCE*

Member, A.I.E.E.

RAPID development of air transportation in this country has continued through the past few years and today established routes connect nearly all important cities. The route mileage of the airways in the United States as shown in Fig. 1 totals over 30,000 miles. Regularly scheduled transport service is given on practically all of these routes and considerable use of them is also made by military and private planes. Figures relating to service of air transport companies seem particularly significant. The Department of Commerce reported approximately 42,800,000 miles flown in passenger, mail, and express service on domestic scheduled lines in 1931, an increase of 35 per cent over the preceding year and more than a fourfold increase since 1928. In the same three-year period passengers carried increased ninefold, reaching a total of around 470,000 in 1931. Along with this growth safety has been increased as indicated by the respective 1928 and 1931 reports of 250,000 and 750,000 miles flown per accident. Reasonable regularity of schedules on air transport lines also has been maintained, the ratio of miles actually flown to scheduled miles last year being in the order of 92 per cent. Communication facilities have been an important contributing factor to all this development and improvement.

It was early recognized that fast and reliable communication would be needed in connection with any extensive development of air transportation. Communication with planes in flight was an obvious requirement and this could be provided only by radio. For land service, however, experience has indicated that wire facilities best meet the general requirements. This paper describes the wire communication facilities in general use today, both by the Government and by transport companies, as aids to air transportation.

Principal airways have been established largely through Federal aid. In addition to marking and lighting airways the Airways Division of the Department of Commerce had provided up to April 1, 1932, 67 radio telephone stations at approximately 200-mile intervals as indicated in Fig. 1, to be used for broadcasting weather reports and similar information to planes in flight and for transmitting directive radio beacon signals for enabling planes to keep on the course. In conjunction with these services it had contracted for 24-hour teletypewriter service along 13,000 miles of main airways connecting some 250 stations, principally for the purpose of transmitting weather reports and to assist in dispatching the planes. All of these facilities have been made available without cost to aircraft operating companies and others using the airways. In addition to this communication service contracted for by the Government, approximately 5,000 miles of teletypewriter circuit are used daily in furnishing private wire communication service to a number of transport companies for transmitting information pertaining to the operation of their own lines. Routes on which these facilities are furnished are indicated by heavy lines in Fig. 1.

When the first air mail service was established, radio telegraph was introduced as a means of point-to-point communication along the New York-Chicago-San Francisco airway route. At each radio station meteorological data were collected from surrounding points by means of long distance telephone and telegraph and these data were exchanged periodically through the day with the other stations over the radio telegraph.

With a rapid expansion foreseen in air transport service it was apparent there would be a large increase in communication requirements not only to equip new routes but to handle increased volume on existing routes. There was the definite requirement for radio telephone communication with planes which would need a number of the radio channels allotted to this service. Considering these factors and the geographic and other conditions applying to probable development of air transportation in the United States, it seemed that regular point-to-point service served by radio telegraph could be provided more satisfactorily in another way.

Arrangements were made in 1928 for teletypewriter communication services at several points connecting radio stations with their local weather bureau offices in order to expedite the delivery of weather reports and other traffic handled by radio telegraph. Shortly afterward, a teletypewriter system was installed on the New York-Cleveland route connecting the Department of Commerce and Weather Bureau stations at Hadley Field, Stelton, N. J., and Cleveland, Ohio, and a number of intermediate points. This type of service seemed ideally fitted for use in weather reporting and plane dispatching and has been extended to replace the service furnished by the radio telegraph system and to provide for communication requirements on other routes. It offers the advantages of simultaneous communication with any number of stations desired, the communications being automatically recorded on machines at each point. A message using code or abbreviations, if desired, can be sent instantly without the necessity of calling in or checking with the receiving stations and the immediate attention of only the sending operator is required. Automatic recording reduces the possibility of human error and permits the most efficient use of operating personnel with resulting savings in labor. Furthermore, as contrasted with radio, this system, utilizing wire

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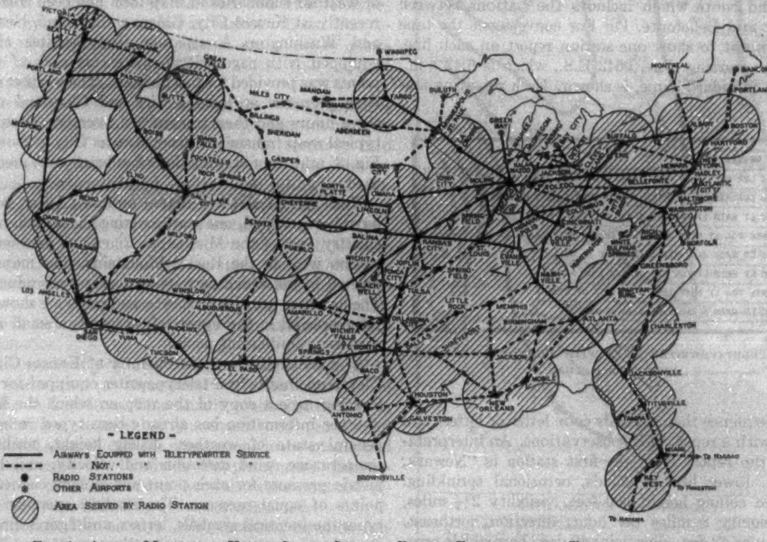
*Long Lines Dept., American Telephone and Telegraph Co., New York.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

transmission, is not so subject to variations in meteorological conditions, is more dependable, and has the advantage of being readily extended to handle large volumes of business. It is also well adapted for carrying on administrative and other work as well as for weather reporting and plane dispatching.

TRANSMISSION OF WEATHER REPORTS

Material progress has been made in reducing the effect of weather hazards to air transportation, through the service rendered by the Department of Commerce and U.S. Weather Bureau in the collection and dissemination of weather reports supplemented by other reports collected by individual transport companies from planes in flight. For this service a system of practically continuous reporting and forecasting for areas along air routes has been developed and weather observations have been extended to include data of particular benefit to air navigation.


VICTORIA
SEATTLE
SPOKANE
PORTLAND
PASCO
MISSOULA
GREAT FALLS
BUTTE
MILES CITY
MANDAN
BISMARCK
WINNIPEG
FARGO
ABERDEEN
BILLINGS
SHERIDAN
MEDFORD
BOISE
IDAHO FALLS
POCATELLO
CASPER
ROCK SPRINGS
ELKO
RENO
SALT LAKE CITY
NORTH PLATTE
CHEYENNE
OAKLAND
DENVER
FRESNO
MILFORD
PUEBLO
LOS ANGELES
KINGMAN
WINSLOW
ALBUQUERQUE
AMARILLO
SAN DIEGO
YUMA
PHOENIX
TUCSON
EL PASO
BIG SPRINGS
WICHITA FALLS
FT. WORTH
DULUTH
MINNEAPOLIS
ST. PAUL
GREEN BAY
MILWAUKEE
MUSKEGON
LANSING
BAY CITY
DETROIT
SIOUX CITY
OMAHA
LINCOLN
IOWA CITY
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KALAMAZOO
JACKSON
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TOLEDO
CLEVELAND
BUFFALO
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PORTLAND
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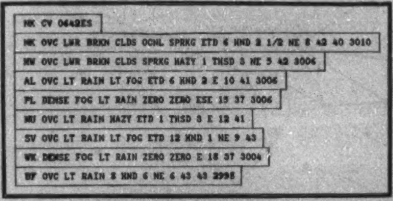
NEW YORK
NEWARK
HADLEY
ATLANTIC CITY
BELLEFONTE
BALTIMORE
WASHINGTON
SALINA
WICHITA
KANSAS CITY
SPRINGFIELD
ST. LOUIS
INDIANAPOLIS
COLUMBUS
PITTSBURGH
CINCINNATI
LOUISVILLE
EVANSVILLE
HUNTINGTON
WHITE SULPHUR SPRINGS
RICHMOND
NORFOLK
JOPLIN
PONCA CITY
BLACKWELL
SPRINGFIELD
TULSA
NASHVILLE
GREENSBORO
LITTLE ROCK
MEMPHIS
SPARTANBURG
OKLAHOMA CITY
BIRMINGHAM
ATLANTA
CHARLESTON
DALLAS
SHREVEPORT
JACKSON
WACO
MOBILE
JACKSONVILLE
HOUSTON
SAN ANTONIO
GALVESTON
NEW ORLEANS
TITUSVILLE
TAMPA
MIAMI
To NASSAU
KEY WEST
To HAVANA
BROWNSVILLE
- LEGEND -
AIRWAYS EQUIPPED WITH TELETYPEWRITER SERVICE
" NOT " " "
RADIO STATIONS
OTHER AIRPORTS
AREA SERVED BY RADIO STATION

FIG. 1—AIRWAY MAP OF THE UNITED STATES SHOWING ROUTES EQUIPPED WITH TELETYPEWRITER SERVICE

The teletypewriter networks furnished the Department of Commerce are devoted largely to this purpose and in conjunction with its radio telephone broadcasting service are the means for providing to pilots information relating to existing conditions and forecasts for both general and local areas.

Twelve selected Weather Bureau airport stations located at strategic points in the country's airway network prepare summaries of weather conditions in their own areas and make area forecasts every three hours based on data collected over the Department of Commerce circuits from connected airway stations and over commercial telegraph lines from other reporting points. These summaries and forecasts are then transmitted over the teletypewriter circuits and made available to all airway stations. While the forecasts include predictions as to storm developments or movements, conditions in specific localities are often likely to change rapidly and it has been necessary to provide additional reports along the air routes on an hourly basis in order to keep pilots continuously advised of conditions likely to be encountered. Consequently, the airway keepers and Weather Bureau observers at the various teletypewriter stations make local observations every hour of general weather conditions, ceiling height, visibility, wind direction and velocity, temperature, and barometric pressure. These data are then sent by teletypewriter and automatically recorded at all points on the circuit in accordance with a predetermined schedule, which is coordinated with the broadcast schedule of radio stations. Since pilots will tune in on particular stations at definite times to obtain reports in accordance with the broadcast schedule, it is important that the schedules be closely adhered to. The following is an illustration of scheduled weather reporting along the Newark-Cleveland route.

At 42 minutes past the hour the observers will begin typing their observations on the circuit beginning with the Newark station, followed successively by Hadley, Allentown, Park Place, Numidia, Sunbury, Winkleblech, Bellefonte, Kylertown, Greenwood Club, Brookville, Mercer, Parkman and Cleveland, with practically no interval between the completion of the report from one station and the beginning of a report from the next. When the Cleveland weather observer has completed typing his report a complete record of weather conditions at all points on the circuit will appear on the teletypewriter tape at each individual station and in the radio broadcasting stations located at Hadley, Bellefonte, and Cleveland. Fig. 2 shows a portion of an actual tape record of an hourly report along the Newark-Cleveland route which includes the stations between Newark and Bellefonte, Pa. For convenience the tape has been cut to show one station report on each line. First the starting time, 0642 E.S., which is 6:42 a.m., Eastern Standard Time, is shown. Each reporting station in sequence then gives its code letter or letters and follows with a report of its observations. An interpretation of the report from the first station is "Newark, overcast, lower broken clouds, occasional sprinkling, estimated ceiling height 600 feet, visibility 2½ miles, wind velocity 8 miles per hour, direction northeast, temperature 42 deg., dew point 40 deg., barometric pressure 30.10 in." The time of actual transmission for all 14 stations, Newark to Cleveland, is generally about four minutes.



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NK CV 0642ES
NK OVC LWR BRKN CLDS OCNL SPRKG ETD 6 HND 2 1/2 NE 8 42 40 3010
HW OVC LWR BRKN CLDS SPRKG HAZY 1 THSD 3 NE 5 42 3006
AL OVC LT RAIN LT FOG ETD 6 HND 2 E 10 41 3006
PL DENSE FOG LT RAIN ZERO ZERO ESE 15 37 3006
NU OVC LT RAIN HAZY ETD 1 THSD 3 E 12 41
SV OVC LT RAIN LT FOG ETD 12 HND 1 NE 9 43
WK DENSE FOG LT RAIN ZERO ZERO E 18 37 3004
BF OVC LT RAIN 8 HND 6 NE 6 43 43 2998
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FIG. 2—TELETYPEWRITER TAPE WITH PORTION OF WEATHER SEQUENCE REPORT

At 50 minutes past the hour the three radio stations will interrupt the beacon signals and broadcast the reports just received. Hadley Station transmits the weather sequence received from stations between Newark and Bellefonte; simultaneously, the Bellefonte radio station transmits the entire sequence received from all points between Newark and Cleveland, and the Cleveland radio station broadcasts reports from points between Bellefonte and Cleveland. All three radio stations include in the sequence, reports of weather at Cleveland and New York. The range beacons are not interrupted for these reports for longer than two minutes, and if the reports require a longer period the beacon signals are restored for one minute and again interrupted to complete the reports.

Based upon the information obtained through the sequence collections, airway weather reporting stations retransmit, generally by teletypewriter, hourly weather reports to the various airway operating companies' offices in that vicinity. Airway companies maintain various arrangements for posting the weather information for the convenience to pilots. Some companies post the information on a series of boards of different color arranged in geographic sequence to represent different airway routes, each board indicating a particular point on that route.

An experimental service involving the transmission of weather summaries in map form has been tried out recently at Kansas City, Chicago, Cleveland, Newark, and Washington. A separate teletypewriter circuit equipped with page teletypewriters at each of these points was provided for this purpose. The weather maps were prepared at Kansas City and Cleveland every three hours and then transmitted over this circuit. A typical map transmitted from Kansas City is shown in Fig. 3, and the following describes briefly the methods used.

Two special airways maps, ordinary letter width, have been printed, one map covering the section of the country east of the Mississippi River and another the section west to the Rocky Mountains. The maps are printed in ink which permits hectograph reproduction. The airways, principal airports, and cities are shown on the map and in the upper left corner is a small circle used as a coordinating point.

At a scheduled time the operator at Kansas City or Cleveland inserts in a teletypewriter equipped for perforating tape a copy of the map on which the latest weather information has already been typed including general state of weather, ceiling height, visibility, temperature, wind direction and velocity, and barometric pressure for each point and isobars connecting points of equal pressure. The sending operator then types the identical symbols, letters, and figures directly over the corresponding ones on the map inserted in his machine, thus making a complete record on perforated tape. On schedule a blank map is inserted in the teletypewriter at each receiving point and positioned so that a type bar will strike the map within the small coordinating circle. The sending operator then releases the tape and the signals transmitted over the circuit reproduce on the map at the receiving stations data similar to those on the original map at the sending station.

The map data are sent in sequence from the two transmitting stations and after they have been received on the map forms a number of duplicate copies can be run off immediately and the two maps fitted together if desired. The maps are then available to pilots at each of the respective airports.

Complete reports of weather are generally maintained by transport companies in dispatching offices. On some lines two-way short-wave radio-telephone equipment has been provided for communicating with planes and periodic contact is maintained during flight. In this way pilots report their positions directly to dispatchers and in addition supplementary weather data are usually exchanged, particularly in respect to local ceiling heights and conditions in the upper air strata.

PLANE DISPATCHING AND OTHER SERVICE

Teletypewriter circuits furnished to air transport companies are used principally for dispatching planes and handling the many traffic matters usual to this type of service. Plane movements including reports of position in flight are transmitted over the teletypewriter system and recorded at various offices. The reports of positions, in many cases, are given by pilots over short wave radiotelephone where this type of equipment has been provided.

To facilitate position reporting some of the companies have superimposed a system of rectangular coordinates over a map of the course cutting the territory into squares or rectangles 10 to 20 miles on a side. The coordinates are numbered so that the pilots and dispatchers can readily establish the location of the plane. The dispatchers generally maintain a typewritten, chronological log of position reports from each plane in the air. Bulletin boards are also used, marked with the stations along the route and with spaces for filling in data such as plane number and license, name of pilot, time of arrival, and departure at each station and final destination.

A considerable volume of information is required to be transmitted in connection with the handling of traffic on large lines. This usually consists of data as to reservations, number of passengers and amount of mail and express carried, connections to be made, and arrangements at terminals. Supplementary instructions to pilots and many administrative matters requiring prompt handling are also transmitted.

Although the airways teletypewriter circuits furnished the Department of Commerce are used mostly for handling weather reports, considerable information is also transmitted over them relating to departure and arrival of planes and their position in flight. Upon request the Department of Commerce will send over its teletypewriter system the license number of a plane, the station from which the plane is departing, its time of departure, and its destination, to stations along the route of the flight. Stations on the route knowing approximately the time the plane will be due watch for it and record the actual time the plane passes so that other stations may be informed.

TELETYPEWRITER CIRCUIT LAYOUT

The teletypewriter networks furnished by the Bell System for service along airways are composed of some 30 separate circuits. Circuit mileage of the longest is about 2,000 miles and the shortest 200 miles. The longer circuits generally connect 15 to 20 intermediate stations. Since airways naturally follow direct air lines the intermediate airway stations are often located at points considerable distances from main communication lines, which, generally, are constructed along routes connecting the industrial and more populous centers, due regard being given to topographic and other conditions. At the larger airports such as Newark and Cleveland, local teletypewriter circuits are also provided to connect the Department of Commerce station with the offices of the various transport companies, the post office, and weather bureau. Automatic transmission equipment is provided so that information received on one circuit can be retransmitted over one or more other circuits if desired.

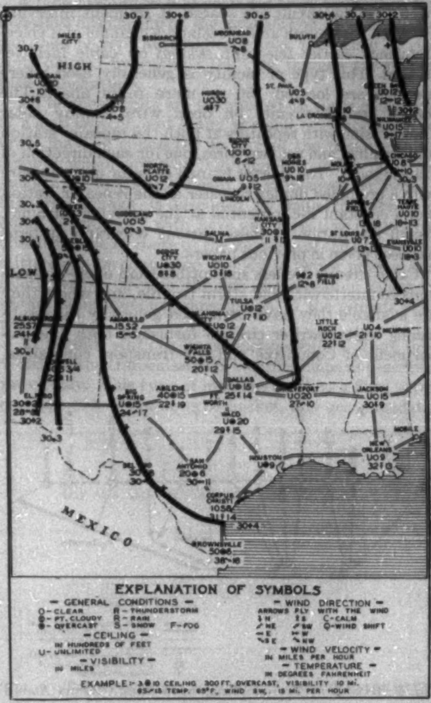

FIG. 3—WEATHER MAP TRANSMITTED BY TELETYPEWRITER

A layout diagram of a typical circuit is shown in Fig. 4. Facilities in the New York-Cleveland long distance cable are used for establishing the main links, totaling 515 miles. Repeater stations on the cable route located approximately every 50 miles afford convenient points from which branch circuits are extended to the intermediate airway stations. The several branch circuits are of the grounded open-wire type and total 331 miles. A total circuit mileage, therefore, of 846 miles is required in this case for connecting all stations along an air route a little over 400 miles long.

The Newark-Pittsburgh section of the main circuit is operated on metallic telegraph cable facilities, a type particularly adapted to use where stations to be connected to the circuit are spaced at frequent intervals. Between Pittsburgh and Cleveland a channel of a voice-frequency carrier telegraph system on cable facilities is used. This type of facility is generally used where stations are located 150 or more miles apart. The longer branch circuits on open wire employ polar transmission with repeaters at both the repeater station and terminal and use two wires, one for each direction of transmission. The shorter branch circuits use one wire with a grounded duplex repeater at the repeater station and a constant d-c. potential at the outlying terminal. Detail description of these various telegraph systems has been given in previous papers.

Cable circuits are less susceptible than open-wire circuits to interference and storm trouble, and where they are available they have been used generally for establishing teletypewriter circuits furnished both the Department of Commerce and the transport companies. At present, over one-third of the mileage of these circuits is in cable.

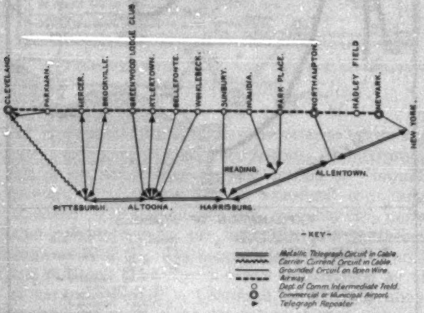

FIG. 4—LAYOUT OF TYPICAL TELETYPEWRITER CIRCUIT ALONG AIRWAY

Facilities on alternate routes are available to be substituted for the regular circuit in the event circuit trouble develops. Spare loops to the teletypewriter stations and spare teletypewriter equipment at each station are provided so that each component part of the service is protected by spare facilities and equipment.

TELETYPEWRITER EQUIPMENT

The theory of teletypewriter operation and descriptions of the machines generally used in this country have been given in other papers but are briefly reviewed here in order to describe some of the specific equipment arrangements used in airways service.

The teletypewriter is designed to perform the functions of an ordinary typewriter with added features that permit the typing units of a number of similar machines located at distant points to be controlled by the operation of the keyboard of any one of them. This is accomplished by the translation of the mechanical action of any key in the keyboard unit to electrical impulses arranged in a code and transmitting them over an electrical circuit to the distant machines where the impulses are translated back to the mechanical action of a type bar in the typing unit corresponding to the key struck in the distant keyboard. Electric motors and electromagnets provide the mechanical power and the means of translation of electrical impulses to mechanical action. It is, of course, necessary that the mechanical action of all of the machines be synchronized. This is provided for by the use of synchronous motors or governed motors regulated to the same speed and a start-stop rather than a continuously rotating system. By the use of the start-stop system the effect of variations in motor speeds is minimized, accurate synchronization being required only during the interval of typing of one letter after which a clutch releases and stops the receiving mechanism momentarily to permit it again to start in synchronism with the sending mechanism. To provide the start-stop feature and sufficient code combinations for the letters and symbols required, a seven impulse code is used consisting of a start pulse, five selecting pulses, and a stop pulse.

Teletypewriters are available to print on an ordinary page or on a narrow strip of tape. Tape machines are generally used in airways service because they are particularly adapted to the handling of short messages and weather sequences where it is generally desirable to rearrange the messages received by cutting and pasting the tape on separate pages to form a continuous weather record for each point. This is preferable to a chronological message record requiring a search through all of the information to obtain the trend of weather at a particular point. The tape machines are also somewhat smaller, less expensive, and more efficient, not requiring the transmission of carriage return and paper feeding signals.

In addition to the ordinary sending and receiving machines supplementary apparatus units may be used so that operators can work at maximum efficiency and the line circuit can be used to its maximum capacity. These units are the perforator, tape transmitter, and reperforator.

The perforator is associated with a keyboard and perforates a tape with one to five perforations for each key struck. The tape is run through a tape transmitter which automatically sends electrical impulses to the circuit corresponding to the perforations in the tape and identical to those that would have been sent from the keyboard direct had it been connected to the circuit for normal keyboard sending. The use of the perforator and tape transmitter permits the circuit to be operated at its maximum speed at all times and permits the operator to work in spurts and rest or do other work while the accumulated tape is running through the transmitter. Also the same tape can be run through several tape transmitters and thus be used for sending the message over several circuits.

Fig. 5—Department of Commerce Teletypewriter Station

The reperforator is a receiving device which records the message on a perforated tape similar to that produced by a perforator unit. This permits storing a received message for immediate or subsequent retransmission to other circuits without retyping it.

The smaller teletypewriter stations of the Department of Commerce and those of the various transport companies are each generally provided with a regular and a spare teletypewriter and a small loop switchboard with jacks and cords to connect the regular or spare machine to the regular or spare loop as desired.

At the larger teletypewriter stations of the Department of Commerce a special arrangement of the equipment has been provided to permit efficient operation of the teletypewriter circuits from the standpoints of requiring a minimum number of operators and of obtaining rapid retransmission of messages received on one circuit to one or more of the other circuits as required. A view of an installation is given in Fig. 5 and a typical floor plan arrangement is illustrated in Fig. 6. The apparatus is mounted on tables specially designed for the purpose, and these are usually arranged on the floor in the shape of a U, the units facing inward so that the operators work inside the U.

A separate reperforator *A* and tape transmitter *B* in addition to a tape teletypewriter *C* are provided for each circuit. Messages are received simultaneously on a printed tape by the tape teletyperwriter and on a perforated tape by the reperforator. Mounted adjacent to the reperforators are the tape transmitters through which the perforated tape can be run to retransmit immediately a received message to another circuit. The reperforators and tape transmitters can be started and stopped individually from the remote control box *H*.

A tape perforator *K* is provided to perforate tape for messages originating at the local station. The messages can then be sent automatically over the circuit or circuits desired by running the tape through the proper tape transmitters.

All of these units are terminated on cords and plugs at a loop switchboard *E* and any unit or combination of units may be connected to any of the teletypewriter circuits which are wired through a number of series jacks in the loop switchboard. A supplementary switching arrangement is provided by radial transmitting board *G* equipped with keys and repeating relays. By operating one or more of the keys one of the tape transmitters can be connected quickly to two or more of the teletypwriter circuits through the repeating relays to obtain simultaneous transmission to the circuits connected.

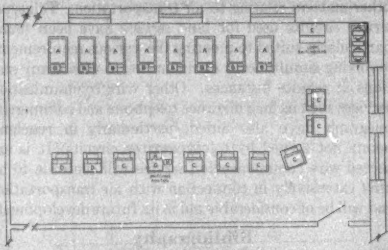

Fig. 6—Teletypewriter Equipment Layout in Large Office

Legend:
A —Reperforator
B —Transmitter distributor
C —Tape teletypewriter
D —Page teletypewriter
E —Loop switchboard
F —Motor-generator set
G —Radial transmitting board
H —Remote control box
K —Perforator

At certain stations page teletypewriters *D* are provided for the transmission of weather maps as described previously. This type of machine employs a fixed paper carriage and movable type basket, and accommodates paper up to 8½ inches wide. It has been equipped with a number of special type characters to provide the symbols required on the maps. These symbols, which are shown in Fig. 3, are provided as upper case characters on the teletypewriters in place of fractions and punctuation marks.

The arrangement of equipment described generally permits one operator to attend all of the circuits. The teletypewriters are all located in a fairly small space, which permits one man to observe the incoming messages and operate the control boards to start and stop the proper transmitter and to relay the messages as required.

RADIO INTERFERENCE

The establishment of teletypewriter stations along the airways brought about the installation of teletypewriter equipment in the same room or in close proximity to short-wave radio receivers, and introduced the problem in specific cases of radio interference caused by the operation of the teletypewriter.

Remedial measures have been designed effectively to reduce this interference, and consist of the use of synchronous motors, rectifiers, and specially designed filters, together with the locating of the apparatus and wiring in such a way as to effect a minimum coupling between the teletypewriter and its associated loop and the radio antenna system.

CONCLUSION

History of air transportation in the past few years indicates that continued growth may be expected, particularly as hazards to flying are mitigated and safety and dependability are recognized by the public. The Government is continuing the extension of airways and weather reporting and other services, and air transport companies are progressing in developing transport business. Fast and reliable communication service has proved the backbone of weather and position reporting and has been a valuable aid in the handling of traffic and other matters relating to air transportation. Teletypewriter circuits used for land service have been found particularly suited to meeting the various requirements involving simultaneous communication with many stations at remote distances. Other wire communication services such as long distance telephone and commercial telegraph have also aided, particularly in reaching points not served by teletypewriter circuits. It is expected wire communication service will continue to be used extensively in connection with air transportation and will be of considerable aid in its future development.

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Discussion

H. T. Killingsworth: Cleveland has one of the most important and best equipped airports in the country. Practically all of the features described in the paper are included in the Cleveland communication equipment. The Department of Commerce has five teletypewriter circuits radiating from Cleveland which parallel the more important air routes. One of these extends from Cleveland to Newark and is shown in Fig. 4 of the paper. One parallels the Cleveland-Chicago-St. Louis Airway via Detroit and Kalamazoo. The third extends from Cleveland to Kansas City via Columbus and St. Louis. The fourth extends from Cleveland to Camden via Pittsburgh and Harrisburg and the fifth from Cleveland to Newark via Buffalo and Utica. All of these circuits have numerous intermediate drops at airports along the route.

This equipment is located in the Administration Building at the Airport. The Administration Building is in turn connected by local teletypewriter circuits to the long wave radio transmitting station which is located about one mile away. One of the most common usages of the reperforating equipment in the Administration Building is in connection with retransmitting to the radio station messages which it is felt should be broadcast immediately to planes in flight.

Short wave radio equipment is used by the airport authorities in dispatching planes. The pilot of incoming planes is contacted as he approaches the port, is advised of weather conditions at the field, movement of planes in the vicinity of the field, where to land, at which gate to discharge his passengers, etc. Similarly, a pilot leaving the field is contacted and is directed off the field.

Characteristics of Electromagnetic Radiation from Aircraft in Flight

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***Synopsis.**—This paper reports the results of an investigation of the effects of altitude, frequency and distance upon the strength of a received radio signal as transmitted from an airplane in flight and received at a point on the earth's surface. Frequencies between 475 kc. and 5,500 kc. are reported on for altitudes up to 6,000 feet and distances up to 60 miles. Representative curves, plotted between the above mentioned variables, are included and the difference in results obtained during daylight and darkness are shown. In general, the data show that, within the limits of this investigation, the higher the frequency the shorter the reliable communication range becomes.*

An appendix is included in which is shown how the data for a given altitude, distance and frequency may be used to calculate the theoretical field strength for the given frequency when the transmitter (or airplane) is at a different distance and/or altitude.

INTRODUCTION

CONSIDERABLE data have been published giving the characteristics of high-frequency radio transmissions from aircraft to ground for distances over 100 miles but, in general, little definite information is given as to the exact conditions at the time the data were collected. Comparatively few data have been published regarding the transmission characteristics from airplane to ground for distances less than 100 miles. Prior to 1929, aircraft operators made little use of the frequencies above 1,500 kc. Transmission tests on frequencies below 1,500 kc. gave consistent results, in that the field strengths seemed to be subject to a uniform attenuation with distance and were apparently little affected by altitude. The use of the higher frequencies on aircraft is desirable as it permits use of small fixed antennas and more compact and lighter radio equipment on the airplane. In the attempt to use these higher frequencies, freakish results were obtained. So many contradictory data were obtained at the shorter distances that it soon became apparent that a thorough investigation would be necessary before complete advantage could be taken of the higher frequencies.

Plans were made in the summer of 1930 to investigate this subject fully. Master-oscillator type, medium-frequency, 50-watt transmitters were available at that time, and some tests on the medium frequencies were made. High-frequency crystal-controlled transmitters were purchased, and field-strength measuring apparatus obtained. Radio receivers of various types were also procured in order that listening tests might be conducted at the same time as the field-strength measurements were being taken.

APPARATUS AND PROCEDURE

In the winter of 1931, two types of tri-motored airplanes were obtained for use in the tests. One of these airplanes was covered with fabric and the other with metal. Identical Western Electric 8A crystal-controlled transmitters were installed on each of these airplanes. Fixed antennas of the same dimensions were erected symmetrically over the fuselage and wing sections of each airplane on stub masts approximately 6 feet high. The antenna structure used approximated a horizontal "T," and was fed at the base of the "T." Due to the wide frequency band covered (1,695 kc. to 5,500 kc.), it was necessary to vary the dimensions of these antennas during the course of the investigation.

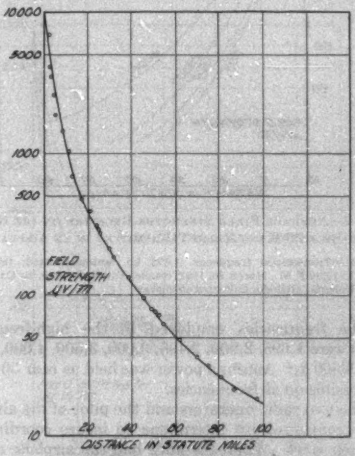

FIG. 1—AVERAGE FIELD STRENGTHS RECEIVED ON THE GROUND FROM A 50-WATT RADIO TRANSMITTER IN AN AIRPLANE

Data—Transmitted frequency, 475 kc.; antenna, trailing wire 194 ft. long; time, 10:08 A.M. to 11:09 A.M., July 31, 1931; course, Indianapolis, Indiana, to Fairfield, Ohio; altitude, 3,000 ft.

A Western Electric type 44A field-strength measuring set was set up in a comparatively level terrain at Fairfield, Ohio. Several radio receivers were set up at selected locations in the vicinity but isolated from each other and the field-strength measuring set.

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*War Department Signal Corps, Wright Field, Dayton, Ohio.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

Two fixed courses were selected for the flights, one from Dayton, Ohio, to Cambridge City, Indiana, and the other from Dayton to Columbus, Ohio. Each course was 60 miles in length. Flights were made over each course at 500 feet, 1,500 feet, 2,500 feet, 3,500 feet, 4,500 feet and 6,000 feet altitude.

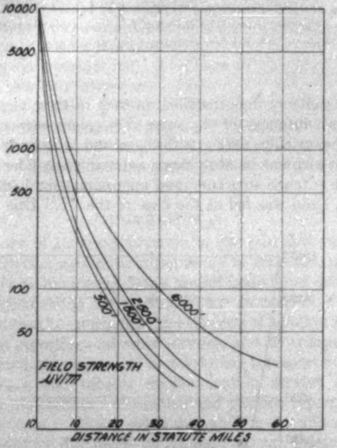

FIG. 2—AVERAGE FIELD STRENGTHS RECEIVED ON THE GROUND FROM A 50-WATT RADIO TRANSMITTER IN AN AIRPLANE

Data—Transmitted frequency, 1,695 kc.; antenna, fixed; time, 9:55 A.M. to 12:30 P.M., March 20, 1931; course, Fairfield, Ohio, to Cambridge City, Indiana; altitudes indicated on curves

The frequencies employed in the high-frequency tests were 1,695, 2,000, 2,495, 3,000, 3,500, 4,000, 5,000 and 5,500 kc. Antenna power was held as near 50 watts as possible on all frequencies.

The two radio operators and the pilot of the airplane were connected by interphone to insure coordination of their work. When the pilot had the airplane on the course at the proper altitude, he notified the first operator. This operator then notified the ground stations of the time and test-run number and thereafter put the carrier frequency on the air for alternate three-minute periods until notified by the pilot that the end of the course had been reached. The second operator kept the meter readings constant by means of the set and power adjustments.

The average speed of the airplane on these test flights was approximately 100 miles per hour. Due to the winds aloft, the airplane speed varied between the limits of 80 and 120 miles per hour. On account of these high speeds, the rate of change of field strength when the airplane was within a short distance of the ground station, was extremely rapid and could not be accurately followed with the field-strength meter.

The field-strength measuring set used was not accurate for measurements of field strengths below 20 microvolts per meter. When fading became severe, it was extremely difficult to secure accurate field-strength measurements. The results shown in the fading zone below the "severe fading line" are the averages for a number of readings taken in rapid succession. The period of the fading was not consistent, on some days being extremely slow and on other days very rapid. Where the curves do not reach the 20 microvolts per meter minimum, it indicates that it was impossible to secure accurate readings due to the rapid and severe fading at that time.

In general, the figures presented in this paper are self-explanatory. A mathematical analysis of these data have been made and is included in the Appendix.

CONCLUSIONS

In concluding, it is desired to emphasize that the tests were conducted over one type of terrain and during the winter and spring seasons, with the exception of that shown in Fig. 1, covering the transmission frequency of 475 kc., which was obtained in the middle of the summer. Results of tests conducted over other terrain and during different seasons of the year will no doubt vary from the data given here.

Under the given conditions, it may be stated that:

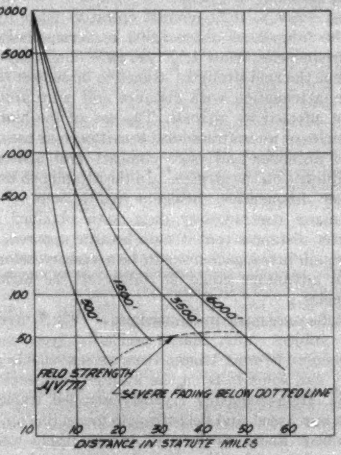

FIG. 3—AVERAGE FIELD STRENGTHS RECEIVED ON THE GROUND FROM A 50-WATT RADIO TRANSMITTER IN AN AIRPLANE

Data—Transmitted frequency, 3,000 kc.; antenna, fixed; time, 1:55 P.M. to 4:50 P.M., April 6, 1931; course, Fairfield, Ohio, to Cambridge City, Indiana; altitudes indicated on curves

1. Field strength is dependent upon altitude, distance and frequency.

2. Fading, and not attenuation, is a controlling factor in the use of these frequencies at short distances.

3. Daylight tests show no greater attenuation than is experienced in the night tests.

4. Fading occurs at lower frequencies and shorter distances at night than during the day.

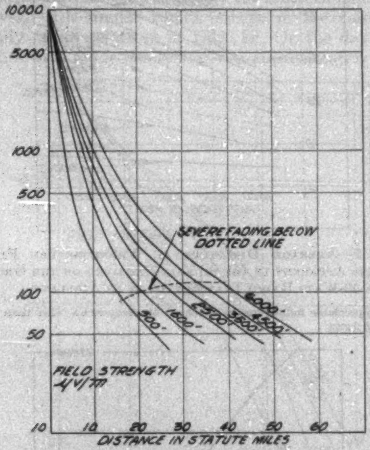

FIG. 4—AVERAGE FIELD STRENGTHS RECEIVED ON THE GROUND FROM A 50-WATT RADIO TRANSMITTER IN AN AIRPLANE

Data—Transmitted frequency, 4,000 kc.; antenna, fixed; time, 11:45 A.M. to 4:14 P.M., March 12, 1931; course, Fairfield, Ohio, to Cambridge City, Indiana; altitudes indicated on the curves

5. Frequencies below 2,000 kc. are more suitable for communication from aircraft to ground for distances up to 60 miles.

6. For any given frequency, the data collected at one altitude and distance may be used as shown in the Appendix to calculate the field strength that will be obtained when flying at another altitude and distance up to the point where fading commences.

7. The similarity of the data obtained when using the trailing wire antenna and when using the fixed antenna indicates that either the field patterns of the two antenna systems are similar or that the difference in field patterns had little effect on the results obtained.

Appendix

LIST OF SYMBOLS

F = measured value of field strength in microvolts per meter.

K = is proportional to the energy radiated in a fixed direction.

f = frequency in kilocycles per second.

λ = wavelength in kilometers.

α' = attenuation factor.

ϵ = 2.718

R = the radius of the earth.

r = distance as measured along the earth.

x = distance as measured along a line tangent to the earth at the receiver.

y = distance the airplane is above or below the tangent line.

ρ = airline distance between the transmitter and receiver.

$\theta = \tan^{-1} y/x$

a = altitude as indicated by altimeter.

All distances are in statute miles, unless otherwise stated.

A typical curve of field strength plotted against ground distance is shown in Fig. 8. A curve showing the variation of the angle θ with distance is also plotted on Fig. 8, and where the two curves intersect, the path of the flight intersects the tangent line to the earth. This occurs at zero degrees and 67.4 miles; for greater distances the path of flight is below the tangent for this altitude.

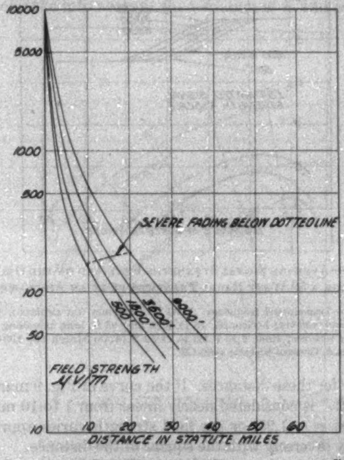

FIG. 5—AVERAGE FIELD STRENGTHS RECEIVED ON THE GROUND FROM A 50-WATT RADIO TRANSMITTER IN AN AIRPLANE

Data—Transmitted frequency, 5,000 kc.; antenna, fixed; time, 8:28 P.M. to 10:53 P.M., April 8, 1931; course, Fairfield, Ohio, to Columbus, Ohio; altitudes shown on curves

The method of increments was used in order to determine if the relationship of field strength to distance was of the form:

$$y = ax + bx^2 + cx^3 + dx^4 \ldots \qquad (1)$$

It was found that neither the first increments, ΔF, nor the second increments, Δ^2F, nor the third, Δ^3F were constant, and there was no indication that any higher increments would be reasonably constant. It was, therefore, concluded that the relationship must be expressed in a form such as:

$$y = ax^{-1} + bx^{-2} + cx^{-3} + dx^{-4} \dots \qquad (2)$$

or as some transcendental function.

If this field strength curve, plotted to logarithmic coordinates, had been linear and expressed as:

$$\log_e y = m \log_e x + \log_e b \qquad (3)$$

then the relationship would have been of the form:

$$y = b/x^m \qquad (4)$$

If that portion between 50 and 100 miles is considered to be approximately linear, then the value of m can be determined by the formula:

$$y - y_1 = m\ (x - x_1) \qquad (5)$$

In this instance m was found to be equal to $-$ 2.32, or the field strength varied with distance to the $-$ 2.32 power for these distances. If the curve in Fig. 9 marked "500 ft." is considered nearly linear from 1 to 10 miles, then $m = -$ 1.95, or the field strength varies approximately inversely with the square of the distance.

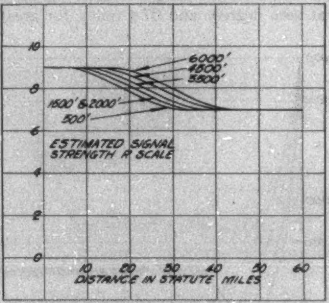

Fig. 6—Average Signal Strengths Received on the Ground from a 50-Watt Radio Transmitter in an Airplane

Data—Transmitted frequency, 2,000 kc.; antenna (on airplane), fixed, (on ground receiver) horizontal, 15 ft. high and 95 ft. long extending N.E. from the receiver; time, 8:20 P.M. to 11:52 P.M. on March 20, 1931; receiver used, General Electric 1496-C

In order to determine if the relationship was of the form of the Austin formula,[1] the unattenuated values of field strength were expressed as:

$$F' = K/r \qquad (6)$$

then if the measured values of field strength could be expressed as:

$$F = (K/r)\ (\epsilon^{-\alpha' r}) \qquad (7)$$

then the ratio of F' to F is:

$$F'/F = \epsilon^{\alpha' r} \qquad (8)$$

also:

$$\log F'/F = \alpha' r \qquad (9)$$

Tabulations made from Fig. 8 of the values of the equations (6) to (9) inclusive and α', show that for ground distances over 15 miles α' remained essentially constant at an average value of 0.0227 for a value of K = 12,800. In Fig. 1 recorded points are shown with a calculated curve having these constants. An equation of the form of (7) fits the data remarkably well, provided that θ is a small angle, two degrees or less, or when the airplane is below the tangent line.

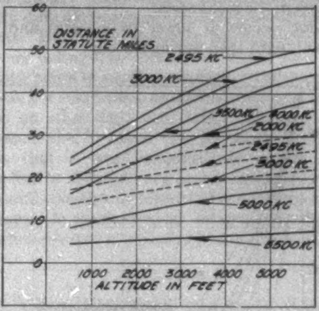

Fig. 7—Average Distances at Which Severe Fading Becomes Apparent in the Signals Received on the Ground from a 50-Watt Radio Transmitter in an Airplane

No appreciable fading was experienced on frequencies other than those shown in figure

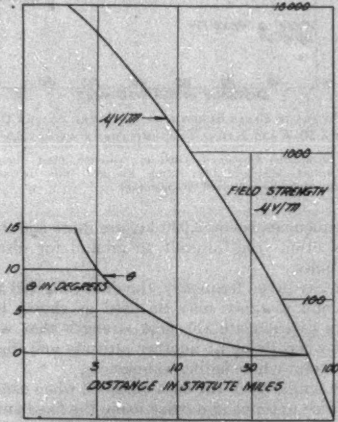

Fig. 8—Curves of Field Strength and θ vs. Ground Distance

The field strength curve is the same as that of Fig. 1 except that log-log coordinates are used for purposes of analysis

This procedure was followed for a large number of curves, and families of curves of which Fig. 9 is typical. In no case was it possible to find one value of K and α' which would apply throughout the entire range of distances. In Fig. 9 α' was practically constant for the

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1. See Bibliography.

6,000-foot test at 0.0396 for ground distances over 10 miles, with a value of K equal to 18,200. With the value of K determined, values of α' were obtained by a similar process as above except that the angle θ was maintained constant, instead of altitude, giving the relationship:

$$F = (K/\rho)\,(\epsilon^{-\alpha'\rho}) \qquad \textbf{(10)}$$

In each case the values of α' decreased as ρ increased, and the product $\alpha'\rho$ was practically constant. This was done for various small angles for the fourteen families of curves, with similar results except in the case of a family of curves taken at 1,695 kc. On the day these flights were made the ceiling was approximately 2,600 feet, and the bank of cumulus clouds extended up to over 5,500 feet above the earth. The 500, 1,500, 2,500, and 6,000-foot flights were made in the usual way, but the 3,500 and 4,500-foot flights, being through clouds, were omitted. The values of $\alpha'\rho$ were approximately 12 per cent higher for the 6,000-foot altitude than for the lower altitudes.

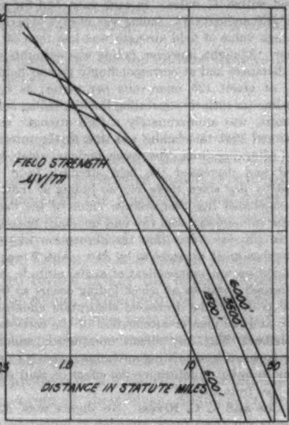

Fig. 9—Curves of Field Strength vs. Ground Distance for Various Altitudes

These curves are the same as those of Fig. 3 except that log-log coordinates are used for purposes of analysis

With values of field strength taken at points along a line making a fixed angle to the tangent line:

$$F_1 = (K_1/\rho_1)\,(\epsilon^{-\alpha'_1\rho_1}) \text{ and } F_2 = (K_2/\rho_2)\,(\epsilon^{-\alpha'_2\rho_2}) \qquad \textbf{(11)}$$

but $K_1 = K_2$ by definition, and with $\alpha_1'\rho_1 = \alpha_2'\rho_2$ then:

$$F_1/F_2 = \rho_2/\rho_1 \qquad \textbf{(12)}$$

or the field strength varies inversely with distance as measured along a line making a fixed angle to the tangent line. This bears out the theory of T. L. Eckersley[2] and the conclusions of Drake and Wilmotte.[3]

From Fig. 9, it is apparent that for very short distances with the particular radiating system used, the field strength decreases with increasing altitudes, and for ground distances of 4 miles or more the field strength increases with altitude. At one mile there is comparatively little change in field strength with altitude. In Fig. 10 (derived from Fig. 9), vertical polar diagrams of field strength for various values of θ are shown, each for a fixed airline distance ρ. The shapes of these curves are determined first, by the values of K which change with θ, and second with α', which is also dependent upon θ. Field strength is practically constant from 20 to 45 degrees. In order for this to be true, $\rho_1 = \rho_2$ of (**11**) so:

$$F_1/F_2 = (K_1/K_2)\,(\epsilon^{-\alpha_1'}/\epsilon^{-\alpha_2'}) \qquad \textbf{(13)}$$

or

$$\log_\epsilon (F_1/F_2) = \log_\epsilon (K_1/K_2) + \alpha_2' - \alpha_1' \qquad \textbf{(14)}$$

but with $F_1 = F_2$ then:

$$\log_\epsilon (K_1/K_2) = \alpha_1' - \alpha_2' \qquad \textbf{(15)}$$

in other words, K and α' must vary in the same sense with variations in θ. As θ is increased, the attenuation and the energy radiated in the direction of the receiver both decrease. For the smaller angles in this particular case, attenuation is predominating factor, while for the larger angles, the directional effect of the transmitting antenna predominates. A complete expression of either field-strength curve of Fig. 8 or 9 must contain terms which take into account the variation of K and α' with θ. However, it is not necessary to know this expression if it is only desired to calculate the values of field strength for various altitudes and distances, having a curve such as Fig. 8, or one of those of Fig. 9.

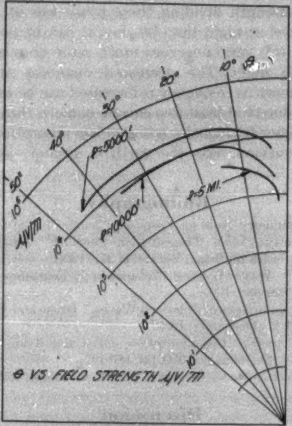

Fig. 10—Curves of Field Strength vs. θ for Various Air-Line Distances

These curves were derived from the same data as Fig. 9

In order to find how α' varied with λ, the method of equations (**3**), (**4**), and (**5**) was used and shows that:

$$\alpha' d = \alpha d/\lambda^{0.612} \qquad \textbf{(16)}$$

where d is the distance in kilometers. This value agrees with the value of 0.6 given by Austin for wavelengths between 300 and 25,000 meters.

There was no difference between the night tests and the daylight tests, provided that the same conditions obtained as to power, antenna, frequency, altitude, etc., except that the distances free from fading were much less than by day. Since the above calculations are all based on data taken in the non-fading zone, it is not feasible to compute the field strengths where there is fading. It was noted that the dividing line separating the non-fading zone from that in which there was fading, was sharply defined, and followed very approximately a field strength contour. There was a time interval between the instant that an outgoing airplane entered the fading zone, and its emergence on the return flight. This was especially true at night at higher frequencies and lower altitudes.

The results of various tests in which fading was encountered at distances under 60 miles leads to the conclusion that the absorption undergone by the reflected or refracted ray is independent of the angle or distance traveled by that ray. It was noted that in that portion of the fading zone nearest the receiver the effect of the sky ray was to add to or subtract from the field strength measured by day when no fading was encountered at this particular altitude and distance. Since the contour of field strength dividing these zones was at a much higher level at night than by day, it can be concluded that the sky ray undergoes much more absorption by day than night. The decreased distances at which fading occurs as frequency is increased can be explained as being due to at least two causes, namely, the sky rays of the higher frequencies undergo less absorption, while the attenuation near the earth is greater, as shown above.

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Discussion

A Member: Is it not possible that effects similar to fading could be obtained when flying was rough? It seems as though changes in altitude and direction due to air current would cause a variation in signal strength at the receiver which would resemble fading.

J. C. Coe and T. C. Rives: In these tests we were fortunate in having good flying conditions for the most part, but occasionally when flying was "bumpy" there were slight changes in field strength. Since large tri-motored airplanes were used that were known to be very stable, this effect was very slight. The variation in field strength would be due more to the change in the orientation of the transmitting antenna with respect to the receiving antenna than to an actual change in altitude or distance. In these tests it was found that the fading zone was sharply defined from the zone in which there was no fading.

A Member: According to these curves the distance free from fading depends upon frequency, altitude, and whether day or night conditions obtain; the distance increasing with altitude, decreasing with frequency, and is less at night than by day. What is your explanation of these facts?

J. C. Coe and T. C. Rives: The attenuation undergone by earth-bound rays depends primarily upon distance, altitude, frequency, terrain, and possibly upon the presence of clouds, but in so far as our data are concerned it is independent on whether daylight or darkness obtains. Fading was encountered when the field strength became less than a certain value, this value depending upon frequency, time of day, power, etc. For example, on the 3,000-kc. daylight tests there was fading when the field strength was less than approximately 50 microvolts per meter. At an altitude of 500 ft. there was no fading within 14 miles, while at an altitude of 6,000 ft. fading was not encountered within 47 miles. In each case, and for all altitudes between 500 and 6,000 ft., fading was always experienced when the average value of field strength was less than 50 microvolts per meter. At night, however, fading was encountered at much shorter distances and at correspondingly greater field strengths, namely, at about 125 microvolts per meter; as by day, the dividing line (or surface) separating the non-fading zone from the fading zone, was approximately a field strength contour. It was assumed that this fading was due to the interaction of a sky ray returning from the upper regions which periodically increased and decreased the field strength from the values measured by day when there was no fading at these distances. It was concluded that the average value of the field strength due to the indirect ray was two and one-half times as much as by day in this case, and that the absorption was only about 40 per cent as much at night as by day. Also it was concluded that absorption was independent of angle, altitude, or distance. The decreased distances at which fading occurs as frequency is increased is due to the decreased absorption undergone by the sky rays, and the greater attenuation of the earth-bound rays.

A Member: Was any direct comparison made between different types of transmitting antennas operating on the same frequency in order to determine the effect on field strength and distance free from fading?

J. C. Coe and T. C. Rives: No flights were made which would bring out these facts, although it should be possible to predict the results in a general way. The two most common types of airplane antennas are the horizontal and the trailing wire. For distances so short that the altitude is comparable to the distance, it is difficult to predict just what the results will be due to the fact that the directional effect of the antenna has a predominating effect. Where the distances are greater such that the altitude is not comparable to the distance, the trailing wire antenna would give a greater field strength, other factors being equal. As to the distance free from fading, this would also favor the trailing wire antenna. The horizontal antenna radiates a greater component upward and a lesser component along the earth than a trailing wire having a greater vertical and a lessor horizontal dimension. If this fading is due to the interaction of the returning sky rays with the earth-bound rays the distance free from fading would be greater when using such a trailing wire antenna.

A Member: Using a fixed antenna system on an airplane, would an increase in power change the distance free from fading, providing that the frequency and current distribution remained the same?

J. C. Coe and T. C. Rives: No tests were made from which any conclusions of this nature could be reached. While an increase in power would increase the radiation along the earth, it would increase the radiation skyward a proportionate amount. The answer to this question would be determined by the relative absorption encountered by the sky rays compared to the earth-bound rays. It seems reasonable to assume that the distance at which fading became appreciable would be the same, but at a higher average value of field strength.

Vertically Cut Sound Records

Recent Fundamental Advances in Recording and Reproducing Sound Using Vertical Undulations on a Disk

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***Synopsis.**—Vertically cut disk sound records possess important advantages over laterally cut records. In their use the recorder and reproducer have been greatly improved. "Cathode sputtering" has replaced graphiting of the wax and a non-abrasive final record has replaced the abrasive record of the past. The improvements which result are:*

Noise has been reduced 25 to 30 db.

Volume range has been increased from about 25 or 30 db. to about 50 or 60 db.

Upper frequency cut-off has been extended nearly an octave (to about 9,000 cycles).

The playing time for a 12-inch record may be increased from about 4 minutes to at least 15 minutes.

The life of the record has been tremendously increased.

A more faithful reproduction has been obtained as a result of flatter frequency characteristics and less non-linear distortion so that sounds are more distinct, life-like, and clean cut.

The results demonstrated are enhanced by the use of better microphones and better and, particularly, much more powerful amplifiers and loud speakers than have usually been used before or are now used commercially.

* * * * *

THE phonograph and the telephone were invented at nearly the same time and have grown side by side. The growing knowledge gained in studies of the telephone has aided the phonograph and much of the advance in the phonograph art has been contributed by men primarily associated with the telephone—Bell, Berliner, and Edison. It is of interest in viewing recent advances to review briefly the history of mechanical recording of sound. With both the telephone and the phonograph many devices were tried and many schemes suggested long before success was attained, which, had their proposers possessed just a little more understanding of the problem or a little more persistence and skill in the execution of their ideas, might have been successful.

HISTORY OF SOUND RECORDING

The recording of sound may be traced back many years. In 1807 Thomas Young described a method of recording the vibrations of a tuning fork on the surface of a drum. The method as described was reduced to practise by Wertheim in 1842. Leon Scott, in 1847, invented the phonautograph, an instrument for the recording of sound which was further improved by König and Barlow, and in 1874 Alexander Graham Bell applied the drum and the bones of the human ear to obtain tracings on smoked glass.

These earlier workers had recorded sound in the form of a wavy line. To Edison goes the credit for making the record in such a form that it could be used to vibrate a diaphragm and thus reproduce sound. Edison's invention was made in 1876, the earliest patent being dated January 1877. The first records were on a cylinder containing a spiral groove, and covered with a thin sheet of tin foil. A steel stylus was attached to a drumhead of gold beater's skin, much like that of Bell's first telephone. The cylinder was rotated and the stylus, moved by the vibrations of the diaphragm, pressed a groove of varying depth into the tin foil. The same device if allowed to again traverse this groove would reproduce the sound.

Bell and Tainter later studied the various methods and concluded that the rubbing process should be abandoned and an engraving process substituted; that is, instead of pushing the record surface down in a spiral groove as in the original phonograph, it should rather be dug out or engraved. As a result of their work they obtained a patent on such a device in 1886, and in 1887 produced the graphophone—the first really practical apparatus of the phonograph type. A critical comment made at the time, however, stated that "the reproduced sound is as loud as that of a good telephone conversation but the distortion is sufficient to make the voice unrecognizable save to a strained imagination added to a previous knowledge of the author of the voice."

The record of this machine was a thin pasteboard cylinder covered with wax. Following this, Edison abandoned the tin-foil type of record and adopted the wax cylinder with an engraving process. In 1887, Emile Berliner patented the gramophone. Berliner concluded that the forces required to cut a groove of varying depth were much greater than those available from the human voice without great distortion of the motions involved. He therefore returned to the earlier ideas of Young, Scott and König, and proceeded to make a lateral record. He concluded in addition that it would be much more convenient if these records were in flat or disk form. It is of interest that Edison in his first disclosures also describes the use of a disk, although he apparently did not consider it practical and abandoned the idea.

Berliner engraved his records on metal disks covered with an extremely thin layer of wax. He tried also a mixture of lamp-black and oil to form "a fatty ink which, when crossed by a stylus, shows even under a

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microscope, a sharply cut transparent line." He recorded with the disk placed above the recording stylus so that any dust that was formed would fall away. Having thus removed the wax or ink from the surface of the metal, he then etched the disk chemically. The lateral type disk record of Berliner has since been greatly developed and improved and has been the most commonly used commercial type of record for about forty years. The very early commercial records which were sold were all original recordings. Subsequently these were copied by playing from them onto other records; that is, by re-recording. About 1900, Edison rendered the original waxes conducting by sputtering or evaporating gold onto the wax at high voltage in a vacuum. About this time also, very fine graphite began to be used to render the original wax conducting. A stamper was plated from the conducting wax and from this a thermoplastic final record was then pressed.

During these early years, many other methods of storing sound were tried. Some of these were optical, some mechanical. For successful operation, however, all of these methods required both a facility in the design and use of vibratory systems and devices not then available.

Electrical recording was often suggested and tried by the various phonograph companies, but in general without much success because of the lack of a satisfactory amplifier. Researches and developments in the communication field paved the way for this application, and development studies were carried on by Bell Telephone Laboratories for a number of years. These reached the point of commercial application in 1924 and a number of companies was licensed to use the improved methods which have now practically displaced the earlier methods in which recording and reproducing were both carried out entirely without electrical means. In 1926 essentially these same methods were extended to the field of sound pictures.

Since that time, although there has been continued development, no outstanding changes or improvements in disk records have been reported until recently. These continuing investigations have now led to considerable further advance. A highly satisfactory process of cutting and of processing records, and of reproducing from them, has been developed. This will be described and demonstrated.

PROBLEMS TO BE SOLVED

Reproduction from disk records of the past has failed in several ways to meet all requirements either for reproduction in rooms, as in the home, or in large halls or theaters. The most serious fault has been the noise often referred to as "needle scratch" or "surface noise." Other shortcomings in their order of importance have been failure to reproduce all components over a broad frequency range particularly the higher frequencies and failure to reproduce an adequate range in loudness, due principally to inability to reproduce weak sounds. This has been caused by masking of weak sounds by surface noise. Next in importance has come distortion due to inability of the reproducing needle to follow the record groove accurately. This has been more marked with loud sounds of certain frequencies and has been a minor cause limiting the volume range. The playing time for a record of reasonable size has been rather inadequate and finally the record life has been too short for some otherwise possible applications.

These different characteristics are closely related. Improvement in one is apt to help the others. Consideration of all these problems has come to indicate strongly the desirability of using a vertically rather than a laterally undulating groove.

SURFACE NOISE

Frequency analyses of surface noise have been made using a variety of reproducers and record materials. In general, the shapes of these frequency characteristics have been found to be very largely influenced by the resonance characteristics of the reproducers whereas the amount of noise is dependent on the record. They do not show any marked differences as between lateral and vertical recordings. Frequency charts of surface noise taken with a vertical reproducer having a very flat frequency characteristic over the audible range have shown the surface noise to be relatively richer in high frequencies. The distribution of energy below 10,000 cycles for a cellulose acetate pressing is shown in Fig. 1. The amount of recorded sound energy in the low frequency range (that is, below about 2,000 or 3,000 cycles) is, however, large relative to that in the higher frequency ranges. Moreover, the characteristics of many lateral reproducers have been such as to accentuate surface noise between 3,000 and 5,000 cycles. Obviously, therefore, the elimination of the higher frequency components of the surface noise must cause a large reduction in such noise without any material reduction in loudness of the sounds of interest. Such elimination of the higher frequencies has another effect which for much commercial work has been of very practical importance. It greatly reduces the audible distortion due to poor recording, poor tracking, overloaded amplifiers, etc. It may serve to give a passable result without the expense involved in a thoroughly high grade system. Although the loss of the desired higher frequencies is serious, it has been held by many that the net result has justified the practise. Such low-pass filter systems, usually of very simple and crude design have been commonly known as "scratch filters."

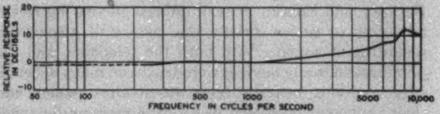

FIG. 1—ENERGY DISTRIBUTION OF SURFACE NOISE FROM A CELLULOSE ACETATE RECORD

Surface noise is caused by a more or less random distribution of impulsive shocks on the needle due to minute irregularities in the record. It has been common practise in lateral recording to use record material containing a certain amount of abrasive in order to grind the needle to fit the groove. The irregularities due to the abrasive would logically be expected to produce a scratchy noise of the character with which we are all familiar. A 5,000-cycle note of the same loudness as a 10,000-cycle band of surface noise from the records whose development is described here, using a reproducer with a flat characteristic, would have a displacement amplitude of only about 0.000001 inch. In order to reduce the surface noise to the point where it is no longer troublesome, it is necessary to eliminate irregularities at least down to this order of magnitude. It has been found that if the usual abrasive record is replaced by an unabrasive record pressed of a very clean homogeneous material such as cellulose acetate, the surface noise caused by the record material itself is greatly reduced. Such a change, however, by itself has been found to give a comparatively minor improvement, for when the noise due to the record material is moved well into the background, other causes of surface noise of nearly the same order of magnitude as that due to the abrasive of a shellac record become controlling.

The next process which it has been found necessary to improve has been that for rendering the surface of the original wax electrically conducting. This is necessary in order that the wax may be electroplated to provide a negative with which to press the final record. The usual methods of graphiting or brushing with fine electrically conducting powders have been found unsatisfactory since the individual particles are reproduced in the plated matrix and also the pressed record, thus introducing noise. In addition the brushing of the wax tends to injure its delicate surface. Recourse has therefore been had to one of the earlier methods used in phonograph practise; namely, cathode sputtering[1] of the wax. Fig. 2 shows graphited and also sputtered groove surfaces as seen under a microscope. By using a very thin layer of wax flowed onto a metal surface it is possible to keep the wax cool, thus simplifying the sputtering operation. It is thus possible to apply an extremely uniform, smooth and tenacious surface of metal of adequate thickness in a very few minutes. This can be electroplated by the ordinary methods and used to press the final record. By the use of this thin flowed wax, it is possible to obtain a surface texture which is extremely smooth, homogeneous, and free from the mechanical strains incident to shaving the waxes by the methods previously used. In addition, waxes of this type possess obvious advantages in ease of transportation, sturdiness, etc.

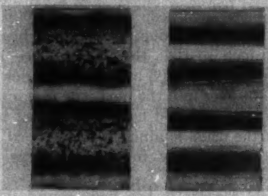

Fig. 2—Record Grooves as Seen Under the Microscope

Left, graphited grooves. Right, sputtered grooves. Note: The microscope is focused on the bottom of the groove

These improvements in the methods of engraving, processing, and in the final record material are more or less applicable to either type of recording, lateral or vertical. However, the non-abrasive record calls for a permanently shaped stylus point. There has appeared to be less difficulty in providing this with the vertical record. Full advantage can, however, only be taken of the inherent quietness of the records described above if the frequency range is extended to the full extent made possible. In amount the reduction in surface noise from that of present commercial records will differ depending on the frequency range reproduced.

The surface noise of the new record when reproduced with a 10,000-cycle band of frequencies if compared with the old record reproduced with a 5,000-cycle band of frequencies, which is the comparison of practical interest, shows a reduction in noise of about 15 db. In addition, it is possible to take advantage of the fact that most sounds to be recorded contain less energy in the high frequency range than in the medium or low frequency range[2] and therefore to record the higher frequencies at somewhat higher than normal level. In reproduction these higher frequencies are, of course, attenuated relative to the lower ones by the proper amount in the reproducing amplifier or circuit. It is thus found that a further reduction of about 10 db. in surface noise can be obtained, the amount depending somewhat on the high frequency cut-off of the reproducer or circuit. This effect is chiefly between 5,000 and 10,000 cycles.

Extension of Frequency Range

The extension of the frequency range for the recorder is a straightforward problem in design of vibratory systems. With the reproducer the problem is one of designing a system which can be accurately driven by the undulations of the groove without undesirably large forces being set up such as would injure the record. To a considerable degree this is a problem in reducing the mass of its vibratory system. This has been found easier with the vertical system. With vertical recording and reproduction the overall frequency range has been extended thus far to about 9,000 cycles.

Volume Range

The volume range for any particular frequency band is usually considered to be the difference in db. between the loudness of the surface noise and the loudness of the maximum recorded sound which the record can accommodate when reproduced faithfully over this frequency range. The volume range of the lateral records of the past when reproduced to 5,000 cycles may be stated as about 25 to 35 db. With vertical recording the volume range for a 5,000-cycle band of frequencies is 55 to 60 db. For 9,000-cycle reproduction the volume range is 50 to 55 db. This is very important in reproduction in large halls or theaters where an amplifier and loud speaker of adequate power capacity must be assumed. For reproduction in the home a smaller volume range may be preferred. For such home reproduction these vertical records should prove practically noiseless.

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[1] Günther Schulze, "Cathode Sputtering," *Zeitsch. f. Physik*, Apr. 1926, Aug. 1926.

[2] "Speech and Hearing," by H. Fletcher, D. Van Nostrand, 1929.

Accuracy of Tracking

In addition to the limitations imposed by surface noise, it is evident that, with the available reproducers for lateral cut records, the needle point may fail to follow the center of the groove accurately when the curvature becomes too sharp and may skid from side to side by varying amounts depending on the record and the characteristics of the reproducer being used. Studies have proceeded relating to the physical characteristics necessary in a reproducer in order that it may faithfully follow a groove. These studies have led us to expect superior performance from a groove cut with vertical undulations to that from one with lateral undulations. With the lateral groove there is distortion due to the fact that the sound is recorded with a chisel shaped stylus and reproduced with a round stylus, also that in reproduction the bearing point of the stylus against the groove shifts forward and backward as the needle rounds a curve. With vertical records the first of these effects, sometimes called the "pinch" effect, is absent but a shifting of the bearing point of the reproducing stylus forward and backward occurs if a round tipped stylus is used. For a given stylus tip radius and for a given recording level this effect increases with frequency.

This failure of a stylus point to follow a vertical record with full accuracy is, of course, due to the finite width of the stylus point along the groove and is somewhat analogous to the slit width in optical reproduction from a film. The fact that speech and music and most of the other sounds which it is of interest to reproduce contain much less energy in the high than in the low frequency range so that the shifting of the bearing point along the groove is considerably less than if relatively more energy were present at the higher frequencies, tends to relieve the situation.

Lateral and vertical records drive the reproducer point quite differently. Lateral records drive the point symmetrically from both sides but the point rarely follows the center of the groove with entire exactitude. It deviates from the center by amounts chiefly dependent upon the mechanical impedance of the reproducer. A vertically cut record, on the other hand, drives in only one direction. The restoring force is due chiefly to the elasticity of the supporting structure of the reproducer, the normal restoring force being equal to the total weight on the needle minus the weight of the moving or vibrating part. The stylus point will always remain in contact with the record unless the forces set up by the record undulations exceed this normal restoring force. Operation should always be below this limiting condition. This sets definite requirements on the mechanical impedance of the vibrating parts and, unless this condition can be met, reproduction of extreme frequencies by vertical records is impossible. With the vertical reproducers which we have used, however, the stylus can follow sudden downward motions of the record groove even to accelerations greater than a thousand times that due to gravity. With lateral records there is no definite limiting condition analogous to the above. However, it appears easier in practical design greatly to reduce the mechanical impedance for vertical than for lateral reproducers. Practical experience has shown that the mass of the vibratory system of a vertical reproducer can be so reduced as to reproduce up to well above 10,000 cycles and the stiffness reduced so as to reproduce down to the order of 20 cycles. In fact, there appears to be considerable margin on this score.

Lateral records have usually been cut with a stylus having a tip radius between 0.002 inch and 0.003 inch. The angle between the two sides has, in this country, commonly been about 90 degrees. The groove has been 0.002 inch to 0.003 inch deep and about 0.006 inch to 0.007 inch wide. The groove spacing has been 0.010 inch to 0.011 inch so that the uncut space between blank grooves has been 0.003 inch to 0.004 inch. If one groove is not cut over into the next, the maximum amplitude which could be used has been limited to about 0.002 inch. If the usual loudness of the record is to be maintained it is necessary to maintain this spacing between grooves.

With vertical records it has been found desirable, particularly where a very loud record is to be made, to use a recording stylus with approximately the same tip radius as used previously with lateral, but to reduce the divergence between the sides of the stylus above the tip. In addition it has not been found necessary to provide any clearance space between grooves. It is therefore feasible to increase the number of grooves per inch from the usual 98 to between 125 and 150 and at the same time to raise the recording level. When using this recording stylus with the lesser divergence to cut a record with 125-150 threads per inch it has been found desirable to make the normal unmodulated groove about 0.007 inch wide and about 0.003 inch deep. The maximum amplitude may, under these conditions, be increased about 4 db. It has been found possible, however, to obtain satisfactory results with most waxes even though the normal depth of the groove is increased to as much as 0.004 inch to 0.006 inch. In this case, the recorded level may be increased 6 db. This increase in the recording level obviously increases the volume range by a like amount. If occasionally, due to a loud crash of sound, the recording stylus completely leaves the wax, the reproducer will still continue in the right groove. The corresponding situation with a lateral record where one groove cuts into another is, of course, fatal since the reproducer will usually cross into the next groove in such a case.

Playing Time of a Record

It has been found desirable with vertical records to use a permanent reproducing stylus in order to reduce the vibrating mass of the reproducer to a satisfactory value. This stylus point remains sharp in contrast with the steel needles used with lateral records and therefore will reproduce satisfactorily for undulations of sharper curvature. In other words, over the same amplitudes the linear speed of the record may be reduced. Practically, it may be undesirable to reduce or change the rate of rotation of a record from what has been commercially in use in theater reproduction. It is, however, feasible to decrease the internal groove diameter recorded on such a 33⅓ r.p.m. record to about 6 inches. By the combination of the various elements mentioned above, it has been found feasible to record for 15 to 20 minutes on a 12-inch record and for 10 to 12 minutes on a 10-inch record. This involves the use of about 200 grooves per inch and a decrease in the recorded level to about the level of laterally recorded records using 98 grooves per inch. Of course, longer recordings can be made in the same space if the recorded level is decreased (more grooves per inch), or if the upper frequency cut-off is decreased (decreased r.p.m. and inner diameter). If only speech or music of small volume range are to be recorded it is feasible to make each face of a 12-inch record play 30 to 40 minutes. However, such changes may introduce tracking difficulties if carried too far and must be well justified by other considerations if carried beyond these limits.

Record Life

The great reduction in the mass and stiffness of the vibrating system of vertical reproducers discussed above makes it possible to reduce the weight with which the reproducer point bears on the record to between 2 and 20 per cent of that which has been used with most commercial lateral reproducers. This reduction in stylus or needle point pressure has been found to decrease the wear on the record very greatly with the result that its life has been vastly increased. Tests have shown that the first few thousand playings cause negligible deterioration and even several hundred thousand playings do not show excessive wear if the record is properly protected from dust and dirt.

The Reproducer

A reproducer for vertically cut records which uses a coil of wire vibrated in a radial magnetic field has been found highly satisfactory. Such a reproducer is simple and sturdy. Its performance is linear over a wide amplitude range; it may be made extremely light and, at the same time, is quite efficient. The coils used have had a diameter of between 0.08 inch and 0.2 inch and the total mass of the vibrating system including the diamond or sapphire stylus has varied with different models from 5 to 35 milligrams. The total force on the record has been reduced from about 150 grams to between 5 and 25 grams, the lighter structure being used when playing from a soft wax. With the larger of these designs it has been found possible to obtain efficiencies which are comparable with the efficiency of the Western Electric oil-damped reproducer used with lateral records. No difficulty has been experienced due to failure to follow the groove if these are mounted on a simple pivoted arm as for lateral reproducers. Due to their very small mass they operate quite satisfactorily even though the record turn table fails to operate in a true plane and though the record be considerably warped.

The response of the moving coil vertical reproducer is practically constant over a very broad frequency range. A typical response characteristic is shown in Fig. 3, which is a characteristic taken with cellulose acetate pressings.

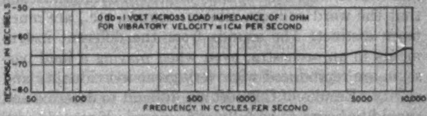

Fig. 3—Response Frequency Characteristic of an Experimental Vertical Reproducer Driven by Cellulose Acetate Records

The Recorder

The design of a recorder for vertical records involves no fundamentally new problems over lateral type recorders which have been described previously.[3] It is still desirable to design the recorder to approximate constant amplitude characteristic for the lower frequency range and constant velocity for the higher range. The same type of recorder which has been used for lateral recording can be converted for vertical recording by the addition of a comparatively simple link system. With a few changes in the masses and stiffnesses of the parts it may be given a quite acceptable frequency characteristic. Such a recorder has been used in making the records which will be demonstrated. Its frequency characteristic is shown in Fig. 4.

Reproducing System Characteristic

The response of the oil-damped lateral reproducer is highest at the very low frequencies. Its response decreases with increasing frequency, this decrease in the higher frequency range more or less compensating for the increase of response with frequency of the recorder. Because of the flat characteristic of the vertical reproducer, it has been found desirable to compensate in the reproducing amplifier or circuit for the low response of the vertical recorder at the lower end of the frequency scale. A frequency characteristic for the combination of recorder, reproducer, amplifier, and network is shown in Fig. 5.

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[3] *High Quality Recording and Reproducing of Music and Speech,* by Maxfield and Harrison, A.I.E.E. Trans., Feb. 1926.

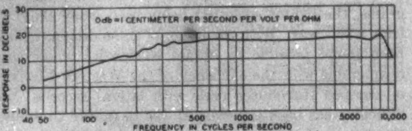

Fig. 4—Response Frequency Characteristic of an Experimental Vertical Recorder

It has been found with vertical records that speech is reproduced with considerably improved naturalness and that the word endings, sibilant sounds, etc., are much more distinct. The sounds from the different instruments in an orchestra, even when playing a loud passage, are reproduced with very great individuality and clarity. Results of this sort are difficult to describe and should be heard to be fully appreciated. If records such as those described are reproduced using various low-pass filters, the elimination of frequencies even above 7,000 cycles is easily noticeable. On the other hand little or no difference in needle scratch or surface noise may be observed, this being almost wholly absent in most cases, whether the records contain speech or music or if blank grooves be reproduced. In listening to these records a loud speaker has been used which is essentially flat over a large portion of the range of audibility, its characteristic being as shown[4] in Fig. 6. The moving coil microphone has been used[5] in making records by this process.

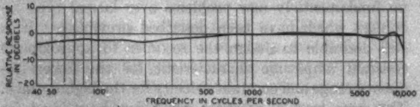

Fig. 5—Overall Frequency Response Characteristic (Recorder + Reproducer + Network + Amplifier)

Discussion

Benjamin Olney: A question sometimes raised after the demonstration of a system of this kind is: How does the performance compare with average radio broadcast reception? The question is of interest because of the wide acceptance of radio as a means of home entertainment; the answer must conclude that the quality delivered by the present system is greatly superior to that of radio reproduction. The reason for the poorer quality of radio reception is not because means for bettering it are unavailable, but because of certain practical limitations which prevent the application of known remedies.

Aside from electrical and mechanical difficulties, an essential acoustical requirement for the reproduction, by a home radio receiver, of frequencies as low as those you may hear in the present demonstration, would be the mounting of the loud speaker in a cabinet or panel comparable in size with the one before you. This, obviously, would be difficult in the average home and constitutes the principal limiting factor to low frequency reproduction.

On account of the spacing of radio channels 10 kilocycles apart in the ether, it is necessary that an adequately selective radio receiver reject frequencies above 5,000 cycles. In the practical case, it is not feasible to secure a sharp cut-off at 5,000 cycles, so the response of even high-grade receivers falls off rapidly somewhere between 4,000 and 5,000 cycles. Some sharply selective receivers may reproduce very little above 3,000 cycles, and many sets are provided with a hand-operated "tone control" by which it is possible virtually to eliminate frequencies above about 1,500 cycles.

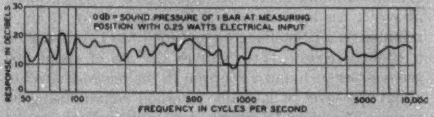

Fig. 6—Response Frequency Characteristic of Combined High and Low Frequency Loud Speakers

Laboratory model receivers whose overall frequency range extends up to 7,000 or 8,000 cycles are found to reveal defects in broadcast station modulation which are inaudible with an ordinary receiver. In addition, there is the expected increase in background noise which accompanies an extension upward of the frequency range.

The volume range displayed in the present demonstration is, of course, much greater than is had with radio receivers, due not only to the limitations of the receiver itself, but also to those of the broadcast transmitter and associated telephone circuits.

It appears that, in order to obtain a high frequency range in radio reception comparable to that given here, the following steps will be necessary aside merely from extending the frequency range of both the radio receiver and the broadcast system:

1. The frequency spacing of adjacent broadcast channels must be at least doubled.
2. The effective power of many stations must be increased so as to provide a greater signal-noise ratio.
3. Carbon button microphones must be used with caution, if at all, because their "carbon hiss" becomes very apparent in systems reproducing the higher frequencies.
4. Over-modulation of broadcast transmitters must be avoided to a much greater extent than at present.

It will thus be seen that fundamental changes must be made in the present radio broadcast system before a frequency range may be enjoyed that is comparable with that afforded by this highly developed recording and reproducing system.

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[4] "An Efficient Loud Speaker at the Higher Audible Frequencies," L. G. Bostwick, *Jl. Acoustical Society of America*, Oct. 1930.

[5] "A Moving Coil Microphone for High Quality Sound Reproduction," by W. C. Jones and L. W. Giles, *Proc. S.M.P.E.*

Adequate Wiring of Buildings, an Essential for Good Illumination

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PROGRESS in securing good illumination in buildings has, within a few years, been seriously impeded by lack of sufficient wiring capacity, much to the disadvantage of owners and managements. This paper reviews the situation from an illuminating engineering standpoint in the hope of stimulating engineering interest in the problem and thus promoting wider exemplification of practises which will insure reasonable prospect of adequacy of lighting circuits to facilitate proper illumination during the life of a structure. No attempt is made to cover applications other than illumination, except so far as they are fed from lighting circuits and thus affect the illumination results.

ILLUMINATING ENGINEERS AND LIGHTING PRACTISE

For the rank and file of ordinary lighting installations such as small stores, work shops, and offices, rather definite rules have been evolved. In fact, these rules have become sufficiently definite as to constitute practises from which the requirements, which lighting imposes upon the wiring, may be predicted within reasonable limits. Contemporaneously with the development of lighting practise there has come into existence a group of illuminating or lighting service engineers associated with the electric utility companies, leading lamp and equipment manufacturers, and some of the large users of light, who, largely through the medium of the Illuminating Engineering Society, have interchanged data and experience with each other as well as with practising consultants.

A large measure of the effort of these engineers has been devoted to the repair and correction of lighting installations, which, for one reason or another, have proved unsatisfactory in operation. They have, therefore, had an unusual opportunity of determining the border lines of satisfactory and unsatisfactory lighting. It is this experience, embodied in the criteria and rules for lighting practise which has rendered the practise so reliable and secured its wide acceptance. While this experience has been dominantly with the smaller installations, in which, for economic reasons, paid consultants are not usually retained, it should be borne in mind that the more elaborate installations have at least as stringent requirements.

The lighting practises, referred to, have been expressed in papers and reports before various associations. Notable among these are the lighting codes of the Illuminating Engineering Society, the American Standards Association, and the International Commission on Illumination. The accepted illumination levels for the most common conditions of building lighting have been compiled in tabular form by leading illuminating engineers. Among other places these tables have been published in the Franklin Red Seal Specification of the Society for Electrical Development, which also prescribes approximate rules for the lighting design of the simpler and more common classes of building interiors. Without recommending the use of these rules beyond their intended scope, it may be pointed out that the fundamental characteristics of illumination upon which the specification is based are representative of good lighting practise as referred to herein. This specification is probably the simplest, most condensed and most comprehensive expression of present lighting practise; it thus furnishes a reasonable expression of the electric power requirements of that practise.

LIGHTING PRACTISE ADVANCING

While, as has been pointed out, electric lighting practise in the more common applications is comparatively definite, it is not static. Illumination levels, that is to say the quantitative elements, have been rising steadily for many years and except for the retardation during periods of business depression, there is no indication that most fields have approached saturation. Moreover, there is an increasing demand for diffusion and reduction of glare, which is generally secured at a sacrifice of light, and, therefore, puts an additional requirement on the amount of electricity to be supplied.

While qualitatively these advances are generally recognized, it is exceedingly difficult to secure any quantitative measure for them. Since the advances are the resultants of various forces, some peculiar to the application, the locality, the time, etc., it follows that they are variable and cannot be simply expressed. Probably the best authoritative examples are illustrated by the illumination values for the industrial lighting field as published in the 1921 and 1930 issues of the "American Standard Code of Lighting for Factories, Mills and Other Work Places."[1] The two issues give the values in different form and in some instances different classifications. However, a few samples have been selected, on the basis of diversity of interest, and incorporated in Table I to facilitate a numerical comparison. It is probably that in most instances the advances are, if anything, greater than here indicated as the later

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Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

[1] For references see Bibliography.

values were presumably more widely exemplified in actual installations at the time of issue. It seems safe to say that, in general, the decade ending in 1930 witnessed an approximate doubling of light requirement for good practise in the principal classes of commercial, industrial and office lighting

TABLE I—ILLUMINATION LEVELS GOOD PRACTISE

A few representative examples extracted from American Standard Code of Lighting Factories, Mills and Other Work Places, Issues of 1921 and 1930

Class of Operation or Interior	Foot Candles	
	1921	1930
Assembly—Rough	2 to 5	8-5
—Medium		12-8
—Medium fine	5 to 10	
—Extra fine	10 to 20	100-25
Chemical works—Hand furnaces, etc	2 to 5	5-3
Cloth products—Light goods	5 to 10	15-10
—Dark goods	10 to 20	100-25
Elevators—Freight and passenger	2 to 5	8-5
Forge shops	2 to 5	10-6
Foundries—Charging floor	2 to 5	8-5
—Fine molding and care making	5 to 10	15-10
Glove manufacturing—Dark goods	10 to 20	100-25
Locker rooms	2 to 5	6-4
Machine shops—Rough work	2 to 5	10-6
—Medium work	5 to 10	15-10
—Fine work	10 to 20	100-25
Offices—Close work		15-10
—No close work		10-8
—Private and general	5 to 10	
—Drafting	10 to 20	25-15

INDICATIONS OF WIRING INADEQUACY

For a number of years illuminating engineers have been encountering installations in which suitable lighting could not be provided because of a lack of capacity in the wiring. This was first evidenced by the blowing of fuses when new lighting equipment was put in operation. It became, therefore, necessary to check up on capacity before placing larger lamps in old installations or recommending suitable lighting even in some new installations. While the advance discovery of the limit saved some embarrassment, it did not produce a happy situation to be unable to provide the illumination needed and desired by the building's occupants.

Even where the safe carrying capacity of the wiring was not exceeded, excessive losses of electrical pressure in the wiring were frequently encountered so that the voltage delivered to lamps was considerably less than it should have been. Such losses were greatest at times when the demand was greatest, and resulted in serious reduction in light output and lamp efficiency. To the user it showed itself in inadequate illumination, a yellowing of the light, and when other loads were switched on and off, a flickering of the lighting. Table II shows a few of the instances of excessive voltage drops found in a brief survey made in 1931 by an electric utility company in Ohio. It is believed that these data are typical and would represent conditions which exist in other communities. Illuminating engineers occasionally report contact with installations where lamps are operating 10 per cent or more below rated volts. Since this means, with tungsten filament lamps, a light output of only about 70 per cent of normal (see curve, Fig. 1), it is easy to understand how unsatisfactory the illumination would be.[2] Oftentimes such conditions have been misinterpreted as being due to poor regulation of the utility's circuits or to defective lamps and have been a source of complaint from this standpoint. In extreme cases building owners have been compelled to incur the costly outlay of rewiring, but in a much larger number of cases, the expense has appeared prohibitive, and unsatisfactory lighting has been continued.

A review of the papers and reports presented before the American Institute of Electrical Engineers and other engineering associations, shows careful treatment of practically every other phase of electrical engineering. Building wiring alone seems to have been neglected, perhaps because it has been considered as an economic problem rather than an engineering problem. Inasmuch as the engineering result of the entire system is in so large a measure vitiated by wiring inadequacy, it seems important that more attention be given to this subject.

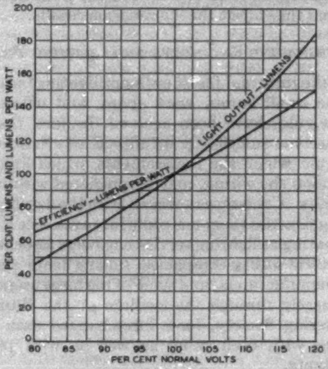

FIG. 1

TABLE II—REPRESENTATIVE EXAMPLES OF INTERIOR WIRING SHOWING EXCESSIVE VOLTAGE DROP

	Sq. ft. area per floor	Volts drop meter to socket	Location
Department store	6,000	0.6	Second floor rear
		5.2	Second floor front
Electric shop	3,000	4.9	First floor rear
		1.8	First floor front
Furniture store	3,500	6.7	First floor
		10.2	First floor front
Jewelry store	3,500	1.8	First floor rear
		7.0	First floor front
Garage	11,000	9.4	First floor rear
		7.6	First floor front
Variety store	4,200	1.5	First floor rear
		5.3	First floor front
Grocery store	3,800	10.9	First floor front
Department store	10,000	4.0	First floor center
		4.0	First floor front
		6.2	Second floor

Moreover, it is not sufficient to provide merely for initial requirements. The wiring should be planned with reference to the expected life of the building anticipating future needs as far as possible. Hence it is also important to consider the trends of lighting practise.

FUTURE LIGHTING

As already pointed out, the present indications are that illumination levels and degrees of light diffusion will continue to advance for a considerable period. There is no indication of retardation in this respect in the near future.

On the other hand there is a number of more or less new lighting applications going into use which bid fair to increase the electrical consumption. Among these may be mentioned the increasing use of light ornaments, that is, light sources to look at rather than for general illumination, ornamental portable lamps, indoor signal indicators, signs, etc. Some of these ornamental light effects, while originating in the home, club or hotel, are spreading to stores, offices and other interiors.

For the past five years, the practise initiated in Europe of using so-called "built-in" lighting, has been spreading rapidly in this country. Such lighting appeared first in lobbies, corridors, and other public spaces of fine buildings, then in the more elaborate stores, but later in less pretentious places. Since such lighting formerly required special construction in the walls and ceilings, it was applicable only to new buildings or those in which extensive reconstruction was being undertaken. It did not at first appear to apply to buildings planned for the older forms of luminaires. However, modified forms of luminaires similar to built-in equipment are beginning to appear, which can be installed without difficulty in finished rooms and supplied from the usual electric outlets. It is, therefore, quite possible and fairly probable that this style of lighting will extend to buildings not especially planned for built-in lighting. This class of lighting is extending because of its artistic merit and in spite of a considerable reduction in efficiency—measured in lumens per watt. It, therefore, represents an increase in electrical consumption which should receive proper consideration.

Another phase of lighting, which is still in the experimental stage, but receiving enough attention to preclude ignoring it, is the so-called "dual purpose" health lighting. In addition to the visible light, these lamps supply the ultraviolet radiation which is depleted from daylight by window glass and city smoke and which is absent from the light of ordinary illuminants. Such equipment does not inherently require more electricity per unit of light than ordinary lighting, but the installations so far made indicate a tendency to employ higher wattage. It is too early to evaluate even roughly the possible effect upon electrical consumption in buildings. It is, however, mentioned as one of the possibilities bearing upon the situation, which may well receive some consideration, especially where large numbers of people are likely to be excluded from normal enjoyment of sunshine.

All of the items mentioned suggest possibilities of increased demand for light and electric power. A compensating factor would be the use of more efficient light sources. The rapid increase in efficiency of incandescent lamps from say 1912 to 1922 just about kept pace with the demand for more light, so that the electrical demand for a given area and purpose held fairly constant throughout that period. Since then the advance in efficiency has slowed down as the art of lamp making approached the perfection possible with present scientific knowledge. As a matter of fact, it seems to be the conclusion of the best scientific investigators that the incandescent lamp and other solid radiators of light do not offer much promise of large increase in efficiency. It is, however, hardly conceivable that we have reached the limit of light generating efficiency. More efficient electric illuminants are hoped for and expected. Illuminants of considerably higher efficiency have been produced in laboratories. So far all of these have one or another objectionable characteristic so that those familar with the developments of the world's leading laboratories,[3] report that no development is yet indicated as having sufficient merit, including practical features—to warrant prophesying the replacement of the best existing forms of illuminants for general lighting of interiors. Since time is required to adapt and apply new forms of illuminants, it appears unsafe to depend upon any great increase in efficiency being widely applicable to today's installations within the next four or five years.

OTHER LOAD ON LIGHTING CIRCUITS

So far, consideration has been given only to lighting applications. Within the past ten years large numbers of small motor and heating appliances have been connected to lighting circuits. These devices include motor fans, bookkeeping and calculating machines, refrigerating equipment, dental, medical and other professional machinery, signals, bell ringers, etc. Such devices are becoming an appreciable part of the load on the lighting circuits, and apparently the next few years will witness considerable increase in the load due to their more extended use. They are valuable services and must be planned for both their own value and for the limits they would otherwise present to good lighting practise.

THE QUANTITATIVE ELEMENT OF WIRING

Interior wiring has received a great deal of attention on the part of code writers, and has been the subject of numerous rules. By many in the electrical industry the National Electric Code has been taken as a standard of good engineering, overlooking the fact that the purpose of The Code is fire prevention and other safety features. Being mandatory in character, the code cannot prescribe wiring on the basis of best economic engineering. Because of the common failure of those responsible for such wiring to provide for later additions to the load on a wiring installation it has been necessary for The Code to anticipate these additions, but still on the basis of safety only. The Code has been the subject of considerable controversy among the various interests involved. Since good engineering would incorporate both the requirements for safety and economic operations, there is good reason to believe that these misunderstandings would disappear and The Code assume its rightful position if good engineering were to prevail in this field.

Early wiring was liberal because of lack of definite knowledge with which to anticipate future demands. More recently, under economic pressure, the latitude has shrunk so that the capacity is often inadequate for the initial lighting. Wiring for a new building is frequently planned and installed before details of the use of different rooms, and therefore, their lighting, are determined. In the absence of generally accepted standards other than the Wiring Code, it has become a widespread practise for designers to establish arbitrary rules and constants of their own making. In the smaller installations, the details of wiring are often determined by competing contractors eager to underbid each other. Where consulting engineers are retained, they are often urged to justify their employment by minimizing investment and so find it difficult to apply best engineering judgment. These varying circumstances all militate against the proper response of wiring practise to the needs of lighting.

To determine investment relations, a committee of illuminating engineers recently made a study of wiring estimates. This committee submitted architectural plans for one floor (9,600 sq. ft.) of a typical building to electrical contractors in several cities throughout the United States. The contractors were asked to prepare bids covering the electrical installation work and material under various specified conditions as to sizes of lamps employed. Eleven different cost estimates were received from the analysis of which conclusions such as the following were drawn:[4] (1) That to cut down the wiring specifications in order to reduce the initial investment was false economy as the greater losses would within a period of a year or two entirely offset the difference in investment. (2) That the investment in the wiring system did not increase in direct proportion to the increase in wiring capacity; doubling the wattage capacity increased the investment only about ⅓ while 50 per cent extra capacity meant an additional investment of only 15 to 18 per cent.

It would appear, therefore, that in the interest of good economic engineering what is needed is a reasonable standard, based upon good lighting practise, with an allowance for advances in the art and probably changes in the use of buildings. Such a standard would do much to overcome the insidious whittling which, in a considerable degree, has been responsible for the unsatisfactory condition.

ADEQUACY SPECIFICATION

A few illumination engineers, with country-wide contacts, became conscious of building wiring as a limitation of good lighting about 1924. Previous to this time these engineers had concentrated their attention on lighting practise, namely, the selection of lamps, luminaires and locations. It became obviously important to give attention to wiring if the advance of illuminating engineering were not to be seriously impeded. After several years of study, investigation and consultation with an ever enlarging circle of engineers, a plan was formed, and in the summer of 1928 the National Electric Light Association was asked to promote a movement for better wiring practise. It is not within the scope of this paper to discuss the educational undertaking which resulted.[5] However, it is pertinent to call attention to the specifications, prepared by illuminating engineers widely checked by other groups, and issued under the auspices of the Association.

Preliminary specification paragraphs were agreed upon in the spring of 1929. Immediately some of the illuminating engineers, especially those associated with electric utility companies began an informal application of the standards so embodied. The results were very gratifying. After a year's experience an extension of these paragraphs for commercial buildings[6] was published by the National Electric Light Association. In the summer of 1931, a corresponding specification for industrial buildings[7] was issued under the same auspices.

Several interested groups individually undertook the preparation of corresponding specifications for residence wiring.[8] In the fall of 1931 an industry committee was organized to coordinate the several undertakings into a single standard specification. This work is not yet completed, although the reports indicate that the end is in sight.

The specifications for commercial and industrial structures are undoubtedly the best available expression of the needs felt by illuminating engineers for building wiring. While especially intended for the smaller installations the quantities and principles are applicable to larger buildings. They have been used as a guide in hundreds and probably thousands of installations and all reports seem to indicate that they represent reasonable and proper standards. They are not presented here as final specifications and it is presumed that they will, ere long, require revision to embody later experience and practise as well as more ready acceptance of the need of adequacy.

CONCLUSION

It is to be hoped that consulting engineers will familiarize themselves with these specifications, subject them to criticism and that out of this there may come generally accepted standards of wiring practise. This would strengthen the weak link in the system of electric lighting, and encourage a normal development along the lines of good economic engineering. The public could then look with confidence to its advisors in the field of electric lighting, and be assured that good illumination, according to its needs, can be had in any building constructed under responsible auspices.

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"House Wiring Standards Soon to be Circulated," *Electrical World*, Vol. 99, No. 6, February 6, 1932, p. 261.

Discussion

Philip Sporn: I do not believe that the engineering profession as a whole has recognized the importance to the electrical industry of the most important subject covered by Messrs. Stickney and Sturrock.

For some reason, unexplainable to me at least, the subject of wiring has been looked down upon as one belonging to the low-brow section of the profession and as one not fit for the application of thinking on the part of the engineers. Certainly it has received not a small fraction of the attention given, say, to heavy electrical apparatus or to high-tension transmission, or to underground distribution, or to any one of a dozen fields of application of electricity. And yet, adequate wiring is important in, and is back of, all these fields of application; and in many cases lack of adequate wiring is responsible for the lack of progress in many other branches of the whole industry.

Perhaps the basic reason for this is a general unfamiliarity with the National Electric Code and the status that that code has legally. For some reason there has permeated a general belief that the code as such has legal status, and that being the equivalent to law, there obviously is nothing that engineers can do about it except comply with it. As a matter of fact, the code has no legal status of any kind except such status as is given to it by municipal or other ordinances. On the other hand, a municipality is entirely free to legislate the terms and conditions under which wiring installations will be made within the legal boundaries of that municipality and such legislation may call for installations in direct contradiction to the code. Such cases have actually happened, and of course all those doing business within those municipalities have to comply with the municipal ordinance regardless of the fact that the code may say otherwise.

There is no question, of course, that a national code is a highly desirable thing, but it is only desirable as long as it does not result in throttling the very thing it is intended to promote. What is needed is a code that will recommend the minimum requirements consistent with safety to property and life and will permit therefore, maximum expansion of electrical service with its attendant benefits to the largest possible number of people. The code, too, ought to be flexible and ought to march hand in hand with technological progress and permit the easy change and experimentation leading toward change when technological progress indicates its safety and desirability. There is no reason to doubt that when a larger percentage of the Institute membership than is doing so at the present time begins to take an active interest in the code, we will get such a set of rules. When we do, we will certainly have gone a long way toward making possible economical wiring, and therefore, will have taken a great step toward providing adequate wiring, the need of which in one branch of the industry, the illumination branch, has been so adequately presented by Messrs. Stickney and Sturrock.

To indicate the effect that the application of well-known engineering principles to wiring can have on the cost of such work, and what it can do toward providing adequate wiring without materially increasing the cost, it might be interesting to cite the development of the bare neutral conductor method of wiring. Bare neutral conductors are at present approved by the code for service entrances only but even in that case are restricted to the use of bare neutral conductors in metallic rigid conduit, the code does not permit bare neutral conductors in armored cable construction. The latter construction is, however, fully approved by many municipalities and their accredited inspection agencies for use not only in services but in interior wiring as well. It used to cost $18.50 for labor and material to install a 30-ampere service, using a 30-ampere service switch and ¾-inch conduit with two No. 8 conductors (Code approved methods) and we found that to employ the same methods of wiring, using a 1-inch conduit and three No. 6 conductors, the lowest cost possible was $28.00; this, too, after a great deal of trimming and compromising in methods and materials, but still keeping the whole within the minimum code requirements. On the other hand, by going to bare neutral armored cable construction we were able to reduce this cost to $21.50, a reduction of over 23 per cent against the standard method, and an increase of only a little over 16 per cent over the standard 30-ampere, two-wire service. For this increase, however, the capacity of the service and its ability to permit electrical development in any home fed through the service is increased 230 per cent; that is, it brings up the possible electrical development within that home from 3.4 kw to 11.5 kw.

Morgan Brooks: The paper by Messrs. Stickney and Sturrock is a timely and impressive appeal for vision in planning building wiring to meet the possible illumination demands of the future. With decreasing costs of the materials for wiring this appeal should bear immediate fruit.

The authors state that improvement in illumination efficiency seems to be approaching a limit, but do not stress the present tendency of public service companies to reduce electricity rates as a spur to further rapid increase of lamps and appliances. With the predicted installation of air-conditioning apparatus in our homes just what wiring will prove adequate is a debatable question.

A field of considerable importance to the house owner is revision of inadequate wiring without recourse to complete replacement. The use of vacuum cleaners and of laundry equipment has usually caused no objectionable disturbance of illumination since their use is confined to daylight hours. Where coffee urns, toasters and chafing-dishes affect dining-room illumination adversely a single additional circuit, often inexpensively wired from the basement to convenience outlets for the offending utensils, and purposely connected to the side of the distribution system opposite that serving the lighting circuit,—will often make the use of such devices a positive illumination advantage by causing an increase of the lighting voltage.

In too many instances the distribution of circuits between the two sides of a three-wire system is determined solely by their direction of approach to the cutout cabinet. Outlets may thus be equally balanced, yet it may happen that nearly all the lamps in use at any given instant are connected on the same side of the system resulting in unnecessary reduction of voltage. Sometimes judicious reconnection of circuits at the cutout cabinet will save the expense of rewiring. The illumination engineer should therefore insist on an intelligent distribution of circuits to maintain current balance under all probable lighting conditions.

F. C. Caldwell: The paper calls attention to the approximate doubling of the demand for illumination during the past decade, but the extension of this trend in both directions may be considered. Forty years ago, following the examples of gas lighting one foot-candle was considered good general illumination where ten or more are called for now. Again, looking toward the future, we see no natural limit to this desire for higher illumination, unless it be daylight with values around one hundred foot candles or even more.

As illustrating the opportunity for real engineering in connection with the wiring of buildings, a simple application of differential calculus shows that the greatest economy of copper will be attained for a two-wire system, when the permitted voltage drop is divided between a feeder and its branches in proportion to the length of the feeder and the average length of the branches. Similar relations can be worked out for three-wire feeders with any given relation of neutral to outside wire. I have never seen this simple principle stated in treatises on wiring.

Too much emphasis cannot be placed on the importance of the specification by the architect of conduit and wire sizes, so that all contractors will bid on the same installation. Too frequently, this important part of the design of the building is passed on to the contractors by the practise of specifying the load and the permitted drop only. Compliance with these values is seldom checked.

J. M. Bryant: This paper is a timely one to give an added impetus to a work which has been started by the leading illuminating engineers of this country during very recent years. It is fitting and necessary for the American Institute of Electrical Engineers to lend its influence towards a change in the existing codes both national and local to assist in the rapid improvement of building wiring systems. One of the greatest steps already made in this direction has been in the education of the architect in providing for adequate building illumination in the design of new structures. What is now needed is to secure the interest of electrical engineers in securing by all legitimate means an increase in the standards for wiring so that the conductors in new structures may have sufficient capacity to permit increasing the illuminating standards of all buildings in line with modern tendencies.

There is probably no field of building illumination which better illustrates the changing tendencies than that of the public school and buildings in institutions of higher education. Before the advent of night school classes and classes in extension it was possible to schedule the work so as to miss the hours when it was thought that illumination was needed in such rooms. However, in modern illumination it is found that many rooms need artificial illumination at any hour of the day to conserve the eyesight as well as to permit more rapid work by the students.

I can best illustrate this by two or three cases. The first is that of a University which had adopted a 220/440-volt three-wire system of distribution derived from a 440-volt two-phase power system. In the design of new buildings farther from the power plant it was necessary to adopt a higher voltage for power distribution and at the same time the more efficient 115-volt incandescent lamp on a 115/230-volt system. This made it desirable to change over the older buildings to the use of the 115-volt lamps to prevent accidents in the use of the wrong lamp. The advent of the mazda lamp at that time saved the day to some extent although it did not permit of increasing the standard of illumination without the use of open wiring in many of the rooms. Another set of buildings illustrating this point was an old school for the blind which was to be adapted for the "Eyes of the Army" during the World War. The only lighting in these buildings was in the living quarters of the Superintendent and the staff. The old wards and classrooms, of course, needed no light for blind pupils. A third set was in a University in which the business manager held the opinion that if we provided adequate illumination in the buildings that the rooms would be used during the evening and the lighting bills of the University would increase. In spite of this opposition, a compromise plan was worked out by providing adequate illumination in some of the buildings and in providing adequate wiring and lamp spacing in all new buildings so that modern tendencies could be followed.

Since the architects of this country are taking such an active interest in adequate illumination as well as in the use of light for decorative purposes, the American Institute of Electrical Engineers should cooperate with standardizing bodies in the adoption of new codes of wiring. It should also do all in its power to see that there is a proper consideration given to the illumination and wiring of all buildings which are constructed in such a manner that it is extremely difficult as well as costly to make changes in the wiring and distribution equipment after the building has been put in use. In this line public school buildings should receive especial attention.

L. W. W. Morrow: There are three aspects of wiring and lighting that should receive serious consideration by every man in the industry. These are humanitarian, business, and technical. Man is a seeing machine and the new conception of lighting is to add to lighting for efficiency more light for the humanitarian effects we desire. Our ideas of illumination are so far ahead of practise and our social values involved in illumination are so great that industry attention should be focussed on the topic.

Then again, wiring and lighting offer enormous business possibilities to the industry. Few engineers realize that a load of 20 watts per sq. ft. for lighting means that each floor of an office building offers a load of 1,200 kw. and thus a 30-story building offers a load of 36,000 kw. High grade engineering and detailed attention is devoted to a 500-kw. industrial power load yet the lighting load is infinitely larger and receives little attention. Each home market offers from 10 to 15 kw. of load if adequately sold and engineered. Thus from the business point of view adequate wiring for lighting is a major task and requires high grade engineering.

In a strictly engineering sense adequate wiring and lighting deals with equipment and apparatus—some thousands of items. A piecemeal development has occurred and empirical standards exist. Undoubtedly engineering attention could be focussed on this aspect with resultant simplification, standardization and development. By way of contrast, consider the high grade engineering that has been focussed upon the relays and other devices used in telephony and telegraphy. These devices are precise and reliable to an advanced degree and carry voltage and current ratings very frequently comparable to those found in wiring for light and power. But consider the relative size, weight and amount of material used for devices in the two fields. Undoubtedly engineering attention to equipment used in wiring will pay enormous dividends.

All engineers should be interested in this subject and it is incorrect to consider the engineering of wiring and wiring devices too "low brow" for serious attention. No more fruitful field for engineering exists in the industry and it should have serious attention.

Taylor Reed: A revised code on adequate wiring of buildings should set limits of permissible momentary dip of voltage. A case in point would be to fix adequate residence wiring to preclude excessive momentary dip of voltage and consequent violent flicker of lights due to the starting of refrigerator motors.

P. L. Alger: This paper has presented very clearly the need for more liberal wiring systems to improve lamp efficiencies and reduce the power consumed in I^2R losses, as well as to anticipate future requirements.

It appears to me that any standard wiring code can properly only call for the minimum requirements consistent with safety, and that the provision of facilities in excess of the minimum must be left optional with the builder. However, if builders generally appreciated the direct savings in power bills obtainable by improved wiring, and especially if these figures were generally available in some authoritative form, I feel sure that more liberal wiring systems would generally be called for.

I, therefore, suggest that the wiring code committee prepare a supplement to the standard code, which would explain the minimum character of the standard code provisions, and give detailed figures on the possible savings to be made by the use of larger wire sizes and more numerous outlets. In particular, this pamphlet should give tables of the annual costs of the power consumed in resistance drop in the wiring for various wire sizes under typical load conditions for various assumed power rates. Also, the costs of the power losses due to poorer lamp efficiencies at low voltages should be tabulated. Besides these definite figures, estimates of the excess cost for additional wiring done after building is completed over the cost of installation during construction should be cited. Such a pamphlet, giving a convenient means for the layman to calculate the economic benefits of installing improved wiring systems should be very helpful in the promotion of more liberal installations; and, if attached as a supplement to every copy of the wiring code, it should prove a most effective means of advancing the objects which Messrs. Stickney and Sturrock so effectively advocate.

E. B. Murray: I wish to express my approval of this paper. There is one point, however, that I wish to criticize. The paper does not insist strongly enough that the architect or engineer connected with the designing of a building should take into consideration the importance and necessity of installing adequate copper in the circuits, and that if they fail to do so, they are not giving the best service to their clients.

I have had the honor of being retained in a consulting capacity on numerous large office buildings throughout the United States and I have yet to find an electrical contractor on an undertaking of this nature who has not called attention to the savings which he can make in the electrical setup by the reduction in circuit size.

Where every economy was necessary, I have insisted that at least oversize conduits be installed to facilitate the pulling of large wires when it became necessary.

Speaking from the standpoint of the building operator, statistics indicate that there is a very marked increase in the demand of tenants for higher lumen output at desk level. It is impossible for a building manager or operator constantly to check lamp sizes in a tenant's space for quite frequently the tenant will purchase and install at his own expense larger lamps without the knowledge of any of the building employees. In addition to that, we find that tenants are constantly adopting electrically operated office equipment which is naturally connected to the lighting circuit.

It is apparent, therefore, without any suspicion of what is going on, the building operators quite frequently find themselves confronted with a succession of blown fuses, which an investigation discloses is due to no other reason than an overloaded circuit.

Unless ample provisions are made on the plans for an increase of copper at not less than 50 per cent of the anticipated load, it is impossible to meet the increasing demands later on due to the fact that the conduit sizes are fixed and cannot be changed.

Careful determining of the wattage requirements per square foot of rentable area is one of the most important computations which should be made at the outset. The size of the circuits thus determined should then be increased arbitrarily at least 50 per cent.

Andrew Steers: After reading the paper with a great deal of interest, I endorse the reasoning and conclusions that it sets forth.

One of the weaknesses in modern business-building design and construction which has resulted in loss of operating efficiency and increased expense, has been the inadequacy of both power and lighting wiring. I believe it will not be difficult to get experienced business property owners and managers to make more adequate provision in this respect in all types of buildings they will build in the future.

The very great increase in the use of electrical appliances in the offices of today, coupled with the constant changes in the field of illumination, demand that all interested in this phase of building operations give serious consideration to the provision of sufficient conductors to meet modern conditions and provide for substantial load increases during the future life of the property.

My suggestion is that these studies be continued and that data, as it is prepared, be distributed to the leading architectural concerns and owners of business properties.

C. H. Roe: This paper serves a very important purpose in inviting further attention to the relation of adequate wiring and good illumination, a subject which is very apt to receive less attention than it deserves because of our present-day economic situation. The subject is so broad that obviously the authors can do no more than treat it in very general terms.

The paper recognizes the scope of the subject as including buildings of all types. It would seem, however, that residence wiring should be recognized as being of special importance in this connection because of the prominence of the residence load in the future business estimates of the various components of the electrical industry.

All that the paper says about tendencies to cut expenditures for wiring, on the part of consulting engineers, architects and wiring contractors, applies with particular force to residential work. Because of the greater prevalence of such practises in residence wiring, it follows that in this field lies that much greater opportunity for improvement in wiring adequacy.

Adequacy of residence wiring has at least two principal components; the first is adequacy of copper as measured by gage size of conductors. In most residences this is of less importance since limits of the actual physical carrying capacities of wiring most commonly used in residences have not yet been approached. Of considerably greater importance is the footage of wires and the provision of numerous outlets. In most homes greater convenience and greater current consumption would follow the installation of a few more feet of No. 14 wire than would result from the expenditure of the same amount of money in providing larger conductors in a smaller installation. Too many appliances are still used through lamp sockets. In fact the expression "lamp-socket appliances" is still frequenctly used in the industry. As for illumination, with which the paper is chiefly concerned, it is only necessary to visualize the all too common handling of flexible cords in living room, library and bedroom, in order to permit the use of the number of table, floor and bridge lamps which the modern home has come to regard as essential. A few baseboard outlets for the connection of portable lamps would seem to be of far greater importance than calculations of lamp sizes and carrying capacity of wires.

When considering adequacy of outlets for lighting purposes, one should not overlook the desirability of adequate switching facilities. Three- and four-point switches are conveniences to infrequently found in the modern home. Switch control of baseboard outlets adds considerably to the convenience of the householder. One of the best protective devices in the world is the master switch in the bedroom for switching on lights throughout the house. All of these things have to do with adequacy of control of lighting rather than with the provision of adequate illumination through consideration of lamps or copper sizes.

It is not proposed that the phases of this subject presented above are of any greater importance than several other aspects of the general question. It is only suggested that they should be given suitable attention in connection with any study of the subject.

Walter Sturrock: From the tone of the discussion the authors of this paper feel very much encouraged. I don't think there is a single thought expressed which is in any way contradictory to the authors' analysis of the problem.

Several of the discussors referred to the National Electric Code and in order that there will be no misunderstanding I would like to emphasize that the National Electric Light Association Minimum Specification for Adequate Wiring not only meets the requirements of this safety code but recommends heavier conductors in so far as they are justified from the standpoint of their being economically practical. The Minimum Specification does not discuss materials to use or methods to employ as such questions enter into an entirely different phase of the problem which was considered as being beyond the scope of the work carried on by those preparing the specification. It might also be mentioned that the Minimum Specification for Adequate Wiring as referred to was designed primarily for smaller buildings where a competent engineer's advice is not usually secured. It is applicable to larger buildings and will be found helpful to consulting engineers who may be employed on the job. Copies of the specifications are available through the headquarters of the association as indicated in the bibliography.

One of the discussors commented on the fact that few consulting engineers give serious thought to the wiring of lighting circuits, because there are more complicated problems in the distribution of power which have capitalized their time. As mentioned by him, this condition should not exist because proper lighting is an essential for every interior and its design demands serious consideration all the way back to the utility company's generator.

In stores, offices and other commercial buildings the relative importance of light and power circuits is no doubt readily recognized, but in industrial plants it is not so evident. A recent survey in 20 typical industrial plants in Detroit revealed the fact that approximately one-third the total connected load was lighting and two-thirds power. In this particular city, engineers have given a great deal of valuable advice on lighting with resulting satisfaction to all concerned. In contrast with the Detroit survey, light and power ratios were obtained from typical plants in an industrial section of Ohio where very little or no engineering work had been done on the design of lighting circuits. This latter survey indicated only 13 per cent of the total connected load as being used on the lighting circuit. These Ohio industrial plants were all inadequately lighted and presumably were so inadequately wired that up to date lighting could not be obtained without major changes. It cannot therefore be overemphasized that engineering advice on lighting circuits wherever they may be found is unquestionably worthwhile. And this advice, when put into practise, will assure the owner that he will not only have adequate wiring capacity to meet the demands for the initial connected load but also to permit a reasonable increase in load which may be expected during the next few years to come.

G. H. Stickney: After reviewing the discussion, we are highly gratified by the splendid response which emphasizes the importance of wiring to the progress of electrical engineering works. Practically the only criticisms are with reference to omissions or lack of stress on certain applications. This was intentional as we considered it best to concentrate our treatment on those aspects which came within our intimate experience, and concerning which we could speak from first hand knowledge.

The discussion has realized our hope that others would emphasize other phases of major importance out of their own viewpoints. As we have mentioned in the paper, the problems extend beyond the lighting field, and any solutions must give proper weight to these other applications.

The relation of the National Electric Code came in for considerable attention, and considerable diversity of opinion is reflected. It is not necessary to repeat our own views expressed in the paper. Obviously it is desirable to carry this question further towards unanimity of opinion as to what is right and best.

The paper, strengthened by the discussion, is merely a start in calling engineering attention to problems of wiring which needed more study, investigation, and coordination.

It is our hope that a suitable technical committee will take hold of the subject and provide a sound engineering basis, on which useful advances of the art can be assured. We believe that the unbiased interest of the A.I.E.E. can provide the kind of leadership under which the wiring of buildings can be developed to serve its proper part in the electric systems of the country.

Induction Motor Versatility

Nature of Its Applications

BY E. W. HENDERSON[1]
Member, A.I.E.E.

THE polyphase induction motor was first announced by Tesla in 1888. It is thus some forty-four years old. It demanded, for its operation, a polyphase system of power distribution. Prior to its inception, alternators and systems were single-phase, frequencies high (120 to 133 cycles), and distribution systems limited in many respects.[2] The induction motor ushered in a new era of electrical development which has grown and expanded to be the giant as we now know it.

Today, the induction motor bids fair to bring about another revolution of ideas as regards its use and capabilities. Heretofore, particularly in respect to the squirrel-cage motor, it has been confined to rather restricted fields of application. Its starting characteristics and the need of special starting devices were against it. Now limited-starting-current motors such as those employing double cage rotors are overcoming these starting objections. The use of these across-the-line motors has encouraged the idea of throwing standard motors directly on the line, and has encouraged also the use of induction motors for applications demanding frequent starting or reversing. Formerly, the squirrel-cage motor was looked upon as one to be applied only for constant speed service. Today, there are many applications where the operating range covers wide variations in speed.

There are many new applications which would have been considered impossible a few years ago. We look at things today from a different angle and new conditions demand a change in some of our former opinions. The purpose of this paper is to point out that the induction motor is becoming something of a universal motor, not from the standpoint of the number of motors used or the widespread territory over which these motors are used, but from the standpoint that it is today being applied to operations which are extremely diversified in type.

The demand for adjustable speed has been met, to some extent by the slip-ring motor. The objections to this type of motor are chiefly that the speed is not independent of the load and that the speed reduction is obtained at the expense of efficiency. The commutator type of motor with variable brush spacing is essentially a slip-ring motor with an opposing e.m.f. substituted for a rotor external resistance and offsets, in some degree, these disadvantages of the slip-ring motor. The multispeed squirrel-cage motor is more simple in construction and offers several fixed speeds. It cannot give adjustable speeds in the strict meaning of the word, and yet it is being more widely applied than formerly, and is finding new applications on machine tools and special drives, lifts, planers, etc.

There is a number of applications of single winding, simple squirrel-cage motors which cover a great variety of uses and it is these which it is desired to mention particularly. Probably the best way to classify these applications is from the standpoint of slip and torque, and Fig. 1 is shown to indicate the divisions which can conveniently be made for this purpose. Under division I are indicated applications covering slips of approximately 0 to 20 per cent. Division II covers a wide speed range of slips from 20 to 70 per cent. Division III covers the balance of the speed range, *viz.*, from 70 per cent slip to standstill. Several characteristic torque curves are shown in Fig. 1 as representative of various types of squirrel-cage motors.

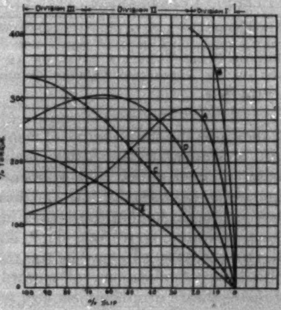

FIG. 1—CHARACTERISTIC TORQUE CURVES OF INDUCTION MOTORS WITH RESPECT TO SLIP OR SPEED

Under division I, curve *A* is that of the standard, low-slip motor. Its chief characteristic is a high pullout torque at a comparatively low slip, and a close speed regulation under load. Its full-load slip will average about 3 per cent. Its applications are so well known that no details are given here.

There are some special applications which demand less slip than 3 per cent, or which require a torque characteristic such that there will be a large increase in torque for a small drop in speed. In the former class belong loom motors as universally used in the textile industry. In the latter class are special centrifugal machines such as babbit pots where the load comes on the motor after it is up to speed. The large inertia load demands a rapidly increasing torque as the speed drops, else the drop in speed will be excessive and valuable production time lost in getting the machine back up to full speed. Certain tapping machines demand this same characteristic for the same reason. Curve *B* of Fig. 1 indicates the type of speed-torque curve covering these applications.

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[1] Reliance Electric & Engineering Co., Cleveland, Ohio.

[2] *Elec. Journal*, Vol. I, page 558.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

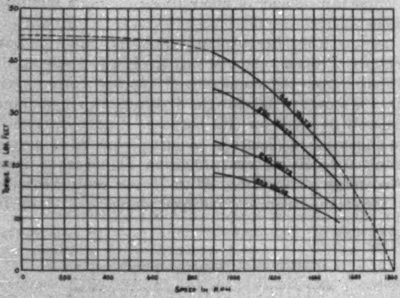

Fig. 2—Set of Speed-Torque Curves of an Induction Motor Used on an Automatic Tension Device. Torque Increased with Slip

Curve *C* indicates the torque curve of a squirrel-cage motor with a comparatively high-resistance rotor. Such a motor is used where high starting-torque is desirable and where the excessive slip is no disadvantage to the work in hand. Applications cover motors for hoists, cranes, elevators, conveyers, skips, etc. On the other hand, this excessive slip may be the thing which is highly desirable. Applications of motors to slow-operating presses, shears, drop hammers, etc., where these tools are equipped with flywheels, demand that the speed drop be sufficient to allow the flywheel to restore part of its kinetically stored energy to the tool drive and thus relieve the motor. This practise allows the use of a low-horsepower-rated motor and makes for a constant power demand.

Curve *D* indicates the torque curve of a motor with a rotor resistance between that of *A* and *C*. It has a slip at full load of approximately 8 per cent and is the motor used for average punch-press service, as its high torque and slip adapt it for this work. It is used extensively for motors which have to start or reverse often, such as tapping machines, threaders, and many special applications.

Division II of Fig. 1 covers slips of 20 to 70 per cent. This range is generally thought of as belonging entirely to the slip-ring motor, yet squirrel-cage motors are used within this range also. One interesting application is that of automatic torque control. For example, in the wire drawing industry it is an advantage to keep a constant tension on the wire as it is reeled. If this is done, then as the reel builds up, the torque of the reel motor must increase due to the increasing radius at which this constant wire tension is to be maintained. Fig. 2 indicates a set of torque curves of a motor used for such a purpose. The automatic rise in torque with decrease in speed fits the motor perfectly for such applications. Only one full torque curve is shown in Fig. 2. The working portion of the lower torque curves are obtained by varying the voltage on the motor by means of a simple auto transformer. Evidently the motor can be designed to give practically any torque variation required.

There has also been a demand for tension control in which the motor torque increases as the speed increases. Such a set of torque curves is indicated in Fig. 3 where again the motor may be designed to meet the conditions required. The decreasing torque of the spooler motor, as the spool fills up in this case, provides constant tension on the wire but the increased torque corresponding to this constant tension is more than offset by the decreased friction and windage of the spool, so that the total desired torque falls off with the speed. If the speeds run to a high slip or cover a wide range, very special motors may be necessary. The motor copper losses vary with the slip and special ventilating schemes may be necessary, or recourse may be had to the slip-ring motor. Since the maximum torque point can be moved about at will, any desired slope of torque curves can be had with close approximation to a straight line over the operating range.

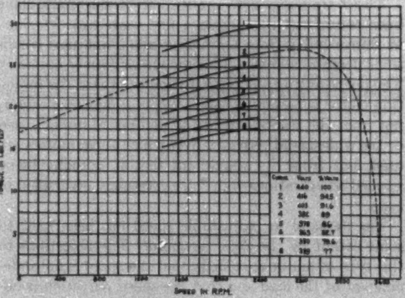

Fig. 3—Speed-Torque Curves of Induction Motor Used for Automatic Tension Control. Torque Decreases with Slip

Division III, covering slips of 70 to 100 per cent, covers applications of induction motors where the motor makes only a few revolutions or crawls at a low speed. Such applications cover valves, screw-downs, certain door-lifts such as for steel mill furnaces or hoppers, special stalled torque-motors for various types of control, etc. The value of the starting or low-speed torque will depend on the type of motor used, with respect to the torque at higher speeds, as indicated in the curves of Fig. 1. The starting torque can be made to vary over wide limits by adjusting the rotor resistance. Some applications will demand a high starting torque, others a low starting torque. Of particular interest in the latter type are motors of especially low starting and reversing torque as used for laundry machines. Such motors have an inherent high reactance. There are other special applications where too quick a start may result in damage to the product and which demand low starting torque, and of the opposite type are special high-torque, short-time ratings for such applications as theater-lifts, tool shifts, etc., which require abnormally high starting torques.

Division III might be extended to include stopping as well as starting characteristics. The stopping of squirrel-cage motors by means of direct current offers some interesting considerations. Figs. 4 and 5 show tests on a standard 7½-hp., 60-cycle motor stopped by applying direct current to the stator. Since it would take an infinitely large current to stop a motor in zero time, the curve of Fig. 4 is asymptotic to the vertical axis. It is not asymptotic to the horizontal axis due to the action of bearing-friction. As the rotor slows down, the secondary frequency changes from that of line frequency less slip frequency, to zero frequency, so that conditions are somewhat similar to those when starting from standstill and running up to speed, where the rotor frequency changes from that of the line to slip frequency.

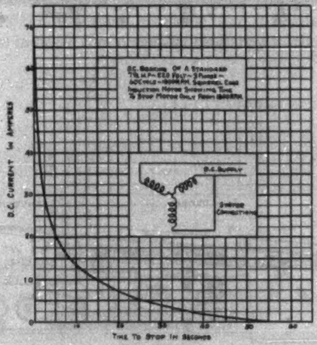

FIG. 4—D-C. BRAKING CHARACTERISTIC OF A SQUIRREL-CAGE INDUCTION MOTOR, SHOWING TIME TO BRING MOTOR TO STOP

The braking effort for a given value of direct current will depend on the manner in which the terminals are connected to the d-c. supply. For a Y-connected stator and connection as shown in Figs. 4 and 5, the d-c. ampere-turns per pole[3] = M_1 such that

$$M_1 = \frac{N}{3} I_1 + 2\left(\frac{N}{3} \times \frac{I_1}{2} \times \cos 60°\right) = 0.5\, NI_1 \qquad (1)$$

Where only two terminals are used, the ampere-turns per pole are

$$M_2 = 2\left(\frac{N}{3} I_2 \cos 30°\right) = 0.578\, NI_2 \qquad (2)$$

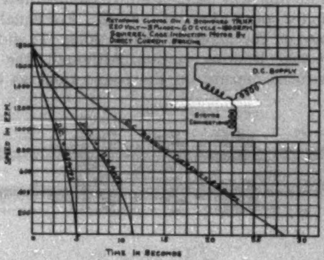

FIG. 5—RETARDATION CURVES OF A SQUIRREL-CAGE INDUCTION MOTOR WHEN STOPPED BY DIRECT CURRENT

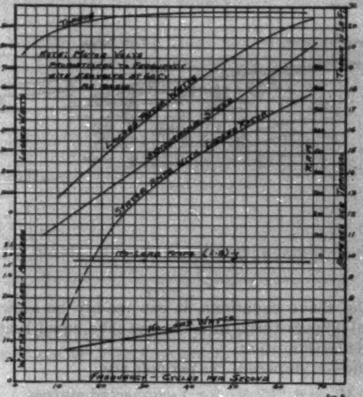

FIG. 6—TEST DATA ON A SQUIRREL-CAGE MOTOR WITH A WIDE SPEED RANGE BY VARIABLE FREQUENCY CONTROL

Three-phase, 16.7/36.7 cycles, 153/336 volts, 241/532 r.p.m.

For a delta-connected stator and two terminals connected to a common d-c. line, the third being connected to the other d-c. line, the ampere-turns per pole are

$$M_3 = 2\,\frac{N_0}{3}\,\frac{I_3}{2} \cos 30° = 0.289\, N_0 I_3 \qquad (3)$$

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[3] See *Measurement of Stray Load Loss in Polyphase Induction Motors*, C. J. Koch, A.I.E.E. TRANS., Sept. 1932, p. 756.

For a delta-connected stator using two terminals only, the ampere-turns per pole are

$$M_4 = \frac{N_0}{3} \times \frac{2\,I_4}{3} + 2\,\frac{N_0}{3} \times \frac{I_4}{3} \cos 60°$$

$$= 0.333\,N_0 I_4 \qquad (4)$$

where N and N_0 represent the turns per pole, and I_1, I_2, I_3, and I_4 represent the total direct current. It will be noted that for equal currents approximately 15.6 per cent more flux will be created by connections ((2) and (4) than for connections (1) and (3). For equal voltages impressed on the different connections, (1) and (3) give the greater braking action. Although the braking ampere-turns depend on the connections as indicated in the above equations, it can readily be shown that equal ampere-turns in each case give equal heating or power consumption.

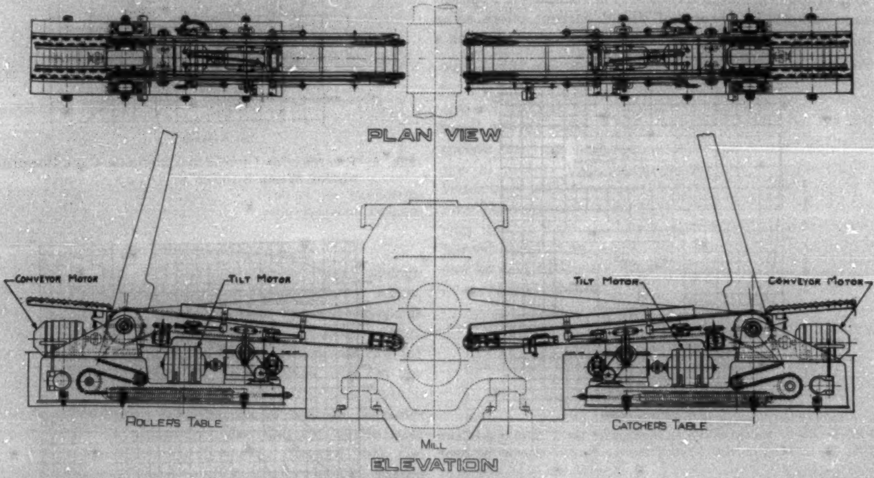

FIG. 7—DETAILS OF MECHANISMS AND MOTOR MOUNTING ON ONE MAKE OF AUTOMATIC TABLES USED IN ROLLING SHEET STEEL OR PLATE

The a-c. ampere-turns per pole are

$$M_5 = 0.707\,NI \text{ for Y-connected stators} \qquad (5)$$

and

$$M_6 = 0.707\,N_0 \frac{I}{3} = 0.408\,N_0 I \text{ for } \Delta$$

$$= \text{connected stators} \qquad (6)$$

where again N and N_0 equal the turns per pole and I represents the effective terminal alternating current. The ratio between d-c. ampere-turns and a-c. ampere-turns is thus apparent.

When a squirrel-cage motor is stopped by a-c. plugging, the alternating current rises anywhere from 5 to 10 times the full-load current of the motor. To stop a squirrel-cage motor by d-c. braking also requires considerably more than the value of rated alternating current, if the stopping is to be done in a reasonable time. The losses, however, are less for d-c. stopping than for a-c. plugging, with the stop accomplished in the same time in each case.

If a squirrel-cage motor is stopped from different speeds by applying a direct voltage proportional to the speed, the stopping time will decrease as the speed increases. For a two-to-one range of speed, experiments indicate that the time to stop from the high speed will be about 75 per cent of the time to stop from the low speed.

An application of a squirrel-cage motor to cover a wide speed range is that of a gang of such motors driving individual units of a conveyer system where the speed range is obtained by applying variable frequency to the motors. Such cases generally require constant torque characteristics, and by applying a voltage proportional to the frequency, the torque is practically constant over a wide range of speed. Fig. 6 shows the characteristics of a squirrel-cage motor when operated under these conditions.

Squirrel-cage motors have been applied where extremely low speeds are required by running them on frequencies as low as 3 to 5 cycles. When operated on such low frequencies, the characteristics differ immensely from those for standard frequencies. One peculiar fact is that under these conditions the locked current, no-load current, and running-load current are almost of equal value. The plugging current may actually be less than the running current if the plugging is done at the instant of low voltage.

Applications of squirrel-cage motors for rapid and oft-repeated reversing or starting duty have been exceedingly active in the last few years. First came such motors as applied to tapping machines, drills, and tools of a similar nature. Small planers have been equipped with 10- to 15-hp., squirrel-cage, multi-speed motors reversing as often as fifteen times a minute. Squirrel-cage motors have been used for special machine operations where a reversing duty as high as 30 to 40 reversals a minute has been demanded, in order to eliminate complicated mechanical reversing mechanisms. Quite recently, motors for rapid reversing duty have been demanded for a special application known as catcher tables, one type of which is indicated in Fig. 7. The author presented a paper before the Pittsburgh Section, A.I.&S.E.E., in March, 1932, on this particular application and its allied problems, and it is cited here simply to indicate the increasing tendency in the use of squirrel-cage motors for reversing duty. Figs. 8, 9 and 10 show characteristic curves of a 25-cycle motor for such service.

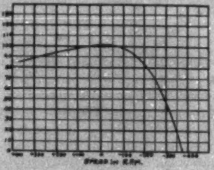

FIG. 8—TORQUE CURVE OF 25-CYCLE—375-R.P.M. REVERSING MOTOR FOR CATCHER-TABLE SERVICE

A very interesting study of a squirrel-cage motor starting from rest occurs when a crank action is interposed between the motor and the final mechanism, particularly if the crank moves through 180 deg. in completing the movement desired. Where the crank is actuated by a motor, either directly or through some reduction unit, and starts with the crank on dead center, the equivalent reduction ratio between mechanism and motor continually changes from infinity to some minimum value. The acceleration of the motor must be combined with the acceleration of the mechanism, and this changing reduction ratio must be taken into account in the calculation of the total equivalent WR^2 at the motor. This condition, combined with a torque characteristic which changes with speed, leads to a solution of the velocity and acceleration of the mechanism which can be obtained only by the method of assumption and trial. In cases where material is conveyed by such a mechanism in a vertical plane, it is usually essential that the acceleration at no time exceeds that of gravity. The minimum time of such a cycle is thus rather definitely fixed.

Assuming that the mechanism is given harmonic motion, the velocity will follow the law

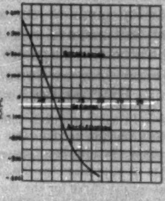

FIG. 9—RETARDATION AND ACCELERATION CURVE OF MOTOR SHOWN IN FIG. 8 USED ON 25-CYCLE SERVICE

$$V = \omega R \sin \omega t$$

Where

ω = angular velocity of crank in radians per second,

R = radius of crank.

With the maximum velocity equal to ωR, the maximum acceleration will be $\omega^2 R$.

For example, assume a total movement of 3 feet. Here $R = 1.5$ (crank to make 180 deg. for total movement).

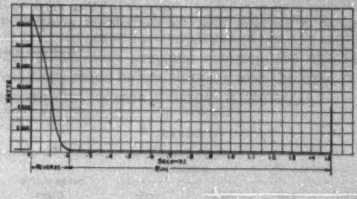

FIG. 10—ENERGY LOSS DURING A CYCLE OF REVERSAL AND RUNNING FOR 25-CYCLE CATCHER TYPE MOTOR OF FIG. 8

$$\omega^2 R = \omega^2 \times 1.5 = 32.2\ (= g)$$

$$\omega^2 = 21.5$$

$$\omega = 4.62 \text{ radians per second.}$$

$$\text{Time to traverse } \pi \text{ radians} = \frac{\pi}{4.62} = 0.68 \text{ seconds.}$$

The time of the movement could thus not be less than 0.68 second on the assumption that the motor ran at constant speed. Since the motor must start from rest, the minimum time of the cycle will be greater than 0.68 second if gravity is never to be exceeded. Such applications, therefore, demand considerable study both from the builder of the mechanism and the motor application designer.

There are many other special applications of the induction motor which might be mentioned. Enough have been indicated to show the widely diverse type of applications which can successfully be met by this motor. This diversity gives to the motor a certain universality which will make it even more popular in the future than it has been in the past.

Discussion

E. W. Henderson: In the paragraph following equation (6) some general remarks are made on the value of d-c. braking current. Fig. 11 is more specific and shows some interesting details; it also covers test data on a standard two-hp., 1,200-r.p.m. motor. Curve *A* is the speed-torque curve for the motor between standstill and synchronism *i. e.*, between rotor slips of 1 to 0. Curve *C* is the speed-torque curve of this same motor with direct current applied to brake it from 1,200 r.p.m. to standstill. Under these conditions the rotor frequencies correspond to slips of 1 to 0. This curve was obtained by using the dynamometer as an adjustable speed motor to drive the two-hp. induction motor under the conditions of a fixed d-c. current in the stator. The resulting torque was read on the dynamometer. The conditions in the rotor as regards frequency are identical in curves *A* and *C* but the shape of the torque curves are quite different. The stator currents to produce these torque curves are indicated by the dotted lines of Fig. 11.

When an induction motor is plugged to standstill by reversing the a-c. phase rotation the slip of the rotor changes from $s = 2$ to $s = 1$. The torque curve for these conditions is indicated in curve *B* of Fig. 11. As a comparison of braking capabilities under a-c. and d-c. plugging, therefore, we must compare curves *B* and *C*. The calculations shown to the right of Fig. 11 indicate that for the same average retarding torque, or in other words to stop the motor in the same time, the d-c. current would be 14.55 amperes whereas the a-c. current would have an r.m.s. value of 27.8 amperes. The heating of the stator under retardation due to stator current is indicated in the calculations. A better a-c. speed-torque curve, as regards available torque for retardation, could be produced by this motor by the use of a higher resistance rotor. The stator heating would still be in favor of the d-c. braking and the running losses of the motor would be materially increased.

D-c. braking thus gives a cooler motor under conditions of starting and stopping not only from the question of stator heating produced during the retardation period indicated above but also due to the absence of the high-frequency iron losses, both in rotor core and teeth, which are present under a-c. plugging.

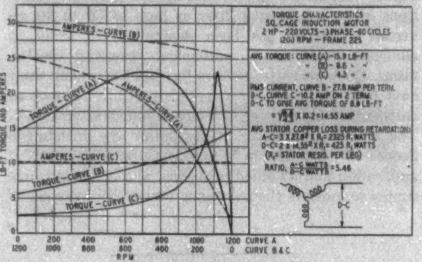

FIG. 11

With regard to the question of the relative merits of d-c. and a-c. motors for rapid stopping and reversing service, each type has advantages and disadvantages with respect to the other. I believe the demand for a-c. motors has come chiefly due to the fact that the a-c rotor is more sturdy, demands less maintenance, operates with lower-cost control and more rapid-acting control. The a-c. rotor lends itself better to lower values of WR^2 but the torque advantage of the d-c. motor together with its reduced stator heating and the locating of the secondary loss outside of the motor tend to restore a balance between the two types.

Dynamic Braking of Synchronous Machines

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and

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INTRODUCTION AND SCOPE

THERE is a number of applications of synchronous machines where it is desirable to stop the rotation as quickly as possible. The electric methods of accomplishing this are plugging and dynamic braking. Plugging consists of reversing the phase rotation of the voltage applied to the armature winding. Dynamic braking consists of short-circuiting the armature through an external resistor and maintaining field excitation. In this paper these two methods are compared and the results of an analytical treatment of dynamic braking are given. The paper is new to the extent that it treats variable speed short-circuits.

GENERAL DISCUSSION

Independent of the type of braking used, it is necessary to convert the stored energy in the rotating system into some other form, usually heat. Analyses of a number of particular applications of synchronous motors to steel and rubber mills, where quick stops are necessary, have shown that practically all of the stored energy in such systems is in the motor rotor. The normal means of dissipation of this energy are the windage, friction, copper and core losses of the motor, and the friction of the rolls. Generally, unless the rolls are loaded, the amount of energy dissipated in them is negligible, being of the same order of magnitude as their stored kinetic energy. The neglect of both these factors is therefore a compensating error. The other means of energy dissipation are functions of the method used.

FIG. 1—TYPICAL APPLICATION WHERE BRAKING IS NECESSARY

In plugging, the phase rotation of the supply voltage is reversed, the performance then being similar to that during starting except that the motor slip varies from two to one instead of from one to zero. At the time the speed becomes zero, the power must be removed to prevent a reversal of rotation. This method of phase reversal is subject to several outstanding objections, the most prominent of which are: first, unless the power supply is large, the system disturbance resulting from the switching operations may be very undesirable; second, the torque developed for a usual motor is small, thus limiting the rate at which the system can be stopped unless a specially designed amortisseur winding is used. This special design of amortisseur winding, beneficial to stopping, frequently is detrimental to other machine characteristics; third, if reversal is to be prevented, the power supply must be accurately controlled so that it may be removed at the instant the motor reaches zero speed. This introduces complications in the control circuits which can be avoided by other stopping methods.

In dynamic braking the armature is short-circuited through an external resistor and field excitation is maintained. The operation is that of a generator short-circuited through an external resistor. The theory of such operation from the standpoint of constant speed has been thoroughly treated in previous publications. The dynamic braking cycle differs from these treatments in that as the rotational energy of the system is dissipated, the speed of the machine decreases. The rate of this dissipation depends greatly upon the resistance used, as will be shown in the section on analytical results.

In order to present a comparison of the two methods, Table I has been prepared showing the number of revolutions made before stopping and the time in seconds required for each of the two methods.

TABLE I

Motor speed r.p.m.	Plugging		Dynamic braking	
	Rev.	Time	Rev.	Time
1,200	15.60	1.72	9.00	.98
600	5.50	1.21	4.32	.95
300	2.90	1.28	2.40	1.05
150	1.65	1.45	1.41	1.24
72	1.02	1.87	.85	1.56

The figures are based on an average 1,000-hp. unity-power-factor, mill-type motor. They represent calculated results based on the theory which follows.

ANALYTICAL RELATIONS

To determine the relations during the braking cycle an expression for the instantaneous armature current is

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obtained from the vector diagram of a suddenly short-circuited generator with known speed decrease. The instantaneous torque in synchronous kilowatts is obtained from the armature current and effective circuit resistance. This is equated to the rate of change of kinetic energy due to speed reduction. There results a differential equation which when solved gives the speed-time curve in terms of the machine constants. Experience has shown that there are two satisfactory methods of solving this equation. The first, based upon the assumption of constant effective excitation, gives an equation for the speed-time curve

$$t = \frac{2H}{e_d'^2 r}\left\{\frac{x_d'^2}{2}(1-n^2) - r^2 \log n + \frac{r^2}{2}\left(\frac{x_d'}{x_q} - 1\right)^2 \log\left(\frac{n^2 x_q^2 + r^2}{x_q^2 + r^2}\right)\right\}$$

where n is the per-unit speed.

The number of revolutions made by the machine upon reaching a given speed is then obtained by changing the variable in the speed-time equation and integrating. This is

$$R = \frac{N_s H}{30\, e_d'^2 r}\left\{ r^2(1-n) + \frac{x_d'^2}{3}(1-n^3) - r^2\left(\frac{x_d'}{x_q} - 1\right)^2 \left[(1-n) - \frac{r}{x_q}\left\{\tan^{-1}\left(\frac{x_q}{r}\right) - \tan^{-1}\left(\frac{n x_q}{r}\right)\right\}\right]\right\}$$

The total revolutions made to stop are found by placing correct boundary conditions in the above equation, and solving. The result is:

$$R_s = \frac{N_s H}{30\, e_d'^2 r}\left\{ r^2 + \frac{x_d'^2}{3} - r^2\left(\frac{x_d'}{x_q} - 1\right)^2 \left(1 - \frac{r}{x_q}\tan^{-1}\frac{x_q}{r}\right)\right\}$$

The optimum value of resistance to be used in order that the revolutions to stop are a minimum, considering all of the machine quantities to remain constant, is obtained by differentiating R_s with respect to r and neglecting the less significant terms. This value is:

$$r_o = \frac{x_d'}{\sqrt{3}}$$

The second method of solution is based upon variable excitation with an assumption regarding the manner of variation. Analysis of a number of oscillograms taken during the dynamic braking cycle has shown that the field current can be represented very closely by a single exponential form, the time constant however being shorter than that of the field under short-circuited armature conditions. Using this assumption there results a relation between the machine quantities, speed and time

$$\frac{E_f^2}{r_f^2} t - \frac{2E_f\left(I_B - \frac{E_f}{r_f}\right)}{r_f \alpha}(\epsilon^{-\alpha t} - 1) - \frac{\left(I_B - \frac{E_f}{r_f}\right)^2}{2\alpha}(\epsilon^{-2\alpha t} - 1) = \frac{2HI_{fo}^2}{r}\left\{\frac{x_d^2}{2}(1-n)^2 - r^2 \log n + \frac{r^2}{2}\left(\frac{x_d}{x_q} - 1\right)^2 \log\left(\frac{n^2 x_q^2 + r^2}{x_q^2 + r^2}\right)\right\}$$

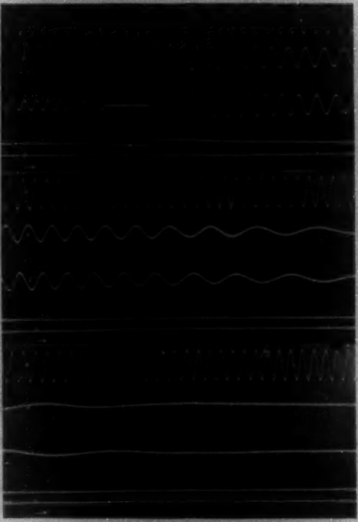

FIG. 2—DYNAMIC BRAKING CYCLE

500 hp., 40 pole, 180 r.p.m., 0.8 power factor
Curve A —60-cycle timing wave
Curve B —terminal voltage
Curve C —armature current
Curve D —field current

In this equation I_B is the field current after disconnecting from the line. All subscripts f refer to the field.

The left hand side of the equation has only time functions, and the right hand side has only speed functions. Being of transcendental character, no explicit solution is available, but the desired result may be obtained by plotting the two sides of the equation and finding the time and corresponding speed for which they are equal.

Discussion of Results

The theoretical results are based on the premise that saturation may be neglected, but their application to a practical machine must include saturation effects in the determination of the initial conditions and in the determination of the instantaneous excitation. This is evident, because regardless of the initial terminal voltage applied, and the initial field current, there is some point during the speed decrease at which normal flux is reached. As the machine passes this point, saturation plays an increasingly greater part in the effective excitation. Since this excitation enters the determination of the stopping revolutions as the square, its accurate calculation is imperative. Satisfactory results would not be possible in many practical applications if saturation were omitted.

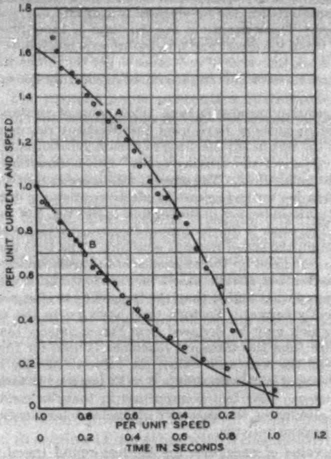

Fig. 3—Current and Speed Characteristics During the Braking Cycle Shown in Fig. 2

Curve *A*—per-unit current *vs.* unit speed

$$i_n = e_d' \left\{ \frac{x_q^2 + \left(\frac{r}{n}\right)^2}{\left(\frac{r}{n}\right)^2 + x_d' x_q} \right\}^{1/2}$$

Test points indicated by small circles

Curve *B*—per-unit speed *vs.* time

$$t = \frac{2\,H}{e_d'^2 r} \left[\frac{x_d'^2}{2}(1 - n^2) - r^2 \log n + \frac{r^2}{2}\left(\left(\frac{x_d'}{x_q}\right) - 1\right)^2 \log \left\{ \frac{n^2 x_q^2 + r^2}{x_q^2 + r^2} \right\} \right]$$

Test points indicated by small circles

The assumption of constant effective excitation will embrace a large majority of applications, in fact will be sufficiently accurate for most calculations.

In the case of a braking cycle being initiated with the machine under load, it is interesting to note that the revolutions to stop, due to the rate of electrical energy dissipation will usually be greater than would occur under initial no-load conditions. The presence of the shaft load, if it continues, will of course aid in the stopping and bring the machine to rest sooner than the no-load condition in spite of the fact that the rate of electrical energy dissipation is less.

The effect of power factor is to increase the effective excitation and consequently improve the stopping cycle. If, however, the power factor necessitates a larger diameter machine than could otherwise be used, the increased mechanical energy storage may more than offset the gain obtained from a greater rate of dissipation. The proper proportioning of a particular machine depends upon these two factors, and to obtain good results a careful balance must be found.

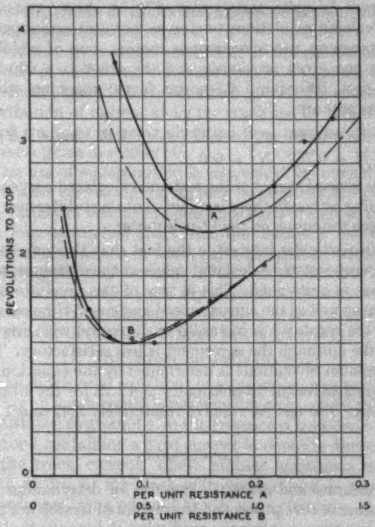

Fig. 4—Effect of External Resistance on Stopping Revolutions

Curves *A*—600 hp., 12 pole, 600 r.p.m., 0.8 power factor
Curves *B*—500 hp., 40 pole, 180 r.p.m., 0.8 power factor

$$R_s = \frac{N_0 H}{30\, e_d' r}\left[r^2 + x_d'^2 - r^2 \left(\frac{x_d'}{x_q} - 1 \right)^2 \left\{ 1 - \frac{r}{x_q} \tan^{-1}\left(\frac{x_q}{r}\right) \right\} \right]$$

Test curves in full lines
Calculated curves broken lines

COMPARISON BETWEEN TEST AND CALCULATED RESULTS

Fig. 2 shows an oscillogram of the relations during the dynamic braking of a 500-hp., 40-pole, 180-r.p.m., 0.8-power factor synchronous motor whose constants are:

$x_d = 1.45$ $x_q = 1.02$ $x_d' = 0.65$ $e_d' = 1.34$ $N_o = 180$
H = stored energy per kva. = 0.493 seconds
r = total per-unit armature resistance = 0.53

Fig. 3 shows a comparison between test and calculated current-speed and speed-time curves from the oscillogram of Fig. 2. The agreement between the curves is quite satisfactory. Fig. 4 shows test and calculated revolutions to stop *vs.* per-unit armature resistance on two synchronous motors, one of which is the one given previously and the other is a 600-hp., 12-pole, 600-r.p.m., 0.8-power factor machine whose constants are:

$x_d = 0.89$ $x_q = 0.52$ $x_d' = 0.22$ $e_d' = 1.27$
$H = 0.762$ $N_o = 600$

The general shapes of the test and calculated curves are similar although there is a fairly constant difference between the curves on the 12-pole motor.

Of particular interest is the fact that the agreement between test and calculated values at the optimum resistance is quite good. This is true of the revolutions to stop as well as the current-speed and speed-time curves. As the resistance is increased or decreased with respect to the optimum the agreement is less satisfactory. The minimum revolutions as determined by the equation for r_o are 2.45 for the 12-pole motor and 1.22 for the 40-pole motor. The test curves give respectively 2.42 and 1.20.

CONCLUSIONS

Accurate and practical formulas for determining the number of revolutions and time required to stop are presented, thus enabling the prediction of dynamic braking performance comparable with that of other machine characteristics.

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Three-Phase Short-Circuit Synchronous Machines—V, R. E. Doherty and C. A. Nickle, A.I.E.E. TRANS., Vol. 49, April 1930.

Discussion

C. C. Shutt: The adaptability of synchronous motors to dynamic braking is doubtless one factor which has influenced their rather general application to rubber mills and similar drives. An additional point in favor of dynamic braking over plugging is that the time required for the control to function, in making the transfer from running to braking possible, is usually less for dynamic braking.

For several years the writer has used an approximate set of equations based on the round rotor theory, which gives the speed-time characteristics, the revolutions to stop and the best value of resistance. These equations are the same as those given by the authors when the direct and quadrature axis reactances are made equal. Also, the value of resistance to get the minimum number of revolutions to stop is the reactance divided by the square root of 3.

The question has always been, what value of reactance to use. The present authors have used the transient reactance. This assumes the flux in the machine to remain substantially constant from the start of the braking cycle. This assumption does not appear to be justified. The current will decrease for two reasons. The first is due to the increase in effective resistance of the armature circuit as indicated in the formula under Fig. 3 of the present paper. The other is due to normal decrement which would occur even though the speed remains constant. The time constant is, of course, modified by the resistance in the armature circuit.

The writer had always based the resistance on a value of reactance somewhat higher than the transient as being more nearly theoretically correct. Factory tests have led to the same conclusion which may be deduced from Fig. 4, namely that when a little more than the transient reactance is used for the basis of arriving at the resistance value, the value itself may be varied ± 30 or 40 per cent with only a 6 to 12 per cent in total revolutions to stop. This fact is important in that for practical purposes it eliminates the necessity of precise determination of the value of braking resistor.

If the value of resistance for shortest round travel could be obtained from the authors' expression based on variable excitation, it should be very nearly the correct value, as the expression itself appears to be correct in form. The solution of this equation and the comparison of the results with those obtained by the previous method would be very interesting.

C. E. Kilbourne: Mr. Shutt has raised the question of what is the correct value of reactance to use in the braking cycle and has suggested a value somewhat higher than the transient used in this presentation. He is correct in saying that the effective reactance is larger than the transient reactance, but the difference between a reactance found by taking a value which is determined from the minimum braking travel and the transient value is small. This is so because the majority of the speed decrease occurs in the first fraction of a second at the start of the cycle and is nearly governed by transient conditions. It is true that a higher reactance dominates the end parts of the action but as far as total effect is concerned it does not have much weight. Fig. 3 bears out this statement.

As stated in the paper, the assumption of constant effective excitation will be sufficiently accurate for most calculations. A number of comparisons made between results obtained by the approximate and more accurate equations seldom showed more than 10 per cent difference in total revolutions as obtained by the two methods, thereby again indicating that the assumption of transient reactance is nearly correct.

Current Propagation in Electric Railway Propulsion Systems

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I. INTRODUCTION

THE study of current propagation in electric railway propulsion systems is important to the telephone engineer engaged in inductive coordination of railway and telephone facilities; it also is of interest, of course, to the railroad engineer because of its effect on network impedances and voltages between track and ground. The present paper presents a systematic treatment of current propagation in the tracks and associated conductors of electric railway propulsion systems. The tracks, being continuously in contact with the earth through the track ballast, allow current to leak into the earth, and track currents and voltages to ground vary along the tracks. The present treatment is limited to systems in which the tracks and other leaky conductors may be represented by a single conductor; for convenience the single conductor will be referred to as a track, with the understanding that it may be the representative of a number of tracks or of tracks with associated ground wires having frequent connections to tracks or ground, whenever this is permissible.

The general equations of propagation in a track in the presence of a conductor carrying a fixed current used herein have been known and in use for some time,[1] they are given in section II, together with a discussion of their interpretation and limitations. In section III they are used to develop the properties of a basic circuit consisting of a conductor of finite length connected to the track, which is continuous and of infinite length. By means of superposition complex systems of circuits may be built up of basic circuits and the currents in the conductors determined by the self and mutual impedances of basic circuits in conjunction with the impedance diagram of the remaining railway network. The current distribution in the track due to each conductor current is given by the basic circuit solution and the resultant may then be obtained by superposition. Track discontinuities may also be included by superposition and details are given for single series discontinuities formed by a series track impedance, and for double shunt discontinuities, formed by connecting impedances between track and ground. The latter includes the special case in which the track is terminated outside the conductor span by arbitrary impedances, giving in the two limits of zero and infinite terminal impedances, the cases of tracks solidly grounded and without grounds at the terminals. The self- and mutual-impedances of propulsion circuits involving track discontinuities are formulated and may be employed in the railway network diagram when discontinuities are involved. The analysis given is applicable to any railway system in which the tracks or tracks and associated ground wires having frequent connections to track or grounds may be represented by a single conductor. A description of a method of employing the basic circuit in determining cumulative induction curves which include the effect of distribution of current along the track and by means of which voltages in communication lines may be determined is given in section III.

The use of superposition is convenient for numerical work since curves and tables may be prepared applying to a variety of conditions, and since parts of a given system may be studied apart from the remainder and the effect of alterations in these parts determined without re-solving the whole system. It also simplifies the study of formal properties since any desired complexity may be secured from elementary circuits whose properties can thus be determined without loss of generality.

II. EQUATIONS OF PROPAGATION

The differential equations of propagation in a track 2 having an earth-return series impedance z_{22}, and a shunt admittance to ground g, in the presence of a second conductor 1 containing a fixed current in opposite direction to the track current, whose mutual impedance with earth return to the track with earth return is z_{12}, all impedances being per unit length, are as follows:

$$z_{22}I_2 - z_{12}J = -\frac{dV_2}{dx} \tag{1}$$

$$gV_2 = -\frac{dI_2}{dx}$$

where

J and I_2 = currents in conductor 1 and track, respectively

V_2 = track voltage to ground

x = distance along track in the direction of the track current.

The solutions of these equations may be written in the following form:

$$\begin{aligned} I_2 &= ae^{-\gamma x} - be^{\gamma x} + \mu J \\ V_2 &= kae^{-\gamma x} + kbe^{\gamma x} \end{aligned} \tag{2}$$

where

a,b = integration constant to be determined from boundary conditions

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[1] These equations were apparently first published by H. Pleijel as Part II of a report to the Swedish Royal Railway Administration "Undersokningar rorande Svagstromsstorningar vid med Einfasstrom drivna Electriska Baner," *A. B. Svenska Teknologforeningens*, Stockholm, 1919.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

k = characteristic impedance = $\sqrt{z_{22}/g}$

γ = propagation constant = $\sqrt{z_{22}g}$

μ = z_{12}/z_{22}

The differential equations (**1**) are the equations of classical transmission line theory and equations (**2**) are thus subject to the restrictions that the propagation constant γ be small compared to unity, in c.g.s. units, and that the circuit terminals be avoided; that is, the equations do not hold near the terminals. Even if these restrictions are maintained, the equations are not theoretically rigorous, since the voltage between points on the earth coextensive with the track is either ignored or supposed to vary linearly with V. It should also be noted that the impedance z_{22} of the track circuit contains a component dependent upon the rail permeability and thus upon the current density, which varies with length. The external component of z_{22} is dependent upon the circuit length as with any earth return circuit.

Thus, the use of the equations requires a justification apart from their theoretical background. For present purposes, it is sufficient to note that it has been found experimentally that the track current is propagated according to the exponential law in equations (**2**) within engineering accuracy. Equations (**2**) may be adopted, in the lack of more rigorous solutions, on the grounds of formal simplicity and sufficient accuracy for most engineering purposes.

The ratio μ, the propagation constant γ, and the characteristic impedance k depend on the impedances z_{12}, z_{22} and the shunt admittance g. Because of the semi-empirical character of the equations of propagation, values of μ, γ and k should be determined from measurements on a track or system of tracks in actual working position, suitably chosen mean values being taken if necessary; the values of z_{12}, z_{22} and g then follow from them, since $z_{22} = k\gamma$, $g = \gamma/k$, $z_{12} = \mu z_{22}$. It is outside the scope of this paper to discuss the measurement or calculation of these quantities; it may be added, however, that g is, in general, purely conductance (at low frequencies) and its range is from 0.5 to 2.0 mhos per mile per track. Estimates of z_{12} and z_{22} may be made from the configuration of the conductors and the size of the tracks, employing the formulas for self- and mutual-impedances of ground-return circuits,[2] and measured values of the rail constants.

It may be noted here that the integrals of track current and voltage with respect to x may be written from equations (**1**) as follows:

$$\int_{x_1}^{x_2} I_2 dx = \mu(x_2 - x_1)J + \frac{1}{z_{22}}(V_2(x_1) - V_2(x_2))$$

$$\int_{x_1}^{x_2} V_2 dx = g^{-1}(I_2(x_1) - I_2(x_2)) \qquad \textbf{(1a)}$$

III. The Basic Conductor-Track/Earth Circuit*

Derivation of Basic Circuit Properties

The circuit consisting of a conductor of length s carrying a current J and connected to a continuous infinite track is taken as the basic circuit since it involves the greatest degree of continuity in the track and is easily adapted to modifications for discontinuities, such as track terminations, track feeders, etc. The circuit is shown in Fig. 1.

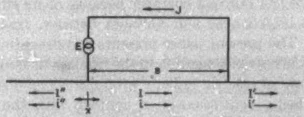

Fig. 1—Basic Conductor—Track/Earth Circuit

The track currents and voltages to ground in the three sections are taken as follows:

$$I = ae^{-\gamma x} - be^{\gamma x} + \mu J$$

$$V = kae^{-\gamma x} + kbe^{\gamma x} \qquad 0 \leqq x \leqq s \qquad \textbf{(3)}$$

$$V' = kI' = ka'e^{-\gamma(x-s)} \qquad s \leqq x \qquad \textbf{(4)}$$

$$V'' = kI'' = ka''e^{-\gamma|x|} \qquad 0 \geqq x \qquad \textbf{(5)}$$

The boundary conditions are the continuity of current and voltage at points 0 and s, that is:

$$\begin{aligned} J - I(o) &= I''(o) \qquad \textbf{(6)} \\ V(o) &= V''(o) \\ J - I(s) &= -I'(s) \\ V(s) &= V'(s) \end{aligned}$$

which when written out by means of equations (**3**), (**4**) and (**5**) are as follows:

$$\begin{aligned} (1-\mu)J - a + b &= a'' \qquad \textbf{(6a)} \\ a + b &= a'' \\ (1-\mu)J - ae^{-\gamma s} + be^{\gamma s} &= -a' \\ ae^{-\gamma s} + be^{\gamma s} &= a' \end{aligned}$$

The solutions of these equations are:

$$a = \frac{1}{2}(1-\mu)J \qquad \textbf{(7)}$$

$$b = -\frac{1}{2}(1-\mu)e^{-\gamma s}J = -ae^{-\gamma s}$$

$$a' = -a'' = -\frac{1}{2}(1-\mu)(1-e^{-\gamma s})J$$

All voltages, currents, and properties of the basic circuit may now be written out; the formulas are conveniently collected in Table I.

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[2] "Wave Propagation in Overhead Conductors with Ground Return," John R. Carson, *Bell System Technical Journal*, V. 5, October 1926, pp. 539-554. "Mutual Impedance of Grounded Wires Lying on the Surface of the Earth," Ronald M. Foster, *Bell System Technical Journal*, V. 10, pp. 408-419, July 1931.

*The dash (—) is used to denote series connection and the slant (/) parallel connection. Thus, the expression above means a circuit consisting of "conductor" as one side with return by track and earth in parallel. This notation will be followed throughout.

TABLE I—CURRENTS AND VOLTAGES IN BASIC CIRCUIT
(SEE FIG. 1)

Track Currents:

$$I = \mu J + \frac{1}{2}(1-\mu)\left(e^{-\gamma x} + e^{-\gamma(s-x)}\right) J$$

$$I' = -\frac{1}{2}(1-\mu)\left(1 - e^{-\gamma s}\right) e^{-\gamma(x-s)} J$$

$$I'' = \frac{1}{2}(1-\mu)\left(1 - e^{-\gamma s}\right) e^{-\gamma|x|} J$$

Earth Currents:

$$i = J - I = \frac{1}{2}(1-\mu)\left(2 - e^{-\gamma x} - e^{-\gamma(s-x)}\right) J$$

$$i' = -I'$$
$$i'' = -I''$$

Minimum and Average Track Current:

$$I\,min = \mu J + (1-\mu)\, e^{-\gamma s/2} J$$

$$\bar{I} = \frac{1}{s}\int_0^s I\,dx = \mu J + (1-\mu)\left(\frac{1-e^{-\gamma s}}{\gamma s}\right) J$$

Track Voltages to Ground:

$$V = \frac{1}{2}k(1-\mu)\left(e^{-\gamma x} - e^{-\gamma(s-x)}\right) J$$

$$V' = -\frac{1}{2}k(1-\mu)\left(1 - e^{-\gamma s}\right) e^{-\gamma(x-s)} J$$

$$V'' = \frac{1}{2}k(1-\mu)\left(1 - e^{-\gamma s}\right) e^{-\gamma|x|} J$$

Integrals of Track Currents:

$$\int_{x_1}^{x_2} I\,dx = \mu(x_2 - x_1) J + \frac{1}{z_{22}}[V(x_1) - V(x_2)]$$

$$\int_{x_1}^{x_2} I'\,dx = \frac{1}{z_{22}}[V'(x_1) - V'(x_2)]$$

$$\int_{x_1}^{x_2} I''\,dx = \frac{1}{z_{22}}[V''(x_1) - V''(x_2)]$$

$$\int_0^s I\,dx + \int_s^\infty I'\,dx - \int_0^\infty I''\,dx = \mu s J$$

Conductor—Track/Earth Impedance:
(Per Unit Length)

$$\begin{aligned} Z_{11} &= E/(sJ) \\ &= Z(1-\nu 2)(1-2) \\ &\qquad \left(\nu = \frac{1}{sJ}\int_0^s I\,dx\right) \\ &= z_{11} - z_{12} + \nu(z_{22} - z_{12}) \\ &= z_{11} - \mu z_{12} + z_{22}(1-\mu)^2\left(\frac{1-e^{-\gamma s}}{\gamma s}\right) \end{aligned}$$

$$\lim_{s\to 0} Z_{11} = z_{11} + z_{22} - 2z_{12}$$

$$\lim_{s\to\infty} Z_{11} = z_{11} - \mu z_{12} = z_{11} + z_{22} - 2z_{12} - (1-\mu)^2 z_{22}$$

Before discussing the formulas it may be noted that the track current may be resolved into three components (*i*) a constant current μJ in the section coextensive with the conductor (*ii*) exponentially propagated currents $ae^{-\gamma|x|}$ flowing each way from point 0, x being measured from this point (*iii*) exponentially propagated currents $(-be^{\gamma s})\,e^{-\gamma|y|}$ flowing each way from point s, y being measured from s. Since $a = -be^{\gamma s}$, components (*ii*) and (*iii*) are of equal maximum value. The resolution is suggested by the relations $a' = ae^{-\gamma s} + be^{\gamma s}$ and $a'' = a + b$ and by the form of the current equations. The current μJ is the limit approached by the track current as the section length is made infinite, the currents $ae^{-\gamma x}$ and $-be^{\gamma(s-x)}$ are the "direct" and "reflected" waves produced by the terminals. Another decomposition which is physically suggestive[3] is the division into component currents J introduced in the track at the conductor terminals, and a component driven by induction from the trolley. The former flow each way from the terminals propagated exponentially, that is, the components are

$$\frac{1}{2} Je^{-\gamma|x|} \text{ and } \frac{1}{2} Je^{-\gamma|y|}$$

where x and y are measured as above. The component driven by induction from the trolley is:

$$I = \frac{1}{2}\mu\left(2 - e^{-\gamma x} - e^{-\gamma(s-x)}\right) J$$

$$I' = \frac{1}{2}\mu\left(1 - e^{-\gamma s}\right) e^{-\gamma(x-s)} J$$

$$I'' = -\frac{1}{2}\mu\left(1 - e^{-\gamma s}\right) e^{-\gamma|x|} J$$

Table I gives the formulas for track currents, track voltages to ground, earth currents, minimum, average and integral track currents, maximum and average earth currents in the section spanned by the conductor, 0 to s, and finally the conductor-track/earth voltage and unit impedance.

The integral of track current between the limits $-\infty$ and $+\infty$ is $\mu s J$, since both direct and reflected waves, each of which flows in both directions as noted above, are cancelled over this range; the same value of the integral of track current is obtained as will be shown later when one or more track shunt discontinuities is present. The integral of track current over the same infinite region for any number of conductors individually connected to the track is the sum of their separate contributions, that is:

$$\int_{-\infty}^{\infty} I\,dx = \mu_1 s_1 J_1 + \mu_2 s_2 J_2 + \ldots\ldots + \mu_n s_n J_n$$

The conductor-track/earth unit impedance is derived from the following formula:

$$Z_{11} = z_{11} - z_{12}\frac{1}{sJ}\int_0^s I\,dx + z_{22}\frac{1}{sJ}\int_0^s I\,dx - z_{12} \qquad (8)$$
$$= z_{11} - z_{12} + (z_{22} - z_{12})\,\nu$$

---

[3] The first of the above resolutions of currents is given by H. Pleijel *loc cit.*; the second by E. R. Benda and H. Voightlander, "Der Spannungsabfall in Fahrleitungen und Schienen elektrischer Bahnen," *Wiss. Veroff. a. d. Siemens-Konzern*, 10, 78-90, who also note the first.

where

$$\nu = \frac{1}{sJ}\int_0^s I\,dx$$

This formula can be written in the more compact form

$$Z_{11} = Z_{12:12} = Z_{(1-\nu 2)(1-2)} \qquad \textbf{(8a)}$$

where the second line of (**8**) is the expanded form of (**8a**)[4]; the subscripts being obtained by multiplying the parenthesis algebraically with $z_{\nu jk}$ interpreted as νz_{jk}.

For the basic circuit the averaging factor ν is given by

$$\nu = \mu + (1-\mu)\left(\frac{1-e^{-\gamma s}}{\gamma s}\right) \qquad \textbf{(9)}$$

and

$$Z_{11} = z_{11} - \mu z_{12} + z_{22}(1-\mu)^2\left(\frac{1-e^{-\gamma s}}{\gamma s}\right) \qquad \textbf{(10)}$$

which has the following limits

$$\lim_{s=0} Z_{11} = z_{11} + z_{22} - 2z_{12} \qquad \textbf{(10a)}$$

$$\lim_{s=\infty} Z_{11} = z_{11} - \mu z_{12} = z_{11} + z_{22} - 2z_{12} - (1-\mu)^2 z_{22} \qquad \textbf{(10b)}$$

The first of the limits is physically evident as the impedance of a metallic loop; all of the conductor current returns in the track. In the second the track current is μ (for unit conductor current) and the track voltage drop is zero since $z_{22}I - z_{12}J = 0$, or, otherwise, since there is no leakage along the track. The second form of (**10b**) shows the maximum difference in Z_{11} due to change of s to be $(1-\mu)^2 z_{22}$.

In systematic calculation of currents, it is convenient to deal with earth currents; either quantity, track or earth current, determines the other: by subtraction from the conductor current within the conductor span, that is, $I + i = J$, or by reversing sign $(I + i = 0)$, outside the conductor span. The earth current itself and its average as dependent on x are each determined in terms of a single parameter, namely:

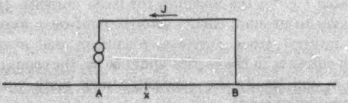
$$\alpha(x) = 1 - e^{-\gamma x} \qquad \text{– earth current} \qquad \textbf{(11)}$$

$$\beta(x) = \frac{1}{x}\int_0^x \alpha(x)dx = 1 - \frac{1-e^{-\gamma x}}{\gamma x} \text{ – average earth current} \qquad \textbf{(12)}$$

The current at any point is the sum of the products, each with proper sign, of two values of $\alpha(x)$ by $(1/2)(1-\mu)J$, as shown in Fig. 2. The formula for average currents in terms of $\beta(x)$ is shown in Fig. 3. The formula in Fig. 2 may be verified by inspection of the proper equations in Table I; the formula in Fig. 3 follows directly by application of the integral formula

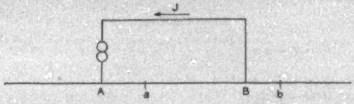
$$\beta(x) = \frac{1}{x}\int_0^x \alpha(x)\,dx$$

after performing the integration of the current in terms of $\alpha(x)$.



$$i(x) = \frac{1}{2}(1-\mu)\,[\alpha(\overline{Ax}) \pm \alpha(\overline{Bx})]\,J$$

Fig. 2—Earth Current in Terms of Parameter $\alpha(x)$; $\alpha(x) = 1 - e^{-\gamma x}$; Plus Sign in Formula when x is Between A and B, Minus Sign Otherwise; Distances $\overline{Ax}$ and $\overline{Bx}$ Positive



$$i = \frac{1}{\overline{ab}}\int_a^b i(x)dx$$

$$= \frac{1}{2\,\overline{ab}}(1-\mu)\,[\overline{Ab}\,\beta(\overline{Ab}) - \overline{Aa}\,\beta(\overline{Aa}) + \overline{Ba}\,\beta(\overline{Ba}) - \overline{Bb}\,\beta(\overline{Bb})]\,J$$

Fig. 3—Average Earth Current Over Distance $\overline{ab}$ in Terms of Parameter $\beta(x)$, $\beta(x) = 1 - (1 - e^{-\gamma x})/\gamma x$; All Distances, $\overline{Aa}$, $\overline{Ab}$, $\overline{Ba}$, $\overline{Bb}$ Positive; a and b Arbitrary Points on the Track, b to the Right of a

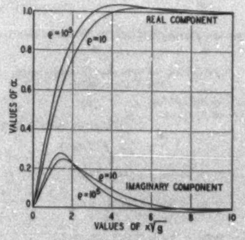


Fig. 4—Earth Current Parameter $\alpha(x)$ for Single Track Railroad, 130-Lb. Rails, as a Function of $x\sqrt{g}$, x Being Distance in Miles and g Conductance from Track to Ground in Mhos per Mile;] Earth Resistivity ρ in Meter-Ohms

Curves of $\alpha(x)$ and $\beta(x)$ for a 25-cycle single-track railroad are shown in Figs. 4 and 5; these curves apply for ground resistivities of 10 and 100,000 meter-ohms to a track having rails weighing 130 lb. per yard, the track impedances being respectively $0.177 + j\,0.453$ and $0.177 + j\,0.693$ ohm per mile.

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[4] This notation is a modification of a notation for Neumann integrals used by G. A. Campbell: "Mutual Impedance of Grounded Circuits," *Bell System Tech. Journal* 2, 1-30 (October 1923).

Mutual Impedance of Basic Circuits

The mutual impedance of basic circuits as shown in Fig. 6 is given by the following formula:

$$\begin{aligned} Z_{12:32} &= Z\,[s(1-\nu_1 2) - (s_3 - s)\,\nu_1' 2]\,(3-2) \\ &= Z(1-2)\,[s\,(3-\nu_3 2) + (s_1 - s)\,\nu_3' 2] \\ &= s\,(z_{13} - \mu_3 z_{12}) - \frac{1}{2}\,k\,(1-\mu_1)\,(1-\mu_3) \\ &\quad \times [e^{-\gamma s} + e^{\gamma s}\,(e^{-\gamma(s_1+s_3)} - e^{-\gamma s_1} - e^{-\gamma s_3})] \quad (13) \end{aligned}$$

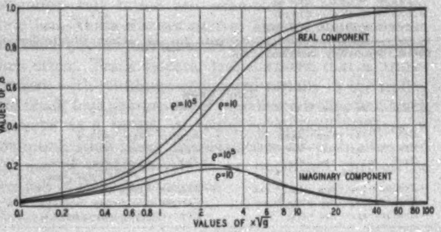

Fig. 5—Average Earth Current Parameter $\beta(x)$ as a Function of $x\sqrt{g}$, Conditions and Units as on Fig. 4

where

s_1, s_3 and s = lengths of wires 1, 3 and their overlapping portion, respectively.

$\nu_1, \nu_1', \nu_3, \nu_3'$ = average unit track currents over sections s, $s_3 - s$ with wire 1 energized and over sections s and $s_1 - s$ with wire 3 energized, respectively.

$$\mu_1 = z_{12}/z_{22},\ \mu_3 = z_{23}/z_{22}$$

The first line of equations (**13**) gives the mutual impedance with wire 1 energized, the second with wire 3 energized; their equality follows from the reciprocal theorem, that is, $Z_{12:32} = Z_{32:12}$.

The mutual impedances of the non-coextensive portions of the wires are not included in equations (**13**) on the assumption that they are negligibly small; this is consistent with the usual practise of using limiting values for infinite length of wire-earth self- and mutual-impedances.

The current direction is the same in each wire and the voltage drop in one section of zero current, which with unit current in the other section gives the mutual impedance of the circuits, is taken positive when it is in the positive sense defined for the wire currents.

This general formula reduces to equation (**8**) or its alternates (**8a**) and (**10**) when the wires are coextensive, $s_1 = s_3 = s$, and $3 = 1$, the result being expressed per unit length. The general formula for non-overlapping circuits of lengths s_1 and s_3 separated by distance s is as follows:

$$Z_{12:32} = -\frac{1}{2}\,k(1-\mu_1)(1-\mu_3)(1-e^{-\gamma s_1})(1-e^{-\gamma s_3})e^{-\gamma s} \quad (14)$$

The general formula for circuits of lengths s_1 and s_3, such that the former spans the latter with distance s between the terminals on the left, is as follows:

$$\begin{aligned} Z_{12:32} &= s_3\,(z_{13} - \mu_1 z_{23}) + \frac{1}{2}\,k\,(1-\mu_1)\,(1-\mu_3) \\ &\quad \times (e^{-\gamma s} - e^{-\gamma(s_1-s)} - e^{-\gamma(s+s_3)} + e^{-\gamma(s_1-s-s_3)}) \quad (15) \end{aligned}$$

The mutual impedance of metallic circuits in the presence of the track may be derived from the above results by making up the energizing circuit of two circuits, one from each of the wires of the metallic loop, with track return and equal and opposite currents. An example of mutual impedances involving metallic circuits will be given later in connection with the three-wire system of electrification.

Application of the Basic Circuit

General. The basic circuit can be applied directly by means of superposition to building up systems of circuits in which the track is continuous and infinite; or, conversely, any system of circuits involving continuous, infinite track may be regarded as made up of basic circuits. The propulsion circuit impedances for use in railway network impedance diagrams may be determined in this way, the basic circuits being made sections of the trolley in which the current has one value. Examples will be given in a section following. The corresponding impedances when the track has discontinuities are also given later, in connection with track discontinuities. A basic circuit of length s may also be subdivided into n subcircuits of length s/n, since the currents flowing in the vertical connections between conductor and track will be in opposite direction in adjacent sections, and all vertical connections except the terminal ones carry no current and may thus be removed or retained as desired. This process is employed in the construction of a cumulative induction curve for communication lines, to be described below.

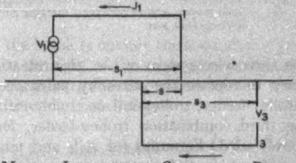

Fig. 6—Mutual Impedance of Overlapping Basic Circuits

Propulsion Circuit Impedances in Railway Network Diagrams. In cases involving single conductors or single-conductor equivalents of multiple conductors with track return, the formulas for self- and mutual-impedance of basic circuits given above may be employed directly in determining the propulsion circuit impedances, the sections being made up as described above. It may be noted that the mutual impedance of adjacent, *i.e.*, non-coextensive, basic circuits is generally small and possibly negligible. When a number of outgoing and return conductors are involved, the propulsion circuit impedances may be obtained by superposition of the component basic circuits energized as they are to be connected in the network. An example is shown in Fig. 7 in which there are two outgoing and two return coextensive conductors. Fig. 7 also shows the simultaneous equations in terms of the self- and mutual-basic circuit impedances by means of which conductor currents may be determined, when the system is energized as shown. The self-impedance of the system per unit length is $V \div s\,(J_0 + J_1)$; the mutual impedance with any other system of conductors may be determined from the currents and component mutual basic circuit impedances, the total system current $J_0 + J_1$ being set equal to unity.

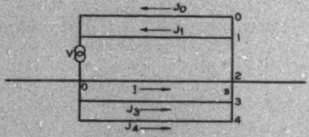

FIG. 7—A SYSTEM OF COEXTENSIVE BASIC CIRCUITS

$$
\begin{aligned}
Z_{00}J_0 + Z_{01}J_1 - Z_{03}J_3 - Z_{04}J_4 &= s^{-1}\,V\\
Z_{01}J_0 + Z_{11}J_1 - Z_{13}J_3 - Z_{14}J_4 &= s^{-1}\,V\\
-Z_{03}J_0 - Z_{13}J_1 + Z_{33}J_3 + Z_{34}J_4 &= 0\\
-Z_{04}J_0 - Z_{14}J_1 + Z_{34}J_3 + Z_{44}J_4 &= 0
\end{aligned}
$$

where

$$
\begin{aligned}
Z_{jk} &= z_{jk} - z_{j2} + \nu_j\,(z_{22} - z_{k2}) \quad (j, k = 0, 1, 3, 4)\\
&= z_{jk} - z_{k2} + \nu_k\,(z_{22} - z_{j2})\\
\nu_j &= \frac{1}{sJ_j}\int_0^s I_{2j}\,dx
\end{aligned}
$$

I_{2j} = track current with conductor j carrying current J_j, all other currents zero

In the three-wire system of a-c. electrification, it is customary to replace the wires by pairs of circuits, either trolley-feeder, trolley-rail or trolley-rail, feeder-rail, the third combination trolley-feeder, feeder-rail being rarely used. Formulas for self- and mutual-impedance per unit length for coextensive portions of these circuits are shown in Table II; these formulas do not include the voltage ratios necessary because of the different working voltages of the circuits. The relations of the impedances are also shown; of the six impedances, only three are independent.

Basic Circuit Induction Curves. Cumulative induction curves are ordinarily used in the calculation of induced voltages in communication lines affected by a railroad electrification. These curves usually give the continuous sum of the induced voltages in elements of the communication line due to unit current in a railway circuit with earth-return and thus the cumulated voltage between any points on the line. In this form they are adapted to cases in which current in the railway circuit is constant along the circuit length; the variation of track current may be taken into account approximately by assuming the average track current constant over the length of track involved.

TABLE II—PROPULSION CIRCUIT IMPEDANCES, THREE-WIRE SYSTEM

$$
\begin{aligned}
Z_{TR} &= Z_{(T-\nu_T R)(T-R)} && = z_{TT} - z_{TR} + \nu_T\,(z_{RR} - z_{TR})\\
Z_{FR} &= Z_{(F-\nu_F R)(F-R)} && = z_{FF} - z_{FR} + \nu_F\,(z_{RR} - z_{FR})\\
Z_{TF} &= Z_{(T-\nu_T R-F+\nu_F R)(T-F)} && = z_{TT} + z_{FF} - 2z_{TF}\\
&&& \quad - (\nu_T - \nu_F)\,(z_{TR} - z_{FR})\\
-Z_{TR:RF} &= Z_{(T-\nu_T R)(F-R)} && = z_{TF} - z_{TR} + \nu_T\,(z_{RR} - z_{FR})\\
&= Z_{(T-R)(F-\nu_F R)} && = z_{TF} - z_{FR} + \nu_F\,(z_{RR} - z_{TR})\\
Z_{TR:TF} &= Z_{(T-\nu_T R)(T-F)} && = z_{TT} - z_{TF} - \nu_T\,(z_{TR} - z_{FR})\\
&= Z_{(T-R)(T-\nu_T R-F+\nu_F R)} && = z_{TT} - z_{TF} - z_{TR} + z_{FR}\\
&&& \quad + (\nu_T - \nu_F)\,(z_{RR} - z_{TR})\\
-Z_{FR:TF} &= -Z_{(F-\nu_F R)(T-F)} && = z_{FF} - z_{TF} + \nu_F\,(z_{TR} - z_{FR})\\
&= -Z_{(F-R)(T-\nu_T R-F+\nu_F R)} && = z_{FF} - z_{TF} + z_{TR} - z_{FR}\\
&&& \quad - (\nu_T - \nu_F)\,(z_{RR} - z_{FR})
\end{aligned}
$$

$$
\begin{aligned}
Z_{TR:TF} &= Z_{TR} + Z_{TR:RF}\\
-Z_{FR:TF} &= Z_{FR} + Z_{TR:RF}\\
Z_{TF} &= Z_{TR:TF} - Z_{FR:TF} && = Z_{TR} + Z_{FR} + 2Z_{TR:RF}
\end{aligned}
$$

ν_T and ν_F are average unit track currents (for unit section) with trolley and feeder, respectively, energized alone.

Limit Cases

$\nu_T = \nu_F = 1$		$\nu_T = \mu_T,\ \nu_F = \mu_F$	
Z_{TR}	$= z_{TT} + z_{RR} - 2z_{TR}$	Z_{TR}	$= z_{TT} - \mu_T z_{TR}$
Z_{FR}	$= z_{FF} + z_{RR} - 2z_{FR}$	Z_{FR}	$= z_{FF} - \mu_F z_{FR}$
Z_{TF}	$= z_{TT} + z_{FF} - 2z_{TF}$	Z_{TF}	$= z_{TT} + z_{FF} - 2z_{TF}$ $- (\mu_T - \mu_F)^2\, z_{RR}$
$-Z_{TR:RF}$	$= z_{TF} - z_{FR} + z_{RR} - z_{TR}$	$-Z_{TR:RF}$	$= z_{TF} - \mu_T z_{FR}$
$Z_{TR:TF}$	$= z_{TT} - z_{TF} - z_{TR} + z_{FR}$	$Z_{TR:TF}$	$= z_{TT} - z_{TF} - \mu_T(z_{TR} - z_{FR})$
$-Z_{FR:TF}$	$= z_{FF} - z_{TF} + z_{TR} - z_{FR}$	$-Z_{FR:TF}$	$= z_{FF} - z_{TF} + \mu_F(z_{TR} - z_{FR})$

The basic circuit induction curve includes the effect of the variation of track current with length and is found by calculating the induced voltage in a given communication line taken as a whole (except, of course, for parts beyond the locality of the exposure at distances such that the induced voltage is negligible) for selected positions of a basic circuit of unit length carrying unit conductor (trolley) current. The results of these calculations are plotted with the position of the basic circuit represented by its midpoint as abscissa. The integral of the resulting curve between two points on the axis of abscissas then gives the induced voltage in the telephone line for unit conductor current between the two points, exactly as with the usual cumulative induction curve. This procedure rests on the fact noted above that a basic circuit of length s may be subdivided into n circuits of length s/n.

It should be pointed out that the induction coefficients on which this curve is based should include the effects of ground potential; the study of ground potentials is not at present sufficiently developed for this purpose and the voltages calculated may thus be less than the actual voltages. The curve is of greatest use and importance when there is a number of loads or short circuits to be considered. When there are relatively few loads and a number of exposures, the preparation of these curves may be more difficult than a direct use of average earth currents prepared for each situation it is desired to consider, and taken from curves of the average earth current parameter β, combined with a cumulative induction curve per unit trolley-earth current.

IV. TRACK DISCONTINUITIES

General

Track discontinuities, in general, are of two kinds, "series," in which an impedance is connected serially in the track, or "shunt," in which the impedance is connected between track and ground. The most common example of a series discontinuity is finiteness of track length, that is, connection of infinite series impedances; grounded rail buses are examples of shunt discontinuities. These discontinuities may be superposed on the basic circuit just as basic circuits are superposed on each other. Track booster transformers, that is, transformers with windings connected serially in the trolley and track for the purpose of equalizing trolley and track currents as a means of reducing induced voltages in communication lines, are also series discontinuities and the use of superposition is an important part of the analysis of their performance.

Series Discontinuities

Fig. 8 herewith shows a series discontinuity at point s_1 within the conductor span; for generality, the discontinuity is represented as a generator. The conductor current is taken from right to left, and the track current through the additional generator following the convention used before is in opposite direction. The coordinate system for track currents produced by the conductor current starts at the generator point with variable x; for the track generator the variable is y with zero at the generator point. The simultaneous equations relating the voltages and currents of the generators are as follows:

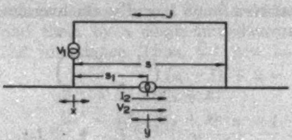

FIG. 8—CONDUCTOR—TRACK/EARTH CIRCUIT WITH TRACK SERIES DISCONTINUITY WITHIN CONDUCTOR SPAN

$$\begin{aligned} Z_{11}'J + Z_{12}I_2 &= V_1 \\ Z_{12}J + Z_{22}I_2 &= V_2 \end{aligned} \qquad (16)$$

The impedances in these equations are defined as follows: Z_{11}' is the impedance of the conductor-track/earth circuit with the track broken at s_1 ($I_2 = 0$); Z_{12} is the mutual impedance of conductor-track/earth and track-earth circuits, that is, the voltage drop across a break in the track at s_1 ($I_2 = 0$) in the direction of I_2 with J unity, or the voltage drop across a break in the trolley ($J = 0$) with I_2 unity; Z_{22} is the self-impedance of the infinite track-earth circuit, as looked into from the break at s_1.

Since with zero trolley current the track current is propagated exponentially in each direction Z_{22} and Z_{12} from the first definition may be written down directly as follows:

$$Z_{22} = 2k$$

$$\begin{aligned} Z_{12} &= (z_{22} - z_{12}) \left\{ \int_0^{s_1} e^{-\gamma y}\,dy + \int_0^{s-s_1} e^{-\gamma y}\,dy \right\} - Z_{22} \\ &= k\,(1-\mu)\,(2 - e^{-\gamma s_1} - e^{-\gamma(s-s_1)}) - 2k \end{aligned}$$

It may be noted that the formulation of this impedance is easier than that corresponding to the second definition to which it is equal by the reciprocal theorem.

Z_{11}' may be formulated indirectly as follows. With $V_2 = 0$, the basic circuit is restored and

$$\begin{aligned} I_2 &= -Z_{12}J/Z_{22} \\ &= \left[\mu + \frac{1}{2}\,(1-\mu)\,(e^{-\gamma s_1} + e^{-\gamma(s-s_1)}) \right] J \end{aligned}$$

which agrees with the basic circuit track current as given in Table I. Conversely, the product of the known basic circuit track current by the track-earth series impedance gives the mutual impedance Z_{12} with sign reversed and is a third means of finding this quantity.

Z_{11}' is now given by the first equation of (**16**), since with $V_2 = 0$ and hence $I_2 = -Z_{12}J/Z_{22}$, $V_1 = Z_{11}J$ where Z_{11} is the basic circuit conductor-track/earth self-impedance. The result is

$$\begin{aligned} Z_{11}' &= Z_{11} + Z_{12}^2/Z_{22} \\ &= s \left\{ z_{11} - \mu z_{12} + z_{22}\frac{(1-\mu)^2\,(1-e^{-\gamma s})}{\gamma s} \right. \\ &\qquad \left. + z_{22}\,\frac{[2\mu + (1-\mu)\,(e^{-\gamma s_1} + e^{-\gamma(s-s_1)})]^2}{2\gamma s} \right\} \end{aligned} \qquad (\mathbf{17})$$

When the break is outside the conductor span at distance s_1 to the right of the origin of coordinates for the conductor current, the impedance of the conductor-track/earth circuit is as follows:

$$\begin{aligned} Z_{11}' = s &\left\{ z_{11} - \mu z_{12} + z_{22}\,(1-\mu)^2 \left(\frac{1-e^{-\gamma s}}{\gamma s} \right) \right. \\ &\left. \times \left[1 + \frac{1}{2}\,e^{-2\gamma(s_1-s)}(1-e^{-\gamma s}) \right] \right\} \end{aligned} \qquad (\mathbf{18})$$

The mutual impedance of collinear conductor-track/earth circuits, one of which contains a track break within the conductor span as shown on Fig. 9 is as follows:

$$\begin{aligned} Z'_{12:32} = -\frac{1}{2}\,k(1-\mu_3)(1-e^{-\gamma s_3})e^{-\gamma s} \{ & (1-\mu_1)(1-e^{-\gamma s_1}) \\ & + e^{-\gamma(s_1-x)}\,[\,2\mu_1 + (1-\mu_1)\,(e^{-\gamma x} + e^{-\gamma(s_1-x)})\,]\} \end{aligned} \qquad (\mathbf{19})$$

These impedances may be used in the propulsion circuit network diagram in problems involving a track break.

The second equation of (**16**) is the equation of voltage across the track break. With $I_2 = J$, that is, with zero earth current at the track generator point, $V_2 = (Z_{12} + Z_{22})\,J$, which is the voltage to be supplied by a perfect track booster transformer, that is, a transformer with zero exciting current. The voltage to be supplied by an actual transformer is a function of its exciting current, say $f\,(I_x)$, and

$$V_2 = Z_{12}J + Z_{22}I_2 = f\,(I_x) \qquad \textbf{(20)}$$

an equation of fundamental importance in the treatment of track booster transformers, since it is essential to finding the exciting current. With a unity ratio transformer, the exciting current is approximately equal to the earth current at the transformer point, that is $I_x = -\,I_2$, the impedance to flow of exciting current in the conductor winding usually being quite large. Although this equation cannot be solved formally without an explicit algebraic expression for $f\,(I_x)$ the solution may be carried out graphically or numerically when $f\,(I_x)$ is specified over a sufficient range.

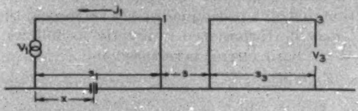

Fig. 9—Mutual Impedance of Conductor—Track/Earth Circuits with Track Break

The substitution $I_2 = I_2' - Z_{12}J/Z_{22}$ in equation (**16**) gives the equations in which the track generator current is superposed on the basic circuit; which are:

$$\begin{aligned} Z_{11}J + Z_{12}I_2' &= V_1 \qquad \textbf{(16a)}\\ Z_{22}I_2' &= V_2 \end{aligned}$$

where $Z_{11} = Z_{11}' - Z_{12}^2/Z_{22} =$ basic circuit self impedance. These equations show that the voltage of the track generator superposed on the basic circuit may be altered independently of the conductor current, and is thus at disposal to satisfy boundary conditions imposed by a discontinuity at a given point. This is a general result holding for more than one discontinuity of either kind and offers a convenient means of formulating the effects of a number of discontinuities; an example will be given in the case of two shunt discontinuities which follows.

Shunt Discontinuities

Fig. 10 shows two shunt discontinuities consisting of impedances Z_1 and Z_2 connected to ground at s_1 and s_2, respectively. The currents $2I_1$ and $2I_2$ which are to be superposed on the basic circuit are shown flowing from ground through the impedances Z_1 and Z_2, respectively, on to the track where they divide, half in each direction. On the track the currents are propagated exponentially. The equations determining these currents are the voltage equations at the two points, that is, the voltages between track and ground required by the currents I_1 and I_2 must be met by the voltages existing in the basic circuit. The equations are as follows:

$$-\frac{1}{2}\,kJ\,(1-\mu)\,(1-e^{-\gamma s})\,e^{-\gamma(s_1-s)} + kI_1 + kI_2e^{-\gamma(s_1+s_2)} = -\,2Z_1I_1 \qquad \textbf{(21)}$$

$$\frac{1}{2}\,kJ\,(1-\mu)\,(1-e^{-\gamma s})\,e^{-\gamma s_2} + kI_1e^{-\gamma(s_1+s_2)} + kI_2 = -\,2Z_2I_2$$

from which

$$I_1 = \frac{1}{2}\,kJ\,(1-\mu)\,(1-e^{-\gamma s})\;\frac{e^{-\gamma(s_1-s)}(2Z_2+k) + ke^{-\gamma(s_1+2s_2)}}{(2Z_1+k)\,(2Z_2+k) - k^2e^{-2\gamma(s_1+s_2)}} \qquad \textbf{(22)}$$

$$I_2 = -\,\frac{1}{2}\,kJ\,(1-\mu)\,(1-e^{-\gamma s})\;\frac{e^{-\gamma s_2}\,(2Z_1+k) + ke^{-\gamma(-s+2s_1+s_2)}}{(2Z_1+k)(2Z_2+k) - k^2e^{-2\gamma(s_1+s_2)}}$$

I_1 and $-\,I_2$ are opposite in direction to the basic track current between 0 and s, and so the average unit track current is:

$$\nu = \frac{\bar{I}}{J} = \mu + (1-\mu)\left(\frac{1-e^{-\gamma s}}{\gamma s}\right) - \frac{1-e^{-\gamma s}}{\gamma sJ}\,[e^{-\gamma(s_1-s)}\,I_1 - e^{-\gamma s_2}\,I_2] \qquad \textbf{(23)}$$

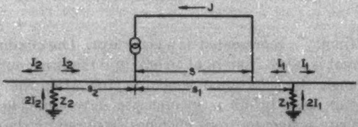

Fig. 10—Conductor—Track/Earth Circuit with Two Arbitrary Impedances Connected to Ground Outside the Conductor Span

In the limiting case $Z_1 = Z_2 = \infty$, both currents I_1 and I_2 vanish, as they should, leaving the basic circuit unaltered. In the limiting case $Z_1 = Z_2 = 0$:

$$I_1 = \frac{1}{2}\,J\,(1-\mu)\,(1-e^{-\gamma s})\,e^{-\gamma(s_1-s)}\;\frac{1+e^{-\gamma(s+2s_2)}}{1-e^{-2\gamma(s_1+s_2)}}$$

$$I_2 = -\,\frac{1}{2}\,J\,(1-\mu)\,(1-e^{-\gamma s})\,e^{-\gamma s_2}\;\frac{1+e^{-\gamma(-s+2s_1)}}{1-e^{-2\gamma(s_1+s_2)}}$$

The corresponding physical circuit consists of solid grounds at track terminals at s_1 and s_2, since the tracks beyond s_1 and s_2 where Z_1 and Z_2 are connected carry no current and may be removed; this is apparent physically since the voltages to ground at s_1 and s_2 are reduced to zero, leaving no driving voltage for current beyond; it also follows from the identities

$$I_1 + I_2 e^{-\gamma(s_1+s_2)} = \frac{1}{2} J (1-\mu)(1-e^{-\gamma s}) e^{-\gamma(s_1-s)}$$

$$I_1 e^{-\gamma(s_1+s_2)} + I_2 = -\frac{1}{2} J (1-\mu)(1-e^{-\gamma s}) e^{-\gamma s_2}$$

which are equal and opposite to the basic circuit currents at s_1 and s_2, respectively, thereby canceling them, and therefore the currents beyond.

The integral track current between the limits $-\infty$ and $+\infty$ is $\mu s J$ as with the basic circuit since the superposed currents flow each way from points s_1 and s_2, just as the direct and reflected waves in the basic circuit. This holds also for the particular case $Z_1 = Z_2 = 0$ in which the tracks are terminated and solidly grounded at s_1 and s_2, as described above.

A further particular case of the general type of shunt discontinuity consists of track termination through arbitrary impedances, which may represent sections of track of arbitrary characteristic impedance and propagation constant as well as a single lumped impedance. This case may be derived from the case above by replacing the impedances Z_1 and Z_2 and the sections of track beyond them by a single impedance, which is their parallel impedance. Thus, the new impedances defined by

$$Z_1' = kZ_1/(Z_1 + k)$$
$$Z_2' = kZ_2/(Z_2 + k)$$

are the arbitrary terminal impedances at s_1 and s_2, respectively. The substitution required in equations (**22**), dropping subscripts, is

$$\frac{2Z + k}{k} = \frac{k + Z'}{k - Z'}$$

The limit $Z_1' = Z_2' = \infty$ gives the case of a track opened at s_1 and s_2 with the following results:

$$I_1 = -\frac{1}{2} J (1-\mu)(1-e^{-\gamma s}) e^{-\gamma(s_1-s)} \frac{1 - e^{-\gamma(s+2s_2)}}{1 - e^{-2\gamma(s_1+s_2)}}$$

$$I_2 = \frac{1}{2} J (1-\mu)(1-e^{-\gamma s}) e^{-\gamma s_1} \frac{1 - e^{-\gamma(2s_1-s)}}{1 - e^{-2\gamma(s_1+s_2)}} \tag{22a}$$

ACKNOWLEDGMENT

The writer's introduction to the subject came through a study of the early work of Messrs. A. E. Bowen and John R. Carson, and was extended during study of the theory of track and feeder booster transformers developed by Mr. Carson. The final systematization was made under the supervision of Mr. K. L. Maurer. Mr. Erling D. Sunde collaborated in the study of the basic circuit without discontinuities and contributed a number of important suggestions: the compact formula for circuit impedances and the introduction of the earth current parameters α and β are the results of his suggestions, and he and Mr. King E. Gould independently worked out the basic circuit cumulative induction curve in more detail than could be given here. Mr. Howard M. Trueblood first suggested the equation determining track booster transformer exciting current and his critical review of the present paper has resulted in a number of improvements.

Discussion

Keith L. Maurer: The peculiar feature of the type of circuit treated in Mr. Riordan's paper, which distinguishes it from the ordinary power circuit, is that one side is made up of a concentrated metallic conductor in parallel with a distributed conductor, the earth. The effect of the earth as a conductor upon the electrical constants of this type of circuit cannot be neglected as it ordinarily may be in the case of the usual power circuit. Examples of the distributed conductor circuit are the electric railway propulsion circuit discussed in the paper, power or communication circuits in cables with grounded metallic sheaths, and zero-sequence networks of power circuits with ground wires.

As mentioned in the paper, the applicability of transmission line theory to the railway propulsion circuit has been known for some time. However, relatively little use seems to have been made of the equations of propagation in practical work due, no doubt, to the fact that their straightforward application is complicated and laborious. The usual procedure has been to use simplified forms of the general equations obtained by assuming great circuit length. For example, the percentage division of current between the return system and earth for use in determining the self-impedance of a trolley-track circuit or the mutual impedance between such a circuit and exposed communication lines is ordinarily assumed to be independent of circuit length, and the value obtained from the infinite circuit length formula is used. This approximation may be sufficiently good for certain kinds of practical problems involving propulsion systems. Other types of problems, notably those involving the tracks by themselves, often require more detailed consideration of the distribution of current in the system.

The basic circuit treatment described by Mr. Riordan is a convenient method for handling problems of the latter kind. Much of the complication of the straightforward use of the equations of propagation is avoided since these equations are employed only to determine the properties of the basic circuit, after which the procedure is by superposition. The method has been found particularly convenient in studies of the design and performance of booster transformer systems; also in calculations of shielding from grounded cable sheaths in exposures to propulsion systems in which two-directional current feeds were involved. The property of the method which permits portions of a system to be studied apart from the whole, has been found convenient in coordination studies with electrifications yet in the design stage when frequent alterations are made as the design progresses.

With regard to experimental justification for the use of the equations of propagation for electric railway propulsion systems, tests both in this country and abroad have shown a degree of conformity which is perhaps surprising in view of the departures from smooth-line conditions which can be expected in the usual electrification. In a test made several years ago in this country on a four-track electrification the observed current differed from that calculated using smooth-line formulas by about 10 per cent in the current at any point, and by less than one per cent in the average current. In this case the energized overhead wire was eight miles long and the track was continuous for at least eight miles beyond the ends of the overhead wire.

The Boric Acid Fuse

BY A. P. STROM* and H. L. RAWLINS†
Associate, A.I.E.E. Associate, A.I.E.E.

Synopsis.—*Recognition of the part played by gas blasts in interrupting arcs in oil circuit breakers and expulsion fuses has opened new avenues for improvements in these devices. The present paper deals with a new type of fuse in which greatly improved performance is obtained by more effective use of the self-generated gas blast.*

In this fuse a form of construction is used which permits the interruption of the smaller ranges of current within a device that is capable of interrupting also heavy short circuits up to 20,000 amperes or more. A special feature of this design is that the lower ranges of current are automatically transferred to, and interrupted in, a small auxiliary fuse bore.

While the construction may be used to advantage with any gas generating material, the present design embodies a compressed boric acid liner which has been found to be much more effective than fiber in extinguishing arcs.

Since boric acid evolves water vapor, a condensible gas, it becomes possible to construct a totally enclosed fuse in which the gas blast is discharged into a surface condenser. With such construction, currents of from 5 to 20,000 amperes at 13,000 volts have been interrupted with an electrode separation of less than six inches, using a relatively small condenser. Currents of more than 20,000 amperes have been interrupted by similar fuses of open type. This type of fuse may readily be designed for various current and voltage ratings.

* * * * *

I. INTRODUCTION

THE value of gas blasts acting upon the electric arc, as a means of circuit interruption, has been well recognized for some time. Many investigators have shown that such blasts are extremely effective whether they originate from a high pressure source of gas or from the decomposition of insulating materials adjacent to the arc.[1,2] When it was recognized that such self-generated blasts play an important role in the operation of oil circuit breakers, new avenues were opened for improvements in this type of apparatus, the status of which had for many years remained substantially the same. This resulted in the development of the deion grid oil circuit breaker which by utilizing these gas blasts more effectively gives greatly superior performance when compared with conventional oil breakers.[3,4]

That the expulsion fuse is also dependent upon self-generated gas blasts is now well appreciated, many investigations having been made to determine the nature of these gas blasts as they apply in the extinction of arcs in fuses.[2,5,6] These investigations have quite naturally suggested ways and means by which devices of this kind might be improved.

The present paper deals with an improved type of expulsion fuse, the development of which is the outgrowth of these investigations. In this fuse the portions exposed to the arc consist of compressed boric acid which serves as the gas-generating material. In addition to this new gas-generating material the fuse employs a construction which differs radically from the conventional expulsion fuse. This construction utilizes more effectively the principles that have been found to apply in gas blast devices and may be used to advantage with any gas-generating material.

II. EFFECT OF FORM AND MATERIAL ON CHARACTERISTICS OF EXPULSION FUSES

The principles governing the action of gas blast circuit interrupters have been treated quite fully in other publications,[5,1,2,6] and it is proposed to give herein only such a brief review of these characteristics as may be required to readily understand the present development.

The circuit volts per inch of fuse length, at a given current, that may be interrupted by an expulsion fuse are dependent upon the length and diameter of the fuse tube and also upon the material with which the arc comes in contact. Other factors remaining the same, lengthening the tube increases the volts per inch that may be interrupted, since the arc comes in contact with a greater length of tube, liberating more gas which, in turn, is effective throughout a greater length of arc. Similarly, decreasing the bore will increase the volts per inch that may be interrupted since the arc will be in more intimate contact with the tube, thereby causing a greater gas evolution as well as a more concentrated blast. A typical curve, showing how the circuit volts per inch interrupted vary with the tube diameter for one particular case, is shown in Fig. 1. This curve has been reproduced from the paper *The Expulsion Fuse*, by Slepian and Denault.[5]

The material, of which the fuse tube walls are composed, determines the magnitude and nature of the gas blast for given tube dimensions and arc current. Therefore, since the deionizing action upon the arc depends upon both the quantity and kind of gas in the blast, it is evident that the material of the fuse wall may further act to vary the volts per inch that may be interrupted. The material of the fuse wall will also determine the nature of the gases expelled into the atmosphere adjacent to the fuse. Organic materials, such as fiber, will liberate mainly hydrogen and carbon monoxide,

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1. For references see end of paper.

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both of which are inflammable and explosive as well as non-condensible. Such gases also have the property of remaining at low dielectric strength for appreciable lengths of time after expulsion from the fuse, and are, therefore, liable to cause breakdown between live parts in the vicinity of the arc. Gases evolved from inorganic materials may be non-inflammable, and in numerous cases possess the additional advantage that they may be condensed to liquid form, at moderate pressures, a property that can be utilized to great advantage in the construction of totally enclosed fuses. Experiments have further shown that arc gases from such of these materials as give off water vapor are much less liable to cause breakdown between adjacent live parts than gases evolved from organic materials.

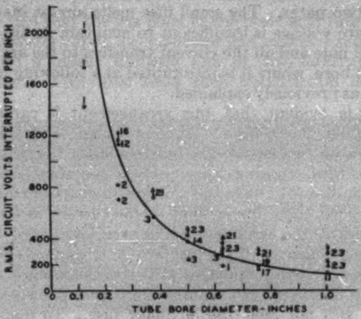


FIG. 1—FIBER EXPULSION FUSE

Effect of varying tube diameter on the circuit volts interrupted at a constant current of 450 amperes. (Numbers indicate half cycles of arcing. Arrows indicate that backup breaker cleared the circuit)

Many inorganic materials have been tested to determine the effectiveness of their gas blasts when used in arc extinguishing devices. While several of these materials have been found to be comparable or superior to fiber, there is one, namely, boric acid whose arc extinguishing properties have been found to be outstanding. This material when used as an expulsion fuse liner will interrupt a much higher voltage than a horn fiber fuse of similar dimensions. Boric acid is readily compressed into hard, firm blocks of good mechanical strength that may be used to line fiber or other insulating fuse tubes.

A comparison of operating characteristics for horn fiber and boric acid fuse tube liners, using a plain expulsion fuse four inches in length and five-eighths inch internal diameter is shown in Fig. 2. These curves which are based on interruptions occurring in ten half-cycles or less, as explained in a previous paper,[5] show that under similar conditions the boric acid fuse is capable of interrupting 660 volts per inch, or more, over the entire current range, as compared to 400 volts per inch, or more, for a fiber fuse.

This set of curves illustrates further the typical voltage-current characteristic of gas blast devices. For very small currents a high voltage can be interrupted since the arc is of very small cross-section and is quickly deionized at a current zero. For somewhat larger currents the arc section increases, yet does not completely fill the tube, so as to produce a vigorous gas blast. The rate of deionization at a current zero is, therefore, much slower than for the smaller currents, thereby creating a range of currents, the value depending mainly upon the diameter of the tube, at which the voltage that can be interrupted decreases and reaches a minimum. Beyond this range the gas blast becomes more and more intense and the voltage that can be interrupted rises very rapidly with increasing current.

III. A NEW FUSE DESIGN UTILIZING THE GAS BLAST MORE EFFECTIVELY

To employ to greater advantage the characteristics of gas blast devices as discussed above in Section II, a particular form of fuse has been designed which is radically different from any previous fuse construction. A sectional view of this fuse is shown in Fig. 3. Essentially, this consists of a short fusible element that links the lower stationary electrode to a spring actuated movable plunger, which upon melting of the fusible element is withdrawn through the boric acid tube. In order to extinguish the lower ranges of current the main fuse bore has been made as small as possible, the cross-section of the hole being only sufficient to provide proper clearance for the plunger designed to carry rated current without appreciable heating. The fusible element, instead of extending throughout the whole length of the fuse tube, is located at one end of the tube and is relatively short. This end of the tube has a bell shaped opening in which the arc is initiated when the fusible section melts. If the current happens to be extremely large it will generate a sufficient gas blast even

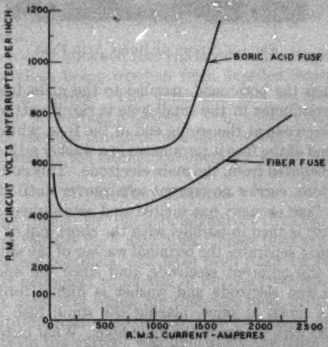


FIG. 2—COMPARISON OF BORIC ACID AND FIBER FUSES

in this enlarged opening to interrupt the circuit at the end of the first half-cycle of arcing. The large diameter of opening together with the short length of arc insures that there will be no excessive pressures, even with extremely high currents. Smaller current arcs which are not interrupted in the first half-cycle in the enlarged portion of the fuse bore are lengthened and drawn into the more restricted sections of the tube by the receding plunger, thus increasing the gas blast until extinction occurs. With this construction, therefore, it is possible to clear the full range of currents in a single tube, without developing excessive pressures on the higher currents or going to impractical lengths to clear the lower ranges of current. However, for currents below six or seven hundred amperes, a hole much smaller even than the main fuse bore would be preferable. It was recognized that if a hole smaller than that required for the main conductor were used to extinguish currents in the above range, the total fuse length could be appreciably shortened. This feature has actually been accomplished by the addition of an auxiliary fuse wire in a small hole through the boric acid, parallel to the main fuse bore. The conductor in the small hole is rigidly attached to the electrode at the spring end of the fuse, while at the bell end of the fuse it terminates in a washer adjacent to, but insulated from, the main electrode. This conductor, therefore, carries no current whatsoever until after the main fuse element has melted and established an arc. The arc is then in parallel with the short gap of insulation that separates the terminal washer of the small fuse from the adjacent electrode, and since the space between the electrode and washer is highly ionized, it breaks down at once, placing the small fuse wire in parallel with the arc in the main hole. What happens at this point is a function of the arc current, the thermal capacity of the auxiliary fuse wire, and the rate at which the plunger is withdrawn. For currents that require one or more half-cycles to melt the auxiliary fuse wire, the parallel arc in the main fuse bore is extinguished at once, since it is shunted by the low resistance auxiliary fuse. The current is, therefore, transferred to, and interrupted by the auxiliary fuse, the plunger having been withdrawn by this time to such a point that an arc will not restrike to it. For somewhat larger currents that melt the auxiliary fuse in a half-cycle or less, the main arc is extinguished for a fraction of a half-cycle but, due to the high arc voltage at the end of the first half-cycle when the arc is extinguished in the small hole, it restrikes to the movable plunger which in this time has traveled only a very short distance. The current is then interrupted as the plunger is withdrawn into the main fuse bore. For large currents the *IR* drop over the small fuse is sufficient actually to maintain an arc in the main fuse chamber, the current being divided between the two paths. The small fuse melts almost instantly, the arc voltage is insufficient to maintain an arc in the small hole and all the current transfers to the arc in the main bore, where it is interrupted at a following current zero as previously explained.

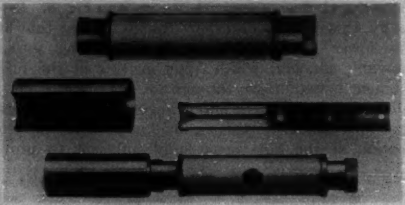


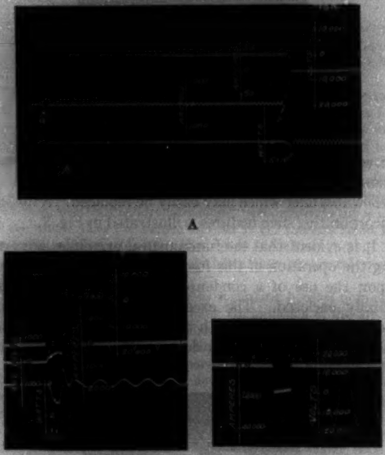
FIG. 3—VIEWS OF BORIC ACID FUSE

It is evident that this arrangement is capable of handling the complete current range, and that the length of the fuse need be only sufficient that the lower current interrupting capacity of the main fuse bore overlaps sufficiently the upper current capacity of the auxiliary bore. The upper limit of the current to be interrupted by the auxiliary fuse bore is determined by the thermal capacity of the fuse wire in it, and may thus be regulated, within limits, at will. Occasions may arise in which it is necessary to interrupt currents so small that they will not melt the auxiliary fuse. To insure the opening of the circuit under these conditions, the auxiliary fuse wire is attached to the movable plunger in such a manner that after the plunger has receded about half way into the main fuse bore the auxiliary fuse wire is mechanically withdrawn into the auxiliary fuse bore where the resultant arc is readily extinguished. This withdrawal can occur only after such time has elapsed that the proper selection of path for the interruption of the current has been determined as previously described, and in no way affects the operation of the fuse throughout the higher ranges of current.

The three oscillograms reproduced in Fig. 4 show how this fuse operates on small, medium and large currents. In oscillogram 4A for a small current the first drop in arc voltage occurs at the time when the gap between the auxiliary fuse terminal washer and the adjacent electrode is bridged by an arc, causing simultaneously the extinction of the arc in the main fuse chamber. This change in arc voltage is shown at point *a* in the magnified arc voltage record of element No. 3. The smaller arc voltage which follows, is that of a short arc from the adjacent electrode to the terminal washer of the auxiliary fuse. In this particular test the auxiliary fuse wire was not mechanically withdrawn and hence a large number of half-cycles elapsed before the small fuse melted. When this occurred the arc voltage rose as seen at point *b*, and the current was extinguished. Oscillogram 4B shows a case of intermediate current value where the arc transfers to the auxiliary hole, is extinguished there, as indicated by the high arc voltage at point *c*, and then restrikes again in the main fuse bore, where it is extinguished as it is drawn into the main fuse bore by the moving plunger. Oscillogram 4C shows the extinction of a heavy current arc. Here there is no evidence of any high arc voltage to indicate that the arc at any time was completely transferred to the small hole. The main arc in this case undoubtedly continued to carry current even while a portion of the total current sufficient to melt the auxiliary fuse was shunted through the auxiliary path.

IV. Totally Enclosed Fuses

The fact that the gas blast in the boric acid fuse consists of water vapor, a condensible gas, makes it possible to build a totally enclosed fuse of high interrupting capacity. With fiber fuses such construction is impractical on account of the enormous pressures that would be developed by the permanent gases liberated during the arcing period.



A

B C

Fig. 4

When boric acid is used as the gas producing medium, the water vapor emerging from the fuse may be discharged into a closed surface condenser of metal tubes or plates. Such a condenser serves to absorb heat energy from the gas, which causes it to condense, and thus eliminates excessive pressures within the closed fuse structure. A condenser of this kind is shown in the sectioned fuse of Fig. 3.

Tests on condensers of this type show that about 80 per cent of the total arc energy can be accounted for as temperature rise in the condenser. The remainder of the energy is used in heating other parts of the fuse, and in supplying the energy required for chemical decomposition of the boric acid. In an actual test at 4,000 amperes, the oscillogram shows 57.5 kilowatt-seconds of arc energy, equivalent to 13,700 calories. The condenser, which weighed 3,060 grams, was raised in temperature 40 deg. cent. This represents a heat input of approximately 11,650 calories, or 85 per cent of the arc energy. That a large percentage of this energy is actually absorbed by the condenser during the arcing period is indicated by the fact that a condenser consisting of 2,120 grams of copper plates housed in a 1,500 gram brass case operates satisfactorily at currents of 12,000 to 20,000 amperes. On these operations the arc lasted for one to two half-cycles, and the energy measured from the films in several instances was in excess of 200 kilowatt seconds corresponding to 48,000 calories. Calculations readily show that if this energy were expended in generating uncondensed steam within the available fuse volume of less than 1,000 cubic centimeters, pressures would be developed far in excess of those required to rupture the fuse case. Evidently a large percentage of the energy must have been absorbed by the condenser during the arcing period. These actual test results, furthermore, agree with calculations of heat flow, which show that if copper condenser plates less than 3/64 inch thick have constant temperature maintained at the plate surface, more than half of the total heat capacity of the plate at the surface temperature will have been absorbed in the first half-cycle of a 60-cycle arc. The rapid absorption of arc energy by the condenser is further augmented by the extremely high superheat at which the arc gases impinge on the condensing surfaces as well as by the increase in pressure which permits condensation to occur at higher temperatures.

This totally enclosed fuse has several advantages, the most obvious being freedom from expelled flame and ionized arc gases. The condenser also renders the operation of the fuse noiseless. Even at the maximum currents the fuse produces no audible sound other than that caused by the mechanical operation of the plunger.

V. Test Results

At the time of writing only laboratory tests had been made on the new boric acid fuses. These tests are, however, rather complete, and have been made on circuits with an extremely rapid rise of recovery voltage after current zero, and which are therefore more difficult to interrupt than any circuit likely to be met in service. In all tests the current was limited by the transformer or machine reactance only, or by additional air-core reactance, except for the currents below 400 amperes which were limited by an iron-core reactor, thus giving the most unfavorable conditions possible for interruption of the circuit.

These tests show that with a circuit e.m.f. of 13,800 r.m.s. volts, the fuse is capable of clearing the full current range up to 20,000 amperes using a 6-inch long boric acid filler. In this filler the main fuse bore is large enough to accommodate a 5/16-inch plunger, and the auxiliary fuse hole is about 0.1 inch in diameter. Operating characteristics for a fuse of this kind are shown in Fig. 5. There are two distinct curves, the first representing extinctions occurring in the auxiliary fuse bore, and the second representing extinctions in the main fuse bore. It may be seen that, in this particular design, currents up to about 600 amperes are cleared by the auxiliary fuse, while above this value the arc restrikes and is cleared in the main fuse chamber.

In Fig. 6 is shown an oscillogram of an operation of a totally enclosed fuse of this type at 20,000 r.m.s. amperes, 12,000 r.m.s. restored volts. The circuit was interrupted with one half-cycle of arcing. The average arc voltage was approximately 1,350 volts, and the arc energy was calculated to be approximately 300 kilowatt-seconds.

Fig. 7 shows an operation with a similar fuse of open type. The current in this test on the first half-cycle measures 20,300 r.m.s. amperes, and the restored voltage, 11,800 r.m.s. volts. The interruption occurred in one half-cycle.

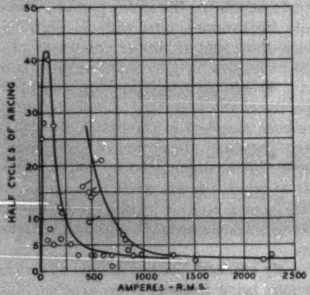


Fig. 5—Performance of Experimental Plunger Type Boric Acid Fuse at 13,800 R.M.S. Volts, 60 Cycles with Maximum Electrode Separation of 6 Inches

VI. Summary and Conclusions

Over 500 tests on plunger type boric acid fuses, made at voltages up to 14,000 r.m.s. volts and currents ranging from 5 to 20,000 amperes show not only that boric acid produces a far more effective and safe gas blast than organic insulation, but also that the particular type of construction employed in this design utilizes the gas blast to better advantage than the conventional expulsion fuse.

By using an auxiliary fuse bore to clear the smaller currents it has been possible to interrupt over 2,000 volts per inch of fuse length throughout the entire current range from 5 to 20,000 amperes. These principles may readily be incorporated in fuses with voltage ratings other than the one described in this paper by appropriate changes in design.

With boric acid as the gas-producing medium in fuses of this type, water vapor is evolved, which permits the construction of a totally enclosed high-capacity fuse. This is accomplished by using a surface condenser to recondense the water vapor, thereby eliminating excessive pressures within the closed fuse structure. This condenser serves not only to prevent escape of flame, but also acts as an excellent muffler.

A fuse embodying the principles which have been discussed in this paper lends itself well to a construction in which the various fuse parts are so assembled that those portions affected by the arc form a separate unit or renewable filler which may easily be replaced. A design incorporating such features is illustrated in Fig. 3.

Fig. 6

It is evident that the fundamental principles governing the operation of this fuse are in no way dependent upon the use of a condenser which renders the fuse totally enclosed. The excellent current interrupting performance obtained with the materials and construction as embodied in this design may thus readily be secured, both in those cases where the totally enclosed feature is desirable and in the larger field where a less expensive open type of fuse is sufficient. Furthermore, a fuse such as described in this paper should open new fields in the application of high-voltage fuses.

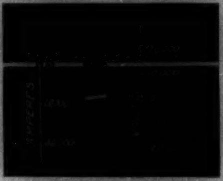


Fig. 7

Acknowledgment

The authors wish to acknowledge their indebtedness to Dr. Slepian and Mr. C. L. Denault for the valuable suggestions they have given, and to Mr. J. W. Huffstutter and Mr. C. J. Witsberger, who have cooperated

in obtaining much of the experimental data used in the preparation of this paper.

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Discussion

T. G. LeClair: The new fuse described in this paper has one feature of material value, *viz.*, the lack of an inflammable liquid filler. This characteristic is a very excellent safety recommendation.

The illustrations do not give a complete picture of the fuse and it is difficult to see whether the inclosed type has any opening for the escape of gases if the internal pressure should increase unexpectedly. I should like to ask the authors if they could tell us from the series of tests performed what takes place when the fuse is called upon to interrupt a current beyond its capacity. In other words, what is the effect of failure? With the inclosed type fuse, does the fuse structure merely burn, under the influence of the arc, or does the sudden increase in gas pressure cause an explosion which scatters the various parts of the fuse?

G. F. Lincks: In reviewing the paper, there are several points which are not entirely clear.

The authors indicate the arc-extinguishing benefits obtained by the use of boric acid, describing the initiation of the arc as being at the open or large chamber end of the tube. Experience with expulsion type fuses using fiber fuse holders, indicates that a much higher interrupting capacity is obtained with this positioning of the fuse links. Is it not possible that the location of the fuse link in the new design is largely responsible for the successful operation at the high currents?

While the paper states that over 500 laboratory tests were made at voltages up to 14,000 and at currents ranging from 5 to 20,000 r.m.s. amperes, it does not specifically cover the combinations of voltage and interrupted current nor does it mention the actual sizes of fuse links used in the tests. The oscillogram in Fig. 4A with the attendant description apparently shows the operation of a main fuse element having a very low current rating, since it was blown by 50 amperes peak current and the plunger drawn away sufficiently to permit arc-over to the auxiliary fuse wire in approximately 3 cycles (0.05 second). In our investigations under somewhat similar conditions, we have found that the use of fuse links of the higher current ratings brings in different and in some ways, more difficult, problems. Will the authors kindly give more detailed information as regards the combinations of the voltage, the interrupted current and the rating of the fuse links used in the tests upon which the reported successful operation is based?

Comparison of the times required to melt the main and the auxiliary fuse elements on the same oscillogram (Fig. 4A) indicates that the auxiliary fuse wire was considerably larger than the main fuse element. Is this necessary for all fuse ratings? If so, how is this to be taken care of without increasing the 0.1 inch auxiliary fuse bore for the larger ratings, which would affect the operating ability at the low currents?

In the description of the operation of the fuse, no mention is made of the effect on the condenser plates of the expelled hot gases, vaporized fuse metal, and decomposed boric acid. Investigation under somewhat similar conditions indicates destructive temperatures and "schooping" of molten materials in the arc gases, especially with the larger fuse links. In the laboratory tests mentioned in the paper, was there any tendency for the hot gases to burn or "schoop" insulating oxides or boric acid on to the condenser plates which would lower the efficiency of the condenser, required to effect rapid condensation?

In the normal operation of a fuse the effect of time under varying atmospheric conditions is important to avoid unnecessary fuse operation. This is particularly true if the fuse element is at the open end of a fuse holder and thus more or less directly exposed to the weather. In the design described what protection is afforded the fuse link and boric acid liner on the open end fuse when the condenser is omitted and how will any means for doing this affect the reported successful results?

One of the important operating requirements of a fuse is that after the fuse link has blown the circuit should be open with no permissible leakage current at rated voltage. The advantages claimed for use of boric acid are premised on its ability to contain and give off more moisture than fiber, thereby permitting shorter spacing between electrodes. What information is available as to the effect of the greater amount of moisture in the compressed boric acid and the shorter spacings on leakage after the circuit has apparently cleared?

Speed of operation is an important requisite of fuse links since it limits the time settings on circuit breaker relays all the way back to the generating station. No mention is made of the time-current characteristics in the paper although referring again to oscillogram Fig. 4A and the description, it would appear that the movement of the plunger is somehow retarded, since it did not mechanically break the auxiliary fuse link in less than approximately one second. It is stated that this should occur when the plunger has traveled one-half the normal movement. How do the time-current characteristics of the boric acid fuse compare with those of relays and other modern fuse links?

H. L. Rawlins: Mr. LeClair raises the question as to what will happen to the totally enclosed fuse if it is called upon to interrupt a current beyond its capacity. Obviously the pressure generated will cause the fuse structure to burst at its weakest point. If the condenser is solidly constructed, this will result in the bursting of the fiber retaining walls and other light parts. It is, then, evidently practical to design the condenser with a weakened section such that if failure does occur the pressure will be released without scattering the various parts of the fuse.

However, if the fuse is properly applied, there will be no failure, for repeated tests at 13,200 volts at currents up to 20,000 amperes have been made without failure of the totally inclosed fuse. At the present time all operating companies know on what parts of their system short circuits in excess of 20,000 amperes will be encountered.

Mr. Lincks raises a number of questions which are taken up in order in the following:

The location of the fusible section at the belled end of the fuse is responsible for a large increase in the interrupting ability of the fuse since it permits the extinction of the heavy currents in a large, freely vented bore.

All tests on the 15,000-volt fuse, as described in this paper, were made at 13,200 volts or more at all currents up to 20,000 amperes.

For convenience in testing most of the lower current tests described were made on fuse elements of lower ratings while most of the higher current tests were made on fuse elements of higher rating. However, the actual fusible section in this fuse is so short that in all cases the arcing is between the moving copper plunger and the lower electrode. It has been determined that the amount of material in this extremely short gap has no influence upon the proper functioning of the fuse.

The size of wire used in the auxiliary fuse bore is independent of the rating of the fuse and is determined so that blowing time on higher currents will be such that reignition and extinction of the arc will occur in the main fuse bore, while on lower currents the blowing time will be long enough to prevent reignition in the main bore, thereby insuring that the lower currents will be interrupted in the auxiliary fuse bore. This means, of course, that the same size of auxiliary fuse wire will be used for all of the current ratings.

In the totally enclosed fuse there is some condensation of the oxide upon the condenser plates and it would seem that upon repeated operation of the fuse, the effectiveness of the condenser plates would be decreased. However, repeated tests have been made using one set of condenser plates without apparently decreasing their effectiveness. This is probably due to the fact that condensation occurs at temperatures above the melting point of boric oxide due to the high pressure at which condensation occurs.

When the fuse is used without a condenser the open end of the fuse is tightly sealed by means of a thin disk to prevent exposure to the atmosphere of the boric acid. Upon operation of the fuse this disk is blown clear.

After operation of the fuse the boric acid remains in a dry state. The boric acid in such a state has leakage characteristics equally good and in some respects better than fiber. Due to its being an inorganic material it will not carbonize and form leakage paths as is the case with fiber. Therefore, the length of time at which voltage is left on the blown fuse has no effect upon breakdown along the boric acid surface. This has been verified by prolonged voltage tests on blown fuses.

The test shown on oscillogram 4A was made upon a fuse without the mechanical pullout feature. Therefore, the time of current flow through the auxiliary fuse wire represents the time to blow the auxiliary fuse wire. This oscillogram brings out very clearly the action of the auxiliary fuse wire in shunting the lower currents away from the main fuse bore. The mechanical pullout feature was adopted to reduce this time of current flow after blowing the main fuse link and in no way affects the selective action of the auxiliary fuse wire in shunting low currents to the auxiliary fuse bore.

Due to the short section of low melting point fusible material used as the fuse element in this fuse, the time characteristics on blowing will be very fast and will line up with the time characteristics of other modern fuse links.

The Torque Balance Telemeter

BY A. J. JOHNSTON[1]

Associate, A.I.E.E.

THE word telemeter, to the electrical industry, has become the name of a complete measuring, transmitting, and receiving apparatus for indicating, recording, or integrating at a distance by electrical translating means the value of a quantity.

The ideal telemeter should consist of apparatus capable of indicating or recording accurately and instantaneously, the value of the quantity being measured. It should require a minimum amount of energy from the source being measured and should not require a closely regulated auxiliary power supply. It should also be flexible, easy to install, and easy to maintain. That high-speed response and high degree of accuracy are considered very important, is shown by the following statement of the committee on automatic stations in its 1931 annual report on telemetering:

> One point emphasized was the lack of high-speed response with accuracy comparable to the usual line of switchboard indicating instruments. There is a tendency, where high speed is desired, to sacrifice somewhat the matter of accuracy.

Telemeters in the past have not been capable of instrument speed and accuracy, partially because engineers looked with disfavor on telemeters which employed electron tubes, even though the use of tubes may have greatly increased the rapidity of response. This, no doubt, was due in part to the fact that tubes change their characteristics with usage. Another reason was a belief that open line wires would most generally be used for telemetering purposes. Thus, if telemeters were to have wide application and be independent of line leakage and induced voltage, they must be of the impulse type. The indicating or recording apparatus of an impulse type telemeter must be heavily damped, and is, therefore, slow to respond.

The annual report of the American Telephone and Telegraph Company shows that approximately 94 per cent of the total telephone wire mileage in the United States is now in lead-sheath cable. It also shows that the open-line wire mileage has decreased approximately 40 per cent since 1925. This, together with the fact that the telephone companies have cooperated with the operating companies in leasing channels for telemetering purposes, at a very reasonable rate, has had the double effect of making available conductors which have very high leakage resistance, are practically free from induced voltages, and at the same time has relieved the operating companies of the initial cost of installing such conductors.

It is now possible, by means of the torque balance telemeter, to indicate or record, up to a distance of one hundred miles, the value of such quantities as watts, amperes, volts, temperature, pressure and flow with instrument speed and accuracy. The value of direct current which is used as the translating means of the telemeter is well within the limits of voltage, current and noise-units stipulated by the telephone companies. The telemeter automatically compensates for variation in line wire resistance, auxiliary supply power voltage, and variation in tube characteristics. In general the torque balance telemeter approaches, in almost all respects, the requirements of the ideal telemeter.

DESCRIPTION

The receiving and indicating apparatus of the torque balance telemeter is one and the same device and will hereafter be referred to as the receiver. Likewise the measuring and transmitting apparatus is a single unit and will be referred to as the transmitter. The measuring element may be one of two general classes, depending upon the quantity to be measured. Pressure gages and selsyn motors are classed as the position type, and voltmeters, ammeters, and wattmeters, are classed as the torque type. Of these only the wattmeter will be discussed in detail in this paper.

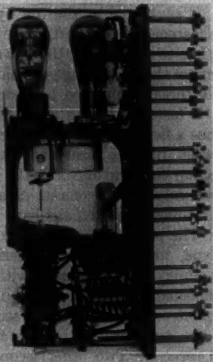

FIG. 1—TRANSMITTER FOR MEASURING POSITIVE AND NEGATIVE WATTS

The *receiver* may be any form of indicating or recording d-c. milliammeter. If desired two or more receivers may be operated in series by one transmitter. Likewise, when it is desired to totalize the values of two or more quantities, the transmitters may be connected in parallel to a single receiver.

The *transmitter*, shown in Fig. 1, is composed of four

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major parts; the measuring or operating element, the balancing or restraining element, the universal coupling, and the electron tube circuit. The apparatus, with the exception of the auxiliary power transformer and condenser, is contained in a standard 5½-in. by 16-in. case. The tubes and instrument parts are mounted on a cast bracket.

The *operating element* shown in Fig. 1, is a standard indicating wattmeter, except the control springs, pointer and damping disk are removed, and a yoke is fastened to the shaft of the element to provide a means of coupling it to the shaft of the restraining element. Lead-in spirals are provided to supply electrical connections to the moving coils.

The *restraining element* in all cases is a standard permanent-magnet d-c. milliammeter except the control springs are replaced by lead-in spirals, and the pointer is replaced by a pin to provide a coupling to the operating element.

The *connecting yoke*, upon which an instrument mirror is mounted, is supported at the top by means of the pin which is fastened to the shaft of the restraining element, being free to move about the axis of the pin. The bottom prongs of the connecting yoke hang free through a loop of silk thread supported by prongs of a second yoke which is fastened to the shaft of the operating element. Thus a flexible insulated universal coupling is formed between the two elements. The total weight of the connecting yoke and mirror is 1.25 grams.

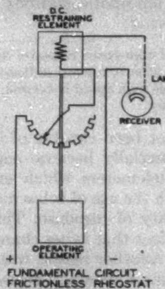
The *electron tube circuit* consists of a pliotron, two photoelectric tubes, and an auxiliary power supply transformer. The photoelectric tubes, which are of the gas filled type, are connected in series across one secondary winding of the transformer with their common junction connected to the grid of the pliotron. The plate of the pliotron is connected to a second secondary winding of the transformer through the restraining element and receiver. Two other secondary windings supply voltage to the filament of the pliotron and lamp.

The pliotron filament, which is of the thoriated tungsten type, is rated at 7.5 volts and the plate current is rated at 28 milliamperes. However, the transmitter is designed to operate the filament at 6.5 volts and to require not more than 10 milliamperes from the plate. The photo tubes are also operated below their normal rating of 20 microamperes. Approximately six microamperes circulate in this circuit. Likewise the lamp bulb is operated at 3.9 instead of 6 volts. This insures a life of approximately one year for the tubes and lamp. The total auxiliary power consumed by the telemeter is approximately 40 watts.

OPERATION

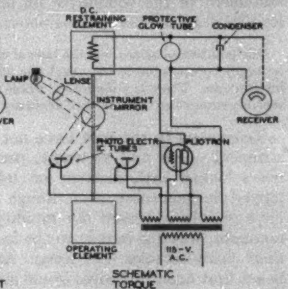
The operation of the telemeter may be best explained by first examining the circuits shown in Fig. 2.

The *fundamental circuit* consists of an operating element, restraining element and a receiver. The receiver is a standard d-c. milliammeter. For purposes of discussion it will be assumed that the rheostat, which is operated by the common shaft of the operating and restraining elements, is frictionless, has a resistance range from zero to infinity, and requires an armature movement of two degrees to cover the complete range. The restraining element is connected in series with the receiver and the rheostat to a direct-current source. The potential is so chosen that the restraining element torque is in opposition to the operating element torque. Since the control springs are removed from both elements, the restraining element torque is the only force opposing the movement of the operating element. Such a circuit is completely self-compensating for changes in potential of the direct-current source and resistance changes of the line wires, because the torque of both elements must always be equal and opposite. Any change in the circuit destroying this balance would cause a movement of the shaft and rheostat armature in the proper direction to reestablish the balance. From the foregoing it will be seen that there is no definite position of the moving system for a given measured quantity. Thus, neglecting bearing friction and line leakage, the deflection of the receiver is a true measure of the quantity being measured by the operating element.



FUNDAMENTAL CIRCUIT
FRICTIONLESS RHEOSTAT
ANALOGY

FIG. 2



SCHEMATIC
TORQUE
BALANCE

FIG. 3

The *photoelectric circuit and optical system* of the torque balance telemeter shown in Fig. 3 are used as a means of maintaining a balance between the two elements instead of the frictionless rheostat. The optical system is so arranged that light from the lamp source is condensed by a 25 diopter plano-convex lens on a stationary mirror which reflects it to the instrument mirror mounted on the connecting yoke, and thence to the photoelectric tubes. The photoelectric tubes are placed so that the beam of light may fall entirely on either tube or may fall partially on both tubes. The constants of the circuits are such that when light is falling only on the left hand photoelectric tube the grid potential is increased sufficiently to bias the pliotron to cut-off, in other words, the pliotron has infinite resistance. When all of the light is falling on the right hand photoelectric tube the grid potential of the pliotron is reduced to a point where the plate resistance is approximately 10,000 ohms, thereby permitting a plate current of more than 10 milliamperes through a total external loop resistance of 8,800 ohms. When the light beam falls anywhere between these two extremes the pliotron resistance accordingly will be somewhere between the extremes of 10,000 ohms and infinity. Thus, since the light beam has no inertia the optical system and electron circuit form a frictionless rheostat.

When the telemeter is used with external resistance less than 3,500 ohms, the transformer is used to supply power directly to the tubes. In this case the translating means is half-wave rectified alternating current. A condenser is connected across the line wires at the transmitter to smooth out the wave.

When the external resistance is more than 3,500 ohms full-wave, copper oxide rectifiers are inserted to supply direct current to both the plate and grid circuits of the pliotron. This addition is made to reduce the peak values of the voltage impressed on the line wires.

CHARACTERISTICS

Of primary importance in the design of any electron-tube circuit for use with measuring apparatus, is self-compensation for changes in tube characteristics. The circuit shown in Fig. 3 is not only compensated for changes in tube characteristics, but also for changes in auxiliary power supply voltages and line wire resistance.

Variation in auxiliary supply voltage affects the current output of the electron tubes as well as the illumination of the lamp. If the voltage is reduced there will be a reduction in the torque of the restraining element which will cause a movement of the shaft in the direction of the torque of the operating element. This movement will increase the light on the right hand photoelectric tube. The shaft will take up a new position at a point where the increased light on the right hand photoelectric tube will sufficiently reduce the grid bias on the pliotron to allow it to pass the original value of current. The action is of course reversed when the supply power voltage is increased.

Changes in electron tube characteristics will occur very gradually. However, the compensation for this change is accomplished in exactly the same way as for changes in supply power voltage. As long as the electron tubes are capable of passing sufficient current to balance the restraining element against the full load torque of the operating element and to prevent any current from flowing when there is no power in the operating element the tubes are satisfactory for service.

Changes in line wire resistance are compensated for in the same manner. The shift of the light on the photoelectric tubes has the effect of changing the pliotron plate resistance by an amount equal and opposite to the change in line wire resistance.

Changes in illumination due to dust on the lens and mirrors or to deterioration of the lamp filament are fully compensated for as long as enough light strikes the photoelectric tubes to give them control of the pliotron. The commercial models will usually operate considerably less than 50 per cent normal illumination.

The *sensitivity and responsiveness* of the telemeter is comparable to that of the usual line of switchboard instruments. Field tests have shown that the transmitter and receiver together actually respond more quickly than an indicating wattmeter connected directly to the measured source in spite of the fact that the responsiveness of the receiver is in the order of two seconds while the responsiveness of the indicating wattmeter is in the order of 1.5 seconds. This can be explained by the fact that there is no damping in the transmitter circuits, and that the moving system has to move not more than three degrees to cause a change in the line current from zero to maximum. Laboratory tests show that the line current reaches a maximum value, which is in the order of twice full-scale current, approximately one-tenth second after full scale power is suddenly applied to the operating element and that the line current is settled down to a steady value within one second. Thus, the receiver is accelerated rapidly during the first half second.

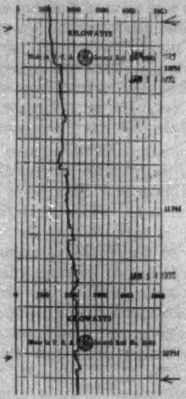


Fig. 4—Chart from Type C-4 Recording Wattmeter
Multiply Scale Reading by 1.5

The charts shown in Figs. 4 and 5 were taken in field test and show conclusively that the receiver was capable of recording any changes that could be recorded by the local wattmeter.

The *accuracy* of the transmitter is usually given the same as a standard switchboard type indicating instrument; *i. e.*, 1 per cent of full scale. By examination of Fig. 3 it can be seen that the complete telemeter is theoretically capable of very high degrees of accuracy.

This is especially true when the receiver is calibrated with the transmitter, because the receiver scale can be marked directly in terms of the values of power in the measured source.

The scale of the receiver is directly proportional and is therefore more easily read than the scale of a standard wattmeter. It will be recalled that the moving system of the transmitter has a maximum deflection of approximately two degrees and that the current field of a wattmeter is practically uniform over that range. Thus the torque produced by the operating element is directly proportional to the watts in the measured source. This feature is very important because it allows the telemeter to be readily used for totalizing alternating-current watts or any other quantity which will produce a torque in the operating element directly proportional to the measured quantity. Some operating elements, such as a-c. ammeters, do not have a direct proportion between current and torque even though the shaft is allowed to turn through only a narrow range.

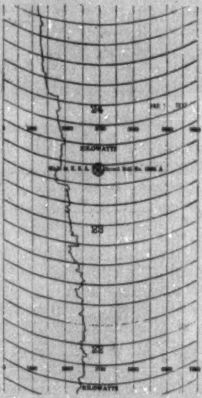


Fig. 5—Chart from Type C-19 Recorder Operating as the Receiver for a Torque Balance Telemeter Transmitter

Connected to the same source as the C-4 recording wattmeter shown in Fig. 4

The factors which are most likely to affect the accuracy of the telemeter are line leakage, bearing friction, and balance of the transmitter moving system.

Line leakage is in effect a shunt across the receiver. If the leakage resistance is constant, it can be compensated for by calibration, but if the leakage resistance is variable the percentage error introduced will be directly proportional to the percentage of current shunted from the receiver. Line leakage can be expressed in general as directly proportional to length of line and inversely proportional to leakage resistance. The calculated error due to leakage in 100 miles of No. 19 B. & S. gage lead-sheath cable is less than one-tenth of one per cent. The calculated error for 100 miles of No. 8 B. & S. gage open line wires is less than five per cent when the receiver is an indicating instrument. This calculation is based on wet weather conditions estimated to occur one day out of one hundred. During dry weather the error is negligible.

Bearing friction is present in any type of instrument, and since the transmitter is composed of standard instrument parts it has no more effect than in standard indicating instruments.

Proper balance of the transmitter moving system is very important if the telemeter is to be perfectly self-compensating. It must be free to move throughout its entire range so that the current in the restraining element will be exactly the same for balancing a given torque of the operating element regardless of the position of the moving system. No trouble of obtaining a satisfactory balance has been experienced in the commercial models.

Application of Various Types of Transmitters

The foregoing discussion applied only to the transmitter for measuring single-direction watts. When it is desired to measure directional quantity, such as positive and negative watts, a second pliotron is added to the transmitter in such a way as to make the circuit function as a frictionless voltage divider instead of a frictionless rheostat, so that positive and negative values of current can be used as the translating means. In this case, of course, the receiver is of the zero center type. When designed to measure such quantities as pressure or the position of a selsyn motor, the position of the gage or selsyn motor is converted into torque by means of spiral springs. For measuring temperature, the restraining and operating elements are combined into a single unit which is a standard differential type temperature instrument.

Conclusion

In combining standard instrument parts and electron tubes into a single unit which is self-compensating and which is capable of indicating and recording at a distance with instrument speed and accuracy, the usual type of measurements such as amperes, volts, watts, gas and liquid flow, temperature, pressure, and positions such as hydraulic head, water-wheel gate openings, and governor settings, a distinct forward step has been made in the field of telemetering.

The Application and Performance of Automatic Equipment

At Stations and Substations on the American Gas and Electric Company System

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INTRODUCTION

THE first urge behind the idea of automatic stations and substations was an economic one. Operators at many electric power stations were performing important functions, it is true, but the performance of these functions, frequently, was relatively simple and required but little of the operators' time. With the first real development of the relay art some fifteen years ago, it was realized that many operators' duties could be performed satisfactorily by relay arrangements. About the first and a most logical application of the automatic idea was to synchronous converters supplying railway load, followed closely by application to isolated hydro plants. The development of an economical and reliable universal motor mechanism made simple automatic switching stations possible; more complex automatic stations naturally followed with the development of synchronizing relays.

The developments just outlined were all based on the economic advantages derived. With the growth of automatic switching stations, however, it became apparent that they possessed another fundamental advantage. Under many circumstances they can act more quickly, more accurately, and more reliably than manual operators. Consequently, in some stations where for other reasons operators were required, automatic equipment took over many of the operators' functions.

This accounts for the parallel development of supervisory control equipment, automatic coal handling equipment, automatic combustion control, automatic frequency control, tie-line load control, etc. In all of these fields numerous applications have been made on the system with which the authors have been associated.

THE AUTOMATIC STATION AND ITS ELEMENTS

At almost any modern station certain features of station operation are automatic; thus a rotary will automatically pick up load called for by the trolley. Similarly many stations called "automatic" are really not entirely so. For example, standard reclosing circuits are generally made to lock out if the fault persists after a certain number of predetermined reclosures, after which manual attention is necessary.

An automatic station may then be defined as one which will place equipment on the line when a predetermined set of conditions (but they can be made as extensive and complicated or as simple as the designer chooses) has been reached, will continue to keep equipment on the line indefinitely or as long as there is a demand for it but will automatically remove any equipment that is at fault or feeding a fault, and, depending on a pre-arranged interpretation of various relay functions, will either lock it out at once or after a predetermined number of trials at reclosure. There are, of course, some exceptions to this definition.

It is evident that before automatic practise could be developed to its present state, much preliminary and basic work had to be done. This work was divided into two parts. First, a foundation of sound ideas was required and second, certain items of equipment were needed and had to be developed because they were either of a totally different nature than any required before, or because their application was different. These ideas and this equipment constitute the elements of the automatic station and from them all types and combinations of automatic stations in service today have been built up.

The development of the economic aspect in automatic station engineering was one of the most important of the ideas referred to above. The very early engineering layouts for automatic stations because of the limited amount of experience that was available with such equipment, were very naturally highly conservative. An attempt was made to cover and protect for all contingencies that had ever been experienced and a great many that could theoretically be experienced in the operation of a system. This naturally resulted in a multiplicity of protective devices that might have worked and justified themselves, if they were ever called upon to function, but which probably could not be justified economically in the vast majority of cases. The insistence on the part of some of the early sponsors of automatic stations on the necessity of protecting for all these contingencies with the resulting necessity for all these devices, not only produced a cost that worked against the automatic installation in many cases, but also resulted in an excessive amount of caution on the part of utility system engineers before approving automatic applications. When it was insisted upon that every piece of apparatus specified meet an economic as well as an engineering test (which was in reality, therefore, only a full engineering test), a great many devices were eliminated. This not only simplified the average automatic board, but also

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*American Gas and Electric Company, New York, N. Y.

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TABLE I—SUMMARY OF AUTOMATIC STATIONS AND EQUIPMENT ON PROPERTIES OF AMERICAN GAS AND ELECTRIC COMPANY

Installation No.	Type of service	Company	Date installed	Location	Station	Voltage Kv.	Transformer capacity Kva.	Equipment controlled
1	Distribution and sectionalizing station on urban high-voltage belt	Appalachian Elec. Power Co.	1931	Huntington, W. Va	East 18th St	33/4	5,000	3—44-kv. supply lines; 5—44-kv. feeders; 11—4-kv. feeders
2			1931	Huntington, W. Va	Johnson Lane	33/4	7,500	2—33-kv. supply lines; 2—33-kv. feeders
3			1928	Roanoke, Va	Mason Creek	11	630	2—11-kv. supply lines; 2—11-kv. feeders
4		Atlantic City Elec. Co.	1927	Atlantic City, N. J	Inlet	22/4	6,000	2—22-kv. supply lines; 4—4-kv. feeders
5		Indiana & Michigan Elec. Co.	1927	So. Bend, Ind	East Side	27/4	3,000	2—27-kv. supply lines; 4—4-kv. feeders
6			1928	So. Bend, Ind	South Side	27/4	3,000	2—27-kv. supply lines; 3—27-kv. feeders; 4—4-kv. feeders
7			1930	So. Bend, Ind	West Side	27/4	12,000	2—27-kv. supply lines; 5—4-kv. feeders
8		Indiana General Service Co.	1928	Muncie, Ind	Blaine St	33/4/2.3	6,750	2—33-kv. supply lines; 5—4-kv. feeders; 2—2.3-kv. feeders
9			1928	Muncie, Ind	Sampson Ave	33/4	12,000	2—33-kv. supply lines; 8—4-kv. feeders
10		Ohio Power Co.	1925	Canton, Ohio	Eighth St	22/4	7,000	2—22-kv. supply lines; 1—22-kv. feeder; 5—4 kv. feeders
11			1927	Lima, Ohio	South Side	33/4	3,000	2—33-kv. supply lines
12			1927	Spencerville, O	Spencerville	33/4	2,000	2—33-kv. feeders
13	Sectionalizing station on urban high-voltage belt	Ohio Power Co.	1925	Canton, Ohio	Bryan Ave	22	..	4—22-kv. supply lines
14			1925	Canton, Ohio	Harrisburg Road	22	..	3—22-kv. supply lines; 2—22-kv. industrial feeders
15	Distribution and sectionalizing station on high-voltage transmission belt	Appalachian Elec. Power Co.	1929	Allen, Ky	Allen	44/6.6	450	2—44-kv. supply lines; 1—44-kv. feeder; 1—6.6-kv. feeder
16			1930	Bartley, W. Va	Bartley	13.2/2.3	2,700	2—13.2-kv. supply lines
17			1930	Caples, W. Va	Caples	13.2/2.3	1,500	3—13.2-kv. supply lines
18			1928	Radford, Va	Radford	33/4	1,500	2—33-kv. supply lines; 2—4-kv. feeders
19			1930	Bluefield, W. Va	Winding Gulf	13.2/2.3	600	2—13.2-kv. supply lines
20			1930	Yukon, W. Va	Yukon	13.2/2.3	1,500	2—13.2-kv. supply lines
21		Atlantic City Elec. Co.	1928	Mullica Hill, N. J	Mullica Hill	33/4	150	1—33-kv. supply line; 1—33-kv. feeder
22		Ind. & Mich. Elec. Co.	1928	Mishawaka, Ind	Mishawaka City	27/4	3,000	1—27-kv. supply line
23		Ohio Power Co.	1930	East Sparta, O	East Sparta	22/6.6	1,800	2—22-kv. supply lines; 3—6.6-kv. feeders
24			1928	Martins Ferry, O	Martins Ferry	66/2.3	2,000	2—66-kv. supply lines
25			1930	Van Wert, Ohio	Van Wert	33/4	4,500	1—4-kv. feeder to condenser
26			1927	Yorkville, Ohio	Yorkville	66/4	1,750	2—66-kv. supply lines; 1—4-kv. feeder
27	Sectionalizing station on high-voltage transmission belt	Appalachian Elec. Power Co.	1930	Elkhorn City, Ky	Elkhorn City	44/88	5,000	Ties 44-kv. and 88-kv. systems together
28			1929	Elwood, Ky	Elwood	44	..	3—44-kv. supply lines
29			1930	Pekin, Ohio	Pekin	22	..	2—22-kv. supply lines; 1—22-kv. feeder
30	Urban distribution station served by at least two high-voltage feeders	Appalachian Elec. Power Co.	1928	Ashland, Ky.	Ashland	33/4	8,100	2—33-kv. supply lines; 9—4-kv. feeders
31			1930	Charlestown, W. Va	Brooks St	44/4	7,500	2—44-kv. supply lines; 7—4-kv. feeders
32			1928	Roanoke, Va	Campbell Ave	33/11/4	6,000	2—33-kv. supply lines; 1—33-kv. feeder; 8-4-kv. feeders; 1-11-kv. feeder
33			1930	Charlestown, W. Va	Central Ave	44/4	2,250	2—44-kv. supply lines; 4—4-kv. feeders
34			1928	Huntington, W. Va	Four Pole Creek	33/4	1,500	2—33-kv. supply lines; 4—4-kv. feeders
35		Atlantic City Elec. Co.	1931	Beach Haven, N. J	Beach Haven	22/4	2,000	2—22-kv. supply lines; 3—4-kv. feeders
36			1927	Atlantic City, N. J	Chelsea	22/4	6,000	2—22-kv. supply lines; 4—4-kv. feeders
37			1924	Ventnor, N. J	Ventnor	22/4	3,000	2—22-kv. supply lines; 4—4-kv. feeders
38			1929	Williamstown, N. J	Williamstown	66/33/4	11,500	2—66-kv. supply lines; 3—33-kv. supply lines; 1—4-kv. feeder
39		Ind. & Mich. Elec. Co.	1926	Benton Harbor, Mich	Benton Harbor	27/4/0.6	6,750	3—600 volt feeders
40		Indiana General Service Co.	1930	Marion, Ind	Marion Steam Pl't	33/4	15,000	3—33-kv. feeders; 9—4-kv. feeders
41			1928	Marion, Ind	South Side	33/4	2,000	2—33-kv. supply lines; 3—4-kv. feeders
42		Ohio Power Co.	1929	Ironton, Ohio	Center St	33/4	3,000	2—33-kv. supply lines; 4—4-kv. feeders
43			1930	Zanesville, Ohio	Cooper Mill	22/4	3,000	3—22-kv. supply lines; 4—4-kv. feeders
44			1930	Zanesville, Ohio	Linden Ave	22/4	3,000	2—22-kv. supply lines; 3—4-kv. feeders
45			1930	Canton, Ohio	Meyers Lake	22/4	1,500	1—22-kv. feeder
46			1930	Zanesville, Ohio	North St	22/2.3	6,000	2—22-kv. supply lines; 6—2.3-kv. feeders
47			1929	Ironton, Ohio	Pleasant St	33/4	3,000	2—22-kv. supply lines; 2—22-kv. feeders
48			1929	Steubenville, Ohio	Stanton	22/4	3,000	2—22-kv. supply lines; 3—4-kv. feeders
49			1931	Scranton, Pa	South Side	22/4	9,000	2—22-kv. supply lines; 4—4-kv. feeders; 1—4-kv. emergency feeder
50			1930	Moundsville, W. Va	Moundsville	66/4	4,000	2—66-kv. supply lines; 1—66-kv. feeder; 5—4-kv. feeders
51	High-voltage sectionalizing and industrial distribution station	Appalachian Elec. Power Co.	1930	Belle, W. Va	Belle	44	..	4—44-kv. supply lines; 4—44-kv. industrial feeders
52			1928	Charlestown, W. Va	Chemical	44	..	4—44-kv. supply lines; 2—44-kv. industrial feeders
53			1927	Leewood, W. Va	Leewood	44	..	3—44-kv. supply lines; 1—44-kv. feeder
54			1928	Russell, Ky	Raceland	33/2.3	6,750	2—33-kv. supply lines; 1—33-kv. feeder; 2—2.3-kv. feeders
55			1927	Russell, Ky	Russell	33/2.3	3,000	2—33-kv. supply lines; 1—33-kv. feeder; 5—2.3-kv. feeders
56			1927	Smithers Creek, Va	Smithers Creek	44/2.3	1,000	2—44-kv. supply lines
57			1930	Van Lear, Ky	Van Lear	44/40	4,500	Ties 44-kv. and 40-kv. systems together
58		Atlantic City Elec. Co.	1931	Bridgeton, N. J	Laurel St	66/13.2	12,500	2—66-kv. supply lines
29			1931	Paulsboro, N. J	Paulsboro	33/4	9,000	2—33-kv. supplt lines; 2—33-kv. feeders; 4—4-kv. feeders
60		Indiana & Michigan Elec. Co.	1930	St. Joseph, Mich	Auto Specialties	27/4	6,000	2—27-kv. supply lines
61			1928	Niles, Mich	Niles	27	..	4—27-kv. supply lines; 1—27-kv. feeder
62			1928	Elkhart, Ind	Northwest	27/4	3,000	2—27-kv. supply lines; 4—4-kv. feeders
63			1929	Mishawaka, Ind	Rubber Regen	27/0.46	2,000	1—27 supply line
64		Indiana General Service Co.	1928	Anderson, Ind	Delco-Remy	33/4	4,500	2—33-kv. supply lines
65			1930	Muncie, Ind	Elmridge	33/4	4,500	2—33-kv. supply lines; 2—4-kv. feeders
66			1926	Gas City, Ind	Gas City	33/2.3	2,000	2—33-kv. supply lines; 4—2.3-kv. feeders
67			1929	Anderson, Ind	Madison	33/13.2	15,000	2—33-kv. supply lines; 2—33-kv. feeders; 2—13.2-kv. feeders
68		Scranton Elec. Co.	1927	Avoca, Pa	Avoca	22/4	3,000	3—22-kv. supply lines; 4—4-kv. feeders
69			1927	Old Forge, Pa	Old Forge	66/22/4	15,000	3—22-kv. feeders
70		Wheeling Elec. Co.	1927	Benwood, W. Va	Benwood	66/4	4,500	2—66-kv. supply lines; 5—4-kv. feeders
71			1929	Wheeling, W. Va	Elm Grove	4	2,000	4—4-kv. supply lines; 2—625 v. d.c. feeders
72		The Ohio Power Co.	1929	Steubenville, Ohio	Fort Steuben	66	25,000	4—66-kv. supply lines; 2—66-kv. feeders

TABLE I—SUMMARY OF AUTOMATIC STATIONS AND EQUIPMENT ON PROPERTIES OF AMERICAN GAS AND ELECTRIC COMPANY—*Continued*

Installation No.	Type of service	Company	Date installed	Location	Station	Voltage Kv.	Transformer capacity Kva.	Equipment controlled
73	High-voltage sectionalizing station	Appalachian Elec. Power Co.	1927	Ashland, Ky	Bellefonte	33/6.9	10,000	1—6.9-kv. feeder. High-voltage switching under "Supervising Control"
74			1928	Bowyer Falls, W. Va	Bowyer	44/13.2	4,500	Ties 44-kv. and 13.2-kv. system together
75			1927	Near Cranberry, W. Va	Prosperity	44	..	2—44-kv. supply lines
76			1927	Sophia, W. Va	Sophia	44	..	4—44-kv. supply lines
77		The Ohio Power Co.	1927	Rutland, Ohio	Rutland	132/33	4,500	2—33-kv. supply lines; 1—33-kv. feeder
78	Low-voltage distribution station	Appalachian Elec. Power Co.	1930	Burton, Ky	Burton	44/6.6	750	3—6.6-kv. feeders
79			1928	Williamson, W. Va	Williamson	44/2.3	4,000	10—2.3-kv. feeders
80			1931	Garrett, Ky	Garrett	44/6.6	750	3—6.6-kv. feeders
81		Atlantic City Elec. Co.	1927	Linwood, N. J	Belle Haven	22/4	1,750	2—4-kv. feeders
82			1927	Berlin, N. J	Berlin	22/4	5,500	3—4-kv. feeders
83			1927	Pennsgrove, N. J	Pennsgrove	33/2.3	600	2—2.3-kv. feeders
84			1930	Sea Isle City, N. J	Sea Isle	22/4	500	2—4-kv. supply lines
85			1929	Stone Harbor, N. J	Stone Harbor	22/4	600	2—4-kv. feeders
86		Indiana & Michigan Elec. Co.	1931	Lakeside, Mich	Lakeside	27/6.9	1,000	2—6.9-kv. feeders
87			1931	New Buffalo, Mich	New Buffalo	27/2.3	450	2—2.3-kv. feeders
88		Wheeling Elec. Co.	1929	Wheeling, W. Va	County Line	66/4	7,500	6—4-kv. feeders
89			1930	Wheeling, W. Va	Fulton	66/4	3,000	4—4-kv. feeders
90	Rotary converter station	Appalachian Elec. Power Co.	1924	Near Welch, W. Va	Boissevain	13.2/0.41/0.2	505	1—500-kv. rotary converter
91			1924	Near Welch, W. Va	Cherokee	13.2/0.41	300	1—300-kw. rotary converter
92			1921	Eckman, W. Va	Eckman	13.2/.206	900	2—300-kw. rotary converters
93			1925	Near Faraday, W. Va	Faraday No. 1	13.2/.206	600	2—300-kw. rotary converters
94			1929	Near Faraday, W. Va	Faraday No. 30	13.2/.206	300	1—300-kw. rotary converter
95			1923	Near Welch, W. Va	Itman No. 1	13.2/.206/.105	510	1—300-kw. rotary converter; 1—200-kw. rotary converter
96			1931	Near Newhall, W. Va	Jacobs Fork No. 1	13.2/.206	550	1—500-kw. rotary converter
97			1931	Near Newhall, W. Va	Jacobs Fork No. 2	13.2/.206	550	1—500-kv. rotary converter
98			1926	Jenkins Jones, W. Va	Jenkins Jones	13.2/0.41	1,230	1—500-kw. rotary converter; 1—600-volt d.c. feeder
3			1928	Roanoke, Va	Mason Creek	11/.41	630	2—300-kw. rotary converter
1	Rectifier station		1931	Huntington, W. Va	East 18th St	33/4	5,000	1—500-kw. rectifier; 2—600-volt d.c. feeders
99			1927	Rolfe, Va	Rolfe	13.2/2.3	900	1—300-kw. rectifier; 1—600-volt d.c. feeder
2			1931	Huntington, W. Va	Johnson Lane	33/4	7,500	1—500-kw. rectifier
69		Wheeling Elec. Co.	1929	Wheeling, W. Va	Elm Grove	4/.625	2,000	2—1,000-kw. rectifier; 8—600-volt d.c. feeders
70	Supervisory control	Appalachian Elec. Power Co.	1928	Ashland, Ky	Bellefonte	33/6.9	10,000	2—33-kv. supply lines; 8—33-kv. feeders
100			1930	Ivanhoe, Va	Ivanhoe	88/13.2	30,000	3—88-kv. supply lines; 2—13.2-kv. supply lines; 5—13.2-kv. feeders
101			1927	Kingsport, Ky	Cherokee	22/4	9,500	2—22-kv. supply lines; 3—4-kv. feeders
102		Ohio Power Co.	1925	Lima, Ohio	Rockhill	33/4/2.3	7,500	2—132-kv. supply lines; 2—33-kv. feeders
103	Coal handling	Appalachian Elec. Power Co.	1927	Glenlyn, Va	Glenlyn	132/88/13.2	135,300	1 crusher; 1 coal screen; 3 belt conveyers; 2 pan conveyers
104			1926	Logan, W. Va	Logan	132/44/2.3	72,000	1 crusher; 8 conveyers
105		Atlantic City Elec. Co.	1930	Deepwater, N. J	Deepwater	66/33/11	199,000	1 crusher; 10 conveyers
106		Ind. & Mich. Elec. Co.	1925	Mishawaka, Ind	Twin Branch	132/11/2.3	96,000	6 crushers; 8 conveyers
107		Ohio Power Co.	1924	Philo, Ohio	Philo	132/11/2.3	241,000	4 crushers; 8 conveyers; 2 coal screens
108			1917	Beech Bottom, W. Va	Windsor	132/66/11	190,000	12 conveyers
109		Scranton Elec. Co.	1927	Pittston, Pa	Stanton	66/11	107,500	2 crushers; 4 conveyers; 2 coal screens
110	Combustion control	Appalachian Elec. Power Co.	1929	Cabin Creek, W. Va	Cabin Creek	132/44/6.9	136,500	1 boiler, stoker fired; 2 boilers, pulverized fuel (Leeds & Northrup)
103			1931	Glenlyn, Va	Glenlyn	132/88/13.2	135,300	1 boiler, pulverized fuel (Bailey)
111			1928	Kenova, W. Va	Kenova	33/11	71,000	5 boiler, stoker fired (Smoot)
105		Atlantic City Elec. Co.	1930	Deepwater, N. J	Deepwater	66/33/11	199,000	6 boilers, pulverized fuel (Smoot)
112			1929	Atlantic City, N. J	Missouri Ave	66/22/11	107,500	2 boilers, stoker fired (Leeds & Northrup)
107		Ohio Power Co.	1931	Philo, Ohio	Philo	132/11/2.3	241,000	2 boilers, pulverized fuel (Leeds & Northrup)
73	Synchronous condenser	Appalachian Elec. Power Co.	1928	Ashland, Ky	Bellefonte	33/6.9	10,000	1—7,500-kva. synchronous condenser
1			1931	Huntington, W. Va	East 18th St	33/4	5,000	1—5,000-kva. semi-automatic synchronous condenser
39		Indiana & Michigan Elec. Co.	1926	Benton Harbor, Mich	Benton Harbor	27/4	6,750	1— 5,000-kva. semi-automatic synchronous condenser
113			1930	Benton Harbor, Mich	Riverside	132/27/11	30,000	1—15,000-kva. synchronous condenser
7			1930	South Bend, Ind	West Side	27/4	12,000	1— 7,500-kva. synchronous condenser
114		Ohio Power Co.	1930	New Boston, Ohio	Millbrook Park	33/2.3	8,500	1—7,500-kva. synchronous condenser
46			1930	Zanesville, Ohio	North St	22/2.3	6,000	1—7,500-kva. synchronous condenser
25			1930	Van Wert, Ohio	Van Wert	33/4	4,500	1—2,500-kva. semi-automatic synchronous condenser
115	Generating station	Appal. Elec. Pr. Co.	1928	Rocky Mt., Va	Rocky Mt	2.3	..	1—187-kva. water wheel generator (vertical)
107	Frequency control	The Ohio Power Co.	1931	Philo, Ohio	Philo	132/11/2.3	241,000	G. E. (Warren)*; (Leeds and Northrup)*
108			1929	Beech Bottom, W. Va	Windsor	132/66/11	190,000	G. E. (Warren)*; (Leeds and Northrup)*

Note: Since the preparation of this paper 6 additional automatic substations have been installed.

*Experimental equipment.

greatly reduced its cost. From both standpoints the natural result was the opening up of the field of application of automatic stations.

Some of the principal problems which had to be worked out in the development of the equipment elements, are given in the following paragraphs.

Relays—General. It was necessary, in general, to redesign relays before they could meet the requirements of automatic work. For this application, where their unsupervised performance had to be depended upon for proper operation of a station, they had to be absolutely reliable and accurate in operation and require very little maintenance. In addition, many new relays had to be developed to perform functions which hitherto had been taken care of manually.

Motor Mechanisms. To make the simple automatic switching station possible, sturdy, compact, and reliable devices for operating oil circuit breaker mechanisms had to be developed with the essential requirement of operation from an a-c. source.

Reclosing Relays. It was necessary to develop a-c. energized timing relays for use in combination with breaker motor mechanisms to make definite reclosing cycles possible. These relays had to provide not only for reclosures at adjustable time intervals, but also lock out features after a definite number of unsuccessful reclosures.

Synchronizing and Check Synchronizing Relays. To make complex automatic switching possible devices were developed which would perform the functions of synchronizing automatically; which would check conditions of voltage magnitude and phase relation and difference in frequency of two systems, and when conditions were all proper, supply the starting impulse to close a selected breaker to synchronize these systems. Check synchronizing devices were also needed for supervising the closing of two parts of the same system under conditions when these parts were already tied together through another circuit.

Energizing Transformers. One of the most important elements of an automatic switching station is the energizing transformer. The reliability of the station depends on the reliability of this element. Transformers with improved design and construction of windings and bushings and proper coordination between these had to be developed from the distribution and potential types to furnish from a line or bus the necessary power for energizing the automatic equipment.

The reliability of these present day automatic station elements has been made possible only by much labor and as the result of much experience. The history of this development is by itself very interesting, but cannot be covered in this paper.

However, the manner in which these various fundamental elements and ideas are combined in the development of automatic stations and systems on the properties of the various subsidiaries of the American Gas and Electric Company is described below.

DEVELOPMENT OF AUTOMATIC EQUIPMENT ON THE AMERICAN GAS AND ELECTRIC COMPANY SYSTEM

The development of automatic equipment on the system of the American Gas and Electric Company consisted of two distinct phases. The first phase consisted principally of the development and installation of automatic synchronous converter substations for mine service and an installation for serving traction load, all confined to two divisions of the Appalachian Electric Power Company. The second phase has consisted of a very extensive and general development over the whole American Gas and Electric Company system of automatic substation switching equipment, automatic control for station and substation equipment, the development of supervisory control, the development of automatic frequency and tie-line control, and, in the steam end of generating stations, the development of automatic coal handling and combustion control. The extensiveness of the application is shown in Table I. It will be seen that a total of 115 automatic stations or applications has been made covering practically the entire field of automatic work.

A classification of all automatic stations or equipment according to type of service is given in Table I.

The development and the use of the listed automatic equipment on the American Gas and Electric Company system, with the exception only of the first synchronous converter equipments, have been under the supervision of the authors. There will be described below the successive steps in this development, from the engineering and operating standpoints, with particular reference to automatic switching for transmission and distribution substations. Automatic synchronous converter installations have been standardized and well understood for some time and are indicated only to show the extent to which they have been used. Coal handling, combustion control and frequency control, too, have been included in Table I to show the extent to which applications of such equipment have been made and to point them out as fields in which automatic control is rapidly being extended.

DEVELOPMENT ON THE SYSTEM OF THE OHIO POWER COMPANY

Canton 22-Kv. System. The first extensive use of automatic switching equipment by the American Gas and Electric Company was on the 22-kv. system of The Ohio Power Company serving the city of Canton and environs. The load in this area is approximately 80,000 kw. It is divided into two principal classes, in the first of which are included several very large industrial loads, totaling approximately 50,000 kw., served directly at 22 kv.; in the second are included residential, commercial, and small industrial loads, totaling approximately 30,000 kw. All this load is served from the Sunnyside Substation, which is both an important 132-kv. switching station and a 132/22-kv. step-down station of approximately 90,000 kva. capacity.

Originally load in this area was served from the Sunnyside Substation at 22,000 volts by means of radial feeders terminating through disconnect switches in step-down distribution substations which could, however, be connected with one another by means of emergency circuits switched through disconnect switches. With this arrangement, it was necessary, particularly in times of trouble or during severe storms, to place operators at the various substations to operate the disconnect switches as one or another circuit became affected by trouble, so as to maintain service to all the distribution points.

After thorough study it was decided to install automatically operated oil switches at all these stations and by a proper grouping of the various feeders and interconnecting emergency circuits to provide a loop system. The installation of these automatic substations not only solved the service problems, but, by permitting complete parallelism of high-voltage feeders greatly improved the voltage situation. It permitted, of course, the elimination of emergency operators.

A map of the city of Canton and environs on which has been spotted the location of the various substations is shown in Fig. 1. Fig. 2 is a one-line diagram showing the loop circuits connecting the automatic substations and the location of oil switches. The attended stations are (1) Sunnyside Substation and (2) Third Street Substation. The former is the principal dispatching point for a large number of 132-kv. circuits terminating there. The latter is a part of a station used to supply steam heat for a considerable area from turbine or engine exhaust. Except for these two stations, the entire area is served by the group of eight automatic stations indicated. Each of these, as will again be seen from Fig. 2, has at least two sources of supply.

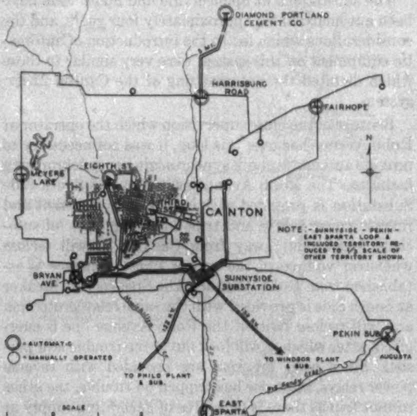
DIAMOND PORTLAND CEMENT CO.
3 MI.
HARRISBURG ROAD
FAIRHOPE
N
MEYERS LAKE
EIGHTH ST.
THIRD ST.
CANTON
NOTE:—SUNNYSIDE – PEKIN – EAST SPARTA LOOP & INCLUDED TERRITORY REDUCED TO 1/3 SCALE OF OTHER TERRITORY SHOWN.
BRYAN AVE.
SUNNYSIDE SUBSTATION
132 KV.
132 KV.
= AUTOMATIC
= MANUALLY OPERATED
PEKIN SUB.
AUGUSTA
TO WINDSOR PLANT & SUB.
TO PHILO PLANT & SUB.
EAST SPARTA
SCALE
1 0 1 2
MILES

Fig. 1—Map Showing Locations of the Automatic Substations on the 22-Kv. Transmission Loop Surrounding the City of Canton, Ohio

The automatic equipment and relaying used at Bryan Avenue is typical of the substations on this particular system. This early installation of reclosing equipment represented a departure from the then recommended practise since none of the ordinary undervoltage and overvoltage relays were employed. Because of the arrangement of the loop circuits shown in Fig. 2, it was not necessary to provide automatic synchronizing and consequently, it was possible to use very simple reclosing circuits and devices. A direct line from Sunnyside is the principal feed and the Sunnyside switch at Bryan Avenue is controlled so that it will reclose after tripping, provided the line from the Sunnyside Substation is energized at normal voltage and with the proper phase sequence; this, consequently, puts the operation of this switch at Bryan Avenue under limited control of the Sunnyside operator. The Harrisburg Road and the Eighth Street lines at this station tie, back through other automatic stations to the Sunnyside Substation. The switches on these two feeders are controlled by a motor-driven lockout timing relay, which will reclose each feeder twice and then lock out upon the occurrence of trouble which persists for longer than the time for which the reclosing relay provides. A fourth circuit is located at Bryan Avenue, operating as a stub feeder, with equipment similar to that on the Harrisburg Road and Eighth Street feeders, to act as an emergency feed to the Alloy Substation. It can also, by suitable switching, be used as an additional source of power from Sunnyside to the Bryan Avenue Substation.

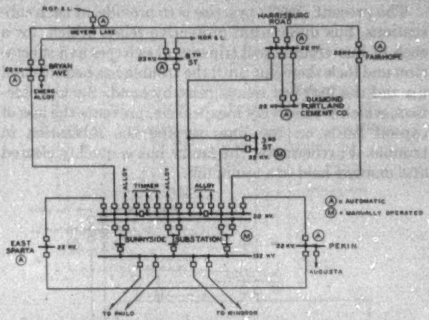
HOP & L.
MEYERS LAKE
HOP & L.
HARRISBURG ROAD
A
22 KV.
22 KV.
FAIRHOPE
22 KV.
BRYAN AVE
8TH ST
EMERG. ALLOY
22 KV.
DIAMOND PORTLAND CEMENT CO.
3RD ST
22 KV.
M
ALLOY
TIMKEN
ALLOY
ALLOY
22 KV.
A = AUTOMATIC
M = MANUALLY OPERATED
EAST SPARTA
22 KV.
SUNNYSIDE SUBSTATION
22 KV.
PEKIN
132 KV.
AUGUSTA
TO PHILO
TO WINDSOR

Fig. 2—Diagram of the Canton 22-Kv. System Showing the Switching Arrangement at the Various Substations

All breakers at Bryan Avenue are protected by induction overload and reverse power relays, which will trip a breaker for power flow (of sufficient magnitude) away from the bus. Bus trouble here will trip the switches at Sunnyside, Eighth Street, and Harrisburg Road. The switches at Bryan Avenue and Harrisburg Road, if the trouble persists, will, after two reclosures, lock out; the Sunnyside operator, after trying his line once or twice, will leave his Bryan Avenue switch open until the trouble has been cleared up.

The present (1932) practise is to provide, at such substations, bus differential protective relays which, for a case of bus trouble will trip out all switches at a substation and lock them out until the trouble has been cleared up and the lock out relays reset by hand. Such protection, which is relatively inexpensive, prevents the loss of tap-off loads on the lines serving the substation in trouble. Furthermore, the faulty bus is quickly cleared and damage held to a minimum.

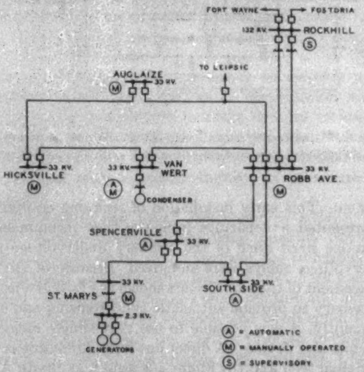

FIG. 3—SWITCHING DIAGRAM OF THE LIMA AND NORTHWESTERN DIVISIONS OF THE OHIO POWER COMPANY

In later installations the scheme employed for such lines as the one from Sunnyside, serving Bryan Avenue, has been somewhat modified. The reclosing relay now employed at Bryan Avenue on this line will reclose the Sunnyside breaker as soon as the line is energized and the voltages have the correct phase rotation, but in more recent practise a relay with a lockout feature is used and set to allow the switch to reclose twice only, after which it will lock out. This was done, since with the older type of relay, if the breaker mechanism should fail to latch properly, the breaker would continue to be reclosed so long as the line was energized.

In the original Canton automatic equipments, as has been pointed out, phase checking relays were provided. Experience has shown, however, that on loops of this type there is such a small likelihood of phase reversal that the expense of protection is hardly warranted. These relays are not being used therefore, in present practise, in situations of that sort.

The Lima 33-Kv. System. Fig. 3 shows the 33-kv. transmission system in the Lima District. A number of cities and large towns in an area of approximately 2,000 square miles is served from this system. The service is for the most part made up of residential, commercial, and small industrial loads.

The 33-kv. system is served from the 132-kv. step-down substation at Rockhill, Lima. The Rockhill Substation, the main point of supply, is controlled through supervisory equipment by the operators at the Robb Avenue Substation, the principal control and dispatching point for this system.

Automatic substations are located at Spencerville and South Side on one 33-kv. loop. The substations at Van Wert, Hicksville, and Auglaize on the other 33-kv. loop are manually controlled, because of present operating conditions, but they are so designed that they can easily be converted to automatic stations when the need for it arises. In view of this possibility, the synchronous condenser which was recently installed at Van Wert was made completely automatic with the exception of the initial starting impulse. This is at present provided from a control switch, but it is so arranged that when the station is made automatic, the control switch can be replaced by relays.

The substations at Spencerville and South Side have been automatized for approximately four years, and the considerations which led to the introduction of automatic equipment on this system were very similar to those which dictated the automatizing of the Canton 22-kv. system.

Because of the close supervision which the operator at Robb Avenue has over this loop, it was not necessary to provide automatic check synchronizing or synchronizing facilities. The Robb Avenue switch at the South Side Substation is provided with induction overcurrent and reverse power relays and trips for power flow (of sufficient magnitude) away from the bus. It will reclose whenever voltage of the proper phase sequence is established from Robb Avenue. The Spencerville breaker at South Side is provided with the same relay protection and will reclose twice if the Robb Avenue line is energized, after which it will lock out if the trouble still persists. Since both breakers are provided with reverse power relays, after one has tripped on trouble, the other cannot trip as there is no source of generating supply at the station. This makes it possible for the operator at Robb Avenue to tell from his indicating instruments what the setup of the loop is. At Spencerville the Robb Avenue and South Side switches are protected in the same way as the Robb Avenue and Spencerville switches at South Side, with the exception that provision is made at Spencerville by suitable interlocks so that the St. Mary's generating plant cannot be closed in out of parallel with the Lima system.

It will be noted that here again it was possible to set up the entire system on an automatic basis without resorting to synchronizing relays.

AUTOMATIC DEVELOPMENT ON THE 66-KV. SYSTEM OF THE WHEELING ELECTRIC COMPANY AND THE 66-KV. SYSTEM OF THE SUNNYSIDE DIVISION OF THE OHIO POWER COMPANY

The territory of the Wheeling Electric Company served by a 66-kv. transmission system which is contiguous to the 66-kv. system of The Ohio Power Company, presents an interesting example of automatic station practise.

The installation of automatically operated substations has facilitated the development of the Wheeling Electric Company system and the 66-kv. portion of The Ohio Power Company system along the Ohio River south of the Windsor Plant. Fig. 4 is a map showing geographically the extent of this system. Fig. 5 is a one-line diagram showing the arrangement of the substations and the switching.

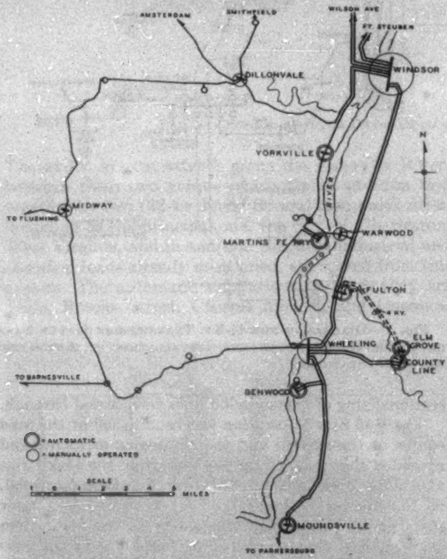
AMSTERDAM
SMITHFIELD
WILSON AVE
FT. STEUBEN
DILLONVALE
WINDSOR
YORKVILLE
N
MIDWAY
TO FLUSHING
RIVER
WARWOOD
MARTINS FERRY
OHIO
FULTON
4 KV.
ELM GROVE
WHEELING
COUNTY LINE
TO BARNESVILLE
BENWOOD
= AUTOMATIC
= MANUALLY OPERATED
SCALE
MILES
MOUNDSVILLE
TO PARKERSBURG

FIG. 4—MAP SHOWING THE EXTENT OF THE TERRITORY SERVED BY THE 66-KV. TRANSMISSION SYSTEM SOUTH OF WINDSOR TO MOUNDSVILLE AND THE LOCATION OF THE AUTOMATIC SUBSTATIONS

The first automatic substation to be installed in this division was at Benwood, followed closely by installations at Martins Ferry and Yorkville. A short time later County Line and Elm Grove, and finally the Moundsville Substation, were installed.

When the Benwood Substation was first installed, it was fed by one 66-kv. circuit directly from the 42nd Street Substation at Wheeling, and by a tap to another 66-kv. circuit extending from the 42nd Street Substation and interconnecting with the system of the Monongahela-West Penn Power Company at Parkersburg. It was accordingly essential that some means for synchronizing the switches at Benwood be employed; hence, the automatic equipment at this substation embodied the first installation of automatic synchronizing relays on the American Gas and Electric Company system. Each breaker at Benwood was provided with induction overload and reverse power relays arranged to trip for power flow (of sufficient magnitude) away from the Benwood bus. The automatic equipment was designed to reclose the breakers at Benwood (after the occurrence of trouble) upon reestablishment of voltage provided the Benwood bus itself was deenergized, or to reclose the breakers only through the synchronizing relays if the bus were energized.

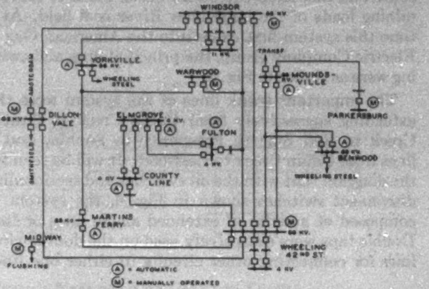
WINDSOR
66 KV
M
11 KV
YORKVILLE
A
66 KV
WHEELING STEEL
AMSTERDAM
WARWOOD
M
TRANSF
MOUNDSVILLE
66 KV.
A
M
PARKERSBURG
66 KV
DILLONVALE
ELMGROVE
4 KV.
A
FULTON
4 KV
SMITHFIELD
COUNTY LINE
4 KV
A
A
66 KV
BENWOOD
WHEELING STEEL
MARTINS FERRY
66 KV
A
M
66 KV.
MIDWAY
M
FLUSHING
WHEELING 42ND ST.
4 KV
A = AUTOMATIC
M = MANUALLY OPERATED

FIG. 5—ONE-LINE DIAGRAM OF THE TRANSMISSION SYSTEM SOUTH OF WINDSOR SUPPLYING WHEELING, WEST VIRGINIA, AND THE SURROUNDING TERRITORY

Later, when the Moundsville Substation, located south of Benwood and serving the city of Moundsville was installed, the Benwood Substation was tapped to the two lines between the 42nd Street Substation and the Moundsville Substation. The principal automatic features of the Benwood Substation, however, remained the same. The operation of the Moundsville Substation is practically the same as that of the Benwood Substation.

The other automatic substations on this system do not use synchronizing or check synchronizing relays since they are not necessary except for the rare operating condition of losing all tie lines between Windsor and Wheeling. Experience has shown that this condition is so seldom encountered that provisions for taking care of it are not justified. Automatic reclosing is provided on the 4-kv. feeder circuits at Elm Grove and County Line Substations.

DEVELOPMENT ON THE SYSTEM OF THE APPALACHIAN ELECTRIC POWER COMPANY

44-Kv. System South and East of Cabin Creek. The Cabin Creek Plant of the Charleston Division of the Appalachian Electric Power Company is located 16 miles up the Kanawha River from the city of Charleston. An extensive 44-kv. transmission system extends south and east from the Cabin Creek Plant to serve the mining loads of the large New River coal field. At the time this system first came into the American Gas and Electric Company group, the principal lines and switching were as shown in Fig. 6.

The important trunk lines of the system were those extending successively from Cabin Creek to Smithers Creek to Red Star to Prosperity to Sophia, and the lines from Cabin Creek to Leewood. It will be seen from the diagram that with the oil switches and sectionalizing disconnect switches shown in Fig. 6, the system was composed of a series of extended single lines or loops. Double taps were extensively used on the double-circuit lines for connecting other circuits to either one line or the other. It was practically impossible to obtain selective relaying with the switching scheme employed, and consequently the occurrence of trouble, particularly on the principal trunk lines, often involved loss of all sources of feed to large portions of the system. With some switching set-ups, as much as 200 miles of line were operated as a single circuit. Voltage conditions, particularly at remote points on the system, were poor and service was subjected to a great deal of interruption since trouble on one part of such a circuit was widespread in its effect.

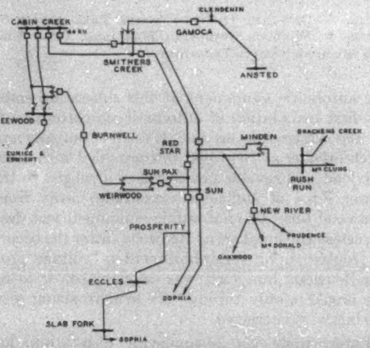
CABIN CREEK
44 KV.
CLENDENIN
GAMOCA
SMITHERS CREEK
ANSTED
LEEWOOD
EUNICE & ESWIGHT
BURNWELL
RED STAR
MINDEN
BRACKENS CREEK
Mc CLUNG
RUSH RUN
SUN PAX
WEIRWOOD
SUN
NEW RIVER
PROSPERITY
PRUDENCE
Mc DONALD
OAKWOOD
ECCLES
SOPHIA
SLAB FORK
SOPHIA

FIG. 6—44-KV. TRANSMISSION SYSTEM EAST OF CABIN CREEK AS IT WAS BEFORE THE INSTALLATION OF AUTOMATIC SWITCHING STATIONS

The territory through which these lines run is very mountainous and inaccessible, and communication which was carried on entirely by wire telephone, was available only at certain points on the system and was entirely inadequate. All switching had to be done manually at scattered points and in times of trouble this presented a very serious problem.

It was apparent that this particular system was an ideal one for the application of automatic switching. The physical location of the transmission lines forming the backbone of the system made it possible, by the use of automatic substations strategically located, to group these lines and to provide reliable sources of feed with good automatic sectionalizing in times of trouble. Fig. 7 is a diagram of this system after the regrouping and sectionalizing of transmission lines was carried through.

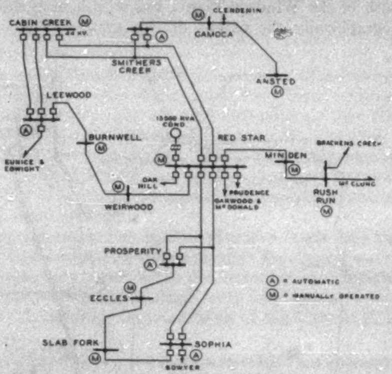
CABIN CREEK
M
44 KV.
M
CLENDENIN
A
GAMOCA
SMITHERS CREEK
ANSTED
M
LEEWOOD
A
EUNICE & EDWIGHT
BURNWELL
M
15000 KVA COND
RED STAR
M
BRACKENS CREEK
MINDEN
M
OAK HILL
M
WEIRWOOD
PRUDENCE
OAKWOOD & Mc DONALD
Mc CLUNG
RUSH RUN
M
PROSPERITY
A
A = AUTOMATIC
M = MANUALLY OPERATED
M
ECCLES
SLAB FORK
M
SOPHIA
A
BOWYER

FIG. 7—DIAGRAM OF THE 44-KV. TRANSMISSION SYSTEM EAST OF CABIN CREEK AFTER THE INSTALLATION OF AUTOMATIC SWITCHING STATIONS

The Red Star Substation was established at the load center of the system and the circuits so arranged and extended that it is fed by two lines from the 44-kv. bus at Cabin Creek and by one 44-kv. line from Leewood. A 15,000-kva. hydrogen cooled condenser was later installed at this station adequately to control system voltage. Full time operators are employed at this station, which is the only one on the system at which this practise is followed. Fully automatic substations were built at Smithers Creek, Prosperity, Sophia, and Leewood. An analysis of Fig. 7 will clearly show to what an extent the reliability and ease of operation of the system have been increased, and how small a portion of the system is affected by trouble on any one of the lines.

44-Kv. System West of Cabin Creek. Whereas the transmission system east of Cabin Creek is rather extensive and covers many hundreds of square miles of mountainous coal region, the system west of Cabin Creek is confined to the comparatively narrow valley of the Kanawha River. On both sides of the valley there are located large groups of chemical and other bulk power consuming industries, industries which are particularly susceptible to interruptions in service and consequently require service of the highest type. Automatic stations have been applied with great success in the development of the transmission supply system to this valley.

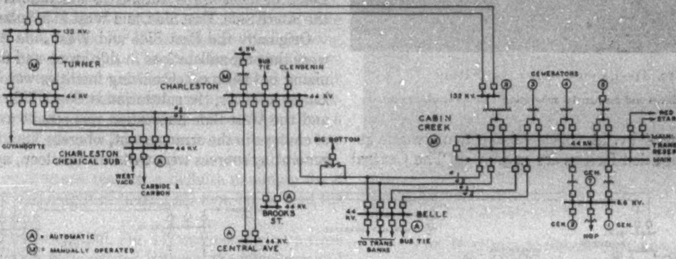
132 KV.
TURNER
44 KV
GUYANDOTTE
CHARLESTON CHEMICAL SUB.
44 KV.
WEST VACO
CARBIDE & CARBON
4 KV.
BUS TIE
CLENDENIN
CHARLESTON
44 KV.
BIG BOTTOM
BROOKS ST.
44 KV.
CENTRAL AVE
44 KV.
44 KV.
BELLE
TO TRANS BANKS
BUS TIE
132 KV.
GENERATORS
CABIN CREEK
RED STAR
MAIN BUS NO. 1
TRANSFER & RESERVE BUS
MAIN BUS NO. 2
44 KV
GEN.
6.6 KV.
GEN.
GEN.
H&P
Ⓐ = AUTOMATIC
Ⓜ = MANUALLY OPERATED

FIG. 8—ONE-LINE SYSTEM DIAGRAM OF THE DISTRICT WEST OF CABIN CREEK TO TURNER

The system of the Charleston Division of the Appalachian Electric Power Company, which extends between Cabin Creek and Turner Substation is shown in Fig. 8. The 44-kv. system extends along the Kanawha River, between these two points, which are in addition tied together by two 132-kv. lines. Several towns and cities, including as the principal one, the city of Charleston, West Virginia, and in addition the large industrial and chemical loads already mentioned, are served from this system. The automatic substations in this valley are Belle, Brooks Street, Central Avenue, and Chemical Substations. The Belle Substation, one view of which is shown in Fig. 9, is one of the most important and feeds a large industrial chemical load. It has a present capacity of 45,000 kva. with provision for additional capacity later. Two 44-kv. incoming lines feed this station directly from Cabin Creek, and two other sources of feed are provided by tapping the two Charleston-Cabin Creek lines. To provide maximum security of service, the preferred source of feed is over the two lines direct from Cabin Creek. The two tap-off lines are normally open at Belle, but in case of failure of voltage on the two lines from Cabin Creek, the Cabin Creek breakers at Belle trip and the tap-off breakers close to reenergize the substation bus. A view of the automatic board is shown in Fig. 10.

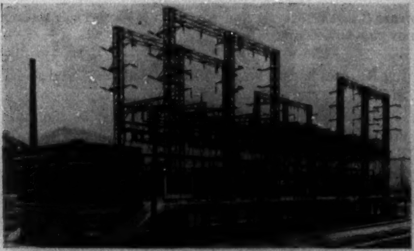

FIG. 9—THE BELLE SUBSTATION

The Chemical Substation in Charleston also supplies a large chemical load area. It is fed by two lines direct from the Turner-Charleston 44-kv. lines. The preferred scheme of reclosing is not employed at this substation, but provision is made so that any breaker will reclose from its line after the occurrence of trouble when the line has been reenergized.

The Brooks Street Substation, Fig. 11, and the Central Avenue Substation, are step-down distribution substations serving the 4-kv. distribution system in the city of Charleston. The Brooks Street station is fed directly by two lines from the Capitol Hill Substation, and the Capitol Hill breakers at Brooks Street are

FIG. 10—AUTOMATIC SWITCHBOARD AT BELLE SUBSTATION

arranged to close automatically when the lines are energized from Capitol Hill. A 7,500-kva. transformer steps down to 4 kv. at which voltage 6 automatic reclosing distribution feeders are employed. The Central Avenue Substation is fed by one line from Capitol Hill Substation and a tap connection from the Turner-Charleston No. 1 line. A preferred reclosing arrangement is used. For normal operation, the Capitol Hill breaker at Central Avenue is closed and the tap connection, which is used as an emergency supply, is automatically connected to the bus if the Capitol Hill feeder fails. When the Capitol Hill line is again energized, however, the Capitol Hill switch will reclose, and after a short period, will trip the emergency supply breaker.

Fig. 11—Brooks Street Substation

A typical business and residential substation automatically operated

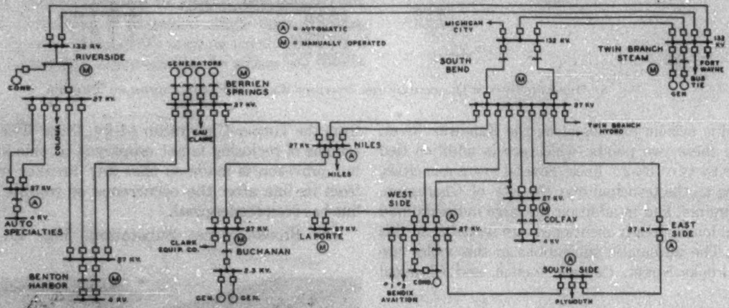
Ⓐ = AUTOMATIC
Ⓜ = MANUALLY OPERATED
132 KV.
RIVERSIDE
Ⓜ
COND.
27 KV.
COLOMA
27 KV.
Ⓐ
4 KV.
AUTO SPECIALTIES
27 KV.
BENTON HARBOR
Ⓜ
4 KV.
GENERATORS
Ⓜ
BERRIEN SPRINGS
27 KV.
EAU CLAIRE
27 KV.
NILES
Ⓐ
NILES
27 KV.
Ⓜ
CLARK EQUIP. CO.
BUCHANAN
2.3 KV.
GEN.
GEN.
27 KV.
LA PORTE
Ⓜ
MICHIGAN CITY
132 KV.
SOUTH BEND
Ⓜ
27 KV
TWIN BRANCH HYDRO
TWIN BRANCH STEAM
Ⓜ
132 KV
FORT WAYNE
BUS TIE
GEN
WEST SIDE
Ⓐ
27 KV.
#1 #2
BENDIX AVAITION
COND.
27 KV.
Ⓜ
COLFAX
4 KV
Ⓐ
SOUTH SIDE
PLYMOUTH
27 KV.
EAST SIDE
Ⓐ

Fig. 12—One-Line Diagram of the Indiana & Michigan Electric Company System North and West from Twin Branch Plant Showing the Location of the Automatic Stations

Development of Automatic Equipment on the System of the Indiana & Michigan Electric Company

The Indiana & Michigan Electric Company presents a number of very interesting applications of automatic switching.

Fig. 12 is a diagram showing the 27-kv. loop of the Indiana & Michigan Electric Company serving the city of South Bend and the 27-kv. system serving the surrounding territory. The city of South Bend has a population of approximately 110,000 and the load in this area is approximately 40,000 kw., supplied principally from the 132-kv. switching and step-down substation with a capacity of 60,000 kva. The service consists of residential, commercial and small industrial loads in the city of South Bend, together with a number of large industrial plant loads.

The 27-kv. loop extending around the city of South Bend includes three automatic substations known as the South Side, East Side, and West Side Substations.

Originally the East Side and West Side Substations were the only substations in this group and no synchronizing or check synchronizing facilities were provided. Later, however, the substation at South Side was added and the West Side Substation was rebuilt and because of changes in the arrangement, whereby lines from other generating sources were tied into this loop, synchronizing features had to be provided at West Side and South Side.

At the East Side Substation, the feeder to the main step-down substation closes automatically with establishment of voltage on that line and the South Side switch closes when the 27-kv. bus is energized from the main step-down substation. A 3,000-kva. transformer bank steps down to 4 kv. and feeds four automatic reclosing 4-kv. distribution feeders. Fig. 13 is a front view of this substation, showing the type of building used in purely residential sections.

The South Side Substation on this loop is connected by a 27-kv. circuit to both the East Side and to the West Side Substations, and in addition is connected by a 27-kv. circuit to the system of the Northern Indiana Power Company at Plymouth, Indiana. Two 27-kv. stub reclosing circuits are also provided to feed a large industrial load. A 3,000-kva. transformer bank steps down to 4 kv. at which voltage four automatic 4-kv. feeders are employed. The East Side and West Side breakers at South Side are provided with check synchronizing only. Full automatic synchronizing is provided for the Plymouth circuit.

The rebuilt West Side Substation is one of the most up-to-date of the fully automatic stations. Seven 27-kv. breakers at this station are automatically controlled. The 4,500-kva. step-down transformer feeds five automatic reclosing 4-kv. distribution circuits and a 7,500-kva. transformer supplies a synchronous condenser. The 27-kv. feeder switches going to the main step-down substation, the Laporte switch and the South Side switch are all equipped for full automatic synchronizing and check synchronizing. The two Bendix circuits are equipped to reclose automatically from the bus when it is energized. The condenser equipment is fully automatic. It starts up when the voltage varies more than a predetermined amount from normal (plus or minus one volt), and shuts down when the corrective kilovolt-ampere requirement reaches a definite minimum for a predetermined length of time. (200 kva. lag or lead for 3 minutes.)

The Niles Substation, Fig. 14, was installed at a time when no 132-kv. tie existed between the Northern and Southern parts of the Indiana & Michigan Electric Company system. This substation was an important tie point between the hydro plants at Buchanan and Berrien Springs and the South Bend 132-kv. substation. A large local industrial load is served by a 27-kv. feeder out of this substation. The switch on this circuit is provided with the usual automatic reclosing features. The other four circuits, *i. e.*, the two circuits to South Bend and the two circuits to Berrien Springs and Buchanan hydro plants, are provided with fully automatic synchronizing and check synchronizing relays. In the two years that this substation operated before the 132-kv. tie was installed between the Benton Harbor and South Bend districts of the Indiana & Michigan Electric Company, it functioned perfectly.

FIG. 13—EAST SIDE SUBSTATION, SOUTH BEND, INDIANA

A typical automatically operated substation in a purely residential district

There are several other automatic high-voltage installations on this system, such as the Mishawaka City Station (installation No. 22, Table I), the Rubber Regenerating Company station (installation No. 63, Table I), and the Northwest Station, Elkhart (installation No. 62, Table I), but none of these can be gone into beyond the extent covered in Table I.

KENTUCKY AND WEST VIRGINIA POWER COMPANY

The territory of the Kentucky and West Virginia Power Company is very different from that of many of the other companies of the American Gas and Electric Company group. This company serves principally coal mine loads in the southeastern section of Kentucky. The topography of the country is unusually rough. A 44-kv. transmission network serves individual coal mines from substations tapped directly to the 44-kv. transmission line. It is necessary, in order to maintain the proper grade of service, to provide some form of loop service, but owing to the type of territory the transmission loops quite frequently have to be very long. From an operating standpoint, this has generally meant long and very expensive patrols upon the occurrence of a transmission line failure. The installation of a small number of sectionalizing stations automatically operated has not only greatly improved the service, but also made possible many operating economies.

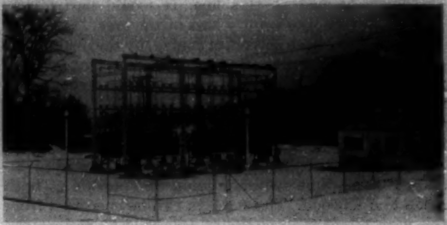

FIG. 14—NILES AUTOMATIC SWITCHING STATION

In Fig. 15 is shown a diagram of the switching arrangement of that portion of the Kentucky and West Virginia Power Company supplied from the Beaver Creek Substation and from the Hazard Plant. The substations at Allen, Elwood, Elkhorn City, and Van Lear are all provided with automatic reclosing features. Approximate transmission distances are indicated to give a better idea of the type of system involved.

The Beaver Creek Substation is a 132/44-kv. switching and step-down station manually controlled. This station employed one of the first of the new type miniature switchboards.

The Allen Substation is connected directly to Beaver Creek by one 44-kv. line. Another line from Allen extends to the Elwood Station and back to Beaver Creek. Automatic reclosing and check synchronizing were provided on each of these switches.

The Substation at Van Lear feeds a coal company system on which generating capacity is normally operating. The substation at Elkhorn City ties in with the system of the Appalachian Electric Power Company. Both of these substations, accordingly, are provided with check synchronizing and synchronizing relays.

A very interesting modification in the automatic equipment has recently been made to take care of certain operating conditions. The line between Elwood and Allen is a long one and if trouble occurs on this line and persists, the line is locked out at both ends, which would interrupt service to a considerable number of customers tapped to this line. It would particularly affect Pikeville, the county seat of the area. The problem could have been solved, of course, by the installation of an additional automatic sectionalizing point at Pikeville. It was felt, however, that such an expenditure was not warranted. Air break sectionalizing switches were therefore installed and the following modification in the automatic equipment at Elwood and Allen was made. Small telephone ringer circuit relays were installed on the line switches going to Pikeville at both Allen and Elwood. These relays can be energized from the wire telephone station at Pikeville to reclose the breakers on which they are installed and the breaker will stay in if the particular section is sound. The telephone relay is so wired that it will only act to reclose the breaker provided the breaker has previously gone through its reclosing cycle and locked out, and provided further that the substation bus at which the breaker is installed is energized.

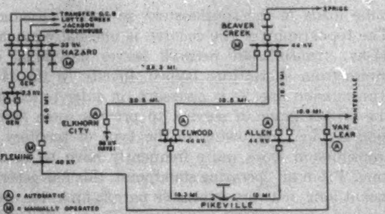

Fig. 15—One-Line Switching Diagram of the Kentucky and West Virginia Power Company 44-Kv. Transmission System Showing Automatic Installations

Low-Voltage Reclosing Circuits

There are on the American Gas and Electric Company system approximately 50 substations at which low-voltage automatic reclosing equipment has been employed. There are approximately 260 breakers controlled in this manner and the voltage range of the circuits so controlled is from 2,300 to 6,900 volts. The breaker duty cycle usually employed is the 3 reclosure cycle. A first reclosure takes place after 15 seconds, a second reclosure 30 seconds later and a third reclosure 75 seconds later. This reclosing cycle is, of course, in some instances modified to meet individual requirements. The interrupting capacity of the breakers used in this class of service ranges approximately from 75,000 to 250,000 kva.

Summary

The systems discussed, it is believed, have demonstrated the fact that there is practically nothing in the way of switching, whether switching of feeders, stations or systems involving one or many generating stations, that cannot be carried out and that has not been carried out completely and satisfactorily by automatic equipment.

Special Automatic Installations

In the foregoing description of automatic applications, stress has been laid on switching arrangements, the installations described covering practically every switching requirement. In addition to these, a considerable number of special equipments and applications has been made over the system, but because of lack of space, they cannot be described in detail in this paper. The following are some of those of particular interest:

Synchronous Condensers. While synchronous converters have been controlled automatically for over ten years, automatic control of synchronous condensers is comparatively recent. As shown in Table I, fully automatic or semi-automatic condensers are installed at eight places on the system. On these installations, protection is provided for failure of a-c. voltage, machine windings (differential), overload on starting compensator, overvoltage, continual overload of moderate amount, field failure, unbalanced phase currents, unbalanced or reverse phase incoming voltage, overheated bearings and rapid restarting.

Two of the completely automatic installations, *viz.*, those at West Side, South Bend, and North Street, Zanesville, come on the line automatically when the voltage is either too low or too high by a certain percentage for a definite length of time, and they are shut down when the need for corrective kilvoltampere of the machines is no longer present. This shutdown takes place when the condenser is supplying a minimum value of corrective kilvoltamperes at the proper voltage for a predetermined (but variable) length of time.

The essential difference between the full automatic and the semi-automatic installations is that the semi-automatic depends on a manually given starting impulse.

Mercury Arc Rectifiers. Although the development of the mercury arc rectifier is comparatively recent, a number of installations as shown in Table I has been very successfully automatized on the American Gas and Electric Company system. The installation at Elm Grove consists of two 1,000-kw. mercury arc rectifiers. The first unit is started from a clock switch, and the second when the load demand is beyond the capacity of the first. The two 500-kw. installations at Johnsons Lane and East 18th Street feed the railway load in the city of Huntington. The Johnsons Lane rectifier is given the starting impulse by a manual control switch. The rectifier at East 18th Street starts automatically on low trolley voltage.

A 300-kw. fully automatic rectifier is installed at the Rolfe Substation and serves a mining load. It is supplied at 13.2 kv. and converts to 600 volts direct current. The station is operated without any attendance except the periodic inspections.

Automatic Synchronizer. The relay development that preceded and accompanied automatic station practise has resulted in the transfer of a considerable number of operators' functions to automatic equipment at attended stations and substations. One of these functions has been the task of synchronizing. The advantage of an automatic synchronizer at such a substation lies in the fact that a switch is closed the very first time that the frequency of the two systems to be tied together is within the synchronizing range. With manual synchronizing an operator may miss a number of good opportunities to tie the lines in while waiting for still better conditions. Loss of time from such delay is sometimes costly from the service standpoint. Installations of this kind have been made at six places on the system, two of these at generating stations. These installations are arranged so that an operator simply has to put in the synchronizing plug on the circuit to be synchronized. From this point the automatic synchronizer takes control, but at the same time, an indication of system conditions is given on the synchroscope and the operator is left free to use his communication system to give instructions regarding the frequency to the proper generating station.

In addition to the applications outlined above, there is, of course, a considerable number of installations of automatic synchronizers at completely automatic substations. At one of these substations a departure from standard practise has been made, and one synchronizing relay and one check synchronizer take care of synchronizing all the switches at the station.

Supervisory Equipment. The straight automatic station has had the most general application on the system dealt with, but there have been a number of cases where, because of the nearness of a substation to a dispatching point, supervisory control offered the best solution. As shown in Table I, four substations on the system have been provided for this type of control. The systems are of either the synchronous visual type, or of the synchronous selector type.

At Lima four 132-kv. oil circuit breakers and five 33-kv. motor-operated air break switches are controlled, and at Cherokee six 22-kv. and four 4-kv. breakers. Twelve 33-kv. oil circuit breakers and one 7,500-kva. automatic synchronous condenser are supervised at Bellefonte, and at Ivanhoe four 88-kv. and ten 13.2-kv. breakers are controlled. An illustration of the Ivanhoe Substation given in Fig. 16 shows clearly the substation development and the nature of the country in which it is situated.

OPERATING EXPERIENCE

Beginning with the earliest automatic installations on the system and continuing from that time, care has been taken to follow closely the operation of each installation. A great deal of operating experience has, as a result, been gathered, covering both general experience with automatic applications and specific experience with automatic substation components. This is outlined below.

This procedure is responsible in a major degree for the uniformly successful operation of automatic equipments on this system.

GENERAL EXPERIENCE WITH AUTOMATIC APPLICATIONS

Reduction in Time of Outages. An automatic substation, when trouble occurs, will function in a minimum of time both to isolate the trouble and to restore normal conditions. The time of an outage to service is consequently reduced to an absolute minimum.

Reliability. The automatic substation is reliable both from the standpoint of mechanical reliability of its components and the reliability of its response to the predetermined conditions for which it is designed to operate.

FIG. 16—THE IVANHOE SUPERVISORY CONTROL SUBSTATION

Automatic Versus Attended Substations. Since an automatic substation responds at once when a predetermined set of conditions obtain it is not subject to the errors of judgment to which even the best of operators are prone and this is a particularly important advantage in times of emergency. In one case, for example, while two of four ties between a large generating plant and a large city were out for maintenance, trouble occurred on one of the remaining ties leaving this load on a single line. This line had an automatic sectionalizing station on it. At this particular time an operator happened to be at this substation and when he noticed the indicating ammeters off scale he decided the line was in trouble and tripped it, causing the line breaker to lock out and resulting in a prolonged interruption. If left to the automatic equipment, this interruption would not have taken place.

Quality of Automatic Service. By its reliability, speed of action, and freedom from errors of judgment the automatic substation gives a brand of service which cannot be approached by even the best attended substations. This applies both to the service given to the feeders at a substation and to the service which the transmission system gives to the substation.

Reduction in Operating Expense. On the whole, the automatic substation is often less expensive to operate than an equivalent attended one. The increased investment cost with the automatic installation is frequently outweighed by the savings in the direct operating expenses of attendance and maintenance. Automatic equipment is designed to require a minimum of maintenance and when such work is necessary, it is carried out by men particularly well qualified and in the least amount of time.

Suitability of Automatics for Isolated Locations. It is difficult to obtain good operators for manual substations in isolated locations. Even if they are obtained, the isolation and lack of sufficient duties to keep them fully occupied usually results in a breakdown of their morale in varying degrees with consequent loss of efficiency. Automatic substations meet the requirements of such locations admirably.

SPECIFIC EXPERIENCE WITH AUTOMATIC SUBSTATION COMPONENTS

The closeness with which the operation of each automatic installation was followed soon showed up certain weaknesses and troubles with some of the component devices and apparatus. These are outlined briefly.

Relays. Plunger type relays provided with bellows for time delay and used on some of the early automatic stations were not adequate for the service. The bellows became stiff either from becoming too dry or because subjected to low temperatures, and this combined with the small air outlet greatly altered their time characteristics or made them inoperable. This led to their replacement at an early stage by induction type relays.

Contacts on many of the automatic relays were not sufficiently sturdy to handle repeatedly rated currents without considerable maintenance. More sturdy contacts with larger current carrying capacity had to be designed.

Motor Mechanisms. Trouble was experienced with motor mechanisms of the centrifugal type caused by binding of the operating links and clevises. A modification of the linkage system resulted in a trouble free design.

The latches of some of the early breaker mechanisms failed to hold when the motor had closed the breaker. A redesign of these latches eliminated this trouble also.

Motor-Operated Timers. A number of these devices on the early automatic equipments burned out because they did not have sufficient leeway in their voltage rating. They were redesigned for a normal rating of 230 volts with a tolerance of 30 volts above or below normal, which solved the problem.

Synchronizing Relays. These devices, though complicated have been remarkably free from trouble. About the only difficulty encountered was from high contact resistance because of too low an operating torque at synchronism. This was remedied by readjusting the shading pole pieces to increase the torque.

Energizing Transformers. The earlier automatic equipments used two distribution transformers on each incoming line for energizing the reverse power relays, for closing the breaker, and for providing reverse phase protection. As the use of automatic equipment was extended to the higher voltages, the cost of these components became a considerable factor in the cost of the equipment. It was finally decided to eliminate the second transformer on the incoming lines and install instead two transformers on the bus. Although this eliminated the reverse phase protection of the early installations, operating experience has justified the omission of such protection.

Tripping Batteries. At the first automatic substations 12-volt batteries were used for tripping. Difficulties were encountered because 12-volt trip coils required too much current and too much voltage was consumed in control wiring and contact resistance to leave a safe margin to operate the trip coil. Consequently, 24-volt batteries were substituted and trip coils developed to require less current for operation. At some substations where supervisory control is used, or where large apparatus has been automatized, 48-volt batteries have been employed. This has been done because some of the circuits at such substations include so many devices that their increased resistance makes the higher voltage necessary.

CONCLUSIONS

The authors believe that the history of the development and operation of automatic station equipment on the system they are associated with shows that certain conclusions can be drawn very definitely with regard to the effect of automatic applications to certain phases of electric power system development and operation. These are:

Flexibility and Adaptability of the Automatic Station and Contribution to Improvement of Service. The automatic station is one of the most flexible of tools which a system planner and designer has available. In the proper hands the automatic station in its various forms can be used with great benefit on almost any system; it is highly adaptable to any local conditions.

Reliability. The reliability of automatic equipment is at least as good as that of the very best type of operator. In general its reliability is better than that of the average operator, provided the equipment is reasonably maintained. Furthermore, automatic equipment will always repeat its performance. When it fails it will invariably leave a tell-tale mark behind it which makes possible the correction of the fault and eliminates the possibility of its recurrence. Many of our most efficient operators today prefer to leave certain functions to automatic equipment, *e. g.*, synchronizing, watching of bearing temperatures, etc. For carrying out definite operations under infrequent emergencies automatic equipment is, too, as a general rule, more reliable than an operator.

Use at Isolated and Lonely Locations. Electric power system development frequently does not and cannot follow population centers. Switching stations may have to be located on lonely mountain sides or at points remote from any settled communities. The automatic station is the ideal solution for handling operating problems at all such locations.

Automatic Practise and System Cost. A great many data have been outlined in this paper to show the improvement to service which is possible by the application of automatic stations. Properly applied, however, automatic stations will not only improve service, but will do so at reduced cost. Reduction in the cost of the distribution end of electric service is one of the pressing problems of today. The automatic station application, in proper hands, is a tried tool and a sharp one for carrying out this particular job.

Universality of Application. Automatic practise is not only adaptable to switching stations, distribution stations, sectionalizing stations, etc., on power systems, but has been used in almost every branch of the power system. Its use has spread very successfully to such functions on power systems as coal handling, coal combustion, and frequency control. Other functions now handled on a manual basis will, with the development of the art of automatic practise be taken over by automatic equipments. There does not seem to be any reason to doubt that in the long run this will result in better service at lower cost, if the past experience in automatic practise is taken as a guide.

ACKNOWLEDGMENT

The authors wish to acknowledge the great help received from Mr. R. C. Miller in the preparation of this paper.

Discussion

A. E. Anderson: This paper readily indicates the extensive use that has been made of automatic switchgear on the system of the American Gas and Electric Company. The fact that over one hundred stations of different types are in use on this system permits the authors to obtain a wealth of operating experience and data. Most of the applications described in this paper are power line switching and distribution. However, the factors which enter here are, to a great extent, common to those found in other applications of automatic switchgear.

As stated by the authors, the first urge behind automatic switchgear was an economic one. In some cases it was possible to remove enough feeder copper to pay for a large share of the automatic control. Investments in automatic switchgear permit a reduction in operating costs and in this way meet one of the most important problems of the present time. With the increased reliability and speed over manual operation, together with other attendant developments, the economics of the situation are sometimes of secondary consideration. As a result of the experience gained with automatic switchgear, we now find that entire systems are being laid out for automatic operation. Such progress has been the result of well chosen applications, thoroughly designed control schemes and the careful selection of devices. In spite of these considerations, there still remains the important item of maintenance which is not expensive if well done and readily pays for itself. The items of inspection and maintenance were recognized from the beginning, and it was also realized that such functions should be performed by skilled employees. Devices form the heart of automatic control and must be suitably maintained if the desired character of service is to result.

Considering the extensive application of automatic switchgear on the system described in this paper; one is astonished by the relatively few cases of faulty operation on the part of the equipment. Some of these difficulties have been experienced in other installations as well, and action has been taken to eliminate these difficulties as fast as they occur. Time delay devices with oil dash pots or air bellows (with needle valves) were used in the early automatic stations. This was due to the fact that the choice of devices was limited to that used in manually operated stations. As the application of automatic equipment was extended it was found desirable to develop devices which would give more satisfactory operation. Oil dash pots and air bellows have been almost entirely eliminated by devices using motors, induction disks, escapement mechanisms, copper jacketed coils, etc., as the source of time delay. Only a relatively few devices using air bellows or oil dash pots remain in active production, and the application of such devices is limited to cases where the operating requirements are less severe.

Practically all devices used in automatic switchgear have a 10 per cent overvoltage rating. However, there have been cases where extensive regulation was obtained or other local conditions entered so as to cause the voltage to exceed 110 per cent normal for an appreciable length of time. Once the range is known, and it comes within the usual variation from maximum to minimum, it is possible to meet the requirements with available devices.

The late progress in the art confirms the decision that individual electrically-operated devices give the most flexible and least expensive arrangement.

An interesting comment is the absence of three-phase voltage check in the more recent installation, which action has been ustified from past operating experience. The increase in tripping battery voltage from 12 to 24 volts was found to leave a safer operating margin.

The writer feels that a standard a-c reclosing feeder is automatic in the sense as covered by definitions 26-50 and 26-53 of the Institute Standards. It is true that this type of equipment is not as elaborate as some machine equipments for example, in that it uses fewer devices and has less functions to perform. This, however, is due to the character of service afforded and is not a limitation of the equipment itself. A so-called standard reclosing feeder is:

a. Unattended in the usual installation.

b. Goes into operation by an automatic sequence under predetermined conditions.

c. Maintains the required character of service by automatic means.

d. Goes out of operation by an automatic sequence under other predetermined conditions.

e. Provides protection against all usual operating emergencies.

This paper brings out a number of valuable conclusions that are common to all forms of automatic switchgear. The more important of these conclusions are:

a. Reduced time of outage.

b. Not subject to errors in judgment.

c. Reliable in its operation. Results obtained are at least as good as those of the best type of operator.

d. Permits reduction in operating costs.

e. Applicable to isolated locations where it would be difficult to establish the desired type of operator.

f. Flexibility makes it readily adaptable to operating conditions.

g. Automatic switchgear is designed to require a minimum amount of maintenance which should be carried out by men particularly well qualified for this type of work.

Robert Treat: It has already been noted that in pursuing the paths of progress we sometimes find that a certain course of procedure results in a collateral and perhaps unexpected advantage which is more important or more valuable than the primary objective originally sought. In this connection the following verbatim quotations from the authors' paper are of interest. "The first urge behind the idea of automatic stations and substations was an economic one." "On the whole, the automatic substation is *often* less expensive to operate than an equivalent attended one." "By its reliability, speed of action, and freedom from errors of judgment, the automatic station gives a brand of service which cannot be approached by even the best attended substations." The authors found from experience that they *often* (presumably not always) achieved what was announced as the first urge behind the idea, but they put no such limitation on the improvement of service which resulted.

A further advance in the adaptation of automatic devices which is being introduced by a few operators lies in the automatic reclosure of circuit breakers instantly instead of after a time delay, as mentioned in the paper. Under certain conditions it has been found that breakers can be reclosed so promptly as to produce on the customer the impression that there was no interruption at all. It seems quite logical that such a procedure should secure quite widespread acceptance in the near future.

There is a tendency not only in the operation of electric substations, but apparently in all walks of life, to replace human effort which is expended in doing uniform repetitive tasks, with automatic machinery. We have already come a long way from the time when about the only automatic operation in a power station was performed by the speed governor. But there are still many tasks performed in power stations and substations which in the future will probably be taken from the operator. For example, one seldom visits a station without witnessing the operator making his half-hourly rounds taking meter readings to be added to the millions which he has already collected—for what purpose we sometimes wonder. May it not be that in the near future some genius will conceive a simple and inexpensive movie camera which can be permanently set up and trained on the entire switchboard; which automatically changes one frame and takes a picture of the board and all its instrument indications once every half-hour or other desired interval. Such records would at least have the merit of compactness.

Frequently it is not the values of voltage, current, kilowatts, etc., during normal steady system conditions in which we are most interested, but the values during a second or so of transient caused by a momentary abnormal condition. Here again the human operator is limited in performance because even if the instrument needles correctly followed the rapid changes in the measured magnitudes, the operator can hardly read even one, let alone the several in which we are interested. To supply this deficiency the automatic oscillograph has been developed. This constitutes still another tool for the further improvement of service.

Vibration and Fatigue in Electrical Conductors

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***Synopsis.**—This paper deals exclusively with the phenomenon "vibration." No reference is made to the allied phenomenon "dancing." The purpose of the paper is to stimulate further research in an endeavor to provide a fundamental preventive of vibration and the resulting fatigue. Economic features such as reducing tensions and spans, are not treated. Published information is referred to and the need of attacking the fundamentals causing vibration is investigated and demonstrated. Experiments using water as a medium on some special cable sections are described in detail. The paper shows that certain peculiar but not altogether impractical sections minimize the tendency of vibration and the resulting fatigue. The ideas underlying the design of these sections are discussed and conclusions drawn. Practical objections to these proposed cable sections are included. Further research is indicated.*

BECAUSE of a general increase in the unit working loads as well as in sizes of conductors, since about 1920, vibration has now become a problem for transmission organizations.

This very rapid vibration of overhead conductors (sometimes resulting in a musical note) was probably first reported, as deteriorating the mechanical characteristics of conductors, from California in 1923. These vibrations seldom have an amplitude exceeding two inches; the node lengths vary considerably and the frequency is of the order of 10 to 100 cycles per second.

Various organizations and interests, such as the State Electricity Commission of Victoria, Australia, the manufacturers of electrical conductors, and the electric utilities of Southern California, soon became interested in reports of these phenomena.

The need for fundamental preventives was made evident because of the following reports of results of vibration as found in the technical press. Bolts in towers became loosened and members were likely to become inoperative. Flat straps supporting arms of the towers seemed to vibrate on their own account. Insulator hardware has been damaged. In other cases, cables have had numerous strands cracked.

REMEDIAL MEASURES

To overcome all these difficulties numerous mechanisms and appliances have been suggested and some of them function very well indeed. They are, however, always subject to the criticism that they may ultimately account for more harm than good. On the other hand, if the cause of the problem can be determined, and preventives applied at the source, there will then be little danger of the trouble showing up later at some unexpected point.

BIBLIOGRAPHY OF VIBRATION

A bibliography of the subject referring largely to cables and wires, and consisting of some 350 items, has been assembled, a large part of which was supplied by cable manufacturers, the American Society of Mechanical Engineers, and the Engineering Foundation. Of these, 65 articles were reviewed and studied. These articles may be divided into two sections, (1) those which deal with reinforcements, absorbers, and generally with curative measures; and (2) those which approach the problem from a fundamental standpoint, by determining the cause and proposing and reporting upon preventive measures.

REVIEW OF INVESTIGATIONAL WORK DONE ON VIBRATION IN CABLES

The more important researches were carried out by Relf and Ower, Karman, Shiba, Varney, Stockbridge, Thoma and Bate. Articles by Relf and Ower[1] and by Karman,[2] dealt with the theory of formation of eddies at the lee-side of stream flow and their relations to the periodicity of the resulting vibrations. Shiba of Japan has photographed the eddy formations, using smoke streams and a special camera taking 12,000 to 20,000 pictures per second by which the relation of eddy frequency to that of the vibrations might be more readily studied. Varney,[3] Stockbridge[4] and Bate[5] discussed various dampers, and generally put forward curative measures and absorbing devices.

EFFECTS OF CONDUCTOR SHAPES

Following reports of a field observation from the Pacific Coast that a three-strand cable did not vibrate as heavily as a standard cable,[6] also that a single strand wrapped about a cable seemed to reduce the vibration,[7] it was thought, although a somewhat impractical cable cross-section might result, that some important principle might be involved which had not been investigated sufficiently. Again, there appeared to be an opportunity of providing fundamentally a preventive of vibration failure by modifying the section of the conductor rather than that afforded remedially by adding reinforcements and absorbers. Accordingly it was decided to make further studies of specially stranded and deformed cables.

Attention was soon called to Thoma's[8] findings, namely, that the section of the cable did play an important part in the vibration characteristics. It was thought that by upsetting the symmetry of the section, the eddies would be disrupted. This is explained in the following theory.

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*Engg. Dept., Hydro-Electric Power Commission of Ontario, Toronto, Canada.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

1. For references see Bibliography.

Ideas Governing the Design of Non-Symmetrical Sections

The successive cross-sections of a 5/8-in. diameter round rod with a 1/8-in. diameter wire spiralled about it, as shown in Fig. 1, have been selected to indicate the complicated system of frequencies brought about by the eddy formation at each of these successive cross-sections along the cable. Balancing of eddy forces is demonstrated at individual pairs of cross-sections. A 5/8-in. plain round rod is examined for comparison and it will be seen that there is no balancing action in this case. Examination of stream line flow was carried out by referring to photographs of somewhat similar cases as obtained by Shiba.

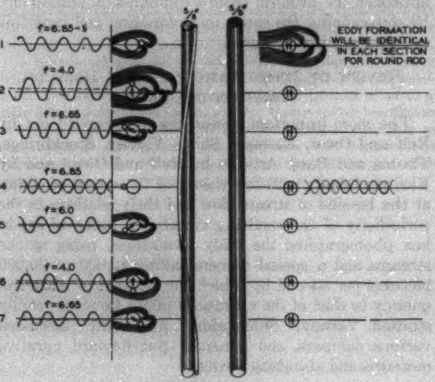
5/8"
5/8"
f=6.65-5
f=4.0
f=6.65
f=6.65
f=6.0
f=4.0
f=6.65
1
2
3
4
5
6
7
EDDY FORMATION WILL BE IDENTICAL IN EACH SECTION FOR ROUND ROD

Fig. 1—Study of Eddy Formation in Lee of Uniform and Non-Uniform Specimens

Comparison of normal 5/8-in. round rod and 5/8-in. rod with 1/8-in. wire spiralled about it

$$\text{Frequency} = \frac{V}{D} \times \phi\left(\frac{VD}{\epsilon}\right)$$

Relf and Ower. Aeronautical Research Committee No. 825

Four variables which modify the resultant eddy action behind the rod and wire are introduced in each individual cross-section, namely:

(a) Frequency
(b) Phase
(c) Amplitude
(d) Neutral vibration plane

Each of these is studied in detail, as follows:

(a) Frequencies were calculated from the Relf and Ower formula. Assuming constant velocity of flow, the frequency is proportional to the diameter of the section. By varying this diameter, different frequencies will occur. In this case, variation is brought about by the spiralling of the wire about the rod. The effective diameter of the combination will be the diameter of the rod plus in cases such as sections 1, 2, 3, 5, 6, etc., Fig. 1, a percentage of the diameter of the wire. Only the diameters were considered in determining the frequencies and the theoretical curves were plotted from these values. These curves will then represent, to some scale, the actual frequency of the section.

(b) The phase of the vibration of an individual section will be governed by the position of the outer wire and has been plotted accordingly in the diagram.

(c) The force acting in the direction of vibration will be:

$$p = KV^2 \times D \text{ per ft. run per lb.}^5$$

Where P = force, V = velocity, D = diameter Assuming velocity constant, the force will be proportional to the diameter. The amplitude of the vibration will, therefore, be proportional to the diameter. For each section in the diagram, the maximum amplitude has been plotted equal to the effective diameter and will, therefore, represent to scale the true amplitude.

(d) Vibration will take place about the center of impact of the section which will be offset from the horizontal diameter of the rod. This has been indicated by a broken line in the diagram.

The eddy formation and resulting forces have been plotted for an instantaneous position of the rod and will reverse at periods according to the frequencies involved, as indicated by the double arrows in the case of the round rod and double wave at cross-section No. 4, Fig. 1. No attempt has been made here to establish the exact mathematical value of the quantities involved. This is being investigated. The diagram illustrates the principle qualitatively, but not quantitatively.

It will be seen that at sections 3 and 7, and at 2 and 6, Fig. 1, the frequencies and amplitudes (therefore forces) are equal but are 180 deg. out of phase. They will, therefore, tend to balance one another in the direction of vibration, thus reducing this objectionable feature. In the case of the symmetrical round rod, the eddy formation will be identical at each section and no balancing tendency is evident.

At section 4, Fig. 1, of the combination of rod and wire, an unstable condition is introduced due to the fact that the forces on the sections on either side of it tend to move the conductor in opposite vertical directions.

It must be noted that this discussion deals with a very small length of cable—with 8 in. lay of the wire; the two boundary sections are separated 8 inches. Similarly, with three-strand cable the boundary sections would be separated by only two and two-third inches. In practise it is highly probable that over such a short section when compared with the length of the span, wind velocity would be approximately constant.

Special Sections Available for Study

The single wire spiralled about the rod is impracticable, on the application of tension, as the load on the cable becomes eccentric. This analysis, however, points the way to modifying, in a practical way, the fundamentals of vibration in cables. For instance, in a three-strand cable, this same complication of frequencies would be introduced, but the tension would be uniformly balanced over the cable section. The four-strand cable is also a good example and is in common use, more especially in steel wire rope. Almost any sector or acorn shape may be made without much loss of mechanical strength by rolling or hammering the round cable as in insulated three-phase cables.

Fig. 2—Sideview of Apparatus Used in Recording Vibrations of Experimental Specimens in Water

As a basis on which to work, it was decided to make sample specimens of sections of cables which were statically balanced so that they would not pull eccentrically. The section should, however, be unsymmetrical about the horizontal axis of the cable and at the same time be unsymmetrical for successive sections along the cable; or in other words, simulate those characteristics which seemed to interfere with vibration in the cable with extra wrapping referred to above. It was evident that as uniform an eddy action could not be expected over a length of these special cables, as appears to be the case with standard cables or round rod, with a consequent reduction of the power input from the wind.

Laboratory Work

Experimental work, based on these assumptions, was carried out in the Hydraulic Laboratory of the University of Toronto. Dr. Thoma's experiments and methods were followed quite closely. Water was used as a medium. The effect of change of shape on the amplitude of the vibration was sought. The specimen conductors of various peculiar shapes were tried at 5 different velocities of water. The range of velocities was from 4 to 7 inches per second. The sample was fastened to a flat steel strap one inch by one thirty-second inch and 5 ft. long, Fig. 2. This strap was used in preference to the elastic steel rod which Thoma used as it restricted movement in the direction of the flow of water most effectively. A light pointer was attached to the strap. This pointer traced the wave motion on a smoke chart, placed on the drum of a vibrograph.

The samples were divided into five groups. Each group had a circular sample as a standard and a number of irregular cross-section samples was compared with it. Projected dimensions, or effective diameters, of all the samples within each group were approximately the same, the object being to obtain the same frequencies of vibrations. In order that the vibrating system in each group should be as nearly as possible the same, the samples were made equal in weight and with centers of mass at the same point as that of the standard.

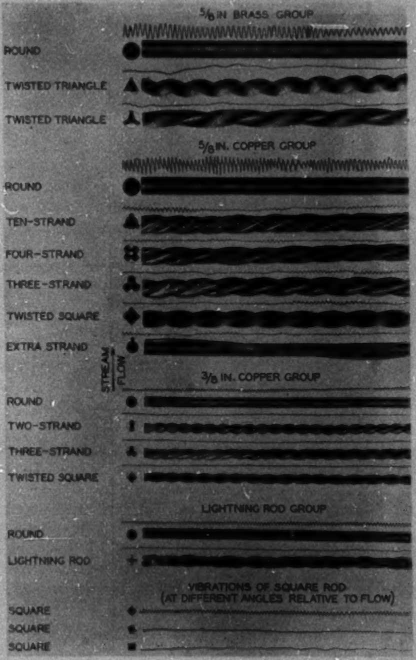

Fig. 3—Vibrations in Water Recorded Comparatively for Various Sections and Types of Conductor

The adjustment for resonance was brought about by varying the length of the strap between the sample and the clamp till the maximum amplitude of vibration of the strap was obtained. Other sections in the corresponding group were similarly treated.

That the required similarity was obtained was evidenced by the fact that in each group adjustment for fundamental resonance of the various samples was negligible and was seldom necessary at all so that the natural frequencies of the systems were approximately identical.

Owing to variations in speed of the smoke chart, no exact check could be kept on the observed frequencies, but the fact that no important adjustments were required within the groups leads to the inference that approximately identical frequencies were obtained in each case. In any case, the final purpose of the experiments was to derive qualitative rather than quantitative results. The design of specimens giving the same vibrating systems within each group (*i. e.*, the mechanical impedance) permitted the use of amplitude as a measure of the relative damping.

NAME	CROSS SECTION	AVERAGE % REDUCTION IN AMPLITUDE
TWISTED CONCAVE TRIANGLE		90%
TWISTED TRIANGLE		90%
TWISTED SQUARE		83%
LIGHTNING ROD WITH HOLES		80%
ROUND, EXTRA STRAND TWISTED ON		70%
THREE-STRAND		70%
TWO-STRAND		70%
FOUR-STRAND		70%
TEN-STRAND SPECIAL		60%
ROUND		0%

Fig. 4—Tabulation of Percentage Reductions of Vibrations as Observed in Water

List of shapes which reduce vibrations. Order of sequence: shapes giving maximum reduction lead the list

Conclusion Drawn from Hydraulic Experiments

Qualitatively the following conclusions may be drawn pending more practical experiments using air as a medium:

1. As evidenced by the charts, Fig. 3, it will be seen that so far as hydraulic experiments are concerned, the twisted triangular section overcomes most satisfactorily the tendency to vibrate.

2. It can be also noticed that practically all the sections tried, decreased the amplitude by at least 50 per cent when compared with the round rod, Fig. 4. From the results, it would seem advisable to construct a cable approximating in cross-section a triangle. Fig. 5 shows a series of cable sections derived from the basic sections which in the laboratory evidenced a considerable reduction in tendency to vibrate.

3. Even if these special triangular and rectangular sections prove impractical in cables, they may be used in outdoor buses where some difficulties of this nature may arise in future.

Later Steps in the Investigation of Vibration in Cables

1. At present special cables approximating triangular cross-sections have been made and are being strung in air to see how closely the hydraulic experiments are in agreement with the practical case. Three sections have been suggested; first, the ordinary three-strand cable which was casually observed in the West as being comparatively free from vibration; second, a nine-strand cable which is made up of 3 strands of three-strand cable with rope lay; and third, a special nine-strand cable using two sizes of strands, the three larger strands so placed in the outside layer as to give approximately the desirable triangular effect, Fig. 6.

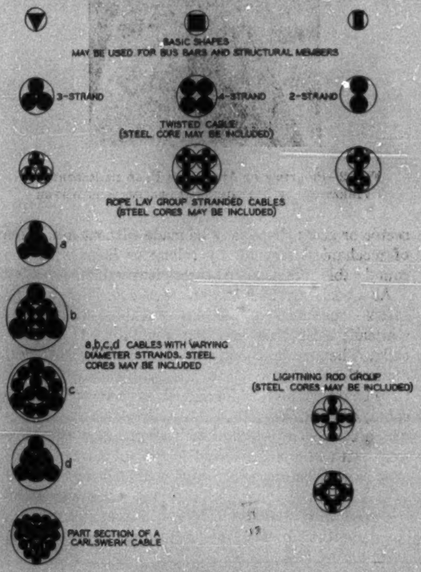
BASIC SHAPES
MAY BE USED FOR BUS BARS AND STRUCTURAL MEMBERS
3-STRAND
4-STRAND
2-STRAND
TWISTED CABLE
(STEEL CORE MAY BE INCLUDED)
ROPE LAY GROUP STRANDED CABLES
(STEEL CORES MAY BE INCLUDED)
a
b
a,b,c,d CABLES WITH VARYING DIAMETER STRANDS. STEEL CORES MAY BE INCLUDED
c
LIGHTNING ROD GROUP
(STEEL CORES MAY BE INCLUDED)
d
PART SECTION OF A CARLSWERK CABLE

Fig. 5—Desirable Cross-Sections for Cables and Buses Which Are Exposed to Light Winds

Any combination of stranding, or grouping of strands of various diameters, which will approximate the basic shapes, *i.e.*, triangular, square, or rectangular, can be included in this set of cross-sections

The special rope lay cable introduces a feature which may be of great importance. Due to a characteristic of the stranding, air gaps, which will not collapse under tension, have been introduced. The presence of these air-gaps may serve to further upset the sequence of eddies behind the cable.

2. Acting conjointly with so-called "stiction" and "interstrand" friction, the elastic properties of the individual strands of a standard cable will tend to modify the vibration. Investigation should be made with the object of segregating these and to determine the comparative importance of the damping in each case. Does interstrand friction absorb any energy at all at the comparatively long radii of curvature found in vibration phenomena?

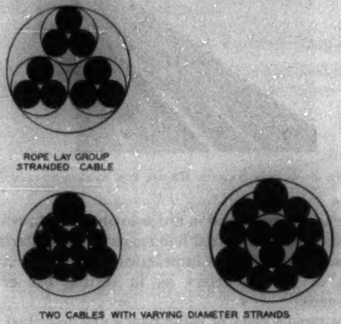
ROPE LAY GROUP STRANDED CABLE
TWO CABLES WITH VARYING DIAMETER STRANDS

Fig. 6—Most Practical Cross-Sections for Suppression of Vibration

3. Experiments should be carried out to verify the following predictions which were made on two cables (6 by 0.2108; 7 by 0.0705, 266,800 cir. mils A.C.S.R., Owl, and 30 by 0.1059; 7 by 0.1059, 336,400 cir. mils A.C.S.R., Oriole). An attempt was made mathematically to predict the characteristic dimensions of vibration in these two cables and to balance energy input against energy dissipated, the remainder of the energy doing work on the cable at points of discontinuity, such as clamps. Many assumptions had to be made. Initial attempts at measurement in the field were unsatisfactory and this work was temporarily abandoned.

4. A determination of the actuating force per foot length of cable should be carried out. This could be done in a suitable wind-tunnel with adjustable elastic supports for the specimen under test.

PRACTICAL DIFFICULTIES

There are many objections to any departures from standard cables such as are indicated by these experiments. It is hoped that so far as vibration is concerned the proposed sections will indicate some new properties which in some cases at least will outweigh practical difficulties, such as standardization.

The corona loss may be increased to some extent. Also the increased diameters would account for greater wind pressures and sleet loadings. There would be some difficulty in making joints. Two sizes of strands in one cable would be a digression from the standards. If more than one pass through the stranding machine is required on account of these various sizes of strand, then the result would be increased cost.

Discussions and studies of these details are deferred until there is an opportunity to confirm in a practical way in air that the amplitudes of vibrations are suppressed.

ACKNOWLEDGMENTS

Thanks are extended to Messrs. W. P. Dobson and W. B. Buchanan of the Laboratories of the Hydro-Electric Power Commission for cooperation in this work and for making several valuable suggestions.

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Four appendixes, including bibliography, have been assembled during experimental work and the preparation of this paper. They are available upon requests to the authors. They are,

A. Visualizing vibrations in small wires.
B. Bibliography.
C. Treatment of resonance by electrical analogies verifying laboratory method of investigation.
D. Mathematical investigation of vibration of 5/0 and 6/0 cables. (6 by 0.2108; 7 by 0.0705, 266,800 cir. mils A.C.S.R., Owl and 30 by 0.1059; 7 by 0.1059, 336,400 cir. mils A.C.S.R., Oriole).

Discussion

For discussion of this paper see page 1076.

Stress-Strain Studies of Transmission Line Conductors

BY G. W. STICKLEY*
Associate, A.I.E.E.

INTRODUCTION

THE actual tensile behavior of materials can be shown graphically by curves plotted from stress-strain tests, with stresses plotted as ordinates and strains (or extensions) as abscissas. Although it is recognized that stress-strain curves of stranded cables are appreciably different from those of single wires or rods, there is a comparatively small amount of information on this subject published in technical literature. With the advances being made in the solution of engineering problems, especially in the design of electrical transmission lines, these differences are becoming of increasing importance, and it is recognized that appreciable errors in design may result from neglect of them.

The purpose of this paper is to present the results of typical tensile stress-strain tests, illustrating the actual mechanical behavior of various types and kinds of cables used in overhead transmission lines, and also to present the results of similar tests illustrating the manner in which this behavior is affected by certain factors such as repeated stressing and lay.†

FIG. 1—100,000-LB. CAPACITY AMSLER CABLE TESTING MACHINE

CABLE TESTING EQUIPMENT AND TEST PROCEDURE

Perhaps one reason that more information has not been published regarding the actual mechanical behavior of stranded conductors has been that most physical testing laboratories do not have suitable testing machines and accessory equipment. The fact that a cable is composed of a number of spiralled wires makes it necessary to use a long test sample in order to minimize the effects of any disturbance of strands during preparation of the test sample and in order to obtain data which are representative of actual line spans of the material. For the same reason it is impractical to use the ordinary types of extensometers as in testing bar or sheet specimens.

FIG. 2—CABLE SAMPLE READY FOR STRESS-STRAIN TEST

The tensile tests of cable which are described in this article were made in an Amsler horizontal cable testing machine having four capacity ranges varying from 10,000 to 100,000 lb. and in which the samples tested usually are 50 feet long. When making a stress-strain test the cable sample was suitably supported in a trough throughout the entire gage length in order to eliminate sag and to insure straightness of the sample at zero

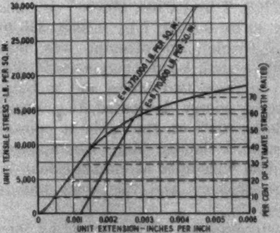

FIG. 3—REPEATED STRESS-STRAIN CURVES

266.800-cir. mil aluminum cable (7 × 0.1953 in.)

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†The lay of a cable is the length expressed in inches for each complete turn of the wire around the axis, measured along its axis.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

stress. The elevation of the trough was such that when tension was applied the cable barely touched it. The apparatus used in obtaining the extension measurements consisted of telescopes mounted above steel scales fastened to the cable at each end of the gage length. Fig. 1 shows the testing machine, and Fig. 2 the trough, scales, and telescopes in place for making a stress-strain test.

Because one of the original reasons for making the stress-strain tests used as illustrations in this article was to show the behavior of transmission line cables both before and after loading, each test contained two stress cycles. In the first, the cable was stressed to some value equal to the highest maximum loading condition which might be used in design, this value varying from 50 to 70 per cent of the nominal ultimate strength. The stress then was removed, and in the second cycle the sample was stressed to a much higher value.

Results of Typical Tests of Various Cables

During the last several years tests as described in the preceding paragraphs have been made on more than eighty different sizes, types, and kinds of cable, including aluminum, aluminum alloy, copper, steel, A.C.S.R. (aluminum cable steel reinforced), and copperweld cables. Figs. 3 to 11 inclusive show typical repeated stress-strain curves which were obtained, and Table I contains average physical data from the tests of certain sizes and types of cable. Stress-strain curves of single wires from certain of these cables are shown in Figs. 12 and 13.

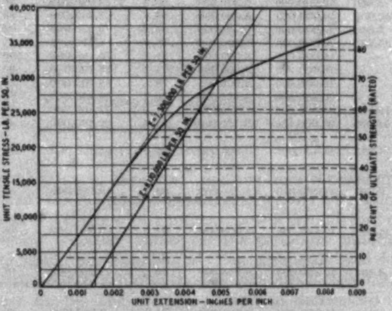

Fig. 4—Repeated Stress-Strain Curves

745,000-cir mil Aldrey (aluminum alloy) cable (61 × 0.1105 in.)

In Table I the theoretical values for the modulus of elasticity of aluminum, aluminum alloy, copper, and steel cables are the generally accepted values for these materials when in the form of bar, rod, or sheet. The corresponding values for the various types of A.C.S.R. are calculated from the theoretical modulus values for aluminum and steel and the proportionate cross-sections of the two metals in the respective cables. In the table there are two values for the actual modulus of each cable, the initial value being that which was obtained when the cable had not previously been stressed, and the permanent value the one after the cable had been stressed to from 50 to 70 per cent of its ultimate strength.

The fact that a cable is not perfectly elastic throughout the range of stress to which it may be subjected is overlooked frequently in calculating sags and tensions of transmission line conductors, and appreciable errors may result. As shown in some of the stress-strain curves included in this article, the base of the curve for the first stress cycle contains a small "foot" as a result of a small amount of unequal stressing of the individual wires, no doubt caused by a slight unavoidable looseness of the strands. The curves also show that the proportional limit is much lower than the stress at the assumed maximum loading condition, this in some cases occurring at a stress not greater than half of the latter value. As has been stated in a preceding paragraph, the assumed maximum loading condition frequently is from 50 to 70 per cent of the ultimate strength of the cable. Further examination of the stress-strain curves shows that application of some stress such as that equal to the assumed maximum loading condition eliminates the "foot" which originally occurred at low stresses and raises the proportional limit to a value near or equal to the stress applied.

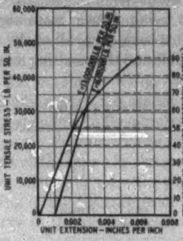

Fig. 5—Repeated Stress-Strain Curves

4/0 medium hard drawn copper cable (7 × 0.1739 in.)

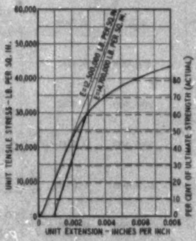

Fig. 6—Repeated Stress-Strain Curves

300,000-cir. mil medium hard drawn copper cable (19 × 0.1256 in.)

As will be noted in studying Table I, and in examining the stress-strain curves, the actual modulus of elasticity the first time that stress was applied to each cable was lower than that obtained the second time. The difference was less pronounced in cables composed of a small number of wires and in those made of high strength material. The first time that stress is applied it seems that the extension of the cable is not entirely the result of actual extension in length of the individual wires because of the tensile stress in them, but is partly caused by the individual wires being drawn more tightly in place. After this has once been done, all of the wires act more as a unit in the cable and the resulting modulus is higher.

As will be shown in several of the following paragraphs, tests have proved that the actual increase in modulus resulting from the application of stress depends, to a certain extent, on the value of stress applied, and that after a cable has once been stressed to a certain tension, the same load can be removed and applied again repeatedly without causing any additional change in modulus. In another paragraph it will be shown that coiling or otherwise disturbing a cable after its modulus has been increased by stressing to some tension will result in the modulus being lowered appreciably to some value usually greater than the initial value. This is considered evidence that the extension of a cable is not en-

TABLE I—MODULUS OF ELASTICITY OF SOME TRANSMISSION LINE CABLES

					Modulus of elasticity				
						Actual			
						Initial		Permanent	
Kind of cable	Size -Cir. mil or No.	Diam. -In.	Stranding	Nominal ultimate strength -Lb.	Theoretical* Lb. per sq. in.	Lb. per sq. in.	% of theo.	Lb. per sq. in.	% of theo.
Aluminum	266,800	0.586	7 × 0.1953	5,030	10,000,000	6,770,000	67.7	8,770,000	87.7
Aluminum	300,000	0.628	19 × 0.1256	5,650	10,000,000	8,330,000	83.3	9,080,000	90.8
Aluminum	1,590,000	1.454	61 × 0.1615	30,000	10,000,000	6,000,000	60.0	8,350,000	83.5
Aluminum alloy	195,700	0.502	7 × 0.1672	6,550	10,000,000	8,420,000	84.2	9,250,000	92.5
Aluminum alloy	745,000	0.995	61 × 0.1105	24,935	10,000,000	7,300,000	73.0	8,170,000	81.7
M. H. D. copper	4/0	0.522	7 × 0.1739		16,000,000	13,000,000	81.2	16,100,000	100.6
M. H. D. copper	250,000	0.600	12 × 0.1443		16,000,000	10,300,000	64.4	13,700,000	85.6
M. H.;D. copper	300,000	0.628	19 × 0.1256		16,000,000	12,500,000	78.1	14,700,000	91.9
M. H. D. copper	500,000	0.813	37 × 0.1162		16,000,000	9,450,000	59.1	13,200,000	82.5
Galvanized steel		0.364	7 × 0.1214	15,370	30,000,000	27,300,000	91.0	27,300,000	91.0
Galvanized steel		0.703	37 × 0.1004	55,600	30,000,000	25,400,000	84.6	26,100,000	87.0
A.C.S.R	6	0.198	6 × 0.0661 Al 1 × 0.0661 St (14.3% steel)	1,045	12,860,000	10,800,000	84.0	11,240,000	87.4
A.C.S.R	101,800	0.276	12 × 0.0921 Al 7 × 0.0921 St (36.8% steel)	9,365	17,370,000	14,450,000	83.2	15,550,000	89.5
A.C.S.R	203,200	0.714	16 × 0.1127 Al 19 × 0.0977 St (47.2% steel)	26,600	19,440,000	16,000,000	82.3	16,800,000	86.4
A.C.S.R	4/0	0.563	6 × 0.1878 Al 1 × 0.1878 St (14.3% steel)	8,435	12,860,000	8,890,000	69.1	11,260,000	87.6
A.C.S.R	266,800	0.609	18 × 0.1217 Al 1 × 0.1217 St (5.3% steel)	6,890	11,050,000	8,750,000	79.2	9,580,000	86.7
A.C.S.R	266,800	0.633	6 × 0.2109 Al 7 × 0.0703 St (11.5% steel)	9,385	12,300,000	10,000,000	81.3	11,260,000	91.6
A.C.S.R	336,400	0.721	26 × 0.1138 Al 7 × 0.0885 St (14.1% steel)	13,230	12,800,000	9,380,000	73.3	11,110,000	86.8
A.C.S.R	378,800	1.074	21 × 0.1343 Al 37 × 0.1151 St (56.4% steel)	70,700	21,300,000	16,650,000	78.2	17,000,000	79.8
A.C.S.R	397,500	0.806	30 × 0.1151 Al 7 × 0.1151 St (18.9% steel)	19,170	13,780,000	9,380,000	68.1	11,220,000	81.4
A.C.S.R	795,000	1.093	54 × 0.1214 Al 7 × 0.1214 St (11.5% steel)	27,950	12,300,000	8,110,000	66.0	10,100,000	82.1
A.C.S.R	795,000	1.140	30 × 0.1628 Al 19 × 0.0977 St (18.6% steel)	37,770	13,720,000	8,365,000	61.0	10,600,000	77.2
Standard tensile copperweld	3 No. 8	.0.276	3 × 0.128		23,000,000	20,800,000	90.4	24,080,000	104.6
Standard tensile copperweld	3/8 in. (7 No. 8)	.0.384	7 × 0.128		23,000,000	22,170,000	96.4	22,600,000	98.2
Xtra-hi-tensile copperweld	3 No. 8	.0.276	3 × 0.128		23,000,000	22,470,000	97.7	23,200,000	100.8
Xtra-hi-tensile copperweld	3/8 in. (7 No. 8)	.0.384	7 × 0.128		23,000,000	21,800,000	94.8	23,000,000	100.0

*Values for aluminum and aluminum alloy from *Aluminum Industry*, Vol. II; value for copper from *National Metals Handbook*; value for copperweld from Wire Tables, Copper Weld Steel Co.

tirely the result of actual extension in length of the individual wires, as explained in the preceding paragraph.

Comparisons of the modulus values in Figs. 12 and 13 with those in Figs. 3 and 8, respectively, show that even after increasing the modulus of elasticity of a cable by first applying some appreciable stress, the resulting value is less than the actual modulus of a representative wire from the cable, this difference in one case being as much as 12 per cent.

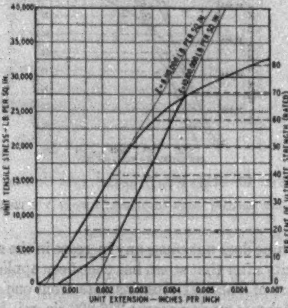

Fig. 7—Repeated Stress-Strain Curves

795,000 cir. mil A.C.S.R. $\frac{54 \times 0.1214 \text{ in. aluminum}}{7 \times 0.1214 \text{ in. steel}}$

In addition to the errors resulting from the assumption that the modulus of elasticity of a cable is the same as that of a single wire, another source of errors is that of assuming incorrect values for the particular material being used. For example, a value commonly used for galvanized steel cables in transmission lines is 30,000,000 lb. per sq. in. The average modulus as determined in more than fifty tests of various sizes of galvanized steel wire varying from 0.0525 in. to 0.188 in. diameter, and in which a small stress had first been applied which straightened the wire, was 28,000,000 lb. per sq. in. Another example is the modulus of elasticity of aluminum, for which the values found in different handbooks vary from 9,000,000 to 11,000,000 lb. per sq. in. although the actual value, based on numerous tests of bar, rod, and sheet is 10,000,000 lb. per sq. in.*

Because A.C.S.R. is a composite cable made of aluminum and steel wires, some additional phenomena occur which are not found in tests of cable composed of a single material. As shown in Fig. 7, the part of the curve obtained in the second stress cycle contains a comparatively large "foot" at its base, which is caused by the difference in extension at the elastic limits of the aluminum and steel. In the curves shown in Fig. 12 the proportional limit, which for the purposes of this explanation may be considered practically the same as the elastic limit, is approximately 8,000 lb. per sq. in., the corresponding extension being 0.080 per cent. In the test of the steel wire, as shown in Fig. 13, the corresponding values are about 60,000 lb. per sq. in. and 0.215 per cent. When a sample of each of these two wires is gripped so that both will have to extend equal amounts, as when these two materials are used in A.C.S.R., a load which will produce a stress just equal to the 60,000 lb. per sq. in. in the steel will not give the steel any permanent set, but obviously will produce a pronounced permanent set in the aluminum. Then, if the stress is gradually removed, there will be a certain tension below which the steel wire will be carrying the entire load. In the test shown in Fig. 7 the stress applied during initial loading gave the aluminum wires a permanent set corresponding to the abscissa of the point at which there is a very definite change in direction of the straight sections of the curve in the second stress cycle (0.230 per cent), and gave the steel wires a permanent set corresponding to the distance from the origin to the base of the curve in the second stress cycle (0.070 per cent).

Fig. 9 is of interest because it contains an analysis showing the distribution of stress between the steel and aluminum wires in A.C.S.R., the analysis having been calculated from Figs. 7 and 8. As shown in Fig. 9 the actual unit stress in the steel core when the cable had not previously been stressed was never less than six times the corresponding actual unit stress in the aluminum wires. Most of this difference was the result of the difference in modulus of elasticity of the two materials, and the rest was caused by the "foot" which occurred at low stress in the initial part of the stress-strain curve. The analysis also shows that, because of the difference in extensions at the elastic limits of the two materials, as explained in the preceding paragraph, a different distribution of stress occurs after a stress greater than the elastic limit of the aluminum has been applied. In this

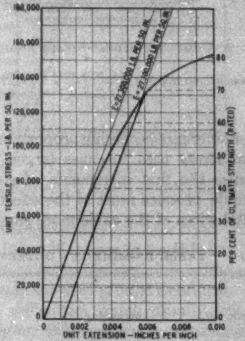

Fig. 8—Repeated Stress-Strain Curves

Steel core of 795,000 cir. mil A.C.S.R. (7 × 0.1214 in.)

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*The Aluminum Industry, Vol. II.

test the tension applied was such that when the cable stress subsequently was less than 6,500 lb. per sq. in. (approximately 15 per cent of the ultimate strength) the steel core was carrying all of the load.

RESULTS OF TESTS ILLUSTRATING EFFECT OF SEVERAL FACTORS

As has already been shown in this article, the application of stress of between 60 and 70 per cent of the nominal ultimate strength of a cable increases its actual modulus of elasticity materially. However, the actual increase depends on the magnitude of the stress applied, the amount of increase becoming proportionately less as the value of stress used becomes larger. This seems logical because the first increments of stress applied naturally would produce most of the effect of drawing the individual wires of the cable more snugly in place.

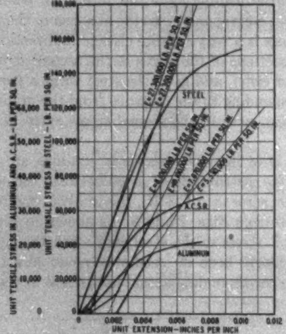

Fig. 9—Computed Stress-Strain Analysis

795,000 cir. mil A.C.S.R. $\frac{54 \times 0.1214 \text{ in. aluminum}}{7 \times 0.1214 \text{ in. steel}}$

Data from a test of 795,000 cir. mil A.C.S.R. (54 aluminum, 7 steel) which illustrate the increase in modulus resulting from certain values of maximum stress previously applied are included in the first part of Table II.

As stated in a preceding paragraph, tests have been made which prove that repeatedly stressing a cable to a certain value does not cause any increase in the modulus of elasticity after that stress has once been applied. The latter part of Table II contains data from one of these tests which was made on some 266,800 cir. mil A.C.S.R. (24 aluminum, 7 steel) in which a stress equal to 66 per cent of the ultimate strength was applied and removed eight times, and was maintained constant at least 30 minutes each time before being removed. Although slight differences in modulus were obtained, the variation was not significant or definite.

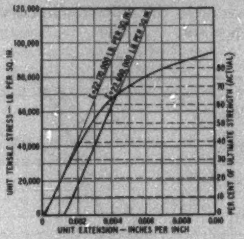

Fig. 10—Repeated Stress-Strain Curves

3/8-inch standard tensile copperweld cable (7 × 0.128 in.)

Another factor which slightly affects the actual initial modulus of a cable is the diameter of the coil or of the reel drum around which the cable has been wrapped, the reason for this being that the cable is disturbed more by wrapping it around the smaller diameter. In tests of two samples of 795,000 cir. mil A.C.S.R. (30 aluminum, 19 steel) from the same length of cable but from inner and outer coils on the reel the modulus was 400,000 lb. per

TABLE II—MODULUS DETERMINATIONS—REPEATED STRESS-STRAIN TESTS OF A.C.S.R.

Cable size—cir. mil.	Stranding	Stress cycle	Max. stress previously applied—% of ultimate strength	Modulus of elasticity	
				Lb./sq. in.	% increase
795,000	$\frac{54 \times 0.1214 \text{ in. Al.}}{7 \times 0.1214 \text{ in. St.}}$	1	0	7,960,000	0
		2	38	9,320,000	17.1
		3	55	9,677,000	21.5
		4	60	9,820,000	23.4
		5	81	9,840,000	23.6
		6	86	9,840,000	23.6
266,800	$\frac{24 \times 0.1063 \text{ in. Al.}}{7 \times 0.0677 \text{ in. St.}}$	1	0	8,450,000	0
		2	66	10,200,000	20.7
		3	66	10,450,000	23.6
		4	66	10,350,000	22.5
		5	66	10,450,000	23.6
		6	66	10,200,000	22.5
		7	66	10,450,000	23.6
		8	66	10,450,000	23.6

sq. in. less for the former sample than for the latter, the respective coil diameters being 25 inch and 63 inch. This difference in modulus was 3.7 per cent.

As the length of lay of the wires of a cable is increased the cable tends to behave more as a bundle of parallel wires or a single wire. In other words, as the lay is increased the actual modulus of the cable approaches the modulus of a single wire of the same metal or metals. This effect is illustrated in Fig. 14 which contains the stress-strain curves of three samples of 4/0 A.C.S.R. having different lengths of lay but made from the same original spools of wire. As would be expected these curves also show that the "foot" at the base of each curve was greatest for the cable having the shortest lay.

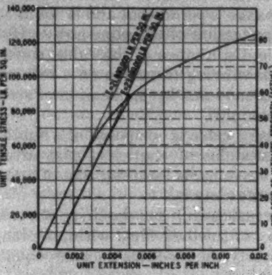

Fig. 11—Repeated Stress-Strain Curves

3/8-inch xtra-hi-tensile copperweld cable (7 × 0.128 in.)

In the design of a composite cable such as A.C.S.R. consideration is given to this effect of lay upon modulus by stranding the steel core with the longest practical lay and the aluminum wires with the shortest practical lay. By this procedure the core has a maximum modulus and the spiralled envelope of aluminum wires a minimum one, which results in the core carrying a maximum share, and the aluminum a minimum share, of the total stress in the cable. In the tests of the three samples of 4/0 A.C.S.R. discussed in the preceding paragraph the stress-strain curves for the aluminum envelopes were obtained in the same manner as was the similar curve in Fig. 9. The three curves are shown in Fig. 14, and as can readily be seen by comparing the stress in the aluminum envelope of each at a certain stress in the complete cable, there is an appreciable difference caused by the difference in lay.

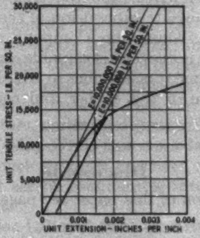

Fig. 12—Repeated Stress-Strain Curves

Hard drawn aluminum wire—0.1953 in. (from 266,800 cir. mil aluminum cable)

An understanding of the mechanical behavior of cables from data such as presented in this article enables transmission line engineers to evaluate the effects of loading. Using this kind of data some engineers have designed lines in which during erection the conductors were stressed for several minutes to a tension equal to that which would result from their assumed maximum loading condition. One result of this procedure is that the cable is given its permanent modulus and its permanent set, and the stringing sag then is the permanent one, unless, of course, the assumed maximum loading conditions are exceeded at some later time. It is evident that this treatment if otherwise feasible would make it unnecessary to take up the sag after a line is built, in order to maintain specified ground clearance.

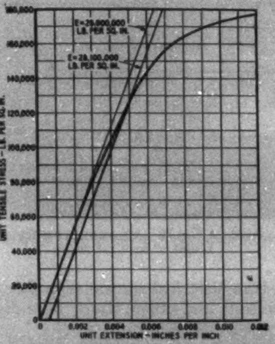

Fig. 13—Repeated Stress-Strain Curves

Galvanized steel wire—0.1214 in. (from core of 795,000 cir. mil A.C.S.R.)

Because of difficulties which usually arise in temporarily applying this maximum tension during the erection of a line, the feasibility of performing this operation using permanent equipment at the mill was investigated. However, stress-strain tests showed that winding the cable for shipment resulted in the loss of a large part of the effect which had been produced. Because of the handling and bending to which the cable is subjected when being wound on a reel, a certain amount of looseness of wires results. In two tests of A.C.S.R. which were made in order to illustrate this loss, the initial and permanent moduli of the first sample were determined. In the second test a stress equal approximately to 70 per cent of the ultimate strength was applied for 30 minutes, the stress was removed, the cable was wound in a coil having a diameter twenty times that of the cable, and then the initial and permanent moduli of the sample after this treatment were determined. The two moduli values obtained in the test of the first sample were 8,350,000 and 10,000,000 lb. per sq. in. respectively, and the corresponding values in the second test were 8,750,000 and 9,850,000 lb. per sq. in.

CONCLUSIONS

The first part of this article describes typical stress-strain tests illustrating the actual mechanical behavior of various bare electrical transmission line conductors. From these tests and others which were discussed, the following conclusions seem evident:

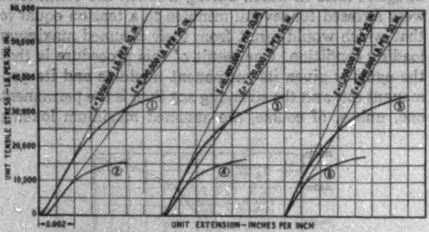

FIG. 14—REPEATED STRESS-STRAIN CURVES

Three samples of 4/0 A.C.S.R. $\frac{6 \times 0.1878 \text{ in. aluminum}}{1 \times 0.1878 \text{ in. steel}}$

NOS. 1 AND 2—A.C.S.R. AND ALUMINUM RESPECTIVELY
Length of lay 5 3/4 inches

NOS. 3 AND 4—A.C.S.R. AND ALUMINUM RESPECTIVELY
Length of lay 7 1/4 inches

NOS. 5 AND 6—A.C.S.R. AND ALUMINUM RESPECTIVELY
Length of lay 9 1/8 inches

1. Cables which have not previously been stressed are not elastic throughout the range of stress to which they ordinarily are subjected when used in transmission lines. The elastic limit usually is considerably less than the assumed maximum loading condition, and the actual extensions of a cable at low stresses are greater than those which would result from the elasticity of the solid material.

2. The proportional limit is raised by application of stress beyond this point. This also results in the cable then being perfectly elastic throughout all or most of the range of stress applied.

3. The application of stress to a cable also increases its actual modulus of elasticity. This increase depends to a certain extent on the magnitude of stress applied, the greatest increases occurring at the lower stress values. After the modulus has been increased in this way, the change is permanent and is not affected by the repeated application of the same stress. However, any considerable disturbance of the cable, such as that resulting from winding the cable on a reel, after having increased its modulus by application of stress, will decrease the modulus.

4. Because a cable is composed of a number of wires, its actual modulus of elasticity, even after being increased by stressing, is less than that of a single wire or rod of the same material.

5. The actual modulus of elasticity of a cable depends upon the lay of the individual wires, the longer the lay the more nearly the modulus becomes equal to that of solid material.

6. Although the temporary application of an initial stress equal to the maximum loading condition produces certain permanent conditions in the mechanical behavior of cables it is not advisable to do this at any place other than in the line during erection, because winding the cable on a reel, or any other handling after application of the stress, results in considerably decreasing the effects originally obtained.

ACKNOWLEDGMENTS

The tests described in this paper were made by the Physical Testing Division of the Aluminum Research Laboratories. This work is under the direction of Mr. R. L. Templin, Chief Engineer of Tests, Aluminum Company of America, and the tests were conducted under the immediate supervision of the writer. To Mr. M. E. Noyes, Aluminum Company of America, acknowledgment is due for his valuable suggestions.

Acknowledgment is also due to Allied Engineers who furnished some of the copper and copperweld cables used.

Discussion

For discussion of this paper see page 1076.

Vibration of Overhead Transmission Lines

BY R. A. MONROE* and R. L. TEMPLIN*
Associate, A.I.E.E. Non-member

INTRODUCTION

FAILURE of conductor strands from fatigue induced by vibration has occurred in widely separated locations throughout the world. Although but a small percentage of the total mileage has been affected, a study of the nature of the phenomenon and means of combating it is warranted.

Any suspended cable, irrespective of material, span length, tension, size, or character of supports will vibrate under certain conditions. This natural phenomenon has always occurred, but only in recent years has it been recognized as the cause of fatigue failures formerly described as crystallization.

About 1925 when the importance of vibration began to be appreciated, Aluminum Company of America started an investigation both in field and laboratory. The early results were presented to the Institute in two papers by Theodore Varney. In the first[4] a theory for the cause of vibration was presented which has received general acceptance, and in the second[5] reinforcement by armor rods was proposed as a means of protection. Recently a number of papers dealing with vibration has been published, particularly abroad.

This paper summarizes and interprets the results of laboratory and field work conducted by Aluminum Company of America since publication of Varney's papers.

Laboratory Work

The problem of eliminating vibration troubles involves the consideration of a number of factors. Determination of these factors demands a better understanding of (1) the actual stresses occurring in the conductors under field conditions; (2) the fatigue or endurance limits of the conductor materials; (3) the effects of various types of conductor accessories; and (4) practical means for minimizing the effects of or preventing harmful vibration. With these considerations in mind a large number of laboratory tests has been carried out during the past seven years, under conditions quite comparable to those in the field. While this work is by no means completed, so much progress has been made that the results obtained are of considerable interest to transmission engineers.

In the Vibration Laboratory at Massena, N. Y., large concrete piers are provided so that thirteen different spans of conductors may be tested simultaneously. The nominal length of each of these spans is 120 ft. Total tensions up to 20,000 lb. can be maintained on each specimen by means of suitable levers and weights. In addition to these, four 50-ft. spans are available for tests of single wires and the smaller stranded conductors. A general view of one wing of the laboratory, Fig. 1, shows some of the longer spans on the left and the four short spans on the right. A more recent installation of vibration equipment is shown in Figs. 2 and 3 where tests at higher frequency can be conducted. The frequencies used for single wires on the four shorter spans are still higher, sometimes being 7,000 per minute. These higher frequencies are made possible by mounting an unbalanced flywheel directly on a specimen as shown in Fig. 4. The motion due to the unbalanced condition of the flywheel when rotating is restricted to the vertical plane only. The use of variable speed friction drives permits the use of constant speed motors. For such tests, however, it is necessary that the speed of the driving unit remain constant within very narrow limits. To accomplish this it was necessary to install a special inverted motor-generator set.

FIG. 1—VIEW OF ONE WING OF VIBRATION LABORATORY

FIG. 2—FRONT VIEW OF VIBRATION APPARATUS

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*Aluminum Company of America, Pittsburgh, Pa.

4. For references see Bibliography.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

Many of the vibration tests have been made primarily to determine the comparative life of various sizes of conductors under very severe conditions. A large mass of data is available but the summary results in Tables I and II for two sizes of A.C.S.R. are sufficient to illustrate a number of interesting facts. In the case of the 795,000 cir. mil A.C.S.R. vibrated in seven loops on 120 ft. span, each loop having an amplitude of 1⅛ in., the marked increase in number of cycles causing failure as the cable tension is decreased should be noted. By reducing the tension from 18,000 to 15,000 lb. the life of the cable is almost doubled for the conditions of vibration used. Similar increases occur with still lower tensions. The data in Table II for 397,500 cir. mil A.C.S.R., vibrated under somewhat different conditions, emphasize this same fact. Further consideration of the data indicates that the outer strands must be severely stressed. This is especially evident in the case of the tests made using the square faced clamps. The repeated bending imposed on these specimens irrespective of the tensions used was undoubtedly more severe than occurs in the field.

FIG. 3—VIEW SHOWING METHOD OF LOADING CABLES IN TENSION

DETERMINATION OF STRESSES

An adequate analysis of stresses in vibration conductors includes a consideration of the general phenomenon of vibration. The studies of such investigators as Varney,[4,5] Relf and Ower,[2] and Ryle[11] have certainly helped to a better understanding of this problem.

It is necessary to know the location and magnitude of the maximum stresses in any conductor under any given set of conditions to determine whether or not the fatigue limit of the material in the strands will be exceeded. If the stresses exceed the fatigue limits of the metals of which the conductor is composed and if the stresses are repeated many times, failure will occur eventually, the time depending upon the magnitude of the stresses.

The most direct method of evaluating the stresses in a cable under given conditions of vibration is by actual measurement. While at first thought this appears impossible because of limitations of available strain measuring apparatus, it has been found feasible by preparing and testing special large specimens, geometrically similar to those of usual size. So far two such specimens of seven-strand conductors have been fabricated and tested. One is a seven-strand hard drawn aluminum cable, each wire being 0.375 in. in diameter and possessing mechanical and electrical properties identical with those of standard conductors. The other is A.C.S.R. composed of six strands of the same wire stranded over a single steel wire. These specimens have been tested using a span of 120 ft. with various tensions and frequencies.

It was found that these specimens could be set in a very steady state of vibration and the position of the top and bottom of the cable at any desirable distance from the end support determined accurately by measuring with micrometer calipers the distance to suitable reference planes. The position of the cable for different test runs could be checked within a few thousandths of an inch, and from the measurements the deflection curve for the cable could be accurately determined when vibrating as well as when not vibrating. It is then comparatively easy to displace the cable under suitable static loads to the same extreme positions it took while vibrating. The strains measured with the cable in the deflected positions should correspond very closely with those in the cable during vibration. Fig. 5 shows a comparison between the measured and computed positions of the cable near the clamped end during vibration. The positions produced by static loading are so nearly identical that the differences cannot be shown in this figure.

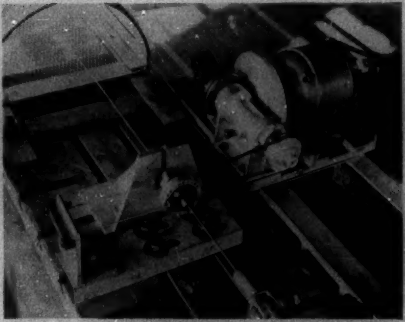

FIG. 4—UNBALANCED FLYWHEEL FOR HIGH FREQUENCY VIBRATION OF WIRES AND SMALL CABLES

By using specimens having such large strands it was possible to attach Huggenberger tensometers, using a ½-in. gage length, to the outside strands at various positions with respect to (1) the distance from the supports, (2) the circumferences of each strand, and (3) the circumference of the conductor as a whole. Fig. 6 shows two of the tensometers attached to a specimen for measuring strains at two particular points. Successive applications of load together with attachment of the tensometers at various points permitted a detailed evaluation of the stresses (strains × modulus), occurring throughout the conductor. Fig. 7 shows a representative set of such data for three positions of the cable. Looking at a cross-section of the conductor from the support end, the top strand is numbered 1, then proceeding clockwise around the central strand, the other strands are numbered consecutively to six, the center strand being designated as 7. The conditions of the test are noted on the graph. The strains measured under static loading were further checked at the lower frequencies by reading the tensometers while the cable was vibrating.

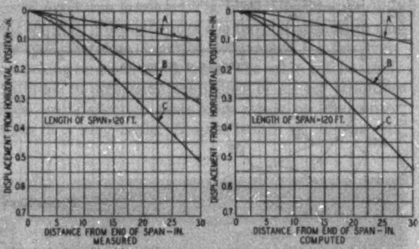

FIG. 5—COMPARISON OF MEASURED AND COMPUTED DEFLECTIONS OF CABLE

A—Upper position when displaced either statically or by vibration (7 loops—1 ⅛ in. amplitude)
B—Natural or free position of cable
C—Lower position of cable when displaced either statically or by vibrations (7 loops—1 ⅛ in. amplitude)

Formulas for the general behavior of a vibrating cable have already been given in papers by Theodore Varney. The mathematical analysis which follows has been developed by R. G. Sturm of the Aluminum Research Laboratories. The frequency at which a cable will vibrate in a loop length, L, is equal to

$$f = \frac{6}{L}\sqrt{\frac{Tg}{W}} \tag{1}$$

where

f = frequency in cycles per sec.,
L = loop length or distance between node points in inches,
T = total tension in cable in lb.,
W = weight of cable in lb. per ft.,

and

g = acceleration due to gravity in ft. per sec. per sec.

Stresses from each of the following sources will be considered independently and the total stress taken as the sum:

1. The stress resulting from the direct tension in the cable.
2. The bending stress resulting from the static sag of the cable.
3. The bending stress resulting from the deformation of loops during vibration.
4. The increased tensile stress caused by the increase in length of the arc of the vibrating cable.

In order to facilitate following the steps in arriving at theoretical values of stress, each source of stress is considered separately.

1. The stress resulting from direct tension depends upon the ratio of steel to aluminum and to a certain degree upon the stranding of the cable. For a solid rod or for a cable in which the strands are all of the same metal this stress may be found by simply dividing the total load by the area of cross-section, *i. e.*,

$$S = \frac{T}{\text{area}} \tag{2}$$

If the cable is bimetallic the stresses in the different materials and the effect of stranding may be determined as outlined by G. W. Stickley.[16]

2. The bending stresses resulting from the weight of the cable will be practically negligible throughout the central portion of the span but might be quite severe in the immediate vicinity of the clamps. These stresses in general are neglected but in some transmission lines they may be far from negligible. If the cable is clamped rigidly in a horizontal position at the end (which is probably a more severe condition than generally exists in actual service) the bending moment at the clamp may be expressed by the equation:

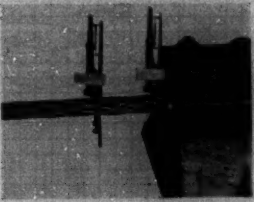

FIG. 6—TENSOMETERS FOR MEASURING STRAINS IN STRANDS

$$M_o = w \cdot \left(\frac{L_s}{2K} - \frac{EI}{T}\right), \tag{3}$$

where

M_o = bending moment at the clamp in in.-lb.,
w = weight of cable in lb. per in.,

L_s = span length of the cable in inches,

EI = flexural rigidity of the cable which depends upon the stranding conditions, but for a solid rod would be the modulus of elasticity times the moment of inertia of cross-section in lb.-in.²,

T = total tension in the cable in lb.,

and

$$K = \sqrt{\frac{T}{EI}}.$$

The stress resulting from the bending moment, M_o, may be figured by the bending moment formula,

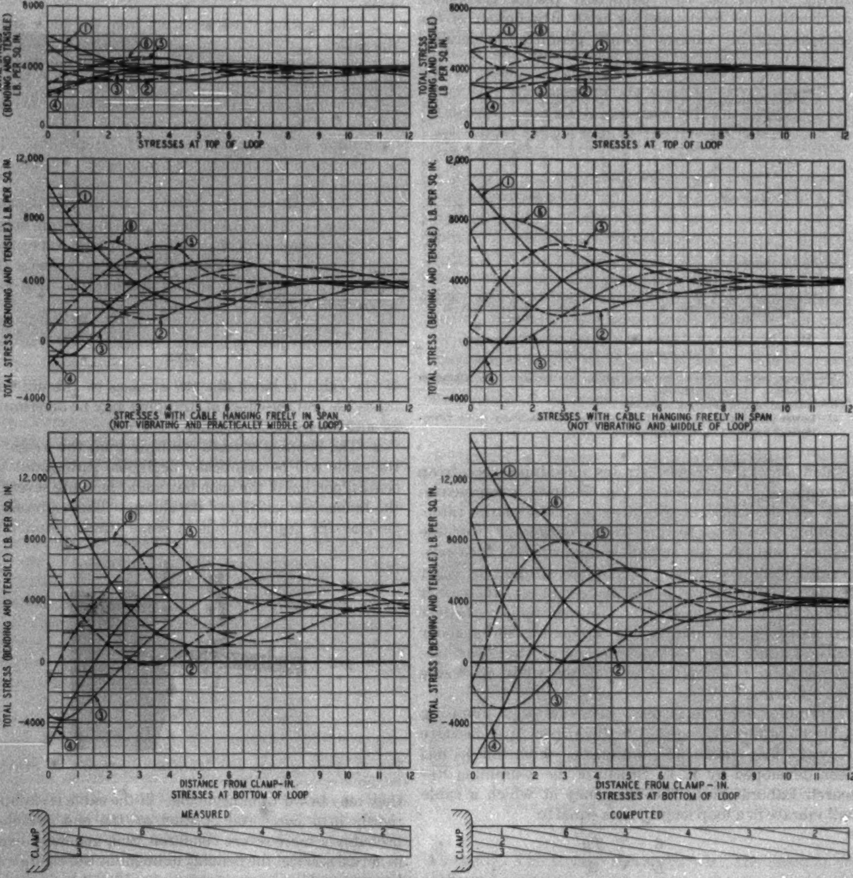

Fig. 7—Distribution of Stress in Cable

Note: Cable tension 6,000 lb.
Tensile stress in aluminum 4,000 lb. per sq. in.
Length of span 120 ft.
Length of lay of cable 12 in.
Clamp at end of span square

The stress in the different strands of the cable at the position of top of loop and bottom of loop was determined for the cable deflected statically to the same position as when it was being vibrated in 7 loops and with 1 1/8 in. amplitude

$$S = \frac{M_o c E_o}{3\,EI}, \tag{4}$$

where

E_o = modulus of elasticity of the material in the outer strands in lb. per sq. in.,

EI = flexural rigidity of the cable in lb.-in.^{2},

and

c = half the outside diameter of the cable in inches.

The value of EI for a stranded cable can not be determined analytically because of the interplay of the strands upon one another. The effect of the stranding is not entirely independent of the tension applied to the cable and, therefore, must be determined for the average working tensions under which the cable will be used.

In order to determine the values of EI for stranded cables, load-deflection tests were made on the center section of a long cable. A span length of at least six times the lay of the cable was chosen in the center portion of the span. Supports were blocked up under the cable and the cable clamped to the supports after the tension had been applied. (See Fig. 8.) Transverse load was then applied to the midpoint of this secondary span and deflection readings taken for increments of load. The ordinary beam formulas for determining these deflections no longer hold because of the relatively large direct stress in the member. Setting up the differential equation representing equilibrium and continuity in the beam, it is found, however, that

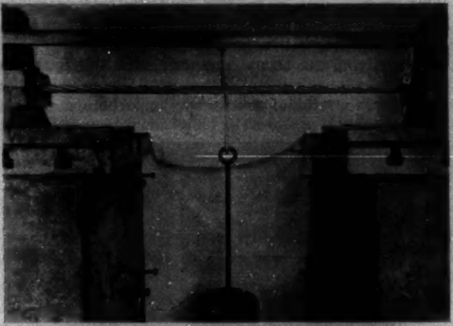

FIG. 8—SHOWING METHOD USED TO DETERMINE *EI* VALUE OF CABLES

$$\frac{d^2y}{dx^2} - \frac{T}{EI}\,y = \frac{M_o}{EI} - \frac{PL}{4\,EI} + \frac{Px}{2\,EI}, \tag{5}$$

where

P = load at the center of the intermediate span in lb.,

y = deflection in the direction of the load in inches,

x = distance from the left end of the intermediate span in inches,

and the other terms are as previously defined.

A solution of this equation gives not only the shape of the deflected cable but also the equation for bending moment at every point in the cable since

$$M = EI\,\frac{d^2y}{dx^2} \tag{6}$$

The boundary conditions that must be satisfied are that the deflection at the clamped ends is zero, that the slope at the clamped end is zero, and the slope at the middle of the span is zero. The resulting solution gives

$$y = \frac{P}{2\,KT\left(e^{\frac{-KL}{2}} + 1\right)}\left[-e^{-Kx} + e^{-K\left(\frac{L}{2} - x\right)} - 1 + e^{\frac{-KL}{2}}\right] + \frac{PL}{4\,T} - \frac{Px}{2\,T}, \tag{7}$$

and the maximum deflection at the center is

$$y_m = \delta = \frac{P}{KT}\left[\frac{KL}{4} - \frac{1 - e^{\frac{-KL}{2}}}{1 + e^{\frac{-KL}{2}}}\right] \tag{8}$$

Knowing the load, P, at the midspan, the tension, T, on the cable and the deflection, δ, for any given span length, it is possible to find the corresponding value of K and from this the value of EI for the cable. The values of EI thus obtained may be tabulated for use in the stress formulas.

3. The stresses at the clamped ends or points of support caused by vibration may be computed on the basis of the static conditions which will produce a deformation of the cable identical with that occurring during vibration. It may be shown that the deflection of a span of length, L, (the loop length) loaded with a load, P_1, which will give a deflection equal to 0.7 times the amplitude will give the conformation desired at the clamped end. From the equation

$$\delta = \frac{P_1}{KT}\left[\frac{KL}{4} - \frac{1 - e^{\frac{-KL}{2}}}{1 + e^{\frac{-KL}{2}}}\right], \tag{9}$$

the value of P_1 which will give a deflection, δ, equal to 0.7 of the amplitude is first found. Then the bending moment at the edge of the clamp may be computed from the formula

$$M_o = \frac{P_1}{2\,K}\cdot\frac{1 - e^{\frac{-KL}{2}}}{1 + e^{\frac{-KL}{2}}} \tag{10}$$

where the terms are as previously defined. The stress resulting from this bending moment may be computed by means of equation (4). It should be noted that this stress is oscillating and produces equal stress components in opposite directions on top and bottom, both being subjected to alternate tension and compression. The stresses throughout the length of cable resulting from the bending during vibration may be computed from the formula

$$S = \frac{1}{2}\pi^2 E_e \cdot \frac{c}{L} \cdot \frac{A}{L} \qquad (11)$$

where A = amplitude of vibration in inches, and the other terms are as previously defined.

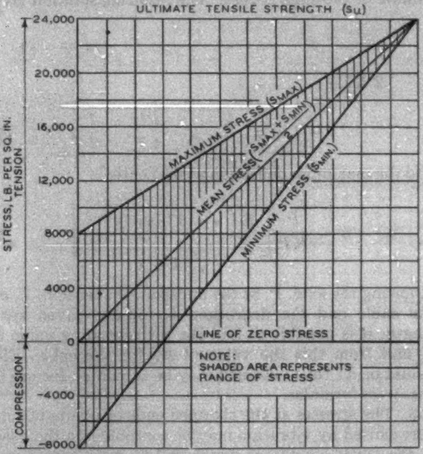

FIG. 9—GOODMAN DIAGRAM FOR ALUMINUM CONDUCTOR WIRE

4. As the cable distorts into waves the length around the arcs will be greater than the length of the cable when not vibrating. The additional length thus required will result in a stretching of the cable because, in general, the clamped ends can not yield rapidly enough. This additional length results in a unit deformation which is quite closely expressed by the formula derived by Bechtold and Folkerts:[8]

$$\epsilon_e = \frac{8}{3} \cdot \frac{A^2}{L^2}, \qquad (12)$$

where ϵ_e = unit elongation of the cable, and the other terms are as previously defined.

The resulting stress then may be computed as

$$S = \frac{8}{3} E_e \cdot \frac{A^2}{L^2} \qquad (13)$$

in which E_e = modulus of the cable as a whole.

This stress is simply an increase in tension of the cable which in general is quite small but which may not be negligible in many cases.

By proper combination of all of these stresses the total stresses in the vibrating cable may be computed to a fair degree of accuracy. Fig. 7 shows the stresses actually measured in the cable and those computed on the above basis.

It may be noted that the stresses measured on the top part of the cable, *i.e.*, where tensile stresses are great, are slightly lower than the theoretical stresses, whereas on the under side of the cable, where compressive stresses exist, the actual measured stresses are somewhat greater than the computed stresses. This is due to the fact that the tensile stress tends to pull the strands closer together, whereas the compressive stress causes them to push out, tending to move the neutral axis toward the top.

It has been found quite feasible to measure with suitable accuracy the deflection curves and determine EI values for all sizes of commercial conductors. Following the general procedure just outlined measurements of stress can be made on the smaller sizes of cable. This work is now in progress and the limited results obtained to date indicate that the theoretical analysis may be accepted with considerable assurance.

In applying this study to transmission line problems it is important to remember that no allowance has been made here for relief of stresses by sympathetic motion of the supports of the cable. The supports used for tests were practically rigid. Supporting clamps and insulator ties used on actual transmission lines are far from rigid.

FIG. 10—VIBRATION DAMPER FATIGUE TESTING MACHINE

FATIGUE PROPERTIES OF MATERIALS

Assuming that the limits of the stress ranges for the critical section in a cable can be determined, it is desirable to know the safe limits of the stress ranges for the materials in the cable. This information is best obtained from vibration tests of single wires under various known conditions. Such tests are essentially fatigue tests in which both the mean stress and the stress ranges may be varied throughout wide ranges. Tests show that when the mean stress is zero the endurance limit based on 500 million complete reversals is 8,000 lb. per sq. in. for hard drawn aluminum conductor metal. Using this value and the Goodman diagram which, although not checked very thoroughly by experimental data, has been found to give safe values, we obtain the limiting ranges of stress shown by Fig. 9. This shows that as the mean stress increases from zero to the ultimate strength of the material (24,000 lb. per sq. in.), the safe range of stress decreases from 16,000 lb. per sq. in. to zero at the ultimate strength. Tests on single wires are now in progress for the purpose of checking the Goodman diagram both for hard drawn aluminum and steel but because of practical difficulties in carrying out the tests, results are being obtained rather slowly. Experience has shown that with single wires vibrating at high speeds it is extremely difficult to maintain uniform vibration. Recent improvements in testing apparatus, however, indicate that the major troubles have been overcome.

EFFECTS OF CONDUCTOR ACCESSORIES

Many tests have been made for the purpose of determining effects of various types of fittings on the vibration or fatigue life of electrical conductors. Space limitations prevent inclusion of all results in this paper.

ARMOR RODS AND VIBRATION DAMPERS

The effects of bell-mouth clamps in increasing the life of vibrating cable is well illustrated in Tables I and II where the number of cycles for failure is shown for cable with square clamps and with bell-mouth clamps. It will be noted that the life of the cable has been practically doubled in every case. This is explained by the fact that the bell-mouth clamp prevents the severe concentration of stress that occurs at the square clamp.

TABLE I—SUMMARY OF VIBRATION TESTS

795,000 cir. mils A.C.S.R. $\left(\frac{54 \times 0.1214 \text{ in. aluminum}}{7 \times 0.1214 \text{ in. steel}}\right)$

Span: 120 ft. No. of loops: 7. Amplitude: 1-1/8 in.

Cable tension, lb.	Cycles for first failure of a strand in the cable		
	Square clamp	Bell-mouth clamp	Bell-mouth clamp with armor rods
18,000	433,000	1,148,000	24,000,000
15,000	796,000	2,078,000	40,538,000
10,000	1,166,000	6,242,000	95,400,000*
7,000	9,770,000	16,211,000	110,700,000*
5,000		75,200,000*	84,900,000*

*Still running.

If armor rods are used with bell-mouth clamps the life of the cable is increased many times, as illustrated in Tables I and II. While these tests have not been completed they have run long enough to demonstrate the effectiveness of armor rods as a means of overcoming vibration troubles.

TABLE II—SUMMARY OF VIBRATION TESTS

397,500 cir. mils A.C.S.R. $\left(\frac{30 \times 0.1151 \text{ in. aluminum}}{7 \times 0.1151 \text{ in. steel}}\right)$

Span: 120 ft. No. of Loops: 14 Amplitude: 9/16 in.

Cable tension, lb.	Cycles for first failure of a strand in the cable		
	Square clamp	Bell-mouth clamp	Bell mouth clamp with armor rods
7,000	3,775,000	11,289,000	49,910,000
5,000		26,380,000	176,100,000*
4,000	45,316,000	106,210,000	377,000,000*

*Still running.

It is possible by means of dampers to practically eliminate the vibration of a cable. The laboratory work on vibration dampers has included extensive tests to determine the relative life of various dampers of the Stockbridge type using the special fatigue testing machine shown in Fig. 10. The machine can be operated at various speeds from about 600 to 1,020 r.p.m. using any amplitude from zero to ½ inch. Provision is made to test 8 dampers simultaneously. Tests with this machine permit the selection of proper cable for supporting the weights and assist in refinements of design.

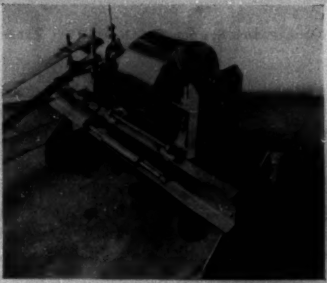

FIG. 11—VIBRATION DAMPER EFFICIENCY MACHINE

The special testing machine shown in Fig. 11 was intended to measure the efficiency of various types and sizes of vibration dampers by measuring the rate of deceleration of the flywheel after it and the attached damper were brought to a definite high speed and then disconnected from the motor. So far the test data have not been satisfactory for determining the efficiency of dampers but the machine has given valuable information relative to critical frequencies of dampers.

Dampers of the Stockbridge type have two critical speeds or natural frequencies which are dependent upon the material, size, length and stranding of the damper cable as well as the size, shape and method of attachment of the weights. The computed frequencies have been found to agree with the observed frequencies within the limits of experimental error. The amplitude of oscillation of the damper weights for any given forced vibration of the center clip depends upon the frequency of the forced vibration and the damping characteristics of the damper cable.

Knowing the damping characteristics of the damper cable this amplitude may be computed and from that the energy absorbed per cycle or per second. This value together with the forces developed by the vibrating weights will give criteria for selecting the most suitable damper for any given conditions. Limitations of space will not permit giving the mathematical analysis.

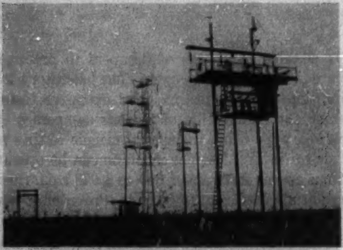

FIG. 12—OBSERVATION STRUCTURES, ROYSE CITY EXPERIMENT STATION

Field Work

Laboratory research has been coordinated with field observations on operating lines in various locations. Early in 1930 it was decided to establish a field laboratory equipped for accurate study. The station was established near Royse City, Texas, about 35 miles east of Dallas. Chief among the considerations which led to this selection were the flat unobstructed topography and the remarkably steady wind conditions. A vacant position on an existing steel tower line was made available for experimental work by the Texas Power and Light Company and additional spans nearly at a right angle to the existing line were erected on wood H-frames. This site has proven to be excellent, since hardly a day has passed without vigorous vibration being recorded on at least one of the two test lines.

In addition to the field studies at Royse City, observations have been conducted on operating lines as follows:

The Public Service Electric and Gas Company of New Jersey on the 795,000-cir. mil A.C.S.R. conductor and 203,000-cir. mil A.C.S.R. ground wire of the 220-kv. Roseland-Bushkill Line.

The Pennsylvania Power and Light Company on the 184,000-cir. mil ground wire of the 220-kv. Plymouth Meeting-Siegfried Line.

The Idaho Power Company on the 4/0-A.C.S.R. conductor of the 132-kv. Caldwell-Ontario Line.

The New England Power Company has observed that under certain conditions a charged conductor tends to vibrate, presumably from the effect of corona. While allied to the general problem of conductor vibration its occurrence is probably rare. The Pacific Gas and Electric Company has made field observations of conductor vibration, and also laboratory experiments on vibration dampers.

Space limitations prevent inclusion of more than reference to the experimental work of these and other companies. It is hoped that discussions will be presented by engineers of these companies.

ARRANGEMENT OF TEST SPANS

At Royse City two test lines are available at an angle of 74 deg. thus insuring one line favorably exposed for vibration regardless of wind direction. The spans are 650 ft. and 1,225 ft. on the steel towers and 416 ft. and 1,226 ft. on the H-frames. Tension is adjustable at the anchorages. Observation platforms extend 12 ft. out under the conductors, Fig. 12. The ground wire on the steel towers is 3/8-inch diameter 7-strand steel, and the conductors of the 132-kv. circuit on the towers are 4/0 A.C.S.R. equipped with armor rods.

Other sizes and special types of conductors have been installed and observed for short periods.

EQUIPMENT

A satisfactory instrument for obtaining 24-hour vibration records was made by using an 8-inch diameter wax-coated, clock-driven chart on which the vertical motion of the conductor was recorded by a stylus attached to a pivoted arm. (See Fig. 13.) For small conductors the inertia and friction of the pivoted arm damped the vibration considerably and a very light stylus attached directly to the cable was substituted. This latter arrangement is satisfactory only if longitudinal motion of the conductor is restrained.

FIG. 13—VIBRATION RECORDER

A continuous record of wind direction and velocity is obtained by an anemometer which records electrically the passage of each mile of wind on a strip chart, and a weather vane with a cam-actuated pen which records direction simultaneously on the same chart. The velocity for short periods is determined with a stop-watch and buzzer which indicates one-sixtieth of a mile. Fig. 14 shows two typical vibration records and a corresponding wind record.

The frequency recorder consists of a uniform speed, motor-driven strip chart on which a trace of the vibration is recorded by a pencil connected to the line by a cord and spring.

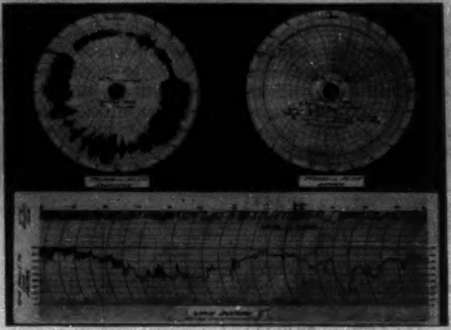

Fig. 14—Typical Vibration and Wind Records

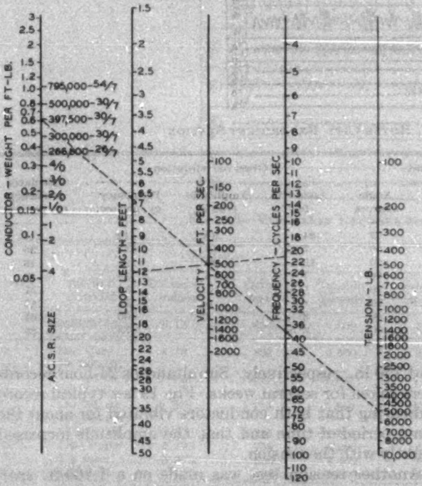

Fig. 15—Relation Between Weight, Tension, Loop Length, and Frequency of a Vibrating Cable

A thermograph records temperature and provision is made for observing sag and tension in the conductors.

Theoretical Characteristics of Vibration

Fig. 15 gives a quick solution of equation (1) showing the theoretical relations between frequency, loop length, tension, and weight of a vibrating cable for common conductor sizes and tensions.

The following formula expressing theoretical frequency of vibration was developed experimentally by Relf and Ower:

$$f = k\frac{V}{D} \tag{14}$$

where

V = velocity of wind in mi. per hr.,

D = outside diameter of the cable in inches,

and

k = a function of $\left(\frac{VD}{e}\right)$ where e depends upon the medium, e being 0.000159 for air. For ordinary sizes of conductor k is a constant having a value of 3.26.

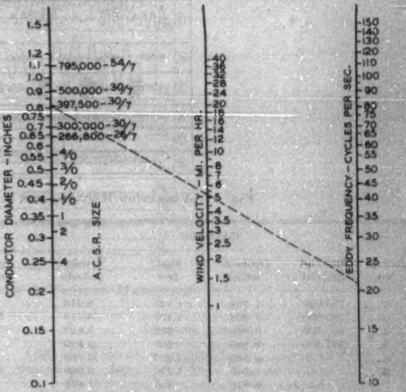

Fig. 16—Wind Eddy Frequency for Circular Cables (Relf and Ower Formula)

A nomographic solution of this formula is given by Fig. 16.

This formula applies to a uniform wind velocity normal to the axis of the wire. Such ideal wind conditions rarely if ever exist over a transmission span. A wide variation in both velocity and direction even in short spans is shown by Professor Sherlock[12,13] at the University of Michigan. Velocity varies with elevation, and in a long span the height of conductor is not constant. Direction shifts rapidly as indicated by any weather vane. The normal component of velocity should probably be used in computing frequency when direction of the wind is oblique. This assumption seems to be in error for very acute angles. A cable has considerable inertia and once vibrating in any particular mode will not quickly change frequency to suit slight changes in wind direction or velocity. For these reasons observed frequency sometimes differs greatly from theoretical, but with steady wind and obliquity not under 45 deg. the agreement is usually good.

Field Records of Vibration

Numerous records of frequency and amplitude for varying sizes of conductor have been obtained. Fig. 17 shows characteristic vibration records for each of four sizes of A.C.S.R. with pertinent data. The correspondence between observed and calculated frequency is fairly close. These records all show recurring beats, possibly caused by variations of wind velocity. With span length and frequency known, the time between recurrence of any particular beat determines velocity of wave propagation, tension, and theoretical loop length.

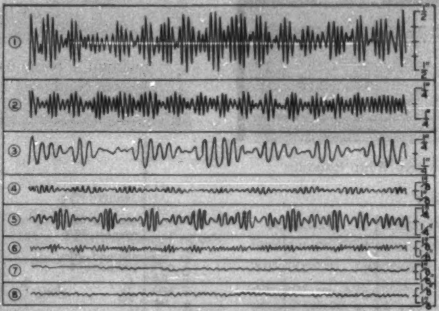

Fig. 17—Vibration Records for A.C.S.R., Royse City Experiment Station

Record No.	Conductor				Wind		Observed vibration			Calculated frequency cy. per sec.
	Size and material	Diameter inches	Span feet	Tension pounds	Velocity m.p.h.	Angle degrees	Loop lth feet	Amplitude 1/32 in.	Frequency cy. per sec.	
1	795,000	1.093	1,125	8,216	8	37	24	34	12	11
2	795,000	1.093	1,125	8,216	18	68	$7^{1}/_{2}$	18	43	35
3	397,500	0.806	650	3,822	$5^{1}/_{2}$	90	15	22	21	16
4	397,500	0.806	650	3,822	$8^{3}/_{4}$	90	7	5	34	33
5	4/0	0.563	1,226	2,869	$5^{1}/_{2}$	70	$7^{1}/_{2}$	14	32	37
6	4/0	0.563	1,226	2,869	10	70	6	4	51	47
7	No. 2	0.316	650	676	5	60	$3^{5}/_{8}$	2	60	61
8	No. 2	0.316	650	676	8	60	$2^{2}/_{3}$	2	75	85

Observation proves the statement that cables of any material will vibrate when conditions are favorable. One of the earliest recorded instances of damage from vibration occurred over 20 years ago on the Carquinez Straights Crossing of the Pacific Gas and Electric Company. A few years after installation breakage of strands was discovered in the heavy plough steel conductors. It is interesting to note that the use of parallel reinforcing or stiffening cables extending some distance out from the support, which was developed here, has been repeated on many long river crossing spans.

Fig. 18 shows records of frequency and amplitude of vibration with pertinent data for conductors of materials other than A.C.S.R.

Influence of Tension

Frequency is a function of wind velocity and diameter of conductor and is independent of tension. From equation (**1**) it is seen that for a given frequency the loop length varies directly as the square root of the tension. Therefore, an increase in tension increases loop length and theoretically should render it more difficult for the conductor to fall into resonance with the wind eddy frequency.

Field records to determine the effect of tension were obtained at Royse City. Two No. 2 A.C.S.R. conductors were erected on a 416-ft. span with tensions of 500 and 750 lb., respectively. Simultaneous 24-hour records were taken for several weeks. Fig. 19 is a typical record indicating that both conductors vibrated for about the same period of time and that the amplitude increased slightly with the tension.

Another tension test was made on a 1,160-ft. span of 795,000 cir. mil A.C.S.R. in New Jersey. The tension was 5,400 lb. in one of the parallel conductors and 9,000 lb. in the other, a difference of over 50 per cent. Comparative records were obtained for several weeks. Vibration occurred on a few occasions only and for short periods only. Fig. 20 is typical. The recorders were set the same distance out from the supports, so the amplitudes are not directly comparable on account of difference in loop lengths.

From these tests it appears that vibration will occur at any practical tension when wind conditions are favorable. A reduction of tension decreases the direct stress in the conductor but does not decrease the tendency to vibrate.

INFLUENCE OF SHAPE

It is obvious that shape of conductor should influence vibration and field tests prove this. At Royse City the relatively smooth No. 2 and 397,500 cir. mil A.C.S.R. are observed to vibrate more frequently than the 4/0 which has a comparatively rough exterior due to larger strands.

A special test to show influence of shape was made with three No. 2 all aluminum conductors. One was single wire, one of 3 strands, and one of 7 strands. Twenty-four hour vibration charts were obtained over a period of several weeks and for several tensions. The recorders were set the same distance from the supports, and the tensions being the same, the amplitudes are comparable. Fig. 21 shows typical charts indicating that the three-strand cable vibrated least frequently and with the smallest amplitude, the seven-strand was second, and the solid wire vibrated the most. In no case was vibration prevented.

Another test was conducted on a special eccentric A.C.S.R. (418,000 cir. mils of aluminum). The outer layer of strands, being graduated in size, formed an enlargement which wound helically around the conductor axis. This cable was installed on a span of 1,125 ft. parallel to a standard 397,500 cir. mil A.C.S.R. of approximately the same diameter and proportion of steel. Fig. 22 shows a comparative record.

These experiments show that amplitude can be reduced by variation in shape but it is doubtful if a practical vibrationless conductor can be thus obtained.

ENERGY TRANSMITTED TO SUPPORTS

Severe conductor vibration sometimes causes supporting structural members to vibrate noticeably. Fatigue failures at bolt holes in small alloy steel cross-arm members have been reported. To measure the variation in vertical load transmitted to the structure a small rod of high elastic limit steel was inserted between the suspension clamp and the insulator string on a 1,125-foot span of 795,000 cir. mil A.C.S.R. An accurate load-deformation curve for this rod had previously been determined with a Huggenberger tensometer. A number of readings was made under varying conditions

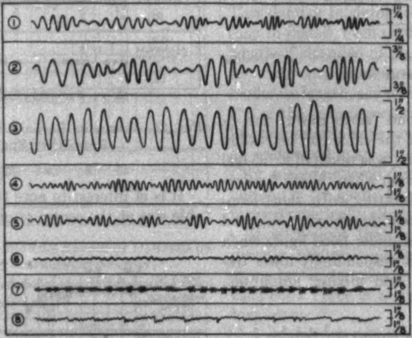

FIG. 18—VIBRATION RECORDS FOR CABLES OF VARIOUS MATERIALS

Record No.	Conductor				Wind		Observed vibration			Calculated frequency cy. per sec.
	Size and material	Diameter inches	Span feet	Tension pounds	Velocity m.p.h.	Angle degrees	Loop lth. feet	Amplitude 1/32 in.	Frequency cy. per sec.	
1	650,000 cm. Cu. Calson Bze. Core	1.13	835	9,500	4.5	90	11.5	12	16	14
2		1.13	835	9,500	2	85	17.7	10	10	9
3	400,000 cm. 19 str. cu.	0.728	1,200	5,100	4	70		32	21	18
4	250,000 cm. 19 str. cu.	0.575	1,125	5,550	5.75	66	8.	9	29	30
5	250,000 cm. 19 str. cu.	0.575	1,125	5,550	6.	66	8.	9	25	31
6	4/0 Copper	0.528	750	3,200	6.8	80	6.9	6	25	29
7	3/8 Steel	0.375	650	3,800	8.75	90	3.75	4	88	89
8	3/8 Steel	0.375	650	3,800	10	75	3.	3	99	102

of vibration. The variation in vertical load was of the order of 100 to 150 lb.

Moving Pictures of Vibrating Cable

Sixteen exposures per second with an ordinary moving picture camera is too slow to show vibration at a frequency of 25 cycles or more. A high-speed camera taking 128 exposures per second "slows down" the vibration so that the motion of the cable can be readily followed on the screen. One reel was taken of a 795,000 cir. mil A.C.S.R. in the afternoon when the wind velocity was high and the cable vibrating with short loop-length, high frequency, and small amplitude. A second reel was taken about midnight under flood lights when wind velocity was low with correspondingly lower frequency, longer loop length, and greater amplitude. Rocking of the suspension clamp is clearly shown and beat pulsations are evident.

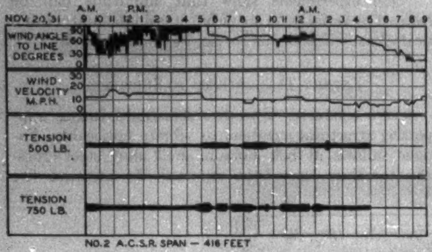

Fig. 19—Vibration Records Showing Effect of Tension, Royse City Experiment Station

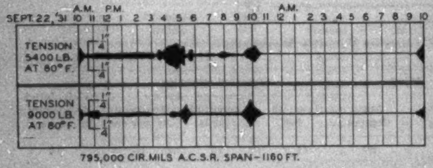

Fig. 20—Vibration Records Showing Effect of Tension, Roseland-Bushkill Line

Effect of Dampers

The problem of damping a vibrating conductor differs from many vibration problems in that the damper must be effective over a wide range of frequencies. This precludes the use of a tuned dynamic vibration absorber for which the theory was presented by J. Ormondroyd and J. P. DenHartog.[7] Such an absorber theoretically consumes no energy and it seems that it can not damp vibration where there is a continuing input of energy as on a transmission line. The dynamic absorber with damping is effective over a broad band of frequencies and also consumes energy. The theoretical formulas in the Ormondroyd-DenHartog paper for this type of damper are applicable to a vibrating cable in principle only and require extensive modification and laboratory determination of certain constants.

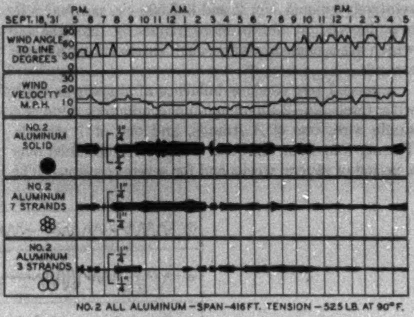

Fig. 21—Vibration Records Showing Effect of Stranding, Royse City Experiment Station

It is of interest to note that a simple weight of suitable size at the center of one loop of a vibrating cable is somewhat effective as a damper. The added weight alters the natural period of that one loop and, with the forced vibration which then ensues in that particular loop, the vibration is damped by consumption of energy in the conductor itself in that loop. The practical difficulty is that a node is soon established at the weight and vibration is renewed.

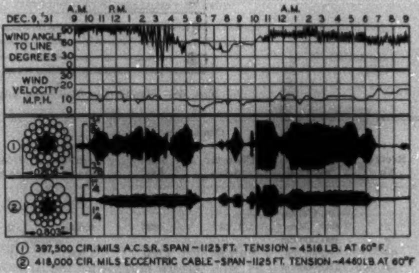

Fig. 22—Vibration Records Showing Effect of Eccentric Shape, Royse City Experiment Station

The energy which must be consumed to damp vibration is undoubtedly very small in amount. This is evident from the theoretical formula developed by Bate[10] for the input of energy to a cable from the wind eddies. His formula involves several assumptions which have not been proved but the method of derivation is logical and dimensionally it is probably correct. The formula gives the energy input per loop per cycle, G in ft.-lb., as follows:

$$G = 0.000022\, V^2 DAL' \qquad (15)$$

where, as previously,

V = velocity of wind in mi. per hr.,

D = outside diameter of the cable in inches,

A = amplitude of vibration in inches,

and

L' = loop length or distance between node points in ft.

This formula shows the input of energy proportional to the amplitude, hence if amplitude can be restrained to a small value the energy input will be small. This agrees with the observed fact that vibration builds up slowly and is easily damped at first.

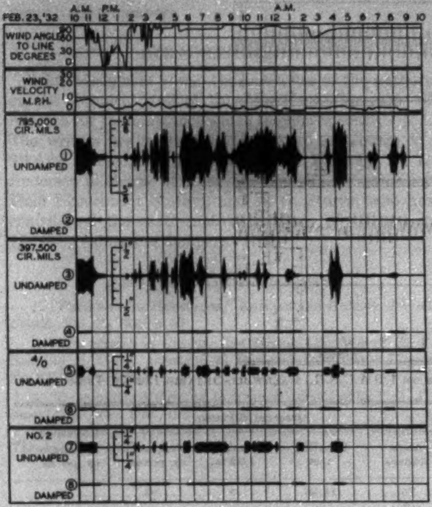

FIG. 23—VIBRATION RECORDS SHOWING EFFECT OF DAMPERS ON A.C.S.R., ROYSE CITY EXPERIMENT STATION

	A.C.S.R. Conductors				Recorder
Record No.	Size in cm. or B & S gage	Diam. in inches	Span in feet	Tension in lb. @ 60° F.	Distance from support
1	795,000	1.093	1,125	6,755	5 ft. 2 in.
2	795,000	1.093	1,125	6,810	5 ft. 0 in.
3	397,500	0.806	1,125	4,200	4 ft. 5 in.
4	397,500	0.806	1,125	4,085	2 ft. 9 in.
5	4/0	0.563	416	2,025	2 ft. 9 in.
6	4/0	0.563	416	2,000	4 ft. 0 in.
7	No. 2	0.316	416	740	3 ft. 1 in.
8	No. 2	0.316	416	720	3 ft. 1 in.

One reason for the outdoor laboratory was to study the action of dampers. The usual method of observation is by comparison of 24-hour vibration records from two conductors identical in size, span length, and tension, one damped and the other undamped.

Many different dampers have been tested at Royse City, hundreds of comparative records being obtained. The most successful type is essentially a mass resiliently attached to the conductor in the loop nearest the support and so designed that energy is consumed by friction or hysteresis.

This damper was developed by Geo. H. Stockbridge of the Southern California Edison Company. The usual form consists of two weights connected by a short piece of stranded steel cable, this assembly being attached to the conductor a short distance out from the insulator. This damper is simple and practical to construct and easy to install, even on hot lines. The weight, size and length of stranded cable, and location in the span vary for different diameters of conductor. For spans of moderate length, say under 1,000 ft. one damper at each end of the span is required. Fig. 23 shows comparative 24-hour records with pertinent data for a damped and an undamped span of four sizes of A.C.S.R. at Royse City, including wind direction and velocity recorded at one end of the span. It is evident that the dampers suppressed practically all visible vibration.

It is obvious that energy input from the wind is directly proportional to length of the span. In long spans better damping is obtained with two dampers at each end. Three or more may be required on extremely long spans.

A damper cannot act effectively until the conductor vibration acquires a certain amplitude. This is manifested by a slight quiver which can be felt although barely visible. The 24-hour charts do not show this very clearly. Recently a series of records from a free and a damped span of 4/0 A.C.S.R. was obtained by engineers of the Idaho Power Company using a carbon pile oscillograph. Fig. 24 shows two of their oscillograph records. The ripple in the damped cable is scarcely visible even at the enlarged scale.

Many tests of Stockbridge dampers have been made to determine the best dimensions and location. The dampers are not critical either as to size or spacing. A variation of 25 per cent in any factor will not seriously affect performance. Fig. 25 shows the style of Stockbridge damper used. To avoid corona discharge this design embodies cylindrical weights which shield the damper cable and a clamp having no sharp corners.

ARMOR RODS

Armor rods are primarily reinforcement but they are also effective as dampers. Numerous comparative records have indicated that they reduce amplitude from

10 to 20 per cent. They reduce stresses by distributing the bending from vibration, reinforce the cable at the point where the stresses are greatest, and furnish valuable protection against flashover burns.

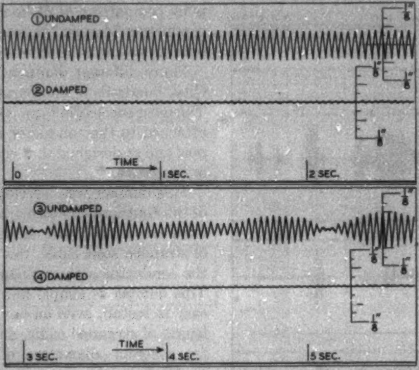

FIG. 24—VIBRATION RECORDS SHOWING EFFECT OF DAMPERS ON 4/0 A.C.S.R., CALDWELL-ONTARIO 132-KV. LINE

Conductor					Wind		Observed vibration			Calculated frequency cy. per sec.
Record No.	Size in cm. or B & S gage	Diameter inches	Span feet	Tension pounds	Velocity m.p.h.	Angle degrees	Loop lth. feet	Amplitude 1/32 in.	Frequency cy. per sec.	
1	4/0 A.C.S.R	0.563	550	2,630	3	75	11.95	6	23	22
2	4/0 A.C.S.R	0.563	550	2,630	3	75				
3	4/0 A.C.S.R	0.563	550	2,650	4	75	11.7	8	23	22
4	4/0 A.C.S.R	0.563	550	2,650	4	75				

Conclusions

1. Satisfactory methods have been developed for measuring actual stresses in vibrating conductors under conditions closely simulating field conditions.

2. A mathematical analysis of the stresses in a stranded conductor under vibration is given which checks closely with measured values.

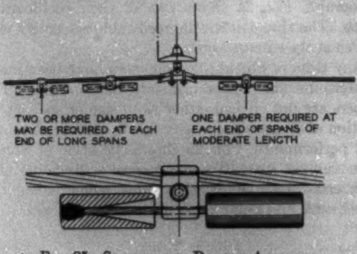
TWO OR MORE DAMPERS MAY BE REQUIRED AT EACH END OF LONG SPANS
ONE DAMPER REQUIRED AT EACH END OF SPANS OF MODERATE LENGTH

FIG. 25—STOCKBRIDGE DAMPER ASSEMBLY

3. Tests indicate that safe limits for stress ranges of conductor materials can be satisfactorily determined.

4. Certain types of cable accessories have marked effects on the life of vibrating conductors.

5. Theoretical formulas[2] expressing relationship between frequency, loop length, tension and weight of a vibrating conductor are in good agreement with field observations.

6. The Relf and Ower formula[2] for the frequency of a vibrating cable is substantially correct.

7. Any suspended cable will vibrate when conditions are favorable.

8. Any feasible reduction in tension will not prevent vibration.

9. Modifications in shape of cross section and stranding have some beneficial effect in reducing amplitude but the results do not support the idea that a vibrationless cable can be evolved.

10. The variation in vertical load transmitted to the insulator is not large.

11. Slow-motion moving pictures of vibrating cable enable the motion to be followed by eye but have been of little assistance.

12. A properly designed damper will prevent the building up of any appreciable amplitude and since the input of energy from the wind varies directly as the amplitude an effective damper is required to absorb very little energy.

13. The Stockbridge damper gives best results, is simple in design and construction, has no moving parts and is easy to install.

14. Armor rods are primarily reinforcement although they reduce the amplitude of vibration from 10 to 20 per cent.

16. Adequate practical protection against damage from vibration of conductors is afforded by proper use of armor rods or dampers.

ACKNOWLEDGMENTS

Acknowledgment is due the associates of the authors on the technical staff of the Aluminum Company of America for the assistance which they have contributed. The writers are particularly indebted to George W. Stickley in charge of the test work at Massena, H. L. Anderson in charge of observation work at Royse City Field Experiment Station, and R. G. Sturm, who has contributed the mathematical analysis of stresses in vibrating cables and dampers.

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Discussion

For discussion of this paper see page 1076.

Transmission Line Vibration Due to Sleet

BY J. P. DEN HARTOG*
Non-member

***Synopsis.**—With sleet, a temperature just below the freezing point, and with a heavy wind blowing transversely across a transmission line, vibrations of amplitudes up to 20 feet have been observed An explanation of this phenomenon is given.*

INTRODUCTION

THE subject of vibration in transmission lines due to the action of a transverse wind has been frequently discussed in the technical literature of the last few years. The phenomena encountered mostly are those occurring at moderate wind velocities (about 5 miles per hr.), characterized by rather high frequencies (5 to 15 cycles per second) and small amplitudes, up to a few cable diameters. This type of vibration is caused by the Kármán vortexes forming behind the wire and is well understood.

Another type of disturbance has been observed, which seems to occur with sharp winds of above 20 miles per hour, with great amplitudes (20 feet), slow frequencies (one cycle per second or slower), and mostly associated with sleet formation on the conductors. This phenomenon has been described in detail in a paper by Davison,† who rightly believed it to be caused by the change in shape of the conductor due to the ice coating and by the consequent change in aerodynamic lift. Davison measured the lift of some specimens in the wind tunnel and found that the lift under certain conditions can even exceed the weight of the cable. However the mere fact that the wind causes a lift on the unsymmetrical conductor does not yet cause it to vibrate, so that Davison's discussion of the phenomenon is incomplete.

It is the object of this paper to give an explanation of the effect, and to show how the stability of a transmission line in this respect can be calculated from the results of an aerodynamic test, whereby both the lift and the drag are determined for various angles of attack.

QUALITATIVE EXPLANATION

Consider an element of the cable of unit length subjected to a lateral wind of constant velocity V and vibrating up and down according to $x = a \sin \omega t$. On account of its vibrating motion the cable experiences a vertical wind component in addition to V.

For instance at the instant that the cable is moving upward with the velocity v the "relative wind" striking it appears to come from above under an angle

$$\Delta \alpha = \tan^{-1} \frac{v}{V},$$

Fig. 1. Since during the vibration the velocity v varies all the time it is seen that the cable is subjected to a wind of which the direction also varies with the time. For a cable of circular cross-section the force exerted by the wind on it will be always exactly in the direction of the wind, or, in aerodynamical parlance, there is only drag, and no lift. In case the cross-section becomes non-circular due to sleet, this ceases to be true and lift as well as drag appears.

Plotting the drag (= wind force in the direction of the wind) as well as the lift (= wind force perpendicular to wind direction) against the angle of attack for an elliptic cross-section Fig. 2 is obtained. The drag is a minimum for $\alpha = 0$, and a maximum for $\alpha = 90$ deg. Due to symmetry the lift must be zero at $\alpha = 0$ and $\alpha = 90$ deg. and it reaches a maximum in between. It is an experimental fact that this maximum is closer to 0 deg. than to 90 deg. For α between 90 and 180 deg. there is negative lift or downpush.

Now consider a cable element having an elliptical section in the position of $\alpha = 0$ deg. in Fig. 2 and let it vibrate vertically. While it is moving upwards the relative wind strikes it obliquely from above (Fig. 1) and the angle $\Delta\alpha$ is negative (Fig. 2). In other words, while the cable is moving upwards it experiences an aerodynamic downpush. In the same manner it can be seen that during downward motion there is lift, so that always the force is directed against the motion. Whenever such a vibration starts the wind will damp it out.

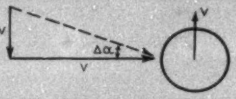

FIG. 1

The conditions are quite different however when the cable has the position $\alpha = 90$ deg. in Fig. 2. Incidentally, such a cable has some resemblance to a line with icicles hanging down vertically from it. While the cable is moving upwards the angle of attack is 85 deg. say instead of 90 deg. and there is *lift;* similarly, during downward motion the angle α is 95 deg. and there is downpush. The forces are in the direction of the velocity of vibration and therefore put energy into the motion. The cable is unstable; any vibration, however small, will be increased in amplitude by the wind.

It is seen that this instability is due to the *negative slope of the lift curve*; a cable in a position $\alpha = 60$ deg. in

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*Research Laboratories, Westinghouse Elec. & Mfg.Co., East Pittsburgh, Pa.

†*Dancing Conductors*, C. E. Davison, TRANS. A.I.E.E., Vol. 49, 1930, p. 1444.

Presented at the Summer Convention of the A.I.E.E., Cleveland, Ohio, June 20-24, 1932.

Fig. 2 would be unstable also. There would be only a constant lift superposed on the variable lift due to vibration. This constant lift only would relieve the towers of some of the load, but would have no other effects.

So far we have not considered the drag force. Fig. 3 represents the forces acting on a cable element in the position $\alpha = 90$ deg. of Fig. 2 while in its upward stroke during the vibration. It is seen that not the entire lift force is pushing the cable up, but only its vertical component, and it is also seen that the drag force has a *downward* component. Therefore *instability occurs when the effect of the negative slope of the lift curve is greater than the damping action due to the drag.*

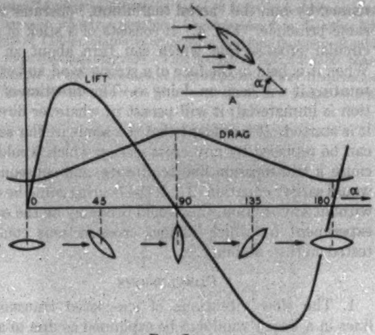

Fig. 2

Quantitative Theory for Small Vibrations

Assume that for the cross-section under investigation a lift and drag diagram like Fig. 2 has been determined experimentally by a wind tunnel test. The diagram holds for a particular wind velocity V_0. The lift and drag forces for other wind velocities can be calculated by observing that they are proportional to the square of V.* Therefore the lift is:

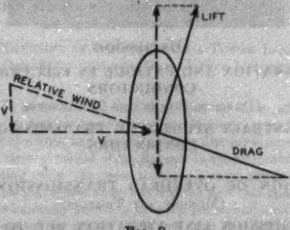

Fig. 3

$$L = \frac{V^2}{V_0^2} \cdot L_0 \qquad (1)$$

where L_0 is the ordinate of Fig. 2. A corresponding relation is true for the drag.

Let the cable element execute small vibrations $a \sin \omega t$. The vertical velocity v then is $a\,\omega \cos \omega t$ and the variation in the angle of attack (Fig. 1) is:

$$\Delta\alpha = \frac{a\omega}{V} \cos \omega t$$

For small $\Delta\alpha$ the lift curve is a straight line and the drag can be considered constant. Then the lift is:

$$\frac{dL}{d\alpha} \cdot \Delta\alpha$$

where $dL/d\alpha$ is the negative slope of the lift curve. Let D be the drag at the point considered; then according to Fig. 3 the vertical component of the drag is $D \cdot \Delta\alpha$. Consequently the driving force is:

$$\left(\frac{dL}{d\alpha} - D\right)\Delta\alpha = \frac{a\,\omega}{V}\left(\frac{dL}{d\alpha} - D\right)\cos \omega t$$

In this expression L and D are still functions of the wind velocity; with (1) the driving force becomes:

$$\frac{a\omega V}{V_0^2}\left(\frac{dL_0}{d\alpha} - D_0\right)\cos \omega t \qquad (2)$$

This driving force is of the nature of a negative "viscous" damping. Sustained vibrations occur when

$$\frac{dL_0}{d\alpha} > D_0 \qquad (3)$$

The expression $dL_0/d\alpha$ is the vertical distance between the two points of the tangent to the L_0-curve whose horizontal distance apart is 1 radian or 57 degrees. When this distance in Fig. 2 is greater than the ordinate of D_0 at that point the cable is dynamically unstable. The formula also shows that the effect, whether it be input or damping, is proportional to the velocity of the wind.

In order to get a quantitative idea about the order of magnitude of the effect it is necessary to possess experimental lift-drag-α diagrams, like Fig. 2, for the various ice-coated shapes that are likely to occur. Such diagrams do not exist at present. Davison only gives the lift curves in his paper. There are lift and drag curves for a number of airfoil sections,† which, when held vertically, resemble to a certain extent a cable with icicles. These airfoil curves show several ranges of α where the $dL/d\alpha$ term is considerably greater than the D-term.

Taking Fig. 5 of Davison's paper, it can be calculated that the part of the driving force due to the $dL/d\alpha$ term is more than 20 per cent of the maximum inertia force of the cable during vibration. Though from this the damping force due to the *D*-term has to be subtracted, it is clear that the forces are of the right order of magnitude to explain the field observations.

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*It is understood that this is only approximately so; for airfoil sections in aeronautics the approximation is very good, but for irregular sections as discussed here the error may be considerable.

†"Lift and Drag of Aerofoils Measured Over 360 Deg. Range of Incidence," Lock and Townsend. Reports and Memoranda of the Aeron. Res. Committee, No. 958, London 1914. Also Prandtl and Betz, "Ergebnisse der Aerodyn. Versuchsanstalt zu Göttingen," Vol. 3, p. 78.

CALCULATION OF THE AMPLITUDE

The theory given so far is limited to small amplitudes, because the slope of the lift curve as well as the amount of drag were assumed to be constant. For large vibrations this ceases to be true, and then it becomes necessary to divide the path of motion of the conductor into a number of sections and to determine for each of these sections the driving force in much the same manner as equation (**2**). Each of these forces has to be multiplied by its respective path in order to obtain the work done. For small vibrations the work during one cycle will be positive, so that the motion will increase. For larger amplitudes, however, certain parts of the motion will have a negative driving force or damping. This occurs for instance, in Fig. 2 when α becomes 90 deg. ± 75 deg., so that we penetrate into the region of positive slopes of the lift curve.

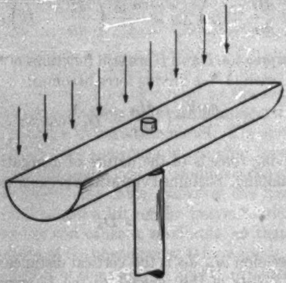

FIG. 4

The final amplitude reached will be such that the total amount of work done during a cycle is zero. In an actual case, where there is additional damping in the towers, insulators, etc., the amplitude will be somewhat smaller. This is so, assuming that all elements of the line have the same amplitude. In reality this is obviously not the case, and then the total work input by the wind has to be calculated for all elements of a span. Assuming that the span vibrates somewhat like a string, *i.e.*, in a sinusoidal form, this calculation is a straightforward process involving no particular difficulties.

RELATED PHENOMENA

The effect can be observed by means of a very simple experiment. Take a stick of rectangular cross-section, a common yardstick for instance; hold one end in the hand and dip the other end vertically into the bath tub. When the stick is pulled through with the narrow side in front, it is in the stable position, $\alpha = 0$ deg. of Fig. 2, and nothing will happen. When it is pulled with the broad side in front, however, $\alpha = 90$ deg. and the condition is unstable. It will be found difficult to pull the stick through along a straight path; it will rather vibrate back and forth just like the transmission line.

A toy, described and explained by Lanchester[*] and named by him the "aerial tourbillion," operates on the same principle. (Fig. 4.) It consists of a stick of semicircular cross-section which can turn about an axis. When it is held in the face of a strong wind and started rotating it will keep on doing so. The direction of rotation is immaterial; it will persist in whatever direction it is started. It is evident that the semi-circular section can be replaced by any cross-section which would also cause a transmission line to vibrate, and consequently would satisfy equation (**3**). If the bearing could be made without any friction, this would probably be the easiest experiment by which various cross-sections could be tested on their stability.

CONCLUSIONS

1. The slow vibrations of ice-coated transmission lines in a heavy wind can be explained as due to a certain aerodynamic instability.
2. The phenomenon is entirely different from and has no connection with the rapid vibrations observed in fair weather at moderate wind velocities.
3. It is shown that instability occurs when the negative slope of the "lift curve" is greater than the amplitude of the "drag curve."
4. It follows from the theory that a change in the span length or in the tension of the cable does not affect the phenomenon.
5. The behavior of a toy, called the "aerial tourbillion" is shown to afford a simple test by which the stability of various ice-coated sections can be determined.

Discussion

VIBRATION AND FATIGUE IN ELECTRICAL CONDUCTORS

(DAVISON, INGLES AND MARTINOFF)

STRESS-STRAIN STUDIES OF TRANSMISSION LINE CONDUCTORS

(STICKLEY)

VIBRATION OF OVERHEAD TRANSMISSION LINES

(MONROE AND TEMPLIN)

TRANSMISSION LINE VIBRATION DUE TO SLEET

(DEN HARTOG)

John A. Koontz: A careful study of the vibration problem was first undertaken after trouble had developed in some of the early Stockbridge type damper designs. This trouble was due to fatigue of the supporting messenger strand. Life tests were made on the early assemblies and it was soon found that it was necessary to substitute high strength steel strand in place of the ordinary guy strand or Siemens-Martin grade. This substitution changed the characteristics of the damper slightly, but later tests proved that they were still effective. The life tests were made on the damper assemblies using a vibration machine similar to that shown in Fig. 10 of the paper by Messrs. Monroe and Templin. The revised damper design, using the high strength steel, was given a life test of over one hundred-million cycles without failure. The failures in the supporting strand occurred not at the center clamp or point of maximum stress but where the cable enters the weight, or at a point where the strand is subjected to a reversal of stress.

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[*] F. W. Lanchester, *Aerodynamics*, London, 1907 and 1923, p. 45.

Messrs. Monroe and Templin seem to favor two dampers per wire per span wherever possible. I prefer using four dampers for any line of 1.00-in. diameter or larger. The four dampers should have their spacing staggered. This is done so that not more than one damper would be located at a node point for any frequencies most generally experienced. This method as a rule permits three dampers to be so located that they may be fairly effective as dampers. A damper has its greatest efficiency when located at the center of a loop, and the Stockbridge damper is most efficient when the natural period of the damper is approximately that of the vibrating cable. In order to make a more careful study of damper spacing, a test span was erected and this span was vibrated mechanically from 4 to 22 cycles per second. After studying the damper spacing on this short span the test equipment was transferred to the field and the actual spans on the towers were vibrated and the laboratory results checked. These tests all seem to indicate that this staggered spacing and the use of four dampers were much more satisfactory than when only two dampers were used. With modern design dampers of the Stockbridge type properly designed and located, I believe it is possible to take care of vibration so that no damage will be done to the conductor.

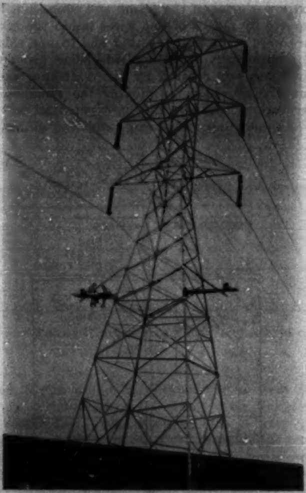

FIG. 1—MOUNTING OF RECORDERS ON A TOWER LINE HAVING ONE ALUMINUM AND ONE COPPER CIRCUIT

In connection with our study of this problem, the majority of the testing was done on an aluminum line which is one circuit on a double circuit 220-kv. tower line. The aluminum circuits consist of three 795,000-cir-mil steel core aluminum cables having a conductivity equal to 500,000-cir-mil copper. On the opposite side of this same tower there are 3 hollow core copper cables of 1-in. diameter having a section of 509,400 cir. mils. Both sets of cables are erected to give the same ground clearance, and hence have the same sag. This line represents almost ideal conditions to check relative vibration on these two types of conductors. The cables are parallel with only 27 ft. of horizontal separation at equal height above ground and supported in the same manner so that all spans are alike. A study of the relative vibration of the two conductors was undertaken and vibration recording equipment was installed, as shown in Fig. 1. This equipment was arranged to take simultaneous records. The observations were carried on for several weeks at a time when conditions were rather favorable for vibration. Fig. 2 shows the actual field records as taken and gives a rather clear conception of the relative vibration. It will be observed that the copper and aluminum cables vibrate at approximately the same time. Loop lengths were observed to be of the same magnitude and the conductors vibrate at approximately the same frequency, but the amplitude of the vibration of the copper conductor is less. This is clearly shown in Fig. 3.

It may be of interest to operating engineers to know that these records were taken while the lines were in operation; the aluminum line operating at 165,000 volts and the copper line at 220,000 volts. These records were taken by attaching the arm of the vibration recorders to the line wires by specially treated cords. The recording instruments were placed approximately 20 ft. directly below the conductor. The records were traced on a Bristol smoked chart.

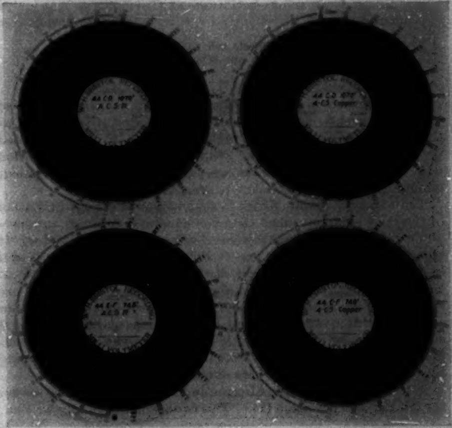

FIG. 2—RECORD OF VIBRATION

There is one other point in connection with the vibration of transmission line conductors, that is, vibration in general occurs during the night hours, late in the evening, or quite early in the morning. The wires seldom vibrate during the usual working hours of the day. This point has led some engineers to believe their lines free from vibration.

F. E. Andrews and H. T. Eddy: The subject discussed in Mr. Den Hartog's paper is that which is generally known as "whipping" rather than that which is generally known as "vibration." The paper would be considerably improved by restricting the use of the term "vibration" to disturbances such as are described in the first paragraph.

The statement is made in the second paragraph that disturbances of greater amplitude (20 ft.) are mostly associated with ice. If the theory advanced by this paper is correct, and it is the same theory advanced by Mr. Davison and which is generally accepted, the disturbances cannot arise without ice. I should like to have this statement checked by Mr. Den Hartog.

The cause of the vertical lift shown in Fig. 3 of the paper is not clear. The lift is given as a function of the velocity but it is shown with a greater force than the drag. Is this vertical lift the vertical component of the wind acting upon the conductor or is it the result of eddy currents?

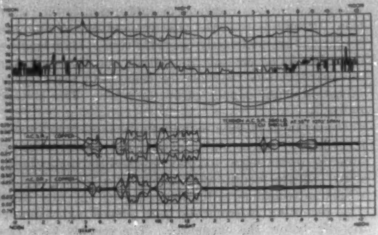

FIG. 3—SIMULTANEOUS RECORDS OF WIND VELOCITY AND DIRECTION, AIR TEMPERATURE, AND THE VIBRATIONS ARISING FROM THESE CONDITIONS IN PARALLEL COPPER AND ALUMINUM CONDUCTORS

——— A.C.S.R., 795,000 cir. mil, 1.093-in. diam.
– – – – Copper 500,000 cir. mil, 4CS hollow core, 1-in. diam.

At other places in the discussion, vibration, presumably caused by eddy currents, is assumed as the originator of action and as the cause of variation of the angle of attack. If this is accepted we would be able to prevent whipping by preventing vibration with dampening devices, but anyone who has seen the violent whipping action would be slow to accept such a solution.

The conclusion that change in span length or change in tension of the cable does not affect the phenomenon is contrary to generally established formulas for traveling waves in a conductor. It also raises doubt as to the practicability of the theory. It is well known that there may be no disturbance in spans adjacent to those in which violent whipping is in process and which are seemingly similar in all respects, except for slight difference in span length or in tension. Such considerations and the additional complications of friction, inertia of clamps and suspension insulators, stiction, etc., make it seem impractical to predict whipping by application of formulas here developed.

In the paper by Messrs. Monroe and Templin we ask the reason for the Fig. 6 in equation (1). This equation is normally written;

$$f = \frac{1}{L}\sqrt{\frac{Tg}{W}}.$$

In discussing the paper by Messrs. Davison, Ingles, and Martinoff: there are several 3-strand conductors now on the market, notably one of high strength material made by the Aluminum Company. Has there been any experience with the use of sleeves on this conductor and is any special sleeve necessary?

C. B. Basinger: It has been suggested that there is a cause of conductor vibration which is closely allied with the phenomenon of corona discharge, and which, though affected by it, is not wholly dependent upon wind for its action.

This discussion presents the results of experiments made by Mr. H. L. Richardson and the writer, at the Massachusetts Institute of Technology, to study the forces which act on a moving conductor which is surrounded by corona discharge. These studies show that a force exists which is a function of the voltage on the conductor and of the velocity of the conductor relative to the air. In general it is necessary for the conductor to be energized above the corona voltage and a further increase in voltage, over the range studied, increases the force. For this reason, in this discussion the force is called "corona force."

A glass frame, shown diagrammatically in Fig. 4, was constructed of pyrex tubing and supported at the top from a wood structure by means of a small precision ball bearing. A direct-current watt-hour meter was placed under the frame and mechanically connected with it by means of a fiber rod, so that it could be used to drive the rotating structure with easily measurable torques. Polished conductors were placed between the upper and lower arms of the frame and connected to the high-voltage source by means of fine wires inside the tubing. Both conductors were connected together and to one side of a high potential transformer, the other side being grounded. Thus, when the frame was rotated by the watt-hour meter, an arrangement was provided for moving the conductors through the air at various speeds while high potential was applied to them. The connecting wires being inside the tubing, the corona discharge from them had no effect upon the turning of the frame, and whatever force was observed could be attributed only to the exposed conductors held between the four projecting arms of the frame.

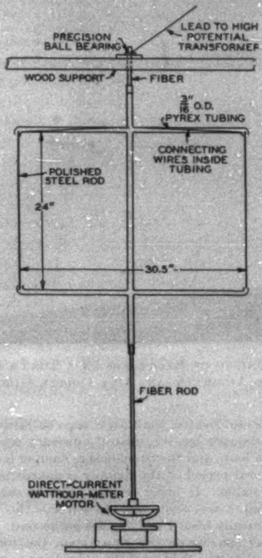
PRECISION BALL BEARING
LEAD TO HIGH POTENTIAL TRANSFORMER
WOOD SUPPORT
FIBER
3" 16 O.D. PYREX TUBING
CONNECTING WIRES INSIDE TUBING
POLISHED STEEL ROD
24"
30.5"
FIBER ROD
DIRECT-CURRENT WATTHOUR-METER MOTOR

FIG. 4—DIAGRAM OF GLASS FRAME AND DRIVING MOTOR

The watt-hour meter motor was calibrated for torque by means of a spring; thereafter torques were determined from the meter field and armature currents. It was therefore possible to plot speed-torque curves for the combination of the motor and the frame, with various voltages applied to the conductors; from the speed-torque curves the force due to the voltage was readily derived.

At any given speed the friction inside the meter, as well as the outside friction on the frame, was the same with and without voltage on the conductors. Also, at any given speed the internal torque supplied by the meter was the same for these two conditions. Therefore, the difference in torque required, with and without voltage, to drive the frame at any given speed is the torque supplied by the electrical force on the conductors at that speed.

Throughout the tests three different sizes of conductor were used: 2.4 and 1.6 mm. steel rods, and 0.8 mm. copper wire. Three voltages, 76.7, 62.4 and 48.0 r.m.s. kv. were used in connection with each of the three sizes of conductor. In all cases the recorded speeds of the conductors through the air were determined by timing the revolutions of the frame after the steady state had been established. Speed-torque curves were plotted in terms of the speed of the conductors through the air in centimeters per second, and the torque required, in gram-centimeters, and in each case a curve was plotted for both directions of rotation.

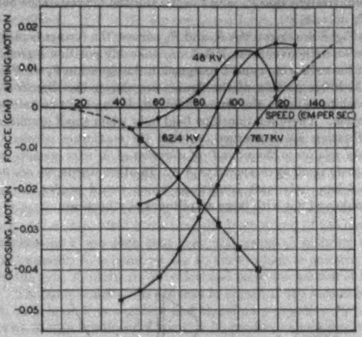

FIG. 5—SPEED FORCE CURVES FOR A 61-CM. LENGTH OF 0.8 MM. WIRE

The corona force speed curves were determined from the difference between the speed-torque curve with no voltage and the average of the speed-torque curves in each direction with voltage applied. The averaging of the torques in each direction balanced out any inherent dissymmetry.

In Figs. 5, 6 and 7 are shown the curves of force on a 61-cm. length of conductor as derived from the speed-torque curves. The force is shown in grams and the speed of the conductor in centimeters per second. The force over the range of velocities studied seems to be somewhat cyclic in nature, being negative or such as to retard the motion of the wire at very low speeds, and positive or such as to accelerate the motion at somewhat higher speeds. Furthermore, in the case of the 0.8 mm. wire, the maximum magnitude of the force in the negative region is considerably greater than that in the positive region. Another interesting point is that the speed for the maximum positive force becomes less as the size of the conductor is increased. In each figure the curve of air resistance for the size of wire in question is also plotted, so that the order of magnitude of the force due to voltage, or the corona force, may be compared with it.

It is realized that the method of arriving at the corona force *versus* speed, which was followed, is an inherently inaccurate one. That is, the difference between two fairly large quantities is used in obtaining a smaller quantity. During the course of taking the readings, however, it was found that the force is at best a rather erratic quantity. The presence of even a slight roughness on the surface of one of the conductors changes the readings a great deal. Furthermore, although the conductors were polished with a cloth at frequent intervals, the accumulation of deposit on the conductors affected the readings, so that they were in a measure dependent upon the time of application of the voltage between each successive reading. For these reasons it is not thought that any detailed conclusions may be drawn from the curves. They do, however, show the existence of a force, its general order of magnitude and manner of variation.

The experimental work which has been briefly described above was entered into in the course of a study of the vibration of small conductors in the laboratory under the influence of high voltage. The most unexpected result shown by the curves which have been presented, is the presence of a relatively strong retarding force at certain speeds of the conductor through the air. Further studies of vibrating spans showed that under certain conditions a span of wire may be unstable and tend to vibrate even though its velocity through the air is such as always to produce a corona force in the direction to retard the motion. The conditions for such vibration, the proof of which is given below, are as follows:

1. The wire must be in a uniform current of air, moving perpendicular to it, of such a velocity that it falls opposite a portion of the force curve which has a positive slope (that is, a portion which shows an increase in the positive value of the force with an increase in air velocity).

2. The magnitude of the slope of the force curve must be greater than the magnitude of the slope of the air resistance curve, at this point. (It is assumed that the air resistance is the only damping force.)

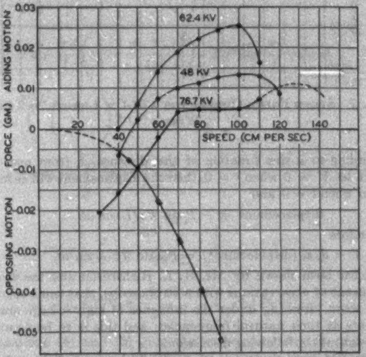

FIG. 6—SPEED FORCE CURVES FOR A 61-CM. LENGTH OF 1.6 MM. WIRE

If these two conditions are fulfilled, the wire will be unstable if vibratory motion is considered to be in a plane parallel to the motion of the air. In other words, for one cycle of vibration the energy input will be greater than the energy output. An analysis leading to a proof of the two conditions above may be given as follows:

Consider a span of wire the length of which is so large as compared with its diameter that the wire may be considered as a flexible string of certain mass per unit length. The natural fundamental frequency of vibration may be easily calculated by means of Lord Rayleigh's formula:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}, \qquad (1)$$

where f = frequency,
L = length,
T = tension,
and ρ = mass per unit length.

If it is further assumed that the wire vibrates harmonically and that its configuration during vibration is a sine wave, it is possible to relate the velocity at any point on the wire at any time, t, with the maximum amplitude at the center of the span. For, considering one point on the wire,

$$V = (\text{max. velocity}) \sin \omega t$$

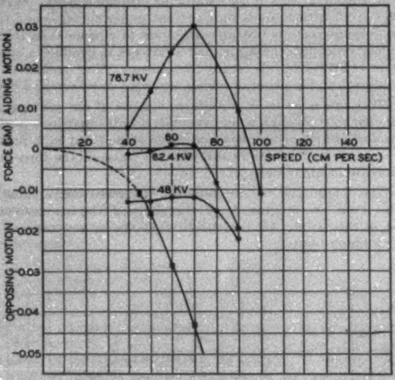

Fig. 7—Speed Force Curves for a 61-Cm. Length of 2.4 Mm. Wire

But the maximum velocity varies over the length of the span according to a sine function of the distance, x, from one end. If we let $V = 0$ when $t = 0$, then the displacement will be maximum when $t = 0$, and

$$\text{displacement} = -P \sin \pi x/L \cos \omega t.$$

Therefore

$$V = d/dt\,(\text{displacement}) = P\omega \sin \pi x/L \sin \omega t. \qquad (2)$$

It has been shown by Mr. H. L. Richardson and the writer, by means of observations with a stroboscope, that small conductors when vibrating under the influence of high voltage always vibrate at their natural frequencies, and this is to be assumed throughout the following discussion.

The first requisite for a mathematical treatment of the problem is some sort of algebraic equations representing the variation of corona force with velocity, and the variation of air damping force with velocity.

Looking at the corona force and air resistance curves which have already been shown, it is seen first that certain portions of the corona force curves are practically straight lines, for instance, the curve for the 0.8 mm. wire at 76.7 kv. has quite a long straight portion. In other cases parabolas might fit quite well, and there are other possibilities. In the second place, it may be found with a little experimentation that the air resistance curves can be very closely represented by a portion of a parabola, $F = -k_0V^2$.

However, it has been found possible, by assuming general shapes for the corona force curves, such as the straight line $F = mV + C$, to arrive at conclusions which may be applied in a general way to all of the force curves. Actual numerical values need not be used to derive these results, although they may be placed in the results afterward and definite answers obtained.

In general considering a variable force acting upon a moving body,

$$\text{energy} = \int FV\,dt. \qquad (3)$$

If the force acts in the direction of motion it will impart energy to the body, or the energy change will be an energy input to the body. The opposite will be true if the force acts counter to the direction of motion.

Now let us assume that the wire is in a uniform current of air of velocity A cm./sec., moving perpendicular to the wire. Also assume that the corona force line may be represented by the general straight line $F_c = mV + C$, and the air resistance curve by $F_d = -k_0V^2$.

In Fig. 8 are shown the F_c and F_d curves; the former is plotted as shown for convenience, but it may take any position depending upon the values of m and C assigned in the equation $F_c = mV + C$. In the lower portion of the figures are shown the direction relationships between the forces involved, during the two halves of the cycle. These relationships hold only when small amplitudes of vibration are concerned, so that *the maximum velocity (the velocity relative to the ground) of vibration of the wire remains comparatively small as compared with A*. In other words, we are concerned here only with conditions which would render the wire unstable and not with large amplitudes of vibration.

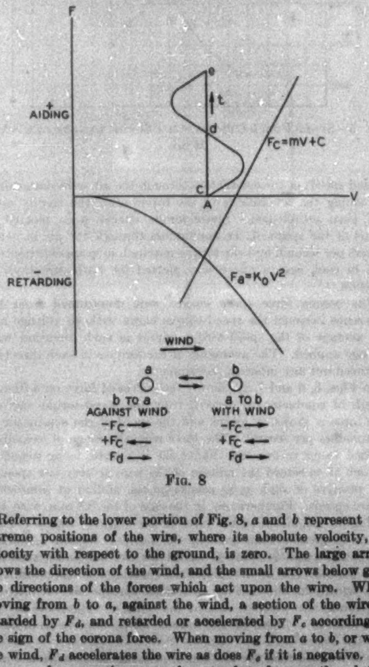

Fig. 8

Referring to the lower portion of Fig. 8, a and b represent the extreme positions of the wire, where its absolute velocity, or velocity with respect to the ground, is zero. The large arrow shows the direction of the wind, and the small arrows below give the directions of the forces which act upon the wire. When moving from b to a, against the wind, a section of the wire is retarded by F_d, and retarded or accelerated by F_c according to the sign of the corona force. When moving from a to b, or with the wind, F_d accelerates the wire as does F_c if it is negative. In other words, a negative corona force tends to decrease the relative velocity of the wire and the air. When the wire is moving with the air, but not as fast as the air, the only way it can do this is to tend to accelerate the wire.

In Fig. 8 is also shown the sine wave of absolute velocity, with the time axis vertical. It is obvious that this velocity curve should be plotted at A, since during one-half of the cycle it adds to the velocity A, giving a velocity $A + V$ relative to the air, and during the other half cycle it subtracts, giving a velocity $A - V$ with respect to the air.

Starting from c and moving to d, on the sine wave of absolute velocity, corresponds to the motion of the wire from b to a in the lower diagram, and from d to e, corresponds to motion from a to b. This is true since motion against the wind increases the relative velocity of the wire and the air, and motion with the wind decreases this velocity.

Now return to equation (3) and consider the signs of the quantities involved. If F is the air resistance force, F_d, its sign will always be negative. Over the first half of the cycle, from c to d on the sine wave, V will be positive and consequently the product, F_dV will be negative and will represent an energy output. During the second half of the cycle, from d to e, both F_d and V will be negative and their product will be positive, representing an energy input.

If F in the equation is F_c, the force due to corona, during the first half cycle the product, F_cV, will be positive or negative according to whether F_c is positive or negative. This agrees with what is seen should be true, because as long as F_c is negative it acts toward the right (Fig. 8, lower portion) and retards the motion of the wire. But if the magnitude of A and of the maximum value of V are such that they add up so as to bring the relative velocity to a point on the F_c curve which is positive, or above the axis, F_c shifts its direction and helps the motion of the wire. The same conclusions are true in regard to the second half cycle, only here V is negative and therefore a negative F_c produces an energy input and a positive F_c produces an energy output. Throughout the integration, therefore, the sign of F_c takes care of itself and the result is a net energy change, positive or negative.

Having clearly in mind the fact that a positive sign for the energy in any case represents an energy input and a negative sign, an energy output, we shall proceed to integrate over one cycle to find the net energy due to corona and the net energy due to air resistance.

As the equations for the corona force and the air resistance have been given, the V in each is measured from the origin and represents the relative velocity of the air and the wire. If we take V as the absolute velocity, then the relative velocity becomes $A + V$, where V is the V of equation (2). This amounts simply to shifting the origin on the diagram to the point A. The equations for the two force curves, then, are

$$F_c = m\,(A + V) + C,$$

$$F_d = -\,k_e\,(A + V)^2.$$

Let E_c be the net energy change per cycle due to the corona force, and E_d be the net energy change per cycle due to the air resistance.

Determination of E_c:

$$E_c = \int F_c\,V\,dt = \int (m\,AV + mV^2 + CV)\,dt.$$

Substituting from equation (2) and placing limits,

$$E_c = \int_{x=0}^{x=L}\int_{t=0}^{t=\frac{2\pi}{\omega}} \left(mAP\omega \sin\frac{\pi x}{L} \sin \omega t + mP^2\omega^2 \sin^2\frac{\pi x}{L} \sin^2 \omega t \right.$$
$$\left. + CP\omega \sin\frac{\pi x}{L} \sin \omega t \right) dx\,dt,$$

$$= \int_{x=0}^{x=L} \left| \left(mP^2\omega \sin^2\frac{\pi x}{L} \right)\left(\frac{t}{2} - \frac{\sin 2\omega t}{4\omega} \right) + \left(mAP\omega \sin\frac{\pi x}{L} \right.\right.$$
$$\left.\left. + CP\omega \sin\frac{\pi x}{L} \right)\left(\frac{1}{\omega} \cos \omega t \right) \right|_{t=0}^{t=\frac{2\pi}{\omega}} dx,$$

$$= \int_{x=0}^{x=L} \left[\left(mP^2\omega^2 \sin^2\frac{\omega x}{L} \right)\left(\frac{\pi}{\omega} \right) - 0 - 0 + 0 \right) + \left(mAP\omega \sin\frac{\pi x}{L} \right.$$
$$\left. + CP\omega \sin\frac{\pi x}{L} \right)\left(\frac{1}{\omega} - \frac{1}{\omega} \right) \Bigg] dx,$$

$$= \int_{x=0}^{x=L} \left(mP^2\,\omega\pi \sin^2\frac{\pi x}{L} \right) dx,$$

$$E_c = mP^2\omega\pi\,\frac{L}{2} = mP^2\pi^2 fL. \qquad (4)$$

This is independent of A and its sign depends upon the sign of m, the slope of the F_c line.

Determination of E_d:

$$E_d = \int F_dVdt = \int (-\,k_eV^3 + 2AV^2 + A^2V)\,dt.$$

Substituting from equation (2) and placing limits,

$$E_d = \int_{x=0}^{x=L}\int_{t=0}^{t=\frac{2\pi}{\omega}} \left[\left(-\,k_eP^3\omega^3 \sin^3\frac{\pi x}{L} \sin^3 \omega t \right) \right.$$
$$\left. + \left(2AP^2\omega^2 \sin^2\frac{\pi x}{L} \sin^2 \omega t \right) + \left(A^2P\omega \sin\frac{\pi x}{L} \sin \omega t \right) \right] dx\,dt,$$

$$= -\,k_e \int_{x=0}^{x=L} \left| P^3\omega^3 \sin^3\frac{\pi x}{L} \left(-\,\frac{1}{\omega} \cos \omega t + \frac{1}{3}\,\omega \cos^3 \omega t \right) \right.$$
$$\left. + 2AP^2\omega^2 \sin^2\frac{\pi x}{L} \left(\frac{t}{2} - \frac{\sin 2\,\omega t}{4\omega} \right) - A^2P \sin\frac{\pi x}{L} \cos \omega t \right|_{t=0}^{t=\frac{2\pi}{\omega}} dx,$$

$$= -\,k_e \int_{x=0}^{x=L} \left(\frac{\pi}{\omega} \right) 2AP^2\omega^2 \sin^2\frac{\pi x}{L}\,dx,$$

$$= -\,k_0\,2AP^2\omega\pi \int_{x=0}^{x=L} \sin^2\frac{\pi x}{L}\,dx,$$

$$= -\,k_e\,2AP^2\omega\pi \left| \frac{x}{2} - \frac{\sin^2\frac{\pi x}{L}}{4\pi/L} \right|_{x=0}^{x=L}$$

$$= -\,k_e\,2AP^2\omega\pi\,\frac{L}{2} = -\,k_0 \cdot 2AP^2f\omega^2L. \qquad (5)$$

Equations (4) and (5) give the expressions for E_c and E_d over a complete cycle of vibration for the whole wire, under the conditions which have been assumed in this case. If m in the equation for the corona force line is positive, the net energy due to corona is an energy input to the wire; if m is negative the energy is an energy output. The net energy due to air resistance is always an energy output, since k_e is always negative.

Now let us take the ratio of the magnitude of E_c to the magnitude of E_d.

$$\frac{|\,E_c\,|}{|\,E_d\,|} = \frac{mP^2\pi^2 fL}{k_e\,2AP^2\pi^2 fL} = \frac{m}{2\,k_eA} \qquad (6)$$

The slope of the corona curve, as has already been stated, is

$$|\,S_c\,| = m\,. \qquad (7)$$

The slope of the air resistance curve is

$$S_d = dF_d/dV = -\,2\,k_eV,$$

and its slope at the point A is

$$S_d = -2\,k_0A.$$

Or since the slope is always negative, taking the magnitude only,

$$|\,S_d\,| = 2\,k_0A. \tag{8}$$

Substituting (7) and (8) in (6),

$$\frac{|\,E_c\,|}{|\,E_d\,|} = \frac{|\,S_c\,|}{|\,S_d\,|} \tag{9}$$

Therefore, under the conditions assumed, the question as to whether or not the wire will vibrate depends only upon the relation between the slopes of the two curves for F_c and F_d at the point A. Since the sign of S_c determines the sign of E_c, the two conditions which must be met in order that the wire may be in an unstable state and tend to vibrate are those stated above. A numerical example follows:

Taking the corona force curves for the 0.8 mm. wire, and considering the curve for 76.7 kv. in particular, it may be found that the straight portion of this curve, *i. e.*, from about 69 cm./sec. to about 115 cm./sec. has a slope $m = S_c = +0.000778$ and that the constant in the equation of the air resistance curve is $k = 3.5 \times 10^{-6}$ both in grams per cm. per sec.

Assuming an 0.8 mm. copper wire 30 ft. (= 915 cm.) long with a tension of 5 lb. the calculation of the fundamental frequency of vibration by means of formula (1) may be easily made, and the result is: $f = 3.8$ cycles/sec.

Also assume

$P = 1$ cm.
$A = 1.5$ miles/hr. $= 67.2$ cm./sec.

(If $P = 1$ cm., $P\omega$ = max. absolute velocity $= 1 \times 2\pi \times 3.8 = 22$; this is well under A, and therefore the assumed condition is fulfilled.)

The length, L, in the formulas for E_c and E_d, is "unit" length, or the unit of length of the wire for which the force on the curves was taken. The unit length in this case is 61 cm. Therefore the expressions for E_c and E_d must be divided by 61. Also since force is given in grams, the units of E_c and E_d will be in joules $\times \dfrac{10^7}{980}$. Therefore, in order to obtain joules in this case we must multiply by $\dfrac{980}{10^7}$.

Then

$$E_c = \frac{mP^2\pi^2FL}{61} \times \frac{980}{10^7} = \frac{0.000778}{61} \times 1 \times \pi^2 \times 3.8$$
$$\times 915 \times \frac{980}{10^7} = 0.0000431 \text{ joules}$$

and

$$E_d = \frac{-2k_0AP^2f\pi^2L}{61} \times \frac{980}{10^7} = \frac{-2}{61} \times 3.5 \times 10^{-6} \times 67.2$$
$$\times 1 \times 3.8 \times \pi^2 \times 915 \times \frac{980}{10^7} = -0.0000260 \text{ joules.}$$

The energy input per cycle is, then, 0.0000431 joules, and the energy output per cycle is 0.0000260 joules. A maximum amplitude of 1 cm. was assumed for convenience. Obviously, for this same value of A, regardless of amplitude E_c will always exceed E_d, as shown by the equations. But the corona force curve is a straight line only within very definite limits and therefore the amplitude would not increase very much beyond 1 cm. before the energy input would fall off very quickly. We cannot predict the maximum amplitude from these results, over the range covered, but we can say that the wire would tend to vibrate under these conditions.

If A were taken still larger, the difference between E_c and E_d for any amplitude would be less, and finally if A were taken opposite the point where the two slopes are equal, E_c would equal E_d and the wire would not vibrate.

E. W. Dillard: Messrs. Davison and Monroe have described in detail two commonly recognized types of conductor vibration which are now familiar to many engineers. Mr. Monroe has also briefly touched on a third type of vibration with which we have had some experience in New England. Although this vibration has been shown to be harmless, it has aroused considerable interest because of its apparently close relation with the phenomenon of corona.

This phenomenon was first observed on the 220-kv. Fifteen Mile Falls transmission line during a snow storm a few weeks after the line was put in operation. Subsequent field tests established the following facts, which seemed to indicate that its origin was different from that discussed in the two papers mentioned above:

1. Vibration occurred only when the line was energized.
2. Vibration occurred only under atmospheric conditions where either fog, rain, or wet snow were present.
3. Vibration occurred only when the wind velocity did not exceed three miles per hour, and most cases occurred with a lower value.
4. The character of the vibration is different from either of the well known types. In amplitude, wave length, and frequency it lies between "wind vibration" and "galloping." The frequency is of the order of 75 cycles per minute with corresponding loop length.

The presence of voltage as a necessary condition was thoroughly demonstrated by the field tests which have been confirmed by subsequent observations. Vibrations die out within fifteen minutes after the line is deenergized and build up in about twenty minutes after sufficient voltage is restored. The presence or absence of current flow is apparently of no significance. Furthermore, the field tests indicated that there is a critical voltage below which vibration will not take place even during the most propitious conditions.

The Fifteen Mile Falls transmission line has been fully described in the September 6, 1930 issue of the *Electrical World*. The line consists of two circuits, 795,000 cir. mils aluminum, on single-circuit towers. It was definitely designed for a low normal tension in the conductors and a low average height above ground which resulted in a normal tension of 2,900 lb. and the use of 8.94 towers per mile. The insulation consists of 15 disks with grading rings. Tapered armor rods were installed to further protect the conductors from burning and from mechanical injury at the clamp. It is believed that the low conductor tension and the damping action of the armor rods greatly reduce the possibility of fatigue failure from high frequency wind vibration of the type described in the preceding papers.

An analysis of the field tests led to the belief that disruptive corona might be a third cause of vibrating conductors. A test span, the exact duplicate of one of the spans on the line, was erected to confirm this hypothesis by further experiments under controlled conditions. Power was supplied from a single-phase 300-kv. grounded neutral testing transformer, and a row of sprinklers installed above the span provided a means of supplying precipitation at will.

It was found that no vibration could be produced if the wind velocity exceeded four miles per hour. Difficulties were experienced in carrying on the tests because of the fact that wind velocities are usually greater than this limit excepting for very short periods at sunrise and at sunset. We also found that vibration will occur with a lower voltage during precipitation than when the conductor is dry. This explains the rare occurrence of vibration on the line since the simultaneous conditions of low wind-velocity and precipitation are required.

Under the test conditions we were able to produce vibrations in all sizes of conductors from No. 14 A.W.G. stranded copper antenna wire to 795,000 cir. mils A.C.S.R. Fig. 9 shows the relation between the voltage required to sustain vibration and the conductor diameter. Within the precision of the set-up the points fall on a smooth curve which closely resembles the "storm corona" voltage curve which is plotted in the same figure. A single exception was encountered. We were unable, with the voltage available, to produce vibration in a No. 4/0, seven-strand, hard drawn copper conductor which seemed to have inherent mechanical characteristics which suppressed any tendency toward vibration.

On the smaller sizes of wire; that is, up to No. 4 A.W.G., it was possible to cause vibration without any precipitation; that is, when the wire was dry. On the larger sizes of copper and aluminum cables precipitation was necessary with the voltages available. In the case of the smaller wires, where voltage enough to cause vibration in the dry state was available, vibration took place at voltages about twice the voltage necessary for vibration when wet.

In general, the amplitudes which were developed in the test span were less than those reported on the line under operating conditions. This is probably accounted for by the relative length of span. In one case, however, an amplitude of 40 in. was obtained by impressing 235 kv. on a No. 4 A.C.S.R. when dry.

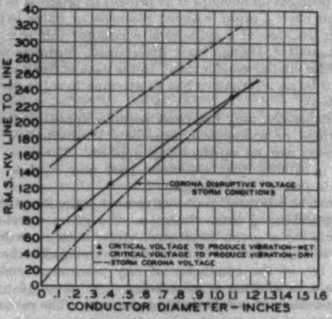

Fig. 9

Vibrations appeared to build up slowly after the application of a voltage higher than the critical voltage; ten minutes to half an hour (depending upon the conductor size) were required before the final steady state was established. For voltages only slightly higher than the critical voltages the motion is likely to be a short traveling wave, approximately one-twentieth the span length, and it is reflected at the points of support. At higher voltages the motion is more likely to take the form of a standing wave with nodes usually not less than 20 ft. apart. When standing waves are established, an increase in voltage causes an increase in amplitude and, frequently, an increase in wavelength together with a decrease in frequency.

The Subcommittee on Conductor Vibration of the Pacific Coast Electrical Association reported, *Electrical West*, May 15, 1930, a case of vibration on the California-Oregon Power Company system which very closely resembled that noted on the Fifteen Mile Falls line. Correspondence with the engineering department of this company has developed the fact that vibration occurs on the two transmission systems under identical conditions. The voltage of the California-Oregon line is 120 kv. and the conductors are No. 2/0 copper. The vibration point of this conductor is closely checked by the results which were obtained in the New England test span.

It is believed that the foregoing data indicate the following new principles in regard to conductor vibration:

1. Any line which is subjected to voltages above the storm corona limit may develop vibrations.

2. Such vibrations are not caused by wind pressures; on the contrary, the presence of even moderate velocities of wind suppresses the tendency toward motion.

3. It is probable that the formation of corona is closely allied with the forces producing vibration.

4. That such forces can exist is clearly shown in the cooperative investigation made by Messrs. Basinger and Richardson of the Massachusetts Institute of Technology whose results are presented in a separate discussion.

5. Frequent vibration of this type is unlikely since it usually occurs only when there is precipitation and never when there is any appreciable wind.

We feel that the possibility of damage from this type of vibration on the Fifteen Mile Falls line is practically negligible. The unusually low stringing tension that was used in the design of this line is a preventative to wind vibration, and armor rods were also installed as a further safeguard against any incipient motion or arcing. Vibration from corona is of such rare occurrence and low frequency that lines with materially less favorable characteristics should not be affected adversely.

We do not feel that our investigation has been exhaustive and it is desirable for academic reasons that it be supported by further observations and experiments. It is urged that all engineers operating transmission lines at or near the limiting corona voltage be particularly watchful for vibrations of this type, especially during periods of precipitation when wind is absent or of low velocity.

L. L. Perry: March 21, 1932 in Illinois and Indiana was an extraordinary day in weather. Conditions produced sufficient dancing of conductors to trip out 132-kv. circuits of both A.C.S.R and copper. Circuits tripped out at points about 130 miles west of Chicago, a few miles south of Chicago, and at a point about 150 miles east of Chicago.

At Chicago the total temperature range was only from 29 deg. F. minimum to 30 deg. F. maximum, with a maximum wind of 35 miles per hour. At one point in Illinois four parallel circuits of 447,000 A.C.S.R. on 2 two-circuit tower lines tripped out within a few seconds of one another when the nearest weather bureau station recorded 19 miles per hour wind, slight rain, freezing, a temperature 31 deg. F. to 30 deg. F., with a range over the day from 32 deg. F. at 1:00 a.m., to 28 deg. F. at midnight.

Apparently all these trip-outs occurred at one point, as evidenced by burns on the conductors and ground wires. At this point, this transmission has a diagonal run that presented the only case of about 90 deg. angle of line with the wind. The circuits danced less in other spans, but gave no evidence of tripping out. Sleet thickness was estimated at 3/8 in. radial. The Springfield, Illinois, weather bureau records read: "From LaSalle south as far as Carlinville the day produced almost everything in the way of weather, *viz.*, rain, snow, sleet, glaze, wind and thunder."

In Alabama that day the tornados, numbering 7, were the worst on record. Since the weather bureau report states that "a bounding tornado" was reported in Illinois with a path 25 to 50 ft. wide, striking the ground intermittently, it is fair to assume winds may easily have been spotty.

The first 132-kv. line in the Chicago District went into service in 1924. This circuit was 300,000 cir. mil copper strung to rather low tension on 500 ft. spans. Very little lightning disturbance occurred in 8 years in that transmission, and none that would prevent the circuit immediately being closed and kept in after trip-out. This line gave no dancing difficulties until about 5 years had elapsed, and then only momentary interruptions resulted. But after another year, however, a day came when interruptions were so frequent that one circuit was kept out of service for some hours.

Commenting on Mr. Monroe's claim that dampers would not be injured by dancing: why then are insulators injured by dancing conductors?

J. A. Ingles: Discussions, such as Mr. Den Hartog's, of vibrations resulting from sleet load or other causes are very opportune. It is highly desirable that this phenomenon be kept prominently before transmission engineers and executives, since there is no convenient method of controlling the hazard and the interruptions in the service, which may be expected. About the only known remedy is to keep the cables warm by artificially heating them slightly above surrounding temperatures during sleet forming periods. We know that this method is effective. The paper does not offer any suggested remedial measures.

While admitting that the so-called "galloping" due to sleet and other causes, is a vibration and is associated with all other vibration phenomena, efforts have been heretofore made to segregate, if possible, in the minds of those who have not studied the matter extensively, the considerable differences which there are between the problems of controlling interruptions and failures due to this "galloping" and the troubles which develop on account of the finer vibrations. Conditions which result from sleet loading and winds at 5 to 15 miles per hour, cannot very well be imagined. They have to be seen in the field. "Galloping" so-called of various conductors, including small telephone wires, is fairly frequent in the district of 100 miles radius centering on Niagara Falls. It is not peculiar to that district.

Mr. Den Hartog quite correctly points out that in earlier references to "lift," "drag" studies were omitted and that it is desirable to study the resultant forces at the various angles which the flopping cable may present to the wind. Vibration is always extremely fortuitous and almost anything can happen if the combination of long icicles and moderate winds continue, for a very long time.

Referring to Mr. Stickley's paper, it has been known for some considerable time that a cable when strung takes some time before reaching its final sag conditions. The reason for this is made quite evident in the paper.

Mr. Stickley presents the view that tension in the cable tends to wrap the strands of the various layers closer together. It would be interesting to know if during tests, he has experienced any basketing or "birdcaging" of the cables, for instance, as at end conditions or connections, as indicated by Dr. Ing. T. Gröble, or if he has taken any record of the rotation of the cables under test.

On the question of vibration, it would be most advantageous to tabulate the fatigue limits of the various metallic cables. Vibration failures are largely due to combined stresses exceeding a safe limit. Until recently the only indication that these stresses were exceeded was the fact that failures occurred. With Stickley's values of a cable a fairly reasonable value of stresses could be calculated. Then reference to some authorative tabulation of fatigue limits, or fatigue curves, should be made to enable the designer to know whether he is within safe limits or not. The fatigue limit is in every way as important as the ultimate stress of a cable and it would be to the advantage of the transmission engineer if these values could be published by the manufacturer.

M. E. Noyes: The suggestion is made by Messrs. Davison, Ingles and Martinoff that different types of stranding of cables should be compared to determine their relative ability to dissipate energy by interstrand friction. The Aluminum Company of America has done this successfully in its laboratory on 120 ft. spans using a decrement method suggested by Mr. Kimball, of General Electric Company. The decrement of amplitude is measured from a trace taken after the source of forced vibration has been suddenly disconnected from the cable. This value of decrement is then substituted in the Kimball formula.

In considering new cross section shapes and new types of stranding as means for reducing amplitude of vibration, it is very important to remember that no mathematical or laboratory study has taken into account the importance of severe stress concentrations which may occur at the ends of the span due to abrasions of the surfaces of the wires. The result of surface deformation is commonly called "notch effect" when encountered in laboratory testing for fatigue of metals. It is well known that a surface indentation produces a concentration of stress which greatly lessens the life of the fatigue test specimen. For transmission line conductors we must not overlook unavoidable localized stresses, from this cause, at the supporting points. Any cross-section shape of conductor which causes certain strands to stand out sharply would, in my opinion, be liable to higher stress concentrations than the usual round cable.

E. Bate: The paper by Messrs. Davison, Ingles and Martinoff is devoted principally to a comparison of effects produced by deliberate departure from the conventional cylindrical form of conductors, the intention being to prevent or minimize the production of eddies identical in phase or period. This is a very sound line of attack on the problem, since it aims at a diminution of the energy which can be transferred to the conductor by the wind. The converse problem is to increase the resistance of the conductor to bending, *i. e.*, to increase greatly its ability to dissipate energy and so reduce the amplitude of vibrations.

To both of these problems the writer, together with many other engineers, has directed some effort, and in the case of the former problem an attempt is at present in progress to determine the wind energy input without resource to water experiments. Even in the case of a cylindrical conductor this is a somewhat difficult matter, though once determined, formulas can be developed for similar conductors. In the case of freak shapes there seems to be no alternative but to attempt to solve each case by experiment.

Unfortunately, very few freak sections appeal to the engineer as practical or economical, especially if alternative means of suppression can be satisfactorily applied. There is, nevertheless, a possibility that assiduous trial may lead to a radical change in present conventions.

It must be admitted that our present estimate of the danger to be apprehended from vibrations of a given wavelength and amplitude for a given conductor, stressed to a predetermined extent, is vague, so that it is difficult to determine the influence on the life of the conductor of any percentage reduction in amplitude, or whether at some point short of complete suppression a condition of indefinite life is reached. The rationalization of these questions to a state where they are susceptible of calculation would greatly inform the minds of transmission engineers.

Strand failure due to fatigue resulting from vibration is not confined to A.C.S.R. conductors. In the writer's experience steel cable though lightly stressed has given much trouble, in short, every cable must be considered in the light of its inherent characteristics and the conditions to which it may be exposed. At present the decision or design depends too greatly on judgment.

In addition to the two directions above mentioned in which a diminution of vibration is to be sought, there is another important provision which can be made—namely dampers of various kinds. A purely frictional device may be used in lieu of inherent damping in the cables, though most of those proposed are clumsy and unsuitable for use on transmission lines.

It should be possible, however, to devise an arrangement which, actuated by the initial movements of the cable before full resonance is attained, generates and allows to be reflected along the span, an interfering wave, which causes an immediate reduction in energy input to the conductor by the wind.

In the ideal state, when resonance is building up on the span, a complete set of loops, practically sinusoidal in shape is formed. This condition is essential for maximum energy transfer, and the energy transferred will then vary directly as the amplitude of the oscillation. If, however, this symmetrical sinusoidal arrangement be upset by interfering waves, it may be possible to reduce the magnitude of energy transfer, and so inhibit resonant oscillation. To such interference the transmission span is very susceptible in view of the minute quantities of energy involved.

An action such as this, is the cause, in the writer's opinion, of the efficacy of one or two of the services or arrangements which are used and designated as dampers. Experiment shows that they could not be expected to dissipate in friction sufficient energy justly to account for the great reductions in amplitude which they cause.

V. M. Martinoff: The users of transmission materials and conductors appreciate the great amount of work which Messrs. Monroe and Templin, and their associates, have done, and as well the cooperation which has been manifested where problems have arisen.

Reference is made to the relative life of certain materials when used at 15,000 to 18,000 lb. This is only a small variation but it would seem that the life has been materially altered. Speed-up tests probably increased the elastic hysteresis in the cable as compared with natural conditions and probably reduced the life of the cable considerably. Is there any information available to prove or disprove quantitatively this statement?

As the experimental determination of the flexural rigidity of cables at discontinuities is of great importance, I would like to ask the writers if they know an empirical equation by means of which an approximation of section modulus of a stranded cable may be obtained at various distances from the clamp. Have any experiments been carried out to determine such an equation?

Have the experiments differentiated between the effects of winds blowing at the more acute angles to a conductor under observation as compared with perpendicular light winds? Were there any curves obtained showing variations in amplitudes for winds of equal velocity but blowing at different angles to the line?

A. E. Davison: It is desirable in case further studies of vibration are made that someone prepare a statement for the direction of other engineers covering as near as possible all those factors which affect vibration and fatigue tendencies, both adversely and beneficially, discuss each briefly and correlate them as to importance.

It is thought by some that the direct and immediate solution would be by way of minor reductions in the economics of a transmission system—that is, add some 2 to 5 per cent to the cost of a transmission system and thereby reduce the mechanics in the conductor by as much as 25 or more per cent.

Apart from this which is an entirely economic feature, some of the adverse factors are:

1. *Location of Line with Regard to Terrain.* Open, level areas, free from wind obstructions such as trees, buildings and hills probably give rise to excessive fatigue on account of heavier vibration.

2. *Relative Smoothness of Surface of Conductors.* It is assumed in this connection that a thoroughly polished and perfectly round cylinder would provide maximum opportunity for the effects which a given wind might have on vibration.

3. *Wind Velocity.* Low wind velocities give rise to approximately stream line flow, and formation of steady eddies in lee of cable. High velocities give turbulence disrupting the eddies. There is probably a critical velocity giving the worst fatiguing effects.

4. *Wind Uniformity.* Non-uniform winds give rise to beats which are probably more serious than uniform vibration. Turbulence as pointed out above may be beneficial in that vibrations could be broken up.

5. *Length of Cable Without Discontinuity Which Could Act as a Reflector, Thereby Introducing a Source of Fatigue.* Where long spans are used, larger amounts of energy are carried to the clamps.

The word "discontinuity" is used to cover splices as well as suspension clamps and other points of attachment. This would require a discussion on methods of suspension, etc.

6. *Tensions.* Higher tensions increase both direct and bending stresses, therefore tend to produce fatiguing more rapidly.

7. *Fatigue or Endurance Limit of Metals Used in Conductors.* What are the correct limits to use for various metals? It may be assumed that most conductors in present practise are stressed beyond the fatigue limit. Is any one type of metal or alloy better than another? Resilience limits require attention.

8. *Load per Clamp.* Should double suspension or double clamps be used to reduce combined stresses so that they will be within fatigue limits?

9. *Inertia of Cable.* Resists setting up of vibration but tends to sustain it once it is built up. Extremely light weight cables if pulled to unnecessarily high tensions should account for generation of greatest mechanical energy.

10. *Resonance Point of Cable or System.* If resonance point can be moved outside governing practical limits, vibration will be overcome. (See Ferrier.)

11. *Diameter of Cable, That Is, Exposure versus Mass.* Frequencies also are inversely proportional to diameters.

Among some of the beneficial factors are:

1. Fatigue resisting qualities of metal used—toughness, etc.

2. Internal friction—interstrand and stiction (static friction). What are the relative damping qualities of each?

3. Viscosity of medium in which cable is vibrating; "head" and "dynamic" resistances.

4. Size of strands making up cable. Distribution of stress in small as compared with large strands. Large strands give irregular profile.

5. Variation of surface from standard round rod.

It is desirable that someone who is interested in the subject give some attention to the analysis and weighting of these and other factors.

Messrs. Andrews and Eddy submitted that "disturbance (greater than ½ inch amplitude) cannot arise without ice," and ask that the statement be checked.

The facts submitted by Mr. Dillard in his discussion confirm the general statement that relatively long catenaries are naturally quite too unstable to limit causes of major vibrational disturbances to glaze, although glaze may account for a very large part of the associated interruptions and their duration.

The horizontal vibration which Mr. Monroe mentioned at Cleveland and vibrations associated with corona as described by Mr. Dillard are phenomenal, but as Mr. Dillard suggests are not sufficiently frequent, as yet, to be considered troublesome.

All vibration curative and preventive measures for various types of vibrations of relatively small amplitude, can scarcely be expected, as Mr. Andrews infers, to solve the troubles associated with glaze, and frequently called galloping or dancing.

Until the latter part of 1932, little had been heard of that type of vibration described by Mr. Dillard. Recently a patrolman called in quite disturbed, saying that on a damp dull afternoon he observed wires which were subject to the ordinary fine vibrations, demonstrating extraordinary amplitudes of the order of 3 or 4 inches. His statement was "approaching galloping," with which phenomenon he was familiar. Conditions did not continue long enough for any one other than the patrolman to observe the phenomenon.

Mr. Dillard's statement that they were unable with the voltage available to produce vibration in No. 4/0, 7-strand, hard drawn copper conductor which seemed to have inherent mechanical characteristics which suppressed any tendency towards vibration, is intriguing. Considerable attention has been given here to relative susceptibilities.

It is desirable that Mr. Dillard, or another, follow up this peculiarity, because it is felt that such fortuitous phenomena as vibration may be checked and wholly cured by some minor inherent mechanical or other characteristic yet to be discovered.

Suggestions made by Mr. Noyes regarding deformed conductors are quite opportune. Very little attention is being given to splicing methods, clamping surface and such details until it is recognized that the suppressing effects are of sufficient magnitude to warrant extra expense in connection with such details. The study is not completed.

Concluding the writer wishes to thank those who have given a great deal of attention to this subject by way of discussion. There is, doubtless, sufficient material in some of the discussions for separate papers, but for the fact that the discussors were anxious to place the information before interested engineers and operators, who have recognized some of the problems.

G. W. Stickley: Referring to Mr. Ingles' discussion, we have never experienced any basketing or "bird-caging" of cables during stress-strain tests. The testing procedure and the care necessary in making such tests practically eliminates the possibility of such occurrences. Rotation of the cable test specimen during any test was not possible because only one head of the testing machine was free to move, and that one could move in one direction only.

R. A. Monroe and R. L. Templin: In the discussion several questions have been raised which can be definitely answered only after further investigational work. This paper was intended as a progress report showing the results of work done to date, and it is planned to present the results of further investigations later.

Two problems discussed were not covered in our investigational work, namely, the phenomenon generally known as "dancing," which was discussed in a separate paper by Mr. Den Hartog and the vibration occasioned by corona, which was discussed from the theoretical and practical standpoint by Messrs. Basinger and Dillard.

Certain direct questions have been asked relative to the data and formulas presented in our paper, some of which can be answered in the light of present information.

Mr. Koontz states that he prefers using four dampers at a staggered spacing for any conductor of 1 in. diameter or larger. Our experience has been that, with cable of sizes up to 795,000 cir. mils (1.1 in. diameter) A.C.S.R., one damper at each end of the span at equal distances from the supports will satisfactorily damp vibration. Considerations of economy would dictate the use of only two dampers instead of four, providing the damping is satisfactory. The use of four dampers is, however, frequently necessary on spans exceeding 1,000 ft. in length.

The question raised by Messrs. Andrews and Eddy concerning equation (1) is explained by the statement that L is the loop length rather than the wave length and is expressed in inches instead of feet, whereas g is in feet per second.

Mr. Perry's question as to why we believe that dampers would not be injured by dancing whereas insulators are frequently broken may be answered as follows: The insulator is firmly attached to a stationary tower and consequently has to withstand the impact resulting from the reversal of the motion of the entire mass of the vibrating cable. The damper, being attached to the moving line, is free to move with it; its mass is also small and the stress due to any differential motion between the cable and the damper cannot be large.

With reference to the first question raised by Mr. Martinoff relative to the effect of tension on the life of the cable, a comprehensive series of tests is now in progress which will cover this subject more completely than was possible in the paper. A further study of the flexural rigidity of cables is also in progress. It has been found that if the component of the wind normal to the cable is used, the theoretical and observed frequencies agree satisfactorily. The relation between amplitude and wind direction, if any exists, has not been determined.

Many of the subjects listed by Mr. Davison are being studied in the investigational work which we now have in progress and it is hoped that we will be able to cover them in future papers. Undoubtedly other investigators will cover the other factors mentioned.

In conclusion, the authors wish to express their appreciation of the interest with which their paper has been received and to thank all of those who have contributed to the discussion.

J. P. Den Hartog: The model shown in Fig. 10 was built since the writing of my paper and was demonstrated at the Cleveland meeting. On the left, clamped to the laboratory stand is the Lanchester tourbillion, carefully constructed with a small ball bearing and a spring clamp on its face into which wooden models of various cross sections can be inserted. In the illustration a semi-circular section is shown. Placed in front of an ordinary fan the various sections behaved in conformity with the theory.

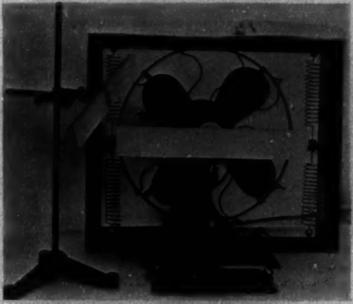

Fig. 10

To the right a wooden section is suspended in springs in a heavy steel frame, thus creating a vibratory system quite similar to a short section of the galloping transmission line.

A semicircular section in this apparatus placed in front of the fan will build up vibrations of two diameters amplitude in a few minutes. Other sections, like the rectangular one for instance require the air stream of a wind tunnel to make them respond.

The difficulty in making a model of this kind is to keep the damping sufficiently small. This has been accomplished by soldering the joints of the springs with careful fillets, and moreover by mounting the whole frame on springs again. In the absence of these latter springs the frame will vibrate the table on which it is placed and thus destroy the effect. Besides offering a rather striking demonstration the model has given a complete verification of the theory propounded in the paper.

Research

ANNUAL REPORT OF THE COMMITTEE ON RESEARCH*

DURING the year 1931 the Committee on Research has encouraged the presentation of research papers and has reviewed many for sessions on research before the A.I.E.E. On account of the broad field which must be covered by this committee it has been impracticable to organize subcommittees under any specific classification. The burden of reviewing papers has fallen to practically all members and excellent cooperation has been obtained by correspondence and without committee meetings.

The following topics are worthy of mention to show the progress that has been made during the year. The subjects reported have all been suggested by committee members.

PHYSICS

Dr. R. J. Van de Graaf of Princeton University and Massachusetts Institute of Technology has developed static machines of a very simple construction using endless silk belts with which he has generated as high as 1,500 kv. This work is being extended to much higher voltages.

Dr. F. Lange and A. Brasch of the University of Berlin have built a simple X-ray tube consisting of alternate rings of paper, rubber, and aluminum which they hope to operate with voltages up to 2,500 kv.

A number of developments have taken place during the year in connection with X-ray tubes and auxiliaries. Some of these are described in the January issue of the *General Electric Review*. The application of X-rays commercially has been extended during the year to the sterilization of food and food products.

Experiments have recently been made in this country on a sodium vapor lamp, originally developed in Germany. This lamp has been made possible by a new kind of glass which is not blackened by the sodium vapor. This lamp is said to be several times as efficient as the ordinary mercury or gas filled lamp.

In ferro-magnetic materials changes in magnetization do not occur instantaneously throughout the mass of the material, but start locally and are propagated with finite velocity. This was demonstrated on a large scale by K. F. Sixtus and L. Tonks and the velocity of propagation was measured in nickel-iron wires under tension. (*Physical Review*, 37, 1930, p. 1958, April 1931.)

From the Westinghouse Laboratories a new magnetic phenomenon has been reported by F. Bitter, which consists in the discovery of magnetic inhomogeneities of a definite and regular pattern in the individual crystals of ferro-magnetic materials. These are of quite a different nature, depending on the type of material. No satisfactory explanation has yet been obtained for this phenomenon.

Members of the staff of the Alabama Polytechnic Institute have announced the discovery of the last missing element, No. 85.

From theoretical considerations, it was believed that atoms of hydrogen might exist which are twice as heavy as the ordinary atoms. These have now been discovered experimentally by H. C. Urey and G. M. Murphy of Columbia University and F. G. Brickweede of the Bureau of Standards.

Walter Bathe is reported by A. H. Compton to have produced gamma rays by bombarding beryllium metal with alpha rays. His experiments indicate a possible synthesis in which a heavier element, carbon, is formed and if proved correct it may greatly affect all ideas of how the solar system and our earth originated. (*Sc. News Letter*, p. 223, Nov. 21, 1931.)

Vacuum tubes are continually extending the ranges of possible electrical measurements and thereby opening new fields of physical research. L. A. DuBridge (*Physical Review*, Vol. 37, page 392, 1931) reports measurements of small currents of the order of 10^{-18} amperes which corresponds to six electrons per second.

Fowler (*Physical Review*, Vol. 38, page 45, 1931) has made important theoretical investigations of the photoelectric threshold values of clean metals at various temperatures.

Newton Harvey (*Jl. of General Physiology*, Vol. 15, p. 147) has demonstrated that living cells are disintegrated by ultra-sonic waves in less than $^1/_{1000}$ of a second.

R. W. G. Wyckoff (*Radiology*, Vol. 17, p. 1171, 1931) has obtained important data on the destruction of micro-organisms by X-rays and cathode rays in continuation of his previous work in this field.

Boer and Teves (*Zeitsch. f. Phys.*, Vol. 73, page 192, 1931) have obtained further important results in connection with their investigation of the photoelectric properties of caesium oxide films in conjunction with halide salt layers.

Bergmann (*Zeitsch. f. Phys.*, April 1, 1931) has described an important photoelectric cell involving essentially the photoelectric properties of iron selenide.

DIELECTRICS

The Committee on Electrical Insulation of the Division of Engineering and Industrial Research of the National Research Council reports very substantial progress in the study of dielectrics and dielectric phenomena. Its report, which summarizes a considerable number of recently presented papers, should be consulted by those interested in this subject. The following items, some of which were reported by the National Research Council, are of particular interest. Some of these results have been presented in the form of papers before the Institute.

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* **COMMITTEE ON RESEARCH:**

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F. W. Peek, Jr.,
H. H. Race,
C. W. Rice,
D. W. Roper,
T. Spooner,
J. B. Whitehead,
R. J. Wiseman.

A further reference which may be consulted with profit to those interested is monograph No. 5 of the American Chemical Society, by Professor C. P. Smythe of Princeton University, giving an extensive summary of the results of the work in determining molecular structure.

The following four A.I.E.E. papers from The Johns Hopkins University on this subject by Dr. Whitehead and his associates are particularly valuable, namely, on the *Conductivity of Insulating Oils*, *The Fundamental Properties of Impregnated Paper*, *Residual Air and Moisture in Impregnated Paper*, and *The Predetermination of the A-C. Characteristics of Dielectrics*. Other valuable A.I.E.E. papers on dielectrics are *Insulation Variability*, by M. C. Holmes; *On the Theory of Thermal Breakdown of Solid Dielectrics*, by P. H. Moon, and another in the same field by Kenney, Luery, and Moriarty.

In one of the above mentioned papers by Whitehead and Banos there is announced a most remarkable agreement between loss, power factor, and capacity values at 60 cycles for solid and liquid dielectrics as determined experimentally and as calculated from the d-c. characteristics as obtained by the oscillograph.

By very careful distillation and purification as reported by L. A. Welo it has been possible to produce insulating oils having conductivities of 10^{-16} to 10^{-19} ohms.

The Bell Telephone Laboratories have reported very interesting results on ordinary insulation in which they point out that it is the metallic salt content of considerable solubility in water which controls the insulation or conductivity characteristics of such materials. Washing with water improves the insulation properties decidedly and improvement can be obtained by replacing the soluble salts with more insoluble salts by chemical reaction.

Work continues in an increased amount on the subject of polar molecules in dielectrics. This subject, originally considered of no practical value, is now being studied for viscous liquids and even solids, with very important results. Development of polar molecules in insulating oils gives a measure of deterioration. The work of W. N. Stoops points out the very definite correlation and the fact that the measure of the polar characteristics under accelerated test may furnish a measure of the useful life of an oil.

Dielectric losses in highly refined insulating oils at commercial frequencies and operating temperatures have been shown to result from conduction, but at high frequencies and low temperatures the major loss may be accounted for by the orientation of polar molecules. (H. H. Race, *Physical Review*, Vol. 37, p. 430-436, Feb. 1931.)

At Johns Hopkins University and the Bell Telephone Laboratories further studies on abietic acid and rosin have been made as to the part played by dipoles in the s.i.c. and loss characteristics of material over ranges of temperature and frequency. This type of work, directed toward an understanding of the mechanism of the dielectric in detail, is most important. It is interesting to note that both sets of data can be explained by the generalized theories of Schweidler and Wagner, which consider the dielectric mass as a whole, or by the dipole theory of Debye, which considers the microscopic structure, namely, the individual atoms, molecules, or ions as dipoles.

Studies of cable oil and cable conditions at a number of laboratories were reported last year, particularly on the effect of ion bombardment on oils. The necessity for a completely filled, solid, compact cable to prevent internal ionization is further shown. For example, it has been shown by the Detroit Edison Company that the power factor of an oil is directly correlated with the amount of hydrogen freed by ion bombardment. In the studies of the fundamental properties of paper, oil, and impregnated paper at Johns Hopkins University, it was found that with high grade oil the larger portion of the dielectric loss was attributable to the paper. Further, it has been shown by the Commonwealth Edison Company that tackiness of adhesion of the compound is correlated with long life of cables.

J. A. Scott of the General Electric Company has shown that oil pressures of several atmospheres applied to impregnated paper insulated cable give a decided increase in the life of the cable as measured by an endurance test.

The increased use of the oil filled type of cable and the growing use of oil for filling joints in ordinary impregnated paper insulated cable have greatly increased the importance of the integrity of the lead sheath. Several research projects dealing with lead are in progress. Work at the University of Illinois has shown that the creep of lead and several common lead alloys occurs at stresses below 200 lb. per square inch; that is, at stresses which may occur in cable sheaths during normal operation.

Researches on transmission cables show a tendency to depend on accelerated aging tests as a measure of the life quality. Both utilities and cable manufacturers are conducting researches on long lengths of impregnated paper insulated cable to duplicate in a short time what takes place over a period of years in service. The present indications are that, by subjecting the cable to about 2.5 times normal voltage, and at the same time superposing loading cycles resulting in a maximum temperature somewhat above the limiting temperature for the cable, and by making measurements of power factor and ionization factor at suitable intervals throughout a test continuing a week or two, it is possible to predict the effect of many years of normal service on the stability of insulation.

Considerable interest is taking place as to the influence of pressure on a cable and its relationship to the migration of the oil in the cable as it goes through its temperature cycle.

The Detroit Edison Research Department reports that chemical reactions of organic compounds in electrical discharges are being studied as a part of a program of investigation of the deterioration of oil-impregnated paper insulation of high-tension cables. The results of an experimental survey covering 57 different hydrocarbons has been published (*Jour. Phys. Chem.*, Vol. 35, 3649 (1931)).

In the field of insulation, J. B. Whitehead summarizes the accomplishments for the year as follows:

This research has been characterized by a definite extension of our knowledge of the behavior of insulating liquids and its causes. This is a result largely of the development of amplifier and oscillograph methods for measuring short-time behavior of liquids when used for the impregnation of paper with the result that a much closer insight is now possible as to the origin of the properties of high-voltage impregnated paper insulation. A continuation of the accelerated life studies on laboratory samples of impregnated paper is clearing up the uncertainties heretofore existing as to the relative importance of residual air and moisture in impregnated paper. In connection with these researches, an important method of analysis of the results of a-c. measurements has been developed and its limitations explored.

CIRCUIT INTERRUPTION

Vacuum relays developed in Germany have been introduced into this country which make use of the flexibility of glass to transmit sufficient motion to operate contacts. This is possible because of the very small contact separation necessary in a vacuum.

Slepian and Strom in an Institute paper (ELECTRICAL ENGINEERING, Dec. 1931) have given a valuable analysis of the factors which govern arc extinction in low-voltage network cables. It is shown that the gas blast, as the result of the action of the arc on cable insulating materials, is largely responsible for the clearing of faults. Data are given on the extinguishing characteristics of various types of chemical compounds.

By intensive research, the possibilities of boric acid in connection with circuit interrupting devices have been developed to a point where commercial applications are in sight. The production of water vapor from the boric acid by the action of the arc causes a powerful deionizing action. The water vapor may be condensed, thus making possible a very efficient enclosed fuse for high interrupting capacities.

MISCELLANEOUS

H. B. Dwight has given a very useful analysis of the proximity effects in cable sheaths (A.I.E.E. TRANSACTIONS for September). This is supplementary to the formulas published by him some years ago in the *Electric Journal*.

In the same issue of the TRANSACTIONS there is a group of papers on the measurement and analysis of noise in electrical machinery which in some aspects represents contribution of new knowledge as the direct result of careful research. The science of acoustics is becoming an important phase of electrical engineering. Since the world has become noise conscious the elimination of unnecessary noises will be only a matter of time. Some of the new portable noise analyzers which have been placed on the market this last year make the measurement of the magnitude and the analysis of noises much easier.

By studies at the Westinghouse Laboratories, of car and train models in wind tunnels, fundamental data have been obtained on the advantages of streamlining. These results will be of great value in reducing power and making possible high-speed land transportation.

R. M. Baker has discovered that the presence of small quantities of mercury vapor in a non-oxidizing atmosphere very greatly reduces the contact resistance for certain types of brushes and slip rings. This may be of some commercial value, especially where it is wished to use very high current densities. (*Elec. Journal*, February 1932.)

The development of the helical groove on slip rings and commutators by G. M. Little promises to be of considerable value in connection with the problem of current collection and commutation. Field tests have shown decided improvements in brush wear, sparking and distribution of current between parallel brushes. (ELECTRICAL ENGINEERING, June 1931, p. 427.)

Researches on the conditions desirable for the greatest human comfort, namely, the effect of humidity, wall temperature, air flow, and ionic condition of the air, etc., have been very active during the past year. Also the usefulness of reversed refrigeration for the heating of houses and buildings is being tried out commercially in several installations. Air conditioning is apparently to become soon a very considerable industry of great importance to the electrical interests.

Very sensitive research measurements have shown that all metals flow to some extent at high temperatures and that this flow can be reduced by using proper heat treatment of certain alloys. Such studies are used as a basis for new mechanical design of high temperature steam and electrical turbines so that proper clearance will be maintained throughout long life. (Paper presented by F. P. Coffin and T. H. Swisher at the National Applied Mechanics Meeting of the American Society of Mechanical Engineers, June 15, 1931.)

In the field of radio, Bruce's work on directive short wave antennas is of importance.

There are two recent developments in piezo-electricity worthy of note. Thin plates of tourmalin have, within recent months, shown themselves to be excellent piezo-oscillators, particularly for very short waves down to about two meters wavelength. An interesting circumstance in this connection is that, although tourmalin was one of the very first piezo-electric crystals to be studied, nevertheless the practical applications are of very recent date. Another recent application of piezo-electric crystals is their use in filter circuits of very high selectivity.

The researches on the fundamental electrical units which have been under way at the Bureau of Standards for several years have reached a stage such that preliminary values can be given, subject to some minor corrections. The results confirm earlier determinations in showing that the international ohm is 5 parts in 10,000 larger than it should be, while the ampere has very nearly the correct value as determined by measurement of mechanical forces between coils. Comparisons of fundamental standards in recent years have shown differences between the standards of different countries large enough to be troublesome in precise scientific work. Direct comparisons between silver voltameters of the Bureau of Standards, the British National Physical Laboratory and the German Physikalisch-Technische Reichsanstalt were made during the year at Berlin; these proved that existing discrepancies arose in large part from changes in the German standard cells. Several years ago the International Committee on Weights and Measures decided that eventually the electrical units should be based upon absolute (mechanical) measurements rather than upon standards arbitrarily defined. The International Committee meets in 1933, and it is hoped that definite plans can then be approved for introducing electrical units on the new basis, assuring international uniformity of values, and removing the discrepancy between electrical and mechanical units.

Electrical Machinery

ANNUAL REPORT OF THE COMMITTEE ON ELECTRICAL MACHINERY*

Activities of the Committee

THE committee during the year has reviewed approximately 50 papers, of which 24 have actually been accepted and published. Work on the preparation of test codes has been carried on during the year, and the Transformer Test Code was published in preliminary form in October. Favorable comments have been received on this, and work on the test codes for induction machines, synchronous machines, and d-c. machines is now approaching completion.

While direct responsibility of the machinery committee for standards, except for transformers, has now been transferred to the Sectional Committee of the A.S.A., the machinery committee has participated indirectly in the revision of the rotating machinery standards, and in particular has sponsored proposals to determine accurate efficiencies of induction and direct-current motors. Mr. C. J. Koch's paper on *Measurement of Stray Load Loss in Polyphase Induction Motors*, presented at the Providence Meeting of the Institute in May 1932, is an important contribution in this field.

The transformer subcommittee has been very active in revising the Transformer Standards No. 13, and it is now undertaking the very important task of standardizing impulse voltage tests.

Revisions in the standards for constant-current transformers and in the Standards Report on Capacitors have also been prepared.

Synchronous Machines

One of the 200,000-kva., 0.8-power factor, 1800-r.p.m. steam turbine-driven alternators (GE)[1] in the Brooklyn Edison Company's plant has been operated successfully at its rated kilowatt output, and the second unit is being installed. A 121,000-kva., 18,000-volt, 1,800-r.p.m. (AC) unit was placed in operation at Waukegan, and a 147,000-kva., 22,000-volt unit (AC) for the State Line Station is under construction. A 25,000-kva., 13,800-volt, 3,600-r.p.m. machine (W) was shipped to the Public Service Electric and Gas Company and is the largest unit yet undertaken at this speed. A 99,000-kva., 13,800-volt, 1,800-r.p.m. generator (W) is under construction for this company and is the largest unit so far built with internal fans.

Five vertical-shaft waterwheel-driven alternators rated at 48,500 kva., 13,800 volts, 25 cycles, 750 r.p.m. (CGE) are under construction for the Abitibi Cangon Development near James Bay.

A 1,250-kva., 960-cycle, single-phase generator (GE); a 1,250-kva., 1,000-cycle, single-phase generator (GE); and a 3,000-kva., 420-cycle, single-phase generator (GE) represent increases in capacity over high-frequency machines of this type built before the year 1931.

A 75,000-kva., 12,600-volt, 514-r.p.m., synchronous condenser (W) is under construction for the Commonwealth Edison Company. This is an increase of fifty per cent over the previous largest synchronous condenser.

During the year there has been considerable activity in the development of designs to improve the starting performance of industrial synchronous motors, particularly as to higher torque per kva. inrush. Several motors of 200 to 600 hp. at 300 to 900 r.p.m. have been built with a starting torque efficiency (per cent starting torque over per cent kva. inrush or unity power factor base) of the order of 55 per cent.

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* COMMITTEE ON ELECTRICAL MACHINERY:

P. L. Alger, Chairman, General Electric Co., Schenectady, N. Y.,
B. L. Barns,
E. S. Bundy,
H. E. Edgerton,
J. B. Goodale,
T. T. Hambleton,
A. L. Harding,
C. F. Harding,
E. W. Henderson,
S. L. Henderson,
L. F. Hickernell,
J. Allen Johnson,
J. J. Linebaugh,
H. C. Louis,
A. M. MacCutcheon,
O. K. Marti,
V. M. Montsinger,
R. W. Owens,
R. H. Park,
E. B. Paxton,
H. V Putnam,
K. A. Reed,
A. M. Rossman,
O. E. Shirley,
R. G. Warner,
C. A. M. Weber,
R. B. Williamson

[1] Manufacturer Designation:

AC—Allis-Chalmers Manufacturing Company
BB—American Brown Boveri Company
CGE—Canadian General Electric Company
CW—Canadian Westinghouse Co., Ltd.
ED—Electric Dynamic Company
EM—Electric Machinery Manufacturing Company
GE—General Electric Company
W—Westinghouse Electric & Manufacturing Company

Two motors for pulp grinder drive (EM) will use stator windings in four parallels, and these motors will be started by applying successively the parallel circuits all on the reduced voltage of the autotransformer. These suitably timed increments will reduce the kva. inrush to a minimum.

The twelve 5,500-hp. motors (GE) for the U. S. Mail S.S. Company have been completed, and two ships with two motors each are in service.

The S.S. *President Hoover* (GE) and the S.S. *President Coolidge* (W) each with two 13,250-hp. motors were placed in service during the year.

Two 30,000-kilowatt frequency changers (GE) with generators rated 42,860 kva., 0.70 power factor, single-phase, 25 cycles, and motors rated 36,000 kva., 0.9 power factor, three-phase, 60 cycles, and operating at 300 r.p.m. are being installed at the Richmond Station of the Philadelphia Electric to supply power to the Pennsylvania Railroad electrification. These sets have twice the capacity of single-phase sets previously built. The generators are equivalent mechanically to 61,000-kva., three-phase machines. The frequency changers are designed for outdoor operation in steel enclosing housings.

INDUCTION MACHINES

There has been considerable activity during the last year in induction machines. The demand for squirrel-cage motors for rapid and oft-repeated reversing duty, of a nature heretofore confined to d-c. mill type motors, has led to the development of special motors with low inertia rotors and liberal radiating surfaces. This demand came chiefly from the steel mill industry and concerned motors for use on furnace conveyers of the "stroke" type and motors for newly developed automatic sheet handling apparatus as used for hot finishing mills. The simple squirrel-cage motor, while it has some disadvantages as regards disposition of loss, nevertheless offers many advantages for this type of service, *e. g.*, a low-inertia, sturdy, dependable, simple, rotating element, simplified control; faster-operating control; lower-cost motor and lower-cost control. The opening up of this field to squirrel-cage motors will undoubtedly add to the wider application of this type of motor.

The use of totally enclosed fan-cooled motors has increased enormously. One company reports that over half of its induction motor production for last year was of the fan-cooled type. This type of motor is available in mounting dimensions the same as those for open motors of equal rating up to fairly large sizes, and has been well received by the trade where open type motors were subject to aggravated dirt conditions.

Selsyn motors as mediums of control formed a successful means of maintaining levels on both ends of a lift bridge on the Missouri, Kansas, Texas Ry. Other novel applications of this type of motor as a control or as an indicator are under way.

Induction frequency converters of the commutator type have, during the year, been built for use at frequencies as low as $2^1/_2$ cycles. Machines of this type with outputs at ten cycles have been supplied by a large manufacturer to increase the range of his slip-ring frequency converters. These latter, driven by a d-c. adjustable-speed motor, have a normal range of 120 to 180 cycles. By supplying 10 cycles to these converters at approximately $^1/_6$ normal voltage, the frequency range of these converters became 70 to 130 cycles, and a range of 70 to 180 cycles was made possible. These latter frequencies were used to supply small high-speed squirrel-cage motors and give them adjustable-speed characteristics for improvement in manufactured product.

DIRECT-CURRENT MACHINES

A triple unit motor has been built (ED) for ship propulsion. One auxiliary and two main motors are placed in line with their armatures on one shaft. The auxiliary motor is designed for low cruising speed with higher efficiency than could be obtained in the main motors at low speed.

Two 2,500-hp., 282/708-r.p.m., constant-torque, induction motors have been built (AC) for driving reciprocating, 1,500-lb. per sq. in., boiler feed pumps. The Rossman system of varying speed is used. This is the first case where the d-c. armature has been mounted directly on the rotatable induction motor primary.

A 12,500-hp., 44-inch, slabbing mill drive (W) has been placed in operation. Each main roll is separately driven, without pinions, by a 5,000-hp. double-unit motor; the edging rolls by a 2,500-hp. single motor. Power is supplied by a 10,500-kw., 700-volt, d-c. set composed of one induction motor, 180,000-lb. flywheel, and three 3,500-kw. generators.

A roll grinder has been equipped (W) with nine d-c. motors for controlling separately every motion and adjustment.

TRANSFORMERS

Manufacturers have given increased attention to the design of both power and distribution transformers to better withstand lightning surges and to developing methods of impulse testing to indicate weakness in transformer insulation to such surges.

Several large shell-type surge-proof power transformers (W) of the new type were built, among them units of 42,000 kva., 220 kv. which were subjected to unusually severe impulse tests, 4,500-kva. units for the Pennsylvania Railroad and a 70,000-kva. unit for the United Electric Light and Power Company, which is thought to be the largest of the self-cooled type in this country.

Self-contained features for protection against lightning independently of transformer connections or grounding practise, and for prevention of outages, were accomplished by incorporating high-voltage deion gaps and coordinated low-voltage bushings in standard distribution transformers (W).

Cover-roof type bushings are a feature of a new line of distribution transformers (AC) intended particularly for service on rural lines.

Two transformers (GE) of new design were fur-

Table I. Mercury Arc Rectifier Units Placed in Operation During 1931 or on Order December 31, 1931

Purchaser	No. of Sets	D-c. Volts	Kilowatts per Set	Total Kilowatts	Type of Control	Service	Placed in Service	Manufacturer
Amer. Gas & Elec. Co., New York	2	610	500	1,000	Automatic	Railway	1931	BB
Boston Elevated Railway	2	600	3,000	6,000	Automatic Remote Control	Railway	1931	GE
Cleveland Railway Company	x1	600	500	500	Semi-Automatic	Railway	1931	W
Clinton, Davenport & Muscatine Rwy	1*	700	500	500*	Automatic	Railway	1931	BB
Commonwealth Edison Co.	3	625	3,125	9,375	Manual	Railway	1931	BB
Commonwealth Edison Co.	1	625	2,080	2,080	Automatic	Railway	1931	BB
Commonwealth Edison Co.	1	625	3,125	3,125	Automatic	Railway	1931	BB
Commonwealth Edison Co.	1	625	3,125	3,125	Manual	Railway	1931	GE
Commonwealth Edison Co.	1	625	3,125	3,125	Manual	Railway	1931	GE
Commonwealth Edison Co.	1	625	3,125	3,125	Manual	Railway	1931	GE
Consol. Mining & Smelting Co. of Can.	1	650	6,500	6,500	Manual	Electrolytic	1931	GE
Consol. Mining & Smelting Co. of Can.	2	650	6,500	13,000	Manual	Electrolytic	1931	BB
Detroit, City of	1	600	2,000	2,000	Automatic	Railway	1931	GE
I. G. Farbenindustrie for Standard Oil Co. of Louisiana	°1	3,500/ 9,600	2,200	2,200	Manual	Electrochemical	Being erected	BB
Illinois Power & Light Company	1	650	540	540	Automatic	Railway	1931	W
Indiana Railroad Company	2	625	850	1,700	Manual	Railway	1931	BB
Long Island Railroad Company	x1	650	3,000	3,000	Automatic Remote Control	Railway	1931	W
Long Island Railroad Company	5	650	3,000	15,000	Automatic	Railway	1931	BB
Long Island Railroad Company	4	650	3,000	12,000	Automatic	Railway	On order	AC
Los Angeles Railway Corporation	2	600	1,500	3,000	Automatic Remote Control	Railway	1931	GE
Louisville Gas & Electric Co.	2	600	1,000	2,000	Automatic	Railway	1931	GE
Montreal Tramways Company	2	600	1,500	3,000	Automatic Remote Control	Railway	1931	GE
Mason City & Clear Lake Railway	1*	650	500	500*	Automatic	Railway	1931	BB
N. Y. Board of Transportation	10	625	3,000	30,000	Automatic Remote Control	Subway	1931	GE
N. Y. Board of Transportation	13	625	3,000	39,000	Automatic Remote Control	Subway	Being erected	GE
N. Y. Board of Transportation	15	625	3,000	45,000	Automatic Remote Control	Subway	On order	GE
N. Y. Board of Transportation	13	625	3,000	39,000	Automatic Remote Control	Subway	On order	GE
N. Y. Central R.R. Company	1	666	3,000	3,000	Automatic Remote Control	R.R. Electrif'n.	1931	GE
Northern Indiana Pub. Service Co.	1	1,500	3,000	3,000	Automatic	Railway	1931	BB
Paris Orleans Rwy.—France	1	1,500	1,500	1,500	Automatic	R.R. Electrif'n.	1931	GE
Philadelphia, City of	2	630	3,150	6,300	Manual	Railway	On order	AC
Piedmont & Northern Railway	1	1,500	750	750	Auromatic	R.R. Electrif'n.	1931	W
Quebec Power Company	1	550	1,200	1,200	Automatic	Railway	1931	BB
Regina, City of	1	575	1,200	1,200	Manual	Railway	1931	BB
Sun Life Assurance Co. of Canada	2	250	400	800	Manual	Power	1931	BB
Trenton Transit Company	3	600	1,400	4,200	Manual	Railway	1931	BB

* In addition, one spare rectifier (without transformer) was supplied.

x Temporary installation.

° Also reported 1930.

Totals*	1931		1930	
	No. of Sets	Kilowatts	No. of Sets	Kilowatts
Placed in service	57	127,845	33	75,225
Being installed	14	41,200	26	71,400
On order	34	102,300	29	79,275
Grand total for year	105	271,345	88	225,900
Total number units in service, Dec. 31st	180	305,379	123	177,534

* Spare rectifier tanks without transformers not included.

nished to operate an X-ray tube for cancer research in the California Institute of Technology. Each unit is rated 0.03 ampere at 700,000 volts to ground continuously, and operates in series with mid-point grounded, giving 1,400,000 volts effective at the X-ray tube.

Magnetic oil level indicators were made available (GE and AC) for power, large distribution, and high-voltage instrument transformers. This gage has numerous advantages over a glass oil gage because it is more easily read, and more easily maintained oil tight. It has no glass to become dirty on the inside or break and allow oil to run out.

The number and capacity of load-ratio control transformers exceeded all previous records. The development of a design (GE) suitable for lower-rated circuits up to 15,000 volts, single-phase or three-phase, resulted in an economical application of load-ratio control to distribution transformer sizes. This type of equipment was applied to the new three-phase 1,500-kva. transformers for high-voltage networks. A new line of standard tap changers (W) using standard parts suitable for wide range of capacities as low as 200 kva. were completed.

A new type of inertaire protection for power transformers (W) using nitrogen from a gas cylinder was perfected. A uni-directional breather (AC) for use on power transformers, which induces a natural circulation of air over the oil in the main transformer tank or expansion tank to reduce condensation and carry off condensed moisture, was developed.

MERCURY-ARC RECTIFIERS

The use of rectifiers has continued to expand during the year as indicated by information contained in Table I.

The large number of additional 3,000-kw. 625-volt units ordered by the New York Board of Transportation is the outstanding feature of this year. Twenty-three of these rectifiers were ordered last year and 28 this year, making a total of 51 sets (GE) with a total capacity of 151,000 kw.

The three 6,500-kw., 650-volt electrolytic rectifiers listed in this and last year's report were placed in service during the year.

A 3,000-kw., 650-volt sectional rectifier (W), consisting of four 750-kw. separate rectifiers arranged in a compact unit, was developed and placed in service.

The first complete portable mercury-arc rectifier substation, consisting of a 540-kw., 650-volt (W) rectifier with transformer and control, cooling system, car, etc., was placed in service by the Illinois Power & Light Company.

Protective Devices

ANNUAL REPORT OF THE COMMITTEE ON PROTECTIVE DEVICES*

THE work of the Committee on Protective Devices during the past year was handled largely through subcommittees as has been the practise for several years. These subcommittees are listed below together with their chairmen:

Relays	O. C. Traver
Oil Circuit Breakers, Switches, and Fuses	T. G. LeClair[1]
Lightning Arresters	H. K. Sels
Fault Current Limiting Devices	R. T. Henry
Interconnections[2]	F. C. Hanker

GENERAL

The committee continued its activities in following the preparation of standards, and review of research and development, although the main activity during the year was fostering the preparation and presentation of papers before the Institute.

While the technical achievement during the year is very satisfactory in view of the general business conditions, these conditions of course had a depressing effect on research and developmental activities. Progress in this respect consisted largely of refinements and extensions of fundamentals previously established.

Difficulty in securing agreement relative to certain matters concerning the standards for fuses and the revised standards for disconnecting, horn-gap and knife switches has prevented completing these standards. Plans have been made for a course of action which it is believed will permit of putting these standards into report form. Every effort is being made to place these standards in such shape that complete agreement may be secured which will permit their adoption as Institute Standards.

RELAYS

Report of Subcommittee

The development of high-speed relays continued, resulting in marked improvements to primary power-system relays generally, and in new developments in the a-c. railway and secondary distribution network system fields.

One of the manufacturers has developed so-called straight-line relays, particularly for application on 4,000-volt networks which are capable of discriminating and operating on very small differences in current, although these currents may be quite heavy.

An important advance during the year was the redesign of all types of relays by at least one manufacturer in order to accommodate the relays in a new universal case, with standard stud arrangement, which is being used to a large extent for front of panel switchboard devices.

A high-speed current differential relay for transformer protection having an operating speed of 0.004 second has been developed.

Numerous refinements in mechanical details contributing to the reliability and accuracy of relays of all types have been carried forward.

Completed developments in coupling and protective equipment for all carrier-current applications include cable-type coupling-capacitor assemblies for all voltages from 115 to 230 kv., inclusive.

In order to secure a consensus of present day practise in regard to protection of apparatus of all classes, a questionnaire on this subject has been sent to a number of engineers, and a number of replies has already been received.

An important advance in relay application during the past year has been the wide increase in the use of the method of symmetrical components for determination of not only the fault current but also the voltage and current at the relay location, particularly for distance relay applications.

The following papers appearing under the auspices of the committee have been presented before the Institute:

A New High-Speed Distance Relay, S. L. Goldsborough and W. A. Lewis, Westinghouse Electric & Manufacturing Co.

Application of High-Speed Relays, G. W. Gerrell, Union Electric Light & Power Co.

A New High-Speed Distance Relay, A. R. Van C. Warrington, General Electric Co.

Relay Operation From Bushing Potential Devices, P. O. Langguth, Westinghouse Electric & Manufacturing Co., and V. B. Jones.

Operation of Relays From Carrier-Current Coupling Capacitors and Capacitance Transformer Bushings, J. E. Clem and R. E. Cordray, General Electric Co.

Effect of Wave Form on Operation of Induction Relays, P. H. Robinson, Houston Lighting & Power Co.

A Telegraphic Pilot-Wire Relay System, C. H. Frier, Oklahoma Gas & Electric Co.

OIL CIRCUIT BREAKERS, SWITCHES, AND FUSES

Report of Subcommittee

Intense interest has been shown during the year in the fundamental theories underlying the various circuit breaker developments of the last few years. These discussions have resulted in clarifying conceptions of the theories on which these new types of breakers operate and have laid ground work for further development in the future. The amount of fundamental development which this intensive scrutiny reveals will, if supported by satisfactory performance records, do much to increase the confidence of the user in the ability of oil circuit breakers to perform adequately.

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* **COMMITTEE ON PROTECTIVE DEVICES:**

Raymond Bailey, Chairman,
L. F. Hickernell, Vice-chairman,
H. W. Collins,
A. W. Copley,
W. S. Edsall,
L. E. Frost,
S. L. Goldsborough,
R. T. Henry,
E. A. Hester,
T. G. LeClair
J. B. MacNeill,
J. P. McKearin,
H. A. McLaughlin,
D. M. Petty,
H. J. Scholz,
H. K. Sels,
H. P. Sleeper,
L. G. Smith,
E. R. Stauffacher
H. R. Summerhayes,
O. C. Traver,
R. M. Wood,
H. B. Wood.

[1] Mr. LeClair was appointed chairman of the subcommittee on oil circuit breakers, switches, and fuses on January 4, 1932, when Mr. Hickernell resigned as chairman of this subcommittee.

[2] The interconnections subcommittee is a joint subcommittee having representation from the following committees: Protective Devices, Power Generation, Transmission and Distribution, Electric Machinery.

The metal-clad switchgear has been improved in a number of details and the development has been extended to interrupting ratings of 500,000 kva. at 15 kv. for indoor breakers and to 1,500,000 kva. at 132 kv. for outdoor breakers.

The oil-blast and deion grid principles of breaker construction were extended to quite high ratings, and both types have appeared in the metal-clad construction. Elaborate tests on a 1,500-ampere 15-kv. high-speed deion circuit breaker were performed for a railway application. Seventy-two short circuits ranging from 2,720 amperes to 50,500 amperes were thrown on the breaker in about five hours time. Some of these short circuits lasted only 0.024 second and the duration of the longer ones was 0.04 second. A series of fifty tests at current values from 11,000 to 49,000 amperes was run on impulse type of oil-blast breaker of a corresponding rating. This breaker clears the circuit in 0.04 second or less over the full current range.

Developments in air circuit breakers include extension of current ratings and improvements in operating speed.

The fuse manufacturers have been active in the development of improved type fuses, which are more economical.

There is a fairly widespread feeling among operating engineers in particular that some improvement could be made on the basis of rating of oil circuit breakers.

The following papers appeared under the auspices of the committee and have been presented before the Institute:

The Theory of Oil-Blast Circuit Breakers, D. C. Prince, General Electric Co.

The Practical Application of the Oil-Blast Principle of Circuit Interruption, R. M. Spurck, General Electric Co.

Recent Developments in Arc Rupturing Devices, R. C. Van Sickle and W. M. Leeds, Westinghouse Electric & Manufacturing Co.

The Extinction of Alternating Current Arcs in Turbulent Gases, T. E. Browne, Jr., Westinghouse Electric & Manufacturing Co.

Application of Primary Distribution Fuses, F. E. Sanford, Union Gas & Electric Company.

The Expulsion Fuse, J. Slepian and C. L. Denault, Westinghouse Electric & Manufacturing Co.

Fuse Cut-Outs—Their Design and Application for A-C. Distribution Circuits, E. G. Newton, General Electric Co.

The Boric Acid Fuse, A. P. Strom and H. L. Rawlins, Westinghouse Electric & Manufacturing Co.

FAULT CURRENT LIMITING DEVICES

Report of Subcommittee

The committee is not aware of any developments relating to fault current limiting devices of sufficient importance to warrant including in this report. Operating experience with such devices apparently continues to be quite satisfactory, and the need for further development is therefore not particularly urgent.

LIGHTNING ARRESTERS

Report of Subcommittee

As in other lines, the developments on lightning arresters consist principally of extensions of recently developed, fundamental theories to a greater variety of forms of station and line type arresters; also, a marked improvement has been made in the mechanical design and construction of lightning arresters which will improve the maintenance of their electrical characteristics.

Studies of lightning and the effects of lightning on power systems were continued during the year with considerable improvement in the technic of fault location. Newly developed lightning current meters and special low resistors were used to measure the current in the lightning stroke.

Continued field investigations using portable surge generators and cathode ray oscillographs, were made in conjunction with study of applications of lightning arresters to distribution transformer installations and associated line construction.

Considerable experimental work concerning grounds and ground connections resulted in making available information which may be used to improve the effectiveness of lightning arrester applications.

New protective devices have been developed, arranged for installation inside of the distribution transformer. This development marks a new phase in the problem of protection of transformers against the effects of lightning and should result in very much improved operating experience.

A report is in preparation jointly with the N.E.L.A. Subject Committee on Lightning Arresters, outlining the prevailing practise on the installation, operation, and experience of various types of lightning arresters by a number of the larger operating companies. This information should be valuable in observing the present trend of operating practise on transmission and distribution systems relative to arrester installation, operation, and performance as affecting suitable standards, and also whether tests and equipment should be developed to determine the operating condition of arresters in service.

Lightning arrester manufacturers have contributed largely to the development of more powerful testing equipment and more accurate recording apparatus which has been used to carry out the proposed tests outlined in the Report on Standards for Lightning Arresters and has effectively demonstrated the value of these tests to determine the characteristics of lightning arresters.

The report on Standards for Lightning Arresters continues in its tentative form and it will probably not be feasible to revise this report until work now in progress on the characteristics and coordination of system insulation with respect to lightning has progressed further.

INTERCONNECTIONS

Report of Joint Subcommittee

The Joint Subcommittee on Interconnections includes representatives of the Protective Devices, Power Generation, Transmission and Distribution, and the Electric Machinery Committees.

At the Winter Convention 1932 the session under the auspices of the subcommittee included two subjects of particular interest to the Protective Devices Committee, namely, the "Report of the Subject Committee on Definitions of Terms Used in Power System Studies" and the papers presenting revised decrement curves.

The activity of various engineering groups in studies of interconnection has built up a terminology for which there has been no official recognition. A Subject Committee has been at work on these definitions for some time and presented a report that will be used as the basis for the consideration of such definitions by the responsible standardizing bodies. A large number of these definitions is related to subjects of interest to the Protective Devices Committee.

The papers on decrement curves and short circuit calculations are:

I—Standard Decrement Curves, W. C. Hahn, General Electric Co., and C. F. Wagner, Westinghouse Electric & Manufacturing Co.

II—Calculation of Short Circuits on Power Systems, C. F. Wagner, Westinghouse Electric & Manufacturing Co., and S. H. Wright, Buffalo, Niagara & Eastern Power Corporation.

III—Decrement Curves for Specific Systems, W. C. Hahn, General Electric Co.

During 1918, Messrs. Burnham, Hewlitt, and Mahoney discussed decrement curves for relay and circuit breaker applications before the Institute. The group of papers presented at the last Winter Convention brings the subject up to date, utilizing the further knowledge of machine characteristics accumulated in the intervening period.

Instruments and Measurements

ANNUAL REPORT OF COMMITTEE ON INSTRUMENTS AND MEASUREMENTS[*]

THE Committee on Instruments and Measurements during the year 1931–32 has had an active membership of 23 members. At the meeting held in October 15 members were in attendance, and at the meeting held in January 13 members attended. Considering the geographical distribution of the membership, this attendance is exceptionally good.

The work has been organized through subcommittees to take care of the following subjects:

1. Recording instruments
2. Telemetering
3. Definitions of instruments and testing
4. Instrument transformers
5. Indicating instruments
6. Temperature measurements
7. High-frequency measurements

SUBCOMMITTEE ON RECORDING INSTRUMENTS

This subcommittee has been engaged for several years on the development of standards for recording instruments. During the year its work was completed and a draft of the proposed standards was approved by the Instruments and Measurements Committee. These have been forwarded to the Secretary of the Standards Committee for adoption and publication. This subcommittee has been under the chairmanship of Mr. Kinnard and is to be complimented on the completion of a task which involved such a large amount of painstaking work.

SUBCOMMITTEE ON TELEMETERING

This subcommittee has concluded its work on standard definitions which have been approved by the Instruments and Measurements Committee. These have been submitted to the Secretary of the Standards Committee.

In addition to its work of following developments in this field, it has undertaken to cooperate with a subcommittee of the Automatic Stations Committee in the preparation of a report on telemetering and supervisory control systems and related communication systems. This report is a very comprehensive survey of all the known telemetering systems, which have been critically analyzed so as to provide detailed information in regard to the limitations and operating characteristics of each. It is hoped to have this report for the Summer Convention. Mr. E. D. Doyle is chairman of this subcommittee.

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[*] COMMITTEE ON INSTRUMENTS AND MEASUREMENTS:

E. J. Rutan, Chairman,
H. S. Baker,
R. D. Bean,
O. J. Bliss,
P. A. Borden,
H. B. Brooks,
A. L. Cook,
E. D. Doyle,
W. W. Eberhard,
Marion Eppley,
J. B. Gibbs,
W. N. Goodwin, Jr.,
I. F. Kinnard,
O. A. Knopp,
A. E. Knowlton,
H. C. Koenig,
W. B. Kouwenhoven,
F. A. Laws,
E. S. Lee,
J. B. Lunsford,
Paul MacGahan,
R. T. Pierce,
W. J. Shackelton,
H. L. Thomson,
H. M. Turner.

SUBCOMMITTEE ON DEFINITIONS OF INSTRUMENTS AND TESTING

This subcommittee is an outgrowth of the cooperation of the Instruments and Measurements Committee with the Sectional Committee of the American Standards Association which is preparing definitions on instruments and testing. Mr. E. S. Lee as chairman of this subcommittee has been assisted by the chairmen of the Recording Instruments, Telemetering, Instrument Transformers, and Indicating Instruments subcommittees as well as all of the members of the Instruments and Measurements Committee. Many valuable suggestions and considerable work has been done in the criticism, correction, and rearrangement of several of the drafts of definitions submitted to this subcommittee. Joint meetings were arranged with the members of the Sectional Committee and, as a result, an acceptable draft of these definitions has been completed.

SUBCOMMITTEE ON INSTRUMENT TRANSFORMERS

The Subcommittee on Instrument Transformers has been active for the past two years in the revision of standards No. 14 and has about completed its work and is preparing to submit a final draft of these standards to the members of the Instruments and Measurements Committee. Mr. J. B. Gibbs is chairman of this subcommittee.

SUBCOMMITTEE ON INDICATING INSTRUMENTS

Indicating instruments standards No. 23 which have been available to the industry since 1927 have been under consideration for revision during the past year. Based on considerable progress in the art since the standards were issued and the experience of the Instruments and Measurements Committee, many desirable changes will be made and new data incorporated in these standards. An early report from this subcommittee is expected which, upon approval by the Instruments and Measurements Committee, will be forwarded to the Standards Committee. Mr. H. C. Koenig is chairman of this subcommittee.

SUBCOMMITTEE ON TEMPERATURE MEASUREMENTS

This subcommittee, which is under the chairmanship of Mr. H. C. Koenig, has been working on the preparation of a standard code for temperature measurements. The scope of this work is to cover all the requirements of temperature measurements as specified in the various standards of the Institute. It is expected, when completed, a very comprehensive procedure will be available for uniform measurements of temperature.

SUBCOMMITTEE ON HIGH-FREQUENCY MEASUREMENTS

This subcommittee has been active under the chairmanship of Prof. H. M. Turner. Arrangements were made to cooperate with a committee appointed by the Standards Committee to draft definitions and standards for sound measurements. The subcommittee is also making a study of vacuum tube voltmeters and will prepare a report on this subject during the coming year.

INSTITUTE SESSIONS AND PAPERS

In addition to the Committee work through its subcommittees on definite subjects, arrangements were made for the promotion of a session at the Mid-Winter Convention. Six papers were presented and are listed below:

1. *High Voltage Bridge for Measurements of Cable with Grounded Sheaths*, C. L. Dawes and A. F. Daniel.
2. *A High Sensitivity Power Factor Bridge*, W. B. Kouwenhoven and Alfredo Banos, Jr.
3. *Capacitance and Power Factor Measurement by the Capacitance Bridge*, R. P. Siskind.
4. *An Automatic Oscillograph*, C. M. Hathaway and R. C. Buell.
5. *The Photoelectric Recorder*, C. W. La Pierre.
6. *Aging and Elastic Hysteresis in Instrument Springs*, P. MacGahan and R. W. Carson.

In addition to the papers presented at the session, the following papers were reviewed by the various members of the Instruments and Measurements Committee and commented upon:

1. *The Metering of Symmetrical Components*, G. R. Shuck.
2. *Electrical Measurements of Sound Absorption*, A. L. Albert and W. R. Bullis.
3. *Electrical Solutions of Problems of Regular Scheduled Flight*, C. F. Green.
4. *The Photoelectric Recorder*, C. W. La Pierre.
5. *A Standard of Low Power Factor*, W. B. Kouwenhoven.
6. *Electrical Measurements in the Gas Industry*, E. X. Schmidt.
7. *A Special Application of the Potentiometer Principle*, J. L. Watson and R. C. Gleeson.
8. *A Bridge for Precision Power Factor Measurements on Small Oil Samples*, J. C. Balsbaugh and P. H. Moon.
9. *Skin Effect in Rectangular Conductors*, H. C. Forbes and L. J. Gorman.

CONCLUSION

The Chairman wishes to acknowledge at the completion of his two years of office the hearty cooperation of all of the members of the Instruments and Measurements Committee. The work of the vice-chairman, secretary, and chairmen of the subcommittees was of especial importance in the successful completion of many of the projects which we had before us. From several years contact with this committee as a member and as chairman, the writer has been very much impressed by the interest displayed, and feels highly confident concerning the continued activity in this field of the electrical engineering art as sponsored by this committee.

Transportation

ANNUAL REPORT OF THE COMMITTEE ON TRANSPORTATION*

Railroad Electrification

THE Reading Company began operation on July 26, 1931, of the first portion of its electrified suburban service running out of Philadelphia. Multiple-unit cars are now operated over 64.6 miles of route embracing 156.9 track miles and the electrification of two more branches is in progress, the completion of which will bring the total electrified mileage to 86.9 route miles or 203 track miles. Single-phase, 25-cycle, alternating current is supplied by the Philadelphia Electric Company through outdoor frequency changer sets located on the railroad's property at Wayne Junction, and is distributed to the trains over a 36,000/12,000 three-wire system with 12,000 volts between the overhead contact wires and the rails, and 24,000 volts between the rails and the feeders carried on the catenary structures. Provision has been made on these structures for supporting transmission lines of higher voltage when future extensions of the electrification make it necessary. Over a part of the right-of-way steel towers are to be used jointly by the power company for high-voltage transmission and by the railroad for supporting its catenary system. All of the power switching is handled from a single supervisory control board at Wayne Junction, at which point also are located the repair shops and storage yards for the car equipment.

The Pennsylvania Railroad Company has made very considerable progress on its project for complete electrification between New York and Washington. The overhead contact system has been installed in the New York Terminal zone between Sunnyside Yards on Long Island and Manhattan Transfer in New Jersey, where d-c. third-rail locomotives have been used hitherto, and a-c. locomotives are now handling some of the trains. Construction is progressing between Manhattan Transfer and Trenton, New Jersey, the completion of which, in association with the work already done, will permit electric operation from New York as far as Wilmington. The erection of catenary structures between Wilmington and Washington has been begun. Several locomotives of the new standard designs have been placed in service. The motors on the two types of passenger locomotives (2-B-2 and 2-C-2) and the road freight locomotive (1-D-1) have interchangeable armatures, stators, and other parts, those on the passenger engines being mounted in twin frames, a pair for each driving axle, while those on the freight locomotive are mounted independently, one on each of the four driving axles. The motors all have a continuous rating of 625 hp. per armature. Standardization and interchangeability of parts have been carried out as far as possible throughout the design of these locomotives, both in the mechanical parts and in the electrical equipment. These standard motive power units may be operated in various combinations to provide the tractive effort required for different classes and weights of trains.

The New York Central Railroad has eliminated the use of steam locomotives on its "West Side" line in New York City, full operation by either electric or oil-electric locomotives having been inaugurated in June 1931. Third rails supplying direct current at 650 volts have been installed as far south as 72nd Street, but only to a very small extent in the yards between 72nd and 60th Streets. No external power supply has as yet been provided on the tracks which lead (still largely on the city streets) to the smaller yards and terminals further downtown. The locomotives which handle the switching and transfer work in this territory have oil engine-generator sets and storage batteries to provide their own power, but are also equipped to receive power from the third rail where it is available.

The New York, New Haven & Hartford Railroad has placed in operation the ten new passenger locomotives mentioned in last year's report. They weigh 403,000 lb. and have the axle arrangement 2−C+C−2 with a weight on drivers of 45,600 lb. per axle. Each of the six driving axles is connected by quill and gear to a twin motor with a continuous rating of 550 hp. These locomotives are notable for improved motor design, giving better commutation and increased tractive effort.

Oil-electric locomotives, as well as oil-electric and gasoline-electric rail motor cars, are finding an increasing number of applications. Seven such locomotives recently placed in service by the Bush Terminal Company in Brooklyn, New York, are unique in that their cabs, underframes, and trucks were fabricated entirely from structural shapes and plates, with electric welding and no riveting.

Air conditioning on passenger cars and experiments with rubber-tired rail cars are among recent developments in railroading which contain elements of interest to the electrical engineer.

Railroad Signaling

A number of additional installations of the recently developed methods of centralized traffic control was made during the past year. These, together with the completion of many new interlocking plants, automatic signals, highway crossing signals, and car retarders kept the record of progress in the installation of signaling facilities up to about 65 per cent of that in the previous year in spite of the reduction in the volume of construction in general.

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* COMMITTEE ON TRANSPORTATION:

E. L. Moreland, Chairman,
Reinier Beeuwkes,
A. B. Bettis,
H. A. Currie,
J. V. B. Duer,
H. H. Field,
I. W. Fisk,
K. T. Healy,
H. N. Latey,
John Murphy,
Hippolyte Parodi,
W. B. Potter,
R. H. Rice,
S. A. Spalding,
N. W. Storer,
W. M. Vandersluis,
R. P. Winton,
Sidney Withington,
G. I. Wright.

URBAN TRANSPORTATION

The New York City Board of Transportation has practically completed the equipment of the new Eighth Avenue Subway from Fulton Street to 207th Street, Manhattan, and has received 300 cars which have been tested in trial runs and are ready for the beginning of operation. Five hundred more cars have been ordered in preparation for extension of the service to other sections of the new system which are under construction in the boroughs of Brooklyn, Queens, and the Bronx. The substation equipment for these lines will include 16 converters in five substations with an aggregate capacity of 53,000 kw., and 62 mercury arc rectifiers rated at 3,000 kilowatts each, most of which will be placed in individual subterranean compartments along the subway lines.

Systematic efforts are being continued to design a type of street car which will more successfully meet the competition of the buses and the private automobiles. The tendencies are toward the use of lightweight, double-truck cars, with small wheels and low floors, equipped with small high-speed motors giving rapid acceleration and high car speeds. Control has been improved by the use of a larger number of resistance steps and also by the introduction of "variable-automatic control" in which the operation of the resistance contactors is governed by relays, but the motorman can select the rate of acceleration according to traffic or rail conditions. Braking by means of electromagnetized shoes on the car acting against the running rails is being used to some extent. Higher speed and lighter weight are the tendencies in interurban cars also, and some attention is being given to improvement in body design to reduce wind resistance.

The use of trolley buses is being extended, such vehicles having been purchased for seven cities during 1931, at the end of which year there were 225 trolley buses in operation in 14 cities in the United States.

MARINE TRANSPORTATION

The two largest passenger liners built in the United States, the *President Hoover* and the *President Coolidge,* were placed in operation during the past year in intercoastal and transpacific services. Each of these twin ships has two 10,000-kw. turbine-driven generators which supply the synchronous-induction propulsion motors with three-phase, alternating current at 4800 volts, and also four 500-kw. turbine-driven d-c. generators which very completely serve the auxiliary equipment on board, including steering gear, winches, capstans, air compressors, refrigerating machines, blowers, pumps of all sorts, elevators, cooking equipment, stateroom heaters, fans, etc., in addition, of course, to searchlights and general lighting. Several other smaller motor-propelled vessels with generators driven by steam turbines or oil or gasoline engines were placed in commission during the year. (See also the report of the Committee on Applications to Marine Work.)

AERIAL TRANSPORTATION

The U.S. Navy dirigible *Akron* carries considerable electrical equipment, including a searchlight and general lighting, cooking appliances, radio equipment, a battery charging set and other motors. The energy is supplied by two 11-kw., 115-volt d-c. generators driven by gasoline engines. The huge mooring mast, by which the dirigible is towed into and out of its hangar at Lakehurst, N. J., is propelled by two caterpillar tractors driven by 125-hp., 250-volt, d-c. motors and is steered by a third caterpillar driven by an 8-hp. motor. A 240-hp. gasoline-driven generator supplies these motors as well as floodlights and signaling apparatus on the mast and equipment for conveying water, fuel, and electricity to the dirigible.

A novel type of electric locomotive is used to tow models of sea-plane hulls or pontoons in a testing channel 2,000 ft. long, 22 ft. wide, and 12 ft. deep, recently completed at Langley Field, Va. The towing car spans the tank and is driven by four 75-hp. motors. The remarkable feature is its rapid acceleration, as it is said to reach a speed of 60 miles an hour in 9 seconds.

Important improvements were made during the year in radio equipment for communication between airplanes and the ground and for direction finding. These include automatic volume control, which relieves the pilot of the necessity of making manual adjustment of receiver amplification over a wide range of distances, and also practicable means of giving visual indication in connection with the direction finders.

Among electrical devices perfected for application to aviation may be mentioned an instrument for indicating engine cylinder temperature which is so calibrated and compensated as to show directly in degrees the temperature of various parts of the engine where thermocouples are installed. The leads from several thermocouples are brought to the indicator through a selector switch. Another device is the sonic altimeter in which a compressed air whistle produces signals electrically timed and controlled, which are reflected from the ground. The echo is heard in a stethoscope and, by means of a timing indicator mounted on the instrument panel, the time interval between outgoing and returning signals is interpreted directly in terms of distance above the ground.

VERTICAL TRANSPORTATION

The opening of the Empire State Building in New York City in 1931 placed in service the largest installation of elevators yet made. These are all of the automatic type and those serving the upper part of the tower have higher speeds and longer vertical travel than any previously installed. Improvements in construction and in the control of emergency stopping devices now permit speeds up to 1200 ft. per minute.

An installation of double-deck elevators, serving two floors simultaneously, has been made in the tower of the new Cities Service Building in New York City. In this same structure there is also the first installation of escalators in an office building, a means of transportation which is being used more and more extensively in department stores, railway stations, etc.

The arrangement, mentioned in last year's report, in which two independent elevators are operated in the same shaft, one running express to the upper floors and the other serving the lower floors, with suitable safety interlocking, appears to be a thoroughly practical development. Such elevators are in service in one of the Westinghouse buildings in East Pittsburgh.

Applications to Marine Work

ANNUAL REPORT OF COMMITTEE ON APPLICATIONS TO MARINE WORK*

As in the past few years, the major items of activity of the committee have been:

1. Accumulating, as a result of experience, data for revision of Standards No. 45, Recommended Practise for Electrical Installations on Shipboard.
2. Promoting the idea of establishing a definite status of the electrical operating personnel on shipboard.
3. Following closely installations of electric propulsion and electrification of auxiliaries on shipboard.

STANDARDS NO. 45

These standards are proving to be more and more useful to naval architects, shipbuilders, ship operators, and manufacturers of electrically driven auxiliaries. As a result of this increase in use, it is natural that changes and additions should be found desirable. During the past year the committee has continued to collect data for such revisions and when the volume of this data seems to justify a reissue of the standards, the committee expects to recommend such action to the Institute.

OPERATING PERSONNEL

For some years the committee has felt it important that the personnel in charge of the electrical machinery on ships should have a more definite status than is now in vogue. With the increasing use of electricity for both propulsion and auxiliaries, the necessity for such a recognized status has become more urgent. Our first efforts were based upon improving the status of electrical operating personnel through governmental channels, but as a result of experience, it has become apparent that our efforts will probably meet with more success if we cooperate with the owners and operators of vessels to this end; hence, during the past year we have been in further conferences with the American Steamship Owners Association and we believe that we are about to arrive at such a working agreement with this association as will lead to definite results.

ELECTRIC PROPULSION AND AUXILIARIES

As stated in last year's report, the year 1930 was one of great activity, progress and accomplishment in marine engineering, generally. This was due largely to the passage of the Jones-White Merchant Marine Act in 1928. This activity resulted in contracts being let for a large number of ships with electric propulsion and auxiliaries.

Due to the present financial depression the past year has not been marked by such activity, but the program started the year before has continued to be executed without cancellation of contracts.

The following table shows the status during the year of new ships in the U.S.A. equipped with electric propulsion:

SHIPS COMMISSIONED IN THE U.S.A. DURING 1931

Turbine Electric Drive

Name	Type	Horsepower
PRESIDENT HOOVER	Passenger Cargo	26,500
PRESIDENT COOLIDGE	Passenger Cargo	26,500
TALAMANCA	Passenger Cargo	10,500
CAYUGA	Coast Guard Cutter	3,220
SHOSHONE	Coast Guard Cutter	3,220
	Total	69,940

Diesel Electric Drive

Name	Type	Horsepower
COLUMBINE	Lighthouse Tender	240
LINDEN	Lighthouse Tender	240
GULF MIST	Tanker	400
WHITE FLASH	Tanker	375
SAN DIEGO	Ferry	750
DUPLEX	Dredge	600
	Total	2,605

SHIPS UNDER CONSTRUCTION IN THE U.S.A. DURING 1931

Turbine Electric Drive

Name	Type	Horsepower
*SEGOVIA**	Passenger Cargo	10,500
CHIRIQUI	Passenger Cargo	10,500
ANTIGUA	Passenger Cargo	10,500
QUIRIGUA	Passenger Cargo	10,500
VERAGUA	Passenger Cargo	10,500
	Total	52,500

* Destroyed by fire Dec. 1931.

*** COMMITTEE ON APPLICATIONS TO MARINE WORK:**

R. A. Beekman, Chairman,
B. C. Alger,
H. C. Coleman,
E. M. Glasgow,
H. F. Harvey, Jr.
C. J. Henschel,
W. Hetherington, Jr.,
H. L. Hibbard,
J. E. Kearns,
Alexander Kennedy, Jr.,
J. B. Lunsford,
I. H. Osborne,
G. A. Pierce,
W. H. Reed,
H. M. Southgate,
W. E. Thau,
John Van der Dussen,
A. E. Waller,
O. A. Wilde,
J. L. Wilson,
R. L. Witham,
W. N. Zippler.

Diesel Electric Drive

Name	Type	Horsepower
———	3 Tugs	1,600
———	1 Yacht	600
	Total	2,200

Gasoline Electric Drive

Name	Type	Horsepower
JOHN T. HARVEY	Fireboat	2,130

Production and Application of Light

ANNUAL REPORT OF THE COMMITTEE ON PRODUCTION AND APPLICATION OF LIGHT*

PRODUCTION OF LIGHT

Statistics on Lighting Progress as Indicated by Incandescent Lamp Sales

THE production and use of artificial light, as indicated by the sale of incandescent lamps, decreased slightly during 1931 as compared with previous years. In actual numbers of lamps sold the decrease was not great. The average wattage decreased from 61.2 to 60.9 watts, lamp efficiency from 13.3 to 13.4 lumens per watt and average lumens from 814 to 817.

The continued increase in the use of 120 volts for lighting circuits is notable. The percentage of 120-volt lamps sold increased from 41.3 per cent in 1930 to 43.9 per cent in 1931, while 110- and 115-volt lamps decreased from 5.2 per cent to 3.7 per cent and 48.9 per cent to 48.0 per cent, respectively. The adoption of the three-phase four-wire network system of distribution is probably the major factor influencing this trend.

Street lighting continues to show an increase both in the number of lamps used and in the average light output.

NEW INCANDESCENT LAMPS

Photoflood Lamps

A companion lamp for the photoflash lamp has been developed—the photoflood lamp. The photoflash lamp is intended for use in making snapshots, while the photoflood lamp is intended for time-exposures and for home or amateur motion picture photography. The new lamp has a bulb of the same size and shape as the standard 60-watt lamp but is rated at 250 watts and operates at an extremely high efficiency. In light output it is equivalent to a standard 500-watt lamp but in photographic effect, due to a high percentage of actinic light, it compares with a 750-watt lamp.

Neon-Filled Christmas Tree Lamps

Christmas trees are illuminated usually by strings of eight 15-volt lamps burning in series. The failure of one extinguishes the entire string and it is necessary to try each successive lamp in the string until the burned-out lamp is located. Neon-filled lamps have been developed which operate in the usual manner until one fails. This impresses the full line voltage across that particular lamp and causes it to glow with the characteristic neon red. As the current producing the glow is not sufficient to light the remaining lamps, the burned-out lamp is identified readily.

Inside Frosting 150- to 500-Watt Lamps

The inside frosted finish has been made available for lamps of 150 to 500 watts. This finish is desirable for lamps used in light density enclosing globes and in indirect lighting units since it diffuses the light and eliminates sharp shadows.

Addition of 1,500-Watt Lamps

Increasing use of high intensity illumination produced a demand for 1,500-watt lamps sufficient to warrant their inclusion in the regular lamp schedules.

Elimination of 600- and 800-Lumen Lamps

Authorities on street lighting practise consider the use of lamps of less than 1,000 lumens output inadvisable. The 600- and 800-lumen, 6.6-ampere lamps have been removed, therefore, from the regular schedules.

IMPORTED INCANDESCENT LAMPS

Electric lighting in America has enjoyed a favorable lamp situation in that a great majority of the lamps supplied to the public have been of well-controlled quality and of highest practicable efficiency. This condition is threatened now with impairment due to the advent of inferior lamps. Notable among these are lamps imported from Japan and sold through irregular channels of distribution. The committee is informed that aside from miniature lamps, about twenty millions of Japanese lamps were imported during the last year. These were characteristically of such low efficiency that at prevailing rates for current the public could not afford to use them.* Defects which interfere with satisfactory service were unduly prevalent among them and their life performance was very irregular and inconsistent.

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* COMMITTEE ON PRODUCTION AND APPLICATION OF LIGHT:

W. T. Blackwell, Chairman,
J. W. Barker,
H. S. Broadbent,
J. M. Bryant,
W. T. Dempsey,
H. E. Dorting,
L. A. Hawkins,
H. H. Higbie,
C. L. Kinsloe,
R. D. Mailey,
G. S. Merrill,
P. S. Millar,
P. H. Moon,
R. E. Simpson,
C. J. Stahl,
G. H. Stickney.

In drawing attention to this development, the committee bespeaks cooperation of electrical engineers generally in doing all that may be practicable to prevent the impairment of American lighting by lamps of inferior quality.

BASE AND SOCKET STANDARDIZATION

At the present time the screw bases and sockets standard in Europe differ slightly from the original American Edison standard in such a way that American lamps are not assured of fitting in European sockets, but European lamps can be put in American sockets, though failing to meet American safety requirements. Although many more American sockets and bases are in actual use, it was proposed to adopt the European design as the international standard. At a meeting of the International Electrotechnical Commission's Committee, held concurrently with the 1931 sessions of the International Commission on Illumination, the American standards were recommended on a par with the European and plans were advanced looking toward a single universal standard of intermediate dimensions.

PROPER OPERATING VOLTAGE

The voltage at which incandescent lamps are operated affects their input, output, efficiency, rate of depreciation, and cost of producing light to such an extent that it cannot be overlooked in any consideration of the production and application of light involving such sources.

The N.E.L.A. Lamp Committee Report for 1930–31* contains a study of operating voltages in use in some 24,590 communities. The detailed figures presented in the report reveal an undesirable tendency toward the increased use of more than one standard voltage in individual communities. Such dual standards make it difficult, if not impossible, to insure that lamps of proper voltage are used. During the five-year period from 1923 to 1928 there was a marked reduction (13.2 per cent to 5.4 per cent) in the extent to which two lighting service voltages were used by any one community, but in the last years this has increased again.

WIRING SPECIFICATION

Of the specifications mentioned in last year's report, the "Minimum Specification for Wiring of Lighting Circuits in Industrial Structures" was issued during the summer.

Regarding residence wiring, several associations undertook independently the development of specifications. An industry committee was organized to coordinate the work and prepare amplified specifications acceptable to all groups. This work is reported now to be complete, subject to final approval by the several associations. It is understood that the proposed specification sets No. 12 wire as the smallest size to be recommended for any lighting circuit. This limitation, corresponding as it does to that of the commercial and industrial specifications, should help eliminate smaller sized wire from good standing with the designers of electric light wiring, and thus advance good engineering in this field.

CARBON ARCS

Recent careful control tests of various sources of illumination with respect to their speed and color reproduction in photography have shown that the white flame photographic carbon almost exactly duplicates the results obtained with natural sunlight

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* *Transactions*, Illuminating Engineering Society, November 1931.

* N.E.L.A. Bulletin—Sept. 1931, p. 627.

Table I—Data on Incandescent Type of Ultra-Violet Sources

Source	Type S-1	Type S-2	Type G-1 Lamp	CX Lamps 60-watt	CX Lamps 300-watt	CX Lamps 500-watt
Approximate lamp watts	400	130	36	60	3000	500
Approximate total watts with auxiliaries	500	175	50	60	300	500
Approximate transformer primary amperes	9	3	1			
Approximate operating volts	14–15	15	18–20	110–115–120*	110–115–120*	110–115–120*
Approximate operating amperes	27.5	8.5	2	0.5	2.6	4.3
Approximate lumens	8400	1850	150	785	6000	10,750
Approximate lumens per watt (lamp)	21	14.2	3.7	13.1	20	21.5
Approximate over-all lumens per watt	16.8	10.5	3.0	13.1	20	21.5
Bulb	PS-22 inside frosted	A-17 clear	A-21 inside frosted	A-21 inside frosted	PS-30 inside frosted	G-38 inside frosted
Base	Mogul screw	Special screw	Medium screw	Medium screw	Mogul screw	Mogul screw
Maximum over-all length—inches	$6\,{}^{7}/_{16}$	$4\,{}^{3}/_{8}$	$4\,{}^{1}/_{4}$	$4\,{}^{15}/_{16}$	$7\,{}^{1}/_{2}$	$7\,{}^{3}/_{8}$
Light center length—inches	5	$3\,{}^{3}/_{4}$	$2\,{}^{3}/_{4}$**	$3\,{}^{3}/_{8}$	5	5
Foot-candle minutes for MPE	3500	6500	1300†	175,000†	120,000	100,000
Approximate life—hours	400	400	500	500	500	500
List price	$7.50	$3.75	$3.50	$1.00	$3.00	$3.50

N. B. Burning the Types S-1 and S-2 lamps at an angle of 45 degrees reduces their ultra-violet outputs 30 and 40%, respectively. Eliminating the wire screen on the Type S-1 units increases their output about 20%.

* Also available for 28–32 volt circuits.

** Since the entire bulb glows this dimension is base contact point to center of bulb which is also the center of the electrodes.

† Estimated.

and gives a nearly perfect separation of color values in effective shades of gray.

In the field of motion picture projection, carbons have been developed which are steadier in burning and, in certain sizes, even higher in intrinsic brilliancy than those formerly obtained.

Precratering of high intensity positives has resulted in a more rapid burning in of natural craters and full intensity of illumination, as well as a resultant saving in costs by reducing the possibility of mirror or condenser pitting.

For searchlights used in airport floodlighting and military purposes, carbons have been developed which will give from 50 to 100 per cent more light at 225 and 250 amperes than has been possible previously with regular searchlight carbons.

ULTRA-VIOLET—INCANDESCENT TYPE

Last year's report mentioned the introduction of four new sources of ultra-violet radiation emitting little or no wavelength shorter than 2,800 and, therefore, safe for use in the home, the school, office, et cetera; *i. e.*, the Types S-1 and S-2 lamps, the Type G-1 or glow lamp and the CX lamps.

Considerable improvement has been made in these sources since their introduction. The essential technical data pertaining to them are given in Table I.

During the year very complete lines of fixtures have been placed on the market for the Types S-1 and S-2 lamps. Suspension and ceiling type units have been developed which have a spread of both the ultra-violet radiation and the visible light flux. Such units are finding wide application in places where the exposure periods are somewhat limited, such as in swimming pools, gymnasiums, hospital solariums, lunch and recreation rooms, et cetera.

For use in offices, schools, et cetera, where it is desired to combine the health maintenance value of ultra-violet radiation with the lighting over exposure periods of 6 to 10 hours, a third line of equipment, known as dual-purpose lighting fixtures, has been developed. Most of these utilize regular incandescent lamps for indirect or semi-indirect lighting and also an ultra-violet lamp and reflector so designed as to spread a mild intensity of ultra-violet over a wide area.

GASEOUS TUBE ELECTRIC ILLUMINANTS

It has long been felt that possibilities of materially higher efficiency of light production lie in the domain of electrical discharge through gases rather than in incandescence. While incandescent lamps, with which most artificial lighting is accomplished, are serving the purpose exceedingly well, despite limitations of theoretical low efficiency of light production, and are undergoing gradual evolution which appears to bring them closer and closer to the limits of the possibilities of production of light by incandescence, the engineering world is favorably predisposed toward the idea of achieving efficiencies of a higher order from gaseous tube illuminants.

The present low-voltage gaseous and mercury-vapor arc tubes make available to the illuminating engineer additional lighting tools having characteristics that possess value for particular purposes. Thus the mercury arc lamp, with its characteristic green-blue light, provides a "work light" which has peculiar value for certain industrial applications. The arc tubes of neon or other gases, with their characteristic instantaneous response to electrical discharge, are especially useful in television, stroboscopic work, et cetera. Further, the vivid colors of these tube illuminants make them applicable for certain special uses, such as aircraft beacons, and for publicity purposes. These sources, however, are of the same order of efficiency as tungsten filament lamps.

Beyond these developments the committee has information only of experimental developments in low voltage gaseous arc tube sources. Among these the highest efficiencies of light production attained are in tubes of high power. Pirani's oft-cited sodium tube of special sodium-resistant glass, heated independently to 350 deg. cent. and enclosed within an outer evacuated tube, is said to attain 70 lumens per watt applied to the tube itself. It produces light of a brilliant yellow color. No data are available as to the power required for the complete equipment. Neon and mercury-vapor low-voltage arc tubes, three or more feet in length and consuming 1,000 to 2,000 watts, are said to attain to 35 to 50 lumens per watt. In smaller sizes of lower power input very much lower efficiencies of light production are attained.

In the opinion of the committee attempts to develop gaseous arc tube light sources of a materially higher efficiency than modern incandescent lamps are interesting and perhaps promising. Of practical achievement in the way of commercially applicable illuminants the committee has nothing new to report at this time.

APPLICATION OF LIGHT

New and Improved Incandescent Lamp Equipment

The standard dome reflector and the glassteel diffuser remain as standard lighting equipment for industrial areas. Recent developments have improved the methods of hanging these reflectors so as to reduce the time required and the cost of cleaning and relamping. The high intensities of light required on certain working areas can be secured by the use of recently developed industrial projectors which can be mounted on the ceiling, side walls or columns and adjusted to project the light in any desired direction.

A totally enclosed semi-indirect lighting unit has been developed with the bottom of the bowl of high density glass and the top portion practically clear. This type of glass makes a highly efficient unit and one easily maintained.

Appearance of the lighting equipment is playing a very important part in commercial installations. The contours of enclosing globe units are more pleasing in design and hangers are receiving considerable attention from the design standpoint. An interesting development in the indirect type of unit is one constructed entirely of metal louvers which can be obtained in a variety of color finishes.

Outdoor lighting such as the floodlighting of buildings and areas devoted to outdoor sports is increasing in popularity. This is partly due to the development of more efficient floodlight projectors and the introduction of open type floodlights. Many manufacturers have discarded the plain parabolic mirror for compound curves forming 2 and 3 surface mirrors. This increases the efficiency and permits accurate control of the beam.

The floodlighting of homes for holiday decorations, and gardens during the summertime, is now made possible by the development of small floodlight projectors designed for use with 100- and 150- watt lamps.

Street Lighting

Street lighting has been referred to frequently, by engineers, as the application of electric light where the practise has lagged most behind growing needs. During the year the demand for smaller municipal expenditures has been reflected in numerous proposals for reduced street lighting. Considerable apprehension was felt by experts lest serious increases in traffic and crime hazards should be incurred for only negligible savings. So far no extensive reduction has been made and several cities, where reduced lighting was tried, returned promptly to former illumination. On the other hand there has been a definite arrest in the advance of street lighting.

Group Replacement of Lamps

Since incandescent lamps operate with almost no attention and a substantial proportion of them last beyond their economic efficiency, many installations fall below their proper illumination performance. The obvious cure is systematic cleaning and replacement at suitable intervals. A recent awakening to the importance of such practises is showing good results. Already about thirty utility companies are applying "group replacement" to their street lighting. Noticeable improvement in illumination and reduced outages are being secured at little, if any, increased expense. Similar methods are applicable to traffic signals, large buildings, large signs, and wherever lamps are grouped in sufficient numbers to permit economic organization of maintenance.

In connection with group replacement, it is desirable to segregate used lamps which have considerable life expectancy from those which are approaching failure. This is facilitated for clear bulb lamps by a new inspection box, which reveals filament defects.

Railway Signals in Locomotive Cabs

Many serious accidents on railroads have resulted from failure of the engine crew to see signals. A new system, being applied extensively by a leading railroad, has a light signal in a conspicuous position in the locomotive cab, where it is independent of weather conditions and its view not limited to a fleeting glance. A sound device calls special attention to changes. The enhancement of safety is obvious.

Sports Lighting

The increasing popularity of artificial illumination in this field is due to the use of more adequate levels of lighting. For example, in football fields about 15 foot-candles have been found to be desirable in contrast to the 5 to 7 foot-candles often recommended in the past. For baseball fields a consumption of about 300 kw. is considered necessary now, whereas 200 kw. was about the maximum of former attempts.

Lighting of Buildings Exteriors

The application of artificial light to the exterior of buildings is making rapid progress, especially in the endeavor to create artistic night-time effects. Lighting effects formerly obtaining only by floodlighting are, in some of the newer buildings, being improved upon by novel methods. Today architects are designing buildings with the idea of artistically lighting the exteriors at night. They are paying attention to the direction of light projection and so designing as to create night-time as well as day-time effects on their buildings. One development is the use of luminous panels built into the façade of the building. Pilasters of diffusing glass behind which are mounted various colored lamps connected to dimmers produce changing color effects. Entire façades are covered with panels of this sort or constructed of glass brick which can be illuminated from behind at night. The development is following gradually that of built-in lighting for interior use.

Built-In Lighting

It is illogical to wait until a building is completed before considering lighting equipment, yet in the past this was generally the case. Now, fortunately, in most new structures the artificial lighting is designed as a component part of the interior. Manufacturers of diffusing material, such as glass, molded plaques and translucent substances, have made available many new varieties and these are finding application. Conventional fixtures designed on traditional lines are passing rapidly from the picture.

Residential Lighting

While the number of newly electrified homes in 1931 decreased to one-seventh the number added in 1930 and to one-seventeenth the number added in 1929, the consumption of electrical energy in homes showed an increase of approximately 8 per cent in 1931, only three or four points less than the increase in previous years when new residential building was at high levels. Decreasing hours spent in the various places of entertainment outside of the home have meant increasing evening hours at home for all members of the family, calling for a greater use of electric light.

The portable lamp has grown in popularity as is evidenced by the increasing number of styles and designs which have been produced. The indirect portable lamp using a 200- or 300-watt lamp has not usurped this field although it has continued to grow in numbers. Decorative floor and table lamps using the Type S-2 lamp have made their initial appearance.

Lighting fixtures providing indirect or semi-indirect lighting and selling at very low prices have been received favorably in various parts of the country for the relighting of certain rooms of the homes of "minimum bill" customers. The design of these fixtures is such that they may be fastened directly to existing single sockets.

The President's Conference on Home Building and Home Ownership devoted a section of its Fundamental Equipment Report to the Wiring and Lighting of the Home, giving attention to these home modernizers equal to that devoted to such items as heating, ventilating, and plumbing.

Automobile Lighting

By means of a three-filament headlamp, a lighting system has been developed giving an asymmetrical distribution of light with reference to the car axis in which more light is directed to the right side of the road, particularly when passing another car. This system is so arranged that a city-driving and a country-driving range of light control is provided.

The advantages of directing more light to the right when passing are well recognized and several auxiliary lighting units are available which when used in connection with the present standard dual beam system provide this desirable passing light.

On some cars the fender lights serve a dual purpose, the switch control being so arranged that in addition to acting as marker lights when the car is parked these fender lamps are lighted when the beam is depressed, thus showing the oncoming driver that the depressed beam is being used and giving in addition an indication of the extreme width of the car.

Two rear lamps are being supplied for many current model motor cars. These lamps are generally a combination rear and signal unit. The majority of rear units are equipped with reflex glass which gives an indication from the reflected light thrown on it. The use of reflex materials in conjunction with incandescent lamps is considered desirable from a safety standpoint. Incandescent lamps are being used also to indicate when the engine needs oil and whether or not the battery is being charged.

Present day headlamp equipment, which is mostly of the fixed focus type, is capable of furnishing good road illumination with a minimum of glare but in order to obtain the maximum advantage from this equipment additional educational work is still needed which should be directed along the lines of encouraging car drivers to use the lower or depressed beam when meeting or signaled by oncoming cars.

Daylight vs. Artificial Light

Recently developed lighting equipment provides artificial light with supplementary ultra-violet radiation, exceeding that received from daylight filtered through window glass. This, coupled with improved electrical air conditioning, is rendering building construction independent of natural illumination. In a number of buildings recently completed or under construction, daylight has been purposely excluded.

Architectural Relations

Each year architects show a growing interest in the decorative as well as utilitarian possibilities of modern lighting. The Illuminating Engineering Society, in cooperation with the Beaux-Arts Institute of Design, has established an Illuminating Engineering Society Prize for which architectural students throughout the country compete. This year's project is designing the lighting for an exhibition hall at a world's fair. It is planned to continue this prize competition for the next decade.

The American Institute of Architects is cooperating with the Illuminating Engineering Society in the revision of the American Standard Code of School Lighting.

Light Control

The photoelectric light control relay is a device which regulates automatically the turning on and off of artificial lighting as the intensity of daylight decreases or increases. During the year such control was applied to the lighting of a school room where children in a special "sight-saving" class receive their instruction. Tests conducted in this room indicated that in order to maintain an intensity of 15 foot-candles, it is necessary to use the artificial lights at least 30 per cent of the time the room is in use.

Thyratron control of lighting is being applied to theaters, floodlighting of buildings, show windows, and electric fountains.

Thermionic tubes appear especially applicable for automatic dimming controls, where the only moving part can be small potentiometers or phase shifters actuated by very small motors. Applications of mobile color lighting with these controls include a floodlighted building, dance halls, night clubs, restaurants, and a number of theaters.

One of the most recent developments utilizes a continuous belt for controlling the potentiometer adjustment by providing a conducting path for each color or group of lights to be controlled. Any desired combination can be secured by varying the position of the conducting paths on the belt, so that any predetermined lighting effect can be repeated at will, or a new combination can be secured by "changing the roll." It is reported that the first installation of this arrangement will be to control the lighting of Buckingham Fountain in Chicago.

With the new thermionic tube controls the manual switchboard for theater and stage lighting can be very small, located in the orchestra pit, on the stage, or even in the front rows of the orchestra. Now the "lighting conductor" becomes a reality and the lighting of the show is played by the conductor who sees and hears the entire action.

Illuminometer

A new portable illuminometer employing the photoelectric principle has become available. The photonic cell consists of a disk on which has been deposited light-sensitive material so that, when exposed to light, a slight but apparently constant electrical potential is generated. The readings are made on a microammeter connected to the cell by means of a flexible conductor cord.

For increased deflections two or more cells are sometimes mounted in a single housing and connected in multiple. Measurements and checks under incandescent lighting of the range ordinarily found indoors seem to show comparatively close agreement with the usual types of visual illuminometers.

Lighting Plans for 1933 Chicago Fair

A wide interest and curiosity has been exhibited by the public regarding the illumination of the 1933 Fair. It is not appreciated generally that the interiors of the Fair buildings will have no natural light. They will be illuminated both day and night by electricity. In general there are no windows or other openings through which natural light may enter.

One novelty will be shimmering effects on exterior walls such as is produced by reflection of the sun's rays from bodies of water in the daytime. A lighting projector built upon this principle will be used to throw fantastic moving color patterns on building walls. By agitation of the water in different ways, patterns of various forms may be produced. So-called scintillators will be used also. In this case squares of polished metal trembling on delicate supports will flash dancing color patterns over large areas of frosted glass. This is one of the few lighting effects that can be utilized in the daytime. The rays of the sun are allowed to penetrate the glass at many points and are reflected back by the metal squares in lively fashion, somewhat similar to the heliograph used by the Army for signal purposes.

Fluorescence will be used also. Many objects such as flowers, foliage, statuary, et cetera, will be treated with fluorescent materials and under so-called invisible light will appear in contrast with the surrounding foliage.

The possibility of fluorescent fountains has not been overlooked. There are several organic substances that may be added to water to produce fluorescence, such as eosine, fluorescene, and aesculin. Very small quantities of these substances added to water in a circulating fountain are sufficient to impart a mysterious glow to the water under so-called invisible light.

During the last few months the Fair has been experimenting with a great so-called flaming ladder arc, using a potential of 33,000 volts. Rising from a huge outdoor transformer will be two electrodes about thirty feet high. A flaming arc will start from the bottom of these electrodes and travel upward. As this intense flame dissipates at the top, another automatically starts at the bottom, so that there is a succession of fiery bridges or rungs in constant motion. Hence the term "ladder arc." As these arcs travel upward, dry chemical salts are injected by compressed air. These salts impart changing colors to the arc stream.

International Commission on Illumination

The International Commission on Illumination met in Cambridge, England, the week of September 13, 1931. In connection with the meeting a Congress was held in which meetings assembled in London, Glasgow, Edinburgh, Sheffield, Buxton, and Birmingham. Many notable demonstrations of illumination were made for these events and the subsequent celebration of the Faraday Centenary, among which were the lighting of the public buildings and streets of London and Edinburgh Castle. Excellent progress was made toward international understanding on illumination problems.

A new topic was Aviation Lighting in which international unity is especially important.

General

Revised Nomenclature and Standards

During 1931 there has been widely circulated for discussion and criticism a revised set of terms and definitions to supersede the American Standard "Illuminating Engineering Nomenclature and Photometric Standards" issued in 1925. The changes which have resulted from this discussion are largely matters of form rather than of substance. The new edition of the nomenclature will show, therefore, few radical departures from the established practise, but is believed to be somewhat more clear and logical.

Primary Standards of Light

International discussion of fundamental photometric problems has continued and several laboratories abroad are reported to have in progress experimental tests of the form of primary standard proposed by the Bureau of Standards. This standard consists of a refractory tube immersed in molten platinum at its freezing temperature. The tube is so constructed and observed as to be a "complete radiator" or "black body." Its brightness as found at the Bureau of Standards is 58.85 candles per square centimeter. The question of accepting this standard is before the International Commission on Illumination and the International Committee of Weights and Measures, but a decision must await independent confirmation of the Bureau's results by some other national laboratories.

Death of Thomas A. Edison

The Committee on Production and Application of Light cannot conclude its report without allusion to the passing of the founder of the electric lighting industry. His death on October 18, 1931, was the signal for an expression of worldwide appreciation of the services and of the life of the man whom electrical engineers have held in high regard by reason of his inauguration of electric light and power as a widespread service to the public. Edison passes but his work goes on. He lives in the esteem of those who carry on under the banner which he raised.

Applications to Mining Work

ANNUAL REPORT OF THE COMMITTEE ON APPLICATIONS TO MINE WORK[*]

THE past year has been one of very greatly reduced activity in all classes of mines. New machines and new applications of electricity in mining work have been few and, generally, of a minor nature. There has, of course, been no incentive to force production or to increase the capacity of mines, and labor-saving devices have not been in demand because of an unwillingness on the part of operators to aggravate the bad unemployment situation. However, there have been some installations and devices worthy of comment.

A West Virginia coal mine has installed a passenger elevator to expedite the movement of men in and out of the mine and to relieve the production hoist of this service so that it can continue in its regular duty. This elevator has all of the safety and control features of a high-speed traction-drive office elevator. It handles twenty men per trip in a shaft 347 feet deep at a speed of 450 feet per minute. As this elevator travels in the intake air compartment, the car has certain special structural features to permit an uninterrupted flow of air.

Some companies have found it profitable to operate their mines only at night to secure a lower rate for electric power at the off-peak period. Power economies have also been effected by the use of power demand limiters. An installation of capacitors for power factor correction has shown a return of approximately 7 per cent per month.

Two new electric cap lamps for use in gaseous mines have been developed. One of these, weighing only 63 ounces, gives a beam candlepower of 26 at the beginning of a shift and the other, which weighs only 87 ounces, gives a beam candlepower of 55 at the beginning of the shift. A flood lamp, particularly designed for lighting the face for cutting and loading, gives a beam candlepower of approximately 350. All of these lamps have been approved by the U. S. Bureau of Mines.

Improved performance is being regularly obtained from cutting and loading machines. One operator reports that a cutting machine regularly cuts 1,000 to 1,200 tons in 6 to 7 foot coal in a shift, and that he is regularly obtaining 375 tons per day from his loading machine.

A Western Pennsylvania mine has installed a system of automatic door control, using "the electric eye" as the controlling relay. The mine doors are caused to open and close in proper sequence for either direction of movement of the trip.

A wound-rotor induction motor with direct-current excitation of the primary has been installed as a braking generator to replace mechanical brakes, holding a rope haul trip at proper speed down grade. The high expense of brake maintenance has been practically eliminated.

Several large induction motors and synchronous motors with automatic line-start control have been installed for pumping in the anthracite field. The controls for several of these are housed in cubicles and completely erected at the factory.

A large, high-speed, heavy-duty, variable-voltage hoist, taking its power from a synchronous motor-generator set, has been installed in Canada. The rotating equipment consists of a 1,250-kw. generator, an 1,800-hp., 750-r.p.m. synchronous motor and a 1,550-hp., 68-r.p.m. hoist motor. The skip has a capacity of 6 tons of rock per trip and travels at a speed of 2,200 feet per minute.

A new light-weight self-propelled cutting machine has shown favorable results over several months of trial. The machine has a very high-speed cutter which resembles in its operation a band saw more than the usual type of cutter.

The construction of new coal-cleaning equipment kept pace with progress of previous years in this line. It is estimated that 8,000 tons per hour of complete mechanical cleaning capacity were installed in 1931 in addition to 20,000 tons of capacity for cleaning and sizing. All of these tipples and cleaning plants were of course, electrically driven.

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[*] COMMITTEE ON APPLICATIONS TO MINING WORK:

D. E. Renshaw, Chairman,
A. R. Anderson,
Graham Bright,
J. H. Edwards,
B. J. Gealy,
L. C. Ilsley,
L. H. James,
J. B. Kearns,
Carl Lee,
W. H. Lesser,
C. W. Parkhurst,
F. L. Stone,
J. F. Wiggert,
C. D. Woodward.

Transmission and Distribution

ANNUAL REPORT OF COMMITTEE ON POWER TRANSMISSION AND DISTRIBUTION[*]

THE activities of the Committee on Power Transmission and Distribution have continued through the past year along the lines organized in previous years. Various subjects of interest to the Institute membership have been treated in a number of papers presented before the conventions and district meetings.

A brief outline of the activities of the several subcommittees follows:

SUBCOMMITTEE ON STEEL TRANSMISSION TOWERS AND CONDUCTORS

Subjects relating to steel towers have been studied and certain items of general interest are now being given particular attention preliminary to the preparation of future reports.

A report has been drafted outlining the requirements for modern steel tower transmission lines which, after review, it is expected will be published. Conductor vibration has been given intensive study. A direct result of this work has been the publishing of a bibliography including all available data on the subject of vibration. A session at the Summer Convention is devoted to this important subject.

SUBCOMMITTEE ON DISTRIBUTION

Continued consideration has been given to the various broader aspects of distribution, including developments in secondary and primary networks.

Of particular interest to distribution engineers is the symposium presented at the Winter Convention consisting of six papers on distribution circuit lightning protection. This symposium summarized the results of intensive field and laboratory experimental work and analyses of operating experience. It is believed that this material will be productive of improvement in distribution equipment and construction practises and in service reliability.

SUBCOMMITTEE ON CABLE DEVELOPMENTS

There have been no marked innovations in the cable art during the past year, although study has been continued on the developments indicated in previous reports. In the high voltage transmission cable field, the highest voltage submarine cable crossing in the world, across the Columbia River near Portland, Oregon, is also the first of the oil-filled type used in submarine work. The cables are 750,000-cir. mil., single-conductor, rated at 115 kv., and an insulation thickness of 0.550 inch. An unusual feature is the use of hard-drawn-copper armor wires, instead of the usual steel, in order to reduce the losses in the armor and thereby increase the rating to 114,000 kva.; over 30 per cent more than if steel armor had been utilized.

The performance of experimental installations of 132-kv. cables, one of the oil-filled type with reduced insulation and the other of the solid type, in Chicago and in Newark, respectively, has continued to be successful.

SUBCOMMITTEE ON INTERCONNECTION AND STABILITY

The joint subcommittee on interconnection is formed from representatives of the Power Generation, Protective Devices and Power Transmission and Distribution committees. During the year the scope of the committee was increased and representatives were added from the Committee on Electrical Machinery.

This joint subcommittee sponsored a session at the Winter Convention on the subject of Decrement Curves and Power System Stability. Two other stability papers were presented during the year.

SUBCOMMITTEE ON LIGHTNING AND INSULATORS

The necessity for establishing a definite wave or group of waves by which to measure impulse strength of insulators has been recognized for some time. In the interest of greater uniformity in test practises, agreement has been reached on a tentative group of waves (1) which would closely approximate the effects of natural lightning voltages imposed on apparatus connected to transmission systems; (2) which could be produced with available laboratory equipment; and (3) on which considerable impulse data have already been secured by various laboratories which have been investigating the performance of insulation under impulse voltages. The following three waves have been agreed upon as preferred test waves for impulse testing. These waves are designated as $^1/_2$ x 5, 1 x 10, and $1^1/_2$ x 40. The first number indicates the microsecond time to crest, and the second number the time in microseconds from zero to 50 per cent of crest on the tail.

Another development in the lightning field has been continued this year in the factory test of commercial transformers with artificial impulse waves. It appears that the manufacturers are now in a position to test commercial transformers in the higher voltage ranges with artificial impulse waves, and a recommendation has been made that a committee formulate the procedure to be followed in applying impulse waves to transformers. This work is already in progress.

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[*] **COMMITTEE ON POWER TRANSMISSION AND DISTRIBUTION:**

P. H. Chase, Chairman,
L. N. Conwell, Vice-Chairman,
T. A. Worcester, Secretary,
Sydney Alling,
F. E. Andrews,
A. B. Campbell,
C. V. Christie,
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Edwin Hansson,
K. A. Hawley,
L. F. Hickernell,
D. C. Jackson, Jr.,
J. P. Jollyman,
A. H. Lawton,
I. E. Moultrop,
L. L. Perry,
T. F. Peterson,
D. W. Roper,
A. E. Silver,
D. M. Simmons,
C. T. Sinclair,
L. G. Smith,
H. H. Spencer,
Philip Sporn,
W. K. Vanderpoel,
C. F. Wagner,
H. S. Warren.

Field investigation of natural lightning which has been in progress for the last few years has been continued on a decreased scale.

Discrepancies in 60-cycle flashover values of insulators mentioned in last year's report as due to humidity conditions have been further studied this year. Papers fully covering this subject have been presented before the Institute during the year.

Reference has been made above to the field, laboratory, and operating investigations relating particularly to 2,300- and 4,000-volt distribution lightning protection. Of particular interest is the apparent benefit to be derived from interconnecting the primary arrester ground with the secondary neutral, if the latter is well grounded.

CONCLUSIONS

In conclusion, the committee wishes to express its gratification at the interest evidenced in the field of Power Transmission and Distribution by the papers submitted and by the attendance and discussion when presented.

This interest together with that displayed by the members of the committee, and by those associated in the work of its subcommittees, have made possible the accomplishments of the committee.

Power Generation

ANNUAL REPORT OF THE COMMITTEE ON POWER GENERATION*

ALTHOUGH the past year has recorded no major strides in power plant design and construction not heretofore discussed in committee reports, the committee has been successful in obtaining a variety of papers of a critical nature for Institute presentation, surveying present practise, with the object of indicating probable lines of advance when the construction of new or additional power projects again becomes active. Following the custom of preparing progress reports only at biennial intervals, the committee report for the past year will merely recount briefly the results accomplished by the several subcommittees engaged in planning and securing technical papers.

The Joint Interconnection Subcommittee, of which F. C. Hanker served as chairman, was instrumental in presenting at the Winter Convention six papers embracing several phases of the subject of stability of power systems, including that of *Stability of Conowingo Hydroelectric System*, by R. A. Hentz and J. W. Jones. A record of the most recent technical advances in power system design and operation was given in that paper. Another valuable paper concerned the standardization of nomenclature regarding stability, being a report by H. K. Sels.

Another session at the Winter Convention was sponsored by the subcommittee on Power Station Auxiliaries, composed of H. W. Leitch and F. H. Hollister. The comparative merits of steam and electric drives for steam power stations were analyzed in papers by W. Poole Dryer and L. W. Smith, advocating opposing views, and in a summary statement by F. H. Hollister.

A subcommittee headed by N. E. Funk has arranged a symposium of four papers on *Combined Economy and Reliability in Operation of Large Electric Systems*, for the Summer Convention. Current practise in the Boston, Chicago, Detroit, and Philadelphia areas will be discussed at that time.

Other work consisted of a review by I. E. Moultrop and J. B. Crane of the economics of the design and use of high-temperature and high-pressure steam equipment, with a supplementary discussion of the use of welding in modern fabrication methods. A paper on this subject awaits opportunity for presentation. The subcommittee on Hydroelectric Practise, J. P. Hogan, chairman, will offer a paper by A. V. Karpov on *Low-Head Hydroelectric Developments* at a Fall District Meeting featuring other papers on hydroelectric subjects. F. A. Annett of the subcommittee on the Design of New Plants has reviewed papers describing the Waukegan and Ashtabula plants, and has cooperated in having the subject of power generation adequately represented on the program of the Pacific Coast Convention this summer.

The Committee recommends that the recent innovations in the design and application of switching structures at power plants be given major attention during the coming year. For that purpose a subcommittee, A. E. Silver, chairman, was appointed which has been assembling data and information for a discussion of that topic next winter.

The Committee wishes to take this opportunity to record its thanks for the assistance given by the several authors and the many discussers of papers to the end that the progress in the field of Power Generation be adequately reported by the committee.

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* COMMITTEE ON POWER GENERATION:

J. R. Baker, Chairman,
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J. B. Crane,
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R. D. DeWolf,
W. P. Dryer,
N. E. Funk,
W. S. Gorsuch,
F. C. Hanker,
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J. P. Hogan,
F. H. Hollister,
A. H. Hull,
H. W. Leitch,
A. H. Lovell,
E. B. Meyer,
I. E. Moultrop,
F. A. Scheffler,
F. O. Schnure,
R. E. B. Sharpe,
A. E. Silver,
A. R. Smith,
E. C. Stone.

General Power Applications

ANNUAL REPORT OF COMMITTEE ON GENERAL POWER APPLICATIONS*

DURING the past year the committee has not sponsored a session at any of the meetings or conventions, but has reviewed and reported upon a number of industrial papers. Two of these papers were presented at the Pacific Coast convention at Lake Tahoe, August 1931, and as far as we know these were the only industrial papers presented at district meetings before the Institute during the year. These two papers are as follows:

Electrical Equipment for Oil Field Operations, by H. C. Hill and J. B. SeLegue

Electric Power in the Wood Products Industry, by C. E. Carey and K. L. Howe

Although the general activity of practically all industrial plants and machinery manufacturers has been greatly curtailed during the year, nevertheless, this extremely competitive situation is responsible to some extent for a number of developments and improvements in industrial equipment which have been brought out. Last year we mentioned the interest that was being shown in connection with the use of electronic tubes for industrial applications. The various characteristics which these tubes possess which are not otherwise obtainable are almost daily finding new industrial uses. A large number of these applications fall under the classification of process control and are in line with the trend of the times toward elimination of waste, reduction of cost, and more accurate control of all manufacturing processes. Most of these electronic developments are given publicity in the various trade papers or in *Electronics* and to keep posted on such activities constant contact with such literature is necessary. A comprehensive review in a report of this kind is entirely impossible.

During the year considerable attention has been given by various electrical machinery manufacturers to the design and construction of motors, controllers, safety switches, push-buttons, and other details to meet the requirements for Class II Hazardous Locations as defined by the Underwriters' Laboratory. Such equipment, although required for such locations in flour mills, grain elevators, starch plants, coal pulverizing plants, etc., finds numerous other desirable applications where its installation results in lower maintenance cost and improved operating conditions.

Instead of attempting to abstract information regarding the numerous developments in industrial apparatus or application, the committee feels that members interested in such equipment can readily obtain such information directly and more completely from the various periodicals as, for instance, *The Electrical World*, *Maintenance Engineering*, *G. E. Review* or *Electric Journal*, together with the various specific industry magazines. The January issue of such magazines generally contains a review of the year's achievements in a much more complete and interesting form than could be given in a report of this kind.

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Iron and Steel Production

ANNUAL REPORT OF COMMITTEE ON APPLICATIONS TO IRON AND STEEL PRODUCTION*

GENERAL

IT is estimated that the steel industry expended approximately 35 million dollars in 1931 for the purchase and installation of new apparatus, the maintenance of old equipment, and in the purchase or generation of electric power.

During the year there were purchased 63 main-drive motors of more than 300 rated hp. each. The total rated horsepower of these drives aggregated 119,325 hp. These figures compare with the installation of 163,490 hp. which occurred in the year 1930.

A major trend toward increased use of refined control equipment for automatically performing tasks which had previously required manual labor was continued during this year. While such equipment was installed at a reduced rate in 1931, the percentage of men whose duties were no longer required in the steel industry due to such automatic devices increased.

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MAIN AND AUXILIARY DRIVES

The 44-inch universal slabbing mill in the Chicago District, referred to in last year's report as being under construction, was completed during the early part of 1931. The upper and lower main horizontal rolls are driven by two 5,000-hp., 40/80-r.p.m., 700-volt, double-armature reversing motors, and the auxiliary vertical rolls are driven by a 2,500-hp., 79/225-r.p.m., 700-volt, single-armature reversing motor. The total continuous capacity of the three motors is 12,500 hp., and the maximum emergency capacity about 35,000 hp., nearly a third greater than any similar mill. The three motors are connected in parallel and are supplied with variable-voltage direct-current power from a 10,500-kw., 5-unit, six-bearing flywheel motor-generator set, consisting of three 3,500-kw., 700-volt generators, a 6,500-hp., three-phase, 25-cycle, 6,600-volt, 370-r.p.m. wound-rotor induction motor and a 180,000-lb. flywheel. When running at synchronous speed, the total stored energy in the rotating parts of the motor-generator set is approximately 270,000 hp-seconds. Latest operating data indicate this installation has proved quite successful.

A recent installation of induction motors and control for operating "catcher tables" has fulfilled the expectations of not only the operators, but of the mill designers and electrical manufacturers as well. These tables are used in the rolling of sheet steel and take the place of the older manual methods of handling the sheets and packs during the rolling process. The cycle of operations includes carrying the material into the mill by means of the roller table, receiving it on the catcher table, tilting the tables upward, transferring the sheet over the top roll back to the roller table, returning the tables to the down position, and reversing the direction of travel back into the mill. This cycle of operation must be completed under some circumstances in three seconds, which means that the motors must be capable of reversing or starting at the rate of 40 times per minute.

Special low-inertia, low-speed motors have been developed to cover this type of application. The speeds used are 375 r.p.m. for 25-cycle supply and 450 r. p. m. for 60-cycle supply. The use of squirrel-cage motors for such duties as this has been attempted only within the past two years, but it is an indication of the changing trend toward the application of simplified motor and control apparatus. Formerly, direct-current mill-type motors were thought the only solution for such problems.

During 1931, the first American installation for producing seamless steel tubes by the "push bench method" was completed. In the production of seamless steel tubing by this method, a heated blank is placed in a hydraulic press and formed into the shape of a thick walled thimble. The thimble is then placed on the end of a mandrel and forced through a series of stationary ring dies, which gradually decreases the wall thickness, and at the same time elongates the thimble, thus forming a long tube with a closed end. After the elongated thimble has been forced by the mandrel through the various dies, the mandrel is withdrawn and the closed end of the tube sawed off. Sizing and finishing is carried on subsequently in mills of the usual design. The power required on this application is approximately the same as that used by the piercing process.

In the installation just completed, the mandrel is forced forward through the dies by means of a rack and pinion which is driven by a 1,500-hp. variable-speed d-c. reversing motor. The control for this equipment is so arranged that upon indications from the operator, the motor will start, accelerate the ram, force the material through the dies at a predetermined rate of speed, then stop, reverse, and return the mandrel to the initial position. Means are provided for adjusting the speed of the stroke over a wide range, and extremely fast acceleration and retardation are necessary to meet the operating cycle. Devices are arranged in connection with this control to prevent the ram from overrunning in either direction. The manufacture of tubes by this process is expected to produce a small seamless tube at very low cost and for this reason it is probable that additional installations of similar equipment will follow.

Two steam engine drives in the Pittsburgh District were replaced by motor drives. A three-high roughing mill driven by a 1,500-hp. steam engine has been re-equipped with a 1,600-hp. wound-rotor induction motor. Likewise a reversing steam engine on a two-high finishing mill has been replaced by a 1,250-hp., d-c. reversing motor. This reversing equipment is said to be one of the smallest applications in the steel industry of reversing main roll drive equipment.

There have been two major hot strip mill installations made during the year. One of these consists of a 72-inch continuous hot strip mill and the other a 76-inch continuous hot strip mill. It is understood that both of these mills are equipped with anti-friction bearings on the main rolls throughout, and combine the use of induction motors on the preliminary stands, synchronous motors on the intermediate stands and d-c. adjustable-speed motors on the finishing stands. It is unusual to find the three types of main roll drive combined in one mill installation, but it was possible on these particular mills to secure the necessary relations between peripheral roll speeds without the use of expensive adjustable-speed d-c. drives throughout the entire mill.

The largest motor equipment ever sold for driving a cold strip mill was also installed during the year. Each of the two stands of this cold strip mill are driven by a 1,500-hp. adjustable-speed, d-c. motor. A tension reel driven by a 400-hp., adjustable-speed, d-c. motor, receives the material from the two cold roll stands. It is apparent that the year has witnessed a change in the electrical equipment used in the rolling of both hot and cold strip.

New designs of electric contactors were announced by one of the leading manufacturers during the year. Decided progress was made by a committee of steel mill engineers toward the standardization of control equipment. It is felt that it will be but a short time until a satisfactory steel mill control standard will be an accomplished fact.

Several new tension-limiting devices were designed for automatic reeling applications. Some are designed to place a predetermined tension on reels coiling strips after they are rolled, while others are between stands of the finishing passes. Automatic screw-down equipment for two-high and universal mills of simplified design appeared. This apparatus uses electron tubes to replace cumbersome limit switches. However, no installations of this type of screw-down control have been completed at the time this is written. Automatic screw-down control assures uniformity in rolling practise, independent of the whim of the operators, and has resulted in a material improvement in the quality of the product of rolling mills where it has been in service. The apparatus used in the past has been quite complicated and it is felt that the new devices tending to simplify this type of equipment will prove to be a worthwhile development and result in more general use.

SYNCHRONOUS MOTOR CONTROL

The recent remarkable increase in the use of synchronous motors in steel mills has made their characteristics, operation, and control a subject of great interest to steel mill electrical engineers. The favorable status of the modern synchronous motor was not achieved, however, until definite limitations of the earlier designs were overcome. Modern design and construction have removed most of these earlier limitations until now the synchronous motor starting, pull-in, and pull-out torques are exceptionally favorable for applications heretofore reserved for induction and direct-current motors.

A recent development of unusual interest has taken place in control systems for synchronous motors. This development is the culmination of investigations started many years ago which have done much to extend the field of application of the synchronous type motor. Recent systems of control automatically start the motors under the best conditions, in the shortest time, and keep them running through nearly all power supply disturbances of a transient nature. In short, modern synchronous motor control eliminates many of the starting and operating difficulties, which prevented their extensive acceptance in industrial fields.

Modern synchronous motor-control systems comprise the following separate features:

1. Part winding starting, resulting in reduced starting kva.
2. Transfer to full voltage by means responsive to the electrical condition of the field winding.
3. Synchronizing by means responsive to the electrical condition of the field winding.
4. Delayed low-voltage release.
5. Field kick-off and resynchronizing (where the load permits).
6. Unloading and reapplication of load to driven machine.

Not all of these features are required for every application, but those are employed that accomplish best possible starting under all conditions and continuous operation through momentary power or load disturbances.

ELECTRIC FURNACES

Several installations of electric furnaces for bright annealing coiled steel strip were completed and placed in operation during the year. Each furnace consists of a bell type heating chamber and several bases or cars upon which the coiled strip is stacked. An inner hood or cover of heat-resisting alloy is placed over the stack and serves to exclude air both during the heating period and while the coils are being cooled. An atmosphere of illuminating gas or other suitable gas is maintained under the hoods to prevent oxidation.

A new type of furnace, applicable especially to annealing steel sheets, has been developed in which a load of material can be cooled in the furnace in a comparatively short time. With previous types, the cooling rate of a loaded furnace is so slow as to make furnace cooling impracticable. The new furnace is lined with light weight refractory brick so that the heat storage in the furnace walls is comparatively small. It employs a circulating system for the furnace atmosphere, with an external cooler by means of which the heat may be extracted and the furnace again charged at a more expeditious rate than was possible heretofore.

MATERIAL HANDLING

There has been installed in the Pittsburgh District a large ore bridge using a bucket of 17 tons capacity. The bridge span is approximately 190 ft. The outstanding feature of the bridge is the unusual use of compressed air for control. Ten motors, 275 to $7^1/_2$ hp., are controlled by electropneumatic type control, similar in design to standard railway equipment. This arrangement, it was found, required less space and lower maintenance. The brakes are spring set and air released for quick, positive stopping. Due to the large volume of air required, there are three sets of compressors and reservoirs, one in the operating cab and one in each leg of the bridge. The bridge in normal service can handle 1,000 tons an hour and serves several blast furnaces located nearby.

ELECTRONIC DEVICES IN THE STEEL INDUSTRY

The past year has seen a rapid increase in the number, diversity, and ingenuity of applications of electronic devices in the iron and steel industry. These devices are now controlling shears, soaking pit covers, repeaters, and various mill table operations.

In one plant, a flying shear is located between the intermediate and finishing stands of a 10-inch hot strip mill. This shear is to cut off the relatively cool front end of the bar before it enters the finishing stands. Formerly an operator was necessary to trip the shear when the strip was in the proper position, so as to cut off a definite crop length. Now a photoelectric tube automatically trips the shear. An adjustable time-delay relay is used to determine the time interval between the interruption of the beam of light and the starting of the shear, thus compensating for the changes in the strip speed.

In another plant, considerable trouble was encountered in maintaining a flag switch on a back blooming mill table, used to control the operation of a recording pyrometer. A photoelectric tube with the required amplifier unit and relays now performs the operation in a more reliable and accurate manner.

Several installations have been made which permit crane operators to control heating furnace doors by means of photoelectric equipment. A number of other steel mill applications of photoelectric devices have been made, such as skip-hoist limit stops, stock-bin level indicators, spotting of material in reheating furnaces, pyrometers, etc.

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All in all, 1931 has, in spite of adverse conditions, brought into being a number of new ideas and electrical developments in the iron and steel industry. Cooperative exchange of information has continued, and permitted the industry to benefit from individual as well as collective effort and development.

The Chairman wishes to express his appreciation for the assistance rendered by the members of the committee in the preparation of this report.

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The committee submits a list of the principal papers that have been presented or published by various engineering societies covering more completely many of the subjects reviewed in this report.

1. "Twin Motor Drive," R. H. Wright and H. E. Stokes, *Iron and Steel Engineer*, June 1931, Page 246.

2. "Motor and Control Equipment," F. Mohler, *Iron and Steel Engineer*, June 1931, Page 272.

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4. "Electrical Developments in 1931," *Iron and Steel Engineer*, January 1932, Page 7.

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6. "Electrical Developments in 1931," *General Electric Review*, January 1932, Page 30.

7. "Electrical Developments in 1931," *Electric Journal*, January 1932, Page 25.

8. "A Review of Electrical Developments in the Iron and Steel Industry," W. B. Shirk, *Iron and Steel Engineer*, February 1932, Page 107.

9. "The Application of Electron Tubes in the Steel Industry," D. W. Dean, *Iron and Steel Engineer*, March 1932, Page 124.

10. "Vacuum Tubes and Their Industrial Application," W. C. White, *Iron and Steel Engineer*, March 1932, Page 131.

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Communication

ANNUAL REPORT OF COMMITTEE ON COMMUNICATION*

COMMITTEE AFFAIRS

THE Committee on Communication continued the practise of outlining at the beginning of the Institute year tentative communication programs for each of the subsequent scheduled Institute meetings. In making up the year's outline, the Committee strove to select material which would be of broad interest to the Institute as a whole and, in particular, to foster the presentation of papers on subjects that had not been covered before or that had received but little recent attention. A successful endeavor was made to cover such subjects in a rather complete and thorough way. The advance planning of programs greatly aided this by allowing the Committee ample time to solicit authoritative papers discussing all the major aspects of each subject selected for extended treatment.

The more important groups of coordinated papers which in accordance with this general policy have been presented at Institute meetings during the past year are:

Symposium on Communication Services for Power Companies—1931 Summer Convention:

This was prepared with the cooperation of the Committee on Transmission and Distribution and is headed by a general review of the subject prepared by a joint subcommittee of the Committee on Communication and the Committee on Transmission and Distribution, followed by a group of individual papers prepared by power and communication engineers, discussing the solution of this problem in specific cases.

Symposium on Time and Time Services—1932 Winter Convention:

This is headed by two papers discussing the astronomical aspects of time and its measurement, followed by discussions of the time services given by the Naval Observatory, the telegraph companies, and the power companies. In arranging this symposium the Committee on Communication had the cooperation of the Committee on Transmission and Distribution.

Symposium on Traffic Control—Northeastern District Meeting, May 1932:

This is headed by a general discussion of traffic control followed by papers describing various electrical systems now having important fields of application.

Planning for Telephone Toll Service—Southwestern District Meeting, October 1931:

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A group of three papers discussing, respectively, the forecasting of population, the general advance planning of telephone toll facilities, and certain technical problems involved in their design.

In addition, a number of individual papers has been presented under the auspices of the Committee at various national and regional meetings during the year, discussing important phases of new developments in the telephone, telegraph, and radio arts. The total number of communication papers, including the symposiums, presented at these meetings during the year was 33.

The communication sessions have been very well attended. The Communication Committee proposes to continue during the coming year its plan of developing programs involving groups of coordinated papers covering subjects of broad general interest.

ADVANCE OF THE ART

In line with the aim to keep this report as short as practicable there are listed below only a few of the outstanding advances made in the field of communication last year.

During the year the Bell System introduced teletypewriter exchange service whereby any subscriber to this new service may secure a direct connection to any other subscriber over which the two may carry on instantaneously reproduced two-way typewritten communication with one another. The methods and switching equipment employed in establishing connections are similar to those employed in telephone practise.

Also during the year the Western Union Telegraph Company and the Postal Telegraph-Cable Company jointly inaugurated a new form of telegraph service known as "Timed Wire Service." This is available to customers who have simplex printers, and provides for the transmission of communications from any printer to any other at rates based on the time the facilities are in use, rather than on the number of words transmitted. Printer customers desiring this service will obtain connection at the central office with an automatic reperforator, which will directly prepare a perforated tape in form for retransmission to the city of destination without additional manual handling. At this point a second perforated tape will be automatically prepared and used for retransmitting the message to the customer addressed. The service is somewhat analogous to a one-way private wire connection, although a direct connection is not established. The Timed Wire Service routine permits of a more economical utilization of the trunk wire plant than would be possible were a direct connection of printers established.

An automatic concentrator was developed for use on printing telegraph circuits to provide faster and more economical service on customers' lines. The Postal Telegraph Company has installed several of these equipments.

A four-wire voice-frequency carrier telegraph system adapted to meet the requirements of a strictly telegraph plant was developed and an initial installation was made on the lines of the Postal Telegraph Company.

A new cable and radio operating center, serving jointly the Commercial Cable Company, the Mackay Cable and Telegraph Company, and All America Cables, was placed in operation last year. Circuits linking the United States with Europe, Central and South America, and the Far East terminate in this center and connect 85,000 miles of under-sea telegraph wires through 19 separate channels. Three cables to Central and South America and two New York-London cables have been equipped with recently developed printer apparatus which has greatly increased the accuracy of reception and has also effected substantial economies. A recently developed method for converting the non-uniform length signals of the Cable Morse Code into a modified 5-unit code, so as to permit of the operation of printers on long non-loaded submarine cables, is employed in connection with these printers.

A pneumatic tube carrier routing device has been developed by the Western Union Telegraph Company for automatically selecting and diverting tube carriers from a main tube into the proper branch tubes. Pneumatic tubes are used to carry telegrams between main and branch offices and between points in large main offices.

Recent achievements in cable making have included the standardization of cables of less than 1,800 pairs employing 26-gage wires, and the application of a new process of insulating small-gage conductors for exchange cables. Under the new method of insulating the cable wires, a coating of wood pulp is applied to each conductor from a bath of water-suspended pulp. This coating when it dries becomes a continuous uniform sleeve of porous paper surrounding the wire. Cables made from this material are of the same size and have practically the same operating characteristics as cables of the older type, for which wires are insulated by helical wrappings of thin, narrow manila paper ribbon. Four machines for producing pulp-insulated wire were placed in operation last year with an output capacity of about 200,000,000 feet of wire a week.

Bermuda, Hawaii, Rio de Janeiro, Java, Sumatra, and the Canary Islands were brought into telephone communication with the interconnected telephone system of North America by means of radio links established in the past year. The first three places are reached by direct radio circuits from the United States. Communication between the sending and receiving stations at Oahu, and the other islands of the Hawaiian archipelago, is secured by an ultra-high-frequency radiotelephone system put into service by the Mutual Telephone Company. This system is also used regularly in inter-island telephone service.

Single side-band transmission of short-wave radiotelephony was successfully demonstrated by the International Telephone and Telegraph Laboratories. The spread side-band system for use on short-wave radio telephone links has also been developed by means of which distortion effects due to selective fading can be largely eliminated. By an extension of the principles involved, a number of side bands can be placed on one carrier in such relative positions to each other and the carrier that harmonic distortion as well as crosstalk between channels is eliminated.

A successful demonstration of two-way radiotelephony on an 18-cm. wavelength was given across the English Channel by the International Telephone and Telegraph Laboratories. This system, called "micro-ray," employs an antenna only 2 cm. in length. The greater directivity of these short waves makes it possible to employ the same wavelength for a number of separate channels closely spaced as to geographical separation, without mutual interference. Because of this, and the wide frequency band thus opened up, the commercial application of the "micro-ray" would make available nearly a quarter of a million additional radio channels.

Improvements have been made during the year in controllers for pre-timed traffic signals. One type is arranged so that signals may be automatically operated on different time cycles at different periods of the day or week. Another type is primarily intended to operate in connection with a flexible progressive system, but may be also used to meet the special requirements of detached street intersections. This provides for several functions in addition to the usual automatic stop-and-go control: manual control, flashing amber on one street and flashing red on the cross streets, lengthened green on the main street, pedestrian period, fire lane at one or more of the intersections and varying go periods at different intersections, while preserving the total cycle for progression.

1932 Index—A.I.E.E. TRANSACTIONS

Papers and reports contained in the 4 quarterly issues of the 1932 TRANSACTIONS are covered in this annual reference index which is published herewith in an enlarged and improved form. This index embraces all material presented at the Pacific Coast (Lake Tahoe) convention, August 25–28, 1931; the South West District (Kansas City) meeting, October 22–24, 1931; the winter convention, New York, January 25–29, 1932; the Great Lakes District (Milwaukee) meeting, March 14–16, 1932; the North Eastern District (Providence) meeting, May 4–7, 1932; and the summer convention, Cleveland, June 20–24, 1932. Papers informally presented at these meetings (and not published in the TRANSACTIONS) are included, with references to issues of ELECTRICAL ENGINEERING containing these papers in whole or part.

Page Allocation by Issues

March	1–286
June	287–578
September	579–838
December	839–1124

Subject Index

A

A-C Networks in Portland, Oregon. Clark. Electrical Engineering, August 1931, 644
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Aircraft in Flight, Characteristics of Electromagnetic Radiation From. Rives, Coe. (Dec.)............... 981
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Aluminum, Oxide Coatings on. Edwards, Tosterud, Work. (Dec.)... 900
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Arc Welder, An Improved A-C. Candy. (June)........................... 567
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Arcs in Turbulent Gases, Extinction of A-C. Browne. (March).......... 185
Armatures, Temperature Rise of Ventilated Railway Motor. Lightband. (June)........................... 502
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Automatic Equipment, The Application and Performance of. Sporn, Lanphier, Turner. (Dec.)......... 1031
Automatic Oscillograph, An. Hathaway, Buell. (March)................. 222
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Auxiliaries for Power Plants, Steam-Driven. Dryer. (June)........... 331
Auxiliaries for Steam Power Stations, Electrically Driven. Smith. (June)........................... 337
Auxiliary Drive for Steam Power Stations. Hollister. (June).......... 329
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B

Backfires in Mercury Arc Rectifiers. Slepian, Ludwig. (March)........ 92
Boric Acid Fuse, The. Strom, Rawlins. (Dec.)........................... 1020
Braking of Synchronous Machines, Dynamic. Kilbourne, Terry. (Dec.)........................... 1007
Breakdown of Glass With Alternating Potentials. Kenney, Luery, Moriaty. (June)................. 404
Bridge, A High Sensitivity Power Factor. Kouwenhoven, Banos. (March)........................... 202
Bridge for Measurements of Cables With Grounded Sheaths, High-Voltage. Dawes, Daniel. (March)......... 198